7.6 Content for Stage 4

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Mathematics • Stage 4

Number and Algebra
Computation with Integers

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- applies appropriate mathematical techniques to solve problems — MA4-2WM
- recognises and explains mathematical relationships using reasoning — MA4-3WM
- compares, orders and calculates with integers, applying a range of strategies to aid computation — MA4-4NA

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-4NA, MALS-5NA, MALS-6NA, MALS-9NA, MALS-10NA*

Students:

Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)

- use an appropriate non-calculator method to divide two-and three-digit numbers by a two-digit number
  - compare initial estimates with answers obtained by written methods and check with a calculator (Problem Solving) [N, CCT]
- apply a practical understanding of commutativity to aid mental computation, eg $3 + 9 = 9 + 3 = 12$, $3 \times 9 = 9 \times 3 = 27$ [N, CCT]
- apply a practical understanding of associativity to aid mental computation, eg $3+8+2=(3+8)+2=3+(8+2)=13$, $2\times7\times5=(2\times7)\times5=2\times(7\times5)=70$ [N, CCT]
  - determine by example that associativity holds true for multiplication of three or more numbers but does not apply to calculations involving division, eg $(80\div8)\div2$ is not equivalent to $80\div(8\div2)$ (Communicating) [N, CCT]
- apply a practical understanding of the distributive law to aid mental computation, eg to multiply any number by 13, first multiply by 10 and then add 3 times the number [N, CCT]
- use factors of a number to aid mental computation of multiplication and division, eg to multiply a number by 12, first multiply the number by 6 and then multiply by 2 [N]

Compare, order, add and subtract integers (ACMNA280)

- place directed numbers on a number line [N]
- order directed numbers [N]
- add and subtract directed numbers

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- ‣ determine, by developing patterns or using a calculator, that subtracting a negative number is the same as adding a positive number (Reasoning) [N, CCT]
- ‣ apply directed numbers to problems involving money and temperature (Problem Solving) [N, CCT]

Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183)

- multiply and divide directed numbers
  - ‣ investigate, by developing patterns or using a calculator, the rules associated with multiplying and dividing directed numbers (Reasoning) [N, CCT]
- use a calculator to perform operations with directed numbers
  - ‣ decide whether it is more appropriate to use mental strategies or a calculator when performing certain operations with directed numbers (Communicating) [CCT]
- use grouping symbols as an operator with directed numbers
- apply the order of operations to evaluate expressions involving directed numbers mentally, including where an operator is contained within the numerator or denominator of a fraction, eg $\frac{15+9}{6}$, $\frac{15+9}{15-3}$, $5+\frac{18-12}{6}$, $5+\frac{18}{6}-12$, $5\times(2-8)$
  - ‣ investigate whether other calculators, such as those found in computer software and on mobile devices, use the order of operations (Problem Solving) [N, CCT]

Background information

To divide two- and three-digit numbers by a two-digit number, students may be taught the long division algorithm or, alternatively, to transform the division into a multiplication,
eg $88 \div 44 = 2$ because $2 \times 44 = 88$
$356 \div 52 = \square$ becomes $52 \times \square = 356$. Knowing that there are two fifties in each 100, students may try 7 so that 52 × 7 = 364 which is too large. Try 6 so that 52 × 6 = 312. Answer is $6\frac{44}{52} = 6\frac{11}{13}$.

Students also need to be able to express a division in the following form in order to relate multiplication and division: 356 = 6 × 52 + 44; and then division by 52 gives: $\frac{356}{52} = 6 + \frac{44}{52} = 6\frac{11}{13}$

Complex recording formats for directed numbers such as raised signs can be confusing. The following formats are recommended: $-2-3=-5$, $-7+(-4)=-7-4$, $-2--3=-2+3$
$=-11$ $=1$

Brahmagupta, an Indian mathematician and astronomer (c598–665 AD) is noted for the introduction of zero and negative numbers in arithmetic.

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Number and Algebra
Computation with Integers

Language

The introduction of operations with negative numbers may highlight some students' misunderstandings of mathematical terms: an operation that leads to a negative result can no longer be used as a cue that the operation has been performed with the numbers in the wrong order. Teachers can assist these students by drawing their attention to the different ways in which the same idea can be expressed, eg '3 take away 9' is the same as 'take 9 from 3', 'divide 6 by 2' is the same as 'how many 2s in 6?', '3 minus 9' is the same as '9 less than 3'.

Mathematics • Stage 4

Number and Algebra
Fractions, Decimals and Percentages

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- applies appropriate mathematical techniques to solve problems — MA4-2WM
- recognises and explains mathematical relationships using reasoning — MA4-3WM
- operates with fractions, decimals and percentages — MA4-5NA

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-7NA, MALS-8NA*

Students:

Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152)

- determine highest common factors and lowest common multiples
- generate equivalent fractions
- reduce a fraction to its simplest form
- express improper fractions as mixed numerals and vice versa
- place positive and negative fractions and mixed numerals on a number line [N]
  - ‣ interpret a given scale to determine fractional values represented on a number line (Problem Solving) [N]
  - ‣ choose an appropriate scale to display given fractional values on a number line, eg when plotting thirds or sixths, a scale of 3 cm for every one unit value is easier to use (Communicating, Reasoning) [N]

Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)

- add and subtract fractions, including mixed numerals, using written and calculator methods
  - ‣ recognise and explain incorrect operations with fractions, eg explain why $\frac{2}{3}+\frac{1}{4}\neq\frac{3}{7}$ (Communicating, Reasoning) [L, N, CCT]
  - ‣ interpret fractions and mixed numerals on a calculator display (Communicating) [N, CCT]
- subtract a fraction from a whole number, eg $3-\frac{2}{3}=2+1-\frac{2}{3}=2\frac{1}{3}$

Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)

- determine the effect of multiplying or dividing by a number with magnitude less than one [N]
- multiply and divide decimals using written methods, limiting operators to two digits
  - ‣ compare initial estimates with answers obtained by written methods and check with a calculator (Problem Solving) [CCT]

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Number and Algebra
Fractions, Decimals and Percentages

- multiply and divide fractions and mixed numerals
  - demonstrate multiplication of a fraction by another fraction using a diagram to illustrate the process (Communicating, Reasoning) [L, N]
  - explain why division by a fraction is equivalent to multiplication by its reciprocal using a numerical example (Communicating, Reasoning) [L, N, CCT]
- calculate fractions and decimals of quantities [N]
  - choose the appropriate equivalent form for mental computation, eg 0.25 of \$60 is equivalent to $\frac{1}{4}$ of \$60 which is equivalent to \$60 ÷ 4 (Communicating) [N, CCT]

Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)

- express one quantity as a fraction of another [N]
  - choose appropriate units to compare two quantities as a fraction, eg 15 minutes is $\frac{15}{60}=\frac{1}{4}$ of an hour (Communicating) [N, CCT]

Round decimals to a specified number of decimal places (ACMNA156)

- round decimals to a given number of places
- use symbols for approximation, eg $\approx$ or $\doteqdot$ [L]

Investigate terminating and recurring decimals (ACMNA184)

- use the notation for recurring (repeating) decimals, eg $0.33333...=0.\dot{3}$, $0.345345345...=0.\dot{3}4\dot{5}$, $0.266666...=0.2\dot{6}$ [L]
- convert fractions to terminating or recurring decimals as appropriate
  - recognise that calculators may show approximations to recurring decimals, and explain why, eg $\frac{2}{3}$ displayed as 0.666666667 (Communicating, Reasoning) [CCT]

Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)

- convert fractions to decimals (terminating and recurring) and percentages
- convert terminating decimals to fractions and percentages
- convert percentages to fractions and decimals
  - evaluate the reasonableness of statements in the media that quote fractions, decimals or percentages, eg ‘the number of children in the average family is 2.3’ (Communicating, Problem Solving) [N, CCT]
- order fractions, decimals and percentages

Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)

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Number and Algebra
Fractions, Decimals and Percentages

- calculate percentages of quantities [N]
  - choose an appropriate equivalent form for mental computation, eg 20% of \$40 is $\frac{1}{5}\times\$40$ which is equivalent to $\$40 \div 5$ (Communicating) [N, CCT]
- express one quantity as a percentage of another, eg 45 minutes is 75% of an hour [N]

Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187)

- increase and decrease a quantity by a given percentage [N]
  - recognise equivalences when calculating, eg multiplication by 1.05 will increase a number/quantity by 5%; multiplication by 0.87 will decrease a number/quantity by 13% (Reasoning) [N]
- interpret and calculate percentages greater than one hundred, eg an increase from \$2 to \$5 is an increase of 150% [N]
- solve a variety of real-life problems involving percentages, including percentage composition (Communicating) [N]
  - interpret calculator displays in formulating solutions to problems by appropriately rounding decimals
  - use the unitary method to solve problems involving percentages, eg find the original value, given the value after an increase of 20% (Problem Solving) [N]
  - interpret and use nutritional information panels on products (Problem Solving) [L, N]
  - interpret and use media and sport reports involving percentages (Problem Solving) [N, CCT]
  - interpret and use statements about the environment involving percentages, eg energy use for different purposes such as lighting (Problem Solving) [N, CCT, SE]

Background information

Students are not likely to have had any experience with rounding to a given number of decimal places prior to Stage 4. The term ‘decimal places’ may need to be defined. Students should be aware rounding is a process of ‘approximating’ and that a rounded number is an ‘approximation’.

The earliest evidence of fractions can be traced to the Egyptian papyrus of Ahmes (about 1650 BC). In the seventh century AD the method of writing fractions as we write them now was invented in India, but without the fraction bar (vinculum), which was introduced by the Arabs. Fractions were widely in use by the 12th century.

One cent and two cent coins were withdrawn by the Australian Government in 1990. Prices can still be expressed in one-cent increments but the final bill is rounded to the nearest five cents (except for electronic transactions), ie totals ending in 3, 4, 6 and 7 are rounded to the nearest 5 cents, and totals ending in 8, 9, 1 and 2 are rounded to the nearest 10 cents.

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Number and Algebra
Fractions, Decimals and Percentages

Language

When expressing fractions in English, the numerator is said first, followed by the denominator. However, in many Asian languages (eg Chinese, Japanese) the opposite is the case: the denominator is said before the numerator. Students from such language backgrounds should be encouraged to think in English when they are speaking about/expressing fractions.

In questions that require calculating a fraction/percentage of a quantity, some students may benefit from firstly writing an expression using the word 'of', before replacing it by '×'.

Students may need assistance with the subtleties of the English language when solving word problems. The different processes required by the words 'to' and 'by' in questions such as 'find the percentage increase if $2 is increased to $3' and 'find the percentage increase if $2 is increased by $3' should be made explicit. Students may also need to be made aware of the difference between '10% of $50' and '10% off $50'.

The word 'cent' comes from the Latin word *centum* meaning 'one hundred'. Percent means 'out of one hundred' or 'hundredths'.

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Number and Algebra
Financial Mathematics

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- solves financial problems involving purchasing goods MA4-6NA

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-11NA, MALS-12NA, MALS-13NA, MALS-14NA*

Students:

Investigate and calculate Goods and Services Tax (GST), with and without digital technologies

- calculate GST and GST-inclusive prices for goods purchased in Australia [N, PSC]
  - interpret GST information contained on receipts (Communicating) [N]
  - investigate efficient methods of computing GST and GST-inclusive prices (Problem Solving) [N]
  - explain why the value of the GST itself is not equivalent to 10% of the GST-inclusive price (Communicating, Reasoning) [N]
- determine pre-GST prices for goods given the GST-inclusive price [N]
  - explain why the pre-GST price is not equivalent to 10% off the GST-inclusive price (Communicating, Reasoning) [N]

Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)

- solve problems involving discounts, including calculating the percentage discount [N]
  - evaluate special offers such as percentage discounts, buy-two-get-one-free, buy-one-get-another-at-half-price, etc to determine how much is saved (Communicating, Problem Solving) [N, PSC]
- calculate 'best buys', eg 500 grams for $4.50 compared with 300 grams for $2.75 [N, PSC]
  - investigate 'unit pricing' used by retailers and use this to determine the best buy (Problem Solving) [N, PSC]
  - recognise that in practical situations there are considerations other than just the 'best buy', eg the amount required, waste due to spoilage (Reasoning) [N, CCT, PSC]
  - use price comparison websites to make informed decisions related to purchases under given conditions (Problem Solving) [N, ICT, CCT, PSC]

Solve problems involving profit and loss, with and without digital technologies (ACMNA189)

- calculate the selling price given the percentage profit/loss on the cost price [N]
- express profit/loss as a percentage of the cost price [N]
- calculate the cost price given the selling price and percentage profit/loss [N]

Mathematics • Stage 4

Number and Algebra
Financial Mathematics

Background information

The Goods and Services Tax (GST) in Australia is a value added tax on the supply of goods and services. It was introduced by the Federal Government and took effect from 1 July 2000. Prior to the GST, Australia operated a wholesale sales tax implemented in the 1930s when Australia had an economy dominated by goods. The GST is levied at a flat rate of 10% on most goods and services, apart from GST exempt items (usually basic necessities such as milk and bread).

Language

GST stands for 'Goods and Services Tax'.

Student understanding may be increased if students write percentage calculations in words first before substituting the appropriate values, eg $\text{percentage discount} = \frac{\text{discount}}{\text{retail price}} \times 100\%$.

Students should be encouraged to write a few key words on the left-hand side of the equals sign to identify what is being found in each step in their working when solving percentage/financial problems. For example:

Find the GST-inclusive price of an item that costs \$350 before GST is added.

$$\begin{aligned}
\text{Pre-GST price} &= \$350 \\
\text{GST} &= 10\% \text{ of } \$350 \\
&= 0.1 \times \$350 \\
&= \$35 \\
\text{GST-inclusive price} &= \$350 + \$35 \\
&= \$385
\end{aligned}$$

Students may need assistance with the subtleties of language used in relation to financial transactions, eg the difference between '\$100 has been discounted by \$10' and '\$100 has been discounted to \$10'.

In calculating best buys, students may benefit from being explicitly taught to associate 'per' with $\div$, ie 'price per gram = money $\div$ grams'.

Mathematics • Stage 4

Number and Algebra
Proportion

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- operates with ratios and rates, and explores their graphical representation MA4-7NA

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-16NA*

Students:

Recognise and solve problems involving simple ratios (ACMNA173)

- use ratio to compare quantities measured in the same units [N]
- write ratios using the symbol ':', eg $4:7$ [L]
  - express one part of a ratio as a fraction of the whole, eg in the ratio $4:7$ the first part is $\frac{4}{11}$ of the whole (Communicating)
- simplify ratios, eg $4:6=2:3$, $\frac{1}{2}:2=1:4$, $0.3:1=3:10$
- apply the unitary method to ratio problems [N]
- divide a quantity in a given ratio [N]

Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188)

- interpret and calculate ratios that involve more than two numbers
- solve a variety of real-life problems involving ratios, eg scales on maps, mixes for fuels or concrete [N]
- distinguish between ratios, where quantities are measured in the same units, and rates, where quantities are measured in different units [N]
- convert given information into rates, eg 150 kilometres travelled in 2 hours = 75 km/h [N]
- solve a variety of real-life problems involving rates, including problems involving rates of travel [N, CCT]

Investigate, interpret and analyse graphs from authentic data (ACMNA180)

- interpret distance/time graphs (travel graphs) made up of straight line segments [N]
  - write or tell a story which matches a given travel graph (Communicating) [L, N]
  - match a travel graph to a description of a particular journey and explain reasons for the choice (Communicating, Reasoning) [L, N]
  - compare travel graphs of the same situation, decide which one is the most appropriate and explain why (Communicating, Reasoning) [L, N, CCT]

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Number and Algebra
Proportion

- recognise concepts such as change of speed and direction in distance/time graphs [N]
  - describe the meaning of different slopes for the graph of a particular journey (Communicating) [N]
  - calculate speeds for straight line segments of given graphs (Problem Solving) [N]
- recognise the significance of horizontal lines in distance/time graphs [N]
- determine which variable should be placed on the horizontal axis in distance/time graphs [N]
- draw distance/time graphs made up of straight line segments [N]
- sketch informal graphs to model familiar events, eg noise level during the lesson [N]
  - record the distance of a moving object from a fixed point at equal time intervals and draw a graph to represent the situation, eg move along a measuring tape for 30 seconds using a variety of activities that involve a constant rate such as walking forwards or backwards slowly, and walking or stopping for 10-second increments (Problem Solving) [N]
- use the relative positions of two points on a graph, rather than a detailed scale, to interpret information [N]

Background information

Work with ratio may be linked with the Golden Rectangle. Many windows are Golden Rectangles, as are some of the buildings in Athens such as the Parthenon. The ratio of the dimensions of the Golden Rectangle was known to the ancient Greeks: $\frac{\text{length}}{\text{width}} = \frac{\text{length} + \text{width}}{\text{length}}$

In this Stage, the focus is on examining situations where the data yields a constant rate of change. It is possible that some practical situations may yield a variable rate of change. This is the focus in Stage 5.3.

It is the usual practice in Mathematics to place the independent variable on the horizontal axis and the dependent variable on the vertical axis. This is not always the case in other subjects, eg Economics.

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Number and Algebra
Proportion

Language

In understanding rates, students may benefit from being explicitly taught that 'per' means $\div$, eg 150 kilometres travelled in 2 hours as a rate in kilometres per hour, can be calculated using kilometres $\div$ hours.

Students should be encouraged to write a few key words on the left-hand side of the equals sign to identify what is being found in each step in their working, and conclude with a statement in words when solving ratio/rate problems. For example:

Divide \$400 in the ratio 2 : 3

$$\begin{aligned}
\text{Total parts} &= 2 + 3 \\
&= 5 \\
\text{1 part} &= \$400 \div 5 \\
&= \$80 \\
\text{2 parts} &= \$80 \times 2 \\
&= \$160 \\
\text{3 parts} &= \$80 \times 3 \\
&= \$240
\end{aligned}$$

$\therefore$ The parts are \$160 and \$240 respectively.

When describing travel graphs, supply a modelled story and graph first and/or jointly construct a story before independent work is required. When describing stories students can use present tense, 'The man travels…' or past tense 'The man travelled…'.

Mathematics • Stage 4

Number and Algebra

Algebraic Techniques 1

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- recognises and explains mathematical relationships using reasoning — MA4-3WM
- generalises number properties to operate with algebraic expressions — MA4-8NA

Related Life Skills outcomes: *MALS-1WM, MALS-3WM, MALS-15NA*

Students:

Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)

- use letters to represent numbers and develop the concept that pronumerals (letters) are used to represent numerical values [N]
- use concrete materials to model the following:
  - expressions that involve a pronumeral, and a pronumeral plus a constant, eg a, $a + 1$ [N]
  - expressions that involve a pronumeral multiplied by a constant, eg $2a$, $3a$ [N]
  - sums and products, eg $2a + 1$, $2(a + 1)$ [N]
  - equivalent expressions, such as $x+x+y+y+y=2x+2y+y=2(x+y)+y$ [N]
  - and to assist with simplifying expressions [N],
    eg $\begin{aligned}(a+2)+(2a+3)&=(a+2a)+(2+3)\\&=3a+5\end{aligned}$ [N]
- recognise and use equivalent algebraic expressions [L],
  eg $y+y+y+y=4y$
  $w\times w=w^2$
  $a\times b=ab$
  $a\div b=\dfrac{a}{b}$
- use algebraic symbols to represent mathematical operations written in words and vice versa, eg the product of x and y is xy; $x + y$ is the sum of x and y [L, N]

Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

- recognise like terms and add and subtract them to simplify algebraic expressions,
  eg $2n + 4m + n = 4m + 3n$
  - ‣ determine equivalence of algebraic expressions by substituting a number for the pronumeral (Reasoning) [N]
  - ‣ connect algebra with the commutative and associative properties of arithmetic to determine that $a+b=b+a$ and $(a+b)+c=a+(b+c)$ (Communicating) [CCT]
- recognise the role of grouping symbols and the different meanings of expressions, such as $2a + 1$ and $2(a+1)$ [CCT]
- translate from everyday language to algebraic language and vice versa [L, N]

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Number and Algebra
Algebraic Techniques 1

- ‣ use algebraic symbols to represent simple situations described in words, eg write an expression for the number of cents in x dollars (Communicating) [L, N]
- ‣ interpret statements involving algebraic symbols in other contexts, eg cell references when creating and formatting spreadsheets (Communicating) [L, N, ICT]

- simplify algebraic expressions that involve multiplication and division, eg $12a \div 3$, $4x \times 3$, $2ab \times 3a$, $\frac{8a}{2}$, $\frac{2a}{8}$, $\frac{12a}{9}$
  - ‣ recognise the equivalence of algebraic expressions involving multiplication, eg $3bc = 3cb$ (Communicating) [CCT]
  - ‣ connect algebra with the commutative and associative properties of arithmetic to determine that $a \times b = b \times a$ and $(a \times b) \times c = a \times (b \times c)$ (Communicating) [CCT]
  - ‣ recognise whether particular algebraic expressions involving division are equivalent or not, eg $a \div bc$ is equivalent to $\frac{a}{bc}$ and $a \div (b \times c)$ but is not equivalent to $a \div b \times c$ or $\frac{a}{b} \times c$ (Communicating) [CCT]

Simplify algebraic expressions involving the four operations (ACMNA192)

- simplify a range of algebraic expressions including those involving mixed operations
  - ‣ apply the order of operations to simplify algebraic expressions (Problem Solving) [N]

Background information

It is important to develop an understanding of the use of pronumerals (letters) as algebraic symbols to represent different numerical values.

The recommended approach is to spend time over the conventions for the use of algebraic symbols for first-degree expressions and to situate the translation of generalisations from words to symbols as an application of students' knowledge of the symbol system rather than as an introduction to the symbol system.

The recommended steps for moving into symbolic algebra are:

- – the variable notion, associating letters with a variety of different numerical values
- – symbolism for a pronumeral plus a constant
- – symbolism for a pronumeral times a constant
- – symbolism for sums and products.

Thus if $a = 6$, $a + a = 6 + 6$ but $2a = 2 \times 6$ and not 26.

To gain an understanding of algebra, students must be introduced to the concepts of pronumerals, expressions, unknowns, equations, patterns, relationships and graphs in a wide variety of contexts. For each successive context, these ideas need to be redeveloped. Students need gradual exposure to abstract ideas as they begin to relate algebraic terms to real situations.

It is suggested that the introduction of representation through the use of algebraic symbols precede Linear Relationships Stage 4, since this topic presumes students are able to manipulate algebraic symbols and will use them to generalise patterns.

Mathematics • Stage 4

Number and Algebra

Algebraic Techniques 1

Language

For the introduction of algebra in Stage 4, the term ‘pronumeral’, rather than ‘variable’ is preferred when referring to unknown numbers. In an algebraic expression such as $2x + 5$, x can take any value (ie x is variable and a pronumeral). However, in an equation such as $2x + 5 = 11$, x represents one particular value (ie x is not a variable but it is a pronumeral). In equations such as $x + y = 11$, x and y could take any values that sum to 10 (x and y are variables and pronumerals).

‘Equivalent’ is the adjective (describing word) for ‘equal’; although, ‘equal’ can also be used as an adjective, ie ‘equivalent expressions’ or ‘equal expressions’.

Some students may confuse the order with which terms/numbers are to be used when a question is expressed in words. This is particularly apparent for word problems that involve subtraction or division to obtain the result, eg ‘$5x$ less than x’, ‘take $5x$ from x’ both require the order of the terms to be reversed to $x – 5x$ in the solution.

Students need to be familiar with the terms sum, difference, product and quotient as words used to describe the results of adding, subtracting, multiplying and dividing respectively.

Mathematics • Stage 4

Number and Algebra

Algebraic Techniques 2

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- generalises number properties to operate with algebraic expressions MA4-8NA

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-15NA*

Students:

Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)

- substitute into algebraic expressions and evaluate the result
  - calculate and compare the value of x^2 for values of x with the same magnitude but opposite sign (Reasoning) [CCT]
  - determine and justify whether a simplified expression is correct by substituting numbers for pronumerals (Communicating, Problem Solving, Reasoning) [CCT]
- generate a number pattern from an algebraic expression [N]

eg

x	1	2	3	4	5	6	…	10	…	100
$x+3$	4	5	6							

- replace written statements describing patterns with equations written in algebraic symbols, eg 'you add five to the position in the pattern to get the term in the pattern' could be replaced with '$y = x + 5$' [L, N]
  - determine whether a particular pattern can be described using algebraic symbols (Problem Solving) [CCT]

Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190)

- expand algebraic expressions by removing grouping symbols, eg $3(a+2)=3a+6$, $-5(x+2)=-5x-10$, $a(a+b)=a^2+ab$
  - connect algebra with the distributive property of arithmetic to determine that $a(b+c)=ab+ac$ (Communicating) [CCT]

Factorise algebraic expressions by identifying numerical factors (ACMNA191)

- factorise a single algebraic term, eg $6ab=3\times 2\times a\times b$
- factorise algebraic expressions by finding a common factor, eg $6a+12=6(a+2)$, $-4t-12=-4(t+3)$
  - check expansions and factorisations by performing the reverse process (Reasoning) [CCT]

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Number and Algebra

Algebraic Techniques 2

Factorise algebraic expressions by identifying algebraic factors

- factorise algebraic expressions by finding a common algebraic factor,
  eg $x^2 - 5x = x(x-5)$, $5ab + 10a = 5a(b+2)$

Background information

When evaluating expressions, there should be an explicit direction to replace the pronumeral by a number to ensure full understanding of notation occurs.

Language

The meaning of the imperatives ‘expand’, ‘remove the grouping symbols’, ‘factorise’ and the expressions ‘the expansion of ’ and ‘the factorisation of’ should be made explicit to students.

Mathematics • Stage 4

Number and Algebra
Indices

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- applies appropriate mathematical techniques to solve problems — MA4-2WM
- recognises and explains mathematical relationships using reasoning — MA4-3WM
- operates with positive-integer and zero indices of numerical bases — MA4-9NA

Students:

Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)

- describe numbers written in index form using terms such as base, power, index, exponent [L]
- use index notation to express powers of numbers (positive indices only), eg $8 = 2^3$ [L]
- evaluate numbers expressed as powers of whole numbers, eg $2^3 = 8$, $(-2)^3 = -8$
  - investigate and generalise the effect of raising a negative number to an odd or even power on the sign of the result (Communicating) [N, CCT]
- apply the order of operations to evaluate expressions involving indices with and without a calculator, eg $3^2 + 4^2$, $4^3 + 2 \times 5^2$ [N, ICT]
- determine and apply tests of divisibility [CCT]
  - verify the various tests of divisibility using a calculator (Problem Solving)
  - apply tests of divisibility mentally as an aid to calculation (Problem Solving) [CCT]
- express a number as a product of its prime factors, using index notation where appropriate
  - find the highest common factor of large numbers by first expressing the numbers as products of prime factors (Communicating, Problem Solving) [CCT]

Investigate and use square roots of perfect square numbers (ACMNA150)

- use the notation for square root $\left(\sqrt{\ }\right)$ and cube root $\left(\sqrt[3]{\ }\right)$ [L]
- recognise the link between squares and square roots and cubes and cube roots, eg $2^3 = 8$ and $\sqrt[3]{8} = 2$ [CCT]
- determine through numerical examples that:
  $(ab)^2 = a^2b^2$, eg $(2 \times 3)^2 = 2^2 \times 3^2$
  $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, eg $\sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4}$
- express a number as a product of its prime factors to determine whether its square and/or cube root is an integer [N]
- find square roots and cube roots of any non-square whole number using a calculator, after first estimating [N]
  - determine the two integers between which the square root of a non-square whole number lies (Reasoning) [N, CCT]

Mathematics • Stage 4

Number and Algebra
Indices

- ‣ determine whether it is more appropriate to use mental strategies or a calculator to find the square root of a given number (Problem Solving) [N, CCT]

- apply the order of operations to evaluate expressions involving square and cube roots with and without a calculator, eg $\sqrt{16+9}$, $\sqrt{16}+9$, $\dfrac{\sqrt{100-64}}{9}$, $\sqrt{\dfrac{100-64}{9}}$ [N]

  - ‣ explain the difference between similar pairs of numerical expressions, eg 'Is $\sqrt{36}+\sqrt{64}$ equivalent to $\sqrt{36+64}$?' (Communicating, Reasoning) [N, CCT]

Use index notation with numbers to establish the index laws with positive integral indices and the zero index (ACMNA182)

- develop index laws arithmetically by expressing each term in expanded form [L],
  eg $3^2 \times 3^4 = (3\times 3)\times(3\times 3\times 3\times 3) = 3^{2+4} = 3^6$
  $3^5 \div 3^2 = \dfrac{3\times 3\times 3\times 3\times 3}{3\times 3} = 3^{5-2} = 3^3$
  $\left(3^2\right)^4 = (3\times 3)\times(3\times 3)\times(3\times 3)\times(3\times 3) = 3^{2\times 4} = 3^8$

  - ‣ verify the index laws using a calculator, eg use a calculator to compare the values of $\left(3^4\right)^2$ and 3^8 (Reasoning)
  - ‣ explain the incorrect use of index laws, eg why $3^2 \times 3^4 \neq 9^6$ (Communicating, Reasoning) [L, CCT]

- establish the meaning of the zero index, eg by patterns [CCT]

3^5	3^4	3^3	3^2	3^1	3^0
243	81	27	9	3	1

  - ‣ verify the zero index law using a calculator (Reasoning)

- use index laws to simplify expressions with numerical bases, eg $5^2 \times 5^4 \times 5 = 5^7$

Background information

Students have not used indices prior to Stage 4 and so the meaning and use of index notation will need to be made explicit. However, students should have some experience in multiplying more than two numbers together at the same time from Stage 3.

In Stage 3, students use the notion of factorising a number as a mental strategy for multiplication. Teachers may like to make an explicit link to this in the introduction of the prime factorisation of a number in Stage 4, eg in Stage 3, 18×5 would have been calculated as $9 \times 2 \times 5$; in Stage 4, 18×5 is factorised as a product of primes as $3 \times 3 \times 2 \times 5$.

The square root sign signifies a positive number (or zero). Thus $\sqrt{9} = 3$ (only). However, the two numbers whose square is 9 are $\sqrt{9}$ and $-\sqrt{9}$ ie 3 and –3.

Mathematics • Stage 4

Number and Algebra
Indices

Language

Students need to be able to express the concept of divisibility in different ways such as ‘12 is divisible by 2’, 2 divides (evenly) into 12’, ‘2 goes into 12 (evenly)’.

A ‘product of prime factors’ can also be referred to as ‘a product of primes’.

Students are introduced to indices in stage 4. The different expressions used when referring to indices should be modelled by teachers. Initially teachers should use fuller expressions before shortening them, eg 2^4 should be expressed as ‘2 raised to the power of 4’, before ‘2 to the power of 4’ and finally ‘2 to the 4’. Students are expected to use the words ‘squared’ and ‘cubed’ when saying expressions containing indices of 2 and 3 respectively, eg 4^2 is ‘four squared’, 4^3 is ‘four cubed’.

Words such as ‘product’, ‘prime’, ‘power’, ‘base’ and ‘index’ have different meanings outside of mathematics. Words such as ‘base’, ‘square’ and ‘cube’ also have different meanings within mathematics, eg ‘the base of the triangle’ vs ‘the base of 3^2 is 3’; ‘the square of length 3 cm’ vs ‘the square of 3’. The relationship between the use of the words ‘square’ and ‘cube’ in indices and the use of the same words in geometry may assist some students with their understanding.

Mathematics • Stage 4

Number and Algebra
Equations

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- applies appropriate mathematical techniques to solve problems — MA4-2WM
- recognises and explains mathematical relationships using reasoning — MA4-3WM
- uses algebraic techniques to solve simple linear equations — MA4-10NA

Students:

Solve simple linear equations (ACMNA179)

- distinguish between algebraic expressions where pronumerals are used as variables, and equations where pronumerals are used as unknowns [CCT]
- solve simple linear equations using concrete materials, such as the balance model or cups and counters, stressing the notion of performing the same operation on both sides of an equation [N]
- solve linear equations using algebraic methods that involve one or two steps in the solution process and may have non-integer solutions,

  eg $x-7=15$ $\quad \frac{x}{7}=5$

  $2x-7=15$, $\quad \frac{2x}{7}=5$

  $7-2x=15$

  - compare and contrast strategies to solve a variety of linear equations (Communicating, Reasoning) [CCT]
  - generate equations with a given solution, eg find equations that have the solution $x = 5$ (Problem Solving) [N]

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194)

- solve linear equations using algebraic methods that involve at least two steps in the solution process and may have non-integer solutions,

  eg $3x+4=13$, $\quad 3x+4=x-8$, $\quad \frac{x}{3}+5=10$, $\quad \frac{x+5}{3}=10$

  $3(x+4)=13$ $\quad 3x+4=8-x$

- check solutions to equations by substituting [CCT]

Background information

The solution of simple equations can be introduced using a variety of models. Such models include: using a two-pan balance with objects such as centicubes and a wrapped ‘unknown’, or using some objects hidden in a container as an ‘unknown’ to produce a number sentence.

Mathematics • Stage 4

Number and Algebra
Equations

Language

Describing the steps in the solution of equations provides students with the opportunity to practise using mathematical imperatives in context, eg ‘add 5 to both sides’, ‘increase both sides by 5’, ‘subtract 3 from both sides’, ‘take 3 from both sides’, ‘decrease both sides by 3’, ‘reduce both sides by 3’, ‘multiply both sides by 2’ and ‘divide both sides by 2’.

Mathematics • Stage 4

Number and Algebra
Linear Relationships

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- recognises and explains mathematical relationships using reasoning — MA4-3WM
- creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane — MA4-11NA

Related Life Skills outcomes: *MALS-1WM, MALS-3WM, MALS-29MG, MALS-30MG, MALS-31MG*

Students:

Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)

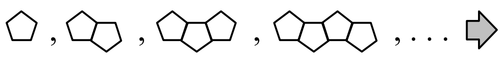
- read, plot and name ordered pairs on the Cartesian plane including those with values that are not whole numbers [N]

Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181)

- use the notation P' to name the 'image' resulting from a transformation of a point P on the Cartesian plane [L]
- plot and determine the coordinates for P' resulting from translating P one or more times
- plot and determine the coordinates for P' resulting from reflecting P in either the x- or y-axis
  - investigate and describe the relationship between the coordinates of P and P' following a reflection in the x- or y-axis, eg if P is reflected in the x-axis, P' has the same x-coordinate, and its y-coordinate has the same magnitude but opposite sign (Communicating) [CCT]
  - recognise that a translation can produce the same result as a single reflection and vice versa (Reasoning) [CCT]
- plot and determine the coordinates for P' resulting from rotating P by a multiple of 90° about the origin
  - investigate and describe the relationship between the coordinates of P and P' following a rotation of 180° about the origin, eg if P is rotated 180° about the origin, the x- and y-coordinates of P' have the same magnitude as P but are opposite in sign (Communicating) [CCT]
  - recognise that a combination of translations and/or reflections can produce the same result as a single rotation and that a combination of rotations can produce the same result as a single translation and/or reflection (Reasoning) [CCT]

Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193)

- use objects to build a geometric pattern, record the results in a table of values, describe the pattern in words and algebraic symbols and represent the relationship on a number grid [N],

eg 

number of pentagons (p)	1	2	3	4	…
number of matches (m)	5	9	13	17	…

  - check pattern descriptions by substituting further values (Reasoning) [CCT]

Mathematics • Stage 4

Number and Algebra
Linear Relationships

- ‣ describe the pattern formed by plotting points from a table and suggest another set of points that might form the same pattern (Communicating, Reasoning) [CCT]
- ‣ describe what has been learnt from creating patterns, making connections with number facts and number properties (Communicating) [CCT]

- recognise a given number pattern (including decreasing patterns), complete a table of values, describe the pattern in words and algebraic symbols, and represent the relationship on a number grid [L]
  - ‣ generate a variety of number patterns that increase or decrease and record them in more than one way (Communicating) [N]
  - ‣ explain why a particular relationship or rule for a given pattern is better than another (Communicating, Reasoning) [CCT]
  - ‣ distinguish between graphs that represent an increasing number pattern and those that represent a decreasing number pattern (Communicating, Reasoning) [CCT]
  - ‣ determine whether a particular number pattern can be described using algebraic symbols (Communicating) [CCT]
- use a rule generated from a pattern to calculate the corresponding value for a larger number [N]
- form a table of values for a linear relationship by substituting a set of appropriate values for either of the pronumerals and graph the number pairs on the Cartesian plane, eg given $y = 3x + 1$, form a table of values using $x = 0$, 1 and 2 and then graph the number pairs on a Cartesian plane with an appropriate scale [N]
  - ‣ explain why 0, 1 and 2 are frequently chosen as x-values in a table of values (Communicating, Reasoning) [CCT]
- extend the line joining a set of points to show that there is an infinite number of ordered pairs that satisfy a given linear relationship [N]
  - ‣ interpret the meaning of the continuous line joining the points that satisfy a given number pattern (Communicating, Reasoning) [CCT]
  - ‣ read values from the graph of a linear relationship to demonstrate that there are many points on the line (Communicating) [CCT]
- derive a rule for a set of points that has been graphed on a Cartesian plane [N]
- graph more than one line on the same set of axes using ICT and compare the graphs to determine similarities and differences, eg parallel, pass through the same point [N, ICT, CCT]
  - ‣ identify similarities and differences between groups of linear relationships (Reasoning) [CCT], eg $y = 3x$, $y = 3x + 2$, $y = 3x - 2$
    $y = x$, $y = 2x$, $y = 3x$
    $y = -x$, $y = x$
  - ‣ determine which term of the rule affects the slope of a graph, making it increase or decrease (Reasoning) [CCT]
- use ICT to graph linear and simple non-linear relationships such as $y = x^2$ [ICT]
  - ‣ recognise and explain that not all patterns form a linear relationship (Communicating) [CCT]
  - ‣ determine and explain differences between equations that represent linear relationships and those that represent non-linear relationships (Communicating) [L, CCT]

Mathematics • Stage 4

Number and Algebra
Linear Relationships

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194)

- recognise that each point on the graph of a linear relationship represents a solution to a particular linear equation [N]
- use graphs of linear relationships to solve a corresponding linear equation with or without ICT, eg use the graph of $y = 2x + 3$ to find the solution of the equation $2x + 3 = 11$ [N, ICT]
- graph two intersecting lines on the same set of axes and read off the point of intersection [N]
  - explain the significance of the point of intersection of two lines in relation to it being the only solution that satisfies both equations (Communicating, Reasoning) [L, CCT]

Background information

When describing number patterns algebraically, it is important that students develop an understanding of the use of letters as algebraic symbols for numbers of objects rather than for the objects themselves.

In 'Linear Relationships', linear refers to straight lines.

Language

In Stage 3, students use 'position in pattern' and 'value of term' when describing a patter from a table of values, eg the value of the term is three times the position in the pattern.

Students will need to become familiar with and be able to use new terms including coefficient, constant term and intercept.

Mathematics • Stage 4

Measurement and Geometry
Length

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- calculates the perimeter of plane shapes and the circumference of circles MA4-12MG

Related Life Skills outcomes: *MALS-1WM*, *MALS-2WM*, *MALS-22MG*, *MALS-23MG*

Students:

Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (ACMMG196)

- find the perimeter of a range of plane shapes including parallelograms, rhombuses, kites and simple composite figures [N]
  - compare perimeters of rectangles with the same area (Problem Solving) [CCT]
- solve problems involving the perimeter of plane shapes, eg find the dimensions of a rectangle given its perimeter and the length of one other side [N, CCT]

Investigate the concept of irrational numbers, including π (ACMNA186)

- identify and name parts of a circle and related lines, including arc, tangent, chord, sector and segment [L]
- demonstrate by practical means that the ratio of the circumference to the diameter of a circle is constant, eg measure and compare the diameter and circumference of various cylinders or use dynamic geometry software to measure circumferences and diameters [N, ICT, CCT]
- define the number π as the ratio of the circumference to the diameter of any circle [L]
  - compare the various approximations for π used throughout the ages and investigate the concept of irrational numbers (Communicating) [CCT]
  - recognise that the symbol π is used to represent a constant numerical value (Communicating) [N]

Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197)

- develop and use the formulas to calculate the circumference of circles in terms of the radius r or diameter d: Circumference of circle = πd and Circumference of circle = $2\pi r$ [N, CCT]
  - use mental strategies to estimate the circumference of circles, using an approximate value of π such as 3 (Problem Solving) [N]
  - find the diameter and/or radius of a circle given its circumference (Problem Solving) [N]
- find the perimeter of quadrants and semi-circles [N]
- calculate the perimeter of simple composite figures consisting of two shapes including quadrants and semicircles [N]
- calculate the perimeter of sectors [N]
- solve problems involving circles and parts of circles, giving an exact answer in terms of π and an approximate answer using the calculator's approximation for π [N]

Mathematics • Stage 4

Measurement and Geometry

Length

Background information

Graphing of the relationship between the length of a rectangle with a constant perimeter and possible areas of the rectangle links to non-linear graphs.

Students should develop a sense of the levels of accuracy that are appropriate to a particular situation, eg the length of a bridge may be measured in metres to estimate a quantity of paint needed but would need to be measured far more accurately for engineering work.

The number π is known to be irrational (not a fraction) and also transcendental (not the solution of any polynomial equation with integer coefficients). At this stage, students only need to know that the digits in its decimal expansion do not repeat (all this means is that it is not a fraction), and in fact have no known pattern.

Pi (π) is the Greek letter equivalent to ‘p’, and is the first letter of the Greek word *perimetron,* meaning perimeter. In 1737, Euler used the symbol for pi to represent the ratio of the circumference to the diameter of a circle.

Language

Perimeter comes from the Greek word *perimetron,* meaning perimeter.

The names for some parts of the circle (centre, radius, diameter, circumference, sector, semi-circle and quadrant) are first introduced in Stage 3. The terms arc, tangent, chord and segment are first introduced in Stage 4.

Some students may find the use of the terms length/long, breadth, width/wide and height/high difficult. Teachers should model the use of these terms in sentences when describing diagrams, both verbally and in written form. Students should be encouraged to speak about, listen to, read about and write about the dimensions of given shapes using various combinations of these words, eg ‘The length of this rectangle is 3 metres and the width is 4 metres’, ‘The rectangle is 3 metres long and 4 metres wide’. Students may also benefit from drawing and labelling a shape given a description of its features in words, eg ‘The base of an isosceles triangle is 6 metres long and its perimeter is 20 metres. Draw the triangle and label the measurements of the three sides’.

Some students may need to be taught explicitly that the phrase ‘dimensions of a rectangle’ refers to the length and breadth of a rectangle.

Mathematics • Stage 4

Measurement and Geometry
Area

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- uses formulasÏ to calculate the area of quadrilaterals and circles, and converts between units of area MA4-13MG

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-26MG*

Students:

Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195)

- choose an appropriate unit to measure the areas of different shapes and surfaces, eg floor space, fields [CCT]
    - use the areas of familiar surfaces to assist with the estimation of larger areas (Problem Solving) [N, CCT]
- convert between metric units of area: 1 cm^2 = 100 mm^2, 1 m^2 = 1000 000 mm^2, 1 ha = 10 000 m^2, 1 km^2 = 1 000 000 m^2 = 100 ha

Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)

- develop and use the formulas for the area of squares and rectangles:
  Area of rectangle = lb where l is the length and b is the breadth of the rectangle
  Area of square = s^2 where s is the side length of the square [N, CCT]
    - explain the relationship that multiplying, dividing, squaring and factoring have with the areas of squares and rectangles with integer side lengths (Communicating) [N, CCT]
    - explain the relationship between the formulas for the areas of squares and rectangles (Communicating) [CCT]
    - compare areas of rectangles with the same perimeter (Problem Solving) [N, CCT]
- develop, with or without digital technologies, and use the formulas for the area of parallelograms and triangles:
  Area of parallelogram = bh where b is the length of the base and h is the perpendicular height
  Area of triangle $= \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height [N, ICT, CCT]
    - identify the perpendicular height of triangles and parallelograms in different orientations (Reasoning) [CCT]
- find the areas of simple composite figures that may be dissected into squares, rectangles, parallelograms and triangles [N]

Mathematics • Stage 4

Measurement and Geometry
Area

Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (ACMMG196)

- develop, with or without digital technologies, and use the formulas for the areas of a kite or rhombus:

  Area of rhombus/kite $= \frac{1}{2}xy$ where x and y are the lengths of the diagonals [N, ICT, CCT]

- develop and use the formula to find the area of a trapezium:

  Area of trapezium $= \frac{1}{2}h\left(a+b\right)$ where h is the perpendicular height and a and b are the lengths of the parallel sides [N, CCT]

  - identify the perpendicular height of various trapeziums in different orientations (Reasoning) [CCT]

- select and use the appropriate formula to calculate the area of any of the special quadrilaterals [N, CCT]
- solve problems relating to areas of triangles and quadrilaterals [N]
  - convert between units of length and area as appropriate when solving area problems (Problem Solving) [N]

Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197)

- develop, with or without digital technologies, the formula to calculate the area of a circle: Area of circle = πr^2 where r is the length of the radius [N, ICT, CCT]
  - find radii of circles given their circumference or area (Problem Solving) [N]
- calculate the area of quadrants, semicircles and sectors [N]
- solve problems involving circles or parts of circles, giving an exact answer in terms of π and an approximate answer using the calculator's approximation for π [N]

Background information

Dynamic geometry software or prepared applets are useful tools for developing the formulas for finding the areas of two-dimensional shapes.

Area formulas for the triangle and parallelogram need to be developed from the area of a rectangle. Dynamic geometry software or applets may be useful in demonstrating to students how the formula $A = \frac{1}{2}bh$ is applied in obtuse-angled triangles where the given perpendicular height lies outside the triangle.

The formula for finding the area of a rhombus or kite depends upon the fact that the diagonals are perpendicular, and so is linked with the geometry of special quadrilaterals. The formula applies to any quadrilateral in which the diagonals are perpendicular. Students should also be aware that the rhombus can be treated as a parallelogram and the area found using the formula $A = bh$.

The formula for finding the area of a circle may be established by using one or both of the following dissections:

- cut the circle into a large number of sectors, and arrange them alternately point-up and point-

Mathematics • Stage 4

Measurement and Geometry
Area

down to form a rectangle with height r and base length πr
- inscribe a number of congruent triangles in a circle, all with a vertex at the centre, and show that the area of the inscribed polygon is half the length of the perimeter times the perpendicular height.

Students should be made aware that the perpendicular height of a triangle is the shortest distance from the base to the opposite angle. Likewise, students may need to be explicitly taught that the shortest distance between parallel sides of a quadrilateral is the perpendicular distance between the parallel sides.

Finding the areas of rectangles and squares with integer side lengths is an important link between geometry and multiplication, division, factoring and squares. Factoring a number into the product of two numbers is equivalent to forming a rectangle with these side lengths, and squaring is equivalent to forming a square.

Language

Teachers should reinforce with students the use of the term 'perpendicular height' rather than simply 'height' when referring to this attribute of a triangle. Students may also benefit from drawing and labelling a triangle when given a description of its features in words.

The use of the term 'respectively' in measurement word problems should be modelled and the importance of the order of the words explained, eg in the sentence 'the perpendicular height and base of a triangle are 5 metres and 8 metres respectively', the first attribute (perpendicular height) mentioned relates directly to the first measurement (5 metres) and so on.

The abbreviation m^2 is read 'square metre(s)' and not 'metre(s) squared' or 'metre(s) square'.
The abbreviation cm^2 is read 'square centimetre(s)' and not 'centimetre(s) squared' or 'centimetre(s) square'.

Mathematics • Stage 4

Measurement and Geometry
Volume

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- applies appropriate mathematical techniques to solve problems — MA4-2WM
- uses formulas to calculate the volume of prisms and cylinders, and converts between units of volume — MA4-14MG

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-25MG, MALS-27MG MALS-28MG*

Students:

Draw different views of prisms and solids formed from combinations of prisms (ACMMG161)

- represent three-dimensional objects in two dimensions from different views [CCT]
- identify and draw the cross-section of a prism [L, CCT]
- visualise, construct and draw various prisms from a given cross-sectional diagram [CCT]
- determine if a solid has a uniform cross-section [CCT]
  - ‣ recognise solids with uniform and non-uniform cross-sections (Reasoning) [CCT]

Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195)

- choose an appropriate unit to measure volume or capacity of different objects, eg swimming pools, household containers, dams [L, N, SE]
  - ‣ use the capacity of familiar containers to assist with estimation of larger capacities (Reasoning) [N, CCT]
- convert between units of volume and capacity, 1 cm^3 = 1000 mm^3, 1 L = 1000 mL = 1000 cm^3, 1 m^3 = 1000 L = 1kL, 1000 kL = 1 ML

Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198)

- develop the formula for volume of prisms by considering the number and volume of layers of identical shape:
  Volume of prism = base area × height leading to $V = Ah$ [N, CCT]
- calculate the volume of a prism given its perpendicular height and the area of its cross-section [N]
- calculate the volume of prisms with cross-sections that are rectangular and triangular [N]
- solve practical problems involving volume and capacity of right prisms [N]

Calculate the surface area and volume of cylinders and solve related problems (ACMMG217)

- develop and use the formula to find the volume of cylinders:
  Volume of cylinder = $\pi r^2 h$ where r is the length of the radius of the base and h is the perpendicular height [N, CCT]
  - ‣ recognise and explain the relationship between the volume formulas for cylinders and prisms (Communicating) [N, CCT]

Mathematics • Stage 4

Measurement and Geometry
Volume

- solve problems involving volume and capacity of right prisms and cylinders, eg calculate the capacity of a cylindrical can of drink or a water tank [N, SE]

Background information

When developing the volume formula, students require an understanding of the idea of a uniform cross-section and should visualise, for example, stacking unit cubes layer by layer into a rectangular prism, or stacking planks into a pile.

Oblique prisms, cylinders, pyramids and cones are those that are not right. The focus here is on right prisms and cylinders, although the formulas for volume also apply to oblique prisms and cylinders provided the perpendicular height is used. In a right prism, the base and top are perpendicular to the other faces. In a right pyramid or cone, the base has a centre of rotation, and the interval joining that centre to the apex is perpendicular to the base (and thus is its axis of rotation).

The volumes of rectangular prisms and cubes are linked with multiplication, division, factorisation and powers. Factoring a number into the product of three numbers is equivalent to forming a rectangular prism with these side lengths, and to forming a cube if the numbers are all equal. Some students may be interested in knowing what fourth and higher powers, and the product of four or more numbers, correspond to.

The abbreviation for megalitres is ML. Students will need to be careful not to confuse this with the abbreviation mL used for millilitres.

Language

The word ‘base’ may cause confusion for some students. ‘Base’ in two-dimensional shapes is linear, whereas in three-dimensional objects ‘base’ refers to a face/area. In everyday language, the word ‘base’ is used to refer to that part of an object closest to the ground. In the mathematics of three-dimensional objects, the term ‘base’ is used to describe the face by which a prism or pyramid is named, even though it may not be the face closest to the ground. In Stage 3, students were introduced to the naming of prisms and pyramids according to the shape of their base. In Stage 4, students should be encouraged to make the connection that the name of a particular prism refers not only to the shape of its base, but also to the shape of its uniform cross-section.

Students should be made aware that a cube is a special square prism that has six congruent faces.

The abbreviation m^3 is read ‘cubic metre(s)’ and not ‘metre(s) cubed’ or ‘metre(s) cube’.
The abbreviation cm^3 is read ‘cubic centimetre(s)’ and not ‘centimetre(s) cubed’ or ‘centimetre(s) cube’.

Mathematics • Stage 4

Measurement and Geometry
Time

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- applies appropriate mathematical techniques to solve problems — MA4-2WM
- performs calculations of time that involve mixed units, and interprets time zones — MA4-15MG

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-17MG, MALS-18MG, MALS-19MG, MALS-20MG, MALS-21MG*

Students:

Solve problems involving duration, including using 12- and 24-hour time within a single time zone (ACMMG199)

- add and subtract time mentally using bridging strategies, eg from 2:45 to 3:00 is 15 minutes and from 3:00 to 5:00 is 2 hours, so the time from 2:45 until 5:00 is 15 minutes + 2 hours = 2 hours 15 minutes [N, CCT]
- add and subtract time with a calculator using the ‘degrees, minutes, seconds’ button
- round calculator answers to the nearest minute or hour [N]
- interpret calculator displays for time calculations, eg 2.25 on a calculator display for time means $2\frac{1}{4}$ hours [N]
- solve problems involving calculations with mixed time units, eg ‘How old is a person today if he/she was born on 30/6/1999?’ [N]

Solve problems involving international time zones

- compare times and calculate time differences between major cities of the world, eg ‘Given that London is 10 hours behind Sydney, what time is it in London when it is 6:00 pm in Sydney?’ [N, CCT]
  - interpret and use information related to international time zones from maps (Problem Solving) [N, CCT]
  - solve problems about international time relating to everyday life, eg determine whether a particular soccer game can be watched live on television during normal waking hours (Problem Solving) [N]

Background information

The calculation of time can be done on a scientific calculator and links with fractions and decimals.

It is believed that the Babylonians thought that the Earth took 360 days to travel around the Sun (last centuries BC). This is why there are 360° in one revolution and hence 90° in one right angle. There are 60 minutes (60') in one hour and 60 minutes in one degree.

Mathematics • Stage 4

Measurement and Geometry
Time

Language

The word ‘minute’ (meaning ‘small’) and minute (time measure), although pronounced differently, are really the same word. A minute (time) is a minute (small) part of one hour. A minute (angle) is a minute (small) part of a right angle.

Mathematics • Stage 4

Measurement and Geometry
Right-Angled Triangles (Pythagoras)

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- applies appropriate mathematical techniques to solve problems — MA4-2WM
- applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems — MA4-16MG

Students:

Investigate Pythagoras' theorem and its application to solving simple problems involving right angled triangles (ACMMG222)

- identify the hypotenuse as the longest side in any right-angled triangle and also as the side opposite the right angle [L]
- establish the relationship between the lengths of the sides of a right-angled triangle in practical ways, including using digital technologies [N, ICT, CCT]
    - describe the relationship between the sides of a right-angled triangle (Communicating) [CCT]
- use Pythagoras' theorem to find the length of sides in right-angled triangles [N]
- write answers to a specified or sensible level of accuracy, using an 'approximately equals' sign, ie $\approx$ or $\doteqdot$ [L, N, CCT]
- solve practical problems involving Pythagoras' theorem, approximating the answer as a decimal [N]
    - apply Pythagoras' theorem to solve problems involving perimeter and area (Problem Solving) [N]
- identify a Pythagorean triad as a set of three numbers such that the sum of the squares of the first two equals the square of the third [L, N]
- use the converse of Pythagoras' theorem to establish whether a triangle has a right angle [N, CCT]

Investigate the concept of irrational numbers, including π (ACMMG186)

- use technology to explore decimal approximations of surds
    - recognise that surds can be represented by decimals that are neither terminating nor have a repeating pattern (Communicating)
- solve practical problems involving Pythagoras' theorem, giving an exact answer as a surd, eg $\sqrt{5}$ [N]

Mathematics • Stage 4

Measurement and Geometry
Right-Angled Triangles (Pythagoras)

Background information

Students should gain an understanding of Pythagoras' theorem, rather than just being able to recite the formula. By dissecting and rearranging the squares, they will appreciate that the theorem is a statement of a relationship amongst the areas of squares.

Pythagoras' theorem becomes, in Stage 5, the formula for the circle on the Cartesian plane. These links can be developed later in the context of circle geometry and the trigonometry of the general angle.

Pythagoras' theorem was probably known many centuries before Pythagoras (c580–500 BC), to at least the Babylonians.

In the 1990s, Wiles finally proved a famous conjecture of Fermat (1601–1665), known as 'Fermat's last theorem', that states that if n is an integer greater than 2, then $a^n + b^n = c^n$ has no positive integer solutions.

Language

The meaning of 'exact' answer will need to be taught explicitly.

Students may find some of the terminology/vocabulary encountered in word problems involving Pythagoras' theorem difficult to interpret, eg 'foot of a ladder', 'inclined', 'guy wire', 'wire stay', vertical and horizontal. Teachers should provide students with a variety of word problems and explain such terms explicitly.

Mathematics • Stage 4

Measurement and Geometry
Angle Relationships

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown side lengths and angles MA4-17MG

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-27MG, MALS-28MG*

Students:

Use the language, notation and conventions of geometry

- label and name points, lines and intervals using capital letters [L]
- label the vertex and arms of an angle with capital letters [L]
- label and name angles using $\angle B$ and $\angle NPW$ notation [L]
- use the common conventions to indicate right angles and equal angles on diagrams [L]

Recognise the geometric properties of angles at a point

- use the words 'complementary' and 'supplementary' for angles adding to 90° and 180° respectively, and the associated terms 'complement' and 'supplement' [L, N]
- identify and name adjacent angles (two angles with a common vertex and a common arm), vertically opposite angles, straight angles and angles of complete revolution, embedded in a diagram [L, CCT]
  - recognise that adjacent angles can form right angles, straight angles and angles of complete revolution (Communicating, Reasoning) [CCT]

Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163)

- use the common conventions to indicate parallel lines on diagrams [L]
- identify and name a pair of parallel lines and a transversal [L]
  - identify parallel lines in the environment (Reasoning) [N]
- use common symbols for 'is parallel to' (||) and 'is perpendicular to' ($\perp$)
- identify, name and measure alternate angle pairs, corresponding angle pairs and co-interior angle pairs for two lines cut by a transversal [CCT]
  - use dynamic geometry software to investigate angle relationships formed by parallel lines and a transversal (Problem Solving, Reasoning) [N, ICT]
- recognise the equal and supplementary angles formed when a pair of parallel lines are cut by a transversal [CCT]

Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (ACMMG164)

Mathematics • Stage 4

Measurement and Geometry
Angle Relationships

- use angle properties to identify parallel lines [CCT]
  - explain why two lines are either parallel or not parallel, giving a reason (Communicating, Reasoning) [CCT]
- find the unknown angle in a diagram using angle relationships, including angles at a point and angles associated with parallel lines, giving a reason [N]

Background information

Students should give reasons when finding unknown angles. For some students formal, setting out could be introduced. For example,

$\angle PQR = 70°$ (corresponding angles, $PQ \| SR$).

Eratosthenes' calculation of the circumference of the Earth used parallel line results.

Dynamic geometry software or prepared applets are useful tools for investigating angle relationships; angles and lines can be dragged to new positions while angle measurements automatically update.

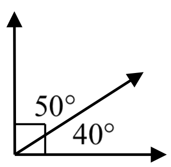
Students could explore the results about angles associated with parallel lines cut by a transversal by starting with corresponding angles – move one vertex and all four angles to the other vertex by a translation. The other two results then follow using vertically opposite angles and angles on a straight line. Alternatively, the equality of the alternate angles can be seen by rotation about the midpoint of the transversal.

Language

At this stage in geometry, students should write reasons without the use of abbreviations to assist them in learning new terminology, and in understanding and retaining geometrical concepts, eg 'co-interior angles are supplementary on parallel lines' or 'co-interior angles add up to $180°$ on parallel lines'.

Some students may find the use of the terms 'complementary'/'supplementary' (adjectives) and 'complement'/'supplement' (nouns) difficult. Teachers should model the use of these terms in sentences, both verbally and in written form,

eg



$50°$ and $40°$ are complementary angles. The complement of $50°$ is $40°$.

Mathematics • Stage 4

Measurement and Geometry
Properties of Geometrical Figures 1

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- identifies and uses angle relationships, including those related to transversals on sets of parallel lines MA4-18MG

Students:

Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165)

- label and name triangles (eg ΔABC) and quadrilaterals (eg *ABCD*) in text and on diagrams [L]
- use the common conventions to mark equal intervals on diagrams [L]
- recognise and classify types of triangles on the basis of their properties (acute-angled triangles, right-angled triangles, obtuse-angled triangles, scalene triangles, isosceles triangles and equilateral triangles) [L, CCT]
  - ‣ recognise that a given triangle may belong to more than one class (Reasoning) [CCT]
  - ‣ explain why the longest side of a triangle is always opposite the largest angle (Reasoning) [CCT]
  - ‣ explain why two sides of a triangle must together be longer than the third side (Communicating, Reasoning) [CCT]
  - ‣ sketch and label triangles from a worded or verbal description (Communicating) [N]
- distinguish between convex and non-convex quadrilaterals (the diagonals of a convex quadrilateral lie inside the figure) [L, CCT]
- investigate the properties of special quadrilaterals (trapeziums, kites, parallelograms, rectangles, squares and rhombuses). Properties to be investigated include: [L, CCT]
  - – the opposite sides are parallel
  - – the opposite sides are equal
  - – the adjacent sides are perpendicular
  - – the opposite angles are equal
  - – the diagonals are equal
  - – the diagonals bisect each other
  - – the diagonals bisect each other at right angles
  - – the diagonals bisect the angles of the quadrilateral
  - ‣ use techniques such as paper folding, measurement or dynamic geometry software to investigate the properties of quadrilaterals (Problem Solving, Reasoning) [ICT, CCT]
  - ‣ sketch and label quadrilaterals from a worded or verbal description (Communicating) [N]
- classify special quadrilaterals on the basis of their properties [L, CCT]
  - ‣ describe a quadrilateral in sufficient detail for it to be sketched (Communicating) [N, CCT]

Mathematics • Stage 4

Measurement and Geometry
Properties of Geometrical Figures 1

Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181)

- investigate the line symmetries and the order of rotational symmetry of polygons, including the special quadrilaterals [CCT]
  - determine if particular triangles and quadrilaterals have line and/or rotational symmetry (Problem Solving) [CCT]
- investigate the line symmetries and the rotational symmetry of circles and of diagrams involving circles, such as a sector or a circle with a marked chord or tangent [CCT]
- identify line and rotational symmetries in pictures and diagrams, eg artistic and cultural designs [CCT, AHC, A]

Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166)

- justify informally that the interior angle sum of a triangle is 180°, and that any exterior angle equals the sum of the two interior opposite angles [L, CCT]
  - use dynamic geometry software or other methods to investigate the interior angle sum of a triangle, and the relationship between any exterior angle and the sum of the two interior opposite angles (Reasoning) [ICT, CCT]
- use the angle sum of a triangle to establish that the angle sum of a quadrilateral is 360° [L, CCT]
- find the unknown angle in a triangle and/or a quadrilateral, giving a reason [N, CCT]

Use angle properties and relationships to solve problems with appropriate reasoning

- find the unknown side and/or angle in a diagram, using angle relationships and the properties of special triangles and quadrilaterals, giving a reason [N, CCT]
  - recognise special types of triangles and quadrilaterals embedded in composite figures or drawn in various orientations (Reasoning) [CCT]

Mathematics • Stage 4

Measurement and Geometry

Properties of Geometrical Figures 1

Background information

The properties of special quadrilaterals are important in Measurement and Geometry. For example, the perpendicularity of the diagonals of a rhombus and a kite allow a rectangle of twice the size to be constructed around them, leading to formulas for finding area.

At this stage, the treatment of triangles and quadrilaterals is still informal, with students consolidating their understandings of different triangles and quadrilaterals and being able to identify them from their properties.

Students who recognise class inclusivity and minimum requirements for definitions may address this Stage 4 content concurrently with content in Stage 5 Properties of Geometrical Figures, where properties of triangles and quadrilaterals are deduced from formal definitions.

Students should give reasons orally and in written form for their findings and answers. For some students, formal setting out could be introduced.

A range of examples of the various triangles and quadrilaterals should be given, including quadrilaterals containing a reflex angle and figures presented in different orientations.

Dynamic geometry software or prepared applets are useful tools for investigating properties of geometrical figures. Computer drawing programs enable students to prepare tessellation designs and to compare these with other designs such as those of M.C. Escher.

When using examples of Aboriginal rock carvings and art it is recommended, wherever possible, that local examples should be used. Consult with local Aboriginal communities and education consultants for such examples.

Language

At this stage, students should use full sentences to describe the properties of plane shapes, eg ‘The diagonals of a parallelogram bisect each other’. Students may not realise that in this context, the word ‘the’ implies ‘all’ and so this should be made explicit. Using the full name of the quadrilateral when describing its properties will assist students in remembering the geometrical properties of each particular shape.

Some students may use the word ‘corner’ rather than ‘vertex’, and the phrase ‘lines of symmetry’ rather than ‘axes of symmetry’, because the plurals of vertex and axis are more difficult: vertices and axes. Note: vertex/vertices has the same form as index/indices.

Scalene means ‘uneven’ (Greek word *skalenos*: uneven). Isosceles comes from the two Greek words *isos*: equals and *skelos*: leg; equilateral comes from the two Latin words *aequus*: equal and *latus*: side; and equiangular comes from *aequus* and another Latin word *angulus*: corner.

Mathematics • Stage 4

Measurement and Geometry
Properties of Geometrical Figures 2

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- identifies and uses angle relationships, including those related to transversals on sets of parallel lines MA4-18MG

Students:

Define congruence of plane shapes using transformations (ACMMG200)

- identify congruent figures by superimposing them through a combination of rotations, reflections and translations [L, CCT]
  - recognise congruent figures in tessellations, art and design work (Reasoning) [N, CCT]
  - recognise that area, length of matching sides and angle sizes are preserved in congruent figures (Reasoning) [N, CCT]
- match sides and angles of two congruent polygons [N, CCT]
- name the vertices in matching order when using the symbol ≡ in a congruence statement [L]
- determine the condition for two circles to be congruent (equal radii) [CCT]

Develop the conditions for congruence of triangles (ACMMG201)

- investigate the minimum conditions needed, and establish the four tests, for two triangles to be congruent: [L, CCT]
  - if three sides of one triangle are respectively equal to three sides of another triangle, then the two triangles are congruent (SSS rule)
  - if two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent (SAS rule)
  - if two angles and one side of one triangle are respectively equal to two angles and the matching side of another triangle, then the two triangles are congruent (AAS rule)
  - if the hypotenuse and a second side of one right-angled triangle are respectively equal to the hypotenuse and a second side of another right-angled triangle, then the two triangles are congruent (RHS rule)
  - use dynamic geometry software and/or geometrical instruments to investigate what information is needed to show that two triangles are congruent (Problem Solving) [ICT, CCT]
  - explain why the angle in the SAS test must be the included angle (Communicating, Reasoning) [CCT]
  - demonstrate that three pairs of equal matching angles is not a sufficient condition for congruence of triangles (Communicating, Reasoning) [CCT]
- use the congruency tests to identify a pair of congruent triangles from a selection of triangles or from triangles embedded in a diagram [CCT]

Mathematics • Stage 4

Measurement and Geometry

Properties of Geometrical Figures 2

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202)

- apply the properties of congruent triangles to find an unknown side and/or angle in a diagram, giving a reason [N, CCT]
- use transformations of congruent triangles to verify some of the properties of special quadrilaterals, including properties of the diagonals, eg the diagonals of a parallelogram bisect each other [CCT]

Background information

For some students, formal setting out of proofs of congruent triangles could be introduced.

Congruent figures are embedded in a variety of designs, eg tapa cloth, Aboriginal designs, Indonesian ikat designs, Islamic designs, designs used in ancient Egypt and Persia, window lattice, woven mats and baskets.

Dynamic geometry software or prepared applets are useful tools for investigating properties of congruent figures through transformations.

Language

The term ‘corresponding’ is often used in relation to congruent and similar figures to refer to angles or sides in the same position, but it also has a specific meaning when used to describe a pair of angles in relation to lines cut by a transversal. This syllabus has used ‘matching’ to describe angles and sides in the same position; however, the use of the word ‘corresponding’ is not incorrect.

The term ‘superimpose’ is used to describe the placement of one figure upon another in such a way that the parts of one coincide with the parts of the other.

Mathematics • Stage 4

Statistics and Probability

Data Collection and Representation

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols — MA4-1WM
- recognises and explains mathematical relationships using reasoning — MA4-3WM
- collects, represents and interprets single sets of data, using appropriate statistical displays — MA4-19SP

Related Life Skills outcomes: *MALS-1WM, MALS-3WM, MALS-32SP, MALS-33SP, MALS-34SP*

Students:

Investigate techniques for collecting data, including census, sampling and observation (ACMSP284)

- define 'variable' in the context of statistics as something measurable or observable that is expected to change either over time or between individual observations [L]
- recognise variables as numerical (either discrete or continuous) or categorical [L]
  - identify examples of categorical variables (eg colour, gender), discrete numerical variables (eg number of students, shoe size) and continuous numerical variables (eg height, weight) (Communicating) [N, CCT]
  - recognise that data collected on a rating scale (Likert-type scale) is categorical, eg 1 = dislike, 2 = neutral, 3 = like (Communicating)
- investigate and determine the differences between collecting data by observation, census and sampling [L, N, CCT]
  - identify examples of variables for which data could be collected by observation, eg direction travelled by vehicles arriving at an intersection, native animals in a local area
  - identify examples of variables for which data could be collected by a census and a sample, eg a census to collect data about the income of Australians; a sample for TV ratings (Communicating) [N, CCT]
  - discuss the practicalities of collecting data through a census compared to a sample, including limitations due to population size, eg in countries such as China and India, a census is conducted only once per decade (Communicating, Reasoning) [CCT, A]

Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes (ACMSP206)

- collect data using a random process, eg numbers from a page in a phone book, or from a random number generator [N]
- identify issues for which it may be difficult to obtain representative data either from primary or secondary sources [CCT, EU]
  - discuss constraints that may limit the collection of data or result in unreliable data, eg proximity to the location where data could be collected, access to ICT or cultural sensitivities that may influence the results (Communicating, Reasoning) [CCT, EU, IU]
- investigate and question the selection of data used to support a particular viewpoint, eg the selective use of data in product advertising [CCT, EU]

Mathematics • Stage 4

Statistics and Probability
Data Collection and Representation

Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)

- identify the difference between data collected from primary and secondary sources, eg data collected in the classroom compared with data drawn from a media source [L]
- explore issues involved in constructing and conducting surveys such as sample size, bias, type of data required and ethics [L, N, EU]
  - discuss the effect of different sample sizes (Communicating, Reasoning) [N, CCT]
  - describe, in practical terms, how a random sample may be selected to collect data about an issue of interest (Communicating, Problem Solving) [CCT]
  - detect and discuss bias, if any, in the selection of a sample (Communicating, Reasoning) [N, CCT]
- construct appropriate survey questions and a related recording sheet to collect both numerical and categorical data about an issue of interest [L, CCT, PSC]
  - construct a recording sheet that allows efficient collection of the different types of data expected (Communicating, Problem Solving) [L, CCT, PSC]
  - refine questions in a survey after a trial (Communicating) [L, CCT, PSC]
  - decide whether a census or sample is more appropriate to collect the data required to investigate the issue (Problem Solving) [L, CCT, PSC]
- collect and interpret information presented as tables and/or graphs about an issue of interest from secondary sources, eg the relationship between wealth or education and the health of populations from different countries; sporting data [L, N, IU, EU]
  - interpret and use scales on graphs including those where abbreviated measurements are used, eg ‘50’ on a vertical axis measured in thousands is interpreted as ‘50 000’ (Reasoning) [N]
  - analyse a variety of data displays used in the print or digital media and in other school subject areas, eg share movement graphs, sustainable food production (Problem Solving) [N, ICT, CCT, SE]
  - identify features on graphical displays that may lead to misleading interpretation, eg displaced zeros, the absence of labelling on one or more axes or misleading units of measurement (Communicating, Reasoning) [L, N, CCT]
- use spreadsheets or statistical packages to tabulate and graph data [ICT]
  - discuss ethical issues that may arise from collecting and representing data (Reasoning) [CCT, EU]

Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170)

- use a tally to organise data into a frequency distribution table [N]
- draw and interpret frequency histograms and polygons [N]
  - select and use appropriate labels and scales on the horizontal and vertical axes (Communicating, Problem Solving, Reasoning) [N, CCT]
- construct and interpret dot plots and stem-and-leaf plots, including stem-and-leaf plots with two-digit stems [N]

Mathematics • Stage 4

Statistics and Probability
Data Collection and Representation

- ‣ explain the importance of aligning data points/values when constructing dot plots and stem-and-leaf plots (Communicating, Reasoning) [CCT]

- interpret a variety of graphs, including divided bar graphs, sector graphs and line graphs [L, N]
  - ‣ calculate the percentage of the whole represented by different categories in a divided bar graph or sector graph (Problem Solving) [N, CCT]
  - ‣ compare the strengths and weaknesses of different forms of data display (Reasoning)
  - ‣ identify and explain which graph types are suitable for the type of data being considered, eg sector graphs and divided bar graphs are suitable for categorical data, but not for numerical data (Communicating, Reasoning) [N, CCT]
  - ‣ draw conclusions from data displayed in a graph, eg 'The graph shows that the heights of all children in the class are between 140 cm and 175 cm and that most are in the group 151−155 cm.' (Communicating, Reasoning) [N, CCT]
- construct divided bar graphs, sector graphs and line graphs with and without ICT [L, N]
  - ‣ calculate the angle at centre/length of bar required for each sector/section of sector graphs/divided bar graphs (Problem Solving) [N, CCT]

Background information

Students studying in Stage 4 can be expected to have some prior knowledge of both dot plots and line graphs as these types of graphs are first introduced in Stage 3. Students construct, describe and interpret column graphs in Stages 2 and 3; however, Stage 4 is the first Stage in which histograms, divided bar graphs and sector (pie) graphs are encountered.

Statistical data is part of everyday life. Data may be displayed in tables and/or graphs, and may appear in all types of media. Graphs provide a visual overview of the topic under investigation. Students should be aware that while many graphs are accurate and informative, some can be misleading. Students need to experience interpreting a wide variety of graphical representations, including column graphs, line graphs, dot plots, stem-and-leaf plots, divided bar graphs and sector (pie) graphs. Students need to be able to select the appropriate graph to represent the collected data.

Language

In everyday language the term 'pie chart' is often used in reference to sector graphs.

Mathematics • Stage 4

Statistics and Probability

Single Variable Data Analysis

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- analyses single sets of data using measures of location and range MA4-20SP

Students:

Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171)

- calculate the mean, $\bar{x}$, of a set of data and using $\bar{x} = \dfrac{\text{sum of scores}}{\text{number of scores}}$ [N]
  - recognise that the mean is often referred to as the 'average' in everyday language (Communicating)
  - use the statistical functions of a calculator to determine the mean (Problem Solving) [N, ICT]
  - use the statistical functions of a spreadsheet to calculate the mean for large sets of data (Communicating, Problem Solving) [ICT]
- calculate the mode, median and range for sets of data [N]
  - recognise which statistical measures are appropriate for the data type, eg the mean, median and range are meaningless for categorical data (Reasoning) [N, ICT]
  - use the statistical functions of a spreadsheet to calculate median, mode and range for large sets of data (Communicating, Problem Solving) [ICT]
- identify and describe the mean, median and mode as 'measures of location' or 'measures of centre', and the range as a 'measure of spread' [L]
- describe, in practical terms, the meaning of the mean, median, mode and/or range in the context of the data, eg when referring to the mode of shoe size data: 'The most popular shoe size is size 7.' [N]

Investigate the effect of individual data values, including outliers, on the mean and median (ACMSP207)

- identify any clusters, gaps and outliers in a data set [L, N]
- investigate the effect of outliers on the mean, median, mode and range by considering small data sets and calculating each measure with and without the inclusion of the outlier [N, CCT]
  - explain why it is more appropriate to use the median than the mean when the data contains one or more outliers (Communicating, Reasoning) [N, CCT]
  - determine situations when it is more appropriate to use the mode or median, rather than the mean, when analysing data, eg median for property prices, mode for shoe sizes (Communicating, Reasoning) [N, CCT]
- analyse collected data to identify any obvious errors and justify the inclusion of any scores that differ remarkably from the rest of the data collected [N, CCT, EU]

Mathematics • Stage 4

Statistics and Probability

Single Variable Data Analysis

Describe and interpret data displays using median, mean and range (ACMSP172)

- calculate measures of location (mean, mode, median) and the range for data represented in a variety of statistical displays, including frequency distribution tables, frequency histograms, stem-and-leaf plots and dot plots [N, CCT]
- draw conclusions based on the analysis of data displays using the mean, mode and/or median, and range [N, CCT]

Explore the variation of means and proportions of random samples drawn from the same population (ACMSP293)

- investigate ways in which different random samples may be drawn from the same population, eg random samples from a census may be chosen by gender, postcode, state, etc
- calculate and compare summary statistics (mean, mode, median and range) of at least three different random samples drawn from the same population [N]
  - use a spreadsheet to calculate and compare summary statistics of different random samples drawn from the same population (Communicating, Problem Solving) [ICT]
  - recognise that summary statistics may vary from sample to sample (Communicating) [N]
  - recognise that the way in which random samples are chosen may result in significant differences between their respective summary statistics, eg a random sample of girls compared to a random sample of boys from the population; or random samples chosen by postcode (Communicating, Reasoning) [N, CCT]
  - suggest reasons why different random samples drawn from the same population may have different summary statistics (Communicating, Reasoning) [N, CCT]

Background information

Many opportunities occur in this topic for students to strengthen their skills in:

- collecting, analysing and organising information
- communicating ideas and information
- planning and organising activities
- working with others and in teams
- using mathematical ideas and techniques
- using technology, including spreadsheets

Language

The term ‘average’ used in everyday language generally refers to the mean and describes a ‘typical value’ within a set of data.

Students need to be provided with opportunities to discuss what information can be drawn from the data presented. Students need to think about the meaning of the information and to put it into their own words.

Language to be developed would include superlatives, comparatives and other language expressions such as ‘prefer … over’ etc.

Mathematics • Stage 4

Statistics and Probability

Probability 1

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- represents probabilities of simple and compound events MA4-21SP

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-35SP, MALS-36SP*

Students:

Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)

- use the term 'chance experiment' when referring to occurrences such as tossing a coin, rolling a die, randomly selecting an object from a bag [L]
- use the term 'outcome' to describe a possible result of a chance experiment and list all possible outcomes included in a single-step experiment [N]
- use the term 'sample space' to describe a list of all possible outcomes of a chance experiment, eg if a fair six-sided die is rolled once, the sample space is $\{1,2,3,4,5,6\}$ [L, N]
- distinguish between equally likely outcomes and outcomes that are not equally likely in single-step chance experiments [L, N]
  - describe single-step chance experiments in which the outcomes are equally likely, eg the outcomes of a single toss of a fair coin (Reasoning) [N]
  - describe single-step chance experiments in which the outcomes are not equally likely, eg 'The outcomes of a single roll of a six-sided die labelled 1, 2, 3, 4, 4, 4 are not equally likely since the outcome '4' is three times more likely to occur than any other outcome.' (Communicating, Reasoning) [N, CCT]
  - design a spinner given the relationships between the likelihood of each outcome, eg design a spinner with three colours red, white and blue, so that red is twice as likely to occur than blue, and blue is three times more likely to occur than white (Problem Solving) [N, CCT]

Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168)

- use the term 'event' to describe either one outcome or a collection of the outcomes in the sample space of a chance experiment, eg in the experiment of rolling a single fair-sided die once, getting the number '1' is an 'event'; similarly, getting a number divisible by three is also an event [L]
  - explain the difference between an experiment, outcomes, the sample space and events in chance situations (Communicating) [L]
- assign a probability of zero to events that are impossible and a probability of one to events that are certain [L, N]
  - explain the meaning of a probability of 0, $\frac{1}{2}$ and 1 in a given situation (Communicating) [N, CCT]

Mathematics • Stage 4

Statistics and Probability
Probability 1

- assign probabilities to simple events by reasoning about equally likely outcomes, eg the probability of randomly drawing a diamond card from a standard pack of 52 playing cards is $\frac{13}{52}=\frac{1}{4}$ [N]
- express the probability of an event A given a finite number of equally likely outcomes as $P(A)=\frac{\text{number of favourable outcomes}}{n}$
  where n is the total number of outcomes in the sample space [L, N]
  - interpret and use probabilities expressed as percentages or decimals (Communicating) [N, CCT]
- solve probability problems involving single-step experiments such as card, dice and other games [N]
- establish that the sum of the probabilities of all possible outcomes of a single-step experiment is 1 [N]

Identify complementary events and use the sum of probabilities to solve problems (ACMSP204)

- identify and describe the complement of an event, eg the complement of the event 'rolling a six' when a die is thrown is 'not rolling a six' [L]
- establish that the sum of the probability of an event and its complement is 1,
  ie $P(\text{event A})+P(\text{complement of A})=1$ [N]
- calculate the probability of a complementary event using the fact that the sum of the probabilities of complementary events is 1, eg calculate the probability of 'rolling a six' when a die is thrown is $\frac{1}{6}$ the probability of the complement 'not rolling a six' is $1-\frac{1}{6}=\frac{5}{6}$ [N]

Background information

In a chance experiment, such as rolling a fair six-sided die, an event is one or a collection of outcomes. For instance an event might be that we roll an odd number, which would include the outcomes 1, 3 and 5. A simple event has outcomes that are equally likely.

It is important that students learn the correct terminology associated with probability.

Mathematics • Stage 4

Statistics and Probability

Probability 2

Outcomes

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- represents probabilities of simple and compound events MA4-21SP

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-35SP, MALS-36SP*

Students:

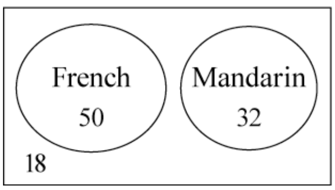
Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and' (ACMSP205)

- recognise the difference between mutually exclusive and non-mutually exclusive events, eg when a die is rolled, ‘rolling an odd number’ and ‘rolling an even number’ are mutually exclusive events; however, ‘rolling an even number’ and ‘rolling a 2’ are non-mutually exclusive events [L]
- describe compound events using the following terms: [L, N, CCT]
  - ‘at least’, eg rolling a 4, 5 or 6 on a single six-sided die is described as rolling at least 4
  - ‘at most’, eg rolling a 1, 2, 3 or 4 on a six-sided die is described as rolling at most 4
  - ‘not’, eg choosing a black card from a pack is described as choosing a card that is not red
  - ‘and’, eg choosing a card that is black and a king means the card must have both attributes
  - ‣ pose problems that involve the use of these terms, and solve problems posed by others (Communicating, Problem Solving) [L, N, CCT]
  - ‣ describe the effect of the use of ‘and’ and ‘or’ when using internet search engines (Communicating, Problem Solving) [L, ICT]
- classify compound events using inclusive ‘or’ and exclusive ‘or’, eg ‘choosing a male or a female’ is exclusive as one cannot be both, whereas ‘choosing a male or someone left-handed’ could imply exclusivity or inclusivity [L, CCT]
  - ‣ recognise that the word ‘or’ on its own often needs a qualifier, such as ‘both’ or ‘not both’, to determine inclusivity or exclusivity (Reasoning) [L, CCT]

Represent events in two-way tables and Venn diagrams and solve related problems (ACMSP292)

- interpret Venn diagrams involving two or three attributes [L, N]
  - ‣ describe regions in Venn diagrams representing mutually exclusive attributes, eg the Venn diagram represents the languages studied by Year 8 students

Language studied in Year 8



There are 50 people who study French; 32 people study Mandarin; 18 people study neither language; no one studies both languages (Communicating, Problem Solving, Reasoning) [L]

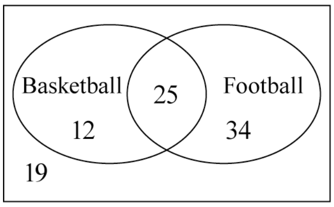
Mathematics • Stage 4

Statistics and Probability

Probability 2

- describe individual or combinations of regions in Venn diagrams representing non-mutually exclusive attributes using the language of ‘and’, exclusive ‘or’, inclusive ‘or’, ‘neither’ and ‘not’, eg the Venn diagram represents the sports played by Year 8 students.

Sports played by Year 8



There are 25 people who play both basketball and football; 46 people who play basketball or football but not both; there are 19 people who play neither sport; there are 71 people who play basketball or football or both (Communicating, Problem Solving, Reasoning) [L, N]

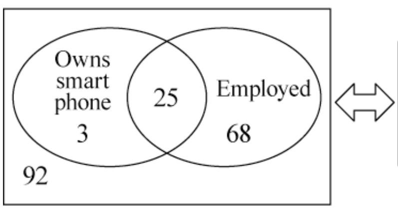
- construct Venn diagrams to represent all possible combinations of two attributes from given or collected data [N, CCT]
  - use given data to calculate missing values in a Venn diagram, eg the number of members that satisfy both attributes or the number of members that satisfy neither attribute (Problem Solving, Reasoning) [N, CCT]
- interpret given two-way tables representing non-mutually exclusive attributes [L, N, CCT]
  - describe relationships displayed in two-way tables using the language ‘and’, exclusive ‘or’, inclusive ‘or’, ‘neither’ and ‘not’ to, eg the table compares gender and handedness of students in Year 8

	Left-handed	Right-handed	
Female	7	46	53
Male	5	63	68
	12	109	121

There are 63 male, right-handed students, ie 63 students are neither female nor left-handed; there are 114 students who are male or right-handed or both. (Communicating, Problem Solving, Reasoning) [L, N]

- construct two-way tables to represent the relationship between attributes [L, N]
  - use given data to calculate missing values in a two-way table (Problem Solving) [N]
- convert representations of the relationship between two attributes in Venn diagrams to two-way tables [L, N, CCT],

eg Smart phone ownership compared to employment status of Year 10 students



	Employed	Not employed	
Owns smart phone	25	3	28
Does not own smart phone	68	92	160
	93	95	188

Mathematics • Stage 4

Statistics and Probability

Probability 2

Background information

John Venn (1834−1923) was a British mathematician best known for his diagrammatic way of representing sets, their unions and intersections.

Students are expected to be able to interpret Venn diagrams involving three attributes; however, students are not expected to construct Venn diagrams involving three attributes.

Language

A compound event is an event which can be expressed as a combination of simple events, eg drawing a card that is black or a King; throwing at least 5 on a fair six-sided die.