

# MATHEMATICS FOR MANAGEMENT QUESTIONS AND ANSWERS

KLINIK MATEMATIK PENGURUSAN  
BBMP 1103  
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## BAHAGIAN A

1. Differentiate the following functions:

a)  $y = 2x^2(3x - 1)$

b)  $y = \frac{2x^3 - 1}{x^2}$

**Answer:**

$$\begin{aligned} \text{a)} \quad &= 2x^2(3x - 1) \\ &= 6x^3 - 2x^2 \\ &= \frac{dy}{dx} 6x^3 + \frac{dy}{dx} 2x^2 \\ &= 18x^2 + 4x \end{aligned}$$

$$\begin{aligned} \text{b)} \quad &= \frac{dy}{dx} \left( \frac{2x^3 - 1}{x^2} \right) \\ &= \frac{dy}{dx} \left( \frac{2x^3}{x^2} - x^{-2} \right) \\ &= \frac{dy}{dx} (2x) - \frac{dy}{dx} (x^{-2}) \\ &= 2 - (-2x^{-3}) \\ &= 2 + \frac{2}{x^3} \end{aligned}$$

2. Integrate the following functions:

a)  $\int (2x^3 + 5x - 1) dx$

b)  $\int \left( 1 - \frac{1}{x^3} \right) dx$

**Answer:**

a)  $= \int (2x^3 + 5x - 1) dx$





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$$\begin{aligned} &= \int 2x^3 + \int 5x - \int 1dx \\ &= \frac{2x^{3+1}}{3+1} + \frac{5x^{1+1}}{1+1} - x + c \\ &= \frac{2}{4}x^4 + \frac{5}{2}x^2 - x + c \\ &= \frac{1}{2}x^4 + \frac{5}{2}x^2 - x + c \end{aligned}$$

$$\begin{aligned} \text{b)} \quad &= \int \left(1 - \frac{1}{x^3}\right) dx \\ &= \int 1 - \int x^{-3} dx \\ &= x - \left( \frac{x^{-3+1}}{-3+1} \right) + c \\ &= x - \left( -\frac{1x^{-2}}{2} \right) + c \\ &= x + \frac{1}{2x^2} + c \end{aligned}$$

3. The revenue function of a product is given by  $R = 1200q - 3q^2$
- a) Find the quantity that will maximize the revenue
  - b) What is the maximum revenue?

**Answer:**

- a) Quantity that will maximise the revenue.

To maximise the revenue,  $\frac{d(R)}{dq} = 0$  and  $\frac{d^2(R)}{d^2q} < 0$

$$\frac{d(R)}{dq} = 1200q - 3q^2$$

$$\frac{d(R)}{dq} = 1200 - 6q$$

$$q = 200$$

$$\frac{d^2(R)}{d^2q} = 1200q - 3q^2$$

$$= -6$$

Therefore,  $q = 200$  is maximised

- b) Calculate the maximum revenue





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$$\begin{aligned}TRF(200) &= 1200q - 3q^2 \\&= 1200(200) - 3(200)^2 \\&= 240,000 - 120,000 \\&= RM120,000\end{aligned}$$

4. The world population in 1960 was 3 billion. In 2011, it has increased to 7 billion people. Find the rate of growth of the world population.

**Answers:**

$$\begin{aligned}7P &= 3P_o e^{rt} \\7P_o &= 3P_o e^{r(51)} \\7 &= 3e^{51r} \\\log_e 7 &= \log_e e^{51r} \\\ln 7 &= 51r \\r &= \frac{\ln 7}{51} \\r &= 0.038155101 \\\therefore \text{The rate of growth is } 3.812\%.\end{aligned}$$

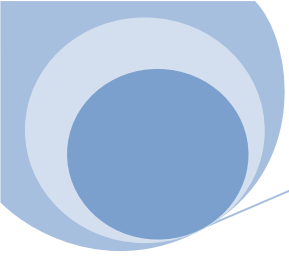
5. A company produces a product which it sells at RM600 per unit. The labor cost and material cost per unit to produce this product is RM30 and RM20 respectively. The company has fixed cost of RM120,000.
- a) Find the profit function  
b) Determine whether the company makes a profit or loss by selling 200 units of the product.

**Answer:**

$$\begin{aligned}\text{a) Total Revenue} &= (\text{Unit Price}) \times (\text{Total Quantity Sold}) \\&= 600 \times q \\&= 600q \\\text{Total Cost} &= \text{Variable Costs} + \text{Fixed Costs} \\&= (30 + 20)q + 120,000 \\&= 50q + 120,000 \\\text{Profit} &= \text{Total Revenue} - \text{Total Cost} \\&= 600q - (50q + 120,000) \\&= 600q - 50q - 120,000 \\&= 550q - 120,000\end{aligned}$$

- b) Total Profit Function





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$$\begin{aligned}TPF &= 550q - 120,000 \\&= 550(200) - 120,000 \\&= 110,000 - 120,000 \\&= -10,000 \\Loss &= 10,000\end{aligned}$$



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## BAHAGIAN B

1. a) The average cost function of a product produced by Syarikat Sejati Sdn Bhd is given by
- $$\bar{C} = \frac{25,000}{q} - 75 + 0.25q.$$
- i) Find the cost function.  
ii) Find the quantity of the product that will minimise the cost.  
iii) Find the minimum cost to produce the product.
- b) Syarikat Sejati Sdn Bhd has decided to invest RM200,000 into a bank which offers an interest rate of 8% compounded quarterly. How much will be accumulated over a period of 15 years.
- c) How long does it takes for RM200,000 to double at an interest rate of 12% compounded monthly?

### Answers:

- a) i) Cost Function

$$\begin{aligned}C(q) &= \bar{C}(q) \times q \\&= \left( \frac{25,000}{q} - 75 + 0.25q \right) \times q \\&= 25,000 - 75q + 0.25q^2\end{aligned}$$

- ii) Quantity of the product that will minimise the cost

To minimise the cost,  $\frac{d(C)}{dq} = 0$  and  $\frac{d^2(C)}{d^2q} > 0$

$$\frac{d(C)}{dq} = 25,000 - 75q + 0.25q^2$$

$$\frac{d(C)}{dq} = -75 + 0.5q$$

$$q = 150$$

$$\frac{d^2(C)}{d^2q} = 25,000 - 75q + 0.25q^2$$

$$= 0.5$$

Therefore,  $q = 150$  is minimised

- iii) Calculate the minimum cost



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$$\begin{aligned}
 TCF(150) &= 25,000 - 75q + 0.25q^2 \\
 &= 25,000 - 75(150) + 0.25(150)^2 \\
 &= 25,000 - 11,250 + 5,625 \\
 &= RM19,375
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } S &= P \left( 1 + \frac{r}{k} \right)^{nk} \\
 S &= 200,000 \left( 1 + \frac{0.08}{4} \right)^{(15)(4)} \\
 S &= 200,000(1.02)^{60} \\
 S &= RM 656,206.16
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } 2P &= P \left( 1 + \frac{r}{k} \right)^{nk} \\
 2P &= P \left( 1 + \frac{0.12}{12} \right)^{12n} \\
 2 &= 1.01^{12n} \\
 \log 2 &= \log 1.01^{12n} \\
 12n &= \frac{\log 2}{\log 1.01} \\
 n &= 5.81
 \end{aligned}$$

2. a) Differentiate the following functions:
- $y = (2x+1)^3(3x^2-1)$ . Using the Product Rule.
  - $y = \sqrt{(2x+1)^3}$ . Using the Chain Rule.

- b) The demand function for a product is given by  $p = 2400 - 3q^2$ .
- Obtain the revenue function
  - Obtain the marginal revenue function

## Answer:

- a) Differentiate the following functions:
- $y = (2x+1)^3(3x^2-1)$   
 Let  $g(x) = (2x+1)^3$  and  $h(x) = (3x^2-1)$   
 Then  $g'(x) = 6(2x+1)^2$  and  $h'(x) = 6x$   
 Therefore;



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$$\begin{aligned} f'(x) &= h(x)g'(x) + g(x)h'(x) \\ &= (3x^2 - 1)6(2x+1)^2 + (2x+1)^3 6x \end{aligned}$$

ii)  $y = \sqrt{(2x+1)^3}$ . Using the Chain Rule.

$$y = ((2x+1)^3)^{\frac{1}{2}}$$

$$y = (2x+1)^{\frac{3}{2}}$$

**Step 1:** Introduce one new variable,  $u$  so that  $\frac{dy}{du}$  and  $\frac{du}{dx}$  are easy to calculate.

$$\text{Let } u = 2x+1 \text{ then } y = u^{\frac{3}{2}}$$

**Step 2:** Calculate  $\frac{dy}{du}$  and  $\frac{du}{dx}$

$$\text{When } u = 2x+1 \text{ and } y = u^{\frac{3}{2}}$$

$$\text{Then } \frac{du}{dx} = 2 \text{ and } \frac{dy}{du} = \frac{3}{2}u^{\frac{1}{2}}$$

**Step 3:** Use the Chain Rule to calculate  $\frac{dy}{dx}$

$$\begin{aligned} y'(x) &= \frac{dy}{dx} = \left(\frac{dy}{du}\right)\left(\frac{du}{dx}\right) \\ &= \frac{3}{2}u^{\frac{1}{2}}(2) \\ &= 3u^{\frac{1}{2}} \end{aligned}$$

**Step 4:** Calculate  $\frac{dy}{dx}$  into expressions of  $x$ .

Substitute  $u = 2x+1$  into  $\frac{dy}{dx}$ , gives

$$\begin{aligned} \frac{dy}{dx} &= 3u^{\frac{1}{2}} \\ &= 3(2x+1)^{\frac{1}{2}} \end{aligned}$$

b)  $p = 2400 - 3q^2$ .

i) Revenue function.



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$$\begin{aligned}TRF &= p \times q \\&= (2400 - 3q^2)q \\&= 2400q - 3q^3\end{aligned}$$

ii) Marginal revenue function.

$$\begin{aligned}\frac{d(R)}{dq} &= 2400q - 3q^3 \\&= 2400 - 9q^2\end{aligned}$$

3. a) Integrate the following:

i)  $\int (2x - 1)(x + 1)dx$

ii) Find the value of  $\int_1^2 \left( \frac{3x^2 - 2}{x^2} \right) dx$

b) The marginal cost of a product produced by Sinar Suria Sdn Bhd is given by  $C' = 30q - 6$  and its fixed cost is RM10,000. The revenue function of the product is given by

$$R = 20q^2 + 35q$$

i) Find its cost function.

ii) What is the cost of production if the company decides to produce 10 units of the product?

iii) Find the profit made by the company when  $q = 20$  units.

### Answers:

a) i)  $\int (2x - 1)(x + 1)dx$

$$\begin{aligned}&= \int 2x^2 + 2x - x - 1dx \\&= \frac{2x^{2+1}}{2+1} + \frac{2x^{1+1}}{1+1} - \frac{x^{1+1}}{1+1} - x + c \\&= \frac{2}{3}x^3 + x^2 - \frac{1}{2}x^2 - x + c\end{aligned}$$

ii)  $\int_1^2 \left( \frac{3x^2 - 2}{x^2} \right) dx$





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$$\begin{aligned}
 &= \int_1^2 \left( \frac{3x^2 - 2}{x^2} \right) dx \\
 &= \int_1^2 3 - \frac{2}{x^2} dx \\
 &= \left[ 3x - 2 \left( \frac{x^{-2+1}}{-2+1} \right) \right]_1^2 \\
 &= \left[ 3x + 2 \left( \frac{1}{x} \right) \right]_1^2 \\
 &= \left( 3(2) + 2 \left( \frac{1}{2} \right) \right) - \left( 3(1) + 2 \left( \frac{1}{1} \right) \right) \\
 &= 7 - 5 \\
 &= 2
 \end{aligned}$$

b)  $C' = 30q - 6$  and its fixed cost is RM10,000.

i) Determine the cost function.

$$C'(q) = 30q - 6$$

$$\int 30q - 6 dx$$

$$\frac{30q^{1+1}}{1+1} - 6x + 10,000$$

$$15q^2 - 6x + 10,000$$

ii) How much is the cost of producing 10 units of the product?

$$C(10) = 15q^2 - 6x + 10,000$$

$$= 15(10)^2 - 6(10) + 10,000$$

$$= 1500 - 60 + 10,000$$

$$= RM 11,440$$

iii) Profit made by the company when  $q = 20$  units

$$TPF(20) = TRF - TCF$$

$$= 20q^2 + 35q - (15q^2 - 6x + 10,000)$$

$$= 20(20)^2 + 35(20) - (15(20)^2 - 6(20) + 10,000)$$

$$= 8,000 + 700 - 6,000 + 120 - 10,000$$

$$= RM - 7,180$$



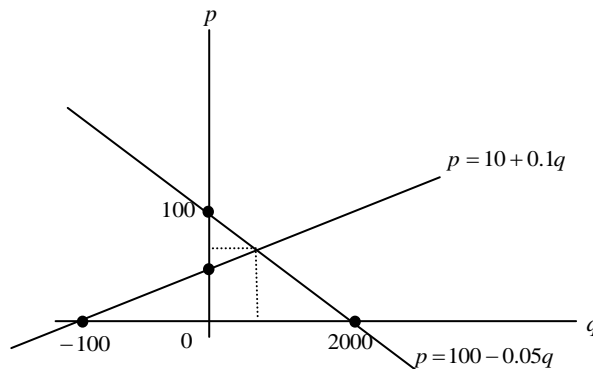
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## BAHAGIAN C

1. a) The demand and supply functions for a particular product are given as  $p = 100 - 0.05q$  and  $p = 10 + 0.1q$  respectively. Find the consumers' and producers' surpluses of the product.
- b) Find the maximum value for  $f(x, y) = x^2 + y^2 - xy$  over a constraint of  $x + y = 8$  using the Lagrange Multiplier Method.

### Answers:

a) **Step 1: Sketch the graphs in the first quadrant only**



$$p = 100 - 0.05q$$

$$\text{If } p = 0, q = 2000$$

$$\text{If } q = 0, p = 100$$

$$p = 10 + 0.1q$$

$$\text{If } p = 0, q = -100$$

$$\text{If } q = 0, p = 10$$

**Step 2: Obtain the market equilibrium point**

$$100 - 0.05q = 10 + 0.1q$$

$$0.1q + 0.05q = 100 - 10$$

$$0.15q = 90$$

$$q = 600$$

Substitute the values of  $q = 600$  into  $p = 100 - 0.05q$

$$p = 100 - 0.05q$$

$$p = 100 - 0.05(600)$$

$$= 70$$

Hence,  $(600, 70)$  is the market equilibrium point.

**Step 3: Find the consumer's surplus**



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$$\begin{aligned}
 CS &= \int_0^{600} (100 - 0.05q) dq - (70)(600) \\
 &= \left[ 100q - \frac{0.05q^{1+1}}{1+1} \right]_0^{600} - 42,000 \\
 &= \left[ 100q - 0.025q^2 \right]_0^{600} - 42,000 \\
 &= \left[ 100(600) - 0.025(600)^2 \right]_0^{600} - 42,000 \\
 &= (60,000 - 9,000) - 42,000 \\
 &= 9,000
 \end{aligned}$$

**Step 4: Find the producer's surplus**

$$\begin{aligned}
 PS &= (70)(600) - \int_0^{600} (10 + 0.1q) dq \\
 &= 42,000 - \left[ 10q + \frac{0.1q^{1+1}}{1+1} \right]_0^{600} \\
 &= 42,000 - \left[ 10q + 0.05q^2 \right]_0^{600} \\
 &= 42,000 - \left[ 10(600) + 0.05(600)^2 \right]_0^{600} \\
 &= 42,000 - (6,000 + 18,000) \\
 &= 42,000 - 24,000 \\
 &= 18,000
 \end{aligned}$$

b)  $f(x, y) = x^2 + y^2 - xy$  over a constraint of  $x + y = 8$

**STEP 1:** Express constraint  $g(x, y) = 0$ .

$$x + y = 8$$

$$x + y - 8 = 0$$

**STEP 2:** Lagrange Function  $f(x, y, \lambda)$ .

$$\begin{aligned}
 f(x, y, \lambda) &= f(x, y) + \lambda g(x, y) \\
 &= x^2 + y^2 - xy + \lambda(x + y - 8) \\
 &= x^2 + y^2 - xy + \lambda x + \lambda y - \lambda 8
 \end{aligned}$$

**STEP 3:** Find  $f_x, f_y, f_\lambda$ .

$$\begin{aligned}
 f(x, y, \lambda) &= x^2 + y^2 - xy + \lambda x + \lambda y - \lambda 8 \\
 f_x &= 2x - y + \lambda \\
 f_y &= 2y - x + \lambda \\
 f_\lambda &= x + y - 8
 \end{aligned}$$



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∴ Using 1 and 2, eliminate  $\lambda$  to obtain relation between  $x$  and  $y$ .

$$2x - y + \lambda = 0$$

$$-x + 2y + \lambda = 0$$

$$\hline 3x - 3y = 0$$

$$3x = 3y$$

$$x = y$$

∴ Substitute  $x = y$  into 3

$$x + y - 8 = 0$$

$$x + x = 8$$

$$2x = 8$$

$$x = 4$$

∴ Substitute  $x = 4$  into  $x = y$

$$x = y$$

$$y = 4$$

**STEP 4:** To obtain the value of  $\lambda$  substitute  $x = 4$ ,  $y = 4$  into 1

$$2(4) - (4) + \lambda = 0$$

$$8 - 4 + \lambda = 0$$

$$\lambda = -4$$

**STEP 5:** To obtain the maximum value

$$f(x, y) = x^2 + y^2 - xy$$

$$f(4, 4) = (4)^2 + (4)^2 - (4)(4)$$

$$= 16 + 16 - 16$$

$$= 16$$

2. a) Given the function  $f(x, y) = 4x^2y^2 - 9xy + 8x^2 - 3y^4$ . Find the following derivatives:

i)  $f_x(x, y)$

ii)  $f_y(x, y)$

iii)  $f_{xx}(x, y)$

iv)  $f_{yy}(x, y)$

v)  $f_{xy}(x, y)$

- b) Given the function  $f(x, y) = x^2 + 4y^3 - 6xy - 1$

i) Find the critical points for  $f(x, y)$

ii) Determine whether the critical point in (i) is maximum or minimum.



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## Answers:

a)  $f(x, y) = 4x^2y^2 - 9xy + 8x^2 - 3y^4$ . Find the following derivatives:

i)  $f_x(x, y) = 8xy^2 - 9y + 16x$

ii)  $f_y(x, y) = 8x^2y - 9x - 12y^3$

iii)  $f_{xx}(x, y) = 8y^2 + 16$

iv)  $f_{yy}(x, y) = 8x^2 - 36y^2$

v)  $f_{xy}(x, y) = 16xy - 9$

b) Given the function  $f(x, y) = x^2 + 4y^3 - 6xy - 1$

i) Find the critical points for  $f(x, y)$

**STEP 1:** Derive first degree differentiation

$$\begin{aligned}f_x &= x^2 + 4y^3 - 6xy - 1 \\&= 2x - 6y\end{aligned}$$

$$\begin{aligned}f_y &= x^2 + 4y^3 - 6xy - 1 \\&= 12y^2 - 6x\end{aligned}$$

**STEP 2:** Obtain the critical points

$$\begin{aligned}2x - 6y &= 0 \\-6x + 12y^2 &= 0 \\ \hline 6x - 18y &= 0 \\-6x + 12y^2 &= 0 \\ \hline 12y^2 - 18y &= 0 \\y(12y - 18) &= 0 \\y &= 0 \\y &= \frac{3}{2}\end{aligned}$$

∴ Substitute the value  $y$  of into the equation  $2x - 6y = 0$





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$$2x - 6y = 0$$

$$2x - 6\left(\frac{3}{2}\right) = 0$$

$$x = \frac{9}{2}$$

∴ The critical point is  $\left(\frac{9}{2}, \frac{3}{2}\right)$

- i) Determine whether the critical point in (i) is maximum or minimum.

**STEP 1:** Obtain  $f_{xx}$ ,  $f_{yy}$ ,  $f_{xy}$  to determine maximum or minimum point.

$$f_x = 2x - 6y$$

$$f_{xx} = 2$$

$$f_y = 12y^2 - 6x$$

$$f_{yy} = 24y$$

$$f_x = 2x - 6y$$

$$f_{xy} = -6$$

$$\text{STEP 2: } M = [f_{xx}(a,b) \times f_{yy}(a,b)] - [f_{xy}(a,b)]^2$$

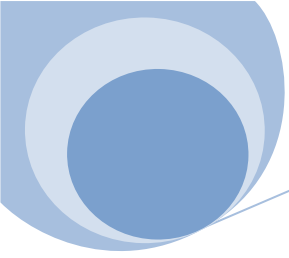
$$\therefore \left[ 2 \times 24\left(\frac{3}{2}\right) \right] - [-6]^2$$

$$= 72 - 36$$

$$= 36$$

∴  $M > 0$  the point  $\left(\frac{9}{2}, \frac{3}{2}\right)$  is a minimum point





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