Grade/Subject	Grade 8/ Mathematics
Unit Title	Unit 4: Functions
Overview of Unit	Using functions, students will define, evaluate, and compare model relationships between quantities. Students will demonstrate their understanding through tables and graphs. Included are examples from real world applications and verbal descriptions.
Pacing	15 days

Background Information For The Teacher

Developing an understanding of functions, primarily linear, gives students a way to evaluate expressions, discuss their properties, and construct and apply operations with functions to have a deeper understanding of real world situations. Function understanding is a fundamental concept in algebra courses; so 8th grade function standards are the foundation for Algebra I.

Students will use their prior knowledge of proportional relationships, lines, and linear equations to define, evaluate, and compare functions. Students will also be able to use functions to model relationships between quantities. This will guide the students to understand that there are many real world applications for rate of change, y-intercepts, and linear relationships.

There has been a shift to ensure that 8th grade students first develop a strong understanding of functions and their properties prior to Algebra I. This understanding will assist the students in developing the Standards for Mathematical Practice, specifically SMP #4: Model with mathematics, SMP #7: Look for and make use of structure and SMP #8: Look for and express regularity in repeated reasoning.

Possible Teacher Misconceptions

Students are not deriving slope-intercept form in this unit. They should be given this form to use for all functions. Students are only identifying non-linear functions, and not deriving them. Teachers should not refer to a curve as a curved line, due to the mathematical definition of a line.

Essential Questions (and Corresponding Big Ideas)	
How can mathematics be used to measure, model and calculate change? • *There are many real world applications for rate of change and solving linear relationships. Linear relationships can be modeled by algebraic, tabular, graphical, or verbal descriptions.* How can linear relationships influence your real world decision-making? • *Rate of change (slope) and the y-intercept define important applications in real world situations.* Why is it valuable to understand a situation in multiple representations? • *The various methods to display linear relationships provide opportunities to understand a situation more thoroughly.*	
Core Content Standards	**Explanations and Examples**
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Note: Function notation is not required for Grade 8. This standard is the students' introduction to functions and involves the definition of function as a rule that assigns to each input exactly one output. Students are not required to use or recognize function notation at this grade but will be able to identify functions using tables, graphs, and	8.F.1 For example, the rule that takes x as input and gives x^2+5x+4 as output is a function. Using y to stand for the output we can represent this function with the equation $y = x^2+5x+4$, and the graph of the equation is the graph of the function. Students are not yet expected use function notation such as $f(x) = x^2+5x+4$.

equations.

A relationship is not a function when there is more than one y value associated with any x value. Using the definition, an example of a table that does not represent a function is as follows:

x	y
2	3
1	4
-1	3
2	5
Not a Function	

X	y
0	1
1	3
-1	-1
0	1
Function	

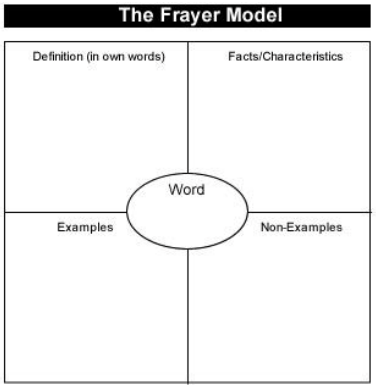
What the Teacher does:

- Provide graphs of relationships, some of which are functions and some not. Each graph should have a context so that students can reason whether or not the graph makes sense. For example, it does not make sense that a plane can be at different heights at the same point in time. Display this graphically and in a table to see that it is not a function.
  Allow students to make sense of other graphs where a rule assigns to each input exactly one out put.
- Present students with tables of relationships, some of which are functions and some are not. Encourage students to reason whether the example is a function or not and justify their conclusion. Do not limit examples to linear relationships.
- Compare graphs of functions and non-functional relationships with their graphs. Discuss what students notice.

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in

What the Students do:

- Reason whether a table or graph models a function or not and defend their reasoning.
- Use an advance organizer such as the Frayer model to clarify the definition of the function.



Misconceptions and Common Errors:

Students sometimes confuse the terms input and output, knowing that each input can have only one output. Function machines may help these students see that if you put in (input) a number in the machine, the rule only allows one number to be put out (output). Students can make or draw their own function machines.

tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

For this standard students will compare the properties of functions. One property of functions is slope. When students are given two different functions, each represented in a different form (algebraically, graphically, in a table, or by a verbal description), students should be able to determine which function has the greater slope. An example follows:

Ruth starts with a $40 gift card for Wal-Mart. She spends $5.50 per week to buy cat food. Let y be the amount left on the card and x represents the number of weeks.

x	y
0	50
1	44.50
2	39.00
3	33.50
4	28.00

Boyce rents bikes for $5 an hour. He also collects non-refundable fee of $10.00 for a rental to cover wear and tear. Write the rule for the total cost (c) of renting a bike as function of the number of hours (h) rented.

Solution: Ruth's story is an example of a function with negative slope. The amount if money left on the card decreases each week. The graph has a negative slope of -5.5, which is the amount the card balance decreases every time Ruth buys cat food.

Boyce's bike rental is an example of a function with a positive slope. This function has a positive slope of 5, which is the amount to rent a bike for an hour. An equation for Boyce's bike could be c = 5h + 10.

What the **teacher** does:

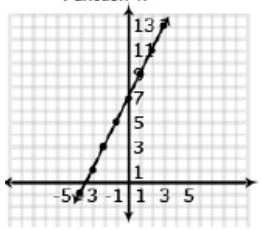
- Present two different linear functions using the same representation (algebraically, graphically, in a table, or by

8.F.2

Examples:

- Compare the two linear functions listed below and determine which equation represents a greater rate of change.

Function 1:



Function 2:

The function whose input x and output y are related by

$y = 3x + 7$

- Compare the two linear functions listed below and determine which has a negative slope.

Function 1: Gift Card

Samantha starts with $20 on a gift card for the book store. She spends $3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks.

x	*y*
0	20
1	16.50
2	13.00
3	9.50
4	6.00

Function 2:

The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (*m*).

Solution:

verbal description). Ask the students if they can explain which has the greater slope (rate of change).

- Present two functions each represented in a different form and ask the students to work in groups to determine which has the greater slope. They may need some time to work in groups to change the representation of the functions. Have groups present their answers to the class along with their reasoning. Facilitate the discussion with questions such as, How did you determine which slope is greater? Why did you select to represent the functions in a different form?
- Present two different functions in similar context so that the question about comparing the slopes has meaning.

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

In this standard students become familiar with the equation y = mx + b as defining a linear function that will graph as a straight line. Students distinguish between linear (functions that graph into a straight line) and nonlinear functions (functions that do not graph into a straight line such a curve). Note that standard form and point-slope form are not studied in this grade.

What the Teacher does:

- Present students with examples of functions that are linear and nonlinear for them to graph. Facilitate a class discussion about the similarities and differences in the graphs. The graphs that are not linear are those with points not on a straight line. The area of a square as function of its side length, $A = s^2$, is an example of a nonlinear function because points (1,1), (2,4) and (3,9) are not on a straight line.
- Present a series of linear equations such as the following:

$$y = \frac{1}{2}x + 7$$

Function 1 is an example of a function whose graph has negative slope. Samantha starts with $20 and spends money each week. The amount of money left on the gift card decreases each week. The graph has a negative slope of -3.5, which is the amount the gift card balance decreases with Samantha's weekly magazine purchase. Function 2 is an example of a function whose graph has positive slope. Students pay a yearly nonrefundable fee for renting the calculator and pay $5 for each month they rent the calculator. This function has a positive slope of 5 which is the amount of the monthly rental fee. An equation for Example 2 could be $c = 5m + 10$.

What the **students** do:

- Compare properties of functions presented in the same and different forms.
- Communicate the reasoning involved in comparing two functions using precise mathmatical language.

Misconceptions and Common Errors:

A common error students make when working with slopes in context understands what the slope represents. If students are having this problem, work with a single function in a context and then, after identifying the slope and its meaning, add a second function in the same context so that students can work a with the second slope separately before comparing the first slope.

8.F.3 Example:

• Determine which of the functions listed below are linear and which are not linear and explain your reasoning.

- o $y = -2x^2 + 3$ non linear
- o $y = 2x$ linear
- o $A = \pi r^2$ non linear
- o $y = 0.25 + 0.5(x - 2)$ linear

$$y = -4x + 8$$
$$y = 6x - 2$$
$$y = 0.5 + 5$$

Ask students to find the similarities and differences among the equations and their graphs. Facilitate a discussion that results in students recognizing the structure and naming y=mx + b as the general equation for a linear function. Point out that when using a graphing calculator, the general equation for a line is usually expressed as y = ax + b.

- Present some linear equations in the form y = b + mx as many contextual problems will present information in this order. Ask students to write examples of linear functions. This may be a group challenge allowing groups to present their work to the class using correct terminology.

- 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Students identify that rate of change (slope) and y-intercept (initial value) from tables, graphs, equations, and verbal descriptions of linear relationships. The y-intercept is the y value when the x value is 0. Interpretation of slope and the initial value of the functions is accomplished using real-world situations.

What a Teacher does:

- Present students with graphs of linear functions and focus a discussion on the y-intercept. From examples, lead students to discover that the y-intercept is the y value when the x value is 0. Provide students with opportunities to identify the

What the Students do:

- Discern the similarities and differences between linear and nonlinear graphs.
- Look for and make use of structure in identifying y = mx + b as the general form of an equation for a straight line.
- Model functions that are nonlinear and explain, using precise mathematical language, how to tell the difference.

Misconceptions and Common Errors:

Some students have difficulty with the general equation y = mx + b for equations presented as subtraction such as y = 5x – 4. Students can be asked to graph a series of such equations to convince themselves that they are linear. In addition, point out that minus 4 is the same as adding -4.

y-intercept on several graphs.

- Pose the following challenge: Show a table for each of the graphs recently presented and identify the y-intercept in the table. For the table below, the y-intercept is (0,4).

X	Y
-2	-2
0	4
1	7

- Ask students to find the equations for the linear functions previously used and see if they can figure out how to find the y-intercept when the function is in equation form. (It is the constant in the equation y = mx + b) Present some equations where the format is y = b + mx. Present some equations where the y-intercept is negative.

- Provide context as much as possible so that students learn to interpret the meaning of the initial value in a function.
- Explain slope of a line by presenting a graph of a linear equation and introducing the slope as the ration of the change in they y values of two points to the change in the x value of the same two points. Relate this back to unit rate fro Grade 6.
- Display tables for students to use to determine rate of change using the rise to run ration, such as the following:

X	Y
-2	-2
0	4
1	7

Select any points for example (-2,-2) and (1,7). The difference between -2 and 7 is -9.

- Provide students with the opportunity to discover that coefficient of x in the equation y = mx + b is the slop by allowing them to look at the tables, calculate slope, and compare to the equations of the lines.

8.F.4

Examples:

- The table below shows the cost of renting a car. The company charges $45 a day for the car as well as charging a one-time $25 fee for the car's navigation system (GPS). Write an expression for the cost in dollars, *c,* as a function of the number of days, *d*.

Students might write the equation $c = 45d + 25$ using the verbal description or by first making a table.

Days (*d*)	Cost (*c*) in dollars
1	70
2	115
3	160
4	205

Students should recognize that the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one time fees vs. recurrent fees will help students model contextual situations.

- When scuba divers come back to the surface of the water, they need to be careful not to ascend too quickly. Divers should not come to the surface more quickly than a rate of 0.75 ft per second. If the divers start at a depth of 100 feet, the equation $d = 0.75t - 100$ shows the relationship between the time of the ascent in seconds (*t)* and the distance from the surface in feet (*d*).
  - Will they be at the surface in 5 minutes? How long will it take the divers to surface from their dive?
- Make a table of values showing several times and the corresponding distance of the divers from the surface. Explain what your table shows. How do the values in the table relate to your equation?

- Provide context as often as possible so that students can interpret the meaning of the slope in a given situation.
- Provide students verbal descriptions of situations where they can create the equation of the function. Identify the slope and initial value and relate them to the y= mx + b general equation.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Given a graph, students will provide a verbal description of the function, including whether the graph is linear or nonlinear or where the function is increasing or decreasing. Given a function's verbal description, students will be able to sketch the graph displaying qualitative properties of that function.

What a Teacher does:

- Model the use of mathematical vocabulary to describe the parts of the graph that are linear, increasing, decreasing, and son on.
- Present students with a graph and ask them to tell/write the story and label the axes. A classic example is to write a story about the height of the water in a bathtub over time to match a graph.
- Provide students with opportunities to sketch graphs given the stories.
- Allow students to create their own graphs and stories to share with the class.
- Select stories where the graph may appear counterintuitive such as the graph of plane's distance from its destination city to its time in the air. This graph has a negative slope since as the time increases, the distance to the destination decreases.

What the Students do:

- Discover the y-intercept (initial value of a function) from a function represented in table, graph, algebraic form and by verbal descriptions.
- Calculate slope of a line using the rise to run ratio.
- Discover slope of a line when the function is presented in a table, graph, algebraic (equation) form, or by verbal descriptions.
- Communicate the meaning of the slope and y-intercept in a given situation using precise mathematical vocabulary.

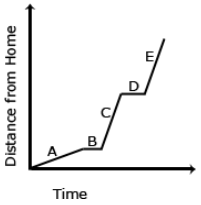
Student Misconceptions and Common Errors:

The most common error students make is confusing the rise and run in the ration for slope. This mistake is easily observed as students calculate slope. Vocabulary foldables using the terms rise and run may help students remember the differences.

8.F.5
Example:

- The graph below shows a student's trip to school. This student walks to his friend's house and, together, they ride a bus to school. The bus stops once before arriving at school.

Describe how each part A-E of the graph relates to the story.



What the Students do:

- Model stories with graphs and vice versa.
- Interpret paths of a story to coincide with parts of the function displayed on a graph.
- Sketch a graph that shows the qualitative features of a function described verbally
- Create a story that matches the qualitative features of a given graph.

Student Misconceptions and Common Errors:
A common error students make is that they do not read the labels on the axes carefully. Eight graders who sketch graphs that appear counterintuitive from the story are making assumptions about the axes without analyzing them. These students should be asked to describe what the axes mean on a graph before they begin to analyze or write story.

Standards for Mathematical Practice	**Explanations and Examples**
Define, evaluate and compare functions. **8.F.1, 8.F.2, 8.F.3** Students are introduced to functions as rules that assign exactly one output to each input. Functions are represented graphically algebraically, numerically in tables, and by verbal descriptions. Function notation (f(x)) is not required in Grade 8. Students compare properties of two functions represented in different forms such as determining which function has a greater rate of change from two functions, one represented graphically and one numerically n tables. Students recognize equations in the form of y = mx + b as defining linear functions as opposed to those that are none linear (quadratic, exponential). **MP2. Reason abstractly and quantitatively.** **MP4. Model with mathematics.** **MP5. Use appropriate tools strategically.** **MP7. Look for and make use of structure.** **Use functions to model relationships between quantities.** **8.F.4, 8.F.5** In this standard students use what they have learned previously and apply it in context to model functional relationships. Students construct functions and determine the slope and y-intercept (initial value) of a function from a verbal description of a relationship or from two (x, y) values, including finding those values in a graph or a table. Students give contextual meaning to the rate of change and y intercept and interpret rate of change and y intercept in terms of the graph or table of the function. Give a graph, students analyze the functional relationship (does the function increase or decrease? Is it linear?) Given a verbal description of a function, students sketch the function showing the qualitative features.	Students determine if a relationship is a function. Students represent linear function in algebraic, graphical, numerical, and verbal forms. Students use technological tools to explore and deepen their understanding of functions. Students apply general mathematical rules such as y = mx + b as the equation for a linear function.

MP4. Model with mathematics. **MP7. Look for and make use of structure.**	Students construct a function to model a linear relationship between two quantities. Students make use of the qualitative features (structure) found in a verbal description of a function and sketch that function.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*

Properties of functions
• One input for one output
• Linear vs. non-linear
 	
Functional Representations (linear and non-linear):
• Algebraic ($y = mx$ and $y = mx+b$)
• Graphic
• Table
• Verbal
 	
Rate of change/slope
	• INTERPRET the form $y= mx + b$ as defining a linear function
	• GIVE example of linear and nonlinear equations
	• COMPARE functions
	  ○ Algebraically
	  ○ Graphically
	  ○ Numerically in tables
	  ○ Verbal descriptions
	• CONSTRUCT functions
	  ○ Algebraically
	  ○ Graphically
	  ○ Numerically in tables

Initial value/y-intercept Slope of a line is a constant rate of change The y-intercept (initial value) is the point at which a line intersects the vertical axis (y-axis) A graph of a linear function is a straight line •	○ Verbal descriptions ○ Model linear relationships • DETERMINE rate of change • INTERPRET the rate of change • DETERMINE initial value of a function • INTERPRET initial value of a linear function ○ in terms of the situation it models ○ in terms of its graph or a table of values • READ a description of a relationship from a table or a graph • DESCRIBE a functional relationship between two quantities • SKETCH a graph (linear or non-linear) from a verbal description
UNDERSTAND *Big ideas, generalizations, principles, concepts, ideas that transfer across situations*	
Students will understand that the various methods to display linear relationships provide opportunities to explain a setting in multiple representations. *Students will understand that there are multiple ways to model and compare linear relationships that arise in everyday life.*	
Common Student Misconceptions for this Unit	
• Students may not recognize when the coefficient is 1 or -1. • Students may think that *x* has a value of 1 when the coefficient is 1. • Students may not recognize that the subtraction sign in front of a coefficient will make that coefficient negative, and recognize the difference between positive and negative slope. • Students may struggle if they do not have a deep understanding of integers. • Students may confuse the x and y axes. • Students may switch the rise and the run. • A common problem when students learn about the slope-intercept equation $y = mx + b$ is that they mechanically substitute	

for *m* and *b* without understanding their meaning.

- Students may not recognize the placement of the negative sign in a fraction may be placed in front, with the numerator or with the denominator.
- Students may think a graph must always go through the origin or cross both axes.
- Students may not realize that a graph may or may not go on forever, depending to the constraints put on it by a given situation.

Unit Assessment/Performance Task	DOK
Unit 4 Test Unit 4 Performance Task "I Dropped an Egg"	

Vocabulary

Academic Vocabulary

- Construct: create
- Interpret: To establish or explain the meaning or significance of something.
- Model: a mathematical representation of a process, device, or concept by means of a number of variables.

Domain-Specific Vocabulary

- Line: A geometrical object that is straight, infinitely long and infinitely thin
- Linear: A relationship in which there is a constant rate of change between two variables; for

each unit increase in one variable, there is a constant change in the other variable; a relationship or function that can be represented by a straight line.

- Ordered Pairs: A pair of numbers of the form (x, y) that gives the location of a point in the coordinate plane. The x term gives the distance left or right from the origin (0, 0), and the y term gives the distance up or down from the origin.
- Slope: The number that expresses the steepness of a line. The slope is the ratio of the vertical change to the horizontal change between any two points on the line.
- Y-intercept: The point where the graph crosses the y-axis. In a linear equation of the form y=mx+b, the y-intercept is the constant, b.

Unit-Specific Vocabulary

- Function**:** A function is a special relationship between values where each input value gives back exactly one output value.
- Initial Value: y-intercept
- Input/Output: Input is a value that you substitute into an equation to get a resulting output.
- Non-Linear: Describes a relationship that is not lying on the same straight line and is not strictly proportional.
- Rate of Change: The rate of change of a linear function is equal to the slope of the graph of the function.

Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Piecewise Functions (optional): Show students cyberchase clip http://www.teachersdomain.org/resource/vtl07.math.data.rep.lpgraphdis/ discuss and answer packet questions (HW: cyberchase worksheet)
Review and Test

Other suggestions for lesson activities:

Students will understand that there are multiple ways to model and compare linear relationships that arise in everyday life.

Students will be given a scenario of two objects falling at different rates, one with a constant speed, and one accelerating due to gravity. Students will compare the graphs of linear and non-linear functions to identify and interpret the differences between the two.

Students will compare the thickness of different types of paper to see how large a multi-page manuscript will be. Students will be writing equations to represent their relationships, and then will use them to create tables and graphs to represent the information.

Students will use the Navigating Through Algebra Series "Walking Strides" experiment with different walking rates. See math08_unit04_LA07
Friel, S., & House, P. A. (Eds.). (2001). *Navigating through algebra in grades 6 – 8*. Reston, VA: National Council of Teachers of Mathematics.

Using the following web link students learn to read distance time graphs and to derive basic information about rate from the graphs. All graphs involve constant speed.
http://www.teachersdomain.org/resource/vtl07.math.data.rep.lpgraphdis/

Students will compare different distance-time graphs to determine which represents a greater speed (including a description of the unit rate in the explanation)
Students will compare two different distance time relationships represented in different ways (including a description of the unit rate in the explanation)

Students will understand that there are multiple ways to model and compare linear

relationships that arise in everyday life.
Mathematics Assessment Project. (2011). *Baseball jerseys*. Retrieved from: http://map.mathshell.org.uk/materials/tasks.php?taskid=362#task362.

Students will compare two functions algebraically to determine better value of buying baseball jerseys in large quantities. A graphical representation could be added to enhance graphing comparisons of linear functions as well as determining the initial value and interpreting it as a y-intercept.

Students will understand that there are multiple ways to model and compare linear relationships that arise in everyday life.
Functional Relationship of Two Quantities in Various Representations

Students will use the Navigating Through Algebra Series "From Stories to Graphs" activity to graph real life data over time on various activities to sketch graphs in coordinate plane, analyze rates of change, write equations and predict results from the equation.
Friel, S., & House, P. A. (Eds.). (2001). Navigating through algebra in grades 6 – 8. Reston, VA: National Council of Teachers of Mathematics.

Students will verbally describe functional relationship of two quantities by analyzing a graph
Students will draw a graph that shows the functional relationship of two quantities from a verbal description.

Constructing and Modeling Functions
An available resource for various graphing and modeling activities can be found at:
http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/exploring-linear-relationships

Given a fundraiser situation, students will need to compare pledge plans and compare the rate of raising funds within equations, graphs, tables, and written scenario. Students will need to determine rates of change and their output, both with and without a flat pledge upfront (the y-intercept).

Supplemental Materials and Resources

Mathematics station activities for Common Core State Standards. (2011). Portland, ME. Walch Education.
This book includes a collection of station-based activities to provide students with opportunities to practice and apply the mathematical skills and concepts they are learning. It contains sets of activities for each of the five Grade 8 Common Core Mathematics strands.

Literature connections:

- Two of Everything by Lily Toy Hong – input/output tables and function machines http://mathwire.com/literature/literature.html
- Math and Literature by Jennifer Bay-Williams
- Math and Nonfiction by Jennifer Bay-Williams

Interdisciplinary connections:

- Science
  - Recording and graphing examining data and then analyzing the graph for information
  - Calculating and graphing speed, distance, time relationship

Tools/Manipulatives

Graph paper Rulers Calculators Computer access Graphing calculator Chart Paper Markers

Suggested Formative Assessment Practices/Processes
Teacher created exit slips, teacher created quizzes

Differentiation and Accommodations
• Provide graphic organizers • Provide additional examples and opportunities for repetition • Provide tutoring opportunities • Provide retesting opportunities after remediation (up to teacher and district discretion) • Teach for mastery not test • Teaching concepts in different modalities • Adjust homework assignments