

## Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as  $3x = y$ ) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They

reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.



# Grade 6 Overview

## Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

## The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

## Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

## Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

## Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understand ratio concepts and use ratio reasoning to solve problems. (6.RP.A)

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

**(6.RP.A.1) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 2)

The students in Mr. Hill's class played games at recess.

6 boys played soccer

4 girls played soccer

2 boys jumped rope

8 girls jumped rope

Afterward, Mr. Hill asked the students to compare the boys and girls playing different games.

Mika said,

*"Four more girls jumped rope than played soccer."*

Chaska said,

*"For every girl that played soccer, two girls jumped rope."*

Mr. Hill said, "Mika compared the girls by looking at the difference and Chaska compared the girls using a ratio."

- a. Compare the number of boys who played soccer and jumped rope using the difference. Write your answer as a sentence as Mika did.
  - b. Compare the number of boys who played soccer and jumped rope using a ratio. Write your answer as a sentence as Chaska did.
  - c. Compare the number of girls who played soccer to the number of boys who played soccer using a ratio. Write your answer as a sentence as Chaska did.
- b. Example: [Solution](#) (DOK 2)  
In a bag of marbles,  $\frac{3}{5}$  of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?
  - c. Example: [Solution](#) (DOK 2)

Ty took the escalator to the second floor. The escalator is 12 meters long, and he rode the escalator for 30 seconds. Which statements are true? Select all that apply.

- a. He traveled 2 meters every 5 seconds.
- b. Every 10 seconds he traveled 4 meters.
- c. He traveled 2.5 meters per second.
- d. He traveled 0.4 meters per second.
- e. Every 25 seconds, he traveled 7 meters.

d. Example: [Solution](#) (DOK 3)

The ratio of the number of boys to the number of girls at school is 4:5. There are 270 students at this school. For each of the following statements, explain whether the statement is true or false and why:

- a. The number of boys at school is  $\frac{4}{5}$  the number of girls.
- b.  $\frac{4}{5}$  of the students in the school are boys.
- c. There are exactly 30 more girls than boys.
- d. There are exactly 30 boys at the school.
- e.  $\frac{5}{9}$  of the students in the school are girls.

e. Example: [Solution](#) (DOK 2)

Alice and Claire go apple picking. When they are done, Claire has 3 times as many apples in her basket as Alice has in hers. All of the apples are whole.

- a. What are three different possibilities for numbers of apples that could be in the baskets?
- b. What is the ratio of Alice's apples to Claire's apples?

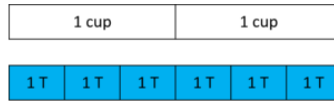
Alice and Claire's mom measures each of their heights in inches, rounded to the nearest whole inch. She remarks, "Wow! Alice's height is exactly three fourths of Claire's height!"

- c. What are three different reasonable possibilities for their heights?
- d. What is the ratio of Claire's height to Alice's height?

f. Example: [Solution](#) (DOK 2)

## Paint

Abigail mixed 2 cups of white paint with 6 tablespoons of blue paint. Which of the following statements describes this situation?

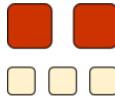


- There is 1 cup of white paint for every 3 tablespoons of blue paint.
- The ratio of the number of cups of white paint to the number of tablespoons of blue paint is 1:3.
- There are 3 tablespoons of blue paint per cup of white paint.
- For each tablespoon of blue paint there are 3 cups of white paint.

## Paste

Clara mixed 8 cups of flour with 2 pints of water to make papier-mâché paste. Describe the relationship between flour and water in the paste in at least four different ways.

- g. Example: [Solution](#) (DOK 2)  
 A recipe calls for 2 cups of tomato sauce and 3 tablespoons of oil. We can say that the ratio of cups of tomato sauce to tablespoons of oil in the recipe is 2:3, or we can say the ratio of tablespoons of oil to cups of tomato sauce is 3:2.



For each of the following situations, draw a picture and name two ratios that represent the situation.

- To make papier-mâché paste, mix 2 parts of water with 1 part of flour.
  - A farm is selling 3 pounds of peaches for \$5.
  - A person walks 6 miles in 2 hours.
- h. Example: A recipe requires  $\frac{3}{4}$  cup of nuts for 1 cake.

Write the maximum number of cakes that can be made using  $7\frac{1}{2}$  cups of nuts.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#8	1	NS	B	1	6.NS.A.1	N/A	10

- Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $\frac{3}{4}$  cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."<sup>1</sup> (6.RP.A.2) (DOK 1,2)

<sup>1</sup> Expectations for unit rates in this grade are limited to non-complex fractions.

- a. Example: [Solution](#) (DOK 3)

A store was selling 8 mangos for \$10 at the farmers market.

Keisha said,

*"That means we can write the ratio 10 : 8, or \$1.25 per mango."*

Luis said,

*"I thought we had to write the ratio the other way, 8 : 10, or 0.8 mangos per dollar."*

Can we write different ratios for this situation? Explain why or why not.

- b. Example: [Solution](#) (DOK 3)

The grocery store sells beans in bulk. The grocer's sign above the beans says,

5 pounds for \$4.

At this store, you can buy any number of pounds of beans at this same rate, and all prices include tax.

Alberto said,

*"The ratio of the number of dollars to the number of pounds is 4:5. That's \$0.80 per pound."*

Beth said,

*"The sign says the ratio of the number of pounds to the number of dollars is 5:4. That's 1.25 pounds per dollar."*

- Are Alberto and Beth both correct? Explain.
  - Claude needs two pounds of beans to make soup. Show Claude how much money he will need.
  - Dora has \$10 and wants to stock up on beans. Show Dora how many pounds of beans she can buy.
  - Do you prefer to answer parts (b) and (c) using Alberto's rate of \$0.80 per pound, using Beth's rate of 1.25 pounds per dollar, or using another strategy? Explain.
- c. Example: [Solution](#) (DOK 2)



Lin rode a bike 20 miles in 150 minutes. If she rode at a constant speed,

- a. How far did she ride in 15 minutes?
- b. How long did it take her to ride 6 miles?
- c. How fast did she ride in miles per hour?
- d. What was her pace in minutes per mile?

d. Example: [Solution](#) (DOK 2)

Ty took the escalator to the second floor. The escalator is 12 meters long, and he rode the escalator for 30 seconds. Which statements are true? Select all that apply.

- a. He traveled 2 meters every 5 seconds.
- b. Every 10 seconds he traveled 4 meters.
- c. He traveled 2.5 meters per second.
- d. He traveled 0.4 meters per second.
- e. Every 25 seconds, he traveled 7 meters.

e. Example: [Solution](#) (DOK 2)

a. Hippos sometimes get to eat pumpkins as a special treat.



If 3 hippos eat 5 pumpkins, how many pumpkins per hippo is that?

b. Lindy made 24 jelly-bread sandwiches with a 16-ounce jar of jelly. How many ounces of jelly per sandwich is that?

c. Purslane bought 350 rolls of toilet paper for the whole year. How many rolls of toilet paper per month is that?

d. In the world's longest running experiment, scientists have tried to capture tar pitch dripping on camera. In the past 86 years, 9 drops have formed. How many years per drop is that?

e. Imagine that 12 goats got into a dumpster behind a pizza parlor and ate 3 pizzas. How many goats per pizza would that be?



f. Example: [Solution](#) (DOK 3)

A school carnival ticket booth posts the following sign:

## TICKET BOOTH

1 Ticket For \$.50  
 12 Tickets For \$5.00  
 25 Tickets For \$10.00  
 50 Tickets For \$25.00  
 120 Tickets For \$50.00

HAVE FUN!

- Which amount of tickets offers the best deal? Explain.
- How would you suggest the students running the ticket booth modify the list of prices?

g. Example: [Solution](#) (DOK 3)

**2 bottles of water cost \$5.00.**

- Fill in the table that shows the costs for 4, 6, and 8 bottles. Find the cost for a single bottle in each case.

Number of bottles	Cost (\$)	Cost per bottle
2	5	
4		
6		
8		

**5 granola bars cost \$4.00**

- Fill in the table that shows the costs for 10, 15, and 20 granola bars. Find the cost for a single granola bar in each case.

Number of granola bars	Cost (\$)	Cost per bar
5	4	
10		
15		
20		

c. Explain why if you can buy  $a$  items for  $b$  dollars, or buy  $2a$  items for  $2b$  dollars, the cost per item is the same in either case.

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  - c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  - d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **(6.RP.A.3) (DOK 1,2)**
    1. Example: [Solution](#) (DOK 2)
 

A mixture of concrete is made up of sand and cement in a ratio of 5 : 3.  
How many cubic feet of each are needed to make 160 cubic feet of concrete mix?
    2. Example: [Solution](#) (DOK 2)
 

Selina bought a shirt on sale that was 20% less than the original price.  
The original price was \$5 more than the sale price. What was the original price? Explain or show work.
    3. Example: [Solution](#) (DOK 2)
      - a. John, Marie, and Will all ran for 6th grade class president. Of the 36 students, 16 voted for John, 12 for Marie, and 8 for Will. What was the ratio of votes for John to votes for Will? What was the ratio of votes for Marie to votes for Will? What was the ratio of votes for Marie to votes for John?
      - b. Because no one got half the votes, they had to have a run-off election. Marie dropped out and convinced all her voters to vote for Will. What is the new ratio of Will's votes to John's?
      - c. John and Will also ran for Middle School Council President. There are 90 students voting in middle school. If the ratio of Will's votes to John's votes remains the same as it was in part (b), how many more votes will Will get than John?
    4. Example: [Solution](#) (DOK 2)

John, Marie, and Will all ran for 6th grade class president. Of the 36 students voting, the ratio of votes for John to votes for Will was two to one. Marie got exactly the average number of votes for the three of them. How many more votes did John get than Marie?

5. Example: [Solution](#) (DOK 2)

John, Marie, and Will all ran for 6th grade class president. The ratio of votes for John to votes for Will was two to one. Marie got exactly the average number of votes for the three of them. John got more votes than Marie. What fraction of the total votes was this difference?

6. Example: [Solution](#) (DOK 3)

Jada has a rectangular board that is 60 inches long and 48 inches wide.

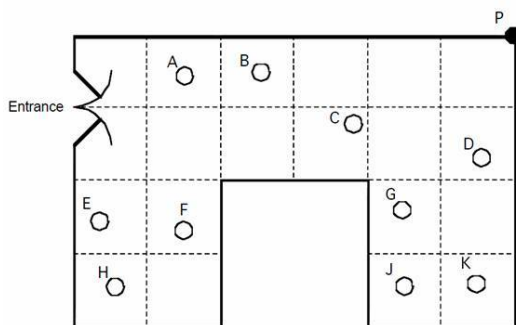
- How long is the board measured in feet? How wide is the board measured in feet?
- Find the area of the board in square feet.
- Jada said,

*To convert inches to feet, I should divide by 12.  
 The board has an area of  $48 \text{ in} \times 60 \text{ in} = 2,880 \text{ in}^2$ .  
 If I divide the area by 12, I can find out the area in square feet.  
 So the area of the board is  $2,880 \div 12 = 240 \text{ ft}^2$ .*

What went wrong with Jada's reasoning? Explain.

7. Example: [Solution](#) (DOK 3)

A shop owner wants to prevent shoplifting. He decides to install a security camera on the ceiling of his shop. Below is a picture of the shop floor plan with a square grid. The camera can rotate 360°. The shop owner places the camera at point P, in the corner of the shop.



- The plan shows where ten people are standing in the shop. They are labeled A, B, C, D, E, F, G, H, J, K. Which people cannot be seen by the camera at P?
- What percentage of the shop is hidden from the camera? Explain or show work.
- The shopkeeper has to hang the camera at the corners of the grid. Show the best place for the camera so that it can see as much of the shop as possible. Explain how you know that this is the best place to put the camera.

8. Example: [Solution](#) (DOK 3)  
The lot that Dana is buying for her new one story house is 35 yards by 50 yards. Dana's house plans show that her house will cover 1,600 square feet of land. What percent of Dana's lot will not be covered by the house? Explain your reasoning.
9. Example: [Solution](#) (DOK 1)  
Kendall bought a vase that was priced at \$450. In addition, she had to pay 3% sales tax. How much did she pay for the vase?
10. Example: [Solution](#) (DOK 2)  
Joe was planning a business trip to Canada, so he went to the bank to exchange \$200 U.S. dollars for Canadian (CDN) dollars (at a rate of \$1.02 CDN per \$1 US). On the way home from the bank, Joe's boss called to say that the destination of the trip had changed to Mexico City. Joe went back to the bank to exchange his Canadian dollars for Mexican pesos (at a rate of 10.8 pesos per \$1 CDN). How many Mexican pesos did Joe get?
11. Example: [Solution](#) (DOK 3)  
Alexis needs to paint the four exterior walls of a large rectangular barn. The length of the barn is 80 feet, the width is 50 feet, and the height is 30 feet. The paint costs \$28 per gallon, and each gallon covers 420 square feet. How much will it cost Alexis to paint the barn? Explain your work.
12. Example: [Solution](#) (DOK 2)  
Taylor and Anya live 63 miles apart. Sometimes on a Saturday, they ride their bikes toward each other's houses and meet somewhere in between. Taylor is a very consistent rider - she finds that her speed is always very close to 12.5 miles per hour. Anya rides more slowly than Taylor, but she is working out and so she is becoming a faster rider as the weeks go by.
- a. On a Saturday in July, the two friends set out on their bikes at 8 am. Taylor rides at 12.5 miles per hour, and Anya rides at 5.5 miles per hour. After one hour, how far apart are they?
- b. Make a table showing how far apart the two friends are after zero hours, one hour, two hours, and three hours.
- c. At what time will the two friends meet?
- d. Taylor says, "If I ride at 12.5 miles per hour toward you, and you ride at 5.5 miles per hour toward me, it's the same as if you stay still and I ride at 18 miles per hour." What do you think Taylor means by this? Is she correct?
- e. A couple of months later, on a Saturday in September, the two friends set out again on their bikes at 8 am. Taylor, as always, rides at 12.5 miles per hour. This time they meet at 11 am. How fast was Anya riding this time?

13. Example: [Solution](#) (DOK 2)  
 A runner ran 20 miles in 150 minutes. If she runs at that speed,
- How long would it take her to run 6 miles?
  - How far could she run in 15 minutes?
  - How fast is she running in miles per hour?
  - What is her pace in minutes per mile?

14. Example: [Solution](#) (DOK 2)  
 Jim and Jesse each had the same amount of money. Jim spent \$58 to fill the car up with gas for a road-trip. Jesse spent \$37 buying snacks for the trip. Afterward, the ratio of Jim's money to Jesse's money is 1 : 4. How much money did each have at first?

15. Example: [Solution](#) (DOK 2)  
 Julianna participated in a walk-a-thon to raise money for cancer research. She recorded the total distance she walked at several different points in time, but a few of the entries got smudged and can no longer be read. The times and distances that can still be read are listed in the table below.

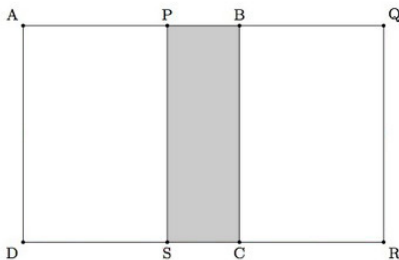
- Assume Julianna walked at a constant speed. Complete the table and plot Julianna's progress in the coordinate plane.
- How fast was Julianna walking in miles per hour? How long did it take Julianna to walk one mile?
- Next year Julianna is planning to walk for seven hours. If she walks at the same speed next year, how many miles will she walk?

Time in hrs	Miles walked
1	
2	6
	12
5	

16. Example: [Solution](#) (DOK 2)  
 The data transfer rate of an Internet connection is the rate in *bytes* per second that a file can be transmitted across the connection. Data transfer is typically measured in kilobytes (KB) per second, or megabytes (MB) where  $1 \text{ MB} = 2^{10} \text{ KB} = 1024 \text{ KB}$ . Suppose the data transfer rate of your internet connection is 500 KB per second.
- How long will it take to download a music file that is 5 MB?
  - How long will it take to download a video file that is 100 MB?

17. Example: [Solution](#) (DOK 2)

Two congruent squares,  $ABCD$  and  $PQRS$ , have side length 15. They overlap to form the 15 by 25 rectangle  $AQRD$  shown. What percent of the area of rectangle  $AQRD$  is shaded?



18. Example: [Solution](#) (DOK 2)

In a bag of marbles,  $\frac{3}{5}$  of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

19. Example: [Solution](#) (DOK 2)

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of \$27.50 for dinner. What is the cost of her dinner without tax or tip?

20. Example: [Solution](#) (DOK 2)

A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?

21. Example: [Solution](#) (DOK 2)

Lin rode a bike 20 miles in 150 minutes. If she rode at a constant speed,

- How far did she ride in 15 minutes?
- How long did it take her to ride 6 miles?
- How fast did she ride in miles per hour?
- What was her pace in minutes per mile?

22. Example: [Solution](#) (DOK 2)

A penny is about  $\frac{1}{16}$  of an inch thick.

- a. In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?
- b. In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?
- c. The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?

23. Example: [Solution](#) (DOK 1)

A store has two different brands of laundry detergent. Brand A can do 80 loads of laundry and costs \$12.75. Brand B does 36 loads of laundry and costs \$6.75. Which laundry detergent costs less per load?

24. Example: [Solution](#) (DOK 2)

Gianna is paid \$90 for 5 hours of work.

- a. At this rate, how much would Gianna make for 8 hours of work?
- b. At this rate, how long would Gianna have to work to make \$60?

25. Example: [Solution](#) (DOK 3)

There are many resources available on the computer to help converting between different units. This task examines a few important aspects of some of these conversions.

- a. According to an online calculator, there are 1728 cubic inches in a cubic foot.
  - i. Explain how to check the calculator's answer. Is it correct?
  - ii. How many cubic inches are there in 5 cubic feet? What about in 200 cubic feet? Explain how to find these answers without the calculator using the number of cubic inches in one cubic foot.
- b. A given website converts kilograms to pounds allowing the user to select how many decimals to provide in the conversion. With no decimals it says "1 kilogram = 2 pounds." For one decimal, it says "1 kilogram = 2.2 pounds."
  - i. According to these two conversion rates, how many pounds are in 5 kilograms?
  - ii. What would be a better way of stating these conversion rates?

26. Example: [Solution](#) (DOK 3)



A school carnival ticket booth posts the following sign:

## TICKET BOOTH

1 Ticket For \$.50  
12 Tickets For \$5.00  
25 Tickets For \$10.00  
50 Tickets For \$25.00  
120 Tickets For \$50.00

## HAVE FUN!

- Which amount of tickets offers the best deal? Explain.
- How would you suggest the students running the ticket booth modify the list of prices?

27. Example: [Solution](#) (DOK 3)

Alyssa took a math test with 20 questions and she answered 16 questions correctly. In order to get an A, she needs to answer at least 90 percent of the questions correctly.

- Did Alyssa get an A on this math test? Explain.
- How many questions would Alyssa need to answer correctly in order to get an A?
- On the same exam, Alyssa's friend Jessie answered three times more questions correct than incorrect. Who got a higher percentage of the test questions correct, Alyssa or Jessie?

28. Example: [Solution](#) (DOK 3)

Jessica sees the following speed limit sign while visiting Australia where the units for speed are kilometers per hour:



a. A conversion table indicates that 1 mile is 1.6 km. With this conversion rate, is the speed limit greater than or less than 65 mph? Explain.

b. Jessica finds out that 1 mile is not exactly 1.6 km. This number has been rounded to the nearest tenth. Does this influence the answer to part (a)?

29. Example: [Solution](#) (DOK 2)

Jessica gets her favorite shade of purple paint by mixing 2 cups of blue paint with 3 cups of red paint. How many cups of blue and red paint does Jessica need to make 20 cups of her favorite purple paint?

30. Example: [Solution](#) (DOK 3)

The 150 students at Skokie School were asked if they prefer seeing the movie *Hunger Games* or *Divergent*. The data showed that 100 preferred *Hunger Games* and 50 preferred *Divergent*.

a. Look at the following statements and decide if each accurately reports the results of the survey and explain *how you know*.

i. At Skokie School,  $\frac{1}{3}$  of the students prefer *Hunger Games*.

ii. Students prefer *Hunger Games* to *Divergent* in a ratio of 2 to 1.

iii. The ratio of students who prefer *Divergent* to students who prefer *Hunger Games* is 1 to 2.

iv. The number of students who prefer *Hunger Games* is 50 more than the number of students who prefer *Divergent*.

v. The number of students who prefer *Hunger Games* is two times the number of students who prefer *Divergent*.

b. Compare statements (iv) and (v) above. In what ways is the information given by these statements similar? In what ways is it different? Explain.

31. Example: [Solution](#) (DOK 3)

When Sam and his friends get together, Sam makes orange soda by mixing orange juice with soda (sparkling water). On Friday, Sam makes 7 liters of orange soda by mixing 3 liters of orange juice with 4 liters of soda. On Saturday, Sam makes 9 liters of orange soda by mixing 4 liters of orange juice with 5 liters of soda.

Make a prediction about how the two orange sodas will compare in taste. Explain your reasoning.

32. Example: [Solution](#) (DOK 3)

Arianna is making origami swans for her friend's birthday party. She wants to make 9 swans, one for each party guest. If Arianna takes 15 minutes to make each swan, will she be able to make 9 swans in 2 hours? Explain.

33. Example: [Solution](#) (DOK 2)

For all of these questions, it may be helpful to draw a diagram.

### A steady speed

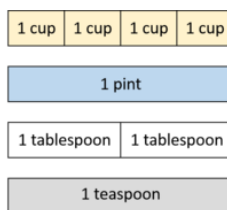
- a. A snail went 6 inches in 2 minutes at a steady speed. How far did it go in one minute?
- b. A caterpillar went 36 inches in 2 minutes at a steady speed. How far did it go in one minute?
- c. A baby turtle went 36 inches in 12 minutes at a steady speed. How far did it go in one minute?
- d. What is the same and what is different about the snail and the caterpillar? The caterpillar and the baby turtle? The baby turtle and the snail?

### Buying in bulk

- a. 30 kilograms of beans costs \$5. How many kilograms of beans can you buy for \$1?
- b. 15 kilograms of rice costs \$5. How many kilograms of rice can you buy for \$1?
- c. 30 kilograms of cornmeal costs \$10. How many kilograms of cornmeal can you buy for \$1?
- d. What is the same and what is different about the beans and the rice? The rice and the cornmeal? The cornmeal and the beans?

34. Example: [Solution](#) (DOK 2)

A recipe for bread calls for 4 cups of flour, 1 pint of water, 2 tablespoons of yeast, and 1 teaspoon of salt. The diagram below represents 1 batch of bread.



- a. How much of each ingredient is needed for 2 batches of bread? Draw a diagram that shows this.
- b. How much of each ingredient is needed for 3 batches of bread? Draw a diagram that shows this.
- c. How much of each ingredient is needed for 5 batches of bread? (Can you answer this without drawing a diagram?)
- d. What is the ratio of flour to yeast in five batches of bread?

35. Example: [Solution](#) (DOK 3)

Callie biked 12 miles in 3 hours. Carter biked 10 miles in 2 hours.

Represent each person's trip with a diagram. Explain how you can see that they are not going the same speed.

36. Example: [Solution](#) (DOK 2)

Malik is using a cookbook to make a recipe, but he can not find his measuring cups! He has, however, found a tablespoon. Inside the back cover of the cookbook, it says that 1 cup = 16 tablespoons.

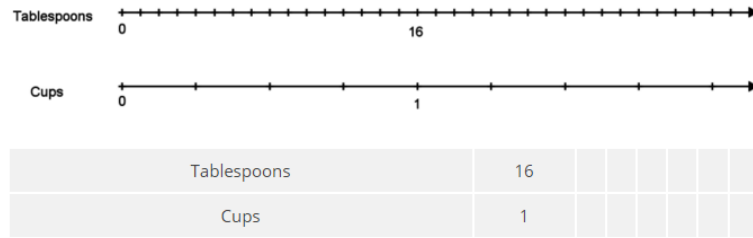
a. Explain how he could use the tablespoon to measure out the following ingredients:

- i. 2 cups of flour
- ii.  $\frac{1}{2}$  cup sunflower seeds
- iii.  $1\frac{1}{4}$  cup of oatmeal

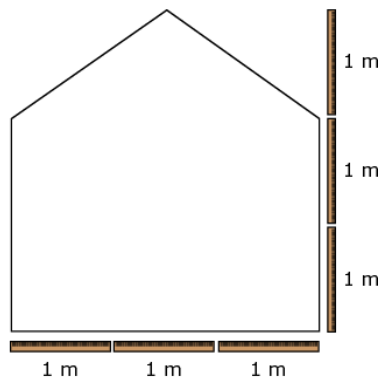
b. Malik also adds the following ingredients. How many cups of each did he add?

- i. 28 tablespoons of sugar
- ii. 6 tablespoons of cocoa powder

Use of the following representations is optional:



37. Example: John needs to paint one wall in his school. He knows that 1 can of paint covers an area of 24 square feet. John uses a meter stick to measure the dimensions of the wall as shown.



[1 meter = approximately 39 inches]

What is the **fewest** number of cans of paint John can use to paint the wall?

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#28	2	G, RP	B, A	2	7.G.B.6, 6.RP.A.3d	1	4

38. Example: George earns \$455 per week. George receives a 20% raise.  
How can George calculate his new weekly pay rate?

Select **all** calculations that will result in George's new weekly pay rate.

- Divide \$455 by 0.20
- Divide \$455 by 1.20
- Multiply \$455 by 0.20
- Multiply \$455 by 1.20
- Solve for x:  $\frac{x}{455} = \frac{120}{100}$
- Solve for x:  $\frac{455}{x} = \frac{20}{100}$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#9	3	RP	D	2	7.RP.A.2, 6.RP.A.3c	1, 2	<input type="checkbox"/> divide \$455 by 0.20 <input type="checkbox"/> divide \$455 by 1.20 <input type="checkbox"/> multiply \$455 by 0.20 <input checked="" type="checkbox"/> multiply \$455 by 1.20 <input checked="" type="checkbox"/> solve for x: $\frac{x}{455} = \frac{120}{100}$ <input type="checkbox"/> solve for x: $\frac{455}{x} = \frac{20}{100}$

39. Example: This table contains x and y values in equivalent ratios. Fill in the missing value in the table.

x	y
2	6
5	
7	21
9	27

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#27	1	RP	A	1	6.RP.A.3a	N/A	15

40. Example: Select the value that completes this expression for converting 10 yards to inches.  
((10 yards)/1)(?/?)((12 inches)/(1 foot))

- a. (1 yard)/(36 inches)
- b. (3 feet)/(1 yard)
- c. (360 inches)/(10 yards)
- d. (120 feet)/(10 inches)

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#26	1	RP	A	1	6.RP.A.3d	N/A	B

41. Example: Carl types 180 words in 2 minutes. Write the number of words Carl types in 5 minutes at this rate.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#13	1	RP	A	2	6.RP.A.3b	N/A	450

42. Example: Enter the unknown value that makes this statement true:  
30% of ? is 60.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#12	1	RP	A	2	6.RP.A.3c	N/A	200

43. Example: ([Former NAEP question](#)) (DOK 1)

The school carnival committee sold a total of 200 tickets for the grand prize drawing. Sue bought enough tickets so that she had a 20 percent chance of winning the grand prize. How many tickets did Sue buy?

- A. 20
- B. 40
- C. 160
- D. 400
- E. 1,000

Answer: B

44. Example: ([Former NAEP question](#)) (DOK 1)

Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored?

- A. 0.91%
- B. 1.10%
- C. 52%
- D. 91%
- E. 95%

Answer: C

45. Example: ([Former NAEP question](#))(DOK 1)

Which of the following is true about 56% of 20?

- A. It is less than 20.
- B. It is greater than 20.
- C. It is equal to 20.
- D. It is more than double 20.

Answer: A

46. Example: ([Former NAEP question](#)) (DOK 1)

On average, thunder is heard in Tororo, Uganda, 251 days each year. What is the probability that thunder will be heard in Tororo on any day? (1 year = 365 days)

Give your answer to the nearest percent.

Answer: 69%

47. Example: ([Former NAEP question](#)) (DOK 2)

In the sequence below, the ratio of each term to the term immediately following it is constant.

What is the next term of this sequence after 2240 ?

35, 280, 2240, \_\_\_\_\_

Answer: 17,920

48. Example: ([Former NAEP question](#)) (DOK 1)

30 is what percent of 60?

- A. 0.5%
- B. 2%
- C. 30%
- D. 50%

Answer: D

49. Example: ([Former NAEP question](#)) (DOK 1)

15 is 25% of what number?

- A. 0.60
- B. 3.75
- C. 37.5
- D. 60

Answer: D

Apply and extend previous understandings of multiplication and division to divide fractions by fractions. (6.NS.A)

**Example:** Darcy likes to eat peanut butter and raisins on apple slices. On each apple slice she puts  $\frac{1}{16}$  cup of peanut butter and 8 raisins.

Darcy has  $\frac{2}{5}$  cup of peanut butter and 80 raisins. She eats a whole number of apple slices until the peanut butter is all gone.

What fraction of 80 raisins did she eat?

Write the fraction.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#14	4	NS, NF	A	3	6.NS.A, 5.NF.B.3	1, 6	$\frac{3}{5}$

- Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for  $(2/3) \div (3/4)$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(2/3) \div (3/4) = 8/9$  because  $3/4$  of  $8/9$  is  $2/3$ . (In general,  $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $3/4$ -cup servings are in  $2/3$  of a cup of yogurt? How wide is a rectangular strip of land with length  $3/4$  mi and area  $1/2$  square mi? (6.NS.A.1) (DOK 1,2)*

- Example: [Solution](#) (DOK 3)

Alice, Raul, and Maria are baking cookies together. They need  $\frac{3}{4}$  cup of flour and  $\frac{1}{3}$  cup of butter to make a dozen cookies. They each brought the ingredients they had at home.

Alice brought  $2\frac{1}{4}$  cups of flour and  $\frac{1}{4}$  cup of butter, Raul brought  $1\frac{1}{2}$  cups of flour and  $\frac{1}{2}$  cup of butter, and Maria brought  $1\frac{1}{4}$  cups of flour and  $\frac{3}{4}$  cup of butter. If the students have plenty of the other ingredients they need (sugar, salt, baking soda, etc.), how many whole batches of a dozen cookies can they make?

- Example: [Solution](#) (DOK 3)

It requires  $\frac{1}{4}$  of a credit to play a video game for one minute.

- Emma has  $\frac{7}{8}$  credits. Can she play for more or less than one minute? Explain how you know.

- How long can Emma play the video game with her  $\frac{7}{8}$  credits?

- Example: [Solution](#) (DOK 3)



Dan observes that

$$\frac{6}{10} \div \frac{2}{10} = 6 \div 2$$

He says,

*I think that if we are dividing a fraction by a fraction with the same denominator, then we can just divide the numerators.*

Is Dan's conjecture true for all fractions? Explain how you know.

- d. Example: [Solution](#) (DOK 3)

One thermos of hot chocolate uses  $\frac{2}{3}$  cup of cocoa powder. How many thermoses can Nelli make with 3 cups of cocoa powder?

- Solve the problem by drawing a picture.
- Explain how you can see the answer to the problem in your picture.
- Which of the following multiplication or divisions equations represents this situation? Explain your reasoning.

$$3 \times \frac{2}{3} =? \quad 3 \div \frac{2}{3} =? \quad \frac{2}{3} \div 3 =?$$

d. Solve the arithmetic problem you chose in part (c) and verify that you get the same answer as you did with your picture.

- e. Example: [Solution](#) (DOK 3)

a. If  $\frac{1}{2}$  cup of water fills  $\frac{2}{3}$  of a plastic container, how many containers will 1 cup fill?

- Solve the problem by drawing a picture.
- Which of the following multiplication or divisions problems represents this situation? Explain your reasoning.

$$\frac{1}{2} \times \frac{2}{3} =? \quad \frac{1}{2} \div \frac{2}{3} =? \quad \frac{2}{3} \div \frac{1}{2} =?$$

- Solve the arithmetic problem you chose in part (3) and verify that you get the same answer as you did with your picture.

b. If  $\frac{1}{2}$  cup of water fills  $\frac{2}{3}$  of a plastic container, how many cups of water will the full container hold?

- Solve the problem by drawing a picture.
- Which of the following multiplication or divisions equations represents this situation? Explain your reasoning.

$$\frac{1}{2} \times \frac{2}{3} =? \quad \frac{1}{2} \div \frac{2}{3} =? \quad \frac{2}{3} \div \frac{1}{2} =?$$

- Solve the arithmetic problem you chose in part (3) and verify that you get the same answer as you did with your picture.

f. Example: [Solution](#) (DOK 2)  
Rosa ran  $\frac{1}{3}$  of the way from her home to school. She ran  $\frac{1}{4}$  mile. How far is it between her home and school?

g. Example: [Solution](#) (DOK 2)  
The distance between Rosa's house and her school is  $\frac{3}{4}$  mile. She ran  $\frac{1}{4}$  mile. What fraction of the way to school did she run?

h. Example: [Solution](#) (DOK 3)  
A recipe for hot chocolate calls for 3 cups of milk. What fraction of the recipe can Nelli make with  $\frac{2}{3}$  cups of milk?

- Solve the problem by drawing a picture.
- Explain how you can see the answer to the problem in your picture.
- Which of the following multiplication or divisions equations represents this situation? Explain your reasoning.

$$3 \times \frac{2}{3} =? \quad 3 \div \frac{2}{3} =? \quad \frac{2}{3} \div 3 =?$$

- Solve the arithmetic problem you chose in part (c) and verify that you get the same answer as you did with your picture.

- i. Example: [Solution](#) (DOK 2)  
Alisa had  $\frac{1}{2}$  liter of juice in a bottle. She drank  $\frac{3}{8}$  liters of juice. What fraction of the juice in the bottle did Alisa drink?
- j. Example: [Solution](#) (DOK 2)  
Alisa had some juice in a bottle. Then she drank  $\frac{3}{8}$  liters of juice. If this was  $\frac{3}{4}$  of the juice that was originally in the bottle, how much juice was there to start?
- k. Example: [Solution](#) (DOK 3)  
Tonya and Chrissy are trying to understand the following story problem for  $1 \div \frac{2}{3}$ :

*One serving of rice is  $\frac{2}{3}$  of a cup. I ate 1 cup of rice. How many servings of rice did I eat?*

To solve the problem, Tonya and Chrissy draw a diagram divided into three equal pieces, and shade two of those pieces.



Tonya says, "There is one  $\frac{2}{3}$ -cup serving of rice in 1 cup, and there is  $\frac{1}{3}$  cup of rice left over, so the answer should be  $1\frac{1}{3}$ ."

Chrissy says, "I heard someone say that the answer is  $\frac{3}{2} = 1\frac{1}{2}$ . Which answer is right?"

Is the answer  $1\frac{1}{3}$  or  $1\frac{1}{2}$ ? Explain your reasoning using the diagram.

- l. Example: [Solution](#) (DOK 3)  
You are stuck in a big traffic jam on the freeway and you are wondering how long it will take to get to the next exit, which is  $1\frac{1}{2}$  miles away. You are timing your progress and find that you can travel  $\frac{2}{3}$  of a mile in one hour. If you continue to make progress at this rate, how long will it be until you reach the exit? Solve the problem with a diagram and explain your answer.
- m. Example: [Solution](#) (DOK 2)

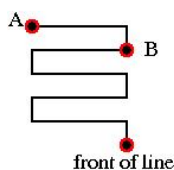
Solve each problem using pictures and using a number sentence involving division.

- How many fives are in 15?
- How many halves are in 3?
- How many sixths are in 4?
- How many two-thirds are in 2?
- How many three-fourths are in 2?
- How many  $\frac{1}{6}$ 's are in  $\frac{1}{3}$ ?
- How many  $\frac{1}{6}$ 's are in  $\frac{2}{3}$ ?
- How many  $\frac{1}{4}$ 's are in  $\frac{2}{3}$ ?
- How many  $\frac{5}{12}$ 's are in  $\frac{1}{2}$ ?

- n. Example: [Solution](#) (DOK 2)

Alysha really wants to ride her favorite ride at the amusement park one more time before her parents pick her up at 2:30 pm. There is a very long line at this ride, which Alysha joins at 1:50 pm (point A in the diagram below). Alysha is nervously checking the time as she is moving forward in the line. By 2:03 she has made it to point B in line.

What is your best estimate for how long it will take Alysha to reach the front of the line? If the ride lasts 3 minutes, can she ride one more time before her parents arrive?



- o. Example: Alex claims that when  $\frac{1}{4}$  is divided by a fraction, the result will be greater than  $\frac{1}{4}$ .

To convince Alex that this statement is only sometimes true:

**Part A:** Write one digit into each box to create an expression that is greater than  $\frac{1}{4}$ .

$$\frac{1}{4} \div \underline{\quad}$$

**Part B:** Write one digit into each box to create an expression that is **not** greater than  $\frac{1}{4}$ .

$$\frac{1}{4} \div \underline{\quad}$$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#10	3	NS	A	2	7.NS.A.2c, 6.NS.A.1	2, 3, 7	<div style="border: 1px solid black; padding: 5px;"> <p><i>Part A: Expression greater than <math>\frac{1}{4}</math></i></p> <math display="block">\frac{1}{4} \div \frac{\boxed{1}}{\boxed{7}}</math> <hr style="border: 1px solid black;"/> <p><i>Part B: Expression not greater than <math>\frac{1}{4}</math></i></p> <math display="block">\frac{1}{4} \div \frac{\boxed{7}}{\boxed{1}}</math> </div>

p. Example: The equation shown has an unknown number.

$$\square \div \frac{2}{3} = \frac{3}{4}$$

Write a fraction that makes the equation true.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#2	1	NS	B	1	6.NS.A.1	N/A	$\frac{1}{2}$

Compute fluently with multi-digit numbers and find common factors and multiples. (6.NS.B)

2. Fluently divide multi-digit numbers using the standard algorithm. **(6.NS.B.2) (DOK 1)**

a. Example: [Solution](#) (DOK 2)

Use the computation shown below to find the products.

$$\begin{array}{r} 189 \\ 16 \overline{)3024} \\ \underline{16} \phantom{00} \\ 142 \phantom{0} \\ \underline{128} \phantom{0} \\ 144 \phantom{0} \\ \underline{144} \\ 0 \end{array}$$

a.  $189 \times 16$

b.  $80 \times 16$

c.  $9 \times 16$

b. Example: [Solution](#) (DOK 3)



Can you find an inconsistency in the information on this box of staples?  
Explain.

c. Example: [Solution](#) (DOK 3)

Before a game, Jake's batting average was exactly 0.350. That is, the decimal expansion of the fraction

$$\frac{\text{number of hits}}{\text{times at bat}}$$

is equal to 0.350. During the game, Jake bats 4 times and get 2 hits. If Jake's batting average after the game is 0.359, how many times had Jake batted before the game? Explain.

- d. Example: Divide.

$$16,536 \div 24$$

What is the quotient?

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#9	1	NS	C	1	6.NS.B.2	N/A	689

- e. Example: ([Former NAEP question](#)) (DOK 2)

In a contest, a prize of 2.72 million dollars was split equally among 32 winners. How much money did each of the 32 winners receive?

- A. \$0.085
- B. \$62,500
- C. \$62,502.25
- D. \$85,000
- E. \$850,000

Answer: D

3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

**(6.NS.B.3) (DOK 1)**

- a. Example: [Solution](#) (DOK 2)

Use the fact that  $13 \times 17 = 221$  to find the following.

a.  $13 \times 1.7$

b.  $130 \times 17$

c.  $13 \times 1700$

d.  $1.3 \times 1.7$

e.  $2210 \div 13$

f.  $22100 \div 17$

g.  $221 \div 1.3$

- b. Example: [Solution](#) (DOK 1)

Jayden has \$20.56. He buys an apple for 79 cents and a granola bar for \$1.76.

- a. How much money did Jayden spend?
- b. How much money does Jayden have now?

- c. Example: [Solution](#) (DOK 1)  
Sophia's dad paid \$43.25 for 12.5 gallons of gas. What is the cost of one gallon of gas?
- d. Example: [Solution](#) (DOK 3)  
Place a decimal on the right side of the equal sign to make the equation true. Explain your reasoning for each.

a.  $3.58 \times 1.25 = 044750$

b.  $26.97 \div 6.2 = 04350$

- e. Example: [Solution](#) (DOK 1)
  - a. Juanita spent \$24.50 on each of her 6 grandchildren at the fair. How much money did Juanita spend?
  - b. Nita bought some games for her grandchildren for \$42.50 each. If she spent a total of \$340, how many games did Nita buy?
  - c. Helen spent an equal amount of money on each of her 7 grandchildren at the fair. If she spent a total of \$227.50, how much did each grandchild get?
- f. Example: [Solution](#) (DOK 2)  
A penny is about  $\frac{1}{16}$  of an inch thick.
  - a. In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?
  - b. In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?
  - c. The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?
- g. Example: [Solution](#) (DOK 2)



Hallie is in 6th grade and she can buy movie tickets for \$8.25. Hallie's father was in 6th grade in 1987 when movie tickets cost \$3.75.

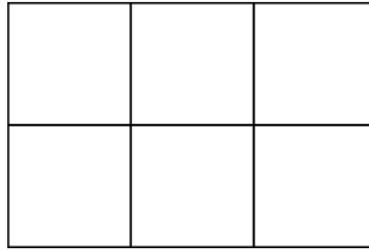
- a. When he turned 12, Hallie's father was given \$20.00 so he could take some friends to the movies. How many movie tickets could he buy with this money?
- b. How many movie tickets can Hallie buy for \$20.00?
- c. On Hallie's 12th birthday, her father said,

*When I turned 12, my dad gave me \$20 so I could go with three of my friends to the movies and buy a large popcorn. I'm going to give you some money so you can take three of your friends to the movies and buy a large popcorn.*

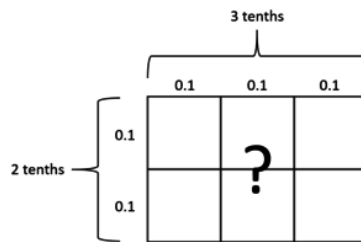
How much money do you think her father should give her?

- h. Example: [Solution](#) (DOK 2)
  - a. Seth wants to buy a new skateboard that costs \$167. He has \$88 in the bank. If he earns \$7.25 an hour pulling weeds, how many hours will Seth have to work to earn the rest of the money needed to buy the skateboard?
  - b. Seth wants to buy a helmet as well. A new helmet costs \$46.50. Seth thinks he can work 6 hours on Saturday to earn enough money to buy the helmet. Is he correct?
  - c. Seth's third goal is to join some friends on a trip to see a skateboarding show. The cost of the trip is about \$350. How many hours will Seth need to work to afford the trip?
- i. Example: [Solution](#) (DOK 2)

Here is a rectangle that is 2 units wide and 3 units long and has an area of 6 square units.



If the unit length is one-tenth, then this rectangle could represent the equation  $0.2 \times 0.3 = ?$ .



The two-by-three rectangle can also represent the following multiplication equations. For each one, draw a new rectangle and label the side lengths so it represents the multiplication equation. What is the area of a single square in the new rectangle? What is the product of the equation?

a.  $2,000 \times 3,000 = ?$

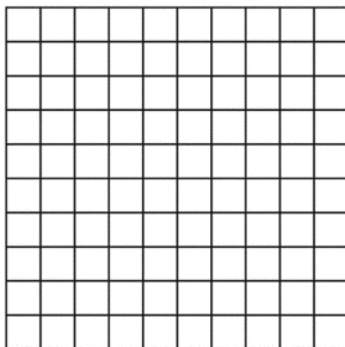
b.  $20 \times 30 = ?$

c.  $0.02 \times 0.03 = ?$

d.  $0.0002 \times 0.0003 = ?$

j. Example: [Solution](#) (DOK 3)

A large square has both sides subdivided into 10 equal pieces.



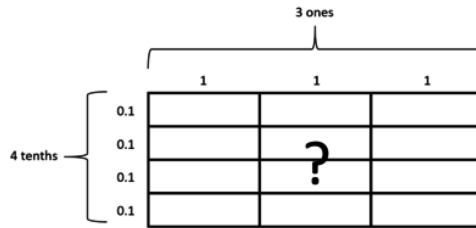
If the large square has a side length of 100 units, then the side length of a small square is  $100 \div 10 = 10$  units. In this case, the area of the large square is  $100 \times 100 = 10,000$  square units and the area of a small square is  $10 \times 10 = 100$  square units.

- a. What if the side length of the large square is 10? What is the area of the large square? What is the area of a single small square?
  - b. What if the side length of the large square is 1? What is the area of the large square? What is the area of a single small square?
  - c. How can you see  $0.1 \times 0.1$  in the diagram? What is  $0.1 \times 0.1$ ?
  - d. Show  $0.01 \times 0.01$  in the diagram, and find this product. Explain how you can see  $0.01 \times 0.01$  in your diagram.
- k. Example: [Solution](#) (DOK 2)

Here is a large rectangle made up of 12 smaller rectangles.



This rectangle could represent the equation  $0.4 \times 3 = ?$  if we label as shown below.



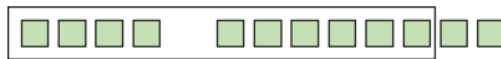
The rectangle can also represent the following multiplication equations. For each one, draw a new rectangle and label the side lengths so it represents the multiplication equation. What is the area of a small rectangle? What is the product of the equation?

a.  $40 \times 300 = ?$

b.  $0.04 \times 0.3 = ?$

l. Example: [Solution](#) (DOK 2)

The diagram below can be used to represent  $40 + 80 = 120$  if a small square represents 10 and a group of ten squares represents 100:



It can also represent  $0.04 + 0.08 = 0.12$  if a small square represents 0.01 and a group of ten squares represents 0.1.

a. Name at least two more sums it can represent. What does a small square represent? What does a group of ten squares represent?

Draw a diagram to represent each of the following sums. Describe what the small squares in your diagram represent.

b.  $0.005 + 0.002$

c.  $0.0008 + 0.0006$

d.  $0.00007 + 0.00003$

m. Example: [Solution](#) (DOK 1)

Add:

a.  $4,000 + 5,000$

b.  $600 + 200$

c.  $20 + 50$

d.  $8 + 1$

e.  $0.3 + 0.4$

f.  $0.07 + 0.02$

g.  $0.001 + 0.006$

h.  $0.0005 + 0.0003$

i.  $4628.3715 + 5251.4263$

Add some more:

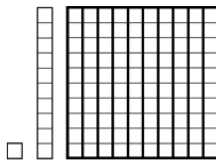
j.  $600 + 700$

k.  $0.005 + 0.008$

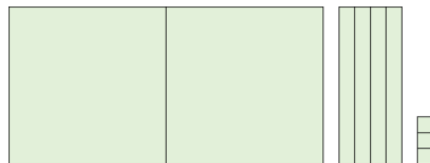
l.  $600.005 + 700.008$

n. Example: [Solution](#) (DOK 2)

In the picture below, a small square represents a unit, a long, thin rectangle represents a bundle of ten units, and a large square represents ten bundles of ten units. A small square can represent any number we want.



So for example, if the small square represents 1,000, then the diagram below represents 243,000:



Draw a picture that represents each of the following sums. What number does a unit represent in your picture? What about a bundle of ten units? What about ten bundles of ten? Make sure you can see both addends and the sum in your picture. For each problem, use two colors: one for each addend.

- a.  $0.016 + 0.082$
- b.  $0.029 + 0.034$
- c.  $0.043 + 0.017$
- d.  $0.081 + 0.045$
- e.  $0.056 + 0.044$

o. Example: [Solution](#) (DOK 2)

- a. A group of 10 scientists won a \$1,000,000 prize for a discovery they made. They will share the prize equally. How much money will each person get?
- b. Two cousins shared 0.006 kilograms of gold equally. How many kilograms of gold did each cousin get?
- c. A barrel contained 160 liters of oil that costs \$51.20. What is the cost for one liter? How many liters can you buy for \$1.00?

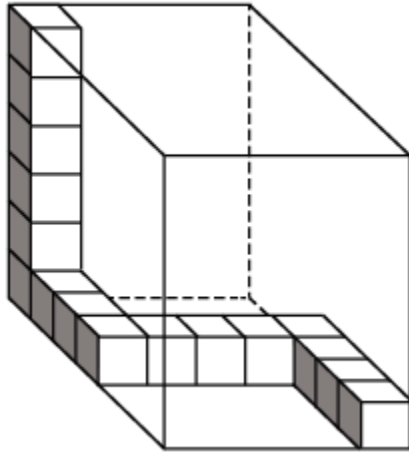
p. Example: [Solution](#) (DOK 1)

- a. How many one-hundred dollar bills do you need to make \$2,000? \$20,000?
- b. How many ten dollar bills do you need to make \$2,000? \$20,000?
- c. How many dimes do you need to make \$0.20? \$2? \$20?
- d. How many pennies do you need to make \$0.02? \$0.20? \$2?
- e. Use the answers to the questions above to fill in the table:

$A$	$B$	$A \div B$
2	0.01	
20	0.1	
200	1	
2,000	10	
20,000	100	

f. What changes (and how does it change) and what stays the same as you move from row to row down the table?

q. Example: Brady started to fill the box shown with some unit cubes.



Write the total number of unit cubes needed to completely fill the box. Include the unit cubes already shown in your total.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#17	2	NS, MD	D	2	6.NS.B.3, 5.MD.C.5	6, 7	210

- r. Example: Carlos needs 1.7 meters of wire for one project and 0.8 meter of wire for another project.  
**Part A:** Shade the model to represent the total amount of wire Carlos needs. Each full row represents 1.0 meter.  
**Part B:** Carlos has 2.4 meters of wire. Does Carlos have enough wire?  
 If he does, answer how much wire he will have left over.  
 If he does **not**, answer how much more he needs.  
 Place the value into one of the boxes.

0.1

Delete ✕

**Part A**

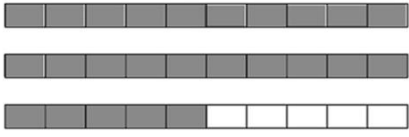
**Each full row = 1.0 meter**

**Part B**

He will have  meters of wire left over.

OR

He needs  more meters of wire.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#4	2	NS	A	2	6.NS.B.3	2, 4	<p><b>Part A</b></p> <p>Each full row = 1.0 meter</p>  <p><b>Part B</b></p> <p>He will have <input type="text"/> meters of wire left over.</p> <p>OR</p> <p>He needs <input type="text"/> more meters of wire.</p>

- s. Example: ([Former NAEP question](#)) (DOK 1)

Sandwich	\$2.70
Drink	\$1.34
Fruit	\$0.80

About how much do the sandwich, drink, and fruit cost altogether?

- A. \$3.00
- B. \$3.50
- C. \$4.00
- D. \$4.25
- E. \$5.00

Answer: E

- t. Example: ([Former NAEP question](#)) (DOK 2)

Add the numbers  $\frac{7}{10}$ ,  $\frac{7}{100}$ , and  $\frac{7}{1,000}$ . Write this sum as a decimal.

Answer: 0.777

4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express  $36 + 8$  as  $4(9 + 2)$ . **(6.NS.B.4) (DOK 1)**

- a. Example: [Solution](#) (DOK 2)

- a. List all the factors of 48.
- b. List all the factors of 64.
- c. What are the common factors of 48 and 64?
- d. What is the greatest common factor of 48 and 64?

- b. Example: [Solution](#) (DOK 2)



- a. List all the multiples of 8 that are less than or equal to 100.
  - b. List all the multiples of 12 that are less than or equal to 100.
  - c. What are the common multiples of 8 and 12 from the two lists?
  - d. What is the least common multiple of 8 and 12?
  - e. Lyle noticed that the list of common multiples has a pattern. Describe a pattern in the list of numbers that Lyle might have seen.
- c. Example: [Solution](#) (DOK 3)  
Nina was finding multiples of 6. She said,

*18 and 42 are both multiples of 6, and when I add them, I also get a multiple of 6:*

$$18 + 42 = 60.$$

Explain to Nina why adding two multiples of 6 will always result in another multiple of 6.

- d. Example: [Solution](#) (DOK 3)
- a. Lindy is having a bake sale. She has 48 chocolate chip cookies to put in bags. How many bags can she fill if she puts the same number in each bag and uses them all? Find all the possibilities. Explain your reasoning.
  - b. Lindy has 64 vanilla wafer cookies to put in bags. How many bags can she fill if she puts the same number in each bag and uses them all? Find all the possibilities. Explain your reasoning.
  - c. How many bags can Lindy fill if she puts the chocolate chip cookies and the vanilla wafers in the same bags? She plans to use all the cookies and wants to include an equal number of chocolate chip cookies and an equal number of vanilla wafers in each bag. Explain your reasoning.
  - d. What is the largest number of bags she can make with an equal number of chocolate chip cookies and an equal number of vanilla wafers in each bag (assuming she uses them all)? Explain your reasoning.
- e. Example: [Solution](#) (DOK 3)  
The florist can order roses in bunches of one dozen and lilies in bunches of 8. Last month she ordered the same number of roses as lilies. If she ordered no more than 100 roses, how many bunches of each could she have ordered? What is the smallest number of bunches of each that she could have ordered? Explain your reasoning.

### Apply and extend previous understandings of numbers to the system of rational numbers. (6.NS.C)

- 5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in

each situation. **(6.NS.C.5) (DOK 1,2)**

a. Example: [Solution](#) (DOK 1)

One morning the temperature is  $-28^{\circ}$  in Anchorage, Alaska, and  $65^{\circ}$  in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?

b. Example: [Solution](#) (DOK 3)

Denver, Colorado is called "The Mile High City" because its elevation is 5280 feet above sea level. Someone tells you that the elevation of Death Valley, California is  $-282$  feet.

a. Is Death Valley located above or below sea level? Explain.

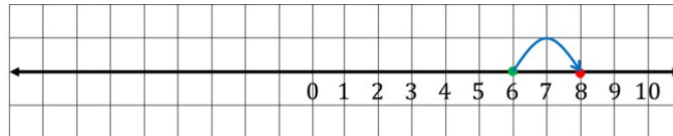
b. How many feet higher is Denver than Death Valley?

c. What would your elevation be if you were standing near the ocean?

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,  $-(-3) = 3$ , and that 0 is its own opposite.
  - Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
  - Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **(6.NS.C.6) (DOK 1)**

1. Example: [Solution](#) (DOK 2)

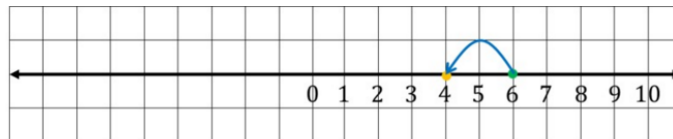
a. Draw a line on graph paper. Make a tick mark in the middle of the line and label it 0. Mark and label 1, 2, 3, ... 10. Since  $6+2$  is 2 units to the right of 6 on the number line, we can represent  $6+2$  like this:



Describe the location of  $3+4$  on the number line in terms of 3 and 4.

Draw a picture like the one above.

b.  $6-2$  is 2 units to the left of 6 on the number line, which we can represent like this:

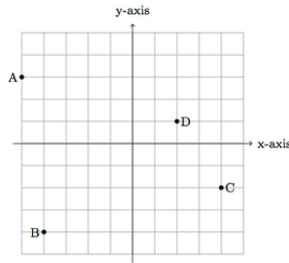


Describe the location of  $3-4$  on the number line in terms of 3 and 4.

Draw a picture like the one above.

2. Example: [Solution](#) (DOK 2)

Below are some points in the coordinate plane:



- Find the coordinates of the points.
- Reflect the points over the  $x$ -axis and find the coordinates of the new points: label the reflection of point  $A$  as  $A'$ , the reflection of  $B$  as  $B'$ , the reflection of  $C$  as  $C'$ , and the reflection of  $D$  as  $D'$ .
- Reflect the points from (b) over the  $y$ -axis: label the reflection of point  $A'$  as  $A''$ , the reflection of  $B'$  as  $B''$ , the reflection of  $C'$  as  $C''$ , and the reflection of  $D'$  as  $D''$ .
- How do the points  $A''$ ,  $B''$ ,  $C''$ ,  $D''$  from (c) relate to the points  $A$ ,  $B$ ,  $C$ , and  $D$ ?

3. Example: [Solution](#) (DOK 2)

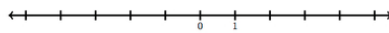
a. For each set of points below, draw and label a set of coordinate axes and plot the points:

- $(1, 2), (3, -4), (-5, -2), (0, 2\frac{1}{2})$
- $(50, 50), (0, 0), (-10, -30), (-35, 40)$
- $(\frac{1}{4}, \frac{3}{4}), (-\frac{5}{4}, \frac{1}{2}), (-1\frac{1}{4}, -\frac{3}{4}), (\frac{1}{4}, -\frac{2}{4})$

b. How do the points influence your choice of scale for the axes?

4. Example: [Solution](#) (DOK 2)

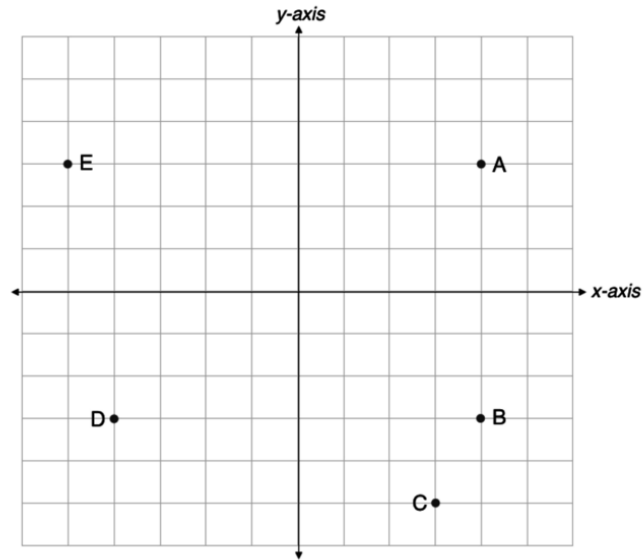
Below is a number line with 0 and 1 labeled:



- Find and label the numbers  $-2$  and  $-4$  on the number line. Explain.
- Find and label the numbers  $-(-2)$  and  $-(-4)$  on the number line. Explain.
- Find and label the number  $-0$  on the number line. Explain.

5. Example: [Solution](#) (DOK 3)

Some points are located on a square-unit grid, and coordinate axes are drawn:



Note:  $x$  and  $y$  are on the same scale. The scale is 1 unit.

a) Choose any two points. Consider their locations in the plane. How are they the same? How are they different? Write down at least three things you notice.

b) Name the coordinates for each point:

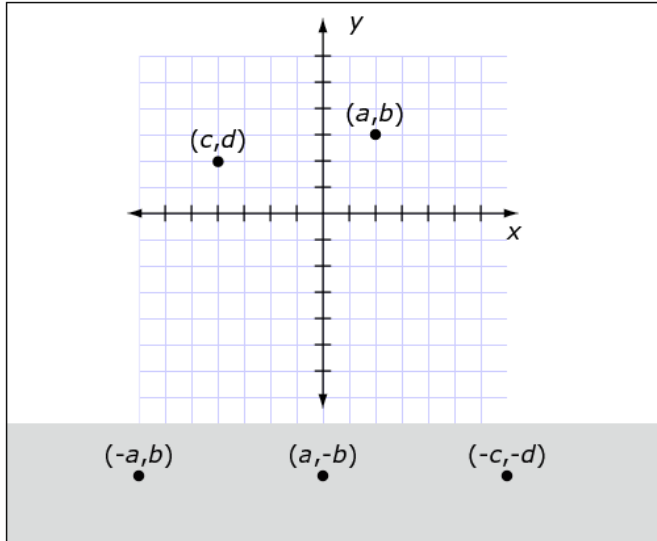
- A (     ,     )
- B (     ,     )
- C (     ,     )
- D (     ,     )
- E (     ,     )

c) Make some observations relating coordinates to locations in the plane. (Here is an example: "When the first coordinate is positive, that point is located to the right of the  $y$ -axis.") Write down at least three things you notice.

d) Consider the rule, "When the first coordinate is positive, that point is located to the right of the  $y$ -axis." Will that always be true or only happen sometimes? Explain your reasoning.

e) Choose one of the things you noticed from part (c) and explain why it will always be true or only happen sometimes.

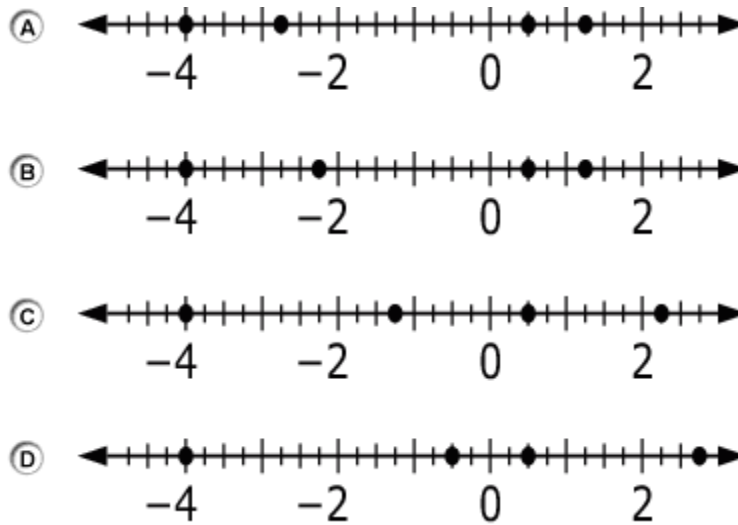
6. Example: Two ordered pairs are shown on a coordinate grid. Place each ordered pair at its correct location on the coordinate grid.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#19	3	NS	F	2	6.NS.C.6b	1, 2	

7. Example: Which number line shows the correct locate of **all** the given values?

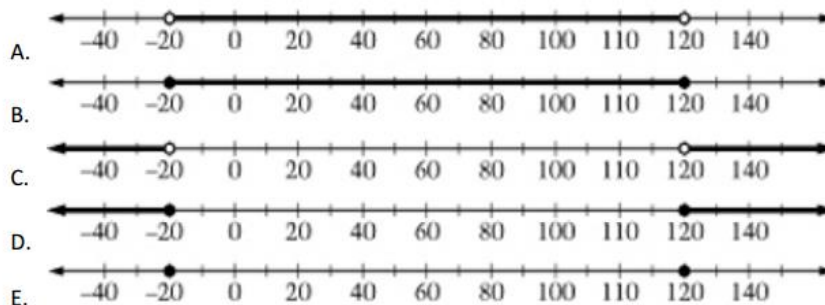
$$\frac{1}{2}, -4, 2\frac{3}{4}, 1\frac{1}{4}$$



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#1	1	NS	D	1	6.NS.C.6c	N/A	A

8. Example: ([Former NAEP question](#)) (DOK 2)

Weather records in a city show the coldest recorded temperature was  $-20^{\circ}\text{F}$  and the hottest was  $120^{\circ}\text{F}$ . Which of the following number line graphs represents the range of recorded actual temperatures in this city?



Answer: B

9. Example: ([Former NAEP question](#)) (DOK 1)

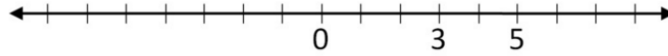
Which of the following is positive?

- A.  $-(-31)$
- B.  $-(31)$
- C.  $-31$
- D.  $0-31$

Answer: A

7. Understand ordering and absolute value of rational numbers.

- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right.*
  - Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write  $-3^{\circ}\text{C} > -7^{\circ}\text{C}$  to express the fact that  $-3^{\circ}\text{C}$  is warmer than  $-7^{\circ}\text{C}$ .*
  - Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of  $-30$  dollars, write  $|-30| = 30$  to describe the size of the debt in dollars.*
  - Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than  $-30$  dollars represents a debt greater than 30 dollars. (6.NS.C.7) (DOK 1,2)*
1. Example: ([Solution](#)) (DOK 3)



- a. Find and label the numbers  $-3$  and  $-5$  on the number line.
- b. For each of the following, state whether the inequality is true or false. Use the number line diagram to help explain your answers.

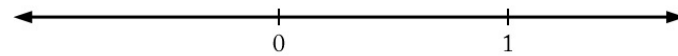
i.  $-3 > -5$

ii.  $-5 > -3$

iii.  $-5 < -3$

iv.  $-3 < -5$

2. Example: [Solution](#) (DOK 3)



- a. Find and label the numbers  $\frac{4}{3}$ ,  $\frac{5}{4}$ ,  $-\frac{2}{3}$ , and  $-\frac{3}{4}$  on the number line.
- b. For each of the following, state which inequality is true. Use the number line diagram to help explain your answers.

i. Is  $\frac{4}{3} > \frac{5}{4}$  or is  $\frac{4}{3} < \frac{5}{4}$ ?

ii. Is  $-\frac{2}{3} > -\frac{3}{4}$  or is  $-\frac{2}{3} < -\frac{3}{4}$ ?

- c. Is  $-\frac{3}{4}$  closer to 0 or is  $\frac{5}{4}$ ? Explain how you know.

3. Example: [Solution](#) (DOK 3)

- a. Here are the low temperatures (in Celsius) for one week in Juneau, Alaska:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
5	-1	-6	-2	3	7	0

Arrange them in order from coldest to warmest temperature.

- b. On a winter day, the low temperature in Anchorage was  $23$  degrees below zero (in  $^{\circ}\text{C}$ ) and the low temperature in Minneapolis was  $14$  degrees below zero (in  $^{\circ}\text{C}$ ). Sophia wrote,

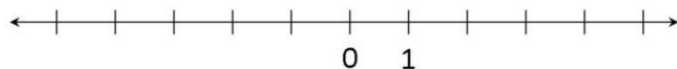
*Minneapolis was colder because  $-14 < -23$ .*

Is Sophia correct? Explain your answer.

- c. The lowest temperature ever recorded on earth was  $-89^{\circ}\text{C}$  in Antarctica. The average temperature on Mars is about  $-55^{\circ}\text{C}$ . Which is warmer, the coldest temperature on earth or the average temperature on Mars? Write an inequality to support your answer.

4. Example: [Solution](#) (DOK 1)

A flea is jumping around on the number line.



- If he starts at 1 and jumps 3 units to the right, then where is he on the number line? How far away from zero is he?
- If he starts at 1 and jumps 3 units to the left, then where is he on the number line? How far away from zero is he?
- If the flea starts at 0 and jumps 5 units away, where might he have landed?
- If the flea jumps 2 units and lands at zero, where might he have started?
- The absolute value of a number is the distance it is from zero. The absolute value of the flea's location is 4 and he is to the left of zero. Where is he on the number line?

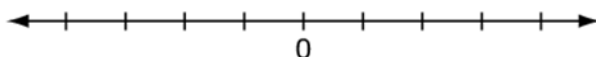
5. Example: [Solution](#) (DOK 3)

The table below shows the lowest elevation above sea level in three American cities.

City	State	Elevation above sea level	Elevation below sea level
Denver	Colorado	5130	
New Orleans	Louisiana	-8	
Seattle	Washington	0	

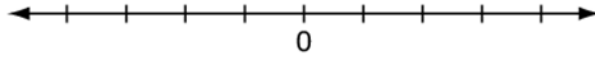
Finish filling in the table as you think about the following statements. Decide whether each of the following statements is true or false. Explain your answer for each one.

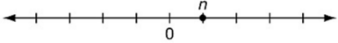
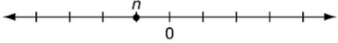
- New Orleans is  $|-8|$  feet below sea level.
  - New Orleans is  $-8$  feet below sea level.
  - New Orleans is  $8$  feet below sea level.
  - Seattle is  $0$  feet above sea level.
  - Seattle is  $|0|$  feet below sea level.
  - Denver is  $-5130$  feet below sea level.
  - Denver is  $|-5130|$  feet below sea level.
  - Denver is  $-|5130|$  feet below sea level.
6. Example: Let  $n$  be an integer. Tracy claims that  $-n$  must be less than 0. To convince Tracy that his statement is only sometimes true:
- Part A: Place  $n$  on the number line so that the value of  $-n$  is less than 0.



Part B: Place  $n$  on the number line so that the value of  $-n$  is greater than 0.





Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#24	3	NS	E	2	6.NS.C.7	3	<p><b>Part A:</b> Value of <math>-n</math> is less than 0</p>  <p><b>Part B:</b> Value of <math>-n</math> is greater than 0</p> 

7. Example: Sea level is 0 feet in elevation. The elevation of land represents its height above or below sea level. This table shows the lowest elevation in some states.

State	Lowest Elevation (ft)
Arizona	72
California	- 282
Louisiana	- 68
Tennessee	178

Determine whether each statement about the lowest elevations is correct. Select **True** or **False** for each statement.

	True	False
The elevation at the lowest point in California is higher than the lowest point in Louisiana.		
The elevation at the lowest point in Tennessee is farther from 0 than the elevation at the lowest point of Louisiana.		
The elevation at the lowest point in Louisiana is higher than the lowest point in California.		

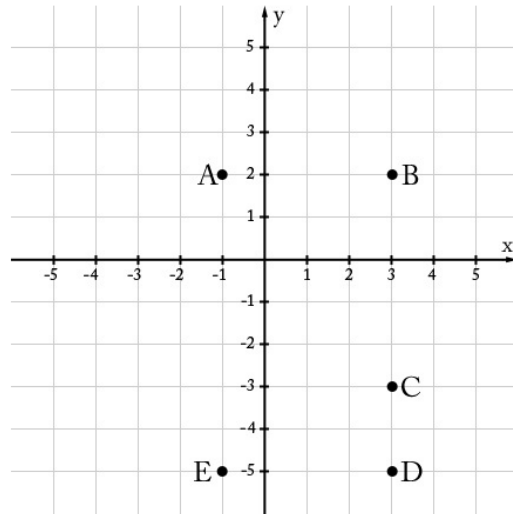
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key								
#3	1	NS	D	2	6.NS.C.7b	N/A	<table border="1"> <thead> <tr> <th>True</th> <th>False</th> </tr> </thead> <tbody> <tr> <td><input type="checkbox"/></td> <td><input checked="" type="checkbox"/></td> </tr> <tr> <td><input checked="" type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> <tr> <td><input checked="" type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> </tbody> </table>	True	False	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
True	False														
<input type="checkbox"/>	<input checked="" type="checkbox"/>														
<input checked="" type="checkbox"/>	<input type="checkbox"/>														
<input checked="" type="checkbox"/>	<input type="checkbox"/>														

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use

of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **(6.NS.C.8) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 1)

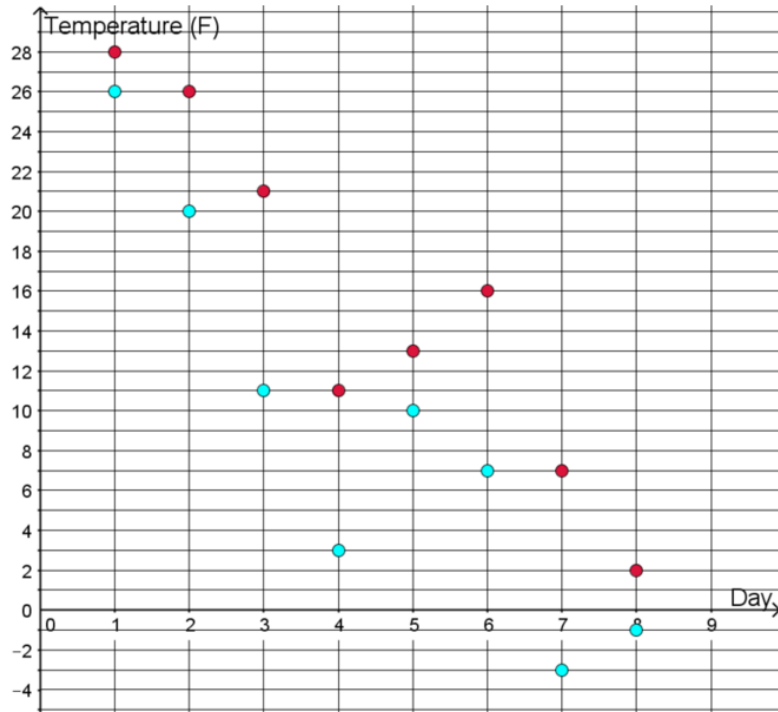
Some points are shown in the coordinate plane below.



- a. What is the distance between points B & C?
- b. What is the distance between points D & B?
- c. What is the distance between points D & E?
- d. Which of the points shown above are 4 units away from  $(-1, -3)$  and 2 units away from  $(3, -1)$ ?

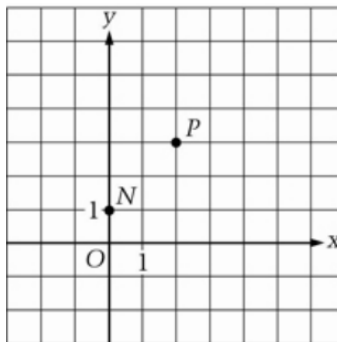
b. Example: Solution (DOK 1)

The high and low temperatures, in degrees Fahrenheit, are plotted in the coordinate plane for 8 days in Nome, Alaska.



- What was the warmest high temperature?
- What was the coldest low temperature?
- What was the biggest same-day difference between the high and low temperature? On what day did it occur?

c. Example: ([Former NAEP question](#)) (DOK 1)



For the figure above, which of the following points would be on the line that passes through points  $N$  and  $P$ ?

- $(-2, 0)$
- $(0, 0)$
- $(1, 1)$
- $(4, 5)$
- $(5, 4)$

Answer: D

Apply and extend previous understandings of arithmetic to algebraic expressions. (6.EE.A)

1. Write and evaluate numerical expressions involving whole-number exponents. (6.EE.A.1) (DOK 1)

- a. Example: [Solution](#) (DOK 2)

After opening an ancient bottle you find on the beach, a Djinni appears. In payment for his freedom, he gives you a choice of either 50,000 gold coins or one magical gold coin. The magic coin will turn into two gold coins on the first day. The two coins will turn into four coins total at the end of two days. By the end of the third day there will be eight gold coins total. The Djinni explains that the magic coins will continue this pattern of doubling each day for one moon cycle, 28 days. Which prize do you choose?

When you have made your choice, answer these questions:

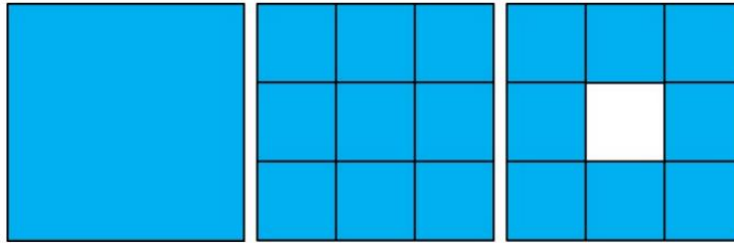
- The number of coins on the third day will be  $2 \times 2 \times 2$ . Can you write another expression using exponents for the number of coins there will be on the third day?
- Write an expression for the number of coins there will be on the 28th day. Is this more or less than a million coins?

- b. Example: [Solution](#) (DOK 2)

a. What is the last digit of  $7^{2011}$ ? Explain.

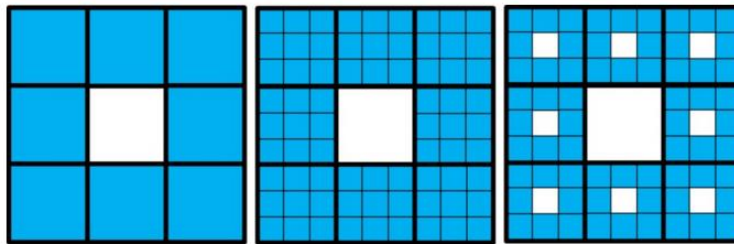
b. What are the last two digits of  $7^{2011}$ ? Explain.

- c. Example: [Solution](#) (DOK 2)
- a. Take a square with area 1. Divide it into 9 equal-sized squares. Remove the middle one.



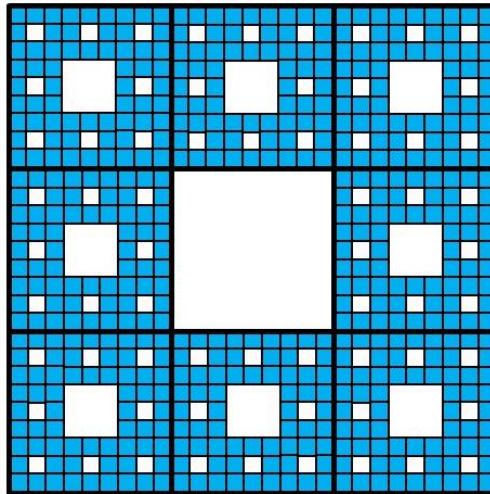
What is the area of the figure now?

- b. Take the remaining 8 squares. Divide each one into 9 equal squares. Remove the middle one from each group of 9.



What is the area of the figure now?

- c. Take the remaining squares. (How many are there?) Divide each one into 9 equal squares. Remove the middle one from each group of 9.



What is the area of the figure now?

- d. Imagine you follow this same process until you have removed "the middle square from each group of 9" 10 times. How many squares will there be? What will the area of each little square be? What will the area of the entire figure be?

- d. Example: [Solution](#) (DOK 2)

Here are some different ways to write the value 16:

$2^4$	$12 - (2^1 + 2^2) + \frac{500}{50}$	$2^3 + 2^3$	$\frac{2}{3} \times 48^1 - (1 + 3)^2$
-------	-------------------------------------	-------------	---------------------------------------

Find at least three different ways to write each value below. Include at least one exponent in all of the expressions you write.

a. 81

b.  $2^5$

c.  $\frac{64}{9}$

- e. Example: [Solution](#) (DOK 3)

Decide whether each equation is true or false, and explain how you know.

a.  $2^4 = 2 \cdot 4$

b.  $3 + 3 + 3 + 3 + 3 = 3^5$

c.  $5^3 = 5 \cdot 5 \cdot 5$

d.  $2^3 = 3^2$

e.  $16^1 = 8^2$

f.  $(1 + 3)^2 = 1^2 + 3^2$

g.  $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 6^3$

- f. Example: [Solution](#) (DOK 2)

In each equation below,  $x$  can be replaced with a number that makes the equation true. Find such a number.

a.  $64 = x^2$

b.  $64 = x^3$

c.  $2^x = 32$

d.  $x = \left(\frac{2}{5}\right)^3$

e.  $\frac{16}{9} = x^2$

f.  $2 \cdot 2^5 = 2^x$

g.  $2x = 2^4$

h.  $4^3 = 8^x$

i.  $x^2 = 25$

j.  $(x + 1)^2 = 25$

- g. Example: ([Former NAEP question](#)) (DOK 1)

A reasonable prediction of the distance  $d$  in feet, that a car travels after the driver has applied the brakes can be found by using the formula  $d = 0.055r^2$ , where  $r$  is the speed of the car in miles per hour.

If Mario is driving at 60 miles per hour and applies the brakes, then according to the formula, how many feet will Mario's car travel before it stops?

- A. 330
- B. 198
- C. 10.89
- D. 6.6
- E. 3.3

Answer: B

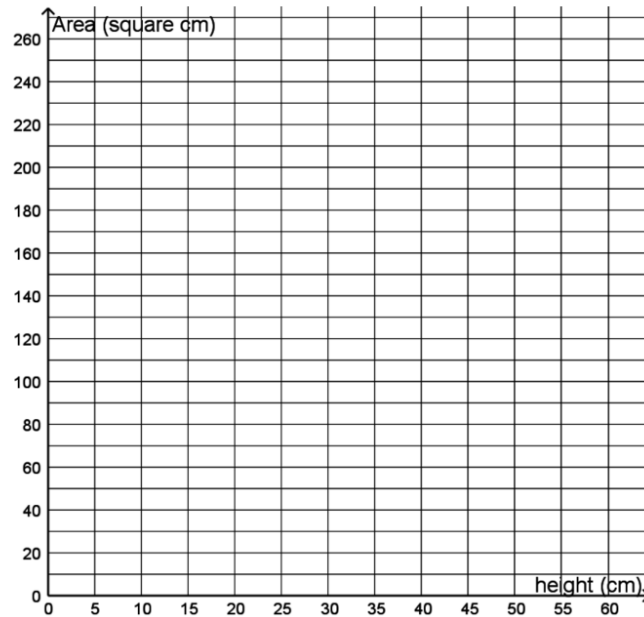
2. Write, read, and evaluate expressions in which letters stand for numbers.
- a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract  $y$  from 5" as  $5 - y$ .*
  - b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression  $2(8 + 7)$  as a product of two factors; view  $(8 + 7)$  as both a single entity and a sum of two terms.*
  - c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of length  $s = 1/2$ . (6.EE.A.2) (DOK 1,2)*
1. Example: [Solution](#) (DOK 2)
- To compute the perimeter of a rectangle you add the length,  $l$ , and width,  $w$ , and double this sum.
- a. Write an expression for the perimeter of a rectangle.
  - b. Use the expression to find the perimeter of a rectangle with length 30 and width 75.
2. Example: [Solution](#) (DOK 2)
- Some of the students at Kahlo Middle School like to ride their bikes to and from school. They always ride unless it rains.
- Let  $d$  be the distance in miles from a student's home to the school. Write two different expressions that represent how far a student travels by bike in a four week period if there is one rainy day each week.
3. Example: [Solution](#) (DOK 2)

There are a bunch of triangles. They all have one side that is 10 centimeters long, which we will consider as the base of the triangle.

a. The triangles each have a different height (as measured off of the 10-centimeter base) and so have different areas. Fill in the table:

Height (centimeters)	Area (square centimeters)
20	
25	
40	
	250

b. Plot the ordered pairs from the table in the coordinate plane and label them with their coordinates.



c. Where can you see the answers to part (a) in the coordinate plane?

d. If  $A$  represents the area and  $h$  represents the corresponding height, write an equation using  $A$  and  $h$  that represents the area of any such triangle.

4. Example: The formula  $C = \frac{5}{9}(F - 32)$  is used to convert the temperature in degrees Fahrenheit ( $F$ ) to the temperature in degrees Celsius ( $C$ ). Write the temperature in degrees Celsius ( $C$ ) equal to 113 degrees Fahrenheit ( $F$ ).

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#22	1	EE	E	1	6.EE.A.2c	6	45

5. Example: ([Former NAEP question](#)) (DOK 2)



The Music Palace is having a sale.

<p style="text-align: center;"><b><i>Music Palace Sale</i></b></p> <p style="text-align: center;">\$12 for the first CD</p> <p style="text-align: center;">\$6 for each additional CD</p> <p style="text-align: center;">(Prices include tax.)</p>
--

Write an expression that shows how to calculate the cost of buying  $n$  CD's at the sale.

Answer:  $12 + 6(n - 1)$  or  $6n + 6$

6. Example: ([Former NAEP question](#)) (DOK 1)

The length of a rectangle is 3 feet less than twice the width,  $w$  (in feet). What is the length of the rectangle in terms of  $w$ ?

- A.  $3 - 2w$
- B.  $2(w + 3)$
- C.  $2(w - 3)$
- D.  $2w + 3$
- E.  $2w - 3$

Answer: E

7. Example: ([Former NAEP question](#)) (DOK 1)

If  $x = 2n + 1$ , what is the value of  $x$  when  $n = 10$ ?

- A. 11
- B. 13
- C. 20
- D. 21
- E. 211

Answer: D

8. Example: ([Former NAEP question](#)) (DOK 1)

If  $m$  represents the total number of months that Jill worked and  $p$  represents Jill's average monthly pay, which of the following expressions represents Jill's total pay for the months she worked?

- A.  $m + p$
- B.  $m \div p$
- C.  $m \times p$
- D.  $p \div m$
- E.  $m - p$

Answer: C

3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression  $3(2 + x)$  to produce the equivalent expression  $6 + 3x$ ; apply the distributive property to the expression  $24x + 18y$  to produce the equivalent expression  $6(4x + 3y)$ ; apply properties of operations to  $y + y + y$  to produce the equivalent expression  $3y$ .* **(6.EE.A.3) (DOK 1,2)**
- a. Example: [Solution](#) (DOK 2)

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of \$27.50 for dinner. What is the cost of her dinner without tax or tip?

- b. Example: (Former NAEP question) (DOK 2)

Which of the following equations has the same solution as the equation  $2x + 6 = 32$ ?

- A.  $2x = 38$
- B.  $x - 3 = 16$
- C.  $x + 6 = 16$
- D.  $2(x - 3) = 16$
- E.  $2(x + 3) = 32$

Answer: E

- c. Example: ([Former NAEP question](#)) (DOK 1)

Consider each of the following expressions. In each case, does the expression equal  $2x$  for all values of  $x$ ?

Fill in one oval to indicate YES or NO for each expression.

- |                  | Yes                   | No                    |
|------------------|-----------------------|-----------------------|
| a) 2 times $x$   | <input type="radio"/> | <input type="radio"/> |
| b) $x$ plus $x$  | <input type="radio"/> | <input type="radio"/> |
| c) $x$ times $x$ | <input type="radio"/> | <input type="radio"/> |

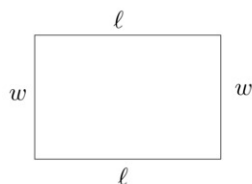
Answer: a = yes; b = yes; c = no

4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.* **(6.EE.A.4) (DOK 1)**

- a. Example: [Solution](#) (DOK 3)

The students in Mr. Nolan's class are writing expressions for the perimeter of a rectangle of side length  $\ell$  and width  $w$ . After they share their answers, the following expressions are on the board:

- Sam:  $2(\ell + w)$
- Joanna:  $\ell + w + \ell + w$
- Kiyoo:  $2\ell + w$
- Erica:  $2w + 2\ell$



Which of the expressions are correct and how might the students have been thinking about finding the perimeter of the rectangle?

- b. Example: [Solution](#) (DOK 3)

Which of the following expressions are equivalent? Why? If an expression has no match, write 2 equivalent expressions to match it.

a.  $2(x + 4)$

b.  $8 + 2x$

c.  $2x + 4$

d.  $3(x + 4) - (4 + x)$

e.  $x + 4$

c. Example: Select **all** the  $t$  expressions that are equivalent to  $8(t + 4)$ .

1.  $2(4t + 2)$

2.  $8t + 32$

3.  $4t + 4 + 4t$

4.  $(8 + t) + (8 + 4)$

5.  $(8 \times t) + (8 \times 4)$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#10	1	EE	E	1	6.EE.A.4	N/A	<input type="checkbox"/> $2(4t + 2)$ <input checked="" type="checkbox"/> $8t + 32$ <input type="checkbox"/> $4t + 4 + 4t$ <input type="checkbox"/> $(8 + t) + (8 + 4)$ <input checked="" type="checkbox"/> $(8 \times t) + (8 \times 4)$

### Reason about and solve one-variable equations and inequalities. (6.EE.B)

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **(6.EE.B.5) (DOK 1)**

a. Example: [Solution](#) (DOK 2)

A theme park has a log ride that can hold 12 people. They also have a weight limit of 1500 lbs per log for safety reasons. If the average adult weighs 150 lbs, the average child weighs 100 lbs and the log itself weighs 200, the ride can operate safely if the inequality

$$150A + 100C + 200 \leq 1500$$

is satisfied ( $A$  is the number of adults and  $C$  is the number of children in the log ride together). There are several groups of children of differing numbers waiting to ride. Group one has 4 children, group two has 3 children, group three has 9 children, group four 6 children while group five has 5 children.

If 4 adults are already seated in the log, which groups of children can safely ride with them?

b. Example: [Solution](#) (DOK 2)

Think about what these equations mean, and find their solutions. Write a sentence explaining how you know your solution is correct.

a.  $x + 6 = 10$

b.  $1000 - y = 400$

c.  $100 = m + 99$

d.  $0.99 = 1 - t$

e.  $3a = 300$

f.  $\frac{1}{2}p = 8$

g.  $10 = 0.1w$

h.  $1 = 50b$

c. Example: [Solution](#) (DOK 1)

In each equation below,  $x$  can be replaced with a number that makes the equation true. Find such a number.

a.  $64 = x^2$

b.  $64 = x^3$

c.  $2^x = 32$

d.  $x = \left(\frac{2}{5}\right)^3$

e.  $\frac{16}{9} = x^2$

f.  $2 \cdot 2^5 = 2^x$

g.  $2x = 2^4$

h.  $4^3 = 8^x$

i.  $x^2 = 25$

j.  $(x + 1)^2 = 25$

d. Example: Select **all** equations that have  $x=3$  as a solution.

1.  $x + 7 = 10$

2.  $3 + x = 3$

3.  $x \cdot 3 = 1$

4.  $4 \cdot x = 12$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#7	1	EE	F	1	6.EE.B.5	N/A	<input checked="" type="checkbox"/> $x + 7 = 10$ <input type="checkbox"/> $3 + x = 3$ <input type="checkbox"/> $x \cdot 3 = 1$ <input checked="" type="checkbox"/> $4 \cdot x = 12$

- e. Example: Consider the inequality  $x > 7$ . Determine whether each value of  $x$  shown in the table makes this inequality true. Select Yes or No for each value.

	Yes	No
22	<input type="checkbox"/>	<input type="checkbox"/>
-7	<input type="checkbox"/>	<input type="checkbox"/>
13	<input type="checkbox"/>	<input type="checkbox"/>
5	<input type="checkbox"/>	<input type="checkbox"/>
-39	<input type="checkbox"/>	<input type="checkbox"/>

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key																		
#5	1	EE	F	1	6.EE.B.5	N/A	<table border="1"> <thead> <tr> <th></th> <th>Yes</th> <th>No</th> </tr> </thead> <tbody> <tr> <td>22</td> <td><input checked="" type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> <tr> <td>-7</td> <td><input type="checkbox"/></td> <td><input checked="" type="checkbox"/></td> </tr> <tr> <td>13</td> <td><input checked="" type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> <tr> <td>5</td> <td><input type="checkbox"/></td> <td><input checked="" type="checkbox"/></td> </tr> <tr> <td>-39</td> <td><input type="checkbox"/></td> <td><input checked="" type="checkbox"/></td> </tr> </tbody> </table>		Yes	No	22	<input checked="" type="checkbox"/>	<input type="checkbox"/>	-7	<input type="checkbox"/>	<input checked="" type="checkbox"/>	13	<input checked="" type="checkbox"/>	<input type="checkbox"/>	5	<input type="checkbox"/>	<input checked="" type="checkbox"/>	-39	<input type="checkbox"/>	<input checked="" type="checkbox"/>
	Yes	No																							
22	<input checked="" type="checkbox"/>	<input type="checkbox"/>																							
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5	<input type="checkbox"/>	<input checked="" type="checkbox"/>																							
-39	<input type="checkbox"/>	<input checked="" type="checkbox"/>																							

- f. Example: ([Former NAEP question](#)) (DOK 2)

The point  $(4, k)$  is a solution to the equation  $3x + 2y = 12$ . What is the value of  $k$ ?

- A. -3
- B. 0
- C. 2
- D. 3
- E. 4

Answer: B

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **(6.EE.B.6) (DOK 1,2)**
- a. Example: [Solution](#) (DOK 2)
- A town's total allocation for firefighter's wages and benefits in a new budget is \$600,000. If wages are calculated at \$40,000 per firefighter and benefits at \$20,000 per firefighter, write an equation whose solution is the number of firefighters the town can employ if they spend their whole budget. Solve the equation.
- b. Example: [Solution](#) (DOK 2)

A penny is about  $\frac{1}{16}$  of an inch thick.

a. In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

b. In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

c. The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?

7. Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  for cases in which  $p$ ,  $q$  and  $x$  are all nonnegative rational numbers. **(6.EE.B.7) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 2)

A town's total allocation for firefighter's wages and benefits in a new budget is \$600,000. If wages are calculated at \$40,000 per firefighter and benefits at \$20,000 per firefighter, write an equation whose solution is the number of firefighters the town can employ if they spend their whole budget. Solve the equation.

- b. Example: [Solution](#) (DOK 2)

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of \$27.50 for dinner. What is the cost of her dinner without tax or tip?

- c. Example: [Solution](#) (DOK 2)

A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?

- d. Example: [Solution](#) (DOK 2)

Sierra walks her dog Pepper twice a day. Her evening walk is two and a half times as far as her morning walk. At the end of the week she tells her mom,

*I walked Pepper for 30 miles this week!*

How long is her morning walk?

- e. Example: Ms. Stone buys groceries for a total of \$45.32. She now has \$32.25 left. Which equation could be used to find out how much money Ms. Stone had before she bought the groceries?

1.  $\$45.32x = \$32.25$

2.  $x - \$45.32 = \$32.25$

$$3. x + \$45.32 = \$32.25$$

$$4. x + \$32.25 = \$45.32$$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#15	1	EE	F	1	6.EE.B.7	N/A	2

- f. Example: ([Former NAEP question](#)) (DOK 1)

Which of the following equations is NOT equivalent to the equation  $n + 18 = 23$ ?

- A.  $23 = n - 18$
- B.  $23 = 18 + n$
- C.  $18 = 23 - n$
- D.  $18 + n = 23$
- E.  $n = 23 - 18$

Answer: A

- g. Example: ([Former NAEP question](#)) (DOK 2)

Robert has  $x$  books. Marie has twice as many books as Robert has. Together they have 18 books. Which of the following equations can be used to find the number of books that Robert has?

- A.  $x + 2 = 18$
- B.  $x + x + 2 = 18$
- C.  $x + 2x = 18$
- D.  $2x = 18$
- E.  $2x + 2x = 18$

Answer: C

8. Write an inequality of the form  $x > c$  or  $x < c$  to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form  $x > c$  or  $x < c$  have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **(6.EE.B.8) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 2)

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 900 pounds of weight for safety reasons.

a. Let  $p$  represent the total number of people. Write an inequality to describe the number of people that a boat can hold. Draw a number line diagram that shows all possible solutions.

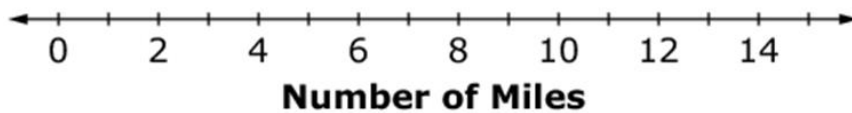
b. Let  $w$  represent the total weight of a group of people wishing to rent a boat. Write an inequality that describes all total weights allowed in a boat. Draw a number line diagram that shows all possible solutions.

- b. Example: [Solution](#) (DOK 2)

At Sea World San Diego, kids are only allowed into the Air Bounce if they are between 37 and 61 inches tall. They are only allowed on the Tide Pool Climb if they are 39 inches tall or under:



- Represent the height requirements of each ride with inequalities.
  - Show the allowable heights for the rides on separate number lines.
  - Using inequalities and a number line, describe the height of kids who can go on the both the Air Bounce and the Tide Pool Climb.
- c. Example: A boat takes 3 hours to reach an island 15 miles away.  
 The boat travels:  
 At least 1 mile but no more than 6 miles during the first hour  
 At least 2 miles during the second hour  
 Exactly 5 miles during the third hour  
 Show the range of miles the boat could have traveled during the second hour, given the conditions above.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#25	4	EE	F, G	3	6.EE.B.8	1, 2, 4	

- d. Example: ([Former NAEP question](#)) (DOK 1)



If  $a > 0$  and  $b < 0$ , which of the following must be true?

- A.  $ab > 0$
- B.  $a - b > 0$
- C.  $b - a > 0$
- D.  $a + b > 0$
- E.  $a + b < 0$

Answer: B

Represent and analyze quantitative relationships between dependent and independent variables.  
(6.EE.C)

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation  $d = 65t$  to represent the relationship between distance and time.* (6.EE.C.9) (DOK 1,2,3)

- a. Example: [Solution](#) (DOK 2)

Stephanie is helping her band collect money to fund a field trip. The band decided to sell boxes of chocolate bars. Each bar sells for \$1.50 and each box contains 20 bars. Below is a partial table of monies collected for different numbers of boxes sold.

Boxes Sold	Money Collected
$b_i$	$m_i$
1	\$30.00
2	
3	
4	
5	\$150.00
6	
7	
8	

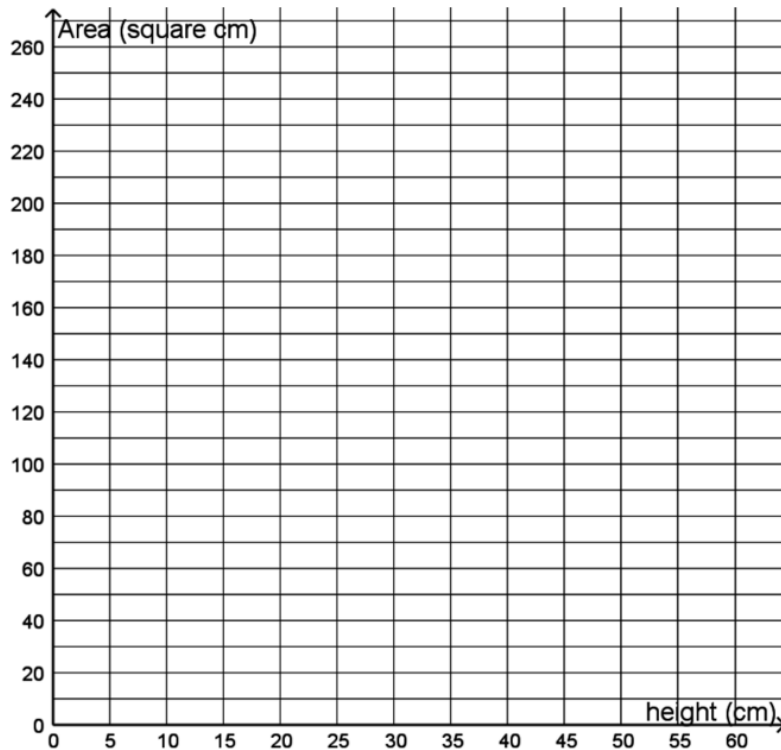
- a. Complete the table above for values of  $m_i$ .
- b. Write an equation for the amount of money,  $m_i$  that will be collected if  $b_i$  boxes of chocolate bars are sold. Which is the independent variable and which is the dependent variable?
- c. Graph the equation using the ordered pairs from the table above.
- d. Calculate how much money will be collected if 100 boxes of chocolate bars are sold.
- e. The band collected \$1530.00 from chocolate bar sales. How many boxes did they sell?
- b. Example: [Solution](#) (DOK 2)

There are a bunch of triangles. They all have one side that is 10 centimeters long, which we will consider as the base of the triangle.

a. The triangles each have a different height (as measured off of the 10-centimeter base) and so have different areas. Fill in the table:

Height (centimeters)	Area (square centimeters)
20	
25	
40	
	250

b. Plot the ordered pairs from the table in the coordinate plane and label them with their coordinates.



c. Where can you see the answers to part (a) in the coordinate plane?

d. If  $A$  represents the area and  $h$  represents the corresponding height, write an equation using  $A$  and  $h$  that represents the area of any such triangle.

c. Example: In the morning, Emily studied for 40 minutes for a math exam. Later that evening, Emily studied for  $x$  more minutes.

Enter an **equation** that represents the total number of minutes,  $y$ , Emily studied for the math exam.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#16	1	EE	G	1	6.EE.C.9	N/A	$40 + x = y$ or an equivalent equation

- d. Example: ([Former NAEP question](#)) (DOK 1)



Mika and her mother noticed the road sign shown above while in their car on their way to Rockville. If their speed is about 65 miles per hour, approximately how many more hours are needed to finish the trip?

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

Answer: B

- e. Example: ([Former NAEP question](#)) (DOK 1)

An airplane climbs at a rate of 66.8 feet per minute. It descends at twice the rate that it climbs. Assuming it descends at a constant rate, how many feet will the airplane descend in 30 minutes?

- A. 96.8
- B. 133.6
- C. 1,002
- D. 2,004
- E. 4,008

Answer: E

- f. Example: ([Former NAEP question](#)) (DOK 1)

The formula  $d = 16t$  gives the distance  $d$ , in feet, that an object has fallen  $t$  seconds after it is dropped from a bridge. A rock was dropped from the bridge and its fall to the water took 4 seconds. According to the formula, what is the distance from the bridge to the water?

- A. 16 feet
- B. 64 feet
- C. 128 feet
- D. 256 feet
- E. 4,096 feet

Answer: B

- g. Example: ([Former NAEP question](#)) (DOK 2)

Each figure in the pattern below is made of hexagons that measure 1 centimeter on each side.



Figure 1  
Perimeter = 6 cm

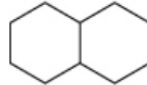


Figure 2  
Perimeter = 10 cm

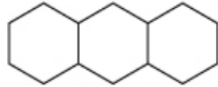


Figure 3  
Perimeter = 14 cm

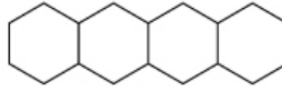


Figure 4  
Perimeter = 18 cm

If the pattern of adding one hexagon to each figure is continued, what will be the perimeter of the 25th figure in the pattern?

Show how you found your answer.

Answer:  $6 + 4(24) = 102$

h. Example: ([Former NAEP question](#)) (DOK 1)

Sarah has a part-time job at Better Burgers restaurant and is paid \$5.50 for each hour she works. She has made the chart below to reflect her earnings but needs your help to complete it.

(a) Fill in the missing entries in the chart.

Hours Worked	Money Earned (in dollars)
1	\$5.50
4	
	\$38.50
$7\frac{3}{4}$	\$42.63

(b) If Sarah works  $h$  hours, then, in terms of  $h$ , how much will she earn?

Answer: 1)  $22$ ; 2)  $7$ ; 3)  $5.5h$

Solve real-world and mathematical problems involving area, surface area, and volume. (6.G.A)

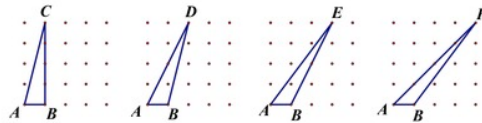
1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **(6.G.A.1) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 3)

Make the following triangles on a geoboard. Remember that the pegs are equally spaced in a square grid. Compare the areas of the triangles.

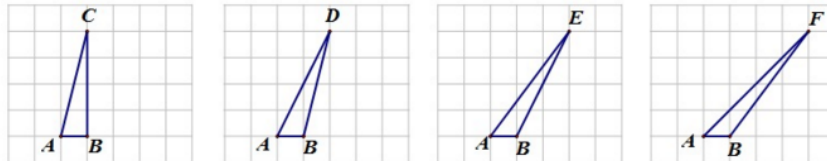
- Which has the greatest area?
- Which has the least area?
- Do any of the triangles have the same area?
- Are some areas impossible to compare?

Explain why you answered as you did.



- b. Example: [Solution](#) (DOK 2)

Which of the triangles,  $\triangle ABC$ ,  $\triangle ABD$ ,  $\triangle ABE$  or  $\triangle ABF$ , has the largest area?

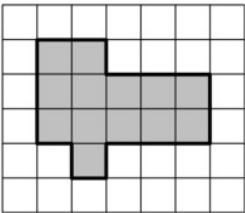


- a.  $\triangle ABC$
- b.  $\triangle ABD$
- c.  $\triangle ABE$
- d.  $\triangle ABF$
- e. They all have the same area
- f. There is not enough information to answer.

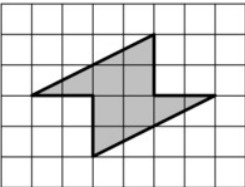
- c. Example: [Solution](#) (DOK 2)

Find the area that is shaded in each figure in at least two different ways.

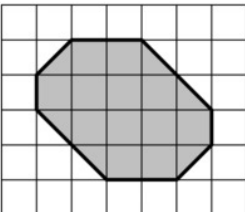
a.



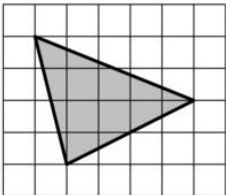
b.



c.

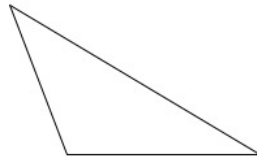


d.



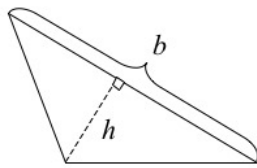
d. Example: [Solution](#) (DOK 3)

Mrs. Lito asked her students to label a base  $b$  and its corresponding height  $h$  in the triangle shown.

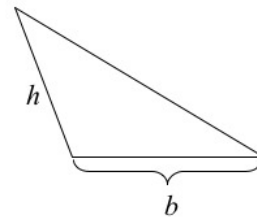


Three students drew the figures below.

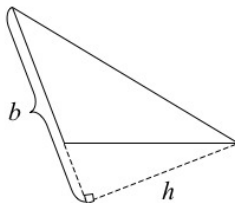
a. Raul



b. Mark



c. Kiki



a. Which students, if any, have correctly identified a base and its corresponding height? Which ones have not? Explain what is incorrect.

b. There are three possible base-height pairs for this triangle. Sketch all three.

e. Example: [Solution](#) (DOK 2)

The vertices of eight polygons are given below. For each polygon:

- Plot the points in the coordinate plane and connect the points in the order that they are listed.
- Color the shape the indicated color and identify the type of polygon it is.
- Find the area.

a. The first polygon is GREY and has these vertices:

$$(-7, 4) (-8, 5) (-8, 6) (-7, 7) (-5, 7) (-5, 5) (-7, 4)$$

b. The second polygon is ORANGE and has these vertices:

$$(-2, -7) (-1, -4) (3, -1) (6, -7) (-2, -7)$$

c. The third polygon is GREEN and has these vertices:

$$(4, 3) (3, 3) (2, 2) (2, 1) (3, 0) (4, 0) (5, 1) (5, 2) (4, 3)$$

d. The fourth polygon is BROWN and has these vertices:

$$(0, -10) (0, -8) (7, -10) (0, -10)$$

e. The fifth polygon is PURPLE and has these vertices:

$$(-8, -5) (-8, -8) (-5, -8) (-5, -5) (-8, -5)$$

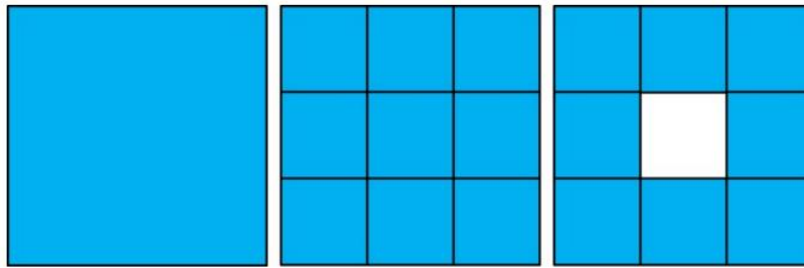
f. The sixth polygon is PINK and has these vertices:

$$(9, -1) (6, 1) (6, -3) (9, -1)$$

f. Example: [Solution](#) (DOK 2)

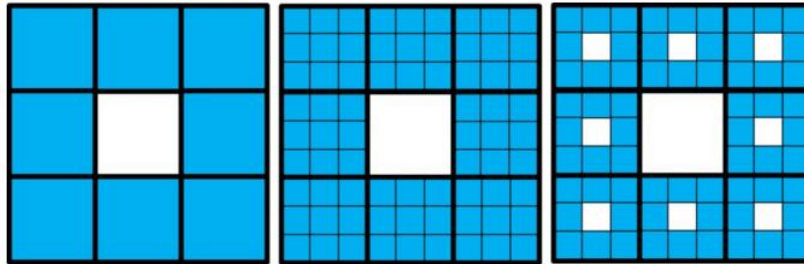


- a. Take a square with area 1. Divide it into 9 equal-sized squares. Remove the middle one.



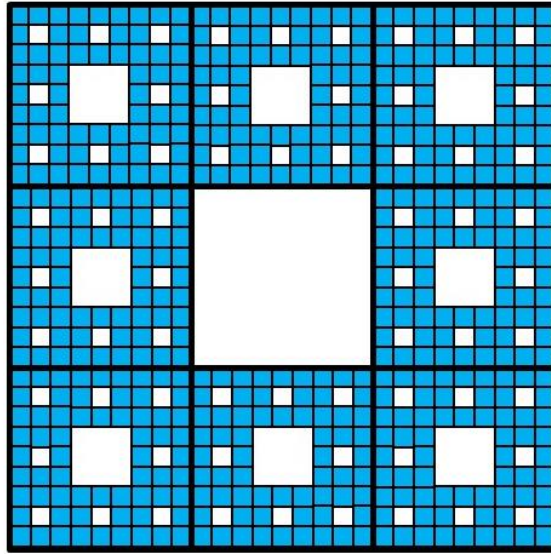
What is the area of the figure now?

- b. Take the remaining 8 squares. Divide each one into 9 equal squares. Remove the middle one from each group of 9.



What is the area of the figure now?

- c. Take the remaining squares. (How many are there?) Divide each one into 9 equal squares. Remove the middle one from each group of 9.

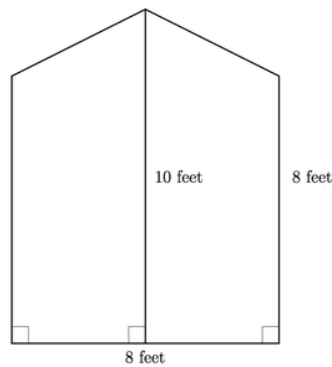


What is the area of the figure now?

d. Imagine you follow this same process until you have removed "the middle square from each group of 9" 10 times. How many squares will there be? What will the area of each little square be? What will the area of the entire figure be?

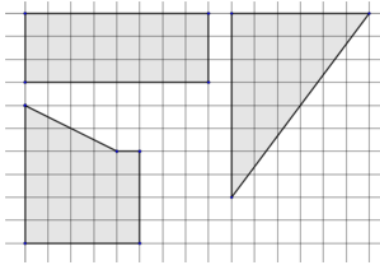
g. Example: [Solution](#) (DOK 3)

Jamie is planning to cover a wall with red wallpaper. The dimensions of the wall are shown below:



- How many square feet of wallpaper are required to cover the wall?
- Wallpaper comes in long rectangular strips which are 24 inches wide. If Jamie lays the strips of wallpaper vertically, can she cover the wall without wasting any wallpaper? Explain.
- If Jamie lays the strips of wallpaper horizontally, can she cover the wall without wasting any wallpaper? Explain.

- h. Example: [Solution](#) (DOK 2)

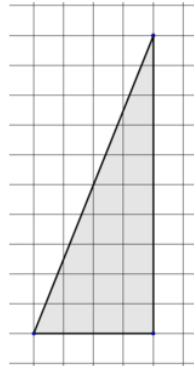


The area of each figure shown is 24 square units.

- a. On grid paper, find a way to draw more figures with an area of 24 square units. Draw at least one of each:
- A rectangle that is different from the one shown.
  - A polygon with more than 4 sides that is different from the one shown.
  - A right triangle that is different from the one shown.
  - A parallelogram.
- b. For each of the figures you draw, explain how you know its area is exactly 24 square units.

i. Example: [Solution](#) (DOK 3)

a. Explain why the right triangle shown has an area of exactly 20 square units.



b. The "legs" of a right triangle are the two sides that form the right angle. If one leg of a right triangle is 5 units long, explain what else would have to be true about the right triangle in order for its area to be 30 square units.

c. Here are leg measurements for more right triangles. What is the area of each?

i. 6 and 3

ii. 12 and  $4\frac{1}{2}$

iii. 3 and 7

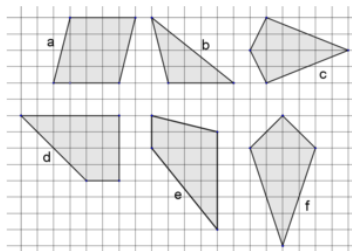
iv. 6.5 and 9

d. Explain in words how you can find the area of a right triangle when you know the lengths of its legs.

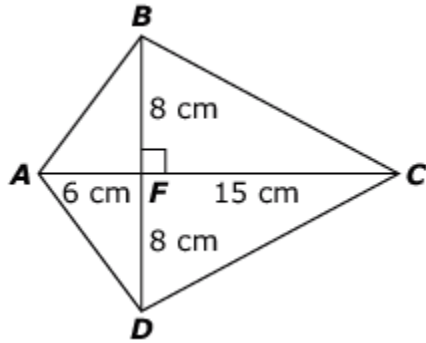
e. Let  $a$  represent the length of one leg of a right triangle and  $b$  represent the length of the other leg of the right triangle. Write a mathematical expression for the area of the right triangle in terms of  $a$  and  $b$ .

j. Example: [Solution](#) (DOK 3)

Of the polygons shown, which have equal areas? Explain how you know.



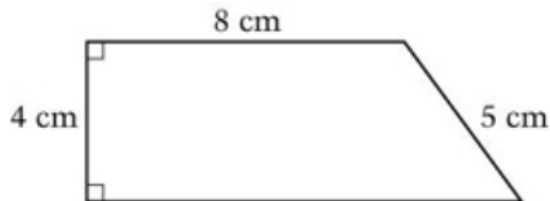
k. Example: Consider this figure.



Write the total area of figure  $ABCD$  in square centimeters.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#20	1	G	H	2	6.G.A.1	N/A	168

- l. Example: ([Former NAEP question](#)) (DOK 2)

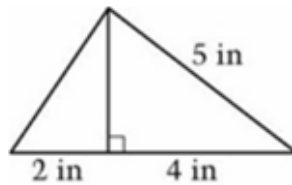


What is the area of the figure shown above?

- A. 28 square centimeters
- B. 32 square centimeters
- C. 38 square centimeters
- D. 44 square centimeters
- E. 64 square centimeters

Answer: C

- m. Example: ([Former NAEP question](#)) (DOK 2)



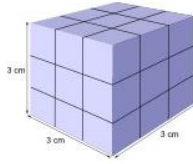
What is the area of the figure above?

ANSWER: \_\_\_\_\_ square in.

Answer: 9 square in.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas  $V = lwh$  and  $V = bh$  to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **(6.G.A.2) (DOK 1,2)**
  - a. Example: [Solution](#) (DOK 2)

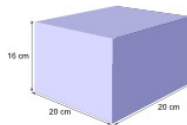
a. Amy wants to build a cube with  $3\text{ cm}$  sides using  $1\text{ cm}$  cubes. How many cubes does she need?



b. How many  $1\text{ cm}$  cubes would she need to build a cube with  $6\text{ cm}$  sides?

b. Example: [Solution](#) (DOK 2)

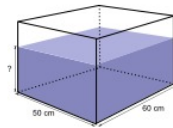
a. Amy has a fish tank shaped like a rectangular prism that is  $20\text{ cm}$  by  $20\text{ cm}$  by  $16\text{ cm}$ . What is the volume of the tank?



b. If Amy only fills the tank  $\frac{3}{4}$  of the way, what will be the volume of the water in the tank?

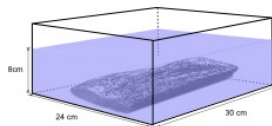
c. Example: [Solution](#) (DOK 2)

A rectangular tank is  $50\text{ cm}$  wide and  $60\text{ cm}$  long. It can hold up to  $126\text{ l}$  of water when full. If Amy fills  $\frac{2}{3}$  of the tank as shown, find the height of the water in centimeters. (Recall that  $1\text{ l} = 1000\text{ cm}^3$ .)



d. Example: [Solution](#) (DOK 2)

A rectangular tank is  $24\text{ cm}$  wide, and  $30\text{ cm}$  long. It contains a stone and is filled with water to a height of  $8\text{ cm}$ . When Amy pulls the stone out of the tank, the height of the water drops to  $6\text{ cm}$ . Find the volume of the stone.



e. Example: [Solution](#) (DOK 2)

Leo's recipe for banana bread won't fit in his favorite pan. The batter fills the  $8.5\text{ inch}$  by  $11\text{ inch}$  by  $1.75\text{ inch}$  pan to the very top, but when it bakes it spills over the side. He has another pan that is  $9\text{ inches}$  by  $9\text{ inches}$  by  $3\text{ inches}$ , and from past experience he thinks he needs about an inch between the top of the batter and the rim of the pan. Should he use this pan?

f. Example: [Solution](#) (DOK 2)

Make sure you have plenty of snap cubes.

a. A *dubsnap* is a length equal to two snap cube edges. Build a cube using 8 snap cubes of one color. Call this a *dubsnap cube*, with side length equal to 1 dubsnap, so it has a volume of

$$1 \times 1 \times 1 = 1 \text{ cubic dubsnap.}$$

i. How long (in dubsnaps) are the side lengths of a single snap cube?

ii. What is the volume of a single snap cube, in cubic dubsnaps?

b. Build a rectangular solid that is 2 snap cubes by 4 snap cubes by 6 snap cubes. What is the volume of this solid in cubic dubsnaps?

i. What are the side-lengths (in dubsnaps) of a solid that is 1 snap cube by 4 snap cubes by 6 snap cubes? What is its volume in cubic dubsnaps?

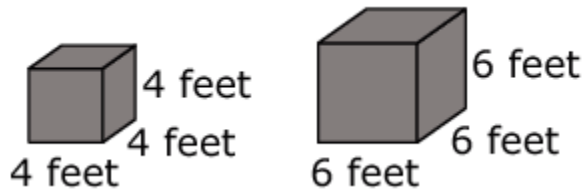
ii. What are the side-lengths (in dubsnaps) of a solid that is 2 snap cubes by 2 snap cubes by 6 snap cubes? What is its volume in cubic dubsnaps?

iii. What are the side-lengths (in dubsnaps) of a solid that is 2 snap cubes by 2 snap cubes by 3 snap cubes? What is its volume in cubic dubsnaps?

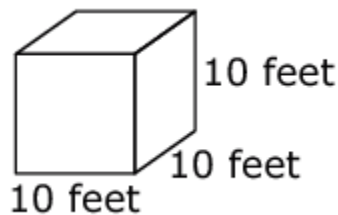
c. Build a solid that has a volume of  $\frac{1}{2}$  cubic dubsnap.

d. Build a solid that has a volume of  $\frac{5}{8}$  cubic dubsnap.

g. Example: Two shaded cubes are shown.



Ben states that the combined volume of these two shaded cubes is equal to the volume of this cube:



**Part A:** Select whether Ben's statement is true or false.

True

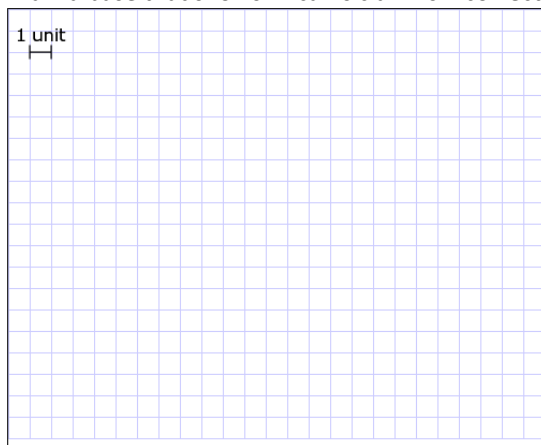
False

**Part B:** Write numbers to show the combined volume of the shaded cubes.

? Cubic feet

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#18	3	G	C	3	6.G.A.2	1, 3	<div style="border: 1px solid black; padding: 10px;"> <p><b>A. Ben's statement</b></p> <p style="text-align: center;">True <input type="checkbox"/> False <input checked="" type="checkbox"/></p> <p><b>B. Combined volume</b></p> <p style="text-align: center;"><input type="text" value="280"/> cubic feet</p> </div>

- h. Example: Micah constructs a rectangular prism with a volume of 360 cubic units. The height of his prism is 10 units. Micah claims that the base of the prism must be a square. Draw a base that shows Micah's claim is incorrect.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#6	3	G	B	3	6.G.A.2	3, 5	<p>Other rectangles with an area of 36 square units will also score correctly and receive full credit. Only a 6 by 6 square will not receive credit.</p>

- i. Example: ([Former NAEP question](#)) (DOK 1)



In order to prepare a piece of ground for building a brick patio, a rectangle measuring 8 feet by 10 feet must be marked off. Then the dirt within the rectangle must be dug out to a depth of 6 inches. Finally, the resulting space must be filled with sand.

(a) What is the volume of sand needed, in cubic feet, to fill the space?

\_\_\_\_\_ cubic feet

Show your work. If you used your calculator, show the numbers and operations that you used to get your answer.

(b) Sand costs \$4 per cubic foot. What is the total cost of the sand needed to fill this space, including a \$35 delivery charge?

\$ \_\_\_\_\_

Show your work. If you used your calculator, show the numbers and operations that you used to get your answer.

Answer: a)  $10 \times 8 \times 6 = 480$  cubic feet; b)  $(480 \times 4) + 35 = \$1,955$

3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **(6.G.A.3) (DOK 1,2)**

a. Example: [Solution](#) (DOK 2)

The vertices of eight polygons are given below. For each polygon:

- Plot the points in the coordinate plane and connect the points in the order that they are listed.
- Color the shape the indicated color and identify the type of polygon it is.
- Find the area.

a. The first polygon is GREY and has these vertices:

$$(-7, 4) (-8, 5) (-8, 6) (-7, 7) (-5, 7) (-5, 5) (-7, 4)$$

b. The second polygon is ORANGE and has these vertices:

$$(-2, -7) (-1, -4) (3, -1) (6, -7) (-2, -7)$$

c. The third polygon is GREEN and has these vertices:

$$(4, 3) (3, 3) (2, 2) (2, 1) (3, 0) (4, 0) (5, 1) (5, 2) (4, 3)$$

d. The fourth polygon is BROWN and has these vertices:

$$(0, -10) (0, -8) (7, -10) (0, -10)$$

e. The fifth polygon is PURPLE and has these vertices:

$$(-8, -5) (-8, -8) (-5, -8) (-5, -5) (-8, -5)$$

f. The sixth polygon is PINK and has these vertices:

$$(9, -1) (6, 1) (6, -3) (9, -1)$$

g. The seventh polygon is BLUE and has these vertices:

$$(-6, -4) (-6, 1) (-9, 1) (-9, -4) (-6, -4)$$

h. The eighth polygon is YELLOW and has these vertices:

$$(-5, 1) (-3, -3) (-1, -2) (0, 3) (-3, 3) (-5, 1)$$

b. Example: [Solution](#) (DOK 2)



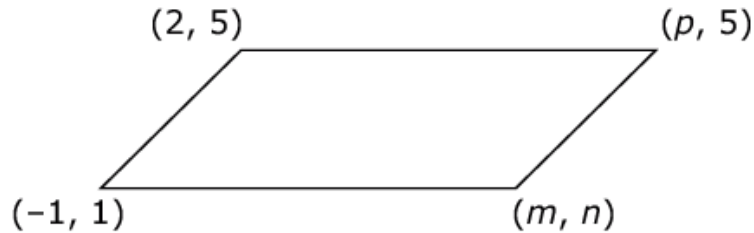
Here is a map of part of Downtown Salt Lake City. You are starting at the corner of 11th Ave. and D St. (on the star).

a. If you walk East to I St., South to 7th Ave., West to D St. and then North to your starting point, how many blocks will you have walked in total? Describe the shape of your path.

b. Draw and describe in words at least two different ways that you can walk exactly 8 blocks and end up where you started.

c. Jessica said the path she took on her walk enclosed a polygon that had an area of 6 square blocks. Draw some possible shapes that her walk could have taken. Was her path necessarily rectangular?

c. Example: The coordinates of this parallelogram are given.

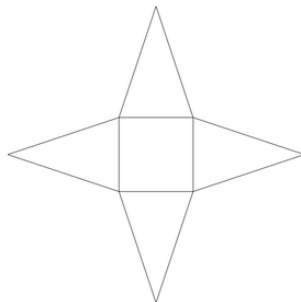


Determine if each statement is True or False.

	True	False
The length of the longer side is $p - 2$ .	<input type="checkbox"/>	<input type="checkbox"/>
The length of the longer side is $n + 1$ .	<input type="checkbox"/>	<input type="checkbox"/>
The short side is 4 units in length.	<input type="checkbox"/>	<input type="checkbox"/>
$n = 5$	<input type="checkbox"/>	<input type="checkbox"/>
$m > n$	<input type="checkbox"/>	<input type="checkbox"/>
$p = 2$	<input type="checkbox"/>	<input type="checkbox"/>

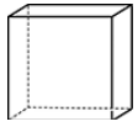
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key																					
#21	3	G	D	3	6.G.A3	2, 3	<table border="1"> <thead> <tr> <th></th> <th>True</th> <th>False</th> </tr> </thead> <tbody> <tr> <td>The length of the longer side is <math>p - 2</math>.</td> <td><input checked="" type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> <tr> <td>The length of the longer side is <math>n + 1</math>.</td> <td><input type="checkbox"/></td> <td><input checked="" type="checkbox"/></td> </tr> <tr> <td>The short side is 4 units in length.</td> <td><input type="checkbox"/></td> <td><input checked="" type="checkbox"/></td> </tr> <tr> <td><math>n = 5</math></td> <td><input type="checkbox"/></td> <td><input checked="" type="checkbox"/></td> </tr> <tr> <td><math>m &gt; n</math></td> <td><input checked="" type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> <tr> <td><math>p = 2</math></td> <td><input type="checkbox"/></td> <td><input checked="" type="checkbox"/></td> </tr> </tbody> </table>		True	False	The length of the longer side is $p - 2$ .	<input checked="" type="checkbox"/>	<input type="checkbox"/>	The length of the longer side is $n + 1$ .	<input type="checkbox"/>	<input checked="" type="checkbox"/>	The short side is 4 units in length.	<input type="checkbox"/>	<input checked="" type="checkbox"/>	$n = 5$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	$m > n$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	$p = 2$	<input type="checkbox"/>	<input checked="" type="checkbox"/>
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$p = 2$	<input type="checkbox"/>	<input checked="" type="checkbox"/>																										

4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **(6.G.A.4) (DOK 1,2)**
- Example: [Solution](#) (DOK 3)
    - Below is a net for a three dimensional shape:

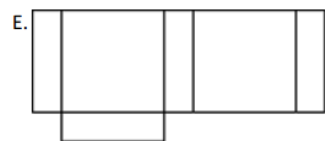
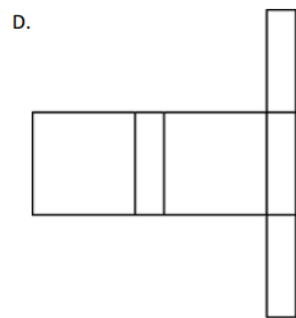
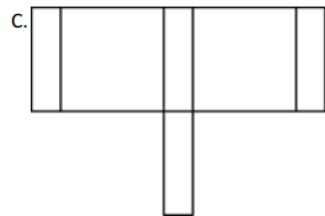
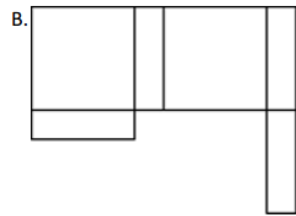
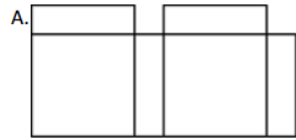


The inner quadrilateral is a square and the four triangles all have the same size and shape.

- What three dimensional shape does this net make? Explain.
  - If the side length of the square is 2 units and the height of the triangles is 3 units, what is the surface area of this shape?
- b. Draw a net for a rectangular prism whose base is a one inch by one inch square and whose faces are 3 inches by 1 inch.
- Is there more than one possible net for this shape? Explain.
  - What is the surface area of the prism?
- b. Example: ([Former NAEP question](#)) (DOK 2)



The box pictured above has six faces that do not overlap. The box will unfold into one of the figures below. Which figure is it?



Answer: D

Develop understanding of statistical variability. (6.SP.A)

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

**(6.SP.A.1) (DOK 1)**

- a. Example: [Solution](#) (DOK 2)

Which of the following are statistical questions? (A statistical question is one that can be answered by collecting data and where there will be variability in that data.)

- a. How many days are in March?
- b. How old is your dog?
- c. On average, how old are the dogs that live on this street?
- d. What proportion of the students at your school like watermelons?
- e. Do you like watermelons?
- f. How many bricks are in this wall?
- g. What was the temperature at noon today at City Hall?

- b. Example: [Solution](#) (DOK 3)

Zeke likes to collect buttons and he keeps them in a jar. Zeke can empty the buttons out of the jar, so he can see all of his buttons at once.



a. Which of the following are statistical questions that someone could ask Zeke about his buttons? (A statistical question is one that anticipates an answer based on data that vary.) For each question, explain why it is or is not a statistical question.

- i. What is a typical number of holes for the buttons in the jar?
- ii. How many buttons are in the jar?
- iii. How large is the largest button in the jar?
- iv. If Zeke grabbed a handful of buttons, what are the chances that all of the buttons in his hand are round?

b. Write another statistical question related to Zeke's button collection.

c. Example: [Solution](#) (DOK 3)

Last night, Jennifer and her family went out for dinner. The questions below came up on their way to the restaurant or during the meal. Decide whether or not each question is a statistical question, and justify your decision.

- a. How far are we from the restaurant?
- b. How long will it be until we get there?
- c. Would Jennifer rather have burgers or pizza?
- d. How much should we leave for the tip?
- e. What was the first dish ordered in the restaurant this evening?
- f. Do customers at the restaurant like pizza?
- g. What is a typical bill for tables at this restaurant?
- h. On average, how many people were sitting at each table this evening?

d. Example: A statistical question is one where you expect to get a variety of answers. Determine whether each question can be classified as a statistical question. Select Yes or No for each question.

How many hours a week do people exercise?	Yes <input type="checkbox"/>
How many hours are there in a day?	No <input type="checkbox"/>
How many rainbows have students seen this month?	Yes <input type="checkbox"/>

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key	
#23	1	SP	I	1	6.SP.A.1	N/A		
							How many hours a week do people exercise?	Yes <input checked="" type="checkbox"/> No <input type="checkbox"/>
							How many hours are there in a day?	Yes <input type="checkbox"/> No <input checked="" type="checkbox"/>
							How many rainbows have students seen this month?	Yes <input checked="" type="checkbox"/> No <input type="checkbox"/>

- 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **(6.SP.A.2) (DOK 1,2)**
  - a. Example: [Solution](#) (DOK 3)

Below are the 25 birth weights, in ounces, of all the Labrador Retriever puppies born at Kingston Kennels in the last six months.

13, 14, 15, 15, 16, 16, 16, 16, 17, 17, 17, 17, 17, 17, 18, 18, 18, 18, 18, 18, 18, 18, 19, 20

- a. Use an appropriate graph to summarize these birth weights.
- b. Describe the distribution of birth weights for puppies born at Kingston Kennels in the last six months. Be sure to describe shape, center and variability.
- c. What is a typical birth weight for puppies born at Kingston Kennels in the last six months? Explain why you chose this value.

- b. Example: [Solution](#) (DOK 3)

Unlike many elections for public office where a person is elected strictly based on the results of a popular vote (i.e., the candidate who earns the most votes in the election wins), in the United States, the election for President of the United States is determined by a process called the Electoral College. According to the National Archives, the process was established in the United States Constitution "as a compromise between election of the President by a vote in Congress and election of the President by a popular vote of qualified citizens."

(<http://www.archives.gov/federal-register/electoral-college/about.html> accessed September 4, 2012).

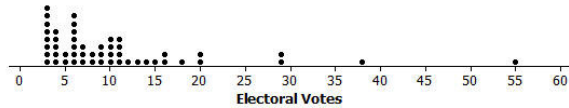
Each state receives an allocation of electoral votes in the process, and this allocation is determined by the number of members in the state's delegation to the US Congress. This number is the sum of the number of US Senators that represent the state (always 2, per the Constitution) and the number of Representatives that represent the state in the US House of Representatives (a number that is directly related to the state's population of qualified citizens as determined by the US Census). Therefore the larger a state's population of qualified citizens, the more electoral votes it has. Note: the District of Columbia (which is not a state) is granted 3 electoral votes in the process through the 23rd Amendment to the Constitution.

The following table shows the allocation of electoral votes for each state and the District of Columbia for the 2012, 2016, and 2020 presidential elections. (<http://www.archives.gov/federal-register/electoral-college/allocation.html> accessed September 4, 2012).

State	Electoral Votes	State	Electoral Votes	State	Electoral Votes
Alabama	9	Kentucky	8	North Dakota	3
Alaska	3	Louisiana	8	Ohio	18
Arizona	11	Maine	4	Oklahoma	7
Arkansas	6	Maryland	10	Oregon	7
California	55	Massachusetts	11	Pennsylvania	20
Colorado	9	Michigan	16	Rhode Island	4
Connecticut	7	Minnesota	10	South Carolina	9
Delaware	3	Mississippi	6	South Dakota	3
District of Columbia	3	Missouri	10	Tennessee	11
Florida	29	Montana	3	Texas	38
Georgia	16	Nebraska	5	Utah	6
Hawaii	4	Nevada	6	Vermont	3
Idaho	4	New Hampshire	4	Virginia	13
Illinois	20	New Jersey	14	Washington	12
Indiana	11	New Mexico	5	West Virginia	5
Iowa	6	New York	29	Wisconsin	10
Kansas	6	North Carolina	15	Wyoming	3

- Which state has the most electoral votes? How many votes does it have?
- Based on the given information, which state has the second highest population of qualified citizens?
- Here is a dotplot of the distribution.





- i. What is the shape of this distribution: skewed left, symmetric, or skewed right?
  - ii. Imagine that someone you are speaking with is unfamiliar with these shape terms. Describe clearly and in the context of this data set what the shape description you have chosen means in terms of the distribution.
  - d. Does the dotplot lead you to think that any states are outliers in terms of their number of electoral votes? Explain your reasoning, and if you do believe that there are outlier values, identify the corresponding states.
  - e. What measure of center (mean or median) would you recommend for describing this data set? Why did you choose this measure?
  - f. Determine the value of the median for this data set (electoral votes).
- c. Example: [Solution](#) (DOK 3)
- The number of siblings for a group of sixth grade students is shown below:

1, 0, 2, 1, 6, 0, 2, 0, 1, 10.

- a. Make a dot plot of the data.
- b. Find the mean and median of the data.
- c. What does the mean tell you about the data? What about the median?
- d. Which measure of average (mean or median) do you think best describes the data? Why?

d. Example: [Solution](#) (DOK 2)

Statistical questions are questions that can be answered by collecting data and where we anticipate that there will be variability in that data. The data collected can be summarized in a distribution that can then be described in terms of center and in terms of spread. For some statistical questions, to answer the question you need to consider center. For other questions you might need to consider spread.

For each of the five statistical questions below, decide if you would answer the question by considering center or considering variability in the data distribution.

**Example 1:** The records office at an elementary school keeps daily attendance records.

**Question 1:** For students at this school, what is a typical number of school days missed in the month of April?

**Example 2:** Suppose that third graders at your school took both a math test and a social studies test. Scores on both tests could be any number between 0 and 100.

**Question 2:** On average, did the students score better on the math test or the social studies test?

**Question 3:** Were the students' scores more consistent (more similar to one another) on the math test or on the social studies test?

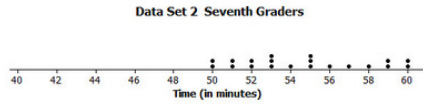
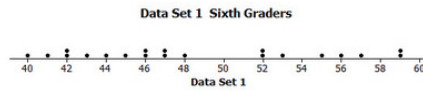
**Example 3:** Bags of M&Ms don't all have exactly the same number of candies in each bag. Suppose you count the number of candies in each of 25 bags of plain M&Ms and in each of 25 bags of peanut M&Ms, and make two dot plots—one for the number of candies in the plain M&M bags and one for the number of candies in the peanut M&M bags.

**Question 4:** If you wanted to give each student in your class a bag of M&Ms and you wanted to try to make sure that each student got the same number of candies, should you give them bags of plain M&Ms or bags of peanut M&Ms?

**Question 5:** If you wanted to give each student in your class a bag of M&Ms and you wanted to try to give students bags with the greatest number of candies, should you give them bags of plain M&Ms or bags of peanut M&Ms?

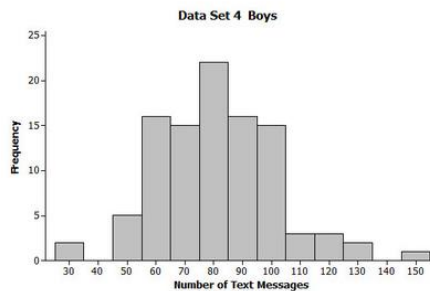
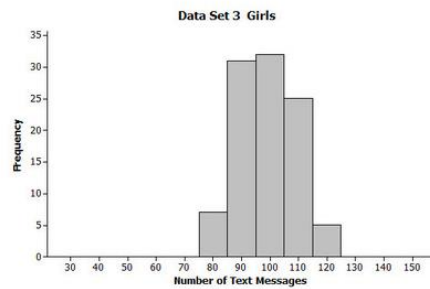
e. Example: [Solution](#) (DOK 2)

Data Set 1 consists of data on the time to complete an assignment (in minutes) for 25 sixth graders. Data Set 2 consists of data on the time to complete an assignment for 25 seventh graders. Dot plots of the two data sets are shown below.



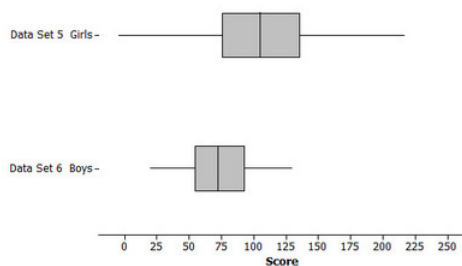
1. Describe the data distribution of times for seventh graders (Data Set 2). Be sure to comment on center, spread and overall shape.
2. Are Data Set 1 and Data Set 2 centered in about the same place? If not, which one has the greater center?
3. Which of Data Set 1 and Data Set 2 has greater spread?
4. Were sixth graders (Data Set 1) or seventh graders (Data Set 2) more consistent in their times to complete the task?

Data Set 3 consists of data on the number of text messages sent in one month for 100 teenage girls who have a cell phone. Data Set 4 consists of data on the number of text messages sent in one month for 100 teenage boys who have a cell phone. Histograms of the two data sets are shown below.



5. Describe the data distribution of number of text messages for the girls (Data Set 3). Be sure to comment on center, spread and overall shape.
6. Are Data Set 3 and Data Set 4 centered in about the same place? If not, which one has the greater center?
7. Which of Data Set 3 and Data Set 4 has greater spread?
8. On average, did the girls (Data Set 3) or the boys (Data Set 4) send more text messages?

Data Set 5 consists of data on the scores on a video game for 100 teenage girls. Data Set 6 consists of the scores on a video game for 100 teenage boys. Box plots of the two data sets are shown below.



9. Describe the data distribution of Data Set 5. Be sure to comment on center, spread and overall shape.
  10. Are Data Set 5 and Data Set 6 centered in about the same place? If not, which one has the greater center?
  11. Which of Data Set 5 and Data Set 6 has greater spread?
  12. On average, did the girls (Data Set 5) or the boys (Data Set 6) tend to have higher scores?
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **(6.SP.A.3) (DOK 1)**
    - a. Example: [Solution](#) (DOK 2)

Statistical questions are questions that can be answered by collecting data and where we anticipate that there will be variability in that data. The data collected can be summarized in a distribution that can then be described in terms of center and in terms of spread. For some statistical questions, to answer the question you need to consider center. For other questions you might need to consider spread.

For each of the five statistical questions below, decide if you would answer the question by considering center or considering variability in the data distribution.

**Example 1:** The records office at an elementary school keeps daily attendance records.

**Question 1:** For students at this school, what is a typical number of school days missed in the month of April?

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#### Summarize and describe distributions. (6.SP.B)

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (6.SP.B.4)

**(DOK 1,2)**

- a. Example: [Solution](#) (DOK 3)

Each of the 20 students in Mr. Anderson's class timed how long it took them to solve a puzzle. Their times (in minutes) are listed below:

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Time (minutes)	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

- a. Display the data using a dot plot.
- b. Find the mean and median of the data. Does it surprise you that the values of the mean and median are not equal? Explain why or why not.
- b. Example: [Solution](#) (DOK 3)

Below are the 25 birth weights, in ounces, of all the Labrador Retriever puppies born at Kingston Kennels in the last six months.

13, 14, 15, 15, 16, 16, 16, 16, 17, 17, 17, 17, 17, 17, 17, 17, 18, 18, 18, 18, 18, 18, 18, 18, 19, 20

- a. Use an appropriate graph to summarize these birth weights.
- b. Describe the distribution of birth weights for puppies born at Kingston Kennels in the last six months. Be sure to describe shape, center and variability.
- c. What is a typical birth weight for puppies born at Kingston Kennels in the last six months? Explain why you chose this value.
- c. Example: [Solution](#) (DOK 3)

The number of siblings for a group of sixth grade students is shown below:

1, 0, 2, 1, 6, 0, 2, 0, 1, 10.

- a. Make a dot plot of the data.
- b. Find the mean and median of the data.
- c. What does the mean tell you about the data? What about the median?
- d. Which measure of average (mean or median) do you think best describes the data? Why?
- d. Example: [Solution](#) (DOK 3)

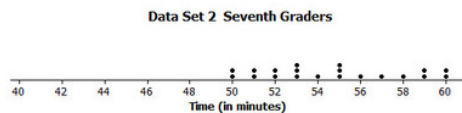
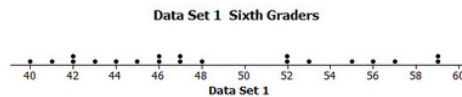
In Mrs. Sanchez' math classroom, more people sit on the right-hand side of the room than the left. The students on the right-hand side of the classroom received the following scores on an exam worth 100 points:

85, 90, 100, 95, 0, 0, 90, 70, 100, 95, 80, 95|

The students on the left received these test scores:

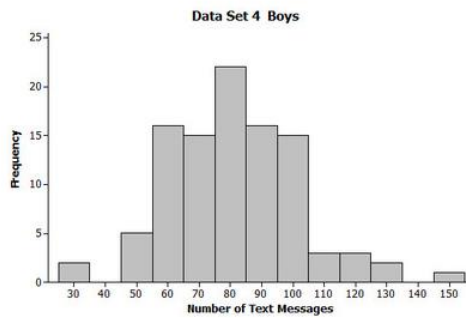
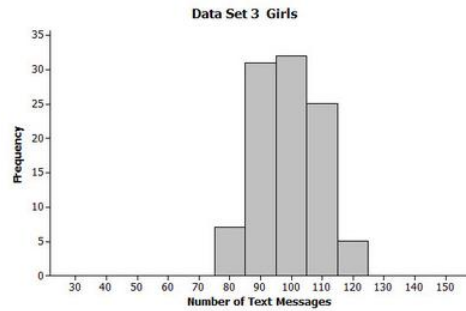
65, 80, 90, 65, 80, 60, 95, 85|

- Make two box plots of the students' scores, one for each side of the room.
  - Make a statistical argument that the students on the right-hand side were more successful.
  - Make a statistical argument that the students on the left-hand side were more successful.
- e. Example: [Solution](#) (DOK 2)
- Data Set 1 consists of data on the time to complete an assignment (in minutes) for 25 sixth graders. Data Set 2 consists of data on the time to complete an assignment for 25 seventh graders. Dot plots of the two data sets are shown below.



- Describe the data distribution of times for seventh graders (Data Set 2). Be sure to comment on center, spread and overall shape.
- Are Data Set 1 and Data Set 2 centered in about the same place? If not, which one has the greater center?
- Which of Data Set 1 and Data Set 2 has greater spread?
- Were sixth graders (Data Set 1) or seventh graders (Data Set 2) more consistent in their times to complete the task?

Data Set 3 consists of data on the number of text messages sent in one month for 100 teenage girls who have a cell phone. Data Set 4 consists of data on the number of text messages sent in one month for 100 teenage boys who have a cell phone. Histograms of the two data sets are shown below.

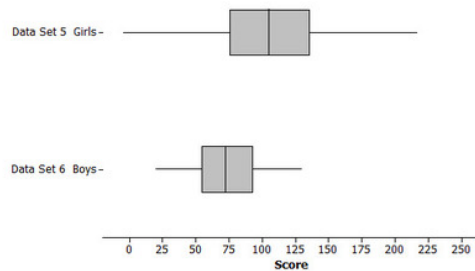


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9. Describe the data distribution of Data Set 5. Be sure to comment on center, spread and overall shape.
  10. Are Data Set 5 and Data Set 6 centered in about the same place? If not, which one has the greater center?
  11. Which of Data Set 5 and Data Set 6 has greater spread?
  12. On average, did the girls (Data Set 5) or the boys (Data Set 6) tend to have higher scores?
5. Summarize numerical data sets in relation to their context, such as by:
    - a. Reporting the number of observations.
    - b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    - c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    - d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. **(6.SP.B.5) (DOK 1,2,3)**
      1. Example: [Solution](#) (DOK 3)

Each of the 20 students in Mr. Anderson's class timed how long it took them to solve a puzzle. Their times (in minutes) are listed below:

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Time (minutes)	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

- Display the data using a dot plot.
- Find the mean and median of the data. Does it surprise you that the values of the mean and median are not equal? Explain why or why not.

2. Example: [Solution](#) (DOK 3)

Unlike many elections for public office where a person is elected strictly based on the results of a popular vote (i.e., the candidate who earns the most votes in the election wins), in the United States, the election for President of the United States is determined by a process called the Electoral College. According to the National Archives, the process was established in the United States Constitution "as a compromise between election of the President by a vote in Congress and election of the President by a popular vote of qualified citizens."

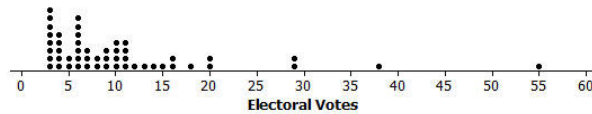
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Each state receives an allocation of electoral votes in the process, and this allocation is determined by the number of members in the state's delegation to the US Congress. This number is the sum of the number of US Senators that represent the state (always 2, per the Constitution) and the number of Representatives that represent the state in the US House of Representatives (a number that is directly related to the state's population of qualified citizens as determined by the US Census). Therefore the larger a state's population of qualified citizens, the more electoral votes it has. Note: the District of Columbia (which is not a state) is granted 3 electoral votes in the process through the 23rd Amendment to the Constitution.

The following table shows the allocation of electoral votes for each state and the District of Columbia for the 2012, 2016, and 2020 presidential elections. (<http://www.archives.gov/federal-register/electoral-college/allocation.html> accessed September 4, 2012).

State	Electoral Votes	State	Electoral Votes	State	Electoral Votes
Alabama	9	Kentucky	8	North Dakota	3
Alaska	3	Louisiana	8	Ohio	18
Arizona	11	Maine	4	Oklahoma	7
Arkansas	6	Maryland	10	Oregon	7
California	55	Massachusetts	11	Pennsylvania	20
Colorado	9	Michigan	16	Rhode Island	4
Connecticut	7	Minnesota	10	South Carolina	9
Delaware	3	Mississippi	6	South Dakota	3
District of Columbia	3	Missouri	10	Tennessee	11
Florida	29	Montana	3	Texas	38
Georgia	16	Nebraska	5	Utah	6
Hawaii	4	Nevada	6	Vermont	3
Idaho	4	New Hampshire	4	Virginia	13
Illinois	20	New Jersey	14	Washington	12
Indiana	11	New Mexico	5	West Virginia	5
Iowa	6	New York	29	Wisconsin	10
Kansas	6	North Carolina	15	Wyoming	3

- Which state has the most electoral votes? How many votes does it have?
- Based on the given information, which state has the second highest population of qualified citizens?
- Here is a dotplot of the distribution.



- What is the shape of this distribution: skewed left, symmetric, or skewed right?
  - Imagine that someone you are speaking with is unfamiliar with these shape terms. Describe clearly and in the context of this data set what the shape description you have chosen means in terms of the distribution.
- Does the dotplot lead you to think that any states are outliers in terms of their number of electoral votes? Explain your reasoning, and if you do believe that there are outlier values, identify the corresponding states.
  - What measure of center (mean or median) would you recommend for describing this data set? Why did you choose this measure?
  - Determine the value of the median for this data set (electoral votes).

3. Example: [Solution](#) (DOK 3)

The number of siblings for a group of sixth grade students is shown below:

1, 0, 2, 1, 6, 0, 2, 0, 1, 10.

- Make a dot plot of the data.
- Find the mean and median of the data.
- What does the mean tell you about the data? What about the median?
- Which measure of average (mean or median) do you think best describes the data? Why?

4. Example: [Solution](#) (DOK 3)

In Mrs. Sanchez' math classroom, more people sit on the right-hand side of the room than the left. The students on the right-hand side of the classroom received the following scores on an exam worth 100 points:

85, 90, 100, 95, 0, 0, 90, 70, 100, 95, 80, 95

The students on the left received these test scores:

65, 80, 90, 65, 80, 60, 95, 85

- Make two box plots of the students' scores, one for each side of the room.
- Make a statistical argument that the students on the right-hand side were more successful.
- Make a statistical argument that the students on the left-hand side were more successful.

5. Example: [Solution](#) (DOK 2)

Bobbie is a sixth grader who competes in the 100 meter hurdles. In eight track meets during the season, she recorded the following times (to the nearest one hundredth of a second).

18.11, 31.23, 17.99, 18.25, 17.50, 35.55, 17.44, 17.85

- What is the mean of Bobbie's times for these track meets? What does this mean tell you in terms of the context?
- What is the median of Bobbie's times? What does this median tell you in terms of the context?
- What information can you gather by comparison of the mean and median?

6. Example: [Solution](#) (DOK 2)

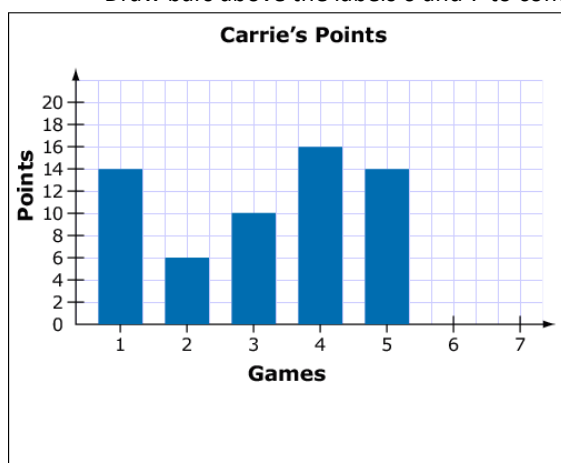
Over a two week period, Jenna had the following number of math homework problems given each day:

20, 0, 7, 10, 1, 11, 0, 25, 15, 1.

- What is the mean number of homework problems Jenna had?
- What is the Mean Absolute Deviation for the number of homework problems?
- What do the mean and Mean Absolute Deviation tell you about the number of homework problems Jenna had over these two weeks?

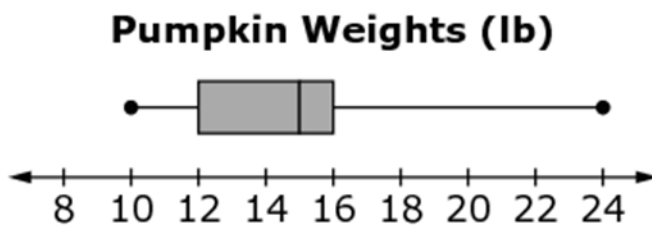
7. Example: Carrie's basketball team has played 5 games. The number of points Carrie scored in each game is shown in the bar graph.

- Determine possible point totals for games 6 and 7 so that the range of data set increases, but the mean and median stay the same.
- Draw bars above the labels 6 and 7 to complete the bar graph.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#29	4	SP	E, C	3	7.SP.B.4, 6.SP.B.5	1, 4	

8. Example: Look at the box-and-whisker plot of pumpkin weights.



What is the **median** pumpkin weight?

- a. 12 lb
- b. 14 lb
- c. 15 lb
- d. 16 lb

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#28	1	SP	J	2	6.SP.B.5c	N/A	C

9. Example: ([Former NAEP question](#)) (DOK 1)

For a school report, Luke contacted a car dealership to collect data on recent sales. He asked, "What color do buyers choose most often for their car?" White was the response. What statistical measure does the response "white" represent?

- A. Mean
- B. Median
- C. Mode
- D. Range
- E. Interquartile range

Answer: Mode

10. Example: ([Former NAEP question](#)) (DOK 1)

TEMPERATURES ON OCTOBER 1ST  
FOR FIVE CITIES (in °F)

	High	Low
City A	72	50
City B	90	75
City C	83	72
City D	50	37
City E	92	72

The table above shows the high and low temperatures on October 1st for five cities. Which city had the greatest temperature range?

- A. City A
- B. City B
- C. City C
- D. City D
- E. City E

Answer: A

***Performance Task Example:***

### CEREAL BOXES

A cereal company uses cereal boxes that are rectangular prisms. The boxes have the dimensions shown.

- 12 inches high
- 8 inches wide
- 2 inches deep

The managers of the company want a new size for their cereal boxes. The new boxes have to be rectangular prisms. You will evaluate one box design the company proposed. Then you will create and propose your own design for the company.

Requirements for the new boxes:

- The new boxes have to use less cardboard than the original boxes.
- The new boxes have to hold the same or a greater volume of cereal as the original boxes.

1.

Determine the volume of the current cereal box with the dimensions 12 inches high, 8 inches wide, and 2 inches deep.

Find the volume,  $V$ , in cubic inches, of each box.

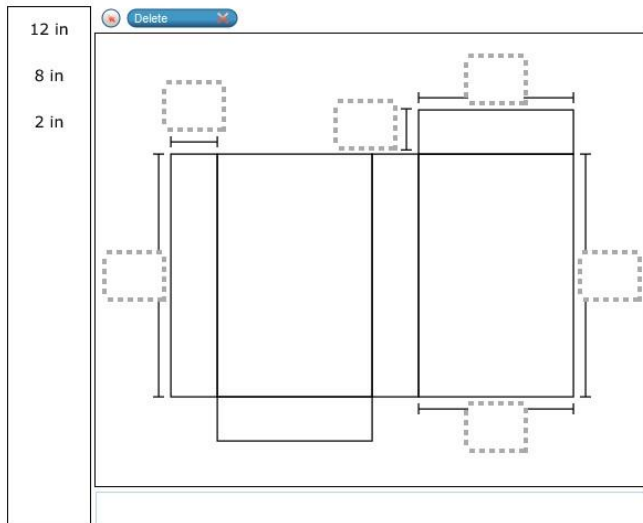
Volume of Original Box:  $V = \underline{\hspace{1cm}} \text{ in}^3$

For this item, a full-credit response (1 point) includes 192.

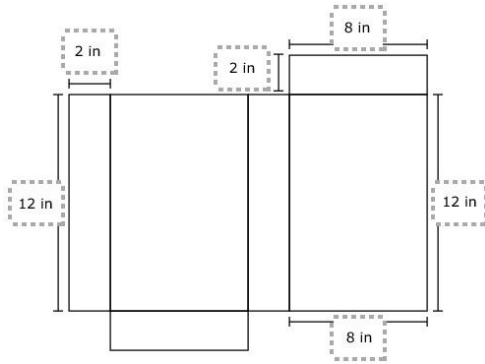
For this item, a no-credit response (0 points) includes none of the features of a full-credit response.

2.

Label the dimensions of the net for the current cereal box with dimensions 12 inches high, 8 inches wide, and 2 inches deep.



For this item, a full-credit response (1 point) includes:



For this item, a no-credit response (0 points) includes none of the features of a full-credit response.

3.

Determine the surface area,  $S$ , in square inches, of the current cereal box with dimensions 12 inches high, 8 inches wide, and 2 inches deep.

$S = \underline{\hspace{1cm}} \text{ in}^2$

For this item, a full-credit response (2 points) includes 272.

For this item, a partial-credit response (1 point) includes all of the following common mistakes:

- 136
- 176
- 248
- 256
- $80 \times 224$
- 240 or square inches consistent with an error in Item 3

For this item, a no-credit response (0 points) includes none of the features of a full- or partial-credit response.

The company proposes a new cereal box with dimensions 10.5 inches high, 7.5 inches wide, and 4 inches deep. The new cereal box is a rectangular prism. Determine if this new box meets each of the requirements. Explain why or why not.

4.

For this item, a full-credit response (2 points) includes:

- comparing the proposed volume to the requirements  
AND
- comparing the proposed surface area to the requirements AND
- judging the proposed dimensions to be inappropriate.
- For example,
- $V = 315$  cubic inches and  $315 > 192$ .  $S = 301.5$  square inches and  $272 < 301.5$ . The box should not be used because the surface area is too large.
- For this item, a partial-credit response (1 point) includes either:
- comparing the proposed volume to the requirements  
AND
- comparing the proposed surface area to the requirements  
(1 point for making the valid comparisons but no judgment call) OR
- judging the proposed dimensions to be inappropriate.  
(1 point for making the correct judgment but not showing evidence to support it)
- For example,
- $V = 315$  cubic inches and  $315 > 192$ .  $S = 301.5$  square inches and  $272 < 301.5$ . OR



- The box should not be used because the surface area is too large
- For this item, a no-credit response (0 points) includes none of the features of a full- or partial-credit response.
- For example, the box can be used because it meets the requirements.
- *This item is not graded on spelling or grammar.*

Design a new cereal box for this company. All cereal boxes are rectangular prisms. Then explain why your design is better for the company, based on the requirements.

In your response,

- give the dimensions of your box;
- explain how your box meets each of the requirements for new boxes.

5.

- For this item, a full-credit response (3 points) includes:
  - giving the dimensions for the cereal box design
  - AND
  - explaining how the design meets the volume requirement AND
  - explaining how the design meets the surface area requirement.
  - For example,
  - “The new box should have a length of 10 inches, a width of 8 inches, and a depth of 3 inches. These dimensions will give a greater volume of 240 cubic inches, and a lesser surface area of 268 inches.”
- For this item, an incorrect response (2 points) includes two of the following:
  - Student gives the dimensions for the cereal box design.
  - Student explains how the design meets the volume requirement.
  - Student explains how the design meets the surface area requirement.
  - For example,
  - “The new box should have a length of 10 inches, a width of 8 inches, and a depth of 3 inches. These dimensions will give a greater volume of 240 cubic inches.”
- For this item, a partial response (1 point) includes one of the following:
  - Student gives the dimensions for the cereal box design.
  - Student explains how the design meets the volume requirement.
  - Student explains how the design meets the surface area requirement.
  - For example,
  - “The new box should have a volume of 240 cubic inches.”
- For this item, a no-credit response (0 points) includes none of the features of a full- or partial-credit response.
  - For example,
  - “The new box should have a length of 10 inches. This will result in a greater volume.”
  - *This item is not graded on spelling or grammar.*