Western Cape Government

Education

Western Cape Education Department

Telematics
Learning Resource 2017

MATHEMATICS
Grade 11

Dear Grade 11 Learner

In 2017 there will be 5 Telematics sessions for grade 11 learners. This workbook provides you with material for sessions 1-5. Please make sure that you bring this workbook along to each and every Telematics session. The table below indicates the number of marks each of the different content areas will be allocated in the grade 11 & 12 end of year paper.

Paper 1 (Grades 12:bookwork: maximum 6 marks)		
Description	**Grade 11**	**Grade. 12**
Algebra and equations (and inequalities)	45 ± 5	25 ± 3
Patterns and Sequences	25 ± 3	25 ± 3
Finance and Growth		
Finance, growth and decay	15 ± 3	15 ± 3
Functions and Graphs	45 ± 3	35 ± 3
Differential Calculus		35 ± 3
Probability	20 ± 3	15 ± 3
Total	**150**	**150**
Paper 2: Grades 11 and 12: theorems and/or trigonometric proofs: maximum 12 marks		
description	**Grade 11**	**Grade. 12**
Statistics	20 ± 3	20 ± 3
Analytical Geometry	30 ± 3	40 ± 3
Trigonometry	50 ± 3	50 ± 3
Euclidean Geometry and Measurement	50 ± 3	40 ± 3
Total	**150**	**150**

Grade 11 is a vital year, 60% of the content you are assessed on in grade 12 next year, will be on the grade 11 content.
Please note the marks allocated for bookwork in paper 2. Ensure you know the proofs to the Area, Sine and Cosine Rule. There are altogether 4 proofs of Geometry theorems you must know. The proofs you are required to know is marked are indicated in the Geometry Session 5 material. Any of these could be assessed in grade 11and 12 in paper 2.

You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and your scientific calculator with you.

You are also encouraged to participate fully in each lesson by asking questions and working out the exercises, and where you are asked to do so, sms or e-mail your answers to the studio.

Remember:" Success is not an event, it is the result of regular and consistent hard work".

GOODLUCK, Wishing you all the success you deserve!

Term 1				
Day	**Date**	**Time**	**Grade**	**Subject**
Monday	6 February	15:00 – 16:00	Grade 11	Mathematics
Monday	20 February	15:00 – 16:00	Grade 11	Mathematics

Term 2				
Day	**Date**	**Time**	**Subject**	**Topic**
Thursday	18 May	15:00 – 16:00	Grade 11	Mathematics

Term 3				
Day	**Date**	**Time**	**Grade**	**Subject**
Monday	7 August	15:00 – 16:00	Grade 11	Mathematics

Term 4				
Day	**Date**	**Time**	**Grade**	**Subject**
Tuesday	10 October	15:00 – 16:00	Grade 11	Mathematics

Session 1:
Exponents and Surds

Exponents:
Def: $a^n = a \times a \times a \times a \times a \ldots \ \ldots\ldots. \ \ n \ times$

Laws:
1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(a.b)^n = a^n.b^n$

Note:
1. $a^0 = 1$
2. $a^{-m} = \frac{1}{a^m}$

Surds:

Note:
1. $a^{\frac{1}{n}} = \sqrt[n]{a}$
2. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
3. $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
4. $\sqrt{a}\,.\sqrt{b} = \sqrt{a.b}$
5. $\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{a.b}$

Calculate:
1. $16 - 2 \times 8$
2. $\sqrt{81 - 9}$

Are the following expressions the same?

- $2^5 \times 2^2/2$
- $\frac{2^5 \times 2^2}{2}$
- $2^5 \times 2^2 \div 2$

What are the order of operations?

Are there patterns in exponent and surd questions?

Write down examples of expression and then examples of equations. What is the difference between an expression and equation?

What are the types of question that could be asked involving expressions?

What are the types of questions that could be asked involving equations?

Some expressions are defined for all real values of the variable. Some expressions are undefined for certain value(s) of the variable.

What is a non-real number?

When do we say an expression is non-real?

Consider the following, try and see if you can identify any patterns?

1. $\frac{[(3x)^{-2}]^{-\frac{2}{5}}}{(3x^{-2})^{-\frac{2}{5}}}$	2. $\frac{11^{n+1} .10^{n+5}}{2^{n+1} .\ 55^{n+1}}$	3. $\sqrt{8y^{10}} + \sqrt{50}\, y^5 - \sqrt{200y^{10}}$
4. $\frac{6^{6x}\ .\ 9^{3x}}{54^{4x}\ .\ \left(\frac{1}{4}\right)^{2-x}}$	5. $\frac{\sqrt{x-3}}{2x+1}$	6. $2^{2009} \times 2^{2008}$
7. $2^{2009} \times 2^{2008}$	8. $\frac{3^{3025} - 3^{3027}}{2^4}$	9. $2^{3x+1} + 2^{3x} = 12$
10. $2^{2015} \times 5^{2019}$	11. $125^{\frac{2}{3}}$	12. $5^x = \frac{1}{125}$
13. $2^{3x+1} + 2^{3x} = 12$	14. $\frac{5^{a-2} .2^{a+2}}{10^a - 10^{a-1} .2}$	15. $\sqrt{x-1} + 3 = x - 4$
16. $\frac{\sqrt{27m^6} - \sqrt{48m^6}}{\sqrt{12m^6}}$	17. $\frac{2^{a+1} - 2^{a-1}}{2^a}$	18. $\frac{2}{1+\sqrt{2}} - \frac{8}{\sqrt{8}} = -2$
19. $2^{x\sqrt{x}} = 2^{27}$	20. $\frac{2^{n+2}\ .\ 4^{n+1}}{8^{n-1}}$	21. $\sqrt{x + \sqrt{2x-1}} . \sqrt{x - \sqrt{2x-1}}$
22. $\frac{3^{x+1} - 3^{x-1}}{2 .3^x}$	23. $\frac{x\sqrt{x\sqrt{x\sqrt{x}}}}{\sqrt[8]{x^7}}$	24. $\sqrt{(x-2)^{-3}} = 64$
25. $\left(\frac{125x^7}{x}\right)^{\frac{2}{3}}$	26. $\frac{3}{\sqrt{3^x - 9}}$	27. $\left(\sqrt{3} + 3\right)^2 - 2\sqrt{27}$
28. $\frac{4^{x-1} + 46^{x+1}}{17 .12^x}$	29. $\frac{10^{2x+3}\ .\ 4^{1-x}}{25^{2+x}}$	30. $(3\sqrt{2} - 12)(2\sqrt{2} + 1)$
31. $\sqrt{27}$; $\sqrt[3]{-27}$; $\sqrt{-27}$	32. $2x^{-\frac{5}{3}} = 64$	33. $x - 3x^{\frac{1}{2}} = 4$
34. $3^{x+2} - 3^{x+2} = 486$	35. $2^{2x} - 6 .2^x = 16$	36. $\sqrt{3} . \sqrt{48} - \frac{4^{x+1}}{2^{2x}}$
37. $\sqrt{\frac{7^{2014} - 7^{2012}}{12}} = a\,(7^b)$	38. $x^{\frac{2}{3}} = 4$	39. $3^x(x-5) < 0$

By examining what is given from 1 – 42, can you tell what the question could possibly be?

Questions from Examination papers:

1. Simplify fully, WITHOUT using a calculator:

1.1 $\frac{2^{n+2} \cdot 4^{n+1}}{8^{n-1}}$

1.2 $\frac{2^{a+1} - 2^{a-1}}{2^{a}}$

1.3 $\frac{5^{a-2} \cdot 2^{a+2}}{10^{a} - 10^{a-1} \cdot 2}$

1.4 $\frac{11^{n+1} \cdot 10^{n+5}}{2^{n+1} \cdot 55^{n+1}}$

1.5 $\frac{x\sqrt{x\sqrt{x\sqrt{x}}}}{\sqrt[8]{x^{7}}}$

1.6 $\sqrt{x + \sqrt{2x-1}} \cdot \sqrt{x - \sqrt{2x-1}}$

1.7 $(\sqrt{3}+3)^{2} - 2\sqrt{27}$

1.8 $(3\sqrt{2} - 12)(2\sqrt{2} + 1)$

2. Solve for x

2.1 $x^{\frac{2}{3}} = 4$

2.2 $2x^{-\frac{5}{3}} = 64$

2.3 $5^{x} = \frac{1}{125}$

2.4 $2^{x\sqrt{x}} = 2^{27}$

2.5 $2^{3x+1} + 2^{3x} = 12$

2.6 $3^{x+2} - 3^{x+2} = 486$

2.7 $\sqrt{(x-2)^{-3}} = 64$

2.8 $3^{x}(x-5) < 0$

3. 3.1 Given: $\frac{\sqrt{27m^{6}} - \sqrt{48m^{6}}}{\sqrt{12m^{6}}}$

For which value(s) of x will the expression be,

a) Undefined
b) Non –real

3.2 Given : $f(x) = \frac{3}{\sqrt{3^{x}-9}}$

3.2.1 Determine the value of $f(3)$. Leave your answer in simplest surd form.

3.2.2 For which value(s) of x is $f(x)$ undefined?

3.2.3 For which value(s) of x is $f(x)$ non-real?

3.3 Which of the following is real, irrational and non-real.

$\sqrt{27}$; $\sqrt[3]{-27}$; $\sqrt{-27}$

4. WITHOUT using a calculator, show that: $\frac{2}{1+\sqrt{2}} - \frac{8}{\sqrt{8}} = -2$

5. Determine the value of a & b. $\sqrt{\frac{7^{2014} - 7^{2012}}{12}} = a\,(7^{b})$

Session 2:
Equations & Inequalities

In this session we will be solving quadratic equations and quadratic inequalities.

The standard form of a quadratic equation is, $ax^2 + b\,x + c = 0$. By completing the square a quadratic equation can be written into the form $a(x+p)^2 + q = 0$.

By completing the square of the quadratic $ax^2 + b\,x + c = 0$, the formulae,

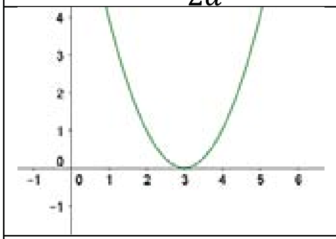
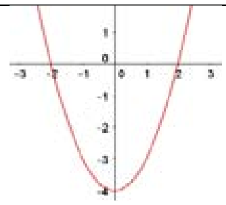
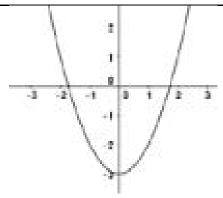
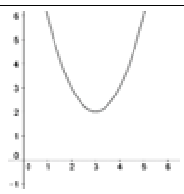
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, is derived.

A quadratic when written in standard form $ax^2 + b\,x + c = 0$, with rational roots, could be solved by either,

- Factorizing
- Using the formula

A quadratic equation with irrational roots can be solved by using the formula.

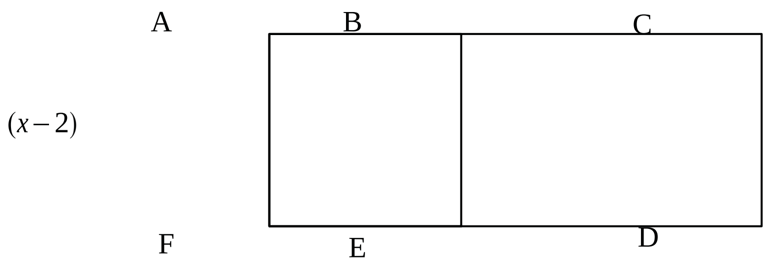
The nature of roots of a quadratic equation is determined by the different values of $b^2 - 4ac$

$b^2 - 4ac = 0$	$b^2 - 4ac > 0$		$b^2 - 4ac < 0$
$x = \frac{-b \pm \sqrt{0}}{2a}$ $x = \frac{-b}{2a}$	$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$		$x = \frac{-b \pm \sqrt{-ve}}{2a}$
			
	$b^2 - 4ac = Perfect\ Square$	$b^2 - 4ac \neq Perfect\ Square$	
One real root, which will be rational	Two real roots, rational	Two real roots, irrational	Roots will be non-real.

Examples:

1. What is the difference between an equation and an inequality?
   Consider a) $x^2 - 4 = 0$ b) $x^2 - 4 > 0$

2. ACDF is a rectangle with an area of $(x^2 + 2x - 8)\,\text{cm}^2$. B is a point on AC and E is a point on FD such that ABEF is a square with sides of length $(x-2)$ cm each.



Calculate the length of ED.

Questions:

1. Solve for *x*:

1.1 $(2x-3)(3x+1)=0$
1.2 $-2x^2+3x+8=0$
1.3 $(2x-3)(3x+1)=0$
1.4 $x(2x-3)=5$
1.5 $(x+1)(x-2)=4$
1.6 $(x-2)^2-9=16$
1.7 $7=1+(x-11)^2$
1.8 $x^2=2(11x-7)$
1.9 $2x-3=\frac{405}{x}$
1.10 $2x^2-4x<0$
1.11 $\sqrt{x-1}+3=x-4$
1.12 $\frac{2x-1}{(x+1)^2}<0$
1.13 $-x^2+2x+150$
1.14 $\sqrt{2x+1}=x-1$
1.14 $x^2\le 2(x+4)$

2. Solve for *x* and *y* simultaneously:

2.1 $x-2y=3$ and $4x^2-5xy=3-6$
2.2 $3^y=81^x$ and $y=x^2-6x+9$
2.3 $3x-\ y=-2$ and $y-8=-x^2+2x$
2.4 $x-2y+3=0$ and $y=\frac{16}{x}$

3. Given: $\frac{x^2-x-6}{3x-9}$

3.1 For which value(s) of x will the expression be undefined?
3.2 Simplify the expression fully.

4. The solution of quadratic equation $x=\frac{3\pm\sqrt{4-8p}}{4}$ where $p\in Q$.
Determine the value(s) of *p* so that, the equation has non-real roots.

5. Show that the roots of $3x^2+(k+2)x=1-k$ are real and rational for all values of *k*.

6. Given: $\sqrt{x+6}=x+4$

6.1 Calculate *x* in the given expression.
6.2 Hence, or otherwise, write down the solution to , $\sqrt{x+8}=x+6$.

7 Given: $(x+2)(x-3)<-3x+2$

7.1 Solve for *x*.
7.2 Hence or otherwise, determine the sum of all the integers satisfying the expression, $x^2+2x-8<0$.

8 Given: $f(x)=5x^2+6x-7$

8.1 Solve for *x* if *f(x)*=0
8.2 Hence, or otherwise, calculate the value *d* for which $5x^2+6x-d=0$ has equal roots.

9 Show that -$x^2+8x-17$ is always negative.

10 Show that $x^2-x+9>5$ for all real values of *x*.

Session 2: FUNCTIONS- Parabola, Hyperbola, Exponential and Straight Line

PROPERTIES RELATING TO ALL FUNCTIONS

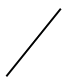
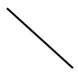
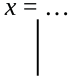
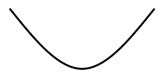
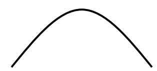
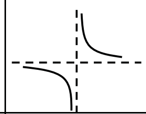
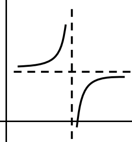
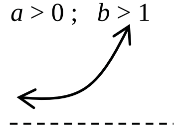
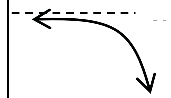
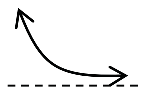
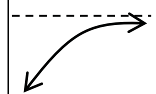
x intercept: Point on the *x*-axis where $y = 0$ (solve for *x* when $y = 0$)   *y*-intercept: Point on the *y*-axis where $x = 0$ (substitute $x = 0$)

Domain: The set of all *x*-values that make the function true (usually $x \in R$, unless there is a vertical asymptote)

Range: The set of all *y*-values that make the function true.

TRANSFORMATIONS IN FUNCTIONS

$g(x) = f(-x)$	Reflection of f about the *y*-axis	
$g(x) = -f(x)$	Reflection of f about the *x*-axis	
$g(x) = f(x) + q$	Translation of f up or down q units	$q > 0$ UP , $q < 0$ DOWN
$g(x) = f(x + p)$	Translation of f left or right p units	$p > 0$ LEFT , $p < 0$ RIGHT
$g(x) = f(ax)$	Changes steepness in a graph (non-trigonometric)	

	Straight Line				Parabola		Hyperbola		Exponential	
Equation	$y = ax + q$				$y = ax^2 + bx + c$ or $y = a(x-p)^2 + q$		$y = \frac{a}{x-p} + q$		$y = a.b^x + q$	
Shape	$a > 0$ 	$a < 0$ 	$a = 0$ $\therefore y = q$	a undefined $\therefore x = \ldots$ 	$a > 0$  $(p ; q)$	$a < 0$ $(p ; q)$ 	$a > 0$ 	$a < 0$ 	$a > 0$; $b > 1$ 	$a < 0$; $b > 1$ 
					(Axis of Symmetry, Min Value) (A/S; Min V)	(Axis of Symmetry, Max Value) (A/S; Max V)			$a > 0$; $0 < b < 1$ 	$a < 0$; $0 < b < 1$ 
Domain	$x \in R$	$x \in R$	$x \in R$		$x \in R$	$x \in R$	$x \in R- \{p\}$	$x \in R- \{p\}$	$x \in R$	$x \in R$
Range	$y \in R$	$y \in R$		$y \in R$	$y \in [q ; \infty)$	$y \in (-\infty ; q]$	$y \in R- \{q\}$	$y \in R- \{q\}$	$y \in (q ; \infty)$	$y \in (-\infty ; q)$
Other Importan t points	a = gradient = $\frac{y_2 - y_1}{x_2 - x_1}$ q: *y*-value of the *y*-intercept				**Turning Point (-p ; q)** **To calculate the turning if** $y = ax^2 + bx + c$ For the *x*-coordinate, (A/S) $x = -\frac{b}{2a}$; this is equation of A/S For *y*-coordinate: substitute the calculated *x*-value into the equation		**Asymptotes:** $y = q$ $x = p$ **Lines of symmetry:** $y = (x - p) + q$ $y = -[(x - p) + q]$		**Asymptotes:** $y = q$	

Question type	Summary of procedure	Example question
1. Sketch any of the graphs.	Identify the shape of graph, intercepts with axes, determine what other information is required i.e turning point or asymptotes or neither.	Sketch *x=2, y=-3*, $y = 3(x-1)^2+4$, $y = \frac{-1}{x+2} - 5$ & $y = 3.2^{-x}+1$
2. Find the equation of a given graph.	Identify the general equation for the graph from the shape and then determine the other variables.	
3. Find the equation of a given graph that has undergone a transformation.	Determine whether the transformation is a horizontal/vertical shift or reflection a about a particular line. It should then be easy to write down the new equation	If $f(x) = 2(x+1)^2 - 1$, find the equation $f(x-3)$, $f(x+5)$

FUNCTIONS

1. Given: $f(x) = -(x-1)^2 + 4$,

   1.1 Sketch the graph of *f* showing the co-ordinates of the turning point and the co-ordinates of any intercepts with the axes.

   1.2 Write down the equation of

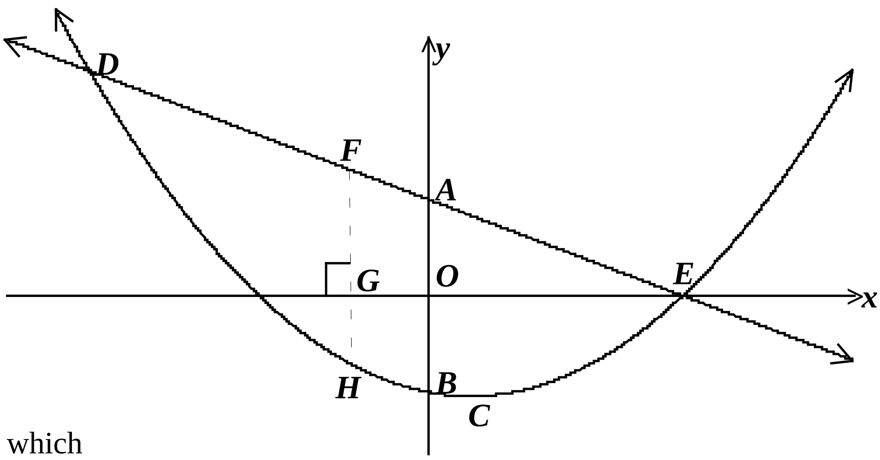
   1.2.1 the reflection of $y = -(x-1)^2 + 4$ in the *y*-axis,

   1.2.2 the reflection of $y = -(x-1)^2 + 4$ in the *x*-axis and

   1.2.3 the graph formed by translating by 1 unit to the right, the graph of $y = -(x-1)^2 + 4$.

   1.3 Does the point $(-2; 3)$ lie on the graph? Why or why not?

2. Sketched below are graphs of $p(x) = 2x^2 - x - 3$ and $q(x) = -2x + 3$



   2.1 Calculate the distance AB.

   2.2 Determine the co-ordinates of C, the turning point of the parabola.

   2.3 Determine the value(s) of x for which $p(x) = q(x)$

   2.4 Determine the co-ordinates of D and E.

   2.5 If G(-1 ; 0), determine the coordinates of F & H.

   2.6 Determine the value(s) of x for which

   a) $p(x) > 0$ b) $p(x).q(x) < 0$

   c) $x.p(x) < 0$

   2.7 For what value(s) of k will the roots of the equation $2x^2 - x = k$ have,

   a) Real roots b) Equal roots

3. Given: $f(x) = -x^2 + 6x + 7$

$g(x) = x + 1$

3.1 Sketch the graph of *f* and *g*, showing clearly all intercepts with the axes and turning points.

3.2 Write down the range of *f*.

3.3 Write down the equation of the axis of symmetry.

3.4 Calculate x where $f(x) = g(x)$.

3.5 Hence, or otherwise, determine the value(s) of x for which $f(x) \geq g(x)$.

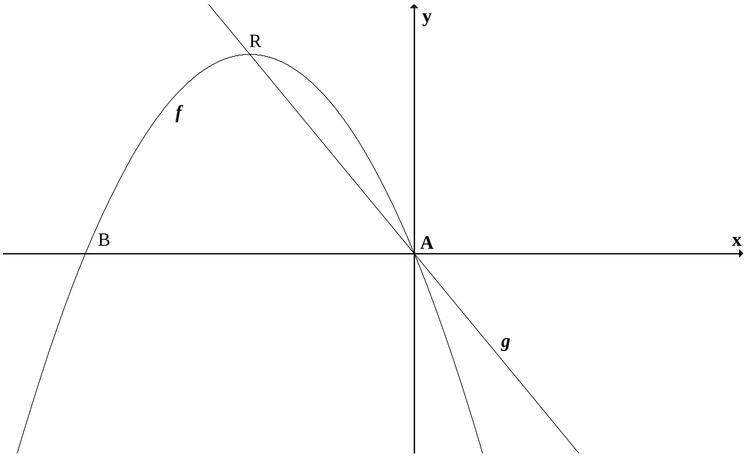
3.6 Determine the average gradient between $x = -1$ and $x = 4$.

3.7 Write down the axis of symmetry of $f(x-2)$.

4. Sketched below are the graphs of

$f(x) = -(x+2)^2 + 4$

$g(x) = ax + q$



4.1 Write down the coordinates of R.

4.2 Calculate the length of AB.

4.3 Determine the equation of *g*.

4.4 For which values of x is,

1) $g(x) > f(x)$?

2) *f* strictly increasing

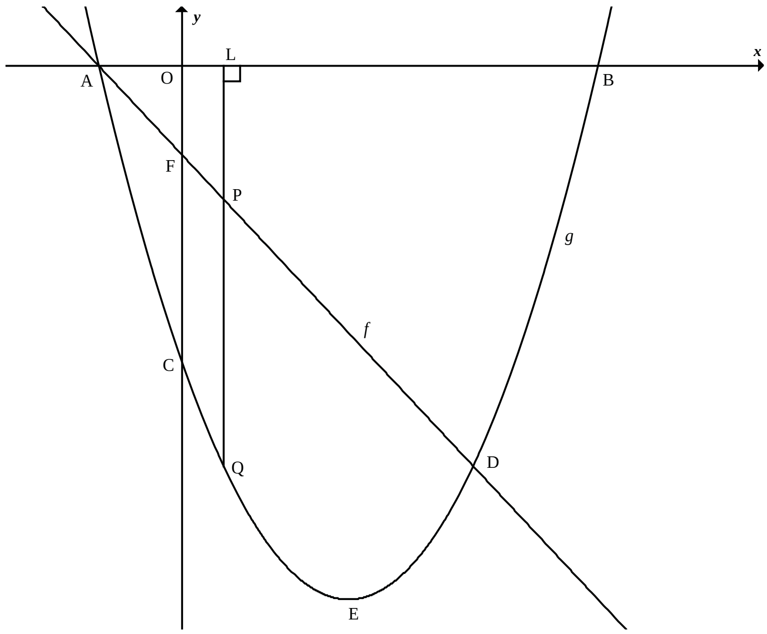
4.5 Write down the equation of the axis of symmetry of *h* if $h(x) = f(-x)$.

4.6 Write down the range of *p* if $p(x) = -f(x)$.

4.7 Determine the equation of a line through B, making an angle of $30°$ with the x-axes.

5. Sketched alongside are the graphs of

$f(x) = -3x - 6$ and $g(x) = x^2 - 8x - 20$



A and B are the x-intercepts of the graph of *g*.

C and F are the y-intercepts of *g* and *f* respectively.

QPL is a straight line with Q in *f* and P in *g* such that QPL $\perp$ x-axis.

A and D are the points of intersection of the graphs of *f* and *g*.

E is the turning point of *g*.

5.1 Write down the length of FC.

5.2 Calculate the length of AB.

5.3 Calculate the coordinates of D.

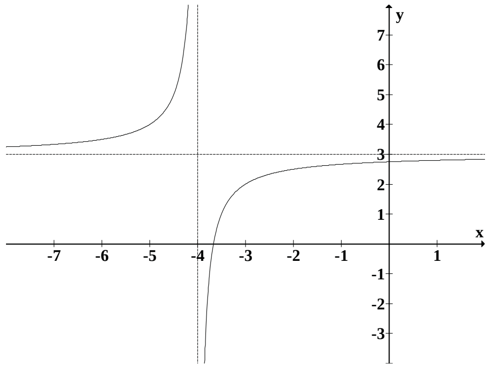
5.4 Determine an expression for the length PQ.

5.5 For which values of x, between the points A and D, will PQ be a maximum?

5.6 Calculate the maximum length of PQ, between the points A and D.

FUNCTIONS- Hyperbola, Exponential & Parabola

1. Sketched below is the graph of $f(x)=\dfrac{a}{x-p}+q$



1.1 Write down values of *p* and *q*.
1.2 Calculate the value of *a* if T(– 2 ; 2,5) lies on the graph of *f*.
1.3 Write down the domain of *f*.
1.4 Write down the equations of the lines of symmetry of *f* .
1.5 For which value(s) of *x* is $f(x) \leq 0$?
1.6 Write down the range of $f(x – 1)$.
1.7 Describe the transformation of *f* to *g* if $g(x) = f(-x)$.

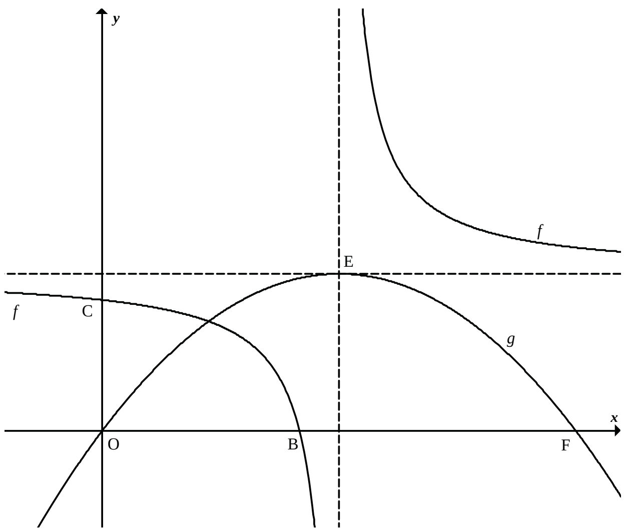
2. Given: $h(x)=2^{x+1}-1$

2.1 Determine the *y*-intercept of the graph of *h*.
2.2 Write down the equation of the asymptote of *h*.
2.3 Draw a sketch graph of *h*, showing all asymptotes and intercepts.
2.4 Write down the range of $g(x) = h(x) +3$
2.5 Describe the transformation of *h* to *g* if $g(x)=-2^{x+1}$

3. Given: $h(x)=\dfrac{2}{x-4}+1$

3.1 Write down the equations of the asymptotes of *h*.
3.2 Determine the *x*- and *y*-intercepts of the graph of *h*.
3.3 Sketch the graph of *h*.
3.4 Write down the domain of *h*.
3.5 Write down the range of *h*.
3.6 Describe the transformation of *h* to *f* if
3.6.1 $f(x) = h(x +3)$ 3.6.2 $f(x) = h(x) – 2$

4. The graph of $f(x)=\dfrac{1}{x-3}+2$ and $g(x)=a(x-p)^2+q$ are drawn. O and F are the *x*-intercepts of the graph of *g*. E is the turning point of *g*. B is the *x*-intercept of the graph of *f*. D is the point of intersection of the graphs of *f* and *g*.



4.1 Write down the coordinates of E.
4.2 Show that, $g(x)=-\dfrac{2}{9}x^2+\dfrac{4}{3}x$
4.3 Calculate the length of OF.
4.4 Write down the equation of the axis of symmetry of *f* that has a negative gradient.
4.5 Write down the equation of *p* if *p* is the reflection of *g* in the line $y = 2$.
4.6 Find the coordinate(s) of a point on *f* which is closest to E.

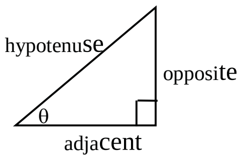
Session 4: Trigonometry

- **Definitions of trigonometric ratios:**
  - In a right-angled Δ

$$\text{Sin}\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cos}\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{Tan}\theta = \frac{\text{opposite}}{\text{adjacent}}$$

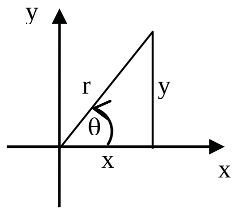


  - On a Cartesian Plane

$$\text{Sin}\theta = \frac{y}{r}$$

$$\text{Cos}\theta = \frac{x}{r}$$

$$\text{Tan}\theta = \frac{y}{x}$$



- **Special Angles**
  - **0°, 90°, 180°, 270°, 360°** can be obtained from the following unit circle

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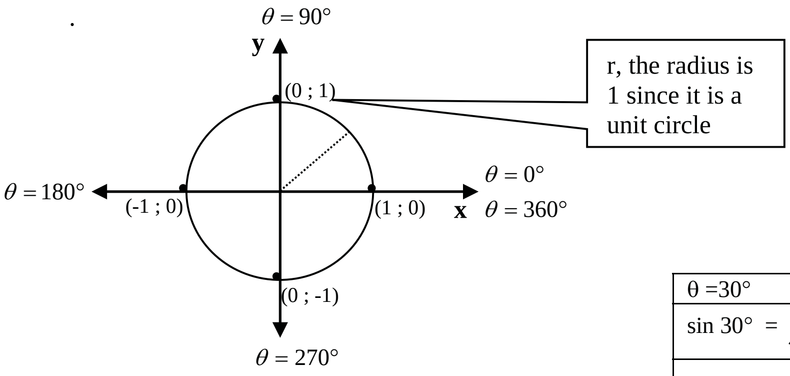
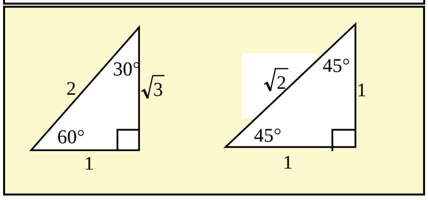
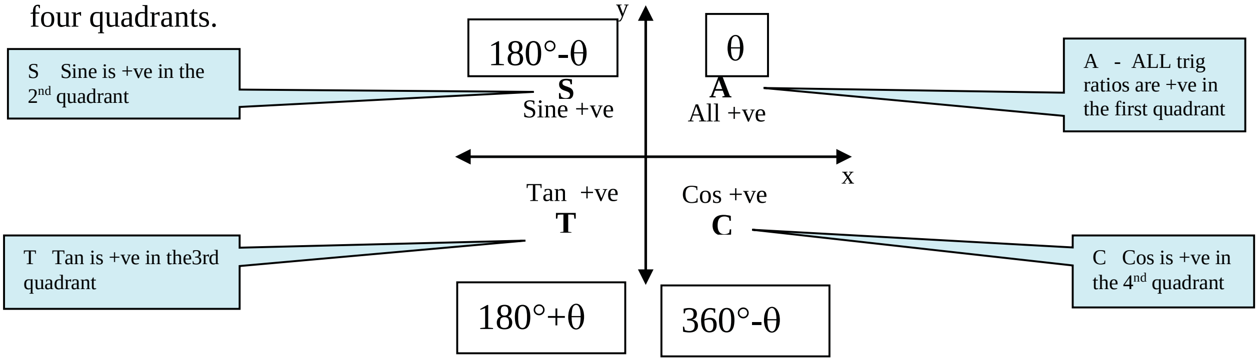


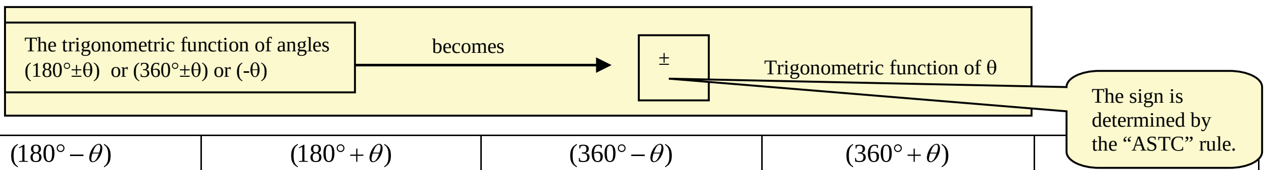
30°, 45° and 60° can be obtained from tthe following two triangles.



θ =30°	θ = 45°	θ = 60°
sin 30° = $\frac{1}{2}$	sin 45° = $\frac{1}{\sqrt{2}}$	sin 60° = $\frac{\sqrt{3}}{2}$
cos 30° = $\frac{\sqrt{3}}{2}$	cos 45° = $\frac{1}{\sqrt{2}}$	cos 60° = $\frac{1}{2}$
tan 30° = $\frac{1}{\sqrt{3}}$	tan 45° = 1	tan 60° = $\sqrt{3}$

- The **"ASTC" rule** enables you to obtain the sign of the trigonometric ratios in any of the four quadrants.





$(180° - \theta)$	$(180° + \theta)$	$(360° - \theta)$	$(360° + \theta)$	
$\sin(180° - \theta) = \sin\theta$	$\sin(180° + \theta) = -\sin\theta$	$\sin(360° - \theta) = -\sin\theta$	$\sin(360° + \theta) = \sin\theta$	$\sin(-\theta) = -\sin\theta$
$\cos(180° - \theta) = -\cos\theta$	$\cos(180° + \theta) = -\cos\theta$	$\cos(360° - \theta) = +\cos\theta$	$\cos(360° + \theta) = \cos\theta$	$\cos(-\theta) = +\cos\theta$
$\tan(180° - \theta) = -\tan\theta$	$\tan(180° + \theta) = +\tan\theta$	$\tan(360° - \theta) = -\tan\theta$	$\tan(360° + \theta) = \tan\theta$	$\tan(-\theta) = -\tan\theta$

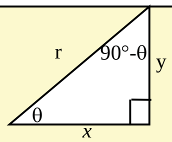
- **TRIGONOMETRIC IDENTITIES**

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $(\cos\theta \neq 0)$

$\sin^2\theta + \cos^2\theta = 1$, $\sin^2\theta = 1 - \cos^2\theta$, $\cos^2\theta = 1 - \sin^2\theta$

- **Co-functions or Co-ratios** →

$\sin(90° - \theta) = \cos\theta$
$\cos(90° - \theta) = \sin\theta$



$\sin(90° + \theta) = +\cos\theta$
$\cos(90° + \theta) = -\sin\theta$

- **Trigonometric Equations**

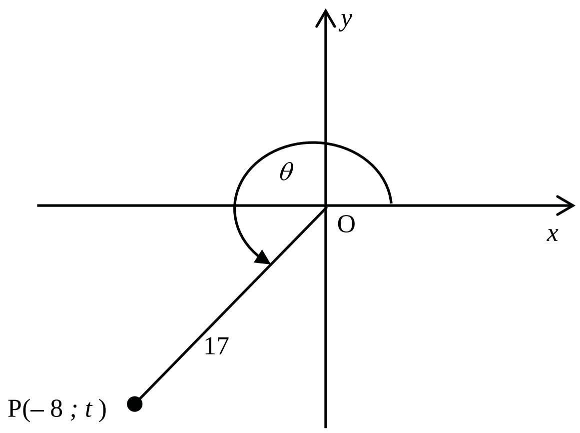
	$\sin\theta = 0{,}707$	$\cos\theta = -0{,}866$	$\tan\theta = -1$
1. Determine the Reference angle	Reference $\angle = \sin^{-1}(0{,}707) = 45°$	Reference $\angle = \cos^{-1}(0{,}866) = 30°$	Reference $\angle = \tan^{-1}(1) = 45°$
2. Establish in which two quadrants θ is.	$\therefore\ \theta = 45°$ or $\theta = 180° - 45°$	$\therefore\ \theta = 180° - 30°$ or $\theta = 180° + 30°$	$\therefore\ \theta = 180° - 45°$
3. Calculate θ in the interval $[0°;\ 360°]$	$\therefore\ \theta = 45°$ or $\theta = 135°$	$\therefore\ \theta = 150°$ or $\theta = 210°$ $\therefore\ \theta = \pm 150°$	$\therefore\ \theta = 135°$
4. Write down the general solution	$\therefore\ \theta = 45° + k360°$ or $\theta = 135° + k360°$ where $k \in Z$	$\therefore\ \theta = \pm 150 + k360°$ where $k \in Z$	$\therefore\ \theta = 135° + k180°$ $k \in Z$

TRIGONOMETRY SUMMARY

Question type	**Summary of procedure**	**Example question**
1. Calculate the value of a trig expression without using a calculator.	Establish whether you need a rough sketch or special triangles or ASTC rules.	1.1 If $13\cos\alpha = 5$ and $\tan\beta = -\dfrac{3}{4}$, $\alpha \in [0°;\ 270°]$ and $\beta \in [0°;\ 180°]$ Determine, without using a calculator, a) $\sin\alpha + \cos\beta$ 1.2 Calculate: a) $\dfrac{\cos(-210°).\sin^2 405°.\cos 14°}{\tan 120°.\sin 104°}$
2. Express trig ratio in terms of the given variable.	Draw a rough sketch with given angle and label 2 of the sides. The 3rd side can then be determined using Pythagoras. Express each of the angles in question in terms of the angle in the rough sketch.	2. If $\sin 27° = q$, express each of the following in terms of q. a) $\sin 117°$ b) $\cos(-27°)$
3. Simplify a trigonometrical expression.	Use the ASTC rule to simplify the given expression if possible. See if any of the identities can be used to simplify it, if not see if it can be factorized. Check again if any identity can be used. This includes using the compound and double angle identities.	3. Simplify: a) $\dfrac{\cos(720° - x)\ .\ \sin(360° + x)\ .\ \tan(x - 180°)}{\sin(-x)\ .\ \cos(90° - x)}$ b) $\dfrac{\sin(90° + x)\ .\ \tan(360° + x)}{\sin(180° + x)\ .\ \cos(90° - x) + \cos(540° + x).\cos(-x)}$
4. Prove a given identity.	Simplify the one side of the equation using reduction formulae and identities until it cannot simplify any further.	4. Prove that a) $\dfrac{\tan x\ .\ \cos^3 x}{1 - \sin^2 x + \cos^2 x} = \dfrac{1}{2}\sin x$ b) $\cos^2(180° - x) + \cos(90° + 2x)\tan(360° - x) = \sin^2 x + 1$
5. Solve a trig equation.	Find the reference angle by ignoring the "-"sign and finding $\sin^{-1}(0{,}435)$ Write down the two solutions in the interval, $x \in [0°;360°]$ Then write down the general solution for the given eq. From the general solution you can determine the solution for any specified interval by using various values of *k*.	5. Solve for $x \in [-180°;\ 360°]$ a) $\sin x = -0{,}435$ b) $\cos 2x = 0{,}435$ c) $\tan \dfrac{1}{2}x - 1 = 0{,}435$

TRIGONOMETRY QUESTIONS

1 In the diagram below, P(–8 ; t) is a point in the Cartesian plane such that OP = 17 units and reflex $X\hat{O}P = \theta$.



1.1 Calculate the value of t. (2)

1.2 Determine the value of each of the following WITHOUT using a calculator:

(a) $\cos(-\theta)$ (2)

(b) $1-\sin\theta$ (2)

2 If $\sin 17° = a$, WITHOUT using a calculator, express the following in terms of a :

2.1 $\tan 17°$ (3)

2.2 $\sin 107°$ (2)

2.3 $\cos^2 253° + \sin^2 557°$ (4)

3 Simplify fully, WITHOUT the use of a calculator:

$$\frac{\cos(-225°).\sin 135° + \sin 330°}{\tan 225°}$$ (6)

4 Prove the identity: $\frac{1}{(\cos x+1)(\cos x-1)} = \frac{-1}{\tan^2 x.\cos^2 x}$ (4)

5 Determine the general solution for $2\sin x.\cos x = \cos x.$ (6)

[31]

Session 5: Grade 11 geometry

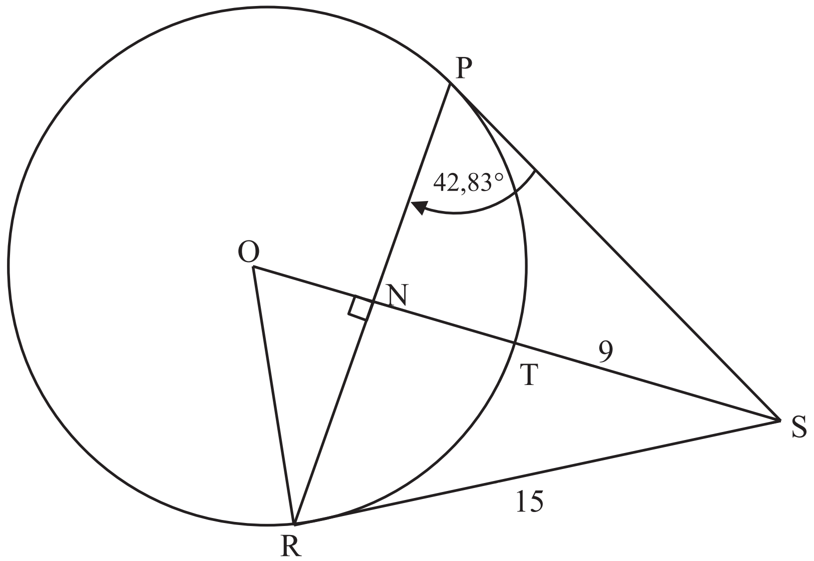
Below are Grade 11 Theorems, Converse Theorems and their Corollaries which you must know. The proofs of the theorems marked with (**) must be studied because it could be examined.

1	**Theorem****	The line drawn from the centre of a circle perpendicular to a chord bisects the chord; **(line from centre ⊥ to chord)**
	Converse	The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord. **(line from centre to midpt of chord)**
		The perpendicular bisector of a chord passes through the centre of the circle; **(perp bisector of chord)**
2	**Theorem****	The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); **(∠ at centre = 2 ×∠ at circumference)**
	Corollary	1. Angle in a semi-circle is 90^0 **(∠s in semi circle)** 2. Angles subtended by a chord of the circle, on the same side of the chord, are equal **(∠s in the same seg)** 3. Equal chords subtend equal angles at the circumference **(equal chords; equal ∠s)** 4. Equal chords subtend equal angles at the centre **(equal chords; equal ∠s)** 5. Equal chords in equal circles subtend equal angles at the circumference of the circles. **(equal circles; equal chords; equal ∠s)**
	Corollary Converse	1. If the angle subtended by a chord at the circumference of the circle is 90^0, then the chord is a diameter. **(converse ∠s in semi circle)** 2. If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.
3	**Theorem****	The opposite angles of a cyclic quadrilateral are supplementary; **(opp ∠s of cyclic quad)**
	Converse	If the opposite angles of a quadrilateral are supplementary then the quadrilateral is a cyclic quadrilateral. **(opp ∠s quad sup OR converse opp ∠s of cyclic quad)**
	Corollary	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral. **(ext ∠ of cyclic quad)**
	Corollary Converse	If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic. **(ext ∠ = int opp ∠ OR converse ext ∠ of cyclic quad)**
4	**Theorem**	The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact. **(tan ⊥ radius)**
	Converse	If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle. **(line ⊥ radius)**
5	**Theorem**	Two tangents drawn to a circle from the same point outside the circle are equal in length. **(Tans from common pt OR Tans from same pt)**
6	**Theorem****	The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. **(tan chord theorem)**
	Converse	If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. **(converse tan chord theorem OR ∠ between line and chord)**

QUESTION 1

O is the centre of the circle PTR. N is a point on chord RP such that $ON \perp PR$. RS and PS are tangents to the circle at R and P respectively.

RS = 15 units; TS = 9 units; $R\hat{P}S = 42{,}83°$.

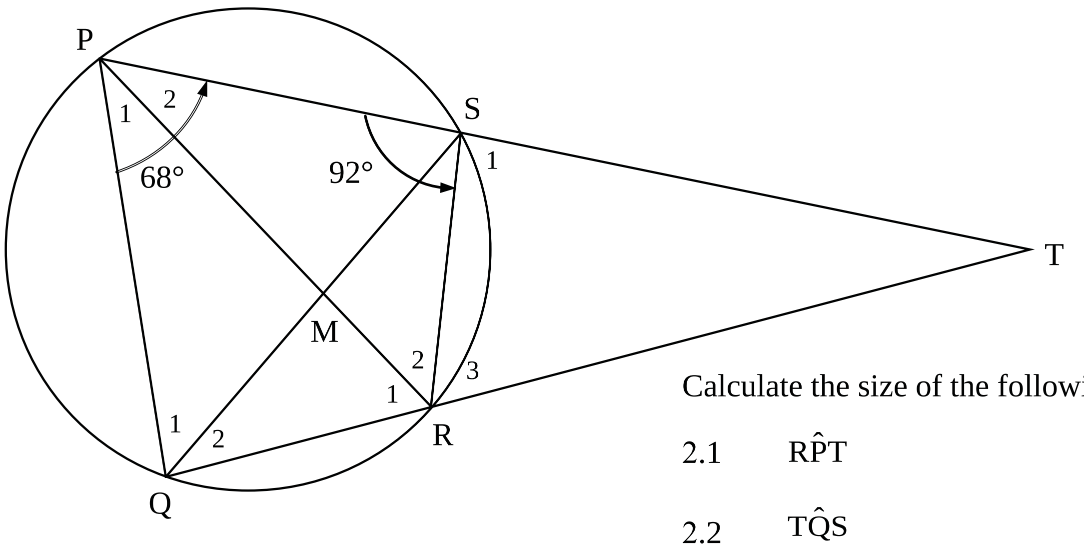


1.1 Calculate the size of $N\hat{O}R$.

1.2 Calculate the length of the radius of the circle.

QUESTION 2

In the diagram, PQRS is a cyclic quadrilateral. PS and QR are produced and meet at T. PR bisects $Q\hat{P}S$. Also, $P\hat{S}R = 92°$ and $Q\hat{P}S = 68°$.



Calculate the size of the following angles:

2.1 $R\hat{P}T$

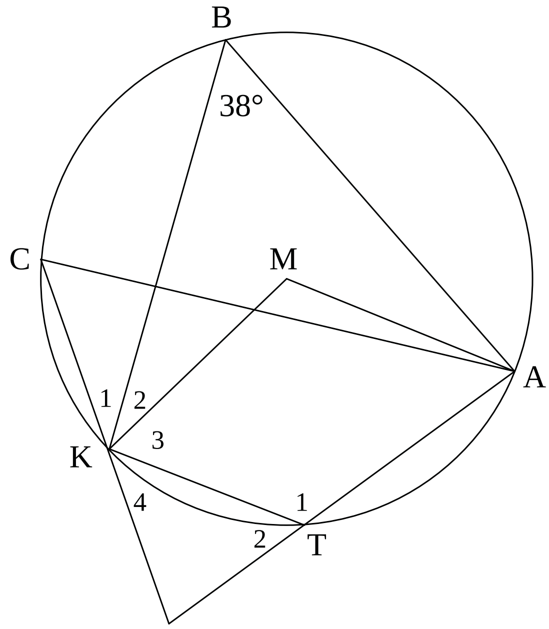
2.2 $T\hat{Q}S$

2.3 $P\hat{Q}S$

2.4 $\hat{T}$

QUESTION 3

In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also NA = NC and $\hat{B} = 38°$.



3.1 Calculate, with reasons, the size of the following angles:

(a) $K\hat{M}A$

(b) $\hat{T}_2$

(c) $\hat{C}$

(d) $\hat{K}_4$

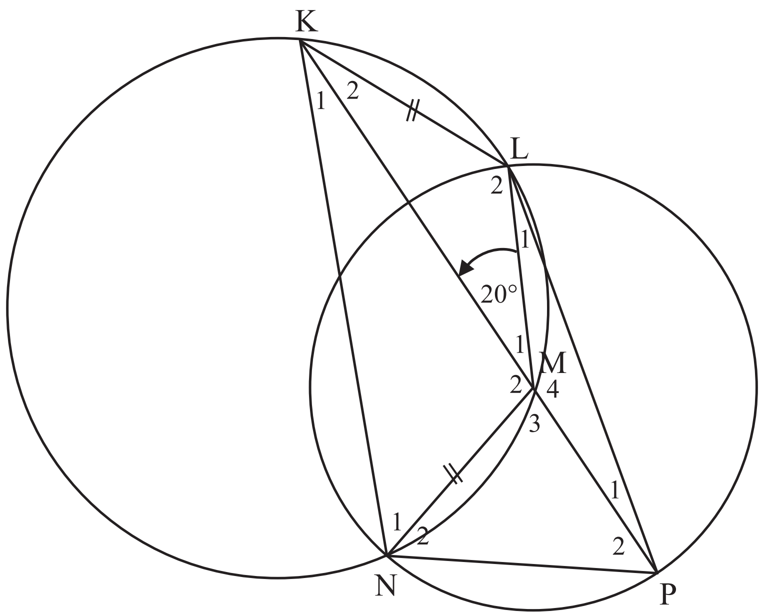
3.2 Show that NK = NT.

3.3 Prove that AMKN is a cyclic quadrilateral.

QUESTION 4

In the diagram M is the centre of the circle passing through points L, N and P.
PM is produced to K. KLMN is a cyclic quadrilateral in the larger circle having KL = MN.

LP is joined. $K\hat{M}L = 20°$



4.1 Write down, with a reason, the size of $N\hat{K}M$.

4.2 Give a reason why KN | | LM.

4.3 Prove that KL = LM.

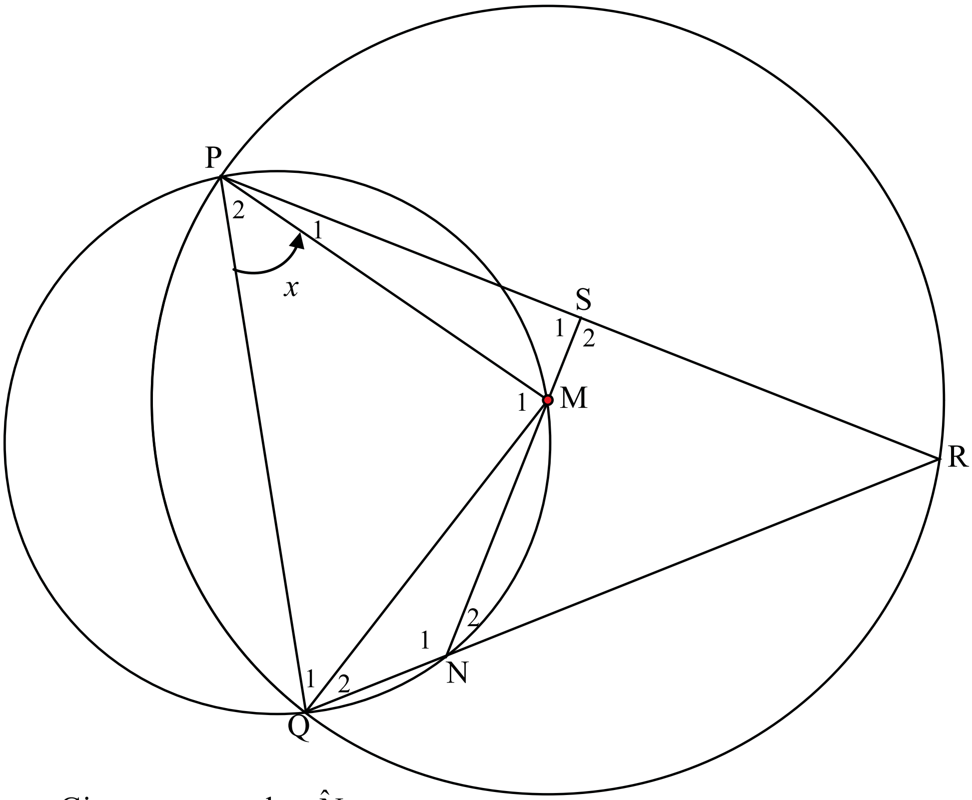
4.4 Calculate, with reasons, the size of:

4.4.1 $K\hat{N}M$

4.4.2 $L\hat{P}N$

QUESTION 5

5.1 In the diagram, QP is a common chord of the two circles. The centre, M, of the larger circle lies on the circumference of the smaller circle. PMNQ is a cyclic quadrilateral in the smaller circle. QN is produced to R, a point on the larger circle. NM produced meets the chord PR at S. $\hat{P}_2 = x$.



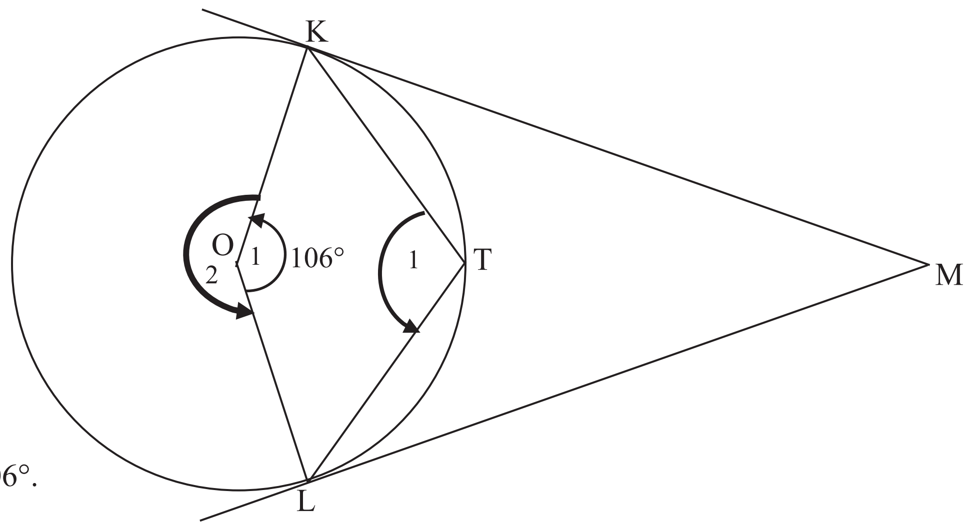
5.1.1 Give a reason why $\hat{N}_2 = x$.

5.1.2 Write down another angle equal in size to x. Give a reason.

5.1.3 Determine the size of $\hat{R}$ in terms of x.

5.1.4 Prove that PS = SR.

5.2 In the diagram O is the centre of the circle. KM and LM are tangents to the circle at K and L respectively. T is a point on the circumference of the circle. KT and TL are joined. $\hat{O}_1 = 106°$.



5.2.1 Calculate, with reasons, the size of $\hat{T}_1$.

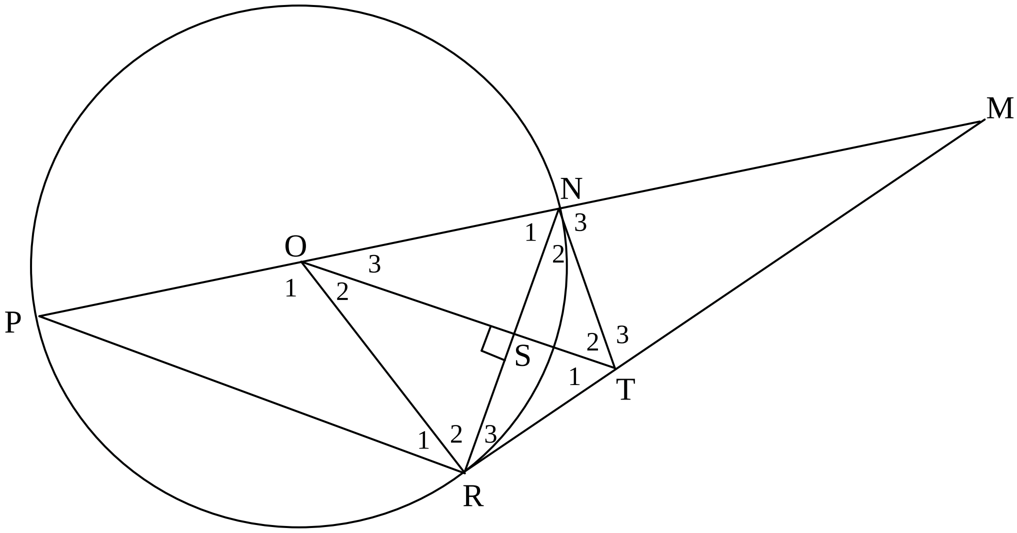
5.2.2 Prove that quadrilateral OKML is a kite.

5.2.3 Prove that quadrilateral OKML is a cyclic quadrilateral.

5.2.4 Calculate, with reasons, the size of $\hat{M}$.

QUESTION 6

In the diagram, PN is a diameter of the circle with centre O. RT is a tangent to the circle at R. RT produced and PN produced meet at M. OT is perpendicular to NR. NT and OR are drawn.



6.2 Prove that TO || RP.

6.2 It is further given that $T\hat{R}N = x$. Name TWO other angles each equal to x

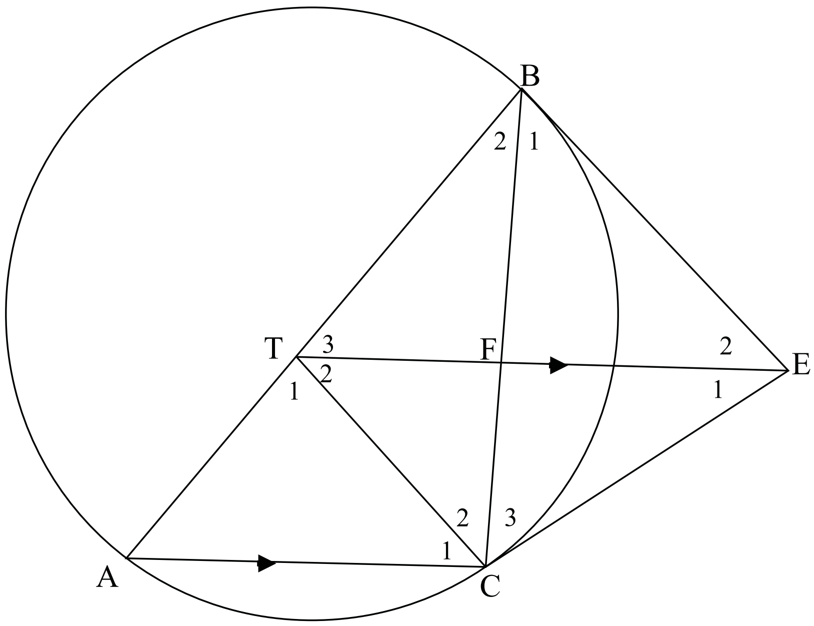
6.3 Prove that NTRO is a cyclic quadrilateral.

6.4 Calculate the size of $\hat{M}$ in terms of x.

6.5 Show that NT is a tangent to the circle at N.

QUESTION 7

In the diagram, the vertices A, B and C of ΔABC are concyclic. EB and EC are tangents to the circle at B and C respectively. T is a point on AB such that TE | | AC. BC cuts TE in F.



7.1 Prove that $\hat{B}_1 = \hat{T}_3$.

7.2 Prove that TBEC is a cyclic quadrilateral.

7.3 Prove that ET bisects $B\hat{T}C$.

7.4 If it is given that TB is a tangent to the circle through B, F and E, prove that TB = TC.

7.5 Hence, prove that T is the centre of the circle through A, B and C.