Mathematics

Upper primary Teachers Guide 2003

Papua New Guinea
Department of Education

Issued free to schools by the Department of Education
Published in 2003 by the Department of Education, Papua New Guinea

Developed by the Curriculum Development Division of the Department of Education and was coordinated by Steven Tandale.

ISBN 9980-930-22-5

Acknowledgements

Many thanks are extended to teachers, students and other education personnel who have conteributed to the development and trialing of this document since 2002. The Department of Education acknowledges the work of members of the 2000, 2001 and 2002 Upper Primary Mathematics Syllabus Advisory Committee. The Department of Education also acknowledges Senior Inspectors, Inspectors, and Education Personnel who were involved from the provinces mentioned below.

The following primary schools and teachers participated in writing workshops, trialing and consultation.

Western Highlands

Holy Trinity, Rambiamul, Hagen United and Tarangau

East Sepik

Kaindi Demonstration, St. Mary's Wiru, Kreer, Kreer Heights and Moem Barracks

East New Britain

Kalamanagunan, Kabaleo Demonstration, St. Paul's, Vunakanau Primary

Western Province, Daru

Chalmers, Edward Baxter Riley and Monford

National Capital District

Butuka, Evedahana, Murray Barracks and Ted Diro

The following organisations also made significant contributions: Mission Education Secretariat, the United Church in Daru and the Catholic Church in Mt. Hagen, Daru High School, Hagen Secondary School, Holy Trinity Teachers' College, Kabaleo Teachers' College, The University of Goroka and the Papua New Guinea Education Institute.

The late Ben Taku also made a significant contribution to the development of this Teachers Guide while attending a writing workshop in Goroka the day before he died in tragic circumstances.

This document was developed with the support of the Australian Government through the Curriculum Reform Implementation Project.

Mathematics

Introduction 1
Key features of the subject 5
Teaching and learning strategies 9
Assessment, recording and reporting 12
Programming 22
Units of work 26
Elaboration of learning outcomes 41
Resources 48
Glossary 56
References 60
Appendix (Time allocations) 61

Inservice Units

A set of inservice units have been written to support the implementation of the upper primary reform curriculum.

These units are:

- self-instructional, so you can access them according to your needs when and where suits you,
- self-paced, so you can study at your own pace,
- outcomes-based, so you can experience outcomes-based approaches to education,
- based on adult learning principles of learning, doing, sharing and reflecting,
- practical and related to your daily work as a teacher or a supervisor,
- collegial, so you can learn together in small groups, whole school or cluster settings,
- accredited with PNG Education Institute, so you can improve your qualifications,
- designed to promote best practice, so you can effectively implement the curriculum,
- applicable across Upper Primary Syllabuses.

These units integrate principles contained in the *National Curriculum Statement* (2002) and the *National Assessment and Reporting Policy* (2003).

These units can be used in conjunction with this Teachers Guide.

Secretary's Message

Primary teachers are generalist teachers and this Teachers Guide is for all teachers in Upper Primary schools. It is one of a set of seven guides written for teachers of Upper Primary, Grades 6-8.

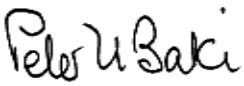
The Upper Primary Syllabuses identify the learning outcomes. The Teachers Guides give more information about what to teach and describe ways of implementing the syllabuses. The Teachers Guides are supported by the In-service Units that have been written to assist the implementation of the Upper Primary Syllabuses and provide valuable information about teaching.

I encourage teachers to work closely with members of their school communities to ensure that local community needs are met.

Important reforms to our education system will only be successful with the support and understanding of teachers. Every Teachers Guide contains detailed information about appropriate subject content, a broad range of ideas and strategies to help teachers use and understand the subject Syllabuses. Each guide is written for a particular subject but many of the ideas and strategies can be used with different subjects or when using an integrated approach to teaching and learning.

Teachers should read each guide carefully and become familiar with the content of each subject as specified in the Elaborations section in each guide.

I also encourage teachers to try out the ideas and strategies that they believe will be effective in their schools with their students. Teachers have the right to modify and amend these ideas to suit their local circumstances.



Peter M. Baki
Secretary for Education

Introduction

Purpose

The Teachers Guide is to be used in conjunction with the Mathematics Syllabus and other Teachers Guides in Upper Primary. The main purpose of the Teachers Guide is to help you to implement the Mathematics Syllabus in Upper Primary. It provides you with information and processes to:

- use the elaborations to identify relevant content and contexts,
- develop units of work or projects relevant to your students' needs, interests and social and economic opportunities,
- select appropriate teaching and learning strategies,
- plan a school based program suitable to your school,
- plan and conduct assessment to monitor students learning and achievement of learning outcomes.

How to use the Teachers Guide

When you receive this book, you need to do the following:

- read it carefully and grasp the flow of the content,
- read it carefully so that you become familiar with the Strands, the Sub-strands, the processes and skills, the Elaborations of learning outcomes and the teaching and learning strategies,
- identify specific projects based on the 10 learning outcomes for Grades 6, 7 & 8,
- consider how to use the information to develop your own programs and units of work.

Some options for developing programs include:

- teaching one of the sample units of work from a particular Strand,
- using the sample units of work as a guide to develop your own units of work relevant to local contexts,
- using the sample unit of work as a guide to develop integrated units of work with other subject outcomes.

The nature of Mathematics

Mathematics is a creative activity that uses reasoning and generalisations to describe patterns and relationships. It is often considered to be one of the greatest cultural and intellectual achievements of humanity.

Knowing Mathematics can be personally satisfying and empowering. The basis of everyday life is increasingly mathematical and technological. For instance, making purchasing decisions, budgeting or considering health plans all require an understanding of Mathematics. In this changing world, those who understand and can do Mathematics will have significantly enhanced opportunities and options for shaping their futures.

Links with other levels of schooling

The Mathematics outcomes covered in the Syllabus, Teachers Guide and the *Worked Examples for Upper Primary Mathematics Outcomes* build on the aspects of Mathematics covered at the Lower Primary level. It is assumed that Mathematics will develop both academic skills for further mathematical studies for those continuing to Grade 9 and beyond, and everyday life skills that students need, to be useful citizens in their communities.

The diagram below outlines this link.

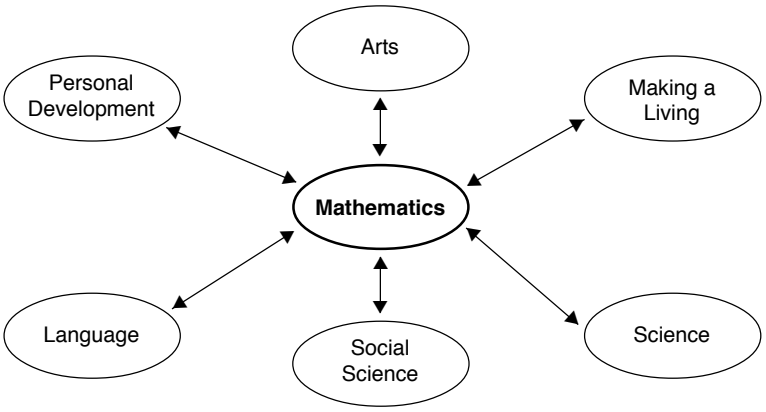
Mathematics Learning Area ↔ **Elementary** Cultural Mathematics ↔ **Lower Primary** Mathematics ↔ **Upper Primary** Mathematics ↔ **Lower Secondary** Mathematics ↔ **Upper Secondary** Mathematics

- Extension (Mathematics A)
- Mathematics core (Mathematics B)

Links with other subjects

The Mathematics learning outcomes covered in the Syllabus and Teachers Guide can also be linked with other subjects. This can be done by collecting outcomes that link naturally together through similar concepts or processes. Units of work can then be planned for these outcomes so that they are taught and learnt in an integrated way.

The diagram below shows how Mathematics can link with other subjects.



Using the Teachers Guide with the Syllabus

The Teachers Guide illustrates key parts of the subject Syllabus. The Teachers Guide provides you with many practical ideas about how to use the Syllabus and why the Teachers Guide and Syllabus should be used together.

The Teachers Guide explains ways you can plan and develop teaching, learning and assessment programs. There are ideas and strategies for writing weekly, term and yearly programs, units of work and sample lesson plans. The Guide also includes recommended knowledge, processes, skills and attitudes for each of the outcomes in the Syllabus. You will also find a separate support document to this Guide titled: *Worked Examples for Upper Primary Mathematics Outcomes*. This document gives you detailed information on the elaborations of the learning outcomes**.** The information you get from the *Worked Examples for Upper Primary Mathematics Outcomes* will help you to decide on the appropriate content to teach. The Teachers Guide also includes examples of how you can assess and report student achievements.

You are encouraged to select and adapt the strategies and processes illustrated in the Guide to meet the needs of your students and their communities.

Using the worked examples with the Teachers Guide

The Teachers Guide and *Worked Examples for Upper Primary Mathematics Outcomes* come in a package with the Upper Primary Mathematics Syllabus. You are to use these books at the Upper Primary level for Grades 6–8. When using these books, you should:

- read the Guide and the *Worked Examples for Upper Primary Mathematics Outcomes* very carefully,
- become familiar with the subject Syllabus and its Strands and Substrands,
- read the outcomes and indicators in the Syllabus,
- read each section of the Teachers Guide again and make notes about these ideas, strategies and processes that you think will be useful to you,
- meet with other teachers, share your ideas, and plan how you will work together to write programs and units of work,
- now be ready to try out some of the units of work in the Teachers Guide,
- now be confident to write your own programs and units of work using one or more of the Teachers Guide, the *Worked Examples for Upper Primary Mathematics Outcomes* and the Mathematics Syllabus.

Language

You will use English as the main language of instruction for Upper Primary. This must not stop you from using local vernacular to facilitate understanding and reinforce meaning. Each Substrand will have a list of new words that have been included in the glossary of this Teachers Guide. You must refer to the glossary or a dictionary whenever you come across new or unfamiliar words.

The first few times students meet these key words, you should:

- say the word with the class a number of times,
- write the word on the board, a chart, or on cardboard,
- explain the meaning of the word using real objects, actions or pictures,
- demonstrate how to use the word in simple mathematical sentences,
- ask the students to use the words in simple mathematical statements,
- tell the students to enter the word in their language vocabulary book or class dictionary.

Key features of the subject

The Strands

The Strands in the Upper Primary Mathematics Syllabus organise the content. The Upper Primary Mathematics Syllabus has five Strands. The Strands are Number and Application, Space and Shapes, Measurement, Chance and Data and Patterns and Algebra. The five Strands are further organised into a number of Sub-strands to describe specifically the development of key ideas.

In each of the Strands, the content is described as learning outcomes. These outcomes identify the knowledge, skills and attitudes to be learnt in each Upper Primary Grade.

The five Strands for Mathematics are outlined below with brief explanations of what is to be covered in each Strand.

Number and Application

In this Strand students learn to use all common forms of number including fractions, decimals, percentage, indices and negative numbers. They learn to apply these to solve problems that might be encountered in everyday life.

Space and Shape

In this Strand students learn to estimate and measure length, area, volume and angles. They learn the language required to discuss shape and direction. They learn to locate points on a plane by way of coordinates. They are presented with practical applications of what they are learning. Throughout they are challenged to apply a broad range of Mathematics to solve problems.

Measurement

This Strand concentrates on the units and practice of measuring weight, temperature and time. Students are required to record, calculate and justify the accuracy of the measurements they make.

Chance and Data

This Strand focuses on the collection, presentation and interpretation of data. This Strand deals with statistical information, graphs, probability and sets. It also considers methods of estimation and issues of accuracy and error.

Patterns and Algebra

This Strand deals with patterns in packing, in number and operations. These are used to link common events to mathematical thought and the idea of abstract representation of numbers and processes that is possible with algebra.

The Substrands

The Substrands are broad topics within the Strands that allow the knowledge, processes and skills to be specific and described as learning outcomes. The Substrands in the five Strands of the Upper Primary Mathematics Syllabus are outlined in the table below.

Strands and Substrands for Upper Primary Mathematics

Strand	Grade 6	Grade 7	Grade 8
Number and Application	Fractions Decimals Fractions and decimals Decimals and percentage Ratios Directed numbers Indices	Fractions Decimals Fractions and decimals Decimals and percentage Ratios Directed numbers Indices	Fractions Decimals Fractions and decimals Decimals and percentage Ratios Directed numbers Indices
Space and Shape	Length Area Volume and capacity Shape Tessellations Angles Nets Direction Maps and coordinates	Length Area Volume and capacity Shape Tessellations Angles Nets Direction Maps and coordinates	Length Area Volume and capacity Shape Tessellations Angles Nets Direction Maps and coordinates
Measurement	Weight Temperature Time	Weight Temperature Time	Weight Temperature Time
Chance and Data	Statistics Sets Probability Accuracy and error Estimation	Statistics Sets Probability Accuracy and error Estimation	Statistics Sets Probability Accuracy and error Estimation
Patterns and Algebra	Packing Algebra	Packing Algebra	Packing Algebra

Due to the interrelated nature of two-dimensional and three-dimensional space, Substrands such as volume and capacity and nets and packaging need to be programmed together. Similarly, the Substrands of shape, tessellations and angles need to be programmed together to maximise learning opportunities and reflect the connected nature of Mathematics.

Developing knowledge, skills and positive attitudes

This Upper Primary Mathematics course is designed to enable students to see Mathematics as an exciting, useful and creative field of study. During these years, many students will solidify or build up stronger ideas about themselves as learners of Mathematics—about their competence, attitude, interest and motivation. Students acquire an appreciation for, and develop an understanding of, mathematical ideas if they have frequent encounters with interesting, challenging problems.

In Grades 6 -8, students should engage in mathematical activities related to their emerging capabilities of conjecturing, that is, investigating doubtful information and verifying it or coming up with solutions, abstracting and generalising. The content addressed in Patterns and Algebra has an obvious focus on students recognising and being able to describe generality.

Problem solving in the Upper Primary Mathematics course should develop the expanding mathematical capabilities of students. This includes problem solving that integrates such topics as probability, statistics and rational number. Well-chosen problems can be particularly valuable in developing or deepening students' understanding of important mathematical ideas. This idea also provides a bridge from the known to the unknown.

Outcomes

The outcomes for each of the Strands and Substrands in this Guide describe what the students know and can do as a result of the learning experiences. They demonstrate the knowledge, skills, understanding and attitudes achieved in Mathematics at the Upper Primary level. These outcomes are numbered with a three-digit code where each number means something. For example in the outcome numbered 6.1.2, the first digit, 6, means Grade 6, the second digit, 1, means Strand number 1 and the third digit, 2, means outcome 2 for that Strand. The list of Mathematics outcomes can be found on pages 12-16 of the Mathematics Syllabus. They can also be found in the *Worked Examples for Upper Primary Mathematics Outcomes* that comes as a support document to this Teachers Guide.

Indicators

The indicators list the kinds of things the students would be able to do, know and understand if they are to achieve a particular outcome. These are examples that you can use to plan your weekly and daily lessons. You can develop other indicators depending on the needs of your students and the resources available within the school or community.

This Mathematics course attempts to place Mathematics into a practical and familiar setting so that students have the opportunity to explore and use mathematical concepts in real life situations. You are encouraged to make use of this opportunity when planning and programming your lessons.

Curriculum principles

This Mathematics course is based on three fundamental learning principles:

- we learn best when we build new learning on what we already know,
- we learn well when we recognise an immediate use or need for what is to be learned,
- we use many ideas and skills in a coordinated way to solve real problems.

The course continuously refers to pre-existing knowledge and sets Mathematics into contexts that are familiar and of interest to the students. This contextual approach leads to real problems in interesting and familiar settings, requiring students to participate in both problem setting and problem solving processes.

The students need to use concepts and skills from many areas of Mathematics and other sources to come up with workable solutions, as in real life. This approach facilitates a student-based mode of learning. By linking new mathematical concepts to existing cultural and scholastic knowledge, the students will integrate the knowledge so that they are more able to use it in their lives.

Catering for diversity

Students' participation in active learning is desirable. It is the teacher's role to therefore ensure all students in the class are given a fair opportunity to demonstrate what they have learnt.

Girls and boys should be:

- given the same access to activities in and outside the classroom,
- encouraged to value individual differences,
- encouraged to have respect and understanding for others within and outside the school community,
- given opportunities to develop positive self-esteem and value life experiences,
- encouraged to participate fully, regardless of their gender, ability, language group, culture and where they live.

Teaching and learning strategies

Teaching and learning strategies

These are some of the many teaching and learning strategies that you can use to teach your students Mathematics. There are many ways you can apply these strategies. You are required to use some of these strategies, along with other strategies you know will work well in the teaching and learning of Mathematics.

Experience-based learning

Mathematical ideas are more likely to be remembered and used if they are based on students' experiences. In this approach Mathematics is a *doing word*. Having students use three steps in most practical lessons can expand this approach. The three steps are:

- predict,
- observe,
- explain.

As an example of this strategy, when developing knowledge of fractions, a teacher might ask a class to **predict** where 1/3 of the way across the blackboard would be. Students would then be invited to make a chalk mark where they think 1/3 of the way across is and write their initials next to the mark. When enough students have made their predictions while other students **observe**, the teacher asks, "How would we work out who is the closest?" Students then **explain** how to determine 1/3 of the way across the board and then carry out the measurements to confirm the results.

Problem-based learning

Using this strategy, the teacher can set a problem or rich task for the class to solve. For example, how could we make a measuring beaker from a 600 mL plastic water bottle?

Steps

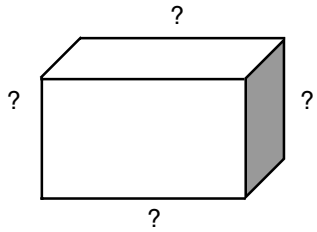
- Brainstorm students' ideas and record them on the board.
- Ask related questions such as, "How could we make a cuboid with a volume of 100 cm^3?"
- Have students carry out the investigation in groups and report back to the class.

It is important that the teacher creates a summary of what has been learnt from solving the problem to make the learning explicit.

Open-ended questions

Closed questions commonly used in Mathematics lessons only have one correct answer. By contrast open-ended questions can have more than one answer and students are encouraged to explore and come up with a variety of answers.

An example of an open-ended question is, "What could be the dimensions of a cuboid with a volume of 240 cm^3?"



One answer could be 2 cm x 12 cm x 10 cm. If a student comes up with one answer and stops, ask the class if anyone had a different answer. How many different answers are possible?

You can record the various answers from the class in a table. If you want to look at all of the possible answers with whole numbers work out the prime factors of 240.

Using prime factors: 240 = 2 x 2 x 2 x 2 x 3 x 5 we can group these to make all of the different sets of three factors of 240. For example (2 x 2) x (2 x 2) x (3 x 5) = 4 x 4 x 15.

One open-ended question can provide many answers for students to find and provide quite a lot of practice at basic skills at the same time.

Cooperative learning

Cooperative learning has students working in groups on common problems. The main difference between group work and cooperative learning is that with cooperative learning all students must contribute to the group's learning. A cooperative learning task might ask a group to find all of the different pentominoes: 5 connected squares, that are nets of open cubes. When groups start to work on the problem, challenge them by asking which group has found the most.

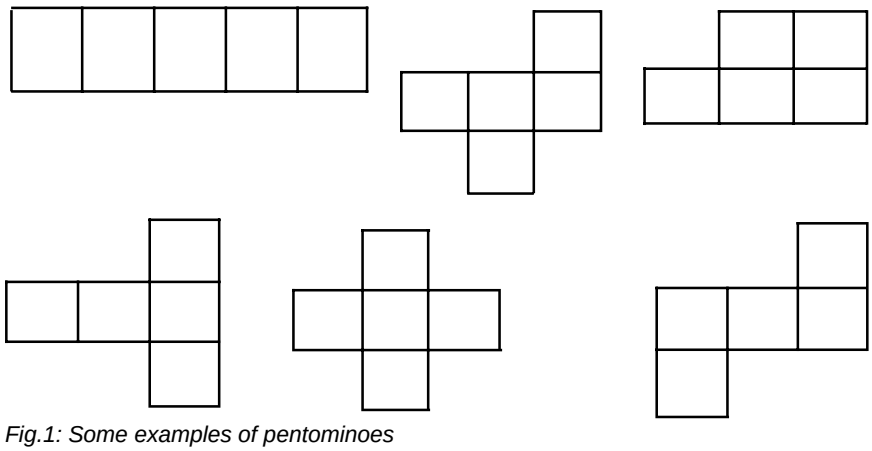


Fig.1: Some examples of pentominoes

Teaching approaches

The teaching approach required for this course is student-centred learning. It promotes the philosophy of 'how' to think and not 'what' to think. Student-centred teaching and learning activities include practical investigations and problem solving as described above. These provide opportunities for students to work cooperatively, to discuss, make decisions, plan, organise, and carry out activities, record results and report findings. Activities should also allow the children to listen to each other's opinions, demonstrate their strategies and critically analyse results. The teaching and learning of concepts promotes the philosophy of 'known to unknown', building on what the students know and teaching concepts using similar contexts to promote better understanding. The teaching and learning approaches must be student-centred and as much as possible, student-directed.

Use of vernacular

While it is recognised that English is the main language of instruction, it must also be recognised that students are still more familiar with their vernacular. You should not only accept the use of these, but encourage their use where it will lead to better understanding. Mathematics is a language in itself and different from any other languages. Therefore, it is believed that the use of the students' first languages will help them to understand better when dealing with mathematical activities.

Integration with other subjects

Some topics or teaching strategies used in the Upper Primary Mathematics course are also dealt with in other subject areas. These topics include measuring, drawing, classifying, time, collecting and presenting data, graphing, money, decimals and percentage. The skills and knowledge taught in Mathematics are used widely in other subjects. Content from other subjects also provides suitable contexts in which to teach Mathematics. For example, if students are studying budgeting or how to run a small business in Making a Living, this would provide a good opportunity to introduce decimals and percentage in Mathematics lessons.

As this course is designed to be student-centred, there is a need for considerable flexibility in programming. The majority of Mathematics topics can be taught in any order that suits the needs of the students. You should take advantage of this flexibility to maximise the links with other subjects. An example of an integrated approach or links with other subjects is provided in the Units of Work section of this document.

Use of multigrade teaching

The contextual approach used for this Syllabus lends itself well to multigrade teaching as the same context can be used for all students in a multigrade classroom. The more advanced students tackle more sophisticated work and others address similar issues at a simpler level. If you are teaching multigrade classes, you need to plan your program of work carefully so that students do not repeat the same contexts. This allows the students to progress through the stages described by the outcomes for each Grade. An example of multigrade teaching is provided in the Units of Work section of this document.

Assessment, recording and reporting

Assessment is the ongoing process of identifying, gathering and interpreting information about student's achievement of the learning outcomes. Assessment using an outcomes-based approach is criterion-referenced and provides information on the actual learning that has taken place. This information is used to enhance further teaching and learning and to provide the basis of reports on students' progress. To do this effectively, it is necessary for teachers to keep accurate records of assessment information. Teacher records must describe the students' achievement of the learning outcomes for the purpose of:

- checking students' progress,
- planning and programming future learning,
- reporting students' achievements to parents, guardians and others,
- informing students about their own progress.

The following information has been extracted from *The National Assessment and Reporting Policy* (2003) that provides guidelines about assessment and reporting for Papua New Guinean schools.

Purpose

The purpose of assessment is to improve students' learning and is focused on students' demonstrations of learning outcomes. The information obtained from this assessment will be used to:

- provide feedback to the individual learner on their progress towards achievement of the learning outcomes,
- make decisions about students' learning and to provide information to improve teaching and learning,
- report students' achievement to parents, guardians, students, teachers and others.

Principles

Assessment and reporting must be culturally appropriate for Papua New Guinea. For assessment to be effective, it should:

- be continuous and based on the learning outcomes,
- be based on a balanced approach,
- be valid and reliable,
- reflect equity principles, by being fair, sensitive and broad enough to cater for differences in gender, culture, language, religion, socio-economic status and geographical locations,
- be an integral part of teaching and learning,
- provide opportunities for students to take responsibility for their own learning and to monitor their own progress.

Upper Primary

Assessment at Upper Primary should:

- be flexible and use a range of assessment methods,
- be continuous and show the development of knowledge, skills and understanding in all school subjects,
- use local cultural approaches to assess and report students' achievement where appropriate,
- be mainly internal but may include external assessment at the end of Grade 8,
- result in the issue of national certificates of basic education approved by the Board of Studies reporting academic achievement, attitudes, values and other relevant achievements.

Responsibilities of teachers

Teachers have a responsibility to:

- develop and implement effective assessment and reporting practices within school assessment and reporting programs,
- discuss with students the assessment recording and reporting procedures that meet the learning needs of individuals and groups of students,
- develop students' knowledge, skills and understanding of effective assessment and reporting methods,
- maintain and share relevant records of students' progress whilst maintaining confidentiality where appropriate,
- plan tasks and activities, which provide sufficient evidence to show that particular learning outcomes have been achieved,
- report students' progress and achievement to students, parents, guardians, teachers and others,
- use assessment information to inform and enhance their teaching and learning practices,
- use assessment information to Guide students to career paths,
- make valid reports on students' achievements of outcomes, attitudes and values using the appropriate reporting or certification system.

Responsibilities of students

Students have the responsibility to:

- use assessment information to improve their learning,
- ensure that they reach their highest potential.

Processes for assessment

The process for assessment

When assessing students' achievements of the outcomes you need to:

- provide students with opportunities to demonstrate what they know, and can do based on the experiences and the required learning outcomes,
- gather and record evidence of students' demonstration of the learning outcomes,
- make judgements about students' achievement of the learning outcomes.

Process for developing a specific assessment task

When you are programming your teaching and learning activities and units of work, you will also need to plan assessment tasks to assess students' achievement of the specified outcomes. When you are doing this:

- identify which part or parts of the outcome you are going to assess; knowledge, skills and attitudes,
- select assessment methods that will best give you the evidence you need to see if students have achieved the outcome or parts of the outcome during the unit of work,
- develop assessment criteria by breaking down the knowledge, skills or attitudes to show what the students will do to complete the activity successfully,
- develop a manageable way of recording your assessment information.

Recommended assessment methods

These are some methods or strategies that you can use to assess students' performance in Mathematics.

Observation class list

When using observation as an assessment method you can have a blank class list which you canuse to annotate record information about each student's achievements. It is a good idea to use this class list to record unexpected or interesting behaviours that individual students demonstrate. For example, students who do not usually answer questions might give some good responses. This should be recorded on the class list. An example of an annotated class list is shown below. Keep this list on your desk and add comments as you observe students during lessons.

Names of Students	Classroom Observations
Neilopal	confused with area of circles
Sarona	having difficulties with mixed fractions
Brian	understood algebraic expressions well
Pearson	
Philip	

Demonstrated Achievement Checklists

In this strategy a checklist is used to record students' achievements. At the completion of a unit of work, for example, you could use a checklist to record which students had demonstrated achievement of the outcomes in that unit of work. List all the expected outcomes for that unit of work against the students' names. As each student demonstrates achievement of the outcome, check that outcome off on the list against that student's name. Also keep consistent and accurate information in a file for that particular student. Use a tick when a student has achieved the outcome and leave the column blank if the student still needs to fully demonstrate the outcome. Go back and tick that outcome off at a later stage when the student has achieved the outcome. An example of this method of recording is shown below.

Names of Students	Grade 6 outcomes from the unit of work					
	6.1.1	6.1.2	6.1.3	6.1.4	6.1.5	6.1.6
Hewane	✓	✓	✓	✓		
Hongone	✓	✓	✓	✓	✓	✓
Egari	✓	✓				✓
Hengene			✓	✓	✓	✓
Helo						
Tailawe						

Samples of students' work

You can collect a student's work as a record of his or her performance. Keep a folder for each student. Every time a student produces a new piece of work you should discuss with the student whether or not it should go into his or her file. Only a piece of work that shows new understanding or improvements on the old skill should go into this file. If it is an improvement on the older work, then the older work can be removed from the file to have enough space in the folders.

Profiles

You may set a series of mini-projects or exercises and keep samples of work from each student on file, maintaining a folder of work for each student. Student exercise books fulfil this function to some extent. Examination of students' exercise books can give a good indication of the students' level of understanding, especially if the work in their books is their own and not copied off the board.

Another form of maintaining such profiles is to only retain the best and latest version of the student's work. For instance, a student may not have demonstrated an ability to carry out long division in March, but in June does do so. Under this approach only the work produced in June will be kept, demonstrating that that particular outcome has been achieved**.** You must make the entries in the checklist progressively and consistently.

Talking with students

You should use both good questioning techniques in class and informal discussions with individual students to get a picture of each student. Relevant points from such talks must be noted down, for example, whether or not the correct mathematical terms were used to answer a question or whether or not the student showed understanding of a concept. You can also watch and listen carefully to what the students are doing and saying. Formal interviews may also provide further information. Sometimes the discussions can be focussed on particular assessment criteria and in other cases the discussions may be more open and informal. A table such as the one below can help you record such information. In this case the teacher has focussed on four assessment criteria to determine if students:

- use mathematical terms to answer questions,
- answer questions accurately,
- demonstrate an understanding of the concepts,
- any other comments.

Sample record sheet to use when talking to students

Names of students	Assessment criteria			
	Use of mathematical terms	Answers questions accurately	Understanding of concepts	Comments
Zinnia	used the term denominator correctly	answered $^2/_3 + {}^1/_3$ correctly		
Rosalind	had difficulty trying to work out the term denominator	did not get $^1/_5 + {}^1/_3$ correct	lacks understanding of fractions	
Norman				
Peabe				
Terence				

Observing students during lessons

You should make informal observations and keep notes of these on a class list or class grid. Choose just one or two students in any one lesson and note down both positive and negative things they do during that lesson. Continue with different students until you have some comments on all students, then repeat the process. Date each observation you make to show how students progress over time.

Sample record sheet for observing students during lessons

Names of students	Positive things done during the lesson	Negative things done during the lesson
Elizabeth	answered $^1/_2 + ^1/_4$ without actually calculating	
Steven		did not respond to $^1/_3 + ^2/_3$. was copying someone's work
Zinnia	answered all questions correctly in addition of common fractions	
Rosalind		
Norman		

Keeping records of practical work

These may include models students have made, assignments they have completed and any other work that demonstrates their ability. This should be labelled with the students' names and the date and displayed in the classroom where everyone could see them.

Tests

These may be short answer or longer exercises. They may be open book or strictly from memory. There are many possibilities. Multiple-choice tests are the most difficult to write well. Those written for examinations undergo rigorous trialling and editing before use and even then they do not always achieve what was intended. You are advised to use other types of questions.

Lots of short tests will provide more relevant and up to date information to alter teaching than longer infrequent tests. An end of unit test does not give you the information needed to alter your approach to that unit until it is too late. A ten-minute quick quiz every week can provide timely clues to the success or failure of a teaching strategy.

Assessment Tasks

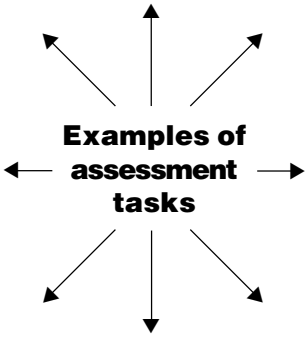
These assessment tasks are applicable at Upper Primary level

Writing
- writing symbols correctly
- setting out work
- explanations
- assignments
- projects
- definitions
- formulas and use of rules

Procedures
- comparing
- putting forward possible solutions
- ordering
- estimating
- presenting information
- inferring
- performing the four basic operations
- critical thinking

Keeping records of practical work
- models
- work samples
- class and group projects

Process
- identifying
- classifying
- investigating
- recognising
- predicting
- questioning
- drawing conclusions
- taking measurements
- observing
- interpreting



Oral tasks
- asking questions
- answering questions
- explaining
- describing
- relaying information
- practical concepts

Visual presentations
- posters
- drawings
- flow charts
- diagrams
- maps
- tables
- graphs
- labels

Practical tasks
- models
- displays
- outside activities
- excursions
- practical activities
- solving problems
- drawing
- measuring
- manipulating materials
- applying knowledge and skills

Attitudes
- respect
- caring
- cooperativeness
- acceptance
- perseverance
- appreciation of subjects
- enjoyment of subject

Research
- small group or independent research
- conducting surveys and interviews
- analysing research
- presenting findings

Assessment tasks with samples

This table expands on the assessment task examples shown above. It takes one example from each of the categories described and provides a sample for that task. These tasks provide examples on how to assess the skills, knowledge and attitudes specified in the Mathematics learning outcomes for Grades 6–8.

Assessment task categories	Examples of assessment tasks	Samples
Writing	Writing symbols correctly	π = Pi $\angle$ = angle > = greater than < = less than $\therefore$ = therefore
Keeping a record of practical work	Class and group projects	Assess according to assessment criteria and display in central place
Visual representation	Drawings	Draw a plan of a house using the correct scale and measurements
Practical tasks	Practical activities	Apply the correct measurements to measure garden plots
Procedures	Presenting information	Measure the dimensions of a football field and report the findings to the teacher and the class
Process	Investigating	Investigate the formulas to work out area of circles, $A = \pi r^2$
Research	Small group research	Small groups of students investigate the lengths of small insects and report back
Oral tasks	Explaining	Students can explain the process they took in their research findings
Attitudes	Appreciation of subjects	Appreciate that Mathematics deals with everyday problems, such as buying and selling of goods

Recording

Record keeping provides accurate evidence drawn from a range of contexts about students' learning related to the demonstration of the learning outcomes. Record keeping must support planning and be manageable and easily maintained.

Records must describe students' achievement of the learning outcomes for the purpose of:

- checking students' progress,
- planning and programming future lessons,
- reporting students' progress or achievements to parents, guardians and others,
- informing students about their progress.

Teachers need to keep appropriate recording methods to gather evidence of students' achievement of the outcomes. Models of recording may include:

- checklists for individuals, groups and the whole class,
- class record sheets,
- profiles,

- annotated class lists,
- portfolios,
- progressive records.

Some examples of different types of recording have been described in the previous section on recommended assessment methods. A further example is provided to you below.

E

Class record sheet for observing students during lessons

Make up a sheet like this to keep an ongoing record of your students' progress.

Code to explain marks

5 indicates the student met the criteria fully

4 indicates the student met the criteria most of the time

3 indicates the student met the criteria some of the time

2 indicates the student rarely met the criteria

1 indicates the student needs further practice to meet the criteria

Date	Names of students	Assessment criteria			
		Uses mathematics terms correctly	Answers questions	Shows understanding of concepts	Total
	Maximum marks	**5**	**5**	**5**	**15**
	Elizabeth	4/5	3/5	3/5	10/15
	Steven	3/5	3/5	3/5	9/15
	Rosalind	4/5	4/5	5/5	13/15
	Zinnia	4/5	4/5	5/5	13/15
	Norman	2/5	3/5	2/5	7/15

Reporting

Reporting is a way of presenting evidence of students' achievements of the demonstration of learning outcomes. It can be drawn from ongoing observations of performance and from assessment tasks. You can make judgements about students' demonstration of learning outcomes when you are satisfied and have sufficient evidence that they have achieved the outcome.

Reporting student's achievement of the learning outcomes should be made:

- without reference to the performance of other students,
- when the teacher is satisfied that the outcomes have been demonstrated consistently to a high standard and in a range of contexts,
- when a variety of assessment, recording and reporting methods have been used.

Students, parents and guardians are entitled to receive feedback about students' progress towards achieving the intended outcomes. This information must be clear, accurate and fair so that all concerned will take part in helping students to improve their standards of achievement. Schools will decide how reports will be presented to best suit the needs of the community.

Some modes of reporting are:

- written report cards,
- records cards,
- certificates,
- letters,
- interviews.

Further information on reporting can be found in *The National Assessment and Reporting Policy* (2003).

Evaluation

Evaluation involves gathering and interpreting information to make judgements about the effectiveness of Mathematics teaching and learning. Evaluation is based on information gained from measurements, descriptions, observations and judgements about students' demonstration of the outcomes. The purpose of evaluation is to improve students' learning and the effectiveness of your teaching and learning program in Mathematics. Through evaluation, you can develop better strategies to teach Mathematics and liaise with the community to more actively participate in the students' learning process.

Evaluation needs to be planned. Too often evaluations ask questions which are irrelevant or collect information which is of no use to the evaluation's desired purpose. It is important that you think carefully about the purpose of conducting any evaluation and the procedures you are going to use. Things such as where to start planning, what to include, and how it will be conducted should be considered during and when planning evaluations.

Programming

Sample yearly breakdown of Strands

When developing a program for the year, you need to balance the time spent on each of the Strands. To assist you with this, an example has been included that shows how schools frequently allocate time across Grades 6–8.

Strand	Weeks
Number and Application	9
Space and Shapes	14
Measurement	6
Chance and Data	5
Patterns and Algebra	4

The school program will need to determine which Sub-strands are taught together and which are taught alone. This will require breaking allocation of time into finer components for the Sub-strands. A sample of this detail is provided below.

Sample term by term breakdown of Strands and Substrands

The sample break down of Strands and Substrands below is only for Number and Application. You can do a similar breakdown using the yearly breakdown of the Strands. This will be entirely up to you and your school or the Upper Primary teachers to decide during programming.

Strand	Sub-strand	Week
Number	Fractions	2
and	Decimals	1
	Fractions and Decimals	1
	Decimals and Percentage	1
Applications	Ratio and Rates	2
	Directed Numbers	1
	Indices	1
		9 weeks

The selected combination of Substrands is then expanded in terms of the appropriate outcomes and learning experiences in the teaching program. A sample of this is provided for you.

Sample weekly breakdown of lessons

Mathematics

Grade: 6 **Term:** 1

Week: 4 and 5 **Year:** 2003

Strand: Number and Application **Sub-strand:** Indices

Week	Outcome	Indicators	No. of lessons	Lesson topics	Time	Assessment tasks	Resources comments
4	**6.1.8** Use indices to the power of 2 and 3	Calculate solutions to numbers such as 3^2 and 4^3	2	Calculate numbers to the second power Calculate numbers to the third power	30 mins 30 mins		Square root table
		Express products like 4 x 4 as 4^2, 9 x 9 as 9^2 and 3 x 3 x 3 as 3^3	1	Express products according to their powers	30 mins	**Quiz** Give a 10 minute quiz on square and cube roots	
		Recognise 1, 4, 9, 16, 25, 36, 49, 64, 81,100 as square numbers	2	List down the square numbers Identify the pattern and list the next square numbers to 300	30 mins 30 mins	**Assignment** Students calculate square numbers to 500	
5		Recognise 1, 8, 27 and 64 as cubed numbers	2	List down the cubed numbers Identify the pattern and list the cubed numbers to 300	30 mins 30 mins	**Project** In groups of 5 students calculate the cube roots to 1000	

The weekly program can be further broken down into daily lesson plans. A sample of this is provided for you. You are given the flexibility to make amendments to the samples provided or to write your own to suit the needs of your students.

Sample daily lesson plan

Date: 24th – 28th February, 2003

Grade: 6 **Term:** 1

Week: 4 **Year:** 2003

Strand: Number and Application **Substrand:** Indices

Outcome: **6.1.8** Use indices to power of 2 and 3
Refer to *Worked Examples for Upper Primary Mathematics Outcomes*

Indicators: Calculate solutions to numbers such as 3^2 and 4^3

Number of Lessons: Two lessons for this indicator

Lesson No 1: Calculate numbers to the second power

Resources and Materials: Square root table, rulers, set of square numbers, pencils and paper

- *Upper Primary Mathematics Syllabus*
- *Worked Examples for Upper Primary Mathematics Outcomes*

Introduction: Identify students' prior knowledge by asking questions such as:

- What is a square number?
- Ask for examples of square numbers from students.
- Ask students to calculate solutions to 2^2 and correct answers together.

Activity: Students calculate solutions to the following:

a. 3^2	b. 5^2	c. 7^2
d. 9^2	e. 4^2	f. 8^2
g. 12^2	h. 10^2	i. 6^2
j. 11^2		

Teacher moves around checking and correcting students' work.

Conclusion

- Ask 2 students to write their answers on the board.
- Correct the next three questions.
- Collect and mark the other 5 questions.

Evaluation

Students found this lesson very interesting and enjoyed it. They understood the activities well. In future square roots and cube roots could be taught together as one rather than as separate lessons.

Suggested teaching time

At present, the total time allocation for teaching Mathematics at the Upper Primary level is 180 minutes per week. You are encouraged to be flexible in your timetabling and programming. You should consider your students and available resources to make the best use of this flexibility.

These are some options you may want to consider when allocating times in your planning and programming:

- teach six 30-minute lessons per week,
- teach five 30-minute lessons and use the extra 30 minutes to complete, revise or extend the activities already taught or to do a special activity,
- teach three 60-minute lessons per week,
- teach four 45-minute lessons per week,
- teach four 40-minute lessons and use the extra 20 minutes for revision exercises, extension work or projects that are integrated with appropriate units or lessons in other subjects.

The above time allocations are only some of the many options available. You should plan your times based on the nature of the teaching activities you are planning for your students. It is highly recommended that you combine some of the Mathematics time allocation with other subjects to teach an integrated topic. For example, when teaching graphing skills, budgeting or profit and loss, combine with appropriate activities in Making a Living. A sample of a unit of work using an integrated approach to planning and teaching is provided in the Units of Work section of this Teachers Guide.

Units of work

What is a Unit of Work?

A unit of work is a series of sequenced learning activities aimed at achieving a learning outcome or outcomes. It has a clear purpose and provides opportunities for all students to engage in furthering their understanding. Consideration should be given to the relationship between Sub-strands when developing a unit. Students need to be provided with more than one opportunity to achieve a Mathematics outcome.

There are two main methods of creating an outcomes-based unit of work: start ing with the outcome or starting with the context. Using the first approach means selecting an outcome or outcomes and planning learning experiences that will assist students to achieve those outcomes. The second approach begins with a context or a rich task**,** such as *Does a square peg fit in a round hole better than a round peg in a square hole?* The related Mathematics outcomes are then determined that relate to this task. In this example the related outcomes are outcomes 8.2.5 and 8.2.9.

Different types of units of work can be planned to assist students to achieve the outcomes in each of the Strands. Four different types of units of work have been included in this Guide. They include a multigrade approach, contextual approach, an integrated approach and a unit designed around a single learning outcome. Remember to be flexible in your planning and programming. Mathematics, unlike other subjects has a large number of outcomes for the five Strands. You are encouraged to use the approaches that focus on more than one outcome where appropriate. You should also look at the sequence of learning described by the outcomes and align the units to the needs of your students.

Sample 1: Starting with a single outcome

Select an outcome and sequence learning experiences that will assist students to achieve this outcome. This unit of work is suitable for Grade 6 and is designed to achieve outcome 6.1.1 in the Fractions Sub-strand of the Number and Application Strand.

Overview of the Unit of Work

This unit focuses on addition and subtraction of mixed numbers and builds on the earlier fraction work from Grade 5. Also covered in this unit are addition and subtraction of proper and mixed fractions with the same denominators and different denominators where lowest common denominators must be calculated.

Concrete and pictorial representation of fractions should be used throughout this unit to help with explanations. Integration of this unit with other Mathematics Strands or other subjects is very much encouraged.

Material	Context	Linking units	Links with other subjects
• fraction kits, improvised if necessary • paper • scissors	• gardens • trade stores • schools • markets	• decimals • percentage • ratio and rates • measurement	• Social science • Making a Living • Arts • Language • Science • Personal Development

Outcome: 6.1.1 Add, subtract, multiply and divide fractions

Indicator: Add and subtract fractions and mixed numbers including those with different denominators

Lesson Overview

Lesson 1: Determine students' prior knowledge (30 minutes)

Lesson 2: Addition and subtraction of simple proper and mixed fractions with the same denominators (30 minutes)

Lesson 3: Lowest common denominators (30 minutes)

Lesson 4: Addition and subtraction of mixed number fractions with different denominators (30 minutes)

Lesson 1: Revision of key fraction words

Teacher's notes	Lesson development
Introduce common fraction words such as proper fraction, improper fraction, equivalent fractions, common denominators, different denominators, mixed numbers, lowest common denominators (LCD) • proper fraction — denominator larger than the numerator • improper fraction — numerator larger than the denominator • mixed number fraction — fractions which contain a mixture of whole numbers and fractions such as $2\frac{1}{2}$ and $3\frac{2}{5}$ • equivalent fractions — fractions with the same value but with different numerators and denominators such as $\frac{1}{2}$ and $\frac{48}{96}$. • common fractions — both the numerator and the denominator are integers such as $\frac{1}{2}$ and $\frac{3}{4}$ • common denominators — denominator of fractions are the same such as $\frac{2}{3}$ and $\frac{1}{3}$ • different denominators — denominators of fractions are different such as $\frac{4}{5}$ and $\frac{5}{7}$ Common denominators should be taught before addition or subtraction is applied • Lowest common denominator (LCD) — number that is divisible by the given denominators. For example for $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{3}{10}$, the LCD is 60	**Objectives** • to revise key fraction words • highlight the key fraction words and applications when working with fractions **Introduction** • Have the words written on a chart or on the blackboard. • Explain that the lesson will focus on fractions and that before we start, we need to see what we already know. • Ask the students to copy the words in their books into a table with three columns — word, meaning 1 and meaning 2. • Ask the students to write down what they think each word means in the column headed meaning 1. • Explain the words to the students using concrete materials where appropriate. • Have students discuss and use them in simple sentences. • Have students rewrite the definitions in their own words in the third column following the explanations. **Development** See *Worked Examples for Upper Primary Mathematics Outcomes*

On completion of Lesson 1, teachers would proceed with Lessons 2–4, taking the time needed for students to understand the processes involved in addition and subtraction of fractions and mixed numbers. When the students have had sufficient practice, but before completing the unit of work, students would be asked to complete the following assessment task.

Assessment Task

Ask your students to write a letter to a friend who is not in the class to explain what they have learnt about fractions. In particular, each student should include some examples of addition and subtraction of fractions including those with different denominators.

Sample 2: A multigrade unit for Grades 7–8

Steps for developing a unit of work

1. Determine what your students currently know.
2. Identify the outcomes and the indicators that your students need to address next.
3. State the purpose of the unit.
4. Identify teaching strategies and learning activities, organised in a logical sequence.
5. Identify ways of assessing your students' learning.
6. Estimate the time needed for the unit of work.
7. Identify resources that may be required.
8. Program the unit of work within your yearly or term-by-term overview.

Identify the outcomes

Grade: 8

Outcome 8.2.9: Make physical models of circles and investigate their properties

Indicators

- draw circles on cardboard and cut them out
- mark diameter, radius, centre, circumference, arc and segment on circles they have drawn and cut them out
- show relationships between parts of circles, such as circumference and diameter, diameter and radius, sector and arc

Grade: 7

Outcome 7.2.9: Draw, investigate and make physical models of quadrilaterals.

Indicators

- make models of quadrilaterals: parallelogram, rhombus, kite and trapezium using paper, cards, sticks, wires
- make models of quadrilaterals and name them, such as parallelograms
- draw quadrilaterals such as parallelograms, rhombuses, trapeziums and kites
- identify the properties of quadrilaterals drawn by students, such as the number of parallel sides, number and position of equal length sides equal angles, complementary angles and axis of symmetry.

Purpose

In this unit Grade 8 students investigate the properties of circles and use them to make physical models.

In this unit Grade 7 students investigate the properties of quadrilaterals and polygons.

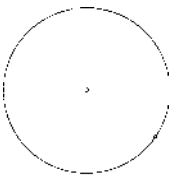
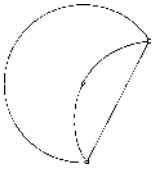
Development of the unit

Activities have been developed largely for the Grade 8 students below, exploring the properties of circles. You would need to plan similar activities and expand upon the activities given for the Grade 7 students about properties of quadrilaterals. Teachers need to manage the students' activities so they can work with each Grade separately as well as some combined lessons where appropriate.

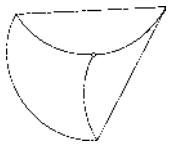
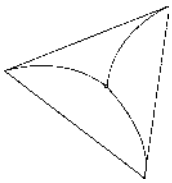
As an introductory Grade 8 activity, ask your students to draw a circle and name as many parts of the circle as they can. Have them put this aside.

Distribute sheets of paper or light card, 1 sheet per person if available. Used sheets of paper are satisfactory. Have each student draw about 4 to 6 circles on the paper or light card. Students can use templates such as lids or a roll of tape to trace the circles. All of the circles on the sheet of paper should be the same size. Have your students cut out the circles they have drawn. Ask each student to select a circle from those they have cut out.

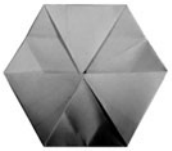
Activity One

Instructions	Teachers questions	Responses
1. Hold your circle.	What do we call the distance around the circle?	circumference
2. Fold the circle in half.	What is the creased line called?	diameter
3. Fold the circle in half again to find the centre. Mark the centre.	What is the distance from the centre to the circle's circumference called?	radius
4. Mark a point on the circumference of the circle. 	What is this creased line called?	chord
5. Fold this point to the centre of the circle. 	What is this folded part of the circle called?	segment
	What is the folded part of the circumference called?	arc
6. On your circle make a summary of the parts of the circle we have covered so far. Your summary should include circumference, centre, diameter, radius, chord, segment and arc.		
7. Compare your summary to the circle you drew earlier.		

Activity Two

Instructions	Teachers questions	Responses
ake another circle. Mark a point on the circumference of the circle. Fold the point to the centre of the circle. Fold again to the centre using one end point of the chord as an endpoint for the new chord. Model this for the students.		
4. Fold the remaining arc to the centre. 	What shape have you made?	Triangle
	What type of triangle is it?	Equilateral
	How do you know?	By folding along the axis of symmetry you can show the sides are the same length.
	What do the angles in the triangle add up to?	180°
	How big are the angles?	60°
	What is an axis of symmetry?	A line where you can fold one half into the other.
	What do you get if you fold the equilateral triangle along an axis of symmetry?	Right-angled triangle
5. Fold the equilateral triangle along an axis of symmetry and then unfold it.	How do you know where the midpoint of one side is?	The two parts are the same. The line of symmetry passes through the midpoint.
6. Point to the vertex opposite the midpoint formed by the fold. Fold the opposite vertex to the midpoint. 	What shape is formed?	Quadrilateral Trapezium
	What do you know about a trapezium?	One pair of parallel sides equal
	As well as the trapezium, what else do you see?	3 equal triangles
7. Fold one of the outside triangles over the middle triangle. 	What polygon has been formed?	Rhombus
	What do you know about a rhombus?	• Like a 'pushed over' square • 4 sides equal. • 2 pairs parallel sides • opposite angles equal
	How many axes of symmetry does a rhombus have?	Two

Activity Three

Instructions	Teachers questions	Responses
1. Take a new circle and make an equilateral triangle.		
2. Fold each vertex of the triangle to the centre of the circle. 	What polygon has been formed?	Hexagon
	How many edges does it have?	6
	How many vertices does it have?	6
	How many axes of symmetry does it have?	6
	How many pairs of parallel sides?	3
	How many equilateral triangles do you see?	6
	What is the size of each angle in an equilateral triangle?	60°
3. Copy this hexagon in your books.	What is the size of each angle in a regular hexagon?	120° (2 lots of 60)
4. Revision of Outcome 6.2.10 Explain how this hexagon shows that equilateral triangles make a regular hexagon?	What is a tessellation?	A tiling that covers the plane without gaps
	Do equilateral triangles tessellate?	Yes
	Do regular hexagons tessellate?	Yes

Assessment tasks for the unit of work

Students can be assessed in different ways from the unit of work. One way would be to have students accurately make physical models of the polygon shapes. This can be done in groups or individually. Another way could be to have the students name the properties of the shapes or angles created by the shapes. The teacher's questions section in the table above contains some questions that can be used to monitor students' learning. Refer to the section on assessment in this Teachers Guide for information on assessment methods and tasks.

Estimated time required for the unit of work

This unit can be planned and taught effectively in three to four lessons. However, the time will vary depending upon the background knowledge of your students and the resources you may have.

Resources

You should have available templates, such as tins, to draw circles. Used paper, rulers, scissors and photocopies or drawings of a variety of polygon shapes would be useful to demonstrate to your students during and after your explanations.

Sample 3: A contextual approach

This unit will take a number of weeks and careful planning is needed to consistently teach the lessons from the context provided.

This unit uses the context of the end of the year class party to introduce a series of real problems associated with the application of numbers.

In describing a class party:

- determine what needs to be done,
- plan the activities to generate money,
- report the plans to the other school staff members, Board of Management and the parents. Recordings of the process might include using photographs, posters and even poetry or songs.

A great many applications of number arise in planning a class party. For example teachers and students can discuss and decide how many students will be involved, the ways to raise the money needed, what needs to be purchased with the money and how to estimate what each person will eat. These processes all use Mathematics. We could use statistics in determining the favourite foods of the class, listing the types of activities that will be carried out to raise money or calculating the total amount of money needed to hold the party.

Links with other Mathematics Substrands

The mathematical content for this unit is applications of number. The context of a class party provides a reason to apply Mathematics in planning and costing. It draws on the Substrands of fractions, decimals and ratios in the Syllabus. However, the further you move on, you can teach almost the entire Strand on number and application, measurement and even space and shapes. The specific outcomes addressed will be determined by the choice of teaching and learning activities arising from the context.

To give you some idea on how you can move from context to focused Mathematics lessons, a sample involving ratios when mixing cordial is provided.

Context: Class party

Grade 6 students planned to have a class party at the end of the school year.

The table below indicates the type of activities they planned to do to raise the money needed and the amount they expect to collect.

No.	Money raising activity	Estimated amount to be raised	Money actually raised	Total
1	sale of fruits, sweets and cooked food	K154.00	K114.00	K114.00
2	…	…	…	…
3	…	…	…	…
	Total amount	…	…	…

The table below was used to determine the cost of items needed for the class party.

No	Food types	Quantity	Amount	Persons responsible
1	6 L cordials, pineapple and strawberry	3x 2 L bottles @ K7.00 each	K21.00	Teacher
2	…	…	…	…
3	…	…	…	…
	Total Amount	…	…	…

A sample of a focused Mathematics lesson from the context

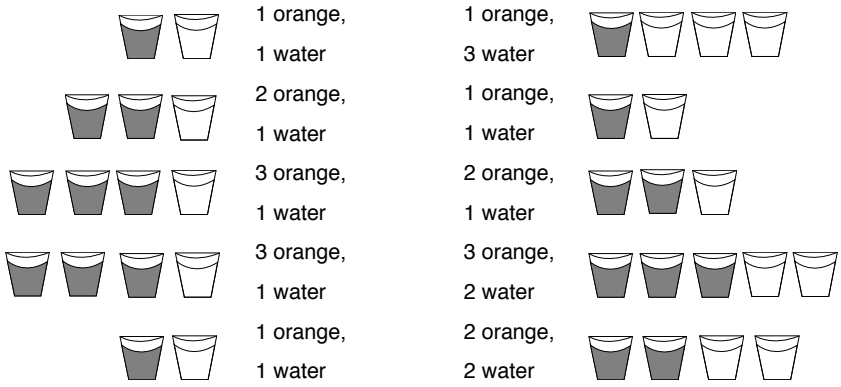
Activity: Mixing cordial

When the class decided that cordial would be bought to provide drinks, the question arose of how much cordial should be bought. Was 6 litres enough or too much? Knowing how many people would be at the party was not enough information to work out this problem. Even allowing two glasses of cordial for each person was not enough information. We needed to use ratios.

Outcome 6.1.5 requires that Grade 6 students recognise ratios. Adding water to cordial to work out a good mix for the cordial helped to achieve this outcome.

1. Make sure that students know what is meant by strong and weak cordial. Mix up two glasses of cordial, one very strong and one glass of weak cordial. Place the different mixtures on two separate tables. Invite one student to pick the strong from the weak cordial by tasting and describing the differences.
2. Have the class select an 'official taster'. Put out three glasses or plastic cups the same size on the teacher's desk. Ask the class to describe what fraction of the glass should be cordial to make the perfect mix. Record the answers on the board. Question any fractions over one-half.
3. See if the class can put the fractions in order. Listen to the explanations as to why one fraction is bigger than another. Invite students to the board to draw a glass and mark the height of the fraction on the drawing.
4. Have students vote on which fraction they think will make the perfect cordial drink.
5. Choose the three most popular fractions and pour these amounts of cordial into the three glasses. Add water to fill the glasses, asking what fraction of each glass should be water.
6. Have the official taster choose the perfect mixture and record the result as a ratio. For example, if the perfect mixture was one-third cordial this would be recorded as one part cordial to two parts water or 1:2.
   One quarter cordial would be one part cordial to three parts water or 1:3.

7. Draw the following on the board and ask which is the stronger cordial mixture in each example, A or B that are shown horizontally, or are they the same.



8. The investigation can be continued to work out how many glasses of the 'perfect mix' can be made from one litre of cordial and then calculate how many bottles are needed, taking into account the other information about numbers of students and number of drinks per person.

Assessment tasks for the unit of work

Your assessment could involve having students draw some equivalent ratios.

The above are not the only activities you can set for your students. There are many that you can do from the context outlined. You may add other outcomes so long as they relate to this context. Make the best out of this flexibility and teach a number of outcomes. Remember that your focus is always the students. If one of your students does not understand, you will have to look at other strategies to help that particular student.

Estimated time required for the unit of work

This unit of work is expected to take three to four weeks, depending on how you plan to carry out the activities outlined. It will very much be determined by what activities you select and your students' ability to understand the concepts involved. You are encouraged to be flexible and adjust your time to meet the needs of your students.

6.4 Sample 4: An integrated approach

How to develop an integrated unit of work

Here is a process and a sample for a unit of work integrating Mathematics, Social Science and Making a Living.

Step 1. Study the content overviews from the selected subject syllabuses that show the Strands, Substrands and outcomes.

Check the syllabuses for the subjects concerned and scan through the Strands and Substrands.

For example, in this sample study the content overview for Mathematics, Social Science and Making a Living.

Step 2. Identify the learning outcomes.

Look through the syllabuses and identify two to four learning outcomes that link naturally together through similar concepts and processes. Pull out the main ideas from each of the selected outcomes. For example, in this unit of work the following outcomes have been grouped together.

Social Science

7.1.4 Describe national and regional sustainable practices related to the natural environment and propose possible solutions toproblems

Making a Living

7.1.1 Investigate and compare consequences of mismanagement of land and water resources and plan, design and undertake a small project using appropriate management practices

Mathematics

7.2.4 Compare areas by estimation

7.2.5 Investigate area rules for quadrilaterals

Select a theme, topic, project or issue that ties these outcomes together. In this case we have selected a project to design and prepare a flower garden.

Step 3. State the purpose of the unit of work by summarising briefly what students will achieve through the selected learning outcomes.

Focus on the key words from each of the outcomes and try to summarise what students will learn in this unit of work to achieve the learning outcomes.

For example, in this unit of work students will consider the importance of sustainable land management practices. They will apply this knowledge by designing and preparing a flower bed. They will compare areas of garden beds by estimation and apply the rules for calculating area.

Step 4. Identify the unit content by stating the knowledge, skills and attitudes that you want students to demonstrate in the unit of work.

Use outcomes and indicators in the Syllabuses, the Teachers Guides and the *Worked Examples for Upper Primary Mathematics Outcomes* to decide relevant content for the unit of work. Write a statement that indicates the knowledge, skills and attitudes that students will learn.

Step 5. Develop and sequence teaching and learning activities.

In this unit of work students will:

- investigate the school surroundings and find out how the land has been used,
- collect information and describe examples of sustainable practices,
- identify problems related to land use such as soil erosion and unused land,
- discuss the causes and effects of the problem identified,
- suggest how to improve the land,

- compare how land is used in two different locations such as two schools, community and school or different parts of the school,
- graphically present the information gathered,
- select a site for their flower garden,
- estimate and measure the area of the flower bed,
- draw a plan of the flowerbed showing where to grow the flowers, spacing between rows and plants, pattern, drainage and the height of the bed,
- dig the soil and prepare a garden bed with drainage,
- apply compost and mulch in preparation for planting.

Step 6. Develop assessment tasks and criteria.

Students will have to perform assessment tasks to demonstrate their understanding of the learning outcomes. Indicate how to assess and record students' achievement of the selected learning outcomes. In this unit of work there are three assessment tasks, one for each subject.

Step 7. Estimate the time required to complete the unit of work.

Consider the duration of the integrated unit of work by looking at the types of activities. You may decide on the unit duration by considering subject time allocation and timetable. For example the activities in this unit of work are estimated for a period of two weeks when considering the time allocations for the subjects:

- Mathematics: 180 minutes,
- Social Science: 180 minutes,
- Making a Living: 360 minutes.

Step 8. List relevant resources.

This is a list of tools, equipment or other materials that are required to teach the specific activities in the unit of work. In this example resources have been identified for each specific activity and incorporated into the teaching program.

Step 9. Develop a teaching program for the unit of work.

The learning activities are organised logically and focused on the project. Consider the time allocation for each subject and the type of project activities.

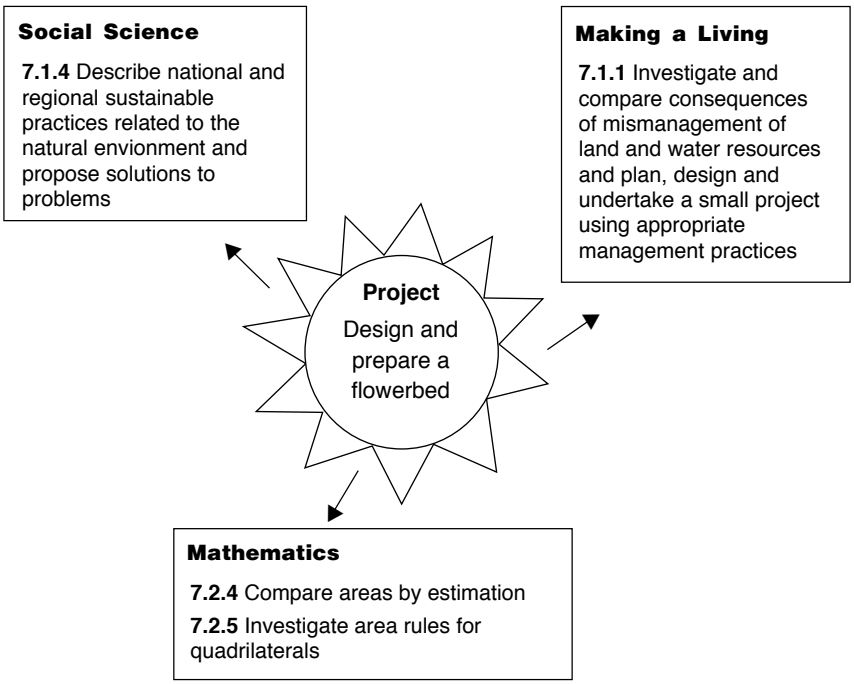
Sample integrated unit of work following the steps outlined

Grade 7 unit of work: Design and prepare a flower bed

Purpose

In this unit of work students consider the importance of sustainable land management practices. They apply this knowledge by designing and preparing a flower bed. They compare areas of garden beds by estimation and applying the rules for calculating area.

Learning outcomes



Content for the unit of work outlining the knowledge, skills and attitudes students are expected to achieve.

In this unit of work students will learn the following knowledge, skills and attitudes.

Knowledge

Students will demonstrate understanding of:

- the cause and effects of mismanagement of land (Making a Living),
- the importance of soil cultivation (Making a Living),
- the meaning of the terms *sustainable practice*, *reuse* and *recycle* (Social Science),
- sustainable practices in relation to land management (Social Science),
- resources that can be reused and recycled (Social Science),
- the rules for calculating area of quadrilaterals (Mathematics).

Skills

Students will develop the skills to:

- produce a plan of a garden bed (Mathematics and Making a Living),
- dig and prepare a garden bed (Making a Living),
- apply composting (Making a Living),
- research relevant sources for information on sustainable practices and resources that be can be reused and recycled (Social Science),
- describe sustainable practices and resources that can be reused and recycled (Social Science),
- apply problem-solving skills (Social Science and Mathematics),
- compare by estimation (Mathematics),
- calculate areas using multiplication and addition(Mathematics),
- apply area formulas in a practical situations (Mathematics).

Attitudes

Students will develop attitudes of:

- caring for land (Making a Living, Social Science),
- appreciation of and respect for careful use of the natural environment (Making a Living, Social Science),
- appreciation of using measurement in practical situations (Mathematics).

U

Sequence teaching and learning activities for the unit

Subject areas	Learning activities	Teaching strategies	Assessment tasks	Materials	Comments
Making a Living	• Investigate the school surroundings and identify problems related to land use sch as soil erosion and unused land. • Discuss the causes and effects of the problem identified. • Compare how land is used in two different locations such as between two schools, between community and school or between different parts of the school.	Excursions		Pen and paper	Collect samples of students' work.
Social science	• Collect information and describe examples of sustainable practices. • Suggest how to improve the land. • Graphically present the information gathered.		Identify a problem on land use.		This is an ongoing activity to be collected at the completion of the unit of work.
Making a Living	• Select a suitable site for their flower garden.	Excursion			
Maths	• Estimate and measure the size of the flower bed. • Draw a plan of the flower bed showing spacing between rows and plants, pattern, drainage and height of the bed. • Use area rules to calculate the actual area of the flower-bed.	Demonstration	Students make a scaled drawing of their flower bed showing correct dimensions and levels.	Tape measure, ropes, strings and metre ruler	Assess students upon completion of the scaled drawing.
Making a Living	• Dig the soil and prepare a garden bed with drainage. • Apply compost and mulch in preparation for planting.	Demonstration	Students prepare the flower bed.	Digging tools	Assess students by observing them as they perform the task.

Assessment plan for the unit of work

There are three assessment tasks for this unit of work. The table below shows how these tasks can be carried out.

Assessment methods	Assessment task	Assessment criteria	Recording method	Comments
Teacher analyses students' work	Mathematics Outcomes 7.2.4 and 7.2.5 Students make a scaled drawing of their flower bed.	Scaled drawing will show: **A** the title **B** an accurate scale **C** where to plant flowers **D** the total area of the actual flower bed	Checklist	
Observation of students performing the task	Making a Living Outcome 7.1.1 Students prepare the flower bed.	Students will: **A** loosen the lumps of soil **B** measure the correct width and length of the bed **C** level the top of the flower bed **D** remove stones, sticks and dead rocks **E** apply compost and add mulch	Ongoing observations on a class grid	
Teacher collects and analyses students' work samples	Students identify a problem about land use.	Students present information that: **A** identifies one land use problem **B** describes the cause and effect of the problem **C** suggests possible solutions **D** names individuals or groups responsible for taking action on the recommended solution	Class checklist	

Estimated time required to do this unit of work

Consider the duration of the integrated unit of work by looking at the types of activities. You may decide on the unit duration by considering subject time allocations and the timetable.

For example, the activities in this unit of work are estimated to take a period of two weeks when considering the time allocations for the subjects:

- Mathematics: 180 minutes,
- Social Science: 180 minutes,
- Making a Living: 360 minutes.

You have 720 minutes in a week to teach this integrated unit. Therefore, if you teach this unit for two weeks, you will have 1 440 minutes to program and teach this unit of work fully.

List of relevant resources

In this section make a list of resources, tools, equipment and other materials that are required to teach each specific activity. In this example resources have been identified for each specific activity and incorporated into the teaching program.

Planning teaching activities from the unit of work

Activity One: Investigation

Divide the class into pairs or groups:

Students will be put in groups of four with two boys and two girls in each group.

Explain the purpose of the activity:

The purpose of the activity is to identify problems related to land use. Students will also find out the causes and effects of the problem and provide possible solutions.

1. The students will:
- walk around the school or the surrounding community and observe how land is used,
- identify problems related to land use such as soil erosion, rubbish dumps, litter or unused land,
- compile the information on the activity worksheet provided below.

Location	Problem	Causes	Effects	Possible solutions

2. Summarise information collected from the students by developing a concept map on land use problems.
3. Discuss the task of making a flower bed and identify the best location for the flower bed.
4. Students give reasons for their choices.

You can fit in the activities in your daily teaching program. How you carry out activities, such as the one above, will be determined by you and your students. You are encouraged to be flexible in your planning and programming based on the needs of your students and the resources available.

Elaboration of Learning Outcomes

Detailed elaborations of learning outcomes for each of the five Strands are outlined in the separate document titled *Worked Examples for Upper Primary Mathematics Outcomes*. It comes as a support document to the Teachers Guide and provides a detailed explanation of steps, processes, formulas, worked examples and samples of most of the outcomes for the five Strands; Number and Application, Space and Shapes, Measurement, Chance and Data and Patterns and Algebra. Please refer to them when planning, programming and teaching your lessons.

You should use the brief elaborations stated below along with the Syllabus and the support document titled, *Worked Examples for Upper Primary Mathematics Outcomes*.

7.1 Number and Application

The application of number reaches across all areas of learning. For example, students can use fractions and decimals to report measurements, to compare survey responses from groups of different sizes or to express probabilities (outcome 8.1.1).

Students need to become fluent in working with fractions, decimals and percentages and moving between these types of numbers (Outcomes 6.1.3, 7.1.3 and 8.1.3). In Grades 6 to 8, students should be able to create and recognise equivalent forms of fractions, decimals and percentages. They need to develop a sense of the relative size of parts of a whole, such as three-quarters, in a variety of forms.

If this diagram shows 3/4 of a rectangle, show where 1/2 of the rectangle would be.

Percentages are a commonly used form of ratio. Adding 10% VAT to the cost of an item is a simple process yet one that is not simply reversible (Outcomes 7.1.4 and 8.1.4). K100 + 10% VAT = K110. A 10% discount on an item marked at K110 is K99!

The idea of proportion connects many of the Mathematics topics studied in Grades 6 to 8, in particular, the Sub-strands of fractions, decimals, fractions and decimals, decimals and percentages and ratios and rates. A student might conclude that 7/8 is greater than 2/3 because each fraction is exactly 'one piece' smaller than 1 and the missing 1/8 piece is smaller than the missing 1/3 piece. This reasoning makes use of a strong sense of proportion. Students develop better understanding of proportions by developing flexibility in working with rational numbers. Students use proportional reasoning when they consider the distance between points on a map drawn to scale and the actual distance between corresponding locations in the world (Outcome 6.1.5), when they determine the rate shown by a conversion graph as its slope (Outcome 7.1.6) or when they use the relationship between the circumference of a circle and its diameter (Outcome 7.2.3). Proportional reasoning is also applied in determining the 'best buy' (Outcome 8.1.6). Which is the better buy: 12 for K15.00 or 20 for K23.00?

Students' informal knowledge of negative integers coming from everyday experiences, such as below-zero temperatures (Outcome 6.1.7), is extended in the directed numbers Sub-strand and formalised in problem solving (Outcomes 7.1.7 and 8.1.7). Problem solving in Grades 6 to 8 can and should correspond to students' questions and engage their interests (Outcomes 8.1.2, 8.1.4, 8.1.5). The essence of problem solving is knowing what to do with unfamiliar problems. Teachers can help students become reflective problem-solvers by frequently and openly discussing with them the critical aspects of the problem-solving process, such as understanding the problem and 'looking back' to reflect on the solution and the process. Through modelling, observing and questioning, you can help your students become aware of their activity as they solve problems.

You should refer to the 'Number and Applications' section of the *Worked Examples for Upper Primary Mathematics Outcomes*. This section provides detailed explanation of steps, processes, formulas, samples and worked examples for the Outcomes in the Number and Application Strand. These should guide and assist you further in your planning, programming and teaching.

7.2 Space and Shape

Central to an understanding of space and shape is the idea of structure. In regards to length, structure relates to identifying units that can be built up into other units, for example 10 millimetres becomes a new unit of one centimetre and 100 centimetres forms a metre (Outcomes 6.2.1, 6.2.2 and 7.2.2). Although students may have developed an initial understanding of area and volume, many will still need additional experiences in measuring directly to deepen their understanding of the area of two-dimensional shapes and the surface area and volume of three-dimensional objects (Outcomes 6.2.4, 6.2.5, 6.2.6, 7.2.4, 7.2.5, 7.2.6 and 6.2.13, 7.2.13, 8.2.13). It is also important to understand what things can be measured as well as the units, systems of units and processes of measurement.

Frequent experiences in measuring surface area and volume can help students develop sound understandings of the relationships between attributes and of the units appropriate for measuring them.

When students measure an object, the result should make sense. Estimates and benchmarks can help students recognise when a measurement is reasonable. The idea of using benchmars is one of the better ways of dealing with mathematical activities and problems, especially in measurement and estimation. The term benchmark was often used by carpenters as explained below.

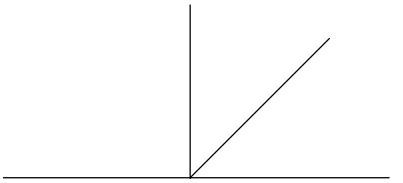
When a carpenter wanted to make several pieces of timber the same length, he drew a mark on his work-bench to help him keep the lengths the same. The next time he wanted to make the same piece he could return to the mark on his bench and have each piece the same size. This idea became known as a 'benchmark'.

In Mathematics we use the idea of a benchmark in work on measurement and estimation. If you want to estimate the height of something, you might have in mind the image of a door. If the height looks to be about the same height as a door then you could say that it is about 2 metres. This is a personal benchmark used to estimate height.

To estimate area, you might imagine two football fields side by side and think that this is about one hectare. In this case, your benchmark is based on knowing how big a football field is.

Personal benchmarks are important for all types of estimation. If you often handle bags of rice then the weight of a bag of rice can become a benchmark for estimating other weights. Benchmarks help us to make sense of Mathematics because they are based on personal experience.

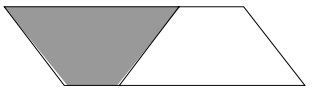
Students can use their sense of the size of common units, or benchmarks, to estimate measurements. For example, a cup will hold about 300 mL so you could use cup measures to estimate capacity. Benchmarks can apply equally well to angles, with typical benchmark angles being right angles, straight angles and 45-degree angles (Outcomes 6.2.11, 7.2.12). Students who have had experience in determining and using benchmark angles are less likely to misread a protractor.



Common benchmark angles.

Whenever possible, students should develop formulas based on investigations and meaning-making sequences. For example, students can use their knowledge of the area of a rectangle to generate a formula for the area of a parallelogram.

Building on this, students can duplicate and rotate a trapezium to determine its area using the known area of a parallelogram.



The trapezium as half the area of the parallelogram formed by duplicating and rotating the trapezium.

Students who have developed the area formulas this way often find them easy to remember (Outcomes 6.2.5 and 7.2.5). Equivalent coverings of the plane by different rotations form a foundation for the techniques of dissection and rearrangement that lie behind the justification of these results. Similarly, determining the interior angle sum of any convex polygon relies on breaking it into a number of triangles and adding up the angle sums of the triangles (Outcome 8.2.11).

Students learn to analyse the properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. For example, in recognising that a rhombus is a special type of kite, a student can argue that they should have the same area formula, half the product of the diagonals.

The nets of solids (Outcomes 6.2.13, 7.2.13, 8.2.13) are introduced to determine the surface areas of solids such as the cylinder. This breaking apart and rearranging moves between three dimensions and two dimensions. Maps and coordinates (Outcomes 6.2.15, 6.2.16, 7.2.15, 7.2.16, 8.2.16) form another way of describing location. They also build on ways of representing the world through mathematical models of space and scale.

You should refer to the 'Space and Shapes' section of the *Worked Examples for Upper Primary Mathematics Outcomes*. This section provides detailed explanation of steps, processes, formulas, samples and worked examples for the Outcomes in the Space and Shape Strand. These should guide and assist you further in your planning, programming and teaching.

7.3 Measurement

The Measurement Strand addresses indirect measurement of weight, time and temperature. Although the mass of an object does not change, its weight can (Outcomes 7.3.2, 8.3.2). While we would have the same mass, we would weigh less on the moon or under water. Temperature is usually measured indirectly, by how much the heat has made something expand (Outcomes 6.3.3, 7.3.3). This is the basis of most thermometers (Outcome 8.3.3). The benchmarks developed for temperature are based on the properties of water. At 0° C water freezes and at 100° C water boils. The work on temperature can link to directed numbers (Outcome 6.1.7).

Measurement of time has been based on the motion of the Earth in respect to the Sun (Outcomes 6.3.4, 7.3.4, 8.3.4). One revolution of the Earth on its axis takes approximately 24 hours and one complete revolution of the Earth around the Sun takes approximately 365.25 days. The history of the measurement of time has meant that not all years have had the same number of days and many different calendars currently exist. In Grade 6, it might be helpful to look at the structure of the calendar month. Although we often teach that there are 4 weeks in a month, only February comes close to this. Record the numbers 1 to 31 on cards as well as the days of the week: Monday to Sunday. Put the cards for the 7 days of the week in order in a row on the floor (Outcome 6.3.4). Distribute the remaining cards among your students.

Ask: 'Who has the same number as today's date?' Have that student place his or her card on the floor in the position it would occur if the cards formed the calendar for the month. Select, the remaining students, one at a time, to position their cards. Ask how each student knew where to locate his or her card. Was the student holding the card with 31 needed? What patterns can students see in the cards?

You should refer to the *Measurement* section of the *Worked Examples for Upper Primary Mathematics Outcomes*. This section provides detailed explanation of steps, processes, formulas, samples and worked examples for the Outcomes in the Measurement Strand. These should guide and assist you further in your planning, programming and teaching.

7.4 Chance and data

The Chance and Data Strand addresses the process of statistical investigation:

- ask a question,
- collect the data,
- organise the data,
- interpret the data,
- answer the question (Outcomes 6.4.1, 7.4.1 and 8.4.1).

It also looks at estimation, accuracy and error (Outcomes 6.4.4, 6. 4.6, 7.4.4, 7.4.5, 7.4.6, 8.4.4, 8.4.5, 8.4.6). The need to develop benchmarks for estimation was described earlier. We use a form of benchmarks as a guide to estimation. If you wish to give an estimated answer to 23.576 x 49.8, you can use numbers that are near multiples of 10 as benchmarks. That is, your answer should be approximately 20 x 50 which equals approximately 1000.

The ideas associated with probability are very important for understanding society (Outcomes 6.4.3, 7.4.3 and 8.4.3). Many things are described as probabilities: the chance of rain, the probability of flood or earthquake or the chance of winning a sporting event. Probability is frequently expressed as a fraction, a percentage or a ratio. Some of the ideas arising from probability do not align with our intuition. For instance, many people incorrectly believe that if a tossed coin has landed on 'heads' five times in a row it is more likely to land on 'tails' next time.

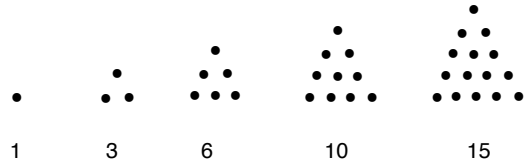
The three different ways of measuring the centre of a group of scores: mean, mode and median, can be confusing to many people (Outcome 6.4.1). To clarify the different uses of these measures it is useful to have students create their own data sets (Outcome 7.4.1) in answer to questions such as, 'Can you be above average and finish in the bottom half of the class?' At first glance, the answer appears to be 'no', but the question is about two different types of averages: the mean and the median. Imagine that there are five students in the class and their final scores are 18, 54, 59, 61 and 63. The average or mean of these scores is 51. If you scored 54 you

would be above the class average but in the bottom half of the class. Ask your students to find other sets of scores like this where one score is above average but in the bottom half of all scores. Sometimes the average or mean is not the best measure of the middle as it can often be influenced by an extreme score. How well does the average of the following four scores represent the scores: 2, 2, 2 and 94? The average of 25 is not particularly close to any of the scores.

You should refer to the 'Chance and Data' section of the *Worked Examples for Upper Primary Mathematics Outcomes*. This section provides detailed explanation of steps, processes, formulas, samples and worked examples for the Outcomes in the Chance and Data Strand. These should guide and assist you further in your planning, programming and teaching.

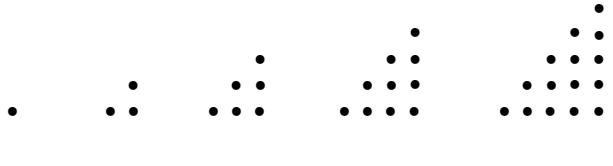
7.5 Patterns and algebra

In Grades 6 to 8, students should investigate numerical and geometric patterns and express them in words or symbols. They should analyse the structure of patterns and how they grow or change (Outcomes 6.5.1, 6.5.2), organise this information systematically and develop generalisations about the mathematical relationships in the pattern (Outcomes 7.5.2, 8.5.2, 6.5.3, 7.5.3). For example, a teacher might ask students to describe patterns they see in the triangular number display.

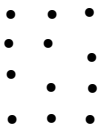


A student might say that the triangles are getting bigger by adding one more onto the bottom row. Another student might say that you add on the number of the triangular number to get the fifth one you add 5 to the fourth one. As well as asking students to work out the sixth triangular number the teacher needs to encourage students in thinking about how to express a generalisation: 'How can we talk about how this pattern works for any triangular number? How would you find the 100th triangular number?' The push to generalise is not a simple transition and it takes time (Outcome 7.5.2). Allow students to struggle with this problem.

One method we use in Mathematics is to rearrange patterns with the aim of making them easier to understand. We can rearrange the triangular numbers so that they look more like staircase numbers. It is a good idea to have students make these numbers using stones, shells or other common objects.



This will not immediately help every student. However, some might see that if you put together two successive triangular numbers you get a square number. This is an important discovery but by itself it will not help us work out the 100th triangular number. What happens if you add a triangular number to itself?



The third triangular number added to itself.

The answer is that we get a rectangular number. The third triangular number added to itself gives a 3 x 4 rectangle, the fourth triangular number added to itself produces a 4 x 5 rectangle and so on. The 100th triangular number added to itself would produce a 100 x 101 rectangle. This gives us a way to work out any triangular number.

The sequence of Outcomes 6.5.2, 7.5.2 and 8.5.2 from the Patterns and Algebra Strand outlines the developing ability of students to describe general patterns. The capacity of students to describe general patterns can be supported by providing a bridge from the known to the unknown. Students should be encouraged to explain these patterns verbally and to make predictions about what will happen if the sequence is continued. Expressed in words, the triangular number pattern is, 'to find any triangular number you multiply the number of the term by one more than the number of the term and then halve it'. In symbols, if 'n' is the number of the term, then, we are well on the way to relating number patterns and algebraic statements (Outcome 7.5.2).

As a follow-up activity you might consider the ' handshake problem'. 'Six men meet at a party and they all shake hands. How many handshakes were there?' To link this problem to the previous work, have students act out the problem. Line up six students and ask the first student to walk down the line shaking hands with everyone and recording the total number of handshakes on the board before sitting down. Have each successive student repeat the process. The total number of handshakes is now the total recorded on the board: 5 + 4 + 3 + 2 + 1, the fifth triangular number. 'How many handshakes would be needed if 10 men had met? How many handshakes would be needed if 100 men had met?'

You should refer to the 'Patterns and Algebra' section of the *Worked Examples for Upper Primary Mathematics Outcomes*. This section provides detailed explanation of steps, processes, formulas, samples and worked examples for the Outcomes in the Patterns and Algebra Strand. These should guide and assist you further in your planning, programming and teaching.

Resources

The Upper Primary Syllabus, Teachers Guide and the *Worked Examples for Upper Primary Mathematics Outcomes* use many different materials. This is due to the fact that students learn to understand new concepts when they use real materials at the same time as they are introduced to the new concept. Many of the materials that you need for this Mathematics course are the things that can be collected by you and your students. Before students use new materials they need to explore them. This will help them to concentrate on the tasks that you set, instead of them wanting to play with the new materials.

The students at this level should continue to use concrete materials collected from their environment. However, it is also important for you to note that progressively, the students should move away from concrete materials and deal directly with symbols.

Students must be given the opportunity to bring in their own materials from their environment to supplement what has been purchased or supplied. You should also allow them to look after their own materials by storing them in a selected corner of their classroom to be used in the future. Provide a duty roster for the students to collect and store any materials that have been purchased or supplied to your school. A safe place, locked and secured should be used to store these types of materials. Some of the materials that you should have at your school are listed below with samples and ideas on how to make some of them yourself.

Collection of objects

In most of the lessons, your students will need objects for counting, estimation and measurement. These can be small stones, buttons, shells, bottle tops, seeds, beads, or similar items. These objects can be easily collected from the students' environment. You should encourage the students to collect them at the beginning of the school year and store them in small containers or boxes.

The table below shows some of the materials, objects and resources that you and your students will need during the lessons. Some of these materials are supplied by the Department of Education, some can be purchased from shops.

Materials in the 'Materials supplied' column are those supplied to all schools free of charge by the Department of Education. Materials marked with an asterisk * can also be purchased locally. Materials in the 'Materials to be made or collected' column can be made or found by you and your students. Materials in the 'Materials to be borrowed' column can be borrowed from other institutions within the community such as the local high school, teachers colleges, aid post or DPI offices.

Most of these materials and resources can be used across the three Upper Primary Grades.

Materials supplied	Materials to be made or collected	Materials to be borrowed
• *linking cubes • *pattern blocks • *mathematics kits • *6A Pupils book • *6B Pupils book • *Grade 6 teacher resource book • *Upper Primary Mathematics Syllabus* • *Worked Examples for Upper Primary Mathematics Outcomes*	• *different sized containers such as cordial bottles, die, balances, square metre measures, fraction boards, clock face, 2-D shapes, 3-D shapes such as cubes, cylinders, prisms and pyramids, 2-D and 3-D nets • angle chart and cardboard spinner • flow chart, timeline • different sized objects such as kaukau, coconuts, rulers, pens and balls • *12- and 24-hour clocks • tessellation patterns • plumb line • *newspapers and magazines	• measuring cylinders and beakers • weights such as 1 g, 100 g, 500 g, 1 kg, 5 kg and 10 kg • weighing scales • *different sized containers such as jugs, cups, spoons, buckets and pots • *balls • thermometers • *calculators • **Melanesian School Mathematics Dictionary*

Materials and equipment that can be made or purchased

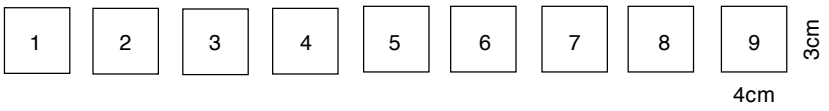
The following materials and equipment can be supplied by the Department of Education. However, it is likely that not all schools will have these materials available. It is recommended that schools can either buy them from local stores or improvise or make them. Some steps are provided for those who wish to make these materials or equipment themselves.

Small number cards

You will need many cards for writing numbers, fractions and decimals. When you write on these cards, cardboard or paper, make sure that you take care of the cards properly. Always store the numbers cards in their correct place. Steps are provided for you to follow when making small number cards. You will need firm cardboard.

1. Cut the cardboard into 4 cm x 4 cm squares. Make as many squares as you can, at least 150 would be recommended.
2. Number the cards so that you have at least ten sets of cards numbered 0 to 9. Leave the remaining cards blank so you can write other numbers on them, as you need them.

This is how they should look.

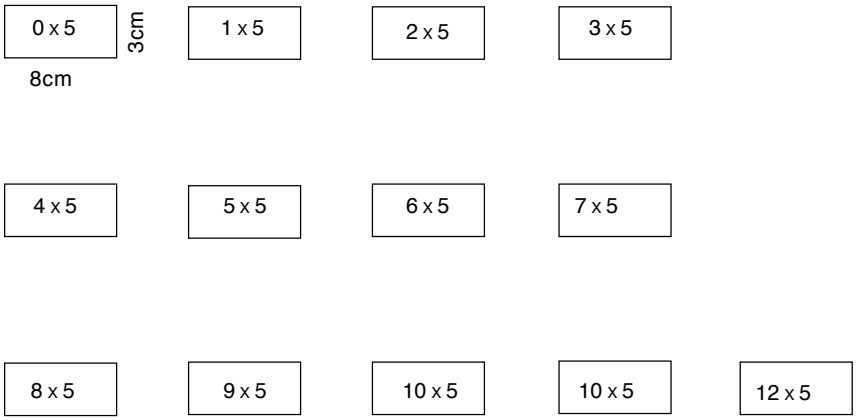


Times table cards

You will need a set of cards for the times table: 2 times to 12 times. These should be no smaller than 3 cm x 8 cm. You can make these cards yourself with cardboard or stiff paper. Here are the steps you could follow.

You will need paper or cardboard.

1. Cut the cardboard into rectangles, at least 3 cm x 8 cm. You will need 13 cards for each times table. If you make a set of each of the times tables, 2 times table to 12 times tables, you will need 143 cards altogether.
2. Write a times table fact on each card. Here are some examples for you to follow.



Division facts cards

You will need a set of cards for the division facts up to 100 divided by 10. These should be no smaller than 3 cm x 8 cm.You can make these cards for students to use. Here are the steps to make the cards.

You will need stiff paper or cardboard.

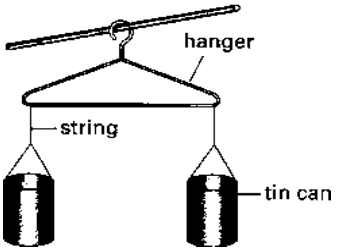
1. Cut the cardboard into rectangles, at least 3 cm x 8 cm. You will need 11 cards for each set of division facts. If you make a set of each of the division tables: division by 2 to division by 10, you will need 99 cards altogether.
2. Write a division fact on each card. Below are examples for the 5 division facts.

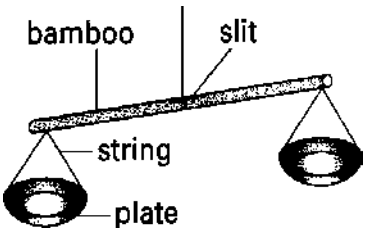
0 ÷ 5 (3cm, 8cm)	5 ÷ 5	10 ÷ 5	15 ÷ 5
20 ÷ 5	25 ÷ 5	30 ÷ 5	35 ÷ 5
40 ÷ 5	45 ÷ 5	50 ÷ 5	

Making balances

These are needed in some of the lessons. If they are not available you will need to make some. Outlined here are some ways you can make balances. Choose the one that you think you should be able to make. Make as many as you need.

You will need strings, bamboo, canes, tin cans, cooking oil containers, plastic plates, halved dry coconut shells and clothes hangers or bamboo and cane can be used instead.





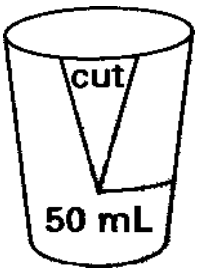
Making containers of mixed volume

You will need to collect a variety of these, especially for the capacity activities. These can be cartons, matchboxes, tins, cans, plastic containers, coconut shells and many more. Some steps are outlined below to collect or make them.

You can use plastic cups to make containers smaller than 200 mL. For example to make a 50 mL container, follow these instructions:

1. Use a measuring cylinder to pour exactly 50 mL of water into the cup. Mark the water level with a pen or pencil on the outside of the cup.
2. Cut a **V** from the top down to the mark.

Now the cup holds only 50 mL. Any extra pours out the **V**



You can do the same for 100 mL or use other containers in the same way to make containers with fixed volumes sucjh as 200 mL and 500 mL. Instead of plastic cups you could use cooking oil, plastic cordial or soft drink bottles of various sizes.

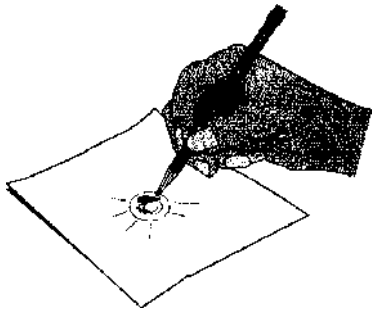
Coin rubbing

Coin rubbings would be useful when teaching parts of the Number and Application Strand. Make many rubbings of each of the coins. These can also be made using coins. Plastic Papua New Guinean play money coins have been supplied to Lower Primary and can also be purchased from school suppliers. If your school does not have them, you need to make some for your class. Here are the steps you could follow to make coin rubbings.

You will need at least one of each type of real Papua New Guinean coins, paper and pencils.

1. Place a piece of paper over one side of the coin.
2. Rub a pencil across the surface until the coin markings show on the paper.
3. Cut around each coin.
4. To make the rubbings stronger, glue them on to cardboard of the same size.

Make as many of each coin as you need.

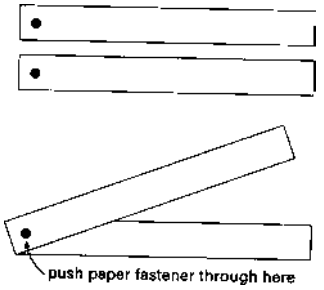


Angle testers

You will need angle testers to support your teaching of the angles section from the Strand on Space and Shapes. Instructions for making these are listed below for your information.

To make angle testers, you will need two thin strips of card about 15 centimetres long, a paper fastener or a drawing pin or nail with plasticine or cork to put over the point.

1. Cut two strips of card as shown in the diagram.
2. Make a hole near the end of each strip.
3. Join them together by pushing a paper fastener through the holes. If you do not have a paper fastener, push a drawing pin or nail through the hole and place a piece of plasticine or cork over the sharp point.

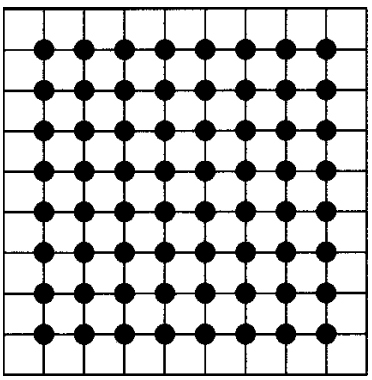


Geoboards

You will need geoboards when teaching the Strand Space and Shape, especially for the section on shapes. Below are some steps you may follow to make some of them.

You will need a piece of board, some small nails and a hammer.

1. Cut the board into a square at least 16 cm x 16 cm. You will need one board for each geoboard that you may want to make.
2. Rule a 2 cm grid on the piece of board.
3. Hammer nails into the intersecting points on the grid as shown here.

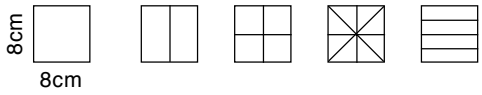


Fraction kits

You will need fraction cards to deal with many activities dealing with fractions as well as other activities in the Number and Application Strand. Some steps are provided for you to follow when making them.

To make a fraction kit, you will need cardboard and rulers.

1. Cut five squares of cards 8 cm x 8 cm.
2. Leave some squares blank and rule lines on the others as shown in the diagrams.
3. Cut the squares into pieces along the lines as shown.
4. Record 1 on the whole square. Record 1/2 on each of the two pieces of the second square. Record 1/4 on each of the four pieces of the third square. Record 1/8 on each of the other pieces from the forth and the fifth squares.



Resource persons

Apart from the materials and resources that are listed, you will have many resource people around you in your communities. Such persons could be carpenters, Aid Post orderlies, nurses, retired teachers, councillors, village leaders, Local Level Government workers and Non-Government Organisation workers, church pastors, academics, business people and many others. You are encouraged to make use of such people whenever it is appropriate.

Resource books

These are some textbooks you may want to borrow from friends, other schools, high schools, teachers colleges or buy them for your own use. You are encouraged to find them to support your planning, programming and teaching.

de Klerk-Cougar, Judith (1986) *Illustrated Maths Dictionary for schools in Papua New Guinea.* Longman, Melbourne, Australia

Musawe, Sinebare (1994) *Melanesian School Mathematics Dictionary.* Dellasta, Victoria, Australia

NDOE (1995) *Secondary School Mathematics 7A & 7B.* Longman, South Melbourne, Australia

NDOE (1995) *Secondary School Mathematics 8A & 8B.* Longman, South Melbourne, Australia

NDOE (2001) *Grade 5 Mathematics Student Book 5A & 5B.* Oxford, South Melbourne, Australia

NDOE (2001) *Grade 5 Mathematics Teachers Guide.* Oxford, South Melbourne, Australia

NDOE (1992) *Community School Mathematics Pupils Book 6A & 6B.* NDOE, Waigani, Papua New Guinea

NDOE (1992) *Community School Mathematics Teacher Guide 6A & 6B.* NDOE, Waigani, Papua New Guinea

NDOE (2003) *Upper Primary Mathematics Syllabus.* NDOE, Waigani, Papua New Guinea

NDOE (2003) *Worked Examples for Upper Primary Mathematics Outcomes.* NDOE, Waigani, Papua New Guinea

NDOE (2002) *Inservice units to support the Implementation of the Lower Primary Curriculum Reform.* NDOE, Waigani, Papua New Guinea

NDOE (n.d.) *Grade 6 Mathematics Student Book 6A & 6B.* Oxford, South Melbourne, Australia

NDOE (n.d.) *Grade 6 Mathematics Teacher Guide.* Oxford, South Melbourne, Australia

Glossary

Mathematical terms used in this Teachers Guide

These are the Mathematical terms used in the Teachers Guide. Some of these terms are explained further in the supporting document called the *Worked Examples for Upper Primary Mathematics Outcomes* that is accompanying the Teachers Guide.

Terms	Definitions
Centicube	a cube which has dimensions measuring 1 cm in length
Circles	sets of points which are a fixed distance from a known point called the centre. When the diameter divides the circle into two equal segments or halves, each segment is called a semi-circle
Circumference	the perimeter of a circle. It is the distance around a circle
Composite numbers	numbers larger than 1 and not a whole number
Cones	solids which have a base and come into a point at the top, similar to the shape of an ice cream cone
Compound shapes	geometrical shapes or figures made by combining more than one shape
Convex polygons	polygons with no curved or rounded shapes. They contain no angles greater than 180°
Cubed numbers	numbers obtained as a result of multiplying a whole number by itself three times
Cube roots	a number when multiplied by itself three times equals a given number. The cube root of 27 is 3 because 3 multiplied by itself three times equals 27
Data	is used to describe a collection of facts, numbers and symbols. It can also be used to make conclusions about something
Decimal system	the counting number system with a base of ten. Decimal points separate the whole numbers from the fractions
Denominator	the bottom number of a fraction. It tells how many parts there are in a whole
Graphs	drawings or diagrams which show mathematical information in a visual form
Imperial system	systems of measurement where feet and inches are used rather than metres and centimetres or gallons are used rather than litres
Improper fraction	a fraction whose numerator is larger than its denominator
Index notation	a short hand way of writing large numbers such as, 1 000 000 000, which can be written as 10^6 in index notation
Integers	any positive or negative whole numbers or zero

Terms	Definitions
Irrational numbers	numbers that can not be expressed as a proper fraction
Lowest common denominator (LCD)	a number that is divisible by the given denominators
Lowest common multiple (LCM)	the smallest multiple which is common to a set of given numbers
Metric system	the decimal system of weights and measures
Metric units	units of measurement used in the metric system
Mixed number	a fraction which contains both whole numbers and fractions
Notation	a form of representing numbers as expressions
Numerator	the top number in a fraction. It tells how many parts of the whole there are
Pi (symbol π)	the ratio of the circumference of a circle to its diameter. Pi is represented by approximately 3.14 or $^{22}/_{7}$
Powers	when numbers are raised to the power of the other
Prime numbers	numbers larger than 1 and divisible only by 1 and the number itself
Pyramids	three dimensional shapes which have a polygon for a base and all other faces are triangular
Proper fraction	a fraction whose denominator is larger than its numerator
Quadrilaterals	plane shapes with four sides
Quotient	the answer obtained in a division problem
Ratio	a comparison of two quantities by expressing one as fraction of the other
Rational number	numbers which can be expressed as a proper fraction
Rates	a comparison of two quantities of a different kind
Scales	measuring devices such as a thermometer, ruler, balance and weighing scales
Shapes	physical outlines of objects such as squares or triangles
Squares numbers	numbers that can be written as a product of two equal numbers. Square numbers can be represented by dots in a shape to give a square
Square roots	a number when multiplied by itself two times equals a given number. The square root of 9 is 3 because 3 multiplied by itself two times equals 9
VAT	stands for Value Added Tax. These are Government taxes on goods and services
Zero index	when any quantity is raised to the power of zero, the answer is always one. For example $A^0 = 1$ and $100^0 = 1$

Other general terms used in the Teachers Guide

Terms	Definitions
Accurate	to be exact, correct, right, or without error
Align	to position in respect to another object or thought
Assessment	the ongoing process of identifying, gathering and interpreting information about students' achievement of learning Outcomes
Assessment criteria	statements that are used to judge the quality of students' performance
Assessment method	ways of assessing students
Assessment task	activities specially designed to find out about students' achievement of particular skills, processes and knowledge
Balanced assessment	requires teachers to use a variety of assessment methods that give students many opportunities in different contexs to demonstrate what they know, understand and can do in relation to the learning outcomes. This approach enables teachers to make well-informed judgements about students' achievement
Benchmarks	in the past when a carpenter wanted to make several pieces of timber the same length, he drew a mark on his work bench to help him keep the lengths the same. The next time he wanted to make the same piece he could return to the mark on his bench and have each piece the same size. This idea became known as a benchmark. In the mathematics activities similar benchmarks can be applied
Conjecturing	when students are conjecturing in Mathematics learning, it means that they are investigating unknown or doubtful activities and are coming up with correct answers or strategies. A lot of mathematics activities will lead students from the known to the unknown
Displacement	occurs when there is a change in a position of an object or of a quantity of material
Criterion-referenced assessment	uses specific assessment criteria derived from the learning outcomes to judge a student's individual performance. It does not compare the performance of one student to another
Dissection	breaking information or an object into its component parts
Essence	the key features that gives something its identity
Evaluation	involves gathering and interpreting data to make judgements about the effectiveness of aspects such as the school curriculum, teaching and learning programs or assessment and reporting practices
Norm-referenced assessment	compares students' achievements with assessments of respective samples of other students. This sample is usually a National sample. The purpose of norm-referenced assessment is usually to sort and rank students
Recording	accurate record of evidence drawn from a range of contexts about students' learning related to the demonstration of the learning outcomes. Record keeping must support planning and be manageable and easily maintained.

Terms	Definitions
Reporting	presenting evidence of students' achievements of the learning outcomes. It can be drawn from ongoing observations of performance and from assessment tasks.
Rich task	one which provides many opportunities for students to learn
Scholastic	means academic or learned
Solidify understanding	means students build stronger understanding and ideas about themselves as learners of Mathematics: their competence, their attitude and their interest and motivation
Verifying	a process of proving and confirming answers

References

de Klerk-Cougar, Judith (1986) *Illustrated Maths Dictionary for schools in Papua New Guinea.* Longman , Melbourne, Australia

Musawe, Sinebare (1994) *Melanesian School Mathematics Dictionary.* Dellasta, Victoria, Australia

NDOE (1992) *Community School Mathematics Pupils Book 6A.* NDOE, Waigani, Papua New Guinea

NDOE (1992) *Community School Mathematics Pupils Book 6B.* NDOE, Waigani, Papua New Guinea

NDOE (1992) *Community School Mathematics Teacher Guide 6A*. NDOE, Waigani, Papua New Guinea

NDOE (1992) *Community School Mathematics Teacher Guide 6B*. NDOE, Waigani, Papua New Guinea

NDOE (1995) *Secondary School Mathematics 7A*. Longman, South Melbourne, Australia

NDOE (1995) *Secondary School Mathematics 7B*. Longman, South Melbourne, Australia

NDOE (1995) *Secondary School Mathematics 8A*. Longman, South Melbourne, Australia

NDOE (1995) *Secondary School Mathematics 8B.* Longman, South Melbourne, Australia

NDOE (2001) *Grade 5 Mathematics Student book 5A.* Oxford, South Melbourne, Australia

NDOE (2001) *Grade 5 Mathematics Student book 5B.* Oxford, South Melbourne, Australia

NDOE (2001) *Grade 5 Mathematics Teacher Guide.* Oxford, South Melbourne, Australia

NDOE (2003) *Upper Primary Mathematics Syllabus.* CDD-NDOE, Waigani, Papua New Guinea

NDOE (n.d.) *Grade 6 Mathematics Student book 6A.* Oxford, South Melbourne, Australia

NDOE (n.d.) *Grade 6 Mathematics Student book 6B.* Oxford, South Melbourne, Australia

NDOE (n.d.) *Grade 6 Mathematics Teacher Guide*. Oxford, South Melbourne, Australia

Appendix

Time allocations for upper primary subjects

In Upper Primary the subjects to be taught and their time allocations per week are:

Arts	180 minutes
Language	180 minutes
Making a Living	360 minutes
Personal Development	240 minutes
Social Science	180 minutes
Science	180 minutes
Mathematics	180 minutes

All subjects are core subjects and must be allocated the required number of minutes per week. Each subject is equally important for Integral Human Development. Making a Living and Personal Development have more time allocated because of their practical orientation.

All subjects can be externally assessed (*National Assessment and Reporting Policy*, *2003*).