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MATHEMATICS EXTENSION 1
HSC Exam* Questions by Topic
2021 - 2017

v2022.1

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Year 11 Course

Functions

Trigonometric Functions

Calculus

Combinatorics

Year 12 Course

Proof

Vectors

Trigonometric Functions

Calculus

Statistical Analysis

Complete Papers

Question Difficulty

Easy

Mid-range

Difficult

Mathematics Advanced, Ext 1, Ext 2 Reference Sheet (2021 HSC)

Questions by Topic from ...

- 2017 – 2020 HSCs (MX1: Mathematics Extension 1, M: Mathematics)
- NESA Sample HSC examination [SP]
- Additional NESA sample questions [SQ]
- NESA Topic Guidance [TG] – selected questions

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Year 11: Calculus
C1.2 Exponential growth and decay

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Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- construct, analyse and manipulate an exponential model of the form $N(t) = Ae^{kt}$ to solve a practical growth or decay problem in various contexts (for example population growth, radioactive decay or depreciation) **AAM**
  - establish the simple growth model, $\frac{dN}{dt} = kN$, where N is the size of the physical quantity, $N = N(t)$ at time t and k is the growth constant
  - verify (by substitution) that the function $N(t) = Ae^{kt}$ satisfies the relationship $\frac{dN}{dt} = kN$, with A being the initial value of N
  - sketch the curve $N(t) = Ae^{kt}$ for positive and negative values of k
  - recognise that this model states that the rate of change of a quantity varies directly with the size of the quantity at any instant
- establish the modified exponential model, $\frac{dN}{dt} = k(N - P)$, for dealing with problems such as 'Newton's Law of Cooling' or an ecosystem with a natural 'carrying capacity' **AAM**
  - verify (by substitution) that a solution to the differential equation $\frac{dN}{dt} = k(N - P)$ is $N(t) = P + Ae^{kt}$, for an arbitrary constant A, and P a fixed quantity, and that the solution is $N = P$ in the case when $A = 0$
  - sketch the curve $N(t) = P + Ae^{kt}$ for positive and negative values of k
  - note that whenever $k < 0$, the quantity N tends to the limit P as $t \to \infty$, irrespective of the initial conditions
  - recognise that this model states that the rate of change of a quantity varies directly with the difference in the size of the quantity and a fixed quantity at any instant
- solve problems involving situations that can be modelled using the exponential model or the modified exponential model and sketch graphs appropriate to such problems **AAM**

Reference Sheet

21 MX 1 **12 b**

A bottle of water, with temperature 5°C, is placed on a table in a room. The temperature of the room remains constant at 25°C. After t minutes, the temperature of the water, in degrees Celsius, is T. **Solution**

The temperature of the water can be modelled using the differential equation

$$\frac{dT}{dt} = k(T - 25) \text{ (Do NOT prove this.)}$$

where k is the growth constant.

(i) After 8 minutes, the temperature of the water is 10°C. **3**

By solving the differential equation, find the value of t when the temperature of the water reaches 20°C. Give your answer to the nearest minute.

(ii) Sketch the graph of T as a function of t. **1**

NESA 2021 Mathematics Extension 1 HSC Examination

TG **1**

The growth rate of a population of bacteria is 10% of the population. [Solution]

At $t = 0$, the population is 1.0×10^6.

Sketch the graph of population against time and determine the population after 3.5 hours, correct to four significant figures.

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG **2**

On an island, the population in 1960 was 1732, and in 1970 it was 1260. [Solution]

Find the annual growth rate to the nearest percent, assuming it is proportional to the population.

In how many years will the population be half of what it was in 1960?

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG **3** / **12 M** **14 c**

Professor Smith has a colony of bacteria. Initially, there are 1000 bacteria. The number of bacteria, $N(t)$, after t minutes is given by $N(t) = 1000e^{kt}$. [Solution]

(i) After 20 minutes there are 2000 bacteria. Show that $k = 0.0347$ correct to four decimal places. **1**

(ii) How many bacteria are there when $t = 120$? **1**

(iii) What is the rate of change of the number of bacteria per minute, when $t = 120$? **1**

(iv) How long does it take for the number of bacteria to increase from 1000 to 100 000? **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2012 Mathematics HSC Examination

TG **4** / **07 M** **8a**

One model for the number of mobile phones in use worldwide is the exponential growth model, $N = Ae^{kt}$, where N is the estimate for the number of mobile phones in use (in millions), and t is the time in years after 1 January 2008. [Solution]

(i) It is estimated that at the start of 2009, when $t = 1$, there will be 1600 million mobile phones in use, while at the start of 2010, when $t = 2$, there will be 2600 million. Find A and k. **3**

(ii) According to the model, during which month and year will the number of mobile phones in use first exceed 4000 million? **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2007 Mathematics HSC Examination

TG **5** / **05 MX 1** **2d**

A salad, which is initially at a temperature of 25°C, is placed in a refrigerator that has a constant temperature of 3°C. The cooling rate of the salad is proportional to the difference between the temperature of the refrigerator and the temperature, T, of the salad. That is, T satisfies the equation $\dfrac{dT}{dt} = -k(T - 3)$, where t is the number of minutes after the salad is placed in the refrigerator. [Solution]

(i) Show that $T = 3 + Ae^{-kt}$ satisfies this equation. **1**

(ii) The temperature of the salad is 11°C after 10 minutes. Find the temperature of the salad after 15 minutes. **3**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2005 Mathematics Extension 1 HSC Examination

19 MX 1 **12 d** A refrigerator has a constant temperature of 3°C. A can of drink with temperature 30°C is placed in the refrigerator. **Solution**

After being in the refrigerator for 15 minutes, the temperature of the can of drink is 28°C.

The change in the temperature of the can of drink can be modelled by $\frac{dT}{dt} = k(T - 3)$, where T is the temperature of the can of drink, t is the time in minutes after the can is placed in the refrigerator and k is a constant.

(i) Show that $T = 3 + Ae^{kt}$, where A is a constant, satisfies $\frac{dT}{dt} = k(T - 3)$. **1**

(ii) After 60 minutes, at what rate is the temperature of the can of drink changing? **3**

NESA 2019 Mathematics Extension 1 HSC Examination

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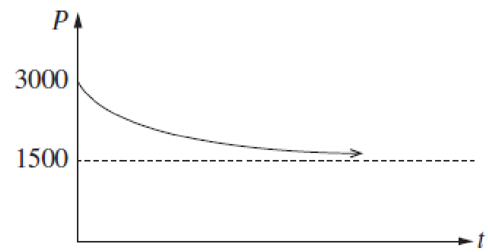
18 MX 1 **5** The diagram shows the number of penguins, $P(t)$, on an island at time t. **1** **Solution**

Which equation best represents this graph?



A. $P(t) = 1500 + 1500e^{-kt}$

B. $P(t) = 3000 - 1500e^{-kt}$

C. $P(t) = 3000 + 1500e^{-kt}$

D. $P(t) = 4500 - 1500e^{-kt}$

NESA 2018 Mathematics Extension 1 HSC Examination

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17 M **14 c** Carbon-14 is a radioactive substance that decays over time. The amount of carbon-14 present in a kangaroo bone is given by $C(t) = Ae^{kt}$, where A and k are constants, and t is the number of years since the kangaroo died. **Solution**

(i) Show that $C(t)$ satisfies $\frac{dC}{dt} = kC$. **1**

(ii) After 5730 years, half of the original amount of carbon-14 is present. Show that the value of k, correct to 2 significant figures, is -0.00012. **2**

(iii) The amount of carbon-14 now present in a kangaroo bone is 90% of the original amount. Find the number of years since the kangaroo died. Give your answer correct to 2 significant figures. **2**

NESA 2017 Mathematics HSC Examination

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Year 12: Vectors
V1.3 Projectile motion

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Students:

- understand the concept of projectile motion, and model and analyse a projectile's path assuming that:
  - the projectile is a point
  - the force due to air resistance is negligible
  - the only force acting on the projectile is the constant force due to gravity, assuming that the projectile is moving close to the Earth's surface
- model the motion of a projectile as a particle moving with constant acceleration due to gravity and derive the equations of motion of a projectile **AAM**
  - represent the motion of a projectile using vectors
  - recognise that the horizontal and vertical components of the motion of a projectile can be represented by horizontal and vertical vectors
  - derive the horizontal and vertical equations of motion of a projectile
  - understand and explain the limitations of this projectile model
- use equations for horizontal and vertical components of velocity and displacement to solve problems on projectiles
- apply calculus to the equations of motion to solve problems involving projectiles (ACMSM115) **AAM**

Reference Sheet

21 MX 1 **13 b** (4 marks) Solution

When an object is projected from a point h metres above the origin with initial speed V m/s at an angle of $\theta°$ to the horizontal, its displacement vector, t seconds after projection, is

$$\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (-5t^2 + Vt\sin\theta + h)\,\underset{\sim}{j}\text{. (Do NOT prove this.)}$$

A person, standing in an empty room which is 3 m high, throws a ball at the far wall of the room. The ball leaves their hand 1 m above the floor and 10 m from the far wall. The initial velocity of the ball is 12 m/s at an angle of 30° to the horizontal.

Show that the ball will NOT hit the ceiling of the room but that it will hit the far wall without hitting the floor.

NESA 2021 Mathematics Extension 1 HSC Examination

SP MX 1 **11 a** Solution

A particle is fired from the origin O with initial velocity 18 ms^{-1} at an angle 60° to the horizontal.

The equations of motion are $\dfrac{d^2x}{dt^2} = 0$ and $\dfrac{d^2y}{dt^2} = -10$

(i) Show that $x = 9t$. **1**

(ii) Show that $y = 9\sqrt{3}\,t - 5t^2$. **2**

(ii) Hence find the Cartesian equation for the trajectory of the particle. **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

TG **1** A ball is thrown in the air with speed 12 ms^{-1} at an angle of 70° to the horizontal. Solution

The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

Using g = 9.8 ms^{-2}, find

(a) its position vector* after 1.5 seconds

(b) its velocity vector* after 1.5 seconds.

* projectmaths included the word 'vector'.

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **2** A golf ball is driven with a speed of 45 ms^{-1} at 37° to the horizontal across a horizontal fairway. The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.** Solution

Use g = 9.8 ms^{-2}.

(a) How high above the ground does the ball rise?

(b) How far away from the tee does it first land?

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **3** A famine relief aircraft, flying over horizontal ground at a height of 160 metres, drops a sack of food. The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.** Solution

Use g = 10 ms^{-2}.

(a) Calculate the time that the sack takes to fall.

(b) Calculate the vertical component of the velocity with which the sack hits the ground.

(c) If the speed of the aircraft is 70 ms^{-1}, at what distance before the target zone should the sack be released?

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **4** A projectile reaches its greatest height after two seconds, when it is 35 metres from its point of projection. Solution

The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

Using g = 9.8 ms^{-2}, determine the initial velocity.

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **5** A particle is projected upwards at velocity u and an angle of θ to the horizontal from a point A. Find an expression for the horizontal distance away from point A at which the particle reaches its greatest height, in terms of u and θ. Solution

The position vector is given as $\underset{\sim}{r}(t) = ut\cos\theta\,\underset{\sim}{i} + (ut\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 6 In a cricket match a batsman hits the ball for six, and it is measured by the cameras after two seconds as moving at a horizontal velocity of 30 ms^{-1} and a vertical velocity of 10 ms^{-1} upwards. Calculate the initial velocity and angle of projection at which the ball was hit by the batter. Use g = 9.8 ms^{-2}. **Solution**

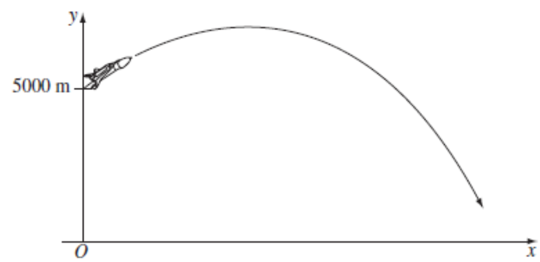
The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 7
05 MX 1 6b

An experimental rocket is at a height of 5000 m, ascending with a velocity of $200\sqrt{2}$ m s^{-1} at an angle of 45° to the horizontal, when its engine stops. **Solution**



After this time, the equations of motion of the rocket are:

$x = 200t$

$y = -4.9t^2 + 200t + 5000,$

where t is measured in seconds after the engine stops. (Do NOT show this.)

(i) What is the maximum height the rocket will reach, and when will it reach this height? **2**

(ii) The pilot can only operate the ejection seat when the rocket is descending at an angle between 45° and 60° to the horizontal. What are the earliest and latest times that the pilot can operate the ejection seat? **3**

(iii) For the parachute to open safely, the pilot must eject when the speed of the rocket is no more than 350 m s^{-1}. What is the latest time at which the pilot can eject safely? **2**

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors
NESA 2005 Mathematics Extension 1 HSC Examination

TG 8 (a) Prove that the range on a horizontal plane of a particle projected upwards at an angle α to the plane with velocity V metres per second is $\dfrac{V^2\sin 2\alpha}{g}$ metres, where g metres per second per second is the acceleration due to gravity. **Solution**

(b) A garden sprinkler sprays water symmetrically about its vertical axis at a constant speed of V metres per second. The initial direction of the spray varies continuously between angles of 15° and 60° to the horizontal.

(i) Prove that, from a fixed position O on level ground, the sprinkler will wet the surface of an annular region with centre O and with internal and external radii $\dfrac{V^2}{2g}$ metres and $\dfrac{V^2}{g}$ metres respectively.

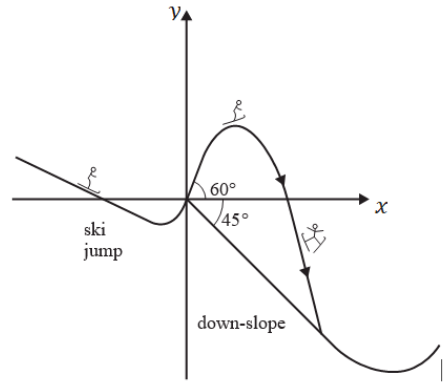
(ii) Deduce that, by locating the sprinkler appropriately relative to a rectangular garden bed of size 6 metres by 3 metres, the entire bed may be watered provided that $\dfrac{V^2}{2g} \geq 1 + \sqrt{7}$.

The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\alpha\,\underset{\sim}{i} + (Vt\sin\alpha - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **9** A skier accelerates down a slope and then skis up a short ski jump (see diagram below). The skier leaves the jump at a speed of 12m/s and at an angle of 60° to the horizontal. The skier performs various gymnastic twists and lands on a straight line section of the 45° down-slope T seconds after leaving the jump.



Solution

Let the origin O of a Cartesian coordinate system be at the point where the skier leaves the jump, with $\underset{\sim}{i}$ a unit vector in the positive x direction, and $\underset{\sim}{j}$ a unit vector in the positive y direction. Displacements are measured in metres, and time in seconds.

(a) Show that the initial velocity of the skier when leaving the jump is $6\underset{\sim}{i} + 6\sqrt{3}\,\underset{\sim}{j}$.

(b) The acceleration of the skier, t seconds after leaving the ski jump, is given by

$$\underset{\sim}{\ddot{r}}(t) = -0.1t\,\underset{\sim}{i} - (g - 0.1t)\,\underset{\sim}{j},\ 0 \le t \le T.$$

Show that the position vector of the skier, t seconds after leaving the jump, is given by $r(t) = (6t - \frac{1}{60}t^3)\underset{\sim}{i} + (6t\sqrt{3} - \frac{1}{2}gt^2 + \frac{1}{60}t^3)\underset{\sim}{j}$, $0 \le t \le T$.

(c) Show that $T = \frac{12}{g}(\sqrt{3} + 1)$.

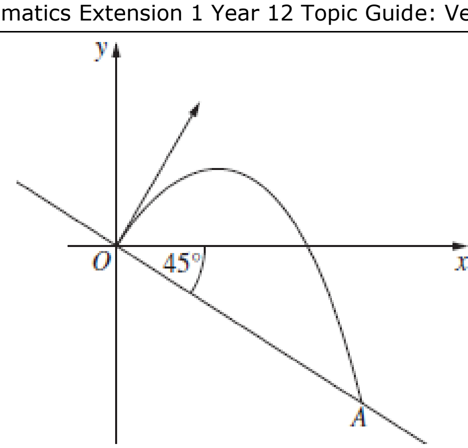
(d) At what speed, in metres per second, does the skier land on the down-slope?

Give your answer correct to one decimal place.

(Source: Question 4, VCE Specialist Mathematics 2, 2005 © **VCAA**, reproduced by permission.)

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

19 MX 1 **13 d** The point O is on a sloping plane that forms an angle of 45° to the horizontal. A particle is projected from the point O. The particle hits a point A on the sloping plane as shown in the diagram.



Solution

The equation of the line OA is $y = -x$.

The equations of motion of the particle are

$x = 18t$ and $y = 18\sqrt{3}\,t - 5t^2$, where t is the time in seconds after projection.

Do NOT prove these equations.

(i) Find the distance OA between the point of projection and the point where the particle hits the sloping plane. **2**

(ii) What is the size of the acute angle that the path of the particle makes with the sloping plane as the particle hits the point A? **3**

NESA 2019 Mathematics Extension 1 HSC Examination

18 MX 1 **13 c** [Solution]

An object is projected from the origin with an initial velocity of V at an angle θ to the horizontal. The equations of motion of the object are

$x(t) = Vt\cos\theta$ and $y(t) = Vt\sin\theta - \dfrac{gt^2}{2}$. (Do NOT prove these.)

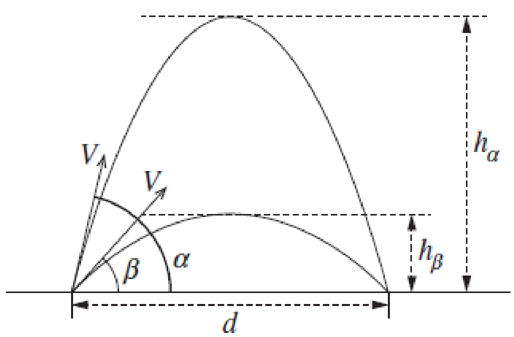
(i) Show that when the object is projected at an angle θ, the horizontal range is $\dfrac{V^2}{g}\sin 2\theta$. **2**

(ii) Show that when the object is projected at an angle $\dfrac{\pi}{2} - \theta$, the horizontal range is also $\dfrac{V^2}{g}\sin 2\theta$. **1**

(iii) The object is projected with initial velocity V to reach a horizontal distance d, which is less than the maximum possible horizontal range. There are two angles at which the object can be projected in order to travel that horizontal distance before landing. **3**

Let these angles be α and β where $\beta = \dfrac{\pi}{2} - \alpha$.

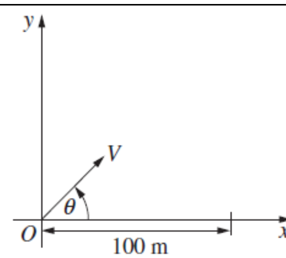
Let h_α be the maximum height reached by the object when projected at an angle α to the horizontal.
Let h_β be the maximum height reached by the object when projected at an angle β to the horizontal.
Show that the average of the two heights, $\dfrac{h_\alpha + h_\beta}{2}$, depends only on V and g.



NESA 2018 Mathematics Extension 1 HSC Examination

17 MX 1 **13 c** [Solution]

A golfer hits a golf ball with initial speed V ms^{-1} at an angle θ to the horizontal. The golf ball is hit from one side of a lake and must have a horizontal range of 100 m or more to avoid landing in the lake. Neglecting the effects of air resistance, the equations describing the motion of the ball are



$$x = Vt\cos\theta \text{ and } y = Vt\sin\theta - \frac{1}{2}gt^2,$$

where t is the time in seconds after the ball is hit and g is the acceleration due to gravity in ms^{-2}. Do NOT prove these equations.

(i) Show that the horizontal range of the golf ball is $\dfrac{V^2\sin 2\theta}{g}$ metres. **2**

(ii) Show that if $V^2 < 100g$ then the horizontal range of the ball is less than 100 m. **1**

It is now given that $V^2 = 200g$ and that the horizontal range of the ball is 100 m or more.

(iii) Show that $\dfrac{\pi}{12} \leq \theta \leq \dfrac{5\pi}{12}$. **2**

(iv) Find the greatest height the ball can achieve. **2**

NESA 2017 Mathematics Extension 1 HSC Examination

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Year 12: Calculus
C3.1 Further area and volume of solids

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Students:

- calculate area of regions between curves determined by functions (ACMSM124)
- sketch, with and without the use of technology, the graph of a solid of revolution whose boundary is formed by rotating an arc of a function about the x-axis or y-axis **AAM**
- calculate the volume of a solid of revolution formed by rotating a region in the plane about the x-axis or y-axis, with and without the use of technology (ACMSM125) **AAM**
- determine the volumes of solids of revolution that are formed by rotating the region between two curves about either the x-axis or y-axis in both real-life and abstract contexts **AAM**

Reference Sheet

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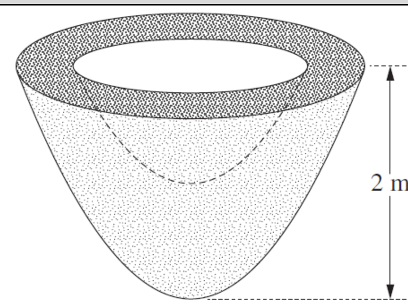
21 MX 1 **13 a** **3** Solution

A 2-metre-high sculpture is to be made out of concrete.

The sculpture is formed by rotating the region between $y = x^2$, $y = x^2 + 1$ and $y = 2$ around the y-axis.

Find the volume of concrete needed to make the sculpture.



NESA 2021 Mathematics Extension 1 HSC Examination

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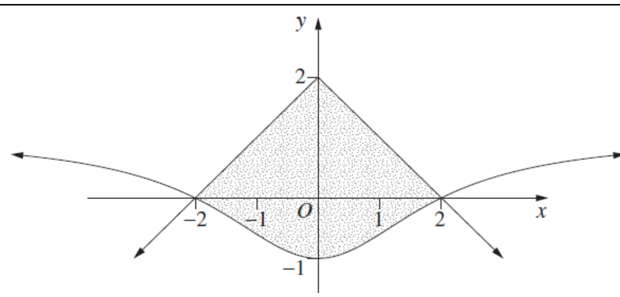
21 MX 1 **13 c** **3** Solution

The region enclosed by

$y = 2 - |x|$ and $y = 1 - \dfrac{8}{4 + x^2}$ is

shaded in the diagram.

Find the exact value of the area of the shaded region.



NESA 2021 Mathematics Extension 1 HSC Examination

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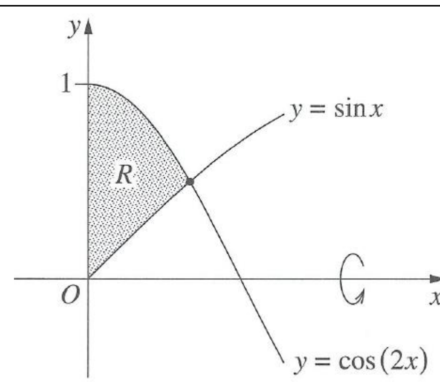
20 MX 1 **13 b** **4** Solution

The region R is bounded by the y-axis, the graph of $y = \cos(2x)$ and the graph of $y = \sin x$, as shown in the diagram.

Find the volume of the solid of revolution formed when the region R is rotated about the x-axis.



NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **14 a** (i) Sketch the graph of $y = x\cos x$ for $-\pi \le x \le \pi$ and hence explain why **3** Solution

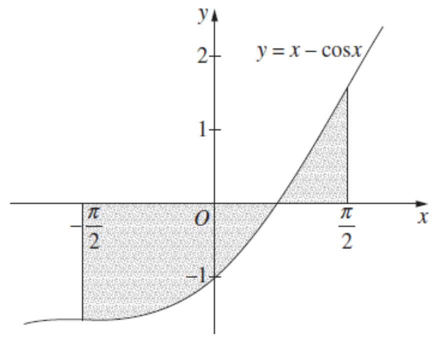
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x \, dx = 0.$$

(ii) Consider the volume of the solid of revolution produced by rotating about the x-axis the shaded region between the graph of **3**

$y = x - \cos x$, the x-axis and the lines

$x = -\dfrac{\pi}{2}$ and $x = -\dfrac{\pi}{2}$.

Using the results of part (a), or otherwise, find the volume of the solid.



NESA Mathematics Extension 1 Sample Examination Paper (2020)

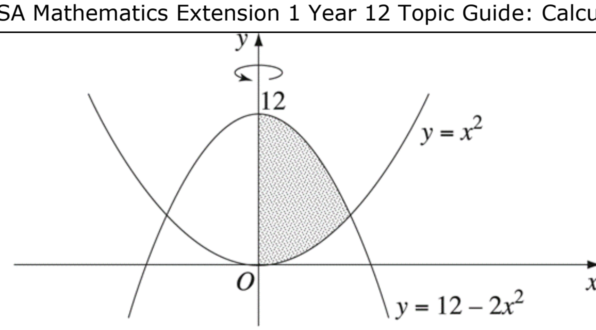
TG **1** Sketch the region bounded by the curve $y = x^2$ and the lines $y = 4$ and $y = 9$. Evaluate the area of this region. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **2** The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram. Solution

(a) Find the points of intersection of the two curves.

(b) The shaded region between the curves and the y-axis is rotated about the y-axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed.



NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **3** The region bounded by the curve $y = (x - 1)(3 - x)$ and the x-axis is rotated about the line $x = 3$ to form a solid. Solution

When the region is rotated, the horizontal line segment at height y sweeps out an annulus.

(a) Find the area of the annulus as a function of y.

(b) Find the volume of the solid.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **4** The region enclosed by the curve $y = 4\sqrt{x}$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the x-axis. Solution

Find the volume of the solid of revolution.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **5** A curved funnel has a shape formed by rotating part of the parabola $y = 2\sqrt{x}$ about the y-axis, where x and y are given in cm. Solution

The funnel is 4 cm deep.

Find the volume of liquid which the funnel will hold if it is sealed at the bottom.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **6** (a) Sketch the region bounded by the curve $y = \sin x + \cos x$ and the coordinate axes in the first quadrant, taking the upper limit of x as $\frac{3\pi}{4}$. Show the intercepts on the axes, and calculate the area of the region. **Solution**

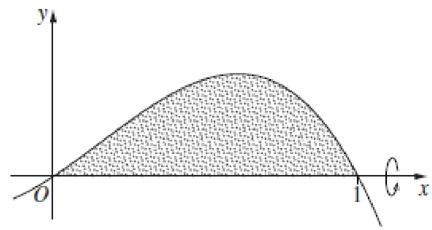
(b) Find the volume of the solid formed if the region is rotated about the x-axis to form a solid of revolution.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

19 M **13 d** The diagram shows the region bounded by the curve $y = x - x^3$, and the x-axis between $x = 0$ and $x = 1$. **3** **Solution**

The region is rotated about the x-axis to form a solid.

Find the exact value of the volume of the solid formed.



NESA 2019 Mathematics HSC Examination

18 M **14 b** The shaded region shown in the diagram is bounded by the curve $y = x^4 + 1$, then y-axis and the line $y = 10$. **3** **Solution**

Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis.

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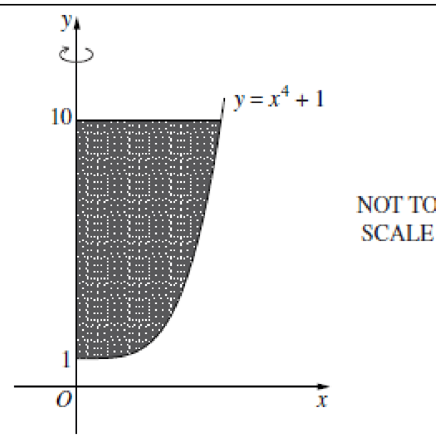
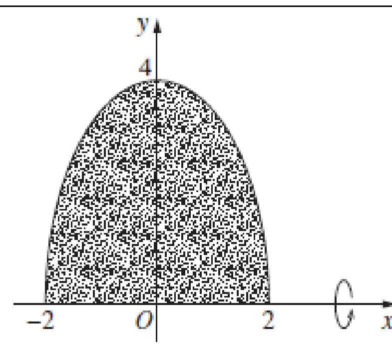


NESA 2018 Mathematics HSC Examination

17 M **12 b** The diagram shows the region bounded by $y = \sqrt{16 - 4x^2}$ and the x-axis. **3** **Solution**

The region is rotated about the x-axis to form a solid.

Find the exact volume of the solid formed.



NESA 2017 Mathematics HSC Examination

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

$A = \frac{1}{2}ab\sin C$

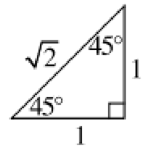
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

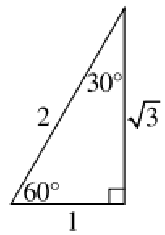
$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \frac{1}{2}r^2\theta$





Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$\cos A = \frac{1 - t^2}{1 + t^2}$

$\tan A = \frac{2t}{1 - t^2}$

$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

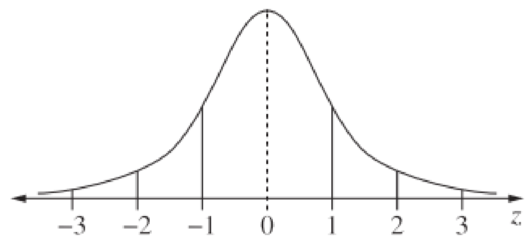
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \le x) = \int_a^x f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2021 HSC Paper

Back

21 MX 1 **1**

Given $\overrightarrow{OP} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, what is $\overrightarrow{PQ}$? **1**

Solution

A. $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ B. $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$ C. $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ D. $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$

NESA 2021 Mathematics Extension 1 HSC Examination

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21 MX 1 **2**

Solution

Which of the following integrals is equivalent to $\int \sin^2 3x \, dx$? **1**

A. $\int \frac{1+\cos 6x}{2} \, dx$ B. $\int \frac{1-\cos 6x}{2} \, dx$ C. $\int \frac{1+\sin 6x}{2} \, dx$ D. $\int \frac{1-\sin 6x}{2} \, dx$

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21 MX 1 **3**

What is the remainder when $P(x) = -x^3 - 2x^2 - 3x + 8$ is divided by $x + 2$? **1**

Solution

A. -14 B. -2 C. 2 D. 14

NESA 2021 Mathematics Extension 1 HSC Examination

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21 MX 1 **4**

Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$. **1**

Solution

Which of the following equations best represents this relationship between x and y?

A. $y^2 = x^2 + c$ B. $y^2 = \frac{x^2}{2} + c$

C. $y = x \ln|y| + c$ D. $y = \frac{x^2}{2} \ln|y| + c$

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21 MX 1 **5**

For the two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ it is known that $\overrightarrow{OA} \cdot \overrightarrow{OB} < 0$. **1**

Solution

Which of the following statements MUST be true?

A. Either, $\overrightarrow{OA}$ is negative and $\overrightarrow{OB}$ is positive, or, $\overrightarrow{OA}$ is positive and $\overrightarrow{OB}$ is negative.

B. The angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is obtuse.

C. The product $|\overrightarrow{OA}||\overrightarrow{OB}|$ is negative.

D. The points O, A and B are collinear.

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21 MX 1 **6**

The random variable X represents the number of successes in 10 independent Bernoulli trials. The probability of success is $p = 0.9$ in each trial. **1**

Solution

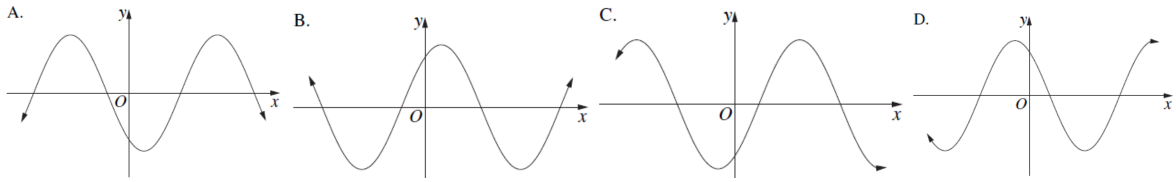
Let $r = P(X \geq 1)$.

Which of the following describes the value of r?

A. $r > 0.9$ B. $r = 0.9$ C. $0.1 < r < 0.9$ D. $r \leq 0.1$

NESA 2021 Mathematics Extension 1 HSC Examination

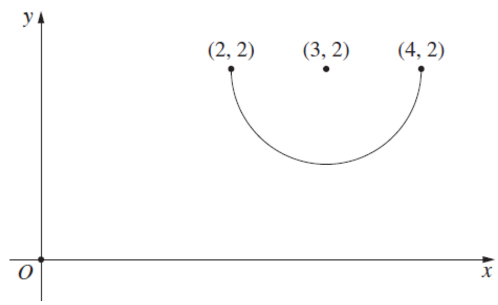
21 MX 1 **7** Which curve best represents the graph of the function $f(x) = -a\sin x + b\cos x$ given that the constants a and b are both positive? **1** Solution



NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1 **8** The diagram shows a semicircle. **1** Solution

Which pair of parametric equations represents the semicircle shown?



A. $\begin{cases} x = 3 + \sin t \\ y = 2 + \cos t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

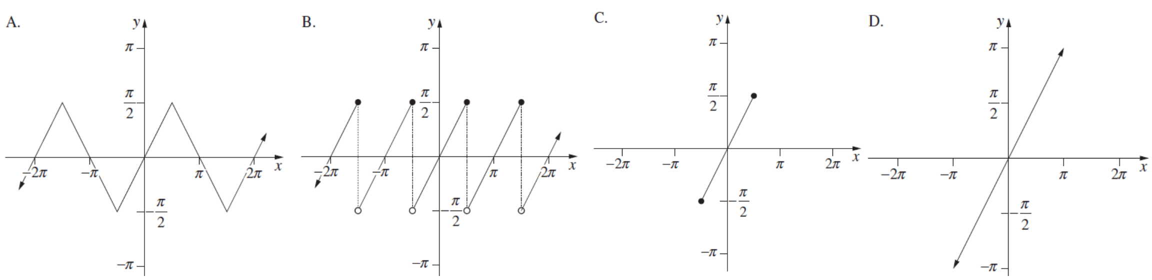
B. $\begin{cases} x = 3 + \cos t \\ y = 2 + \sin t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

C. $\begin{cases} x = 3 - \sin t \\ y = 2 - \cos t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

D. $\begin{cases} x = 3 - \cos t \\ y = 2 - \sin t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1 **9** Which graph represents the function $y = \sin^{-1}(\sin x)$? **1** Solution



NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1 **10** The members of a club voted for a new president. There were 15 candidates for the position of president and 3543 members voted. Each member voted for one candidate only. **1** Solution

One candidate received more votes than anyone else and so became the new president.

What is the smallest number of votes the new president could have received?

A 236 B 237 C 238 D 239

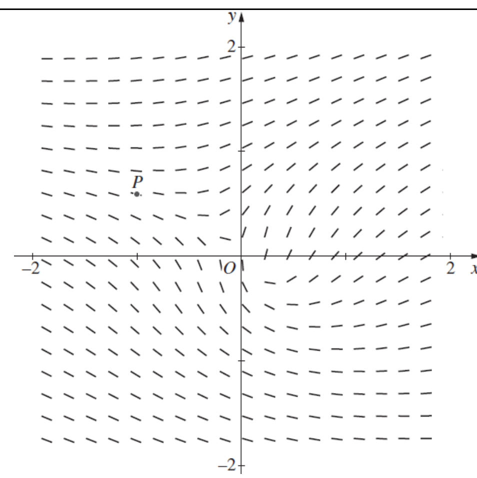
NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1 **11 a** Find $(\underset{\sim}{i} + 6\underset{\sim}{j}) + (2\underset{\sim}{i} - 7\underset{\sim}{j})$. **1** Solution

NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1 **11 b** Expand and simplify $(2a - b)^4$. **2** Solution

NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1	**11 c**	Use the substitution $u = x + 1$ to find $\int x\sqrt{x+1}\,dx$. NESA 2021 Mathematics Extension 1 HSC Examination	**3**	**Solution**
21 MX 1	**11 d**	A committee containing 5 men and 3 women is to be formed from a group of 10 men and 8 women. In how many different ways can the committee be formed? NESA 2021 Mathematics Extension 1 HSC Examination	**1**	**Solution**
21 MX 1	**11 e**	A spherical bubble is moving up through a liquid. As it rises, the bubble gets bigger and its radius increases at the rate of 0.2 mm/s. At what rate is the volume of the bubble increasing when its radius reaches 0.6 mm? Express your answer in mm^3/s rounded to one decimal place. NESA 2021 Mathematics Extension 1 HSC Examination	**2**	**Solution**
21 MX 1	**11f**	Evaluate $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$. NESA 2021 Mathematics Extension 1 HSC Examination	**2**	**Solution**
21 MX 1	**11 g**	By factorizing, or otherwise, solve $2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$ for $0 \le x \le 2\pi$. NESA 2021 Mathematics Extension 1 HSC Examination	**3**	**Solution**
21 MX 1	**11 h**	The roots of $x^4 - 3x + 6 = 0$ are α , β , γ and δ . What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$? NESA 2021 Mathematics Extension 1 HSC Examination	**2**	**Solution**
21 MX 1	**12 a**	The direction field for a differential equation is given on page 1 of the Question 12 Writing Booklet. The graph of a particular solution to the differential equation passes through the point P. Sketch the graph of the particular solution that passes through the point P.  NESA 2021 Mathematics Extension 1 HSC Examination	**1**	**Solution**

21 MX 1 **12 b**

A bottle of water, with temperature 5°C, is placed on a table in a room. The temperature of the room remains constant at 25°C. After t minutes, the temperature of the water, in degrees Celsius, is T. [Solution](#)

The temperature of the water can be modelled using the differential equation

$$\frac{dT}{dt} = k(T - 25) \text{ (Do NOT prove this.)}$$

where k is the growth constant.

(i) After 8 minutes, the temperature of the water is 10°C. **3**

By solving the differential equation, find the value of t when the temperature of the water reaches 20°C. Give your answer to the nearest minute.

(ii) Sketch the graph of T as a function of t. **1**

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21 MX 1 **12 c**

Use the principle of mathematical induction to prove **3** [Solution](#)

$$\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

that for all integers $n \geq 1$.

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21 MX 1 **12 d**

A function is defined by $f(x) = 4 - \left(1 - \frac{x}{2}\right)^2$ for x in the domain $(-\infty, 2]$. [Solution](#)

(i) Sketch the graph of $y = f(x)$ showing the x- and y-intercepts. **2**

(ii) Find the equation of the inverse function, $f^{-1}(x)$, and state its domain. **3**

(iii) Sketch the graph of $y = f^{-1}(x)$. **1**

NESA 2020 Mathematics Extension 1 HSC Examination

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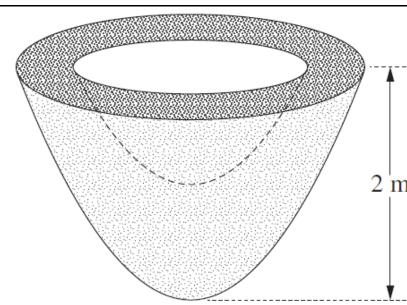
21 MX 1 **13 a**

A 2-metre-high sculpture is to be made out of concrete. **3** [Solution](#)

The sculpture is formed by rotating the region between $y = x^2$, $y = x^2 + 1$ and $y = 2$ around the y-axis.

Find the volume of concrete needed to make the sculpture.



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21 MX 1 **13 b**

When an object is projected from a point h metres above the origin with initial speed V m/s at an angle of θ° to the horizontal, its displacement vector, t seconds after projection, is **4** [Solution](#)

$$\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (-5t^2 + Vt\sin\theta + h)\,\underset{\sim}{j}\text{. (Do NOT prove this.)}$$

A person, standing in an empty room which is 3 m high, throws a ball at the far wall of the room. The ball leaves their hand 1 m above the floor and 10 m from the far wall. The initial velocity of the ball is 12 m/s at an angle of 30° to the horizontal.

Show that the ball will NOT hit the ceiling of the room but that it will hit the far wall without hitting the floor.

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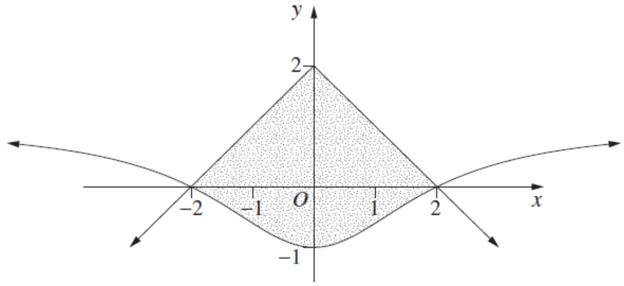
21 MX 1 **13 c** **3** Solution

The region enclosed by

$y = 2 - |x|$ and $y = 1 - \dfrac{8}{4 + x^2}$ is

shaded in the diagram.

Find the exact value of the area of the shaded region.



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21 MX 1 **13 d** Solution

(i) The numbers A, B and C are related by the equations $A = B - d$ and $C = B + d$, where d is a constant. **2**

Show that $\dfrac{\sin A + \sin C}{\cos A + \cos C} = \tan B$.

(ii) Hence, or otherwise, solve $\dfrac{\sin \frac{5\theta}{7} + \sin \frac{6\theta}{7}}{\cos \frac{5\theta}{7} + \cos \frac{6\theta}{7}} = \sqrt{3}$, for $0 \le \theta \le 2\pi$. **2**

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21 MX 1 **14 a** **3** Solution

A plane needs to travel to a destination that is on a bearing of 063°. The engine is set to fly at a constant 175 km/h. However, there is a wind from the south with a constant speed of 42 km/h.

On what constant bearing, to the nearest degree, should the direction of the plane be set in order to reach the destination?

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21 MX 1 **14 b** **4** Solution

In a certain country, the population of deer was estimated in 1980 to be 150 000.

The population growth is given by the logistic equation $\dfrac{dP}{dt} = 0.1P\left(\dfrac{C - P}{C}\right)$ where t is the number of years after 1980 and C is the carrying capacity.

In the year 2000, the population of deer was estimated to be 600 000.

Use the fact that $\dfrac{C}{P(C - P)} = \dfrac{1}{P} + \dfrac{1}{C - P}$ to show that the carrying capacity is approximately 1 130 000.

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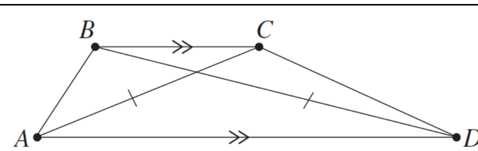
21 MX 1 **14 c** Solution

(i) For vector $\underset{\sim}{v}$, show that $\underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2$. **1**



NOT TO SCALE

(ii) In the trapezium $ABCD$, BC is parallel to AD and $|\overrightarrow{AC}| = |\overrightarrow{BD}|$. **3**

Let $\underset{\sim}{a} = \overrightarrow{AB}$, $\underset{\sim}{b} = \overrightarrow{BC}$ and $\overrightarrow{AD} = k\overrightarrow{BC}$, where $k > 0$.

Using part (i) or otherwise, show that $2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1 - k)|\underset{\sim}{b}|^2 = 0$.

NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1 **14 d**

At a certain factory, the proportion of faulty items produced by a machine is $p = \frac{3}{500}$, which is considered to be acceptable. **3** **Solution**

To confirm that the machine is working to this standard, a sample of size n is taken and the sample proportion $\hat{p}$ is calculated.

It is assumed that $\hat{p}$ is approximately normally distributed with $\mu = p$ and $\sigma^2 = \frac{p(1-p)}{n}$.

Production by this machine will be shut down if $\hat{p} \geq \frac{4}{500}$.

The sample size is to be chosen so that the chance of shutting down the machine unnecessarily is less than 2.5%.

Find the approximate sample size required, giving your answer to the nearest thousand.

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21 MX 1 **14 e**

The polynomial $g(x) = x^3 + 4x - 2$ passes through the point $(1, 3)$. **2** **Solution**

Find the gradient of the tangent to $f(x) = xg^{-1}(x)$ at the point where $x = 3$.

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