Mathematics | Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Contents

Grade 7 Overview

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Analyze proportional relationships and use them to solve real-world and mathematical problems. (7.RP.A)

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.* **(7.RP.A.1) (DOK 1,2)**

    a. Example: Solution (DOK 2)

    > Angel and Jayden were at track practice. The track is $\frac{2}{5}$ kilometers around.
    >
    > • Angel ran 1 lap in 2 minutes.
    >
    > • Jayden ran 3 laps in 5 minutes.
    >
    > a. How many minutes does it take Angel to run one kilometer? What about Jayden?
    >
    > b. How far does Angel run in one minute? What about Jayden?
    >
    > c. Who is running faster? Explain your reasoning.

    b. Example: Solution (DOK 2)

    > Travis was attempting to make muffins to take to a neighbor that had just moved in down the street. The recipe that he was working with required $\frac{3}{4}$ cup of sugar and $\frac{1}{8}$ cup of butter.
    >
    > a. Travis accidentally put a whole cup of butter in the mix.
    >
    > > i. What is the ratio of sugar to butter in the original recipe? What amount of sugar does Travis need to put into the mix to have the same ratio of sugar to butter that the original recipe calls for?
    > >
    > > ii. If Travis wants to keep the ratios the same as they are in the original recipe, how will the amounts of all the other ingredients for this new mixture compare to the amounts for a single batch of muffins?
    > >
    > > iii. The original recipe called for $\frac{3}{8}$ cup of blueberries. What is the ratio of blueberries to butter in the recipe? How many cups of blueberries are needed in the new enlarged mixture?
    >
    > b. This got Travis wondering how he could remedy similar mistakes if he were to dump in a single cup of some of the other ingredients. Assume he wants to keep the ratios the same.
    >
    > > i. How many cups of sugar are needed if a single cup of blueberries is used in the mix?
    > >
    > > ii. How many cups of butter are needed if a single cup of sugar is used in the mix?
    > >
    > > iii. How many cups of blueberries are needed for each cup of sugar?

c. Example: Solution (DOK 2)

Molly runs $\frac{1}{3}$ of a mile in 4 minutes.

a. If Molly continues at the same speed, how long will it take her to run one mile?

b. Draw and label a picture showing why your answer to part (a) makes sense.

d. Example: Solution (DOK 2)

Molly ran $\frac{2}{3}$ of a mile in 8 minutes. If Molly runs at that speed, how long will it take her to run one mile? [_____]

e. Example: Solution (DOK 2)

The price of a gallon of apple cider is $7.00. The price of eight 4.23-ounce juice boxes is $2.39.

$7.00/gallon $2.39 for 8
                4.23 oz boxes

a. Suppose the juice was instead packaged like the cider. Approximately what is the cost per gallon of the juice?

b. Suppose the cider was instead packaged like the juice. Approximately what is the cost per eight 4.23-ounce boxes of cider?

c. Peter wants to have at least a gallon of either only cider or only juice. Which product is the better deal?

d. State the unit rate(s) you used to compare the cost of cider versus juice in your answer to Question c.

e. List two or more additional unit rates that could be used to make this comparison.

f. Example: Solution (DOK 2)

$7.00/gallon $2.39 for 8
4.23 oz boxes

a. Assuming you like juice and cider equally, which product is the better deal?

b. Suppose the juice boxes go on sale for $1.79 for the eight 4.23-ounce juice boxes, and the cider goes on sale for $6.50 per gallon. Does this change your decision?

g.   Example: David uses $\frac{1}{2}$ cup of apple juice for every $\frac{1}{4}$ cup of cranberry juice to make a fruit drink. Write the number of cups of apple juice David uses for 1 cup of cranberry juice.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#22	1	RP	A	2	7.RP.A.1	N/A	2

2. Recognize and represent proportional relationships between quantities.
    a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
    c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
    d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate. **(7.RP.A.2) (DOK 1,2)**
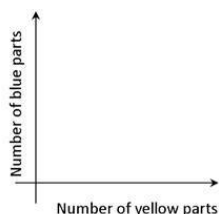   1. Example: Solution (DOK 2)

            BeatStreet, TunesTown, and MusicMind are music companies. BeatStreet offers to buy 1.5 million shares of TunesTown for $561 million. At the same time, MusicMind offers to buy 1.5 million shares of TunesTown at $373 per share. Who would get the better deal, BeatStreet or MusicMind? What is the total price difference?

        2. Example: Solution (DOK 3)

The students in Ms. Baca's art class were mixing yellow and blue paint. She told them that two mixtures will be the same shade of green if the blue and yellow paint are in the same ratio.

The table below shows the different mixtures of paint that the students made.

Yellow	1 part	2 parts	3 parts	4 parts	6 parts
Blue	2 part	3 parts	6 parts	6 parts	9 parts

a. How many different shades of paint did the students make?

b. Some of the shades of paint were bluer than others. Which mixture(s) were the bluest? Show work or explain how you know.

c. Carefully plot a point for each mixture on a coordinate plane like the one that is shown in the figure. (Graph paper might help.)

d. Draw a line connecting each point to (0,0). What do the mixtures that are the same shade of green have in common?

3. Example: [Solution](DOK 2)

The students in Ms. Baca's art class were mixing yellow and blue paint. She told them that two mixtures will be the same shade of green if the blue and yellow paint are in the same ratio.

The table below shows the different mixtures of paint that the students made.

	A	B	C	D	E	F
Yellow	1 part	2 parts	3 parts	4 parts	5 parts	6 parts
Blue	2 part	3 parts	6 parts	6 parts	8 parts	9 parts

a. How many different shades of paint did the students make?

b. Write an equation that relates y, the number of parts of yellow paint, and b, the number of parts of blue paint for each of the different shades of paint the students made.

4. Example: [Solution](DOK 2)

Coffee costs $18.96 for 3 pounds.

a. What is the cost for one pound of coffee?

b. At this store, the price for a pound of coffee is the same no matter how many pounds you buy. Let x be the number of pounds of coffee and y be the total cost of x pounds. Draw a graph of the relationship between the number of pounds of coffee and the total cost.

c. Where can you see the cost per pound of coffee in the graph? What is it?

5. Example: Solution (DOK 3)

Nia and Trey both had a sore throat so their mom told them to gargle with warm salt water.

. Nia mixed 1 teaspoon salt with 3 cups water.

. Trey mixed $\frac{1}{2}$ teaspoon salt with $1\frac{1}{2}$ cups of water.

Nia tasted Trey's salt water. She said,

"I added more salt so I expected that mine would be more salty, but they taste the same."

a. Explain why the salt water mixtures taste the same.

b. Which of the following equations relates s, the number of teaspoons of salt, with w, the number of cups of water, for both of these mixtures? Choose all that apply.

i. $s = \frac{1}{3}w$

ii. $s = 3w$

iii. $s = 1\frac{1}{2}w$

iv. $w = 3s$

v. $w = \frac{1}{3}s$

vi. $w = \frac{1}{2}s$

6. Example: Solution (DOK 3)

Carli's class built some solar-powered robots. They raced the robots in the parking lot of the school. The graphs below are all line segments that show the distance d, in meters, that each of three robots traveled after t seconds.

a. Each graph has a point labeled. What does the point tell you about how far that robot has traveled?

b. Carli said that the ratio between the number of seconds each robot travels and the number of meters it has traveled is constant. Is she correct? Explain.

c. How fast is each robot traveling? How did you compute this from the graph?
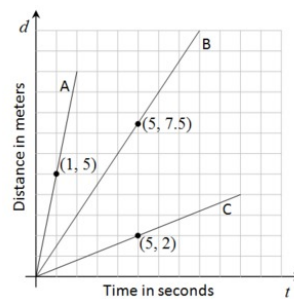

8
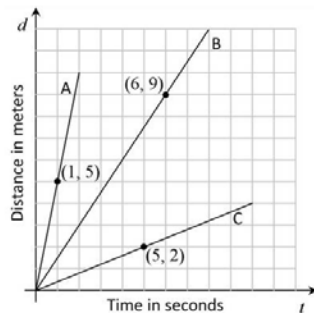
7. Example: Solution (DOK 2)

The students in Carli's class built some solar-powered robots which they raced in the cafeteria of the school.
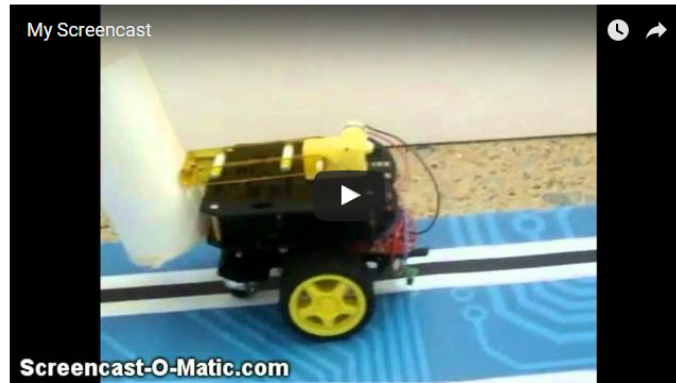
After the race, Carli drew the graphs shown below to represent the distance d, in meters, that each of three robots A, B, and C traveled after t seconds.

a. Which of the following statements about Robot B are true? (Select all that apply.)

 i. Robot B traveled in a different direction than the other two robots.

 ii. Robot B traveled 5 meters in 7.5 seconds.

 iii. Robot B traveled 7.5 meters in 5 seconds.

 iv. Robot B traveled $\frac{2}{3}$ meters per second.

 v. Robot B traveled $\frac{3}{2}$ meters per second.

 vi. None of these are true.

b. How do the speeds of the robots compare? (Choose one.)

 i. The Robots all traveled at the same speed, they just left at different times.

 ii. Robot [▾] is the fastest and Robot [▾] is the slowest.

 iii. There is not enough information given to compare how fast the robots traveled.

8. Example: <u>Solution</u> (DOK 2)

The students in Ms. Baca's art class were mixing yellow and blue paint. She told them that two mixtures will be the same shade of green if the blue and yellow paint are in the same ratio.

The table below shows the different mixtures of paint that the students made.

Amount of Yellow Paint (cups)	0.5	1	1.5	2	3
Amount of Blue Paint (cups)	0.75	2	3	3	4.5

a. How many different shades of paint did the students make? [▾]

b. Which mixture(s) make the same shade as mixture A? [▾]

c. How many cups of yellow paint would a student add to one cup of blue paint to make a mixture that is the same shade as mixture A?
[▾]

d. Let b represent the number of cups of blue paint and y represent the number of cups of yellow paint in a paint mixture. Write an equation that shows the relationship between the number of cups of yellow paint, y, and the number of cups of blue paint, b, in mixture E.
[▾]

9. Example: <u>Solution</u> (DOK 2)

Carlos bought $6\frac{1}{2}$ pounds of bananas for $5.20.

a. What is the price per pound of the bananas that Carlos bought?
[_____]

b. What quantity of bananas would one dollar buy? [_____] pounds

c. Which of the points in the coordinate plane shown below correspond to a quantity of bananas that cost the same price per pound as the bananas Carlos bought? (Select all that apply.)
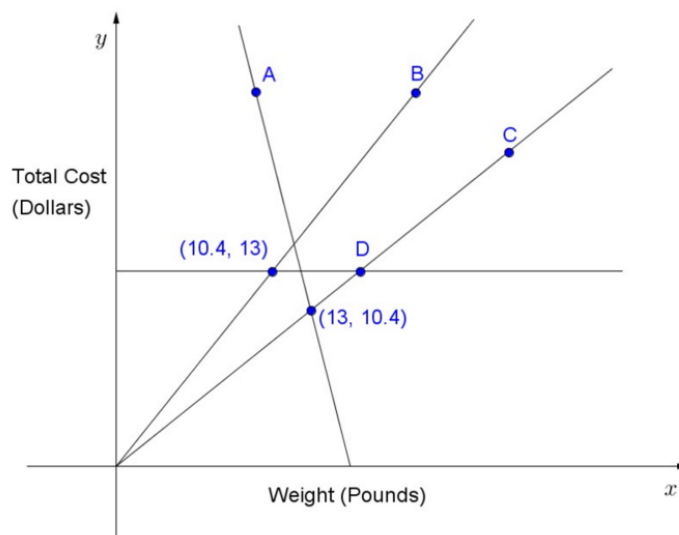
i. A

ii. B
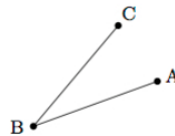
iii. C

iv. D

v. (10.4, 13)

vi. (13, 10.4)

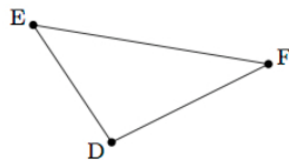vii. There is not enough information to determine this.
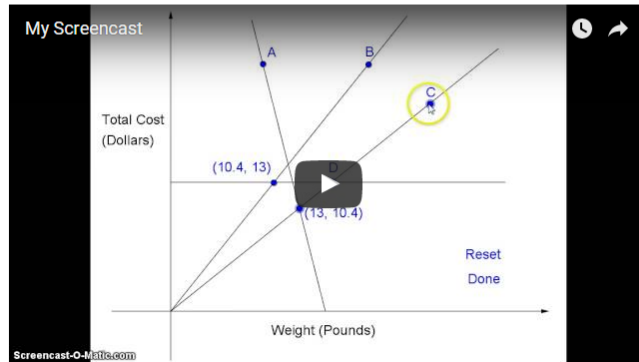
10. Example: <u>Solution </u>(DOK 3)

a. For each shape below, draw a scaled copy with a scale factor of 2. Explain why your drawing is accurate.
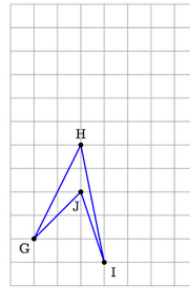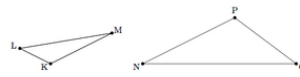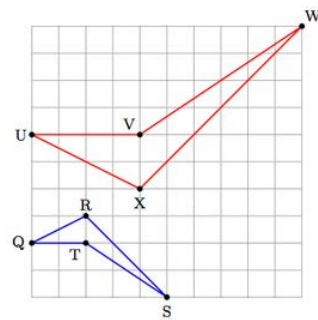
i.

ii.

iii.

b. Below are two pairs of polygons. For each pair, explain whether or not one is a scaled version of the other.

i.

ii.

11. Example: [Solution](link) (DOK 2)

Julianna participated in a walk-a-thon to raise money for cancer research. She recorded the total distance she walked at several different points in time, but a few of the entries got smudged and can no longer be read. The times and distances that can still be read are listed in the table below.

a. Assume Julianna walked at a constant speed. Complete the table and plot Julianna's progress in the coordinate plane.

b. What was Julianna's walking rate in miles per hour? How long did it take Julianna to walk one mile? Where do you see this information on the graph?

c. Write an equation for the distance d, in miles, that Julianna walked in n hours.

d. Next year Julianna is planning to walk for seven hours. If she walks at the same speed next year, how many miles will she walk?

Time in hrs	Miles walked
1	
2	6.4
	8
5	

12. Example: [Solution](link) (DOK 2)

A text book has the following definition for two quantities to be directly proportional:

We say that y is directly proportional to x if $y = kx$ for some constant k.

For homework, students were asked to restate the definition in their own words and to give an example for the concept. Below are some of their answers. Discuss each statement and example. Translate the statements and examples into equations to help you decide if they are correct.

• Marcus:

This means that both quantities are the same. When one increases the other increases by the same amount. An example of this would be the amount of air in a balloon and the volume of a balloon.

• Sadie:

Two quantities are proportional if one change is accompanied by a change in the other. For example the radius of a circle is proportional to the area.

• Ben:

When two quantities are directly proportional it means that if one quantity goes up by a certain percentage, the other quantity goes up by the same percentage as well. An example could be as gas prices go up in cost, food prices go up in cost.

- Jessica:

When two quantities are proportional, it means that as one quantity increases the other will also increase and the ratio of the quantities is the same for all values. An example could be the circumference of a circle and its diameter, the ratio of the values would equal π.

13. Example: [Solution](DOK 2)

The price of a gallon of apple cider is $7.00. The price of eight 4.23-ounce juice boxes is $2.39.

$7.00/gallon $2.39 for 8 4.23 oz boxes

a. Suppose the juice was instead packaged like the cider. Approximately what is the cost per gallon of the juice?

b. Suppose the cider was instead packaged like the juice. Approximately what is the cost per eight 4.23-ounce boxes of cider?

c. Peter wants to have at least a gallon of either only cider or only juice. Which product is the better deal?

d. State the unit rate(s) you used to compare the cost of cider versus juice in your answer to Question c.

e. List two or more additional unit rates that could be used to make this comparison.

14. Example: [Solution](#) (DOK 2)

a. Assuming you like juice and cider equally, which product is the better deal?

b. Suppose the juice boxes go on sale for $1.79 for the eight 4.23-ounce juice boxes, and the cider goes on sale for $6.50 per gallon. Does this change your decision?

15. Example: [Solution](#) (DOK 3)

In January, Georgia signed up for a membership at Anytime Fitness. The plan she chose cost $95 in start-up fees and then $20 per month starting in February. Edwin also signed up at Anytime Fitness in January. His plan cost $35 per month starting in February, and his start-up fees were waived.

a. Create tables for both Georgia and Edwin that compare the number of months since January to the total cost of their gym memberships. Continue this table for one year.

b. Plot the points from the two tables in part (a) on a coordinate plane.

c. Decide if either or both gym memberships are described by a proportional relationship, and write an equation representing any such relationship. Explain how parts (a) and (b) could be used to support your answer.

16. Example: Select **all** the graphs that show a proportional relationship between x and y.

☐   ☐

☐   ☐
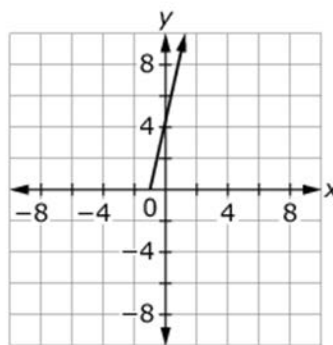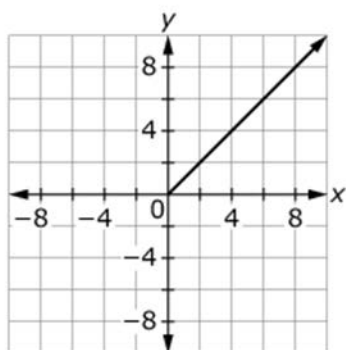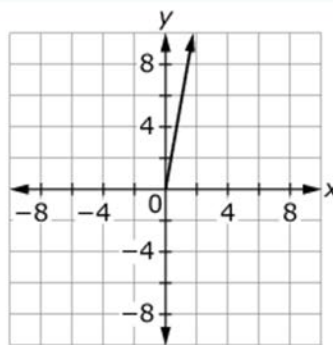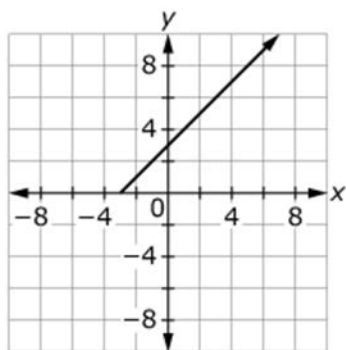
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#25	1	RP	A	1	7.RP.A.2a	N/A	

17. Example: The graph shows a proportional relationship between the number of hours (*h*) a business operates and the total cost (*c*) of electricity.

Cost of Electricity
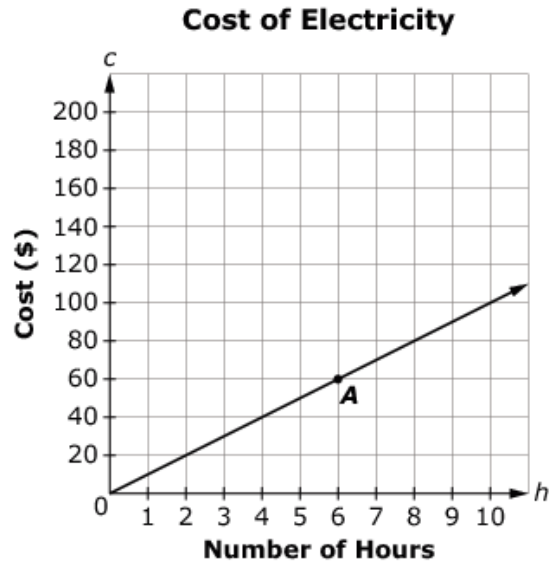
Select **True** or **False** for each statement about the graph.

	True	False
Point *A* represents the total cost of electricity when operating the business for 6 hours.	☐	☐
The total cost of electricity is $8 when operating the business for 80 hours.	☐	☐
The total cost of electricity is $10 when operating the business for 1 hour.	☐	☐

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#18	1	RP	A	2	7.RP.A.2	N/A	True, False, True

18. Example: This table shows a proportional relationship between *x* and *y*.

x	*y*
4	48
5	60
8	96

Find the constant of proportionality (*r*).

Using the value for *r*, enter an equation in the form of y = *rx*.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#16	1	RP	A	2	7.RP.A.2b, 7.RP.A.2c	N/A	12

19. Example: George earns $455 per week. George receives a 20% raise.
How can George calculate his new weekly pay rate?

Select **all** calculations that will result in George's new weekly pay rate.
   a. Divide $455 by 0.20
   b. Divide $455 by 1.20
   c. Multiply $455 by 0.20
   d. Multiply $455 by 1.20

e.   Solve for x: $\dfrac{x}{455} = \dfrac{120}{100}$

f.   Solve for x: $\dfrac{455}{x} = \dfrac{20}{100}$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#9	3	RP	D	2	7.RP.A.2, 6.RP.A.3c	1, 2	☐ divide $455 by 0.20 ☐ divide $455 by 1.20 ☐ multiply $455 by 0.2 ☑ multiply $455 by 1.2 ☑ solve for x: $\frac{x}{455} = \frac{12}{10}$ ☐ solve for x: $\frac{455}{x} = \frac{2}{1}$

20. Example: ([Former NAEP question](#)) (DOK 2)

If $\dfrac{P}{41} = 64$, what does $\dfrac{P}{82}$ equal?

   A.   32
   B.   64
   C.   128
   D. 5248

     Answer: A

3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **(7.RP.A.3)** **(DOK 1,2)**

    a.   Example: [Solution](#)(DOK 2)

        BeatStreet, TunesTown, and MusicMind are music companies. BeatStreet and MusicMind are teaming up together to make an offer to acquire 1.5 million shares of TunesTown worth $374 per share. They will offer TunesTown 20 million shares of BeatStreet worth $25 per share. To make the swap even, they will offer another 2 million shares of MusicMind.

        What price per share (in dollars) must each of these additional shares be worth?

    b.   Example: [Solution](#)(DOK 3)

        The sales team at an electronics store sold 48 computers last month. The manager at the store wants to encourage the sales team to sell more computers and is going to give all the sales team members a bonus if the number of computers sold increases by 30% in the next month. How many computers must the sales team sell to receive the bonus? Explain your reasoning.

    c.   Example: [Solution](#)(DOK 2)

18

After eating at your favorite restaurant, you know that the bill before tax is $52.60 and that the sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much should you leave for the waiter? How much will the total bill be, including tax and tip? Show work to support your answers.

d. Example: Solution (DOK 2)

Taylor and Anya are friends who live 63 miles apart. Sometimes on a Saturday, they ride toward each other's houses on their bikes and meet in between. One day they left their houses at 8 am and met at 11 am. Taylor rode at 12.5 miles per hour. How fast did Anya ride?

e. Example: Solution (DOK 2)

Historically, different people have defined a year in different ways. For example, an Egyptian year is 365 days long, a Julian year is $365\frac{1}{4}$ days long, and a Gregorian year is 365.2425 days long.

a. What is the difference, in seconds, between a Gregorian year and a Julian year?

b. What is the percent decrease, to the nearest thousandth of a percent, from a Julian year to a Gregorian year?

c. How many fewer days are there in 400 years of the Gregorian calendar than there are in 400 years of the Julian calendar?

f. Example: Solution (DOK 2)

There were 24 boys and 20 girls in a chess club last year. This year the number of boys increased by 25% but the number of girls decreased by 10%. Was there an increase or decrease in overall membership? Find the overall percent change in membership of the club.

g. Example: Solution (DOK 1)

5,000 people visited a book fair in the first week. The number of visitors increased by 10% in the second week. How many people visited the book fair in the second week?

h. Example: Solution (DOK 2)

Tom wants to buy some protein bars and magazines for a trip. He has decided to buy three times as many protein bars as magazines. Each protein bar costs $0.70 and each magazine costs $2.50. The sales tax rate on both types of items is 6½%. How many of each item can he buy if he has $20.00 to spend?

i. Example: Solution (DOK 2)

The 7th graders at Sunview Middle School were helping to renovate a playground for the kindergartners at a nearby elementary school. City regulations require that the sand underneath the swings be at least 15 inches deep. The sand under both swing sets was only 12 inches deep when they started.

The rectangular area under the small swing set measures 9 feet by 12 feet and required 40 bags of sand to increase the depth by 3 inches. How many bags of sand will the students need to cover the rectangular area under the large swing set if it is 1.5 times as long and 1.5 times as wide as the area under the small swing set?

j.   Example: Solution (DOK 3)

Juan wants to know the cross-sectional area of a circular pipe. He measures the diameter which he finds, to the nearest millimeter, to be 5 centimeters.

a. How large is the possible error in Juan's measurement of the diameter of the circle? Explain.

b. As a percentage of the diameter, how large is the possible error in Juan's measurement?

c. To find the area of the circle, Juan uses the formula $A = \pi r^2$ where A is the area of the circle and r is its radius. He uses 3.14 for π. What value does Juan get for the area of the circle?

d. As a percentage, how large is the possible error in Juan's measurement for the area of the circle?

k.   Example: Solution (DOK 2)

The taxi fare in Gotham City is $2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate $0.20 for each additional 0.1 mile. You plan to give the driver a $2 tip. How many miles can you ride for $10?

l.   Example: Solution (DOK 2)

There are 270 students at Colfax Middle School, where the ratio of boys to girls is 5:4. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is 4:5. The two schools hold a dance and all students from both schools attend. What fraction of the students at the dance are girls?

m.   Example: Solution (DOK 2)

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of $27.50 for dinner. What is the cost of her dinner without tax or tip?

n. Example: [Solution](DOK 3)

Inflation is a term used to describe how prices rise over time. The rise in prices is in relation to the amount of money you have. The table below shows the rise in the price of bread over time:

YEAR	Cost of 1 lb. of Bread	Percent Increase
1930	$0.09	N/A
1940	$0.10	
1950	$0.12	
1960	$0.23	
1970	$0.25	
1980	$0.50	
1990	$0.75	
2000	$1.99	
2010	$2.99	

For the price in each decade, determine what the increase is as a percent of the price in the previous decade. Is the percent increase steady over time?

Under President Roosevelt, the Fair Labor Standards Act introduced the nation's first minimum wage of $0.25 an hour in 1938. The table shows the rise in minimum wage over time:

http://www.census.gov/compendia/statab/2012/tables/12s0652.pdf))

YEAR	Federal Minimum Wage	Percent Increase
1930	None	N/A
1940	$0.30	N/A
1950	$0.75	
1960	$1.00	
1970	$1.60	
1980	$3.10	
1990	$3.80	
2000	$5.15	
2010	$7.25	

For hourly wage in each decade, determine what the increase is as a percent of the hourly wage in the previous decade. Is the percent increase steady over time?

Consumers are not affected by inflation when the amount of money they make increases proportionately with the increase in prices. Complete the last column of the table below to show what percentage of an hour's pay a pound of bread costs:

YEAR	Cost of 1 lb. of Bread	Federal Minimum Wage	Percentage Cost
1930	$0.09	None	
1940	$0.10	$0.30	
1950	$0.12	$0.75	
1960	$0.23	$1.00	
1970	$0.25	$1.60	
1980	$0.50	$3.10	
1990	$0.75	$3.80	
2000	$1.99	$5.15	
2010	$2.99	$7.25	

In which decade were people who earn minimum wage most affected by inflation? Explain.

o.   Example: Solution (DOK 2)

Jamaican sprinter Usain Bolt won the 100 meter sprint gold medal in the 2012 Summer Olympics. He ran the 100 meter race in 9.63 seconds. There are about 3.28 feet in a meter and 5280 feet in a mile. What was Usain Bolt's average speed for the 100 meter race in miles per hour?

p.   Example: Solution (DOK 2)

If 100 dollars in one year gain $3\frac{1}{2}$ dollars interest, what sum will gain $38.50 cents in one year and a quarter?

q.   Example: Solution (DOK 3)

Emily has a coupon for 20 percent off of her purchase at the store. She finds a backpack that she likes on the discount rack. Its original price is $60 but everything on the rack comes with a 30 percent discount. Emily says

Thirty percent and twenty percent make fifty percent so it will cost $30.

a. Is Emily correct? Explain.

b. What price will Emily pay for the backpack?

r.   Example: An empty corn silo in the shape of a cylinder is being filled with corn.

Silo

19 feet

32 feet
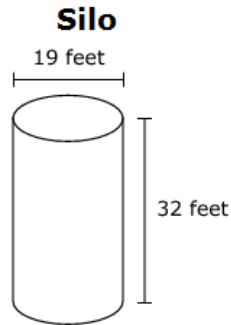
The silo is filled at a constant rate for a total of 10 hours. The table shows the amount of corn, in cubic feet, in the silo at the given number of hours after filling started.

Number of Hours	Amount of Corn (cu ft)
0	0
3	2475
5	4125
8	6600

Write the **percent** of the silo that is filled with corn at 10 hours.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#25	4	G, RP	A	2	8.G.C.9, 7.RP.A.3	1, 4	90.9 – 91

s. Example: A store is having a sale. Each customer receives either a 15% discount on purchases under $100, or a 20% discount on purchases of $100 or more. Kelly is purchasing some clothes for $96.60 before the discount. She decides to buy the fewest packs of gum that will increase her purchase to over $100. The price of each pack of gum is $0.79.
After the discount, how much less will Kelly pay by purchasing the clothes and the gum instead of purchasing only the clothes? (Assume there is no sales tax to consider.)
   a. $1.05
   b. $1.67
   c. $3.69
   d. $3.87

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#23	4	RP	A	2	7.RP.A.3	1, 2	B

t. Example: Dave buys a baseball for $15 plus an 8% tax. Mel buys a football for $20 plus an 8% tax. Write the difference, in dollars, of the amounts Dave and Mel pay, including tax. Round your answer to the nearest cent.
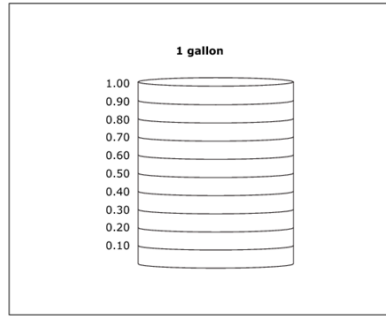
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#17	1	RP	A	2	7.RP.A.3	N/A	5.40

u. Example: Tim makes 80 gallons of paint by mixing 48 gallons of gray paint with 32 gallons of white paint.

What part of every gallon is gray paint?

The model represents 1 gallon of mixed paint.
Shade in the bars to show how much of the gallon is gray paint.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#13	2	RP	A	2	7.RP.A.3	1, 2	

v. Example: ([Former NAEP question](#)) (DOK 1)

Which of the following is true about 200% of 30?

A. It is $\frac{1}{2}$ times 30.

B. It is 2 times 30.

C. It is 200 times 30.

D. It is 200 times greater than 30.

Answer: B

w. Example: ([Former NAEP question](#)) (DOK 2)

If $\frac{A}{40} = 120$, what does $\frac{A}{80}$ equal?

A.   60

B.   120

C.   240

D. 9,600

Answer: A

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. (7.NS.A)

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers. **(7.NS.A.1) (DOK 1,2)**
      1. Example: Solution (DOK 3)

         A number line is shown below. The numbers 0| and 1| are marked on the line, as are two other numbers a| and b|.
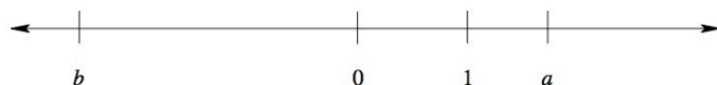
         Which of the following numbers is negative? Choose all that apply. Explain your reasoning.

         a. $a - 1|$

         b. $a - 2|$

         c. $-b|$

         d. $a + b|$

         e. $a - b|$

         f. $ab + 1|$

      2. Example: Solution (DOK 3)
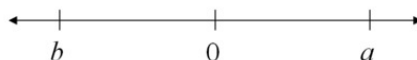
         On the number line above, the numbers a| and b| are the same distance from 0|. What is $a + b$|? Explain how you know.

      3. Example: Solution (DOK 2)

Ocean water freezes at about $-2\frac{1}{2}°C$. Fresh water freezes at $0°C$. Antifreeze, a liquid used in the radiators of cars, freezes at $-64°C$.

Imagine that the temperature has dropped to the freezing point for ocean water. How many degrees more must the temperature drop for the antifreeze to turn solid?

4. Example: Solution (DOK 2)

Conner and Aaron are working on their homework together to find the distance between two numbers, a and b, on a number line. Conner counts the units between the numbers, while Aaron subtracts the least number from the greatest. While both methods can give the correct answer, Conner and Aaron do not always apply them correctly.

a. In the first question $a = 1\frac{1}{3}$ and $b = 5\frac{1}{4}$.

Conner finds the difference $b - a$.

$$
\begin{aligned}
b - a &= 5\frac{1}{4} - 1\frac{1}{3} \\
&= \frac{21}{4} - \frac{4}{3} \\
&= \frac{63}{12} - \frac{16}{12} \\
&= \frac{47}{12} \\
&= 3\frac{11}{12}
\end{aligned}
$$

So Conner says that the distance between the two points is $3\frac{11}{12}$.

Aaron marks the two numbers on the number line and counts 3 whole units between them. Then he adds $\frac{1}{4}$ and $\frac{1}{3}$ to account for the additional fractional distances. Since

26

$$\frac{1}{3} + \frac{2}{5} = \frac{11}{15}\Big|$$

he says that the distance between the two numbers is $5\frac{11}{15}\Big|$.

a=-3⅓   b=2⅖

(number line from -3 to 2)

Which, if either of them, is correct? Find and correct any incorrect work.

c. After talking with each other and understanding their mistakes, Aaron and Conner each obtain the correct answer with their preferred method when $a = -8\frac{2}{3}\Big|$ and $b = -1\frac{1}{2}\Big|$.

Show how each student might have arrived at his answer.

a=-8⅔   b=-1½

(number line from -8 to -2)
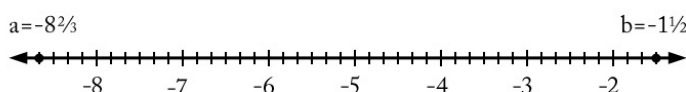
5.   Example: [Solution](#) (DOK 2)

Aakash, Bao Ying, Chris and Donna all live on the same street as their school, which runs from east to west.

- Aakash lives $5\frac{1}{2}\Big|$ blocks to the west.

- Bao Ying lives $4\frac{1}{4}\Big|$ blocks to the east.

- Chris lives $2\frac{3}{4}\Big|$ blocks to the west.

- Donna lives $6\frac{1}{2}\Big|$ blocks to the east.

a. Draw a picture that represents the positions of their houses along the street.

b. Find how far is each house from every other house?

c. Represent the relative position of the houses on a number line, with the school at zero, points to the west represented by negative numbers, and points to the east represented by positive numbers.

d. How can you see the answers to part (b) on the number line? Using the numbers (some of which are positive and some negative) that label the positions of houses on the number line, represent these distances using sums or differences.

6.   Example: [Solution](#) (DOK 2)

Xiaoli was estimating the difference between two positive numbers x and y (where $x > y$). First she rounded x up by a small amount. Then she rounded y down by the same amount. Finally, she subtracted the rounded values. Which of the following statements is correct?

a. Her estimate is larger than $x - y$.

b. Her estimate is smaller than $x - y$.

c. Her estimate equals $x - y$.

d. Her estimate equals $y - x$.

e. Her estimate is 0.

f. There is not enough information to compare $x - y$ with her estimate.

7. Example: Solution (DOK 3)

a. At the beginning of the month, Evan had $24 in his account at the school bookstore. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then represent each transaction on a number line. What is the unknown quantity in each case?

   i. First he bought some notebooks and pens that cost $16.

   ii. Then he deposited some more money and his account balance was $28.

   iii. Then he bought a book for English class that cost $34.

   iv. Then he deposited exactly enough money so that he paid off his debt to the bookstore.

b. Explain why it makes sense to use a negative number to represent Evan's account balance when he owes money.

8. Example: Solution (DOK 3)
Ojos del Salado is the highest mountain in Chile, with a peak at about 6900 meters above sea level. The Atacama Trench, just off the coast of Peru and Chile, is about 8100 meters below sea level (at its lowest point).

a. What is the difference in elevations between Mount Ojos del Salado and the Atacama Trench?

b. Is the elevation halfway between the peak of Mount Ojos del Salado and the Atacama Trench above sea level or below sea level? Explain without calculating the exact value.

c. What elevation is halfway between the peak of Mount Ojos del Salado and the Atacama Trench?

9. Example: Two water tanks are shown. Tank A is a rectangular prism and Tank B is a cylinder. The tanks are not drawn to scale.
   - In **Figure 1**, Tank A is filled with water to the 10-meter mark.
   - In **Figure 2,** Tank A is filled with water to the 5-meter mark and the

excess water has been transferred to Tank B.
- Find the approximate radius of Tank B.
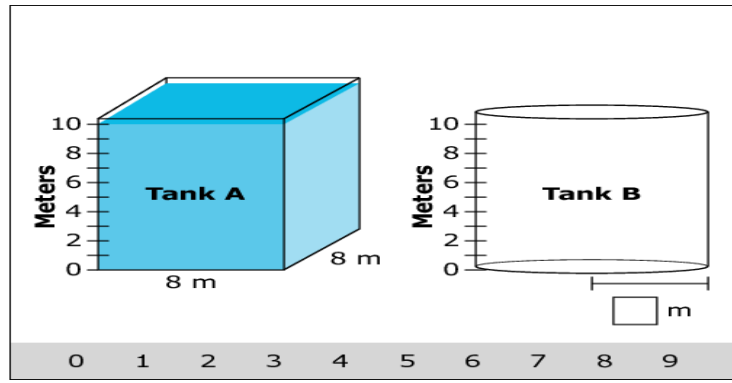
Figure 1
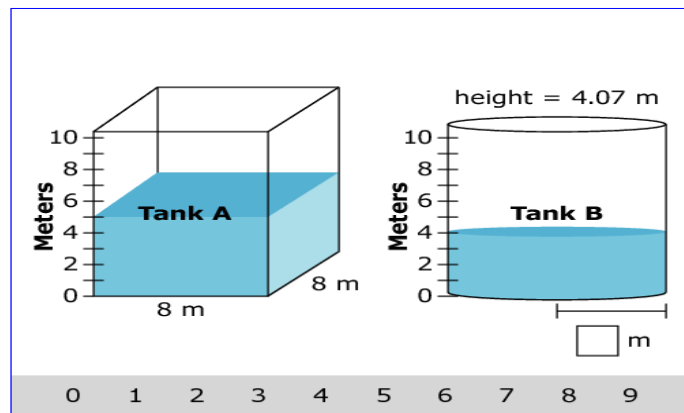
Figure 2

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#2	1	NS	B	1	7.NS.A.1d	N/A	$5\frac{1}{3}$

10. Example: The point on the number line shows the location of $3\frac{1}{2}$. Write each expression into the appropriate box to show its correct location on the number line.

The number line shows $-3\frac{1}{2}$ marked with a point, with four empty boxes above and arrows pointing to positions, and 0 marked on the right. Below the number line:

$$-3\frac{1}{2} - 3\frac{1}{2} \qquad\qquad -3\frac{1}{2} - (-5)$$

$$-3\frac{1}{2} + 3\frac{1}{2} \qquad\qquad -3\frac{1}{2} + (-5)$$
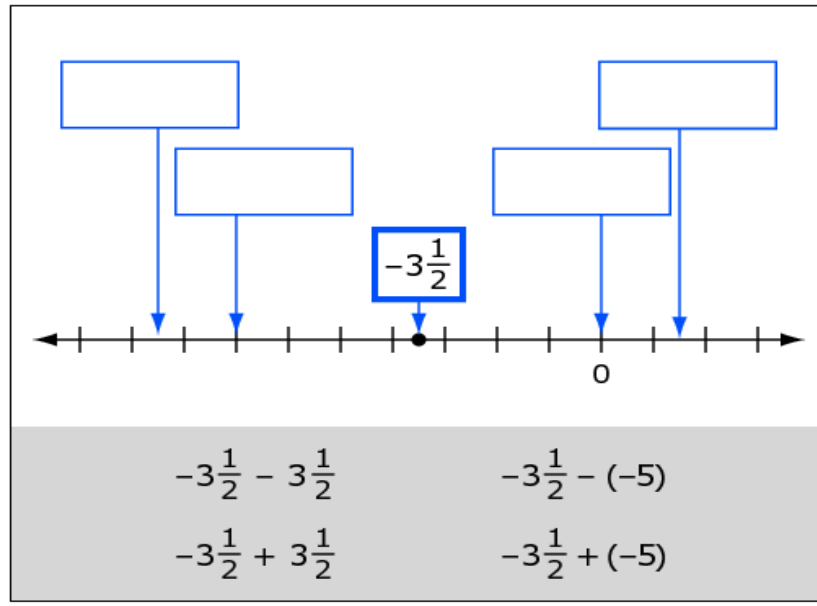
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#31	1	NS	1B	1	7.NS.1	N/A	

11. Example: ([Former NAEP question](#)) (DOK 1)

Add: $+8 + (-16)$

    A. -24
    B.   -8
    C. +8
    D. +24

Answer: B

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

30

c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **(7.NS.A.2)** **(DOK 1,2)**

1. Example: Solution (DOK 2)

   a. Use long division to find the repeating decimal that represents $\frac{29}{13}$

   b. Take the number obtained by including only the first two digits after the decimal point, and multiply that by 13.

   c. Take the number obtained by including only the first four digits after the decimal point, and multiply that by 13.

   d. Take the number obtained by including only the first six digits after the decimal point, and multiply that by 13.

   e. What do you notice about the product of 13 and decimal approximations of $\frac{29}{13}$ as more and more digits are included after the decimal point?

   f. How does what you observed in Part (e) help make sense of what it means for $\frac{29}{13}$ to be equal to the repeating decimal expression you found in the Part (a)?

2. Example: Solution (DOK 2)

   Malia found a "short cut" to find the decimal representation of the fraction $\frac{117}{250}$. Rather than use long division she noticed that because $250 \times 4 = 1000$

   $$\frac{117}{250} = \frac{117 \times 4}{250 \times 4} = \frac{468}{1000} = 0.468.$$

   a. For which of the following fractions does Malia's strategy work to find the decimal representation?

   $$\frac{1}{3}, \frac{3}{4}, -\frac{6}{25}, \frac{18}{7}, \frac{13}{8} \quad \text{and} \quad -\frac{113}{40}.$$
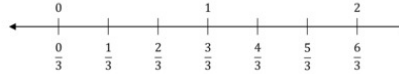
   For each one for which the strategy does work, use it to find the decimal representation.

   b. For which denominators can Malia's strategy work?

3. Example: [Solution](#) (DOK 3)

Tiffany said,

I know that 3 thirds equals 1 so $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$.

I also know that $\frac{1}{3} = 0.333\ldots$ where the 3's go on forever. But if I add them up as decimals, I get $0.999\ldots$.

$$
\begin{array}{r}
0.333\ldots \\
0.333\ldots \\
+0.333\ldots \\
\hline
0.999\ldots
\end{array}
$$

I just added up the tenths, then the hundredths, then the thousands, and so on. What went wrong?

a. Write $0.999\ldots$ in the form of a fraction $\frac{a}{b}$ where a and b are whole numbers. Are Tiffany's calculations consistent with what you find? Explain.

b. Use Tiffany's idea of adding decimals to write $\frac{1}{3} + \frac{1}{6}$ as a repeating decimal. Can this also be written as a terminating decimal?

4. Example: [Solution](#) (DOK 2)

Sarah learned that in order to change a fraction to a decimal, she can use the standard division algorithm and divide the numerator by the denominator. She noticed that for some fractions, like $\frac{1}{4}$ and $\frac{1}{100}$ the algorithm terminates at the hundredths place. For other fractions, like $\frac{1}{8}$ she needed to go to the thousandths place before the remainder disappears. For other fractions, like $\frac{1}{3}$ and $\frac{1}{6}$, the decimal does not terminate. Sarah wonders which fractions have terminating decimals and how she can tell how many decimal places they have.

a. Convert each of the following fractions to decimals to help Sarah look for patterns with her decimal conversions:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{15}.$$

b. Which fractions on the list have terminating decimals (decimals that eventually end in 0's)? What do the denominators have in common?

c. Which fractions on the list have repeating decimals? What do the denominators have in common?

d. Which fractions $\frac{p}{q}$ (in reduced form) do you think have terminating decimal representations? Which do you think have repeating decimal representations?

5. Example: [Solution](#) (DOK 2)

A water well drilling rig has dug to a height of –60 feet after one full day of continuous use.

a. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

b. If the rig has been running constantly and is currently at a height of –143.6 feet, for how long has the rig been running?

6. Example: <u>Solution</u> (DOK 2)

a. If we use the same definition for multiplication, what should the value of $3 \times (-5)$ be?

b. Here is an example of the distributive property:

$$3 \times (5 + 4) = 3 \times 5 + 3 \times 4$$

If the distributive property works for both positive and negative numbers, what expression would be equivalent to $3 \times (5 + (-5))$?

If we use the fact that $5 + (-5) = 0$ and $3 \times 5 = 15$, what should the value of $3 \times (-5)$ be?

c. We can multiply positive numbers in any order:

$$3 \times 5 = 5 \times 3$$

Use what you know from parts (a) and (b). If we can multiply signed numbers in any order, what should the value of $(-5) \times 3$ be?

If the distributive property works for both positive and negative numbers, what expression would be equivalent to $(-5) \times (3 + (-3))$?

d. Use what you know from parts (a), (b), and (c). What should the value of $(-5) \times (-3)$ be?

7. Example: <u>Solution</u> (DOK 3)

Lucia uses the picture below to explain the distributive property for the expression (3+1) ×(5+1):
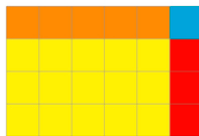
a. Find (3+1) ×(5+1) using the distributive property.

b. Explain how Lucia's picture relates to your calculation in (a).

c. How can Lucia use a picture to find (3-1) ×(5-1) using the distributive property? Explain.

8. Example: <u>Solution</u> (DOK 2)

Each expression below gives the rate of change of temperature in degrees Celsius over a certain period of time. Interpret each expression below in terms of the temperature change.

a. $-\left|\dfrac{2 \text{ degrees}}{3 \text{ hours}}\right|$

b. $\left|\dfrac{-2 \text{ degrees}}{3 \text{ hours}}\right|$

c. $\left|\dfrac{2 \text{ degrees}}{-3 \text{ hours}}\right|$

9. Example: Alex claims that when $\dfrac{1}{4}$ is divided by a fraction, the result will be greater than $\dfrac{1}{4}$.

To convince Alex that this statement is only sometimes true:

Part A: Write one digit into each box to create an expression that is greater than $\dfrac{1}{4}$.

$$\frac{1}{4} \div \underline{}$$

Part B: Write one digit into each box to create an expression that is **not** greater than $\dfrac{1}{4}$.

$$\frac{1}{4} \div \underline{}$$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#10	3	NS	A	2	7.NS.A.2c, 6.NS.A.1	2, 3, 7	**Part A: Expression greater than** $\frac{1}{4}$ $\dfrac{1}{4} \div \dfrac{\boxed{1}}{\boxed{7}}$ **Part B: Expression not greater than** $\frac{1}{4}$ $\dfrac{1}{4} \div \dfrac{\boxed{7}}{\boxed{1}}$

10. Example: In the given equation, *a, b,* and *c* are nonzero rational numbers.

$$a \cdot b = c$$

Given this equation, write one number into each box to complete four true equations.

a
b
c
−a
−b
−c

Delete

$$-a \bullet \boxed{} = c$$

$$\boxed{} \bullet \boxed{} = -c$$

$$\frac{\boxed{}}{-b} = a$$

$$\frac{\boxed{}}{\boxed{}} = -a$$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#8	3	NS	C	3	7.NS.A.2	2, 7	$-a \bullet \boxed{-b} = c$ $\boxed{a} \bullet \boxed{-b} = -c$ $\dfrac{\boxed{-c}}{-b} = a$ $\dfrac{\boxed{-c}}{\boxed{b}} = -a$

11. Example: ([Former NAEP question](#)) (DOK 1)

$$\text{Divide: } -36 \div (+4)$$

A. -9
B. +9
C. $\dfrac{-1}{9}$
D. $\dfrac{+1}{9}$
E. -32

Answer: A

3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1]
    (7.NS.A.3) **(DOK 1,2)**

---

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

a. Example: Solution (DOK 2)

> The three seventh grade classes at Sunview Middle School collected the most boxtops for a school fundraiser, and so they won a $600 prize to share among them. Mr. Aceves' class collected 3,760 box tops, Mrs. Baca's class collected 2,301, and Mr. Canyon's class collected 1,855. How should they divide the money so that each class gets the same fraction of the prize money as the fraction of the box tops that they collected?

b. Example: Solution (DOK 2)

> A water well drilling rig has dug to a height of –60 feet after one full day of continuous use.
>
> a. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?
>
> b. If the rig has been running constantly and is currently at a height of –143.6 feet, for how long has the rig been running?

c. Example: Mark buys a wooden board that is 7 1/2 feet long. The cost of the wooden board is $0.50 per foot, including tax. What is the total cost, in dollars, of the wooden board?

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#2	1	NS	B	1	7.NS.A.3	N/A	3.75

d. Example: (Former NAEP question) (DOK 2)

> The manager of a company has to order new engines for its delivery trucks after the trucks have been driven 150,000 miles. One of the delivery trucks currently has 119,866 miles on it. This truck has the same delivery route each week and is driven an average of 40,000 miles each year. At this rate, the manager should expect this truck to reach 150,000 miles in approximately how many months?
> A. Less than 4 months
> B. Between 4 and 6 months
> C. Between 6 and 8 months
> D. Between 8 and 10 months
> E. More than 10 months

Answer: D

e. Example: (Former NAEP question) (DOK 2)

> The cost to mail a first-class letter is 33 cents for the first ounce. Each additional ounce costs 22 cents. (Fractions of an ounce are rounded up to the next whole ounce.)
>
> How much would it cost to mail a letter that weighs 2.7 ounces?
> A. 55 cents
> B. 66 cents
> C. 77 cents
> D. 88 cents
> E. 99 cents

Answer: C

Use properties of operations to generate equivalent expressions. (7.EE.A)

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **(7.EE.A.1)** **(DOK 1)**
   a. Example: Solution (DOK 1)

   Write an expression for the sequence of operations.

   a. Add 3 to x, subtract the result from 1, then double what you have.

   b. Add 3 to x, double what you have, then subtract 1 from the result.

   b. Example: Look at each expression. Is it equivalent to $\frac{x+3y}{2}$?

   Check Yes or No for expressions A –

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#30	1	NS	1C	1	7. EE. 2	N/A	A. No, B. No, C. Yes, D. Yes

   c. Example: Write the value of *n* so the expression $(-y + 5.3) + (7.2y - 9)$ is equivalent to $6.2y + n$.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#15	1	EE	C	1	7.EE.A.1	7	-3.7

2.

   Example: Which expression is equivalent to -8(10x - 3)?
   a. $-80x + 24$
   b. $-80x - 24$
   c. $-80x - 3$
   d. $-80x + 3$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#12	1	EE	C	1	7.EE.A.1	N/A	A

   e. Example: Write the value of p so the expression 5/6-1/3 n is equivalent to p (5-2n).

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#6	1	EE	C	1	7.EE.A.1	7	$\frac{1}{6}$

3. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **(7.EE.A.2)** **(DOK 1,2)**
   a. Example: Solution (DOK 2)

Malia is at an amusement park. She bought 14 tickets, and each ride requires 2 tickets.

a. Write an expression that gives the number of tickets Malia has left in terms of x, the number of rides she has already gone on. Find at least one other expression that is equivalent to it.

b. $14 - 2x$ represents the number of tickets Malia has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?

$$14$$

$$-2$$

$$2x$$

c. $2(7 - x)$ also represents the number of tickets Malia has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?

$$7$$

$$(7 - x)$$

$$2$$

Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (7.EE.B)

4. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **(7.EE.B.3) (DOK 1,2,3)**
   a. Example: Solution (DOK 3)

When working on a report for class, Catrina read that a woman over the age of 40 can lose approximately 0.06 centimeters of height per year.

a. Catrina's aunt Nancy is 40 years old and is 5 feet 7 inches tall. Assuming her height decreases at this rate after the age of 40, about how tall will she be at age 65? (Remember that 1 inch = 2.54 centimeters.)

b. Catrina's 90-year-old grandmother is 5 feet 1 inch tall. Assuming her grandmother's height has also decreased at this rate, about how tall was she at age 40? Explain your reasoning.

b.   Example: Solution (DOK 2)
Katie and Margarita have $20.00 each to spend at Students' Choice book store, where all students receive a 20% discount. They both want to purchase a copy of the same book which normally sells for $22.50 plus 10% sales tax.

- To check if she has enough to purchase the book, Katie takes 20% of $22.50 and subtracts that amount from the normal price. She takes 10% of the discounted selling price and adds it back to find the purchase amount.

- Margarita takes 80% of the normal purchase price and then computes 110% of the reduced price.

Is Katie correct? Is Margarita correct? Do they have enough money to purchase the book?

c.   Example: Solution (DOK 2)
The taxi fare in Gotham City is $2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate $0.20 for each additional 0.1 mile. You plan to give the driver a $2 tip. How many miles can you ride for $10?

d.   Example: Solution (DOK 2)
Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of $27.50 for dinner. What is the cost of her dinner without tax or tip?

e.   Example: Solution (DOK 2)

The students in Mr. Rivera's art class are designing a stained-glass window to hang in the school entryway. The window will be 2 feet tall and 5 feet wide. They have drawn the design below:
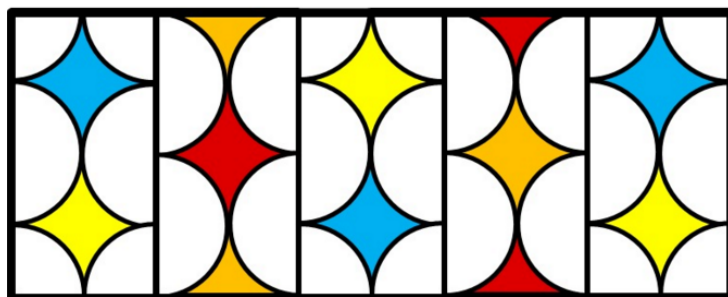
They have raised $100 for the materials for the project. The colored glass costs $5 per square foot and the clear glass costs $3 per square foot. The materials they need to join the pieces of glass together costs 10 cents per foot and the frame costs $4 per foot.

Do they have enough money to cover the costs of the materials they will need to make the window?

f. Example: Solution (DOK 3)

Below is a table showing the number of hits and the number of times at bat for two Major League Baseball players during two different seasons:

Season	Derek Jeter	David Justice
1995	12 hits in 48 at bats	104 hits in 411 at bats
1996	183 hits in 582 at bats	45 hits in 140 at bats

A player's *batting average* is the fraction of times at bat when the player gets a hit.

a. For each season, find the players' batting averages. Who has the better batting average?

b. For the combined 1995 and 1996 seasons, find the players' batting averages. Who has the better batting average?

c. Are the answers to (a) and (b) consistent? Explain.

g. Example: Aimee has $10.00 to spend on school supplies. The following table shows the price of each item in the school store. No sales tax is charged on these items.

Item	Price
Eraser	$0.89
Folder	$1.29
Notebook	$2.35
Pen	$0.70

Determine if Aimee can buy the combination of items with her $10.00. Check **Yes** or **No** for each combination of items.

	Yes	No
5 folders and 5 pens	☐	☐
6 pens and 6 erasers	☐	☐
1 pen and 4 notebooks	☐	☐
3 folders and 7 erasers	☐	☐
4 folders and 2 notebooks	☐	☐

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#24	1	EE	D	1	7.EE.B.3	N/A	(see below)

Key:

	Yes	No
	☑	☐
	☑	☐
	☐	☑
	☐	☑
;	☑	☐

h. Example: Shelly incorrectly solves the equation $\frac{1}{2}(c + 6) = 7$. Her work is shown.

Part A: Circle **all** the steps that show an error based on the equation in the previous step.

Part A:

$$\frac{1}{2}(c + 6) = 7$$

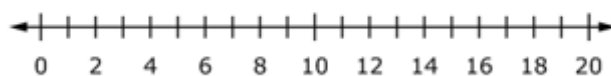Step 1: $\frac{1}{2}c + 6 = 7$

Step 2: $\frac{1}{2}c = 7 + 6$
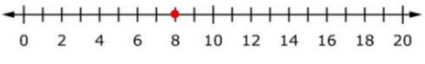
Step 3: $\frac{1}{2}c = 13$

Step 4: $c = 13 \div 2$

Step 5: $c = 6\frac{1}{2}$

Part B: Draw a point on the line to show the correct solution of the given equation.

Part B: Correct solution

0   2   4   6   8   10   12   14   16   18   20

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key

#21	3	EE	E	2	7.EE.B.3	6	

Part A: $\frac{1}{2}(c + 6) = 7$

Step 1:	$\frac{1}{2}c + 6 = 7$
Step 2:	$\frac{1}{2}c = 7 + 6$
Step 3:	$\frac{1}{2}c = 13$
Step 4:	$c = 13 \div 2$
Step 5:	$c = 6\frac{1}{2}$

Part B: Correct solution

Select locations of points.

     i.   Example: What is the value of the expression?

$$2.3 \bullet (4 + 12)$$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#5	1	EE	D	1	7.EE.B.3	N/A	36.8

5.   Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

    a.   Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*`

    b.   Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* **(7.EE.B.4)** **(DOK 1,2,3)**

      1. Example: Solution (DOK 2)

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs of gear for the boat plus 10 lbs of gear for each person.

a. Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.

b. Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? What is the maximum number of people that may rent a boat?

      2.   Example: Solution (DOK 2)

The taxi fare in Gotham City is $2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate $0.20 for each additional 0.1 mile. You plan to give the driver a $2 tip. How many miles can you ride for $10?

3. Example : Solution (DOK 2)

Jonathan wants to save up enough money so that he can buy a new sports equipment set that includes a football, baseball, soccer ball, and basketball. This complete boxed set costs $50. Jonathan has $15 he saved from his birthday. In order to make more money, he plans to wash neighbors' windows. He plans to charge $3 for each window he washes, and any extra money he makes beyond $50 he can use to buy the additional accessories that go with the sports box set.

Write and solve an inequality that represents the number of windows Jonathan can wash in order to save at least the minimum amount he needs to buy the boxed set. Graph the solutions on the number line. What is a realistic number of windows for Jonathan to wash? How would that be reflected in the graph?

4. Example: Solution (DOK 3)

a.

At the beginning of the month, Evan had $24 in his account at the school bookstore. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then represent each transaction on a number line. What is the unknown quantity in each case?

i. First he bought some notebooks and pens that cost $16.

ii. Then he deposited some more money and his account balance was $28.

iii. Then he bought a book for English class that cost $34.

iv. Then he deposited exactly enough money so that he paid off his debt to the bookstore.

b. Explain why it makes sense to use a negative number to represent Evan's account balance when he owes money.

5. Example: Solution (DOK 2)

A water well drilling rig has dug to a height of –60 feet after one full day of continuous use.

a. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

b. If the rig has been running constantly and is currently at a height of –143.6 feet, for how long has the rig been running?
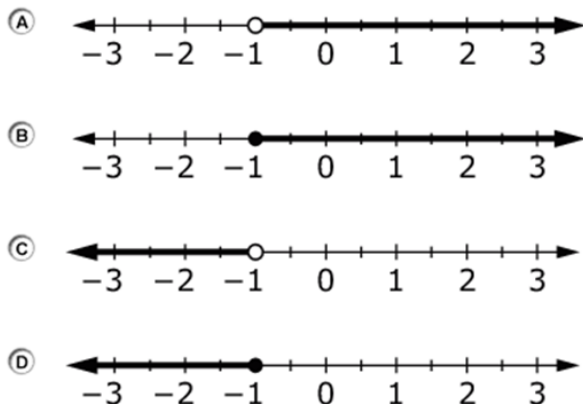
6. Example: The entry fee to the fair is $4.00. Each ride requires a ticket that costs $0.50. Heidi spent a total of $12.00.

How many tickets did Heidi purchase?

    a. 6
    b. 16
    c. 24
    d. 32

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#20	1	EE	D	1	7.EE.B.4a	1	B

7. Example: Which number line shows the solution to the inequality -3x-5 < -2?

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#3	1	EE	D	1	7.EE.B.4b	N/A	A

8. Example: ([Former NAEP question](#)) (DOK 2)

The admission price to a movie theater is $7.50 for each adult and $4.75 for each child. Which of the following equations can be used to determine T, the total admission price, in dollars, for x adults and y children?

    A. T = (7.50 + 4.75)(x + y)
    B. T = 7.50x + 4.75y
    C. T = 7.50y + 4.75x
    D. T = (7.50x)(4.75y)
    E. T = (7.50 + 4.75) + (x + y)

Answer: B

9. Example: ([Former NAEP question](#)) (DOK 2)

A rectangle has a width of m inches and a length of k inches. If the perimeter of the rectangle is 1,523 inches, which of the following equations is true?

    A. 2(m + k) = 1,523
    B. 2m + k = 1,523
    C. m + k = 1,523
    D. mk = 1,523
    E. m^2k^2 = 1,523

Answer: A

10. Example: ([Former NAEP question](#)) (DOK 1)

If 15 + 3 *x* = 42, then *x* =
- A. 9
- B. 11
- C. 12
- D. 14
- E. 19

Answer: A

11. Example: ([Former NAEP question](#)) (DOK 2)

The temperature in degrees Celsius can be found by subtracting 32 from the temperature in degrees Fahrenheit and multiplying the result by $\frac{5}{9}$. If the temperature of a furnace is 393 degrees Fahrenheit, what is it in degrees Celsius, to the nearest degree?
- A. 649
- B. 375
- C. 219
- D. 201
- E. 187

Answer: D

12. Example: ([Former NAEP question](#)) (DOK 2)

At the school carnival, Carmen sold 3 times as many hot dogs as Shawn. The two of them sold 152 hot dogs altogether. How many hot dogs did Carmen sell?
- A. 21
- B. 38
- C. 51
- D. 114
- E. 148

Answer: D

13. Example: ([Former NAEP question](#)) (DOK 2)

Angie has a bag containing n apples. She gives 4 to her brother and keeps 5 for herself. She then divides the remaining apples equally among 3 friends. Which of the following expressions represents the number of apples each friend receives?
- A. $\frac{n}{3} - 4 - 5$
- B. $\frac{n - 4 - 5}{3}$
- C. $\frac{4 + 5 - n}{3}$
- D. $\frac{n - 4}{3} - 5$
- E. $\frac{n - 5}{3} - 4$

Answer: B

Geometry   7.G

Draw, construct, and describe geometrical figures and describe the relationships between them. (7.G.A)

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **(7.G.A.1) (DOK 1,2)**
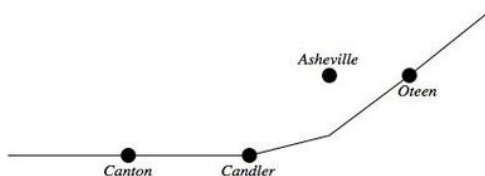
    a. Example: Solution (DOK 2)

    > Mariko has an $80 : 1$ scale-drawing of the floor plan of her house. On the floor plan, the dimensions of her rectangular living room are $1\frac{7}{8}$ inches by $2\frac{1}{2}$ inches.
    >
    > What is the area of her real living room in square feet?

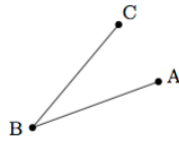   b. Example: Solution (DOK 2)

    > On the map below, $\frac{1}{4}$ inch represents one mile. Candler, Canton, and Oteen are three cities on the map.


    > a. If the distance between the real towns of Candler and Canton is 9 miles, how far apart are Candler and Canton on the map?
    >
    > b. If Candler and Oteen are $3\frac{1}{2}$ inches apart on the map, what is the actual distance between Candler and Oteen in miles?

    c. Example: Solution (DOK 3)

46

a. For each shape below, draw a scaled copy with a scale factor of 2. Explain why your drawing is accurate.
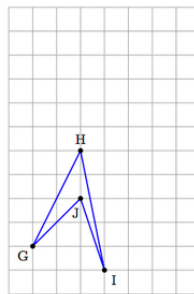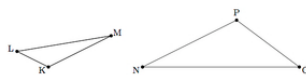
    i.

    ii.

    iii.
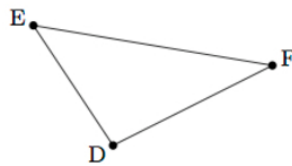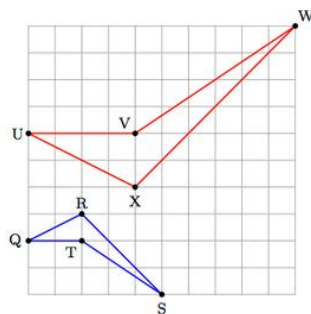

b. Below are two pairs of polygons. For each pair, explain whether or not one is a scaled version of the other.

    i.

    ii.

d. Example: <u>Solution</u> (DOK 3)

Your computer shows you a map of Washington Park in Eugene, Oregon. The scale in the bottom right corner of the map tells us that the length of the bar represents 200 feet on the map.
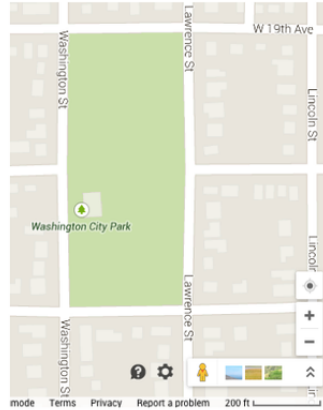
Clicking a resizing button on the computer screen will result in an image of Washington Park where the exact same-sized bar now represents 150 feet.

a. Do you think the size of Washington Park under the 150 ft scale will appear smaller or larger than it was under the 200 ft scale?

b. Draw an accurate picture/map of Washington Park under this new 150 ft scale.

c. Was your guess in part a correct? Can you explain why the size of the map changed as it did?

e. Example: <u>Solution</u> (DOK 3)

a. Draw circles with diameters as indicated below and measure their circumferences to complete the following table.

| Diameter of Circle (inches)	Circumference of Circle (inches)	$\frac{\text{Circumference of Circle}}{\text{Diameter of Circle}}$
1		
2		
3		
$\frac{1}{2}$		

b. The number π can be defined as the circumference of a circle with diameter 1 (unit). Using your knowledge about circles (that is, *without measuring*), complete the following table. Explain how you know the circumferences of the different circles.

Diameter of Circle (inches)	Circumference of Circle (inches)	$\frac{\text{Circumference of Circle}}{\text{Diameter of Circle}}$
1	π	π
2		
3		
$\frac{1}{2}$		

c. How does the information in the two tables compare? Explain.

f.   Example: [Solution](#) (DOK 3)

Below is a picture of a circle of radius r and a square of side length r.
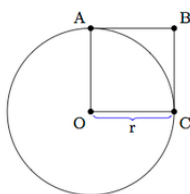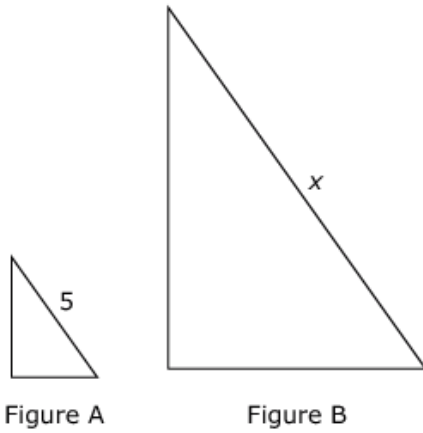

a. Show that

$$2 \leq \frac{\text{Area(Circle)}}{\text{Area(Square)}} \leq 4.$$

b. How can we find a more accurate estimate of $\frac{\text{Area(Circle)}}{\text{Area(Square)}}$ than the one in part (a)?

c. Explain why the quotient $\frac{\text{Area(Circle)}}{\text{Area(Square)}}$ does not depend on the radius r.

g.   Example: A scale factor of 3.5 maps Figure A onto Figure B.

Figure A   Figure B

Enter the value of x.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#26	1	G	E	1	7.G.A.1	N/A	17.5

h.   Example: ([Former NAEP question](#)) (DOK 1)
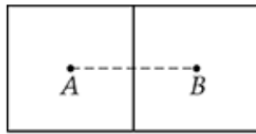
Each square above is 10 units on a side. Points A and B are the centers of the squares. What is the distance between A and B?
  A. 5 units
  B. 10 units
  C. 15 units
  D. 20 units

    Answer: B

i.   Example: ([Former NAEP question](#)) (DOK 2)

☐ represents 25 square meters

On the scale drawing above, the shaded area represents a piece of property along the river.
Which of the following measurements is the best estimate of the area of the property?

    A. 750 square meters
    B. 850 square meters
    C. 900 square meters
    D. 1,050 square meters
    E. 1,200 square meters

Answer: D

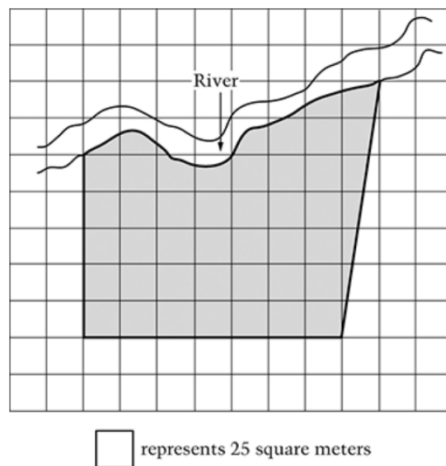2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **(7.G.A.2) (DOK 1,2)**
    a. Example: Solution (DOK 2)

Starting at the origin, a ladybug walked 4 units east. Then she walked a distance of 3 units in an unknown direction. At that time she was 30 degrees to the north of her original walking direction.

The diagram shows one possibility for the ladybug's final location. Find a different final location that is also consistent with the given information, and draw the ladybug there.
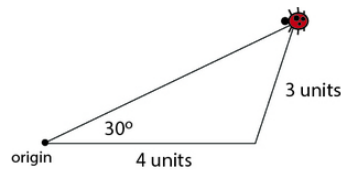
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **(7.G.A.3) (DOK 1,2)**
   a. Example: Solution (DOK 2)

   Imagine you are a ninja that can slice solid objects straight through. You have a solid cube in front of you. You are curious about what 2-dimensional shapes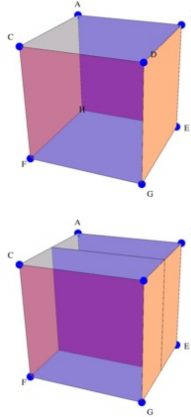 are formed when you slice the cube. For example, if you make a slice through the center of the cube that is parallel to one of the faces, the cross section is a square:

   For each of the following slices, (i) describe using precise mathematical language the shape of the cross section. (ii) draw a diagram showing the cross section of the cube.
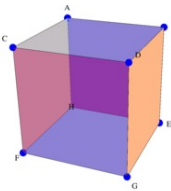
   a. A slice containing edge AC and edge EG b. A slice containing the vertices C, B, and G. C. A slice containing the vertex A, the midpoint of edge EG, and the midpoint of edge FG.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (7.G.B)

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **(7.G.B.4) (DOK 1,2)**
   a. Example: Solution (DOK 2)

The figure below is composed of eight circles, seven small circles and one large circle containing them all. Neighboring circles only share one point, and two regions between the smaller circles have been shaded. Each small circle has a radius of 5 cm.
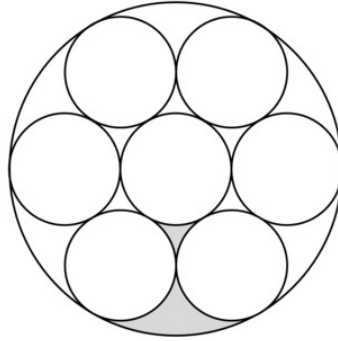
Calculate:

a. The area of the large circle.

b. The area of the shaded part of the figure.

b. Example: <u>Solution</u> (DOK 3)

Juan wants to know the cross-sectional area of a circular pipe. He measures the diameter which he finds, to the nearest millimeter, to be 5 centimeters.

a. How large is the possible error in Juan's measurement of the diameter of the circle? Explain.

b. As a percentage of the diameter, how large is the possible error in Juan's measurement?

c. To find the area of the circle, Juan uses the formula $A = \pi r^2$ where A is the area of the circle and r is its radius. He uses 3.14 for π. What value does Juan get for the area of the circle?

d. As a percentage, how large is the possible error in Juan's measurement for the area of the circle?

c. Example: Solution (DOK 2)

Find the area and perimeter of the colored part of each of the six figures below.
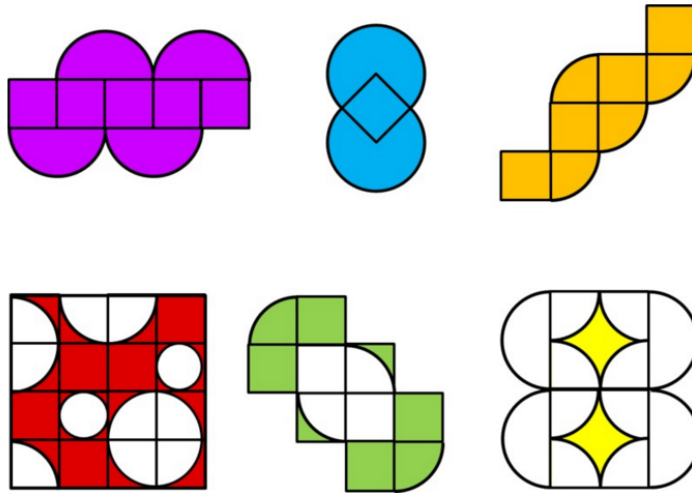
The purple, blue, orange, red, and green figures are composed of small squares with side-length 1 unit and curves that are an arc of a circle. The squares in the yellow figure are larger than the others and have a side-length of 2 units.

d. Example: Solution (DOK 2)

The students in Mr. Rivera's art class are designing a stained-glass window to hang in the school entryway. The window will be 2 feet tall and 5 feet wide. They have drawn the design below:
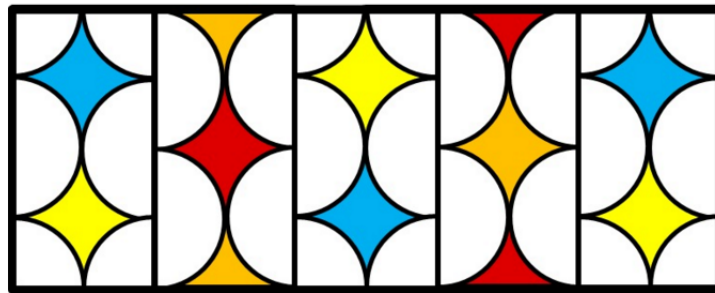
They have raised $100 for the materials for the project. The colored glass costs $5 per square foot and the clear glass costs $3 per square foot. The materials they need to join the pieces of glass together costs 10 cents per foot and the frame costs $4 per foot.

Do they have enough money to cover the costs of the materials they will need to make the window?

e. Example: Solution (DOK 3)

a. Draw circles with diameters as indicated below and measure their circumferences to complete the following table.

Diameter of Circle (inches)	Circumference of Circle (inches)	$\frac{\text{Circumference of Circle}}{\text{Diameter of Circle}}$
1		
2		
3		
$\frac{1}{2}$		

b. The number π can be defined as the circumference of a circle with diameter 1 (unit). Using your knowledge about circles (that is, *without measuring*), complete the following table. Explain how you know the circumferences of the different circles.

Diameter of Circle (inches)	Circumference of Circle (inches)	$\frac{\text{Circumference of Circle}}{\text{Diameter of Circle}}$
1	π	π
2		
3		
$\frac{1}{2}$		

c. How does the information in the two tables compare? Explain.

f. Example: (DOK 3)

Below is a picture of a circle of radius r and a square of side length r:

a. Show that

$$2 \le \frac{\text{Area(Circle)}}{\text{Area(Square)}} \le 4.$$

b. How can we find a more accurate estimate of $\frac{\text{Area(Circle)}}{\text{Area(Square)}}$ than the one in part (a)?

c. Explain why the quotient $\frac{\text{Area(Circle)}}{\text{Area(Square)}}$ does not depend on the radius r.

g. Example: (DOK 3)

Martin and Muriel finished a project for class showing one way to see why the area of a circle is given by $A = \pi r^2$, if r is the radius of the circle. Muriel is not in class today and Martin is trying to understand the following page of pictures from their project.
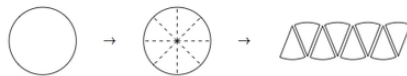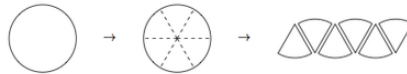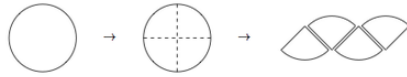
Help Martin by writing up an explanation of how these pictures could be used to derive the formula for the area of a circle.

h. Example: A corner shelf is $\frac{1}{4}$ of a circle and has a radius of 10.5 inches.
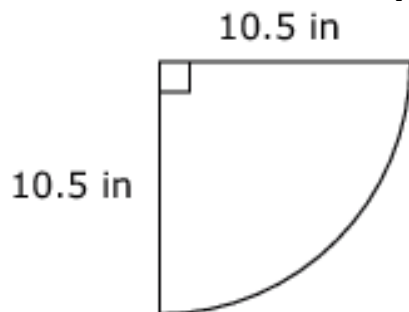
What is the area of the shelf, in square inches? Round your answer to the nearest hundredth.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#26	1	G	E	1	7.G.A.1	N/A	17.5

i. Example: (Former NAEP question) (DOK 2)

The distance around a circular pond are shown below. From the snack bar, Jake notices an island in the center of the pond.
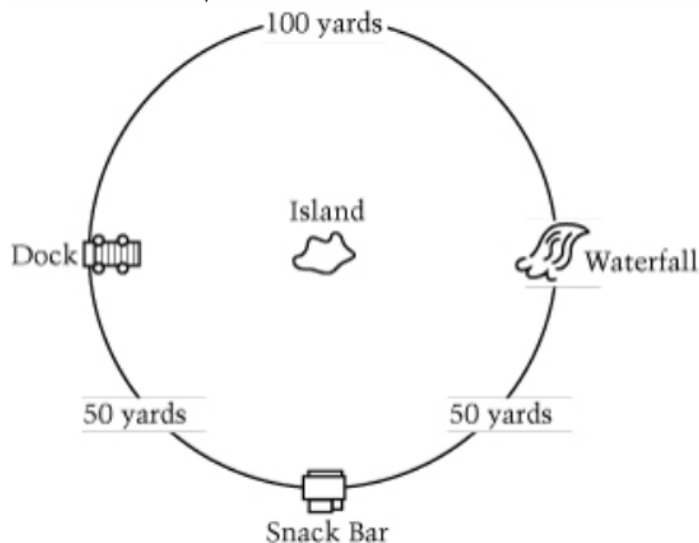
Of the following, which is the best approximation of the distance from the snack bar to the center of the island?

    A. 16 yards
    B. 20 yards
    C. 32 yards
    D. 50 yards
    E. 64 yards

Answer: C

j.   Example: (Former NAEP question) (DOK 1)

What is the radius of the largest circle that can be drawn on a 36-by-36-inch square piece of poster board?

    A. 3 inches
    B. 6 inches
    C. 9 inches
    D. 18 inches
    E. 36 inches

Answer: D

k.   Example: (Former NAEP question) (DOK 3)

Three tennis balls are to be stacked one on top of another in a cylindrical can. The radius of each tennis ball is 3 centimeters. To the nearest whole centimeter, what should be the minimum height of the can?

Explain why you chose the height that you did. Your explanation should include a diagram.

Answer: 18 centimeters because the diameter of each ball equals 6 centimeters and there are 3 balls (6 x 3 = 18)

5.   Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write

and solve simple equations for an unknown angle in a figure. **(7.G.B.5)** **(DOK 1,2)**

    a.   Example: Determine whether each statement is true for all cases, true for some cases, or not true for any case.

	Truefor all cases	Truefor some cases	Not truefor any cases
Two vertical angles form a linear pair.	☐	☐	☐
If two angles are supplementary and congruent, then they are right angles.	☐	☐	☐
The sum of two adjacent angles is 90°.	☐	☐	☐
The measure of an exterior angle of a triangle is greater than every interior angle of the triangle.	☐	☐	☐

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key		
#19	3	G	G	3	7.G.B.5		**True for all cases**	**True for some cases**	**Not true for any cases**
							☐	☐	☑
							☑	☐	☐
							☐	☑	☐
							☐	☑	☐

    b.   Example: ([Former NAEP question](#)) (DOK 1)

In the figure above, what is the measure of angle *DAC*?

    A.   47°
    B.   57°
    C.   80°
    D.   90°
    E.   137°

Answer: A

6.   Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **(7.G.B.6)** **(DOK 1,2)**

    a.   Example: [Solution](#) (DOK 2)

The 7th graders at Sunview Middle School were helping to renovate a playground for the kindergartners at a nearby elementary school. City regulations require that the sand underneath the swings be at least 15 inches deep. The sand under both swing sets was only 12 inches deep when they started.

The rectangular area under the small swing set measures 9 feet by 12 feet and required 40 bags of sand to increase the depth by 3 inches. How many bags of sand will the students need to cover the rectangular area under the large swing set if it is 1.5 times as long and 1.5 times as wide as the area under the small swing set?

b. Example: John needs to paint one wall in his school. He knows that 1 can of paint covers an area of 24 square feet. John uses a meter stick to measure the dimensions of the wall as shown.

1 m

1 m

1 m

1 m   1 m   1 m
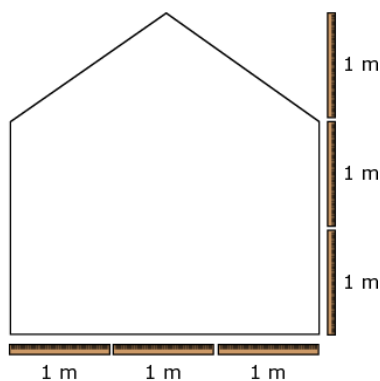
[1 meter = approximately 39 inches]

What is the **fewest** number of cans of paint John can use to paint the wall?

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#28	2	G, RP	B, A	2	7.G.B.6, 6.RP.A.3d	1	4

c. Example: (Former NAEP question) (DOK 1)

John is going to cover an attic floor with insulation. The floor measures 25 feet by 35 feet. If one roll of insulation will cover 64 square feet, how many rolls of insulation does John need?

A. 1
B. 2
C. 8
D. 14
E. 110

Answer: D

Use random sampling to draw inferences about a population. (7.SP.A)

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **(7.SP.A.1) (DOK 2)**

   a. Example: Solution (DOK 3)

   In a poll of Mr. Briggs's math class, 67% of the students say that math is their favorite academic subject. The editor of the school paper is in the class, and he wants to write an article for the paper saying that math is the most popular subject at the school. Explain why this is not a valid conclusion and suggest a way to gather better data to determine what subject is most popular.

   b. Example: A representative sample of 50 students from a high school is surveyed. Each student is asked what science class he or she is taking.
      This table shows the responses.

Science Class	Number of Students
Physics	6
Chemistry	10
Biology	18
Earth Science	4
Health Science	12

   Select **all** of the statements that are valid based on the survey results.

   a. About 20% of students at the high school are taking Chemistry.
   b. About twice as many students are taking Health Science than are taking Physics.
   c. For every 150 students we could predict that at least 18 of the students are taking Physics.
   d. For every 25 students we could predict that at least 4 of the students are taking Earth Science.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#7	1	SP	G	2	7.SP.A.1	N/A	☑ About 20% of students at the high school are taking Chemistry. ☑ About twice as many students are taking Health Science than are taking Physics. ☑ For every 150 students we could predict that at least 18 of the students are taking Physics. ☐ For every 25 students we could predict that at least 4 of the students are taking Earth Science.

   c. Example: (Former NAEP question) (DOK 2)

   A survey is to be taken in a city to determine the most popular sport. Would sampling opinions at a baseball game be a good way to collect this data? Explain your answer.

   Answer: No, this would result in a biased poll since people with certain characteristics would be at the baseball game. Likely, this poll would result in baseball's popularity being inflated.

2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **(7.SP.A.2) (DOK 2,3)**

   a. Example: Solution (DOK 3)

   A hotel holds a Valentine's Day contest where guests are invited to estimate the percentage of red marbles in a huge clear jar containing both red marbles and white marbles. There are 11,000 total marbles in the jar: 3696 are red, 7304 are white. The actual percentage of red marbles in the entire jar ($33.6\% = \frac{3696}{11000}$) is known to some members of the hotel staff.

   Any guest who makes an estimate that is within 9 percentage points of the true percentage of red marbles in the jar wins a prize, so any estimate from 24.6% to 42.6% will be considered a winner. To help with the estimating, a guest is allowed to take a random sample of 16 marbles from the jar in order to come up with an estimate. (Note: when this occurs, the marbles are then returned to the jar after counting.)

   One of the hotel employees who does not know that the true percentage of red marbles in the jar is 33.6% is asked to record the results of the first 100 random samples. A table and dotplot of the results appears below.

Percentage of red marbles in the sample of size 16	Number of times the percentage was obtained
12.50%	4
18.75%	8
25.00%	15
31.25%	22
37.50%	20
43.75%	12
50.00%	12
56.25%	4
62.50%	2
68.75%	1
Total:	100

Percentage of red marbles in a sample of size 16 (from 100 samples)

61
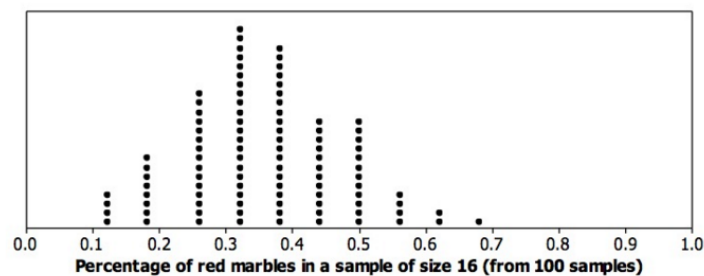
For example, 15 of the random samples had exactly 25.00% red marbles; only 2 of the random samples had exactly 62.50% red marbles, and so on.

a. Assuming that each of the 100 guests who took a random sample used their random sample's red marble percentage to estimate the whole jar's red marble percentage. Based on the table above, how many of these guests would be "winners"?

b. How many of the 100 guests obtained a sample that was *more than* half red marbles?

c. Should we be concerned that none of the samples had a red marble percentage of exactly 33.6% even though that value is the true red marble percentage for the whole jar? Explain briefly why a guest can't obtain a sample red marble percentage of 33.6% for a random sample of size 16.

d. Recall that the hotel employee who made the table and dotplot above didn't know that the real percentage of red marbles in the entire jar was 33.6%. If another person thought that half of the marbles in the jar were red, explain briefly how the hotel employee could use the dotplot and table results to challenge this person's claim. Specifically, what aspects of the table and dotplot would encourage the employee to challenge the claim?

e. Design a simulation that takes a large number of samples of size 16 from a population in which 65% of the members of the population have a particular characteristic. For each sample of size 16, compute the percentage of red items in the sample. Record these percentages, and then summarize all of your sample percentages using a table and dotplot similar to those shown above. In what ways is your dotplot similar to the dotplot used in this task? In what ways does it differ?

Draw informal comparative inferences about two populations. (7.SP.B)

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variability's, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **(7.SP.B.3) (DOK 2,3)**

    a. Example: Solution (DOK 3)

    Below are the heights of the players on the University of Maryland women's basketball team for the 2012-2013 season and the heights of the players on the women's field hockey team for the 2012 season. (Accessed at http://www.umterps.com/sports/w-fieldh/mtt/md-w-fieldh-mtt.html, http://www.umterps.com/sports/w-baskbl/mtt/md-w-baskbl-mtt.html on 1/13/13) Note: it is typical for a women's field hockey team to have more players than a women's basketball team would.

Field Hockey Player Heights (inches)	Basketball Player Heights (inches)
66	75
64	65
66	76
63	75
67	76
62	72
62	67
64	69
64	74
64	68
65	74
66	79
65	
64	
63	
62	
62	
68	
68	
66	
70	
67	
65	
62	
64	

a. Based on visual inspection of the dotplots, which group appears to have the larger average height? Which group appears to have the greater variability in the heights?

b. Compute the mean and mean absolute deviation (MAD) for each group. Do these values support your answers in part (a)?

c. How many of the 12 basketball players are shorter than the tallest field hockey player?

d. Imagine that an athlete from one of the two teams told you she needs to go to practice. You estimate that she is about 65 inches tall. If you had to pick, would you think that she was a field hockey player or that she was a basketball player? Explain your reasoning.

e. The women on the Maryland field hockey team are not a random sample of all female college field hockey players. Similarly, the women on the Maryland basketball team are not a random sample of all female college basketball players. However, for purposes of this task, suppose that these two groups can be regarded as random samples of all female college field hockey players and all female college basketball players, respectiviely. If these were random samples, would you think that female college basketball players are typically taller than female college field hockey players? Explain your decision using answers to the previous questions and/or additional analysis.

b. Example: [Solution](#) (DOK 3)

College football teams are grouped with similar teams into "divisions" (and in some cases, "subdivisions") based on many factors such as game attendance, level of competition, athletic department resources, and so on. Schools from the Football Bowl Subdivision (FBS, formerly known as Division 1-A) are typically much larger schools than schools of any other division in terms of enrollment and revenue. "Division III" is a division of schools with typically smaller enrollment and resources.

One particular position on a football team is called "offensive lineman," and it is generally believed that the offensive linemen of FBS schools are heavier on average than the offensive linemen of Division III schools.

For the 2012 season, the University of Mount Union Purple Raiders football team won the Division III National Football Championship while the University of Alabama Crimson Tide football team won the FBS National Championship. Below are the weights of the offensive linemen for both teams from that season.

(Accessed at http://athletics.mountunion.edu/sports/fball/2012-13/roster, http://www.rolltide.com/sports/m-footbl/mtt/alab-m-footbl-mtt.html on 1/14/13)

Alabama
277
265
292
303
303
320
300
313
267
288
311
280
302
335
310
290
312
340
292

Mount Union
250
250
290
260
270
270
310
290
280
315
280
295
300
300
260
255
300

Offensive Linemen - Weight (in pounds)

a. Based on visual inspection of the dotplots, which group appears to have the larger average weight? Does one group seem to have greater variability in its weights than the other, or do the two groups look similar in that regard?

b. Compute the mean and mean absolute deviation (MAD) for each group. Do your measures support your answers in part (a)?

c. Choose from the following to fill in the blank: "The average Alabama offensive lineman's weight is about _____ than the average Mount Union offensive lineman's weight."

    i. 20 pounds lighter

    ii. 15 pounds lighter

    iii. 15 pounds heavier

    iv. 20 pounds heavier

"This difference in average weights is approximately _____ of either team."

    i. About half of the MAD

    ii. Slightly more than 1 MAD

    iii. Twice the MAD

d. The offensive linemen on the Alabama team are not a random sample from all FBS offensive linemen. Similarly, the offensive linemen on the Mount Union Team are not a random sample from all Division III offensive linemen. However, for purposes of this task, suppose that these two groups can be regarded as random samples of offensive linemen from their respective divisions/subdivisions. If these were random samples, would you think that offensive linemen from FBS schools are typically heavier than offensive linemen from Division III schools? Explain your decision using answers to the previous questions and/or additional analysis.

c. Example: Mr. Anthony wants to know how some student athletes are improving in the number of push-ups they can do.
These dot plots show the number of push-ups each student was able to do last month and this month.

10   15   20   25   30   35   40   45

Number of Push-ups Last Month

10   15   20   25   30   35   40   45
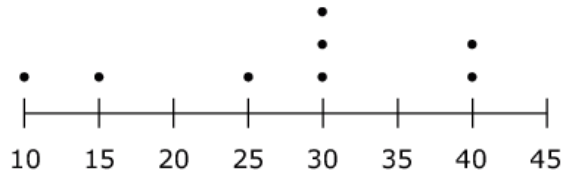
Number of Push-ups This Month

What is the increase in the mean number of push-ups from last month to this month?

4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **(7.SP.B.4) (DOK 2,3)**
   a. Example: Solution (DOK 3)

   Below are the heights of the players on the University of Maryland women's basketball team for the 2012-2013 season and the heights of the players on the women's field hockey team for the 2012 season. (Accessed at http://www.umterps.com/sports/w-fieldh/mtt/md-w-fieldh-mtt.html, http://www.umterps.com/sports/w-baskbl/mtt/md-w-baskbl-mtt.html on 1/13/13) Note: it is typical for a women's field hockey team to have more players than a women's basketball team would.

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64	74
64	68
65	74
66	79
65	

64
63
62
62
68
68
66
70
67
65
62
64

Heights (in inches)

a. Based on visual inspection of the dotplots, which group appears to have the larger average height? Which group appears to have the greater variability in the heights?

b. Compute the mean and mean absolute deviation (MAD) for each group. Do these values support your answers in part (a)?
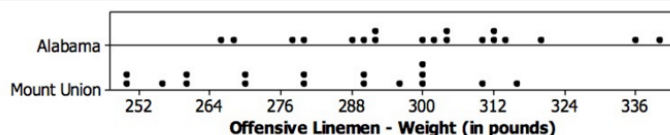
c. How many of the 12 basketball players are shorter than the tallest field hockey player?

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e. The women on the Maryland field hockey team are not a random sample of all female college field hockey players. Similarly, the women on the Maryland basketball team are not a random sample of all female college basketball players. However, for purposes of this task, suppose that these two groups can be regarded as random samples of all female college field hockey players and all female college basketball players, respectiviely. If these were random samples, would you think that female college basketball players are typically taller than female college field hockey players? Explain your decision using answers to the previous questions and/or additional analysis.

b. Example: Solution (DOK 3)

Alabama
277
265
292
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280
302
335
310
290
312
340
292

Mount Union
250
250
290
260
270
270
310
290
280
315
280
295
300
300
260
255
300

Offensive Linemen - Weight (in pounds)

a. Based on visual inspection of the dotplots, which group appears to have the larger average weight? Does one group seem to have greater variability in its weights than the other, or do the two groups look similar in that regard?

b. Compute the mean and mean absolute deviation (MAD) for each group. Do your measures support your answers in part (a)?

c. Choose from the following to fill in the blank: "The average Alabama offensive lineman's weight is about _____ than the average Mount Union offensive lineman's weight."

    i. 20 pounds lighter

    ii. 15 pounds lighter

    iii. 15 pounds heavier

    iv. 20 pounds heavier

"This difference in average weights is approximately _____ of either team."

    i. About half of the MAD

    ii. Slightly more than 1 MAD

    iii. Twice the MAD

d. The offensive linemen on the Alabama team are not a random sample from all FBS offensive linemen. Similarly, the offensive linemen on the Mount Union Team are not a random sample from all Division III offensive linemen. However, for purposes of this task, suppose that these two groups can be regarded as random samples of offensive linemen from their respective divisions/subdivisions. If these were random samples, would you think that offensive linemen from FBS schools are typically heavier than offensive linemen from Division III schools? Explain your decision using answers to the previous questions and/or additional analysis.

c. Example: Carrie's basketball team has played 5 games. The number of points Carrie scored in each game is shown in the bar graph.
- Determine possible point totals for games 6 and 7 so that the range of data set increases, but the mean and median stay the same.
- Draw bars above the labels 6 and 7 to complete the bar graph.

70

Carrie's Points
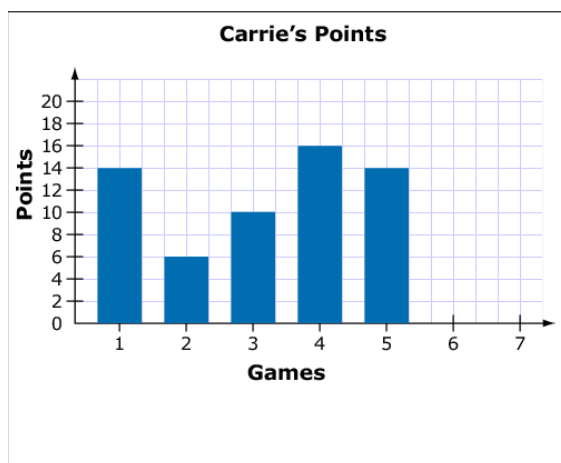
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#29	4	SP	E, C	3	7.SP.B.4, 6.SP.B.5	1, 4	

Investigate chance processes and develop, use, and evaluate probability models. (7.SP.C)

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **(7.SP.C.5) (DOK 1)**

    a. The spinner has 8 equal-sized sections, each labeled 1, 2, 3, or 4. The arrow on the spinner is spun.
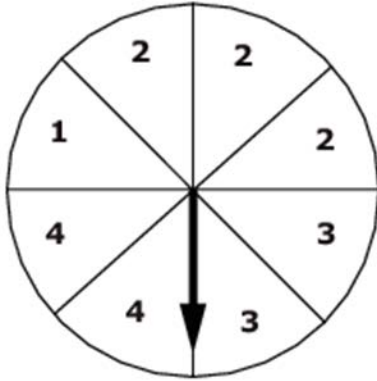
What is the probability of the arrow stopping on a section labeled with a 2?

a. $\dfrac{1}{4}$

b. $\dfrac{1}{8}$

c. $\dfrac{3}{8}$

d. $\dfrac{3}{4}$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#4	1	SP	I	1	7.SP.C.5	N/A	C

b. Example: ([Former NAEP question](#)) (DOK 3)
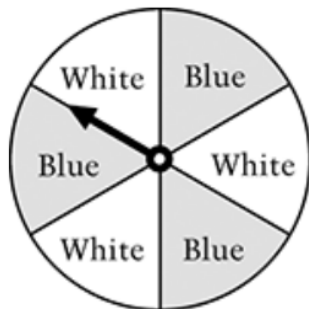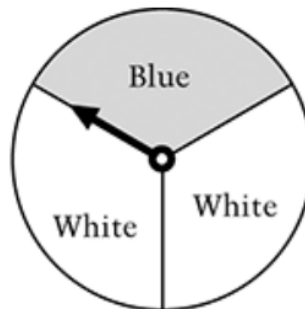
Spinner A   Spinner B

Lori has a choice of two spinners. She wants the one that gives her a greater probability of landing on blue.

Which spinner should she choose?

⬭ Spinner A   ⬭ Spinner B

Explain why the spinner you chose gives Lori the greater probability of landing on blue.

Answer: Spinner A - Because she would have a 50% chance of landing on blue for Spinner A, whereas she would only have a 33% chance for landing on blue for Spinner B.

c. Example: ([Former NAEP question](#)) (DOK 1)

Marty has 6 red pencils, 4 green pencils, and 5 blue pencils. If he picks out one pencil without looking, what is the probability that the pencil he picks will be green?

    A. 1 out of 3
    B. 1 out of 4
    C. 1 out of 15
    D. 4 out of 15

Answer: D

d. Example: ([Former NAEP question](#)) (DOK 1)

A person is going to pick one marble without looking. For which dish is there the greatest probability of picking a black marble?

A.

B.

C.

D.
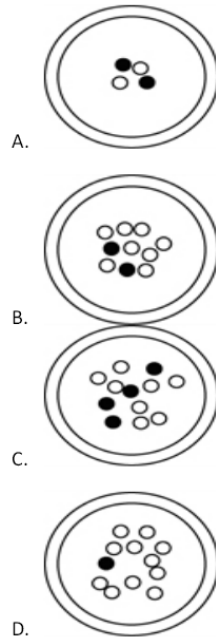
Answer: A

e. Example: ([Former NAEP question](#)) (DOK 1)
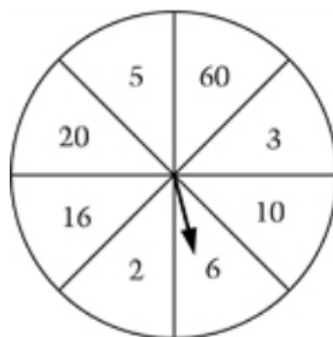
If the arrow is spun and stops in one of the 8 spaces, what is the probability that the arrow will stop in the space labeled 6?

    A. 1 out of 6
    B. 1 out of 8
    C. 1 out of 10
    D. 1 out of 60

Answer: B

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **(7.SP.C.6)** **(DOK 2,3)**

   a. Example: Solution (DOK 2)

   Think about a cylindrical (or cylinder-like) object, such as a bottle lid or a roll of tape. Some possible objects are shown in the picture below. Suppose you were to toss one of these objects into the air and observe its landing position once it reaches the floor.

   a. For your object, what are the possible outcomes of this experiment?

   b. Make a guess – what are the probabilities of each of the possible outcomes?

   c. Toss the object into the air and record the outcome. Repeat this process 25 to 30 times.

   d. Determine the experimental probability of each outcome. How does this experimental probability compare to your guess from part b? Based on this information, would you like to change your guess?

   e. Repeat this activity with a different cylindrical (or cylinder-like) object that you think has a greater probability of landing on its side than your first object.

b. Example: [Solution](#) (DOK 2)
    Roll two dice 10 times. After each roll, note whether any sixes were observed and record your results in the table below.

Roll	Any Sixes? (Y/N)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

a. What fraction of the 10 rolls resulted in at least one six?

b. Combine your results with those of your classmates. What fraction of all the rolls in the class resulted in at least one six?

c. Make a list of all the different possible outcomes that might be observed when two dice are rolled. (Hint: There are 36 different possible outcomes.)

d. What fraction of the 36 possible outcomes result in at least one six?

e. Suppose you and your classmates were able to roll the two dice many thousands of times. What fraction of the time would you expect to roll at least one six?

c. Example: [Solution](#) (DOK 3)

Each of the 20 students in Mr. Anderson's class flipped a coin ten times and recorded how many times it came out heads.

a. How many heads do you think you will see out of ten tosses?

b. Would it surprise you to see 4 heads out of ten tosses? Explain why or why not.

c. Here are the results for the twenty students in Mr. Anderson's class. Use this data to estimate the probability of observing 4, 5 or 6 heads in ten tosses of the coin. (It might help to organize the data in a table or in a dot plot first.)

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number of heads	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

d. Example: (Former NAEP question) (DOK 1)

A quarter is flipped 50 times. Which of the following is most likely to be the number of times heads comes up?
- A. 2
- B. 3
- C. 11
- D. 26
- E. 50

Answer: D

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **(7.SP.C.7)** **(DOK 2,3)**
      i. Example: Solution (DOK 2)

Look at the shirt you are wearing today, and determine how many buttons it has. Then complete the following table for all the members of your class.

	No Buttons	One or Two Buttons	Three or Four Buttons	More Than Four Buttons
Male				
Female				

Suppose each student writes his or her name on an index card, and one card is selected randomly.

a. What is the probability that the student whose card is selected is wearing a shirt with no buttons?

b. What is the probability that the student whose card is selected is female and is wearing a shirt with two or fewer buttons?

ii. Example: Solution (DOK 2)

Roll two dice 10 times. After each roll, note whether any sixes were observed and record your results in the table below.

Roll	Any Sixes? (Y/N)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

a. What fraction of the 10 rolls resulted in at least one six?

b. Combine your results with those of your classmates. What fraction of all the rolls in the class resulted in at least one six?

c. Make a list of all the different possible outcomes that might be observed when two dice are rolled. (Hint: There are 36 different possible outcomes.)

d. What fraction of the 36 possible outcomes result in at least one six?

e. Suppose you and your classmates were able to roll the two dice many thousands of times. What fraction of the time would you expect to roll at least one six?

iii. Example: (Former NAEP question) (DOK 1)
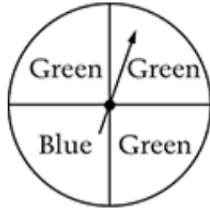
Ms. Livingston's class spins the arrow on the spinner 92 times. Of the following, which is the most likely result?

A. 66 green, 26 blue
B. 46 green, 46 blue
C. 23 green, 69 blue
D. 2 green, 90 blue

Answer: A

iv.   Example: ([Former NAEP question](#)) (DOK 1)

One student will be chosen at random from the list above. What is the probability that the student's name begins with the letter *P*?

A. 1 out of 3
B. 1 out of 10
C. 3 out of 7
D. 3 out of 10

Answer: D

v.   Example: ([Former NAEP question](#)) (DOK 1)

There are 6 cubes of the same size in a jar.
 2 cubes are yellow.
 3 cubes are red.
 1 cube is blue.

Chuck is going to pick one cube without looking. Which color is he most likely to pick?

What is the probability of this color being picked?
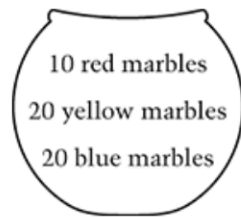
Answer: Red; 1 out of 2 chance of red being picked

vi.   Example: ([Former NAEP question](#)) (DOK 1)

A pencil, 2 pens, and a marker are in a drawer. Leah picks one of these without looking. What are her chances of picking out the pencil?

A. 1 out of 1
B. 1 out of 2
C. 1 out of 3
D. 1 out of 4

Answer: B

vii.   Example: ([Former NAEP question](#)) (DOK 3)

The bowl above contains the indicated number of marbles. The marbles are well mixed in this bowl. Juan will randomly pick a marble from the bowl. Juan believes that his chance of picking a blue marble is the same as his chance of picking a yellow marble. Is Juan correct?

Fill in the correct oval below.

◯ Yes   ◯ No

Explain your answer.

Answer: Yes, because there are equal numbers of yellow and blue marbles (yellow = 20; blue = 20)

viii.   Example: ([Former NAEP question](#)) (DOK 2)

Each of the 6 faces of a fair cube is painted red, yellow, or blue. This cube is rolled 500 times. The table below shows the number of times each color landed faced up.

Color	Red	Yellow	Blue
Total	100	340	60

Based on these results, what is the most likely number of yellow faces on the cube?
A. One
B. Two
C. Three
D. Four
E. Six

Answer: D

ix.   Example: ([Former NAEP question](#)) (DOK 1)

$$(1, 1) \quad (2, 1) \quad (3, 1)$$
$$(1, 2) \quad (2, 2) \quad (3, 2)$$
$$(1, 3) \quad (2, 3) \quad (3, 3)$$

A pair of numbers will be chosen at random from the list above. What is the probability that the first number in the pair will be less than the second number in the pair?

Answer: 1 out of 3 chance

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
     a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
     b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
     c. Design and use a simulation to generate frequencies for compound events. *For example, use*

random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? **(7.SP.C.8)** **(DOK 1,2,3)**

1. Example: [Solution](DOK 2)

   Suppose each box of a popular brand of cereal contains a pen as a prize. The pens come in four colors, blue, red, green and yellow. Each color of pen is equally likely to appear in any box of cereal. Design and carry out a simulation to help you answer each of the following questions.

   a. What is the probability of having to buy at least five boxes of cereal to get a blue pen? What is the mean (average) number of boxes you would have to buy to get a blue pen if you repeated the process many times?

   b. What is the probability of having to buy at least ten boxes of cereal to get a full set of pens (all four colors)? What is the mean (average) number of boxes you would have to buy to get a full set of pens if you repeated the process many times?

2. Example: [Solution](DOK 2)

   Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the theoretical probability that Angie and Carlos are seated opposite each other?

3. Example: [Solution](DOK 2)

   A fair six-sided die is rolled twice. What is the theoretical probability that the first number that comes up is greater than or equal to the second number?

4. Example: [Solution](DOK 2)

   Many games use dice which are six-sided and fair (meaning each face on the die is equally likely to land face up). Many games also use the sum of two dice rolled at the same time to determine movement of game pieces. However, not all dice are six-sided. Imagine a game in which two fair four-sided (tetrahedral) dice are rolled simultaneously. These dice are in the shape of a pyramid, and when a die is rolled, the outcome is determined by the side that lands face down. Suppose that for these two dice, the possible values (corresponding to the four sides of the die) that can be obtained from each die are as follows:

   Die #1: 1, 2, 3, or 4
   Die #2: 2, 4, 6, or 8

   a. A certain game determines the movement of players' game pieces based on the SUM of the numbers on the face down sides when two dice are rolled. There are 10 distinct sum values that can occur, and some of those sums occur more often than others.

   i. Using an organized list, table, tree diagram, or method of your choosing, develop a list of all 16 possible outcomes (for example, Die #1 = 1 and Die #2 = 2 for a sum of 3; Die #1 = 1 and Die #2 = 4 for a sum of 5; and so on).

   ii. From your work in part i, determine the 10 **distinct sum values** that are possible and calculate the probability of obtaining each sum value. Note: as mentioned above, some values will occur more frequently than others.

iii. Using your work in part ii, answer the following questions:
What is the probability of obtaining a sum of 5?
What is the probability of obtaining a sum that is more than 5?
What is the probability of obtaining a sum that is at most 5?
What is the probability of obtaining a sum that is at least 5?
What is the probability of obtaining a sum that is no less than 5?

b. Now consider the case where the DIFFERENCE in the numbers on the face down sides when two dice are rolled is important to the game. Unless the two die values are the same (in which case the difference is 0), the difference for purposes of this game will always be computed as the larger number value rolled minus the smaller number value rolled. In this way, the difference value for any roll of the two dice will always be positive or 0.

i. Using an organized list, table, tree diagram, or method of your choosing, develop a list of all 16 possible outcomes (for example, Die #1 = 1 and Die #2 = 2 for a difference of 1; Die #1 = 1 and Die #2 = 4 for a difference of 3; and so on).

ii. From your work in part i, determine the 8 distinct difference values that are possible and calculate the probability of obtaining each difference value. Note: as mentioned above, some values will occur more frequently than others.

iii. Using your work in part e, answer the following questions:
What is the probability of obtaining a difference of 5?
What is the probability of obtaining a difference that is more than 5?

What is the probability of obtaining a difference that is less than or equal to 5?

5. Example: [Solution](DOK 3)

This is a game for two people.

You have three dice; one is red, one is green, and one is blue. These dice are different than regular six-sided dice, which show each of the numbers 1 to 6 exactly once. The red die, for example, has 3 dots on each of five sides, and 6 dots on the other. The number of dots on each side are shown in the table and picture below.

Red	3	3	3	3	3	6
Green	2	2	2	5	5	5
Blue	1	4	4	4	4	4
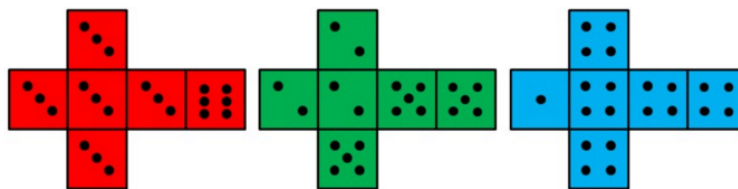
To play the game, each person picks one of the three dice. However, they have to pick different colors.

• The two players both roll their dice. The highest number wins the round.

• The players roll their dice 30 times, keeping track of who wins each round.

• Whoever has won the greatest number of rounds after 30 rolls wins the game.

a. Who is more likely to win when a person with the red die plays against a person with the green die? What about green vs. blue? What about blue vs. red?

b. Would you rather be the first person to pick a die or the second person? Explain.

6. Example: ([Former NAEP question]) (DOK 1)

How many different three-digit whole numbers can be written using each of the digits 4, 5, and 6 exactly once?

    A.   3
    B.   6
    C.   9
    D. 24
    E. 27

Answer: B

Performance Task Example:

You are a volunteer at International Food Assistance. This organization delivers "food baskets" to help people around the world. The requirements for each food basket are shown below.

Here are the requirements for each food basket:
- Contains grains such as rice, wheat or oatmeal
- Contains legumes such as kidney beans, nuts, or lentils
- Contains exactly 35 grams (g) of oil for cooking
- Contains exactly 50 grams (g) of Super Cereal
- Has a minimum of 2100 total calories
- At least 8% of the total calories come from protein
- At least 10% of the total calories come from fat
- The cost of each basket cannot exceed $0.75

Here are the contents and quantities of a **Sample Food Basket:**

Food	Quantity	Calories	Protein (1 g = 4 calories)	Fat (1 g = 9 calories)	Cost per kilogram
Rice	800 g	920	9 g	2 g	$0.58
Lentils	240 g	812	34 g	2 g	$0.90
Oil	35 g	315	0 g	35 g	$1.20
Super Cereal	50 g	200	10 g	5 g	$0.12

This assessment has four questions about planning food baskets. You will examine factors such as nutrition and food prices. The final question requires you to design a food basket using the interactive simulation table. Read and answer each question.

Nutritional Value and Cost of Wheat and Oatmeal

Grain (Oatmeal ▼)

Quantity (g) (100 ▼)

[Start]

[🗑 Clear All Rows]

Grain	Quantity (g)	Calories	Protein (g) (1 g = 4 calories)	Fat (g) (1 g = 9 calories)	Cost per kilogram
					🗑

1. Create an expression to calculate the number of calories from fat in the **Sample Food Basket.**
   For this item, a full-credit response (1 point) includes

   $(2 + 2 + 35 + 5) \times 9$ (and equivalent expressions).

   For this item, a no-credit response (0 points) includes none of the features of a full-credit response.

2. Create an expression to calculate the percent of total calories from protein in the **Sample Food Basket.**

   For this item, a full-credit response (1 point) includes

$$100 \times \frac{((9 + 34 + 10) \times 4)}{(920 + 812 + 315 + 200)}$$ (and equivalent expressions).

For this item, a no-credit response (0 points) includes none of the features of a full-credit response.

3. Explain how the **Sample Food Basket** does not meet all of the requirements for a food basket. Write your answer in the space provided. Use specific numbers in your explanation.

For this item, a full-credit response (1 point) includes

- confirming that there are enough quantities of each of the four requirements for the food basket AND

- referring to the quantity (correctly or incorrectly) determined in item 1496 as well as the percentage of total calories from fat.
For example,

- The food basket meets the requirements. The percentage of protein calories is about 9%. The percentage of fat is about 18%. 9% is greater than 8% and 18% is greater than 10%. There is also enough oil and super cereal.

For this item, a no-credit response (0 points) includes none of the features of a full-credit response.
For example,

- The food basket meets the requirements. There is enough protein, fat, oil and super cereal.

This item is not graded on spelling or grammar.

4.

Bad weather is damaging rice crops, so you need to use wheat **or** oatmeal as the grain requirement in the food baskets. Enter different quantities in the table **Nutritional Value and Cost of Wheat and Oatmeal** to explore the changes in calories, protein, fat, and cost of replacing rice with wheat or oatmeal.

Using your information from exploring in the table **Nutritional Value and Cost of Wheat and Oatmeal**, you need to make a new food basket.

Part A
Determine the contents of a new basket that uses wheat **or** oatmeal instead of rice and meets all of the requirements. Enter your information in all six blank cells in the table.

Food	Quantity	Calories	Protein (1 g = 4 calories)	Fat (1 g = 9 calories)	Cost per kilogram
Lentils	240 g	812	34 g	2 g	$0.90
Oil	35 g	315	0 g	35 g	$1.20
Super Cereal	50 g	200	10 g	5 g	$0.12

Part B
Explain how your new basket meets all of the requirements for a food basket.

For this item, a full-credit response (2 points) includes:
Part A

- entering a correct quantity of wheat (100 g to 600 g) or oatmeal (100 g to 400 g).
For example:

Food	Quantity	Calories	Protein (1 g = 4 calories)	Fat (1 g = 9 calories)	Cost per kilogram
Wheat	400 g	1360	56 g	12 g	$0.75
Lentils	240 g	812	34 g	2 g	$0.90
Oil	35 g	315	0 g	35 g	$1.20
Super Cereal	50 g	200	10 g	5 g	$0.12

Part B

● explaining that the values in the table meet the requirements for the food basket, by comparing the protein, fat, and cost values to those required.

For example,

● "My basket contains 100 grams of protein and 54 grams of fat. 100 grams of protein is equal to 400 calories. There are 2687 calories total in my basket. 400/2687 = 14.89% calories from protein. 54 grams of fat is equal to 486 calories. 486/2687 = 18.09% calories from fat. 14.89 > 8 and 18.09 > 10. The total cost of my basket should be around $0.56, so it meets the cost requirement."

For this item, a partial-credit response (1 point) includes

● not completing the table in Part A, but noting the correct quantities in Part B

OR

● not making ALL necessary comparisons in Part B, but completing the table in Part A (with consistent quantities) OR

● Entering an incorrect set of quantities in the table in Part A, but making consistently incorrect comparisons in Part B.

For this item, a no-credit response (0 points) includes none of the features of a full- or partial-credit response.

This item is not graded on spelling or grammar