MATHEMATICS
CLASS - IX

CHAPTER 1- NUMBER SYSTEMS

In your earlier classes, you have learnt about the number line and how to represent various types of numbers on it (see Fig. 1.1).

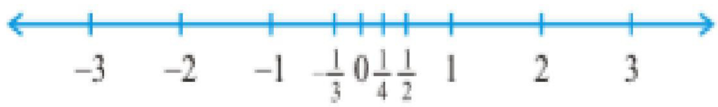


Fig. 1.1 : The number line

Let's recall different types of numbers and their definitions:

- **Natural numbers**: The counting numbers 1, 2 , 3 .... are called Natural numbers. It is denoted by ' N'.

- **Whole numbers**: The natural numbers along with 0 i.e, 0, 1, 2 , 3 .... are called whole numbers. It is denoted by ' W'.

  REMARKS:
  a) Every natural number is a whole number.
  b) 0 is a whole number which is not a natural number.

- **Integers**: The collection of all whole numbers along with negative numbers is called integers. It is denoted by ' Z '.

  REMARKS:
  a) Every natural number is an integer.
  b) Every whole number is an integer.

Now, let's talk about Rational numbers –

- **Rational numbers**: It is a number which can be expressed in the form of p/q , where p and q are co-prime integers and q ≠ 0. It is denoted by ' Q '.

Examples: 5 7 , – 2 3 , 0 , - 100

Note:

p and q are called co-prime integers if they have no common factors other than 1.

Equivalent rational numbers (or fractions) :

Example: 2 1 = --- 24 = 10 20 = 25 50 = 47 94 = ...

These are known as equivalent rational numbers.

Now, watch the following videos based on types of numbers:

https://youtu.be/6ptpoI4E-vA

https://youtu.be/wZW5XzmAvk0

Now, do example 1, Exercise 1.1- Q. No. 1 and 4.

Example 1 : Are the following statements true or false? Give reasons for your answers.

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

EXERCISE 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

4. State whether the following statements are true or false. Give reasons for your answers.
   (i) Every natural number is a whole number.
   (ii) Every integer is a whole number.
   (iii) Every rational number is a whole number.

Example 2: Find five rational numbers between 1 and 2.

Solution:

Method 1:

Five rational numbers between 1 and 2 are $\frac{9}{8}$, [illegible] and [illegible]

The rational number between a and b is $(a + b)/2$

Hence rational number between 1 and 2 is $(1+2)/2 = 3/2$

Rational number between 1 and 3/2 is $[1+(3/2)]/2 = 5/4$

Rational number between 1 and 5/4 is $[1+(5/4)]/2 = 9/8$

Rational number between 3/2 and 2 is $[(3/2) + 2]/2 = 7/4$

Rational number between 7/4 and 2 is $[(7/4) + 2]/2 = 15/8$

Method 2:

Number of rational numbers between 1 and 2 needed $n = 5$

$n + 1 = 5 + 1 = 6$

$1 = \frac{1}{1} \times \frac{6}{6} = \frac{6}{6}$ and $2 = \frac{2}{1} \times \frac{6}{6} = \frac{12}{6}$

Five integers between 6 and 12 are 7, 8, 9, 10 and 11

Therefore, five rational numbers between 1 and 2 are $\frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}$ and $\frac{11}{6}$

$= \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}$ and $\frac{11}{6}$

REMARK: There are infinitely many rational numbers between any two given rational numbers.

Let us do one more example.

Find a rational number between $-\frac{1}{2}$ and $\frac{2}{3}$.

Solution:

Method 1:

Let a = $-\frac{1}{2}$ and b = $\frac{2}{3}$

$\therefore$ Required rational number = $\frac{1}{2}\left(-\frac{1}{2} + \frac{2}{3}\right)$

$-\frac{1}{12}$

$-\frac{1}{12}$ $\therefore$ Ans =

Method 2:

LCM of 2 and 3 = 6

$\therefore -\frac{1}{2} = \frac{-1\times 3}{2\times 3} = -\frac{3}{6}$

$\therefore \frac{2}{3} = \frac{2\times 2}{3\times 2} = \frac{2}{6}$

$\therefore$ Required rational number = $\frac{1}{6}$

$\therefore$ Ans = $\frac{1}{6}$

Now, watch the following video based on finding rational numbers between given two rational numbers-

https://youtu.be/fysueeUG-FE

Do Exercise 1.1- Q. No. 2 and 3.(By both the methods)

2. Find six rational numbers between 3 and 4.
3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Now, practice the following worksheet:

WORKSHEET

1. What are rational numbers? Give ten examples of rational numbers.
2. Write four rational numbers equivalent to $\frac{3}{7}$.
3. Find a rational number between $\frac{1}{4}$ and $\frac{1}{3}$.
4. Find two rational numbers between - 1 and $\frac{1}{2}$.
5. Find three rational numbers between $\frac{-3}{4}$ and $\frac{-2}{5}$.
6. Find two rational numbers between 1.3 and 1.4.
7. Find five rational numbers between $\frac{3}{8}$ and $\frac{2}{5}$.

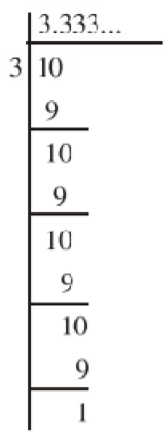
CLASS-IX NUMBER SYSTEMS

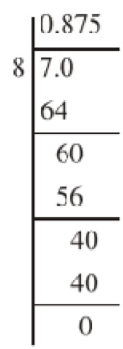
SUB- TOPIC: REAL NUMBERS AND THEIR DECIMAL EXPANSIONS

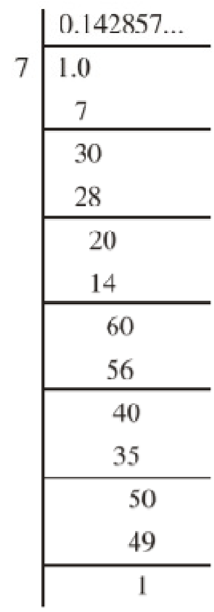
In this section, we will study about rational and irrational numbers from a different point of view. We will look at the decimal expansions of real numbers and see if we can use the expansions to distinguish between rationals and irrationals.

Let us take three examples **(RATIONAL NUM BERS) : 10/3 , 7/8 , 1/7**

Pay special attention to remainders and see if you can find and pattern







Remainders : 1, 1, 1, 1, 1... **Remainders : 6, 4, 0** **Remainders : 3, 2, 6, 4, 5, l.**

Divisor: 3 **Divisor : 8** **3, 2, 6, 4, 5,**

Divisor : 7

OBERVATIONS:

(In 1/3, one number repeats itself and the divisor is 3, in 1/7, there are six entries 326451 in the repeating string of remainders and 7 is the divisor).

3. **If the remainders repeat, then we get a repeating block of digits in the quotient.** **(In 1/3, 3 repeats in the quotient, in 1/7, we get the repeating block 142857 in the quotient).**

Above pattern is true for all rationals of the form p/q (q 0). On dividing p by q, two main thing happen which are :

Case 1: remainder becomeszero Remainder becomes zero after a few steps	**Case 2: remainder never becomeszero** Remainders repeat after a certain stage forcing the decimal expansion to go on for ever.
Decimal expansion of such numbers is called *terminating*	Decimal expansion of such numbers is called *non terminating recurring*
Example :Y2= 0.5 , 639/250 = 2.556 etc	Example:14/11 = 1.272727 .... 233/990= 0.235353535 .... etc

Thus, we we that decimal expansion of rational numbers have only two choices: either they are terminating or non-terminating recurring.

Now, let us suppose, you come across a number ike 3.142678 (terminating decimal expansion) or a number like 1.272727 (**non-terminating decimal expansion).**

CAN YOU CONCLUDE THAT IT ISA RATIONAL NUMBER?

THE ANSWER IS YES ! LET'S LEARN

Example 6 : Show that 1142678 is a rational number. In other words. express 3.142678 in the form $\frac{p}{q}$ where p **and q** are integers and q $\neq 0$**.**

Solution We have 3.142678 = $\frac{3142678}{1000000}$ **and hence is a rational number.**

Now, let us consider the case when the decimal expansion is non- terminating recurring.

Example 7. **Show that 0.3333... = $\overline{0.3}$ can he expressed in the form** $\frac{p}{q}$ where p **and q** are integers and q $\neq$0.

Solution **Since we do not know what $\overline{0.3}$ let us call it `x' and so**

x= 0,3333...

w here is where the trick comes in. Look at

l 0x = 10 x (0.333—) = 3.333...

Now,

3.3333... = 3 + x, since x = 0.3333...

Thererore,

10x= 3 +x

Solving for x. we get

9x = 3, i.e., x = $\frac{1}{3}$

Alternative method:-

Let x = 0.33.. eqn 1

10 x = 10 x 0.33.. ...... eqn 2

10 x = 3.3....

Subtracting eqn 1 from eqn 2, we get

10x - x = 3.3.... — 0.3.....

9x = 3

x = 3/9

Example 8 : Show that 1.272727... = 1.27 can be expressed in the form P/q , where *p* and *q are* integers and *q* ≠O.

Solution Let x = 1.272727... Since two digits are repeating, we multiply x by 100 to get

$$100x = 127.2727...$$

So, $100x = 126 + 1.272727... = 126 + x$

Therefore, $100x - x = 126$, i.e., $99x = 126$

$$x = \frac{126}{99} = \frac{14}{11}$$

You can check the reverse that $\frac{14}{11} = 1.2727..$.

Example 9: Show that 0.2353535... = 0.235 can be expressed in the form _P,

where p and q are integers and q 0.

Solution: Let x= 2.353535…, Over here, note that 2 does not repeat, but the block 35 repeats. Since two digits are repeating, we multiply x by 100 to get

$$100x = 23.53535...$$

So, $100x = 23.3 + 0.23535... = 23.3 + x$

Therefore, $99x = 23.3$

$$99x = \frac{233}{10} \text{ which gives } x = \frac{233}{990}$$

You can also check the reverse that $\frac{233}{990} = 01\bar{}35$

Let us summarize :

The decimal expansion of a rational number is either terminating R or non-terminating recurring. Moreover, a number whose decimal expansion is terminating or non-terminating recurring is rational.

The decimal expansion of an irrational number is non-terminating non-recurring. Moreover, a number whose decimal expansion is non-terminating non-recurring is *irrational.*

Decimal Expansion

Terminating

Eg
2.5, 3,28, 9.81

All numbers are **rational**

Non-Terminating

4

Non Terminating Repeating

Eg;

235555...
1.323232...

These numbers are **rational**

Non Terminating Non Repeating

Eg:

1 3,14159265m
Nam 1.4142135...

These numbers are irrational

Now, let's take an example:

Example 10 **Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.**

Solution **We saw that $\frac{1}{7} = 0.\overline{142857}$. So. you can easily calculate $\frac{2}{7} = 0.\overline{285714}$.**

To find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$ we find a number which is non-terminating non-recurring lying between them. Of course, you can find infinitely many such numbers.

An example of such a number is 0.150 I 50015000150000...

PRACTICE QUESTIONS : (to be done in cw register)

Express each of the following as a fraction in simplest form:

(a) 0.177777777................

(b) 0.163636363 ....

(c) 0.001001001001.................

EXERCISE 1.3

L Write the following in decimal form and say what kind of decimal expansion each has

U) $\frac{36}{100}$ (h) (iii) ⁴i

(iv) 3 (v) 2 (vi) $\frac{329}{400}$

. you know that = 0.142857 Can you predict what the decimal expansions of $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ are, without actually doing the long division? If so, how?

[Hint Study the remainders while finding the value of $\frac{}{7}$ carefully.]

3. Express the following in the form , where p and q are integers and q 0.

(I) 0i) (ii) 0.47 QOM

Express 0.99999 . in the formJ1 Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of ? Perform the division to check your answer,

6. Look at several examples of rational numbers in the form (q *0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

7. Write three in whose decimal expansions arc non-terminating non-recuring.

S. Find three different irrational numbers between rational numbers $\frac{5}{7}$ and $\frac{9}{I}$

9. Classify the following numbers as rational or irrational

(ii) N1 (iii) 037%

(iv) 7,478478— (v) I_101(l1)1(XX11001X11