Mathematics 3
Guide

NOVA SCOTIA
NOUVELLE-ÉCOSSE

2013

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Mathematics 3

Prepared by the Department of Education and Early Childhood Development

This is the most recent version of the current curriculum materials as used by teachers in Nova Scotia.

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CURRICULUM GUIDE

NOVA SCOTIA

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Implementation Draft
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Contents

Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

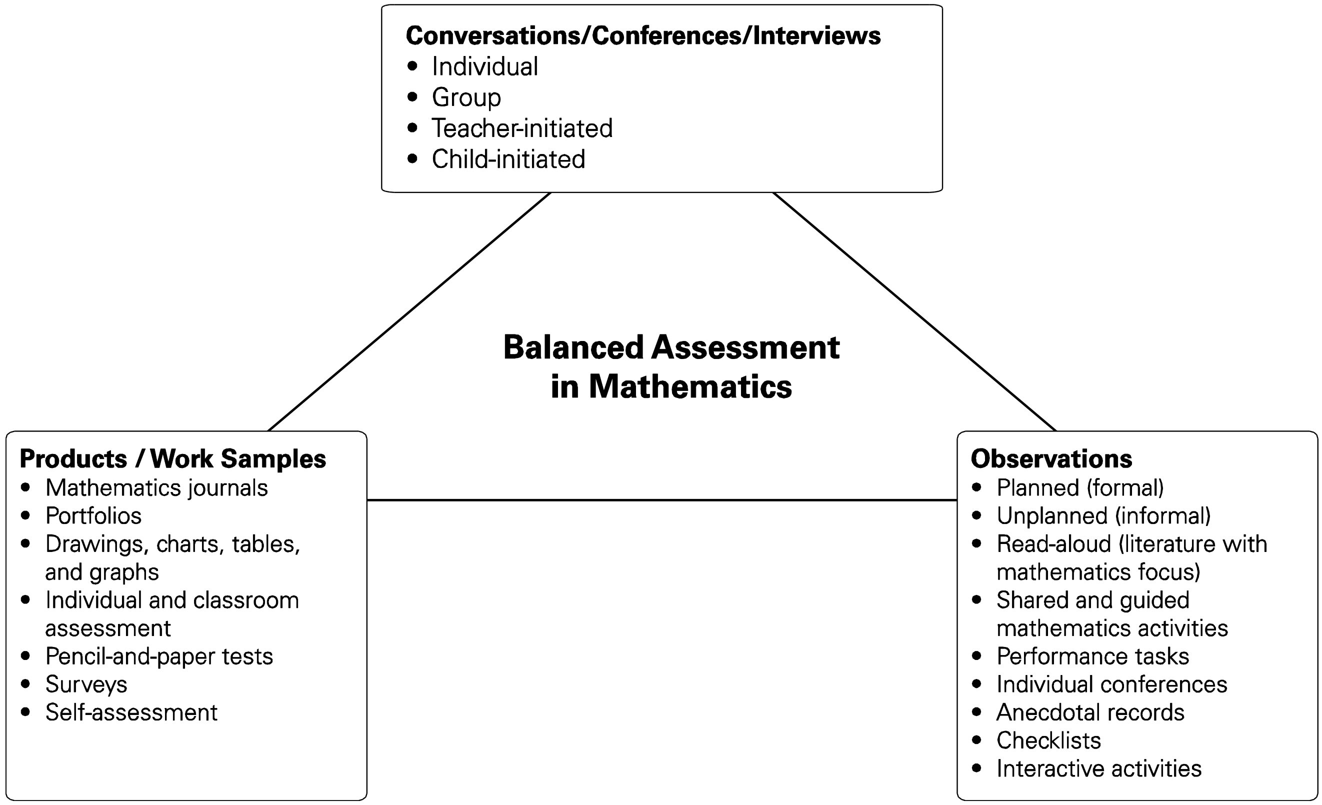
Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should
- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Time to Learn for Mathematics

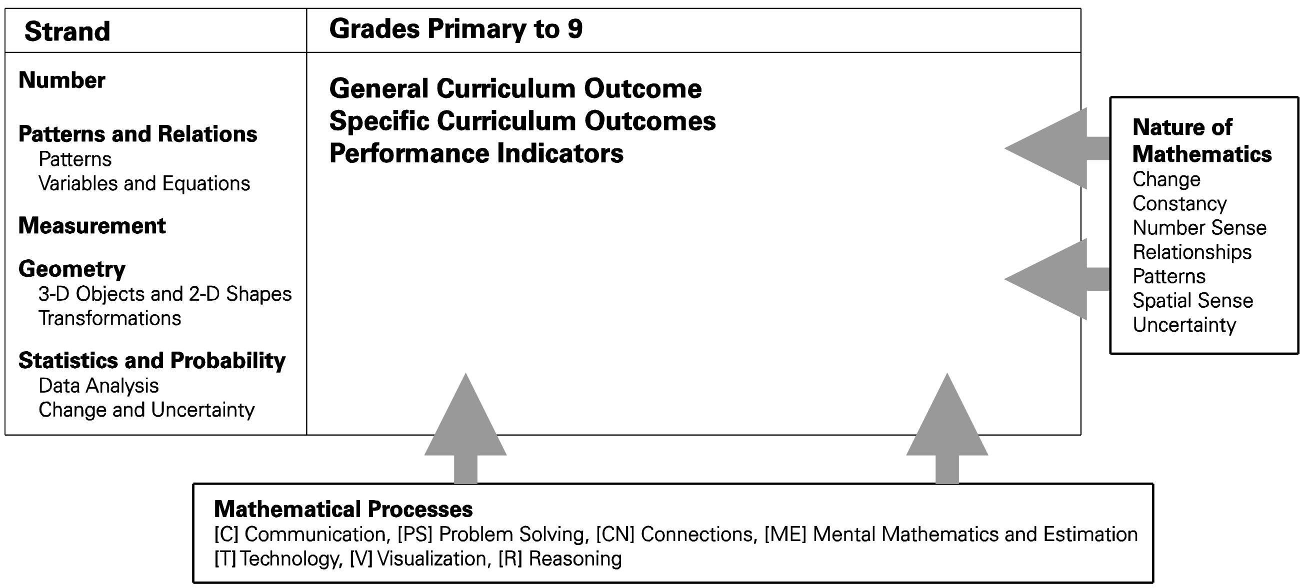
The Time to Learn Strategy *Guidelines for Instructional Time: Grades Primary*–6 includes time for mathematics instruction in the “Required Each Day” section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at
www.ednet.ns.ca/files/ps-policies/semestering.pdf
www.ednet.ns.ca/files/ps-policies/instructional_time_guidelines_p-6.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns
GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations
GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes
GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations
GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis
GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty
GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

NUMBER (N)

N01 Students will be expected to say the number sequence forward and backward by

- 1s through transitions to 1000
- 2s, 5s, 10s, or 100s, using any starting point to 1000
- 3s, using starting points that are multiples of 3 up to 100
- 4s, using starting points that are multiples of 4 up to 100
- 25s, using starting points that are multiples of 25 up to 200. [C, CN, ME]

Performance Indicators

N01.01 Extend the number sequence by 1s, particularly through transition from decade to decade and century to century.
N01.02 Extend a given skip counting sequence by 2s, 5s, 10s, or 100s, forward and backward, using a given starting point.
N01.03 Extend a given skip counting sequence by 3s, forward and backward, starting at a given multiple of 3 up to 100.
N01.04 Extend a given skip counting sequence by 4s, forward and backward, starting at a given multiple of 4 up to 100.
N01.05 Extend a given skip counting sequence by 25s, forward and backward, starting at a given multiple of 25 up to 200.
N01.06 Identify and correct errors and omissions in a given skip counting sequence.
N01.07 Determine the value of a given set of coins (nickels, dimes, quarters, and loonies) by using skip counting.
N01.08 Identify and explain the skip counting pattern for a given number sequence.

N02 Students will be expected to represent and partition numbers to 1000. [C, CN, V]

Performance Indicators

N02.01 Read a given three-digit numeral without using the word **and**.
N02.02 Read a given number word (0 to 1000).
N02.03 Represent a given number as an expression.
N02.04 Represent a given number concretely and pictorially in a variety of ways.
N02.05 Write number words for given multiples of ten to 90.
N02.06 Write number words for given multiples of a hundred to 900.
N02.07 Record numerals for numbers expressed orally, concretely, or pictorially.

N03 Students will be expected to compare and order numbers up to 1000. [CN, R, V]

Performance Indicators

N03.01 Place a given set of numbers in ascending or descending order and verify the result using a number chart or other models.
N03.02 Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
N03.03 Identify errors in a given ordered sequence.
N03.04 Identify missing numbers in parts of a given number chart and on a number line.
N03.05 Identify errors in a given number chart and on a number line.
N03.06 Place numbers on a number line containing benchmark numbers for the purpose of comparison.
N03.07 Compare numbers based on a variety of methods, and record the comparison using words and symbols (=, > and <).

N04 Students will be expected to estimate quantities less than 1000 using referents. [ME, PS, R, V]

Performance Indicators

N04.01 Estimate the number of groups of ten in a given quantity using 10 as a referent (known quantity).
N04.02 Estimate the number of groups of a hundred in a given quantity using 100 as a referent.
N04.03 Estimate a given quantity by comparing it to a referent.
N04.04 Select an estimate for a given quantity by choosing among three possible choices.
N04.05 Select and justify a referent for determining an estimate for a given quantity.

N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]

Performance Indicators

N05.01 Record, in more than one way, the number represented by given proportional and non-proportional concrete materials in traditional and non-conventional formats.
N05.02 Represent a given number in different ways using proportional and non-proportional concrete materials and explain how they are equivalent; e.g., 351 can be represented as three 100s, five 10s, and one 1s; or two 100s, fifteen 10s and one 1s; or three 100s, four 10s, and eleven 1s.
N05.03 Record a given number in additive expanded form.
N05.04 Record a number represented by base-ten blocks arranged in a non-conventional format.

N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals. [C, ME, PS, R, V]

Performance Indicators

N06.01 Explain mental mathematics strategies that could be used to determine a sum.
- Ten and some more
- Tens and some more
- Quick addition
- Addition facts to 10 applied to multiples of 10
- Addition on the hundred chart
- Adding on

- Make ten
- Compensation
- Compatible numbers

N06.02 Use and describe a personal strategy for determining a sum.
N06.03 Determine a sum of two 2-digit numerals efficiently, using mental mathematics strategies.

N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals. [C, ME, PS, R, V]

Performance Indicators

N07.01 Explain mental mathematics strategies that could be used to determine a difference.
- Facts with minuends of 10 or less applied to multiples of 10
- Quick subtraction
- Subtraction on the hundred chart
- Compensation
- Back through ten

N07.02 Use and describe a personal strategy for determining a difference.
N07.03 Determine a difference of two 2-digit numerals efficiently, using mental mathematics strategies.

N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, and 3-digit numerals in a problem-solving context. [C, ME, PS, R]

Performance Indicators

N08.01 Explain estimation strategies that could be used to determine an approximate sum or difference.
N08.02 Use and describe a strategy for determining an estimate.
N08.03 Estimate the solution for a given story problem involving the sum or difference of up to two 3-digit numerals.

N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000 by
- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically [C, CN, ME, PS, R]

Performance Indicators

N09.01 Model the addition of two or more given numbers using concrete or visual representations and record the process symbolically.
N09.02 Model the subtraction of two given numbers using concrete or visual representations and record the process symbolically.
N09.03 Create an addition or subtraction story problem for a given solution.
N09.04 Determine the sum of two given numbers using a personal strategy, e.g., for 326 + 48, record 300 + 60 + 14.
N09.05 Determine the difference of two given numbers using a personal strategy, e.g., for 127 – 38, record 2 + 80 + or 127 – 20 – 10 – 8.
N09.06 Solve a given problem involving the sum or difference of two given numbers.

N10 Students will be expected to apply mental mathematics strategies and number properties to develop quick recall of basic addition facts to 18 and related basic subtraction facts.
[C, CN, ME, R, V]

Performance Indicators

N10.01 Describe a mental mathematics strategy that could be used to determine a given basic addition fact up to 9 + 9.
N10.02 Explain how the commutative (order-doesn't-matter) property and the identity (no-change-with-zero) property can assist in addition fact learning.
N10.03 Describe a mental mathematics strategy that could be used to determine a given basic subtraction fact with minuends up to 18 and subtrahends up to 9.
N10.04 Recognize which facts could be determined by a given strategy.
N10.05 Quickly recall basic addition facts to 18 and related subtraction facts in a variety of contexts.

N11 Students will be expected to demonstrate an understanding of multiplication to 5 × 5 by

- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involves multiplication
- modelling multiplication using concrete and visual representations and recording the process symbolically
- relating multiplication to repeated addition
- relating multiplication to division [C, CN, PS, R]

Performance Indicators

N11.01 Identify events from experience that can be described as multiplication.
N11.02 Represent a given story problem (orally, shared reading, written) using manipulatives or diagrams and record in a number sentence.
N11.03 Represent a given multiplication expression as repeated addition.
N11.04 Represent a given repeated addition as multiplication.
N11.05 Create and illustrate a story problem for a given number sentence and/or expression.
N11.06 Represent, concretely or pictorially, equal groups for a given number sentence.
N11.07 Represent a given multiplication expression using an array.
N11.08 Create an array to model the commutative property of multiplication.
N11.09 Relate multiplication to division by using arrays and writing related number sentences.
N11.10 Solve a given problem in context involving multiplication.

N12 Students will be expected to demonstrate an understanding of division by

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication

(Limited to division related to multiplication facts up to 5 × 5.) [C, CN, PS, R]

Performance Indicators

N12.01 Identify events from experience that can be described as equal sharing.
N12.02 Identify events from experience that can be described as equal grouping.
N12.03 Illustrate, with counters or a diagram, a given story problem involving equal sharing, presented orally or through shared reading, and solve the problem.

N12.04 Illustrate, with counters or a diagram, a given story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
N12.05 Listen to a story problem, represent the numbers using manipulatives or a diagram and record the problem with a number sentence and/or expression.
N12.06 Create and illustrate with counters, a story problem for a given number sentence and/or expression.
N12.07 Represent a given division sentence and/or expression as repeated subtraction.
N12.08 Represent a given repeated subtraction as a division sentence.
N12.09 Relate division to multiplication by using arrays and writing related number sentences.
N12.10 Solve a given problem involving division.

N13 Students will be expected to demonstrate an understanding of fractions by

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole with like denominators [C, CN, ME, R, V]

Performance Indicators

N13.01 Describe everyday situations where fractions are used.
N13.02 Represent a given fraction concretely or pictorially.
N13.03 Identify, model, and explain the meaning of numerator and denominator.
N13.04 Sort a given set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.
N13.05 Name and record the fraction represented by the shaded and non-shaded parts of a given region.
N13.06 Compare given fractions with the same denominator using models.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

Performance Indicators

PR01.01 Identify and describe increasing patterns.
PR01.02 Describe a given increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
PR01.03 Extend a pattern, using the pattern rule, for the next three terms.
PR01.04 Compare numeric patterns.
PR01.05 Identify and explain errors in a given increasing pattern.
PR01.06 Create a concrete, pictorial, or symbolic representation of an increasing pattern for a given pattern rule.
PR01.07 Create a concrete, pictorial, or symbolic increasing pattern and describe the pattern rule.
PR01.08 Solve a given problem using increasing patterns.
PR01.09 Identify and describe the strategy used to determine a missing term in a given increasing pattern.
PR01.10 Use ordinal numbers (to 100th) to refer to or to predict terms within an increasing pattern.

PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

Performance Indicators

PR02.01 Identify and describe decreasing patterns.
PR02.02 Describe a given decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
PR02.03 Extend a pattern using the pattern rule for the next three terms.
PR02.04 Compare numeric patterns.
PR02.05 Identify and explain errors in a given decreasing pattern.
PR02.06 Create a concrete, pictorial, or symbolic representation of a decreasing pattern for a given pattern rule.
PR02.07 Create a concrete, pictorial, or symbolic decreasing pattern and describe the pattern rule.
PR02.08 Solve a given problem using decreasing patterns.
PR02.09 Identify and describe the strategy used to determine a missing term in a given decreasing pattern.
PR02.10 Use ordinal numbers (to 100th) to refer to or to predict terms within a decreasing pattern.

PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]

Performance Indicators

PR03.01 Explain the purpose of the symbol in a given addition and in a given subtraction equation with one unknown.
PR03.02 Create an addition or subtraction equation with one unknown to represent a given combination or separate action.
PR03.03 Provide an alternative symbol for the unknown in a given addition or subtraction equation.
PR03.04 Solve a given addition or subtraction equation that represents combining or separating actions with one unknown using manipulatives.
PR03.05 Solve a given addition or subtraction equation with one unknown using a variety of strategies including guess and check.
PR03.06 Explain why the unknown in a given addition or subtraction equation has only one value.

MEASUREMENT (M)

M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]

Performance Indicators

M01.01 Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice.
M01.02 Identify activities that can or cannot be accomplished in minutes, hours, days, weeks, months, and years.
M01.03 Provide personal referents for minutes and hours.
M01.04 Select and use a standard unit of measure, such as minutes, hours, days, weeks, and months to measure the passage of time and explain the choice.

M02 Students will be expected to relate the number of seconds to a minute, the numbers of minutes to an hour, the numbers of hours to a day, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]

Performance Indicators

M02.01 Determine the number of days in any given month using a calendar.
M02.02 Solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, the number of hours in a day, or the number of days in a given month.
M02.03 Create a calendar that includes days of the week, dates, and personal events.

M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by

- selecting and justifying referents for the units centimetre or metre (cm, m)
- modelling and describing the relationship between the units centimetre or metre (cm, m)
- estimating length using referents
- measuring and recording length, width, and height [C, CN, ME, PS, R, V]

Performance Indicators

M03.01 Provide a personal referent for one centimetre and explain the choice.
M03.02 Provide a personal referent for one metre and explain the choice.
M03.03 Match a given standard unit to a given referent.
M03.04 Show that 100 centimetres is equivalent to 1 metre by using concrete materials.
M03.05 Estimate the length of an object using personal referents.
M03.06 Determine and record the length and width of a given 2-D shape.
M03.07 Determine and record the length, width or height of a given 3-D object.
M03.08 Draw a line segment of a given length using a ruler.
M03.09 Sketch a line segment of a given length without using a ruler.

M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by

- selecting and justifying referents for the units gram and kilogram (g, kg)
- modelling and describing the relationship between the units gram and kilogram (g, kg)
- estimating mass using referents
- measuring and recording mass [C, CN, ME, PS, R, V]

Performance Indicators

M04.01 Provide a personal referent for one gram and explain the choice.
M04.02 Provide a personal referent for one kilogram and explain the choice.
M04.03 Match a given standard unit to a given referent.
M04.04 Explain the relationship between 1000 grams and 1 kilogram using a model.
M04.05 Estimate the mass of a given object using personal referents.
M04.06 Measure, using a balance scale, and record the mass of given everyday objects using the units gram (g) and kilogram (kg).
M04.07 Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg.
M04.08 Determine the mass of two given similar objects with different masses and explain the results.
M04.09 Determine the mass of an object, change its shape, re-measure its mass and explain the results.

M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by

- estimating perimeter using referents for centimetre or metre (cm, m)
- measuring and recording perimeter (cm, m)
- create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter [C, ME, PS, R, V]

Performance Indicators

M05.01 Measure and record the perimeter of a given regular shape and explain the strategy used.
M05.02 Measure and record the perimeter of a given irregular or composite shape and explain the strategy used.
M05.03 Construct a shape for a given perimeter (cm, m).
M05.04 Construct or draw more than one shape for the same given perimeter.
M05.05 Estimate the perimeter of a given shape (cm, m) using personal referents.

GEOMETRY (G)

G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]

Performance Indicators

G01.01 Identify the faces, edges, and vertices of given 3-D objects, including spheres, cones, cylinders, pyramids, cubes and other prisms.
G01.02 Identify the shape of the faces of a given 3-D object.
G01.03 Determine the number of faces, edges, and vertices of a given 3-D object.
G01.04 Sort a given set of 3-D objects according to the number of faces, edges, or vertices.

G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides. [C, CN, R, V]

Performance Indicators

G02.01 Classify a given set of regular and irregular polygons according to the number of sides.
G02.02 Identify given regular and irregular polygons having different dimensions.
G02.03 Identify given regular and irregular polygons having different positions.

STATISTICS AND PROBABILITY (SP)

SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. [C, CN, V]

Performance Indicators

SP01.01 Record the number of objects in a given set using tally marks.
SP01.02 Determine the common attributes of line plots by comparing line plots in a given set.
SP01.03 Organize a given set of data using tally marks, line plots, charts, or lists.
SP01.04 Collect and organize data using tally marks, line plots, charts, and lists.
SP01.05 Answer questions arising from a given line plot, chart, or list.
SP01.06 Answer questions using collected data.

SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems. [PS, R, V]

Performance Indicators

SP02.01 Determine the common attributes, title, and axes of bar graphs by comparing bar graphs in a given set.
SP02.02 Create bar graphs from a given set of data including labelling the title and axes.
SP02.03 Draw conclusions from a given bar graph to solve problems.
SP02.04 Solve problems by constructing and interpreting a bar graph.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*." (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

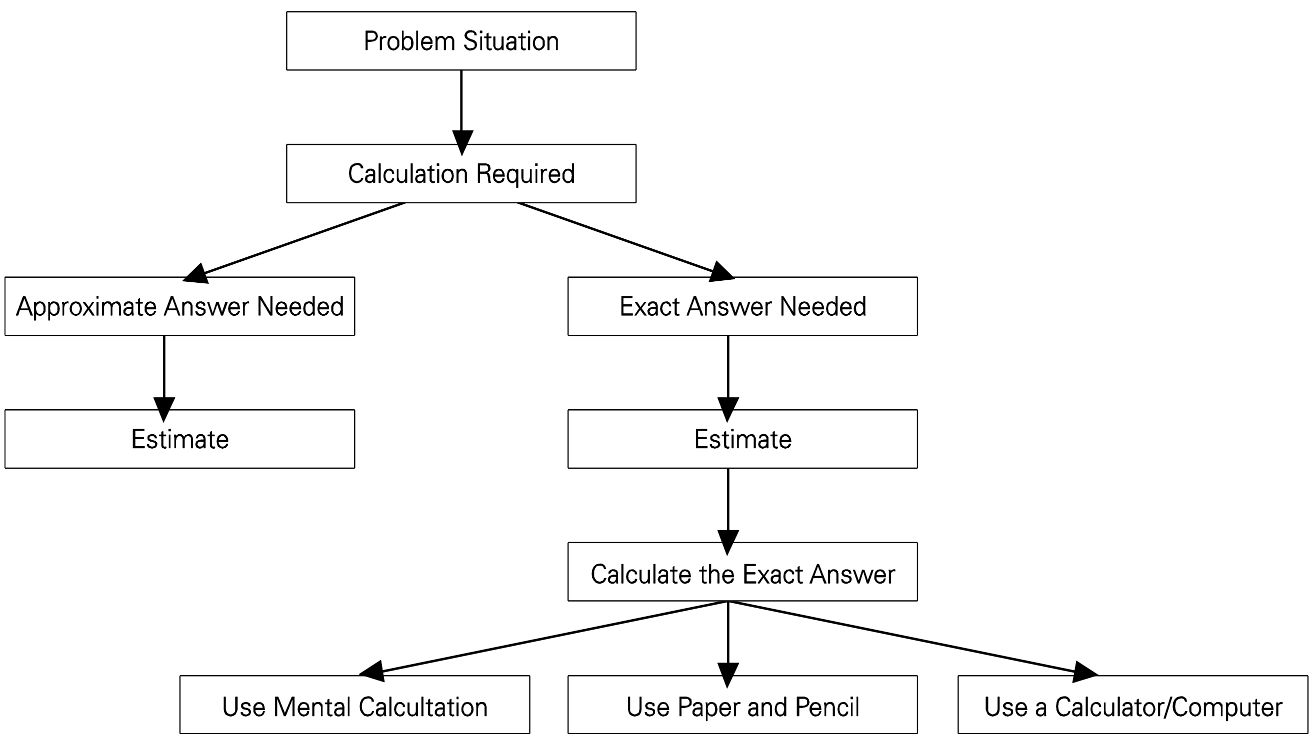
Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. “Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers.” (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Techonology can be used to
- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary to 3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.” (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes
[C] Communication **[PS]** Problem Solving **[CN]** Connections
[ME] Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the "big ideas" to be learned and how they relate to work in previous grade and work in subsequent courses.

Additional Information

A reference to Appendix A, which contains further elaborations for the performance indictators.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Sample tasks that can be used to determine students' prior knowledge.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Some suggestions for specific activities and questions that can be used for both instruction and assessment

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Correlations to related resources.

Planning for Instruction

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Suggested strategies for planning daily lessons.

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES

MATHEMATICAL LANGUAGE

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will
- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

*"*No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn."* (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today's classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student's potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include
- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are
- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A Gender-Inclusive Curriculum

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

Valuing Diversity: Teaching with Cultural Proficiency

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. "Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students' engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995)." (Herzig 2005)

Students with Language, Communication, and Learning Challenges

Today's classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

Students who Demonstrate Gifted and Talented Behaviours

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

Specific Curriculum Outcomes

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

N01 Students will be expected to say the number sequence forward and backward by
- 1s through transitions to 1000
- 2s, 5s, 10s, or 100s, using any starting point to 1000
- 3s, using starting points that are multiples of 3 up to 100
- 4s, using starting points that are multiples of 4 up to 100
- 25s, using starting points that are multiples of 25 up to 200. [C, CN, ME]

N02 Students will be expected to represent and partition numbers to 1000. [C, CN, V]

N03 Students will be expected to compare and order numbers up to 1000. [CN, R, V]

N04 Students will be expected to estimate quantities less than 1000 using referents. [ME, PS, R, V]

N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]

N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals. [C, ME, PS, R, V]

N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals. [C, ME, PS, R, V]

N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, and 3-digit numerals in a problem-solving context. [C, ME, PS, R]

N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000 by
- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically [C, CN, ME, PS, R]

N10 Students will be expected to apply mental mathematics strategies and number properties to develop quick recall of basic addition facts to 18 and related basic subtraction facts. [C, CN, ME, R, V]

N11 Students will be expected to demonstrate an understanding of multiplication to 5 × 5 by
- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involves multiplication
- modelling multiplication using concrete and visual representations, and recording the process symbolically
- relating multiplication to repeated addition
- relating multiplication to division [C, CN, PS, R]

N12 Students will be expected to demonstrate an understanding of division by

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication

(Limited to division related to multiplication facts up to 5 × 5.) [C, CN, PS, R]

N13 Students will be expected to demonstrate an understanding of fractions by

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole with like denominators [C, CN, ME, R, V]

SCO N01 Students will be expected to say the number sequence forward and backward by ▪ 1s through transitions to 1000 ▪ 2s, 5s, 10s, or 100s, using any starting point to 1000 ▪ 3s, using starting points that are multiples of 3 up to 100 ▪ 4s, using starting points that are multiples of 4 up to 100 ▪ 25s, using starting points that are multiples of 25 up to 200 [C, CN, ME]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Extend the number sequence by 1s, particularly through transition from decade to decade and century to century.
N01.02 Extend a given skip counting sequence by 2s, 5s, 10s, or 100s, forward and backward, using a given starting point.
N01.03 Extend a given skip counting sequence by 3s, forward and backward, starting at a given multiple of 3 up to 100.
N01.04 Extend a given skip counting sequence by 4s, forward and backward, starting at a given multiple of 4 up to 100.
N01.05 Extend a given skip counting sequence by 25s, forward and backward, starting at a given multiple of 25 up to 200.
N01.06 Identify and correct errors and omissions in a given skip counting sequence.
N01.07 Determine the value of a given set of coins (nickels, dimes, quarters, and loonies) by using skip counting.
N01.08 Identify and explain the skip counting pattern for a given number sequence.

Scope and Sequence

Mathematics 2	**Mathematics 3**	**Mathematics 4**
N01 Students will be expected to say the number sequence by ▪ 1s, forward and backward, starting from any point to 200 ▪ 2s, forward and backward, starting from any point to 100 ▪ 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100 ▪ 10s, starting from any point, to 100	**N01** Students will be expected to say the number sequence forward and backward by ▪ 1s through transitions to 1000 ▪ 5s, 10s, or 100s using any starting point to 1000 ▪ 3s using starting points that are multiples of 3 up to 100 ▪ 4s using starting points that are multiples of 4 up to 100 ▪ 25s, using starting points that are multiples of 25 up to 200	—

Background

Students continue to develop an understanding of number and counting. Students should count with proficiency forward and backward by 1s, 2s, 5s, 10s, or 100s starting at any number from 0 to 1000. “The focus of skip counting in these early years seems to be on helping students see the patterns in our place value system as well as prepare students for work with money.” (Small 2009, 86). Skip counting by 3s, 4s, 5s, 10s, 25s, and 100s is a cornerstone for later multiplicative understanding.

When exploring numbers over 100, spend considerable time focusing on the numbers between 100 and 200. Students need to think about how the counting pattern sounds when they count from 20 to 29 and 30 to 39 and apply it when counting from 120 to 129, and 130 to 139, and when counting through all the decades that follow. Students need multiple opportunities to count the numbers through transition from decade to decade and century to century. For example, when counting from 98 … 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 … a common misconception for students is to think that the next number after the one that ends in “9” is the next big number name. As a result many believe that 200 comes after 109. Students need to see that this same transition will apply for counting into all of the centuries. For example, on a regular basis, they should be asked questions such as, what would come after 199? or count on from 389.

Being able to skip count forward and backward by 5s, 10s, and 100s from any given number; by 3s and 4s, starting at any multiple of 3 and 4 respectively up to 100; and by 25s starting at any multiple of 25 up to 200, will help students be successful with many upcoming concepts such as patterns, money, and place value. Students in Mathematics 2 were skip-counting by 2s forward and backward to 100. Number charts may be used to explore these patterns.

Students should also investigate these skip counting patterns and whether the similar patterns occur when counting by 100s or when counting on hundreds charts beyond 100. A specific focus on looking at the numbers between 100 and 200 can help students to develop a sense of the repeating patterns in the number system. A hundred chart can easily be extended to a 200 chart by adding rows for the next 100 numbers.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to start at
  - 92 and count forward by 1s (stop at 121)
  - 42 and count forward by 2s (stop at 60)
  - 13 and count by 2s (stop at 35)
  - 78 and count backward by 2s (stop at 58)
  - 30 and count by 10s (stop at 100)
  - 8 and count by 10s (stop at 58)
  - 100 and count backward by 10s (stop at 40)
  - 15 and count by 5s (stop at 60)
  - 85 and count backward by 5s (stop at 55)

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to skip count following directions, such as
  - Start at 95 and count forward by 5s to 140
  - Start at 349 and count by 100s without going over 1000, but get as close as you can
  - Start at 635 and count forward by 10s to 725
  - Start at 64 and count backward by 2s to 38
  - Start at 399 and count forward by 1s to 420
  - Start at 708 and count backward by 1s to 690
  - Start at 0 and count forward by 25s from 200
  - Start at 4 and count forward by 4s to 32
  - Start at 30 and count backward by 3s to 12
- Provide students with a number of beans (e.g., 60). Ask them to separate the beans from the pile as they count the beans by 3s and then by 4s.
- Ask, Why do you say fewer numbers when counting to 100 by 10s than when counting by 5s?
- Provide students with a skip counting pattern, such as 40, 36, 32, 28, 24, 20, 16, … Ask them to identify the pattern and then to continue the pattern until they reach 0.
- Ask students to decide which starting point, 6 or 7, is easier when counting by 3s. Ask students to explain their choice.
- Write and say, 25, 50, 60, 65, 70. Ask, What coins am I counting?
- Have students identify and correct the error in a given skip counting sequence, such as
  - 12, 16, 21, 24, 28, 32
  - 27, 30, 33, 35, 39, 42
- Provide students with a set of nickels and ask them to skip count to find the total value of those coins.
- Provide students with a set of coins containing loonies, quarters, dimes, and nickels. Ask them to count the coins and to tell you the total.
- Have students count by 5s until they reach 60. Ask, What other numbers can you count by and still land on 60?

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 1, Tasks 1, 2, and 3, pp. 20–21

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Skip Counting

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Give students frequent opportunities to count materials (large quantities) in a variety of ways.
- Highlight the numbers on a number line or hundred chart that occur when skip counting and have students describe the patterns they see. Ensure that the numbers extend beyond 100.
- Provide students with multiple experiences counting both forward and backward, with various starting points.
- Use the calculator constant feature (described below) to count by 2s, 3s, 4s, 5s, 10s, 25s, and 100s. Ask the students to predict what number will come next, before it appears on the display.
- Be sure to use many examples including numbers greater than 100.
- Use situations, such as school fundraisers, as opportunities to count money by skip counting.

SUGGESTED LEARNING TASKS

- Provide students with a hundreds chart and have them colour in the pattern for a given skip counting sequence.
- Provide students with many number patterns to encourage skip counting; for example
  - 25, 50, _, _, 125, _, _
  - 752, 652, _, 452, _, 252, _
  - 110, 105, _, 95, 90, _, _, 75, _, _, _
  - 12, _, 18, _, 24, 27, 30
  - 39, 36, _, 30, _, 24, _, _,
  - 4, _, _, 16, _, 24, 28, 32
- Ask students to count a large collection of objects, such as beans in a jar. Ask them how they grouped the beans (e.g., by 2s, 5s, 10s) for ease of counting.
- Use the *constant* (repeat) function (press 0, +, 5, =, =, =, …) on the calculator to skip count to a target number. For example, if you start at 0 and want to end at 400, by which number(s) could you skip count? What if you started at a different point? What if you wanted to end at a different point?
- Play "What's in the Can?" Tell students that you are going to drop nickels (or dimes or quarters) into a can. Have the students listen as the coins drop and count to find the total. As an extension, tell the students that there is, for example, 45 cents in the can. Tell them that you are going to add nickels (or dimes) and ask them to keep track to find the total.
- Provide students with a hundreds chart. Point to one of the squares (e.g., 56) and ask, If you start at 28 and count by 4s, will you say this number? Explain. Repeat using other numbers and starting points.
- Provide coins for the students. Ask, Can you use six coins to make 87 cents? Can you make a total of one dollar and 45 cents with only six coins? What are the coins?
- Provide students with play coins. Tell them that you have, for example, five coins in your hand that total 81 cents. Ask, What coins am I holding? (This is a problem situation and may require time.) Have students create puzzle questions like this.
- Provide students with a skip-counting pattern containing an error or an omission. Ask students to correct the error or omission.

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- coins
- hundred chart
- number line
- open number lines
- various objects for counting (e.g. beans, counters)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ forward, backward ▪ nickels, dimes, quarters, loonies ▪ numbers: zero to one thousand ▪ skip counting patterns	▪ forward, backward ▪ nickels, dimes, quarters, loonies ▪ numbers: zero to one thousand ▪ skip counting patterns

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 86–87, 144
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 142–143
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 56–58, 61–62, 138–139, 144, 150–152, 285–286
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 51–52

Videos

- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)

Notes

SCO N02 Students will be expected to represent and partition numbers to 1000. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Read a given three-digit numeral without using the word **and**.
N02.02 Read a given number word (0 to 1000).
N02.03 Represent a given number as an expression.
N02.04 Represent a given number concretely and pictorially in a variety of ways.
N02.05 Write number words for given multiples of ten to 90.
N02.06 Write number words for given multiples of a hundred to 900.
N02.07 Record numerals for numbers expressed orally, concretely, or pictorially.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N04 Students will be expected to **r**epresent and partition numbers to 100.	**N02** Students will be expected to represent and partition numbers to 1000.	**N01** Students will be expected to represent and partition whole numbers to 10 000 concretely, pictorially, and symbolically.

Background

Students must be able to record numbers heard, read written numbers, write numbers in words, and represent written numbers symbolically. They must be able to translate a number from the written to the oral.

Although some students will have a clear understanding of the base-ten pattern of our place value system, many will still be in the early stages of development. It is important that students be provided with regular opportunities to represent numbers concretely, pictorially, symbolically, and in words to strengthen their knowledge.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

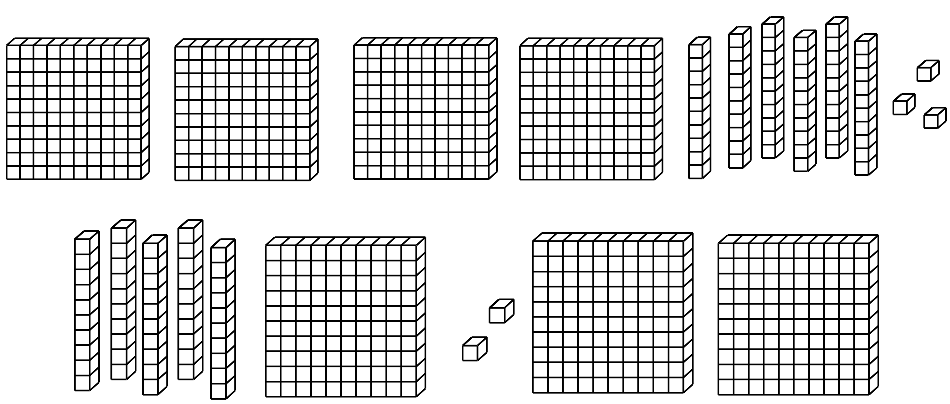
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to represent 52 (or any 2-digit number) with
  - ten-frames
  - tallies
  - coins
  - base-ten blocks
  - a picture
  - an expression

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to choose a number less than 1000 and represent it in at least three ways using base-ten blocks. For example, if a student has chosen 527, he or she may represent it with 5 flats, 2 rods, and 7 small cubes; 4 flats, 12 rods, and 7 small cubes; 3 flats, 20 rods, and 27 small cubes or any other combination of blocks that totals 527.
- Present students with a pictorial representation of base-ten blocks illustrating a 3-digit number. Ask students to record the numeral represented by the picture. Ensure that both conventional and non-conventional displays of base-ten blocks are included. For example,



- Ask students to record numbers read orally, both symbolically and with words, making sure to include numbers that have a zero such as 902 or 370.
- Present students with cards on which are written 3-digit numerals. Ask students to read the numbers to you orally and to represent them using base-ten blocks.
- Ask students, Which of the expressions below represents 360? Ask them to explain their thinking.
  - 200 – 160
  - 380 – 30
  - 400 – 40
  - 300 + 60
  - 100 + 100 + 100 + 50 + 10
  - 260 + 75 + 25
  - 357 + 3
  - 260 + 10
- Provide students with an open number line and have them place the benchmark numbers, such as 250, 500, 750, and 1000, on it.
- Present students with cards on which are written numbers in words, such as four hundred eighteen or nine hundred seven. Ask students to read the numbers to you and to record the numeral.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 1, Tasks 1, 2, and 3, pp. 20–21

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Representing Number

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have many opportunities to use a variety of concrete materials.
- Have a mathematics word wall available for students to assist with the correct spelling of number words.
- Provide students with frequent opportunities to represent numbers using words, pictures, and symbols.
- Have students create different expressions for the same numbers, ensuring students understand when describing 3-digit numbers there are more than 99 ones and more than 9 tens.

SUGGESTED LEARNING TASKS

- Have students work in pairs or in small groups. Provide each group with a sheet of chart paper. Ask each group to select a 3-digit number and to represent that number in as many different ways as they can using base-ten blocks. As each base-ten model is created, groups should record it on their chart paper using pictorial representations. Groups should then record the expressions that correspond to each pictorial display. After each group has completed as many different representations for their chosen number as they can, post the chart paper from each group. Have the class examine each chart paper to determine the number represented by the pictorial representations and the expressions. Ask them to explain why all of the pictures and number expressions on a sheet of chart paper are equal.
- Have students create a “thousand” chart by writing the number sequence in 10 blank hundred charts.
- Have students find numbers up to 1000 from different sources, such as newspapers, the Internet, signs, etc., and then ask them to read and model pictorially the numbers they found.
- Ask students to rename a number, less than 1000, as the sum of other numbers.
- Draw a number line labelled 0 and 100 at opposite ends (or 200 and 400, 100 and 600, 100 and 1000, etc.). Mark a few different points on this number line, and ask students what number they think each point might be and why they think that.
- Have students place benchmark numbers on the number line labelled 0 and 1000, for example 250, 500, 750.
- Have students create and solve number riddles, such as, I have written a secret number between 600 and 800. It is an odd number. What might it be?
- Ask students to record a series of numbers that are read to them. Include examples such as “ten less than 652” and numbers that contain a zero.
- Model a number using base-ten materials in an unconventional order and have students say the number.
- Tell students that a number has at least 15 tens and 3 ones. Ask students to decide what number that could be and to explain their thinking.
- Have students work with a partner and record a number with words, exchange with their partner, record that number symbolically, then say that number to their partner.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- cards with digits
- counters
- hundred chart
- hundred frame
- linking cubes
- money
- number lines (including walk-on and open number line)
- place-value cards
- place-value chart
- place-value dice

MATHEMATICAL LANGUAGE

Teacher	Student
▪ multiples ▪ number expression ▪ number lines, hundreds charts ▪ number words, symbols, digits ▪ ones, tens, hundreds ▪ represent, partition numbers	▪ number expression ▪ number lines, hundreds charts ▪ number words, symbols, digits ▪ ones, tens, hundreds ▪ represent, partition numbers

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 137–144, 146–147
- *Making Math Meaningful to Canadian Students K–8,* Second Edition (Small 2013), pp. 193
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 132–136, 145–148
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 39–45, 51–55

Notes

SCO N03 Students will be expected to compare and order numbers to 1000. [CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Place a given set of numbers in ascending or descending order and verify the result using a number chart or other models.
N03.02 Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
N03.03 Identify errors in a given ordered sequence.
N03.04 Identify missing numbers in parts of a given number chart and on a number line.
N03.05 Identify errors in a given number chart and on a number line.
N03.06 Place numbers on a number line containing benchmark numbers for the purpose of comparison.
N03.07 Compare numbers based on a variety of methods, and record the comparison using words and symbols (=, > and <).

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N05 Students will be expected to compare and order numbers up to 100.	**N03** Students will be expected to **c**ompare and order numbers to 1000.	**N02** Students will be expected to **c**ompare and order numbers to 10 000.

Background

Students should encounter a variety of numbers in context. These contexts help them develop an understanding of number size. In Mathematics 2, students compared and ordered numbers to 100 using benchmarks, number lines, hundred charts, and ten-frames. In Mathematics 3, students will compare numbers to 1000 and will order a set of numbers in ascending and descending order using a variety of methods including number charts, number lines, and place value materials. Visual models encourage reasoning, as students consider how to compare and order numbers. As with all concepts, begin with concrete models before moving to more pictorial and symbolic representations.

Students should recognize that every 2-digit whole number is greater than every 1-digit whole number. Thus, when they compare 2-digit numbers, they should understand that the tens digit is the more vital element of the number; and when they compare two numbers with the same tens digit, they should compare the ones digit. Students should also apply this logic to 3-digit numbers. Given a set of numbers, students should be able to place the numbers in ascending or descending order, and verify the result using a hundreds chart or by drawing a number line.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

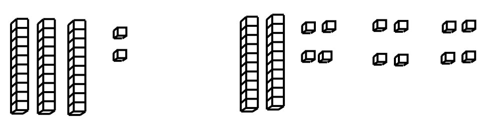
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with two representations of the same number. Point to one and ask, Is this number more, less, or are they the same? Have students explain their thinking.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask the student to use models to show why 243 is less than 324.
- Ask students to rearrange the digits 1, 4, and 5 to create the number that is closest to 500.
- Ask the student to find a number between 312 and 387 that can be represented using 8 base-ten blocks.
- Ask, What do you do to compare the value of two numbers?
- Show the students two numbers (e.g., 501 and 398) and ask which is greater. Have the students explain their answers. Encourage them to use a variety of models in their explanations.
- Ask, Why are there more numbers greater than 123 than less than 123?
- Ask, If __39 is greater than 422, what do you know about __ (the missing digit)? If __39 is greater than __87, what do you know about the missing digits?
- Ask the student to write a number that is
  - greater than 165 but less than 200
  - a little less than 300
  - between 463 and 474
  - greater than 348 but less than 360, etc.
- Ask students to explain why a 3-digit whole number is always greater than a 2-digit whole number. Provide students with specific examples to use in their explanation (e.g., 560 and 56).
- Ask students to select five numbers between 600 and 630, and to write them in increasing order.
- Ask students to make as many numbers as they can using the digits 2, 3, and 4, but using each digit only once. Have them list them in order from least to greatest or greatest to least.
- Provide students with an ordered sequence of numbers that contain an error. Have students identify and correct the error (e.g., 123, 132, 213, 231, 321, 312).

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Compare and Order Number

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students plot numbers on an open number line (horizontal and vertical) to show their relative positions. Have students share their thinking.
- Give students many opportunities to explore the magnitude of similar digits. For example, How are the digits in 777 similar? How are they different?
- Give students opportunities to build visual representations of numbers. For example, What does 35 look like compared to 353?
- Show students the “greater than” (>) and “less than” (<) symbols. Tell them that mathematicians use these symbols to replace the words “greater than” and “less than” when comparing numbers. Have students discuss why these symbols may have been chosen.

SUGGESTED LEARNING TASKS

- Ask students to model two different 3-digit numbers using twelve of the base-ten blocks. For example, students might chose to model 642 using 6 flats, 4 rods, and 2 small cubes and 381 using 3 flats, 8 rods, and 1 small cube. After modelling the two numbers, ask students to record the number in symbolic form. Then, ask students to identify which number is greater and to explain their thinking.
- Have students use a prepared deck of 40 numeral cards (4 sets of 0 to 9). Have each student select three of the cards and arrange them to make the greatest possible number and the least possible number. Ask the students to use base-ten blocks to prove they are correct.
- Prepare a deck of cards that contain 2- and 3-digit numerals. Have the students deal all the cards face down to the players. Have each player turn their top card over. The player whose card represents the greater (greatest) numeral wins both or all the cards in play. The winner is the one who has collected the most cards when all the cards have been turned over.
- Provide a set of cards (10 to 15) with each card having a 2- or 3-digit numeral on it. Ask students to order the cards from least to greatest and to explain their thinking using hundreds charts, a number line, or base-ten blocks.
- Have students work in pairs. Provide a set of 10 to 15 cards with each card having a 2- or 3-digit numeral on it. One partner selects 5 cards and secretly decides whether he or she will place them in order from least to greatest or from greatest to least and whether he or she will make an error in the order or not. After arranging the cards according to the decisions made, he or she invites his or her partner to explain how the cards are ordered and whether or not an error has been made. If an error has been made, the partner must correct it and explain his or her thinking.
- As a class activity, repeatedly roll a die and have the students fill in the digits, one at a time, on a place value chart. Alternate by having them try to make the greatest number or the least number. Model the task by placing your digits on an overhead chart or interactive whiteboard. Regularly ask questions such as, What do you need? What don’t you want me to roll?
- Play “Guess My Number,” in partners, with numbers less than 1000. Use greater than, less than, and is equal to in the response (e.g., Is your number 489? No. My number is greater than that.). Continue the game until the number is guessed, and then change roles and have the other partner guess.
- Give each pair of students two spinners each with 10 numbers that are in the hundreds. Have them spin at the same time. The one who spins the greater number gets a token. The students play until someone has gathered 10 tokens. Select numbers according to the students’ level of understanding.
- Provide students with hundreds charts that contain missing numbers. Ask them to fill in the missing numbers and to explain how they know.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- calculator
- dice (including place value dice)
- hundred charts
- number cards
- number lines
- open number lines
- place-value charts
- spinners

MATHEMATICAL LANGUAGE

Teacher	Student
▪ benchmark numbers ▪ compare, order, ascending, descending order ▪ hundreds chart, number line ▪ least, greatest ▪ less than, more than, closer to, greater than, ▪ missing numbers, errors	▪ benchmark numbers ▪ compare, order ▪ hundreds chart, number line ▪ least, greatest ▪ less than, more than, closer to, greater than, ▪ missing numbers, errors

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 87–89, 143–144, 145
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 144–145, 196–200
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 142–143
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 45–47

Notes

SCO N04 Students will be expected to estimate quantities less than 1000 using referents. [ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Estimate the number of groups of ten in a given quantity using 10 as a referent (known quantity).
N04.02 Estimate the number of groups of a hundred in a given quantity using 100 as a referent.
N04.03 Estimate a given quantity by comparing it to a referent.
N04.04 Select an estimate for a given quantity by choosing among three possible choices.
N04.05 Select and justify a referent for determining an estimate for a given quantity.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N06 Students will be expected to estimate quantities to 100 using referents.	**N04** Students will be expected to estimate quantities less than 1000 using referents.	—

Background

The ability to estimate, a key reasoning skill in mathematics, should develop with regular practise over the course of the year. To develop estimation skills, students should be provided with collections of objects and be asked to estimate the size of the group, using a referent. Estimation helps students develop flexible, intuitive ideas about numbers, further developing number sense.

This outcome should be dealt with in conjunction with outcome N02 as students work toward developing a conceptual understanding of numbers. Making an estimate is often a very difficult task for children. They often do not understand the concept of “about” or “estimate.” Spend a lot of time working with children to help them understand the term **about**. There are many possible estimates for any given estimating situation, and this notion needs to be reinforced with students. Encourage estimation in real-world contexts and emphasize the reasoning underlying the estimate.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

Show students a jar containing about 100 beads. Tell them that one student estimated there were 90 beads in the jar. Another student estimated there were 25 beads in the jar. Ask, Which estimate is closer to the actual number of beads in the jar? Explain your thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show students a group of items and ask them to choose between three given estimates. Have students explain their reasoning.
- Place a pile of objects on a desk (e.g., paper clips, linking cubes, base-ten units, buttons). Ask students to estimate the number. Observe and interview students to determine if they are using a referent. Guiding questions should include, How did you pick that number? About how many groups of 10 (or 100) are there in the pile?
- Show students a group of objects (e.g., pennies, markers, crayons, stickers, sheets of paper, marbles, etc.) or pictures showing groups (e.g., people in a gym, cars in a parking lot, etc.). Ask students, About how many groups of 10 (or 100) are in the whole group?
- Ask students to describe a strategy used to find an estimate.
- Show students 25 buttons, all buttons touching sides. Say, Susan said, "643 buttons will fit on top of a desk." Do you agree or disagree? Explain.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- No Checkpoint for this outcome

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- No Pathway for this outcome

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to count groups of 10 and 100 objects in order to develop a sense of these benchmarks. Students should discover that these quantities are not always the same size (e.g., 100 raisins versus 100 oranges).
- Estimate a given quantity by comparing it to a referent (known quantity).
- Estimate the number of groups of 10 and 100 in a given quantity using 10 and 100 as a referent.
- Select between three possible estimates for a given quantity and explain the choice.
- Provide students with opportunities to build referents themselves to determine how many objects are in a specific group.
- Use children's literature as a context for estimation such as *Counting on Frank* by Rod Clement and *Betcha!* by Stuart Murphy. Discuss how the characters in the stories used estimation.

SUGGESTED LEARNING TASKS

- Show 100 paper clips as a visual referent for the students. Next display a larger group of paper clips. Ask students to estimate how many paper clips there are. Have students explain their thinking.
- Do activity 2.13 of *Teaching Student-Centered Mathematics, Grades 3–5.* Volume Two (Van de Walle and Lovin 2006, 50). Ask students to estimate how many
  - candy bars would cover the floor of your room
  - steps a student would take to walk around the school
  - quarters could be stacked in one stack, floor to ceiling
  - pennies can be laid side by side down an entire room or hallway
  - pieces of notebook paper would cover the gym floor
  - pieces of cereal are in the cereal box

For each scenario, help students identify an appropriate referent and discuss how this referent could be used to determine the total estimate.

- Collect some type of object as a class, with the objective of reaching 1000 (e.g., stickers, pennies, marbles, toy cars, rocks, leaves, buttons).
- Show a quantity of objects such as linking cubes. Ask, If this is 10 linking cubes, what might 143 linking cubes look like?
- Tell students, 100 counters takes up this much space. How much space would 783 counters take up if you placed them flat on the table? If they were in a milk jug? Explain your thinking?
- Have students put some items in a large jar or plastic container. Estimate how many items there are and then count to check. Ask, How far away was your estimate? Change the items in the container each day and repeat the activity. Have students share and discuss their estimation strategies with the class.
- Provide students with different quantities (e.g., 50, 100, 500) of objects of varying sizes and ask them to determine a referent and justify their choice.

SUGGESTED MODELS AND MANIPULATIVES

- variety of containers and objects (e.g., baggies, buckets, beads, marbles, cubes, paper clips, linking cubes, 500-sheet package of photocopy paper)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ about, estimate ▪ estimate quantities ▪ groups of ten, hundred	▪ about, estimate ▪ groups of ten, hundred

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 144–145, 200
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 132
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), p. 279

Notes

SCO N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Record, in more than one way, the number represented by given proportional and non-proportional concrete materials in traditional and non-conventional formats.
N05.02 Represent a given number in different ways using proportional and non-proportional concrete materials and explain how they are equivalent; e.g., 351 can be represented as three 100s, five 10s, and one 1s; or two 100s, fifteen 10s and one 1s; or three 100s, four 10s, and eleven 1s.
N05.03 Record a given number in additive expanded form.
N05.04 Record a number represented by base-ten blocks arranged in a non-conventional format.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100.	**N05** Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.	**N01** Students will be expected to represent and partition whole numbers to 10 000 concretely, pictorially, and symbolically.

Background

The development of place-value strategies will help students make sense of larger numbers. Learning to group items in order to make them easier to count is the foundation of place value. When students examine large numbers, they develop a greater sense of the patterning in the place-value system. This exploration will help students to recognize the regularity of the patterns that are inherent in the place-value system. Students should be able to explain that the digits 0–9 are used cyclically to indicate the number of units in any given place. They should also be able to explain the relationship between each place-value position and its neighbour positions, namely a group of ten in one position makes a group of one in the position to the left and a group of one in any position makes a group of ten in the position to the right. Students have used this principle to regroup and trade in previous grades and are now able to state that this pattern continues to work regardless of the size of the number.

Students will need many opportunities to explore the value of the digits in a number using proportional and non-proportional materials. Proper introduction and use of these materials will move student thinking from counting strategies to a deeper understanding of numbers. It is important to understand that students must construct their own understanding of number. This is best accomplished through a variety of materials and through the introduction of those materials as a representation of student thinking.

Students should have had previous experiences with grouping in different ways in preparation for the standard base-ten groupings. Students' initial experience should be with proportional models so that students can see that the piece that represents the 10 is actually 10 times the size of the unit piece.

Initially, students should work with their own personally made proportional materials before moving on to commercially developed materials. Personally made proportional models include toothpicks or small sticks bundled by students into tens and hundreds, beads or buttons strung in tens and hundreds, blocks that can be connected together, and beans glued to wooden stir sticks. Grouping and ungrouping these materials can help prepare students for working with commercially developed materials and to understand the concept of trading 10 little cubes for a rod worth 10. After many experiences with these personally made proportional materials, students should begin to use commercially pre-grouped materials, such as base-ten blocks, ten-frames, and hundreds charts as these models show the magnitude of the number. For example, a base-ten rod is ten times the size of a small cube and 1000 can be represented by ten flats. It is important that this development not be rushed because later problems with number can often be traced back to a poor development of place value.

After extensive work with proportional models, various coloured counters can be used as a non-proportional base-ten model. For example, a red counter can be worth 1, a blue counter can be worth ten times more, and a green counter can be ten times more than the blue. The counters are all the same size but assigned different values.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to choose a 2-digit number and to record it symbolically. Ask them to represent their 2-digit number using base-ten blocks. Ask them to explain the value of each digit in relation to the base-ten blocks. Then, show the students a 2-digit numeral with both digits the same (e.g., 55). Have students model the value of each digit. Ask students to explain why these digits do not represent the same value.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe 3-digit numbers without using the word **hundred** (e.g., 324 as thirty-two tens, four ones).
- Give students a 3-digit numeral and ask them to represent it with base-ten blocks or other models. Ask students to explain their representation. Ask, Can you represent it another way?
- Ask students to represent numbers with base-ten blocks in different ways and to record each representation using numerals and words (e.g., 132 is 1 hundred, 3 tens, 2 ones; or 13 tens, 2 ones; or 132 ones).
- Ask students to describe 1000 in as many ways as they can. They can use words, materials, pictures, and/or symbols.
- Ask students to explain using words, numbers, and/or pictures how they know that 1000 is the same as 100 tens or that 100 is the same as 10 tens.
- Tell students that pencils can be bought in packages of 1, 10, and 100. Ask students to use stir sticks grouped in 10s, 100s, and 1s to show as many ways as possible to buy 132 pencils.
- Ask students to choose any 3-digit number and tell everything they know about that number.
- Ask how 480 and 680 are the same and how they are different (focus should be on place value).
- Ask how 97 and 907 are the same and how they are different. Ask, Do you think zero (0) is an important number? Why or why not?
- Ask students to select a 3-digit number and to represent that number using base-ten blocks. Then, ask them to represent the number in expanded form.
- Show students a 3-digit number represented in additive expanded form. Ask them to represent that number with base-ten blocks and to record the numeral.
- Ask students to draw pictures of base-ten blocks that would show each of the following numbers: 302, 786, 950, and 878.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Representing Number

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

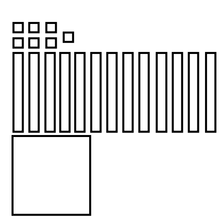
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have opportunities to use both proportional and non-proportional concrete materials. Students should begin working with personally made proportional concrete materials, before working with commercially made proportional concrete materials. Finally, they should work with non-proportional concrete materials.
- Have students represent the same number with different partitions. For example, 254 can be represented using 2 flats and 54 small cubes or 1 flat, 15 rods, and 4 small cubes.
- Provide multiple opportunities for students to show they understand that the position of a digit within a number determines its value.
- Provide students with many experiences modelling numbers with zeros as digits. It is important that students develop a good understanding of the meaning of zero in numbers. For some students, the number 406 looks like 46.

Suggested Learning Tasks

- Provide students with a large collection of objects, such as beans or paper clips. Ask them to group the objects in 10s and 100s and to record the numeral that represents the number of objects in the collection.
- Have students use a number that has all 3-digits the same (e.g., 111). Ask students to use models such as beans, counters, blocks, etc., to explain and show the meaning of each digit.
- Ask students to record the value of the base-ten blocks shown below.



- Ask student to record the number that is made up of 15 tens and 15 ones.
- Ask students to build a model or draw a picture using base-ten blocks. Ask, What is the value of the drawing or model?

- Give each group of students 12 base-ten rods and 16 units. Have them record the numeral the blocks represent.
- Model numbers such as 421 and 139. Discuss which number has more tens and how they know. Students should recognize that 421 has more tens, although it has a smaller digit in the tens place.
- Ask the students to enter a certain number on a calculator (e.g., 235). Ask, How can you, without clearing the calculator, make the number 255? (35? 205? 261?)
- Ask students to record a specific 3-digit number, with all digits different. Ask students to remove the value of one of the digits or the value of one of the places with only one operation. For example, to remove the value of the 3 from the number 734, the student would need to subtract 30.
- Have students create non-proportional concrete models and explain their values.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- calculators
- counters
- digit cards
- hundred chart
- hundred frame
- money
- number lines
- place-value chart

MATHEMATICAL LANGUAGE

Teacher	Student
▪ ones, tens, hundreds ▪ place value ▪ small cubes, rods, flats	▪ ones, tens, hundreds ▪ small cubes, rods, flats

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 138–144, 146–148
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 193–202
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 127–134, 136–137, 145–148

Videos

- *Teaching Place Value 20–99* (29:08 min) (ORIGO Education 2010)

Notes

SCO N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals. [C, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Explain mental mathematics strategies that could be used to determine a sum.
- Ten and some more
- Tens and some more
- Quick addition
- Addition facts to 10 applied to multiples of 10
- Addition on the hundred chart
- Adding on
- Make ten
- Compensation
- Compatible numbers

N06.02 Use and describe a personal strategy for determining a sum.
N06.03 Determine a sum of two 2-digit numerals efficiently, using mental mathematics strategies.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.	**N06** Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals.	**N03** Students will be expected to demonstrate an understanding of addition of numbers with answers to 10000 and subtraction (limited to 3- and 4-digit numerals) by ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction

Background

When a problem requires an exact answer, students' first consideration should be whether or not they can calculate it mentally. The development of mental mathematics needs to be a major goal of any mathematics program for two major reasons. First, in their day-to-day activities, most people's computational, measurement, and spatial needs can be met by having well-developed mental mathematics strategies. Secondly, because technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well-developed mental mathematics strategies to be alert to the reasonableness of the results generated by this technology.

Mental calculation refers to getting exact answers by using strategies to do the calculations in one's head. In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Note: The addition facts discussed in SCO N10 should be addressed prior to working on these mental mathematics strategies.

Situations must be regularly provided to ensure that students have sufficient practice with mental mathematics strategies and that they use their skills as required. Using mental mathematics will allow a student to focus on the relationships between numbers and operations rather than relying on completing a traditional algorithm. For example, students may solve 49 + 99 mentally by adding 100 to 49, then subtracting 1. This method involves using benchmark numbers then compensating by adding or subtracting, whichever operation is necessary.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to describe in as many ways as possible why 8 + 7 = 15.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask, How does knowing 8 + 8 = 16, help you solve 58 + 8?
- Ask the student to add mentally as you draw numbers from a bag and to stop you when the sum is greater than 40. Have the student explain their thinking process to reach the sum of 40 or more.
- Ask students to explain how many different ways they can add 49 + 34 in their heads. Ask them to explain which strategy is easier to use.
- Tell students that when adding 9 and 57, Jon said that he would rather add 10 to 57 and subtract 1. Ask them if this works and why.
- Ask students to describe a strategy for solving 76 + 11 (or other 2-digit addition questions) mentally using models, numbers, words, or pictures.

- Provide students with a sheet of 10 addition practice items, such as
  - 33 + 12
  - 71 + 24
  - 98 + 42
  - 56 + 34
  - 25 + 65
  - 87 + 13
  - 15 + 75
  - 44 + 52
  - 76 + 19
  - 82 + 17

  Ask them to circle all the questions they can solve mentally. For the questions they've circled, ask them to explain the strategy they used.
- Ask the student to describe a strategy for solving 68 + 39 mentally using models, numbers, words, or pictures.
- Ask students how many different ways they can mentally subtract 19 from 43? Which way was easiest?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 7, pp. 40–41

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Mental Math

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students recognize that mental mathematics is an approach that they should use every time they are required to calculate.
- Require that students recall addition and subtraction facts to 18 with automaticity (see SCO N10).
- As a class, share, discuss, and explore strategies used by individual students. This allows for exposure to a variety of strategies for students to choose ones that make sense to them and are more efficient.
- Review "making a ten" with students. For example, for 28 + 4, one might think 28 and 2 make 30, and 2 more is 32. This can be extended to the addition of 2-digit numbers. For example, for 38 + 24, 38 plus 20 is 58, and 2 more is 60, plus 2 is 62.
- Have students add two 2-digit numbers, and explain their thinking. For example, 24 + 31. Students might say, "20 and 30 makes 50, 4 more makes 54, and 1 more makes 55—the answer is 55."
- Have students explain their thinking using number lines (e.g., to solve 28 + 37, think 20 + 30 + 15).
- Have students use the hundred chart to solve a variety of 2-digit addition problems.
- Have students use two metre sticks and place one under the other so that the numbers are the reverse of the one on the top. This model can be used to explore compatible numbers.

SUGGESTED LEARNING TASKS

- Present calculations such as in the following example, orally (or on an overhead), and ask the student to write only the answer (e.g., 30 + 60; 20 + 40; 20 + 80). They should be able to do this quickly.
- Provide a set of computation practice items and ask students to circle the questions that they could solve mentally and describe the strategy they would use.
- Have students explain how they would use mental mathematics strategies to solve 2-digit addition questions, such as 34 + ☐ = 69 or 39 + ☐ = 64.
- Have the student make a list of calculations involving 2-digit numbers that would be quicker to do mentally than to do using paper and pencil or a calculator.
- Have students explain how changing the addend will affect the answer (e.g., if 31 + 48 is changed to 31 + 50, the sum would be two more than the actual sum).
- Have the student list the doubles facts that might help him or her solve expressions such as 88 + 89 and 49 + 51 or 39 + 38.
- Present students with a variety of mental mathematics questions, such as
  21 + 43
  37 + 59
  63 + 41
  74 + 46

  Have students share different ways to solve each question.

  To solve 21 + 43 students might
  - start with the tens (20 + 40 = 60), then add the ones (1 + 3 = 4) and then add those sums together (60 + 4 = 64)
  - start with 43, add on 20 to get 63, and then add on 1 to get 64
  - start with 21, add on 40 to get 61, and then add on 3 to get 64

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- hundred chart
- numeral cards
- open number line

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 1-digit, 2-digit ▪ addition facts to 10 applied to multiples of 10: small cubes, rods ▪ addition on the hundred chart ▪ mental math ▪ quick addition: tens ▪ strategy ▪ ten and some more ▪ tens and some more	▪ 1-digit, 2-digit ▪ small cubes, rods ▪ mental math ▪ tens ▪ strategy ▪ ten and some more

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 33, 162
- *Making Math Meaningful to Canadian Students K–8,* Second Edition (Small 2013), pp. 29–30, 218
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 51–52, 158, 160–161
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), p. 44

Videos

- *Comparing Mental Strategies: Addition* (14:42 min.) (ORIGO Education 2010)
- *Powerful Models to Help Struggling Students: Number Lines* (17:37 min.) (ORIGO Education 2010)
- *Powerful Strategies to Help Struggling Students: Bridge to Ten* (13:23 min.) (ORIGO Education 2010)
- *Questions for Developing Mental Computation Strategies* (13:42 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using Mental Strategies to Add* (26:15 min.) (ORIGO Education 2010)

Notes

SCO N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals. [C, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Explain mental mathematics strategies that could be used to determine a difference.
- Facts with minuends of 10 or less applied to multiples of 10
- Quick subtraction
- Subtraction on the hundred chart
- Compensation
- Back through ten

N07.02 Use and describe a personal strategy for determining a difference.

N07.03 Determine a difference of two 2-digit numerals efficiently, using mental mathematics strategies.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.	**N07** Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals	**N03** Students will be expected to demonstrate an understanding of addition of numbers with answers to 10000 and subtraction (limited to 3- and 4-digit numerals) by ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction

Background

When a problem requires an exact answer, students' first consideration should be whether or not they can calculate it mentally. Situations must be regularly provided to ensure that students have sufficient practice with mental mathematics strategies and that they use their skills as required. Using mental mathematics will allow a student to focus on the relationships between numbers and operations rather than relying on completing a traditional algorithm. Most mental mathematics strategies provide a more efficient computation method as opposed to pencil and paper. Students should be presented with the horizontal form of an equation to encourage the use of mental mathematics.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

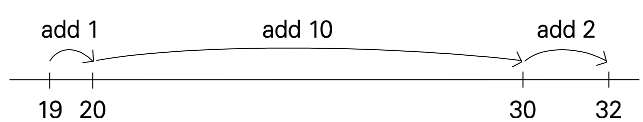
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to describe in as many ways as possible why 15 – 8 = 7.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that when subtracting 7 from 51, Jon said that he would rather subtract 6 from 50. Ask them if this works and why.
- Ask students to describe a strategy for solving 76 – 11 mentally using models, numbers, words, or pictures.
- Ask students to describe a strategy for solving 68 – 39 mentally using models, numbers, words, or pictures.
- Have students explain what is wrong with Lisa's method for solving 45 – 26. Lisa said, "45 – 25 = 20 and 20 + 1 = 21. The answer is 21."
- Ask students to explain different ways that someone could subtract 19 from 43 in their head? Ask them to identify which way was easiest.
- To solve 47 – 29, Beth said, "47, 27, 20, 18. The answer is 18." Explain the strategy Beth used to solve the problem.
- To solve 32 – 19, Jonah showed the following jumps on a number line and said the answer was 13.



Ask students to explain how Jonah solved the problem.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 7, pp. 40–41

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Mental Math

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Ensure students recognize that mental mathematics is an approach that they should use every time they are required to calculate.
- Require that students recall addition and subtraction facts to 18 with automaticity (see SCO N10).
- As a class, share, discuss, and explore strategies used by individual students. This allows for exposure to a variety of strategies for students to choose ones that make sense to them and are more efficient.
- Have students subtract using quick subtraction and explain their thinking.
- Relate addition to subtraction regularly so students are better able to use this understanding to solve addition and subtraction problems and to check their work. Use missing addends to encourage this.
- Have students model subtraction and explain their thinking using number lines, hundred charts, and base-ten blocks.

Suggested Learning Tasks

- Provide a set of computation practice items and ask students to circle the questions that they could solve mentally and describe the strategy they would use.

- Present students with a variety of mental mathematics questions, such as
  - 43 – 21
  - 59 – 37
  - 92 – 73
  - 74 – 46

  Have students share different ways to solve each question.
- Present calculations, such as in the following example, orally (or on an overhead), and ask the student to write only the answer (e.g., 60 – 30, 40 – 20, 80 – 30). They should be able to do this quickly. Ask students to share their strategies.
- Have students make a list of calculations involving 2-digit numbers that would be quicker to do mentally than using paper and pencil or with a calculator.
- Have students explain how changing the subtrahend will affect the answer (e.g., for 100 – 48 to 100 – 50, the difference would be two more).
- Have students explain how they would use mental mathematics strategies to solve 2-digit subtraction questions, such as 34 – ☐ = 19 or 69 – ☐ = 54.

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- number cards
- open number line

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 1-digit, 2-digit ▪ hundred chart ▪ mental mathematics ▪ quick subtraction: tens, minus ▪ small cubes, rods ▪ strategy	▪ 1-digit, 2-digit ▪ hundred chart ▪ mental mathematics ▪ tens, minus ▪ small cubes, rods ▪ strategy

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 33, 162
- *Making Math Meaningful to Canadian Students K–8,* Second Edition (Small 2013), pp. 29–30, 218
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 51–52, 158, 160–161
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), p. 44

Videos

- *Powerful Models to Help Struggling Students: Number Lines* (17:37 min.) (ORIGO Education 2010)
- *Questions for Developing Mental Computation Strategies* (13:42 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)

Notes

SCO N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, or 3-digit numerals in a problem-solving context. [C, ME, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Explain estimation strategies that could be used to determine an approximate sum or difference.
N08.02 Use and describe a strategy for determining an estimate.
N08.03 Estimate the solution for a given story problem involving the sum or difference of up to two 3-digit numerals.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
—	**N08** Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, or 3-digit numerals in a problem-solving context.	**N03** Students will be expected to demonstrate an understanding of addition of numbers with answers to 10000 and subtraction (limited to 3- and 4-digit numerals) by ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction

Background

Estimating is a critical skill in today's world. For most people in their daily lives, an estimate is all that is needed to make decisions, and to think about the reasonableness of numerical claims and answers generated by others. In the technology-rich society, it is essential to encourage student thinking and reasoning skills based on estimation, rather than the acceptance of answers generated by computers or calculators.

Before attempting pencil-and-paper or calculator computations, students must find estimates, so they think about the reasonableness of those pencil-and-paper or calculator answers. Always model estimating before personally doing any calculations in front of the class, and constantly remind students to estimate before calculating. After the calculation, students can use their estimate to check their solutions, asking themselves, Does my answer make sense? Help students develop this skill by frequently asking questions such as, Is your answer reasonable? How do you know?

While teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are **about**, **approximately**, **between**, **a little more than**, **a little less than**, **close**, **close to**, and **near**.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to tell whether 44 is closer to 40 or 50 and to explain how they know.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to identify situations in which an exact answer would be required and some in which an estimate is sufficient.
- Ask students to explain two different ways to estimate the difference for 54 – 26.
- Ask, Do you think that 48 might ever be used as an estimate for a sum or difference? Explain your answer.
- Ask students to explain why a good estimate for a subtraction might be greater than the actual answer sometimes, but less other times. Encourage students to use examples to help them explain.
- Tell students that Jason knew there were 35 members in his Karate Club and about 28 in the club in the neighbouring town. When asked to estimate the number of name tags to make for members of both clubs, Jason said, "I think I should make 65." Ask, How do you think Jason estimated? Was it a good estimate?
- Have students toss two dice and create a 2-digit number. Ask them to estimate how much should be added to the number to get a sum of about 200 or ask them to tell how much could be subtracted to get a difference of about 10?
- Tell the student that 4□ + □8 is about 70. Ask what digits might go in the blanks.
- Ask students to identify which of the following questions would have an answer close to 150 and to explain their thinking.
  92 + 37
  69 + 82
  77 + 87

- Show students the number of sports cards in James' collection. Baseball: 48, Football: 19, Hockey: 84. Ask students to estimate the total number of cards in the collection and to describe the strategy they used.
- Tell students that Marc wants to buy a new bike that costs $135. He has saved $48. About how much more will he need to save? Ask students to explain how they solved the problem.
- Tell students that a number between 30 and 40 is added to a number between 40 and 50. Ask, What might be a good estimate for the answer? Why?

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets 3 (Bauman 2011)
- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- No Pathway for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Ensure students recognize that estimation should be used every time they are required to solve a problem, make predictions, or check answers.
- Use left-to-right or front-end method. The following is an example:
  - 138 + 149 (130 + 140 is 270, 8 + 9 is close to 20, for an estimate of 290)
- Use ten-frames for relatively small numbers. Displaying 23 on ten-frames, for example, clearly shows that 23 is closer to 20 than to 30.

- Use base-ten blocks or a hundred chart to help students as they begin estimating with larger numbers. For example, using base-ten blocks will help a student see that 37 (3 rods and 7 ones) is closer to 4 rods than to 3 rods. Eventually, students should realize that estimating can easily be performed without the base-ten blocks.
- Use quarters (25¢) as a model to think of multiples of 25. Have students consider multiples of 25 when estimating numbers.

SUGGESTED LEARNING TASKS

- Have students use estimation in story problem situations, such as
  - Tali baked 49 whole wheat rolls and Miranda baked 158. Do they have enough to feed two hundred parents coming to Mathematics Night?
  - Play “A Fast Ten” with students. Students turn over two playing cards (a deck of cards numbered 1–9 only) to build a 2-digit number. The student who determines to which multiple of ten that number is closest gets the cards. This game could be extended to add or subtract estimates of two pairs of cards.
  - Tell the student that the sum of two numbers has been estimated to be about 120. Ask the student to list four possible pairs of numbers that might have been added.
  - Have students explore open-ended questions, such as, The difference between two numbers was estimated to be 50. What could the numbers have been? The sum of two numbers was estimated to be 700. What could the two numbers have been?
  - Have students explain whether an estimate is reasonable. For example, tell students that Juan estimated the difference for 689 – 276 to be 500. Ask whether that estimate is reasonable or not.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- metre sticks
- money
- number lines
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 1-digit, 2-digit, 3-digit ▪ about, between, a little more than, a little less than, close, close to, and near. ▪ estimate ▪ front-end estimation ▪ front-end estimation adjusted	▪ 1-digit, 2-digit, 3-digit ▪ about, between, a little more than, a little less than, close, close to, near ▪ estimate

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 160–161, 173
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 29–30, 218
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 144–145

Notes

SCO N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000 by ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically. [C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Model the addition of two or more given numbers using concrete or visual representations and record the process symbolically.
N09.02 Model the subtraction of two given numbers using concrete or visual representations and record the process symbolically.
N09.03 Create an addition or subtraction story problem for a given solution.
N09.04 Determine the sum of two given numbers using a personal strategy, e.g., for 326 + 48, record 300 + 60 + 14.
N09.05 Determine the difference of two given numbers using a personal strategy, e.g., for 127 – 38, record 2 + 80 + 7 or 127 – 20 – 10 – 8.
N09.06 Solve a given problem involving the sum or difference of two given numbers.

Scope and Sequence

Mathematics 2	**Mathematics 3**	**Mathematics 4**
N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems that involve addition and subtraction ▪ explaining and demonstrating that the order in which numbers are added does not affect the sum ▪ explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference.	**N09** Students will be expected to demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2-, and 3-digit numerals) by ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically.	**N03** Students will be expected to demonstrate an understanding of addition of numbers with answers to 10000 and subtraction (limited to 3- and 4-digit numerals) by ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction

Background

This outcome involves the development of two critical abilities—the ability to solve the full range of addition and subtraction story problems efficiently, and the ability to add and subtract up to 3-digit numbers efficiently. For the most part, these two abilities should be taught simultaneously; however, there will be times when some lessons should focus on one or the other.

Students should be presented with addition and subtraction story problems of all structures.

- Join (result, change, and start unknown)
- Separate (result, change, and start unknown)
- Part-part-whole (part and whole unknown)
- Compare (difference, smaller, and larger unknown).

Join story problems all have an action that causes an increase, while separate story problems have an action that causes a decrease. Part-part-whole story problems, on the other hand, do not involve any actions, and compare story problems involve relationships between quantities rather than actions.

Examples of these various types of problems appear in the table provided below.

Join			Part-Part-Whole	Compare
Result Unknown	**Change Unknown**	**Start Unknown**	**Whole Unknown**	**Difference Unknown**
Mike earned $328 last year selling newspapers. This year he earned $415. How much money did he earn in all? 328 + 415 = ?	Last week Katie picked 115 kg of blueberries. She picked some more blueberries this week giving her a total of 236 kg. How many kilograms of blueberries did she pick this week? 115 + ? = 236 or 236 – 115 = ?	The grade 4 class is fund raising for a community centre. A donor just gave them $563 and now they have $998. How much money did they have before the donation? ? + 563 = 998 or 998 – 563 = ?	There are 317 boys and 248 girls in a school. How many students are in the school? 317+ 248= ?	Mary sold 278 greeting cards for the school fund raiser. Chantella sold 195. How many more greeting cards did Mary sell than Chantella sold? 195 + ? = 278 or 278 – 195 = ?
Separate			**Part-Part-Whole**	**Compare**
Result Unknown	**Change Unknown**	**Start Unknown**	**Part Unknown**	**Smaller or Larger Unknown**
Gavin collected 239 toy cars in his bucket. He gave his brother 103 of those toy cars. How many toy cars does he have left? 239 – 103 = ?	Kayla had 156 g of sugar. She used some to make cookies and has 83 g left. How much sugar did she use? 156 – ? = 83 or 156 – 83 = ?	A company had some books to donate to schools. They gave the first school 256 of them. They still have 517 books to give away. How many books did they have to begin with? ? – 256 = 517 or 256 + 517= ?	There were 735 people at the community dinner. If 352 of them were children, how many were adults? 352 + ? = 735 or 735 – 352 = ?	Joe collected 387 bottles for the recycling project. Sue collected 185 more bottles than Joe. How many bottles did Sue collect? 387 + 185 = ? If Joe collected 74 more than Dan, how many did Dan collect? 387 – 74 = ?

In the table, there are number sentences that students may generate depending upon how they think about the problem. For example, consider the Join (change unknown) problem: Last week Katie picked 115 kg of blueberries. She picked some more blueberries this week giving her a total of 236 kg. How many kilograms of blueberries did she pick this week? If students solved this by starting with 115, adding on until they reached 236, and determining what they added on, they would represent the problem in symbols as 115 + ? = 236. On the other hand, if students solved it by starting with 236, removing 115, and determining what was left, they would represent the problem as 236 – 115 = ? Students should be encouraged to model the story problems with base-ten blocks and write number sentences that reflect their thinking. All these story problems can also be modelled using a variety of pictorial representations including student-generated pictures, those described on page 67 of *Teaching Student-Centered Mathematics, Grades K–3* by John Van de Walle and LouAnn Lovin, or strip diagrams as described in Appendix A. It is important that the pictures students draw represent their thinking and should mirror their work with models.

When students have to compute sums and differences involving numbers with up to 3-digits, they should use strategies that are reliable, accurate, and efficient. Through the sharing of strategies, students will be exposed to a variety of possible addition and subtraction strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as "personal strategies." The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

While some of these strategies may have emerged directly from students work with base-ten blocks, other strategies should be modelled by students using the base-ten blocks to help understand the logic behind them. Research has shown that children who create personal strategies almost universally start by adding from the left. Many students in Mathematics 3 are not developmentally ready to work with groups of 10 and groups of 100 with deep understanding; therefore, these students are more likely to understand alternative strategies that highlight the actual quantities represented by the digits in the tens and hundreds places.

The paper-and-pencil recording of students' personal strategies should reflect their thinking and must be reliable, accurate, and efficient. The symbolic recording used need not be the standard algorithm. Regardless of the strategy used, the teacher must monitor each student's symbolic recording of the strategy to ensure that the recording is mathematically correct, organized, and efficient.

Two examples of strategies and symbolic recordings are shown below. Additional examples are provided in Appendix A.

If students are asked to add 237 and 478, students could determine the sum by

- Start by writing 237 as 200 + 30 + 7 and 478 as 400 + 70 + 8.
- Add 200 and 400 to get a sum of 600.
- Add 30 and 70 to get a sum of 100.
- Add 7 and 8 to get a sum of 15.
- Add 600, 100, and 15 to get a sum of 715.

This may be recorded on paper as

237 + 478 = 200 + 30 + 7 + 400 + 70 + 8
200 + 400 = 600
30 + 70 = 100
7 + 8 = 15
600 + 100 + 15 = 715

or

$$\begin{array}{r} 237 \\ \underline{+\,478} \\ 600 \\ 100 \\ \underline{+\;\;15} \\ 715 \end{array}$$

If we introduce subtraction using word problems, students can begin modelling their solutions. Consider the following problem: On our vacation, we went to visit our aunt in Fredericton. We drove 239 km and stopped for lunch. If the distance to our aunt's house is 526 km, how much further do we have to drive?

Students could explain and record their solution as shown below:

- We knew we had to subtract 239 from 526. So we started with 5 flats, 2 rods, and 6 small cubes to show 526. We removed 2 flats. Then, we had to remove 3 rods, so we changed one flat to 10 rods. Finally we removed 9 small cubes, after we traded 1 rod for 10 small cubes. This could be recorded on paper as
  526 – 239 = ?
  526 – 200 = 326
  326 – 30 = 296
  296 – 9 = 287
  We have to travel 287 km more.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Give students an addition or a subtraction number sentence involving 1- and 2-digit numbers. Ask them to create a story problem that would be solved using the number sentence. Ask them to model the story problem using concrete materials or pictures.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell a student that someone told you that you do not have to learn to subtract if you know how to add. Ask, Do you agree? Why or why not?

- Observe the student as he or she adds 125 and 134 or subtracts 134 from 217 using base-ten blocks or an open number line.
- Provide the following addition or subtraction calculations for the student to complete. Ask the student to explain and record symbolically his or her strategy.
  38 + 97
  98 – 44
  400 – 255
- Show the student a number of addition and subtraction questions, some of which require regrouping and some of which do not. Ask him or her to circle the questions they could do quickly and explain why they made those choices.
- Have the student explain in writing why someone might first subtract 30 from 74 in order to calculate 674 – 26. Ask what would be done next.
- Display the numbers 124 and 75 with base-ten blocks. Ask the student to describe the addition process as he or she manipulates the models.
- Tell the student that Sue had to add 36 + 59 and said, “36, 96, 95.” Have the student explain Sue’s thinking.
- Ask why someone might find it easier to subtract 123 – 99 than 123 – 87.
- Ask the student to prepare a display showing a variety of ways to calculate 287 + 162 indicating his or her preference and the reason for it.
- Ask the students to use a sales flyer to create some problems for his or her classmates. Have them record both problems and solutions.
- Using the numbers 811 and 543 create a subtraction problem that can be solved using addition. Then, ask students to solve the problem.
- Ask, How does knowing 13 – 6 = 7, help you solve 153 – 6?
- Ask students to add 125 and 78 and describe the process using an open number line.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Addition
- Subtraction

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide examples of situations in which students will have to devise some method for regrouping. For example, tell them that one student found that there were 155 M & M'S in one bag and 258 in another. Ask them to determine how many there were in the two bags. Ask them to model the question and to explain how they solved it.
- Create a sheet with 10 ten-frames to represent 100. Provide each student with two sheets and have him or her show each of the two addends (e.g., 67 + 76). This will help students visualize how the numbers could be combined (e.g., 60 + 70, 7 + 6). Students may need to explore moving the amounts to be on the same sheet if they are having trouble finding the sum.
- Examine number patterns to help students understand the connection between addition and subtraction facts and 2-digit plus 2-digit and 3-digit plus 3-digit addition and subtraction. For example, 6 + 7 = 13, so 60 + 70 = 130 and 600 + 700 = 1300 and 13 tens minus 6 tens = 7 tens.
- Have students model their thinking on an open number line.

SUGGESTED LEARNING TASKS

- Tell students that Fran had 187 stickers. She gave 59 of them to her friend. Ask students to explain how they go about solving the problem. Providing a model for the students, such as a number line, may be helpful.
- Set up a "store" within the classroom and have the students take turns being the cashier. Model for them how to "count on" when making change.
- Create sheets or overhead transparencies containing completely and partially filled ten-frames representing one part of a target number. Students apply strategies that make sense to them to determine the missing part. Since this is essentially a subtractive problem, many students will use "think addition" to work their way up to the target number.
- Use the following digits to create two, 2-digit numbers that have the greatest possible sum: 2, 3, 4, 5. Use the same digits to create the greatest difference.
- Have pairs of students roll place value dice to create pairs of 2- and 3-digit numbers and find the sum or difference. As a variation, give students a "target number" (e.g., 100) that they try to reach by adding or subtracting the numbers they create with the dice.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- calculator
- dice
- hundred chart
- hundred frames
- money
- open number line
- place-value dice
- place-value mats
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ addition, subtraction ▪ number sentence ▪ small cubes, rods, flats ▪ strip diagram: part, whole ▪ sum, difference ▪ trading, grouping ▪ unknown	▪ addition, subtraction ▪ number sentence ▪ small cubes, rods, flats ▪ strip diagram: part, whole ▪ sum, difference ▪ trading, grouping ▪ unknown

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 104–110, 162–172
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 160–166, 215–217
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 66–70, 70–75, 157–172
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 53, 108–113

Videos

- Using Language Stages to Develop Addition Concepts (15:38 min.) (ORIGO Education 2010)
- Using Language Stages to Develop Subtraction Concepts (18:32 min.) (ORIGO Education 2010)
- Using Static Problems to Relate Addition and Subtraction and Introduce Equality (13:25 min.) (ORIGO Education 2010)
- Using Static Problems to Relate Addition and Subtraction and Introduce Functions (18:59 min.) (ORIGO Education 2010)

Notes

SCO N10 Students will be expected to apply mental mathematics strategies and number properties, to develop quick recall of basic addition facts to 18 and related basic subtraction facts. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Describe a mental mathematics strategy that could be used to determine a given basic addition fact up to 9 + 9.
N10.02 Explain how the commutative (order-doesn't-matter) property and the identity (no-change-with-zero) property can assist in addition fact learning.
N10.03 Describe a mental mathematics strategy that could be used to determine a given basic subtraction fact with minuends up to 18 and subtrahends up to 9.
N10.04 Recognize which facts could be determined by a given strategy.
N10.05 Quickly recall basic addition facts to 18 and related subtraction facts in a variety of contexts.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.	**N10** Students will be expected to apply mental mathematics strategies and number properties, to develop quick recall of basic addition facts to 18 and related basic subtraction facts.	—

Background

It is expected that, by the end of Mathematics 2, students will have achieved quick recall of addition facts up to 9 + 9. In Mathematics 3, therefore, the majority of time should be spent on the related subtraction facts. It is important to note, however, that students will struggle with subtraction facts if the related addition facts are not well known. Quick recall for most students should be three seconds, or less, but there may be some students who need one or two additional seconds to mentally determine some of the facts. Knowing that students develop facility at different times of the year, we should not promote competition between students but should instead focus on individual self-improvement. Whatever the time limit, to achieve this outcome students should not be counting on their fingers or using other cumbersome strategies. They should be processing the facts in their minds to get the sums and differences. Through understanding of relationships, repetition, and practise, however, students may achieve automaticity (instant recall) for some or all of the facts without any strategy processing whatsoever.

Why should students learn the basic facts? Having an instant recall of the basic facts

- means students have one of the building blocks for mental computation that is the most common means of computation in society today
- gives students a sense of empowerment
- means that students can concentrate on learning newer concepts and procedures that use these facts, rather than devoting neural energy to finding these sums and differences

It is important to provide opportunities for practice using games and meaningful contexts, rather than just learning the facts for their own sake. Furthermore, students should be expected to use quick recall of facts in their everyday work in mathematics and other subjects, not just during the times allocated for fact learning. Once facts are mastered, there should be no need to use counting strategies when two numbers are combined in addition or subtraction. Many students continue to use counting strategies from force of habit. These students need to be encouraged and challenged to use facts in order to sustain those facts and to help students develop new habits.

"Memorizing basic facts, perhaps with the use of flash cards, is very different from internalizing number combinations. *Memorized* knowledge is knowledge that can be forgotten. *Internalized* knowledge can't be forgotten because it is a part of the way we see the world. Children who memorize addition and subtraction facts often forget what they have learned. On the other hand, children who have internalized a concept or relationship can't forget it; they know it *has* to be that way because of a whole network of relationships and interrelationships that they have discovered and constructed in their minds." (Richardson 1999, 43)

Students should continue to practise quick recall of addition facts that were expected in Mathematics 2, by using appropriate strategies or by instant recall. While there are 100 addition facts up to 9 + 9, students should understand that the order-doesn't-matter property (commutative property) of addition means that 90 facts are in 45 pairs, such as 5 + 8 and 8 + 5, so there are really only 55 addition facts to be learned (10 double facts and 45 commutative pairs).

Students should review addition fact strategies and the clusters of facts to which these strategies apply, sustaining the three-second or less response time expected in Mathematics 2. These strategies include

- Associations for the Double Facts (1 + 1, …, 9 + 9)
- Next Number for the Plus-1 Facts (2 + 1, 1 + 2, 3 + 1, 1 + 3, …, 9 + 1, 1 + 9)
- Double, Next Number for the 1-Apart Facts (2 + 3, 3 + 2, 3 + 4, 4 + 3, …, 8 + 9, 9 + 8)
- Next Even or Odd for the Plus-2 Facts (4 + 2, 2 + 4, 5 + 2, 2 + 5), …, 9 + 2, 2 + 9)
- No Change for the Plus-0 Facts (0 + 0, 1 + 0, 0 + 1, 2 + 0, 0 + 2, …, 9 + 0, 0 + 9)
- Make-10 for the Plus-9 Facts (9 + 3, 3 + 9, 9 + 4, 4 + 9, …, 9 + 7, 7 + 9)
- Make-10 for the Plus-8 Facts (8 + 3, 3 + 8, 8 + 4, 4 + 8, 8 + 5, 5 + 8, 8 + 6, 6 + 8)
- Make-10 for the Plus-7 Facts (7 + 3, 3 + 7, 7 + 4, 4 + 7, 7 + 5, 5 + 7)
- Double the Number between for the 2-Apart Facts (5 + 3, 3 + 5, 6 + 4, 4 + 6, 7 + 5, 5 + 7)
- Variety of Strategies for the Last Two Facts (6 + 3, 3 + 6)

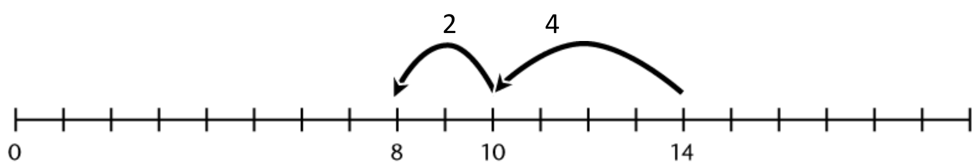
Students may also share other very effective strategies besides the ones listed above. Sharing and discussing alternative strategies should be part of fact learning. It opens eyes and minds to the flexibility of numbers and operations. Having such flexibility is a critical aspect of the development of number sense and operation sense.

While facts involving zero appear to be the easiest of all, they are often a source of error. Because students have generalized that the addition of one number to another, changes the number, they resist the idea that no change has occurred even though there has been an addition. Furthermore, students have not likely experienced reading a story problem that results in writing a number sentence with zero as an addend. To develop the strategy for the Plus 0 facts, it is suggested that such story problems are created. Students will find them funny, and this will help them internalize the no-change aspect of these facts. Zero is the additive identity because under the operation of addition it does not affect a change.

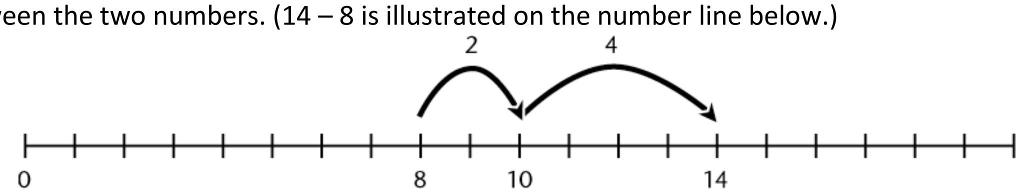
Students will already have had experiences with subtraction fact strategies in Mathematics 2. Now, the focus will be on using the strategies to efficiently develop quick recall of these facts. Perhaps the most useful strategy for subtraction facts is the think-addition strategy, in which students get answers to subtraction facts by recalling the related addition facts. For example, for 15 – 7, students think of what they would add to 7 to get 15, recall that 7 + 8 = 15, and so know that 15 – 7 must be 8. This strategy rests on students' understanding the inverse relationship between addition and subtraction, and on knowing addition facts. Also, this strategy focuses on the difference meaning of subtraction (how far apart the two numbers are) rather than a take-away meaning.

Besides the think-addition strategy, there are two other very useful subtraction strategies that not only can be used for quick fact recall, but also can be applied in subtraction situations involving larger numbers. These strategies are as follows.

- **Back-through-10:** This strategy involves subtracting in two steps—one part of the subtrahend is subtracted to get to ten and the other part of the subtrahend is then subtracted from 10. This strategy is most effective when only 1 or 2 has to be subtracted from 10 in the second step. Modelling this strategy on a number line as a "take-away" would help students visualize the steps—Mark the minuend, show the leap from the minuend to 10, and then the final leap to the answer. (14 – 6 is illustrated on the number line below.)



- **Up-through-10:** This strategy involves finding the difference between the two numbers in steps: first, the difference between the subtrahend and 10 is found and then the difference between 10 and the minuend is found, and finally these two differences are added to give the total difference. Modelling this strategy on a number line will also help students visualize the steps: Mark both the minuend and the subtrahend on the number line, show the leap from the subtrahend to 10, show the leap from 10 to the minuend, and see that the two leaps together represent the total difference between the two numbers. (14 – 8 is illustrated on the number line below.)



Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Orally and visually present a series of addition facts, one at a time, to students allowing 3–5 seconds for them to recall each fact. Ask them to record each sum before proceeding to the next question.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to explain, using a model, why he or she knows that 3 + 4 has to equal 4 + 3 even before finding the total.
- Ask, Why is it easy to add or subtract 0 to numbers? (interview student or write response in a journal)
- Ask, Why is it easy to add the numbers 5 + 5 + 6 + 4 + 8 + 2?
- Ask, How can you use addition to solve 16 – 7?
- Have students write all of the number facts they can for a provided sum or difference (e.g., 6 as a difference: 6 – 0, 7 – 1, 8 – 2, 9 – 3, 10 – 4, 11 – 5, 12 – 6, 13 – 7, 14 – 8, 15 – 9).
- Show students a mathematics fact. Ask students to record answers on individual white boards (or use plastic plates or plastic sheet protectors). Have students share their personal strategies. This activity could be extended to include open frame questions (e.g., 7 + __ = 13) as explored in outcome PR03.
- Ask students to describe as many different ways as possible to solve 18 – 9.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 3, pp. 26–27
- Checkpoint 7, pp. 40–41

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Mental Math

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of models to practise and help visualize the basic facts. Students can also use other strategies, such as drawing pictures and role-playing to represent various sums and differences in a problem-solving context.
- Use mathematical vocabulary with students, including **sum**, **difference**, and **number sentence**.
- Use the open number line to explore "making ten" or "bridging through 10."
- Ensure students have the opportunity to discuss their strategies with others. The focus should be on the efficiency of the strategy.
- Provide lots of opportunities for practise (visually/orally) with immediate feedback over an extended period of time.
- Use the addition table to explore patterns and help students identify the facts that they have mastered. The known facts can be coloured in with the goal of having the entire table coloured.
- Have students create realistic word problems related to addition and subtraction.
- Provide many opportunities where the focus is on the relationship between the numbers.

Suggested Learning Tasks

- Have students roll two number cubes (dice). They either add or subtract these values. For example, if a 5 and a 2 are rolled, they will work with either 5, 2, 7, or 5, 2, 3. Ask the students to make up a subtraction story based on these numbers, and write the corresponding number sentence. (**Note:** Ten-sided dice work well, as do prepared numeral cards.) If students are proficient with the addition facts, adapt this activity so that they must focus on subtraction.
- Play "Missing Part" game for two students to practise their fact recall. One student places a number of counters in front of them (e.g., 16) and then the student covers some of the counters with their hand. The other student must determine how many counters are hidden as quickly as possible.
- Use a "Looping Activity" where every student is given a card with a basic fact number sentence in which one of the numbers is missing, written as "Who has …?" (e.g., Who has 5 + __ = 11). The card also has the answer from someone else's card written as "I have …" Students take turns reading their cards in sequence by responding when their card answers someone else's question.
- Provide students with cards with a subtraction number sentence (e.g., 13 – 7 = __). Have students rewrite the sentence as a missing addend number sentence (e.g., 7 + □ = 13) and solve it.

SUGGESTED MODELS AND MANIPULATIVES

- 10-sided dice
- addition chart
- counters
- dominoes (double nine)
- double ten-frames
- linking cubes
- number cards
- number cubes
- open number line

MATHEMATICAL LANGUAGE

Teacher	Student
▪ addition, subtraction facts ▪ double, plus-0, plus-1, plus-2, 1-apart, 2-apart, make-ten, back-through-10, up-through-10 ▪ mental mathematics strategy ▪ order, no change, order does not matter	▪ addition, subtraction facts ▪ double, plus-0, plus-1, plus-2, 1-apart, 2-apart, make-ten, back-through-10, up-through-10 ▪ mental mathematics strategy ▪ order, no change, order does not matter

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 166–172
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 20, 94–119
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 74–99

Videos

- An Introduction to Teaching Addition Number Facts (15:51 min.) (ORIGO Education 2010)
- Teaching the Bridge-to-10 Strategy for Addition Number Facts (17:11 min.) (ORIGO Education 2010)
- Teaching the Count-on Strategy for Addition Number Facts (17:49 min.) (ORIGO Education 2010)
- Teaching the Think-Addition Subtraction Fact Strategy (13:41 min.) (ORIGO Education 2010)
- Teaching the Use- Dobules Strategy for Addition Number Facts (14:20 min.) (ORIGO Education 2010)

Notes

SCO N11 Students will be expected to demonstrate an understanding of multiplication to 5 × 5 by ▪ representing and explaining multiplication using equal grouping and arrays ▪ creating and solving problems in context that involves multiplication ▪ modelling multiplication using concrete and visual representations and recording the process symbolically ▪ relating multiplication to repeated addition ▪ relating multiplication to division [C, CN, PS, R]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N11.01 Identify events from experience that can be described as multiplication.
N11.02 Represent a given story problem (orally, shared reading, written) using manipulatives or diagrams and record in a number sentence.
N11.03 Represent a given multiplication expression as repeated addition.
N11.04 Represent a given repeated addition as multiplication.
N11.05 Create and illustrate a story problem for a given number sentence and/or expression.
N11.06 Represent, concretely or pictorially, equal groups for a given number sentence.
N11.07 Represent a given multiplication expression using an array.
N11.08 Create an array to model the commutative property of multiplication.
N11.09 Relate multiplication to division by using arrays and writing related number sentences.
N11.10 Solve a given problem in context involving multiplication.

Scope and Sequence

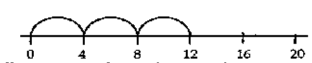
Mathematics 2	Mathematics 3	Mathematics 4
—	**N11** Students will be expected to demonstrate an understanding of multiplication to 5 × 5 by ▪ representing and explaining multiplication using equal grouping and arrays ▪ creating and solving problems in context that involve multiplication ▪ modelling multiplication using concrete and visual representations, and recording the process symbolically ▪ relating multiplication to repeated addition ▪ relating multiplication to division	**N05** Students will be expected to describe and apply mental mathematics strategies to recall basic multiplication facts to 9 × 9 and to determine related division facts. **N06** Students will be expected to demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by ▪ using personal strategies for multiplication with and without concrete materials ▪ using arrays to represent multiplication ▪ connecting concrete representations to symbolic representations ▪ estimating products ▪ applying the distributive property

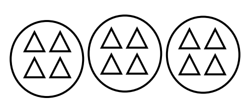
Background

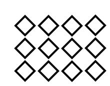
Students should be introduced to multiplication through situations (equal-group story problems) that lend themselves to modelling with sets, arrays, and linear or measurement models, such as number lines. For example, students would likely represent three plates of five cookies by making three groups of five counters (set model), represent three rows of five cadets on parade by making three rows of five counters (array model). After students have modelled story problems, they should explain the connection between their models and the story problems using verbal expressions such as “groups of,” “rows of,” and “jumps of.” Their initial number sentences, based on their prior knowledge, would involve repeated addition, such as 5 + 5 + 5 = 15. It is important not to begin too soon using the word “times” and the multiplication symbol because this may interfere with students’ understanding of multiplication situations. The formal writing of multiplication sentences should be delayed until students understand the meaning of multiplication; that is, they can correctly interpret and create story problems, model them concretely, record them pictorially, and write repeated addition number sentences. Students will need help to translate these repeated addition number sentences to multiplication number sentences. Students should first verbalize repeated addition sentences as “ __ groups of ___ is __” and then learn that the symbol X can be used for “groups of.” For example, 5 + 5 + 5 = 15, would be described as “3 groups of 5 is 15” and then written as 3 × 5 = 15.

In number sentences for multiplication, the numbers being multiplied are called factors and the answers are called products. For example, in 3 × 5 = 15, the factors are 3 and 5, and the product is 15. Students should understand and use these terms, **factors** and **product**, when describing multiplication situations. Students should understand the different ways that factors and products can be represented, such as

- by repeated addition (3 × 4 means 4 + 4 + 4)
- by making sets of equal groups (3 groups of 4 items)
- by making an array (3 rows of 4 columns)
- by showing jumps on a number line (3 jumps of 4)




3 groups of 4 is equal to 12 (3 × 4 = 12)


3 rows of 4 is equal to 12 (3 × 4 = 12)

Students in Mathematics 3 should determine the products of two factors by creating or visualizing one of these representations, and skip counting to get the total. While students may initially use one-to-one counting to get products, they need to be convinced that skip counting is a more efficient and desirable strategy. Ideally, by the end of Mathematics 3, students would use a double-count strategy to find products. For example, for 4 × 5, a student would touch 1 finger and say 5, a second finger and say 10, a third finger and say 15, and a fourth finger and say 20. They used fingers to keep track of one count (the number of groups) while they kept track of the second count (the number in the groups) by orally skip counting.

As students explore the meaning of multiplication with concrete and pictorial models, they will encounter commutative pairs, such as 2 × 3 and 3 × 2, or 4 × 5 and 5 × 4. Through focused questions, students should discover what is alike about these pairs (the same product) and what is different (the number of groups in one expression is the number in each group in the other expression). Array models of these commutative pairs best illustrate this property of multiplication because the arrays are quarter-turn rotated images of each other. Students should begin to realize that this commutative property of multiplication can often make determining products easier. For example, it is easier for most students to skip count by 5s than by 4s; therefore, to find 5 × 4, it would be easier to think of 4 × 5, saying 5, 10, 15, 20. Even though 5 × 4 would actually be 4, 8, 12, 16, 20, it has the same product as 4 × 5.

Students in Mathematics 3 are initially likely to view multiplication and division (SCO N12) as two very distinct concepts, so they will need focused experiences to help them begin to see the inverse relationship between these two operations. It is through the concrete and pictorial representations of the two operations that this relationship is most vivid. For example, if students are asked to use counters to show 3 × 4 and to show 12 ÷ 3, their final displays may both be 3 groups of 4 counters or 3 by 4 arrays, even though the processes they used to create those displays were different. As operations, this inverse relationship means that multiplication undoes division and division undoes multiplication. For example, if 3 is multiplied by 5 to get 15, then 15 divided by 5 is 3. After both concepts are introduced, it is important to have specific experiences to help students make this inverse relationship between multiplication and division.

Note: There is no expectation in Mathematics 3 for students to learn multiplication facts. All multiplication problems should involve either pictures and/or context.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to skip count forward by 2s, starting at 2 and ending at 10.
- Ask students to skip count forward by 5s, starting at 5 and ending at 25.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Have students represent story problems using models or diagrams and record the corresponding number sentences.
- Create and illustrate a story problem for 2 × 4.
- Have students create a real-life story problem that involves multiplication and solve it.
- Have students represent a given multiplication sentence, such as 5 × 3, using an array.
- Have students represent a given repeated addition as multiplication and vice versa.

- Have students represent equal groups for a given number sentence concretely or pictorially.
- Ask students to model as many arrays as possible with 16 counters. Have them write the related multiplication and division facts for each array.
- Solve a contextual problem such as, Jacques has 3 bags of apples. Each bag has 4 apples. How many apples does he have?
- Ask students to put 10 tiles into rows of 5. Ask how many rows there are.
- Show students an array and have them provide the related multiplication and division sentences.
- Use an array to show that 2 × 3 is the same amount as 3 × 2.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 9, p. 48 (Line Master 9.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- No Pathway for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Brainstorm objects that come in sets of 2, 3, 4, and 5, such as hands, eyes, ears, mittens, shoes, wheels on a bicycle, for things that come in 2s; wheels on a tricycle, sides on a triangle, juice packs for things that come in 3s; sides on a square, tires on a car, legs on a horse for things that come in 4s, and fingers on a hand, days in a school week for things that come in 5s. These contexts can be used when students are creating story problems for multiplication.

- Play “Broken Calculator” to relate multiplication and addition. Students use the constant feature of the calculator to find various products without using the multiplication key. Challenge the students to model their product using counters.
- Show an array. Ask students to write the fact family illustrated in the array (multiplication and division).
- Ask students to show multiple representations of a given multiplication fact.
- Ask students to draw pictures showing various situations in which multiplication might be used.
- Give students many opportunities to solve missing factor problems. Example: It takes 4 toothpicks to build a square. How many of the same sized squares can be built with 16 toothpicks? ($4 \times \square = 16$)

Suggested Learning Tasks

- Using counters, have students build as many arrays as possible for a given number, and write the corresponding equations.
- Provide students with a number of multiplication and division story problems. Ask them to represent each story concretely. Have them record their solutions by drawing pictures and writing repeated addition and multiplication sentences.
- Ask students to draw pictures to show what 3×4 means.
- Create arrays on cards and cut off a corner so that some counters are missing but the intended number of rows and columns remains clear. Show cards to students and ask them how many counters the card had initially if all the rows and columns had the same number of counters.
- Have students investigate what happens when you multiply a number by 0, 1, 2, 3, 4, 5. Are there patterns in the products?
- Invite a group of students to act out a skit modelling either a multiplication or division situation. Ask other students to suggest the number sentence being dramatized.
- Have the students create a realistic story problem to go with a given number sentence (e.g., 4×5) or describe a situation for which you might have to find the answer to 5×3. Students may exchange these story problems and solve them using pictures and concrete models.
- Students can create multiplication booklets for a given number (limited to 5) using computer-generated or magazine pictures. For example, students could create a booklet for multiplying by 2s. On the front of a page, students could insert a picture and write a story problem that could be solved with multiplication. On the back of that page, students could write the repeated addition and the multiplication equations that would be used to solve their story problem.
- Present students with pictures of equal groups, arrays, or number lines showing jumps. Ask them to record the repeated addition and the multiplication sentence represented by the picture. Then, ask them to explain how they would find the product.
- Ask students to use concrete materials to model the solution to a multiplication story problem, such as, Tomas and his two friends each have 3 pencils. How many pencils do they have altogether?

Suggested Models and Manipulatives

- array examples
- calculator
- counters
- ruler
- square tiles

MATHEMATICAL LANGUAGE

Teacher	Student
▪ factors, product ▪ groups of, rows of, jumps of ▪ multiplication, division ▪ number line ▪ number sentence, number expression ▪ repeated addition, equal groups, number of groups ▪ sets, arrays	▪ factors, product ▪ groups of, rows of, jumps of ▪ multiplication, division ▪ number line ▪ number sentence, number expression ▪ repeated addition, equal groups, number of groups

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 118–124
- *Making Math Meaningful to Canadian Students Grades K–8*, Second Edition (Small 2013) pp. 172–174, 176–187
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 77–84, 86–89

Notes

SCO N12 Students will be expected to demonstrate an understanding of division by ▪ representing and explaining division using equal sharing and equal grouping ▪ creating and solving problems in context that involves equal sharing and equal grouping ▪ modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically ▪ relating division to repeated subtraction ▪ relating division to multiplication. (Limited to division related to multiplication facts up to 5 × 5.) [C, CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N12.01 Identify events from experience that can be described as equal sharing.
N12.02 Identify events from experience that can be described as equal grouping.
N12.03 Illustrate, with counters or a diagram, a given story problem involving equal sharing, presented orally or through shared reading, and solve the problem.
N12.04 Illustrate, with counters or a diagram, a given story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
N12.05 Listen to a story problem, represent the numbers using manipulatives or a diagram and record the problem with a number sentence and/or expression.
N12.06 Create and illustrate with counters, a story problem for a given number sentence and/or expression.
N12.07 Represent a given division sentence and/or expression as repeated subtraction.
N12.08 Represent a given repeated subtraction as a division sentence.
N12.09 Relate division to multiplication by using arrays and writing related number sentences.
N12.10 Solve a given problem involving division.

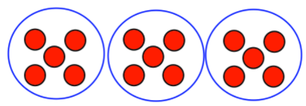
Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
—	**N12** Students will be expected to demonstrate an understanding of division by ▪ representing and explaining division using equal sharing and equal grouping ▪ creating and solving problems in context that involves equal sharing and equal grouping ▪ modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically ▪ relating division to repeated subtraction ▪ relating division to multiplication (Limited to division related to multiplication facts up to 5 × 5.)	**N07** Students will be expected to demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by ▪ using personal strategies for dividing with and without concrete materials ▪ estimating quotients ▪ relating division to multiplication

Background

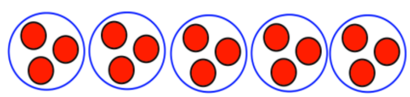
Students should be introduced to division through story problems. For this introduction, there are two types of situations, equal sharing and equal grouping, which need to be considered.

Equal-sharing problems are those in which the number of groups is known and the number in each group needs to be found. For example, Three friends want to share 15 candies. How many candies will each friend get? If students model this problem with counters, they would most likely start with 15 counters and distribute the counters, one at a time, into three groups. After all the counters have been distributed, they see there are 5 counters in each group, indicating that each friend will get 5 candies.



This may be described verbally as, When 15 is divided into 3 groups, there are 5 in each group. (**Note:** Some mathematics educators refer to these problems simply as sharing problems. While students' prior experiences of sharing in the social sense may not have guaranteed the shares were equal, they need to understand that equal sharing is fair sharing and is essential in the mathematics concept of division.)

Equal-grouping problems are those in which the number in each group is known and the number of groups needs to be found. For example, Friends want to share 15 candies by each taking 3 candies. How many friends will get candies? If students model this problem with counters, they would most likely start with 15 counters, take 3 of these counters to give to 1 friend, take another 3 to give to a second friend, and continue doing this until they run out of counters; thus, 5 friends will get candies.

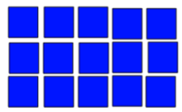


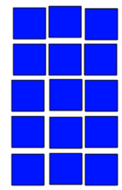
This may be described verbally as, When 15 is divided into groups of 3, there are 5 groups. (**Note:** Some mathematics educators refer to these problems as measurement problems.)

Students should solve several examples of both of these types of division problems by modelling them concretely, recording them pictorially, and describing the division in words before they are introduced to division sentences. Students can be introduced to the symbol for division as a symbolic translation of the verbal descriptions. For example, "When 15 is divided into 3 groups, there are 5 in each group" is translated $15 \div 3 = 5$; "When 15 is divided into groups of 3, there are 5 groups" is translated $15 \div 3 = 5$. Students are surprised that these two very different situations result in the same division sentence. These two interpretations of a division sentence need to get equal attention.

In division sentences, there are three numbers—one is called the dividend, one is called the divisor, and one is called the quotient. The dividend is the quantity being divided, the divisor is the number of groups (equal-sharing situations) or the number in each group (equal-grouping situations), and the quotient is the number in each group (equal-sharing situations) or the number of groups (equal-grouping situations). For example, for $15 \div 3 = 5$, 15 is the dividend, 3 is the divisor, and 5 is the quotient. The interpretations of the divisor and the quotient depend upon whether the situation represented in symbols is an equal-sharing situation or an equal-grouping situation.

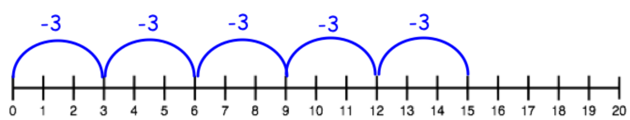
Students should also relate arrays to the two types of division situations. For example, for 15 ÷ 3 = 5, the divisor 3 may represent the number of rows (equal-sharing situations) as in the array on the left, or may represent the number in each row (equal-grouping situations) as in the array on the right.





It is important that students model story problems that do not involve remainders.

Just as multiplication is a short-hand for repeated addition, division is a short-hand for repeated subtraction. This is most apparent in equal-grouping situations. For example, when 15 candies are divided into groups of 3, 3 candies at a time are removed from the set of 15 until there are no candies left. Students are easily convinced that 15 – 3 – 3 – 3 – 3 – 3 = 0 represents the action in this situation, especially if it is also modelled on a number line.



Most students will need help, however, to see equal-sharing situations as repeated subtraction because they usually share the quantities into the groups one at a time. Students need to understand that if they keep track of how many are shared in each round of distribution, they can also write a repeated subtraction sentence for these situations. For example, when 15 candies are divided into 3 groups, sharing them 1 at a time, there are a total of 3 distributed in each round (each time each group gets 1). Thinking about this equal-sharing situation in this way, students can appreciate why the number line picture (above) can also represent it.

Students in Mathematics 3 are initially likely to view multiplication (SCO N11) and division as two very distinct concepts, so they will need focussed experiences to help them begin to see the inverse relationship between these two operations. It is through the concrete and pictorial representations of the two operations that this relationship is most vivid. For example, if students are asked to use counters to show 3 × 4 and to show 12 ÷ 3, their final displays may both be 3 groups of 4 counters or both be 3 by 4 arrays, even though the processes they used to create those displays were different. As operations, this inverse relationship means that multiplication undoes division and division undoes multiplication. For example, if you start with 3 and multiply it by 5, you get 15; then, if you divide 15 by 5, you get back to 3. After both multiplication and division are introduced, it is important to have specific experiences to help students make this inverse relationship between these two operations.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

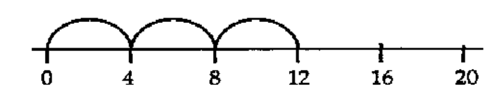
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to skip count backward by 2s, starting at 12 and ending at 0.
- Ask students to skip count backward by 5s, starting at 25 and ending at 0.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to write a division story about 25 ÷ 5.
- Have students draw a picture or use counters to show what 12 ÷ 3 means.
- Ask students to describe a situation for which you might have to find the answer to 15 ÷ 3.
- Ask students to draw pictures showing various situations in which either multiplication or division might be used.
- Show the number line below. Ask students to record what multiplication and division sentences it might be showing.



- Show students the multiplication sentence 5 × 3 = 15. Ask them to write related division sentences.
- Tell students that amusement park rides are priced as follows:
  - $1 for the ferris wheel
  - $2 for the bullet
  - $3 for the twister

  Ask, How many rides, and of which kind, can you have for $12? Are there other possibilities?
- Show students an array of up to 25 counters. Ask students which multiplication and division family is shown by the array.
- Have students listen to a story problem and represent the numbers using models or an illustration and record the problem with a number sentence. For example, Emma has 16 stickers to share with 4 friends. How many stickers will each friend get? (16 ÷ 4 = 4)

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 8, pp. 44–45

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- No Pathway for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Explore various types of division problems: equal share, equal groups, and repeated subtraction. Expect students to use concrete materials and pictures to model these situations.
- Use concrete materials to help students understand the relationship between the meanings of division. Demonstrate that, in sharing 12 items equally among 3 people, for example, the actual giving of 1 item to each person is the same as creating a group of 3. In other words, sharing among 3 people is equivalent to finding how many groups of 3 can be formed.
- Play "Broken Calculator." Students work in groups to find ways to use the calculator to solve division exercises without using the divide key.
- Provide problem solving situations in which solutions can be found using either multiplication or division.

Suggested Learning Tasks

- Provide the student with some toothpicks and ask him or her to use 12 to make 4 identical shapes. Ask the student what division and multiplication sentences could describe the creation of the shapes.
- Set up a 3 × 4 array and ask the student to give two multiplication and two division sentences that describe it by looking at the array from different perspectives.
- Invite a group of students to act out a skit modelling either a multiplication or division situation. Ask other students to identify the number sentence being dramatized.
- Ask the student to write problems in which one has to multiply or divide to find the answer. Have him or her illustrate the solutions and describe the multiplication/division relationship.
- Ask students to solve a division problem in as many ways as possible (including multiplication).
- Ask students to solve division problems using the same numbers; one where the result is "equal shares" and the other where the result is "equal groups." Have students represent these problems with counters or with a diagram.

Suggested Models and Manipulatives

- arrays
- counters
- number lines

Mathematical Language

Teacher	Student
▪ arrays, repeated subtraction ▪ divided into ▪ equal sharing/fair sharing, equal grouping ▪ multiplication, division ▪ number in each group, number of groups ▪ number sentence, number expression	▪ repeated subtraction ▪ divided into ▪ multiplication, division ▪ number in each group, number of groups ▪ number sentence, number expression

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 118–124
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 174–187
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 77–84, 86–89

Notes

SCO N13 Students will be expected to demonstrate an understanding of fractions by ▪ explaining that a fraction represents a part of a whole ▪ describing situations in which fractions are used ▪ comparing fractions of the same whole with like denominators [C, CN, ME, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N13.01 Describe everyday situations where fractions are used.
N13.02 Represent a given fraction concretely or pictorially.
N13.03 Identify, model, and explain the meaning of numerator and denominator.
N13.04 Sort a given set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.
N13.05 Name and record the fraction represented by the shaded and non-shaded parts of a given region.
N13.06 Compare given fractions with the same denominator using models.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
—	**N13** Students will be expected to demonstrate an understanding of fractions by ▪ explaining that a fraction represents a part of a whole ▪ describing situations in which fractions are used ▪ comparing fractions of the same whole with like denominators.	**N08** Students will be expected to demonstrate understanding of fractions less than or equal to one by using concrete, pictorial, and symbolic representations to ▪ name and record fractions for the parts of a whole or a set ▪ compare and order fractions ▪ model and explain that for different wholes, two identical fractions may not represent the same quantity ▪ provide examples of where fractions are used.

Background

Fractions are very complex, and it will take several years for students to develop a full understanding of them. In Mathematics 3, the focus is only on the development of students’ understanding of fractions less than one whole in situations that can be represented by regional (area) models. Students should investigate the more common fraction families such as halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. Students should be introduced to these fractions through contexts that are real to them before investigating other contexts. Students should model these fractions using created and commercial concrete materials, draw and interpret pictures of these fractions, and describe them using oral language before they are carefully introduced to the symbolic representations. Often it is the symbols that are the source of difficulty with fractions, especially if those symbols are introduced too early in the development of the concept.

While this is the first grade in which students are formally introduced to fractions, they would have encountered them and heard them used in their daily lives. One-half, or half, is likely the most familiar fraction for most students, and you may hear them using the word, maybe not always correctly. Because of their familiarity, halves are the family of fractions that you should start with to establish some general understandings about fractions before moving on to other families of fractions. These understandings include the following:

- Fractional parts are equal shares or equal-sized portions of one whole. (For example, if you hear students referring to “the bigger half,” it is an indication that they do not yet understand that one-half is one of two equal portions.)
- A fraction is a number that describes a relationship between a part and a whole. (For example, oral language should include “1 out of 2 equal parts” and “one-half.”)
- The equal parts of the whole do not have to be the same shape. (For example, join the diagonals of one of two congruent squares and the mid-points of two opposite sides of the other square: the two triangles and the two rectangles formed are all one-half of the square and are all equal to one another.)
- When creating fractional parts of a whole, the whole amount must be used. (For example, if a student is asked to cut a given length of string into halves and makes the cut, only to find that one piece is longer than the other, he or she cannot just cut off the extra, throw it away, and claim the two pieces are halves of the original piece of string.)
- Two fractions with the same name are not equal amounts unless they represent the part(s) of the same whole. (For example, one-half of the white board is not equal to one-half of a credit card.)

The concrete materials that students use to model fractions should be a balance between commercially produced models, such as The Fraction Factory, fraction circles, pattern blocks, and tangrams, and models created by the students themselves by cutting and folding paper and creating shapes on geo-boards. When students have to create their own fraction models, they must concentrate on making all the parts equal. If they only use prepared materials, they may concentrate on the number of pieces and/or the colours rather than on the equality of the parts. Also, when using materials, what is designated the whole should vary. For example, if pattern blocks are used, sometimes the hexagon should be designated the whole and other times one of the other blocks should be the whole. This varying of the whole helps students develop flexibility in thinking about fractions, and forces them to focus on the relationship aspect of a part to a whole.

Students’ initial pictures of fractions may involve them tracing the concrete models; however, they should move on to creating their own parts of a whole, such as a square, rectangle, or circle, being careful to make the parts equal in area. When they are asked to identify fractions in pictures with parts of shapes shaded, those pictures should be varied. For example, include some pictures in which the parts are not all the same shape but equal in area, some in which the parts are not equal, and some that will require the students to partition to make the parts.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Give students two pieces of paper. Ask them to fold one piece of paper in half. Ask them to fold the second piece of paper in half, but in a different way.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students, If you are really hungry and want a large piece of vegetarian pizza, would you cut the pizza into thirds, fourths, or tenths? Have them explain their thinking.
- Provide students with a square piece of paper and ask them to show fourths by folding the paper. Have the students compare their fourths. Are they the same shape? Are they all really fourths?
- Show students a region with a shaded part. Ask students to name and record the fraction represented by the shaded part. Ask them to name and record the fraction represented by the unshaded part.
- Ask students to sort various shapes that show equal and unequal parts shaded. Ask students to explain in writing how they sorted the shapes.
- Ask students to use the hexagon from the pattern block as one whole. Ask them to model and name fractions represented by the triangle, rhombus and/or trapezoid.
- Ask students, Is half a lot or a little? Have them explain their thinking.
- Ask students to identify the numerator and denominator of a given fraction.
- Provide students with fractions with the same denominator and have them identify the larger (or smaller) fraction and explain their reasoning using models.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 4, pp. 29–30 (Line Master 4.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Fractions

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Have students explore various models for fractions—part of a region and part of a length.
- Ensure students develop an understanding that “A fraction is a number that describes a relationship between a part (represented by the numerator) and a whole (represented by the denominator). Although students see two numbers, they have to think of one idea, the relationship.” (Small 2009, 196). Students should be able to describe what is the whole and what are the parts.
- Provide students with rectangles and number lines that are the same length. Ask students to colour half of a rectangle, and indicate where half is on the number line. Once students understand the concept of half, this activity could be extended to fourths (quarters) and thirds, etc.

Suggested Learning Tasks

- Ask students to fold a strip of paper into equal parts (e.g., halves, quarters, thirds).
- Give students pieces of scrap paper that are different sizes. Have students tear off a piece and describe what part of the whole it represents. Compare the pieces with classmates, and discuss why some students may have the same fraction, but the sizes of their pieces of paper are different.
- Give students some pattern blocks or Cuisinaire rods. Have them model one-half, one-fourth, and one-third (or other fractions) using various blocks or rods.
- Have students work in groups of four. Provide each student in the group with a piece of string but ensure that each piece of string has a different length. Ask each student to cut his or her piece of string in half. Then, ask students to compare their half with that of other students in their group. Ask, If everyone in your group has half a piece of string, why aren’t the strings all the same length? (Students need to understand that the “whole” was different for each person in the group.)

- Show students three pictures of varying sizes of the same item, all items cut into the same number of pieces. Ask students which of the pieces they would like to have. Explain why they made that choice.
- Ask students to model a specific fraction using five pattern blocks. Draw their model on isometric grid paper and colour the fractional part they have represented with their model.
- Have students model on a number line (0 to 1) where one-half, one-third, one-fourth (or another fraction) would be. Ask them to explain their thinking.
- Show students a 2-D shape. Tell students that the shape is a part of a whole. What could the whole be? Discuss the various possible answers and reasons why there is more than one correct answer.
- Represent a 2-D shape on a geo-board. Tell students that the shape is a one-half (or another fraction) of a whole. Ask them to use geo-boards to show what the whole might look like.
- Give students a sheet on which multiple copies of a particular regular polygon are printed. Have students explore the various fractions that can be represented on their given polygon.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- Cuisinaire rods
- fraction pieces
- geo-boards
- isometric grid paper
- number lines
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ fact families: halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths	▪ halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths
▪ fractions	▪ fractions
▪ greater than, less than	▪ greater than, less than
▪ numerator/top number, denominator/bottom number	▪ top number, bottom number
▪ one-half, half, one-fourth	▪ one-half, one-fourth
▪ part of a whole, equal parts, fair shares	▪ equal parts
▪ whole, one whole, one	▪ whole, one whole, one

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 195–200, 203–204, 206–210
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 249–254, 257–258
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 251–255, 256–258, 267
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 131–135, 136–140

Notes

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

Specific Curriculum Outcomes

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]

SCO PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Identify and describe increasing patterns.
PR01.02 Describe a given increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
PR01.03 Extend a pattern, using the pattern rule, for the next three terms.
PR01.04 Compare numeric patterns.
PR01.05 Identify and explain errors in a given increasing pattern.
PR01.06 Create a concrete, pictorial, or symbolic representation of an increasing pattern for a given pattern rule.
PR01.07 Create a concrete, pictorial, or symbolic increasing pattern and describe the pattern rule.
PR01.08 Solve a given problem using increasing patterns.
PR01.09 Identify and describe the strategy used to determine a missing term in a given increasing pattern.
PR01.10 Use ordinal numbers (to 100th) to refer to or to predict terms within an increasing pattern.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, reproducing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**PR01**Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**PR01** Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart. **PR02** Students will be expected to translate among different representations of a pattern, such as a table, a chart, or concrete materials. **PR03** Students will be expected to represent, describe, and extend patterns and relationships, using charts and tables to solve problems.

Background

Patterns are the foundation for many mathematical concepts. They should be taught throughout the year in situations that are meaningful to students, as patterns are embedded in all areas of mathematics. Providing students with the opportunity to discover and create patterns then describe and extend those patterns will result in more flexible thinking across strands.

In Mathematics 2, students had experiences describing, reproducing, extending, and creating repeating and increasing patterns. They used ordinal numbers (to tenth) to describe elements of repeating patterns. Students in Mathematics 3 will continue to explore increasing patterns, both numerical patterns with numbers to 1000 and non-numerical patterns with concrete materials, pictures, sounds, and actions. They will use ordinal numbers (to 100th) to refer to or to predict terms within an increasing pattern.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use tiles to create a growing pattern. Ask them to explain their pattern rule.
- Ask students to skip count by 10s starting at 23 and ending at 93.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students a diagram showing a square table with 4 chairs (one on each side). Tell students that if 2 tables were put together, you would seat 6 people. Ask, How many people can we seat with 6 tables? 8? 10? What if we started with a table of 6? Have students explain their reasoning.
- Ask students to show you different ways these patterns could be extended.
  20, 40, ___, ___, ___, ...
  1, 4, ___, ___, ...
- Tell students, "I am thinking of a pattern. I have landed on 50. What could I be counting by?" Accept any reasonable answer that includes an explanation.
- Ask students to begin skip counting by 100 from a given 2- or 3-digit number. Ask them to record the numbers that they say.
- Give students an increasing pattern modelled with tiles and ask them to describe, recreate, and extend the pattern in another way.

- Have students identify the pattern rule of the following increasing patterns and extend the pattern 3 more terms.
  4, 7, 10, 13, 16, …
  13, 18, 23, 28, 33, …
- Have students identify the errors in the following increasing patterns and correct them
  3, 6, 9, 12, 15, 19, 21, 24, 28, 30, …
  40, 45, 50, 60, 65, 75, …
- Provide students with an increasing pattern such as 5, 10, 15, 20, … Ask them to predict the 11th term in the pattern.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 10, Task 1, pp. 50–51

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Patterns

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of concrete and pictorial materials such as linking cubes, colour tiles, or pattern blocks, to create and extend increasing patterns.
- Expect students to discuss and write about how patterns increase and can be extended.
- Encourage students to identify the attributes of different increasing patterns (e.g., increases by the same amount).

- Have students describe errors or missing elements within an increasing pattern.
- Expect students to demonstrate their understanding of patterns by representing the same pattern in many different ways—concretely, pictorially, symbolically, orally, rhythmically, and physically.

SUGGESTED LEARNING TASKS

- Show students the first two elements of an increasing pattern made with tiles. Ask them to copy and continue the pattern.
- Have students work in groups of 6. Ask each group to represent an increasing pattern using actions/movements. After groups have practised their patterns, have them present their increasing patterns to the class. Students should then describe each of the increasing patterns in words.
- Have students explore hundreds charts to 1000 (1–100, 101–200, 201–300, etc.). Look for increasing patterns when counting forward by 2s, 5s, 10s, 25s, and 100.
- Give students one of the elements of an increasing pattern (not necessarily the first element). Ask students to model as many possible ways to extend the pattern as they can. For example, if the third element is 12, possible solutions could be
  4, 8, 12, 16, …
  3, 7, 12, 18, …
  2, 6, 12, 20, …
  6, 9, 12, 15, …
- Take students on a "Pattern Hunt" identifying increasing patterns in their school environment. Have them use numbers, pictures, and words to describe the patterns they discover.
- Have students add 2, 10 and/or 25 to a number. Ask students to describe what they notice.
- Provide students with the first 3 or 4 elements of an increasing pattern. Have them use appropriate materials to extend and explain the pattern.

SUGGESTED MODELS AND MANIPULATIVES

- coloured tiles
- grid paper
- hundred charts (up to 1000)
- linking cubes
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ describing, extending, comparing, creating ▪ increasing patterns ▪ pattern rule ▪ starting point, increasing by, … ▪ term	▪ increasing patterns ▪ pattern rule ▪ starting point, increasing by, … ▪ term

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 570–573, 579–581
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 608–615
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 57–58, 138, 281–282, 285–288
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 293–295

Notes

SCO PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Identify and describe decreasing patterns.
PR02.02 Describe a given decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
PR02.03 Extend a pattern using the pattern rule for the next three terms.
PR02.04 Compare numeric patterns.
PR02.05 Identify and explain errors in a given decreasing pattern.
PR02.06 Create a concrete, pictorial, or symbolic representation of a decreasing pattern for a given pattern rule.
PR02.07 Create a concrete, pictorial, or symbolic decreasing pattern and describe the pattern rule.
PR02.08 Solve a given problem using decreasing patterns.
PR02.09 Identify and describe the strategy used to determine a missing term in a given decreasing pattern.
PR02.10 Use ordinal numbers (to 100th) to refer to or to predict terms within a decreasing pattern.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
—	**PR02** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**PR01** Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart. **PR02** Students will be expected to translate among different representations of a pattern, such as a table, a chart, or concrete materials. **PR03** Students will be expected to represent, and describe, and extend patterns and relationships, using charts and tables to solve problems.

Background

Patterns are the foundation for many mathematical concepts. They should be taught throughout the year in situations that are meaningful to students, as patterns are embedded in all areas of mathematics. Providing students with the opportunity to discover and create patterns then describe and extend those patterns will result in more flexible thinking across strands. Students should initially describe non-numerical patterns, such as shape, action, sound, and then incorporate numerical patterns by connecting them to the non-numerical patterns.

A large focus in Mathematics 3 is the introduction and development of decreasing patterns. Students use their knowledge of increasing patterns to make connections to the concept of decreasing patterns, since similar understandings are developed. Several of the same tasks that were suggested with work on increasing patterns can be used with modifications to represent decreasing patterns.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to skip count backward by 10s starting at 123. Ask them to symbolically record the numbers that they say and to describe how the numbers change.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to show you different ways these decreasing patterns could be extended.
  80, 40, …
  925, 825, ….
  1000, 500, …
- Tell students, I am thinking of a decreasing pattern. I have landed on 50. What could I be counting by? Accept any reasonable answer that includes an explanation.
- Ask the student to say a number that is 100 less (10 less) than a 2- or 3-digit number that is provided.
- Give students a decreasing pattern modelled with tiles and ask them to describe, recreate, and extend the pattern in another way.
- Have students identify the pattern rule of the following decreasing patterns and extend the pattern three more terms.
  25, 22, 19, 16, …
  24, 20, 16, 14, 10, 6
  83, 78, 73, 68, 63

- Have students identify the errors in the following decreasing patterns and correct them.
  138, 128, 118, 108, 88, 78
  30, 28, 24, 21, 19, 15, 12, 9, 6, 3
  40, 35, 29, 25, 20, 15, 10, 5
  576, 566, 556, 546, 536, 516, 506, 486

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Patterns

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

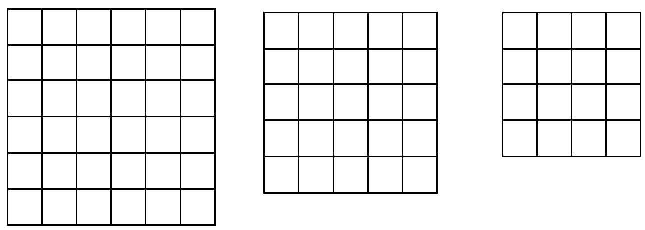
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of concrete and pictorial materials such as linking cubes, colour tiles, or pattern blocks to create and extend decreasing patterns.
- Expect students to discuss and write about how patterns decrease and can be extended.
- Encourage students to identify the attributes of different decreasing patterns (e.g., increases by the same amount). Ask, What is changing in the pattern? What remains the same?
- Have students describe errors or missing elements within a decreasing pattern.
- Expect students to demonstrate their understanding of patterns by representing the same pattern in many different ways—concretely, pictorially, symbolically, orally, rhythmically, and physically.

SUGGESTED LEARNING OPPORTUNITIES

- Provide students with the first three elements of a decreasing pattern, such as the one below,



  Ask students to continue the pattern using tiles and then to record the pattern numerically.
- Have students explore hundreds charts to 1000 (1–100, 101–200, 201–300, etc.). Look for patterns when skip counting backward by 2s, 5s, 10s, 25s, and 100.
- Give students one of the elements of a decreasing pattern (not necessarily the first element). Ask students to model as many possible ways to extend the pattern as they can (e.g., if the third element is 12, possible solutions could be 32, 22, 12, 2, … or 18, 15, 12, 9, … or 20, 16, 12, 8, …).
- Take students on a "Pattern Hunt" identifying decreasing patterns in their school environment. Have them use numbers, pictures, and words to describe the patterns they discover.
- Have students repeatedly subtract 2, 5, or 10. Ask students to describe what they notice.
- Provide students with the first three or four elements of a decreasing pattern. Have them use appropriate materials to extend and explain the pattern.

SUGGESTED MODELS AND MANIPULATIVES

- coloured tiles
- grid paper
- hundreds charts (up to 1000)
- linking cubes
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ decreasing patterns	▪ decreasing patterns
▪ describing, extending, comparing, creating	
▪ pattern rule	▪ pattern rule
▪ starting point, decreasing by …	▪ starting point, decreasing by …
▪ term	▪ term

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 570–573, 579–581
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 608–615
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 57–58, 138, 281–282, 285–288
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 293–295

Notes

SCO PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Explain the purpose of the symbol in a given addition and in a given subtraction equation with one unknown.
PR03.02 Create an addition or subtraction equation with one unknown to represent a given combination or separate action.
PR03.03 Provide an alternative symbol for the unknown in a given addition or subtraction equation.
PR03.04 Solve a given addition or subtraction equation that represents combining or separating actions with one unknown using manipulatives.
PR03.05 Solve a given addition or subtraction equation with one unknown using a variety of strategies including guess and check.
PR03.06 Explain why the unknown in a given addition or subtraction equation has only one value.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). **PR04** Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or the not equal symbol.	**PR03** Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.	**PR05** Students will be expected to express a given problem as an equation in which a symbol is used to represent an unknown number. **PR06** Students will be expected to solve one-step equations involving a symbol to represent an unknown number.

Background

In Mathematics 2, students learned the concepts of equality and inequality and the meaning of the symbols = and ≠. This knowledge is extended in Mathematics 3 to solving equations that include symbols that represent unknowns. An equation is a mathematical statement that includes an equal sign and may have been called a number sentence in the earlier grades. The equal sign tells us that the quantity on the left is the same as the quantity on the right. The equal sign is a symbol of equivalence and balance.

The focus of this outcome is to ask students to develop strategies to help them solve equations when there is a symbol representing an unknown number. Students should solve equations of the following forms with the equal sign and/or the unknown symbol in different locations:
6 + 3 = ◯
5 + ◊ = 8
Δ + 4 = 24
8 – 5 = ?
8 – ? = 3

$\Delta = 12$
$\lozenge - 15 = 5$
$6 = 3 + \Delta$
$6 = ? + 5$
$\Delta = 16 - 12$
$4 + ? = 5 + 7$

Students may find some of the equations above difficult to solve and need to have many opportunities to explore all of the different forms. It is also very important to read and interpret equations in a meaningful way. In reading $9 + \Delta = 16$ you may say, What do I need to add to 9 to get 16? or If 16 is made up of two parts, and one part is 9, how many are in the other part?

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to write number sentences using the equal sign (=) and/or the not equal sign (≠) and then explain their reasoning.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to find the number that makes each equation true.
  $5 + \square = 13$
  $38 = \square + 16$
  $16 - \square = 7$
  $\square = 24 - 18$
  $\square = 6 + 4$
  $\square - 44 = 25$

Have students explain the strategies they used. Ask, Can there be more than one answer for each? Why or why not?

- Ask, How might you use counters (or another model) to find the number to make this equation true? □ + 18 = 25 Have students write a story problem for this equation.
- Have students write the corresponding equation for a word problem and solve it. For example, Gabrielle had some stickers and gave her friend 9. Now she has 8 left. How many did she have at the start? (□ – 9 = 8)
- Present students with two numbers and ask them to create equations where one of the numbers is unknown. For example, for 15 and 8 some possible equations are 15 – 8 = □, 8 + □ = 15, 15 = □ + 8, □ = 15 – 8. Ask students to explain what a symbol represents in an equation (e.g., it represents an unknown).
- Show students a mathematics fact that includes a symbol for the missing number. Ask students to record their answers on individual whiteboards or paper. Have students share their strategies for solving the question.
- Tell students that when Amy solved the equation 13 = 7 + □, she said that the answer was 20. Is she correct? Explain using models, pictures, numbers, and/or words.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- Checkpoint 5, pp. 33–34 (Line Master 5.1 and 5.2)
- Checkpoint 6, pp. 36–37 (Line Master 6.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Equality

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Ensure students see and use a variety of symbols representing the unknown.
- Ensure that students use oral and written language to read and interpret equations in a meaningful way.
- Re-emphasize the part-part-whole relationship of addition and subtraction. This will help students solve a variety of equations by thinking of them in a different way (e.g., 12 – □ = 8 can be thought of as 8 + □ = 12).
- Provide story structures that involve more than basic fact knowledge in the equations (e.g., □ + 15 = 36). Have students explore how to solve for the unknown. They could use the relationship between addition and subtraction to solve equations.
- Have students use models to help solve equations.
- Have students solve equations that originate from word problems. Ensure that students are able to explain how to find the unknown in a variety of equations. Use a variety of forms of equations (e.g., start unknown, change unknown, result unknown).

Suggested Learning Tasks

- Ask students to match equations with word problems where the unknown is in different locations. In the following examples, an addition **or** a subtraction equation could be used to represent each problem.
  - Mia has 15 cherries and eats some. Now she has 6. How many did she eat?
    15 – 6 = □ or __+ 6 = 15
  - Edmond has 6 hockey cards, but he would like to have 15. How many more does he need?
    15 – □ = 6, 6 + …. = 15
  - Zane has 15 markers, but 6 of them no longer work. How many does he have that work?
    6 + □ = 15, 15 – 6 = ___
  - Some cookies are on a plate. Six cookies are in a jar making 15 cookies altogether. How many cookies are on the plate?
    □ + 6 = 15, 15 – 6 = ____
  - Have students create problems to represent equations such as the following:
    4 + 7 = □   □ – 8 = 8   □ + 4 = 13
- Show the students a balance scale using linking cubes to represent an equation and a piece of paper with a question mark on it to represent the unknown. Ask students to record the equation and solve it. Students can replace the paper with linking cubes to help solve the equation or to check their answer.

Suggested Models and Manipulatives

- balance scales
- base-ten blocks
- counters
- linking cubes

Mathematical Language

Teacher	Student
▪ addition and subtraction equation ▪ guess and check ▪ solve ▪ symbol, unknown	▪ addition and subtraction equation ▪ guess and check ▪ solve ▪ symbol, unknown

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 115, 582, 642–643
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 170, 620–621, 624–629
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 297–302
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 306–312

Notes

Measurement (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

Specific Curriculum Outcomes

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]

M02 Students will be expected to relate the number of seconds to a minute, the numbers of minutes to an hour, the numbers of hours to a day, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]

M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by
- selecting and justifying referents for the units centimetre or metre (cm, m)
- modelling and describing the relationship between the units centimetre or metre (cm, m)
- estimating length using referents
- measuring and recording length, width, and height [C, CN, ME, PS, R, V]

M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by
- selecting and justifying referents for the units gram and kilogram (g, kg)
- modelling and describing the relationship between the units gram and kilogram (g, kg)
- estimating mass using referents
- measuring and recording mass [C, CN, ME, PS, R, V]

M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by
- estimating perimeter using referents for centimetre or metre (cm, m)
- measuring and recording perimeter (cm, m)
- create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter [C, ME, PS, R, V]

SCO M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice.
M01.02 Identify activities that can or cannot be accomplished in minutes, hours, days, weeks, months, and years.
M01.03 Provide personal referents for minutes and hours.
M01.04 Select and use a standard unit of measure, such as minutes, hours, days, weeks, and months to measure the passage of time and explain the choice.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years.	**M01** Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).	**M01** Students will be expected to read and record time using digital and analog clocks, including 24-hour clocks.

Background

Time, as a unit of measurement, presents a unique challenge to students because it cannot be seen. Students need the opportunity to explore and discuss daily activities that involve the passage of time and to make connections to their real-world experiences. Through the use of non-standard or standard units, students will understand that time, as a measurement, is about the duration of an event from beginning to end. Personal referents will allow students to better estimate time. The goal is for students to be able to determine the appropriate unit of time to describe an event.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to name the days of the week in order beginning with Sunday. Ask them to name the months of the year beginning with January.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Have students describe the duration of something (e.g., physical education class) using their personal referent.
- Ask students to name an activity that takes minutes (hours, weeks, months, or years) to complete.
- Ask students,
  - What is something you can do in a second? In a minute?
  - What is something you can do about 10 times in a minute? In an hour?
- Give students a set of time cards (minutes, hours, days, months, years) and have students hold up the appropriate card to describe the duration of an event said by the teacher or by another student (e.g., recess—student holds up the "minutes" card).

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 11, pp. 54–55 (Line Master 11.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Time

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Engage students in daily conversations whereby they need to select an appropriate unit of time to describe activities (e.g., Does it take minutes or hours to eat your lunch?).
- Ask students to identify events that take exactly one minute. More than one minute? Less than one minute? This should be extended to other durations of time.
- Have students create their own non-standard unit timers to compare durations, such as plastic water bottles to create a water timer (see *Teaching Student-Centred Mathematics, Grades K–3, Volume 1,* Van de Walle and Lovin, 2006, 242, Fig. 8.14) or a pendulum by using a tennis ball suspended on a long string.
- Discuss the duration of various school events occurring throughout the school day and year.
- Use children's literature that focuses on durations of time to provide connections for students with this outcome.

Suggested Learning Tasks

- Ask students to estimate how many times one can count to ten, while walking heel-to-toe across the classroom. Have the student verify his or her estimate. Ask why another student might get a different result.
- Have students work in pairs to predict which of two specified activities will take longer. One student times the other who is performing the two activities, then the roles are reversed. Activities could include
  - printing your name five times
  - walking the length of the classroom heel to toe
  - making a chain of 25 "links," paper clips, or linking cubes
  - completing 10 jumping jacks
- Ask students to build a timeline reflecting the time and duration of a sequence of events.
- Ask students to identify and correct errors in your statements about time. For example, it will take me about fifteen days to eat my lunch. My favourite TV show is about one second long.

Suggested Models and Manipulatives

- calendar
- links
- linking cubes
- timeline

Mathematical Language

Teacher	Student
▪ estimate, measure ▪ minutes, hours, days, weeks, months, years ▪ time	▪ estimate, measure ▪ minutes, hours, days, weeks, months, years ▪ time

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 441–448
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 487–490
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 242–243, 244
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 269–270

Notes

SCO M02 Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M02.01 Determine the number of days in any given month using a calendar.
M02.02 Solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, the number of hours in a day, or the number of days in a given month.
M02.03 Create a calendar that includes days of the week, dates, and personal events.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years.	**M02** Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context.	**M01** Students will be expected to read and record time using digital and analog clocks, including 24-hour clocks.

Background

Students will solve problems that relate the number of seconds to a minute, minutes to an hour, and days to a month using calendars and personal events. Although it is not an expectation that students will be assessed on their ability to use a clock to tell time, clocks may be referred to during the learning of this outcome. Students have had previous experiences relating the number of days to a week and months to a year.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Point to a date on the calendar. Ask students to read the date. They should tell you the month, the day of the week, and the date (e.g., Tuesday, April 6).

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to tell how many
  - seconds in a minute
  - minutes in an hour
  - days in a given month
- Ask students,
  - What is something you can do in a second? In a minute?
  - What is something you can do about 10 times in a minute? In an hour?
- Tell students that
  - Ashram took 90 seconds to run a race and Logan took 3 minutes and ask, Who was faster?
  - it took Julie 125 minutes to drive to her grandparent's house; and ask, How many hours did it take?
- Give students a set of time cards (minute, hour, day, month, year) and have students hold up the appropriate card to describe the duration of an event said by the teacher (e.g., 60 seconds—students hold up the "minute" card; 52 weeks—students hold up the "year" card).
- Show students a calendar for the year and ask them to
  - identify ways in which months are the same and ways in which they differ
  - point out today's date and to find out what the date will be in six weeks
- Have students create a calendar and include their birth date and three other important dates for them.
- Provide students with a calendar for the year. Ask them to find a date that would become a new month in six days. How do they know?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Time

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Use a calendar throughout the school year. Engage students in discussions about the number of days in any given month, the cycle of days, and the number patterns within the calendar.
- Ask students to solve problems using the calendar.
- Direct students' attention to the analog clock. Count the seconds aloud with the students to verify it takes 60 seconds for the minute hand to move from one tick mark to the next on an analog clock. Ask students to point to the minute hand, tell them that when the minute hand moves from one tick mark to the next, one minute or sixty seconds has passed.
- Use the calendar to plan, keep track of appointments, and measure time. Focus on the structure of the month and numerical patterns.
- Have students build and create their own monthly calendar. They will need to write the months and the days of the week in order, number the days, and fill in any special dates for that month, such as class trips and physical education days.
- Use children's literature containing references to time to provide connections for students with this outcome.

Suggested Learning Tasks

- As a class, watch the second hand on an analog clock count off seconds. Extend this to include watching the second hand count 60 seconds. After these experiences, ask students to estimate the passage of specific amounts of time. For example, students could be asked to raise their hands when they think 10 seconds (30 seconds, 60 seconds) have passed.
- Provide a calendar for the year, and have the students figure out how many school days each month will have. How many Friday 13ths are there in the year? On what days of the week do the birthdays of friends and family fall? Ask the students to write about their findings.
- Ask pairs of students to predict how many weeks there are in a year. Have them use a calendar for the year to check their prediction.

- Show students a calendar for the year. Ask them to point out the day's date and to find out what date it will be in six weeks. Seven weeks?
- Ask students to build a timeline reflecting the time and duration of a sequence of events.
- Create time circles for days of the week and months of the year to demonstrate the cyclical nature of the passage of time.

SUGGESTED MODELS AND MANIPULATIVES

- calendar
- clocks
- linking cubes
- links
- sand timer
- timeline

MATHEMATICAL LANGUAGE

Teacher	Student
▪ clock, calendar ▪ seconds, minutes, hours, days, weeks, months	▪ clock, calendar ▪ seconds, minutes, hours, days, weeks, months

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 441–448
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 487–490
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 242–243, 244
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 269–270

Notes

SCO M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height [C, CN, ME, PS, R, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M03.01 Provide a personal referent for one centimetre and explain the choice.
M03.02 Provide a personal referent for one metre and explain the choice.
M03.03 Match a given standard unit to a given referent.
M03.04 Show that 100 centimetres is equivalent to 1 metre by using concrete materials.
M03.05 Estimate the length of an object using personal referents.
M03.06 Determine and record the length and width of a given 2-D shape.
M03.07 Determine and record the length, width or height of a given 3-D object.
M03.08 Draw a line segment of a given length using a ruler.
M03.09 Sketch a line segment of a given length without using a ruler.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. **M03** Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison. **M04** Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). **M05** Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.	**M03** Students will be expected to demonstrate an understanding of measuring length (cm, m) by ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height.	**M03** Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by ▪ recognizing that area is measured in square units ▪ selecting and justifying referents for the units cm^2 or m^2 ▪ estimating area by using referents for cm^2 or m^2 ▪ determining and recording area (cm^2 or m^2) ▪ constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.

Background

Working with standard units is integral to students' understanding of measurement. Students may start using standard units to measure length when they realize that non-standard units mean different things to different people. For example, if someone says a book is 15 cm long, everyone knows how long that is, but to say the book is 15 cards long would be more difficult to interpret. They need to develop a familiarity with standard units and explore the relationship between them.

Students in Mathematics 3 will be introduced to two basic standard units of length—centimetre and metre. They have had previous experiences with measuring, using non-standard units, the attributes of length, height, and width, which will enable them to measure 3-D objects as well as 2-D shapes.

Estimation in measurement is an essential part of the measurement process that has applications to real-world situations. Estimates are sometimes all that is needed and at other times reassure us about the reasonableness of our answers. Through estimation, students become more familiar with the standard units. It is also engaging for students to challenge themselves to have their estimates as close as possible to the actual measurement.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to estimate the number of cubes it would take to measure the length of a big book. Provide students with a collection of linking cubes and ask them to measure the length of the big book (multiple copies of a non-standard unit). Then, ask them to measure the length of the big book again using only one cube (single copy of the same unit many times). Ask them to explain the results.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to estimate the length of a book using a personal referent for centimetres.
- Ask students to estimate the length of the classroom using a personal referent for metres.
- Ask students to cut a length of about 1 m from a ball of string. Have them verify their estimates.
- Ask students to draw a line segment that is about 7 cm long without using a ruler.
- Show students a line segment that is 95 cm and have students estimate its length and then measure it with a ruler.
- Have students use materials to show that a metre is the same as 100 centimetres.
- Provide students with a shoebox or other box and have them measure its length, width, and height.
- Have students use a ruler to measure the length of a pencil or other object without using zero as the starting point.
- Provide students with a photograph and have the student measure the length and width of the picture.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Length

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide opportunities for students to discover and share their personal referents for centimetres and metres. They should be able to explain their choices and recognize that there are many appropriate referents for each unit.
- Include measurement situations that are of interest to the students and that provide useful information, such as measuring book heights for a new bookcase or determining if a large piece of furniture can fit through the door.
- Have students create their own rulers. Initially numbers should not be included so students need to count the number of units, rather than looking at the number on the ruler. As they become more familiar with its use, numbers can be added.
- Present situations requiring students to choose the most appropriate unit of measure.

SUGGESTED LEARNING TASKS

- Have students relate lengths to their own bodies. For example, “My legs are about half a metre long, my nose is 4 cm long, and 8 of my footprints would make a metre.”
- Compare 100 centimetre cubes (base-ten blocks) linked together to a metre.
- “Challenge students to find different ways to measure the same length with one ruler. Start from either end; start at a point not at the end; measure different parts of the object and add the results.” (Van de Walle and Lovin, 2006, 233).
- Read the book, *How Big Is a Foot?* by Rolf Myller, and relate the story to why standard units of measurement are valuable. As a follow-up, discuss why it is not a good idea to tell someone how long a table is by using pieces of paper as a measurement unit.
- Have students develop a book on measurement that they can add to over time. This could include drawings of their personal referents, pictures of objects that they have estimated and measured, and descriptions of length, width, and height.
- Give each student a metre-long piece of twine and ask them to use it to measure objects in their homes. Have them make lists of items that are less than one metre, one metre, or a little more than one metre. Have the students enter their findings in a table such as the one shown below.

Less Than One Metre	One Metre	More Than One Metre

- Set up a mini-Olympics in which students compete in events such as a tissue kick, a penny thumb toss, and cotton ball puffing. Have students measure all results to the nearest centimetre or metre, and then record and compare them.
- Ask students to explain how they could use the twine to identify objects that are about half a metre in length.

SUGGESTED MODELS AND MANIPULATIVES

- centimetre cubes
- Cuisenaire rods
- metre sticks
- rulers
- string

MATHEMATICAL LANGUAGE

Teacher	Student
▪ estimate, measure ▪ length, width, height ▪ line segment ▪ personal referent ▪ ruler, straight edge ▪ standard units: centimetre, metre	▪ estimate, measure ▪ length, width, height ▪ line segment ▪ referent ▪ ruler, straight edge ▪ centimetre, metre

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 363–368, 376–378
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 411–414, 423–425
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 223–233
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 252–260

Notes

SCO M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by ▪ selecting and justifying referents for the units grams and kilograms (g, kg) ▪ modelling and describing the relationship between the units grams and kilograms (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass [C, CN, ME, PS, R, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M04.01 Provide a personal referent for one gram and explain the choice.
M04.02 Provide a personal referent for one kilogram and explain the choice.
M04.03 Match a given standard unit to a given referent.
M04.04 Explain the relationship between 1000 grams and 1 kilogram using a model.
M04.05 Estimate the mass of a given object using personal referents.
M04.06 Measure, using a balance scale, and record the mass of given everyday objects using the units gram (g) and kilogram (kg).
M04.07 Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg.
M04.08 Determine the mass of two given similar objects with different masses and explain the results.
M04.09 Determine the mass of an object, change its shape, re-measure its mass and explain the results.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. **M03** Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.	**M04** Students will be expected to demonstrate an understanding of measuring mass (g, kg) by ▪ selecting and justifying referents for the units grams and kilograms (g, kg) ▪ modelling and describing the relationship between the units grams and kilograms (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass.	—

Background

When addressing this outcome, note that the terms **mass** and **weight** are similar, but they are not the same. Weight measures how heavy an object is and depends upon gravity, so it will vary with height above sea level. On the other hand, mass measures the amount of matter in an object and will be the same at all heights above sea level.

As with all measurement units, it is important that students have a personal referent for a gram and a kilogram. Students should recognize which mass unit (gram or kilogram) is appropriate for measuring the mass of a specific item. It is helpful for students to investigate how everyday items, such as food items, are measured. Include items that are small and dense, such as a golf ball, as well as those that are large and hollow or porous, such as a beach ball. Students need to understand that grams are used to measure very light objects and kilograms are more appropriate units for heavier objects.

Students have had previous experiences investigating mass using non-standard units. They will now begin to estimate and measure masses, using the gram (g) and kilogram (kg). By lifting and holding a variety of objects that have a mass of 1 kg, such as a bag of sugar, they should also develop a sense of what a kilogram feels like.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Provide a set of objects for students. Ask them to measure the mass of each object using non-standard units and then to order them from heaviest to lightest.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask, Could you eat 1 kg of watermelon? 1 kg of popcorn? Have students explain their thinking.
- Have students discuss which unit (g or kg) is more likely to be used in measuring the mass of:
  - a bag of potatoes
  - a box of paper clips
  - an apple
  - a bicycle
- Ask students to draw a picture of an object that they think would have a mass of about 1 kilogram.
- Display a set of five objects of similar size and a sixth target object. Ask students to sort them into groups with masses less than and greater than the target object.
- Provide students with a golf ball and a Ping-Pong ball. Ask if they can tell which has a larger mass by looking at them (comparing the sizes of the two balls). Have them find the mass of the balls.

- Ask students, Do bigger objects always have greater mass than smaller objects? Explain your thinking.
- Have students measure the mass of a ball of modelling clay. Have them use all of the clay to make a new object. Ask them to predict the mass of the new object and verify their prediction.
- Ask students to say which would be a more reasonable estimate for the mass of an adult cat—50 g or 5 kg—and explain their reasoning?
- Provide students with a collection of objects. Ask students to predict which have a mass of about 1 g, 100 g, and 1 kg. Have students explain their choices.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- Mass

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students compare the mass of objects to an established gram, 100 grams and 1 kg mass.
- Have students create masses of 1 g, 100 g, 1 kg (e.g., ask students to fill containers with various materials until they think a mass of 1 kg is reached).
- Have students find common items that are measured in grams and kilograms. Create a classroom display.
- Have students measure mass on a balance scale or other more accurate scales. Bathroom scales can be harder to interpret.

- Ensure students estimate and measure mass, using grams and kilograms as the units.
- Provide situations in which students make comparisons between the masses of two objects; one in grams, the other in kilograms.
- Provide opportunities for students to explore what happens to the mass of the same object if the shape of the object changes.

SUGGESTED LEARNING TASKS

- Have students estimate and then measure the mass of different objects in the classroom.
- Ask students to predict, from a collection of objects, which one has a mass of about 1 kilogram.
- Ask students to choose a small item. Next have the student estimate and determine how many of the items would be required to make a mass of a kilogram.
- Ask students to find something that has the same mass as two bags of marbles.
- Ask students to find the number of potatoes in 2 kg. Ask, Will the number always be the same? Why or why not?
- Have students predict and measure the number of pennies needed for a mass of 100 grams. Repeat with other coins. Ask students how much 1 kg of that coin would be worth.
- Have students measure 20 g of unpopped popcorn. Have students predict if the mass will be greater than, the same as, or less than 20 g after it has been popped? Have students compare how much space is taken up by the popped versus unpopped popcorn.
- Investigate the number of kilograms students could comfortably carry in their backpack or the total number of kilograms of a group of books on a shelf, etc.
- Use balance scales to have students investigate the mass of different kinds of balls (e.g., Ping-Pong ball vs. golf ball).
- Have students write what they know about the relationship between 1000 grams and a kilogram.
- Have the students select a personal referent for 1 g and 1 kg and explain their choice.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- kitchen scale
- pan balance or beam balance
- sets of standard weights
- variety of objects to weigh

MATHEMATICAL LANGUAGE

Teacher	Student
▪ balance scale ▪ estimate, measure ▪ mass ▪ personal referent ▪ standard units: gram, kilogram	▪ balance scale ▪ estimate, measure ▪ mass ▪ referent ▪ gram, kilogram

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 433–435, 437
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 480–481
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 241–242, 245–248
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 268–268, 274–279

Notes

<table>
<tr><td colspan="4">SCO M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by
▪ estimating perimeter using referents for centimetre or metre (cm, m)
▪ measuring and recording perimeter (cm, m)
▪ create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter
[C, ME, PS, R, V]</td></tr>
<tr><td>[C] Communication
[T] Technology</td><td>[PS] Problem Solving
[V] Visualization</td><td>[CN] Connections
[R] Reasoning</td><td>[ME] Mental Mathematics and Estimation</td></tr>
</table>

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M05.01 Measure and record the perimeter of a given regular shape and explain the strategy used.
M05.02 Measure and record the perimeter of a given irregular or composite shape and explain the strategy used.
M05.03 Construct a shape for a given perimeter (cm, m).
M05.04 Construct or draw more than one shape for the same given perimeter.
M05.05 Estimate the perimeter of a given shape (cm, m) using personal referents.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. **M03** Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison. **M04** Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). **M05** Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.	**M05** Students will be expected to demonstrate an understanding of perimeter of regular, irregular and composite shapes by ▪ estimating perimeter using referents for centimetre or metre (cm, m) ▪ measuring and recording perimeter centimetre or metre (cm, m) ▪ create different shapes for a given perimeter centimetre or metre (cm, m) to demonstrate that many shapes are possible for a perimeter	**M03** Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by ▪ recognizing that area is measured in square units ▪ selecting and justifying referents for the units cm^2 or m^2 ▪ estimating area by using referents for cm^2 or m^2 ▪ determining and recording area (cm^2 or m^2) ▪ constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area

Background

Students should be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region. An understanding that perimeter is not distinct from linear measurement will be key to students’ success when exploring perimeter. Students will need to understand that perimeter is the same as measuring linear distance that is not in a straight line. They should already be proficient using non-standard and standard tools (SCO M03) to measure length. In Mathematics 3, they will develop proficiency with measuring perimeter in standard units (centimetre and metre) using rulers, measuring tapes, metre sticks, and trundle wheels.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to estimate and measure the distance around the top of their desk using non-standard units such as linking cubes.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students regular and irregular, including composite, shapes and have them find the perimeter and explain their strategy. For example,
  - have students construct a shape with a given perimeter using grid paper
  - ask students to construct two different shapes with the same given perimeter using grid paper
- Provide students with a geo-board. Have them create,
  - a rectangle with a perimeter of 12 units
  - a second rectangle of 12 units but with different dimensions
  - a different shape (not a rectangle or triangle) with a perimeter of 12 units
- Ask students to estimate the perimeter of a given shape. Have them measure and record the actual length.
- Ask students to solve the following problem: Farmer Bill has 24 metres of fencing. How many different rectangular chicken coops can he make?
- Provide students with three shapes and ask whether it is possible that they all have the same perimeter. Explain. Have them find the perimeter of each.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- No Pathway for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to predict the perimeter prior to making their measurements.
- Provide students with frequent opportunities to construct, measure, and record perimeter of regular and irregular, including composite, shapes.
- Ask students to construct or draw more than one shape for the same given perimeter.
- Use perimeter problem-solving situations that provide a context for students (e.g., border around rooms or bulletin boards, frames, fences, trim).
- Provide many opportunities for students to measure the perimeter of irregular shapes using indirect measure with materials such as a string and ruler.
- Ask students to make comparisons between the perimeter of various shapes and estimate which shapes have a similar perimeter.

SUGGESTED LEARNING TASKS

- Ask students, How can we find the distance around a shape? (Provide regular and irregular shapes).
- Give each group a metre stick, tape measure and a 30-cm ruler, and string. Ask them to figure out how to find the perimeter of shapes around the classroom. Discuss different results.
- Give students pieces of string (different lengths) and ask, "How many different objects can you find with a perimeter that is equal to the length of your string?"
- Ask students, How many shapes can you find with a perimeter of 10 cm? 30 cm? 1 m? 3 m?
- Provide students with geo-boards or grid paper and ask, How many different shapes can you make with a given perimeter?
- Tell students, I have drawn a shape in grid paper with a perimeter of 24 cm. What might my shape look like?
- Have students trace the outline of their bodies on a large surface with sidewalk chalk and then estimate and measure the perimeter of their bodies.
- Provide students with a set of 12 pentominoes and have them find the perimeter of each piece. Do all of the shapes have the same perimeter? Which perimeter is the most common?
- Use a trundle wheel to find the perimeter of the gym or playground.

SUGGESTED MODELS AND MANIPULATIVES

- 30-cm rulers
- base-ten units and rods
- geo-boards
- grid paper
- metre sticks
- pentominoes
- string
- tape measure
- trundle wheel

MATHEMATICAL LANGUAGE

Teacher	Student
▪ measure, estimate ▪ perimeter, distance around ▪ personal referent ▪ standard units: metre, centimetre	▪ measure, estimate ▪ perimeter, distance around ▪ referent ▪ metre, centimetre

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 380–387
- *Making Math Meaningful to Canadian Students Grades K–8*, Second Edition (Small 2013), pp. 427–428
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 264–265

Notes

Geometry (G)

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

Specific Curriculum Outcomes

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]

G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides. [C, CN, R, V]

SCO G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Identify the faces, edges, and vertices of given 3-D objects, including spheres, cones, cylinders, pyramids, cubes and other prisms.
G01.02 Identify the shape of the faces of a given 3-D object.
G01.03 Determine the number of faces, edges, and vertices of a given 3-D object.
G01.04 Sort a given set of 3-D objects according to the number of faces, edges, or vertices.

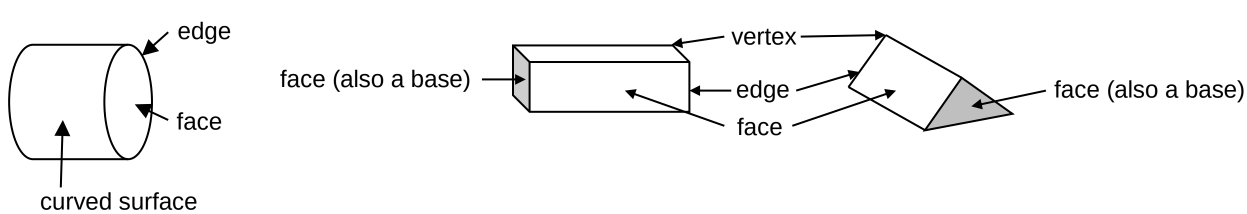
Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule. **G04** Students will be expected to identify 2-D shapes as parts of 3-D objects in the environment.	**G01** Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices.	**G01** Students will be expected to describe and construct rectangular and triangular prisms.

Background

Students in Mathematics 2 had experiences identifying, sorting, comparing, describing, and constructing 2-D shapes and 3-D objects. Students in Mathematics 3 will continue to develop their knowledge by describing and sorting 3-D objects according to their geometric attributes. Students will identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, and cubes and other prisms.

The geometric attributes of 3-D objects are as follows:
- **Face:** A 2-D shape that forms part of a 3-D object. It is a flat surface that can be traced. Both the shape of the face and the number of faces should be considered as an attribute.
- **Edge:** Occurs where two surfaces of a 3-D object join.
- **Vertex (vertices):** A point where three or more edges meet. (**Note:** On a cone and a pyramid the highest point above the base is called the apex. In a pyramid the apex is also a vertex, but for a cone, it is a mistake to refer to the apex as a vertex as there are no edges that meet.)



Provide students with opportunities to explore these attributes through sorting and constructing activities. As they become more familiar with identifying the attributes, students can determine the number of faces, edges, and vertices. Students will use informal language at this stage rather than precise mathematical language. Students may say “corners” rather than “vertices” and “sides” rather than “faces” for 3-D objects.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students’ prior knowledge.

- Tell students that you have a 3-D object in a bag. One of its faces is round (a circle). Ask what the object could be. Repeat for other 3-D objects and faces of different shapes.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe objects according to their attributes, making sure correct mathematical terms are used for names of objects and faces, edges, and vertices.
- Have 3-D objects sorted by attribute, and ask students to identify the sorting rule.
- Place a triangular prism and triangular pyramid beside one another. Ask students to name them. Ask them to tell you some things that are the same about them and some things that are different.
- Ask students to solve riddles such as, I have 5 faces, 8 edges, and 5 vertices. Who am I? Have students create and solve their own 3-D riddles.
- Have students sort a group of objects according to the number of faces, edges, and vertices.
- Have students identify the shape of the faces of a given 3-D object.

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 15, Task 2, pp. 70–71
- Checkpoint 16, Task 2, pp. 73–74

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- 3-D Objects

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Provide students with concrete models of given 3-D objects (geometric solids or other objects), including cubes and other prisms, spheres, cones, cylinders, and pyramids.
- Identify and sort 3-D objects according to the number of faces, edges, and vertices.
- Use cross-curricular opportunities to explore 3-D objects in art and science classes.
- Ask students to identify particular 3-D objects in their environment and in pictures and to justify their answers.
- Read children's literature that includes geometry concepts, such as *Sir Cumference and the Sword in the Cone* by Cindy Neuschwander and *The Greedy Triangle* by Marilyn Burns. Discuss.

SUGGESTED LEARNING TASKS

- Say to students, In a bag I have an object that has flat faces, and straight edges. What might this object be? (Other attributes should be used to extend this activity.)
- Tell students, The object behind my back is able to roll. Ask what might it be? (Other attributes should be used to extend this activity.)
- Have students create a mini book about 3-D objects that include a picture of each object and its attributes.
- Ask students to build a wall using 3-D objects. Discuss what 3-D objects could and could not be used.
- Ask students, What can you tell me about a pyramid? A cone? A prism? Have them focus on the attributes.
- Give each group a collection of 3-D objects. Have them sort the objects according to the geometric attributes and provide the sorting rule.
- Have students choose two different 3-D shapes. Have them write and illustrate three ways they are the same, and three ways they are different.
- Ask students to play a game of “Name That 3-D Shape.” Students must determine the object from the clues given.
- Have students make “Wanted” posters for 3-D objects, describing number of faces, edges, and vertices, and shapes of faces.
- Cut out and label pictures from magazines to build a collage and to identify 3-D objects in the environment.

SUGGESTED MODELS AND MANIPULATIVES

- geometric solids
- marshmallows
- modelling clay
- Polydrons
- straws
- toothpicks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 3-D objects	▪ 3-D objects
▪ apex	
▪ attributes	
▪ cubes, spheres, cones, cylinders, pyramids, prisms	▪ cubes, spheres, cones, cylinders, pyramids, prisms
▪ faces, edges/sides, vertices/corners	▪ faces, edges/sides, vertices/corners
▪ flat surface, curved surface	▪ flat surface, curved surface

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 285–286, 291–292, 303–304
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 343–344, 347–348, 358–359
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 186–193, 195, 204–207, 217
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 220–224, 204–211

Notes

SCO G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides. [C, CN, R, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Classify a given set of regular and irregular polygons according to the number of sides.
G02.02 Identify given regular and irregular polygons having different dimensions.
G02.03 Identify given regular and irregular polygons having different positions.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
G03 Students will be expected to recognize, name, describe, compare and build 2-D shapes, including triangles, squares, rectangles, and circles.	**G02** Students will be expected to name, describe, compare, create, and sort regular and irregular polygons including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides.	—

Background

Students have had many opportunities to explore 2-D shapes through sorting, patterning, and building activities. Students' previous experiences with describing and comparing polygons included squares, triangles, and rectangles. Students in Mathematics 3 will extend their knowledge to include both regular and irregular polygons.

Polygons are closed 2-D shapes with three or more straight sides. They have the same number of sides as angles and are classified by their number of sides. Regular polygons have all equal sides and angles, such as equilateral triangles, squares, and the hexagon pattern blocks. In an irregular polygon, all the sides are not the same length and/or all the angles are not the same size.

Students should focus on comparing the number of sides as the key attribute for classifying polygons. In this outcome, students should be able to name the specific polygons triangle, quadrilateral, pentagon, hexagon, and octagon. In the following diagram, the shaded polygons are regular polygons, and all others are irregular polygons.

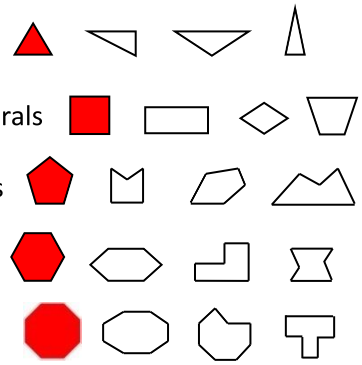
3 straight sides: triangles

4 straight sides: quadrilaterals

5 straight sides: pentagons

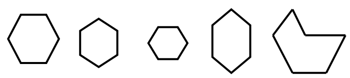
6 straight sides: hexagons

8 straight sides: octagons



Regular polygons are shaded.

Although pattern blocks are frequently used for geometric inquiry, most of these shapes are regular. Students may develop the misconception that only certain familiar polygons meet the criteria for these shapes. For example, students may not initially recognize all of the following shapes as hexagons.



Ask students to find examples of polygons in the world around them, perhaps even collect as many types of a shape as they can find. Sort the shapes according to the number of sides. By sorting polygons according to the number of sides, students can learn the names for the polygons.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Show students a set of triangles in various sizes and positions. Tell them that one student thinks these shapes are all triangles, but another student doesn’t agree. Ask them to explain who is correct.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a sheet that includes a number of different polygons (regular and irregular) that are different sizes, forms, and/or orientations. Have the students sort and name the polygons. Observe that the students recognize the same shape in different positions or orientations.
- Have students create two different pentagons (or other polygons) on a geo-board.
- Provide students with pattern blocks. Have them create new polygons by using two blocks (equal sides should be matched) and trace the shape of the new polygon. Have them write the type of polygon they created.
- Have students explain how an octagon and a hexagon (or other shapes) are similar and different.
- Show students two groups of sorted polygons. Ask, What might the sorting rule have been? Include different types of polygons that are regular and irregular and different sizes.
- Ask students, If you draw a pentagon (or another shape) and your friend draws a pentagon, will the two shapes look exactly the same? Why or why not? What will be the same every time?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)

- No Checkpoint for this outcome.

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)

- 2-D Shapes

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Ask questions that focus on the attributes of polygons. For example, What other shapes look like this one? In what way are the shapes alike? In which ways are they different?
- Provide opportunities for students to develop their own definitions for the different types of polygons. Have students sort shapes such as those included as black line masters in *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006). Have students explain their reasoning to others.
- Have students create different polygons on geo-boards or dot paper. Challenge the students to create different types of triangles (quadrilaterals, pentagons, etc.).
- Have students create a book for a particular polygon. Include a variety of examples (regular and irregular) for each type of polygon included in this outcome. This could be extended to include other types of polygons.
- Use geo-strips or strips of paper of different lengths to create various polygons.
- Use children's literature, such as *The Greedy Triangle* by Marilyn Burns to further explore the attributes of polygons.
- Integrate art activities using these shapes. For example, create a piece of art using only a single 2-D shape, but change the other attributes (e.g., size, orientation, length of sides, colour).

Suggested Learning Tasks

- Ask students to make a triangle on a geo-board that has two pegs inside, then one that has three. Ask, What is the greatest number of pegs that can be inside a triangle on a geo-board? Repeat this activity with other shapes.
- Have students sort a collection of pattern blocks by the type of polygon.
- Provide students with sets of tangrams and pentominoes. Have them sort the shapes into triangles, quadrilaterals, pentagons, hexagons, and octagons. (**Note:** There are no pentagons and there are some shapes that have more than 8 sides.)
- Have groups of students create a "path of polygons" using sidewalk chalk on a large surface and drawing a sequence of different polygons.
- Ask students to create riddles about a chosen polygon. Riddles can be exchanged for other students to solve.
- Ask students to explain why it is possible to have a polygon with more than eight sides but it is not possible to have an octagon with more than eight sides.
- Have students complete a concept map or Frayer model for a given polygon.

Suggested Models and Manipulatives

- dot paper
- geo-board
- geo-strips
- pattern blocks
- pentominoes
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student
▪ dimension, size ▪ position ▪ regular, irregular polygons ▪ triangles, quadrilaterals, pentagons, hexagons, octagons ▪ turning (rotating), flipping (reflection), sliding (translations)	▪ sizes ▪ polygons ▪ triangles, quadrilaterals, pentagons, hexagons, octagons ▪ turning, flipping, sliding

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 287–288, 292–296
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 343–344, 348–352
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 186–193, 193–195, 196–199, 202–204, 206–208
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 204–211, 211–214, 220–222, 225–226

Notes

Statistics and Probability (SP)

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. [C, CN, V]

SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems. [PS, R, V]

SCO SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. [C, CN, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01 Record the number of objects in a given set using tally marks.
SP01.02 Determine the common attributes of line plots by comparing line plots in a given set.
SP01.03 Organize a given set of data using tally marks, line plots, charts, or lists.
SP01.04 Collect and organize data using tally marks, line plots, charts, and lists.
SP01.05 Answer questions arising from a given line plot, chart, or list.
SP01.06 Answer questions using collected data.

Scope and Sequence

Mathematics 2	**Mathematics 3**	**Mathematics 4**
SP01 Students will be expected to gather and record data about self and others to answer questions.	**SP01** Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.	**SP01** Students will be expected to demonstrate an understanding of many-to-one correspondence.

Background

Students will be given opportunities to collect, organize, and display data to answer questions. Previously, students have constructed concrete graphs and pictographs to answer questions and solve problems. In Mathematics 3, students will use tally marks, lists, charts, line plots, and bar graphs to organize data relevant to their everyday life. Opportunities for collecting data to answer questions should naturally occur throughout the year. These opportunities may include putting students' names on a birthday chart to display in the classroom, deciding on a lunchtime activity, comparing shoe sizes, or comparing bedtimes. When working with data, students discover not only answers to questions, but meaningful information that can evoke change in their world. The expectation is that students will collect, organize, and display data to answer questions.

At this level, students should be encouraged to become more independent in the selection of appropriate strategies for collecting and organizing data. For example, ask pairs of students to decide on the strategy they will use to collect and organize data that will show interesting information about classmates.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

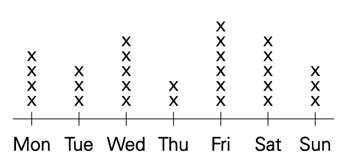
Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a collection of about 25 linking cubes in three or four different colours. Ask them to organize the cubes and record the date in a chart using tally marks or another method. Ask them to write two questions that the tally marks would answer.

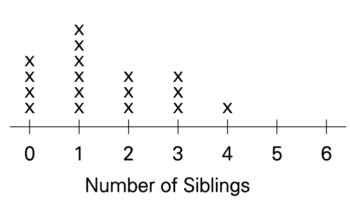
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to select a topic, survey family members and/or neighbours, and present their findings to the class in an organized chart, list, or line plot.
- Ask the students to keep track of weather conditions over the period of one month and to design a way to present the information in an organized chart, list, or line plot.
- Show students a line plot such as the one below and ask what it may represent.



- Ask students how they would represent the sports the children in their class play and how many students play each sport?
- Show students the following line plot and ask questions such as, What is the most common number of siblings? How many students have two siblings or less? How many students have four siblings? (Ensure students know that siblings are what their brothers and sisters are called.)



- Show students a set of data presented in chart form. Ask them to represent the data in another way, such as tally marks or a line plot.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 17, p. 77
- Checkpoint 18, pp. 79–80 (Line Master 18.1)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Sorting and Organizing Data

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have pairs of students decide on the procedure they will use to collect and display data using tally marks, line plots, charts, or lists showing interesting information about class members.
- Have students plan and conduct an in-class survey about a favourite ______ (e.g., toy, television program, hockey player). Ask them to present the results of the survey in an organized chart or list.
- Have students conduct a survey to find out what objects 8- and 9-year-olds like to collect. They will need to decide who to survey and how to organize and present their data.
- Ask small groups of students to brainstorm an interesting question for a possible survey. Then, have each group conduct their survey, and collect and organize their data.

SUGGESTED LEARNING TASKS

- Ask students why it is easier to count the “yes” responses when they are shown like this 𝍸 𝍸 rather than like this || | ||| | || |.
- Have students collect, record, and organize data in a line plot, chart, tally marks, or list to describe the favourite books of their classmates (or other relevant topic).
- Model recording a set of data in a line plot, list, and chart format and discuss the advantages and disadvantages of each type of data display.
- Collect and display data that represent
  - accomplishments of favourite sport figures or friends (e.g., the number of goals, hits, points)
  - the distance class members can throw a ball
  - mass of various fruits or vegetables
  - mass of subject textbooks
  - mass of different breeds of dogs
- Give students a list of questions and have them identify the questions that might be used for a particular graph or set of data.
- Ask students to describe what they would expect to find in a “well-made” line plot.
- Show students an organized list of first-hand data and have them pose relevant questions about the data.

SUGGESTED MODELS AND MANIPULATIVES

- craft sticks
- linking cubes
- paper clips

MATHEMATICAL LANGUAGE

Teacher	Student
▪ collect, organize, display, interpret data	▪ collect, organize, display
▪ first-hand data	
▪ tally marks, line plots, charts, lists	▪ tally marks, line plots, charts, lists
▪ title, label, horizontal axis, dots, crosses	▪ title, label, horizontal axis, dots, crosses

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 471–473, 484, 525–528, 530–532
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 517–518, 568–574, 530
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 310–312, 317–319, 322, 329–330
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 320–322, 329–330, 333

Notes

SCO SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems. [PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP02.01 Determine the common attributes, title, and axes of bar graphs by comparing bar graphs in a given set.
SP02.02 Create bar graphs from a given set of data including labelling the title and axes.
SP02.03 Draw conclusions from a given bar graph to solve problems.
SP02.04 Solve problems by constructing and interpreting a bar graph.

Scope and Sequence

Mathematics 2	Mathematics 3	Mathematics 4
SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.	**SP02** Students will be expected to construct, label, and interpret bar graphs to solve problems.	**SP02** Students will be expected to construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.

Background

A bar graph is a useful tool for organizing data. Students will explore both vertical and horizontal bar graphs, making the connection that the height or length of the bars represents a number. Ensure that all graphing activities are based on one-to-one correspondence. Initially, students will focus on reading and interpreting given bar graphs. Spacing the bars on a graph is necessary because the data is discrete, that is, the data represents separate categories. When reading a bar graph, ask students to use a ruler, index card, or finger to accurately find the number on the appropriate axis that aligns with the top or end of each bar. Once they have explored reading and interpreting given bar graphs, they will then construct their own. Once students have constructed a bar graph, they should make observations about the data and interpret the data to answer questions. They should also be given experiences answering questions about the data displayed on other bar graphs found in newspapers, magazines, and television or on the Internet.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of data about a topic of interest to them. Ask them to create a pictograph and a concrete graph that represents the data.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show students a bar graph on a topic of interest to students. Have them answer questions about the graphs and have them make up questions about the graph.
- Provide students with data. Have them construct a bar graph on grid paper. Ensure that students include a title, and labels on both axes.
- Ask students, What would happen if the bars in a graph were rearranged? Would the graph still give you the same information? Explain.
- Have students answer the following: This is a graph of a survey I did with my Mathematics 3 class. What might the survey be about? Label the graph, make up a title, and then, make up three questions that could be answered with this graph.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2011)
- Checkpoint 18, pp. 79–80 (Line Master 18.2)

Leaps and Bounds toward Math Understanding 3/4 (Small, Lin, and Kubota-Zarivnij 2011)
- Displaying Data

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Emphasize the use of real data when constructing graphs.
- Use grid paper to ensure bar graphs are as accurate as possible.
- Determine common attributes of bar graphs by examining examples from various sources.
- Make use of opportunities to integrate graphing concepts in other areas, such as science, morning message, social studies, etc.

SUGGESTED LEARNING TASKS

- Provide several bar graphs. Have students compare and determine the common attributes, making sure title, axes, and labels are included.
- Provide several bar graphs. Have students draw conclusions and answer questions about the graphs.
- Provide opportunities for students to match created bar graphs with data organized in charts, lists, tally marks, or line plots.
- Ask students to create a bar graph to show the kinds of pets students in the class have at home. Have them write two questions about their graph.
- Create a bar graph for a set of data on a grid on the floor.
- Provide students with a real-life problem to solve such as, What game should we play in physical education? or What special activity should be at the Celebration Assembly? or What book should be read during literacy time? Create a bar graph from collected data, and use it to make decisions or solve problems.
- Give students a bar graph that has been created, but does not contain labels or title. Ask students to explain why the graph is difficult to read and interpret.

SUGGESTED MODELS AND MANIPULATIVES

- grid paper
- pre-made bar graphs

MATHEMATICAL LANGUAGE

Teacher	Student (oral language)
▪ bar graph, bar(s) ▪ data, collect, organize, display, interpret ▪ title, labels, scale, axis, axes ▪ vertical, horizontal	▪ bar graph, bar(s) ▪ data, collect, organize, display ▪ title, labels, scale, axis, axes ▪ vertical, horizontal

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students K–8* (Small 2009), pp. 474–478, 480–481
- *Making Math Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 526–529
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 317–318, 319–321
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 329–331

Notes

Appendices

Appendix A: Performance Indicator Background

Number (N)

SCO N01 Students will be expected to say the number sequence forward and backward by ▪ 1s through transitions to 1000 ▪ 2s, 5s, 10s, or 100s, using any starting point to 1000 ▪ 3s, using starting points that are multiples of 3 up to 100 ▪ 4s, using starting points that are multiples of 4 up to 100 ▪ 25s, using starting points that are multiples of 25 up to 200 [C, CN, ME]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Extend the number sequence by 1s, particularly through transition from decade to decade and century to century.
N01.02 Extend a given skip counting sequence by 2s, 5s, 10s, or 100s, forward and backward, using a given starting point.
N01.03 Extend a given skip counting sequence by 3s, forward and backward, starting at a given multiple of 3 up to 100.
N01.04 Extend a given skip counting sequence by 4s, forward and backward, starting at a given multiple of 4 up to 100.
N01.05 Extend a given skip counting sequence by 25s, forward and backward, starting at a given multiple of 25 up to 200.
N01.06 Identify and correct errors and omissions in a given skip counting sequence.
N01.07 Determine the value of a given set of coins (nickels, dimes, quarters, and loonies) by using skip counting.
N01.08 Identify and explain the skip counting pattern for a given number sequence.

Performance Indicator Background

N01.01 When exploring numbers over 100, spend considerable time focusing on the numbers between 100 and 200. Students need to think about how the counting pattern sounds when they count from 20 to 29 and 30 to 39 and apply it when counting from 120 to 129 and 130 to 139, and when counting through all the decades that follow. Students need multiple opportunities to count the numbers through transition from decade to decade and century to century. For example, when counting from 98—98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, … a common misconception for students is to think that the next number after the one that ends in “9” is the next big number name (29 → 30; 49 → 50). As a result many believe that 200 comes after 109. Students need to see that this same transition will apply for counting into all of the centuries. For example, on a regular basis, they should be asked questions such as, what would come after 199? or count on from 389.

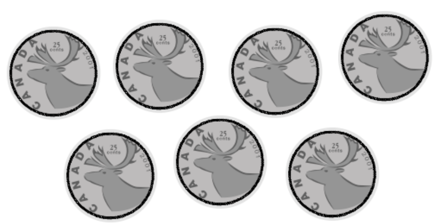
N01.02, N01.03, N01.04, and N01.05 Students in Mathematics 2 skip-counted by 2s forward and backward to 100. In Mathematics 3, students will skip count forward and backward by 5s, 10s, and 100s from any given number; by 3s and 4s, starting at any multiple of 3 and 4 respectively up to 100; and by 25s, starting at any multiple of 25 up to 200. Being able to skip-count will help students be successful with many upcoming concepts such as patterns, money, and place value. Number charts may be used to explore these patterns.

Students should also investigate these skip-counting patterns and whether the similar patterns occur when counting by 100s or when counting on hundreds charts beyond 100. A specific focus on looking at the numbers between 100 and 200 can help students to develop a sense of the repeating patterns in the number system. A hundred chart can easily be extended to a 200 chart by adding rows for the each of the next 100 numbers.

Ask students to use number lines and hundreds charts to skip count forward or backward by 4s starting at different multiples of 4. Ask them to record their jumps on the number line or colour in the number they jump to on the hundred charts.

N01.06 and N01.08 Students should be able to identify skip-counting patterns and any errors and omissions that may occur in these patterns, whether presented forward or backward. Using number lines and hundreds charts are effective visuals to help students identify the patterns and the errors or omissions. For example, display a part of a hundreds chart with numbers missing in a skip-counting sequence. Ask students to identify the skip-counting pattern and to fill in the missing numbers.

N01.07 Students will skip-count to determine the value of a group of one type of coins, such as nickels, dimes, quarters, and loonies. For example, the following set of quarters could be used to count 25, 50, 75, 100, 125, 150, 175. **Note:** The final count is recorded and said as a whole number, 175 cents; not using decimal notation.



SCO N02 Students will be expected to represent and partition numbers to 1000. [C, CN, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Read a given three-digit numeral without using the word **and**.
N02.02 Read a given number word (0 to 1000).
N02.03 Represent a given number as an expression.
N02.04 Represent a given number concretely and pictorially in a variety of ways.
N02.05 Write number words for given multiples of ten to 90.
N02.06 Write number words for given multiples of a hundred to 900.
N02.07 Record numerals for numbers expressed orally, concretely, or pictorially.

Performance Indicator Background

N02.01 and N02.02 The number 205 is read, two hundred five. Students must be able to record numbers heard and read numbers written symbolically. When reading numbers, the word **and** is reserved for the decimal, which will be discussed in Mathematics 4.

N02.03 Students who have a deep understanding of numbers up to 1000 will be able to represent numbers in a variety of ways. It is important to model the correct use of the term **expression** to students. An expression names a number. Sometimes an expression is a number such as 150. Sometimes an expression shows an arithmetic operation, such as 125 + 25. 150 may also be represented by its partitions, such as 80 + 70, 100 + 50, and 50 + 50 + 50. Numbers can also be represented by a difference expression, such as 175 – 25.

N02.04 It is important that students see the numbers up to 1000 in different ways in order to realize a number can cover a big area or a small area, depending on the size of the items being counted. Provide opportunities for students to use hundreds charts and collections of materials such as straws, buttons, commercial counters, kidney beans, and paper clips to represent given numbers. Students will decide on various ways to count the objects, perhaps grouping them in tens and/or hundreds, then presenting their numbers in pictures.

325 represented pictorially with buttons

Students should recognize that 1000 is just another expression for ten hundreds or 100 tens.

N02.05, N02.06, and N02.07 Students will also need to be able to write the number words for the multiples of ten to ninety (twenty, thirty, forty, …) and multiples of a hundred to nine hundred (two hundred, three hundred, …). It should also be noted that when writing one thousand symbolically, there is no space or comma between the thousands and hundreds place; 1000 not 1 000, nor 1,000. We do not use the comma because in many countries using the metric system, the comma is used as the decimal point.

SCO N03 Students will be expected to compare and order numbers to 1000. [CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

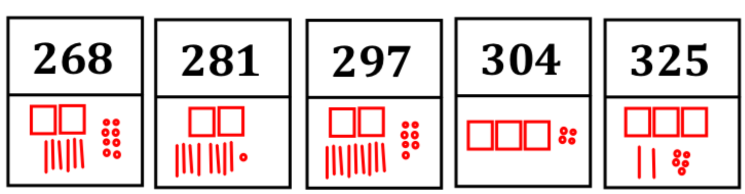
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Place a given set of numbers in ascending or descending order and verify the result using a number chart or other models.
N03.02 Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
N03.03 Identify errors in a given ordered sequence.
N03.04 Identify missing numbers in parts of a given number chart and on a number line.
N03.05 Identify errors in a given number chart and on a number line.
N03.06 Place numbers on a number line containing benchmark numbers for the purpose of comparison.
N03.07 Compare numbers based on a variety of methods, and record the comparison using words and symbols (=, > and <).

Performance Indicator Background

N03.01 Students should be given opportunities to place a given set of numbers in ascending or descending order. For example, a student may be given six or eight base-ten cards with different amounts on them and be asked to sort them from least to greatest, or vice versa. It is important for teachers to mix up the amounts so that the sets do not always represent consecutive numbers. Students could also be given cards with numbers on them and be asked to model and sort the numbers. This type of task reinforces modelling while providing an opportunity to put numbers in order.

The student could verify the order by looking for the numbers on hundreds charts or by drawing a number line.



N03.02 Given any three digits, students should make as many 3-digit numbers as they can, then put them in order from least to greatest. For example, if given cards containing the digits 3, 5, and 2, students can make 235, 253, 325, 352, 523, and 532. Students should be able to explain how they determined all possible numbers and how they ordered them. Students should also be able to arrange them from greatest to least or should be able to place the numbers on an open number line.

N03.03 Students should be able to identify when a given sequence of numbers is not in the correct order and be able to correct it by rearranging them. They should be encouraged to talk about how they made their corrections.

N03.04 and N03.05 Students should have enough familiarity with the hundreds chart that they are able to identify errors or the values of missing numbers. A student could be given a hundreds chart with numbers missing and be asked to fill in the missing values and to explain how they decided what number went in each empty position.

N03.06 When comparing two numbers, students should be encouraged to make use of benchmarks. Students should say that 48 is less than 95 since both numbers are to the left of 100 on a number line but only 48 is to the left of 50. Similarly, 37 is greater than 27 since 37 is to the right of 30 and 27 is to the left of 30 on a number line. This reasoning process is part of having number sense. Students will often refer to the number of tens in a number in order to compare it to another; for example, 47 is greater than 21 since 47 is more than 4 tens, but 21 is only a bit more than 2 tens. This type of language is preferable to 4 is more than 2 so 47 is greater, particularly since students should focus on the fact that the 4 in 47 represents 40, not 4, and the 2 in 21 represents 20, not 2. This work should be connected to the use of base-ten materials, number lines, and hundreds charts.

N03.07 An understanding of place value (explored in greater depth in SCO N05) is essential for students to compare and order numbers. For example, to compare 667 and 607, students should notice that both numbers have 6 hundreds, but that the 667 is greater than 607 because it has more tens in the tens place. The numbers could also be compared by considering their relative position in the counting sequence: 667 comes after 607, so 667 is greater than 607. Students should be able to compare two or more numbers, each less than 1000, to determine relative sizes. Include situations in which numbers are located on hundreds charts and number lines. When numbers are represented in their standard or symbolic form, students can use the number of digits to get a sense of their size in order to compare them. Three-digit numbers are less than a 1000 but greater than any 2-digit number. SCO N05 should be done before or in conjunction with comparing 3-digit numbers.

SCO N04 Students will be expected to estimate quantities less than 1000 using referents. [ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Estimate the number of groups of ten in a given quantity using 10 as a referent (known quantity).
N04.02 Estimate the number of groups of a hundred in a given quantity using 100 as a referent.
N04.03 Estimate a given quantity by comparing it to a referent.
N04.04 Select an estimate for a given quantity by choosing among three possible choices.
N04.05 Select and justify a referent for determining an estimate for a given quantity.

Performance Indicator Background

N04.01, N04.02, and N04.03 Students use reasoning skills to estimate a total using a visual referent. Students may create a mental picture (visualization) of an amount and use that picture to estimate a total. This process of making connections between similar visuals will lead to greater proportional understanding and reasoning. It is essential that students develop referents in order to be effective at estimating.

For example, knowing how much 10 stars is, helps students to estimate the quantity in the larger group of stars.

Students must build on strategies developed in earlier grades involving ten to include a sense of one hundred. Using their knowledge of one hundred, students can then estimate larger quantities. For example, a bag of one hundred counters may be used to determine how many counters are in a larger pile, by estimating how many groups of 100 are in the pile.

N04.04 Students should be able to select an estimate for a given quantity by choosing from among three possible choices. For example, show students a collection of a certain object and have them choose the best estimate for the quantity from three possible choices, and record it. Objects such as beans, raisins, toothpicks, or wooden stir sticks could be used. After recording their estimates, students should begin counting the objects. As they count to certain benchmarks, allow students to change their estimates.

N04.05 Through the process of selecting and using referents, students will be able to justify a referent for determining an estimate for a given quantity. For example, when asked to determine the number of jellybeans in a jar, students select a useful visual referent, such as the number of jellybeans on the top layer, to determine a reasonable estimate.

SCO N05 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Record, in more than one way, the number represented by given proportional and non-proportional concrete materials in traditional and non-conventional formats.
N05.02 Represent a given number in different ways using proportional and non-proportional concrete materials and explain how they are equivalent; e.g., 351 can be represented as three 100s, five 10s, and one 1s; or two 100s, fifteen 10s and one 1s; or three 100s, four 10s, and eleven 1s.
N05.03 Record a given number in additive expanded form.
N05.04 Record a number represented by base-ten blocks arranged in a non-conventional format.

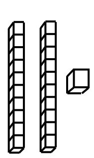
Performance Indicator Background

N05.01 Students should be given many opportunities to record numbers. When provided with a model representation, students need to be able to record numbers in more than one way for the given model. For example, a model or picture of 2 flats, 3 rods, and 4 small cubes can be recorded as, 234; 200, 30, and 4; or 2 hundreds, 3 tens, 4 ones.

It is important to spend time developing a good understanding of the meaning and use of zero in numbers. Students need many experiences using base-ten materials to model numbers with zeros as digits. Teachers should ask students to write the numerals for numbers such as three hundred forty and nine hundred eight. When you write a number in its symbolic form using digits, you call the digit 0 a place holder. If you did not have the digit 0, the number would be recorded as 32, and you would mistakenly think that the 3 represented 30 instead of 300. Students need many experiences using base-ten materials to make connections with the symbols for numbers with zeros as digits.

N05.02 Students must have a deep understanding of numbers up to 1000 and be able to rename numbers in a variety of ways. For example, 842 is the same as 84 tens and 2 ones; 8 hundreds and 42 ones; 8 hundreds, 4 tens, and 2 ones; or 7 hundreds, 14 tens, and 2 ones. Provide opportunities for students to represent each digit in a 3-digit number, using concrete materials, explaining the value of each digit.

Students should come to understand that the position of a digit determines its value. Students should recognize and work with the idea that the value of a digit varies, depending on its position or place, in a numeral. Another area of place value that may cause some confusion for students is that one number, such as 21, can be represented in a variety of ways.



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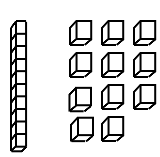

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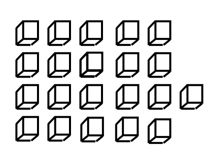


20 + 1 | 10 + 11 | 21 + 0

N05.03 Once students have ample opportunities with concrete, pictorial, and verbal representations of base-ten models, they can record the base-ten partitions in an expression such as 256 is 200 + 50 + 6. This is known as the additive expanded form.

SCO N06 Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals. [C, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Explain mental mathematics strategies that could be used to determine a sum.
- Ten and some more
- Tens and some more
- Quick addition
- Addition facts to 10 applied to multiples of 10
- Addition on the hundred chart
- Adding on
- Make ten
- Compensation
- Compatible numbers

N06.02 Use and describe a personal strategy for determining a sum.
N06.03 Determine a sum of two 2-digit numerals efficiently, using mental mathematics strategies.

Performance Indicator Background

N06.01 and N06.02 Students will develop, apply, and describe mental mathematics strategies to add two 2-digit numbers.

Ten and Some More
Most students know that teen numbers can be made with a ten and some more even with limited knowledge of place value. They need to experience finding sums with a 10 and a single-digit number to be convinced they do not need to count on: the answer is automatic. This strategy should be reinforced before fact-strategy learning is undertaken. For example, for 10 + 5, think: 10 and 5 more makes 15.

Tens and Some More
As students develop understanding of place value, they realize that the answers to the addition of single-digit numbers to multiples of 10 (20, 30, 40, …, 90) are as easy as adding single digits to 10. Answers can be quickly stated without finger counting or counting on. Through place-mat activities and ten-frame activities in regular classroom time, students should become convinced of this easy addition. Afterwards, these addition questions can be reinforced in mental mathematics time. For example, for 30 + 5, think: 30 and 5 more makes 35.

Quick Addition
Initially you should have students add 1-digit numbers to 2-digit numbers for questions that require no regrouping. Students need to be convinced that questions such as 32 + 7 and 74 + 5 are as easy as 2 + 7 and 4 + 5. Therefore, these questions should be modelled using ten-frames so students can see that 32 (3 full ten-frames and a 2) plus 7 just requires the addition of the 2 and 7, or that 74 (7 full ten-frames and a 4) plus 5 just requires the addition of 4 and 5. The single digit facts required are those with sums less than 10. For example, for 34 + 3, think: 4 and 3 is 7, so 30 and 7 more is 37.

Subsequently, they can apply “quick addition” as an addition strategy for solving combinations that require no regrouping and result in answers under 100. For example, for 56 + 23, think and record: 5 tens and 2 tens is 7 tens, and 6 and 3 is 9, so 70 and 9 more is 79.

Addition Facts to 10 Applied to Multiples of 10
Through modelling with small cubes and rods from the base-ten materials, students should be convinced that adding two sets of rods is no different than adding two sets of small cubes. For example, adding 4 rods and 2 rods results in 6 rods in the same way that adding 4 small cubes and 2 small cubes results in 6 small cubes. Therefore, when asked to find sums such as 20 + 30, 40 + 10, and 30 + 50, students should make the connections to the facts 2 + 3, 4 + 1, and 3 + 5.

You should restrict questions to combinations that result in sums to 100. Students would solve these questions by applying their knowledge of facts to 10. Since this is an extension of the facts (three-second response), a response time goal of five seconds would be reasonable for these questions. For example, for 50 + 20, think: 5 tens and 2 tens is 7 tens, or 70.

Addition on the Hundred Chart
Display a hundred chart, present students with addition questions involving two 2-digit numbers, ask them to visualize the additions on the chart, and record (or state) their answers. After many experiences, students may be able to visualize the hundred chart and do the addition completely in their heads.

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
20	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**
50	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**
60	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**
70	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**
80	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**

For example,
- for 45 + 21, think: Starting at 45, go down two spaces to 65 and then go 1 space to the right to 66. The answer is 66.
- for 34 + 63, think: Starting at 34, go down 6 spaces to 94 and then go 3 spaces to the right to get to 97; or Starting at 63, go down 3 spaces to 93 and then go 4 spaces to the right to get to 97. The answer is 97.

N06.03 The objective of this outcome is that students have flexibility with numbers and can efficiently add or subtract two 2-digit numbers mentally. They should have many experiences practising these mental mathematics strategies in a variety of contexts as well as in isolation. Students should become comfortable enough with these strategies that they spontaneously use them whenever appropriate during the time in mathematics, other subjects, and in their daily lives. When students are responding to these questions, it would be reasonable to expect a five- to seven-second response time for most students.

SCO N07 Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals. [C, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Explain mental mathematics strategies that could be used to determine a difference.
- Facts with minuends of 10 or less applied to multiples of 10
- Quick subtraction
- Subtraction on the hundred chart
- Compensation
- Back through ten

N07.02 Use and describe a personal strategy for determining a difference.

N07.03 Determine a difference of two 2-digit numerals efficiently, using mental mathematics strategies.

Performance Indicator Background

N07.01 and N07.02 Students will develop, apply, and describe mental mathematics strategies to subtract two 2-digit numbers.

Facts with Minuends of 10 or Less Applied to Multiples of 10
Through modelling with small cubes and rods from the base-ten materials, students should be convinced that subtracting two sets of rods is no different than subtracting two sets of small cubes. For example, subtracting 3 rods from 9 rods results in 6 rods in the same way that subtracting 3 small cubes from 9 small cubes results in 6 small cubes. Therefore, when asked to find differences such as 50 – 10, 40 – 20, and 90 – 50, students should make the connections to the facts 5 – 1, 4 – 2, and 9 – 5.

Questions should involve minuends of 20, 30, 40, …, 100 so students can solve the questions by applying their knowledge of subtraction facts to 10. A response time goal of 5 seconds is reasonable for these questions. For example, for 50 – 20, think: 5 tens minus 2 tens is 3 tens, or 30.

Quick Subtraction
This strategy is used when two 2-digit numbers are to be subtracted and there is no regrouping needed. Starting at the highest place value, simply subtract and record each place value's digits. For example, for 56 – 12, think about each place value difference: Starting at the front end, 5 tens minus 1 ten is 4 tens, 6 minus 2 is 4, so, 40 and 4 more is 44.

Because this strategy only applies to questions with no regrouping, students must examine each question as a whole to decide whether this strategy can be used. This habit of thinking needs to pervade all mental mathematics lessons. For example, present students with a list of 20 questions, some of which require regrouping, and direct students to apply quick subtraction to the appropriate questions, leaving out the ones for which this strategy cannot be used.

Subtraction on the Hundred Chart

This strategy involves actions on a hundred chart—where going straight up one space represents subtracting 10, and where going left one space represents subtracting 1. Display a hundred chart, present students with subtraction questions involving two 2-digit numbers, ask them to visualize the subtractions on the chart, and record (or state) their answers. After many experiences, students may be able to visualize the hundred chart and do the subtraction completely in their heads.

For example,

- for 45 – 21, think: Starting at 45, go up two spaces to 25 and then go 1 space to the left to 24. The answer is 24.
- for 74 – 65, think: Starting at 74, go up 6 spaces to 14 and then go 5 spaces to the left to get to 9. The answer is 9.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

N07.03 Students need flexibility with numbers to efficiently add or subtract two 2-digit numbers mentally. They should have many experiences practising these mental mathematics strategies in a variety of contexts as well as in isolation. Students should become comfortable enough with these strategies that they spontaneously use them whenever appropriate during the time in mathematics, other subjects, and in their daily lives.

SCO N08 Students will be expected to apply estimation strategies to predict sums and differences of 1-, 2-, or 3-digit numerals in a problem-solving context. [C, ME, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Explain estimation strategies that could be used to determine an approximate sum or difference.
N08.02 Use and describe a strategy for determining an estimate.
N08.03 Estimate the solution for a given story problem involving the sum or difference of up to two 3-digit numerals.

Performance Indicator Background

N08.01 and N08.02 The front-end estimation strategy involves adding or subtracting the values in the highest place-value position to get an estimate. These estimates provide answers that are close enough to judge pencil-and-paper and technology-generated answers. Because the other place values are not considered, the front-end estimates for addition questions will always be less than the actual answers. Therefore, you can always use the phrase “more than” in describing your estimate. For example, to estimate 213 + 347, think: 200 + 300 = 500, so the estimate is 500 and the answer must be more than 500. In subtraction questions, however, without considering the other place values, you can only use the word “about” in describing your estimate. For example, to estimate 423 – 145, think: 400 – 100 = 300, so the estimate is about 300.

The front-end adjusted strategy for addition estimation begins by getting a front-end estimate and then adjusting that estimate to get a better, or closer, estimate; by clustering all the values in the other place values to decide whether there would be enough together to account for an adjustment. For example, to estimate 337 + 545, think: 300 plus 500 is 800, but this can be adjusted by thinking 37 and 45 make almost another 100; so, the adjusted estimate would be 900.

N08.03 Students should be presented with contexts in which an estimate is all that is required. Story problems should be carefully constructed or selected so that students have opportunities to make decisions based on estimates rather than exact answers.

SCO N09 Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000 by ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically. [C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Model the addition of two or more given numbers using concrete or visual representations and record the process symbolically.
N09.02 Model the subtraction of two given numbers using concrete or visual representations and record the process symbolically.
N09.03 Create an addition or subtraction story problem for a given solution.
N09.04 Determine the sum of two given numbers using a personal strategy, e.g., for 326 + 48, record 300 + 60 + 14.
N09.05 Determine the difference of two given numbers using a personal strategy, e.g., for 127 – 38, record 2 + 80 + or 127 – 20 – 10 – 8.
N09.06 Solve a given problem involving the sum or difference of two given numbers.

Performance Indicator Background

N09.01 and N09.02 Students should be able to model the sum and difference of two given numbers up to 3-digits using base-ten blocks, and use symbols to record the processes that reflect their actions with those blocks. For example, to subtract 137 from 265, students may represent 137 with 1 flat, 3 rods, and 7 small cubes in one set; in another set they may place 1 flat saying 237, 2 rods saying 257, and 8 small cubes saying 265; determine that 128 is the total in the second set; and state the difference between 265 and 137 is 128. The students would record 137 + 100 + 20 + 8 = 265 and 100 + 20 + 8 = 128 to reflect the counting-up strategy they did with the blocks.

After students have modelled and solved a number of addition and subtraction situations, they may be introduced to strip diagrams as another way to represent the situations. For example, Bobby was given 63 green stamps. He already had 127 stamps. How many stamps does he have now? The strip diagram for this problem is as follows:

<table><tr><td>127</td><td>63</td></tr><tr><td colspan="2" align="center">?</td></tr></table>

As another example, Bobby had 98 stamps. He was given more by his friend. Then he had 137 stamps. How many stamps did his friend give him? The strip diagram for this problem is as follows:

<table><tr><td>98</td><td>?</td></tr><tr><td colspan="2" align="center">137</td></tr></table>

Because students have to decide where to place in the diagram the two given numbers in the story problem, they have to carefully read the problem to determine whether each given quantity is a part or a whole. If the quantity is a part, it would be placed in one section of the top rectangle; if the quantity is a whole it would be placed in the bottom rectangle. They should put a question mark in the bottom rectangle or one of the sections in the top rectangle, depending upon what is missing (what they are asked to find).

The principal use of strip diagrams is as a strategy to help students interpret story problems. Students will solve the problems using their personal strategies; however, through extensive use of strip diagrams, some students may generalize that subtraction is the operation that will always find a missing part and that addition will always find a missing whole.

N09.03 Students should be able to create story problems given an addition or subtraction number sentence. In order for their story problems to go beyond simple result-unknown types, they will need to have very specific experiences in which they create story problems similar to ones that are modelled. For example, students should be presented with four or five join (change unknown) story problems and, after they solve those problems, they should be asked to create a story problem similar to the join problems they were presented, but in a different context.

N09.04 and N09.05 It is expected that students will be able to symbolically add and subtract two 3-digit numbers using reliable and efficient strategies. Students should be able to explain their strategy and whether their solution is reasonable based on their prior estimate.

Examples of strategies and symbolic recordings for addition and subtraction are shown below.

If students are asked to add 237 and 478, students could determine the sum by doing the following:

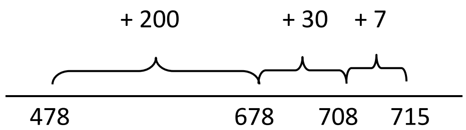
- Start by writing 237 as 200 + 30 + 7 and 478 as 400 + 70 + 8.
- Add 200 and 400 to get a sum of 600.
- Add 30 and 70 to get a sum of 100.
- Add 7 and 8 to get a sum of 15.
- Add 600, 100, and 15 to get a sum of 715.

This may be recorded on paper as follows:

237 + 478 = 200 + 30 + 7 + 400 + 70 + 8	or	$\begin{array}{r} 237 \\ +\ 478 \\ \hline 600 \\ 100 \\ +\ 15 \\ \hline 715 \end{array}$
200 + 400 = 600		
30 + 70 = 100		
7 + 8 = 15		
600 + 100 + 15 = 715		

- Start with the larger number 478.
- Add 200 to get a sum of 678.
- Add 30 to 678 to get a sum of 708.
- Add 7 to 708 to get a sum of 715.

This can be represented as jumps on a number line.



This may be recorded on paper as follows:

237 + 478
478 + 200 = 678
678 + 30 = 708
708 + 7 = 715

- Start by placing one addend below the other.
- Add 8 and 7 and place the sum of 15 below the two addends in line 1.
- Add 30 and 70 and place the sum of 100 in line 2.
- Add 200 and 400 and place the sum of 600 in line 3.
- Add the three lines to get a sum of 715.

This may be recorded on paper as follows:

$$\begin{array}{rl} 237 & \\ \underline{+\ 478} & \\ 15 & \text{(line 1)} \\ 100 & \text{(line 2)} \\ \underline{+\ 600} & \text{(line 3)} \\ 715 & \end{array}$$

This same strategy might also be recorded as follows:

237 + 478 = 7 + 8 + 30 + 70 + 200 + 400
7 + 8 = 15
30 + 70 = 100
200 + 400 = 600
15 + 100 + 600 = 715

- Start by decomposing 237 into 22 and 215.
- Combine the 22 with 478 to get a sum of 500.
- Add 500 and 215 to get a sum of 715.

This may be recorded on paper as follows:

237 + 47 = 22 + 215 + 478
478 + 22 = 500
500 + 215 = 715

- Start by adding 500 to 237 to get a sum of 737.
- Subtract 22 from 737 to get a difference of 715.

This may be recorded on paper as follows:

237 + 478
237 + 500 = 737
737 – 22 = 715
237 + 478 = 715

- Start by placing one addend below the other.
- Add 7 ones and 8 ones to get a sum of 15 ones.
- Regroup the 15 ones into 1 ten and 5 ones.
- Record a 1 in the tens place above the addends.
- Record a 5 in the ones place below the line.
- Add 3 tens, 7 tens, and 1 ten (from regrouping the ones) to get a sum of 11 tens.
- Regroup the 11 tens into 1 hundred (10 of the tens) and 1 ten.
- Record a 1 in the hundreds place above the addends.
- Record 1 in the tens place below the line.
- Add 2 hundreds, 4 hundreds, and 1 hundred (from the regrouping of the tens) to get a sum of 7 hundreds.
- Record a 7 in the hundreds place below the line.

This may be recorded on paper as follows:

$$\begin{array}{r} {\scriptstyle 1\,1\,\,} \\ 237 \\ \underline{+\ 478} \\ 715 \end{array}$$

If we introduce subtraction using word problems, students can begin modelling their solutions. Consider the following problem: On our vacation, we went to visit our aunt in Fredericton. We drove 239 km and stopped for lunch. If the distance to our aunt's house is 526 km, how much further do we have to drive?

Students could use base-ten blocks, number lines, or mental mathematics strategies to solve the problem in different ways. Possible solutions may include the following:

Group 1
We knew we had to subtract 239 from 526. So we started with 5 flats, 2 rods and 6 small cubes to show 526. We removed 2 flats. Then, we had to remove 3 rods, so we changed 1 flat to 10 rods. Finally we removed 9 small cubes, after we traded 1 rod for 10 small cubes.

This could be recorded on paper as follows:

526 – 239 = ?
526 – 200 = 326
326 – 30 = 296
296 – 9 = 287
We have to travel 287 km more.

Group 2
We started with 2 flats, 3 rods, and 9 small cubes to show 239. Then, we added 3 flats, but we knew that was too much because we had 539. So, we removed 9 small cubes, and we had 530. We still needed to remove 4 more small cubes to get to 526. So, we traded 1 rod for 10 small cubes, and removed the 4 small cubes.

This could be recorded on paper as follows:

239 + ? = 526
239 + 300 = 539
539 – 9 = 530
530 – 4 = 526
300 – 13 = 287
So, we know we have 287 km to drive.

Group 3
We used an empty number line. We put 239 and 526 on the line. We made a jump of 1 from 239 to 240. Next, we made a jump of 60 from 240 to 300. Then we made a jump of 200 from 300 to 500. Then we made a jump of 26, from 500 to 526. So, we combined all of our jumps, 1 + 60 + 200 + 26, to get 287. We have 287 km more to travel.

This could be recorded on paper as follows:

239 + ? = 526
239 + 1 = 240
240 + 60 = 300
300 + 200 = 500
500 + 26 + 526
1 + 60 + 200 + 26 = 287

Group 4
We started with 2 flats, 3 rods, and 1 small cube to show 239. We added 2 flats and had 439. We added 6 rods to get to 499. We added 1 small cube to get to 500. Then, we added on 2 rods and 6 small cubes to get to 526. So, we looked at everything we had added on (2 flats, 6 rods, 1 small cube, 2 rods, 6 small cubes) and knew that we had added on 287.

This could be recorded on paper as follows:

239 + ? = 526
239 + 200 = 429
439 + 60 = 499
499 + 1 = 500
500 + 26 = 526
200 + 60 + 1 + 26 = 287

Group 5
We knew we had to subtract 239 from 526. We decided to subtract 240 instead because it was easier to work with. So, we started at 526, jumped back 200 to 326. Then we jumped back 20 to 306, and then jumped back another 20 to 286. But we knew we had jumped back 1 too many and so we moved to 287.

This could be recorded on paper as follows:
526 – 200 = 326
326 – 20 = 306
306 – 20 = 286
286 + 1 = 287

Group 6
We wanted to subtract a friendly number. It would be nice to subtract 300. So, we changed 239 to 300 by adding on 61. Since we added 61 to 239, we had to add 61 to 526 to keep our constant difference. Then, we had a nice question to solve mentally, 587 – 300 = 287.

This could be recorded on paper as follows:

239 + 61 = 300
526 + 61 = 587
587 – 300 = 287
526 – 239 = 587 – 300 = 287

Regardless of the strategy used, the teacher must monitor each student's recording of the strategy to ensure that the recording is mathematically correct, organized, and efficient. For example, to solve 237 + 478, a student could accurately record his or her thinking as follows:

Method A
237 + 478 = 200 + 30 + 7 + 400 + 70 + 8
200 + 400 = 600
30 + 70 = 100
7 + 8 = 15
600 + 100 + 15 = 715

Method B

$$\begin{array}{r} 237 \\ \underline{+\ 478} \\ 600 \\ 100 \\ \underline{+\ \ 15} \\ 715 \end{array}$$

Method C
237 + 478 = 200 + 400 + 30 + 70 + 7 + 8 = 600 + 100 + 15 = 715

However, if a student recorded his or her thinking as
237 + 478 = 200 + 30 + 7 + 400 + 70 + 8 = 200 + 400 = 600 + 30 + 70 = 700 + 7 + 8 = 715,
it would be necessary to work with the student to correct the recording error. Correction would be necessary, as this is an example of the incorrect use of the equal sign. It may result from a student's misunderstanding of the meaning of the equal sign. One way to address this is to have students verify the accuracy of the recording by asking them to read the equal sign as "is the same as." In the example above, it is correct to say

- 237 + 478 is the same as 200 + 30 + 7 + 400 + 70 + 8
- 237 + 478 is the same as 600 + 100 + 15
- 237 + 478 is the same as 715
- 200 + 30 + 7 + 400 + 70 + 8 is the same as 600 + 100 + 15
- 200 + 30 + 7 + 400 + 70 + 8 is the same as 715
- 600 + 100 + 15 is the same as 715

However, it is incorrect to say

- 237 + 478 is the same as 200 + 400
- 237 + 478 is the same as 600 + 30 + 70

- 237 + 478 is the same as 200 + 400
- 200 + 30 + 7 + 400 + 70 + 8 is the same as 200 + 400, etc.

N09.06 Students should be able to solve story problems of different types by writing the most efficient open number sentences and computing the sums or differences to find the solutions. They should be able to do this either directly upon reading the problem, or by drawing or visualizing pictures that represent the problem.

SCO N10 Students will be expected to apply mental mathematics strategies and number properties, to develop quick recall of basic addition facts to 18 and related basic subtraction facts. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Describe a mental mathematics strategy that could be used to determine a given basic addition fact up to 9 + 9.
N10.02 Explain how the commutative (order-doesn't-matter) property and the identity (no-change-with-zero) property can assist in addition fact learning.
N10.03 Describe a mental mathematics strategy that could be used to determine a given basic subtraction fact with minuends up to 18 and subtrahends up to 9.
N10.04 Recognize which facts could be determined by a given strategy.
N10.05 Quickly recall basic addition facts to 18 and related subtraction facts in a variety of contexts.

Performance Indicator Background

N10.01 Students should be familiar with a mental strategy that can be used to quickly recall a given addition fact and be able to explain the strategy using that fact.

N10.02 Students should understand and use the commutative property of addition without an expectation that they know the word **commutative**. It is sufficient that they know the meaning; that is, the order in which numbers are combined in addition does not affect the answer. This knowledge is applied throughout fact learning. For example, 5 + 8 and 8 + 5 are both determined by making 8 a 10 and then adding 3, and 6 + 7 and 7 + 6 are both determined by doubling 6 and adding 1. There are 45 such pairings in the 100 addition facts.

Students should observe that the sum of a number and zero will always result in that number. It is the only number that will not result in a change in either addition or subtraction. If asked, What number can you add to 8 or subtract from 8 and get the same answer? students should realize that number is 0.

N10.03 Students should be familiar with a mental strategy that can be used to quickly recall a given subtraction fact and be able to explain the strategy using that fact. In addition, students should be able to state three other facts (one subtraction and two addition) that are related to this fact. If called on, they can apply the back-through-10 strategy to a fact such as 16 – 7. Similarly, students could apply the up-through-10 strategy to a fact such as 15 – 9.

N10.04 If students are presented with a set of facts, they should be able to identify the ones that could be found by a stated strategy, or sort the facts into groups that could be determined by the same strategy.

N10.05 The ultimate objective of this outcome is that students have quick recall of both addition and subtraction facts. Either the students use a mental strategy to determine the fact or have instant recall (they just know it). They should have many experiences practising these facts in a variety of contexts as well as in isolation. Students should understand that knowledge of addition and subtraction facts replaces counting strategies, and this fact knowledge should be applied all the time in mathematics, other subjects, and in their daily lives.

SCO N11 Students will be expected to demonstrate an understanding of multiplication to 5 × 5 by ▪ representing and explaining multiplication using equal grouping and arrays ▪ creating and solving problems in context that involves multiplication ▪ modelling multiplication using concrete and visual representations and recording the process symbolically ▪ relating multiplication to repeated addition ▪ relating multiplication to division [C, CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N11.01 Identify events from experience that can be described as multiplication.
N11.02 Represent a given story problem (orally, shared reading, written) using manipulatives or diagrams and record in a number sentence.
N11.03 Represent a given multiplication expression as repeated addition.
N11.04 Represent a given repeated addition as multiplication.
N11.05 Create and illustrate a story problem for a given number sentence and/or expression.
N11.06 Represent, concretely or pictorially, equal groups for a given number sentence.
N11.07 Represent a given multiplication expression using an array.
N11.08 Create an array to model the commutative property of multiplication.
N11.09 Relate multiplication to division by using arrays and writing related number sentences.
N11.10 Solve a given problem in context involving multiplication.

Performance Indicator Background

N11.01 Challenge students to think of real-life objects that come in equal groups, such as wheels on bicycles or tricycles, legs on chairs, legs on stools, leaves on three-leaf clovers, legs on animal, eggs in a carton, fingers on hands, and days in a week. Ask them to create situations using these objects that would be described as multiplication situations, such as the number of eyes in a group of 4 people, the number of wheels on 5 bicycles, and the number of shoes in 3 pairs.

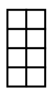
N11.02 The most meaningful way to apply and practice multiplication is in problem-solving contexts. Students should sometimes have story problems read to them, sometimes have them in print, and other times turn situations in story books into multiplication situations. No matter how the story problems are presented, students should model their solutions using concrete materials and/or pictures or diagrams from which they get solutions by skip counting the equal groups. Students will also be able to write the corresponding repeated addition number sentences and, eventually, the appropriate multiplication number sentences.

N11.03 and N11.04 Students should be able to translate any multiplication number expression into a repeated addition number expression. For example, given 5 × 4, students write 4 + 4 + 4 + 4 + 4 because they understand the meaning of the two factors in multiplication expressions. Interpreting the first factor as the number of groups cannot be overemphasized because of students' prior experiences with addition expressions in which both addends were countable quantities of objects. Similarly, students should be able to translate a repeated addition expression into a multiplication expression. For example, given 3 + 3 + 3 + 3 + 3, students can write this as 5 × 3.

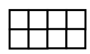
N11.05 Students should be able to create appropriate story problems that correspond to given multiplication number expressions. Having created the problems, they can draw pictures or diagrams that provide the solutions. For example, given 2 × 5, a student’s problem may be, Sam and Mary each had 5 pencils. How many pencils did they have altogether? The student may draw two stick people and show five pencils in each person’s right hand, writing 10 next to the picture.

N11.06 and N11.07 Students should be able to model concretely or pictorially a given multiplication number expression or sentence as equal groups and as an array. For example, given 5 × 2 = 10 or 2 × 5 = 10, students can show this as 5 groups with 2 counters in each group and as 2 rows with 5 counters in each row.

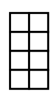
N11.08 The array is a powerful model to illustrate the order or commutative property in multiplication. For example, the first array below has 4 rows of 2 columns and therefore is a model for 4 × 2. The second is a 2 × 4 array. Both have an answer of 8.



$4 \times 2 = 2 \times 4$



N11.09 After students have been introduced to multiplication and division, they should understand that one array represents a multiplication sentence and two corresponding division sentences. For example, the array below is a representation for 4 × 2 = 8 and also a representation for eight divided into groups of two (8 ÷ 2 = 4) and for eight divided into four groups (8 ÷ 4 = 2).



N11.10 Students should have experiences solving equal-group story problems in contexts that are new to them (and perhaps novel as well) using a strategy of their own choosing. Some students may choose to model it with some concrete materials, other students may draw a picture or diagram, while still others may immediately write a number sentence. Seeing the same problem solved in a variety of ways is enriching for all students when the solution strategies are shared. If students solve the problem concretely or pictorially, encourage them to write a multiplication sentence to represent it.

SCO N12 Students will be expected to demonstrate an understanding of division by			
▪ representing and explaining division using equal sharing and equal grouping ▪ creating and solving problems in context that involves equal sharing and equal grouping ▪ modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically ▪ relating division to repeated subtraction ▪ relating division to multiplication. (Limited to division related to multiplication facts up to 5 × 5.) [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N12.01 Identify events from experience that can be described as equal sharing.
N12.02 Identify events from experience that can be described as equal grouping.
N12.03 Illustrate, with counters or a diagram, a given story problem involving equal sharing, presented orally or through shared reading, and solve the problem.
N12.04 Illustrate, with counters or a diagram, a given story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
N12.05 Listen to a story problem, represent the numbers using manipulatives or a diagram and record the problem with a number sentence and/or expression.
N12.06 Create and illustrate with counters, a story problem for a given number sentence and/or expression.
N12.07 Represent a given division sentence and/or expression as repeated subtraction.
N12.08 Represent a given repeated subtraction as a division sentence.
N12.09 Relate division to multiplication by using arrays and writing related number sentences.
N12.10 Solve a given problem involving division.

Performance Indicator Background

N12.01 and N12.02 Students should be encouraged to identify everyday experiences that would be equal-sharing situations and equal-grouping situations. For example, students should see that dividing the class into two groups, sharing 12 pieces of paper with 4 students, and sharing a large bag of candy into 3 small bags are all examples of equal-sharing situations. They should also see that dividing the class into groups of 5, giving each student 4 pencils, and placing books into stacks of 4 are all examples of equal-grouping situations.

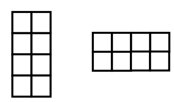
N12.03 and N12.04 The most meaningful way to apply and practise division is in problem-solving contexts. Students should sometimes have story problems read to them, sometimes have them in print, and other times turn situations in story books into division situations. No matter how the story problems are presented, students should model their solutions using concrete materials and/or pictures or diagrams, from which they get solutions. Story problems should be of the two types—equal sharing and equal grouping.

N12.05 Students should be able to interpret a division story problem (equal sharing or equal grouping) that is read three times, model that problem with concrete materials or diagrams, and write the division sentence that represents the problem and its solution.

N12.06 Students should be able to create appropriate story problems that correspond to given division number expressions. Having created the problems, they can draw pictures or diagrams that provide the solutions. For example, given 12 ÷ 4, a student's problem may be, Sam had 12 pencils and shared them with 4 friends. How many pencils did each friend get? The student may draw 4 stick people and draw pencils, one at a time, in each person's right hand, until the 12 are all shared and each person has 3 pencils. The student would write 12 ÷ 4 = 3 to represent this story and its solution.

N12.07 and N12.08 Students should be able to translate repeated subtraction sentences into a division sentence, and to translate a division sentence into a repeated subtraction sentence. For example, given 8 – 2 – 2 – 2 – 2 = 0, students should write 8 ÷ 2 = 4; given 12 ÷ 3 = 4, students should write 12 – 3 – 3 – 3 – 3 = 0. For 12 ÷ 3 = 4, students should think "How many 3s can be subtracted from 12 to get to 0."

N12.09 After students have been introduced to multiplication and division, they should understand that one array represents a multiplication sentence and two corresponding division sentences. For example, the first array below has 4 rows of 2 columns and therefore is a model for 4 × 2 = 8. The second is a 2 × 4 array. Both have an answer of 8.



N12.10 Students should have experiences solving division story problems in contexts that are new to them (and perhaps novel as well) using a strategy of their own choosing. Some students may choose to model it with some concrete material, other students may draw a picture or diagram, while still others may immediately write a number sentence. Seeing the same problem solved in a variety of ways is enriching for all students when the solution strategies are shared. If students solve the problem concretely or pictorially, they should be encouraged to write a division sentence that would represent the problem and its solution.

SCO N13 Students will be expected to demonstrate an understanding of fractions by ▪ explaining that a fraction represents a part of a whole ▪ describing situations in which fractions are used ▪ comparing fractions of the same whole with like denominators [C, CN, ME, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N13.01 Describe everyday situations where fractions are used.
N13.02 Represent a given fraction concretely or pictorially.
N13.03 Identify, model, and explain the meaning of numerator and denominator.
N13.04 Sort a given set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.
N13.05 Name and record the fraction represented by the shaded and non-shaded parts of a given region.
N13.06 Compare given fractions with the same denominator using models.

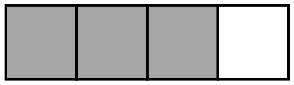
Performance Indicator Background

N13.01 It is likely that students will have heard fractional language in their everyday life, such as half an apple, half moon, one-third cup of flour, etc. Students will benefit from learning about fractions concretely, and in the context of real life.

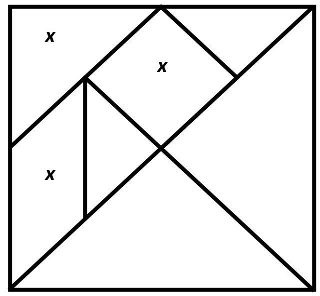
As students are exposed to fractional language in meaningful contexts, they will begin to develop an understanding of fractions. Students in Mathematics 3 should be given the opportunity to discuss simple unit fractions in the context of division stories of regional models, for example, when a pizza is shared by three people, each person has one-third of the original amount of the pizza.

It is also helpful to recognize, when examining a situation involving a regional model, if one-third of a pizza is eaten, then two-thirds of that pizza remains. Informal experiences will help students see that when wholes are divided into a greater number of fair shares, the shares are smaller. This will help later when comparing fractions.

N13.02 and 13.05 Part of a whole—This is when one unit is partitioned into equal parts. The sharing of an apple, or a piece of paper is commonplace to students. The more opportunities they have to partition fairly, the better their visual concept will be for fractions. The emphasis should be on equal parts or fair shares. Students should understand that while the parts are equal in area, they do not need to be identical in shape; this can be a misconception. A tangram set demonstrates this idea clearly where the square, the medium-sized triangle, and the parallelogram all have equivalent area but are not identical in shape. It is important that the representation of the whole, one whole or one, is clear so students understand which region they are taking apart; this concept is essential for comparing fractions.



$\frac{3}{4}$ of the bar is shaded.



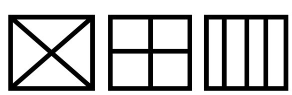
The parallelogram, the square, and the medium-sized triangle each represent $\frac{1}{8}$ of the whole region.

In Mathematics 3, students will be introduced to, and explore, the parts of a whole that results when the whole has been divided into equal-sized portions or “fair shares.” It is important to use the terms **whole**, **one whole**, or **one** to ensure that students have a common language to use regardless of the model used.

Teachers should model the use of language such as “1 of 3 equal parts” and help students connect the language with its symbol $\frac{1}{3}$. Point out to students that $\frac{1}{4}$ should be read “one-fourth” rather than “one-quarter.”

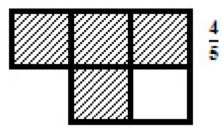
The use of models and concrete representations is essential to their understanding. It is important to use a wide variety of models so that fractions do not simply become apple pieces, granola bars, or pizza slices. Fractional concepts can be strengthened by linking other areas of mathematics such as geometry, money, number, and patterning. It is important that students develop visual images for fractions and be able to tell “about how much” a particular fraction represents and have exposure to common benchmarks, such as one-half. These areas play a key role in consolidating a student’s conceptual understanding of fractions.

Students should see that there are many ways to make the same fractional part. Using pattern blocks where the hexagon is designated as the whole, students could find how many different ways they can make $\frac{1}{2}$, $\frac{1}{3}$, etc. Or using a square, find how many different ways to make $\frac{1}{4}$. This can help with the understanding of equivalence.



Students need to see and explore a variety of models of fractions with a key focus on halves, thirds, fourths, fifths, sixths, eighths, and tenths. Initially, students explore unit fractions such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ before moving onto other proper fractions such as $\frac{2}{3}$, $\frac{5}{8}$, and so on. Beginning with fractional terms such as halves, thirds, fourths, etc., and their pictorial representations, provides a bridge to the more challenging concept of the symbolic representations.

N13.03 The meaning of the numerator (top number) and the denominator (bottom number) needs to be emphasized as these can be misleading to students. This is best accomplished by introducing and focusing on the numerator or denominator separately and using visual models linked to the symbols. For example, ask students to use grid paper to represent a fraction as in the picture below. Ask them to name the fraction ($\frac{4}{5}$), and identify and explain the meaning of numerator and denominator. The 4 is the numerator because it tells how many parts of the shape are shaded. The 5 is the denominator because it tells how many equal parts the whole shape is divided into.



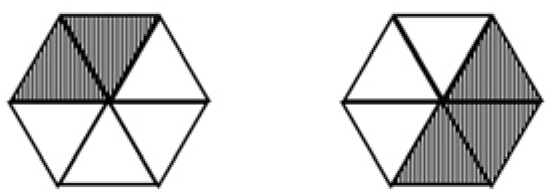
N13.04 It is important that students see and represent non-examples of the area model for fractions as with the rectangle or other shapes.

N13.06 Discuss with students the fact that if two fractions have the same denominator, the fraction with the greater numerator represents the larger piece of the whole. If the denominators of two fractions from the same whole are the same, then the parts are the same.

Students will compare fractions with the same denominator. Students will use language and/or pictures to indicate which fraction is greater than or less than the other. Pattern blocks can be used to demonstrate this concept. For example, the hexagon can be used to represent a pizza and the triangles to represent the slices.

Ask the students, If John ate $\frac{2}{6}$ of the pizza and Gina ate $\frac{3}{6}$ of the pizza, who ate the most pizza?

Ask students to model their answers using the pattern blocks and record the fraction symbols showing which is greater and which is less. Discuss with students how they know.



The shaded part of the first picture ($\frac{2}{6}$) is less than the shaded part of the second ($\frac{3}{6}$).

Patterns and Relations (PR)

SCO PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

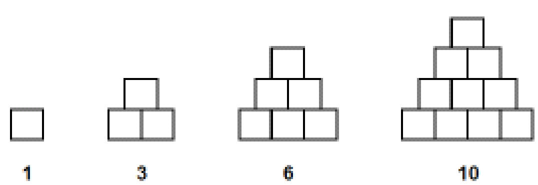
Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Identify and describe increasing patterns.
PR01.02 Describe a given increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
PR01.03 Extend a pattern, using the pattern rule, for the next three terms.
PR01.04 Compare numeric patterns.
PR01.05 Identify and explain errors in a given increasing pattern.
PR01.06 Create a concrete, pictorial, or symbolic representation of an increasing pattern for a given pattern rule.
PR01.07 Create a concrete, pictorial, or symbolic increasing pattern and describe the pattern rule.
PR01.08 Solve a given problem using increasing patterns.
PR01.09 Identify and describe the strategy used to determine a missing term in a given increasing pattern.
PR01.10 Use ordinal numbers (to 100th) to refer to or to predict terms within an increasing pattern.

Performance Indicator Background

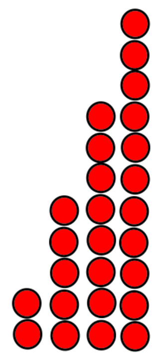
PR01.01 and **PR01.10** Students should be able to describe an increasing pattern. An increasing pattern is a growing pattern where the size of the term increases in a predictable way. The terms in an increasing pattern grow by either a constant amount or by an increasing amount each time. Students need sufficient time to explore increasing patterns through various manipulatives, such as Cube-A-Links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, or buttons, to realize they increase in a predictable way. As students describe increasing shape patterns, help them recognize that each term has a numeric value.



A hundred chart is a valuable tool to use with students when exploring increasing patterns. Students should be able to locate and describe various increasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns. For example, when skip counting by 3, use only starting points that are multiples of 3 (3, 6, 9, 12, ...) and this will result in a diagonal representation on a hundred chart. Skip counting with 5, starting at 0, the pattern is two vertical columns with numbers ending in the digits 5 and 0. Students should also explore hundred charts to 1000 (1–100, 101–200, 201–300, …) and look for patterns when counting by 2s, 5s, 10s, 25s, and 100s.

A counting sequence is an increasing pattern where each number represents a term in the pattern. For example, in the counting sequence 1, 2, 3, 4, …, 1 represents the first term, 2 the second term, 3 the third term … This counting sequence can be connected to ordinal numbers where students should be able to recognize that the 34th term is 34 and that 57 is the 57th term. These ordinal number patterns should be investigated for numbers up to 100.

PR01.02 Students should be able to describe a given increasing pattern by stating the pattern rule. A pattern rule tells how to make the pattern and can be used to extend an increasing pattern. Give students the first three or four terms of an increasing pattern. Ask them to state the pattern rule by identifying the term that represents the starting point and describing how the pattern continues. For example, in the pattern below, the pattern rule is, start with 2 counters and add 3 counters each time.



As students describe concrete or pictorial patterns, help them recognize that each term has a numeric value. For example, the above pattern can be expressed as, 2, 5, 8, 11, … by counting the number of counters in each term. Students may also find it useful to record the change from one number to the next as shown below:

2 5 8 11 ____ ____ ____

+3 +3 +3

PR01.03 Students should be able to extend a pattern by identifying the rule, and use the rule to build and draw the next three terms. Initially, students should replicate the first three terms with concrete materials and then extend the pattern. The use of the concrete materials allows them to make changes if necessary and to build onto one term to make the next term. Students should be able to explain why their extension follows the pattern. It is important to note that for some patterns, there may be more than one way to extend the pattern. For example, If only one term is given, such as the third term 12, some possible solutions could be
4, 8, 12, 16, …
3, 7, 12, 18, …
2, 6, 12, 20, …
6, 9, 12, 15, …

PR01.04 Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing increasing patterns, compare the starting points and how each term increases. For example, one way students may address this is by using a page with four small hundred charts. Ask them to skip count and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart comparing the starting points and the amount of increases.

PR01.05 Students should be provided with a variety of increasing patterns that contain errors, and be able to identify and explain the errors. For example, given the pattern 3, 7, 11, 15, 19, 23, 26, 31, 35, 39, students should state the pattern rule: Start at 3 and add 4 each time. Therefore, 26 is an error since it is only adding on 3 not 4. A second error is 31 since it is adding 5 and not 4. To help students visualize this pattern, they can shade the numbers on a hundred chart and look for the mistakes. Students may see that 26 and 31 do not fit the number pattern; therefore, they are errors.

PR01.06 Students should be able to create various representations of an increasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

PR01.07 Students should be able to create increasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating increasing patterns, initially students need to choose a starting point and then decide on the amount of increase. The amount of increase may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

PR01.08 Students should have frequent experiences with solving real-world problems that interest and challenge them using increasing patterns. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

PR01.09 Students should be able to identify and describe the strategy used to determine a missing term in a given increasing pattern. Since patterns increase in a predictable way, to determine a missing term, students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies may include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

SCO PR02 Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

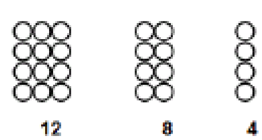
PR02.01 Identify and describe decreasing patterns.
PR02.02 Describe a given decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
PR02.03 Extend a pattern using the pattern rule for the next three terms.
PR02.04 Compare numeric patterns.
PR02.05 Identify and explain errors in a given decreasing pattern.
PR02.06 Create a concrete, pictorial, or symbolic representation of a decreasing pattern for a given pattern rule.
PR02.07 Create a concrete, pictorial, or symbolic decreasing pattern and describe the pattern rule.
PR02.08 Solve a given problem using decreasing patterns.
PR02.09 Identify and describe the strategy used to determine a missing term in a given decreasing pattern.
PR02.10 Use ordinal numbers (to 100th) to refer to or to predict terms within a decreasing pattern.

Performance Indicator Background

PR02.01 and **PR02.10** Students should be able to describe a decreasing pattern. A decreasing pattern is a shrinking pattern where the size of the term decreases in a predictable way. The terms in a decreasing pattern shrink by either a constant amount or by an increasing amount each time. Students need sufficient time to explore decreasing patterns through various manipulatives, such as Cube-A-Links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, and buttons. Sometimes students are more comfortable during the exploration stage if they can experiment first, using manipulatives, then pictures, and eventually numbers.

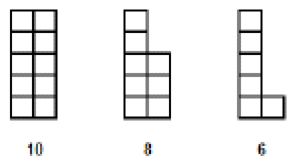
As students begin to investigate patterns, they sometimes confuse repeating patterns with decreasing patterns. Remind them to look for a core first. If they cannot find a core, then the pattern is not a repeating pattern.

Earlier, students became familiar with assigning a numeric value to each element in an increasing pattern. This expectation also applies to decreasing patterns.



Students should be able to identify and describe various decreasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns. This can be connected to skip counting in outcome N01. Provide copies of hundred charts. Ask students to begin with 100 and skip count backward, shading in the number for each count all the way to 1. Then they write a description of the pattern. For example, if they chose 5, the pattern is two vertical columns, with numbers ending in the digits 5 or 0.

PR02.02 Students should be able to describe a given decreasing pattern by stating the pattern rule. A pattern rule includes a term representing a starting point and a description of how the pattern continues. A pattern rule tells how to make the pattern and can be used to extend a pattern. For example, in the pattern below, the pattern rule is to start with 10 squares and decrease by 2 squares each time.



As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 10, 8, 6, … by counting the number of squares in each term. Students may also find it useful to record the change from one term to the next as shown below:

10 8 6 ____ ____ ____

-2 -2 -2 -2 -2

Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 10, 8, 6, ... as a decrease by 2 pattern without indicating that it starts at 10, the pattern rule is incomplete.

PR02.03 Students should be able to extend a pattern by identifying the rule, and use the rule to build and draw the next three terms. Students should replicate the first three terms with concrete materials and then extend the pattern. The use of the concrete materials allows them to make changes if necessary and to build onto one term to make the next term. Students should be able to explain why their extension follows the pattern. It is important to note that for some patterns, there may be more than one way to extend the pattern and this is quite acceptable.

PR02.04 Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing decreasing patterns, compare the starting points and how each term decreases using a variety of representations such as shape patterns, hundred charts, and number patterns. For example, give students a page with four small hundred charts. Ask them to skip count backward and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart indicating the starting point and the amount of decrease.

PR02.05 Students should be provided with a variety of decreasing patterns that contain errors and be able to identify and explain the errors. For example, given the pattern 89, 86, 83, 80, 77, 75, 71, …, they would state the pattern rule: Start at 89 and subtract 3 each time. Therefore, 75 is an error since it is only subtracting 2 not 3. A second error is 71 since it is subtracting 4 and not 3. To help students visualize this pattern they can shade the numbers on a hundred chart and look for the mistakes. Students may see that there are fewer than three numbers between 77 and 75 and more than three numbers between 75 and 71, therefore, it is an error.

PR02.06 Students should be able to create various representations of a decreasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

PR02.07 Students should be able to create decreasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating decreasing patterns, initially students need to choose a starting point and then decide on the amount of the decrease. The amount of decrease may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

PR02.08 Students should have frequent experiences with solving real-world problems that interest and challenge them using decreasing patterns. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

PR02.09 Students should be able to identify and describe the strategy used to determine a missing term in a given decreasing pattern. Since patterns decrease in a predictable way, to determine a missing term the students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies may include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

SCO PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Explain the purpose of the symbol in a given addition and in a given subtraction equation with one unknown.
PR03.02 Create an addition or subtraction equation with one unknown to represent a given combination or separate action.
PR03.03 Provide an alternative symbol for the unknown in a given addition or subtraction equation.
PR03.04 Solve a given addition or subtraction equation that represents combining or separating actions with one unknown using manipulatives.
PR03.05 Solve a given addition or subtraction equation with one unknown using a variety of strategies including guess and check.
PR03.06 Explain why the unknown in a given addition or subtraction equation has only one value.

Performance Indicator Background

PR03.01 The unknown value in an equation can be shown using a variety of symbols, such as a circle, a triangle, and an open box. It is important that a variety of symbols are used so students do not develop the misconception that an unknown can only be represented by one of these symbols. Students should be able to explain that in the equation 15 + Δ = 18, the triangle symbol represents the missing part, 3.

PR03.02 Students are expected to create an addition or subtraction equation with one unknown to represent a given combination or separation action. For example, when students are presented with the following story problem they are expected to create an addition or subtraction equation. Mary has 24 stickers on a roll and gives 10 stickers to her friend Betty. How many stickers are left on the roll? One student wrote, 24 – 10 = ? as his number sentence to represent the problem. Another student wrote, 24 – ? = 10 as his number sentence. Have several students explain the reasoning for each number sentence.

PR03.03 When students create equations, they should know that they can use different symbols to represent the unknown to help solve story problems. For example, Josh has some marbles and he bought 12 more. Now he has 33 marbles. How many marbles did he have at the start? This can be represented with the equation: ◊ + 12 = 33 or ? + 12 = 33.

PR03.04 and PR03.05 To solve given addition and subtraction problems, students should initially use manipulatives. For example, Ms. Best needs 18 pieces of construction paper for art class. She has 7 pieces, how many more pieces of construction paper does she need? Therefore, the equation for this problem would be 18 – 7 = ? or 7 + ? = 18. Students may use counters to model the problem. Observe to see if students start with 18 counters and separate 7 from the group to find the unknown or if they start with 7 counters and add up to 18.

To solve addition or subtraction equations with one unknown, students need to explore and explain different strategies. Some examples of strategies may include, but are not limited to, the following:

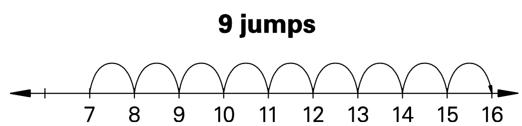
Guess and Check Strategy: This strategy is based on trying different numbers. The key is to think after each try and change or revise the guess when necessary. For example, 7+ Δ = 16.

- Think 7 + 7 = 14, that is too low.
- Think 7 + 8 = 15, that is too low but closer to 16.
- Think 7 + 9 = 16. So the missing number is 9.

Mental Mathematics Strategy: For example, 7+ Δ = 16.

- Think doubles. I know 7 + 7 = 14, and 14 is only 2 away from 16, so the missing number must be 9.

Number Line Strategy: Create a number line with the start point being 7. Then count up to 16, keeping track of the jumps on the number line.



It is important that students read and solve equations when the unknown number is on either the left side or the right side of the equal sign.

PR03.06 Students should be able to explain why the unknown in a given addition or subtraction equation has only one value. Present students with an equation such as 8 + Δ = 17. Starting with 8 counters on the table, secretly place 9 counters under a cup. Ask students to tell you how many you put under the cup by viewing how many more are needed to make 17. Once the students have determined that there are 9 counters under the cup, ask if there could be any other answer. Could the number be anything else? How do you know? Listen carefully to students' reasoning and explanations. This will provide insight as to how the students are thinking.

Measurement (M)

SCO M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice.
M01.02 Identify activities that can or cannot be accomplished in minutes, hours, days, weeks, months, and years.
M01.03 Provide personal referents for minutes and hours.
M01.04 Select and use a standard unit of measure, such as minutes, hours, days, weeks, and months to measure the passage of time and explain the choice.

Performance Indicator Background

M01.01 Initially, it is essential that students are able to choose non-standard units, such as pendulum swings, television shows, sand-timers, or recesses that measure the passage of time in a uniform and appropriate manner. Hand claps do not always ensure uniformity of time from person to person. Ask students to select and justify an appropriate non-standard unit to estimate how long it would take them to do activities such as tying their laces, writing their names, walking down the hall, or going to a movie.

M01.02 Students will explore the concept of the passage of time and use time vocabulary such as minutes, hours, days, weeks, months, and years. It is useful to discuss throughout the day the duration of long and short events to develop a sense of the various standard units of time. Students will identify activities that can or cannot be completed in a given amount of time. For example, ask students to describe something they do that takes a minute, an hour, a week, a month, or a year.

Engage students in daily conversations whereby they need to select an appropriate unit of measurement for activities such as brushing their teeth, walking to school, reading a story, extracurricular activities, sleeping, summer vacation, or building a highway. Ask questions such as, Would it take hours or minutes to tie your shoes? Can a house be built in days, weeks, or months? Do we measure the growth of trees by days or years?

M01.03 Personal referents, such as the length of a favourite television show or how long it takes to properly wash their hands, allow students to better estimate time. Students can also use activities with known durations as referents for estimating the duration of other activities. For example, I know that it takes one hour to watch my favourite TV show, which is about the same time it takes to read a chapter in my book.

SCO M02 Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M02.01 Determine the number of days in any given month using a calendar.
M02.02 Solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, the number of hours in a day, or the number of days in a given month.
M02.03 Create a calendar that includes days of the week, dates, and personal events.

Performance Indicator Background

M02.01 Using a calendar throughout the school year strengthens the students' sense of time. Each month brings a new calendar page to explore. As students are examining the calendar to determine the number of days in any given month, some students may find it easy to remember, using the jingle, Thirty days hath September, April, June, and November. All the rest have 31, … Others may enjoy the knuckle method for remembering the number of days in each month. Make a fist showing four knuckles, start by pointing to the first knuckle and saying, January. The space between knuckles is February, the second knuckle is March, and so on. After saying, July, go back to the beginning making August land on the first knuckle and continue until year end. The months that land on the knuckles each has 31 days.

M02.02 Before engaging in problem-solving activities, students need to consolidate their understanding of number of seconds in a minute and minutes in an hour. Some examples follow.

- Direct students' attention to the analog clock. How many big numbers are on the clock? Ask students to point to the hour hand, tell them that when the hour hand moves from one number to the next, one hour has passed, or sixty minutes. Ask students to point to the minute hand, tell them that when the minute hand moves from one tick mark to the next, one minute has passed, or sixty seconds.
- Count the seconds aloud with the students to verify it takes 60 seconds for the minute hand to move from one tick mark to the next on an analog clock.
- Challenge students to estimate how long one minute is by having them place their heads on their desks. When they think one minute is up, they should raise their hand without looking up. At the end of one minute, tell the students that one minute is up.

M02.03 In the real world, the calendar is used to plan, keep track of appointments, and measure time. This is how it should be used in the classroom. In order to focus on the structure of the month and numerical patterns, have students build and create their own monthly calendar. They will need to write the months and the days of the week in order, number the days, and fill in any special dates for that month, such as birthdays, class trips, and physical education days. Having a one page, year-long calendar nearby will help students see and understand where the current month fits into a year's progression. Let students take their calendar home for scheduling personal activities.

SCO M03 Students will be expected to demonstrate an understanding of measuring length (cm, m) by ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M03.01 Provide a personal referent for one centimetre and explain the choice.
M03.02 Provide a personal referent for one metre and explain the choice.
M03.03 Match a given standard unit to a given referent.
M03.04 Show that 100 centimetres is equivalent to 1 metre by using concrete materials.
M03.05 Estimate the length of an object using personal referents.
M03.06 Determine and record the length and width of a given 2-D shape.
M03.07 Determine and record the length, width or height of a given 3-D object.
M03.08 Draw a line segment of a given length using a ruler.
M03.09 Sketch a line segment of a given length without using a ruler.

Performance Indicator Background

M03.01 and M03.02 When introducing centimetres and metres, it is important for students to have the opportunity to discover personal referents for these standard units of length. Students should think about how they could tell if something is about 1 cm or 1 m long if they did not have a ruler or a metre stick. They should identify and explain why the width of their finger is a personal referent for 1 cm and why the height of a doorknob from the floor is a personal referent for 1 m. Having these personal referents helps students visualize measurements and estimate more accurately. Personal referents also make the units easier for students to remember.

M03.03 Students should identify objects from around the classroom, that would be an appropriate referent for a centimetre or a metre; for example, a pencil, a garbage can, a teacher desk, or a glue stick.

M03.04 Students should recognize that a metre is 100 centimetres long. Although many metre sticks are marked up to 100, it is often still not clear to students. Working in groups with base-ten materials, students need to explore how many small cubes would line up along a metre stick to consolidate their understanding of the equivalence of 100 cm to 1 m.

M03.05 Students need to estimate the length of an object using personal referents. Students should find items in the classroom that are close to given centimetre or metre lengths using their personal referents as a measurement.

M03.06 This is the first year where students will begin to use a standard tool to measure length. It is valuable to initially use simpler rulers that are created by the students. Then move on to tools that are easy for students to read. Students should use rulers (or the side of the ruler) that show only numbered centimetres and not millimetres.

It is important for students to line up the 0 mark with one end of the shape being measured. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Lining up small cubes from base-ten materials along the ruler will demonstrate that the numbers on the ruler correspond to the number of small cubes, starting at 0.

It is also important to observe how students use a ruler to measure a shape that is longer than the ruler. Show students how to measure something that is longer than a ruler by marking, recording, and starting again.

M03.07 Using a centimetre ruler, students should measure the length, width, or height of a given 3-D object in the classroom, such as a lunch box, their desk, or a cereal box. Students can record their measurements using both the number and the measurement unit; for example, 3 cm or 3 centimetres. Ensure students are clear about the distance they should be measuring.

M03.08 Before creating a specific length, students should practise drawing a variety of straight lines, such as drawing a triangle, rectangle, or house. Once they establish comfort with the ruler, students should draw lines of given lengths. Students need to attend to the starting point when drawing a line segment of a given length.

M03.09 Once students have ample opportunities to measure and draw line segments of a particular length, they should be able to draw a line segment that is about a certain measure using a straight edge but not a ruler. Students should develop personal referents to aid them in this skill. For example, my thumb is 1 cm wide, and my hand-span is 15 cm wide. Students should sketch line segments of 6 cm, 10 cm, and 20 cm, and discuss how the use of their personal referent could help them with this activity.

SCO M04 Students will be expected to demonstrate an understanding of measuring mass (g, kg) by ▪ selecting and justifying referents for the units grams and kilograms (g, kg) ▪ modelling and describing the relationship between the units grams and kilograms (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M04.01 Provide a personal referent for one gram and explain the choice.
M04.02 Provide a personal referent for one kilogram and explain the choice.
M04.03 Match a given standard unit to a given referent.
M04.04 Explain the relationship between 1000 grams and 1 kilogram using a model.
M04.05 Estimate the mass of a given object using personal referents.
M04.06 Measure, using a balance scale, and record the mass of given everyday objects using the units gram (g) and kilogram (kg).
M04.07 Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg.
M04.08 Determine the mass of two given similar objects with different masses and explain the results.
M04.09 Determine the mass of an object, change its shape, re-measure its mass and explain the results.

Performance Indicator Background

M04.01 Using their understanding of a kilogram, ask students to brainstorm items that may have a mass of 1 gram. They may also use a small base-ten cube as a personal referent of a gram. You may wish to provide students with items such as a raisin, bean seed, jelly bean, or a paper clip, to conceptualize the sense of how a gram feels.

M04.02 It would be beneficial for students to have an opportunity to make a kilogram mass of their own. Provide students with materials such as sand, flour, sugar, and small cubes from base-ten materials to fill a container until it exactly balances with a 1 kg mass on a balance scale. Using this kilogram container they can now compare its mass to items in the classroom to help them find a personal referent for 1 kg.

M04.03 Using objects from the classroom, for example a counter, a raisin, a paper clip, a textbook, a sneaker, or a lunch box, ask students to identify whether the object is an appropriate referent for grams or kilograms.

M04.04 It is important for students to know that 1000 grams is equal to a kilogram. Using food items of various benchmark masses, such as 2 bags of 500 g, 4 boxes of 250 g, or you may wish to have a pre-counted bag of 1000 jellybeans, model how 1000 g is equal to 1 kg using a balance scale.

M04.05 Estimating mass is more difficult than estimating other measures, as the object's size and shape is not directly related to its mass. Once students have established a personal referent for 1 g and 1 kg, they can now use their referents to estimate the mass of common objects such as an eraser, an apple, a juice box, or a textbook, or to estimate whether an object is heavier or lighter than 1 kg.

M04.06 Model how a balance scale can be used to determine the mass of everyday objects. Provide a variety of objects for students to use as they explore measuring mass. Students can record their measurements using both the number and the measurement unit, for example 3 kg or 3 kilograms.

M04.07 Students need many opportunities to find examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg. With the mass of a gram being so small, it is important to provide students with opportunities to work with masses of varying benchmark sizes in an effort to develop a conceptual understanding for working with grams and kilograms. For example, a shoelace has a mass of 1 g, a nickel has a mass of about 5 g, a rod has a mass of about 10 g, 10 rods could be used to show 100 g, or 10 flats could be used to show 1 kg.

M04.08 Determine and record the mass of two similar items such as a Ping-Pong ball and a golf ball. Ask students to explain why two objects that appear to be so similar can have different masses. For example, a Ping-Pong ball is made of lighter material, is hollow, and intended to move short distances, whereas a golf ball is made of heavier material, is solid, and intended to travel long distances. Measuring and comparing similar items with different masses will help students understand the necessity for using the same unit of measurement when comparing the amount of matter those objects contain.

M04.09 Using manipulatives such as multilink cubes, ask students to create a shape and measure its mass. Then ask them to change the shape of their creation using the exact same material and measure its mass again. Having opportunities to compare objects that have been rearranged will strengthen the understanding that the same object rearranged will maintain its original mass.

SCO M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by ▪ estimating perimeter using referents for centimetre or metre (cm, m) ▪ measuring and recording perimeter (cm, m) ▪ create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter [C, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M05.01 Measure and record the perimeter of a given regular shape and explain the strategy used.
M05.02 Measure and record the perimeter of a given irregular or composite shape and explain the strategy used.
M05.03 Construct a shape for a given perimeter (cm, m).
M05.04 Construct or draw more than one shape for the same given perimeter.
M05.05 Estimate the perimeter of a given shape (cm, m) using personal referents.

Performance Indicator Background

M05.01 Students should find the perimeter of many different regular-, irregular-, and composite-shaped objects, before being introduced to pictorial forms. Students should explore their own methods for determining the perimeter of a shape and should not develop or follow a formula for calculating perimeter. For example, provide students with various regular and irregular polygons (e.g., squares, rectangles, and triangles), some string, and a ruler. Small groups of students should be asked to find the perimeter in a variety of ways. Some may use the string, while others go directly to measuring the sides with the ruler.

Metres are used when measuring the perimeter of large shapes such as a window, a door, or a room. Discuss with students possible strategies for determining the perimeter of the classroom. Students should explore and record the measurements as they go. Students need to be able to explain the strategies they used for finding perimeter as they proceed. Ask students what number sentence could be used to find the perimeter.

M05.02 Pentominoes may be used to illustrate measuring and recording the perimeter of a given composite shape. Pentominoes are shapes each made up of five squares, all of which must have at least one side matching up with the side of another. In addition to composite-shaped objects with straight sides, it is important to expose students to other shapes such as their handprint. Working with a partner, ask students to trace around their closed hand. Using string they can outline their handprint and then cut the string to determine the perimeter of their handprint by measuring the length of the string with their ruler. Again, students need to be able to explain the strategies they used for finding perimeter as they proceed.

M05.03 Students should be given opportunities to construct shapes of a given perimeter. Discuss with students that when constructing shapes for a given perimeter, they must remember that their shapes should be completely enclosed. It would be easier for students to begin their constructions drawing rectangles using centimetre grid paper and horizontal and vertical lines only.

M05.04 Students need to draw more than one shape for the same given perimeter. Students may use a geo-board or centimetre grid paper to explore various shapes with the same perimeter. They may explore various rectangles before exploring other more complicated shapes.

M05.05 Students should use personal referents when estimating perimeter. Through estimation, students can verify whether their measurements are reasonable. For example, provide students with a playing card and ask them how they could find the card's perimeter using the width of their finger. Invite students to estimate the card's perimeter using this personal referent. Then, using a ruler, ask students to find the actual perimeter and compare it to their estimate.

Brainstorm a list of possible referents for a metre, such as a wrapping paper roll, their arm span, or the height of the door knob from the floor. Ask students to select a referent to determine the perimeter of a given shape, such as a bulletin board, a bookshelf, or a table. Estimation in some circumstances may be the only measurement necessary.

SCO G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

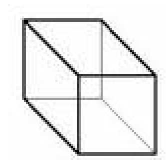
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Identify the faces, edges, and vertices of given 3-D objects, including spheres, cones, cylinders, pyramids, and cubes and other prisms.
G01.02 Identify the shape of the faces of a given 3-D object.
G01.03 Determine the number of faces, edges, and vertices of a given 3-D object.
G01.04 Sort a given set of 3-D objects according to the number of faces, edges, or vertices.

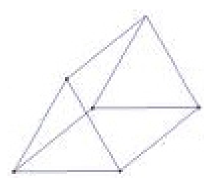
Performance Indicator Background

G01.01, G01.02, and G01.03 Students should be able to identify the faces, edges, and vertices as well as the shape of the faces of a given 3-D object.

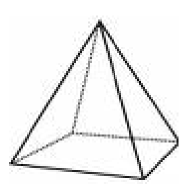
This prism has 6 rectangular faces, 8 edges, and 8 vertices.



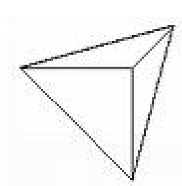
This prism has 3 rectangular faces, 2 triangular faces, 9 edges, and 6 vertices.



This pyramid has 1 rectangular face, 4 triangular faces, 8 edges, and 5 vertices.



This pyramid has 4 triangular faces, 6 edges, and 4 vertices.



Show students models and real-life objects of cylinders, cones, and spheres. Ask students what the difference is between these solids and the prisms and pyramids already studied. Show students the faces, edges, and vertices of each solid. Brainstorm, with the students, what each term means.

Students should be able to determine the number of faces, edges, and vertices of a given 3-D object.

- A cylinder is a 3-D object with 2 faces, 1 curved surface, 2 edges, and 0 vertices.
- A cone is a 3-D object with 1 face, 1 curved surface, 1 edge, and 1 apex.
- A sphere is a 3-D object with 1 curved surface, 0 faces, 0 edges, and 0 vertices.

G01.04 Students should compare and sort 3-D objects by observing the number of faces, edges, and vertices. A student may sort objects in various ways, such as those that have all square faces, those that have circular faces, those that have 8 vertices, or those that have straight edges. Students should play games with their peers where they sort objects and ask their peers to guess the sorting rule according to the number of faces, edges, and vertices.

SCO G02 Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

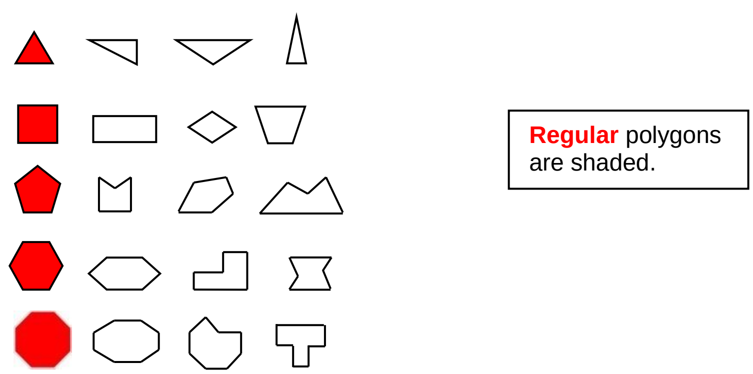
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Classify a given set of regular and irregular polygons according to the number of sides.
G02.02 Identify given regular and irregular polygons having different dimensions.
G02.03 Identify given regular and irregular polygons having different positions.

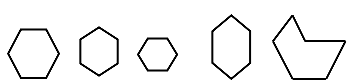
Performance Indicator Background

G02.01 Students should focus on comparing the number of sides as the key attribute for classifying polygons. In this outcome, students should be able to name the specific polygons—triangle, quadrilateral, pentagon, hexagon, and octagon. In the diagram below, the shaded polygons are regular polygons, and all others are irregular polygons.

3 straight sides: triangles
4 straight sides: quadrilaterals
5 straight sides: pentagons
6 straight sides: hexagons
8 straight sides: octagons



Although pattern blocks are frequently used for geometric inquiry, most of these shapes are regular. Students may develop the misconception that only certain familiar polygons meet the criteria for these shapes. For example, students may not initially recognize all of the shapes below as hexagons.



Ask students to find examples of polygons in the world around them, perhaps even collect as many types of a shape as they can find. Sort the shapes according to the number of sides. By sorting polygons according to the number of sides, students can learn the names for the polygons.

G02.02 and G02.03 Students should be given opportunities to explore both regular and irregular polygons varying the positions and dimensions of the shapes. Provide students with a polygon to trace as they experiment with different positions by turning (rotating), flipping (reflection), and sliding (translations). Through many experiences with identifying polygons in a variety of positions, students should begin to realize that a polygon, regardless of its position, remains the same shape.

Provide students with various sizes of a particular polygon. Have students count the number of sides and identify the polygon. Having a variety of these experiences with different polygons, students should begin to realize that a polygon, regardless of its dimensions, remains the same shape.

Statistics and Probability (SP)

SCO SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01 Record the number of objects in a given set using tally marks.
SP01.02 Determine the common attributes of line plots by comparing line plots in a given set.
SP01.03 Organize a given set of data using tally marks, line plots, charts, or lists.
SP01.04 Collect and organize data using tally marks, line plots, charts, and lists.
SP01.05 Answer questions arising from a given line plot, chart, or list.
SP01.06 Answer questions using collected data.

Performance Indicator Background

SP01.01, SP01.03, and SP01.04 Students should be encouraged to collect, organize and record their data using a tally system, line plots, charts and lists to answer questions. Using tally marks is a simple way for students to keep track of information as they collect it. Lists are a way for students to record the objects collected. A list can be made into a chart on which, like last year, students would record their tally marks. Grouping the tally marks in fives makes it easier for students to total the numbers in each category by skip counting. When making a chart, students should always give it a title or heading to inform the reader about the meaning of the data. Students could then organize the data on line plots.

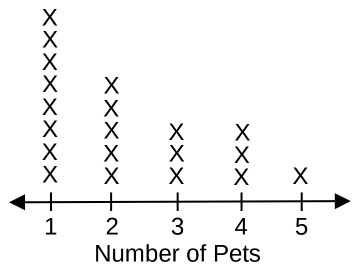
A line plot is a graph that uses a number line as a horizontal axis. Instead of a number line, the horizontal axis could just be a list of the collected data. A line plot provides a bridge from tally charts to bar graphs. At first students should create their line plots using grid paper, with one dot or cross per grid paper square. The dots or crosses are placed one above the other for each tally mark for each item in the list or chart. A line plot offers students a visual comparison of the different quantities of every piece of data.

SP01.02 Students should place different line plots they have made together, and have a discussion about what they might consider their common attributes. For example, they should notice that the attributes that are common include the title, the labels, the horizontal axis, and the use of dots or crosses. They should also notice that the common attributes can differ; for example there could be different titles, different use of the horizontal axis, and different labels. They might also notice that when a line plot does not have a title as in the picture below, it is hard to make sense of the graph.

Tally system

Number of Pets	Number of Students
1	𝍸 III
2	𝍸
3	III
4	III
5	I

Line plot



Chart/List

Number of Pets	Number of Students
1	8
2	5
3	3
4	3
5	1

SP01.05 After a display of the data is constructed, discussing the information that can be obtained from the display is a valuable exercise. Students should work together to formulate questions that can be answered by other students using the data in the line plot, chart, or list. For example, from a line plot that displays the number of letters in your last name, students might formulate questions such as, What is the most common number of letters in a name? How many letters does the longest name in the class have? Shortest?

SP01.06 A good graph should communicate some overall impressions of the data to a reader. Students should be able to answer questions using the display of the collected data. It is also important that each graph accurately represents the data and includes clear labelling and a title.

SCO SP02 Students will be expected to construct, label, and interpret bar graphs to solve problems. [PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP02.01 Determine the common attributes, title, and axes of bar graphs by comparing bar graphs in a given set.
SP02.02 Create bar graphs from a given set of data including labelling the title and axes.
SP02.03 Draw conclusions from a given bar graph to solve problems.
SP02.04 Solve problems by constructing and interpreting a bar graph.

Performance Indicator Background

SP02.01 Present students with vertical and horizontal bar graphs that represent two different sets of data. Discuss what common attributes the two bar graphs have, such as title, axes, labels for the axes, numerical scale, and bars. Discuss how the two bar graphs are different; for example, the titles of the graphs, labels for the axes, lengths and widths and spacing of the bars, and how some graphs have horizontal bars and others have vertical bars.

SP02.02 Once data has been collected, it should be organized and displayed so that questions can be asked and answered. Students have made line plots that have common attributes and the appearance of a bar graph. Students can discuss how they can change their line plots into bar graphs. As they create their bar graphs, have them check to see if all the attributes of the bar graph are visible.

SP02.03 Although students may be able to create bar graphs, some may experience difficulty with drawing conclusions from them. To develop the skill of interpreting graphs, students should be given bar graphs and be asked to draw conclusions. They should be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal questions and inferential questions should be asked, such as, What can you tell about _____ by looking at this graph? How many more/less than …? Based on the information presented in the graph, what other conclusions can you make? Why do you think _____? Eventually, when trying to solve the problem about what foods should be offered in the cafeteria, students would examine a bar graph that has the title, What Foods Should Be Available on the Cafeteria Menu? and be able to tell what food selections were considered the favourites by noticing which bars are the highest or longest. They may also draw conclusions that more students want healthy food for lunch than not healthy food.

SP02.04 Students should understand that to solve some problems, collecting and organizing graphs can help people to reach conclusions. Data is usually collected to answer questions, to discover something of interest, or most importantly, to solve a problem. Some examples of problems students might be interested in include, What should students be allowed to do during the lunch break? What foods should be available on the cafeteria menu? What foods should be removed from the cafeteria menu? What activities would you like to do in the gym? To answer questions like these, or to solve these kinds of problems, students could collect and display data, then interpret it.

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