Mathematics - HG - Nov 2001 National Paper 2 [Grade 12 Mathematics - HG]

Ref: Doe/M_2/1/01	Total pages: 22	Time: 3 hours	Marks: 200

This question paper consists of a cover page, 18 pages, 3 diagram sheets and a formula sheet.

DIAGRAM SHEET

INSTRUCTION

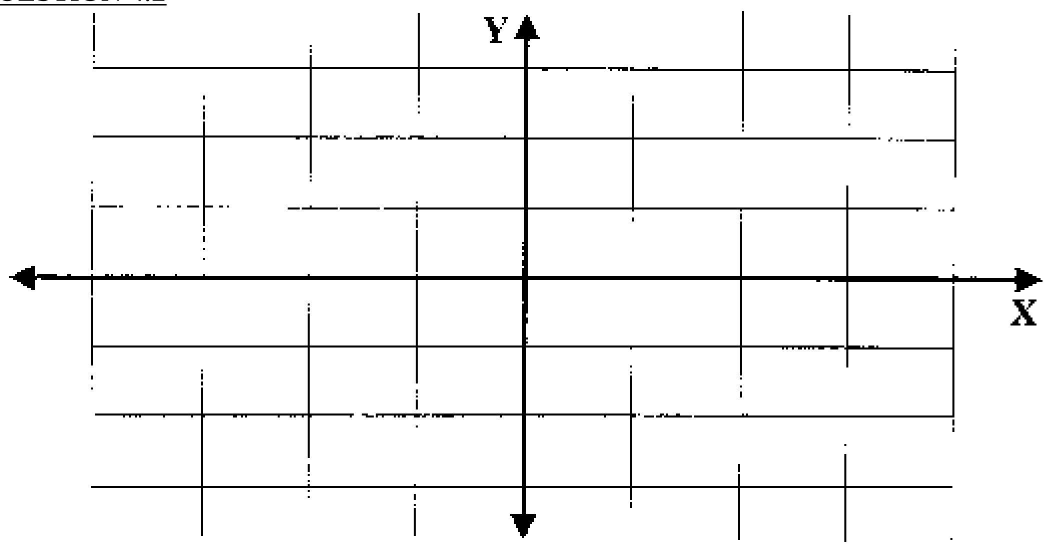
This diagram sheet must be handed in with the answer book. Please ensure that your details are complete.

INSTRUKSIE

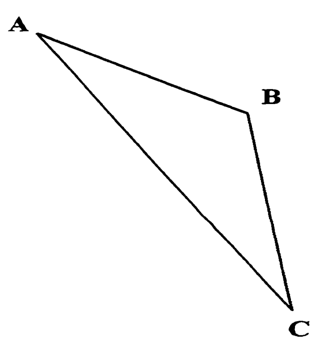
EXAMINATION NUMBER

CENTRE NUMBER

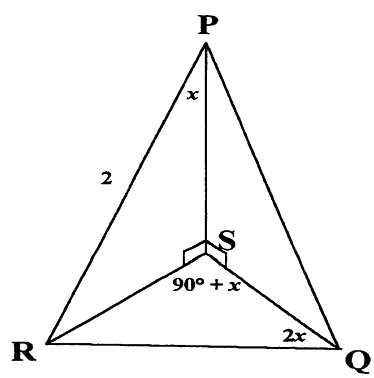
QUESTION 4.2



QUESTION 6.1

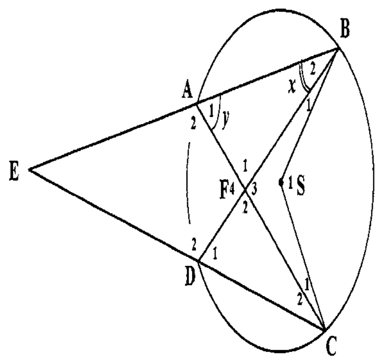


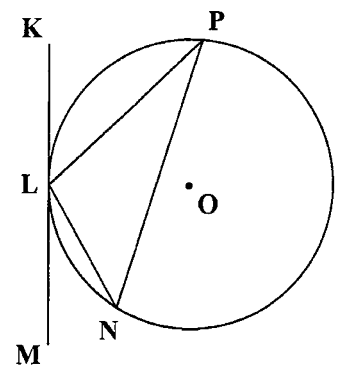
QUESTION 6.2



QUESTION 7

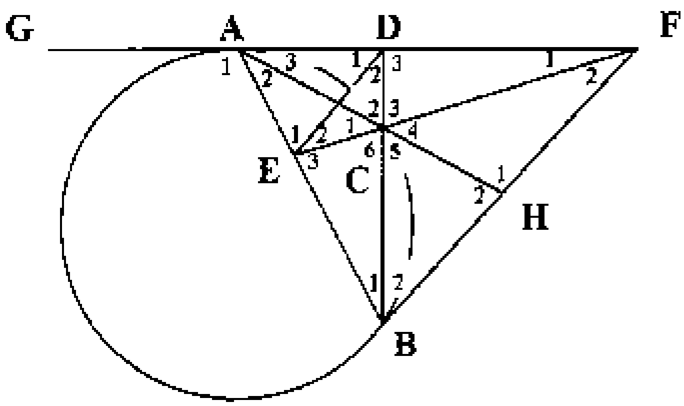
QUESTION 8.1

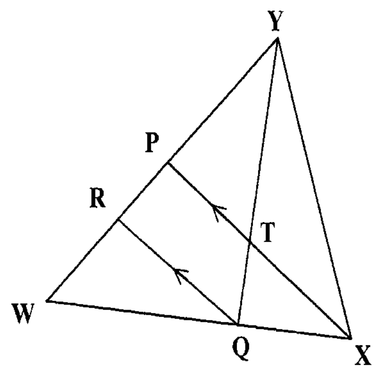




QUESTION 8.2

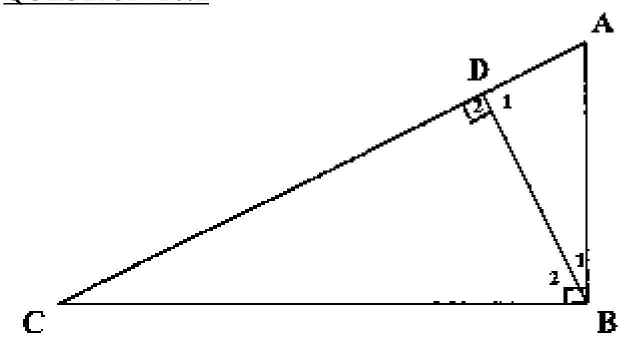
QUESTION 9

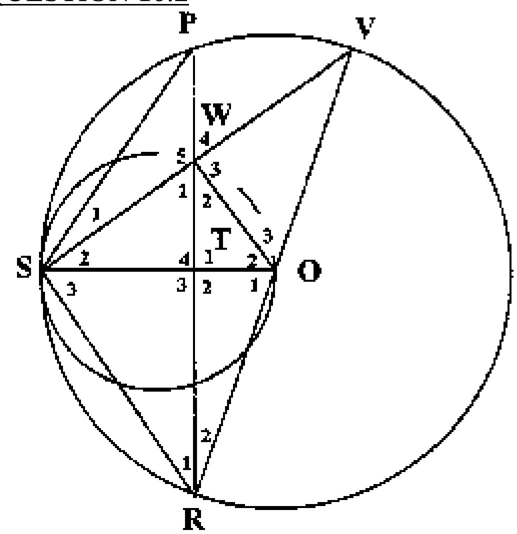




QUESTION 10.1

QUESTION 10.2





INSTRUCTIONS

1. **Answer ALL the questions.**
2. **A formula sheet is included in the question paper.**
3. **Show ALL the necessary calculations.**
4. **Number ALL the answers clearly and correctly.**

5. **The diagrams are not drawn to scale.**

6. **Three diagram sheets are included. Place it in the ANSWER BOOK.**

7. **Non-programmable calculators may be used, unless the question states otherwise.**

ANALYTICAL GEOMETRY

NOTE: IN THIS SECTION ONLY ANALYTICAL METHODS MAY BE USED. ACCURATE CONSTRUCTIONS AND MEASUREMENTS MAY NOT BE USED.

QUESTION 1

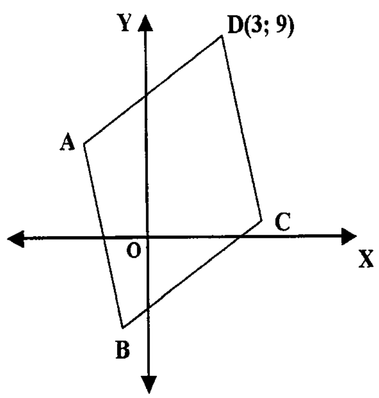
1.1 P(-3; 2) and Q(5; 8) are two points in a Cartesian plane.

1.1.1 Calculate the length of PQ. (2)

1.1.2 Calculate the angle that PQ forms with the x-axis, rounded off to one decimal digit. (3)

1.1.3 Determine the equation of the perpendicular bisector of PQ in the form $ax + by + c = 0$ (5)

1.2 In the diagram below A, B, C and D(3; 9) are the vertices of a rhombus. The equation of AC is $x + 3y = 13$



1.2.1 Show that the equation of BD is $3x - y = 0$ (3)

1.2.2 Calculate the coordinates of K if the diagonals of the rhombus intersect at point K. (4)

1.2.3 Determine the coordinates of B. (2)

1.2.4 Calculate the coordinates of A and C if AD = $\sqrt{73}$ units. (8)

[27]

QUESTION 2

2.1 Write down the equation of the circle with the centre at (-2; 3) and a radius of $\sqrt{13}$ units. (2)

2.2 $x^2 + y^2 + 4x - 12y + 4 = 0$ is the equation of a circle with centre M and radius r.

2.2.1 Calculate the coordinates of the centre M and the length of the radius *r*. (5)

2.2.2 Write down the coordinates of the point(s) where this circle intersects the *x*-axis without any further calculations. (2)

2.2.3 Determine the equation(s) of the tangent(s) to this circle which are parallel to the *y*-axis. (2)

2.3 A circle with centre P(*x; y*) passes through A(4; -1) and touches the line $y = 3$

2.3.1 Determine the equation of the locus of P. (4)

2.3.2 Calculate the gradient of this locus at the point where $x = 1$ (3)

2.3.3 Determine the equation of the tangent to the locus of P where $x = 1$ (3)

[21]

TRIGONOMETRY

QUESTION 3

3.1 If sin 161° = t, express the following in terms of t:

3.1.1 cos 19° (3)

3.1.2 tan 71º (3)

3.1.3 sec (-341°) (2)

3.2 If $p \sin \theta = -3$ and $p \cos \theta = 3$, $p > 0$, determine the value of the following:

3.2.1 θ for θ ∈ [0°; 360°] (4)

3.2.2 *p* (Leave the answer in surd form if necessary.) (3)

3.3 Prove that:

$$\frac{\operatorname{cosec}(-0) + \sec(180° + 0)}{\cot(90° - 0) - \cot(-0 - 180°)} = -(\cos\theta + \sin\theta)$$

3.4 Determine the value of $\frac{3}{2}\cot^2(-60°) - \frac{3}{2}\cos 330° - 2\sin^2(-1035°)$ without using a calculator. (7)

[32]

QUESTION 4

4.1

4.1.1 If 1 + tanθ = 2θ and cosθ, show that sinθ = 0 if sin2θ = -1 (5)

4.1.2 Determine the value(s) of θ∈ [-180°; 90°] for which 1 + tanθ = cos2θ (3)

4.2

4.2.1 Make sketch graphs of f(θ) = 1 + tanθ and g(θ) = cos2θ for θ∈[-180°; 90°] on the set of axes provided on the diagram sheet. (6)

4.2.2 Write down the period of cos 2θ (1)

4.2.3 Determine, by using the graphs, the value(s) of θ for which cos2θ - 1 < tanθ, for θ∈[-180°; 90°] (4)

[19]

QUESTION 5

5.1

5.1.1 Using the formulae for cos(A + B) and sin(A + B), prove that: (3)

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

5.1.2 Determine the general solution of $\frac{\tan 2x + \tan 40°}{1 - \tan 2x \tan 40°} = 1$ (4)

5.2

5.2.1 Prove that $\frac{1}{2}\left(\cot\theta - \frac{\sec\theta}{\operatorname{cosec}\theta}\right) = 2\theta$ using fundamental identities. (5)

5.2.2 Give the general solution of 0, for which the identity is undefined. (2)

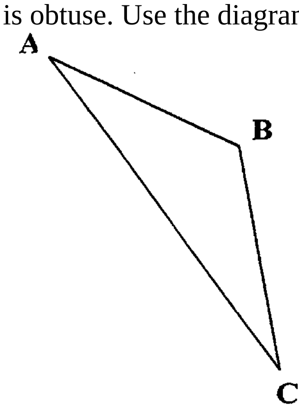
5.2.3 Hence, solve for θ in $\frac{1}{2}\left(\cot\theta - \frac{\sec\theta}{\operatorname{cosec}\theta}\right) = \tan\theta$ (6)

if $\theta \in [-90°; 180°]$. **[20]**

QUESTION 6

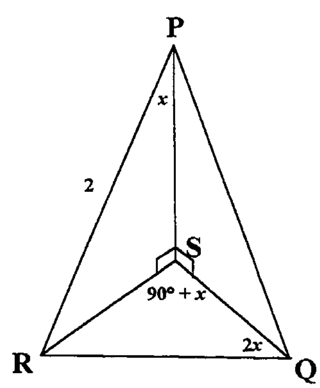
6.1 In the diagram alongside, B of 0 ABC is obtuse. Use the diagram to prove that: (4)

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$



6.2 In the diagram alongside, SP is a vertical tower and the points R and Q are in the same horizontal plane as S, the foot of the tower.

$S\hat{P}R = x$, $R\hat{S}Q = 90° + x$,
$S\hat{Q}R = 2x$ and RP = 2 units.



6.2.1 Given that sin(90° + x) = sin(90° - x), prove that RQ = 1 unit. (6)\

6.2.2 Prove that:

SQ = 2 cos 2x - 1 vir $x \in (0°; 30°)$ (9)

[19]

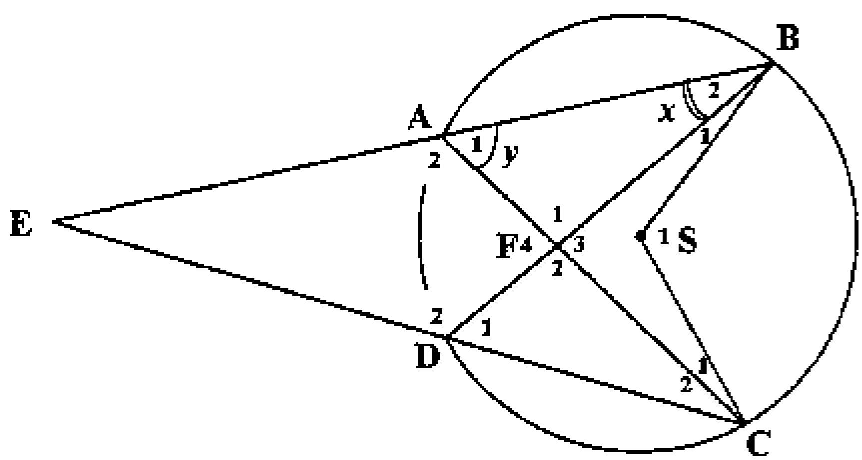
GEOMETRY

NOTE: DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEET OR REDRAWN IN THE ANSWER BOOK.
DETACH THE DIAGRAM SHEET FROM THE QUESTION PAPER AND PLACE IT IN THE ANSWER BOOK.
GIVE A REASON FOR EACH STATEMENT.

QUESTION 7

7.1 In the diagram below A, B and C are on a circle with centre S. Chords BA and CD are produced to meet at E. AC and BD intersect at F and SB and SC are drawn.

$A\hat{B}D = x$ and $B\hat{A}C = y$



7.1.1 Express $D\hat{F}A$ in terms of *x* and *y* (2)

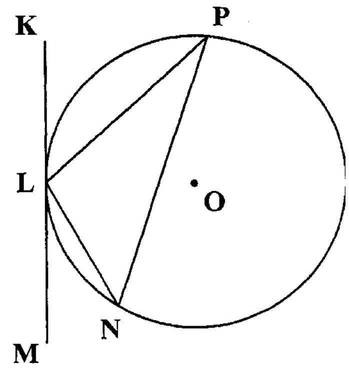
7.1.2 Prove that:

$B\hat{S}C = D\hat{F}A + \hat{E}$ (6)

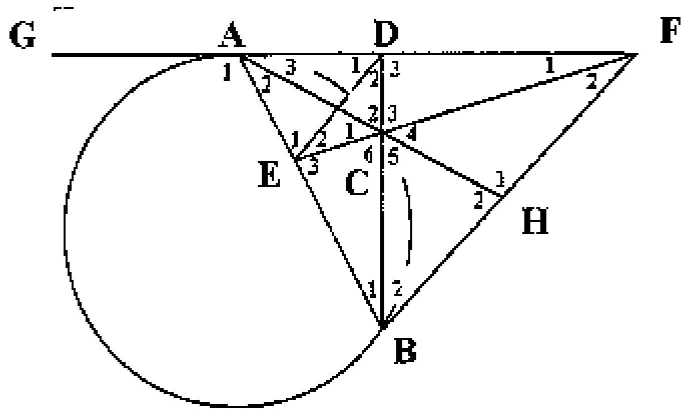
[8]

QUESTION 8

8.1 In the diagram alongside, KM is a tangent to circle O at L. Use the diagram to prove the theorem which states that:
Hip = PNL
$K\hat{L}P = P\hat{N}L$ (7)



8.2 In the diagram alongside, GF is a tangent to the circle at A. AB is a chord and BD ⊥ AF intersects the circle at C. E is on AB such that DE = DA. EC produced meets AF at F. BF is joined but is not a tangent. AC is produced to meet BF at H.



Prove that:

8.2.1 $D\hat{C}A = B\hat{A}D$ (4)

8.2.2 ADCE. is a cyclic quadrilateral (4)

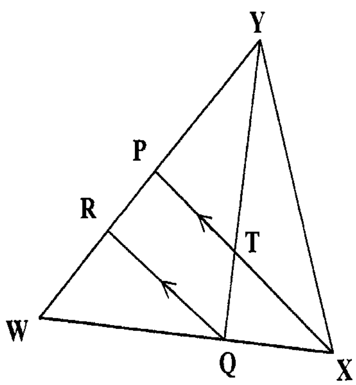
8.2.3 AH ⊥ BF (6)

8.2.4 CD is the bisector of $A\hat{C}F$ (2)

[23]

QUESTION 9

In the diagram alongside, XP is the perpendicular bisector of side WY of ΔWXY.
Q is a point on WX such that W Q : WX = 3 : 5.XP and YQ intersect at T. QR is drawn parallel to XP.



Determine:

9.1 $\frac{YP}{YR}$ (3)

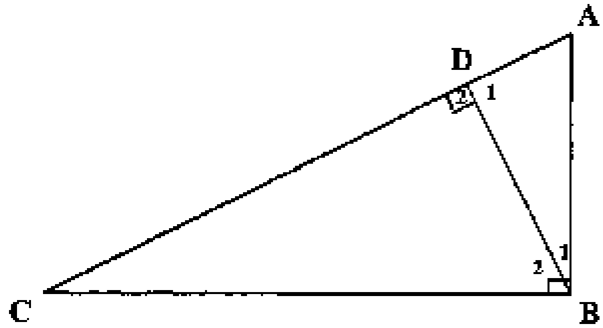
9.2 $\frac{QR}{TP}$ (2)

9.3 $\frac{Area\ \Delta TPY}{Area\ \Delta QRY}$ (3)

[8]

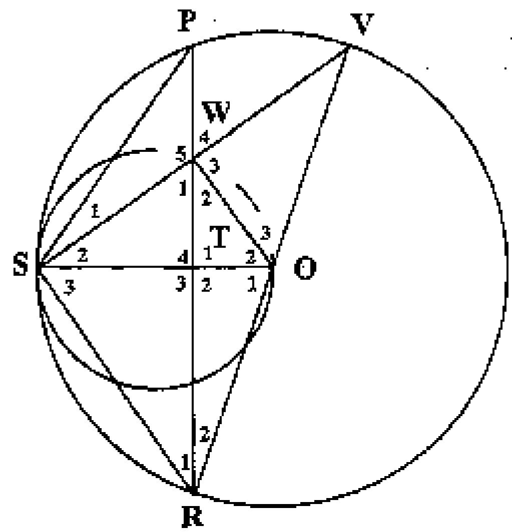
QUESTION 10

10.1 In the diagram alongside, $A\hat{B}C = 90°$ and BD is drawn perpendicular to AC. Use the diagram to prove the theorem which states that:



Δ ABC ||| Δ ABD ||| Δ BDC (5)

10.2 In the diagram alongside, two circles touch internally at S. O is the centre of the bigger circle and OS is a diameter of the smaller circle. PR is a double chord such that PT = TR intersecting the smaller circle at W. SW is produced to meet the bigger circle at V. VOR is a straight line. WO and PS are drawn.



Prove that:

10.2.1 SW = WV (3)

10.2.2 $SW^2 = PW.WR$ (6)

10.2.3 $SW^2 = WT.WR$ (5)

10.2.4 PW : WR =1 : 3 (4)

[23]

TOTAL: 200

Mathematics Formula Sheet (HG and SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + 1) \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = a.r^{n-1} \quad S_n = \frac{a(1 - r^n)}{1 - r} \quad S_n = \frac{a(1 - r^n)}{r - 1} \quad S_\infty = \frac{a}{1 - r}$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2 \qquad (x - p)^2 + (y - q)^2 = r^2$$

$In\ \Delta ABC:$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \tfrac{1}{2}ab.\sin C$$

Mathematics - HG - Nov 2001 National Paper 2 Memorandum [Grade 12 Mathematics - HG]

1.1.1 $PQ^2 = (5+3)^2 - (8-2)^2$

$= 8^2 + 6^2$

$PQ = 10$

1.1.2 $m_{PQ} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{6}{8}$

$\therefore \theta = 36{,}9°$

1.1.3 Mdpt PQ : (1;5)

Grad. line $\perp$: $m = -4/3$

Eq. line $y = (-4/3)x + c$

subst. (1;5) $5 = (-4/3)(1) + c$

$c = 19/3$

Eq. $y = -(4/3)x + 19/3$

and $3y + 4x - 19 = 0$

1.2.1 Eq. AC : $x + 3y = 13$ and $y = -(1/3)x + 13/3$

BD $\perp$ AC and through (3; 9).

Eq. BD : $y = 3x + c$

$\therefore 9 = 3(3) + c$

$\therefore c = 0$

Eq. $y = 3x + 0$ or $3x \; y = 0$

1.2.2 Solve equations for AC and BD simultaneously.

$-(1/3)x + 13/3 = 3x$

$-x + 13 = 9x$

$x = 1{,}3$ [subst.]

$y = 3{,}9$

1.2.3 B(x;y)

$\dfrac{3+x}{2}$ AND $\dfrac{9+y}{2} = \dfrac{39}{10}$

$30 + 10x = 26 \quad 45 + 5y = 39$

$x = -0,4 \quad y = -1,2$

1.2.4

$(3 - x)^2 + (9 - y)^2$	$= 73$	Points equidistant from D.
$[3 - (13 - y)]^2 + (9 - y)^2$	$= 73$	Solve equations for AC and circle
$(100 - 60y + 9y^2 + 81 - 18y + y^2$	$= 73$	simultaneously.
$10y^2 - 78y + 108$	$= 0$	
$5y^2 - 39y + 54$	$= 0$	Discriminant a square, factorise.
$(5y - 9)(y - 6)$	$= 0$	
Thus $y = 9/5$ OR $y = 6$		
and $x = 7,6$ $x = -5$		Thus A(-5;6) and C(7,6; 1,8)

2.1 $(x + 2)^2 + (y - 3)^2 = 13$

2.2.1

$x^2 + 4x + y^2 - 12y + 4$	$= 0$	
$x^2 + 4x + 4 + y^2 - 12y + 36$	$= -4 + 36 + 4$	[Complete the square]
$(x + 4)^2 + (y - 6)^2$	$= 36$	

Thus: M(-2; 6) en r = 6

2.2.2 X-intercept: $x = -2$
r = 6 and M(-2;6)
X-axis touches the circle at x = -2.

2.2.3 Tangent A is 6 units to the left of M.
Tangent B is 6 units to the right of M.
Thus: A: $x = -8$ and B: $x = 4$.

2.3.1

$(\text{radius})^2$	$= (\text{ distance from mdpt. to tangent})^2$
$(x - 4)^2 + (y + 1)^2$	$= (y - 3)^2$
$x^2 - 8x + 16 + y^2 + 2y + 1$	$= y^2 - 6y + 9$
$x^2 - 8x + 8$	$= -8y$
y	$= (-1/8)x^2 + x - 1$

2.3.2

Gradient: D_x	$= (-1/8).2x + 1$	
	$= (-1/8)(2) + 1$	[subst. x = 1]
	$= 3/4$	

2.3.3 Point of tangency: (1; -1/8)

Eq. tangent: y	$= mx + c$	
	$= (3/4)x + c$	
(-1/8)	$= (3/4)(1) + c$	[subst.]
c	$= -7/8$	
Eq. tangent: y	$= (3/4)x - 7/8$	

3.1.1 $\cos 19° = \sqrt{1 - \sin^2} 19° = \sqrt{1 - \sin^2} 161 = \sqrt{1 - t^2}$

3.1.2 tan71° = $\frac{\sin 71°}{\cos 71°} = \frac{\cos 19°}{\sin 19°} = \frac{\sqrt{1-t^2}}{t}$

3.1.3 sec(-341°) = sec341° = sec19° = $\frac{1}{\cos 19°} = \frac{1}{\sqrt{1-t^2}}$

3.2.1 psin θ = -3 ... (i)
pcos θ = 3 ... (ii)
θ tan θ = -1 ... (i)/ (ii)
θ = 180° - 45° OR 2 = 360° - 45°
= 135° = 315°
But sin θ is negative and cos θ is positive; only one answer: θ = 315°

3.2.2 pcos315° = 3 [given]
cos 315° = 3/p

$\frac{1}{\sqrt{2}} = \frac{3}{p}$

Thus: $p = 3\sqrt{2}$

3.3 LHS = $\frac{\operatorname{cosec}(-\theta) + \sec(180° + \theta)}{\cot(90° - \theta) - \cot(-\theta - 180°)}$

$= \frac{-\operatorname{cosec}\theta - \sec\theta}{\tan\theta + \cot(180 + \theta)}$

$= \frac{\frac{-1}{\sin\theta} - \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \frac{-\cos\theta - \sin\theta}{\sin\theta.\cos\theta} \times \frac{\cos\theta.\sin\theta}{\sin^2\theta + \cos^2\theta}$

$= -(\cos\theta + \sin\theta)$

$= RHS$

3.4 (3/2)$\cot^2$(-60) – (3/2)cos330° - 2$\sin^2$(1035°)

$= \frac{3}{2}\left(\frac{-1}{\sqrt{3}}\right)^2 - \frac{3}{2}\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{-1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} - \frac{3\sqrt{3}}{4} - 1$

$= \frac{-3\sqrt{3}}{4} - \frac{1}{2}$

4.1.1 1 + tan θ = cos2 θ
1 + sin θ /cos θ = 1 – 2$\sin^2$θ [cos2 nie nul]
sin θ = -2$\sin^2$θ.cosθ
sin θ (1 + 2sin θ cos θ) = 0
∴ sin θ = 0 OR 1 + 2sin θ cos θ = 0
2sin θ cos θ = -1
∴sin2θ = -1

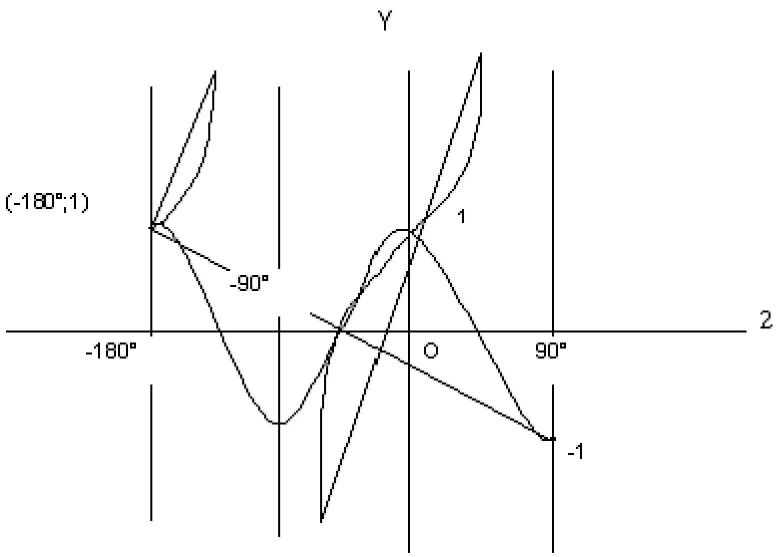
4.1.2 $\sin\theta = 0$ OR $\sin 2\theta = -1$

$\therefore \theta = 0°$ of $-180°$ $\quad \therefore 2\theta = 270° + k.360°$ .. $k \varepsilon Z$

$= 135° + k.180°$

$\therefore \theta = -45°$

4.2.1



4.2.2 Period of $\cos 2\theta$ is 180°.

4.2.3 $\cos 2\theta - 1 \leq \tan\theta$

$\cos 2\theta \leq \tan\theta + 1$ Thus: $-180° \leq \theta \leq -90°$ or $\theta = -45°$ or $0° \leq \theta < 90°$.

5.1.1 Theory.

5.1.2 LHS $= \tan(2x + 40°)$ from 5.1.1. $\quad \therefore \tan(2x + 40°) = 1$

$$\therefore 2x + 40° = 45° + k.180° \quad .. \; k \varepsilon Z$$

$$2x = 5° + k.180°$$

$$x = 2{,}5° + 90°.k \quad .. \; k \varepsilon Z$$

5.2.1

LHS $= \frac{1}{2}\left(\cot\theta - \frac{\sec\theta}{cosec\,\theta}\right)$

$$= \frac{1}{2}\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{1}{2}\left(\frac{\cos^2\theta - \sin^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta.\cos\theta}$$

$$= \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \cot 2\theta = RHS$$

5.2.2 The identity is not defined for $\sin\theta = 0$ OR $\sin 2\theta = 0$ OR $\cos\theta = 0$ i.e. not defined for $\theta = 0° + k.90°$ .. $k \in Z$

5.2.3

$$\cot 2\theta = \tan\theta$$
$$\tan(90 - 2\theta) = \tan\theta$$
$$90 - 2\theta = \theta + k.180° \quad .. \ k \in Z$$
$$-3\theta = -90° + k.180° \quad .. \ k \in Z$$
$$-\theta = -30° + k.60° \quad ... \ k \in Z$$
$$\theta = 30 + k.60° \quad ... \ k \in Z$$

Solution: $\theta = -30°,\ 30°,\ 150°$. [$\theta$ not defined for 90°]

6.1 Proof of Sine Rule.

6.2.1 In triangle PRS:

SR/2 = sinx

$\therefore$ SR = 2sinx

In triangle RSQ:

$$\frac{SR}{\sin 2x} = \frac{RQ}{\sin(90 + x)}$$

$$\therefore RQ = \frac{(2\sin x).\sin(90 + x)}{\sin 2x} = \frac{(2\sin x)\sin(90 + x)}{2\sin x\cos x} = \frac{\sin(90 + x)}{\cos x} = \frac{\sin(90 + x)}{\sin(90 - x)} = 1$$

6.2.2 In triangle RSQ: $\angle R = 90° - 3x$

$$\text{Therefor}: \quad \frac{SQ}{\sin(90 - 3x)} = \frac{1}{\sin(90 + x)}$$

$$\therefore SQ = \frac{\sin(90 - 3x)}{\sin(90 + x)}$$

sin(90 – x) – sin(90 + x), given

$$= \frac{\cos 3x}{\sin(90 - x)}$$

$$= \frac{\cos 3x}{\cos x}$$

Expand cos3x

$$= \frac{\cos 2x\cos x - \sin 2x.\sin x}{\cos x}$$

$$= \frac{\cos 2x\cos x - 2\sin x\cos x.\sin x}{\cos x}$$

$$= \cos 2x - 2\sin^2 x$$

$$= (2\cos^2 x - 1) - 2\sin^2 x$$

$$= 2(\cos^2 x - \sin^2 x) - 1$$

$$= 2\cos 2x - 1$$

7.1.1 $\angle$ DFA = x+ y [ext $\angle$ = sum of int. opp. angles]

7.1.2 $\angle BSC = 2y$ [$\angle$ at centre = 2$\angle$ angle at circum.]

$= y + y$

$= y + (x + \angle E)$ [$\angle C_2 = x$ and $\angle A_1$ ext. angle of triangle]

$= y + x + \angle E$

$= (y + x) + \angle E$

$= \angle DFA + \angle E$ [proved in 7.1.1]

8.1 Proof of tangent-chord theorem.

8.2.1 $\angle \pi C_2 = \angle B_1 + \angle \pi A_2$ [ext.◻ of triangle]

but $\angle B_1 = \angle A_3$ [tangent; chord]]

$\therefore \angle C_2 = \angle A_3 + \angle A_2$

$\therefore \angle DCA = \angle BAD$

8.2.2 $\angle E_1 = \angle A_2 + \angle A_3$ [DA = DE]

$= \angle C_2$ [proved in 8.2.1]

but $\angle E_1$ and $\angle C_2$ are subtended by DA

$\therefore$ ADCE a cyclic quad.

8.2.3 $\angle E_3 = \angle D_3 = 90°$ [ext $\angle$ = int. opp. angle of cyclic quad]

$\therefore$ $AH \perp BF$ [Three altitudes concurrent]

8.2.4 $\angle C_2 = \angle A_2 + \angle A_3$ [proved in 8.2.1]

$= \angle C_3$ [ext$\angle$ = int. opp angle of cyclic quad ADCE]

$\therefore$ CD bisects $\angle ACF$.

9.1 $\frac{WQ}{QX} = \frac{3}{2}$ [given]

$\frac{WR}{RP} = \frac{3}{2}$ [RQ // PX]

$\frac{WP}{PY} = \frac{5}{5}$ [XP perpendicular bisector]

$\therefore \frac{YP}{YR} = \frac{5}{7}$ [RQ //PT]

9.2 In triangle YPT and triangle YRQ:

$\angle Y = \angle Y$

$\angle P = \angle R = 90°$ [corresponding angles]

and $\angle YTP = \angle YQR$ [angles of triangle = 180°]

$\therefore \Delta YPT /// \Delta YRQ$

$\frac{QR}{TP} = \frac{RY}{PY} = \frac{5}{7}$

9.3 $\frac{\Delta TPY}{\Delta QRY}$

$= \frac{0,5\,PT.PY}{0,5\,QR.RY}$

$= \frac{5}{7} \times \frac{5}{7}$

$= \frac{25}{49}$

10.1 Theorem.

10.2.1 $\angle W_1 + \angle W_2 = 90°$ [$\angle$ in semi circle]
$\therefore$ SW = WV [chord perpendicular to radius]

10.2.2 In)PWS en)VWS is
$\angle S_1 = \angle R_2$ [subtended by PV]
$\angle P = \angle \pi V$ [" SR]
$\therefore \angle PWS /// \angle VWS$
and $\frac{SW}{WR} = \frac{PW}{WV}$
SW.WV = PW.WR
SW^2 = PW.WR [SW = WV]

10.2.3 SO $\perp$ PR [PT = TR, given]
$\angle S_2 + \angle S_3 = 90°$ [$\angle$ in semi circle]
$\therefore \Delta SWT /// \Delta RWS$ [Theorem in 10.1]
$\therefore SW^2 = WT.WR$

10.2.4 PW.WR = WT.WR [proved in 10.2.2 and 10.2.3]
$\therefore$ PW = WT [divide by WR]
and PT = TR [given]
$\therefore$ PW:WR = 1:3