Mathematics

Applied Mathematics 2202

Interim Edition

Newfoundland Labrador

Curriculum Guide
2012

Contents

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INTRODUCTION

Background

The curriculum guide communicates high expectations for students.

The Mathematics curriculum guides for Newfoundland and Labrador have been derived from *The Common Curriculum Framework for 10-12 Mathematics: Western and Northern Canadian Protocol*, January 2008. These guides incorporate the conceptual framework for Grades 10 to 12 Mathematics and the general outcomes, specific outcomes and achievement indicators established in the common curriculum framework. They also include suggestions for teaching and learning, suggested assessment strategies, and an identification of the associated resource match between the curriculum and authorized, as well as recommended, resource materials.

Beliefs About Students and Mathematics

Mathematical understanding is fostered when students build on their own experiences and prior knowledge.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in developing mathematical literacy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. Students at all levels benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students' experiences and ways of thinking, so that students feel comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must come to understand that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

Affective Domain

To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting, asssessing and revising personal goals.

Goals For Students

Mathematics education must prepare students to use mathematics confidently to solve problems.

The main goals of mathematics education are to prepare students to:

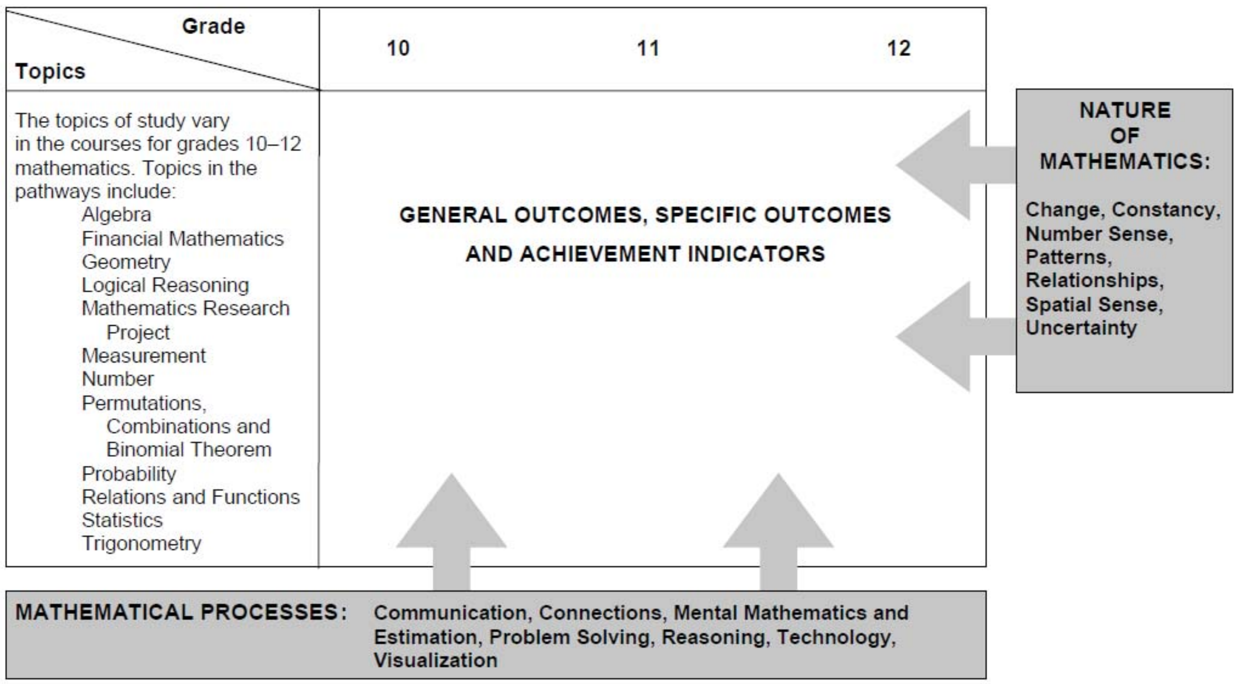
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

- *Communication [C]*
- *Connections [CN]*
- *Mental Mathematics and Estimation [ME]*
- *Problem Solving [PS]*
- *Reasoning [R]*
- *Technology [T]*
- *Visualization [V]*

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.
Students are expected to:

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and for solving problems
- develop visualization skills to assist in processing information, making connections and solving problems.

This curriculum guide incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

Connections [CN]

Through connections, students begin to view mathematics as useful and relevant.

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding … Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p.5).

Mental Mathematics and Estimation [ME]

Mental mathematics and estimation are fundamental components of number sense.

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "... become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "... provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels.

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you know?" or "How could you ...?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity requires students to determine a way to get from what is known to what is unknown. If students have already been given steps to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly seek and engage in a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics.

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

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Technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- create geometric patterns
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

Nature of Mathematics

- *Change*
- *Constancy*
- *Number Sense*
- *Patterns*
- *Relationships*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this curiculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Change is an integral part of mathematics and the learning of mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS-Benchmarks, 1993, p.270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

An intuition about number is the most important foundation of a numerate child.

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p.146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by product of learning rather than through direct instruction. It can be developed by providing rich mathematical tasks that allow students to make connections to their own expereinces and their previous learning.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of mathematics.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships.

Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense offers a way to interpret and reflect on the physical environment.

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

Uncertainty is an inherent part of making predictions.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Essential Graduation Learnings

Essential graduation learnings are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression, citizenship, communication, personal development, problem solving, technological competence* and *spiritual and moral development*.

Aesthetic Expression

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship

Graduates will be able to assess social, cultural, economic and environmental interdependence in a local and global context.

Communication

Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn and communicate effectively.

Personal Development

Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving

Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts.

Technological Competence

Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

Spiritual and Moral Development

Graduates will be able to demonstrate an understanding and appreciation for the place of belief systems in shaping the development of moral values and ethical conduct.

See *Foundations for the Atlantic Canada Mathematics Curriculum*, pages 4-6.

The mathematics curriculum is designed to make a significant contribution towards students' meeting each of the essential graduation learnings (EGLs), with the communication, problem-solving and technological competence EGLs relating particularly well to the mathematical processes.

Outcomes and Achievement Indicators

The curriculum is stated in terms of general outcomes, specific outcomes and achievement indicators.

General Outcomes

General outcomes are overarching statements about what students are expected to learn in each course.

Specific Outcomes

Specific outcomes are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given course.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

Achievement Indicators

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

Specific curriculum outcomes represent the means by which students work toward accomplishing the general curriculum outcomes and ultimately, the essential graduation learnings.

Program Organization

<table>
<tr><th>Program Level</th><th>Course 1</th><th>Course 2</th><th>Course 3</th><th>Course 4</th></tr>
<tr><td>Advanced</td><td rowspan="2">Mathematics 1201</td><td>Mathematics 2200</td><td>Mathematics 3200</td><td>Mathematics 3208</td></tr>
<tr><td>Academic</td><td>Mathematics 2201</td><td>Mathematics 3201</td><td></td></tr>
<tr><td>Applied</td><td>Mathematics 1202</td><td>Mathematics 2202</td><td>Mathematics 3202</td><td></td></tr>
</table>

The applied program is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the workforce.

The academic and advanced programs are designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs. Students who complete the advanced program will be better prepared for programs that require the study of calculus.

The programs aim to prepare students to make connections between mathematics and its applications and to become numerate adults, using mathematics to contribute to society.

Summary

The conceptual framework for Grades 10-12 Mathematics (p. 3) describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should result from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between topics.

ASSESSMENT AND EVALUATION

Purposes of Assessment

What learning is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others about what is really valued.

Assessment techniques are used to gather information for evaluation. Information gathered through assessment helps teachers determine students' strengths and needs in their achievement of mathematics and guides future instructional approaches.

Teachers are encouraged to be flexible in assessing the learning success of all students and to seek diverse ways in which students might demonstrate what they know and are able to do.

Evaluation involves the weighing of the assessment information against a standard in order to make an evaluation or judgment about student achievement.

Assessment has three interrelated purposes:

- assessment *for* learning to guide and inform instruction;
- assessment *as* learning to involve students in self-assessment and setting goals for their own learning; and
- assessment *of* learning to make judgements about student performance in relation to curriculum outcomes.

Assessment *for* Learning

Assessment *for* learning involves frequent, interactive assessments designed to make student understanding visible. This enables teachers to identify learning needs and adjust teaching accordingly. It is an ongoing process of teaching and learning.

Assessment *for* learning:

- requires the collection of data from a range of assessments as investigative tools to find out as mush as possible about what students know
- provides descriptive, specific and instructive feedback to students and parents regarding the next stage of learning
- actively engages students in their own learning as they assess themselves and understand how to improve performance.

Assessment *as* Learning

Assessment *as* learning actively involves students' reflection on their learning and monitoring of their own progress. It focuses on the role of the student as the critical connector between assessment and learning, thereby developing and supporting metacognition in students.

Assessment *as* learning:

- supports students in critically analysing their learning related to learning outcomes
- prompts students to consider how they can continue to improve their learning
- enables students to use information gathered to make adaptations to their learning processes and to develop new understandings.

Assessment *of* Learning

Assessment *of* learning involves strategies to confirm what students know, demonstrate whether or not they have met curriculum outcomes, or to certify proficiency and make decisions about students' future learning needs. Assessment *of* learning occurs at the end of a learning experience that contributes directly to reported results.

Traditionally, teachers relied on this type of assessment to make judgments about student performance by measuring learning after the fact and then reporting it to others. Used in conjunction with the other assessment processes previously outlined, however, assessment *of* learning is strengthened.

Assessment *of* learning:

- provides opportunities to report evidence to date of student achievement in relation to learning outcomes, to parents/guardians and other stakeholders
- confirms what students know and can do
- occurs at the end of a learning experience using a variety of tools.

Because the consequences of assessment *of* learning are often far-reaching, teachers have the responsibility of reporting student learning accurately and fairly, based on evidence obtained from a variety of contexts and applications.

Assessment Strategies

Assessment techniques should match the style of learning and instruction employed. Several options are suggested in this curriculum guide from which teachers may choose, depending on the curriculum outcomes, the class and school/district policies.

Observation (formal or informal)

This technique provides a way of gathering information fairly quickly while a lesson is in progress. When used formally, the student(s) would be aware of the observation and the criteria being assessed. Informally, it could be a frequent, but brief, check on a given criterion. Observation may offer information about the participation level of a student for a given task, use of a concrete model or application of a given process. The results may be recorded in the form of checklists, rating scales or brief written notes. It is important to plan in order that specific criteria are identified, suitable recording forms are ready, and all students are observed within a reasonable period of time.

Performance

This curriculum encourages learning through active participation. Many of the curriculum outcomes promote skills and their applications. In order for students to appreciate the importance of skill development, it is important that assessment provide feedback on the various skills. These may be the correct manner in which to use a manipulative, the ability to interpret and follow instructions, or to research, organize and present information. Assessing performance is most often achieved through observing the process.

Paper and Pencil

These techniques can be formative or summative. Whether as part of learning, or a final statement, students should know the expectations for the exercise and how it will be assessed. Written assignments and tests can be used to assess knowledge, understanding and application of concepts. They are less successful at assessing processes and attitudes. The purpose of the assessment should determine what form of paper and pencil exercise is used.

Journal

Journals provide an opportunity for students to express thoughts and ideas in a reflective way. By recording feelings, perceptions of success, and responses to new concepts, a student may be helped to identify his or her most effective learning style. Knowing how to learn in an effective way is powerful information. Journal entries also give indicators of developing attitudes to mathematical concepts, processes and skills, and how these may be applied in the context of society. Self-assessment, through a journal, permits a student to consider strengths and weaknesses, attitudes, interests and new ideas. Developing patterns may help in career decisions and choices of further study.

Interview

This curriculum promotes understanding and applying mathematics concepts. Interviewing a student allows the teacher to confirm that learning has taken place beyond simple factual recall. Discussion allows a student to display an ability to use information and clarify understanding. Interviews may be a brief discussion between teacher and student or they may be more extensive. Such conferences allow students to be proactive in displaying understanding. It is helpful for students to know which criteria will be used to assess formal interviews. This assessment technique provides an opportunity to students whose verbal presentation skills are stronger than their written skills.

Presentation

The curriculum includes outcomes that require students to analyze and interpret information, to be able to work in teams, and to communicate information. These activities are best displayed and assessed through presentations. These can be given orally, in written/pictorial form, by project summary, or by using electronic systems such as video or computer software. Whatever the level of complexity, or format used, it is important to consider the curriculum outcomes as a guide to assessing the presentation. The outcomes indicate the process, concepts and context for which a presentation is made.

Portfolio

Portfolios offer another option for assessing student progress in meeting curriculum outcomes over a more extended period of time. This form of assessment allows the student to be central to the process. There are decisions about the portfolio, and its contents, which can be made by the student. What is placed in the portfolio, the criteria for selection, how the portfolio is used, how and where it is stored, and how it is evaluated are some of the questions to consider when planning to collect and display student work in this way. The portfolio should provide a long-term record of growth in learning and skills. This record of growth is important for individual reflection and self-assessment, but it is also important to share with others. For all students, it is exciting to review a portfolio and see the record of development over time.

INSTRUCTIONAL FOCUS

Planning for Instruction

Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There should be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

Teaching Sequence

The curriculum guide for Applied Mathematics 2202 is organized by units. This is only a suggested teaching order for the course. There are a number of combinations of sequences that would be appropriate.

Each two page spread lists the topic, general outcome, and specific outcome.

Instruction Time Per Unit

The suggested number of hours of instruction per unit is listed in the guide at the beginning of each unit. The number of suggested hours includes time for completing assessment activities, reviewing and evaluating. The timelines at the beginning of each unit are provided to assist in planning. The use of these timelines is not mandatory. However, it is mandatory that all outcomes are taught during the school year, so a long term plan is advised. Teaching of the outcomes is ongoing, and may be revisited as necessary.

Resources

The authorized resource for Newfoundland and Labrador students and teachers is *Math at Work 11* (McGraw-Hill Ryerson). Column four of the curriculum guide references *Math at Work 11* for this reason. Teachers may use any other resource, or combination of resources, to meet the required specific outcomes.

GENERAL AND SPECIFIC OUTCOMES

GENERAL AND SPECIFIC OUTCOMES WITH ACHIEVEMENT INDICATORS (pages 19-152)

This section presents general and specific outcomes with corresponding achievement indicators and is organized by unit. The list of indicators contained in this section is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used to determine whether or not students have achieved a given specific outcome. Teachers may use any number of these indicators or choose to use other indicators as evidence that the desired learning has been achieved. Achievement indicators should also help teachers form a clear picture of the intent and scope of each specific outcome.

Applied Mathematics 2202 is organized into seven units: *Surface Area*, *Drawing and Design*, *Volume and Capacity*, *Interpreting Graphs*, *Banking and Budgeting, Slope*, and *Right Triangles and Trigonometry*.

Surface Area

Suggested Time: 17 Hours

Unit Overview

Focus and Context

In this unit, students will investigate the surface area of prisms, pyramids, cylinders, spheres and cones. To connect the area of two-dimensional shapes to the surface area of three-dimensional objects, students will begin by using nets to find the surface area. They will continue to build on their understandings of direct and indirect measurement from Mathematics 1202 and characteristics of two-dimensional and three-dimensional shapes from Grade 9.

Applying the concept of surface area to real-world applications will help students see the use of three-dimensional measurement in their everyday lives.

Outcomes Framework

GCO

Develop spatial sense through direct and indirect measurement.

SCO M1

Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.

GCO

Develop number sense and critical thinking skills.

SCO N1

Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

GCO

Develop algebraic reasoning.

SCO A1

Solve problems that require the manipulation and application of formulas related to:

- volume and capacity
- **surface area**
- slope and rate of change
- simple interest
- finance charges.

SCO A3

Solve problems by applying proportional reasoning and unit analysis.

Process Standards

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Measurement		
M3. Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [CN, ME, PS, V] M4. Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. [CN, ME, PS, V]	M1. Solve problems that involve SI and imperial units in surface area measurements and verify the solutions. [C, CN, ME, PS, V]	M1. Demonstrate an understanding of the limitations of measuring instruments including: • precision • accuracy • uncertainty • tolerance and solve problems. [C, PS, R, T, V]
Algebra		
A1. Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R]	A1. Solve problems that require the manipulation and application of formulas related to: • volume and capacity • **surface area** • slope and rate of change • simple interest • finance charges. [CN, ME, PS, R] A3. Solve problems by applying proportional reasoning and unit analysis. [C, CN, PS, R]	not addressed
Geometry		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Measurement

Outcomes

Students will be expected to

M1 Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.

[C, CN, ME, PS, V]

Achievement Indicator:

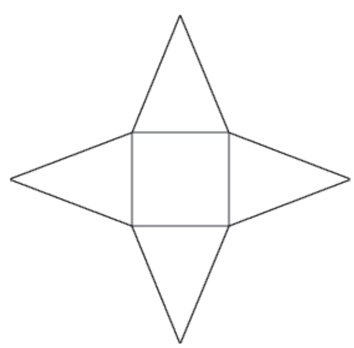
M1.1 *Explain, using examples, including nets, the relationship between area and surface area.*

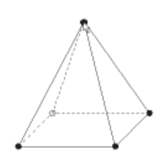
Elaborations—Strategies for Learning and Teaching

In Grade 7, students developed an understanding of 2-D objects and calculated the area of triangles, circles, and parallelograms (7SS2). In Grade 8, they developed a conceptual understanding of the surface area of 3-D objects, using nets, and then calculated the surface area of right rectangular prisms, right triangular prisms and right cylinders (8SS3, 8SS4). In Grade 9, students extended their knowledge of surface area to calculating the surface area of composite 3-D objects (9SS2). In Mathematics 1202, work with surface area was reviewed and extended to include cones (M4). In addition, SI and imperial units were used in estimations and calculations. A review of unit conversions may be necessary. In this unit, the surface area of rectangular prisms, triangular prisms, cylinders and cones will be reinforced. Students will also explore, estimate and calculate the surface area of pyramids and spheres using referents, net diagrams and formulas.

Students have previously investigated the relationship between area and surface area but a review may be necessary. They have worked with area as the amount of two-dimensional space that a shape covers, and surface area as the sum of the areas of all faces or surfaces of a solid, measured in square units. Students should be aware that they will need to identify an object's net and use the appropriate formulas to determine the area of each face. In the beginning, focus should be on right prisms and cylinders. Students have already worked with these three-dimensional shapes. They may need to be reminded that prisms consist of two parallel congruent faces called bases. A prism is named for its bases. To visualize nets, students can deconstruct objects such as tissue boxes, paper towel rolls or Pringles™ containers.

Students should be introduced to pyramids and examine the relationship between area and surface area of this figure. An example of a net diagram for a square pyramid is below. They should recognize the surface area is the sum of the areas of the base and four triangular faces.





Students will work with objects composed of rectangles, circles and triangles. A review of these area formulas may be necessary.

General Outcome: Develop spatial sense through direct and indirect measurement.

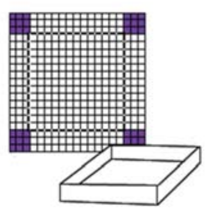
Suggested Assessment Strategies

Performance

- In groups of two, students can play a memory matching game where they must match net diagrams to their corresponding shapes. The images will be on cards turned facedown and students will receive a point when they turn two up that display a shape and its matching net. Include multiple copies of each net. Composite objects could also be included.

(M1.1)

- Provide students with sheets of graph paper, tape and scissors. Ask them to cut out a 2 x 2 square from each corner, fold and tape to create a box. They should create another box by removing a 3 x 3 square from each corner.



Ask students to calculate the surface area of the two boxes created. Discuss with them how many different size boxes could be made and the largest size square that can be removed.

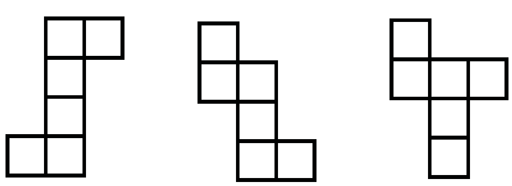
(M1.1)

- Students can colour code a net to show congruent parts. They then use the colours to create a plan to find the surface area.

(M1.1)

Observation

- A hexomino is a shape made of six identical squares connected along their sides. There are 35 different patterns that can be made from six squares. Below are 3 different hexominos.



Ask students to create at least 3 more hexominos. They should decide if any of the created hexominos can be folded to form a closed cube.

(M1.1)

Resources/Notes

Authorized Resource

Math at Work 11

1.1 Nets and Area of Three-Dimensional Objects

Student Book (SB): pp. 6-14

Teacher's Resource (TR): pp. 11-19

Blackline Masters (BLM): 1-5

Web Links

- *http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/3d/index.htm*
- *http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF&lesson=html/object_interactives/surfaceArea/use_it.html*

These interactive sites demonstrate 3-D objects and their nets.

- *http://illuminations.nctm.org/ActivityDetail.aspx?ID=205*

A site for creating nets

- *http://www.learner.org/interactives/geometry/3d_prisms.html*

"Explore and Play" with prisms and pyramids provides an animation of various shapes and their corresponding nets.

Measurement

Outcomes

Students will be expected to

M1 Continued ...

Achievement Indicators:

M1.2 *Explain how a referent can be used to estimate surface area.*

M1.3 *Estimate the surface area of a 3-D object.*

Elaborations—Strategies for Learning and Teaching

Students will use a referent to determine approximate dimensions of objects from which they can estimate the surface area. They have previously used referents to estimate the dimensions of a 2-D shape or 3-D object. Students should work with referents in both SI and imperial units. Some common referents for linear measurements include:

thickness of a dime	≈ 1 mm
width of a paper clip	≈ 1 cm
distance from a door knob to the floor	≈ 1 m
thickness of a hockey puck	≈ 1 in.
length of a standard floor tile	≈ 1 ft
distance from the tip of the nose to the outstretched fingers	≈ 1 yd

Students may relate 1 kilometre to a distance between two well known points in their own communities.

Area referents will also be used to estimate the surface area of an object. Some common referents for area measurement include:

area of a floor tile	≈ 1 ft^2
area of a postage stamp	≈ 1 $in.^2$
area of a fingernail	≈ 1 cm^2
area of an exterior house door	≈ 2 m^2
area of an exercise notebook	≈ 93.5 $in.^2$ or 600 cm^2
area of an ice rink surface	≈ 1500 m^2 or 17 000 ft^2
area of a sheet of gyproc or plywood	≈ 32 ft^2 or 3 m^2
area of a square of shingles (3 packs)	≈ 100 ft^2

These referents are suggestions. Students should be encouraged to select their own personal referents. They may, for example, use their waist height as a referent for one metre. If they determine the height of the seat of a chair to be approximately half of their waist height, then the seat of the chair is about 0.5 metres high. If students are determining the length of a room, they could count the tiles on the floor knowing that the length of a standard tile is one foot. Students should be given an opportunity to estimate measurements of various items using their referents. They should always be encouraged to analyse their answers to determine if they are reasonable.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Performance

- In groups, ask students to use SI and imperial referents to estimate the surface area of objects inside or outside the classroom (e.g., teacher's desk, nearby building, tube slide, basketball, etc.). Ask them to compare estimates with classmates.

(M1.2, M1.3)

Journal

- Ask students to describe a situation in their everyday lives where estimation would be beneficial. They should explain the consequences of overestimating or underestimating in this situation.

(M1.2, M1.3)

Resources/Notes

Authorized Resource

Math at Work 11

1.2 Estimating Surface Area

SB: pp. 15-24

TR: pp. 20-29

BLM: 1-6

Web Link

www.k12pl.nl.ca/mathematics/seniorhigh/introduction/math2202/classroomclips.html

The **Linear Referents** clip demonstrates students estimating linear measurements using referents.

Algebra

Outcomes

Students will be expected to

A1 Solve problems that require the manipulation and application of formulas related to:

- volume and capacity
- **surface area**
- slope and rate of change
- simple interest
- finance charges.

[**CN, ME, PS, R**]

M1 Continued ...

Achievement Indicators:

M1.4 *Solve a problem that involves determining the surface area of 3-D objects, including pyramids and spheres.*

A1.1 *Solve a contextual problem involving the application of a formula that does not require manipulation.*

M1.5 *Solve a contextual problem that involves the surface area of 3-D objects and that requires the manipulation of formulas.*

A1.2 *Solve a contextual problem involving the application of a formula that requires manipulation.*

Elaborations—Strategies for Learning and Teaching

It is intended that this outcome be integrated throughout the course. In this unit, some of the problems will require the manipulation and application of surface area formulas.

The exploration with nets should lead into generalizing formulas and using symbolic representations to calculate surface area. Students have previously used formulas to calculate the surface area of right prisms, right cylinders and cones. They will now extend this to include calculating the surface area of right pyramids with rectangular bases and spheres.

Begin with a review of calculating the surface area of a prism and a cylinder. Calculating the surface area of a prism using its net allows for easy identification of congruent faces, which sometimes avoids the necessity of having to find the area of each face individually. Students may conclude that the surface area of a rectangular prism can be calculated using the formula $SA_{prism} = 2lw + 2lh + 2wh$. Some students may never use this formula but continue to find the sum of the areas of all the faces.

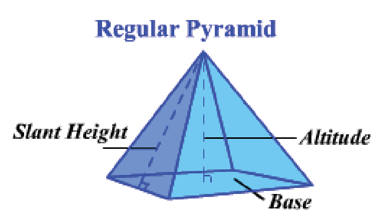
Next, revisit the cylinder. The net of a cylinder consists of two circles and a rectangle. The curved surface opens up to form a rectangle. A good way to demonstrate this is to unpeel the label on a can to show it is a rectangle.

$SA_{cylinder} = 2 \times (\text{area of circle}) + (\text{area of rectangle})$

$SA_{cylinder} = 2\pi r^2 + 2\pi rh$

A pyramid is a 3-D figure with a polygon base. The shape of the base determines the name of the pyramid. Students should work with square-based pyramids.

Students should recognize the difference between the height of a right pyramid and its slant height. The height refers to the perpendicular height from the apex to the base and the slant height is the altitude of the triangular face. The slant height of the pyramid should be provided.



The surface area of the pyramid is the total area of all five sides: the square base and the four triangles.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Performance

- Provide students with several objects, such as a cereal box, ice cream cone, Toblerone™ box, paper towel roll, etc., and ask them to calculate the surface area of each using a ruler or measuring tape. Encourage students to use both imperial and SI units.

  (M1.4, A1.1)

- Ask students to measure the dimensions of a box, such as a toothpaste tube box, and calculate the surface area of the box. They should then disassemble the box and calculate the area of the cardboard used in the box.

  (M1.4, A1.1)

Paper and Pencil

- Ask students to answer the following:

  Stacks of 500 sheets of paper are wrapped in paper. A 500-sheet stack is $8\frac{1}{2}$ inches by 11 inches by 2 inches. The wrapping paper is $22\frac{1}{2}$ inches by 15 inches.

  (i) Calculate the surface area of a 500-sheet stack.
  (ii) Calculate the area of the wrapping paper.
  (iii) How much wrapping paper is used in overlap?

  Alternatively, this could be completed as an activity where students are given a pack of wrapped paper and have to determine the dimensions.

  (M1.4, A1.1)

- Ask students to answer the following:

  You are painting a cylindrical oil tank with a radius of 150 ft. If one gallon of paint covers 350 ft^2 and you have 16 gallons of paint, how far up the tank can you paint?

  (M1.5, A1.2)

Resources/Notes

Authorized Resource

Math at Work 11

1.3 Surface Area of Three-Dimensional Objects

SB: pp. 25-39

TR: pp. 30-43

BLM: 1-7

Measurement

Outcomes

Students will be expected to

M1 and A1 Continued ...

Achievement Indicators:

M1.4, M1.5, A1.1, A1.2
Continued

Elaborations—Strategies for Learning and Teaching

These outcomes provide a good opportunity for students to make connections between mathematics and real world situations. Surface area can be used to determine such things as the amount of:

- vinyl siding for a new house
- paint needed to cover walls and ceiling
- materials for a cylindrical punching bag
- shingles for a roof

In some situations, surface area will be calculated when all measurements are given, requiring no formula manipulation. Alternatively, students may be given the surface area and asked to find another measurement. In such cases, be selective with the missing dimensions students are asked to find. Avoid cases involving factoring. For a cylinder, for example, provide the surface area and radius and ask students to determine the height.

M1.6 *Illustrate, using examples, the effect of dimensional changes on surface area.*

Students will investigate how increasing or decreasing the dimension(s) of an object changes the surface area. Students can be given several objects to calculate the surface area of and then asked to calculate the new surface area once one of the dimensions is doubled, tripled or halved, etc. They should realize that doubling a dimension does not double the surface area.

As an extension, students could explore the effect on surface area of changing all dimensions by the same factor.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Journal

- Ask students to consider if doubling the radius of a cylinder doubles its surface area. They should explain their reasoning.

(M1.4, M1.6, A1.1)

Paper and Pencil

- Lisa builds a shipping crate out of quarter inch plywood. The crate is a cube with a side dimension of 4 ft.

  Ask students to answer the following:

  (i) What is the surface area of the crate?
  (ii) She buys plywood in standard sheet sizes of 4 ft x 8 ft. How many sheets of plywood does she need to build one shipping crate?
  (iii) She builds a second crate that is half the height, but has the same length and width. How many sheets of plywood will she need to build the smaller shipping crate? Explain.

(M1.4, M1.6, A1.1)

- A toy block manufacturer needs to cover its wooden blocks with a non-toxic paint. One block is a right square pyramid with a base of 2 in. and a slant height of $3\frac{1}{2}$ in. A second block is a right cone that has a slant height of $3\frac{1}{2}$ in. and a base radius of 1 in.

  Ask students to answer the following:

  (i) Which block requires the most paint?
  (ii) When the blocks rest on their bases, which block is the tallest? How do you know?

(M1.4, A1.1, M1.5, A1.2)

Resources/Notes

Authorized Resource

Math at Work 11

1.3 Surface Area of Three-Dimensional Objects

SB: pp. 25-39

TR: pp. 30-43

BLM: 1-7

Web Link

http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF&lesson=html/video_interactives/areavolume/areaVolumeInteractive.html

This interactive site demonstrates 3-D objects and their nets.

Algebra

Outcomes

Students will be expected to

A3 Solve problems by applying proportional reasoning and unit analysis.

[**C, CN, PS, R**]

Achievement Indicators:

A3.1 *Explain the process of unit analysis used to solve a problem.*

A3.2 *Explain, using an example, how unit analysis and proportional reasoning are related.*

A3.3 *Solve a problem, using unit analysis.*

A3.4 *Solve a problem within and between systems, using proportions or tables.*

M1.4, M1.5 and M1.6 *Continued*

A1.1 and A1.2 *Continued*

Elaborations—Strategies for Learning and Teaching

Students applied proportional reasoning to problems involving ratios in Grade 8 (8N5) and similar polygons in Grade 9 (9SS3). They also used proportional reasoning as they converted measurements within and between the imperial and SI systems in Mathematics 1201 (M1, M2).

Throughout this unit, students will use proportional reasoning as they solve problems that involve SI and imperial units in surface area measurements. Unit analysis will also be used to verify that units in a conversion are correct.

To convert from one measurement to another, students need to understand the relationship between the units of measurement. To change km/h to km/min, for example, students can multiply by $\frac{1\text{h}}{60\,\text{min}}$ because hours and minutes are proportional. There is a constant relationship between them.

In the context of a problem, it may be necessary to convert given measurements into common units. When converting within or between SI and imperial units, unit analysis should be used to verify that units in a conversion are correct.

Unit analysis should be used, for example, if students have to convert 200 metres to kilometres.

$$200\,\cancel{\text{m}} \times \frac{1\text{km}}{1000\,\cancel{\text{m}}} = 0.200\,\text{km}$$

It is important for students to notice that the units have changed but the actual distance has not. The distance of 200 m is the same as 0.200 km. Prompt students to discuss the value of the ratio $\frac{1\text{km}}{1000\text{m}}$. Since 1 km = 1000 m, the ratio is equal to 1. Students should recognize that when a numerical value is multiplied by 1, the value remains the same. This is the basis of unit conversion. Encourage students to check the reasonableness of their answers when performing conversions. They should ask themselves if the answer makes sense. In this example, students should realize that when they convert 200 metres to kilometres, the answer should be less than 200.

In addition to prisms and pyramids, students also worked with the surface area of cones in Mathematics 1202. Revisit the formula $SA_{cone} = \pi r^2 + \pi rs$, where r is the radius of the base and s is the slant height of the cone. In some cases, students will have to apply the Pythagorean theorem to determine the slant height.

If the cone has no top (like a drinking cup), the formula is simply $SA = \pi rs$. Discuss examples such as an ice cream cone, a traffic pylon and a teepee.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

  You are building a child's wagon. The wagon is in the shape of a rectangular prism with no top. It is to be 3 feet long and 18 inches wide. The sides of the wagon will be 10 inches high. Determine the surface area of the wagon, in square inches.

(A3.3, A3.4, M1.4)

Resources/Notes

Authorized Resource

Math at Work 11

1.3 Surface Area of Three-Dimensional Objects

SB: pp. 25-39

TR: pp. 30-43

BLM: 1-7

Measurement

Outcomes

Students will be expected to

M1 and A1 Continued ...

Achievement Indicators:

M1.4, M1.5 and M1.6 *Continued*

A1.1 and A1.2 *Continued*

Elaborations—Strategies for Learning and Teaching

Finally, students will investigate the surface area of a sphere. The following activity is a great way to investigate the connection between the area of a circle and the surface area of a sphere. Students will need an orange, a calliper or ruler, and a compass.

- Measure and record the circumference of the orange.
- Calculate the radius by dividing the circumference by 2π.
- Use a compass to draw 6 circles with radius equal to the radius of the orange.
- Peel the orange and arrange the peel within the circles. Completely fill one circle before moving to the next.
- Continue filling the circles until all peel has been used.
- Use the filled circles and the formula for the area of a circle to estimate the surface area of the orange.

Students should reach the conclusion that the surface area of a sphere is the area of four circles and is calculated using $4\pi r^2$.

The effect of dimensional changes on surface area should also be explored here. The following is an engaging way to get a class snapshot of student thinking. Give students a probe that requires them to select an answer and provide a justification for their answer.

What's the surface area?

The radius of a sphere is doubled.

Choose the statement that describes how the surface area of the new sphere compares to the surface area of the original sphere:

A. Same size

B. Doubles in size

C. Quadruples in size

D. Not enough information to compare

Explain your reasoning.

After completing the question, students crumple their papers into a ball and toss the paper balls around until they are instructed to stop and pick up or hold on to one paper. They share the answer and explanation that is described on the paper they are holding. Students can form small groups acording to the selected response on their papers and discuss the similarities or differences in the explanation provided.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Journal

- Students could describe how they would estimate the amount of leather required to cover a softball.

(M1.4)

Paper and Pencil

- Emily is placing a gazing ball in one of her customer's gardens. The ball has a diameter of 2 ft and will be covered with reflective crystals. One jar of these crystals covers 10 ft^2. Ask students to answer the following:
  - (i) Estimate the surface area to decide whether one jar of the crystals will cover the ball.
  - (ii) Calculate the surface area, to the nearest square foot.
  - (iii) Was your estimate reasonable? Explain.

(M1.3, M1.4, A1.1)

- A basketball has a diameter of 24.8 cm.

  Ask students to answer the following:
  - (i) How much leather is required to cover this ball?
  - (ii) If one square metre of leather costs $28, how much will it cost to cover the ball?

(M1.4, A1.1)

- The surface area of an official 5-pin bowling ball is approximately 459.96 cm^2. Ask students to determine the diameter of the bowling ball.

(M1.5, A1.2)

- Ask students to answer the following using a sphere with a radius of 15 cm.
  - (i) Predict how much the surface area increases if the radius increases by a factor of 2.
  - (ii) Calculate the change in the surface area, to the nearest square centimetre.
  - (iii) How accurate was your prediction?

(M1.4, A1.1, M1.6)

Resources/Notes

Authorized Resource

Math at Work 11

1.4 Surface Area of Cones and Spheres

SB: pp. 40-49

TR: pp. 44-52

BLM: 1-8

Web Link

www.k12pl.nl.ca/mathematics/seniorhigh/introduction/math2202/classroomclips.html

In the professional learning clip on **Surface Area**, students develop the formula for the surface area of a sphere.

Number

Outcomes

Students will be expected to

N1 Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

[C, CN, PS, R]

Achievement Indicators:

N1.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

N1.2 *Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.*

Elaborations—Strategies for Learning and Teaching

This outcome is intended to be integrated throughout the course by using puzzles and games with a focus on numerical reasoning. In Mathematics 1202, students applied problem-solving strategies to analyze puzzles and games that involved spatial reasoning. Logical reasoning using puzzles and games will be addressed in Mathematics 3202. Students need time to play and enjoy the game before analysis begins. They should then discuss the game, determine the winning strategies and explain them by demonstration, orally or in writing.

A variety of puzzles and games, such as board games, online puzzles and games, appropriate selections for gaming systems and paper and pencil games should be used. It is not intended that the activities be concentrated in a block of time, but rather that they are dealt with periodically throughout the year.

Games provide opportunities for building self-concept, enhancing reasoning and decision-making and developing positive attitudes towards mathematics, through reducing the fear of failure and error. In comparison to more formal activities, greater learning can occur through games due to increased interaction between students, opportunities to explore intuitive ideas and problem-solving strategies. Students' thinking often becomes apparent through the actions and decisions they make during a game, so teachers have the opportunity to assess learning in a non-threatening situation.

Some students will explain their strategy by working backwards, looking for a pattern, using guess and check, or eliminating possibilities. Others plot their move by trying to anticipate their opponents' moves. As students play games and analyze strategies, they explore mathematical ideas and compare different strategies for efficiency.

The following are some tips for using games in the mathematics class:

- Keep the number of players from two to four, so that turns come around quickly.
- Communicate to students the purpose of the game.
- Engage students in post game discussions.

As students play a game it is important for teachers to pose questions and engage students in discussions about the stratgeies they are using.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Observation

- Playing games creates dialogue and provides a tool for informal assessment. Stations could be set up with one or two games at each centre. Teachers should circulate among the groups and assess students' understanding. Puzzles and games involving numerical reasoning could include:

(i) cribbage
(ii) magic square
(iii) Yahtzee™
(iv) Sudoku™
(v) Kakuro™
(vi) Monopoly™
(vii) Game of Life™

Students can work through the puzzles or games individually or with a partner. They could record their progress in a table such as the one shown below.

Puzzle	Solved?	Strategy	Comments/Hints

Journal

- Ask students to write about the puzzles and games they found interesting, and why.

Performance

- Using a game or puzzle of their choice, ask students to write their own description of the game/puzzle, the rules of play, and helpful hints. They should give the information to another classmate as they play the game.
- Students can play Fast Figuring. Using the number cards from an ordinary deck, five cards are dealt to each player. One more card is turned up to reveal the target number. Players race to use their five cards and any four operations (+, -, ×, ÷) to form a statement that results in the target number. The first player to do so wins a point. If, after 3 minutes, no one can find a solution, the players show their hands for checking, then cards are shuffled and play continues.

Resources/Notes

Authorized Resource

Math at Work 11

Games and Puzzles

High Roller SB: p. 55

Surface Area Dice Challenge SB: p. 55

TR: p. 58

Web Link

www.rinkworks.com/brainfood/c/logic.shtml

Brain Food contains Mathematical Reasoning Puzzles that require both logical and mathematical reasoning, as well as Number Puzzles such as Scrambled Equations.

Number

Outcomes

Students will be expected to

N1 Continued ...

Achievement Indicators:

N1.1 and N1.2 *Continued*

Elaborations—Strategies for Learning and Teaching

When introducing games, students have to be taught the rules and procedures of the game. Consider the following:

- Introduce the game to one group of students while others are completing individual work. Then divide the whole class into groups, putting a student from the first group into each of the other groups to teach them the game.
- Choose students to play the game as a demonstration, possibly with assistance in decision making from the whole class.
- Divide the class into groups. Play the game with the whole class, with each group acting as a single player.

When working with puzzles and games, teachers could set up learning centres. Consider the following tips when creating the centres:

- Depending on the nature of the game, some stations may require multiple games, while other stations may involve one game that requires more time to play.
- Divide students into small groups. At regular intervals, have students rotate to the next station.

Timing and integration of this outcome should be included in teacher planning throughout the course. Students can be exposed to three or four games at different times, whether it is at the beginning or end of each unit or a set "game day". Students could engage in a game when they have finished other work. As they work through the different games and puzzles, they will begin to develop effective strategies for solving the puzzle or game.

There may be situations where students are able to play the game and solve problems but are unable to determine a winning strategy. Teachers could work with the group and think through the strategies out loud so the group can hear the reasoning for selected moves. Ask the group's opinions about moves in the game and facilitate discussions around each of the other player's moves and strategies.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Observation

- As students play games or puzzles, ask probing questions and listen to students' responses. Record the different strategies and use these strategies for class discussion. Possible discussion starters include:
  - (i) Thumbs up if you liked the game, thumbs sideways if it was okay, and thumbs down if you didn't like it. What did you like or dislike about it? Why?
  - (ii) What did you notice while playing the game?
  - (iii) Did you make any choices while playing?
  - (iv) Did anyone figure out a way to quickly find a solution?

Performance

- Students can work in groups to complete the activity *Operations*. Each group will need a set of digit cards (two or three copies of the digits 0-9) and a template of the algorithm. A sample is shown below where the algorithm is a two column sum.

$$\begin{array}{r} \square\square \\ +\,\square\square \\ \hline \square\square \end{array}$$

FIND THE SUM The sum of the digits in the second number is 4.	FIND THE SUM The sum of the digits in the top number is 10.
FIND THE SUM The digit 0 is not used in any number.	FIND THE SUM All of the digits are even numbers.
FIND THE SUM The difference between the digits in the answer is 2.	FIND THE SUM The ones digit in the first number is the same as the tens digit in the answer.

- *Magic 15* is a game for two players. Begin with the numbers 1 to 9. Players take turns selecting a number, with each number used only once. The winner is the first player to have exactly three numbers that total 15.
- In *Roll Six*, players roll six dice and use five of the numbers together with any of the four operations to make the sixth number. Points are scored for successful equations.

Resources/Notes

Authorized Resource

Math at Work 11

Games and Puzzles

High Roller SB: p.

Surface Area Dice Challenge SB: p.

TR: p.

Web Link

www.k12pl.nl.ca/mathematics/seniorhigh/introduction/math2202/classroomclips.html

The **Games and Puzzles** videos demonstrate students playing various games and discuss using games and puzzles as formative assessment.

Number

Outcomes

Students will be expected to

N1 Continued ...

Achievement Indicators:

N1.3 *Create a variation on a puzzle or game, and describe a strategy for solving the puzzle or winning the game.*

Elaborations—Strategies for Learning and Teaching

As students work through the games and puzzles, it would be a good idea to keep a checklist of the games and puzzles each student is working on. Students can work in groups where each member has been exposed to a different game or puzzle. Ask the group to do the following:

- Explain the rules of the game in your own words to the other group members and give a brief demonstration of how the game is played.
- What advice would you give to other students trying to solve the puzzle or play the game?
- What did you do when you got stuck?

Invite students to bring in their own games and puzzles involving numerical reasoning. This may involve bringing in board games from home or searching the Internet to find a game or puzzle that interests them. Students can introduce this game/puzzle by providing information such as the following:

- What is the puzzle or game, and where did you find it?
- Describe the puzzle or game. Why did you select it?
- Describe the objective of the game and the rules of play.
- What strategy would you use to solve the puzzle or play the game?

As an alternative, students may have an idea for a game or puzzle that would challenge their classmates. To create a game, they could use the rules of an existing game, but use different materials or add extra materials. They could also use the idea for a game and change the rules. Another option is to use a board game and add math tasks to it. Rather than writing tasks directly onto the boards, they can place coloured stickers on certain spaces and make up colour-coded cards with questions. A game such as Snakes and Ladders™, for example, can be modified to Operation Snakes and Ladders. The board can be used with two dice. On each turn, to determine the number of spaces to move, the player has the option of multiplying, dividing, adding or subtracting the two numbers, with a maximum answer of twenty.

The following guiding questions could be used to help students evaluate their games.

- Can the game be completed in a short time?
- Is there an element of chance built in?
- Are there strategies which can be developed to improve the likelihood of winning?

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Performance / Observation

- Students work in groups to complete the activity *Pieces of the Puzzle*.

  Create cards which contain pieces of each puzzle. Students are each given a clue card and they must find their groups based on which puzzle name their card contains. It is important to have each card titled with the puzzle name to allow students to form their groups easily. The student must hold on to his/her clue card and is responsible for communicating the content to the group. Each student's role is to work with the group to solve the puzzle using the clues. It is important for groups to report on their problem-solving processes, as well as confirming the correctness of the solution.

  A sample puzzle is shown below. In this case, students have to find the mystery number. It may be helpful if members of the group were provided with a hundreds chart.

PETER'S NUMBER The number is a multiple of four.	PETER'S NUMBER The number is in the bottom half of the hundreds chart.
PETER'S NUMBER One of the digits is NOT double the other digit.	PETER'S NUMBER The sum of the digits is an odd number.
PETER'S NUMBER The number is a multiple of six.	PETER'S NUMBER The number is even.

  Prepare an extension question ready for a group that finishes before others. Asking students to invent their own problems is an excellent extension. It is also useful to have one or two extra clues ready. These can be used to allow the inclusion of an extra group member, to give help to a group that is stuck, or to assist checking when a group thinks they have finished the task.

  While observing the activity, focus on questions such as:

  (i) How did your group get started?
  (ii) How did the group "join" the clues together?
  (iii) Were there any challenges?
  (iv) Which clue did you find most helpful?
  (v) What might you do differently next time?

Journal

- Students describe how they constructed their puzzles or games and the criteria used to evaluate it.

Resources/Notes

Authorized Resource

Math at Work 11

Games and Puzzles

High Roller SB: p.

Surface Area Dice Challenge SB: p.

TR: p.

Drawing and Design

Suggested Time: 14 Hours

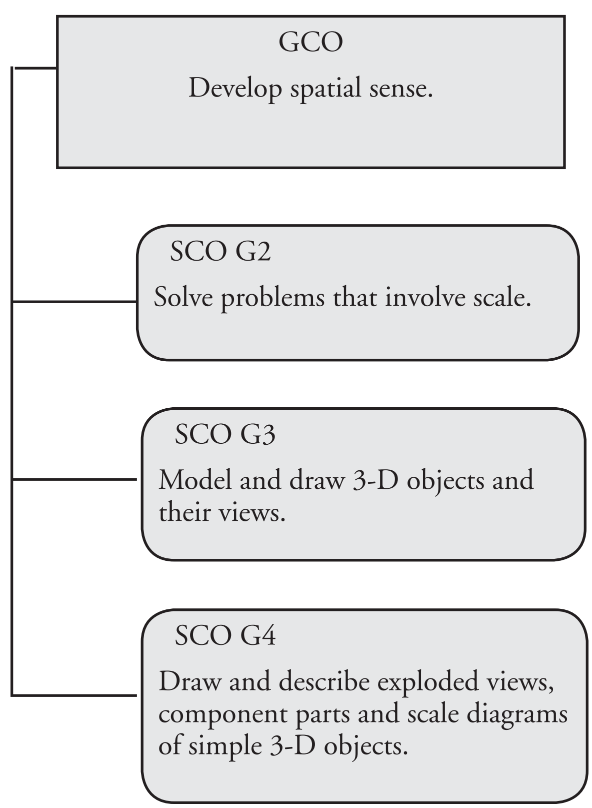
Unit Overview

Focus and Context

In this unit, students read and interpret scale drawings to calculate the full-size measurements of objects. They also apply their knowledge of scale to create different styles of technical drawings and to illustrate objects from different points of view. They work with orthographic and one-point perspective drawings, as well as isometric and exploded view drawings.

Working with scale and representing views and perspectives of 3-D objects provides opportunities to explore contextual problems related to real-life. As consumers, for example, students may need to assemble a piece of furniture, using illustrated instructions. Many careers also require the ability to read, interpret, and follow diagrams.

Outcomes Framework



Process Standards

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Geometry		
not addressed	G2. Solve problems that involve scale. [PS, R, T, V] G3. Model and draw 3-D objects and their views. [CN, R, V] G4. Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects. [CN, V]	G3. Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including: • translations • rotations • reflections • dilations. [C, CN, R, T, V]

Geometry

Outcomes

Students will be expected to

G2 Solve problems that involve scale.

[PS, R, T, V]

Achievement Indicators:

G2.1 *Describe contexts in which a scale representation is used.*

G2.2 *Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model.*

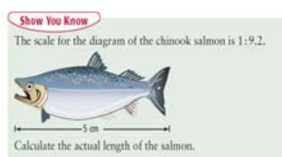
Elaborations—Strategies for Learning and Teaching

In Grade 8, students used models and diagrams to investigate ratios, wrote and compared equivalent ratios and set up proportions to solve problems (8N5). In Grade 9, they were introduced to drawing and interpreting scale diagrams representing enlargements and reductions (9SS4). This outcome reviews these concepts and extends to include 3-D modelling. The basic modelling done here will be further expanded upon in outcome G3.

Although students have already been exposed to scale concepts and proportions, a review of these key concepts is important. Students may be familiar with scale statements but they may not understand exactly what they mean. Examples of real-world applications, such as maps, sewing patterns, car models, and construction blueprints, would be a great way to introduce the concept and capture student interest. Students with experience using scale diagrams for activities such as building models of boats, cars or planes, sewing and orienteering can share their insight on the importance of using a scale.

Opportunities should be given for students to review and practice proportional reasoning. As a review or pre-assessment, students could be asked to find the value of the variable in a proportion such as $5 : x = 40 : 56$.

Using scale, students should be able to determine the dimensions of a reduced or enlarged object. Setting up and solving proportions is crucial to student success.



Students should be made aware of the importance of units in scale diagrams. Review that focuses on converting different units in both the metric and imperial systems may be necessary. It is also important to emphasize the meaning of units so students better understand the concept of scale and can gain a visual appreciation of the object that is being reduced or enlarged.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- Ask students to vocalize a list of real world examples where scales are used.
- Students could generate a list of jobs and hobbies that require the use of scales.

(G2.1)

Paper and Pencil

- A model of a sailboat is 9 inches long and the actual length of the boat is 32 feet.

  Ask students to answer the following:

  (i) Convert 32 feet to inches.

  (ii) On the model, the mast is 6 inches high. What is the height of the mast of the actual boat?

(G2.2)

Resources/Notes

Authorized Resource

Math at Work 11

2.1 Working With Scale

Student Book (SB): pp. 60-71

Teacher's Resource (TR): 70-81

Blackline Masters (BLM): 2-4

Geometry

Outcomes

Students will be expected to

G2 Continued ...

Achievement Indicators:

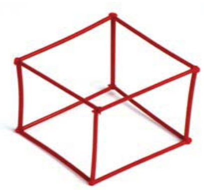
G2.3 *Construct a model of a 3-D object, given the scale.*

G2.4 *Draw, with and without technology, a scale diagram of a given object.*

Elaborations—Strategies for Learning and Teaching

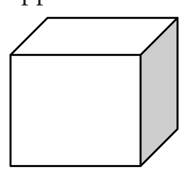
Students will be given a basic 3-D object, such as a cubic box, and asked to construct a scale model. Working in groups, they can choose their media (e.g., linking cubes, cardboard, straws and pipe cleaners, or modelling clay) and use an appropriate scale to produce their model.

The following is an example of a cubic box model constructed with pipe cleaners. The dimensions of the original cubic box could be provided. Alternatively, students could be required to measure the box to determine the dimensions.



Based on their model or directly from the 3-D object, students should construct 2-D scale diagrams with or without technology.

The following is the 2-D representation of the pipe cleaner model. Visually, students' 2-D representations may look quite different depending on how they view the 3-D object. However, if they use the scale properly the drawings will be correct. A class discussion regarding the difference in appearance of their drawings would be helpful.



G2.5 *Solve a contextual problem that involves scales.*

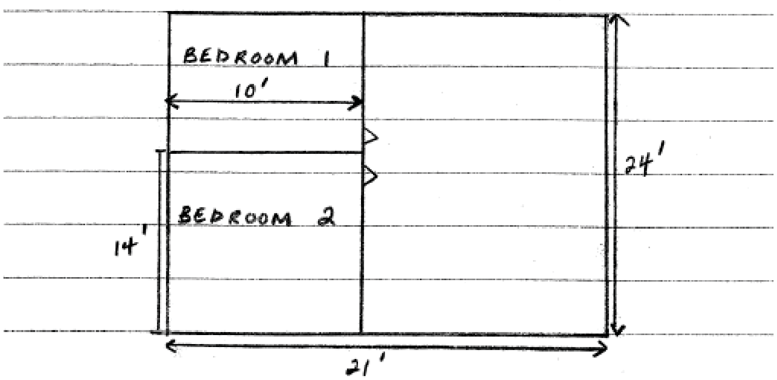
Students will determine dimensions for real world situations using their knowledge of scale diagrams and proportions. Including examples that revolve around student interest, such as cars, video game animations, motorcycles and sport surfaces, will help engage learners.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- Given a sketch of a floor plan, ask students to complete the following:
  - (i) Find the scale factor that will allow the largest possible view of the floor plan to fit on a sheet of $8\frac{1}{2}'' \times 11''$ paper.
  - (ii) Draw a scale floor plan using these measurements.
  - (iii) Create a model of the floor plan using a media of choice.



The diagram is not drawn to scale.

Students should note that the bedroom doors are 30 inches wide and 6 inches from the corner of the room.

(G2.3, G2.4, G2.5)

- Ask students to locate and print a copy of a house (e.g., their own or someone they know) from Google map. Using measurements of their actual house and their house on Google map, they should determine how much the measurements have been reduced.

(G2.5)

Resources/Notes

Authorized Resource

Math at Work 11

2.1 Working With Scale

SB: pp. 60-71

TR: pp. 70-81

BLM: 2-4

Geometry

Outcomes

Students will be expected to

G3 Model and draw 3-D objects and their views.

[CN, R, V]

Achievement Indicators:

G3.1 *Draw a 2-D representation of a given 3-D object.*

G3.2 *Draw to scale top, front and side views of a given 3-D object.*

G3.3 *Construct a model of a 3-D object, given the top, front and side views.*

G3.4 *Draw a 3-D object, given the top, front and side views.*

Elaborations—Strategies for Learning and Teaching

Students will explore multiple ways to draw representations of 3-D objects. They could be asked to draw a variety of 3-D objects, such as tissue boxes, drinking glasses, teacher desks, and bookcases, and then discuss their representations.

Discussion should include such questions as:

- Which diagram best represents the object?
- Was a scale factor used?
- Does the diagram show all parts of the object?
- Which object was most difficult to draw?

In Grade 8, students drew and interpreted top, front and side views of 3-D objects composed of right rectangular prisms. As they continue to explore the various views, the use of concrete models, such as linking cubes, will be beneficial. When building 3-D objects, each view of an object provides some of the information required to build the object. Through exploration, students should come to realize that several views are required to accurately build a unique 3-D object. Some objects can be completely represented with fewer than six views because they are symmetrical, or have sides that are equal in shape and measure.

A view can be drawn to scale or it can be an approximation. Discuss with students when an approximation might provide enough information and when a scale diagram might be needed.

Students will be given a variety of 3-D objects to construct scale orthographic diagrams. Orthographic drawings are 2-D views of 3-D objects. These drawings include front, top and side views of the object. Assembling furniture is great example of a real world situation where 2-D component parts are put together to form a 3-D object. Students should be encouraged to take precise measurements of the object and chose an appropriate scale.

Provide students with various orthographic diagrams and ask them to construct a scale model of the 3-D object. Students can use the following guidelines to help construct their model:

- determine the amount of space and materials available to work with
- determine the measurements of the actual object
- use an appropriate scale
- convert actual measurements to scale measurements

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- In groups, students measure a piece of furniture in the classroom. Using their measurements and an appropriate scale, draw the top, front and side views. Draw a 2-D representation of the chosen piece of furniture.

(G3.1, G3.2)

- Ask students to do the following:
  (i) Make a rectangular prism out of linking cubes. Draw the top, front and side views. How are the views alike?

  (G3.2)

  (ii) Construct two different objects that have the same front, top and side views.

  (G3.3)

- Ask a student to construct an original 3-D object using linking cubes. (Keep hidden from class.)
  (i) Draw front, back, side, and top views of the object.
  (ii) Ask the class to re-construct the object using linking cubes based on the views.

  (G3.3, G3.4)

Paper and Pencil

- Ask students to answer the following:

  Your friend insists that you need to tell her all six views so she can draw your object. Is she correct? Explain your reasoning.

  (G3.4)

Resources/Notes

Authorized Resource

Math at Work 11

2.2 Representing Views of 3-D Objects

SB: pp. 72-83

TR: pp. 82-87

BLM: 2-5

Geometry

Outcomes

Students will be expected to

G3 Continued ...

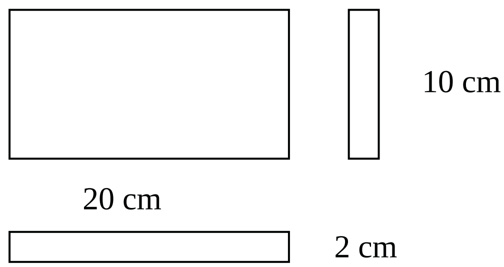
Achievement Indicators:

G3.1, G3.2, G3.3, G3.4 *Continued*

G3.5 *Determine if given views of a 3-D object represent a given object, and explain the reasoning.*

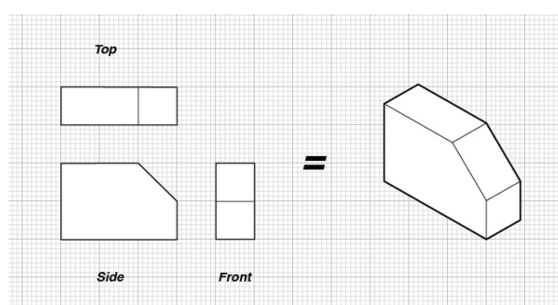
Elaborations—Strategies for Learning and Teaching

An example of a rectangular cushion is shown below:



Students should be able to draw the object given the views above.

Students should be able to apply their knowledge of scale factor and 3-D structure to determine whether given views represent a 3-D object. Students may be given the scale, or they might have to determine the scale by making corresponding measurements on both the object and diagram. Measurements on the scale diagram should correspond to measurements taken of the 3-D object with the correct use of the scale factor. If the object is to be represented by the views, hidden irregularities must be shown.



Though views are useful in representing 3-D objects, they are limited. If an object is hollow, for example, it may be difficult to see its thickness or the positions of materials inside.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to identify an object that can be completely represented with fewer than six views, as well as an object that would need six views to be completely represented. They should explain why they chose these objects.

(G3.5)

Resources/Notes

Authorized Resource

Math at Work 11

2.2 Representing Views of 3-D Objects

SB: pp. 72-83

TR: pp. 82-87

BLM: 2-5

Games and Puzzles

Model It SB: p. 101

TR: p. 98

Geometry

Outcomes

Students will be expected to

G3 Continued ...

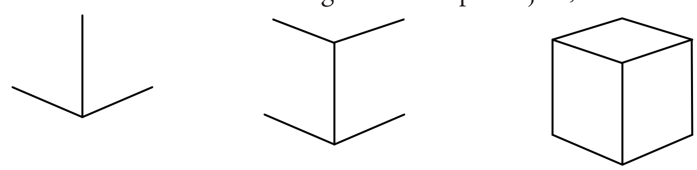
Achievement Indicators:

G3.6 *Draw, using isometric dot paper, a given 3-D object.*

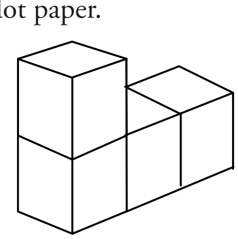
Elaborations—Strategies for Learning and Teaching

An isometric drawing is a representation of a 3-D object where the same scale is used to draw the same object height, width and depth. Lines that are parallel in reality are shown to be parallel in the drawing. Isometric drawings combine the depth of an object with an undistorted view of the object's dimensions. This makes it easy to measure and visualize the assembly of the component parts.

Demonstrate isometric drawing with a simple object, such as a cube.



Students should be shown how to draw the cube using isometric dot paper. After successively drawing a single cube students should try to draw a series of cubes linked together, and then extend to more complex shapes using isometric dot paper.



Discuss with students how views and isometric drawings are different. They should realize that each view shows only one face of the structure. Point out that an isometric drawing can show two faces, such as the front and the side, as well as part of the top structure. An isometric drawing shows the structure in space, whereas a view shows it flat.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to draw the following 3-D objects on isometric dot paper:
  (i) tissue box
  (ii) cereal box
  (iii) Toblerone™ chocolate bar box
  (iv) milk carton

(G3.6)

- Students could discuss the following with a partner, and then write a response.
  (i) Why might architects need to draw the front, top, back and side views when planning the construction of a building?
  (ii) Why might they use an isometric drawing instead of face views to show a structure?

(G3)

Resources/Notes

Authorized Resource

Math at Work 11

2.2 Representing Views of 3-D Objects

SB: pp. 72-83

TR: pp. 82-87

BLM: 2-5

Geometry

Outcomes

Students will be expected to

G3 Continued ...

Achievement Indicator:

G3.7 *Draw a one-point perspective view of a given 3-D object.*

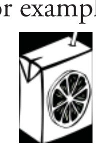
Elaborations—Strategies for Learning and Teaching

Students should gain an understanding of perspective drawing versus isometric drawing and identify situations where one or the other is more appropriate. Both give a representation of a 3-D object. However, in isometric drawings, because parallel lines do not converge in the distance as they do in perspective drawings and in real life, they do not look as realistic.

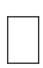
In a one-point perspective drawing, an object is drawn as it really looks from a single point. It is used to create the impression of depth and space. Lines that are parallel in reality appear to converge at a vanishing point and objects in the foreground are represented as larger than those in the background. The vanishing point is the point at which the object appears to disappear into the distance. It is drawn on the page to appear to be behind the object. If the artist wishes the point of perspective to be below and to the left of the object, for example, the vanishing point is drawn on the page below and to the left of the object and appears to be behind the object in the distance. Discuss how one-point perspective drawings can be used to represent products in advertisements or scenes in paintings.

Students should also be able to construct a one-point perspective drawing given a 3-D object. Key concepts include the horizon line which is at the eye level of the observer and the vanishing point located on the horizon line. All horizontal lines should be parallel to the horizon line. All vertical lines need to be perpendicular to the horizon.

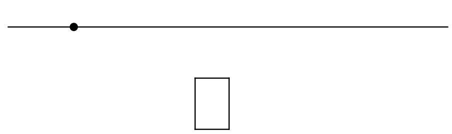
To draw a one-point perspective of a juice box, for example, students would follow the steps outlined below:



Step 1: Draw the front view.



Step 2: Decide the perspective the viewer will have. If students want the perspective to be above and to the left, they select a vanishing point above and to the left.



General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to draw a one-point perspective view of a milk carton.

(G3.7)

Journal

- Students respond to three reflective prompts that describe what they learned about one-point perspective drawings.

  Example of a 3-2-1 reflection sheet:

3 new things I learned

1.
2.
3.

2 things I am still struggling with

1.
2.

1 thing that will help me tomorrow

1.

Provide students with a copy of the reflection sheet and time to complete their reflection. They could also be paired up to share their 3-2-1 reflections.

(G3.7, G3.8)

Resources/Notes

Authorized Resource

Math at Work 11

2.3 Representing Perspectives of 3-D Objects

SB: pp. 84-95

TR: pp. 88-93

BLM: 2-6

Geometry

Outcomes

Students will be expected to

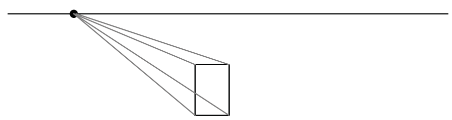
G3 Continued ...

Achievement Indicators:

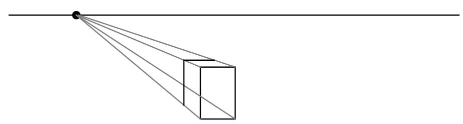
G3.7 *Continued*

Elaborations—Strategies for Learning and Teaching

Step 3: Draw a line from the vanishing point to each of the corners of the object.



Step 4: Draw a horizontal line between the two top lines that end at the vanishing point, and from the left end of this line draw a vertical line.



The viewer can now also see the top and left side of the juice box.

G3.8 *Identify the point of perspective of a given one-point perspective drawing of a 3-D object.*

Students should be given examples of one-point perspective drawing from which they can determine the point of perspective.

The point of perspective is the position from which the viewer is viewing the object drawn.

- If the viewer is viewing the object from below the object, the vanishing point is below the object.
- If the viewer is viewing the object from above the object, the vanishing point is above the object.
- If the viewer is viewing the object from a position to the left of the object, the vanishing point is to the left of the object.

These can also be combined. If the viewer is viewing the object from a position above and to the right of the object, for example, the vanishing point is above and to the right of the object.

This provides a good opportunity for a cross-curricular link with Art and Design 2200/3200, where students make one-point perspective drawings.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

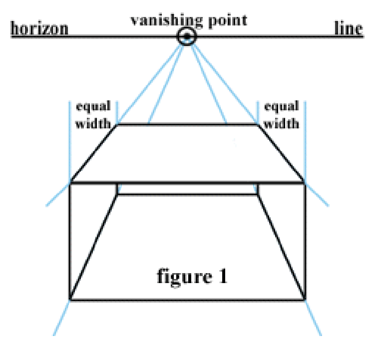
- Provide pictures of a variety of drawings which demonstrate a one-point perspective (e.g., DaVinci's Last Supper). Ask students to identify the point of perspective in each picture. Be sure to include pictures with points of perspective that are not centered.

(G3.8)

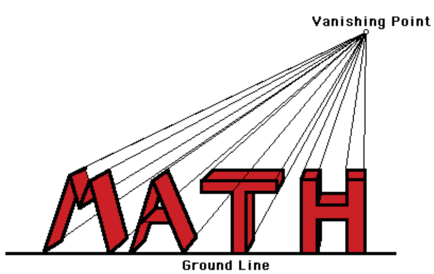
Interview

- Ask students to identify the point of perspective in drawings such as the following.

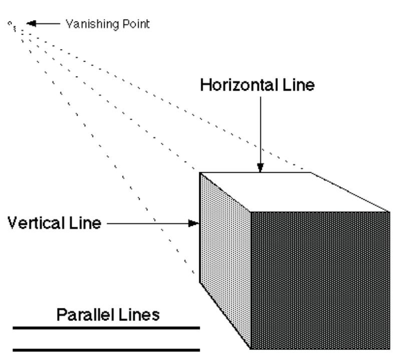
(i)



(ii)



(iii)



They should explain how they decided on the perspective.

(G3.7, G3.8)

Resources/Notes

Authorized Resource

Math at Work 11

2.3 Representing Perspectives of 3-D Objects

SB: pp. 84-95

TR: pp. 88-93

BLM: 2-6

Web Link

http://learningtodraw.net/?Kg6kf8Gr

This site describes how to draw one-point perspective.

Geometry

Outcomes

Students will be expected to

G4 Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.

[CN, V]

Achievement Indicator:

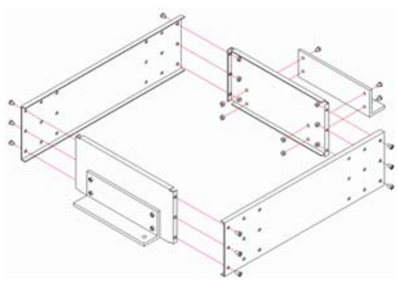
G4.1 *Draw the components of a given exploded diagram, and explain their relationship to the original 3-D object.*

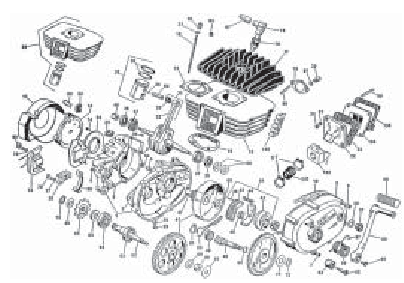
Elaborations—Strategies for Learning and Teaching

Many careers require the ability to read and interpret different types of diagrams. As consumers, students may encounter situations where knowledge of diagrams may be beneficial (e.g., assembling furniture and ordering parts online).

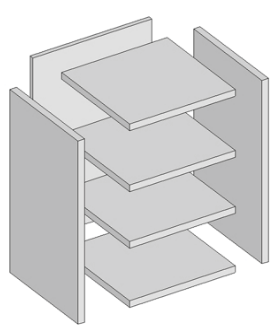
Students should be aware that an exploded diagram can be either a perspective or isometric drawing. In an exploded diagram, the components that make up an object are shown separated, but in their relative positions. This allows the viewer to see where all the parts go and how they fit together. An exploded diagram used in a parts diagram or instruction manual would be a perspective drawing, whereas an isometric exploded diagram would be useful when constructing the object.

Examples of various exploded diagrams should be shown or distributed to students when introducing the concept. Students could be asked what they noticed about the diagrams and when they would be useful. This could lead to a discussion of the advantages and disadvantages of each type of diagram.





Students should be able to draw the components of a given exploded, simple 3-D object. In addition, they should be able to identify the relationship between the individual components and the original 3-D object. Students could be given diagrams like the ones below, for example, and asked to draw and label all components (i.e., front, back, side, shelf).

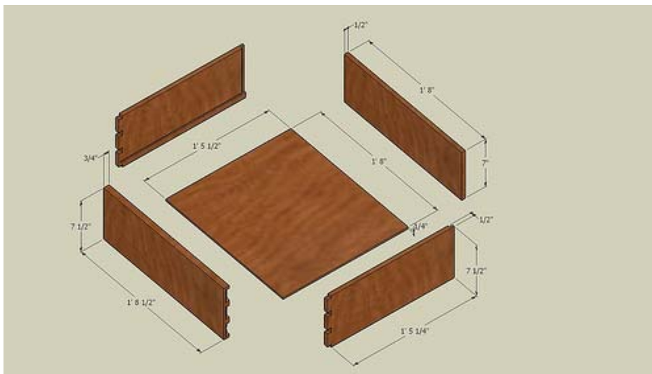


General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Given the exploded view of a drawer, ask students to draw the individual components. They should explain what each component represents with respect to the drawer.



(G4.1)

Resources/Notes

Authorized Resource

Math at Work 11

2.3 Representing Perspectives of 3-D Objects

SB: pp. 84-95

TR: pp. 88-93

BLM: 2-6

Geometry

Outcomes

Students will be expected to

G4 Continued ...

Achievement Indicators:

G4.2 *Sketch an exploded view of a 3-D object to represent the components.*

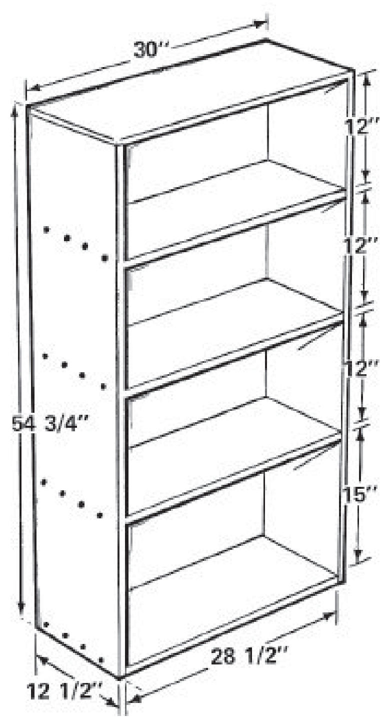
G4.3 *Draw to scale the components of a 3-D object.*

Elaborations—Strategies for Learning and Teaching

Students could be asked to sketch exploded views of 3-D objects found in the classroom or at home. Students should keep in mind that all parts of the object should be shown. Drawing the components isometrically can be helpful. Draw dashed lines to show the relationship between the parts, or how the parts could be assembled together to form the 3-D object. Students should start with simple objects and progress towards more complicated shapes.

Once students are comfortable sketching an exploded view of a 3-D object they should progress to drawing to scale the components of the 3-D object. If students are sketching a real world 3-D object found in the classroom, they must first measure components of the object and then determine the appropriate scale.

Students could also be given problems where a sketch of a 3-D object with measurements is provided and asked to draw its components to scale.



General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to measure a wooden seat such as the one below. (Similar seats could be found around the school.)

(i) Sketch an exploded view of the seat.
(ii) Draw to scale the components of the seat.

(G4.2, G4.3)

Resources/Notes

Authorized Resource

Math at Work 11

2.3 Representing Perspectives of 3-D Objects

SB: pp. 84-95

TR: pp. 88-93

BLM: 2-6

Geometry

Outcomes

Students will be expected to

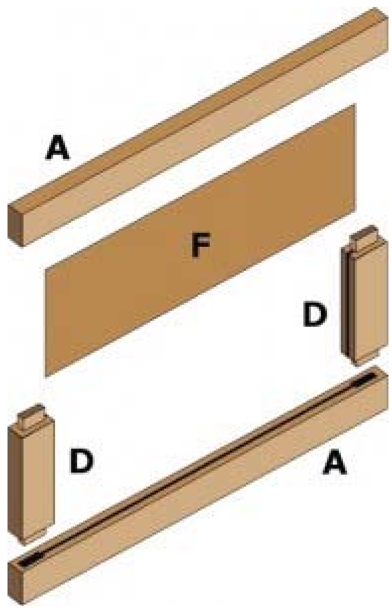
G4 Continued ...

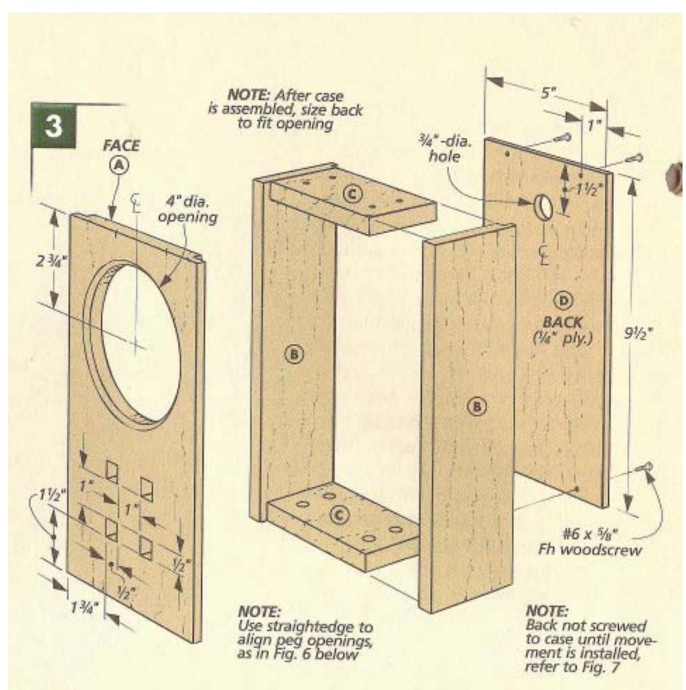
Achievement Indicator:

G4.4 *Sketch a 2-D representation of a 3-D object, given its exploded view.*

Elaborations—Strategies for Learning and Teaching

Finally, students should be ready to sketch a 2-D representation of a 3-D object given an exploded view. They could be asked to sketch a diagram that shows how the pieces of an exploded diagram would fit together.



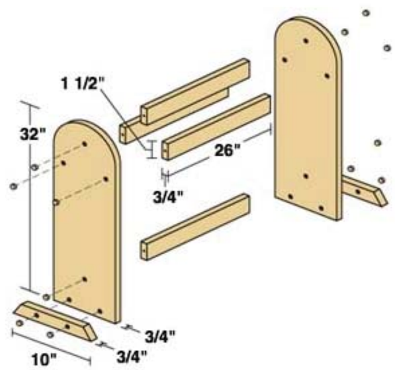


General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to sketch a 2-D representation of a 3-D object with the exploded view given. The exploded view of a quilt rack is shown here:



(G4.4)

Resources/Notes

Authorized Resource

Math at Work 11

2.3 Representing Perspectives of 3-D Objects

SB: pp. 84-95

TR: pp. 88-93

BLM: 2-6

Volume and Capacity

Suggested Time: 17 Hours

Unit Overview

Focus and Context

In this unit, there is an emphasis on the difference between volume and capacity. Students revisit estimating and calculating the volume of prisms and cylinders. They explore the relationship between the volume of prisms and pyramids, as well as cylinders and cones. Finally, they are introduced to the volume and capacity of spheres. Students solve problems that involve the volume of 3-D objects and composite 3-D objects in a variety of contexts.

Outcomes Framework

GCO

Develop algebraic reasoning.

SCO A1

Solve problems that require the manipulation and application of formulas related to:

- **volume and capacity**
- surface area
- slope and rate of change
- simple interest
- finance charges.

SCO A3

Solve problems by applying proportional reasoning and unit analysis.

GCO

Develop spatial sense through direct and indirect measurement.

SCO M2

Solve problems that involve SI and imperial units in volume and capacity measurements.

GCO

Develop number sense and critical thinking skills.

SCO N1

Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

Process Standards

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Measurement		
M3. Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [CN, ME, PS, V] M4. Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. [ME, PS, R, V]	M2. Solve problems that involve SI and imperial units in volume and capacity. [C, CN, ME, PS, V]	M1. Demonstrate an understanding of the limitations of measuring instruments, including: • precision • accuracy • uncertainty • tolerance and solve problems. [C, PS, R, T, V]
Algebra		
A1. Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R]	A1. Solve problems that require the manipulation and application of formulas related to: • **volume and capacity** • surface area • slope and rate of change • simple interest • finance charges. [CN, PS, R, T, V] A3. Solve problems by applying proportional reasoning and unit analysis. [CN, PS, R, T, V]	not addressed
Number		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Measurement

Outcomes

Students will be expected to

M2 Solve problems that involve SI and imperial units in volume and capacity measurements.

[C, CN, ME, PS, V]

Achievement Indicator:

M2.1 *Explain, using examples, the difference between volume and capacity.*

Elaborations—Strategies for Learning and Teaching

In Grade 7, students worked with areas of triangles, circles, and parallelograms (7SS2). In Grade 8, they developed and applied formulas for determining the volume of right rectangular prisms, right triangular prisms, and right cylinders (8SS4). They were also exposed to strategies to convert within and between the imperial and SI measurement systems (10M1, 10M2). Additionally, students approximated linear measurements using referents (10M3). Work with volume will now be extended to include right pyramids, right cones, spheres, and composite 3-D objects. Students will also work with capacity, with a focus on understanding how it differs from volume.

Students will extend their use of referents to estimating the volume and capacity of 3-D objects. Emphasis should be placed on hands-on activities using manipulatives and a variety of measuring tools such as graduated cylinders, callipers and micrometers. Problems should involve a mixture of SI and imperial units and include job related contexts, where possible.

Although students worked with volume in Grade 8 (8SS2), it will be necessary to revisit the concept here. Work should begin with rectangular prisms, triangular prisms and cylinders. They will use the formula, V = (area of base × height), developed in Grade 8 to determine the volume of these 3-D objects. Capacity is sometimes used interchangeably with volume. Since the two are not the same, it is important to emphasize the difference between them.

Volume is the measure of the space an object occupies. Capacity is the amount a three-dimensional object can hold. A milk container, for example, could have a capacity of 250 mL of milk and a volume of 300 cubic centimetres. All objects have volume. An example such as the following may highlight this difference:

- A brick has volume because it occupies space.
- A box has volume because it occupies space, and it also has capacity because it can contain another material within it.

Students should realize that hollow objects have volume and capacity, but solid objects only have volume.

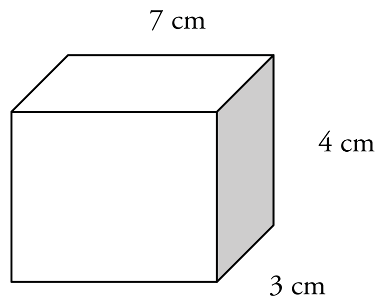
Volume is measured in units such as cubic metres (m^3) or cubic yards (yd^3), reflecting the fact that it is measured in three dimensions. Measures of capacity could include units such as litres (L) or gallons (gal).

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Performance

- The large block of chocolate shown below is being cut into 1 cm cubes. Ask students how many cubes can be cut from the block.



  Using centimeter cubes, ask students to construct an open container with a capacity equal to the volume of the chocolate block.

(M2.1)

Journal

- Ask students to discuss when they would use the concepts of volume and capacity in everyday life. (Responses might include filling a tank with gas, a measuring cup with water, a rectangular flower pot with soil, etc.)

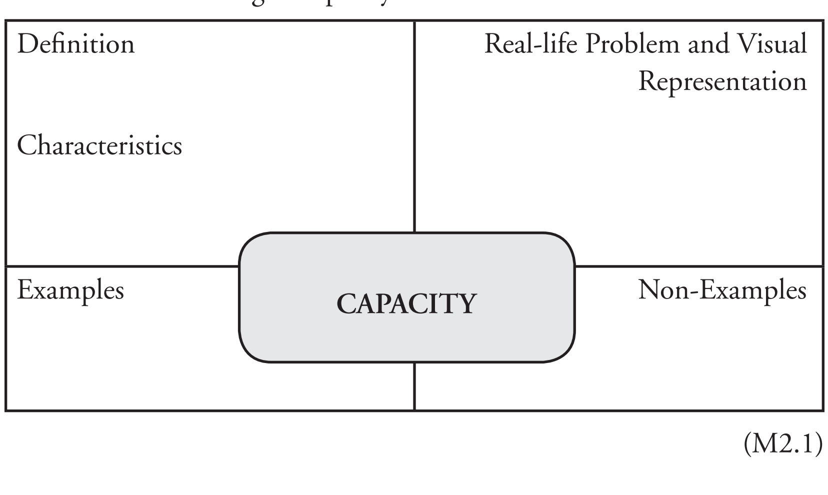
(M2.1)

- Ask students to describe an object which has a volume in cm^3 that is different from its capacity in mL.

(M2.1)

Paper and Pencil

- Provide students with a template of the Frayer Model and ask them to complete the sections individually or as a group to consolidate their understanding of capacity.



(M2.1)

Resources/Notes

Authorized Resource

Math at Work 11

3.1 Volume

3.2 Capacity and Volume

Student Book (SB): pp. 106-117, 118-127

Teacher's Resource (TR): pp. 110-119, 120-127

Blackline Masters (BLM): 3-3, 3-4

Web Link

http://www.funmaths.com/worksheets/downloads/ws0136.pdf

This activity uses math to design a swimming pool. The pool project covers many of the indicators for volume and surface area.

Measurement

Outcomes

Students will be expected to

M2 Continued ...

Achievement Indicators:

M2.2 *Identify and compare referents for volume and capacity measurements in SI and imperial units.*

M2.3 *Identify a situation where a given SI or imperial volume unit would be used.*

M2.4 *Estimate the volume or capacity of a 3-D object or container, using a referent.*

Elaborations—Strategies for Learning and Teaching

From their earliest introduction to linear measurements, students have used personal referents to estimate lengths which would normally be measured in SI or imperial units. The use of referents is now extended to estimate volume and capacity. Students should be encouraged to use a referent that represents approximately one unit of measurement. Consider the following examples:

Item	Approximate Volume / Capacity
sugar cube	1 cm^3
large MacDonald's™ drink	1 L
large can of paint	1 gal

Encourage students to select personal referents that make sense to them and to identify referents with both imperial and SI units. Students should also be given an opportunity to discuss and identify situations where given units would be appropriate.

A cubic unit is used to measure volume. Show students a hollow unit cube for overhead base ten blocks. Fill the unit cube with water and pour the water into a 1 mL measuring spoon ($\frac{1}{4}$ teaspoon) to verify that this amount is 1 millilitre. This connects volume to capacity (i.e., 1 cm^3 occupies the same space as 1 mL of water). One millilitre is used as a standard unit of measure.

Students can estimate the volume and capacity of 3-D objects within the classroom. Examples include estimating the volume/capacity of the classroom (using a filing cabinet as a referent), a filing cabinet (using a text book as a referent), a coffee cup or a tissue box (using sugar cubes as a referent). Students might also find the volume/capacity of a locker (using a Rubik's Cube™ as a referent), a water gun, (using a full paper cup as a referent), or a Toblerone™ chocolate bar box (using sugar cubes as a referent).

Students may be interested in exploring the "handful"; a nonstandard unit for measuring capacity that is related to the body. They could estimate and then measure how many handfuls of peas, beans or corn it takes to fill a margarine container. Students should compare their results using handfuls as a unit of measurement.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Performance

- Ask students to suggest a suitable referent for mL and explain why they think it would work. They should then use their referent for 1 mL to estimate the capacity of a small container in millilitres. Then have them find the capacity of the container by filling it with water, sand or rice and pouring the contents into a graduated cylinder. Alternatively, they could repeatedly use a 1 mL measuring spoon to fill the container. Students should then compare their measurements to their estimated answers.

(M2.2, M2.4)

- Photograph students as they estimate the volume and capacity of 3-D objects within the classroom. Give students the photograph and ask them to describe what they were doing in the picture. Students write about the activity under the photograph, describing what they learned as a result. A collection of class photos with annotations can be displayed as a visual record of students' learning. (Be sure to follow school policies for using digital images of students.)

(M2.4)

Resources/Notes

Authorized Resource

Math at Work 11

3.1 Volume

3.2 Capacity and Volume

SB: pp. 106-117, 118-127

TR: pp. 110-119, 120-127

BLM: 3-3, 3-4

Measurement

Outcomes

Students will be expected to

A3 Solve problems by applying proportional reasoning and unit analysis.

[C, CN, PS, R]

M2 Continued ...

Achievement Indicators:

M2.5 *Write a given volume measurement expressed in one SI unit cubed in another SI unit cubed.*

M2.6 *Write a given volume measurement expressed in one imperial unit cubed in another imperial unit cubed.*

M2.7 *Write a given capacity expressed in one unit as another unit in the same measurement system.*

A3.3 *Solve a problem, using unit analysis.*

Elaborations—Strategies for Learning and Teaching

As with surface area, students will use proportional reasoning as they solve problems that involve SI and imperial units in volume meansurements. Unit analysis will be used to verify that units in a conversion are correct.

It is sometimes more practical to use certain units of volume or capacity. While we may talk about mL of milk in a small carton, for example, it is more practical to talk about L of milk in a container on a transport truck. It is also sometimes necessary to convert volume to capacity. Engine displacements may need to be converted from cubic centimetres (sometimes referred to as cc's) to litres. When calculating the amount of cement needed, cubic inches or cubic feet is converted into cubic yards and perhaps then into cubic meters.

Students should know that to convert cm to m they divide by 100. A common error is to also divide cm^3 by 100 to obtain m^3. To avoid this, ask students by what they would divide 1 000 000 cm^3 to convert it to m^3. Show that the correct answer is 1 000 000.

Similarly, a common error occurs when converting cubic feet to cubic yards. Students should be reminded to divide by 3 for each dimension (i.e., divide by 27). Where possible, converting the linear measurements prior to calculating volume is recommended.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to convert the following:
  (i) 3 000 000 cm^3 = _______m^3
  (ii) 1 ft^3 = _______ in^3
  (iii) 4 L = _______mL
  (iv) 872mL = _______L
  (v) 8 yd^3 = _______ft^3

(M2.5, M2.6, M2.7)

Performance

- Ask students to construct a "Gallonbot" which can be used as a visual reminder of the relationships between the units for capacity in the imperial system. The following site contains a picture of a Gallonbot, the pieces necessary for construction, as well as imperial unit conversion questions for capacity.

  http://www.superteacherworksheets.com/measurement/gallonbot-pictures.pdf

  http://www.superteacherworksheets.com/measurement/gallonbot-assemble.pdf

  http://www.superteacherworksheets.com/measurement/capacity-word problems.pdf

(M2.7)

Resources/Notes

Authorized Resource

Math at Work 11

3.1 Volume

3.2 Capacity and Volume

SB: pp. 106-117, 118-127

TR: pp. 110-119, 120-127

BLM: 3-3, 3-4

Measurement

Outcomes

Students will be expected to

M2 Continued ...

Achievement Indicators:

M2.8 *Determine the volume of* ***prisms****, cones,* ***cylinders****, pyramids, spheres and composite 3-D objects, using a variety of measuring tools such as rulers, tape measures, callipers, micrometers, and displacement.*

M2.9 *Determine the capacity of* ***prisms****, cones, pyramids, spheres and* ***cylinders****, using a variety of measuring tools and methods such as graduated cylinders, measuring cups, and measuring spoons.*

M2.10 *Illustrate, using examples, the effect of dimensional changes on volume.*

Elaborations—Strategies for Learning and Teaching

The emphasis here is on hands-on activities. The intent is that students measure the dimensions of a variety of 3-D objects and substitute those values into the appropriate volume formula. It is strongly recommended that teachers develop volume formulas to promote student understanding of these formulas and minimize formula memorization.

Students may not be familiar with the use of callipers and micrometers so some time may need to be spent on the operation of these tools. For irregularly shaped objects such as balloons and rocks, determining the amount of water they displace might be the best method for finding volume.

To determine the capacity of an object, students may completely fill the object with rice or water and pour these into a container that can measure the amount of the liquid or material accurately and precisely. Students can be provided with nets from which they can create their own 3-D object to determine the capacity.

As students investigate dimensional changes, the expectation is that, initially, only one dimension would be changed and the effect on the volume discussed. This would then be extended, if possible, to two dimensions and then three. The same scale factor should be used when changing dimensions.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Performance

- Provide nets of various sized cubes to students. Ask students to measure the side length of the cube and to find the volume of the cube using the formula. Students should then cut out and fold their nets to create a cube (leave the top open). They can then fill the cube with rice or sand and measure the capacity by pouring the rice or sand into a graduated cylinder. Students should recognize that the volume in cm^3 should be equal to the capacity in mL. (This activity can be done using any of the regular 3-D objects.)

(M2.8, M2.9, M2.10)

- Ask students to use centimetre cubes to answer the following:

  A cube has a volume of $27cm^3$. If the side length of the cube is doubled, will the volume of the cube also be doubled?

  (Note: The class will need 216 centimetre cubes to complete this activity.)

(M2.10)

- Give each student, or group of students, 36 centimetre cubes. Ask the students to make a rectangular prism using all 36 cubes. Ask students to find the volume and surface areas of their prisms. As a class, compare the different shapes of the prisms, the surface areas and the volumes. Ask students to look for relationships between the shapes of the prisms, the surface areas and the volumes. (Note: This activity can be optimized by starting with a perfect cubed number of centimetre cubes, for example, 64 or 125 cubes.)

(M2.10)

Resources/Notes

Authorized Resource

Math at Work 11

3.1 Volume

3.2 Capacity and Volume

SB: pp. 106-117, 118-127

TR: pp. 110-119, 120-127

BLM: 3-3, 3-4

Measurement

Outcomes

Students will be expected to

M2 Continued ...

Achievement Indicator:

M2.11 *Describe the relationship between the volumes of:*

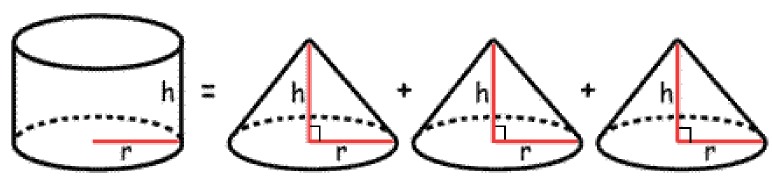
- *cones and cylinders with the same base and height*
- *pyramids and prisms with the same base and height.*

Elaborations—Strategies for Learning and Teaching

When calculating volume, the focus should be on right 3-D objects with regular bases such as squares, rectangles, right triangles and circles and students should then progress to composite 3-D objects.

Students should be given the opportunity to explore the relationships between the volumes of cones and cylinders with the same base and height, and between pyramids and prisms with the same base and height. Use a substance such as rice or water to develop the 1: 3 ratio.

The volume formula for a cylinder ($V = \pi r^2 h$) was developed in Grade 8. To explore the volume of a cone with the same base and height as a cylinder, students can fill three congruent cones, all having the same height and radii as a cylinder, with water. They pour the water from the cones into the cylinder.



Students should see that the water from the cones fills the cylinder entirely. This means it takes the volume of three cones to equal one cylinder, or $V_{cone} = \frac{1}{3}\pi r^2 h$.

The volume of a rectangular prism was also developed in Grade 8 using the formula V = (area of base) × height. Studnets will continue to use this formula for any right prism.

The volume of a pyramid is found by calculating one third of the volume of its related prism. A rectangular pyramid, for example, can be filled with rice and students can investigate how many times it takes to fill a rectangular prism with the same base length, base width and height.

The result of these investigations should be the development of the related formulas.

Three Dimensional Object	Formula
Cylinder or Prism	V = Area of Base × Height
Cone or Pyramid	$V = \frac{\text{Area of Base} \times \text{Height}}{3}$

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

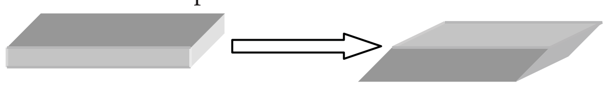
Paper and Pencil

- A cube has a volume of 27 cm^3. Ask students to find the volume of the square pyramid that has the same height and base area as the cube.

(M2.11)

Performance

- Ask students to determine the volume of an open package of paper. Then have students find the volume of the same package which has been skewed and compare the two volumes.



(M2.8, M2.11)

- Ask students to make a cylinder by rolling a 8.5″ by 11″ sheet of paper lengthwise. Make another cylinder by rolling another 8.5″ by 11″ sheet of paper widthwise. Will the two cylinders have the same capacity?

(A1.1, A1.2, M2.1, M2.11)

Resources/Notes

Authorized Resource

Math at Work 11

3.3 Working With Volume and Capacity

SB: pp. 128-136

TR: pp. 128-135

BLM: 3-5

Web Links

- *www.k12pl.nl.ca/mathematics/seniorhigh/introduction/math2202/classroomclips.html*

In the **Volume and Capacity** clip, students investigate the relationship between the volumes of pyramids and prisms and cones and cylinders.

- *http://www.youtube.com/watch?v=QnVr_x7c79w*

This video derives the formula for the volume of a cone.

Measurement

Outcomes

Students will be expected to

A1 Solve problems that require the manipulation and application of formulas related to:

- **volume and capacity**
- surface area
- slope and rate of change
- simple interest
- finance charges.

[CN, ME, PS, R]

M2 Continued ...

Achievement Indicators:

M2.12 *Solve a contextual problem that involves the volume of a 3-D object, including composite 3-D objects, or the capacity of a container.*

A1.1 *Solve a contextual problem involving the application of a formula that does not require manipulation.*

A1.2 *Solve a contextual problem involving the application of a formula that requires manipulation.*

A1.3 *Describe, using examples, how a given formula is used in a trade or an occupation.*

A1.4 *Create and solve a contextual problem that involves a formula.*

A1.5 *Identify and correct errors in a solution to a problem that involves a formula.*

Elaborations—Strategies for Learning and Teaching

It is intended that this outcome be integrated throughout the course. In Grade 8, students were exposed to problems related to volume (8SS4). In this unit, some of the problems will require the manipulation and application of formulas related to volume and capacity. While students have already had experience manipulating the formulas for the volumes of right prisms and right cylinders, this will be their first exposure to manipulating the formulas for the volumes of cones, pyramids and spheres.

The composite 3-D objects students work with would normally consist of two objects, or three objects where two of them are congruent. Initially, students should find the volume by substituting values for the dimensions without having to manipulate the formulas and later progress to problems where the manipulation of formulas is required. In all cases, an effort should be made to contextualize problems using trades and occupational examples. Many construction problems, for example, involve knowing how to use the volume formulas of common geometric shapes:

- How much soil needs to be moved? How many truckloads will actually be hauled? How many trucks are needed on site?
- How much concrete has to be ordered for the pour?
- How much does something weigh? What will it take to lift it?
- How much water will be in that pool? How long will it take to fill it?

Ground maintenance contractors may calculate areas of diseased sod which needs to be treated with fungicide, amounts of dilute solution needed for these jobs and the volumes of fungicide and water needed to make the solutions. Formulas related to volume and capacity are also used by firefighters to answer questions like “How much water is left in the tank?” and “At 15 gallons per minute, how many more minutes before the tank is empty?”. Students should be given the opportunity to investigate other trades and occupations that use volume and capacity formulas.

Students should also be exposed to incorrect solutions to problems and challenged to find and correct the error(s). Common errors occur in the substitution of values for variables, in the rearrangement of the formula, in the order of operations, and in unit conversions within the problem.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

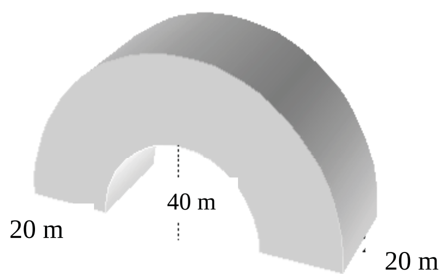
Performance

- Ask pairs of students to select a volume formula. Each student is to rearrange the formula incorrectly and have their partner identify and correct the error.

(A1.5)

Paper and Pencil

- Ask students to answer the following:
  (i) Connie's Concrete Company is making an archway for the entrance to a building. What volume of concrete is needed to make the archway below?

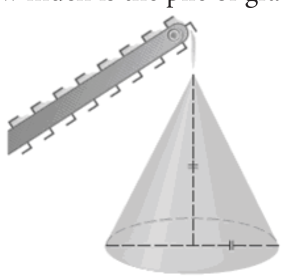


(A1.1, M2.5)

  (ii) An aquarium has a square base with side lengths of 120 cm. It contains 3600 L of water. Ask students to determine the depth of the water in centimeters. (Note: to make this question more challenging, change 120 cm to 1.2 m.)

(A1.2, M2.6, M2.12)

  (iii) A conveyor belt drops gravel into a cone shaped pile. If the radius of the base of the pile is 7 ft, and the pile is 12 ft high, what is the volume of the gravel in the pile? If the gravel costs \$0.66/ ft^3, how much is the pile of gravel worth?



(M2.12, A1.1)

  (iv) A cylindrical tank of foam concrete is 5 feet tall. The tank diameter is 2.5 feet. What is the capacity, in gallons, of the tank?

(M2.12, A1.1, A1.3)

Resources/Notes

Authorized Resource

Math at Work 11

3.3 Working With Volume and Capacity

SB: pp. 128-136

TR: pp. 128-135

BLM: 3-5

Measurement

Outcomes

Students will be expected to

M2 and A1 Continued ...

Achievement Indicator:

M2.12 *Continued*

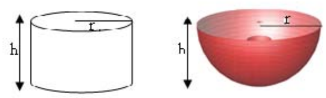
N1 Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

[C, CN, PS, R]

Elaborations—Strategies for Learning and Teaching

The volume formula of a sphere could be introduced using the following activity. The cylinder and hemisphere must have the same height and radius.

- Cut a small basketball (or other spherical object) in half, creating a hemisphere.
- Use a cylinder having the same radius and height as the hemisphere.
- Fill the hemisphere completely with rice. Then pour the rice into the cylinder. It will fill two-thirds of the cylinder.



Students have already worked with the volume of a cylinder, $V = \pi r^2 h$. From the activity, they should see that the volume of the hemisphere is $V = \frac{2}{3}(\pi r^2 h)$. The height is also the radius of the hemisphere, resulting in $V = \frac{2}{3}\pi r^3$. Since the volume of a sphere is twice the volume of a hemisphere, it is $V = \frac{4}{3}\pi r^3$.

Another way to find the volume of a sphere is to use the displacement method. This uses the amount of liquid displaced by an object to determine its volume. Provide pairs of students with a cup, a graduated cylinder, a pan, and several spherical objects to measure (e.g., a tennis ball, a golf ball). To do this activity, students will:

- place the cup on a pan and fill it completely with water
- slowly drop their object into the cup and let the excess water pour into the pan
- pour the displaced water from the pan into the graduated cylinder to see how much water was displaced.

Students can then use the formula to verify the volume of their objects.

To modify this activity, students can place the object into a filled graduated cylinder. They then take the object out to see how much liquid spills out.

This would be a good time to incorporate puzzles and games involving numerical reasoning. Refer back to pp. 34-39 for additional information.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Interview

- Ask students to respond to the following using a numerical example:

  Sam says that tripling the radius of a sphere will also triple its volume. Lydia disagrees with Sam. She thinks the volume will become much larger. Who is correct, and why?

(A1.1, M2.12)

Paper and Pencil

- As students complete the displacement activity for the volume of a sphere, ask them to answer the following:

  (i) Were their volume measurements the same as the volume calculated using the formula? If not, what might be some sources of error?

  (ii) What is an advantage of using this method to calculate volume?

(M2.12)

Resources/Notes

Authorized Resource

Math at Work 11

3.4 The Volume and Capacity of Spheres

SB: pp. 137-143

TR: pp. 136-141

BLM: 3-6

Web Link

www.k12pl.nl.ca/mathematics/seniorhigh/introduction/math2202/classroomclips.html

Students use displacement to find the volume of a sphere.

Games and Puzzles

Fill Up the Cups SB: p. 149

TR: p. 148

BLM: 3-9

Interpreting Graphs

Suggested Time: 16 Hours

Unit Overview

Focus and Context

The focus of this unit is on the representation and interpretation of data. Students will work with bar graphs, histograms, line graphs and circle graphs. They should explore the graphing of data both manually and using graphing technology.

Students will sometimes be asked to represent the same data in different formats, giving consideration to whether one type of graph is a better representation of the data than another. It is important that students are able to interpret data presented to them and identify the potential of misleading information.

Outcomes Framework

GCO

Develop statistical reasoning.

SCO S1

Solve problems that involve creating and interpreting graphs, including:

- bar graphs
- histograms
- line graphs
- circle graphs.

Process Standards

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Statistics		
not addressed	S1. Solve problems that involve creating and interpreting graphs, including: • bar graphs • histograms • line graphs • circle graphs. [C, CN, PS, R, T, V]	S1. Solve problems that involve measures of central tendency, including: • mean • median • mode • weighted mean • trimmed mean. [C, CN, PS, R] S2. Analyze and describe percentiles. [C, CN, PS, R]

Statistics

Outcomes

Students will be expected to

S1 Solve problems that involve creating and interpreting graphs, including:

- **bar graphs**
- **histograms**
- **line graphs**
- **circle graphs.**

[C, CN, PS, R, T, V]

Achievement Indicator:

S1.1 *Determine the possible graphs that can be used to represent a given data set, and explain the advantages and disadvantages of each.*

Elaborations—Strategies for Learning and Teaching

Students encounter vast amounts of information that require organization, interpretation and analysis so that they can process the data and make appropriate conclusions.

In Grade 3, students worked with bar graphs (3SP2). In Grade 6, they were introduced to line graphs and created, interpreted and solved problems in relation to given data sets (6SP1, 6SP3). Circle graphs were constructed and analysed in Grade 7 (7SP3). Work with bar graphs and line graphs was extended in Grade 8 (8SP1). These topics will be further explored in this unit. In addition, histograms will be created, interpreted and analyzed.

Initially, the focus should be on the discussion of the similarities and differences between bar graphs, line graphs, circle graphs and histograms as they relate to a given data set. Actual construction of the graphs will be explored later. This will be the first time that students are exposed to a histogram so a guided discussion may be needed. As an introduction to these graphs, provide students with a relevant data set and the corresponding line graph, bar graph, circle graph and histogram. Data sets may include class test results, weather patterns or favourite television shows. The data set can be developed in class through discussion or surveys or it could be provided. While comparing each type of graph, discussion questions could include:

- Which of the graphs display the data in a manner that is easier to interpret? Why?
- Which of the graphs display the data in a manner that is difficult to interpret? Why?
- Which of the graphs would describe the data in terms of percent? Which of the graphs would describe the data in terms of number of students surveyed?
- How are the graphs similar? How are the graphs different? What are the limitations of each type of graph?
- Which graph best represents the data collected?

Students should realize that the suitability of the graph depends on the data set given. The choice of the graph is determined by what you want to analyse and interpret from the data.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Observation

- In groups, students discuss the characteristics of the different types of graphs using 'Talking Chips'. Each student is given an equal number of game chips. They take turns saying something about the different types of graphs. Each time a student makes a comment about the topic, he/she puts a chip into the centre of the group. The activity ends when each student in the group is out of chips. They should record the characteristics in a foldable. This can be used as a pre-assessment or as formative assessment.

(S1.1)

Paper and Pencil

- Ask students to describe one unique characteristic of each of the graphs discussed. They could use foldables or graphic organizers to present this.

(S1.1)

Resources/Notes

Authorized Resource

Math at Work 11

4.1 Choosing a Graph

Student Book (SB): pp. 154-167

Teacher's Resource (TR): pp. 159-169

Blackline Masters (BLM): 4-5

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicator:

S1.1 *Continued*

Elaborations—Strategies for Learning and Teaching

Sample advantages and disadvantages of each type of graph are shown in the table below. This list is not intended to be exhaustive.

Type of Graph	Advantages	Disadvantages
Circle Graph	• compares "part to whole" • data is displayed as a percent • size of sector can be easily compared to other sectors	• does not show actual amount or number for each category • more difficult to draw • data must have "part to whole" relationship • too many categories makes it look crowded
Line Graph	• shows change over a period of time • can easily see trends • can be used to interpolate and extrapolate • easy to draw	• limited to continuous data • may be difficult to read accurately depending on scale • comparisons between categories are not identified as quickly
Bar Graph	• shows number of items in specific categories • easy to compare data • easy to draw	• may be difficult to read accurately depending on scale • trends are identifiable but not for purposes of interpolating and extrapolating
Histogram	• displays the shape of distribution • trends are easily recognizable • frequency count of items in each bin	• cannot read exact values • more difficult to compare two data sets • used only with continuous data

The advantages and disadvantages of each type of graph should be discussed with students.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to consider the following:

  Over a 2-month period, Michelle collected data about her family.

  (i) How many times each week her dad fell asleep while watching television

  (ii) The weekly height of a tomato plant in the garden

  (iii) How many hours a day her brother spent eating, sleeping, playing video games, doing homework, and chatting on-line

  Michelle wants to display her data for a school project. Which type of graph would you suggest for each data set? Justify your choice.

(S1.1)

Resources/Notes

Authorized Resource

Math at Work 11

4.1 Choosing a Graph

SB: pp. 154-167

TR: pp. 159-169

BLM: 4-5

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicators:

S1.2 *Create, with and without technology, a graph to represent a given data set.*

S1.3 *Solve a contextual problem that involves the interpretation of a graph.*

Elaborations—Strategies for Learning and Teaching

A review of the important characteristics and a description of how to create circle graphs, line graphs and bar graphs will be needed. More detailed instruction will be required to create histograms. Students should develop an awareness that certain data sets are better represented by a particular graph.

Some of the attributes of a circle graph include the title, labels, and legend. Calculating percentages is an important aspect of creating circle graphs. Students may need practice in this area. They should be reminded of the formula to convert percentages to degrees: $\% \times 360° = \#$ of degrees. In addition, they should note that the total measure of the central angles is 360° and the percentages total 100%. It will be important for students to consider rounding to the nearest degree and, because of this, recognizing that at times the angles will not add to exactly 360°. The total should not be off by more than a degree, which will not make much difference in the overall graph.

Similarly, line graphs and bar graphs must have a title and labels. When constructing line graphs special attention must be given to determining an appropriate scale with equal increments on each of the axes. A review of independent and dependent variables may be needed. When creating bar graphs students will sort the information into appropriate categories and construct the bars.

One approach to introducing histograms is to compare them to bar graphs and discuss the similarities and differences. Students can use their knowledge of bar graphs to create histograms as outlined in the following example.

The following data set represents the length, in millimetres, of various fish present in an aquarium: 9, 12, 14, 18, 22, 27, 29, 29, 44, 46, 47, 48, 50, 51, 52, 53, 64.

If students examine the data above, they should realize a bar graph will have too many bars. They should now be introduced to organizing the data into bins. We can create a bin which includes, for example, all the fish measuring between 10 mm and 20 mm. It should be brought to students' attention that histograms have a maximum of 10 bins and the range of the data can be divided by 10 to obtain an approximate bin width.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

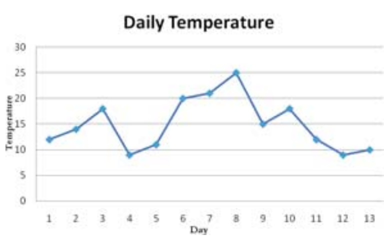
Presentation

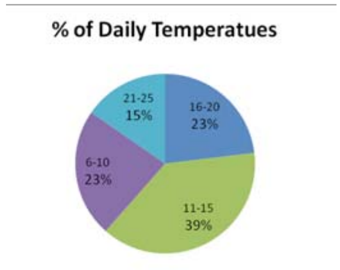
- Students survey classmates on areas of interest, such as favourite colour, sport or video game and create a graph to display the data. They present it to the class, explaining why they chose the particular type of graph. (S1.1, S1.2)

Paper and Pencil

- Ask students to analyze the following data and the accompanied graphs to answer the questions.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13
Temp	12	14	18	9	11	20	21	25	15	18	12	9	10





(i) Which graph can be used to determine the temperature for a particular day?

(ii) Which graph can be used to determine the most frequent temperature range?

(iii) Describe one advantage of using the line graph and one advantage of using the circle graph in this situation. (S1.1, S1.3)

- Chris records the amount of hours he spends on daily activities.

Activity	Hours
sleep	8
school	6
work	3
entertainment	3
meals	2
homework	2

Ask students to create a circle graph for the data and determine what percentage of Chris' day is spent on school and homework.

(S1.2)

Resources/Notes

Authorized Resource

Math at Work 11

4.1 Choosing a Graph

SB: pp. 154-167

TR: pp. 159-169

BLM: 4-5

Web Link

http://illuminations.nctm.org

This site allows students to create bar graphs, circle graphs and histograms from their own data.

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicators:

S1.2 and S1.3 *Continued*

S1.4 *Describe trends in the graph of a given data set.*

S1.5 *Interpolate and extrapolate values from a given graph.*

Elaborations—Strategies for Learning and Teaching

Students should be cautioned when sorting the data into bins. The number 20, for example, is normally included in the 20-30 bin. It is also acceptable to include it in the 10-20 bin. Although both conventions are acceptable, it is important for students to consistently follow one or the other.

Graphing data that has been organized in bins results in a histogram. Students should be made aware that, in histograms, the data elements are grouped and form a continuous range from left to right.

Once students have created various graphs, they will focus on analysing the information displayed through trends, interpolation and extrapolation. Trends in a graph are based on the context of the data set. Descriptors such as increase, decrease, constant rate, and percent of data can be used to describe the trend displayed in the graph. It is important that students relate it to the context of the problem. They are expected to identify values that fall within the given range of data points (interpolation) as well as ones that lie outside (extrapolation), while keeping in mind the trend of the data.

Interpolating is likely to be more accurate since the predicted value is bracketed by two unknown values. Extrapolating, on the other hand, is less reliable because a new trend could occur. Students may find extrapolating more challenging.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

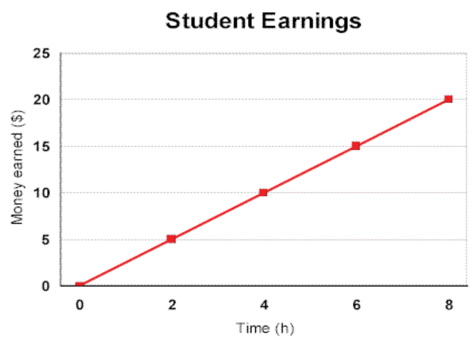
Performance

- Place labels of car models along the back wall of the classroom. Students make a 'human bar graph' by choosing their favourite type of car and lining up in front of the selected label.

(S1.2)

Paper and Pencil

- The graph below depicts a student's earnings over an 8-hour work shift.

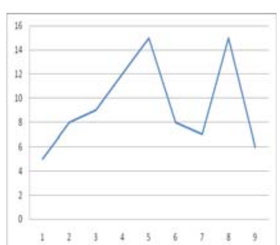


Ask students to answer the following questions:

(i) How much does the student earn after 5 hours?
(ii) How much would you expect the student to make after 12 hours?
(iii) What trend is depicted by the graph?
(iv) What is an example of a situation that may change the general trend of the graph?

(S1.4, S1.5)

- Ask students to describe a situation that could be modelled by the following graph.



(S1.3, S1.4)

Journal

- Ask students to describe a situation in their daily lives where they would need to extrapolate information and explain why it would be useful.

(S1.5)

Resources/Notes

Authorized Resource

Math at Work 11

4.2 Interpolating and Extrapolating Values

SB: pp. 168-181

TR: pp. 170-177

BLM: 4-6

Web Link

www.statcan.gc.ca

This site provides activities and projects for students in the area of statistical reasoning.

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicators:

S1.6 *Explain, using examples, how the same graph can be used to justify more than one conclusion.*

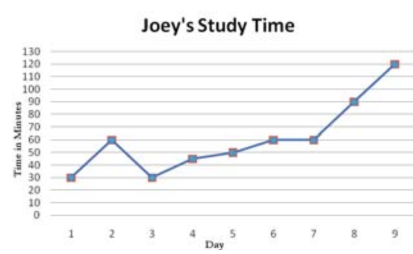
S1.7 *Explain, using examples, how different graphic representations of the same data can be used to emphasize a point of view.*

S1.3 *Continued*

Elaborations—Strategies for Learning and Teaching

Students should be aware that different conclusions can be interpreted from the same graph. If a circle graph shows a large percentage of test scores below 50%, for example, one conclusion could be that students did not study enough for the exam while another could be that the exam was not fair. A guided discussion encouraging students to provide examples of this concept is a possible teaching strategy.

Students should explore how changing certain aspects of a graph may change perception of the information displayed. The scale used, for example, will affect how data is perceived. The two line graphs below depict Joey's study time. Both graphs are based on the same data set, but Joey appears to have studied more in the second graph because of the change in scale.



Students should consider how the change in scale and setting of a starting point can affect their interpretation of data. Discuss with them when they have seen graphs that have been misleading.

Certain points of view may also be emphasized more strongly by using one type of graph over another. Students could be asked to represent the same data in different formats and encouraged to consider whether one type of graph is a better means of representing the data than another.

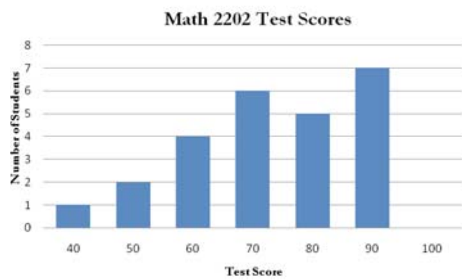
Students should use their knowledge of trends, interpolation, and extrapolation to interpret a graph to solve a contextual problem. They will need to recognize patterns and trends represented in the graph and connect them to the context of the problem. Students are frequently exposed to graphs in daily life. It is important that they are able to interpret the data presented to them to identify the potential of misleading information.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a graph, such as the one shown below, and ask them to make at least two different conclusions about the information displayed.



(S1.6)

Resources/Notes

Authorized Resource

Math at Work 11

4.3 Graphic Representations

SB: pp. 182-195

TR: pp. 178-184

BLM: 4-7

Banking and Budgets

Suggested Time: 15 Hours

Unit Overview

Focus and Context

In this unit, students will explore various financial institution services. As they evaluate various options, the focus will be on understanding how different services can help them manage money, and choosing those that best meet their needs.

Students will create a personal budget, prioritize expenses to balance a budget, and analyze a budget. They will also calculate simple and compound interest.

Credit options, including credit cards and loans, will be examined so that students can make informed decisions about the use of credit and plan ways to use credit effectively.

Outcomes Framework

GCO

Develop algebraic reasoning.

SCO A1

Solve problems that require the manipulation and application of formulas related to:

- volume and capacity
- surface area
- slope and rate of change
- **simple interest**
- **finance charges.**

GCO

Develop number sense and critical thinking skills.

SCO N4

Demonstrate an understanding of financial institution services used to access and manage finances.

SCO N3

Demonstrate an understanding of compound interest.

SCO N2

Solve problems that involve personal budgets.

SCO N5

Demonstrate an understanding of credit options, including:

- credit cards
- loans

SCO N1

Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

Process Standards

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Number		
N2. Demonstrate an understanding of income, including: • wages • salary • contracts • commissions • piecework to calculate gross and net pay. [C, CN, R, T]	N4. Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T] N3. Demonstrate an understanding of compound interest. [CN, ME, PS, T] N2. Solve problems that involve personal budgets. [CN, PS, R, T] N5. Demonstrate an understanding of credit options, including: • credit cards • loans. [CN, ME, PS, T]	N2. Solve problems that involve the acquisition of a vehicle by: • buying • leasing • leasing to buy. [C, CN, PS, R, T] N3. Critique the viability of small business options by considering: • expenses • sales • profit or loss. [C, CN, R]
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]
Algebra		
A1. Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R]	A1. Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • **simple interest** • **finance charges**. [CN, PS, R]	Not Addressed

Number

Outcomes

Students will be expected to

N4 Demonstrate an understanding of financial institution services used to access and manage finances.

[C, CN, R, T]

Elaborations—Strategies for Learning and Teaching

In this unit, students will be introduced to financial institutions, the services they provide, and how to access and manage finances. Financial literacy involves being able to understand money and financial issues, how to “read and write” in the language of dollars and cents. It includes having money knowledge and money skills, and being able to apply them. Financial literacy also means being able to make informed decisions about money and planning for the future.

Bank accounts, credit cards, and debt have become a reality in today’s society. It is important for students to be aware of the financial realities they will face throughout their adult lives. The ability to make informed financial decisions, ranging from daily spending and budgeting to choices involving insurance or saving for post-secondary education, home ownership and retirement, is essential to the well-being of society.

The focus of this outcome is on the various services provided by financial institutions, such as banks and credit unions. Students will investigate and describe the types of accounts available at financial institutions in order to select an account that best meets their needs. The focus will be on three of the most commonly used services available from financial institutions: Automated Teller Machine (ATM), online banking, and debit card purchases. Students will examine ways to protect their personal and financial information.

Achievement Indicator:

N4.1 *Describe the type of banking services available from various financial institutions, such as online services.*

Financial institutions provide a wide variety of services, including various types of savings and chequing accounts, credit cards, lines of credit, mortgages, investments, and online services. The focus should be on those services that are relevant to students, such as special youth bank accounts, debit cards, and online banking.

To begin a discussion about banking services, ask students if they have a bank account and, if so, which type. Discuss why they chose this type of account. Ask what benefits the account offers. Are there a number of free transactions per month? If they have a savings account, what is the interest rate?

Students should be exposed to the different ways to access and manage money.

- Self-service banking (e.g., using an automated teller machine, online banking, telephone banking)
- Full-service banking (banking that is done with the help of a teller)

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Students could create a mind map that shows the types of transactions possible for each service: bank teller, ATM, online banking, telephone banking.

(N4.1)

- Students can use brochures or the Internet to investigate one type of chequing account offered by one bank. They should answer the following questions:
  (i) What is the name of the bank and the name of the account?
  (ii) How much is the monthly fee?
  (iii) How many full-service transactions are included in the monthly fee?
  (iv) How much does each additional full-service transaction cost?
  (v) How many self-service transactions are included in the monthly fee?
  (vi) How much does each additional self-service transaction cost?

(N4.1)

- Ask students to use the following chequing account information:

  For a monthly fee of $4.15, you get
  - 8 self-service transactions
  - 5 full-service transactions

  Additional self-service transactions cost $0.50 each.

  Additional full-service transactions cost $1 each.

  Each use of the ATM not associated with this bank costs $1.25 plus the transaction fee, if applicable.

  Kelly did the following banking in one month:
  - wrote 3 cheques
  - paid 6 bills online
  - used her debit card 6 times
  - withdrew money 3 times from an ATM not associated with her bank

  What was the total monthly charge?

(N4.1)

Resources/Notes

Authorized Resource

Math at Work 11

5.1 Accounts

Student Book (SB): pp. 206-213

Teacher's Resource (TR): pp. 199-206

Blackline Masters (BLM): 5-3, 5-4

Web Link

www.themoneybelt.gc.ca/thecity

The City is a learning program developed by the Financial Consumer Agency of Canada and the British Columbia Securities Commission. It teaches financial skills that students can carry with them throughout their lives.

Number

Outcomes

Students will be expected to

N4 Continued ...

Achievement Indicators:

N4.2 *Describe the type of accounts available at various financial institutions.*

N4.3 *Identify the type of account that best meets the needs for a given set of criteria.*

N4.4 *Describe the advantages and disadvantages of online banking.*

Elaborations—Strategies for Learning and Teaching

Two basic types of deposit accounts are chequing and savings. The two are different in that chequing accounts allow users to write cheques, whereas savings accounts usually offer a higher interest rate. Other characteristics depend on the type of savings or chequing account and on the financial institution. Most banks offer students accounts that have lower monthly fees or no service fees and may not require a minimum balance.

As students begin thinking about the type of account most suitable for them, they should consider whether they would prefer to pay bills online and frequently use bank machines, or if they would expect to make frequent deposits and prefer to do their transactions with a bank teller.

Students could work in small groups to brainstorm the following:

- What are some transactions that students are likely to make?
- Have you ever written a cheque? When might you need to write a cheque?
- Why might you want a savings account?
- What information should you consider about your financial needs so that you can make an informed choice when selecting a bank account?

Financial institutions are now offering online banking as a viable option for their customers. There has been a transition from banking in person to banking over the telephone to banking using a computer. Online banking has both advantages and disadvantages. The convenience it offers simplifies life for some people. For others it may be a little more complex and intimidating. Students could first brainstorm a list of advantages and disadvantages of online banking and then use the Internet to complete their list. Some suggestions follow. These are not intended to be all-inclusive.

Advantages	Disadvantages
fast, effecient and effective	trust / security
convenient	steep learning curve
real-time monitoring of accounts	

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Presentation

- Students could work with a partner to research and compare banking services at a local bank or credit union. They should look for the following:
  (i) What are five types of accounts and other services offered by the financial institution?
  (ii) What types of transactions can be made using Internet or telephone banking?
  (iii) What types of transactions must be done in person?
  They should present their findings.

(N4.1, N4.2, N4.3)

Portfolio

- Ask students to use an online tool to determine the type of account that best meets their needs and compare it to the account they may currently have.

(N4.2, N4.3)

Paper and Pencil

- Ask students to describe three advantages and three disadvantages of online banking.

(N4.4)

Resources/Notes

Authorized Resource

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TR: pp. 199-206

BLM: 5-3, 5-4

Number

Outcomes

Students will be expected to

N4 Continued ...

Achievement Indicators:

N4.5 *Identify and explain various automated teller machine (ATM) service charges.*

N4.6 *Describe the advantages and disadvantages of debit card purchases.*

Elaborations—Strategies for Learning and Teaching

Most financial institutions issue a debit card to account holders. They can be used at automatic teller machines, as well as to make purchases in stores that accept bank cards as payment. Many students may already have bank cards and be familiar with their uses. Nevertheless, consumer awareness about the potential dangers of bank cards is important, particularly for young people.

ATM service charges are the fees many banks and interbank networks charge for the use of their automated teller machines. In some cases, these fees are assessed solely for non-members of the bank; in other cases, they apply to all users. Engage students in a discussion about these fees, prompting them to think about the cost to operate the machines compared to the cost of human resources in a bank. Students should be aware that three types of consumer charges exist:

- regular account fee

  This is a service charge imposed by a consumer's financial institution to withdraw money at an ATM.
- surcharge

  This fee may be imposed by the ATM owner and will be charged to the consumer using the machine.
- foreign fee

  This transaction fee or network access fee is charged by the card issuer to the consumer for conducting a transaction outside of their network of machines.

Since its national launch in 1994, Interac Direct Payment has become so widespread that, as of 2001, more transactions in Canada were completed using debit cards than cash. This popularity may be partially attributed to two main factors: the convenience of not having to carry cash, and the availability of automated bank machines and Direct Payment merchants on the network. Debit cards also have disadvantages for the consumer and the retailer. There is a spending limit, for example, on purchases and withdrawals. User fees can also apply, and the security may not be as high as with some other forms of payment. Students should first brainstorm a list of advantages and disadvantages of debit card purchases and then use the Internet to add to their list.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Using an ATM printout (example below) containing different customer service charges, ask students to identify the fee category and amount(s).

DIRECash ATM

TERMINAL #	02016199
SEQUENCE #	19212
AUTH #	03241 00
DATE	18:45 08/20/2007
CARD NUMBER	XXXXXXXXXXXX0121
DISPENSED AMOUNT	$200.00
REQUESTED AMOUNT	$200.00
FROM ACCOUNT	Chequing
TERMINAL FEE	$1.25
TOTAL AMOUNT	$201.25
BALANCE	$3057.85

As they examine the receipt, ask them to answer the following:

(i) What important information is provided by an ATM receipt?
(ii) Why do you think the card number was printed this way?
(iii) Why is it a good idea to keep your ATM receipt?

(N4.5)

Journal

- On rare occasions, such as when the network system becomes overloaded, it is impossible to pay for purchases using debit cards or to access money at an ATM. Ask students to describe how they could be prepared for such an occasion.

(N4.6)

Resources/Notes

Authorized Resource

Math at Work 11

5.1 Accounts

SB: pp. 206-213

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BLM: 5-3, 5-4

Number

Outcomes

Students will be expected to

N4 Continued ...

Achievement Indicator:

N4.7 *Describe ways that ensure the security of personal and financial information; e.g., passwords, encryption, protection of personal identification number (PIN) and other personal identity information.*

Elaborations—Strategies for Learning and Teaching

Discuss with students how important it is to keep personal records, and why they should also keep track of their bank charges. They should be aware of the dangers of someone else accessing their personal and financial information through online banking. This could lead to identity theft. Personal information includes:

- age, name, ID numbers, income, ethnic origin, or blood type
- opinions, evaluations, comments, social status, or disciplinary actions
- employee files, credit records, loan records, medical records, existence of a dispute between a consumer and a merchant

Financial information includes:

- account numbers and balances
- tax returns
- loan and mortgage information
- credit card and debit card numbers
- credit rating
- passwords
- PIN numbers

Accounts that are accessed on a shared computer or accessed with a simple password are in particular danger of this type of theft. Discuss ways for students to ensure that their personal and financial information remains secure.

Security Feature	Security Measure
Passwords	• disable automatic password-save features in the browsers and software used to access the Internet • use unique passwords for each website visited

Ask students to suggest features of a strong password. Discussion should include the following:

- passwords should be at least 7 characters long, and not contain user name, real name or company name
- passwords should contain upper and lower case letters, numbers and symbols such as %, # or *
- passwords should not contain a complete dictionary word, and should be significantly different from previous passwords

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Journal

- Ask students to discuss ways that they currently ensure the security of their personal and financial information and ways that they can increase its security.

(N4.7)

Paper and Pencil

- Ask students to describe two ways in which someone else could access their personal and financial information.

(N4.7)

Presentation

- Ask students to create a way to present "Tips to Protect Your Personal and Financial Information". They may consider creating a pamphlet, a web page, a slideshow, or a short video.

(N4.7)

Resources/Notes

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Number

Outcomes

Students will be expected to

N4 Continued ...

Achievement Indicator:

N4.7 *Continued*

Elaborations—Strategies for Learning and Teaching

Security Feature	Security Measure
PIN	• PINs should never be shared • PINs should never be given in response to email or telephone requests • shield your PIN when using it • choose a PIN that is not obvious • do not write your PIN on the card • vary your PIN on different cards • contact your bank immediately if your card is lost or stolen
Encryptions	• use the highest level of encryption available in web browsers and in wireless communications

High security web browsers visually verify that the user is logged on securely. Students should take note that when a high security browser verifies the authenticity of a website, it colours the address bar green and displays details about the business (e.g., location for incorporation and country).

Students should also investigate ways that financial institutions ensure security for online and electronic banking. Most use security measures such as firewalls and cookies, several levels of login (e.g., passwords, PINs, authentication questions), instructions on how to report suspicious emails, chip technology in bank cards and credit cards, voiceprint verification for telephone banking and cheque imaging.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Students can work in small groups to research ways that a financial institution protects the personal information of an account holder. (N4.7)

Resources/Notes

Authorized Resource

Math at Work 11

5.1 Accounts

SB: pp. 206-213

TR: pp. 199-206

BLM: 5-3, 5-4

Number

Outcomes

Students will be expected to

N2 Solve problems that involve personal budgets.

[CN, PS, R, T]

Achievement Indicators:

N2.1 *Identify income and expenses that should be included in a personal budget.*

N2.2 *Explain considerations that must be made when developing a budget; e.g., prioritizing, recurring and unexpected expenses.*

N2.3 *Create a personal budget based on a given income and expense data.*

Elaborations—Strategies for Learning and Teaching

In Mathematics 1202, students were expected to demonstrate an understanding of income (i.e., net and gross pay) and deductions (e.g., CPP, EI, income tax, health plans, union dues,) including wages, salary, contracts, commissions, and piecework (N2). Income, deduction, and expense data will now be used by students to prepare budgets. Budgets are useful for determining how money is being made and spent, for preventing the accumulation of large amounts of debt, and for planning for future purchases or emergencies. Students are expected to create a budget based on data that is provided, as well as data they collect themselves. At the start of the unit, students should be asked to record their personal expenses and income for one week in preparation for creating their own budget later in the unit. Once completed, students should modify their budget to achieve immediate personal goals (ex. purchasing a car, and making car and insurance payments) and analyze changes to their budget to achieve future goals.

To create a personal budget, students should identify net pay, as well as income from other sources such as investments, tax credits, and rental properties. Students should then identify fixed expenses (e.g., rent, car payments, telephone bills) and variable expenses (e.g., heat/light, dining out, repairs). It may be useful for students to first identify general categories of expenses (e.g., housing, transportation, insurance, education, food, pets, personal, entertainment, loans, taxes, savings and investments, recreation, miscellaneous) and then give specific expenses under each category.

When developing their personal budget, students need to prioritize their expenses to first ensure that the necessities of life are met and hopefully some money remains for investing and unexpected expenses (ex. auto repair, healthcare, gifts, appliance replacement). Some expenses, such as property tax, are billed annually but can be paid monthly and students need to cognizant of this when making a monthly budget. Students need to realize that, for some expenses, the amount budgeted and the amount actually spent may not match. Initially, it may be better to provide students with the data needed to complete their monthly budget (sample data is given in the table on page 114), and with a budget template on which to enter their data.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a budget and a goal which is currently unattainable. Ask them to decide which expenses to reduce or eliminate in order to make the goal attainable.

(N2.2)

- Ask students to make a list of variable and fixed expenses that should be included in their personal budgets. Based on a weekly income of $250, ask students to prepare a personal budget where they will save at least $25 per week.

(N2.1, N2.2, N2.3)

Resources/Notes

Authorized Resource

Math at Work 11

5.2 Budgets

SB: pp. 214-227

TR: pp. 207-214

BLM: 5-5

Number

Outcomes

Students will be expected to

N2 Continued ...

Achievement Indicators:

N2.3 *Continued*

Elaborations—Strategies for Learning and Teaching

Sample Monthly Family Budget

Income ($)	Expected Expenses ($)	Actual Expenses ($)	Difference
Net Pay (1) - 2300.00	Mortgage - 700.00	700.00	0
Net Pay (2) - 1500.00	Property Tax - 120.00	120.00	0
Rental Income - 500.00	Groceries - 600.00	650.00	-50
	Child Care - 800.00	800.00	0
	Dining Out - 250.00	200.00	50
	Heat & Light - 200.00	230.00	-30
	Telephone - 30.00	70.00	-20
	Mobile Phone - 50.00	30.00	0
	Cable / Internet - 120.00	120.00	0
	Car Payment - 400.00	400.00	0
	Gas - 300.00	350.00	-50
	Insurance (vehicle) - 200.00	200.00	0
	Insurance (house) - 50.00	50.00	0
	Clothing - 150.00	120.00	30
	Personal Care - 50.00	70.00	-20
	Investments - 120.00	120.00	0

N2.4 *Collect income and expense data, and create a budget.*

After completing a budget from given data, students may be provided with income data and some basic expenses and asked to add at least three expenses of their own. Using this data, students could create a second budget. Eventually, students will be expected to collect their own income and expense data and create a budget. This data may be collected from sources such as telephone and electricity bills, credit card statements, catalogues and advertising flyers. Using the data collected at the beginning of the unit, and data from other sources, students can create their personal budget.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to explore options for when they finish high school (e.g., post-secondary studies or employment). They should list the sources of income (e.g., part-time jobs, scholarships, savings, loans) and possible expenses they would incur (e.g., tuition, transportation, rent). Then using the collected data, ask them to develop a personal budget.

(N2.1, N2.2, N2.3, N2.4)

Resources/Notes

Authorized Resource

Math at Work 11

5.2 Budgets

SB: pp. 214-227

TR: pp. 207-214

BLM: 5-5

Number

Outcomes

Students will be expected to

N2 Continued ...

Achievement Indicators:

N2.5 *Modify a budget to achieve a set of personal goals.*

N2.6 *Investigate and analyze, with or without technology, "What if ..." questions related to personal budgets.*

Elaborations—Strategies for Learning and Teaching

Once their personal budget is completed, students should identify some personal goals, the monetary cost of each goal, a timeline for achieving each goal and then determine how to modify their budget to attain these goals. For example, students may set an educational goal (eg., attending a trade program at a local college), purchasing a new vehicle, or starting a family, and determine what budget changes are necessary to fund their goal. These changes may involve a combination of increasing income and decreasing or eliminating expenses.

Students should be helped to realize that a realistic and achievable goal can sometimes be derailed by unexpected and uncontrollable events. This can be investigated by posing "what if" scenarios and determine if previously identified goals are still attainable. Spreadsheets are an excellent tool for investigating the solutions to these scenarios. For example, students could be faced with loss of income, major health expenses, vehicle or property damage, or pregnancy.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a completed budget and a goal and ask them to determine if the goal is attainable/realistic.

(N2.5)

- Michelle is a high school student living at home. She has a part-time job. Provide students with Michelle's budget for a month.

Monthly Net Income	Budget
job	$475
Monthly Expenses	
Fixed Expenses	
savings	$45
Internet access	$20
cell phone	$25
transportation - bus fare	$40
Variable Expenses	
clothing	$250
entertainment	$50
personal items	$55
school expenses	$35
other - gifts	$50

Ask students to answer the following:

(i) Calculate her total expenses.
(ii) How much more are her planned expenses than her income?
(iii) Suggest ways she can alter her spending to keep her planned expenses less than or equal to her income.

(N2.6)

Journal

- It has been suggested that families have a reserve equal to 6 times the family's monthly income for unexpected events such as job loss. List the advantages and disadvantages of this suggestion.

(N2.6)

Resources/Notes

Authorized Resource

Math at Work 11

5.2 Budgets

SB: pp. 214-227

TR: pp. 207-214

BLM: 5-5

Number

Outcomes

Students will be expected to

N3 Demonstrate an understanding of compound interest.

[CN, ME, PS, T]

A1 Solve problems that require the manipulation and application of formulas related to:

- volume and capacity
- surface area
- slope and rate of change
- **simple interest**
- **finance charges.**

[CN, PS, R]

Achievement Indicators:

N3.1 *Solve a problem that involves simple interest, given three of the four values in the formula I = Prt.*

A1.1 *Solve a contextual problem involving the application of a formula that does not require manipulation.*

A1.2 *Solve a contextual problem involving the application of a formula that requires manipulation.*

A1.5 *Identify and correct errors in a solution to a problem that involves a formula.*

Elaborations—Strategies for Learning and Teaching

This may be students' first exposure to simple and compound interest. In this unit, the initial focus is on simple interest and its use in problem solving. This is followed by a comparison of simple and compound interest with a focus on the relationship between the two. Students will then work with compound interest, considering the effect of different compounding periods, and use compound interest in problem solving situations. Finally, they will estimate the doubling time for an investment using the Rule of 72.

It is intended that this algebra outcome be integrated throughout the course. In this unit, some of the problems will require the manipulation and application of formulas related to simple interest and finance charges.

Students are introduced to the concept of simple interest and the use of the simple interest formula. Simpler problems will require that students substitute the values for the principal, rate (as a decimal) and time (in years) into the formula to obtain the interest earned. Other problems may require formula rearrangement, either before or after values are substituted. Students may also be given the time in months and expected to change it into years before substituting into the interest formula.

Compound interest is much more common than simple interest. To understand compound interest, however, it is important to have an understanding of simple interest.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to find the error in each calculation and correct it.
  - (i) A $1000.00 investment earns simple interest at an annual rate of 5.5%. The interest earned in 2 years was calculated.

    $I = P \times r \times t$
    $I = 1000 \times 5.5 \times 2$
    $I = 11\ 000$

  - (ii) A $1000.00 investment earns simple interest at an annual rate of 5.5%. The interest earned in 18 months was calculated.

    $I = P \times r \times t$
    $I = 1000 \times 0.055 \times 18$
    $I = 990$

  - (iii) A $1000.00 investment earns simple interest at an annual rate of 8%. The amount of time needed to earn $400 in interest was calculated.

    $I = P \times r \times t$
    $400 = 1000 \times 0.08 \times t$
    $400 = 80 \times t$
    $400 - 80 = t$
    $320 = t$

  (N3.1, A1.1, A1.6)

- Paul earns $60 in interest. Jessica earns $70 interest. Ask students to discuss some possible reasons why Jessica earns more.

  (N3.1)

- A $1500 investment earns simple interest at an annual rate of 4.5%. Ask students to determine the number of years it will take to earn $270 in interest.

  (A1.2)

Resources/Notes

Authorized Resource

Math at Work 11

5.3 Simple and Compound Interest

5.4 Investing and Borrowing

SB: pp. 228-238, 239-251

TR: pp. 215-223, 224-233

BLM: 5-6, 5-7, 5-8

Number

Outcomes

Students will be expected to

N3 Continued ...

Achievement Indicators:

N3.2 *Compare simple and compound interest, and explain their relationship.*

Elaborations—Strategies for Learning and Teaching

The relationship between simple and compound interest should be investigated using investment vehicles such as savings accounts or guaranteed investment certificates (GIC). Initially, students should use an iterative process to compare the effects of simple and compound interest. The tables below illustrate the effects of simple and compound interest on the value of $1000.00 in a savings account earning annual interest of 5%.

Simple Interest:

Year	Principal	Amount of Annual Interest	Total Amount at the End of the Year
1	$1000	1000 × 0.05 = 50	$1050
2	$1000	1000 × 0.05 = 50	$1100
3	$1000	1000 × 0.05 = 50	$1150
4	$1000	1000 × 0.05 = 50	$1200

Compound Interest:

Year	Principal plus Interest	Amount of Annual Interest	Total Amount at the End of the Year
1	$1000	1000 × 0.05 = 50	$1050
2	$1050	1050 × 0.05 = 52.50	$1102.50
3	$1102.50	1102.50 × 0.05 = 55.13	$1157.63
4	$1157.63	1157.63 × 0.05 = 57.88	$1215.51

To highlight the effect of compound interest in the short term, larger values for the initial amount of money in the savings account can be used. Alternatively, a longer time period can be used.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Journal

- Ask students to explain why the amount of interest earned from the end of year 1 to the end of year 2 is much less than the amount of interest earned from the end of year 29 to the end of year 30.

Year	Principal	Total Amount at End of Year
1	\$1000	$A = 1000(1.08)^1 = 1080$
2	\$1080	$A = 1000(1.08)^2 = 1166.40$
⋮	⋮	⋮
29	\$8627.11	$A = 1000(1.08)^{29} = 9317.27$
30	\$9317.27	$A = 1000(1.08)^{30} = \$10\,062.66$

(N3.3)

Paper and Pencil

- Ask students to answer the following:
  - (i) A student invests \$1000.00 in a GIC at an interest rate of 3%. Calculate the value of the GIC after 20 years.
  (N3.3)
  - (ii) A house is bought today for \$335 000.00. If the value of real estate increases by 7% every year, what is the value of the house after 6 months?
  (N3.3)

Resources/Notes

Authorized Resource

Math at Work 11

5.3 Simple and Compound Interest

5.4 Investing and Borrowing

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Number

Outcomes

Students will be expected to

N3 Continued ...

Achievement Indicators:

N3.3 *Solve, using a formula, a contextual problem that involves compound interest.*

N3.4 *Explain, using examples, the effect of different compounding periods on calculations of compound interest.*

N3.5 *Estimate, using the Rule of 72, the time required for a given investment to double in value.*

Elaborations—Strategies for Learning and Teaching

Once they have explored the iterative process, students can be introduced to the appropriate formulas and use them to calculate the value of an investment after a specific amount of time. In Grade 9, students evaluated powers with integral bases and whole number exponents (9N1). They are now expected to evaluate powers with decimals in both the base and the exponent. It is not an expectation that students rearrange the formula in compound interest problems.

Using the tables on page 120, the total amount of simple interest can be calculated using $I = P \times r \times t$ and $FV = P + I$. The final amount for compound interest can be calculated using $FV = P(1 + r)^t$.

Students should note that in the formula $FV = P(1 + r)^t$, r is the annual interest rate divided by the number of compounding periods per year and t is the total number of compounding periods. For an annual interest rate of 7.2% compounded quarterly for 2 years, $r = \frac{0.072}{4} = 0.018$ and $t = 2 \times 4 = 8$. Students who have difficulty determining the value of r and the value of t may prefer to use the formula $FV = P\left(1 + \frac{r}{n}\right)^{nt}$ where r is the annual interest rate, n is the number of compounding periods and t is the time in years.

Students should compare the value of an investment when interest is compounded annually, semi-annually, quarterly, monthly, and daily. They should realize that the more often interest is compounded, the greater the value of their investment.

In the study of compound interest, students might investigate how long it takes for an amount to double if it is invested at different rates. This should lead to the "rule of 72," which students can then use to quickly approximate doubling time for an investment. If an amount is invested at 7.2%, it will take approximately $\frac{72}{7.2} = 10$ years to double. Similarly, if an amount is invested at 10%, it will double in approximately $\frac{72}{10} = 7.2$ years.

This provides an opportunity to use mental mathematics. Students can use the Rule of 72, for example, to estimate how long it will take for an investment to double at an interest rate of 1.95% compounded annually. They can round the interest rate to 2% and use the formula to determine that half of 72 is 36. They should conclude that an investment invested at a rate of 1.95% would take over 36 years to double.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

  Joe has $1000 to invest. He checks out 5 different banks, each having an annual interest rate of 6.25%. However, each bank uses a different compounding period. Complete the following table to help Joe decide which bank he should choose.

Bank	Compounding Period	Interest Rate	Number of Compounding Periods	Amount of Money After 1 Year
1	annually			
2	semi-annually			
3	quarterly			
4	monthly			
5	daily			

(N3.3, N3.4)

- Ask students to use the Rule of 72 to estimate how long it would take an investment to double at an interest rate of 3.1% compounded annually.

(N3.5)

Journal

- Ask students to respond to the following:

  Two GICs have the same principal, the same rate per annum, and the same term. One is compounded semi-annually and the other is compounded daily. Which do you think would earn more interest? How could you confirm your thinking?

(N3.4)

Resources/Notes

Authorized Resource

Math at Work 11

5.3 Simple and Compound Interest

5.4 Investing and Borrowing

SB: pp. 228-238, 239-251

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BLM: 5-6, 5-7, 5-8

Number

Outcomes

Students will be expected to

N5 Demonstrate an understanding of credit options, including:

- **credit cards**
- **loans.**

[CN, ME, PS, R]

Achievement Indicators:

N5.1 *Compare advantages and disadvantages of different types of credit options, including bank and store credit cards, personal loans, lines of credit, overdraft.*

Elaborations—Strategies for Learning and Teaching

Students will be introduced to various credit options. They should be aware that knowledge of credit is important for financial well-being. Students should have knowledge of the advantages and disadvantages of the different types of credit options such as credit cards, loans, and lines of credit, in order to use credit effectively and make sound financial decisions. Students will investigate various credit card options and use their financial knowledge in problem solving situations involving credit cards and loans.

There are a variety of credit options including credit cards, loans, lines of credit, and overdrafts. Each of these options has a number of advantages and disadvantages and students should be aware of these in order to make wise financial decisions. To lead into a discussion about credit, students could brainstorm different types of credit. They could construct a list of the advantages and disadvantages of using a credit card. Ask them to share ideas on how credit cards can be used effectively. Ask students if they know how old they must be to get a credit card and why this might be. Some suggested advantages and disadvantages of each credit option are included here.

Credit Option	Advantages	Disadvantages
Bank Credit Cards	• convenient • no need to carry large amounts of cash • can be used to pay bills • more secure than some other options • helps establish a credit history • may provide extra product warranty or other rewards	• can lead to large debts • account balance must be checked every month to guard against fraudulent charges • can lead to identity theft
Store Credit Cards	• similar to bank credit cards • introductory offers	• usually higher interest rates than bank credit cards
Loans	• lower interest rates • higher amounts of money	• may need collateral • more difficult to obtain
Lines of Credit	• could be very low interest rates • money always available	• temptation to spend money you don't have
Overdrafts	• prevents bounced cheques or insufficient funds charges	• interest charges can be very high • service charge when used each month

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to decide which credit option they would use to purchase the following items. Students should be able to support their choice.

Item	Cost ($)
iPod Touch™	250.00
laptop computer	600.00
snowmobile	12 000.00
truck	40 000.00
house	250 000.00

(N5.1)

Resources/Notes

Authorized Resource

Math at Work 11

5.4 Investing and Borrowing

SB: pp. 239-251

TR: pp. 224-233

BLM: 5-8

Number

Outcomes

Students will be expected to

N5 Continued ...

Achievement Indicators:

N5.2 *Make informed decisions and plans related to the use of credit, such as service charges, interest, payday loans and sales promotions, and explain the reasoning.*

N5.3 *Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt and timing purchases.*

Elaborations—Strategies for Learning and Teaching

Students should realize that all forms of credit come with a cost. When certain credit offers seem extremely attractive they often come with higher service charges, higher interest rates and/or higher penalties. If students apply for a payday loan they need to pay special attention to the agreement. Interest rates range from 10% - 30% and failing to repay the loan on time will result in a doubling of the interest rate for the next repayment schedule. Sales promotions, such as using a store credit plan, often have hidden administration charges and high interest charges.

The interest rate on a credit card affects finance charges when a balance is carried. The higher the interest rate, the higher the finance charges will be. Students should brainstorm ideas that could convince creditors to give a customer a lower interest rate. Possible ideas include using the number of years he/she has been a customer, the most recent number of consecutive on-time payments, credit score, and actual lower interest rates of other credit cards and offers.

As a general rule, the longer the repayment period for a loan, the greater the amount interest that is paid. Students could investigate amounts of interest paid on $10 000 loan by varying the repayment period (e.g., 24 months versus 60 months).

Reducing debt is not as easy as creating it. Daily living habits, as well as long-term spending mistakes, are two factors that contribute to debt. Students should brainstorm ways to reduce debt load. They can use the Internet to complete their list. Some suggestions follow:

- Start a diary of daily spending habits. Cut back on expenses that are not necessities and find ways to spend less money on the necessities.
- Purchase items on credit that can be repaid in 30 days. By paying your credit card balances in full each month, you can avoid interest charges.
- Eliminate multiple merchant credit cards.
- Transfer debt on high-interest credit cards to low-interest credit cards to save money and reduce debt.
- Write daily reminders and notes in a spending diary, which will serve as a reminder to pay bills on time and avoid late fees.
- Ask credit card issuers if there are programs available to reduce interest, monthly payments or even settle the balance amount completely.

Timing purchases to coincide with manufacturer discounts, clearance sales and off-season discounts will also reduce the amount of credit required.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Journal

- Ask students to describe situations when they would use a payday loan. They should also identify situations when they would not.

(N5.2)

Presentation

- Ask students to research a local store credit plan with special emphasis on the hidden details of the plan, such as administration charges and repayment details. They should present their findings to the class.

(N5.2)

Performance

- Ask students to do a role play where one student represents a credit card company and one represents a customer. The customer should negotiate a lower interest rate for their credit card.

(N5.3)

Resources/Notes

Authorized Resource

Math at Work 11

5.4 Investing and Borrowing

SB: pp. 239-251

TR: pp. 224-233

BLM: 5-8

Number

Outcomes

Students will be expected to

N5 Continued ...

Achievement Indicators:

N5.4 *Compare credit card options from various companies and financial institutions.*

Elaborations—Strategies for Learning and Teaching

Emphasis should be placed on the importance of understanding the conditions of a credit card before signing an agreement. Researching credit options will help students use credit effectively.

When shopping around for a credit card, students should ask themselves:

- How much will I spend on the card each month?
- Will I be able to pay my balance in full each month?
- Would I benefit from reward programs? Will the benefits outweigh any associated fees?
- Am I prepared to pay an annual fee? What other fees apply?
- If I consistently carry a balance, what interest rate am I prepared to pay on my purchases and/or cash advances?
- Could I benefit from a low-rate credit card?
- Do I understand how credit card interest is charged?
- If I am considering a low, "introductory" interest rate, do I understand how that interest rate will change once the introductory period is over?
- Do I understand how payments will be applied to the charges on the card?
- What are my rights and responsibilities with a credit card?

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Performance

- Research a bank-issued credit card to determine the annual fees, income requirements, credit limit, annual rate of interest charged, and other distinguishing features. Compare findings with those of classmates who researched different cards.

(N5.4)

Paper and Pencil

- What is the difference between paying with a debit card and paying with a credit card?

(N4.6, N5.4)

Portfolio

- Ask students to use an online tool to determine the type of credit card that best meets their needs.

(N5.4)

Resources/Notes

Authorized Resource

Math at Work 11

5.4 Investing and Borrowing

SB: pp. 239-251

TR: pp. 224-233

BLM: 5-8

Number

Outcomes

Students will be expected to

N5, A1 Continued ...

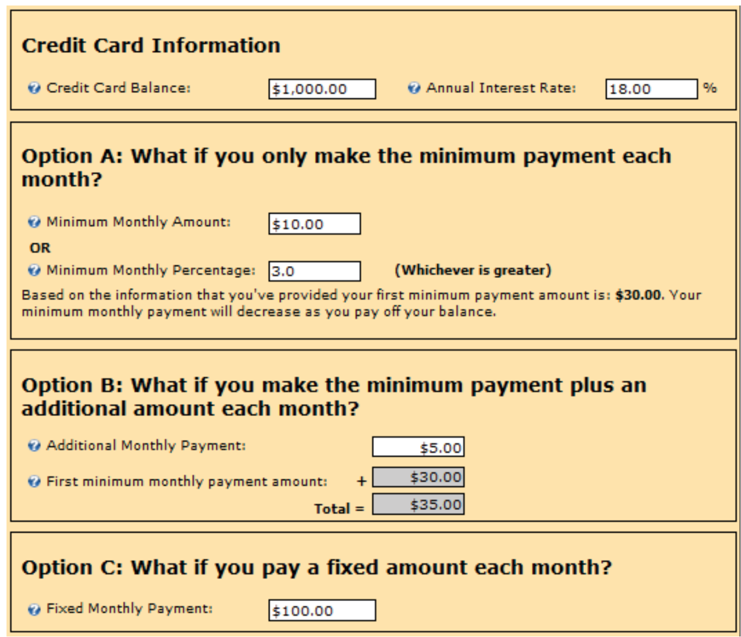
Achievement Indicators:

N5.5 *Solve a contextual problem that involves credit cards or loans.*

A1.1 *Solve a contextual problem involving the application of a formula that does not require manipulation.*

Elaborations—Strategies for Learning and Teaching

Students could apply their knowledge of credit cards and loans in a situation similar to the one below. Students would not be expected to do all of the calculations in this type of problem. They should realize, however, that loans have different repayment options, each resulting in different interest charges and penalties, but in general, shorter repayment periods result in lower interest charges.



Calculation Results

	Option A: What if you only make the minimum payment each month?	Option B: What if you make the minimum payment plus an additional amount each month?	Option C: What if you pay a fixed amount each month?
Time to pay off	10 years	6 years and 2 months	11 months
Original balance	$1,000.00	$1,000.00	$1,000.00
Interest paid	$798.89	$512.52	$91.62
Total paid	Calculation Results	$1,512.52	$1,091.62
Amount saved	-	**$286.37**	**$707.26**
Time saved	-	**3 years and 10 months**	**9 years and 1 month**

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- A credit card has an annual interest rate of 18%. The minimum monthly payment must be $50 or 3% of the balance, whichever is greater. Ask students to answer the following questions.
  - (i) What is the monthly interest rate?
  - (ii) If the balance is $ 5000, what is the monthly interest charge?
  - (iii) What is the balance on the account after the interest is added on, and the minimum payment is made?

(N5.5, N3.1, A1.1)

Resources/Notes

Authorized Resource

Math at Work 11

5.4 Investing and Borrowing

SB: pp. 239-251

TR: pp. 224-233

BLM: 5-8

Web Link

http://www.fcac-acfc.gc.ca/iTools-iOutils/CreditCardCalculator-eng.aspx

Credit Card Payment Calculator Tool

Number

Outcomes

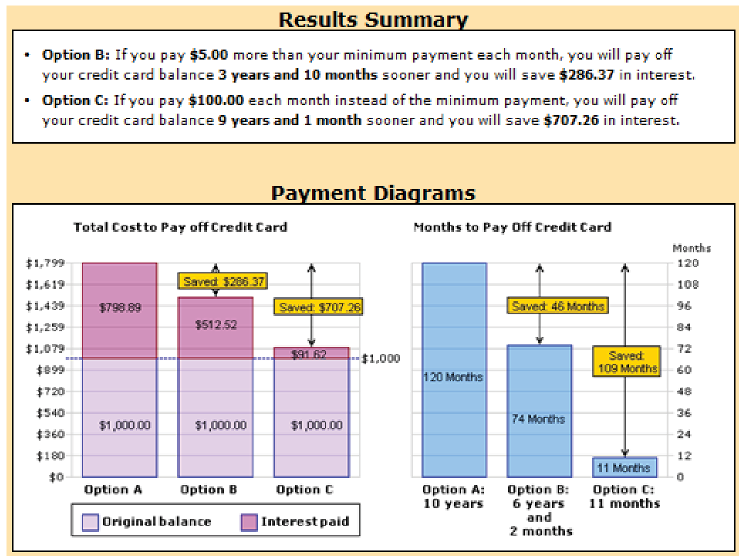
Students will be expected to

N5, A1 Continued ...

Achievement Indicators:

N5.5, A1.1 *Continued*

Elaborations—Strategies for Learning and Teaching



N5.6 *Solve a contextual problem that involves credit linked to sales promotions.*

Some businesses offer sales promotions in the form of deferred payment plans (store credit plans). Deferred payment plans allow customers to make a purchase, with payment due sometime in the future. The use of a deferred payment model is considered one of the more common sales and marketing tools used by many companies. Essentially, the underlying concept is to buy now and pay later. When a consumer is unable to pay for the purchase today, but has a reasonable expectation of being able to provide payment in full by an agreed upon date in the future, the process of assuming deferred debt can make sense. Students should be able to solve problems based on various deferred payment plans.

N1 Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

[C, CN, PS, R]

Revisit the puzzles and games outcome, focusing on the strategies students are using. Refer back to pp. 34-39 for additional information.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the questions related to the following deferred payment plan.

  Billy-Bob's Discount Store: No payments until 2015!!!!

  - I want to buy a $ 2000 big screen TV!
  - Suppose I don't have enough money yet, but I can use a store's credit plan which states "You don't have to make any payments until 2015."

  However, the fine print says:

  - You have to pay taxes on the TV and an administrative fee of $50 up front. This fee is to process the credit application.
  - Then you don't have to make a payment until 2015.
  - After 2015 you can either pay it off, or make 48 monthly payments of $100 each month.

  (i) What amount do I have to pay up front?
  (ii) If I decide to make the 48 monthly payments how much will I have paid, in payments, after 48 months?
  (iii) What is the total amount that I would have paid?
  (iv) What would it have cost if I paid cash?
  (v) How much extra would I have paid if I used the credit plan?
  (vi) Why would someone use a credit plan rather than saving and paying cash?

(N5.6, A1.1)

Resources/Notes

Authorized Resource

Math at Work 11

5.4 Investing and Borrowing

SB: pp. 239-251

TR: pp. 224-23

BLM: 5-8

Games and Puzzles

Math Symbols SB: p. 257

TR: p. 238

Slope

Suggested Time: 17 Hours

Unit Overview

Focus and Context

Slope is a graphical way to show how two quantities change with respect to each other. This unit begins with the ratio of rise over run. Students will discover that a slope value is the ratio of the change in the vertical to the change in the horizontal. They should begin to connect rise to vertical change, or the difference between the y-coordinates, and run to horizontal change, or the difference in x-coordinates.

Students' prior knowledge of trigonometry will be activated to make the connection that rise over run is equivalent to the ratio of the length of the opposite side to the length of the adjacent side, which is the tangent of the angle of elevation.

Finally, slope will be further developed as students solve rate of change problems.

Outcomes Framework

GCO

Develop algebraic reasoning.

SCO A1

Solve problems that require the manipulation and application of formulas related to:

- volume and capacity
- surface area
- **slope and rate of change**
- simple interest
- finance charges.

SCO A2

Demonstrate an understanding of slope:

- as rise over run
- as rate of change

GCO

Develop number sense and critical thinking skills.

SCO N1

Analyze puzzles and games that involve logical reasoning, using problem-solving strategies.

Process Standards

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Algebra		
A1. Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R]	A1. Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • **slope and rate of change** • simple interest • finance charges. [CN, ME, PS, R]	A1.Demonstrate an understanding of linear relations by: • recognizing patterns and trends • graphing • creating tables of values • writing equations • interpolating and extrapolating • solving problems. [CN, PS, R, T, V]
G4. Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: • applying similarity to right triangles • generalizing patterns from similar right triangles • applying the primary trigonometric ratios • solving problems. [CN, PS, R, T, V]	A2. Demonstrate an understanding of slope: • as rise over run • as rate of change • by solving problems. [C, CN, PS, V]	
Geometry		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Algebra

Outcomes

Students will be expected to

A2 Demonstrate an understanding of slope:

- **as rise over run**
- **as rate of change**
- **by solving problems.**

[C, CN, PS, V]

Achievement Indicators:

A2.1 *Describe contexts that involve slope; e.g., ramps, roofs, road grade, flow rates within a tube, skateboard parks, ski hills.*

A2.2 *Explain, using diagrams, the difference between two given slopes (e.g., a 3:1 and a 1:3 roof pitch), and describe the implications.*

A2.3 *Describe the conditions under which a slope will be either 0 or undefined.*

Elaborations—Strategies for Learning and Teaching

Students were introduced to linear relations in Grades 7, 8 and 9. This included work with tables of values and graphs (7PR1, 7PR2, 8PR1, 9PR2). Students discussed the meaning of variables and described how a change in one variable affected a change in the other. The concept of slope has been discussed but the term slope has not yet been used. This outcome will focus on describing slope and eventually solving contextual problem using the rate of change formula. Students will gain a visual perspective of slope through diagrams and illustrations, calculate the rise over run ratio in diagrams and apply the rate of change formula to real world contexts. Throughout the unit, students will solve problems involving slope.

Students should be introduced to slope through discussion of real world examples in order to develop a concrete image of what slope represents. Talk about the importance of slope, for example, when building decks, ramps, stairs and roof trusses. Concepts such as steepness and positive and negative slope should be discussed using a variety of visuals. Providing students with multiple examples of objects with varying slopes and putting them in order based on steepness could be a useful teaching strategy. Discussion of the differences of various slopes should lead students to the concept of zero and undefined slope. The connection should be made that horizontal lines have slope 0 and vertical lines have undefined slope.

When introducing slope, incorporate activities such as the following:

- Students could measure various staircases and compare rise to run. Discuss the steepness of the staircases.
- Ask students to examine the grade of a road. If a highway has a 6% grade for the next 8 km, for example, how far does it drop vertically over the 8 km travelled horizontally?

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Performance

- Ask students to bring in visuals of objects such as ramps, roofs, ski hills, etc., and have them discuss the following in groups:
  (i) Which object has the steepest slope?
  (ii) Which visual demonstrates a negative slope?
  (iii) Arrange the visuals in order from least to greatest with respect to slope.
  (iv) Do any of the visuals depict an undefined or 0 slope?

  (A2.1, A2.3)

- Ask students to create a Quad-fold Foldable with positive, negative, zero and undefined slope as the headings. They should write what they have learned about each, including a sketch and a real-life example.

  (A2.1, A2.3)

Journal

- Ask students to describe objects they encounter in their everyday lives that have slopes.

  (A2.1)

Presentation

- A plumb bob is a weight at the end of a string which is used to determine if an object is vertical (has undefined slope). Ask students to find occupations which use a plumb bob. They should present their findings to the class.

  (A2.1, A2.3)

Resources/Notes

Authorized Resource

Math at Work 11

6.1 What is Slope?

Student Book (SB): pp. 262-273

Teacher's Resource (TR): pp. 247-257

Blackline Masters (BLM): 6-3, 6-4

Web Links

- ***www.bobvila.com/articles/495-the-plumb-bob/pages/1***

This online article describes the uses of a plumb bob.

- ***www.youtube.com/watch?v=ZuRtD9ZOSSE***

This video shows a plumb bob in use.

Algebra

Outcomes

Students will be expected to

A2 Continued ...

Achievement Indicators:

A2.4 *Explain, using examples and illustrations, slope as rise over run.*

Elaborations—Strategies for Learning and Teaching

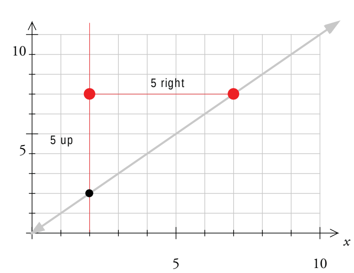
Slope should first be developed in terms of rise over run. An activity such as building staircases will help students develop an understanding of slope. As a warm up, lay one metre stick flat on the ground and another one vertically against a wall. Ask students to explain, to a partner, the positions of these metre sticks. Explain to students that although it is difficult to describe slope using everyday language, it is easier to describe with mathematics. Define the term slope.

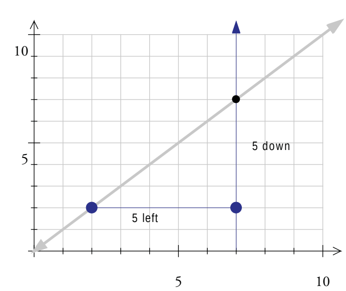
The following can be done as a whole class activity.

- Using blocks, build one staircase with a slope of 1 (by placing together one block, then two blocks, then three, etc.) and one staircase with a slope of 2. Balance a metre stick on each staircase, and ask students how they might calculate the slope of the metre stick.
- Create a diagram on grid paper illustrating the staircases within a coordinate grid.
- Explain how to calculate the slopes of each of the metre sticks.
- Compare the numerical values with the everyday expressions the class came up with during the warm-up activity.

As a follow-up, assign small groups of students a slope and have them build a model of a staircase with that slope. On grid paper, students can illustrate the staircase and show how they calculated the slope.

The connection to rise over run can also be made by providing students with grids depicting line segments of varying slopes and asking them to identify the rise and the run. Students should be cautioned to pay close attention to the direction they are moving and the sign of the rise and the run. The diagrams below show how students can calculate the slope by interpreting the rise and run differently but the slope will still be same in both situations.





Students should also be exposed to graphs of lines with negative slopes.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Observation

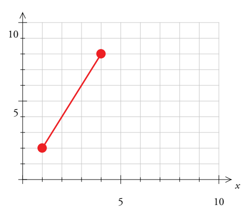
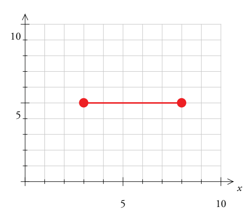
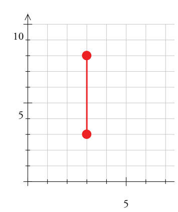
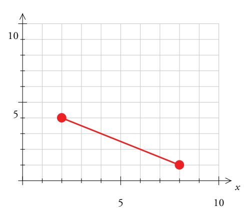
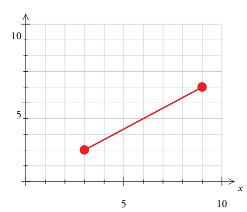
- An in-class activity such as the following provides a quick check of a student's understanding of slope.
  (i) Ask students to draw the coordinate system.
  (ii) Write down a slope of a line.
  (iii) Ask students to position a piece of uncooked spaghetti on their paper to reflect the slope of the line.

  A quick assessment of student understanding can be performed by walking around the classroom and observing the steepness and direction of the spaghetti.

(A2.4)

Paper and Pencil

- Ask students to determine the slopes of the following line segments using the rise over run.

(A2.4)

- Ask students to draw a line segment that has an endpoint at (2, -1) and whose slope is $\frac{2}{3}$. (A2.4)

Portfolio

- Ask students to bring in visuals depicting slopes in real world contexts. Using a grid transparency, have them estimate the slope of each.

(A2.1, A2.4)

Journal

- Ask students to discuss the following:
  (i) If the rise stays the same, what effect would changing the run have on slope?
  (ii) If the run stays the same, what effect would changing the rise have on slope?

(A2.4)

Resources/Notes

Authorized Resource

Math at Work 11

6.1 What is Slope?

SB: pp. 262-273

TR: pp. 247-257

BLM: 6-3, 6-4

Web Link

illuminations.nctm.org

Algebra Lesson: Rise-Run Triangle

Algebra

Outcomes

Students will be expected to

A2 Continued ...

Achievement Indicators:

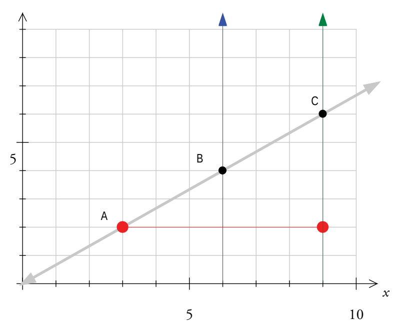
Elaborations—Strategies for Learning and Teaching

A2.4 Continued

At this time, the connection may be made that the slope is equal to the change in the dependent variable divided by the change in the independent variable. This concept will be explored in greater detail when the rate of change formula is developed.

A2.5 *Verify that the slope of an object, such as a ramp or a roof, is constant.*

Observation that the slope of the line is equal to the slope of the line segment joining any two points on the line is important to emphasize. Using a line segment, similar to the one below, enables students to verify that the slope between points A and B is the same as the slope between points A and C and between points B and C.



$$m_{\overline{AB}} = m_{\overline{BC}} = m_{\overline{AC}} = \frac{2}{3}$$

A2.6 *Explain, using illustrations, the relationship between slope and angle of elevation; e.g., for a ramp with a slope of 7:100, the angle of elevation is approximately 4°.*

Students worked with the primary trigonometric ratios in Mathematics 1202 (G4, A1). The idea that the slope of a line is the tangent of the angle of elevation should now be developed. When the tangent ratio (slope) is known, the inverse tangent function can be used to find the angle of elevation.

To introduce slope as angle of elevation, activate students' prior knowledge of the tangent ratio by drawing a right triangle and labelling an acute angle with θ. Ask students to identify its opposite and adjacent sides and the formula they would use to find θ.

To help students visualize the relationship between slope and the tangent ratio, they could draw two congruent triangles. On the first triangle, label the rise and run. On the second triangle, label the angle of elevation and the opposite and adjacent sides of the triangle. By comparing the two triangles, they should see that the slope ratio is the same as the tangent ratio.

A2.7 *Explain the implications, such as safety and functionality, of different slopes in a given context.*

There are many real world contexts where special consideration must be given to slope. Wheelchair ramps, ski slopes and stairs, for example, need to be constructed using an appropriate slope with safety and functionality in mind. Students should discuss situations where steepness is important or where a small slope is more practical.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:
  (i) Describe real world situations where a positive, negative, zero and undefined slope are present.
  (ii) If you went skiing, what slope of the hill would you prefer: $\frac{1}{5}$ or $\frac{4}{5}$? Why?
  (iii) Ask students to demonstrate the relationship between slope and angle of elevation using a detailed drawing from a real world context.

(A2.6, A2.7)

Portfolio

- Ask students to complete the following:
  (i) Explain why the slope value is important for safety in large vehicle transportation.

  (ii) Provide other examples in real world contexts where slope and safety may be related. Include the appropriate slope(s) for each context.

(A2.7)

Resources/Notes

Authorized Resource

Math at Work 11

6.2 Relationship Between Slope and Angle of Elevation

SB: pp. 274-285

TR: pp. 258-269

BLM: 6-5, 6-6, 6-7

Web Links

- *www2.worksafebc.om/Publications/OHSRegulation/GuidelinePart26.asp#SectionNumber:G26.16*

This site contains regulations outlining the maximum slope on which forestry equipment can be operated.

- *wheelchair.ca/ramp.php*

This site contains wheelchair ramp safety and standards.

- *www.gov.ns.ca/just/regulations/regs/ohs296f.htm#roof*

Information on roof slope and fall protection can be found here.

- *nsidc.org/snow/avalanche*

Avalanche awareness information can be found at this site.

Algebra

Outcomes

Students will be expected to

A1 Solve problems that require the manipulation and application of formulas related to:

- volume and capacity
- surface area
- **slope and rate of change**
- simple interest
- finance charges.

[CN, ME, PS, R]

A2 Continued ...

Achievement Indicators:

A2.8 Explain, using examples and illustrations, slope as rate of change.

A1.6 Explain and verify why different forms of the same formula are equivalent.

A1.5 Identify and correct errors in a solution to a problem that involves a formula.

Elaborations—Strategies for Learning and Teaching

It is intended that this outcome be integrated throughout the course. In this unit, some of the problems will require the manipulation and application of the slope formula.

While working with slope as rise over run, students may have developed the connection that slope is equal to the change in the dependent variable divided by the change in the independent variable. To apply the slope formula, students will have to link the dependent variable to the *y*-axis and the independent variable to the *x*-axis of a coordinate grid. Review of these terms and the coordinate grid, including how to plot and read points, may be necessary. They should realize that slope is equal to the change in the *y*-values divided by the change in the *x*-values which can be determined using coordinates. This is referred to as rate of change. This should be developed using real world contexts. For example, if a ski hill has a slope of one quarter students should realize that they drop 1 metre for every 4 metres they travel horizontally.

To develop the concept of rate of change, students could find the slope of a line segment on a coordinate grid using three different forms of the same formula:

- $\frac{\text{rise}}{\text{run}}$
- $\frac{\text{change in dependent variable}}{\text{change in independent variable}}$
- $\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$ (using coordinates of the endpoints)

Students should then be able to make the connection to slope and the formula for rate of change: $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$. Some common student errors that occur when determining rate of change using this formula include inverting the formula, substituting coordinates incorrectly (e.g., substituting x_2 for x_1), and incorrect calculations.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Using the rate of change formula, ask students to find the slope of the line that passes through each pair of points.

  (i) P(3, 2) and Q(-3, 1)
  (ii) A(1, 4) and B(7, 4)
  (iii) S(3, 0) and T(3, 6)
  (iv) M(-2, 1) and N(-3, 5) (A2.8)

- Ask students to identify and correct the errors in the following problem:

  What is the slope of the line segments with endpoints A(4, 1) and B(1, 5)?

  Solution: $\text{slope} = \frac{5-1}{4-1}$

  $= \frac{4}{3}$

  (A1.6, A2.8)

- The values A(4, 12) and B(14, 28) were substituted into the slope formula as follows:

  $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
  $$m = \frac{28-12}{14-4}$$
  $$m = \frac{16}{10}$$
  $$m = \frac{8}{5}$$

  (i) What has been done differently in the following expression?

  $$m = \frac{12-28}{4-14}$$

  Calculate the slope. Is it acceptable to do this? Explain.

  (ii) What has been done differently in the following expression?

  $$m = \frac{12-28}{14-4}$$

  Calculate the slope. Is it acceptable to do this? Explain.

  (iii) What is an important rule to remember about the order of the coordinates when substituting into the slope formula?

  (A1.3, A1.6, A2.8)

Resources/Notes

Authorized Resource

Math at Work 11

6.3 Slope as Rate of Change

SB: pp. 286-297

TR: pp. 270-280

BLM: 6-8

Algebra

Outcomes

Students will be expected to

A1 and A2 Continued ...

Achievement Indicators:

A1.1 *Solve a contextual problem involving the application of a formula that does not require manipulation.*

A2.9 *Solve a contextual problem that involves slope or rate of change.*

A1.4 *Create and solve a contextual problem that involves a formula.*

A1.3 *Describe, using examples, how a given formula is used in a trade or an occupation.*

N1 Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

[C, CN, PS, R]

Elaborations—Strategies for Learning and Teaching

Students should be able to apply the rate of change formula to real world situations. It is important for students to understand the meaning of slope with respect to the context of the given problem. Wheelchair ramps, ski slopes, roof tops, and waterslides are all situations where slope or rate of change can be applied. Students should be able to extract important information from the contextual problem by applying their knowledge of rate of change. They will need to utilize and manipulate the rate of change formula to provide a solution to the problem. Having students create contextual problems is a great way for them to gain a better understanding.

There are many trades and occupations where knowledge and application of the slope formula is commonly used. Some examples are carpentry, construction, professional athletes such as skiers and skateboarders, and the field of ergonomics. Students have learned how to apply the formula in contextual problems but they should research and discuss applications of slope and rate of change with respect to specific occupations.

Having been exposed to various puzzles and games throughout the course, students should have ideas about how to create a variation on a puzzle or game. They can introduce their games to other students, and describe a strategy for playing the game. Refer back to pp. 34-39 for additional information.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

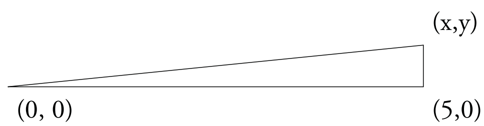
Performance

- Take students on a "math walk" throughout the building or neighbourhood to take photographs of objects depicting slope. Have students write a story or series of problems related to their observations and create a class book.

(A1.5, A2.9)

Paper and Pencil

- Sally is running on a treadmill with no incline. She decides to increase the incline by two levels. Each incline level increases the track height by 1 unit.



Ask students to answer the following questions:

(i) What is the slope of the track before the incline?
(ii) What aspect of slope was changed to create the incline?
(iii) What is the slope of the track after the incline?

(A1.1, A2.9)

Presentation

- Ask students to research the importance of slope within an occupation and describe specific examples where the formula is used.

(A1.4)

Resources/Notes

Authorized Resource

Math at Work 11

6.3 Slope as Rate of Change

SB: pp. 286-297

TR: pp. 270-280

BLM: 6-8

Web Link

www.k12pl.nl.ca/mathematics/seniorhigh/introduction/math2202/classroomclips.html

The classroom clip on **Slope** highlights a cross-curricular link to Skilled Trades.

Games and Puzzles

Slippery Slope SB: p. 303

TR: p. 285

Right Triangles and Trigonometry

Suggested Time: 14 Hours

Unit Overview

Focus and Context

In this unit, students will solve problems with more than one right-angled triangle, using sine, cosine and tangent ratios, as well as the Pythagorean theorem. They will also solve problems involving the angle of elevation and the angle of depression.

The unit provides an opportunity to further develop students' spatial sense, as well as their ability to break down complex problems and analyze situations with multiple components. Breaking down a problem into a set of simple problems that can be solved easily with known skills is an essential one for many trades, from the calculations performed by metal fabricators to the spatial planning of cabinetmakers.

Outcomes Framework

GCO

Develop spatial sense.

SCO G1

Solve problems that involve two and three right triangles.

Process Standards

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Geometry		
G4. Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: • applying similarity to right triangles • generalizing patterns from similar right triangles • applying the primary trigonometric ratios • solving problems. [CN, PS, R, T, V]	G1. Solve problems that involve two and three right triangles. [CN, PS, T, V]	G1. Solve problems by using the sine law and cosine law, excluding the ambiguous case. [CN, PS, V] G2. Solve problems that involve: • triangles • quadrilaterals • regular polygons. [C, CN, PS, V]

Geometry

Outcomes

Students will be expected to

G1 Solve problems that involve two or three right triangles.

[CN, PS, T, V]

Elaborations—Strategies for Learning and Teaching

Students were introduced to the Pythagorean theorem in Grade 8 (8SS1) and used it in problem solving situations in Grade 9 (9N6, 9SS1, 9SS2). In Mathematics 1202, students used real world examples to promote the importance and relevance of the Pythagorean theorem (G2, A1) and they were introduced to the three primary trigonometric ratios (G4). Students solved right triangles and contextual problems using the primary trigonometric ratios but they were limited to working with only one right triangle. This outcome extends their learning to solving problems with two or three right triangles. In this unit, students will work with angles of elevation and depression. They were introduced to angles of elevation in the previous unit. This will be their first exposure to angles of depression.

An extensive review of the primary trigonometric ratios and their applications to right triangles will be necessary. Students apply the sine, cosine and tangent ratios to determine missing sides and angles in right triangles. This is important for later work with angles of elevation and depression and situations that involve more than one right triangle.

When solving problems involving trigonometry, sketching the scenario can help students find the answer. It is easier to decide which trigonometric ratio to apply when the opposite and adjacent sides and the hypotenuse are identified on a diagram. Students sometimes have difficulty correctly identifying the opposite and adjacent sides in relation to the reference angle. They should be exposed to right triangles with reference angles in various locations, so that they recognize that the opposite and adjacent sides are relevant to the reference angle.

The trigonometric ratios can then be used to determine missing side lengths. In Grade 9, students solved equations of the form $a = \frac{b}{c}$ (9PR3). A review of this may be necessary prior to solving equations such as $\tan 30° = \frac{x}{10}$ or $\tan 30° = \frac{5}{x}$.

Students will use the sine, cosine or tangent ratios to determine the measure of a missing acute angle in a right triangle. This requires the use of the inverse of the ratio.

It is expected that they will use a calculator to find trigonometric ratios and angle measurements of given ratios. They will have to be reminded of the need to work in degree mode.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to complete a graphic organizer that includes categories such as examples, non-examples, characteristics and definition with respect to right triangles.

- A questionnaire can be used to determine students' familiarity with the trigonometry terminology. Students select a response based on their level of familiarity with the mathematical term.

  Example:

Opposite	Adjacent
☐ I have never heard of this. ☐ I have heard of this but I'm not sure what it means. ☐ I have some idea what it means. ☐ I clearly know what it means and can describe it.	☐ I have never heard of this. ☐ I have heard of this but I'm not sure what it means. ☐ I have some idea what it means. ☐ I clearly know what it means and can describe it.
Angle of Elevation	**Tangent**
☐ I have never heard of this. ☐ I have heard of this but I'm not sure what it means. ☐ I have some idea what it means. ☐ I clearly know what it means and can describe it.	☐ I have never heard of this. ☐ I have heard of this but I'm not sure what it means. ☐ I have some idea what it means. ☐ I clearly know what it means and can describe it.

  For the third and fourth selected responses, consider leaving a blank space to have students describe their ideas about the term.

  The questionnaire can be administered again as a post-assessment at the end of the unit.

Resources/Notes

Authorized Resource

Math at Work 11

7.1 Right Triangles

Student Book (SB): pp. 308-321

Teacher's Resource (TR): pp. 295-303

Blackline Masters (BLM): 7-3

Suggested Resource

Math at Work 10

Chapter 7

Geometry

Outcomes

Students will be expected to

G1 Continued ...

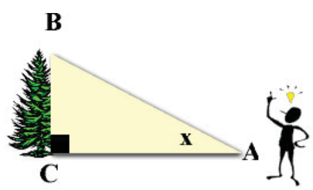
Achievement Indicators:

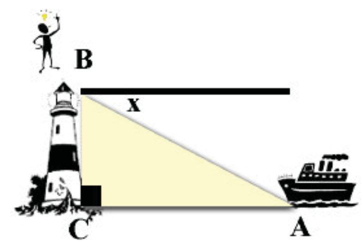
G1.1 *Solve a contextual problem that involves angles of elevation or angles of depression.*

G1.2 *Sketch a representation of a given description of a problem in a 2-D or 3-D context.*

Elaborations—Strategies for Learning and Teaching

Although students solved contextual problems using trigonometric ratios in Mathematics 1202, the terms angle of elevation and angle of depression were not used to describe angles. These angles are always measured relative to the horizontal. Providing students with visuals of real world situations and indicating where the angle of elevation or depression is located should give students an understanding of these terms.





The angle between a horizontal line of sight and the line of sight to an object that is higher than the observer is the angle of elevation. If the object is lower than the observer, the angle between the horizontal line of sight and the line of sight to the object is the angle of depression.

Students should solve contextual problems that include angles of elevation and depression, limited to one right triangle.

In situations where it is inconvenient or impossible to make measurements directly, a clinometer can be used to measure the angle of elevation or depression. Students can measure the angle between the horizontal and the line of sight to the top of the object. Measuring the horizontal distance from the observer to the object should provide the data needed to calculate the height of the object using trigonometry. To link this to the workplace, invite a surveyor to visit the class and describe the requirements of his/her job, show the tools used on the job, and discuss and demonstrate how they would perform a similar task.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

  (i) Two poles on horizontal ground are 60 m apart. The shorter pole is 3 m high. The angle of depression from the top of the longer pole to the top of the shorter pole is 20°. Sketch a diagram to represent the situation.

  (G1.2)

  (ii) A man who is 2 m tall stands 30 m away from a tree. The angle of elevation of the top of the tree from his eyes is 28°. Sketch a diagram and determine the height of the tree.

  (G1.1, G1.2)

Performance

- Set up stations at which groups of students find the height of various objects using trigonometry. They can measure the angle of elevation using a clinometer. Some objects may include the height of a basketball net, the height of a clock on the wall, the height of a gym wall, and the height of a door.

  (G1.1)

Resources/Notes

Authorized Resource

Math at Work 11

7.2 Angles of Elevation and Depression

SB: pp. 322-337

TR: pp. 304-313

BLM: 7-4

Web Links

- *http://illuminations.nctm.org*

  Lesson: Trigonometry for Solving Problems

- *www.k12pl.nl.ca/mathematics/seniorhigh/introduction/math2202/classroomclips.html*

  In the **Trigonometry** clip, students use a clinometer to measure the angle of elevation of various objects. They then use the tangent ratio to determine the heights of the objects.

Geometry

Outcomes

Students will be expected to

G1 Continued ...

Achievement Indicators:

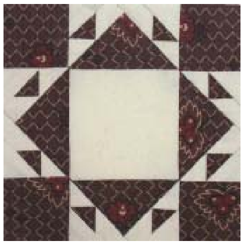
G1.3 *Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.*

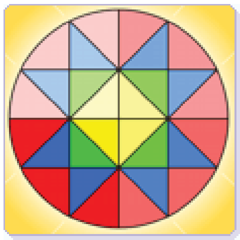
G1.4 *Identify all of the right triangles in a given illustration for a context.*

G1.2 *Continued*

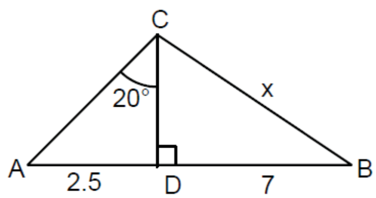
Elaborations—Strategies for Learning and Teaching

Identification of right triangles is an important starting point when solving problems involving trigonometry. Right triangles can be identified by exploring angle measurement and using the Pythagorean theorem. Providing students with diagrams such as the ones below and discussing what information is needed to determine if the triangles are right triangles is a possible teaching strategy.





Students will extend their knowledge of solving contextual problems involving one right triangle to solving problems with two and three right triangles. They should practice determining missing side lengths and angles from given diagrams before solving contextual problems. Trigonometry can be used to solve for lengths or angles in a sequence of triangles by using the solution of one triangle to provide information needed to solve the next triangle. Students could work with a partner to explore this. Ask them, for example, to determine the length of CB in the diagram below.



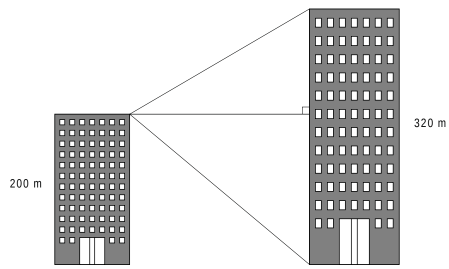
To calculate the length of CB, more than one triangle will be used. Students should recognize that △ACD will be used before △BCD.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Roger is on the top floor of the smallest building and views the base of the other building at an angle of depression of 40°.

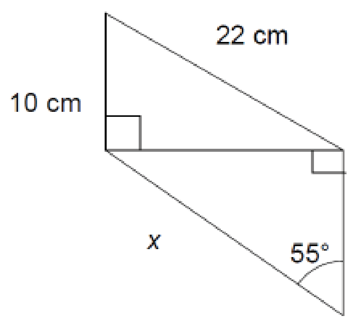


Ask students to answer the following questions:

(i) What is the distance between the two buildings?
(ii) What is the angle of elevation that Roger views the top floor of the other building?

(G1.1, G1.3, G1.4)

- Ask students to determine the value of x in the following diagram.



(G1.3, G1.4)

Performance

- Choose a problem that involves two or three right triangles. Put each step for solving the problem on a card and distribute the cards to small groups of students. Ask students to decide on a logical sequence in which to place the cards. They should justify their reasons for the sequence.

  A blank card could also be included in the set. Students would have to provide the missing card to complete the steps required to solve the problem.

(G1.3, G1.4)

Resources/Notes

Authorized Resource

Math at Work 11

7.3 Multiple Right Triangles

SB: pp. 338-353

TR: pp. 314-323

BLM: 7-5

Geometry

Outcomes

Students will be expected to

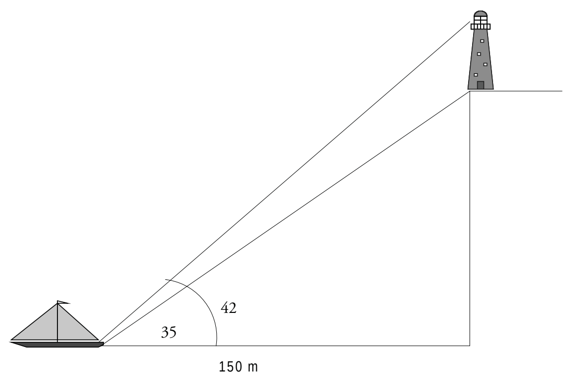
G1 Continued ...

Achievement Indicator:

G1.2, G1.3, G1.4 *Continued*

Elaborations—Strategies for Learning and Teaching

The following is an example of a contextual diagram using two triangles. Students can use trigonometry to determine the height of the lighthouse.



This strategy can also be used to find missing sides and angles of triangles with no right angles by breaking them down into right triangles.

G1.5 *Determine if a solution to a problem that involves two or three right triangles is reasonable.*

Students should always be encouraged to use the properties of a triangle to check the reasonableness of their solutions. The smaller angle, for example, should be located opposite the side with the shortest length, the sum of any two side lengths of a triangle should be greater than the length of the third side and the sum of the angles in a triangle should be 180°. Students should also check if their solution is reasonable within the context of the problem. They should be exposed to problems such as the following:

Two trees are 100 m apart. From a point halfway between the trees, the angle of elevation of the top of the shorter tree is 32° and the angle of elevation to of the top of the taller tree is 50°. What is the height of each tree?

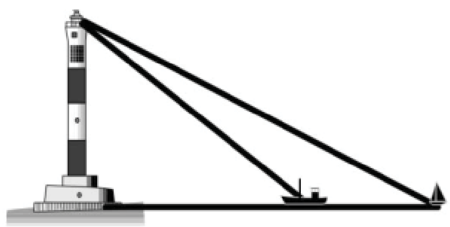
This involves using the tangent ratio to find side lengths. Common student errors, such as using cosine rather than tangent or incorrectly identifying the opposite and adjacent sides, could lead to answers resulting in the shorter tree having a longer length than the taller tree. In such cases, by asking themselves questions such as "Does this make sense?" or "Is this possible?" students should realize they have made an error.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

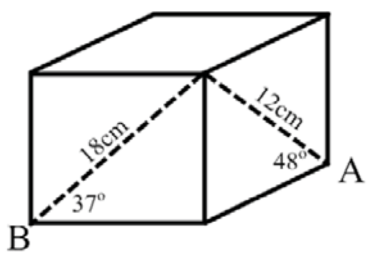
Paper and Pencil

- A tourist at Point Amour sees a fishing boat at an angle of depression of 23° and a sailboat at an angle of depression of 9°. If the tourist is 33.5 m above water, ask students to determine how far apart the two vessels are.



(G1.1, G1.3, G1.4)

- Ask students to determine the shortest distance between A and B in the rectangular prism below.



(G1.3, G1.4)

Journal

- Ask students to describe a work or recreational situation where trigonometry could be used to find a length or distance. Ask them to explain what information is needed to make the calculation and to describe how they could gather this information. They should also provide a method for the calculation.

Resources/Notes

Authorized Resource

Math at Work 11

7.3 Multiple Right Triangles

SB: pp. 338-353

TR: pp. 314-323

BLM: 7-5

Appendix:

Outcomes with Achievement Indicators Organized by Topic (With Curriculum Guide Page References)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Measurement	**General Outcome**: Develop spatial sense through direct and indirect measurement.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
M1. Solve problems that involve SI and imperial units in surface area measurements and verify the solutions. [C, CN, ME, PS, V]	M1.1 Explain, using examples, including nets, the relationship between area and surface area.	p. 22
	M1.2 Explain how a referent can be used to estimate surface area.	p. 24
	M1.3 Estimate the surface area of a 3-D object.	p. 24
	M1.4 Solve a problem that involves determining the surface area of 3-D objects, including pyramids and spheres.	pp. 26-28, 30-32
	M1.5 Solve a contextual problem that involves the surface area of 3-D objects and that requires the manipulation of formulas.	pp. 26-28, 30-32
	M1.6 Illustrate, using examples, the effect of dimensional changes on surface area.	pp. 28, 30-32

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Measurement	**General Outcome**: Develop spatial sense through direct and indirect measurement.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
M2. Solve problems that involve SI and imperial units in volume and capacity. [C, CN, ME, PS, V]	M2.1 Explain, using examples, the difference between volume and capacity.	p. 68
	M2.2 Identify and compare referents for volume and capacity measurements in SI and imperial units.	p. 70
	M2.3 Identify a situation where a given SI or imperial volume unit would be used.	p. 70
	M2.4 Estimate the volume or capacity of a 3-D object or container, using a referent.	p. 70
	M2.5 Write a given volume measurement expressed in one SI unit cubed in another SI unit cubed.	p. 72
	M2.6 Write a given volume measurement expressed in one imperial unit cubed in another imperial unit cubed.	p. 72
	M2.7 Write a given capacity expressed in one unit as another unit in the same measurement system.	p. 72
	M2.8 Determine the volume of prisms, cones, cylinders, pyramids, spheres and composite 3-D objects, using a variety of measuring tools such as rulers, tape measures, callipers, micrometers, and displacement.	p. 74
	M2.9 Determine the capacity of prisms, cones, pyramids, spheres and cylinders, using a variety of measuring tools and methods such as graduated cylinders, measuring cups, and measuring spoons.	p. 74
	M2.10 Illustrate, using examples, the effect of dimensional changes on volume.	p. 74
	M2.11 Describe the relationship between the volume of: • cones and cylinders with the same base and height • pyramids and prisms with the same base and height.	p. 76
	M2.12 Solve a contextual problem that involves the volume of a 3-D object, including composite 3-D objects, or the capacity of a container.	pp. 78-80

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Topic: Geometry	**General Outcome**: Develop spatial sense.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
G1. Solve problems that involve two and three right triangles. [CN, PS, T, V]	G1.1 Solve a contextual problem that involves angles of elevation or angles of depression.	p. 152
	G1.2 Sketch a representation of a given description of a problem in a 2-D or 3-D context.	pp. 152-156
	G1.3 Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.	pp. 154-156
	G1.4 Identify all of the right triangles in a given illustration for a context.	pp. 154-156
	G1.5 Determine if a solution to a problem that involves two or three right triangles is reasonable.	p. 156
G2. Solve problems that involve scale. [PS, R, T, V]	G2.1 Describe contexts in which a scale representation is used.	p. 44
	G2.2 Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model.	p. 44
	G2.3 Construct a model of a 3-D object, given the scale.	p. 46
	G2.4 Draw, with and without technology, a scale diagram of a given object.	p. 46
	G2.5 Solve a contextual problem that involves scale.	p. 46

[C] Communication		**[PS]** Problem Solving	
[CN] Connections		**[R]** Reasoning	
[ME] Mental Mathematics and Estimation		**[T]** Technology	
		[V] Visualization	

Topic: Geometry	**General Outcome**: Develop spatial sense.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
G3. Model and draw 3-D objects and their views. [CN, R, V]	G3.1 Draw a 2-D representation of a given 3-D object.	pp. 48-50
	G3.2 Draw to scale top, front and side views of a given 3-D object.	pp. 48-50
	G3.3 Construct a model of a 3-D object, given the top, front and side views.	pp. 48-50
	G3.4 Draw a 3-D object, given the top, front and side views.	pp. 48-50
	G3.5 Determine if given views of a 3-D object represent a given object, and explain the reasoning.	p. 50
	G3.6 Draw, using isometric dot paper, a given 3-D object.	p. 52
	G3.7 Draw a one-point perspective view of a given 3-D object.	pp. 54-56
	G3.8 Identify the point of perspective of a given one-point perspective drawing of a 3-D object.	p. 56

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Geometry	**General Outcome**: Develop spatial sense.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
G4. Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects. [CN, V]	G4.1 Draw the components of a given exploded diagram, and explain their relationship to the original 3-D object. G4.2 Sketch an exploded view of a 3-D object to represent the components. G4.3 Draw to scale the components of a 3-D object. G4.4 Sketch a 2-D representation of a 3-D object, given its exploded view.	p. 58 p. 60 p. 60 p. 62

Topic: Number	**General Outcome**: Develop number sense and critical thinking skills.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches. N1.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. N1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.	pp. 34-39 pp. 80-81 pp. 130-131 pp. 144-145

[C] Communication	**[PS]** Problem Solving		
[CN] Connections	**[R]** Reasoning		
[ME] Mental Mathematics and Estimation	**[T]** Technology		
	[V] Visualization		

Topic: Number	**General Outcome**: Develop number sense and critical thinking skills.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
N2. Solve problems that involve personal budgets. [CN, PS, R, T]	N2.1 Identify income and expenses that should be included in a personal budget.	p. 110
	N2.2 Explain considerations that must be made when developing a budget; e.g., prioritizing, recurring and unexpected expenses.	p. 110
	N2.3 Create a personal budget based on a given income and expense data.	pp. 110-112
	N2.4 Collect income and expense data, and create a budget.	p. 112
	N2.5 Modify a budget to achieve a set of personal goals.	p. 114
	N2.6 Investigate and analyze with or without technology, "What if ..." questions related to personal budgets.	p. 114
N3. Demonstrate an understanding of compound interest. [CN, ME, PS, T]	N3.1 Solve a problem that involves simple interest, given three of the four values in the formula $I = Prt$.	p. 116
	N3.2 Compare simple and compound interest, and explain their relationship.	p. 118
	N3.3 Solve, using a formula, a contextual problem that involves compound interest.	p. 120
	N3.4 Explain, using examples, the effect of different compounding periods on calculations of compound interest.	p. 120
	N3.5 Estimate, using the Rule of 72, the time required for a given investment to double in value.	p. 120

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Topic: Number	**General Outcome**: Develop number sense and critical thinking skills.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
N4. Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T]	N4.1Describe the type of banking services available from various financial institutions, such as online services.	p. 100
	N4.2 Describe the type of accounts available at various financial institutions.	p. 102
	N4.3 Identify the type of account that best meets the needs for a given set of criteria.	p. 102
	N4.4 Describe the advantages and disadvantages of online banking.	p. 102
	N4.5 Identify and explain various automated teller machine (ATM) service charges.	p. 104
	N4.6 Describe the advantages and disadvantages of debit card purchases.	p. 104
	N4.7 Describe ways that ensure the security of personal and financial information; e.g., passwords, encryptions, protection of personal identification number (PIN) and other personal identity information.	pp. 106-108

[C] Communication		**[PS]** Problem Solving	
[CN] Connections		**[R]** Reasoning	
[ME] Mental Mathematics and Estimation		**[T]** Technology	
		[V] Visualization	

Topic: Number	**General Outcome**: Develop number sense and critical thinking skills.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
N5. Demonstrate an understanding of credit options, including: • credit cards • loans. [CN, ME, PS, T]	N5.1Compare advantages and disadvantages of different types of credit options, including bank and store credit cards, personal loans, lines of credit, overdraft.	p. 122
	N5.2 Make informed decisions and plans related to the use of credit, such as service charges, interest, payday loans and sales promotions, and explain the reasoning.	p. 124
	N5.3 Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt and timing purchases.	p. 124
	N5.4 Compare credit card options from various companies and financial institutions.	p. 126
	N5.5 Solve a contextual problem that involves credit cards or loans.	pp. 128-130
	N5.6 Solve a contextual problem that involves credit linked to sales promotions.	p. 130

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Algebra	**General Outcome**: Develop algebraic reasoning.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
A1. Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • simple interest • finance charges. [CN, ME, PS, R]	A1.1Solve a contextual problem involving the application of a formula that does not require manipulation.	pp. 26-28, 30-32, 78, 116, 128-130, 144
	A1.2 Solve a contextual problem involving the application of a formula that requires manipulation.	pp. 26-28, 30-32, 78, 116
	A1.3 Describe, using examples, how a given formula is used in a trade or an occupation.	pp. 78, 144
	A1.4 Create and solve a contextual problem that involves a formula.	pp. 78-144
	A1.5 Identify and correct errors in a solution to a problem that involves a formula.	pp. 78, 116, 142
	A1.6 Explain and verify why different forms of the same formula are equivalent.	p. 142

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Algebra	**General Outcome**: Develop algebraic reasoning.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
A2. Demonstrate an understanding of slope: • as rise over run • as rate of change • by solving problems. [C, CN, PS, V]	A2.1 Describe contexts that involve slope; e.g., ramps, roofs, road grade, flow rates within a tube, skateboard parks, ski hills.	p. 136
	A2.2 Explain, using diagrams, the difference between two given slopes (e.g., a 3:1 and a 1:3 roof pitch), and describe the implications.	p. 136
	A2.3 Describe the conditions under which a slope will be either 0 or undefined.	p. 136
	A2.4 Explain, using examples and illustrations, slope as rise over run.	pp. 138-140
	A2.5 Verify that the slope of an object, such as a ramp or a roof, is constant.	p. 140
	A2.6 Explain, using illustrations, the relationship between slope and angle of elevation; e.g., for a ramp with a slope of 7:100, the angle of elevation is approximately 4°.	p. 140
	A2.7 Explain the implications, such as safety and functionality, of different slopes in a given context.	p. 140
	A2.8 Explain, using examples and illustrations, slope as rate of change.	p. 142
	A2.9 Solve a contextual problem that involves slope or rate of change.	p. 144

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Algebra	**General Outcome**: Develop algebraic reasoning.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
A3. Solve problems by applying proportional reasoning and unit analysis. [C, CN, PS, R]	A3.1 Explain the process of unit analysis used to solve a problem.	p. 30
	A3.2 Explain, using an example, how unit analysis and proportional reasoning are related.	p. 30
	A3.3 Solve a problem, using unit analysis.	pp. 30, 72
	A3.4 Solve a problem within and between systems, using proportions or tables.	p. 30

Topic: Statistics	**General Outcome**: Develop statistical reasoning.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
S1. Solve problems that involve creating and interpreting graphs, including: • bar graphs • histograms • line graphs • circle graphs. [C, CN, PS, R, T, V]	S1.1 Determine the possible graphs that can be used to represent a given data set, and explain the advantages and disadvantages of each.	pp. 86-88
	S1.2 Create, with and without technology, a graph to represent a given data set.	pp. 90-92
	S1.3 Solve a contextual problem that involves the interpretation of a graph.	pp. 90-94
	S1.4 Describe trends in the graph of a given data set.	p. 92
	S1.5 Interpolate and extrapolate values from a given graph.	p. 92
	S1.6 Explain, using examples, how the same graph can be used to justify more than one conclusion.	p. 94
	S1.7 Explain, using examples, how different graphic representations of the same data can be used to emphasize a point of view.	p. 94

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