Mathematics 9

Unit 2: Rational Numbers

N03, N04

SCO N03 Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers. [C, CN, PS, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Order a given set of rational numbers in fraction and decimal form by placing them on a number line.
N03.02 Identify a rational number that is between two given rational numbers.
N03.03 Solve a given problem involving operations on rational numbers in fraction or decimal form.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
N04 Students will be expected to demonstrate an understanding of ratio and rate. **N05** Students will be expected to solve problems that involve rates, ratios, and proportional reasoning. **N06** Students will be expected to demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically.	**N03** Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers.	**AN02** Students will be expected to demonstrate an understanding of irrational numbers by representing, identifying, simplifying, and ordering irrational numbers. **FM01** Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. **FM02** Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.

Background

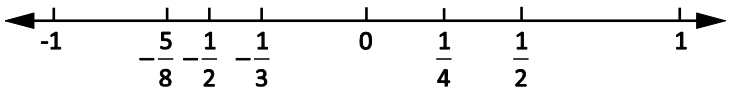
The four operations on negative integers were introduced in Mathematics 8. These operations are extended to rational numbers in Mathematics 9.
A rational number is any number that can be written as a ratio of two integers $\frac{a}{b}$, where *b* does not equal zero. Classifying numbers by sets is introduced in Mathematics 10.

Students have previous experience with ratios, integers, positive decimals, and fraction operations. In Mathematics 6, students were introduced to ordering integers. In Mathematics 7, students compared and ordered positive fractions, positive decimals, and whole numbers. In Mathematics 9 they will compare and order rational numbers, including negative fractions and negative decimals. They have worked with a variety of strategies for comparing fractions and decimals. Such strategies can be revisited here in the context of comparing rational numbers.

The placement of a negative sign in a fraction will be an extension of what students have learned in the past. It is important for students to understand that: $-\frac{6}{2}, \frac{6}{-2}$ and $-\frac{6}{2}$ are all equivalent. However, the first ratio is the preferred placement of the negative sign. This becomes apparent when the division is completed and all equal – 3, regardless of where the negative sign is placed.

Comparing and ordering rational numbers largely draws upon students' number sense. Strategies for ordering numbers should include the following:

- understanding that a negative number is always less than a positive number
- developing a number line with zero marked, and with positioning of positive and negative benchmark fractions for comparison purposes, without conversion to decimals:



- comparing fractions with the same denominator
- comparing fractions with unlike denominator
- comparing fractions with the same numerator (Students should develop a variety of strategies to compare fractions in addition to finding common denominators).
- identifying fractions between any two given fractions, or decimals between any two decimals such as 0.3 and 0.4, $\frac{1}{2}$ and $\frac{1}{3}$, $-\frac{1}{2}$ and $-\frac{1}{3}$.
- reading fractions properly ($\frac{1}{4}$ is read as one-fourth and NOT as one over four, $-\frac{1}{8}$ is read as negative one-eighth and NOT as negative one over eight)

Students have a choice of strategies when they are asked to determine a rational number between a fraction and a decimal. They could convert the fraction to a decimal, or vice versa, and then use the appropriate method.

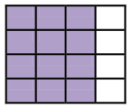
Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- What fractions are represented in the following diagram? Explain your reasoning.



- Order the following fractions without using decimal representations or converting all fractions to a common denominator: $\frac{3}{5}, \frac{8}{9}, \frac{1}{4}, \frac{9}{10}, \frac{13}{7}, \frac{13}{11}, \frac{7}{8}, \frac{1}{3}$

- Ask students which is greater:

0.55 or 0.5 $\qquad$ $\frac{3}{10}$ or $\frac{17}{19}$ $\qquad$ $\frac{2}{3}$ or 0.74

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

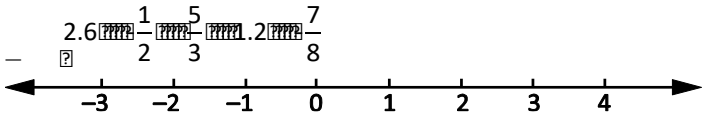
- Use estimation to determine which expression has the greatest quotient.

$\frac{9}{5} \div \frac{11}{12}$ $\quad 2\frac{1}{5} \div 1\frac{6}{8}$ $\quad -3\frac{1}{10} \div \frac{5}{6}$ $\quad -\frac{1}{4} \div -\frac{3}{8}$

- Find three rational numbers that lie between each of the following:

-1 and 0 $\qquad$ $\frac{1}{2}$ and $\frac{1}{3}$ $\qquad$ -3.5 and -3.6 $\qquad$ $-\frac{1}{3}$ and -0.4 $\qquad$ $-\frac{2}{3}$ and -0.6

- Order the following rational numbers on the number line:



- Identify all integers that are between $\frac{11}{5}$ and $-\frac{15}{4}$.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Present examples of positive values with concrete models and pictures before moving to symbolic representation or introducing negative values.
- Generalize and apply a rule for determining the sign of the product or quotient of rational numbers through exploring patterns.
- Extend the common denominator method for division of fractions, taught in grade 8, to negative fractions. When denominators are the same, the numerators can be divided as in the following example:

$$\frac{5}{3} \div \frac{1}{2} = \frac{10}{6} \div \frac{3}{6} = \frac{10 \div (\ 3)}{6 \div 6} = \left(\frac{\frac{10}{3}}{1} \right) = \frac{-10}{3}$$

The answer, read as “negative ten thirds,” can be left as is, unless the context of the question requires it to be expressed in the mixed form: $-3\frac{1}{3}$.

- Compare the multiplication and division of fractions using the meanings of the operations in expressions such as:

$8 \div -\frac{1}{2} = -16$ (How many halves in 8? How does the negative sign affect the answer?)

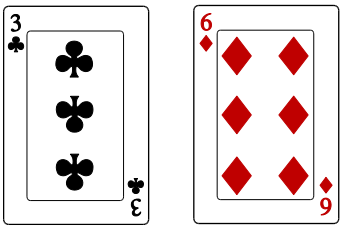
$8 \times -\frac{1}{2} = -4$ (What is half of 8? How does the negative sign affect the answer?)

- Use number lines as models for comparing and ordering rational numbers and for addition and subtraction of rational numbers.

- For this outcome, calculators should be discouraged, as careful attention should be given to the numbers used.

SUGGESTED LEARNING TASKS

- Prepare a set of cards with a variety of rational numbers. Give each student one card. Place them in teams. Students must compare their cards and arrange themselves so that they are in order (ascending or descending). The team who correctly orders the cards first is the winner. A variation of this activity is the Clothesline Game. Students are required to ‘pin’ the cards on a clothesline that has been prepared in advance. The clothesline would be marked off at regular intervals, representing integers on a number line.
- From several decks of cards remove all the cards that are numbered from 2 to 9. These will be the game cards. Pair students off and give each pair a set of game cards. Students should place the set of cards face down and turn over two cards at once.



- The black cards represent positives and the red cards represent negatives. Students will form two fractions using the indicated cards. In the case above, the fractions will be $-\frac{3}{6}$ and $-\frac{6}{3}$. The first student to determine which fraction is closest to zero wins that round. The game continues until all cards have been played. This activity can be modified to work with decimals.
- Use patterning to justify the result for a negative multiplied by a negative using rational numbers. Have students complete the following pattern:

$3 \times \frac{-1}{2} = -\frac{3}{2}$

$2 \times \frac{-1}{2} = -1$

$1 \times \frac{-1}{2} = \frac{-1}{2}$

$0 \times \frac{-1}{2} = 0$

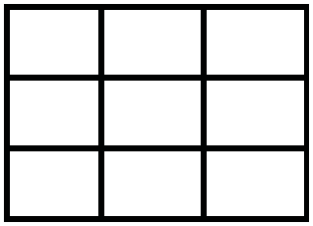
$-1 \times \frac{-1}{2} = \frac{1}{2}$

$-2 \times \frac{-1}{2} = \_\_\_\_$

$-3 \times \frac{-1}{2} = \_\_\_\_$

- Determine three rational numbers between two given decimal numbers, such as 0.6 and 0.61. Choose one number and explain why it is between the two given numbers.

- In a magic square, the sum of each row, column, and diagonal is the same:
  - Create a magic square that uses a mixture of positive and negative rational numbers written in decimal form.
  - Create a magic square that uses a mixture of positive and negative rational numbers written in fractional form.



- A children's wading pool has a small leak. During an afternoon, one-eighth of the water leaks out of the pool. What could the expression below describe about this situation?

$0.75 \times \left(-\frac{1}{8}\right)$

- Using Think-Pair-Share, give individual students time to think about the question. Students then pair up with a partner to discuss their ideas. After pairs discuss, students share their ideas in a small-group or whole-class discussion.

Suggested Models and Manipulatives

- fraction pieces
- integer tiles
- Number Lines (virtual Math Tools)

Mathematical Language

Teacher	Student
▪ benchmarks ▪ compare ▪ greater, greatest ▪ integer ▪ less, least ▪ place value ▪ rational number ▪ simplify	▪ benchmarks ▪ compare ▪ greater, greatest ▪ integer ▪ less, least ▪ place value ▪ rational number ▪ simplify

Resources

Digital

"Ordering Rational Numbers," *SoftSchools.com* (Softschools.com 2015): www.softschools.com/math/rational_numbers/ordering_rational_numbers

Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit3: Rational Numbers
    - > Section 3.1: What is a Rational Number?
    - > Start Where You Are: How Can I Learn from Others?
    - > Section 3.2: Adding Rational Number
    - > Section 3.3: Subtracting Rational Numbers
    - > Game: Closest to Zero
    - > Section 3.4: Multiplying Rational Numbers
    - > Section 3.5: Dividing Rational Numbers
    - > Unit Problem: Investigating Temperature Data
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters

    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 207–209
- *Teaching Student-Centered Mathematics, Grades 5*–8, Volume Three (Van de Walle and Lovin 2006), pp. 148–149

SCO N04 Students will be expected to explain and apply the order of operations, including exponents, with and without technology. [PS, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Solve a given problem by applying the order of operations without the use of technology.
N04.02 Solve a given problem by applying the order of operations with the use of technology.
N04.03 Identify the error in applying the order of operations in a given incorrect solution.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
N06 Students will be expected to demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. **N07** Students will be expected to demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically.	**N04** Students will be expected to explain and apply the order of operations, including exponents, with and without technology.	**AN02** Students will be expected to demonstrate an understanding of irrational numbers by representing, identifying, simplifying, and ordering irrational numbers. **FM01** Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. **FM02** Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.

Background

Order of operations was introduced in Mathematics 6 and practiced in Mathematics 7 and Mathematics 8 when solving problems involving a variety of operations with integers, positive decimals and fractions. In Mathematics 9, students will extend the rules of order of operations to exponents and negative rational numbers. It is important that students have a solid foundation in operations with rational numbers, since it is fundamental to the study of algebra.

The order of operations is:
1. **Brackets**—Brackets group symbols so that we can treat them as a single term.
2. **Exponents**
3. **Multiplication and division** from left to right, in the order in which they appear. This means that multiplication and division are performed before addition or subtraction. It does not mean that multiplication should be done before division. Multiplication and division have the same priority. If both multiplication and division occur within an expression, these operations are performed from left to right.
4. **Addition and subtraction** from left to right, in the order in which they appear

It is important for students to demonstrate their understanding of the order of operations, with and without the use of calculators, and not simply memorize the mnemonic (BEDMAS). The mnemonic is solely a memory aid, and does nothing to promote the understanding that exponents are performed prior to multiplication because exponents represent repeated multiplication; multiplication is performed prior to addition because multiplication can represent repeated addition; and that division and subtraction are considered in the same manner, as there is an inverse relationship between multiplication and division, and between addition and subtraction. To help students gain an understanding of this concept, examine how grouping and order affects answers prior to going to the 'rules' of the mnemonic, by looking at the established order of precedence in which operations must be done. BEDMAS is simply a reminder of this order. Start by looking at an expression to see if order matters. For example, with $2 + 3 \times 5$:

Incorrect:	Correct:
$2+3\times 5$	$2+3\times 5$
$=5\times 5$	$=2+15$
$=25$	$=17$

Since we can write $2 + 3 \times 5$ as $2 + 5 + 5 + 5 = 17$, it is reasonable to do multiplication before addition.

Exponents can be looked at in a similar manner. Since $2 \times 4 \times 4 \times 4 = 128$ can be written as 2×4^3, it is reasonable to do exponents before multiplication.

When there is more than one operation in an expression, it can cause confusion. Students need to understand that it matters what operation is done first. Have students examine expressions when order is important. For example:

Correct:	Incorrect:
$\frac{5}{6} \div \frac{1}{2} \times \left(-\frac{1}{4}\right)$	$\frac{5}{6}, \frac{1}{2} \times \left(-\frac{1}{4}\right)$
$= \frac{5}{6} \times \frac{2}{1} \times \left(-\frac{1}{4}\right)$	$= \frac{5}{6}, \left(-\frac{1}{8}\right)$
$= \frac{5}{3} \times \left(-\frac{1}{4}\right)$	$= \frac{5}{6} \times \left(-\frac{8}{1}\right)$
$= -\frac{5}{12}$	$= -\frac{20}{3}$

The answers are different. Operations need to be performed in the order that they appear, from left to right.
As an indication of understanding, students should be given steps towards an incorrect solution to a problem and be able to identify the step at which the error occurred.

Students should demonstrate a competence in evaluating expressions that include fractions, powers, decimals, and negative integers.

Note: Students can use calculators however, be aware that entry sequences of calculators will vary. An exploration of this variation could offer an opportunity to develop a better understanding of order of operations. It is important for students to know how their personal calculators process the input and that they are able to apply this knowledge.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Play Order of Operations Bingo from NCTM: http://illuminations.nctm.org/Lesson.aspx?id=2583.
- Which operations should be completed first in the following expressions:

$1\frac{2}{5} - \frac{3}{4}, \frac{2}{5}$

$\left(\frac{2}{3} - \frac{1}{4}\right) \times \frac{7}{8}$

$3 + \frac{5}{6} - \frac{4}{5}, \frac{4}{5}$

- Evaluate the expressions above, showing all steps.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Without the use of a calculator, simplify the expression and express your answer as a fraction.
  - By inserting one pair of brackets, how many different answers are possible?

- By inserting two pairs of brackets, is it possible to receive a different answer?

$\frac{1}{2}-\frac{1}{3}\div\frac{1}{2}-\frac{1}{3}$

- Use a calculator to simplify the following:

$$\frac{56.3-22.5}{4.2\times(5.9-10.5)}$$

- Arrange the solutions of the following from least to greatest:

$\frac{-3}{4}-\left(\frac{-3}{4}+\frac{4}{-5}\right)$ $\frac{-3}{5}-\frac{-3}{4}+\frac{9}{-10}$ $6\div\frac{-1}{5}-\frac{1}{-2}$ $\frac{3}{5}-\left(\frac{-3}{5}-\frac{-2}{3}\right)$

- Indicate at which step an error occurred and explain:

$5-2(4+5)^2$

$5-2(9)^2$ Step 1

$3(9)^2$ Step 2

$3(81)$ Step 3

243 Step 4

- Use a calculator to convert the following Fahrenheit temperatures to Celsius, using the given formula: $^\circ C=\frac{5}{9}(^\circ F-32^\circ)$

10°F 15°F –17.2°F

- Explain why it is essential that the rules for order of operations for rational numbers be the same as the order of operations for integers.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Explore a variety of expressions in which brackets, fractions, and negative numbers are used. Through these explorations students will demonstrate that the rules for order of operations ensure a consistent result.
- Students should be encouraged to perform operations without calculators as much as possible. However, some questions lend themselves to calculator use more than others. When evaluating with decimals, it is appropriate to use a calculator for more than 2-digit multipliers or more than 1-digit divisors.
- Have students compare their results when using calculators to simplify expression and, if different from one another, determine how different calculators interpret data inputs differently. This could be an opportunity to establish the importance of order of operations, and the importance of correct calculator use.

- For differentiation: Have students simplify expressions, showing each step in the order of operations. To the right of each step, identify the step as brackets, exponents, multiplication, division, addition, or subtraction.

SUGGESTED LEARNING TASKS

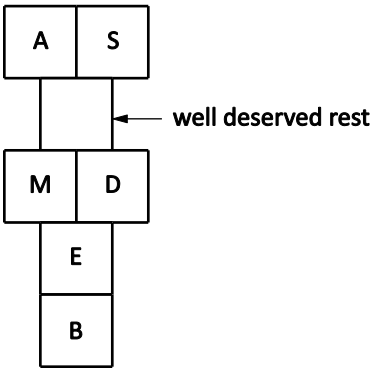
- Simplify expressions that include fractions and a mix of operations, without the use of a calculator, and express the answer as a fraction. Explore how the insertion of one or more sets of brackets at various positions affects the answer. For example:

$\frac{1}{2}, \frac{2}{3} - \frac{4}{3}, \frac{3}{2} = -\frac{5}{3}$ $\frac{1}{2} \times \left(\frac{2}{3} - \frac{4}{3}\right) \times \frac{3}{2} = -\frac{1}{2}$ $\left(\frac{1}{2} \times \frac{2}{3} - \frac{4}{3}\right) \times \frac{3}{2} = -\frac{3}{2}$ $\frac{1}{2} \times \left[\left(\frac{2}{3} - \frac{4}{3}\right) \times \frac{3}{2}\right] = -\frac{3}{2}$

- Add brackets where required to make this a true statement:

$\frac{4}{3}$ x 10 + 5 + (–8.1) = 11.9

- Use a calculator to explore the use of brackets to simplify expressions with multiple terms in the numerator and denominator.
- You have been hired by a company to produce a skill-testing math question using order of operations. Create the question, with the solution, to be used to determine the prizewinner.
- Explain why the following hopscotch analogy works with respect to order of operations.



- Create a game where students are given a strip containing 4 rational numbers and ask students to use $+, -, \times, \div, (\ ), \sqrt{\ }, x^2$ to create expressions. The individual operations could be placed on pieces of paper to allow students to move them around the numbers. Teachers may have students create the expression with the largest solution or solution closets to zero, or any other variation.

  For example, these numbers:

Given

–1.86	–2	5.3	9

Students create

$(-1.86 + 2)^2 \times -5.3 - \sqrt{9}$

A variation of this activity is for a student to choose 4 numbers, create an expression, and ask their partner to simplify the expression.

SUGGESTED MODELS AND MANIPULATIVES

- Calculator

MATHEMATICAL LANGUAGE

Teacher	Student
▪ brackets ▪ cube ▪ integers ▪ negative ▪ order of operations ▪ positive ▪ square	▪ brackets ▪ cube ▪ integers ▪ negative ▪ order of operations ▪ positive ▪ square

Resources

Digital

- "24 [Game]," *4 Numbers* (4 Numbers 2015): www.4nums.com/apps
- "Exploring Order of Operations: Use It," *Math Interactives* (Alberta Education, LearnAlberta.ca 2015) www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.INTE&lesson=html/object_interactives/order_of_operations/use_it.html
- "Order of Operations Bingo," *Illuminations: Resources for Teaching Math* (NCTM 2015): http://illuminations.nctm.org/Lesson.aspx?id=2583

Print

- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 473–474
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit2: Powers and Exponent Laws
    - > Section 2.3: Order of Operations with Powers
    - > Game: Operation Target Practice
  - Unit 3: What Is a Rational Number?
    - > Section 3.6: Order of Operations with Rational Numbers
    - > Unit Problem: Investigating Temperature Data
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
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- *Teaching Student-Centered Mathematics, Grades 5–8*, Volume Three (Van de Walle and Lovin 2006), pp. 132, 134