INTRODUCTION

Ancient Egypt has left us with impressive remains of an early civilization. These remains also directly and indirectly document the development and use of a mathematical culture—without which, one might argue, other highlights of ancient Egyptian culture would not have been possible. Egypt's climate and geographic situation have enabled the survival of written evidence of this mathematical culture from more than 3000 years ago, so that we can study them today. The aims of this book are to follow the development of this early mathematical culture, beginning with the invention of its number notation, to introduce a modern reader to the variety of sources (often, but not always, textual), and to outline the mathematical practices that were developed and used in ancient Egypt. The history of ancient Egypt covers a time span of more than 2000 years, and although changes occurred at a slower pace than in modern societies, we must consider the possibility of significant change when faced with a period of this length. Consequently, this book is organized chronologically, beginning around the time of the unification of Egypt around 3000 BCE and ending with the Greco-Roman Periods, by which time Egypt had become a multicultural society, in which Alexandria constituted one of the intellectual centers of the ancient world.

Each section about a particular period analyzes individual aspects of Egyptian mathematics that are especially prominent in the available sources of this time. Although some of the features may be valid during other periods as well, this cannot simply be taken for granted and is not claimed. Covering a time span this large also means that the material presented can be only a selection of all possible available sources. Similar to other areas of Egyptian culture, the source situation is often problematic. As we will see later, the most detailed sources for Egyptian mathematics, Egyptian mathematical texts, are available only from the time of the Middle Kingdom (2055–1650 BCE)[1] onward, and even then we have only about half a dozen

1 Dates given are taken from Shaw, *History*.

chance finds, which cover a period of about 200 years. From the New Kingdom (1550–1069 BCE), practically no mathematical texts have survived. It is only in the Greco-Roman Periods (332 BCE–395 CE) that a second group of mathematical texts of approximately the same quantity as the earlier Middle Kingdom material is extant. However, not only mathematical texts themselves inform us about Egyptian mathematics. If the available sources in the form of architectural drawings, administrative documents, literary texts, and others are taken into account, a much more complete picture of Egyptian mathematical culture becomes possible. I have chosen those sources that I hope to be the most significant in illustrating the mathematics of ancient Egypt.

The following parts of this introduction shall serve to provide the reader with a background in the historiography of Egyptian mathematics and the problems and possible approaches in the historiography of ancient mathematics (technical aspects vs. contextual, sociological, and cultural aspects), as well as indicate specific difficulties inherent in Egyptian sources.

0.1 PAST HISTORIOGRAPHY

The study of Egyptian mathematics has captured the interest of modern scholars for almost 200 years. Some try to find the foundations of the impressive buildings—still evident today in remains of pyramids, temples, and tombs all over Egypt—which were built under the auspices of Egyptian pharaohs. Others are drawn to the subject by a fascination with mathematics and its earliest foundations: Egypt and Mesopotamia were the first cultures to develop sophisticated mathematical systems, and these have appealed to mathematicians as well as math students interested in the history of their subject. The way these were organized, their distinct characteristic features, which in some respects differ greatly from our modern mathematical habits while being surprisingly similar to what we do today in others, have sparked the fascination of many.

But it is also relevant—and due to the extant sources maybe most interesting of all—to the historians or Egyptologists who study practical aspects of daily life in ancient Egypt. Then, as today, mathematics was needed in daily life.

Consequently, the mathematical system developed in pharaonic Egypt was practically oriented, designed to satisfy the needs of bureaucracy. Therefore, it is "more than mathematics" that we can find in Egyptian mathematical sources.[2] By reading mathematical texts, we

2 See Robson, "More than metrology," p. 361: "It turns out that a wealth of interesting insights can be gained from mathematical material that has traditionally been dismissed as unimportant and trivial."

can learn about the practical backgrounds that required and shaped the evolving mathematical knowledge. They inform us about the exchange of bread and beer, the distribution of rations, work rates of different professions, and other aspects of daily life. Thereby the information found in mathematical texts complements the evidence from administrative documents and archaeological finds.[3]

The first publications about mathematics in pharaonic Egypt appeared at the second half of the nineteenth century, after the now-famous Rhind mathematical papyrus had made its way into the collections of the British Museum and thereby made itself available for study.[4] Even today, the Rhind mathematical papyrus constitutes our most important source, and its initial publication in 1877 was followed by two further editions in 1923 and 1927, as well as numerous articles and some monographs on Egyptian mathematics.[5] The 1877 edition of the Rhind mathematical papyrus was followed in 1898 by the publication of the major mathematical fragments from the Lahun papyri and in 1900 and 1902 by the publication of two fragments of a mathematical papyrus (papyrus Berlin 6619) kept in Berlin.[6] The volume of the *Catalogue général des antiquités égyptiennes du Musée du Caire* of the ostraca published by Georges Daressy in 1901 also included two wooden boards of mathematical content. Daressy published his interpretation in 1906, which was corrected in 1923 by Thomas Eric Peet.[7] The mathematical leather roll (BM 10250), which had arrived at the British Museum together with the Rhind mathematical papyrus, was too brittle to be unrolled immediately. After a method for dealing with this brittle leather was developed, it was unrolled in 1926 and was published in the following year by Stephen R. K. Glanville.[8] The second major source, the Moscow mathematical papyrus, was not published until 1930.[9] Since then, only two further mathematical texts have been found, both of them ostraca with only few (incomplete) lines of text.[10]

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3 This was used, for example, by Dina Faltings in her work on ancient Egyptian baking and brewing (Faltings, *Lebensmittelproduktion*).

4 Even before the Rhind papyrus was available, publications on Egyptian numbers and arithmetic based on the inscriptions of the Edfu temple appeared. See Brugsch, "Rechenexempel."

5 Eisenlohr, *Mathematisches Handbuch*; Peet, *Rhind Mathematical Papyrus*; Chace, Bull, Manning, and Archibald, *Rhind Mathematical Papyrus*. Following the controversial publication of Eisenlohr, the British Museum published its own facsimile in 1898. On this controversy and the quality of the British Museum facsimile, see Imhausen, *Algorithmen*, p. 8, with notes 11 and 12.

6 Lahun fragments: Griffith, *Petrie Papyri*, pp. 15–18. Griffith included only those fragments in his publication that seemed well-enough preserved; therefore, some of the material (including some mathematical fragments) remained unpublished until the complete edition of all Lahun papyri in 2002–2006 (Collier/Quirke, *UCL Lahun papyri*), Vol. 2, pp. 71–96. Berlin 6619 fragments: Schack-Schackenburg, "Berlin Papyrus 6619," and Schack-Schackenburg, "Kleineres Fragment."

7 Daressy, *Catalogue général*; Daressy, "Calculs égyptiens"; Peet, "Arithmetic."

8 Glanville, "Mathematical leather roll." The last part of this publication (pp. 238–39) describes the method used to deal with brittle leather. See also Scott and Hall, "Laboratory notes."

9 Struve, *Mathematischer Papyrus Moskau*.

10 Hayes, *Senmut*, No. 153, and López, *Ostraca Ieratici*, No. 57170.

Two monographs on Egyptian mathematics have been published during the twentieth century. In 1972, Richard Gillings, a historian of mathematics based in Australia, published his *Mathematics in the Time of the Pharaohs*, which is still available from Dover publishers.[11] In 1993, the French monograph *Mathématiques égyptiennes*, by Sylvia Couchoud, was published.[12] Two further book-length studies are the dissertation and habilitation of the German Egyptologist Walter Friedrich Reineke: *Die mathematischen Texte der Alten Ägypter* (Berlin 1965) and *Gedanken und Materialien zur Frühgeschichte der Mathematik in Ägypten* (Berlin 1986). While the dissertation remains unpublished (although photocopies are held in the libraries of some German Egyptological institutes), the habilitation was published in 2014, almost 30 years after its completion. In 2014, a French publication on Egyptian mathematical texts appeared, which—despite its author's knowledge of recent publications on Egyptian mathematics—reverts to writing an assessment of Egyptian mathematics from a modern mathematician's point of view.[13] In addition, a source book of Egyptian mathematics was published in 1999.[14]

Much more numerous, in fact, too numerous even to be listed completely, are works on individual aspects of Egyptian mathematics, mostly in the form of articles in Egyptology and history of mathematics journals, but some also full monographs. Due to the large number of studies, only selected works will briefly be sketched in the following paragraph. An aspect of Egyptian mathematics that has fascinated historians of mathematics is the Egyptian method of fraction reckoning. Expressed in modern mathematical terminology, ancient Egyptian mathematics used only unit fractions, with the only exception being the fraction $\frac{2}{3}$. The first two researchers to publish monographs on aspects of Egyptian fraction reckoning were Otto Neugebauer and Kurt Vogel, who both wrote their doctoral theses on this subject.[15] A central part in all investigations of Egyptian fraction reckoning was the composition of the so-called $2 \div n$ table, a table that must have been essential in handling fractions during standard computations. From a mathematical point of view, the representation of a fraction as a set of unit fractions is not unambiguous; however, the two extant copies of the $2 \div n$ table indicate that a standard representation was used in ancient Egyptian mathematics. This led to the question of how the individual entries were chosen. A controversial contribution was published by Richard Gillings, who tried to establish a set of rules that were used when the table

11 Gillings, *Mathematics in the Time of the Pharaohs*.
12 Couchoud, *Mathématiques égyptiennes*.
13 Michel, *Mathématiques de l'Égypte ancienne*.
14 Clagett, *Egyptian Mathematics*. See the reviews Allen, "Review Clagett," and Spalinger, "Review Clagett".
15 Neugebauer, *Bruchrechnung*, and Vogel, *Grundlagen der ägyptischen Arithmetik*. Other contributions on Egyptian fraction reckoning (in chronological order) are Rising, "Egyptian use of unit fractions," Bruins, "The part," Bruins, "Reducible and trivial decompositions," Rees, "Egyptian fractions," and Knorr, "Techniques of fractions."

was compiled.[16] However, although these rules explain some of the choices, overall it is not possible to predict the answer that is found in the table based on these rules; therefore, mathematicians keep coming back to work on its analysis.[17] The modern description of Egyptian fraction reckoning as being "restricted" to unit fractions is obviously anachronistic (indeed, the Egyptian concept of fractions did not include a numerator, but from a historian's point of view this cannot be criticized on the basis that our modern fractions consist of denominator and numerator). Furthermore, this criticism does not do justice to the development of Egyptian fractions. Finally, the representation of Egyptian fractions in our modern system using the numerator 1 throughout causes Egyptian fraction reckoning to look more cumbersome than necessary to a modern reader, which has led to negative assessments of modern researchers, as for example that of Otto Neugebauer:

> The primitive, strictly additive, Egyptian way of computing with unit fractions had a detrimental effect throughout, even on Greek astronomy.[18]

Fraction reckoning was, without any doubt, a demanding part of Egyptian mathematics; the fact that all the extant Egyptian tables are for fraction reckoning (either for absolute numbers or for metrological systems) bears testimony to the inherent intricacies of this area of Egyptian mathematics. However, the Egyptian system also apparently had its advantages, and the available tables presumably reduced the effort that was needed. It is remarkable that Egyptian fractions prove to be resistant against the contact with Mesopotamian mathematics (which used sexagesimal fractions) and that Egyptian fractions also appear in Greek mathematics as well as the Western mathematics of the Middle Ages.[19]

Other aspects on which researchers have focused are the Egyptian calculation of the area of a circle,[20] the way in which the Egyptian method to calculate the volume of a truncated pyramid was obtained,[21] and the method of solution for a set of problems that—again phrased in anachronistic modern terminology—are similar to our algebraic equations.[22]

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16 Gillings, "Divisions of 2," and Gillings, *Mathematics in the Time of the Pharaohs*, pp. 45–80. His results were questioned in Bruckheimer and Salomon, "Some comments" and defended by Gillings in, "Response."

17 For example, van der Waerden, "(2:*n*) table" and the recent contribution by Abdulaziz, "Egyptian method."

18 Neugebauer, *History of Ancient Mathematical Astronomy*, p. 559.

19 See, for example, Fibonacci's Liber Abaci from the early thirteenth century (Sigler, *Fibonacci-Liber Abaci*).

20 Gillings and Rigg, "Area of a circle," Smeur, "Value equivalent to π," Engels, "Quadrature of the circle," and Gerdes, "Three alternate methods."

21 Gunn and Peet, "Four geometrical problems," Vogel, "Truncated pyramid," Struve, *Mathematischer Papyrus Moskau*, pp. 174–76, Thomas, "Moscow mathematical papyrus, no. 14," Luckey, "Rauminhalt," Vetter, "Problem 14," Gillings, "Volume of a truncated pyramid," Neugebauer, "Pyramidenstumpf-Volumen."

22 For a discussion, see Imhausen, "ꜥ*ḥ*ꜥ-Aufgaben."

The amount of available literature on Egyptian mathematics is even more astonishing if we take into account that the only sources on which almost all these studies were founded are four papyri (of which half consist only of a number of fragments), a leather roll, a wooden board, and two ostraca (stone or pottery shards used as writing material)—almost all from the Middle Kingdom (2055–1650 BCE), with five more papyri and some further ostraca—from the Ptolemaic and Roman Periods (332 BCE–395 CE), more than a thousand years later. The Ptolemaic and Roman sources (the number of which is likely to increase from unpublished texts in museums as well as possible new finds) were rarely taken into consideration. Studies of Egyptian mathematics concentrated mostly on the earlier material. In spite of all these limitations, the sources still allow us new insights and were sufficient material for many studies over the past 125 years. In fact, we seem far from exhausting these sources. Due to developments in the history of mathematics of the last 40 years, it has now become obvious that many "statements" about Egyptian mathematics that were made a long time ago and that have since then been accepted as "truths" need to be reassessed.[23] At the beginning of the twentieth century, a period in which research on Egyptian mathematics reached its first boom, questions of *how* ancient mathematical knowledge related to later developments (e.g., Greek geometry), including our modern mathematical system, determined the research on ancient sources. In the respective works, the unfamiliar ways in which mathematical operations were expressed were translated into their "modern equivalents." The insight that mathematics is not culturally independent, that it is not constantly evolving in a linear way toward the next level, was gained much later—and this insight and its consequences are still debated.[24] As a consequence, "close reading" of ancient mathematical sources evolved, in which the technical language and the formal structures of the original texts are taken seriously.[25] As a further consequence, ancient mathematics was understood as best studied in relation to the culture in which it evolved.[26]

23 For an overview of the most common "myths" in the historiography of Egyptian mathematics, see Imhausen, "Myths."

24 One of the essential articles was Unguru, "Need to Rewrite," which was elaborated in Unguru and Rowe, "Quadratic Equation I" and Unguru and Rowe, "Quadratic Equation II." The importance of the discussion that was sparked by Sabetai Unguru has been acknowledged in the volume Christianidis, *Classics*, which includes a reprint of Unguru, "Need to Rewrite" and its first responses. Twenty-five years later, Sabetai Unguru and Michael Fried have added another monograph to the discussion (Fried and Unguru, *Conica*).

25 This has affected Egyptian mathematics as well as Mesopotamian mathematics. Fundamental in this respect for Mesopotamian mathematics are the works of Jens Høyrup, Jim Ritter, and Eleanor Robson; see, for example, Høyrup, *Lengths, Widths, Surfaces*; Ritter, "Reading Strasbourg 368," and Robson, *Mathematics in Ancient Iraq*.

26 See, for example, Imhausen, "Egyptian mathematical texts and their contexts." Again, this is also true for other areas of ancient mathematics; for Mesopotamian mathematics, see Robson, *Mathematics in Ancient Iraq*; for Greek and Roman mathematics, Cuomo, *Ancient Mathematics*.

At the same time, however, traditional work on Egyptian mathematics continues, as does the publication of unfounded speculation about Egyptian mathematics (or even Egyptian science in general), which also had a recent boom with the Internet as a platform, where this kind of material can be made easily available. The scarce source material has not been, and presumably never will be, able to answer every question asked in modern times. This has encouraged rather speculative theories founded on practically no evidence; in fact, it is exactly the lack of evidence that has enabled speculations of this kind. Classical examples are the methods that were used to align and build the pyramids as well as the way Egyptian mathematical knowledge was discovered.[27] The honest and academically responsible answer to most of these questions (at least at the present moment) would simply be that we don't know. For some of them, at least, possibilities may be discussed, while others may almost certainly never be answered. With the same certainty, however, there will always be some trying to fill these gaps with their individual ideas. In assessing this kind of speculation, the actual available evidence must remain in the picture. Therefore, by presenting a variety of available sources and pointing out the limitations of the available material, one aim of this book is to encourage its readers to judge speculations about Egyptian mathematics with a critical and informed eye.

The progress in the history of mathematics, which has influenced recent studies of Egyptian mathematics, has run parallel to an improvement in our understanding of Egyptian languages and culture. Not many of the mathematical texts, however, have yet benefitted from this development. Color photographs of the Rhind mathematical papyrus, along with a brief overview of Egyptian mathematics, were published in 1987 by Gay Robins and Charles Shute.[28] The third-largest source is the Lahun mathematical fragments, which Jim Ritter and I have reedited in the context of the new edition of the Lahun papyri.[29] It is to be hoped that other sources will follow.

0.2 AIMS OF THIS STUDY

Traditionally, the mathematical texts, especially the hieratic mathematical texts, have been the main sources in works on ancient Egyptian mathematics. There are obvious reasons for this. The mathematical texts indicate specific mathematical problems and also detail how they were solved, giving the most immediate access to ancient mathematical techniques and concepts.

27 For a detailed discussion of the theories and their shortcomings of the architecture of the pyramids and the mathematics involved, see Rossi, *Architecture and Mathematics*.
28 Robins and Shute, *Rhind Mathematical Papyrus*.
29 Imhausen and Ritter, "Mathematical fragments," and Imhausen, "UC32107A."

However, as Jim Ritter has elaborated, the problems that a historian of ancient mathematics faces in the interpretation of these texts are many and varied:

> What does a text mean? A question like this poses particularly acute problems for someone who works on ancient texts. This is not to underplay the complexity and plurality of interpretations in, say, contemporary literary texts, but rather that in the case of Antiquity we are often confronted with the opposite situation: the difficulty of finding even a single possible interpretation. The problem may occur at a very concrete level: lacunae in the texts, *hapaxes*, a technical vocabulary for which it is difficult or impossible to fix the semantic referents. Moreover, we are often confronted with contemporary constraints, not always immediately perceptible to us, concerning, among other things, the rules of the textual genre, the available concepts or techniques of expression, the aims pursued by the authors.[30]

These difficulties are reflected, at least in part, in the amount of available literature on the Egyptian mathematical texts. In this book on Egyptian mathematics, the mathematical texts obviously remain important sources. Following recent developments in the methodology of ancient mathematics, I will try to describe the characteristics of Egyptian concepts and techniques and especially focus on the differences from our modern concepts. Fundamental in this respect is to realize that algebraic equations, an extremely powerful mathematical tool, did not exist in ancient Egypt. Instead, the mathematical solutions are indicated in the form of procedures, step-by-step instructions that—if followed—will lead to the solution of the given problem.

The aim of this study is to go beyond the few hundred years covered by the still available mathematical texts. The beginnings of Egyptian mathematics, in the form of number notation and metrological systems, can be traced much further back if other sources are taken into account. I will attempt to sketch the development of Egyptian mathematics from the invention of number notation, which occurred at approximately the same time as the invention of writing, until the Greco-Roman Periods using a variety of available sources, thereby also describing the context and cultural setting of Egyptian mathematics throughout pharaonic history. In order to achieve this, archaeological sources (such as drawings on the walls of tombs), administrative texts, autobiographies, and various literary texts will be used. Obviously, information

30 Ritter, "Reading Strasbourg 368," p. 177.

to be received about specific mathematical techniques from these sources is very limited indeed; however, they are extremely useful in assessing the role of mathematics within pharaonic culture and in getting glimpses at those people who may have been the authors of the mathematical texts that we have been studying in isolation for the past 150 years.

A chronological order was chosen to help the reader place the individual evidence within its respective time frame. Egyptian history spans a period of more than 3000 years, and it has to be kept in mind that individual developments did take place during that time. It is not legitimate to simply mix evidence from different stages of Egyptian history; however, at least at some times and instances it seems plausible to assume a certain continuity even in the absence of direct evidence. To provide a historical frame, each section will include a very brief sketch of historical and political events for that period.[31]

31 A more detailed overview and introduction to the respective times can be found at Shaw, *History*. For an overall introduction into ancient Egyptian history and culture, see the revised British Museum book of ancient Egypt (Spencer, *Ancient Egypt*).