MATHEMATICS FOR MANAGEMENT BBMP1103

TOPIC 2 : LINEAR AND QUADRATIC FUNCTION

Objectives:

1. Identify linear and quadratic functions
2. Find the slope of a line
3. Determine whether two lines are parallel or perpendicular
4. Sketch graphs of linear and quadratic functions
5. Find intersection point.

INTRODUCTION

1. In general, linear function is related with the previous topic where a linear function is one to one relation
2. There are many applications of linear function in our daily lives especially in economic field.

4.1 LINEAR EQUATIONS AND GRAPH SKETCHING

4.1.1 Linear Equations

1. The general form $y = mx + c$

   Where:
   m is the gradient
   c is the intercept

   Example:

   Obtain the gradient and y intercept for each of the linear equations below:

   (a) $y = \frac{2}{3}x + 1$

   (b) $y = 6 - 3x$

   (c) $y = -\frac{x}{4}$

   (d) $2y + 6x = 9$

   Solutions:
   (Express the following equations in general form $y = mx + c$. Then calculate the value for m (scalar for x) and the y intercept, c).

   (a) $m = \frac{2}{3}$ and $c = 1$

   (b) $m = -3$ and $c = 6$

(c) $m=-\frac{1}{4}$ and $c=0$

(d) $2y=-6x+9$

$y=\frac{9-6x}{2}$

$y=-3x+\frac{9}{2}$

$m=-3$ and $c=\frac{9}{2}$

4.1.2 Slope

1. If two points $A(x_1,y_1)$ and $B(x_2,y_2)$ are given , a slope can be derived by using the formula below:

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Examples:

Find the slope for the following points:

(a) $A(1,4)$ and $B(-2,5)$

(b) $C(0,-3)$ and $D(7,-1)$

(c) $E(-6,6)$ and $F(1,6)$

Solutions:

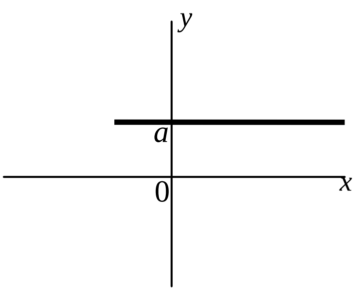
(a)
$$m=\frac{5-4}{-2-1}$$
$$=-\frac{1}{3}$$

(b)
$$m=\frac{-1-(-3)}{7-0}$$
$$=-\frac{2}{7}$$

(c)
$$m=\frac{6-6}{1-(-6)}$$
$$=0$$

4.1.3 Types of Straight Lines

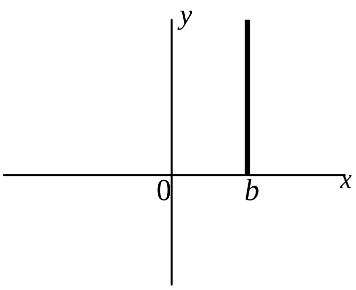
1.



Horizontal Line

- $y = a$
- Symmetrical to x-axis
- Its gradient is zero

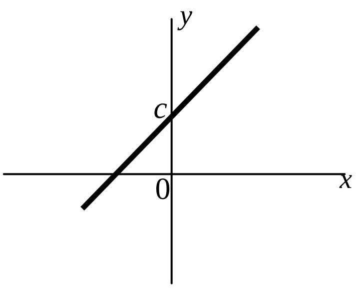
2.



Vertical Line

- $x = b$
- Its gradient is infinite

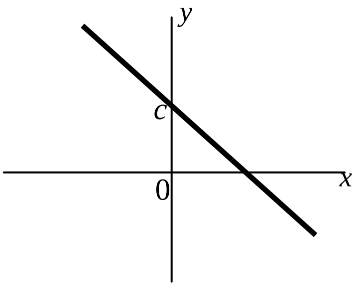
3.



Ascending-Slant Line

- $y = mx + c$
- Ascending line form left to right
- Its gradient is positive

4.



Descending-Slant Line

- $y = mx + c$
- Descending line form left to right
- Its gradient is negative

4.1.4 Graph Sketching (Linear Function)

The following are steps for sketching a linear function graph:

(a) Find two different points of coordinate and plot them

(i) The y intercept can be obtained by letting $x = 0$. Substituting $x = 0$ into the equation and calculate the corresponding value for y

(ii) The x intercept can be obtained by letting $y = 0$. Substituting $y = 0$ into the equation and calculate the corresponding value for x

(b) Connect the two points to form a straight line

Examples:

Sketch graph for each of the following linear functions:

(a) $y = 2x - 1$

(b) $y = -4x$

(c) $2y + 3x = 6$

Solutions:

(a) $y = 2x - 1$

Step 1: Find the y intercept

Let $x = 0$,

$y = 2(0) - 1$

$y = -1$

$\therefore$ Hence, the point of y intercept is $(0,-1)$

Step 2: Find the x intercept

Let $y = 0$,

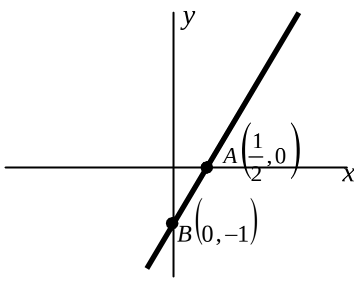
$0 = 2x - 1$

$2x = 1$

$x = \frac{1}{2}$

$\therefore$ Hence, the point of x intercept is $\left(\frac{1}{2},0\right)$

Step 3: Sketch the graph $y = 2x - 1$



(b) $y = -4x$

Step 1: Find the y intercept

Let $x = 0$,

$y = -4(0)$

$y = 0$

$\therefore$ Hence, the point of y intercept is $(0,0)$

Step 2: Find the x intercept

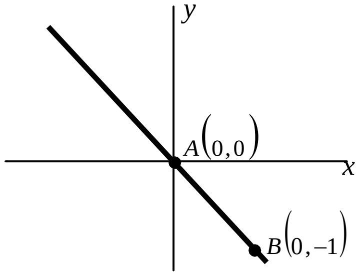
Let $x=2$,

$y=-4(2)$

$y=-8$

$\therefore$ Hence, the point of x intercept is $(2,-8)$

Step 3: Sketch the graph $y=-4x$



(c) $2y+3x=6$

Step 1: Find the y intercept

Let $x=0$,

$2y+3(0)=6$

$2y=6$

$y=3$

$\therefore$ Hence, the point of y intercept is $(0,3)$

Step 2: Find the x intercept

Let $y=0$,

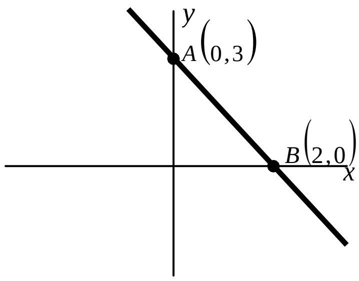
$2(0)+3x=6$

$3x=6$

$x=2$

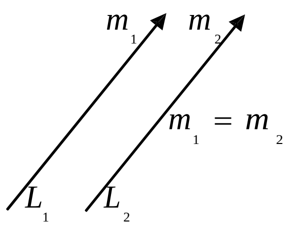
$\therefore$ Hence, the point of x intercept is $(2,0)$

Step 3: Sketch the graph $2y+3x=6$



4.2 PARALLEL AND PERPENDICULAR LINES

1. Two lines are said to be parallel if and only if have the same gradient.



Example 1:

Determine wether a straight line $2y - 3x + 6 = 0$ is parallel to another straight line $4y = 6x + 3$?

Solution:

Step 1:
(Express the equations into the general form)

$$2y - 3x + 6 = 0 \qquad 2y = 3x - 6 \qquad y = \frac{3}{2}x - 6$$

$$4y = 6x + 3 \qquad y = \frac{6}{4}x + \frac{3}{4} \qquad y = \frac{3}{2}x + \frac{3}{4}$$

Step 2:
(Identify the gradient)

$$m_1 = \frac{3}{2}, m_2 = \frac{3}{2}$$

Step 3:

Since the gradients are the same, these two lines are parallel.

Example 2:

Find an equation of a straight line that passes through point $(-2,10)$ and which is parallel to another straight line $5x - y = 0$.

Solution:

Step 1:
(Determine the gradient)

$$5x - y = 0$$
$$-y = -5x$$
$$y = 5x$$
$$m = 5$$

Step 2:
(Substitute $m = 5$ into equation $y = mx + c$)

$$y = 5x + c$$

Step 3:
(Find c. Substitute $x = -2$ and $y = 10$ into equation $y = 5x + c$)

$$y = 5x + c$$
$$10 = 5(-2) + c$$
$$c = 20$$

Step 3:
(Form the equation)

$$y = 5x + 20$$

Example 3:

Find an equation of a straight line that passes through point $(1,2)$ and which is parallel to another straight line $x + 5y = 2$.

Solution:

Step 1:
(Determine the gradient)

$$x + 5y = 2$$
$$5y = -x + 2$$
$$y = -\frac{1}{5}x + \frac{2}{5}$$
$$m = -\frac{1}{5}$$

Step 2:
(Find the gradient of the required line)

$$\left(-\frac{1}{5}\right)(m_2) = -1$$
$$m_2 = 5$$

Step 3:
(Substitute $m_2 = 5$ into equation $y = mx + c$)

$$y = 5x + c$$

Step 4:
(Find c. Substitute $x = 1$ and $y = 2$ into equation $y = 5x + c$)

$$y = 5x + c$$
$$2 = 5(1) + c$$
$$c = -3$$

Step 5:
(Form the equation)

$$y = 5x - 3$$

Exercise 4.1

(a) For each of the following equations, determine their gradient and y intercept:

(i) $y = \frac{x}{2} - 1$

(ii) $y = -5 - 5x$

(iii) $y = -3x$

(iv) $3y = 5 - 2x$

(b) Find the equation of a straight line with gradient of -1 and passes through point $(3,2)$.

(c) Given two points $A(2,4)$ and $B(5,12)$ Determine the equation of a straight line that passes through them.

(d) Find an equation of a straight that passes through point $(2,1)$ and parallel to $2y + x = 5$.

(e) Obtain an equation of a straight line that passes through point $(3,-2)$ and which is perpendicular to line $3x - y + 3 = 0$.

Exercise 4.2

Sketch a graph for each of the linear functions below:

(a) $y = 3x + 2$

(b) $y = \frac{-x}{2}$

(c) $3y + 2x = 2$

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Solution 4.1

(a) For each of the following equations, determine their gradient and y intercept:

(i)
$$y=\frac{x}{2}-1$$
$$m=\frac{1}{2}, c=-1$$

(ii)
$$y=-5-5x$$
$$y=-5x-5$$
$$m=-5, c=-5$$

(iii)
$$y=-3x$$
$$m=-3, c=0$$

(iv)
$$3y=5-2x$$
$$3y=-2x+5$$
$$y=-\frac{2}{3}x+\frac{5}{3}$$
$$m=-\frac{2}{3}, c=\frac{5}{3}$$

(b) Find the equation of a straight line with gradient of -1 and passes through point $(3,2)$.

Step 1: (Determine the gradient)

$$m=-1$$

Step 2: (Substitute $m=-1$ into equation $y=mx+c$)

$$y=-x+c$$

Step 3: (Find c. Substitute $x=3$ and $y=2$ into equation $y=-x+c$)

$$y=-x+c$$
$$2=-3+c$$
$$c=5$$

Step 3: (Form the equation)

$$y=-x+5$$

(c) Given two points $A(2,4)$ and $B(5,12)$ Determine the equation of a straight line that passes through them.

Step 1: (Determine the gradient)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{12-4}{5-2}$$
$$m = \frac{8}{3}$$

Step 2: (Substitute $m = \frac{8}{3}$ into equation $y = mx + c$)

$$y = \frac{8}{3}x + c$$

Step 4: (Find c. Substitute $x = 2$ and $y = 4$ into equation $y = \frac{8}{3}x + c$)

$$y = \frac{8}{3}x + c$$
$$4 = \frac{8}{3}(2) + c$$
$$c = -\frac{4}{3}$$

Step 5: (Form the equation)

$$y = \frac{8}{3}x - \frac{4}{3}$$
$$3y = 8x - 4$$

(d) Find an equation of a straight that passes through point $(2,1)$ and parallel to $2y + x = 5$.

Solution:

Step 1: (Determine the gradient)

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$$2y + x = 5$$
$$2y = -x + 5$$
$$y = -\frac{1}{2}x + \frac{5}{2}$$
$$m = -\frac{1}{2}$$

Step 2: (Find the gradient of the required line)

$$m_1 = m_2 = -\frac{1}{2}$$

Step 3: (Substitute $m_2 = -\frac{1}{2}$ into equation $y = mx + c$)

$$y = -\frac{1}{2}x + c$$

Step 4: (Find c. Substitute $x = 2$ and $y = 1$ into equation $y = 2x + c$)

$$y = -\frac{1}{2}x + c$$
$$1 = -\frac{1}{2}(2) + c$$
$$c = 2$$

Step 5: (Form the equation)

$$y = -\frac{1}{2}x + 2$$
$$2y = -x + 4$$
$$2y + x = 4$$

(e) Obtain an equation of a straight line that passes through point $(3, -2)$ and which is perpendicular to line $3x - y + 3 = 0$.

Step 1: (Determine the gradient)

$$3x - y + 3 = 0$$
$$y = 3x + 3$$
$$m = 3$$

Step 2: (Find the gradient of the required line)

$3(m_2) = -1$

$m_2 = -\frac{1}{3}$

Step 3: (Substitute $m_2 = -\frac{1}{3}$ into equation $y = mx + c$)

$y = -\frac{1}{3}x + c$

Step 4: (Find c. Substitute $x = 3$ and $y = -2$ into equation $y = -\frac{1}{3}x + c$)

$y = -\frac{1}{3}x + c$

$-2 = -\frac{1}{3}(3) + c$

$c = -1$

Step 5: (Form the equation)

$y = -\frac{1}{3}x - 1$

$3y + x + 3 = 0$

Solution 4.2

Sketch a graph for each of the linear functions below:

(a) $y = 3x + 2$

Step 1: Find the y intercept

Let $x = 0$,

$y = 3(0) + 2$

$y = 2$

$\therefore$ Hence, the point of y intercept is $(0,2)$

Step 2: Find the x intercept

Let $y = 0$,

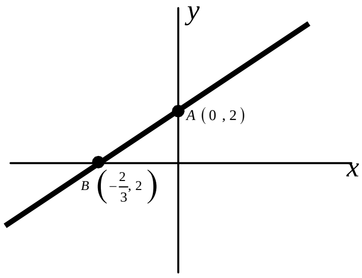
$0 = 3x + 2$

$3x = -2$

$x = -\frac{2}{3}$

$\therefore$ Hence, the point of x intercept is $\left(-\frac{2}{3},0\right)$

Step 3: Sketch the graph $y = 3x + 2$



(b) $y = \frac{-x}{2}$

Step 1: Find the y intercept

Let $x = 2$,

$y = \frac{-2}{2}$

$= -1$

$\therefore$ Hence, the point of y intercept is $(2,-1)$

Step 2: Find the x intercept

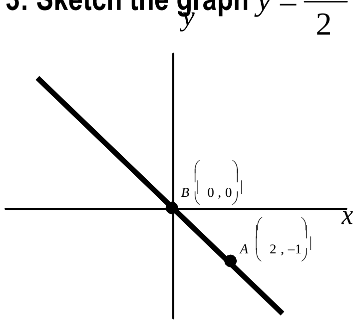
Let $y = 0$,

$0 = \frac{-x}{2}$

$= 0$

$\therefore$ Hence, the point of x intercept is $(0,0)$

Step 3: Sketch the graph $y = \frac{-x}{2}$



(c) $3y + 2x = 2$

Step 1: Find the y intercept

Let $x = 0$,

$3y + 2(0) = 2$

$3y = 2$

$y = \frac{2}{3}$

$\therefore$ Hence, the point of y intercept is $\left(0, \frac{2}{3}\right)$

Step 2: Find the x intercept

Let $y = 0$,

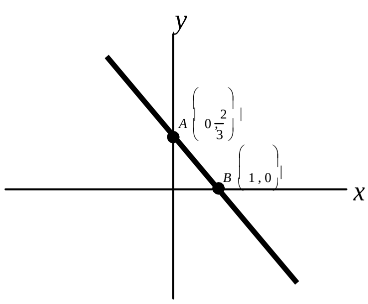
$3(0) + 2x = 2$

$2x = 2$

$x = 1$

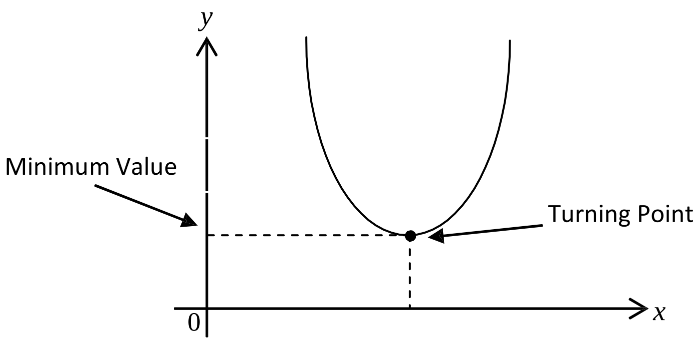
$\therefore$ Hence, the point of x intercept is $(1,0)$

Step 3: Sketch the graph $3y + 2x = 2$

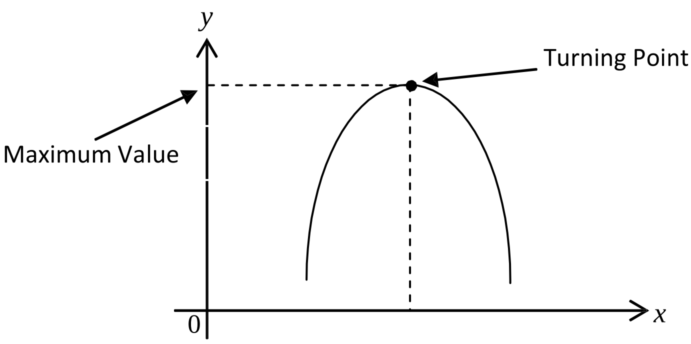


4.3 QUADRATIC EQUATIONS AND GRAPH SKETCHING

1. The general form of a quadratic equation is $y = ax^2 + bx + c$
   Where:
   a, b, c are real numbers
   $a \neq 0$
2. The highest degree for x in this equation is 2.
3. The graph of such function is parabola.



- a is positive
- Parabola opens upward
- Minimum value



- a is negative
- Para opens downward
- Maximum value

4. The following are steps for sketching graph of quadratic function $f(x) = ax^2 + bx + c$.

Step 1:
(Determine the direction of parabola, max or min value)

$$f(x) = ax^2 + bx + c$$

Step 2:
(Find the turning point (x, y))

$$x = -\frac{b}{2a}, y = \frac{4ac - b^2}{4a} \quad \text{or} \quad y = f\left(-\frac{b}{2a}\right)$$

Step 3:
(Find the y–intercept, at which $x = 0$.)
Substitute $x = 0$ into the quadratic function.

$$f(x) = ax^2 + bx + c$$
$$f(0) = a(0)^2 + b(0) + c$$
$$= c$$

Hence, $(0, c)$ is the y–intercept.

Step 4:
(Find the x–intercept (if exist))
Can be solved by using factored method or quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The nature of the graph whether it crosses the x–axis or not, depends on the value of $b^2 - 4ac$.

(a) When $b^2 - 4ac > 0$, the graph crosses the x–axis at two points
(b) When $b^2 - 4ac = 0$, the graph crosses the x–axis at only one point
(c) When $b^2 - 4ac < 0$, the graph does not cross on the x–axis

Step 5:
(Plot all the predetermined points forms. Draw a smooth curve passing through the points)

Example:

Sketch graphs for each of the following quadratic functions:
(a) $f(x) = x^2 - 4x$
(b) $f(x) = 3 - 2x - x^2$
(c) $f(x) = 2x^2 + 2x + 1$

Solutions:

(a) $f(x) = x^2 - 4x$

Step 1:
(Determine the direction of parabola, max or min value)

$$f(x) = x^2 - 4x$$
$$a = 1, b = -4, c = 0$$

$\therefore$ The value of a is positive, minimum value, hence the parabola opens upward.

Step 2:
(Find the turning point (x, y))

$$x = -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2$$

$$y = \frac{4ac - b^2}{4a} = \frac{4(1)(0) - (-4)^2}{4(1)} = \frac{0 - 16}{4} = -4$$

$\therefore$ The turning point is $(2, -4)$

Step 3:
(Find the y–intercept, at which $x = 0$.)
Substitute $x = 0$ into the quadratic function.

$$f(x) = ax^2 + bx + c$$
$$f(x) = a(0)^2 + b(0) + c$$
$$= c$$

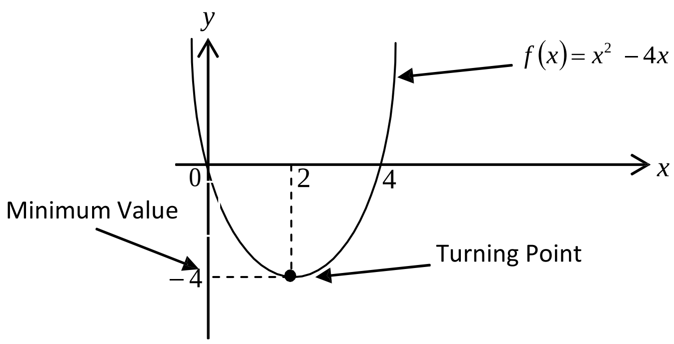
$\therefore\ c = 0$

Step 4:
(Find the x–intercept (if exist))
Can be solved by using factored method or quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(0)}}{2(1)}$$
$$= \frac{4 + \sqrt{16 - 0}}{2}, \frac{4 - \sqrt{16 - 0}}{2}$$
$$= 4, 0$$

$\therefore\ x = 4,\ x = 0$
$\therefore$ Two points are $(4,0), (0,0)$

Step 5:
(Plot all the predetermined points forms. Draw a smooth curve passing through the points)



(b) $f(x) = 3 - 2x - x^2$

Step 1:
(Determine the direction of parabola, max or min value)

$$f(x) = 3 - 2x - x^2$$
$$f(x) = -x^2 - 2x + 3$$
$$a = -1, b = -2, c = 3$$

$\therefore$ The value of a is negative, maximum value, hence the parabola opens downward.

Step 2:
(Find the turning point (x, y))

$$x = -\frac{b}{2a} = -\frac{(-2)}{2(-1)} = -1$$

$$y = \frac{4ac - b^2}{4a} = \frac{4(-1)(3) - (-2)^2}{4(-1)} = \frac{-12 - 4}{-4} = 4$$

$\therefore$ The turning point is $(-1, 4)$

Step 3:
(Find the y –intercept, at which $x = 0$.)
Substitute $x = 0$ into the quadratic function.

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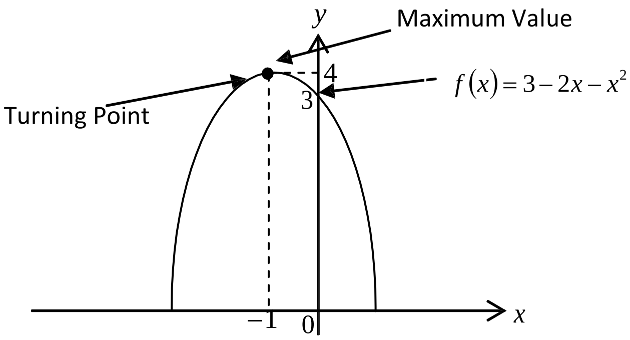
$$f(x) = ax^2 + bx + c$$
$$f(x) = a(0)^2 + b(0) + c$$
$$= c$$
$$\therefore\ c = 3$$

Step 4:
(Find the x–intercept (if exist))
Can be solved by using factored method or quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(3)}}{2(-1)}$$
$$= \frac{2 + \sqrt{4+12}}{-2}, \frac{2 - \sqrt{4+12}}{-2}$$
$$= -3, 1$$
$$\therefore\ x = -3,\ x = 1$$
$\therefore$ Two points are $(-3,3), (1,3)$

Step 5:
(Plot all the predetermined points forms. Draw a smooth curve passing through the points)



(c) $f(x) = 2x^2 + 2x + 1$

Step 1:
(Determine the direction of parabola, max or min value)

$$f(x) = 2x^2 + 2x + 1$$
$$a = 2, b = 2, c = 1$$
$\therefore$ The value of a is positive, minimum value, hence the parabola opens upward.

Step 2:
(Find the turning point (x, y))

$$x = -\frac{b}{2a} = -\frac{2}{2(2)} = -\frac{1}{2}$$

$$y = \frac{4ac - b^2}{4a} = \frac{4(2)(1) - (2)^2}{4(2)} = \frac{8-4}{8} = \frac{1}{2}$$

$\therefore$ The turning point is $\left(-\frac{1}{2}, \frac{1}{2}\right)$

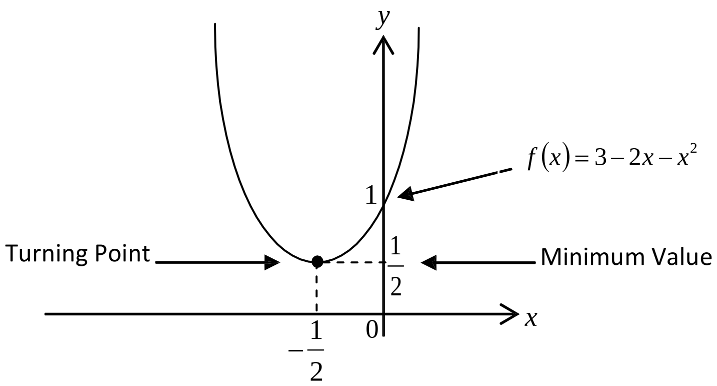
Step 3:
(Find the y–intercept, at which $x = 0$.)
Substitute $x = 0$ into the quadratic function.

$$f(x) = ax^2 + bx + c$$
$$f(x) = a(0)^2 + b(0) + c$$
$$= c$$
$\therefore\ c = 1$

Step 4:
(Find the x–intercept (if exist))
Can be solved by using factored method or quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(1)}}{2(2)}$$
$$= \frac{-2 + \sqrt{4-8}}{4}, \frac{-2 - \sqrt{4-8}}{4}$$
$\therefore\ \sqrt{b^2 - 4ac} \langle 0$
$\therefore$ The graph has no x–intercept

Step 5:
(Plot all the predetermined points forms. Draw a smooth curve passing through the points)



Exercise 5.1

Sketch graphs for each of the quadratic functions below:

(a) $f(x) = x^2 - 6x + 5$
(b) $f(x) = x^2 + 4x$
(c) $f(x) = 4x - x^2 - 3$
(d) $f(x) = -x^2 - 2x - 3$
(e) $f(x) = 3x^2 = 7x + 2$
(f) $f(x) = x^2 - 16$
(g) $f(x) = (x-1)(3-x)$
(h) $f(x) = (x+1)^2 - 2$

Solutions 5.1

Sketch graphs for each of the quadratic functions below:

(a) $f(x) = x^2 - 6x + 5$

Step 1:
(Determine the direction of parabola, max or min value)

$$f(x) = x^2 - 6x + 5$$
$$a = 1, b = -6, c = 5$$

$\therefore$ The value of a is positive, minimum value, hence the parabola opens upward.

Step 2:
(Find the turning point (x, y))

$$x = -\frac{b}{2a} = -\frac{(-6)}{2(1)} = 3$$

$$y = \frac{4ac - b^2}{4a} = \frac{4(1)(5) - (-6)^2}{4(1)} = \frac{20 - 36}{4} = -4$$

$\therefore$ The turning point is $(3, -4)$

Step 3:
(Find the y–intercept, at which $x = 0$.)
Substitute $x = 0$ into the quadratic function.

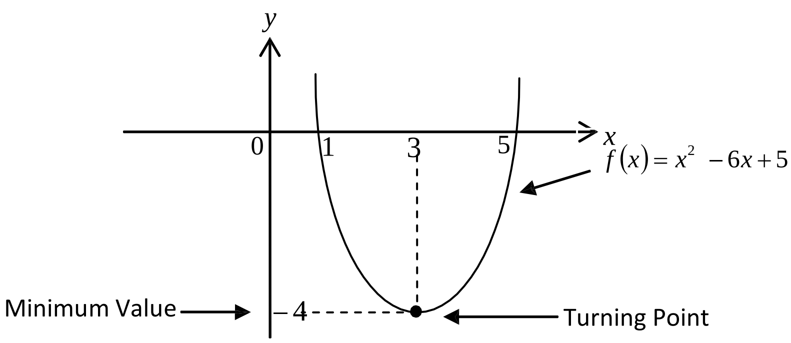
$$f(x) = ax^2 + bx + c$$
$$f(x) = a(0)^2 + b(0) + c$$
$$= c$$

$\therefore\ c = 5$

Step 4:
(Find the x–intercept (if exist))
Can be solved by using factored method or quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{6 + \sqrt{36 - 20}}{2}, \frac{6 - \sqrt{36 - 20}}{2}$$
$$= 5, 1$$
$$\therefore\ x = 5,\ x = 1$$

Step 5:
(Plot all the predetermined points forms. Draw a smooth curve passing through the points)



(b) $f(x) = x^2 + 4x$

Step 1:
(Determine the direction of parabola, max or min value)

$$f(x) = x^2 + 4x$$
$$a = 1, b = 4, c = 0$$

$\therefore$ The value of a is positive, minimum value, hence the parabola opens upward.

Step 2:
(Find the turning point (x, y))

$$x = -\frac{b}{2a}$$
$$= -\frac{4}{2(1)}$$
$$= -2$$

$$y = \frac{4ac - b^2}{4a}$$
$$= \frac{4(1)(0) - (4)^2}{4(1)}$$
$$= \frac{0 - 16}{4}$$
$$= -4$$

$\therefore$ The turning point is $(-2,-4)$

Step 3:
(Find the y–intercept, at which $x = 0$.)
Substitute $x = 0$ into the quadratic function.

$$f(x) = ax^2 + bx + c$$
$$f(x) = a(0)^2 + b(0) + c$$
$$= c$$

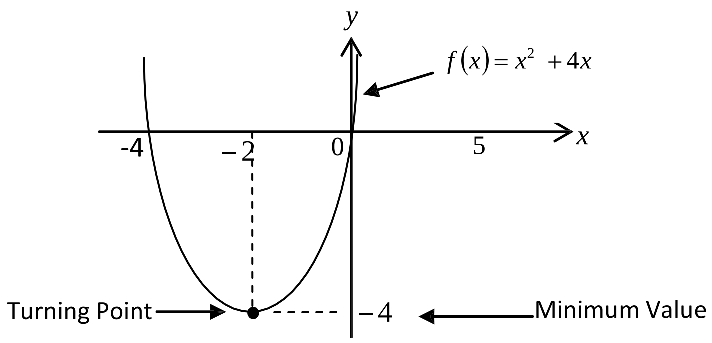
$\therefore c = 0$

Step 4:
(Find the x–intercept (if exist))
Can be solved by using factored method or quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(0)}}{2(1)}$$
$$= \frac{-4 + \sqrt{16 - 0}}{2}, \frac{-4 - \sqrt{16 - 0}}{2}$$
$$= 0, -4$$

$\therefore x = 0,\ x = -4$

Step 5:
(Plot all the predetermined points forms. Draw a smooth curve passing through the points)



(c) $f(x) = 4x - x^2 - 3$

Step 1:
(Determine the direction of parabola, max or min value)

$$f(x) = 4x - x^2 - 3$$
$$f(x) = -x^2 + 4x - 3$$
$$a = -1, b = 4, c = -3$$

$\therefore$ The value of a is negative, maximum value, hence the parabola opens downward.

Step 2:
(Find the turning point (x, y))

$$x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$

$$y = \frac{4ac - b^2}{4a} = \frac{4(-1)(-3) - (4)^2}{4(-1)} = \frac{12 - 16}{-4} = 1$$

$\therefore$ The turning point is $(2,1)$

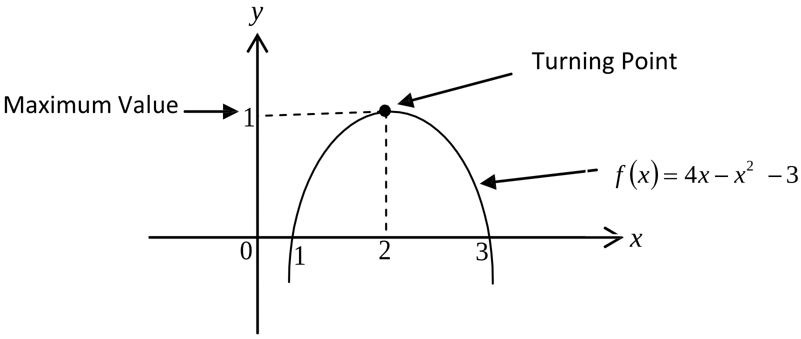
Step 3:
(Find the y–intercept, at which $x = 0$.)
Substitute $x = 0$ into the quadratic function.

$$f(x) = ax^2 + bx + c$$
$$f(x) = a(0)^2 + b(0) + c$$
$$= c$$
$$\therefore \; c = -3$$

Step 4:
(Find the x–intercept (if exist))
Can be solved by using factored method or quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(-1)(-3)}}{2(-1)}$$
$$= \frac{-4 + \sqrt{16 - 12}}{-2}, \frac{-4 - \sqrt{16 - 12}}{-2}$$
$$= 1, 3$$
$$\therefore \; x = 1, \; x = 3$$

Step 5:
(Plot all the predetermined points forms. Draw a smooth curve passing through the points)



(d) $f(x) = -x^2 - 2x - 3$

(e) $f(x) = 3x^2 = 7x + 2$

(f) $f(x) = x^2 - 16$

(g) $f(x) = (x-1)(3-x)$

4.4 INTERSECTION POINT (LINEAR FUNCTIONS)

1. The point of intersection between two straight lines can be obtained by solving the equations of the two lines.

Examples 1:

Find the intersection point for lines $2x + y = 4$ and $x - y = 2$.

Solutions:

Step 1: (Solve the two equations simultaneously. Eliminate x or y)

$$\begin{array}{rr} & 2x+y=4 \\ + & x-y=2 \\ \hline & 3x=6 \\ & x=2 \end{array}$$

Step 2: (Substitute x or y into equation)

$$\begin{aligned} 2-y&=2 \\ y&=0 \end{aligned}$$

Step 3: (Find the point of intersection)

$\therefore$ Therefore, the point of intersection is $(2,0)$

Examples 2:

Find the intersection point for lines $2x + 4y = 6$ and $6x + 3y = 18$.

Solutions:

Step 1: (Solve the two equations simultaneously. Eliminate x or y)

$2x + 4y = 6$ → Multiply by 3 (to equate the scalar of x)

$$\begin{array}{rr} & 6x+3y=18 \\ \hline & 6x+12y=18 \\ - & 6x+3y=18 \\ & 9y=0 \\ & y=0 \end{array}$$

Step 2: (Substitute x or y into equation)

$$\begin{aligned} 2x + 4y &= 6 \\ 2x + 4(0) &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

Step 3: (Find the point of intersection)

$\therefore$ Therefore, the point of intersection is $(3,0)$

Exercise 4.3

(a) For each of the following equations, find their point of intersection:

(i) $2x + y = 10$ and $6x + y = 14$

(ii) $3x + y - 2 = 0$ and $3x - 4y + 8 = 0$

(iii) $2x - 3y = 7$ and $3x + 2y = 4$

(b) Find the equation of a straight line which passes through point of intersection between $x = y$ and $y = 2x - 3$, and with gradient of -2.

(c) Given two straight line $x + y = 5$ and $3x - y = 1$. Determine the equation of a straight line that passes through their intersection point and is perpendicular to $2x + y = 7$.

Solutions 4.3

(a) For each of the following equations, find their point of intersection:

(i) $2x + y = 10$ and $6x + y = 14$

Step 1: (Solve the two equations simultaneously. Eliminate x or y)

$$\begin{array}{rl} & 2x + y = 10 \\ - & 6x + y = 14 \\ \hline & -4x = -4 \\ & x = 1 \end{array}$$

Step 2: (Substitute x or y into equation)

$$2(1) + y = 10$$
$$y = 8$$

Step 3: (Find the point of intersection)

$\therefore$ Therefore, the point of intersection is $(1,8)$

(ii) $3x + y - 2 = 0$ and $3x - 4y + 8 = 0$

Step 1: (Solve the two equations simultaneously. Eliminate x or y)

$$\begin{array}{rl} & 3x + y - 2 = 0 \\ - & 3x - 4y + 8 = 0 \\ \hline & 5y - 10 = 0 \\ & 5y = 10 \\ & y = 2 \end{array}$$

Step 2: (Substitute x or y into equation)

$$3x + 2 - 2 = 0$$
$$3x + 0 = 0$$
$$x = 0$$

Step 3: (Find the point of intersection)

$\therefore$ Therefore, the point of intersection is $(0,2)$

(iii) $2x-3y=7$ and $3x+2y=4$

Step 1: (Solve the two equations simultaneously. Eliminate x or y)

$$2x-3y=7 \longrightarrow \text{Multiply by 3 (to equate the scalar of x)}$$
$$3x+2y=4 \longrightarrow \text{Multiply by 2 (to equate the scalar of x)}$$

$$\begin{aligned} 6x-9y&=21\\ -\;6x+4y&=8\\ \hline -13y&=13\\ y&=-1 \end{aligned}$$

Step 2: (Substitute x or y into equation)

$$\begin{aligned} 2x-3y&=7\\ 2x-3(-1)&=7\\ 2x+3&=7\\ 2x&=4\\ x&=2 \end{aligned}$$

Step 3: (Find the point of intersection)

$\therefore$ Therefore, the point of intersection is $(2,-1)$

(b) Find the equation of a straight line which passes through point of intersection between $x=y$ and $y=2x-3$, and with gradient of -2.

Step 1: (Solve the two equations simultaneously. Eliminate x or y)

$$\begin{aligned} x&=y\\ y&=2x-3\\ \hline x-y&=0\\ -\quad 2x-y&=3\\ \hline -x&=-3\\ x&=3 \end{aligned}$$

Step 2: (Substitute x or y into equation)

$$\begin{aligned} x&=y\\ 3&=y \end{aligned}$$

Step 3: (Find the point of intersection)

$\therefore$ Therefore, the point of intersection is $(3,3)$

Step 4: (Find the equation)

$y = mx + c$
$\therefore\ m = -2$, passes through point $(3,3)$
$y = -2x + c$
$3 = -2(3) + c$
$c = 9$
$\therefore$ Therefore, the equation is $y = -2x + 9$ @ $y = 9 - 2x$

(c) Given two straight line $x + y = 5$ and $3x - y = 1$. Determine the equation of a straight line that passes through their intersection point and is perpendicular to $2x + y = 7$.

Step 1: (Solve the two equations simultaneously. Eliminate x or y)

$$\begin{array}{r} x + y = 5 \\ +\quad 3x - y = 1 \\ \hline 4x = 6 \\ x = \dfrac{6}{4} \\ \hline x = \dfrac{3}{2} \end{array}$$

Step 2: (Substitute x or y into equation)

$x + y = 5$
$\frac{3}{2} + y = 5$
$y = 5 - \frac{3}{2}$
$y = \frac{7}{2}$

Step 3: (Find the point of intersection)

$\therefore$ Therefore, the point of intersection is $\left(\frac{3}{2}, \frac{7}{2}\right)$

Step 4: (Find the equation)

$$-2(m_2) = -1$$

$$m_2 = \frac{1}{2}$$

$\therefore\ m = \frac{1}{2}$, passes through point $\left(\frac{3}{2}, \frac{7}{2}\right)$

$$y = \frac{1}{2}x + c$$

$$\frac{7}{2} = \frac{1}{2}\left(\frac{3}{2}\right) + c$$

$$\frac{7}{2} = \frac{3}{4} + c$$

$$c = \frac{11}{4}$$

$\therefore$ Therefore, the equation is $y = \frac{1}{2}x + \frac{11}{4}$ @ $4y = 2x + 11$

4.5 INTERSECTION POINT (QUADRATIC FUNCTIONS)

1. The point of intersection between two graph can be obtained by solving the equations of the two graphs.

Example 1:

Find the intersection point for curves $y = 4x - x^2$ and $y = x^2 - 6$.

Solution 1:

Step 1:
(Solve the equations)

$$y = 4x - x^2 \text{ , } y = x^2 - 6$$

$$4x - x^2 = x^2 - 6$$

$$x^2 + x^2 - 4x - 6 = 0$$

$$2x^2 - 4x - 6 = 0$$

Step 2:
(Apply the quadratic formula)

$$a = 2, b = -4, c = -6$$

Step 3:

(Find the x values)
Can be solved by using factored method or quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)} \\ &= \frac{4+\sqrt{16+48}}{4}, \frac{4-\sqrt{16+48}}{4} \\ &= 3, -1 \end{aligned}$$

$\therefore\ x = 3,\ x = -1$

Step 4:
(Find the y values by applying the x values into one of the equations)

$$\begin{aligned} &\therefore x = 3 \\ y &= x^2 - 6 \\ &= 3^2 - 6 \\ &= 3 \end{aligned}$$

$$\begin{aligned} &\therefore x = -1 \\ y &= x^2 - 6 \\ &= (-1)^2 - 6 \\ &= -5 \end{aligned}$$

$\therefore\ y = 3,\ y = -5$

Step 5:
(Find the intersection points)

$\therefore$ Hence, the intersection points are $(3,3)$ and $(-1,-5)$.

<u>Example 2:</u>

Find the intersection point for curves $x^2 + y - 3 = 0$ and $2x + y = 0$.

<u>Solution 1:</u>

Step 1:
(Solve the equations)

$$\begin{aligned} x^2 + y - 3 = 0&, 2x + y = 0 \\ 3 - x^2 &= -2x \\ x^2 - 2x - 3 &= 0 \end{aligned}$$

Step 2:
(Apply the quadratic formula)

$$a = 1, b = -2, c = -3$$

Step 3:
(Find the x values)
Can be solved by using factored method or quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{2+\sqrt{4+12}}{2}, \frac{2-\sqrt{4+12}}{2} \\ &= 3, -1 \end{aligned}$$

$\therefore\ x = 3,\ x = -1$

Step 4:
(Find the y values by applying the x values into one of the equations)

$$\begin{aligned} &\therefore x = 3 \\ 2x + y &= 0 \\ 2(3) + y &= 0 \\ &= -6 \end{aligned} \qquad \begin{aligned} &\therefore x = -1 \\ 2x + y &= 0 \\ 2(-1) + y &= 0 \\ &= 2 \end{aligned}$$

$\therefore\ y = -6,\ y = 2$

Step 5:
(Find the intersection points)

$\therefore$ Hence, the intersection points are $(3,-6)$ and $(-1,2)$.

Exercise 4.5

Find the intersection points for each of the followings pair:

(a) $y = 8 - x^2$ and $4x - y + 11 = 0$

(b) $y = 2x^2 - 3x$ and $y = x^2 - 2$

(c) $y = x^2 + 6x + 2$ and $y = 2x^2 + 2x + 5$

GOOD LUCK