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Mathematics in the Primary Curriculum

Why this area of learning is important: Mathematics introduces children to concepts, skills and thinking strategies that are essential in everyday life and support learning across the curriculum. It helps children make sense of the numbers, patterns and shapes they see in the world around them, offers ways of handling data in an increasingly digital world and makes a crucial contribution to their development as successful learners. Children delight in using mathematics to solve a problem, especially when it leads them to an unexpected discovery or new connections. As their confidence grows, they look for patterns, use logical reasoning, suggest solutions and try out different approaches to problems. Mathematics offers children a powerful way of communicating. They learn to explore and explain their ideas using symbols, diagrams and spoken and written language. They start to discover how mathematics has developed over time and contributes to our economy, society and culture. Studying mathematics stimulates curiosity, fosters creativity and equips children with the skills they need in life beyond school.

In this chapter there are explanations of

- the different kinds of reason for teaching mathematics in the primary school;
- the contribution of mathematics to everyday life and society;
- the contribution of mathematics to other areas of the curriculum;
- the contribution of mathematics to the learner's intellectual development;
- the importance of mathematics in promoting enjoyment of learning;
- how mathematics is important as a distinctive form of knowledge;
- how the essential content of the primary curriculum in England is not just about knowledge and skills but also about using and applying mathematics;

- the various components of using and applying mathematics in the primary curriculum in England; and
- the relationship of numeracy to mathematical understanding.

Why teach mathematics in the primary school?

The statement about the importance of mathematical understanding in the primary National Curriculum programme of study quoted at the head of this chapter is packed with worthy intentions and is consequently rather difficult to take in as a whole. I find it helpful, therefore, to identify within this statement at least five different kinds of **aims of teaching mathematics** in primary schools. They relate to the contribution of mathematics to: (1) everyday life and society; (2) other areas of the curriculum; (3) the child's intellectual development; (4) the child's enjoyment of learning and (5) the body of human knowledge. These are not completely discrete strands, nor are they the only way for structuring our thinking about why we teach this subject.

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In shaping, monitoring and evaluating their medium-term planning, teachers should ensure that sufficient prominence is given to each of the five reasons for teaching mathematics:

1. its importance in everyday life and society;
2. its importance in other curriculum areas;
3. its importance in relation to the learner's intellectual development;
4. its importance in developing the child's enjoyment of learning;
5. its distinctive place in human knowledge and culture.

How does mathematics contribute to everyday life and society?

This trand relates to what are often referred to as **utilitarian** aims. We teach mathematics because it is useful for everyone in meeting the demands of everyday living. The National Curriculum importance statement refers, for example, to introducing children to 'concepts, skills and thinking strategies that are useful in everyday life'. Many everyday transactions and real-life problems, and most forms of employment, require confidence and competence in a range of basic mathematical skills and knowledge – such as measurement, manipulating shapes, organizing space, handling money, recording and interpreting numerical and graphical data, and using information and communications technology (ICT).

Teachers themselves, for example, need a large range of such skills in their everyday professional life – for example, in handling school finances and budgets, in organizing their timetables, in planning the spatial arrangement of the classroom, in processing assessment data, in interpreting inspection reports and in using ICT in their teaching. We should note also here the reference to 'ways of handling data in an increasingly digital world': if in teaching mathematics we are to equip young people for the demands of everyday life then our approach to the subject must reflect the availability of ICT applications such as calculators and spreadsheets.

The relationship of mathematical processes to real-life contexts is demonstrated in this book particularly in the process of *modelling* which is introduced in Chapter 5 and which forms the basis of the discussion of addition, subtraction, multiplication and division structures in Chapters 6 and 9.

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Learning experiences for children that reflect the contribution of mathematics to everyday life and society could include, for example: (a) realistic and relevant financial and budgeting problems; (b) meeting people from various forms of employment and exploring how they use mathematics in their work; (c) helping teachers with some of the administrative tasks they have to do that draw on mathematical skills.

How does mathematics contribute to other areas of the curriculum?

This strand relates to the **application** of mathematics. We teach mathematics because it has applications in a range of contexts, including other areas of the curriculum. Much of mathematics as we know it today has developed in response to practical challenges in science and technology, in the social sciences and in economics. So, as well as being a subject in its own right, with its own patterns, principles an d procedures, mathematics is a subject that can be applied. The National Curriculum importance statement for mathematics refers, for example, to mathematical skills that 'support learning across the curriculum'. The primary-school teacher who is responsible for teaching nearly all the areas of the curriculum is uniquely placed to take advantage of opportunities that arise, for example, in the context of science and technology, in the arts, in history, geography and society, to apply mathematical skills and concepts purposefully in meaningful contexts – and to make explicit to the children what mathematics is being applied.

This is a two-way process: these various curriculum areas can also provide meaningful and

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Learning experiences for children that reflect the application of mathematics to other curriculum areas could include, for example: (a) collecting, organizing, representing and interpreting data arising in science experiments or in enquiries related to historical, geographical and social understanding; (b) drawing up plans and meeting the demands for accurate measurement in technology and in design; (c) using mathematical concepts to stimulate and support the exploration of pattern in art, dance and music, and (d) using mathematical skills in cross-curricular studies such as 'transport' or 'a visit to France'.

purposeful contexts for introducing and reinforcing mathematical concepts, skills and principles. With the new primary curriculum in England (statutory from 2011) cross-curricular studies have once again become a feature of primary education. So, for example, the programme of study for Mathematical Understanding refers to 'enhancing children's mathematical understanding through making links to other areas of learning and wider issues of interest and importance' (DCSF/QCDA, 2010b, Mathematical Understanding, Programme of Study, section 3). Cross-curricular studies will inevitably draw on and develop mathematical skills, for example, in organizing, representing and interpreting data – and can be planned with particular mathematical content in mind.

How does mathematics contribute to the child's intellectual development?

This strand includes what are sometimes referred to as *thinking skills*, but I am including here a broader range of aspects of the learner's **intellectual development**. We teach mathematics because it provides opportunities for developing important intellectual skills in problem solving, deductive and inductive reasoning, creative thinking and communication. We may note here, for example, the reference in the importance statement for mathematics to 'thinking strategies', to using mathematics to 'solve a problem' and to 'use logical reasoning, suggest solutions and try out different approaches to problems' – these are all distinctive characteristics of a person who thinks in a mathematical way.

Sometimes to solve a mathematical problem we have to reason logically and systematically, using what is called deductive reasoning. Other times, an insight that leads to a solution may require thinking creatively, divergently and imaginatively. So the importance statement for mathematics quoted at the head of this chapter rightly, if surprisingly, also claims that 'studying mathematics … fosters creativity'. So mathematics is an important context for developing effective problem-solving strategies that potentially have significance in all areas of human activity. But also in learning mathematics, children have many opportunities to 'look for patterns'. This involves inductive reasoning leading to the articulation of generalizations, statements of what is always the case. The process of using a number of specific instances to formulate a general rule or principle, which can then be applied in other instances, is at the heart of mathematical thinking.

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Learning experiences for children in mathematics should include a focus on the child's intellectual development, by providing opportunities to foster: (a) problem-solving strategies; (b) deductive reasoning, which includes reasoning logically and systematically; (c) creative thinking, which is characterized by divergent and imaginative thinking; (d) inductive reasoning that leads to the articulation of patterns and generalizations, and (e) communication of mathematical ideas orally and in writing, using both formal and informal language, and in diagrams and symbols.

Then finally, in this section, in terms of intellectual development we should note that in learning mathematics children are developing a 'powerful way of communicating'. Mathematics is effectively a language, containing technical terminology, distinctive patterns of spoken and written language, a range of diagrammatic devices and a distinctive way of using symbols to represent and manipulate concepts. Children use this language to articulate their observations and to explain and later to justify or prove their conclusions in mathematics. Mathematical language is a key theme throughout this book.

How does mathematics contribute to the child's enjoyment of learning?

This strand relates to what is sometimes referred to as the **aesthetic** aim in teaching mathematics. We teach mathematics because it has an inherent beauty that can provide the learner with delight and enjoyment. I suspect that there may be some readers whose experience of learning mathematics in school may not resonate with this statement! But there really is potential for genuine enjoyment and pleasure for children in primary schools in exploring and learning mathematics. It is emotionally satisfying for children to be able to make coherent 'sense of the numbers, patterns and shapes they see in the world around them', for example, through the processes of classification and conceptualization. 'Children delight in using mathematics to solve a problem' – indeed they will often be seen to smile with pleasure when they get an insight that leads to a solution; when they spot a pattern, discover something for themselves or make connections; when they find a mathematical rule that always works – or even identify an exception that challenges a rule. The extensive patterns that underlie mathematics can be fascinating, and recognizing and exploiting these can be genuinely satisfying. Mathematics can be appreciated as a creative experience, in which flexibility and imaginative thinking can lead to interesting outcomes or fresh avenues to explore for the curious mind. Throughout this book I aim to increase the reader's own sense of delight and enjoyment in mathematics, with the hope that this will be communicated to those they teach.

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Learning experiences for children in mathematics should ensure that children enjoy learning mathematics, by providing opportunities to: (a) experience the sense of pleasure that comes from solving a problem or a mathematical puzzle; (b) have their curiosity stimulated by formulating their own questions and investigating mathematical situations; (c) play small-group games that draw on mathematical skills and concepts; (d) experiment with pattern in numbers and shapes and discover relationships for themselves, and (e) have some beautiful moments in mathematics where they are surprised, delighted or intrigued.

Why is mathematics important as a distinctive form of knowledge?

This strand is what the more pretentious of us would call the **epistemological** aim. Epistemology is the theory of knowledge. The argument here is that we teach mathematics because it is a significant and distinctive form of human knowledge with its own concepts and principles and its own ways of making assertions, formulating arguments and justifying conclusions. This kind of purpose in teaching mathematics is based on the notion that an educated person has the right to be initiated into all the various forms of human knowledge and to appreciate their distinctive ways of reasoning and arguing. For example, an explanation of a historical event, a theory in science, a doctrine in theology and a mathematical generalization are four very different kinds of statements, supported by different kinds of evidence and arguments.

In mathematics, as we have indicated above, some of the characteristic ways of reasoning would be to look for patterns, to make and test conjectures, to investigate a hypothesis, to formulate a generalization and then to justify the generalization by means of a deductive argument (a proof). The most distinctive quality of mathematical knowledge is the notion of a mathematical statement being incontestably true because it can be deduced by logical argument either from the axioms (self-evident truths) of mathematics or from previously proven truths. Of course, children in primary schools will not be able to justify their mathematical conclusions by means of a formal proof, but they can experience many of the other distinctive kinds of mathematical processes and even at this age begin to demonstrate and explain why various things are always true.

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Primary school teachers should include as one of their aims for teaching mathematics: to promote awareness of some of the contributions of various cultures to the body of mathematical knowledge. This can be a fascinating component of history-based cross-curricular projects, such as the study of ancient civilizations.

The primary National Curriculum importance statement for mathematics proposes that children should 'start to discover how mathematics has developed over time'. Mathematics is a significant part of our cultural heritage. Not to know anything about mathematics would be as much a cultural shortcoming as being ignorant of our musical and artistic heritage. Historically, the study of mathematics has been at the heart of most major civilizations. Certainly much of what we might regard as European mathematics was well known in ancient Chinese civilizations. Our number system has its roots in ancient Egypt, Mesopotamia and Hindu cultures. Classical civilization was dominated by great mathematicians such as Pythagoras, Zeno, Euclid and Archimedes. To appreciate mathematics as a subject should also include knowing something of how mathematics as a subject has developed over time and how different cultures have contributed to this body of knowledge.

What mathematics do we teach in the primary school?

The level descriptions for mathematics in the National Curriculum for England (DCSF/QCDA, 2010a), which cover both primary and secondary schools, are organized under four attainment targets. These provide a simple framework for describing the mathematics we teach in primary schools: (1) Using and Applying Mathematics – which is an integral component of the whole of the primary school curriculum description for Mathematical Understanding, as well as the basis of the sections entitled ‘Key Skills’ and ‘Breadth of Learning’; (2) Number and Algebra – which is taught through the sections of the Mathematical Understanding curriculum entitled ‘Number and the number system’, ‘Number operations and calculations’ and ‘Money’; (3) Shape, Space and Measures – which is taught through the sections entitled ‘Measures’ and ‘Geometry’; (4) Statistics – which in the primary curriculum corresponds to the section also called ‘Statistics’.

This mathematics is essentially the content of the rest of this book. Section C covers the curriculum for number and algebra: Chapters 9–19 explain all the various aspects of number, such as different kinds of numbers, our number system, the structures of the four basic number operations, mental strategies and written methods for calculations, remainders and rounding, various properties of numbers, fractions and ratios, calculations with decimals, proportion and percentages; and Chapters 20 and 21 introduce some of the foundations of algebra. Shape, space and measures are the subject of Section D, Chapters 22–26. Statistics, including probability, is dealt with in Section E, Chapters 27–29.

But at the head of the list of attainment targets, given appropriate prominence, is ‘**using and applying mathematics**’. The mathematics curriculum – and therefore this book – contains a huge amount of knowledge to be learnt, and a great number of skills to be mastered and concepts and principles to be understood. But it is a lifeless and purposeless subject if we do not also learn to use and apply all this knowledge and all these skills, concepts and principles. Teachers have to ensure that children get opportunities to learn not just mathematical content but also how to use and apply their mathematics. The opening statement in the programme of study for Mathematical Understanding (DCSF/QCDA, 2010b) makes this clear: ‘Learning

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Using and applying mathematics is not just something for children to do after they have learnt some mathematical content, but should be integrated into all learning and teaching of the subject. Sometimes an appropriate approach to planning a sequence of mathematics lessons might be: introduce some new concept or skill; practise it; apply it in various problems. But not always! Sometimes a real-life problem that draws on a wide range of mathematical ideas can be used as a meaningful context in which to introduce some new mathematical concept or to provide a purposeful stimulus for children to extend their mathematical skills.

in this area should include an appropriate balance of focused subject teaching and well-planned opportunities to use, apply and develop knowledge and skills across the whole curriculum.'

What do children learn in using and applying mathematics in the primary school?

Much of this attainment target is about using and applying mathematics in real-life contexts: 'children use mathematics as an integral part of classroom activities'. This leads on to the development of problem-solving strategies. These are used and developed not just in realistic problems set in real-life contexts, but also through what we might regard as essentially problems within mathematics itself: 'children develop their own strategies for solving problems and use these strategies both in working within mathematics and in applying mathematics to practical contexts'.

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Three areas of skills to be developed in teaching children to use and apply mathematics are (a) problem-solving strategies, (b) reasoning mathematically and (c) communicating with mathematics.

In practice, it makes little sense to categorize problems as either 'within mathematics' or in 'practical contexts'. There is really a continuum of contexts for using and applying mathematics. At one end are problems that are purely mathematical, just about numbers and shapes, in which the outcome is of no particular practical significance. An example is shown in Figure 2.1, where the challenge would be: how many different shapes can you make by joining five identical squares together edge to edge? At the other end of the continuum would be problems that are genuine, real-life situations that need to be solved. An example might be: how much orange squash should we buy to be able to provide three drinks for each player in the inter-school football tournament? But many other problems or investigations are set in real-life contexts, but are perhaps less genuine. An example might be: find out as many interesting things as you can about the way the page numbers are arranged on the sheets of a newspaper.

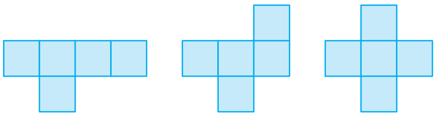

Figure 2.1 *How many different shapes can you make with five squares joined together like these?*

The using and applying mathematics attainment target also includes the development of mathematical reasoning: 'children show that they understand a general

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To develop the key processes involved in using and applying mathematics children should have opportunities to use mathematics in a range of tasks, including:

(a) activities within their everyday experience in the classroom, such as planning their timetable for the day, or grouping children for various activities;
(b) identifying and proposing solutions to genuine problems, such as where in the playground staff should park their cars;
(c) tackling artificial but realistic problems, such as estimating the cost for a family of four to go on a two-week holiday on the Norfolk Broads;
(d) applying mathematics in practical tasks, such as making a box to hold a set of calculators;
(e) solving mathematical problems, such as finding two-digit numbers that have an odd number of factors;
(f) pursuing mathematical investigations, such as 'find out as much as you can about the relationships between different paper sizes (A5, A4, A3, and so on)'.

statement by finding particular examples that match it … they look for patterns and relationships.' The key processes in mathematical reasoning include those associated with recognizing patterns and relationships, making conjectures, formulating hypotheses, articulating and using generalizations.

Then, another clear strand is about the development of skills in communicating with mathematics: 'they explain why an answer is correct … presenting information and results in a clear and organised way … draw simple conclusions of their own and explain their reasoning.' It is in using and applying mathematics that children get the most powerful experience of communicating with mathematical language, symbols and diagrams. This will involve explaining insights, describing the outcomes of an investigation, providing convincing reasons for a conclusion they have drawn, or offering evidence to support a point of view.

Key processes in using and applying mathematics, such as modelling, problem solving, generalizing and creative thinking, are introduced and explained in detail in Section B (Chapters 4 and 5) and then developed as an integral component of subsequent chapters.

How does numeracy relate to mathematical understanding?

There was a time when 'numeracy' was understood to refer to no more than competence with numbers and calculations within the demands of everyday life – a small subset of the mathematics curriculum. The word was often preceded by the word 'basic'. So, it would amount to not much more than knowing your multiplication tables and being able to work out simple everyday calculations with money – most of which in reality would be done with calculators anyway. Then – apparently without any justification – in the early twenty-first century the National Numeracy Strategy in England chose to use the word synonymously with 'mathematics'. So everything in the primary mathematics curriculum suddenly became numeracy, and mathematics lessons in primary schools became 'the numeracy hour'. The 2010 revision of the primary curriculum in England (DCSF/QCDA, 2010b) has sensibly brought back 'mathematics', in the section of the curriculum entitled 'Mathematical Understanding' – and given a new lease of life to the word 'numeracy'.

Significantly, numeracy now appears as one of the Essential for Learning and Life, which are aspects of learning to be embedded and developed across the curriculum. In this new understanding of the term, numeracy is clearly and specifically about the using and applying aspects of mathematics: 'Children use and apply mathematics confidently and competently in their learning and in everyday contexts. They recognise where mathematics can be used to solve problems and are able to interpret a wide range of mathematical data' (DCSF/QCDA, 2010b, Essentials for Learning and Life). So, numeracy takes on a much more substantial meaning, including aspects like problem solving, using mathematical models of real-life situations (see Chapter 5) and communicating with mathematics. Encouragingly, the requirement to develop numeracy, understood in this way, across the curriculum has the potential to make the using and applying aspect of mathematics appropriately prominent in primary learning and teaching – and not just in mathematics lessons.

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In developing numeracy, children in primary schools should learn across the curriculum to:

(a) represent and model situations using mathematics, using a range of tools and applying logic and reasoning in order to predict, plan and try out options;
(b) use numbers and measurements for accurate calculation and an understanding of scale, in order to make reasonable estimations;
(c) interpret and interrogate mathematical data in graphs, spreadsheets and diagrams, in order to draw inferences, recognise patterns and trends, and assess likelihood and risk;
(d) use mathematics to justify and support decisions and proposals, communicating accurately using mathematical language and conventions, symbols and diagrams.

(DCSF/QCDA, 2010b, Essentials for Learning and Life, Numeracy).

Research focus

The programme of study for mathematical understanding in the National Curriculum for primary schools (DCSF/QCDA, 2010b) emphasizes the important part that ICT should play in the learning of mathematics. The key skills that children need to learn to make progress include: 'develop, select and apply a range of mental, written and ICT-based methods and models to estimate, approximate, calculate, classify, quantify, order and compare'. Children should 'use a wide range of practical resources, including ICT'. One of the most useful and accessible ICT devices is, of course, the simple hand-held calculator. There are just two references to calculators in the detail of the curriculum, both of them in the detailed mathematics curriculum for 9–11-year-olds: 'interpret calculator displays and round to an appropriate level of accuracy' and 'develop a range of strategies … for calculating and checking, including using a calculator or computer efficiently'. This might be taken to imply that calculators should be used only with children aged 9 years and over. However, there is a significant note in the explanatory text for the section for 5–7-year-olds: 'Using calculators to explore number patterns and properties is important here.' This shows awareness of the many ways in which calculators can be used right across the primary age range, not just to do calculations, but to promote understanding of mathematical concepts and to explore patterns and relationships between numbers. This stance is supported by research evidence. For example, an Australian research project (Groves, 1993; 1994) explored the results achieved by using a calculator-aware number curriculum with young children from the reception class onwards. When these children reached the age of 8–9 years they were found to perform better in a number of key mathematical tasks than children two years older than them. These included: estimating the result of a calculation; solving real-life problems; understanding of place value, decimals and negative numbers, and interpreting calculator answers involving decimals.

Suggestions for further reading

1. Read the entries on 'Aims of mathematics teaching', 'Deductive and inductive reasoning' and 'Using and applying mathematics' in Haylock with Thangata (2007). Each entry in this book contains a definition of the 'concept', explanation and discussion, practical examples and related reading.
2. The contributors to White and Bramall (2000) explore the varied aims of learning and teaching mathematics, and consider to what extent the subject deserves the privileged status it has traditionally enjoyed in the school curriculum. Recommended for those with a philosophical bent.
3. In a chapter entitled 'Calculators for all?', in Thompson (2003), Williams and Thompson suggest that the National Numeracy Strategy in England failed to take the opportunity to articulate effective calculator practice in mathematics teaching in both Key Stages of primary schooling. Examples are given of exciting ways of using calculators with young children exploring the pattern and application of number.
4. Read an intelligent discussion of numeracy in the twenty-first century in Chapter 1 of Anghileri (2007).

Glossary of key terms introduced in Chapter 2

Aims of teaching mathematics: in describing the importance of mathematics in the primary curriculum a number of different kinds of aims in teaching mathematics can be identified; these can be classified as utilitarian, application, intellectual development, aesthetic and epistemological.

Utilitarian aim in teaching mathematics: mathematics is useful in everyday life and necessary in most forms of employment.

Application aim in teaching mathematics: mathematics has many important applications in other curriculum areas.

Intellectual development aim in teaching mathematics: mathematics provides opportunities for developing important intellectual skills in problem solving, deductive and inductive reasoning, creative thinking and communication.

Aesthetic aim in teaching mathematics: mathematical experiences in the primary school can provide delight, wonder, beauty and enjoyment.

Epistemological aim in teaching mathematics: mathematics should be learnt because it is a distinctive and important form of knowledge and part of our cultural heritage.

Using and applying mathematics: using the skills, knowledge, concepts and principles learnt in mathematics to solve problems, across a continuum from genuine problems in a real-life context to purely mathematical challenges; engaging in investigations and enquiries that develop key processes in mathematical reasoning; and communicating insights, reasoning, results and conclusions with mathematical language, diagrams and symbols.