# MATHEMATICAL SCIENCES NEWS 2020

Carnegie Mellon University
Mellon College of Science
**Mathematical Sciences**

## **MATHEMATICAL SCIENCES** NEWS

Department of Mathematical Sciences

The Mathematical Sciences News is published yearly by the Department of Mathematical Sciences at Carnegie Mellon University for its students, alumni and friends to inform them about the department and serve as a channel of communication for our community. Readers with comments or questions are urged to send them to the Mathematical Sciences News. The department is headed by Tom Bohman.

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**Online Magazine**
Gigi Wiltanger, Web Manager



**Carnegie Mellon University**
**Department of Mathematical Sciences**
Wean Hall 6113
5000 Forbes Avenue
Pittsburgh, PA 15213
www.cmu.edu/math




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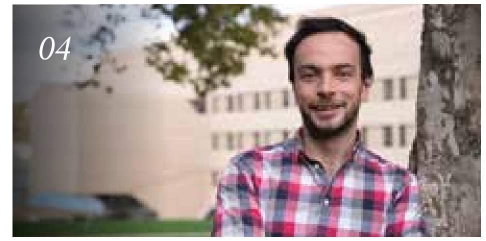
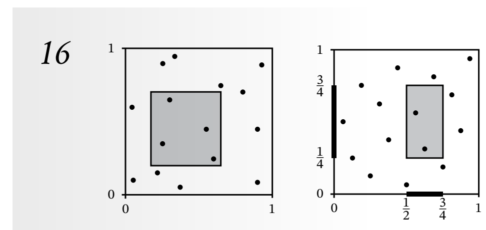


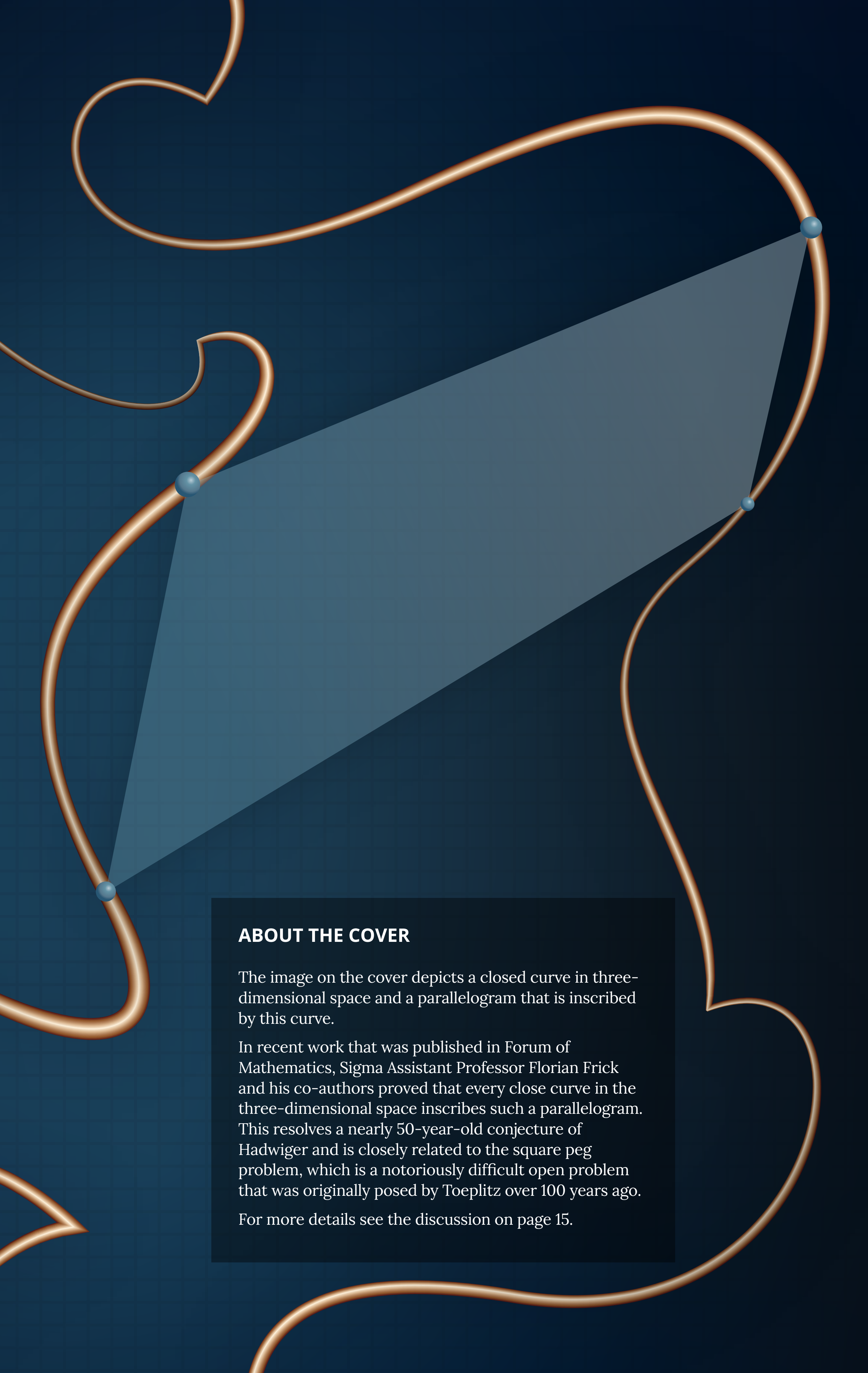












## ABOUT THE COVER

The image on the cover depicts a closed curve in three-dimensional space and a parallelogram that is inscribed by this curve.

In recent work that was published in Forum of Mathematics, Sigma Assistant Professor Florian Frick and his co-authors proved that every close curve in the three-dimensional space inscribes such a parallelogram. This resolves a nearly 50-year-old conjecture of Hadwiger and is closely related to the square peg problem, which is a notoriously difficult open problem that was originally posed by Toeplitz over 100 years ago.

For more details see the discussion on page 15.

## LETTER FROM MATHEMATICS DEPARTMENT HEAD, TOM BOHMAN

This has been a challenging year due to the COVID-19 pandemic, but I am happy to report that the department nimbly adapted to the new reality and responded in a number of productive ways.

After evacuating campus in March and moving all classes online, faculty developed teaching innovations tailored to the new learning format, and we believe that we are delivering a high-quality educational product. While we hope to gradually move operations back to campus as soon as we are able and we believe that in-person learning is generally superior to its online counterpart, some of lessons learned over the last few months will impact and improve our course delivery even after we are teaching fully in-person again.

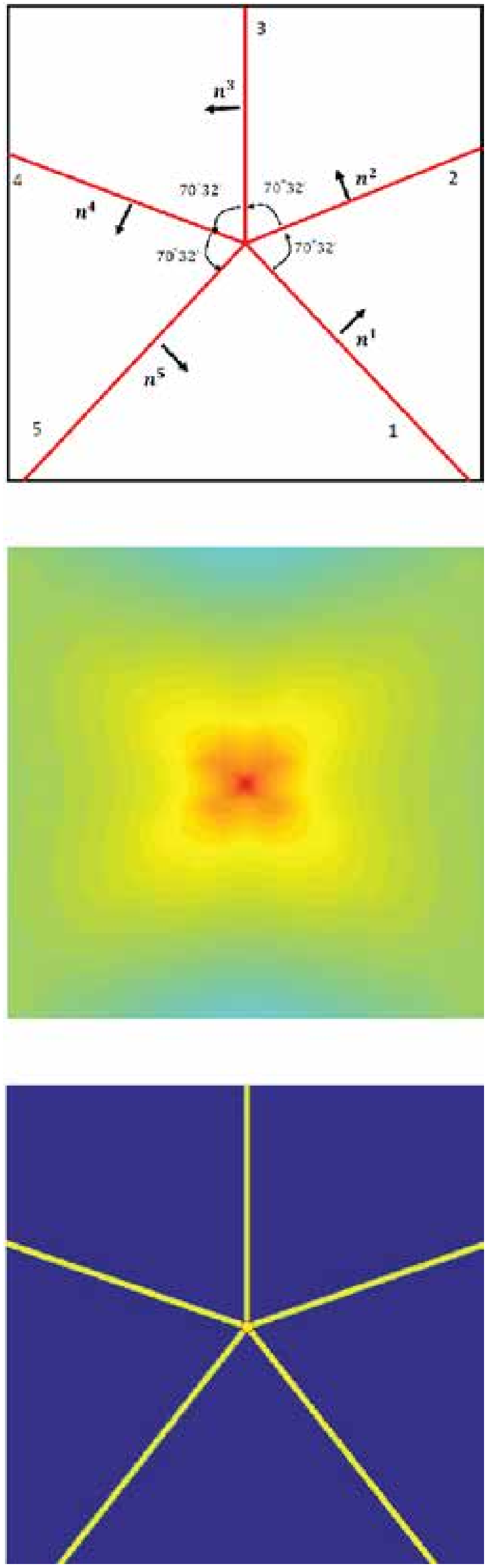
In addition to the adaptation of our course delivery, math faculty launched research and entrepreneurial efforts in response to COVID-19. Po-Shen Loh developed and deployed the COVID-19 tracking app, NOVID, which uses ultrasound to detect anonymous positive cases within your network of recent contacts. The app can discern your social proximity to other NOVID users and the length of time of your interaction with them. NOVID works through self-reporting but does not collect personal information. For more details see the story on page 6.

Wes Pegden and Maria Chikina (Chikina is faculty at the University of Pittsburgh) developed a novel COVID-19 public health response strategy informed by classical mathematical modeling of the pandemic. Two key features of their approach were recognizing that people would not be able to remain quarantined indefinitely, and that the virus would still be present even after the first wave of infections passed. Thus, any strategy has to account for the impact of subsequent waves of infection. Neither one of these notions were prevalent in discussions during the early days of the pandemic when Chikina and Pegden released their potentially life-saving study. You can read more on page 7.

Many of our students had their summer plans canceled due to the pandemic. The department responded by increasing our summer course offerings and expanding our traditional summer research program (SURF). We also introduced the Summer Experiences in Mathematical Sciences (SEMS), a new slate of summer mathematics activities for our majors. The department has been developing a culture of engagement with undergraduate research over the last several summers. This put us in an excellent position to develop the SEMS program on very short notice. For details see pages 10-13.

Despite the COVID-19 upheaval, the department continues to produce impressive research results. This issue of the newsletter features recent work in geometry, which is a significant interest of some of the faculty who have joined the department in the last few years. Much of this work is in discrete and convex geometry, an area of mathematics that features not only a wealth of beautiful classical problems but also close and deepening ties with coding theory and data science.

We extend our deepest appreciation and gratitude for your continued support and engagement with the Department of Mathematical Sciences. Likewise, we would like to hear from you! Please keep the department updated by contacting us directly or joining our LinkedIn group: https://www.linkedin.com/groups/12371860.




**Disclinations and interfaces in a solid (Top to Bottom):** Penta-twin, Finite strain stress field of a penta-twin, Stress-shielding of penta-twin by dislocations.

## CENTER FOR NONLINEAR ANALYSIS RESEARCHERS RECEIVE NSF GRANT TO STUDY TOPOLOGICAL DEFECTS

Researchers from the Department of Mathematical Sciences' Center for Nonlinear Analysis (CNA) have received a grant from the National Science Foundation to better understand the ubiquitous phenomena associated with topological defects.

The interdisciplinary team will aim to bring together knowledge from the fields of engineering, physics and mathematics to create a unifying framework for describing and understanding topological defects and the phenomena associated with them. These phenomena occur when two uniform patterns of organized matter — from a jet turbine blade to a galaxy — are offset from one another, rotated differently or otherwise have some form of deformation on the surface at which they meet. The crack or curve created where the joining surface ends and the two patterns separate is called a topological defect, and depending on the situation and material, its effect can range from interesting to catastrophic.

"Our team envisions a future where new knowledge from our research can be leveraged to seed advances in technology relevant for grand challenges in seismic forecasting, infrastructure renewal and energy-efficient transportation," the researchers wrote in their proposal.

Through mathematically defining how topological defects agree and differ within these different materials, scientists and engineers in civil engineering, advanced manufacturing, astronomy, seismology and countless more fundamental fields will have a universal tool for defining one of the most ubiquitous phenomena in our physical world.

The team of researchers is led by Professor of Civil and Environmental Engineering Amit Acharya and includes CNA Director and Kavčić-Moura University Professor of Mathematics Irene Fonseca; Assistant Professor of Mathematical Sciences Franziska Weber; Professor of Physics Ira Rothstein; Emeritus Professor of Physics Bob Suter; Hamerschlag University Professor Emeritus of Civil and Environmental Engineering Jacobo Bielak; Assistant Professor of Civil and Environmental Engineering Jerry Wang; and researchers from the University of Arizona.

■ *Dan Carroll*



# A Familiar Face

**Mathematical Sciences Ph.D. graduate Clive Newstead returns to the department as an assistant teaching professor**

After Clive Newstead received his Ph.D. in mathematical sciences in 2018 from Carnegie Mellon University, he joined the Department of Mathematics at Northwestern University as a lecturer, teaching courses in multivariable calculus, linear algebra and abstract algebra.

Shortly after, he returned to Carnegie Mellon — this time as a faculty candidate. His visit to campus certainly did not go unnoticed.

Newstead was a bit of a legend among students. Not only had he been a highly respected teaching assistant, winning both the Mellon College of Science's and university's Graduate Student Teaching Award, but he also wrote a free textbook, "An Infinite Descent into Pure Mathematics," that became the main text for two courses in the department.

During his interview, many students attended his job talk and one student even asked him to sign a copy of the front cover.

"A lot of students only know me as the mysterious figure who wrote their textbook, so it was nice to be able to interact with them and show them that I'm a real human being after all," said Newstead.

This fall, Newstead officially joined the department as an assistant teaching professor. He's teaching two courses remotely, Concepts of Mathematics — the very course he wrote the textbook for — and Multivariable Calculus and Vector Calculus for Engineers.

Newstead's research is in category theory and its connections with logic. "I like to think about category theory as the 'mathematics of mathematics,'" Newstead said of his work. "Whereas mathematics tries to identify patterns and structures in the real world and make sense of them abstractly, category theory does the same thing with mathematics itself."

A native of Yorkshire, England, Newstead likes cooking hearty British dishes. Outside of research and teaching, he enjoys biking and relaxing with a good dose of reality TV and is in the midst of planning his upcoming wedding.

■ *Emily Payne*

Visit the online magazine to watch a walkthrough of the NOVID app

# STOP THE SPREAD

## PO-SHEN LOH DEVELOPS NOVID APP TO TRACK CORONAVIRUS WITHIN YOUR USER NETWORK

As the world continues to focus on slowing the spread of COVID-19 and determining how to safely resume everyday life, a team of designers, mathematicians, computer scientists and physicists, led by Mathematical Sciences Professor Po-Shen Loh, developed NOVID — an innovative new app designed to help track the spread of COVID-19.

NOVID presents a new paradigm in exposure notification: it gives users a heads-up when someone in their user network, with up to 12 degrees of separation, has tested positive for the coronavirus.

"I had an idea of how to help the world return to normalcy," said Loh. "And the Carnegie Mellon and broader Pittsburgh community had everything I needed to start making that idea into reality."

Using social media and crowdsourcing, Loh brought together a team, including many CMU students and alumni working in mathematics, human-computer interaction, app development and information security — all areas where CMU excels — to develop the free app. Unlike many COVID-19 apps in development or in use, NOVID is completely anonymous. The notification system doesn't ask for personal information. Instead, each user is assigned a random number, which is used when their phone senses and communicates with other smartphones. The app also doesn't store GPS locations.

NOVID allows people to anonymously self-report if they have tested positive for COVID-19. The app periodically senses if one user comes close to another and anonymously records the distance between users. It later notifies a user if they have come in contact with a person who just reported that they tested positive or a person who has been in contact with someone who reported that they tested positive.

Another way NOVID is different from many other apps is that it doesn't rely solely on Bluetooth to measure distances between smartphones. Instead, NOVID augments Bluetooth with ultrasound — soundwaves with frequencies just outside the range of human hearing, which modern mobile phones can emit and record. By accurately measuring the time sound takes to travel, ultrasound can more accurately measure the distance between devices.

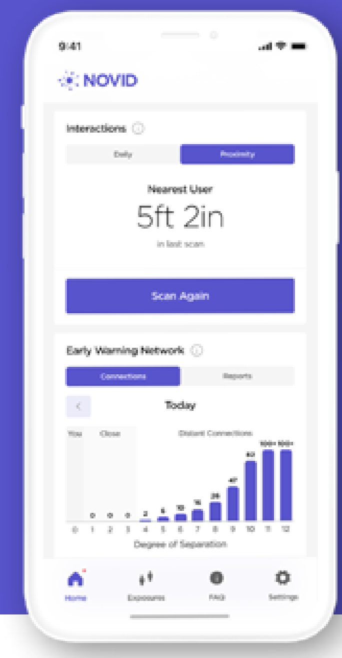
*"I had an idea of how to help the world return to normalcy."*

It is widely known that Bluetooth is unable to accurately calculate distances and can incorrectly register interactions between people in different rooms when signals pass through walls or ceilings. Ultrasonic signals improve accuracy because they can also be detected through bags and pockets and can go around obstacles by reflecting off ceilings without significantly affecting distance measurement.


Since launching, the app has been featured in Wired, Forbes, Fast Company and the New Yorker. NOVID's user-friendly interface allows users to follow their social interactions and see how well they are playing their part to stop the spread.

With its early warning network, you can see how many users in your network you've met through direct and distant contact and receive a notification if any user you've been in contact with has self-reported a positive COVID-19 diagnosis. NOVID has also shown it avoids false exposure alerts with 99.6% accuracy — to date, NOVID is the only COVID app in the world that displays the estimated distance from other devices.

■ *Emily Payne*




## MATHEMATICAL SCIENCES PROFESSOR PUBLISHES COVID-19 MODELING

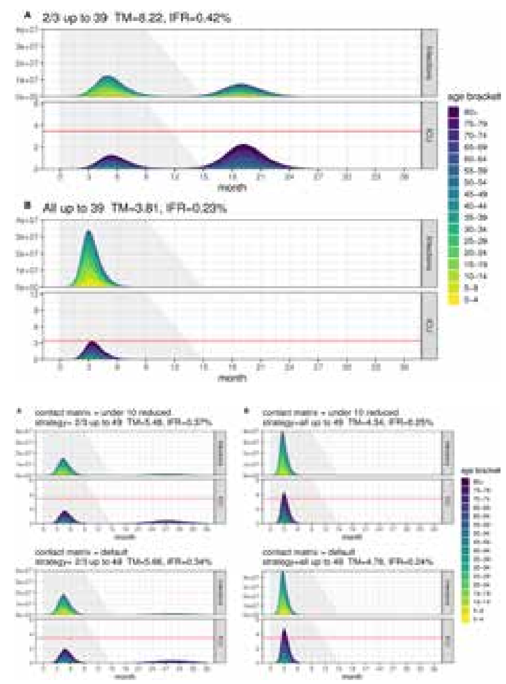
Research from Carnegie Mellon University and the University of Pittsburgh published in the journal PLOS One modeled how age-targeted strategies to mitigate the transmission of COVID-19 could lead to better final outcomes in mortality and hospitalization from the novel coronavirus.

"COVID-19 has a striking age-dependent mortality rate," said Maria Chikina, an assistant professor at the University of Pittsburgh Medical School. "An infected 75-year-old is 1,000 times more likely to die than an infected 15-year-old, and an infected 55-year-old is 20 times more likely to die than an infected 25-year-old."

Chikina co-authored this study with Associate Professor of Mathematical Sciences Wesley Pegden to dig into this "unusual characteristic" of the disease. Using a standard "Susceptible, Infected, Removed" epidemiological model, the pair sought to model the dynamics of how often people in various age groups have contact with each other, how that contact affects the transmission of COVID-19 and how adjusting those contacts would affect the impact of the pandemic.

The researchers concluded that heterogenous strategies focusing on reducing transmission of COVID-19 to and among the highest-risk age groups are most effective at reducing mortality and intensive care unit usage. These strategies were more effective compared to homogeneous strategies targeting all age groups equally, in situations where population immunity plays a role in controlling the epidemic. In particular, their modeling shows that age-targeted mitigations can allow immunity to contribute to epidemic control even while significantly reducing the burden of infection among at-risk groups.

**Charts:** Infection and ICU usage rates under various modeled scenarios for age-targeted COVID-19 mitigation.



Zheng Family

# Postdoctoral Fellowship

The postdoctoral program in the Department of Mathematical Sciences offers fertile ground for budding researchers to flex their scholarly muscles. We are excited to announce a new endowed position to our growing department: the Zheng Family Postdoctoral Fellowship.

The Zheng Family Postdoctoral Fellowship is a generous gift made by Fang Zheng, a longtime admirer of mathematics and a parent of a Carnegie Mellon mathematics student. Zheng is the managing director and chief investment officer of Keywise Capital, an asset management company based in Hong Kong. He regularly hosts Carnegie Mellon faculty, alumni and events in Hong Kong. An engineer by training, Zheng understands the need for strong mathematics research and its applicability to all aspects of human life. Without a solid mathematical foundation, many of society's innovations would not be possible.

Providing such a strong foundation requires extensive preparation and experience. An important element of curating mathematics excellence is through robust postdoctoral training. Once a scholar has completed their dissertation, a postdoctoral position affords them the space and time to invest in their research and gain valuable experience teaching and collaborating with a large network of colleagues. The Zheng Family Postdoctoral Fellowship adds prestige to the department by joining the Zeev Nehari Visiting Assistant Professor position. Together, these named positions offered by the Department of Mathematical Sciences add stature and a competitive edge to the department.

The department offers several avenues for postdoctoral recruitment and training. These opportunities range from funding through individual faculty members' grants, positions supported by income from the Masters of Science in Computational Finance program, general postdoctoral positions funded by the department itself and postdocs affiliated with the Center for Nonlinear Analysis (CNA). The CNA has a long history of garnering research funding that supports postdocs, who bring new expertise, perspectives and energy to the department. As new researchers to the field, our postdocs regularly engage in interdisciplinary and multidisciplinary projects at Carnegie Mellon and with other institutions. This strengthens the reputation of the department nationally and internationally and is conducive to building relationships across institutions, which is necessary to continue working on important and applicable research.

William Chan is the first recipient of this fellowship. Graduating with a Ph.D. in mathematics from the California Institute of Technology in 2017, Chan accepted a National Science Foundation Postdoctoral Fellowship and Research Assistant Professor position at the University of North Texas from 2017-2020. His research, while far-reaching, is centered in descriptive set theory. Topics of interest range from continuity and partition properties at small uncountable cardinals to analysis of Borel equivalence relations. Throughout his research, you can find traces of the application of determinacy in the context of games.

The Fang Zheng Family Postdoctoral Fellowship will deepen the impact of our postdoctoral program for many years to come.

*Colleen Storm*

**William Chan,** *Zheng Family Postdoctoral Fellowship recipient*

**Fang Zheng,** *Managing Director & Chief Investment Officer, Keywise Capital*

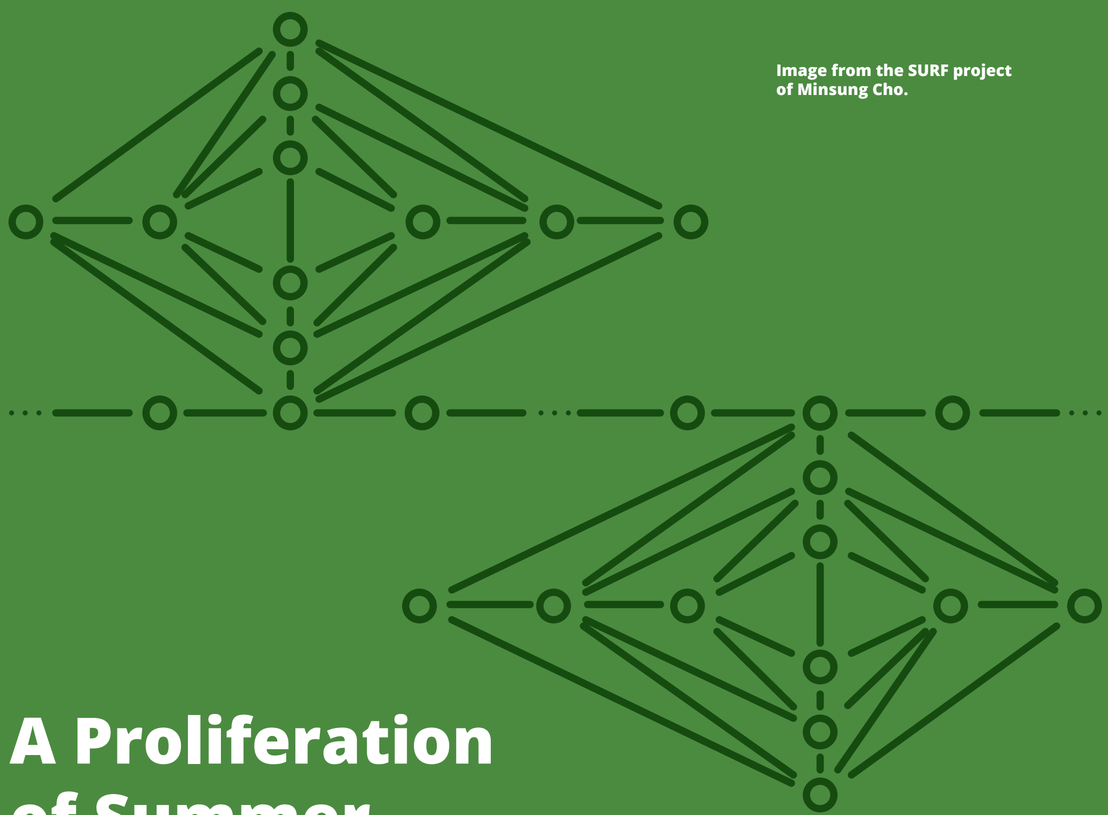


Image from the SURF project of Minsung Cho.

# A Proliferation of Summer Undergraduate Research

Summertime in Pittsburgh traditionally ushers in a flurry of out-of-class activities for students at Carnegie Mellon University. Many of them disperse, traveling home, studying abroad, or for some mathematics undergraduates, interning in the finance industry on Wall Street. But, this year, as the global coronavirus pandemic brought the world to a screeching halt, many students found that their summer programs had been canceled. However, in true Tartan spirit, they were eager to have the productive summer they had been planning. Answering this call, the Department of Mathematical Sciences curated a brand new summer program: Summer Experience in Mathematical Sciences (SEMS).

The SEMS program is an exciting new addition to the Carnegie Mellon menu of undergraduate summer programs in which math students can participate. These include the Summer Undergraduate Research Fellowship (SURF), Summer Undergraduate Applied Mathematics Institute (SUAMI) and Mathematical Finance Summer Undergraduate Research Program (MFSURP). Similar to SUAMI and MFSURP, SEMS is a math-only experience, administered by the Department of Mathematical Sciences. Even more, SEMS is not strictly a research-based program. It also includes in-depth reading groups and weekly seminars, which students can participate in alongside their projects. The variety of experiences afforded students who were expecting to have internships in areas like math finance and software engineering the opportunity to stay engaged in mathematics over the summer. Many of the participants described it to be a meaningful and enjoyable way to spend their time.

$$f(\vec{x}) = x_1x_2x_3 + x_1x_4x_5 + x_1x_6x_7 + x_1x_8x_9$$

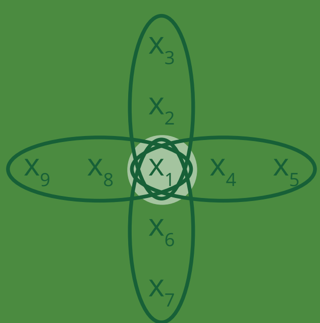



Image from the SURF project of Shanjiawen Zhao, Kevin Zhou and Vinod Krishnamoorthy.

"I applied to the research program because I really wanted to see how math research is conducted, and what questions people think are interesting," said mathematical sciences major Xinjie He. He continued, "I was also introduced to a new area and frontier questions in math, and some advanced techniques that I may not be able to learn in classes."

> *"It's always a pleasure to watch my students grow as researchers over the course of the summer program. Getting to continue this tradition during the chaos of the pandemic was a much needed morale booster for me."*
> **– Ian Tice**

Implementing the SEMS program was seamless. In the last several summers, the department has made a concentrated effort to invest more time in collaborating on compelling research and study projects with undergraduates. Thus, the department was naturally well positioned to execute such a big project with little time to prepare.

"I took on twice as many students as usual this year to help offset the cancellations in other programs due to COVID-19. Obviously, we had to forgo the usual in-person group research meetings and my students were spread across the country, but I was very impressed at how well they managed to work together as a group by using online collaboration tools," said Associate Professor Ian Tice. "It's always a pleasure to watch my students grow as researchers over the course of the summer program. Getting to continue this tradition during the chaos of the pandemic was a much needed morale booster for me."

In line with departmental goals, the research projects were intellectually diverse and included both applications of mathematics and pure mathematics research. While some students were concerned with gaining research experience for potential careers in academia, others were greatly invested in industry-related topics.

"I've had the good fortune to work with Professor [Hayden} Schaeffer, whose guidance has been critical in figuring out my career path, as the research I've done with him always involved practical applications of math and bits of machine learning. It's largely due to my research with Professor Schaeffer that I feel like I know what it means to be an applied math major," said student Hannah Milano.

Participants also found SEMS to lay insightful groundwork for more rigorous research.

"I'm glad I did the program because I used to be really intimidated by math research, but now it seems more approachable, and I feel like I know a little bit more about what I can expect I will be doing when I do research for my master's thesis," junior Carolyn Lee said.

Though the summer of 2020 brought about many changes in procedure for Carnegie Mellon, the Department of Mathematical Sciences found a way to use it to provide more opportunity for our undergraduates. We are looking forward to resuming on-campus research and study when conditions allow.

■ *Colleen Storm*

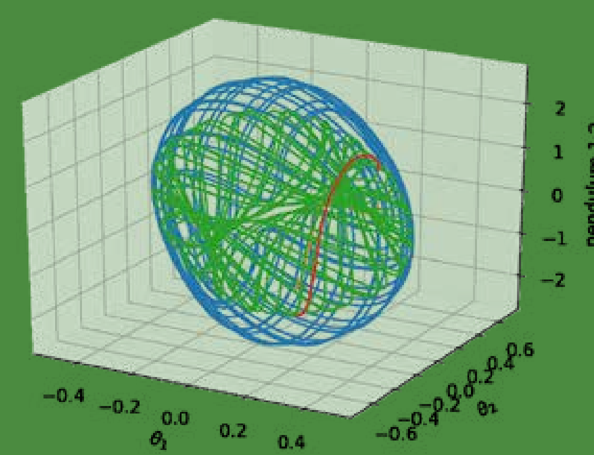



Image from the SURF project of Venessa Jiang and Ben Yuan.

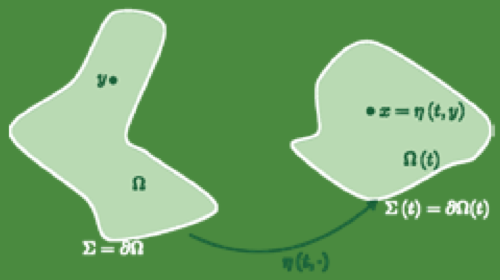



Image from the SURF project of Jonathan Jenkins, Carolyn Lee, Yuxuan Liu, Ethan Lu and Desmond Reed.

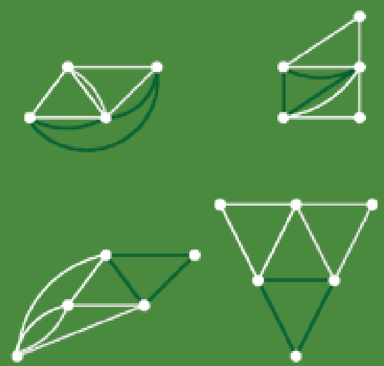


Image from the SURF project of Chloe Ireland.

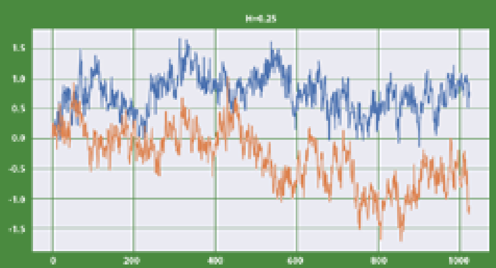



Image from the SURF project of Yidan Hu.

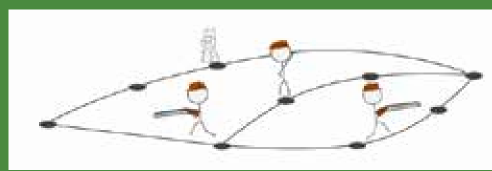


Image from the SURF project of Eugene Lee.

## ANDREEV MEMORIAL ENDOWED SURF

Elizabeth and Konstantin Andreev are lifelong math lovers. In fact, as a high school student Konstantin attended a math-focused school in Sofia, Bulgaria, with the full support of his father. Later, in 2002, Elizabeth and Konstantin met at Carnegie Mellon while enrolled in mathematics degree programs. Elizabeth was an undergraduate in the department, and Konstantin was a Ph.D. student in Algorithms, Combinatorics and Optimization.

Their commitment to mathematics has served as a tether to the academic world, which they departed from after graduation. Always interested in the goings-on of the Department of Mathematical Sciences, the Andreevs make a point to stay informed about the stellar accomplishments of the faculty and students. Specifically, they find student performance in renowned mathematics competitions highly impressive. Our Summer Undergraduate Research Fellowship (SURF) is no exception to their attention. Year after year, the SURF program churns out serious and competitive undergraduate research in mathematics, which is exciting for both the Andreevs and the department.

"In 2016, we were looking to make a more meaningful gift to an organization, and Carnegie Mellon was a top contender from the beginning, due to our engagement with the department and our ability to be involved in how the contribution would be used," Elizabeth said.

The SURF program offered up a lot of potential for this contribution. As previous students, the Andreevs understand the benefit of such a unique, research-based program for budding mathematicians.

"Konstantin in particular, having done a Ph.D., feels it's very valuable for undergrads to have a testing ground for how research works," Elizabeth said.

Every summer, the Andreev Memorial SURF endowment will provide a stipend of $3,500 for a student pursuing summer research in mathematics. Eugene Lee was the inaugural recipient of the award; he completed a research project entitled, "The Invisible Search Game" under the direction of Anton Bernshteyn. The Andreevs made this endowed gift in memory of Konstantin's father, Andrei, who was a researcher at the Bulgarian Academy of Sciences.



# SUMMER 2020 UNDERGRADUATE RESEARCH PROJECTS

## SURF PROJECTS

Neil Chen, Daria Mashanova, Hannah Milano and Mukund Subramaniam
***Neural Network Based Model Approximation for Discovering Dynamical Systems***
*Advisor: Hayden Schaeffer*

Zhijie Chen
***Numerical Scheme for Allen-Cahn Equation and its Energy Stability***
*Advisor: Franziska Weber*

Minsung Cho
***Weakly Cop Win Infinite Graphs***
*Advisor: David Offner*

Yidan Hu
***Self-Avoiding Property of Multi-Dimensional Gaussian Processes***
*Advisor: Jiawei Li*

Chloe Ireland
***Improving the Ramsey Bounds on Berge-K3 Hypergraphs***
*Advisor: Tom Bohman*

Jonathan Jenkins, Carolyn Lee, Yuxuan Liu, Ethan Lu and Desmond Reed
***Surfactant Dynamics from the Arnold Perspective***
*Advisor: Ian Tice*

Eugene Lee
***An Invisible Search Game***
*Advisor: Anton Bernshteyn*

Alexander Lum-Havrilla
***Khinchin-Type Inequalities***
*Advisor: Tomasz Tkocz*

Anrey Peng
***Nonlinear Cantilever Limit Cycle Oscillations***
*Advisor: Jason Howell*

Fei Peng
***Lonely Runners and Coprime Mappings***
*Advisor: Tom Bohman*

Noah Stevenson
***Traveling Wave Solutions to the Multilayer Free Boundary Incompressible Navier-Stokes Equations***
*Advisor: Ian Tice*

Zimu Xiang
***Upper Bound of the Expectation of the Longest Common Sequence for a Large Alphabet***
*Advisor: Boris Bukh*

Zhuqing Yang
***Particle Filtering Method and its Application in Heat Diffusion Problem***
*Advisor: Yu Gu*

Shanjiawen Zhao, Kevin Zhou and Vinod Krishnamoorthy
***On Log-Rank Conjecture for AND Functions***
*Advisor: Kaave Hosseini*

Alexander Zheng
***Entropy-Regularized Time-Discrete Optimal Transport Paths with Multimarginal Constraints***
*Advisor: Robert Pego*

Hongyi Zhou
***Coarsening Regularity of Integer Partitions***
*Advisor: Boris Bukh*

## SEMS PROJECTS

Neil Chen, Daria Mashanova, Hannah Milano and Mukund Subramaniam
***Neural Network Based Model Approximation for Discovering Dynamical Systems***
*Advisor: Hayden Schaeffer*

Yun Cheng, Albert Xu and Yixue Liu
***Combinatorial Structures with Gaussian Weights***
*Advisor: Tomasz Tkocz*

Ari Fiorino, Eric Li, Justin Sun and Xiao Liu
***Detecting Sumsets***
*Advisor: Kaave Hosseini*

Vanessa Jiang and Ben Yuan
***Sparse Approximation of High-Dimensional Functions and its Application to Learning Governing Equations***
*Advisor: Hayden Schaeffer*

Tudor-Dimitrie Popescu, Xinjie He and Ling Hu
***Structure of Complement of Sum-Free Sets***
*Advisor: Kaave Hosseini*

Shanjiawen Zhao, Kevin Zhou and Vinod Krishnamoorthy
***On Log-Rank Conjecture for AND Functions***
*Advisor: Kaave Hosseini*

## MEETING OF THE MINDS POSTER COMPETITION

**Grand Prize Winner**

Fei Peng
*What Can You Draw?*

**Runners Up**

Max Aires
*On the Number of Edges in a Maximally Linkless Graph*

Philip Lamkin
*Log-Convexity of Moments of Averages*

Noah Stevenson
*Characterizations of Screened Sobolev Spaces*

# EXCELLENCE IN DISCRETE & CONVEX GEOMETRY

*The faculty of the Department of Mathematical Sciences have continued to grow their expertise and influence in the field of discrete and convex geometry. Four recent results in particular highlight that success and offer promise for future innovations in mathematics and related fields.*

## KELLER'S CONJECTURE

Teaching Professor John Mackey was first introduced to Keller's Conjecture as a graduate student in Hawaii roughly 30 years ago.

"The conjecture is pretty natural: if you tile a plane with identical square tiles, then some pair will have to share an entire side," explained Mackey. "If you tile three-dimensional space with identical cubes, then some pair will have to share an entire square face. [Eduard Ott-Heinrich] Keller conjectured that this pattern continues in all higher dimensions."

Since his introduction to it, Mackey has regularly returned to the tessellation problem. The conjecture had been proved true up through six dimensions in the 1940s, and other researchers had been able to harness powerful computers to disprove it in ten dimensions and higher. In 2002, Mackey was able to create a counterexample to disprove Keller's conjecture in eight and nine dimensions, leaving only the seventh dimension unresolved.

Last year, in collaboration with alumnus Joshua Brakensiek, now a Ph.D. candidate at Stanford University, and Associate Professor of Computer Science Marijn Heule, Mackey was able to finally show that no counterexample exists for Keller's conjecture in seven dimensions.

## HADWIGER'S COVERING CONJECTURE

More than 60 years ago, famed Swiss mathematician Hugo Hadwiger published a series of intriguing, unresolved problems, including what came to be called his "covering conjecture."

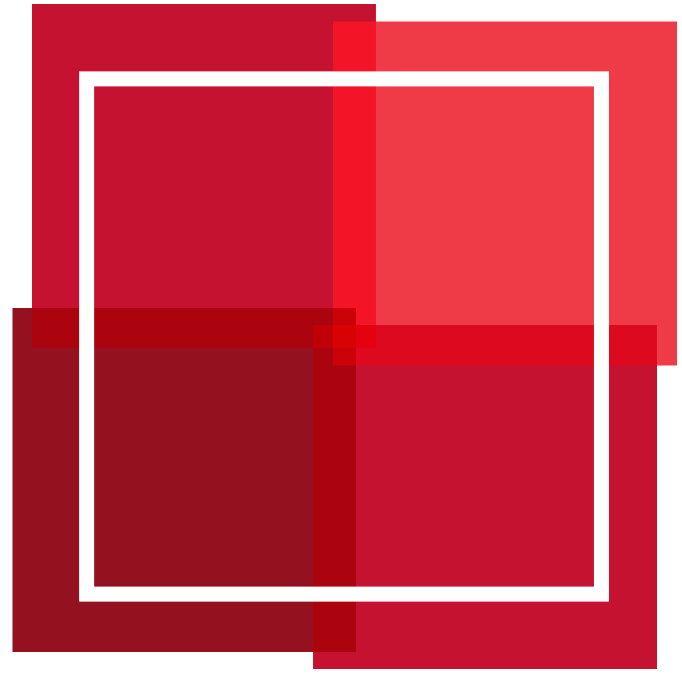
The question asks "can every n-dimensional convex body be covered by $2^n$ smaller copies of itself?" Hadwiger believed it was possible, and since then mathematicians, including Assistant Professor Tomasz Tkocz, have worked to prove that conjecture true or false.

"Besides aiming at understanding geometric properties of convex sets (which play an important role in many areas of mathematics), Hadwiger's conjecture touches upon the idea of a covering, one of the simplest and most fundamental in mathematics," Tkocz said.

For decades, the best upper bound on the conjecture was provided by English mathematician Claude Ambrose Rogers, who proved that for an arbitrary n-dimensional convex body, approximately $4^n\sqrt{n}$ smaller copies of the body are sufficient to cover it. However, in new research published in collaboration with mathematicians from the University of Michigan, the Weizmann Institute of Science in Israel and the University of Alberta, Tkocz was able to use tools from information theory and asymptotic convex geometry to improve on Rogers' upper bound.

**Top Right Image:** It is possible to cover the a closed n-dimensional cube with $2^n$ copies of the interior of the cube. This is achieved by assigning a copy of the interior to each of the $2^n$ vertices of the cube and shifting each copy of the interior a small distance in the direction of its assigned vertex. The image above depicts such a covering of the square (i.e., the two-dimensional cube). The covering conjecture of Hadwiger assets that every convex body in n-dimensional Euclidean space can be covered by $2^n$ copies of its interior.





## RESOLVING A KNOTTY PROBLEM

In 1911, the German mathematician Otto Toeplitz posited that any closed loop on a plane inscribes, or contains, four points that can form a square. While breakthroughs have been made to show this "square peg problem" to be true in certain situations, this deceptively simple conjecture has been resistant to being fully solved. "Even now, more than a century later, this is still an open problem," said Assistant Professor Florian Frick.

Hadwiger proposed a variant of the problem in 1971, asking whether any closed loop in three-dimensional space inscribes a parallelogram. "You could think of a piece of rope that might be knotted in complicated ways such that the two ends of the rope are fused together to make a loop," Frick explained. An illustration of a closed curve with an inscribed parallelogram is given on the cover of this newsletter.

In collaboration with researchers from North Carolina State University, Brandeis University, Cornell University and the Université du Québec á Montréal, Frick was able to prove Hadwiger's conjecture true.

## REDUCING DISCREPANCY

Ongoing work from Associate Professor Boris Bukh aims to make integration in higher-dimensions more feasible and accurate.

"Computing integrals exactly is almost never possible as that would require infinitely many additions," Bukh said. "For that reason, our only choice is to approximate integrals."

While it is relatively easy to approximate integrals of functions defined on low-dimensional domains, higher-dimensional integrals have proved difficult for mathematicians.

One can improve their approximation with a low "discrepancy" set of points to examine in a domain; however, it is not yet known which sets of points have the smallest discrepancy. In the technical feature of this publication, Bukh relates this issue of high-dimensional integration with another area of mathematics — convex geometry.

"By doing so, not only we can leverage decades of research in numerical integration to attack problems in convex geometry but also bring ideas from the latter area," Bukh said.

See the next page for more details on Bukh's work.

*Ben Panko*

# Discrepancy & Holes

How to spread $n$ points uniformly in the unit cube $[0, 1]^d$? A natural impulse is to arrange them into a regular grid. For many applications, this is a poor way. One such application is numerical integration: given a function $f$, the aim is to compute the definite integral $\int_{[0,1]^d} f$. If $f$ has a nice antiderivative, we may compute the integral using the Fundamental Theorem of Calculus. Sadly, most functions have no nice antiderivatives and so one must resort to numerical approximations.

Consider the approximation of $\int_{[0,1]^d} f$ by a finite sum $\frac{1}{n}\sum_{p\in P} f(p)$, where P is an $n$-points set. What happens if P is the grid? The good news is that by the very definition of the Riemann integral, the sum converges to the integral as $n \to \infty$. The bad news is that the convergence is quite slow even for very well-behaved functions.

For example, for the step function $f(x_1, \ldots, x_d) = 1_{x_1 \le \alpha}$ the approximation error is proportional to $n^{-1/d}$ for the most values of $n$. This is impractically large even for $d = 5$. One might be tempted to blame this on the discontinuity in $f$. However even for the function $x_1^2$ the error is proportional to $n^{-2/d}$.

It turns out that the grid points are a poor choice for our integration method because some axis-parallel boxes contain too few points, whereas others contain too many. Here, by *axis-parallel box* we mean a set of the form $B=[a_1,b_1) \times \cdots \times [a_d,b_d)$, and by *too few* and *too many* we mean that the number of points of P that are in B deviates significantly from $n$ vol(B), which is the number of points that we expect to fall into B had it been completely uniformly distributed.

*Discrepancy* of a set B is the difference $\text{disc}_P(B)=|P\cap B|-n\ \text{vol}(B)$. Discrepancy of P is $\text{disc}\,P=\max\ \text{disc}_P(B)$, where the maximum is over all axis-parallel boxes. For example, discrepancy of the regular grid is $\Theta(n^{1-1/s})$. Koksma and Hlawka showed that the approximation error for integration of functions of bounded variation is behaves like $\frac{1}{n}\,\text{disc}\,P$ in the worst case. Intuitively, integration with respect to a high-discrepancy set assigns undue weight to some subregions of $[0, 1]^d$ at expense of the others.

It is a fascinating open problem to find sets of smallest discrepancy in $[0, 1]^d$. The best existent constructions have discrepancy of asymptotic order $\log^{d-1} n$ in dimension $d$. One such construction is the Halton–Hammersley sets. It is easy to define these sets for $d=2$: given a natural number $m$ whose base-2 expansion is $a_t \cdots a_1 a_0$, its reversal is the binary number $r(m)=0.a_0a_1\cdots a_t$. Halton–Hammersley set is $\{(m/n,\ r(m)) : m=0, 1, \ldots, n-1\}$. The higher-dimensional Halton–Hammersley sets are defined similarly using digit reversals of $m$ when written in several prime bases.

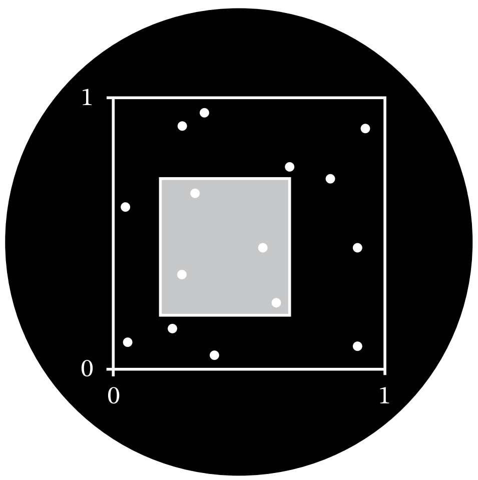



**Figure 1:** Discrepancy of a box is the difference between the expected and the actual number of points inside.

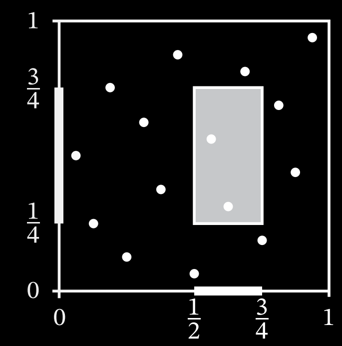



**Figure 2:** Halton-Hammersley set of size 16. The number of points in every not-too-small dyadic rectangle is exactly proportional to its area.

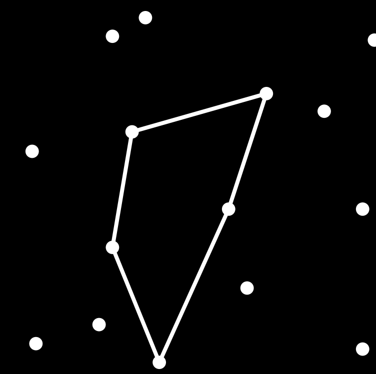


**Figure 3:** A 5-hole in a planar set.

Planar Halton–Hammersley sets with $n=2^k$ points have a peculiar property: they have *zero* discrepancy on every axis-parallel rectangle of the form $[a_1/2^s, b_1/2^s) \times [a_2/2^t, b_2/2^t)$ with $s + t \le k$. In discrepancy theory, a set with this property is called a *net*.

In addition to their uses in numerical algorithms, nets are also useful in combinatorial geometry. One such application is to the problem of convex holes. A *convex hole* in a set $P \subset \mathbb{R}^d$ is a subset $H \subset P$ that are vertices of a convex polytope; it is called $\ell$-hole if $|H|=\ell$. Erdős asked if, for every fixed $\ell$, all sufficiently large planar sets in general position contain an $\ell$-hole. Horton answered Erdős's question in negative: he constructed arbitrarily large planar sets without 7-holes. Valtr generalized Horton's construction to higher dimensions; he showed that there exist arbitrary large sets in $\mathbb{R}^d$ without $f(d)$-holes for some function $f$ that is slightly larger than the factorial. Recently, in a collaboration with Professor Holzman from Haifa, graduate student Ting-Wei Chao and the author showed that the sets of Horton and Valtr are Halton–Hammersley sets in disguise.

To be precise, Horton's set is obtained from the planar Halton–Hammersley by stretching $y$-coordinates; the $y$-coordinate $0.a_0a_1\cdots$ (in binary) becomes $a_0 Y_0 + a_1 Y_1 + \cdots$ where $Y_0 \gg Y_1 \gg \cdots$ is a quickly-decaying sequence of real numbers. A similar transformation turns any net into a set without large holes. Using this relation, new sets in $\mathbb{R}^d$ without $c^d$-holes were found. It is conjectured that the exponential bound is tight but the best lower bound for $d \ge 3$ is mere $2d + 1$.

■ *Boris Bukh*

# AT THE TOP OF HER GAME

## *Elizabeth Andreev is making her mark in municipal finance*

Elizabeth Andreev has sat in many seats in the finance industry — she's been a buyer, a seller, a market maker and an underwriter. She's brought her sharp quantitative skills to these roles at hedge funds, large commercial banks and multinational investment banks.

She credits this expanse of experience to beginning her career just as the most tumultuous financial crisis in modern history hit the United States and the world.

"It was overwhelming and hard to process all that was going on. Not only because I was so new but because it was kind of new to everybody. Things that were breaking down weren't supposed to be able to break down," Andreev recalls.

Andreev had just started her professional career at Susquehanna International Group, a hedge fund headquartered in the Philadelphia area.

When the mathematics major was graduating from Carnegie Mellon University, she never pictured a career in finance, but she did know that she wanted to find a career where she could use math to solve problems in everyday life.

She joined Susquehanna in 2006 as a trader in equity options but because of her quantitative skills, the company quickly moved her to the commercial mortgage-backed securities trading desk. Shortly thereafter, the subprime mortgage crisis completely obliterated that market.

Her position was effectively eliminated, but the group offered her a position as a trader and market-maker for municipal bonds and municipal bond exchange traded funds (ETF).

While at Susquehanna, Andreev developed a proprietary intraday pricing model for municipal bond ETFs and worked on a collaboration with investment management firm VanEck to structure and seed the first High Yield Municipal Bond ETF.

But Andreev soon realized that she wanted to try different things, an outlook she attributes to the lingering effects of the financial crisis. "I think for people of my generation, compared to those before who might have held a job for 20 years at the same firm, there's a little bit more mobility or interest in branching out and trying new things because we all know that situations can change very quickly. Developing breadth and new skill sets feels very important," Andreev said.

> ***"It's different than any other market because we are not just looking at and crunching numbers all day. We're working with people who are trying to get money to do things that often tangibly affect your everyday life."***

In 2010, she transitioned from being a buyer to a seller on the institutional sales and trading team at US Bank, transacting in bonds for a wide range of customers such as hedge funds, mutual funds and insurance companies. At US Bank, Andreev became the first municipal bond salesperson to receive the US Bank Summit Award for performance and leadership in 2014.

After nearly eight years, Andreev took another leap to become an underwriter at UBS, which was expanding its small municipal bonds group after deciding to reenter the realm of public finance.

Andreev finds municipal bond finance fascinating. Investment banks, like UBS, help public entities — governments, nonprofits, etc. — structure and sell municipal bonds to investors to help fund public projects.

"It's different than any other market because we are not just looking at and crunching numbers all day. We're working with people who are trying to get money to do things that often tangibly affect your everyday life," she said.

Andreev sees projects come across her desk from municipal issuers like a hospital trying to finance a new wing to a toll authority trying to build a new road.

Every day is something new, Andreev says. A facet that is both challenging — to know about all these different municipal issuers and their needs — and exciting. "One day, we will be working on financing a minor league ballpark, and the next day, it's something totally different, like financing an airport."

As an underwriter, Andreev works between the worlds of buyers and sellers for these municipal issuers. Her job is to structure and price the bond deal to help her issuer clients meet all their needs and objectives while also creating bonds that the sales force can market to investors. Andreev has impressed not only her clients and investors with her market insights and leadership but also the industry at large.

Last year, the Bond Buyer, one of the most respected and widely circulated daily newspapers for the municipal bond industry, named Andreev as one of its 2019 Rising Stars, an honor recognizing the best and brightest municipal finance professionals under age 40.

"I was honored to be nominated and selected. It was definitely a highlight of my career," said Andreev, who was also promoted to executive director at UBS earlier this year.

■ *Emily Payne*

# QUANT CLUB AND CITADEL HOST
# MARKET MARKING GAME

In February 2020, 150 students gathered to test their financial prowess at a Market Making Game. The event was hosted by Carnegie Mellon University's Quant Club and sponsored by Citadel Securities, one of the world's biggest market makers. It brought together undergraduate and graduate students from all across the university.

The game was a thought-provoking mixture of estimation, optimization and negotiation. Teams of participants took on the role of market makers as they tried to maximize their capital by trading with other teams.

Twenty-five teams started the game with contracts and some (fake) cash. Throughout five rounds of play, teams had to use their cash to buy contracts or sell contracts to make more cash. However, there was a catch. They didn't actually know the value of the contracts. Instead, each round rotated through questions about Citadel securities, the economy, mathematical puzzles and general trivia that teams had to solve to reveal the price of the contracts.

"These were tricky problems," said Bill Hrusa, mathematical sciences professor and faculty advisor to the Quant Club. "For example, one probability-focused question asked the number of times you need to roll a six-sided die in order that the product is going to be bigger than 20200201, the date of the event."

Hrusa was not surprised to see teams using a variety of strategies throughout the game.

One of those strategies is to visit the liquidity station. The liquidity station knows the hidden value of all the contracts. Teams could trade with the station once per round to gain information on the true value of their contracts.

"They can assume we will make the choice that maximizes our profit or minimizes our loss," said Quant Club President Tony Wang, who oversaw one of the stations. Essentially, if the liquidity station buys a contract, the team then knows they undervalued the contract. If the station sells the team a contract, the team knows it overvalued the contract.

It's a lot of uncertainty, but that's what makes the game all the more exciting.

"The most challenging part is the uncertainty that comes from not knowing the exact value of the contract and having to trade it anyways if you want any chance to make money," said Dallas Foster, recent computational finance graduate.

Fellow 2020 computational finance graduate Jason Ren agreed. His team managed that uncertainty by splitting up into different roles based on their strengths. Shengming Luo, a Ph.D. candidate in statistics, and Justin Cao, another computational finance recent graduate, solved the problems to find out the value of the contracts; Ren and mathematical sciences student Wynn Huang drove hard bargains as the team's traders; and computational finance major Cindy Ding also traded with other teams and managed the contracts and communication for the team.

The team took home first prize, ending the day with a 10% return on their contracts.

Ren's favorite part about the event was interacting with other teams. "With a room of 150 people, it can get pretty intense," he said. "Multiple groups are trying to compete at the same time. It almost felt like a trading floor where people are shouting on the floor and the phone."

*"The Market Making Game is not just based on problem-solving but teamwork, too."*

The Quant Club has been hosting the Market Making Game for a few years, but this was their first university-wide event. Former Quant Club executive Max Chen brought the game to the club after participating in a trading competition at another university. Chen, a computational finance graduate, went on to earn his master's in machine learning from CMU before joining Citadel as a trader in August this year. He wrote all the questions for the event and, along with Wang, helped organize the partnership with Citadel. Citadel representatives gave advice to teams during the game and chatted with students at a pre-event lunch.

The event was undoubtedly a success for the Quant Club. "I think the Market Making Game is my favorite Quant Club event," said Lily Qiao, a computational finance student and club executive. "It is a lot of fun because it's so interactive. It's not just based on problem-solving but teamwork, too."

*Emily Payne*


# SENIOR NOAH STEVENSON SELECTED AS GOLDWATER SCHOLAR

Mathematical Sciences student Noah Stevenson has received a 2020 Barry Goldwater Scholarship. Given by the Barry Goldwater Scholarship and Excellence in Education Foundation, the prestigious award supports students who intend to pursue research careers in the natural sciences, engineering and math.

Stevenson is pursuing the Mathematical Sciences Honors Degree Program. He takes both undergraduate and graduate level classes and will graduate from Carnegie Mellon University next year with his B.S. and M.S. in mathematical sciences.

At CMU, Stevenson has molded his interests in both teaching and research. He has served as a teaching assistant for the mathematical sciences' courses Math Studies: Analysis I and II, and he has conducted several research projects studying partial differential equations in fluid mechanics and function spaces, advised by Associate Professor of Mathematical Sciences Ian Tice.

His projects are influenced by his interest in applying abstract mathematical concepts to solve problems in the physical world.

For example, one of Stevenson's projects focused on the rigorous analysis of micropolar fluid equations, which describe the dynamics of viscous fluids with microstructure such as blood, colloidal suspensions or liquid crystals. Mathematicians contribute to the understanding of fluid flow by studying the nonlinear partial differential equations that govern their motion. To that end, a major focus of modern mathematical analysis is determining when solutions to fluid equations exist and studying if and how singularities can form.

"Working in this area is challenging even for graduate students, but Noah's strong background and honors program preparation made it possible for him to do so as a rising junior," said Tice.

Through his analysis of fluid equations, Stevenson discovered and analyzed a particular stability in micropolar fluids. His work has been submitted as a manuscript to be published.

These experiences have shaped Stevenson's career plans to earn his Ph.D. and become a researcher and professor of mathematics. He believes the Goldwater Scholarship will help him achieve this goal not only for its financial support but because of what it means to be chosen as a Goldwater Scholar.

"Receiving the Goldwater Scholarship is reassuring to me that I am traveling along the right academic and career path," said Stevenson. He adds that the scholarship process helped him learn how to communicate mathematical ideas to non-experts, a skill that will come in handy when applying to graduate school.

Stevenson is one of 396 students selected for the scholarship from an applicant pool of over 5,000 sophomores and juniors nationwide. He is also one of four Carnegie Mellon recipients.

Stephanie Wallach, assistant vice provost for undergraduate education, said that CMU can nominate four students annually for the award. Brittany Allison, assistant director in the Office of Undergraduate Research and National Scholarships, oversees the university's Goldwater nomination process, which includes a committee of faculty members who discuss the nominations, and also includes working closely with each nominee before the final submission.

"Dr. Allison did a double take when she saw that all four of our nominees were selected," Wallach said. "This is an award that recognizes our CMU strengths in the fields of engineering, natural sciences and mathematics and the exceptional ability of our faculty to involve our immensely talented and highly motivated students in undergraduate research.

*Emily Payne*

# ANTOINE REMOND-TIEDREZ RECEIVES GRADUATE STUDENT TEACHING AWARD

Mathematical Sciences Ph.D. graduate Antoine Remond-Tiedrez was awarded the 2020 Hugh Young Graduate Student Teaching Award.

"When I assigned TAs to various courses, I knew that I could assign Antoine anywhere, knowing that he would be an asset to any instructor," wrote Deborah Brandon, former associate teaching professor, in supporting his nomination for the award.

As testament to this fact, Remond-Tiedrez was a teaching assistant for seven semesters for nine different courses. He was committed to becoming a better educator with each course he taught and he poured countless hours of preparation into doing so.

"I like how the students' eyes light up when they have an 'aha moment.' I like to hear students voice their sense of accomplishment after completing a difficult yet rewarding class; and I like the challenge of finding the right teaching strategies for a particular cohort in a particular class," Remond-Tiedrez said.

When he became the instructor for the course Concepts of Mathematics in the summer of 2017, he designed the course himself using only the syllabus and the textbook, taking great care with planning lectures, quizzes, homework assignments, midterms and exams to provide repeated opportunities for students to develop their mathematical skills.

"The course is difficult to teach because it is the first proof-based course for most of our students," wrote Irene Gheorghiciuc, associate teaching professor.

However, students had nothing but praise for Remond-Tiedrez.

"Antoine did well in breaking down complicated problems and directing us to find an approach to each step," wrote student Joanna Yao about her experience in Concepts of Mathematics. "His teaching style gave me more chances to improve my mathematical thinking."

If you ask his students for someone who is dedicated and effective in teaching, they say to look no further than at Remond-Tiedrez.

Brandon agrees: "Antoine puts a lot of thought and effort into his teaching, in addition to having a natural talent for it. He is a wonderful (and versatile) teacher."



# CHRISTOPHER COX RECEIVES GRADUATE STUDENT RESEARCH AWARD

Mathematical Sciences Ph.D. graduate Christopher Cox received the 2020 Guy C. Berry Graduate Research Award.

"He is one of two or three best graduate students in Discrete Mathematics that I have seen in the department in my nearly eight years here," wrote Boris Bukh, associate professor of mathematical sciences and Cox's major professor, in nominating him for the award. Cox is now a postdoctoral fellow at Iowa State University.

Describing him as driven and independent, Bukh highlighted the fact that Cox has written five papers as a graduate student, compared to the average of one to three that a mathematical sciences student produces on their way to getting a Ph.D.

"In a nutshell, Chris is one of the most brilliant young combinatorialists that I have encountered over my thirty years of activity in this area," wrote Ron Holzman, a professor of mathematics at the Technion-Israel Institute of Technology, to support the nomination. Cox worked with Holzman in 2018 after receiving a grant allowing him to travel to Israel and collaborate on research there.

"My research focuses on the interplay between algebraic, geometric and probabilistic tools in extremal combinatorics," Cox said.

Specifically, Cox has studied problems relating to information theory and coding theory, fields which focus on solutions to efficiently communicating information across noisy channels.

"While passing through a noisy environment, a message may be corrupted: some parts of the message may be erased and other parts may be completely changed," Cox explained. "The main way to combat this noise is to add redundancy to the message."

Through his work related to the Shannon-Hartley theorem, Holzman noted, Cox and Bukh achieved a breakthrough on a "notoriously difficult problem" of calculating the upper bound of the Shannon capacity of a fixed graph.

In response to winning the Berry award, Cox chose to express his feelings about it in the form of puns.

"I'm Berry pleased to have won this award. I'd like to thank Boris Bukh, my adviser, and Dejan Slepčev, denominator, for their non-trivial support," Cox said. "Being a discrete mathematician, I'm glad I could count on them. Hopefully my research doesn't atrophy after winning a trophy."

# CLASS OF 2020 SPOTLIGHT: **TRAJAN HAMMONDS**

Trajan Hammonds spent his undergraduate years cultivating experiences that would give him as much exposure as possible to research in mathematics, a journey that has prepared him for his next steps as a Ph.D. candidate at Princeton University.

Each summer, Hammonds participated in a Research Experience for Undergraduates (REU) program, always hoping to find a new problem to tackle. He spent his first summer at the Mathematical Sciences Research Institute Undergraduate Program, the next at Williams College and his final summer at the University of Minnesota, Duluth.

"All three REU programs were instrumental in my decision to pursue a research career in mathematics," said Hammonds.

His passion for research culminated in an impressive five papers during his time as an undergraduate, three of which have been published: one in the Houston Journal of Mathematics, one in the Journal of Number Theory and another in the International Journal of Number Theory.

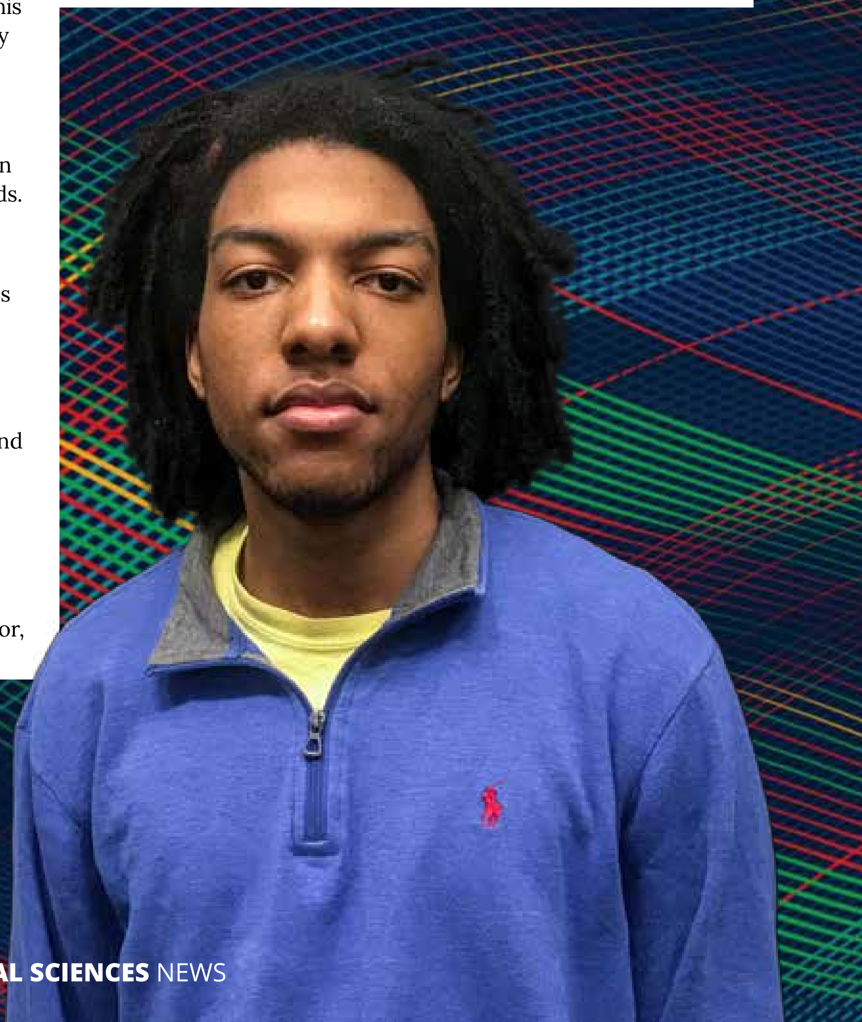
"Trajan has been a vibrant and valuable member of the mathematics community at CMU," said Hammond's advisor, Director of Undergraduate Studies Jason Howell. "He accomplished a great deal during his four years here, and I know that the future for Trajan is very bright."

Number theory, which uses a variety of tools from other subjects like algebraic geometry and harmonic analysis to answer basic questions about whole numbers, is Hammonds overarching area of study. And what led him to continue his studies at Princeton.

"I chose Princeton primarily for its abundance of faculty in number theory, including two recent Fields medal winners, who together cover a wide range of topics in number theory," Hammonds said.

While Hammonds has physically moved to New Jersey, he's starting his graduate studies virtually. He spends his time in classes, seminars, reading groups and in finding his doctoral research path. Currently, much of his research consists of trying to understand what has been done in the field before. A lot of number theory research consists of trying to figure out the right way to think about things, Hammonds notes. And he's looking for the central story that links everything together.

"I want to identify key tools and components in the landscape that would be useful for problems I may try to solve in the future. Moreover, it's important to really understand how things work in a classical setting in order to try to frame things in a more general setting," he said.





# CLASS OF 2020 DESTINATIONS

## Firms that hired more than one member of the class:

amazon

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Capital One

citi

CREDIT SUISSE

Microsoft

PNC

## Doctoral Programs:

University of Chicago, Statistics

Penn State University, Mathematics

Princeton University, Mathematics

University of California, Los Angeles, Mathematics (x2)

University of North Carolina at Chapel Hill, Bio Statistics

University of Wisconsin, Mathematics

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