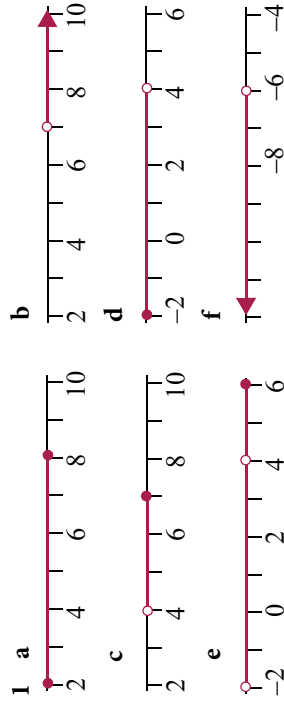


# ANSWERS

## Chapter 2

### Exercise 2.1



- 2 a  $[-2, 7]$  b  $]9, \infty[$  c  $]0, 5]$  d  $]-\infty, 0]$  e  $]-4, 8[$  f  $]-\infty, -1[ \cup ]2, \infty[$   
 3 a  $5\sqrt{5}$  b  $-\sqrt{3}$  c  $\sqrt{3}$   
 4 a 4 b  $4 + \sqrt{6}$  c  $6\sqrt{2}$  d  $31 + 12\sqrt{3}$   
 5 a  $2 - \sqrt{3}$  b  $\sqrt{7} + 2$  c  $2\sqrt{3} + \sqrt{15}$  d  $2 - \sqrt{3} - 4\sqrt{5} - 2\sqrt{15}$   
 e  $\frac{3 + \sqrt{6} + \sqrt{10} + \sqrt{15}}{-2}$  f  $3\sqrt{6} + 2\sqrt{15}$   
 6 a i  $\frac{3\sqrt{5} + \sqrt{3}}{2}$  ii  $10 + \frac{3\sqrt{15}}{2}$  b i  $\frac{14\sqrt{3} + 48}{13}$  ii  $\frac{1344\sqrt{3} + 3230}{169}$   
 7 a  $\{\pm 3\}$  b  $\{\pm 10\}$  c  $\emptyset$  d  $\{-4, 2\}$  e  $\{-12, 8\}$  f  $\{0, 4\}$   
 8 a b c d

- 9 a  $]1, \infty[$  b  $]4, \infty[$  c  $]4, 6[$   
 11 a  $\sqrt{6} + \sqrt{3}$  b  $2\sqrt{2} - 2$

### Exercise 2.2.1

- 1 a 4 b 3 c -6 d  $-\frac{11}{2}$  e  $\frac{1}{10}$  f  $\frac{3}{8}$   
 2 a  $\frac{17}{5}$  b  $\frac{4}{3}$  c  $-\frac{3}{4}$  d  $\frac{4}{3}$  e  $\frac{35}{2}$  f  $\frac{92}{41}$   
 3 a  $-\frac{44}{5}$  b -39 c  $-\frac{1}{7}$  d -3 e 2 f 4

- 4 a  $2b - 2$  b  $b + 1 + \frac{b}{a}$  c  $\frac{ab}{a+b}$  d  $a(a+b)$  e  $ab$  f  $\frac{ab}{a-b}$  g 0

h  $\frac{a+b}{a^2+b^2}$  i  $a+b$

- 5 a -4, 4 b  $-\frac{9}{5}, 3$  c -6, 18 d  $-\frac{11}{2}, \frac{17}{2}$  e  $-\frac{7}{10}, \frac{1}{10}$  f  $-\frac{5}{8}, \frac{3}{8}$  g  $-\frac{7}{5}, \frac{17}{5}$   
 h  $\frac{4}{3}, \frac{20}{3}$  i -3, 0 j  $\frac{a-b}{2}, \frac{b-a}{2}, a \geq b$  k  $\pm(b^2 - ab), a \geq b$  l  $-\frac{b}{a}, \frac{2b}{a}, b \geq 0$

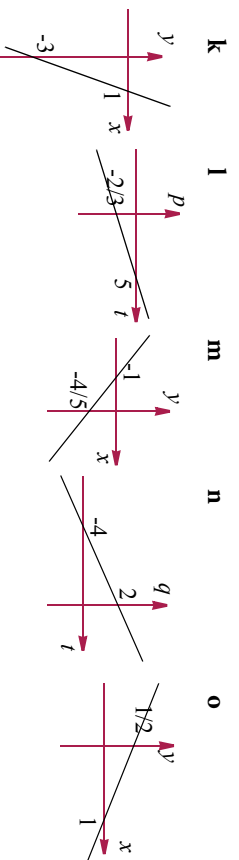
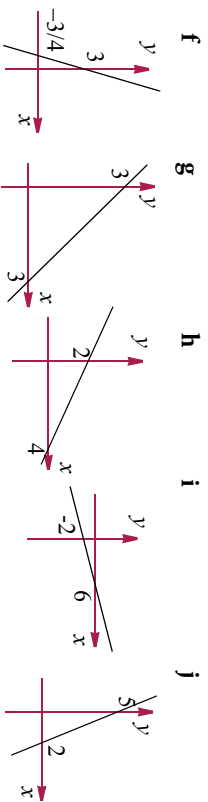
### Exercise 2.2.2

- 1 a  $x < -4$  b  $x \leq -\frac{1}{5}$  c  $x > 1$  d  $x \leq -6$  e  $x > \frac{18}{7}$  f  $x > \frac{3}{8}$   
 2 a  $x > \frac{52}{11}$  b  $x \leq 1$  c  $x \leq \frac{10}{3}$   
 3 a  $x < 1$  b  $x < 2 - a$  c  $x > \frac{2b}{3a}$  d  $x \geq \frac{2}{(a+1)^2}$   
 4 a  $-2 \leq x \leq 1$  b  $-2 \leq x \leq 3$  c  $-\frac{3}{2} \leq x \leq \frac{5}{2}$  d  $x = -\frac{1}{2}$  e  $-7 \leq x \leq 9$   
 f  $-5 \leq x \leq 3$  g  $-4 \leq x \leq 16$  h  $-28 \leq x \leq 44$  i  $-\frac{5}{12} \leq x \leq \frac{1}{12}$   
 5 a  $x < -\frac{3}{2} \cup x > \frac{5}{2}$  b  $x < \frac{3}{2} \cup x > \frac{7}{2}$  c  $x \leq -12 \cup x \geq 16$  d  $x \leq -24 \cup x \geq 6$   
 e  $x < \frac{3}{4} \cup x > \frac{9}{4}$  f  $-6 < x < 14$  g  $x < -28 \cup x > 44$  h  $x < -\frac{5}{12} \cup x > \frac{1}{12}$   
 i  $x \leq -4 \cup x \geq 16$

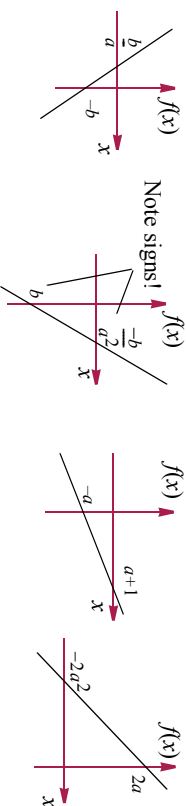
6  $p < 3$

### Exercise 2.3.1

- 1 a b c d e



- 2 a 2 b 3 c  $\frac{5}{3}$   
 3 a  $y = 2x - 1$  b  $y = 3x + 9$  c  $y = -x - 1$   
 4 a  $-\frac{1}{2}$  b  $\frac{1}{3}$  c  $\frac{3}{2}$  d  $-\frac{4}{5}$   
 5  $y = 2x$   
 6  $y = -x + 1$   
 7  $y = \frac{x + 2}{2}$   
 8 2  
 9 a  $y = \frac{5}{2}x$  b  $y = -\frac{3}{2}x + 3$  c  $y = \frac{5}{6}x - \frac{1}{2}$  d  $y = -2x + 1$   
 10 a b c d



- Exercise 2.3.2**  
 1 a  $x = 1, y = 2$  b  $x = 3, y = 5$  c  $x = -1, y = 2$  d  $x = 0, y = 1$  e  $x = -2, y = -3$   
 $f x = -5, y = 1$   
 2 a  $x = \frac{13}{11}, y = \frac{17}{11}$  b  $x = \frac{9}{14}, y = \frac{3}{14}$  c  $x = 0, y = 0$  d  $x = \frac{4}{17}, y = -\frac{22}{17}$   
 e  $x = -\frac{16}{7}, y = \frac{78}{7}$  f  $x = \frac{5}{42}, y = -\frac{3}{28}$

- 3 a -3 b -5 c -1.5  
 4 a  $m = 2, a = 8$  b  $m = 10, a = 24$  c  $m = -6, a = 9$ .  
 5 a  $x = 1, y = a - b$  b  $x = -1, y = a + b$  c  $x = \frac{1}{2}, y = 0$  d  $x = b, y = 0$   
 $e x = \frac{a-b}{a+b}, y = \frac{a-b}{a+b}$  f  $x = a, y = b - a^2$

- Exercise 2.3.3**  
 1 a  $x = 4, y = -5, z = 1$  b  $x = 0, y = 4, z = -2$  c  $x = 10, y = -7, z = 2$   
 d  $x = 1, y = 2, z = -2$  e  $\emptyset$  f  $x = 2t - 1, y = t, z = t$

**Exercise 2.4.1**

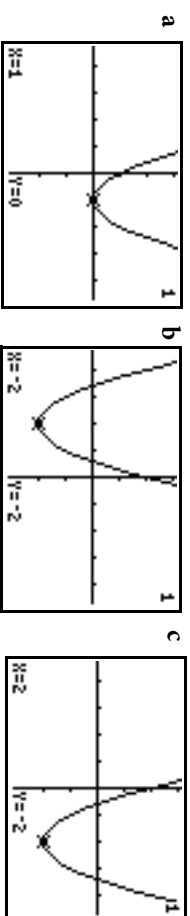
- 1 a -5 b 4, 6 c -3, 0 d 1, 3 e -6, 3 f  $-\frac{2}{3}, \frac{5}{3}$  g 2 h -3, 6 i -6, 1 j  $0, \frac{3}{2}$   
 2 a -1 b -7, 5 c  $-\frac{2}{3}, 3$  d -2, 1 e -3, 1 f 4, 5  
 3 a  $-1 \pm \sqrt{6}$  b  $3 \pm \sqrt{5}$  c  $1 \pm \sqrt{5}$  d  $\frac{-1 \pm \sqrt{33}}{8}$  e  $\frac{9 \pm \sqrt{73}}{4}$  f  $\frac{1 \pm \sqrt{85}}{6}$   
 4 a  $\frac{3 \pm \sqrt{37}}{2}$  b  $\frac{5 \pm \sqrt{33}}{2}$  c  $\frac{3 \pm \sqrt{33}}{2}$  d  $\frac{7 \pm \sqrt{57}}{2}$  e  $\frac{-7 \pm \sqrt{65}}{2}$  f -4, 2  
 g  $-1 \pm 2\sqrt{2}$  h  $\frac{-5 \pm \sqrt{53}}{2}$  i  $\frac{3 \pm \sqrt{37}}{2}$  j no real solutions k  $4 \pm \sqrt{7}$

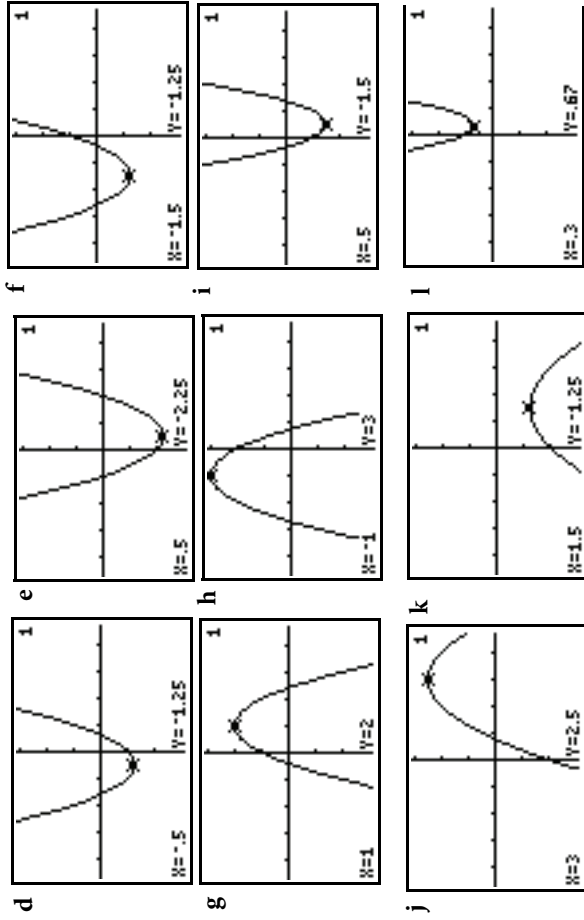
No real solutions m  $\frac{2 \pm \sqrt{13}}{2}$  n  $\frac{3 \pm 2\sqrt{11}}{5}$  o  $\frac{6 \pm \sqrt{31}}{5}$

- 5 a  $-2 < p < 2$  b  $p = \pm 2$  c  $p < -2$  or  $p > 2$   
 6 a  $m = 1$  b  $m < 1$  c  $m > 1$   
 7 a  $m = \pm 2\sqrt{2}$  b  $]-\infty, -2\sqrt{2}[ \cup ]2\sqrt{2}, \infty[$  c  $]-2\sqrt{2}, 2\sqrt{2}[$   
 8 a  $k = \pm 6\sqrt{2}$  b  $]-\infty, -6\sqrt{2}[ \cup ]6\sqrt{2}, \infty[$  c  $]-6\sqrt{2}, 6\sqrt{2}[$

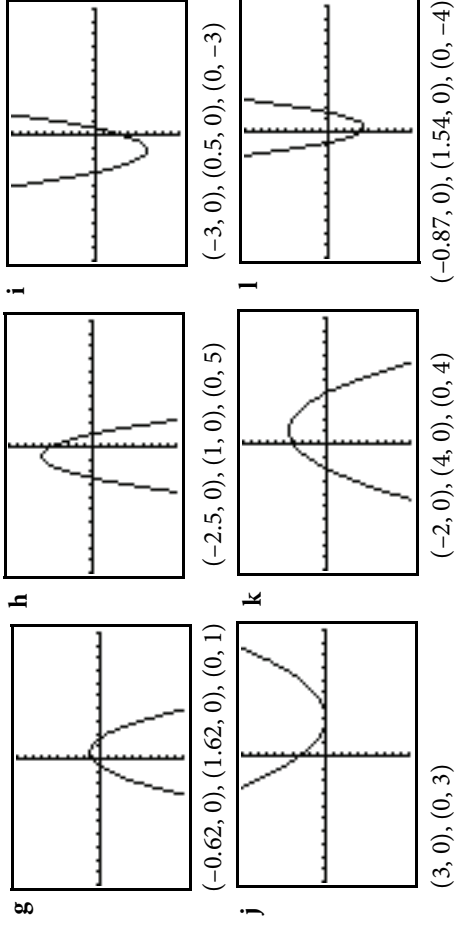
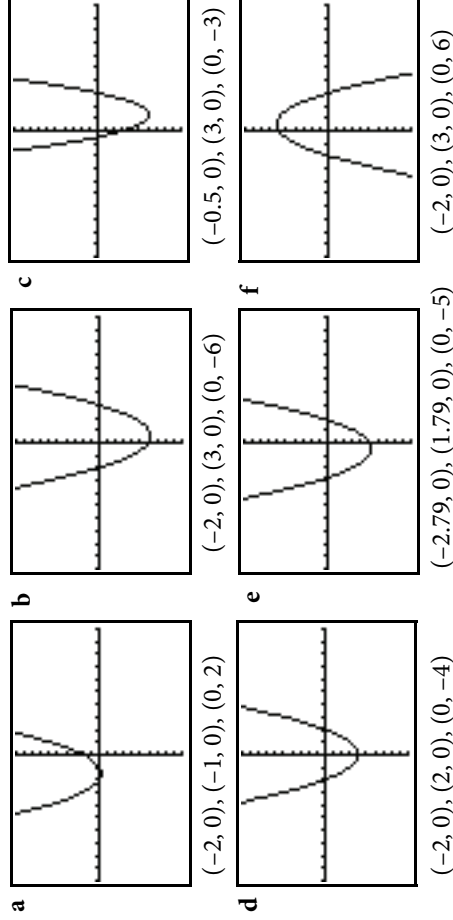
**Exercise 2.4.2**

1 Graphs are shown using the ZOOM4 viewing window:

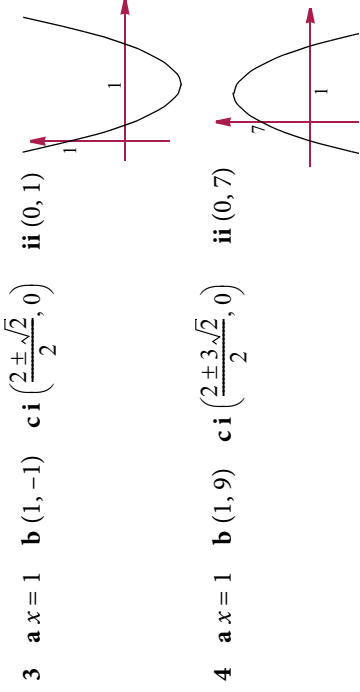




2 Graphs are shown using the ZOOM6 viewing window:



(3, 0), (0, 3) (-2, 0), (4, 0), (0, 4) (-0.87, 0), (1.54, 0), (0, -4)



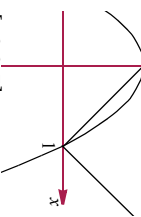
- 5 a  $k = \frac{9}{4}$  b  $k < \frac{9}{4}$  c  $k > \frac{9}{4}$   
 6 a  $k = \frac{25}{8}$  b  $k < \frac{25}{8}$  c  $k > \frac{25}{8}$   
 7 a  $k = \pm 1$  b  $-1 < k < 1$  c  $k < -1 \cup k > 1$   
 8 a  $y = \frac{5}{12}(x-2)(x-6)$  b  $y = -\frac{3}{8}(x+4)^2$  c  $y = \frac{3}{4}(x-2)^2 + 1$  d  $y = 3x^2 - 6x + 7$   
 9 a  $y = -\frac{2}{5}x(x-6)$  b  $y = \frac{3}{4}(x-3)^2$  c  $y = \frac{7}{9}(x+2)^2 + 3$  d  $y = -\frac{7}{3}x^2 - 2x + \frac{40}{3}$

Exercise 2.4.3

- 1 a  $]-\infty, -2[ \cup ]1, \infty[$  b  $[-3, 2]$  c  $]-\infty, 0] \cup [4, \infty[$  d  $]\frac{1}{3}, 3[$  e  $]-\infty, -1.5] \cup [-1, \infty[$  f  $]0.75, 2.5[$

- 2 a)  $]-\infty, -2[ \cup ]-1, \infty[$  b)  $]-2, 3[$  c)  $]-\infty, -0.5[ \cup ]3, \infty[$  d)  $]-2, 2[$  e)  $] = \frac{-1-\sqrt{21}}{2}, -1+\sqrt{21}[$   
 f)  $]-\infty, -2[ \cup ]3, \infty[$  g)  $] = \frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}[$  h)  $]-2.5, 1[$  i)  $]-\infty, -3[ \cup ]0.5, \infty[$  j)  $]1, 3[$   
 k)  $]-1, 0.5[$  l)  $\emptyset$  m)  $\emptyset$  n)  $]-1.5, 5[$  o)  $]-\infty, -2[ \cup ]\frac{1}{2}, \infty[$

- 3 a)  $-1 < k < 0$  b)  $-2\sqrt{2} < k < 2\sqrt{2}$  c)  $n \leq -0.5$   
 4 a) i)  $]-\infty, -1[ \cup ]2, \infty[$  ii)  $]-1, 2[$  b) i)  $]-\infty, 2[ \cup ]3, \infty[$  ii)  $]2, 3[$   
 c) i)  $]1, 3[$  ii)  $]-\infty, 1[ \cup ]3, \infty[$  d) i)  $]-\frac{2}{3}, 1[$  ii)  $]-\infty, -\frac{2}{3}[ \cup ]1, \infty[$   
 e) i)  $]-\infty, -2[ \cup ]2, \infty[$  ii)  $]-2, 2[$  f) i)  $]2-\sqrt{3}, 2+\sqrt{3}[$  ii)  $]-\infty, 2-\sqrt{3}[ \cup ]2+\sqrt{3}, \infty[$   
 5  $]0, 1[$



- 6  $[-2, 0.5]$   
 7 a)  $\{x: x < -3\} \cup \{x: x > 2\}$  b)  $\{x: -1 < x < 4\}$

**Exercise 2.4.4**

- 1 a)  $(-2, -3)$  (2, 5) b)  $(-2, -1)$  (1, 2) c)  $(-\frac{1}{3}, -2)$ , (2, 5) d)  $(-\frac{3}{2}, -\frac{15}{4})$ , (1, 0)  
 e)  $(-\frac{9}{2}, -\frac{19}{4})$ , (1, -2) f)  $(\frac{3+\sqrt{73}}{4}, \frac{-3-\sqrt{73}}{8})$ ,  $(\frac{3-\sqrt{73}}{4}, \frac{-3+\sqrt{73}}{8})$   
 g)  $(\frac{1-\sqrt{13}}{2}, 1-\sqrt{13})$ ,  $(\frac{1+\sqrt{13}}{2}, 1+\sqrt{13})$  h) no real solutions  
 i)  $(\frac{1-\sqrt{17}}{2}, \frac{5-3\sqrt{17}}{2})$ ,  $(\frac{1+\sqrt{17}}{2}, \frac{5+3\sqrt{17}}{2})$  j)  $(-2, -3)$ , (2, 1)  
 k) no real solutions  
 2 a) (1, 4),  $(-7, 84)$  b)  $(\frac{4}{3}, -\frac{56}{9})$ ,  $(\frac{3}{4}, -\frac{7}{4})$  c) (0, 2), (3, 23) d)  $(-a, -a^2)$ ,  $(\frac{a}{2}, \frac{a^2}{2})$   
 e)  $\emptyset$  f) (2, 8) g)  $\emptyset$  h)  $(\frac{1}{2}, \frac{23}{4})$

- 3 a)  $\pm 2\sqrt{6}$  b)  $m < -2\sqrt{6}, m > 2\sqrt{6}$  c)  $-2\sqrt{6} < m < 2\sqrt{6}$   
 4  $\sqrt{80}$   
 5 1.75  
 7  $-\frac{23}{12}$   
 8  $c = \frac{a}{m}$

- 10 a) i) (1, 3),  $(-\frac{14}{3}, \frac{196}{3})$  ii)  $(-2, 12)$ ,  $(\frac{7}{3}, \frac{49}{3})$  c) i) A(1, 3), B(-2, 2) ii) 4 sq. units

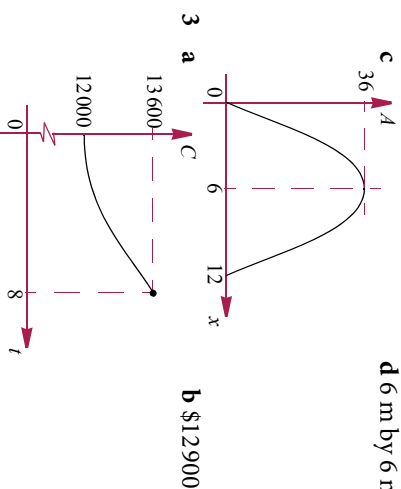
**Chapter 3**

**Exercise 3.1.1**

- 1 8  
 2 4, 0.25  
 3 8, 18  
 4 8 and 11 or -8 and -11  
 5 6, -10  
 6 2 m  
 7 51 kmh<sup>-1</sup>  
 8 11, 13; -11, -13  
 9 25 days  
 10 30  
 11 a) 30 b) \$50 each.  
 12 6 kmh<sup>-1</sup>  
 13 16  
 14 6  
 15 3 hours  
 16 9  
 17 a) 15 hrs b) 10 hrs  
 18 Chair-one: 20; Chair-two: 24  
 19 a) 2 km b) 2.5 km  
 20 7.5 hrs, 10.5 hrs

**Exercise 3.1.2**

- 1 a) i)  $100 - 2x$  ii)  $0 < x < 50$  (Note: if  $x = 0$  or 50,  $A = 0$  and so there is no enclosure)  
 b) i)  $A = 2x(50 - x), 0 < x < 50$  ii) 10 m by 80 m or 40 m by 20 m  
 iii) 1250 m<sup>2</sup> iv) 25 m by 50 m  
 2 a) ii)  $0 < x < 12$  b) i) 20 m<sup>2</sup> ii) 32 m<sup>2</sup> iii) 32 m<sup>2</sup>  
 d) 6 m by 6 m



- 3 a)  $C$  b) \$12900

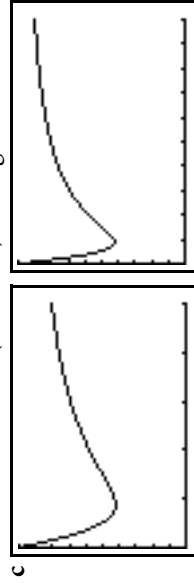
4 a  $R(x) = xp = x(40 - 0.0004x)$ ,  $0 \leq x \leq 100,000$  **b** \$960 000 **ii** 18 377 or 81 622 (as answer must be integer values) **iii** \$1 000 000

5 a  $y = \frac{4}{3}(50 - x)$  **b**  $A = \frac{8}{3}x(50 - x)$  **ii**  $0 < x < 50$  **c**  $\frac{5000}{3} \text{ m}^2$

**ii**  $x = 25$ ,  $y = \frac{200}{3}$ ; dimensions 50 m by  $\frac{100}{3}$  m

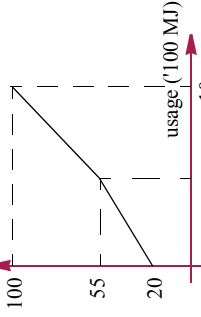
6 35.83  $\text{kmh}^{-1}$

7 a 100% **b**  $t = 0.229$  (first time) then again at  $t = 13.104$



$[0, 10]$  by  $[0, 1]$   
**d** As time increases, oxygen level will be 100%.

8 a **b** \$95.26

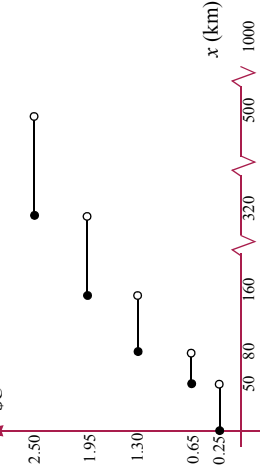


9 B(3254, 1953), C(6146, 3687) units in metres

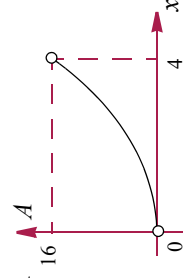
10 a

$$C = \begin{cases} 0.25, 0 \leq x < 50 \\ 0.65, 50 \leq x < 80 \\ 1.30, 80 \leq x < 160 \\ 1.95, 160 \leq x < 320 \\ 2.50, 320 \leq x < 500 \\ 3.40, 500 \leq x < 1000 \end{cases}$$

**b** **a** \$3.40 **b** \$2.50 **c** \$4.70 **d** \$1.88



11 a  $0 < x < 4$  **b**  $A(x) = 3x + 0.25x^2$ ,  $0 < x < 4$  **ci**



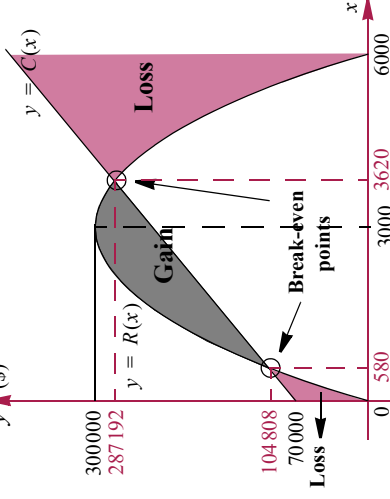
**ii** 3.25 **iii**  $-6 + \sqrt{48}$

12 a **i** 200 m **ii** 320 m **b** 0.34 sec and 11.66 sec **ii** 11.31 s **c** 12 s **d** 360 m

13 a 0.53 s (on the way up) and 9.47 s (on the way down) **b** 10 s **c** 500 m  
**d** 12.07 s **e** 750 m

14 a \$72 500 **b** No. (loss of \$20 000) **c** 2500

15 a **y** (\$) **b** **i** \$70 000 **ii** \$300 000



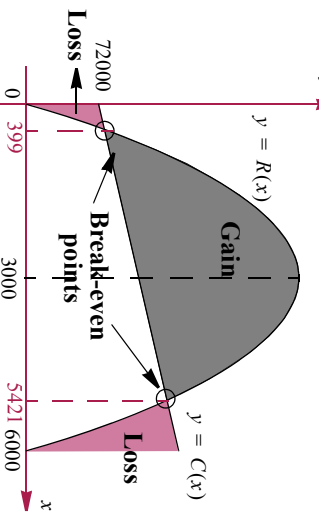
**c** Fixed cost (e.g. salary, electricity, ...) **d** See graph in part a

**e** \$76 667 (to nearest dollar) **fi**  $P(x) = 140x - \frac{1}{30}x^2 - 70000$ ,  $0 \leq x \leq 6000$

**ii** \$77 000 **iii** 2100 **gi**  $0 \leq x \leq 580$  or  $3620 \leq x \leq 6000$  **ii**  $581 \leq x \leq 3619$

**h** See graph in part a

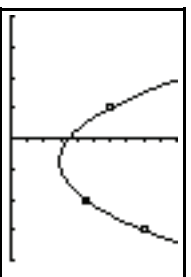
16 a  $y \uparrow$  (\$)  $x$



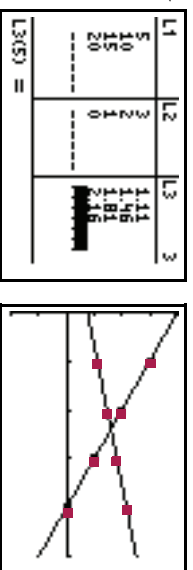
- b i  $P(x) = -\frac{1}{30}x^2 + 194x - 72000$  ii  $0 \leq x \leq 6000$  d The company will break even at 399 radios and 5421 radios. Provided the company sells between 399 and 5421 radios they will make a profit. e 2910

**Exercise 3.1.3**

- 1 a ii  $y = 0.4x + 7.2$  b ii  $y = 6 - 2x$  c ii  $y = 0.5x + 3.2$
- 2 Second difference = 0.64
- 3 b  $y = x^2 + 4x + 2$
- 4 a & c  $y = 2x^2 - 3x + 7$

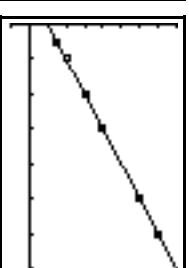
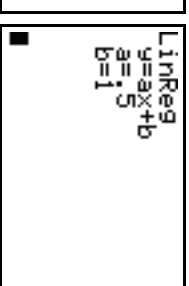
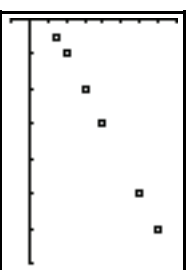


5  $y = 2x^2 - x + 3$

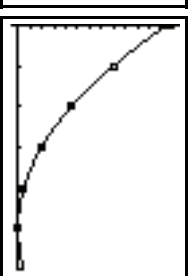
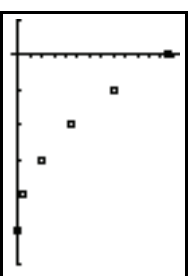


- 6 a
- 7 a  $y = -0.6333x^2 + 8.8333x - 19.2$  b  $-40^\circ\text{C}$  at 11 p.m. The model is not valid outside data range, therefore extrapolation will not necessarily work.
- 8 Equation of path:  $y = -\frac{31}{2400}x^2 + \frac{49}{48}x + 1$ . Greatest height: 21.17 m.

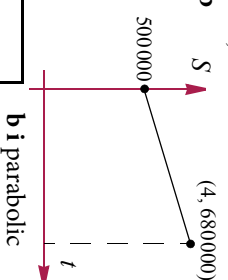
9 a b linear c ii  $M = 0, x = 1$ , i.e. 1 m



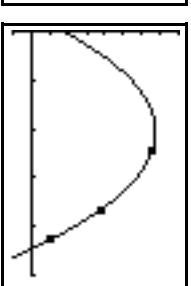
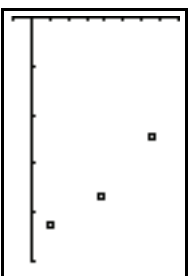
10 a b c parabolad



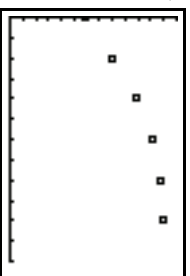
- 11 a  $P(x) = -2x^2 + x + 3$  b  $P(x) = (1-k)x^2 + x + k, x \in \mathbb{R}, k \neq 1$
- 12 a i  $a = 45000, b = 500000$  ii  $k_1 = 0, k_2 = 4$  b  $S$



13 a b i parabolic



- ii  $h(x) = -0.04694x^2 + 0.96518x + 1.7896$  c i 6.75 m ii 22.27 m
- 14 a  $y = -\frac{2}{9}x^2 + \frac{11}{9}x + \frac{9}{2}, 0 \leq x \leq 5.5$  b [BE]:  $y = -\frac{11}{9}x + \frac{121}{18}, 1 \leq x \leq 5.5$ , [BO]:  $y = 5.5x, 0 \leq x \leq 1$  c  $49^\circ 36'$
- 15 a b second difference is constant = -50



- c  $y = -0.25x^2 + 25x + 580$  d \$22500 per car e i \$824750 ii \$19750

16 a i & ii have a constant gradient iii results imply quadratic form

b ii  $p = 10 - 0.001x$ ,  $C(x) = 2x + 7000$ ,  $R(x) = -0.001x^2 + 10x$

c  $P(x) = -0.001x^2 + 8x - 7000$ , max. profit =  $P(4000) = 9000$

**Chapter 4**

**Exercise 4.1.1**

- 1 a  $b^2 + 2bc + c^2$  b  $a^3 + 3a^2g + 3ag^2 + g^3$  c  $1 + 3y + 3y^2 + y^3$
- d  $16 + 32x + 24x^2 + 8x^3 + x^4$  e  $8 + 24x + 24x^2 + 8x^3$  f  $8x^3 - 48x^2 + 96x - 64$
- g  $16 + \frac{32}{7}x + \frac{24}{49}x^2 + \frac{8}{343}x^3 + \frac{1}{2401}x^4$  h  $8x^3 - 60x^2 + 150x - 125$
- i  $27x^3 - 108x^2 + 144x - 64$  j  $27x^3 - 243x^2 + 729x - 729$
- k  $8x^3 + 72x^2 + 216x + 216$  l  $b^3 + 9b^2d + 27bd^2 + 27d^3$
- m  $81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$
- n  $x^5 + 15x^4y + 90x^3y^2 + 270x^2y^3 + 405xy^4 + 243y^5$  o  $\frac{125}{p^3} + \frac{150}{p} + 60p + 8p^3$
- p  $\frac{16}{x^4} - \frac{32}{x} + 24x^2 - 8x^5 + x^8$  q  $q^5 + \frac{10q^4}{p^3} + \frac{40q^3}{p^6} + \frac{80q^2}{p^9} + \frac{80q}{p^{12}} + \frac{32}{p^{15}}$
- r  $x^3 + 3x + \frac{2}{x} + \frac{1}{x^3}$

**Exercise 4.1.2**

- 1 a  $160x^3$  b  $21x^5y^2$  c  $-448x^3$  d  $-810x^4$  e  $216p^4$  f  $-20412p^2q^5$
- g  $-22680p$
- 2 a  $-1400000$  b  $6000$  c  $540$  d  $-240$  e  $81648$  f  $40$
- 3 1.0406 0.0004%
- 4 a  $64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625$
- b 19750 c 20.6 d 0.1%
- 5 19
- 6  $-\frac{63}{8}$
- 7  $\frac{231}{16}$
- 8  $-\frac{130}{27}$
- 9  $-20$
- 10 a =  $\pm 3$
- 11 n = 5
- 12 n = 9
- 13 a 0 b  $-59$
- 14 a = 3, n = 8
- 15 a =  $\pm 2$ , b =  $\pm 1$

**Chapter 5**

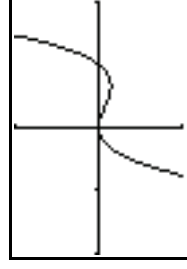
**Exercise 5.1**

- 1 a dom =  $\{2, 3, -2\}$ , ran =  $\{4, -9, 9\}$  b dom =  $\{1, 2, 3, 5, 7, 9\}$ , ran =  $\{2, 3, 4, 6, 8, 10\}$
- c dom =  $\{0, 1\}$ , ran =  $\{1, 2\}$
- 2 a  $1, \infty[$  b  $[0, \infty[$  c  $]9, \infty[$  d  $] - \infty, 1[$  e  $[-3, 3]$  f  $] - \infty, \infty[$  g  $] - 1, 0[$  h  $[0, 4[$
- i  $[0, \infty[$  j  $]1, 5[$  k  $]0, 4[$  l  $] - \infty, -1[ \cup [1, \infty[$
- 3 a  $r = [-1, \infty[$ ,  $d = [0, 2[$  b  $r = \{y : y \geq 0\} \setminus \{4\}$ ,  $d = \mathbb{R}$
- c  $r = [0, \infty[ \setminus \{3\}$ ,  $d = [-4, \infty[ \setminus \{0\}$  d  $r = [-2, 0[$ ,  $d = [-1, 2[$
- e  $r = ] - \infty, \infty[$  d =  $] - \infty, -3[ \cup [3, \infty[$  f  $r = [-4, 4[$ ,  $d = [0, 8[$
- 4 a one to many b many to one c many to one d one to one
- e many to many f one to one
- 5 a  $\mathbb{R} \setminus \{-2\}$  b  $] - \infty, 9[$  c  $[-4, 4[$  d  $] - \infty, -2[ \cup [2, \infty[$  e  $\mathbb{R} \setminus \{0\}$  f  $\mathbb{R}$
- g  $\mathbb{R} \setminus \{-1\}$  h  $] - a, \infty[$  i  $[0, \infty[ \setminus \{a^2\}$  j  $] - \infty, -a[ \cup [a, \infty[$  k  $\mathbb{R} \setminus \{-a^{-1}\}$
- 6 a  $] - \infty, -a[$  b  $]0, ab[$  c  $] - \infty, \frac{1}{4}a^3[$  d  $] \frac{1}{4}a^3, \infty[$  e  $\mathbb{R} \setminus \{a\}$  f  $] - \infty, a[$
- g  $] - a, \infty[$  h  $] - \infty, 0[$

**Exercise 5.2**

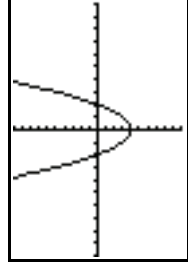
Graphs with graphics calculator output have standard viewing window unless otherwise stated.

- 1 a 3, 5 b  $i 2(x+a) + 3$  ii  $2a$  c 3
- 2 a  $0, \frac{10}{11}$  b  $-\frac{5}{4}$  c  $[\frac{10}{11}, 0[$
- 3 a  $-\frac{1}{2}x^2 - x + \frac{3}{2}$ ,  $-\frac{1}{2}x^2 + x + \frac{3}{2}$  b  $\pm\sqrt{2}$  c no solution
- 4 a  $x = 0, 1$

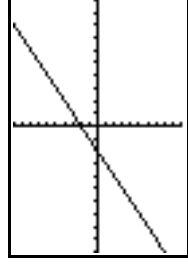


Window  $[-2, 2], [-1, 1]$   
Range:  $[-12, 4]$

5 a i



ii

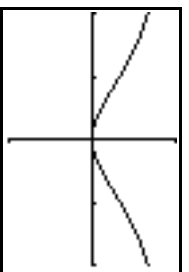


**b**  $\{2\sqrt{2}, -2\sqrt{2}\}$     **ii**  $\{3, -2\}$

**6** b, c, d, e

**8** a, d, e, f

**9** **a** Window  $[-2, 2], [-1, 1]$     **b**  $[0, 1]$



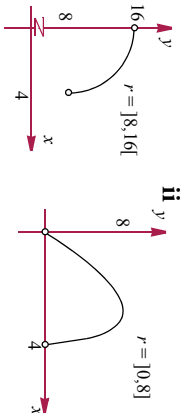
**10** **a**  $\{y: y > 1\} \cup \{y: y \leq -1.25\}$     **b** 10

**11** b 1

**12** a only – it is the only one with identical rules and domains

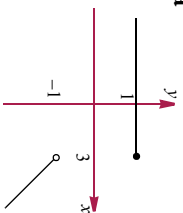
**13** **a**  $[-3, \infty[$     **b**  $[-3, 0]$     **c**  $[3, \infty[$     **d**  $[1.5, 3[ \cup ] 3, \infty[$

**14** **a**  $i$   $p(x) = 8 + 2\sqrt{16 - x^2}, 0 < x < 4$     **ii**  $f(x) = x\sqrt{16 - x^2}, 0 < x < 4$

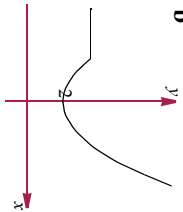


**Exercise 5.3.1**

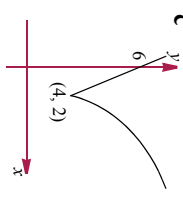
**1** **a**



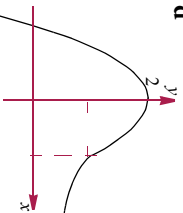
**b**



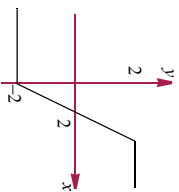
**c**



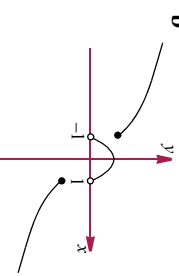
**d**



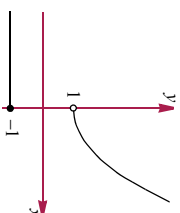
**2** **a**



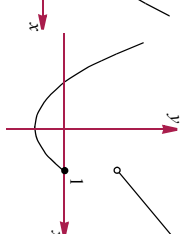
**b**



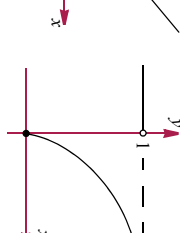
**3** **a**



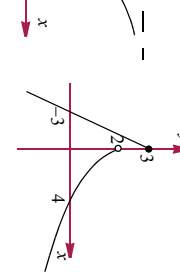
**b**



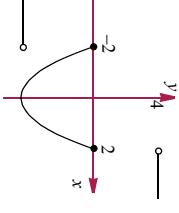
**c**



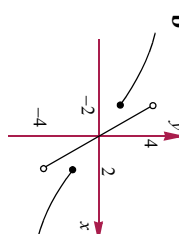
**d**



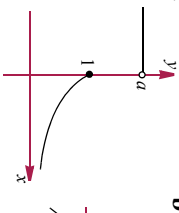
**4** **a**



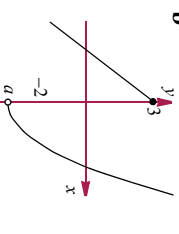
**b**



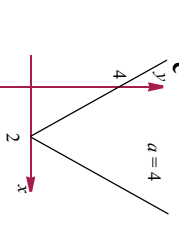
**5** **a**



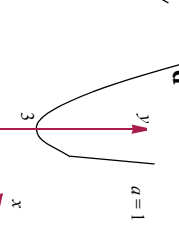
**b**



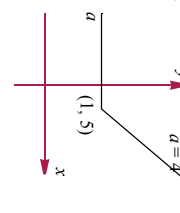
**c**



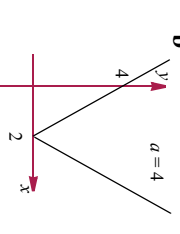
**d**



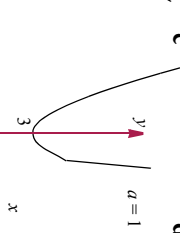
**6** **a**



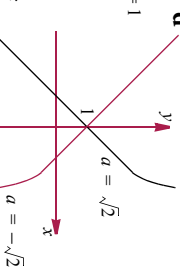
**b**



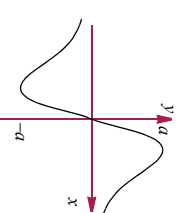
**c**



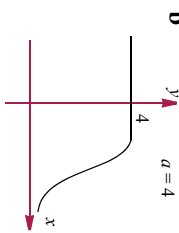
**d**



**7** **a**

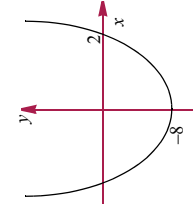
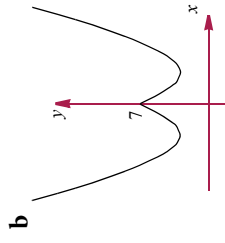
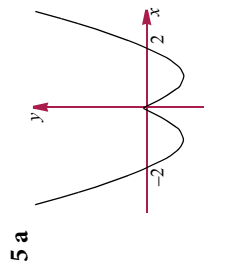
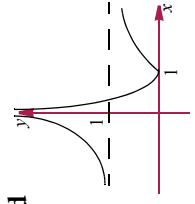
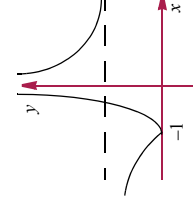
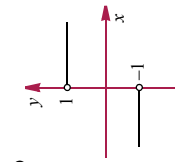
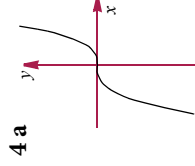
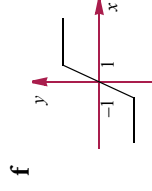
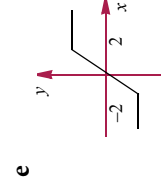
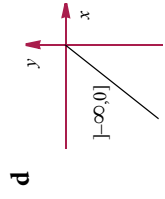
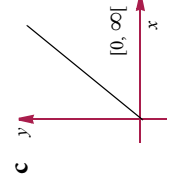
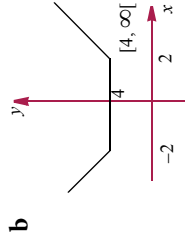
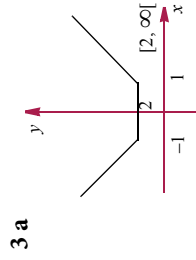
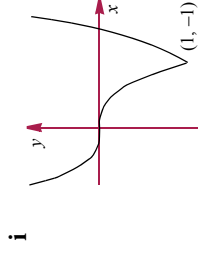
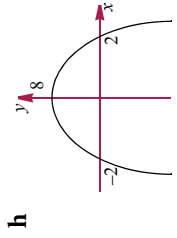
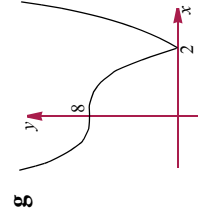
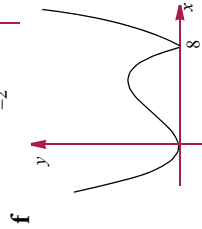
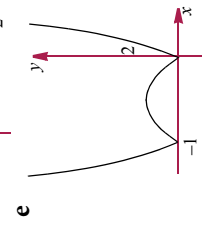
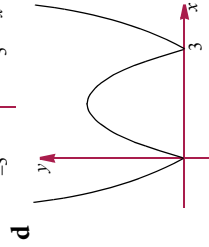
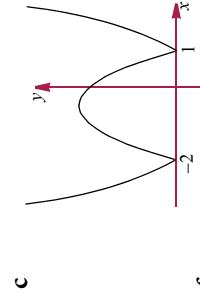
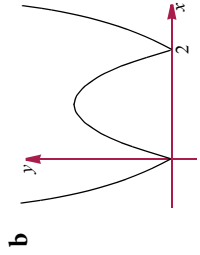
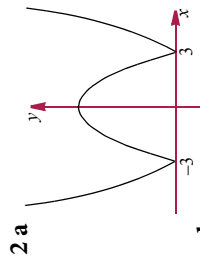
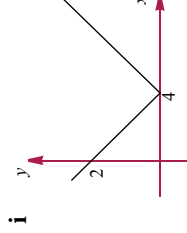
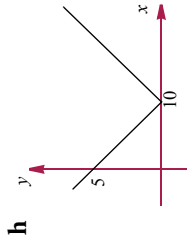
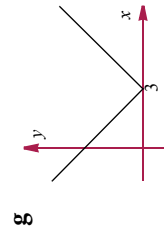
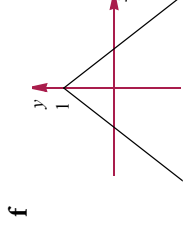
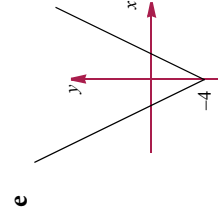
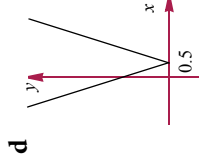
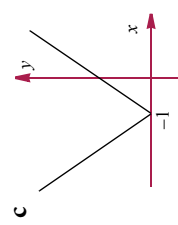
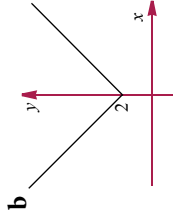
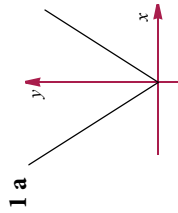


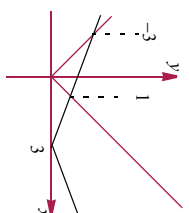
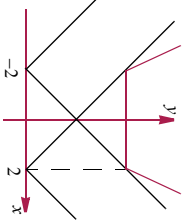
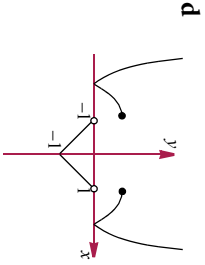
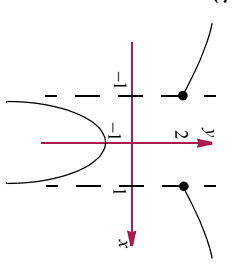
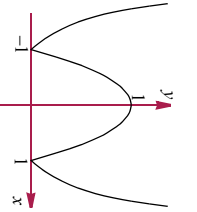
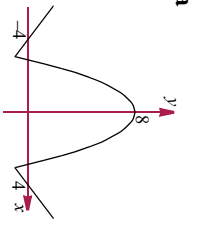
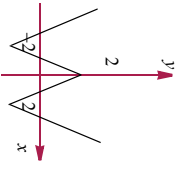
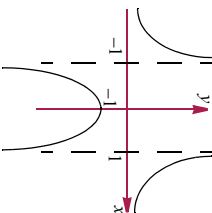
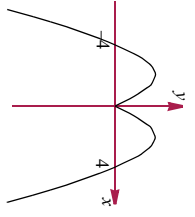
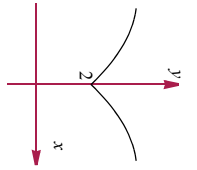
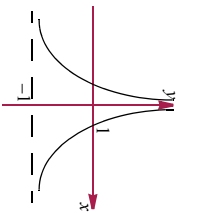
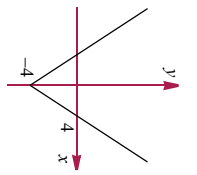
**b**





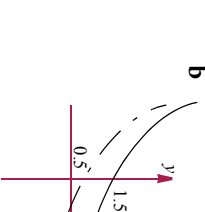
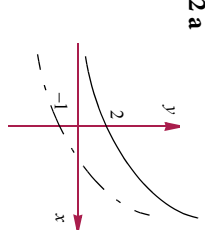
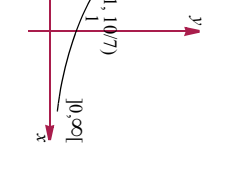
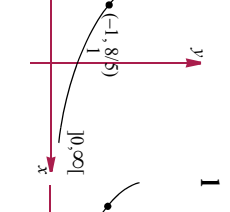
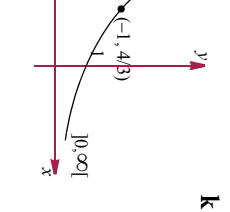
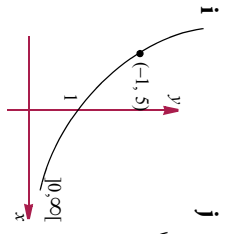
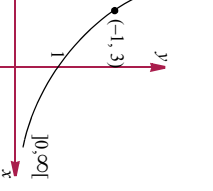
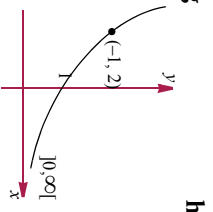
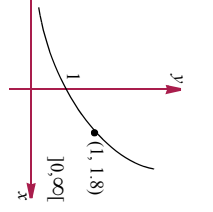
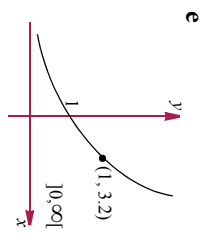
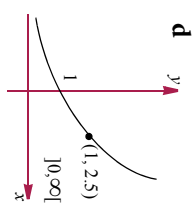
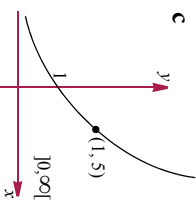
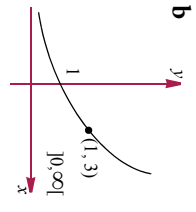
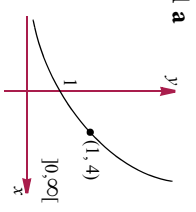
Exercise 5.3.2





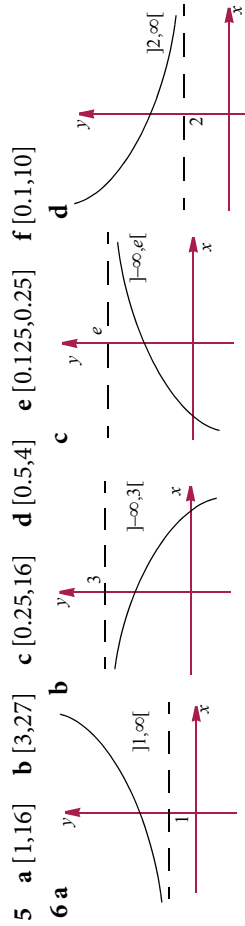
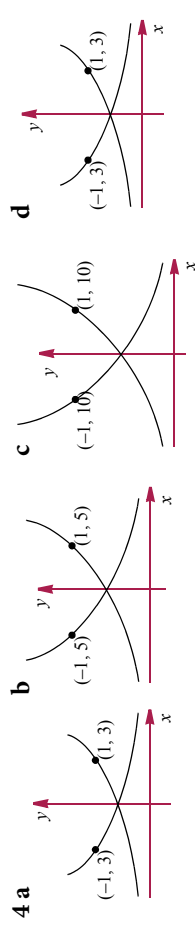
$\{x: x \leq -3\} \cup \{x: x \geq 1\}$

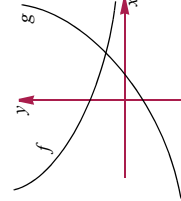
**Exercise 5.3.3**



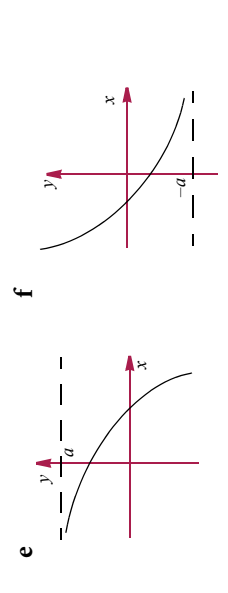
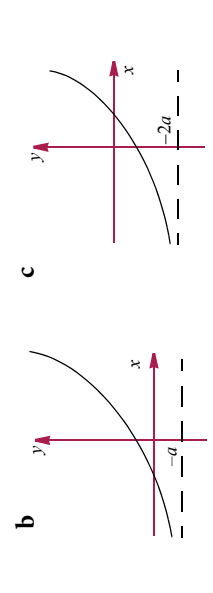
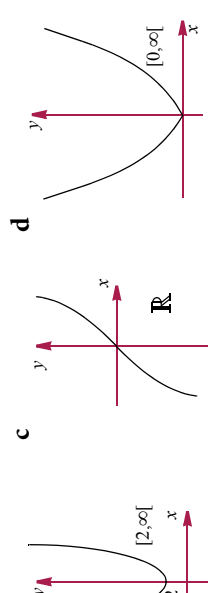
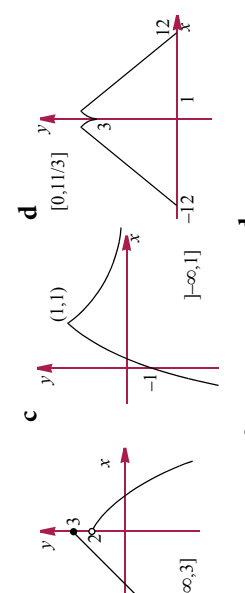
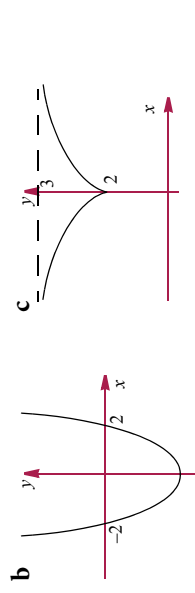
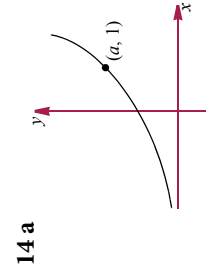
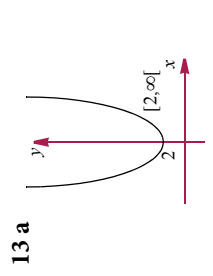
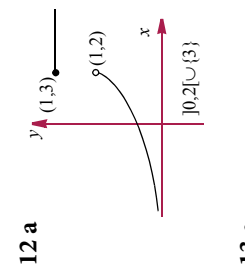
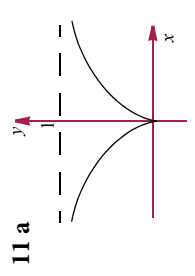
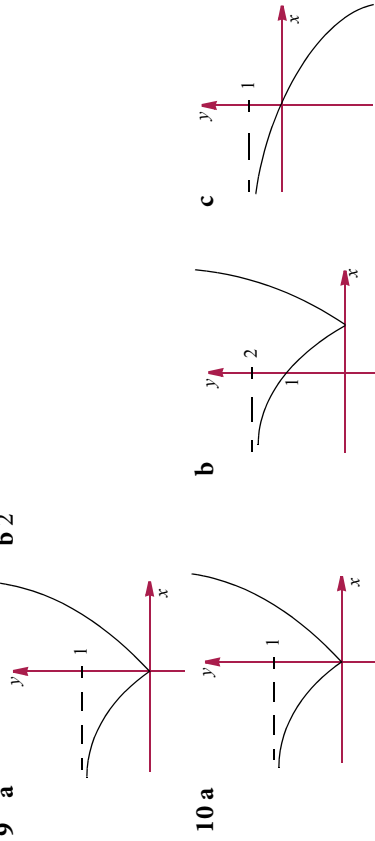
- b i  $\emptyset$  ii  $[-2, 2]$  iii  $\{\pm 4\}$

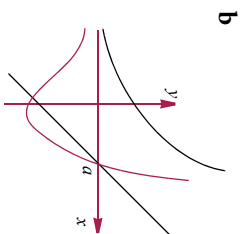
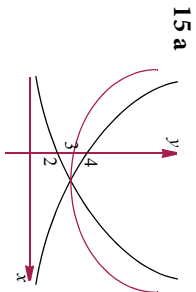
3 'b' has a dilation effect on  $f(x) = a^x$  (along the y axis).



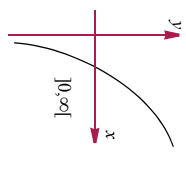
**7 a**  $-1.5$  **b**  **c**  $i$   $f = g; x = 1$  **ii**  $f > g; x < 1$

**8 a**  $[2, 2 + e^{-1}]$  **b**  $[-1, 1]$  **c**  $[1 - e, 1 + e^{-1}]$

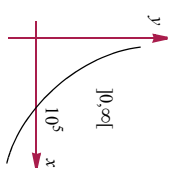




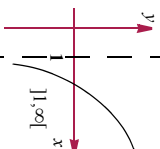
2 a



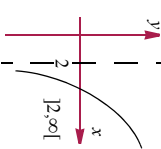
b



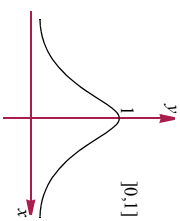
c



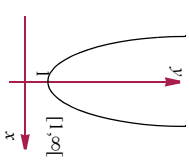
f



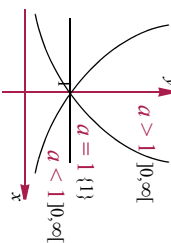
16 a



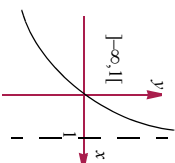
b



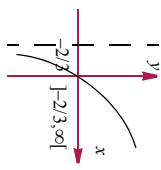
c



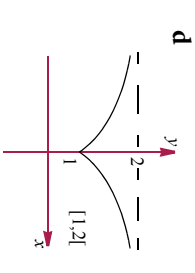
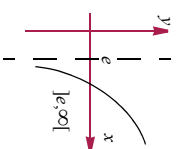
3 a



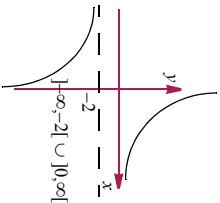
b



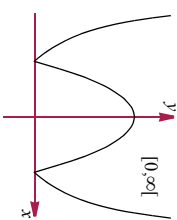
c



e

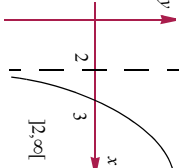


f

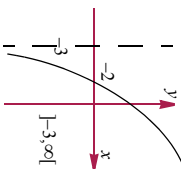


Exercise 5.3.4

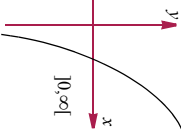
1 a



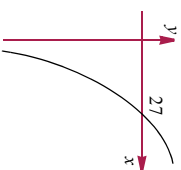
b



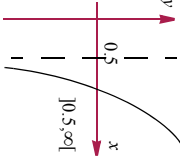
c



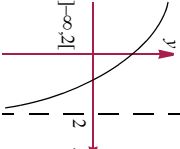
d



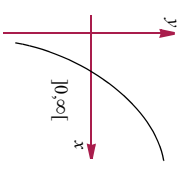
e



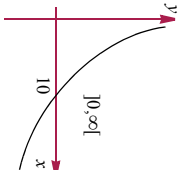
f



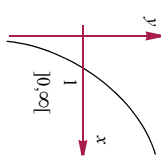
g



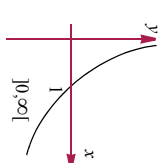
h



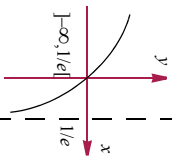
4 a



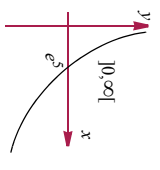
d



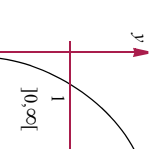
d



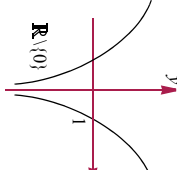
e



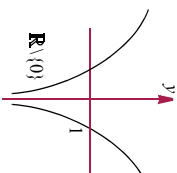
f



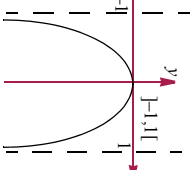
b



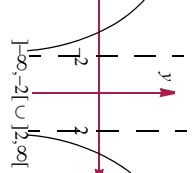
c

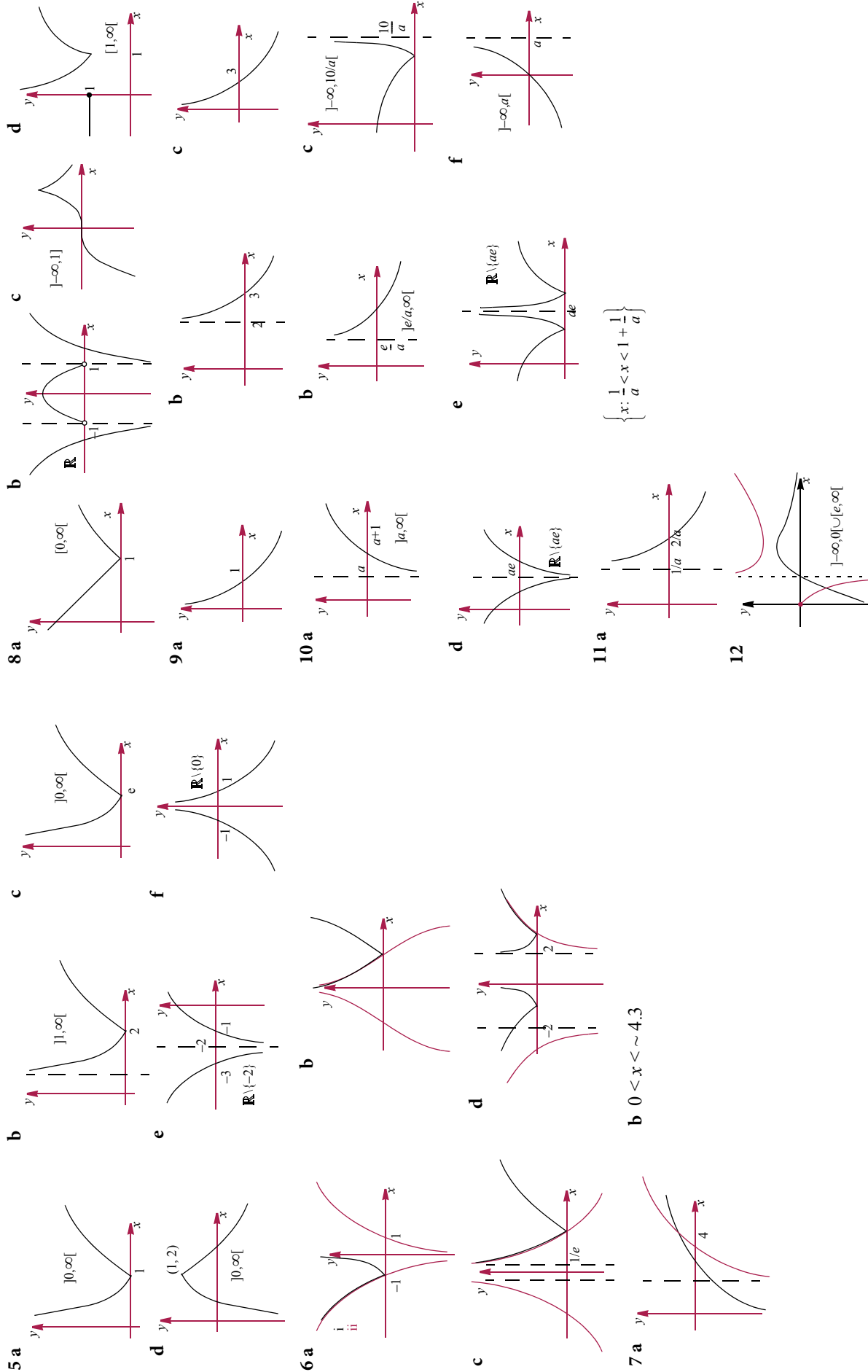


e

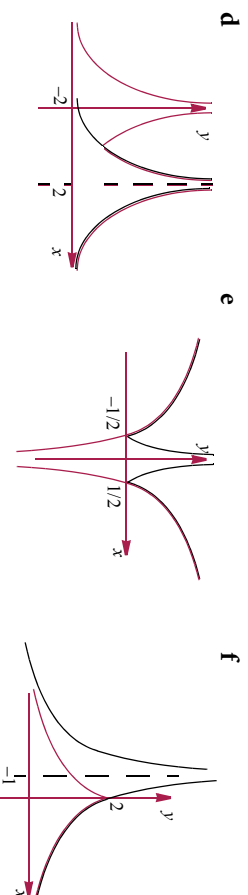
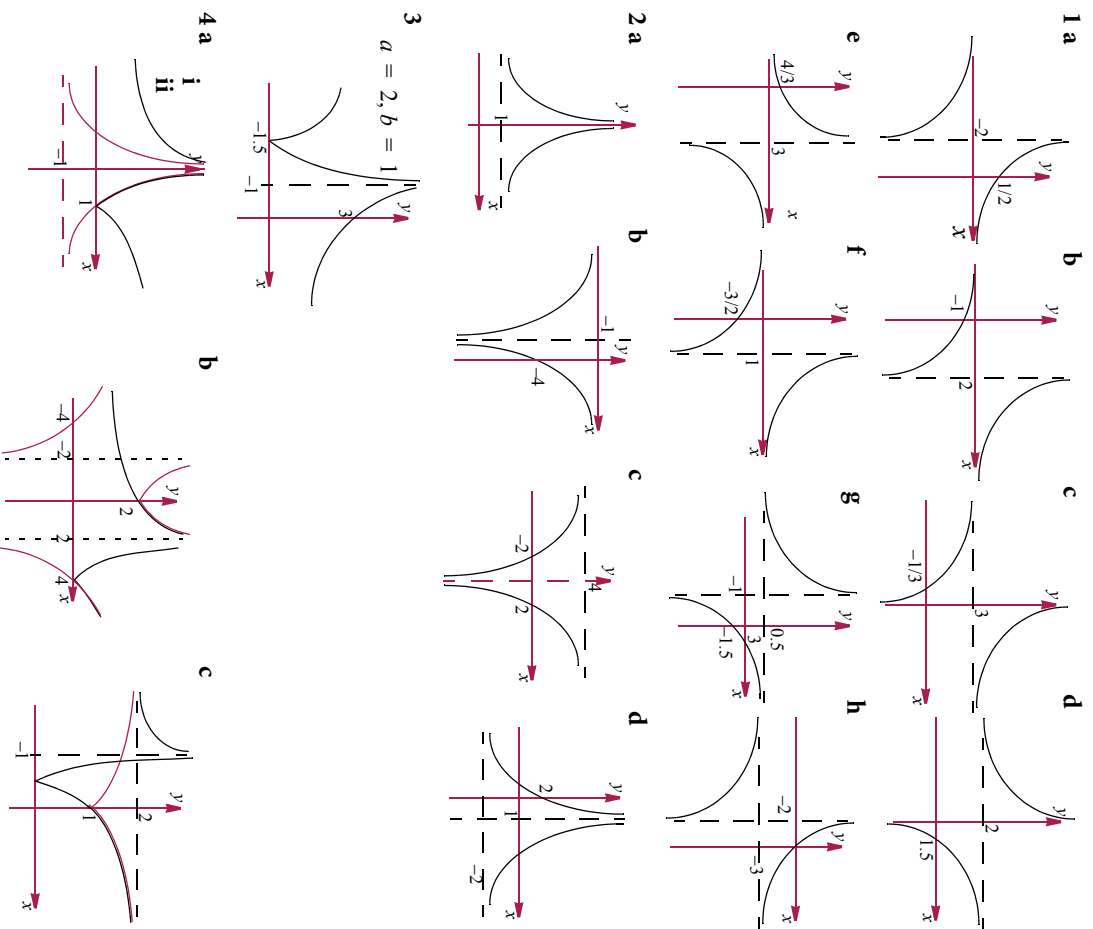


f





Exercise 5.3.5



Exercise 5.4.1

- 1 a i  $f+g: [0, \infty[ \mapsto \mathbb{R}$  where  $(f+g)(x) = x^2 + \sqrt{x}$   $[0, \infty[$   
 ii  $f+g: ]0, \infty[ \mapsto \mathbb{R}$  where  $(f+g)(x) = \frac{1}{x} + \ln(x)$   $[1, \infty[$   
 iii  $f+g: [-3, -2] \cup [2, 3] \mapsto \mathbb{R}$  where  $(f+g)(x) = \sqrt{9-x^2} + \sqrt{x^2-4}$   $[\sqrt{5}, \sqrt{10}]$   
 b i  $fg: [0, \infty[ \mapsto \mathbb{R}$  where  $(fg)(x) = x^2\sqrt{x} = x^{5/2}$   
 ii  $fg: ]0, \infty[ \mapsto \mathbb{R}$  where  $(fg)(x) = \frac{\ln(x)}{x}$   
 iii  $fg: [-3, -2] \cup [2, 3] \mapsto \mathbb{R}$  where  $(fg)(x) = \sqrt{(9-x^2)(x^2-4)}$   
 2 a i  $f-g: ]-\infty, \infty[ \mapsto \mathbb{R}$  where  $(f-g)(x) = 2e^x - 1$   $]-1, \infty[$   
 ii  $f-g: ]-1, \infty[ \mapsto \mathbb{R}$  where  $(f-g)(x) = (x+1) - \sqrt{x+1}$   $]-0.25, \infty[$   
 iii  $f-g: ]-\infty, \infty[ \mapsto \mathbb{R}$  where  $(f-g)(x) = |x-2| - |x+2|$   $[-4, 4]$   
 b i  $f/g: \mathbb{R} \setminus \{0\}, \mapsto \mathbb{R}$  where  $(f/g)(x) = \frac{e^x}{1-e^x}$   
 ii  $f/g: ]-1, \infty[ \mapsto \mathbb{R}$  where  $(f/g)(x) = \sqrt{x+1}$   
 iii  $f/g: \mathbb{R} \setminus \{-2\} \mapsto \mathbb{R}$  where  $(f/g)(x) = \frac{|x-2|}{x+2}$   
 3 a i  $fofg(x) = x^3 + 1$ ,  $gof(x) = (x+1)^3$  ii  $]-\infty, \infty[$ ,  $]-\infty, \infty[$   
 b i  $fofg(x) = x+1$ ,  $x \geq 0$ ,  $gof(x) = \sqrt{x^2+1}$  ii  $[1, \infty[$ ,  $[1, \infty[$   
 c i  $fofg(x) = x^2$ ,  $gof(x) = (x+2)^2 - 2$  ii  $[0, \infty[$ ,  $[-2, \infty[$   
 d i  $fofg(x) = x$ ,  $x \neq 0$ ,  $gof(x) = x$ ,  $x \neq 0$  ii  $\mathbb{R} \setminus \{0\}$ ,  $\mathbb{R} \setminus \{0\}$   
 e i  $fofg(x) = x$ ,  $x \geq 0$ ,  $gof(x) = |x|$  ii  $[0, \infty[$ ,  $[0, \infty[$   
 f i  $fofg(x) = \frac{1}{x^2} - 1$ ,  $x \neq 0$ ,  $gof(x)$  does not exist. ii  $]-1, \infty[$   
 g i  $fofg(x) = x^2$ ,  $x \neq 0$ ,  $gof(x) = x^2$ ,  $x \neq 0$  ii  $]0, \infty[$ ,  $]0, \infty[$

**h i**  $f \circ g(x) = |x - 4|$ ,  $g \circ f(x) = |x - 4|$    **ii**  $[-4, \infty[$ ,  $[0, \infty[$

**ii**  $f \circ g(x) = |x + 2|^3 - 2$ ,  $g \circ f(x) = |x^3|$    **iii**  $[-2, \infty[$ ,  $[0, \infty[$

**j i**  $f \circ g(x)$  does not exist,  $g \circ f(x) = (4 - x)$ ,  $x \leq 4$    **ii**  $[0, \infty[$

**k i**  $f \circ g(x) = \frac{x^2}{x^2 + 1}$ ,  $g \circ f(x) = \left(\frac{x}{x+1}\right)^2$ ,  $x \neq -1$    **ii**  $[0, 1[$ ,  $[0, \infty[$

**li**  $f \circ g(x) = x^2 + |x| + 1$ ,  $g \circ f(x) = |x^2 + x + 1|$    **iii**  $[1, \infty[$ ,  $[0.75, \infty[$

**mi**  $f \circ g(x) = 2^{x^2}$ ,  $g \circ f(x) = 2^{2x}$    **iii**  $[1, \infty[$ ,  $[0, \infty[$

**ni**  $f \circ g(x)$  does not exist,  $g \circ f(x) = \frac{1}{x+1} - 1$ ,  $x \neq -1$    **iii**  $\mathbb{R} \setminus \{-1\}$

**oi**  $f \circ g(x)$  does not exist,  $g \circ f(x) = \frac{4}{x-1} + 1$    **iii**  $[1, \infty[$

**pi**  $f \circ g(x) = 4^{\sqrt{x}}$ ,  $x \geq 0$ ,  $g \circ f(x) = 4^{0.5x}$    **iii**  $[1, \infty[$ ,  $[0, \infty[$

**4 a**  $f \circ g(x) = 2x + 3$ ,  $x \in \mathbb{R}$    **b**  $g \circ f(x) = 2x + 2$ ,  $x \in \mathbb{R}$    **c**  $f \circ f(x) = 4x + 3$ ,  $x \in \mathbb{R}$

**5 g(x) = x^2 + 1,  $x \in \mathbb{R}$**

**6 a**  $f \circ g(x) = \frac{1}{x} + x + 1$ ,  $x \in \mathbb{R} \setminus \{0\}$ ,  $]-\infty, -1[ \cup ]3, \infty[$    **b**  $g \circ f(x)$  does not exist.

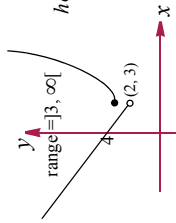
**c**  $g \circ g(x) = x + \frac{1}{x} + \frac{x}{x^2 + 1}$ ,  $x \neq 0$ ,  $]-\infty, -2.5[ \cup ]2.5, \infty[$

**7 a 9 b 3**

**9 a**  $x = \pm 1$    **b**  $x = 1, -3$

**10 a**  $\frac{1}{x}$    **b**  $\frac{-x}{2x+1}$

**11**  $h \circ f(x) = \begin{cases} (x-1)^2 + 4, & x \geq 2 \\ 5 - x, & x < 2 \end{cases}$



**12 a**  $r_f \subseteq d_g$  and  $r_{g \circ f} \subseteq d_h$    **b**  $g(x) = 4(x+1)^2$ ,  $x \in \mathbb{R}$

**13 a**  $f \circ g(x) = x$ ,  $x \in ]0, \infty[$    **range** =  $]0, \infty[$

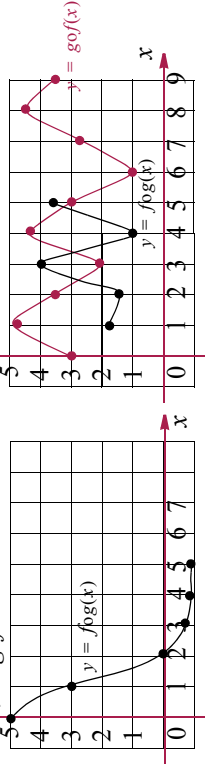
**b**  $g \circ f(x) = \frac{1}{2}(\ln(e^{2x-1}) + 1)$ ,  $x \in \mathbb{R}$  ( $= x$ )   **range** =  $]-\infty, \infty[$

**c**  $f \circ f(x) = e^{2(e^{2x-1})-1}$ ,  $x \in \mathbb{R}$    **range** =  $]e^{-1}, \infty[$

**14 a**  $h \circ k$  does not exist.   **b**  $k \circ h(x) = 4 \log(4x - 1) - 1$ ,  $x > \frac{1}{4}$ ,  $\mathbb{R}$

**15 a**  $S = \mathbb{R} \setminus ]-3, 3[$ ;  $T = \mathbb{R}$    **b**  $T = \{x : |x| \geq 6, x = 0\}$ ;  $S = ]-\infty, -3[ \cup ]3, \infty[$

**16** **y**  $g \circ f$  does not exist



**17 a**  $\text{Dom } f = ]0, \infty[$ ,  $\text{ran } f = ]e, \infty[$ ,  $\text{Dom } g = ]0, \infty[$ ,  $\text{ran } g = \mathbb{R}$

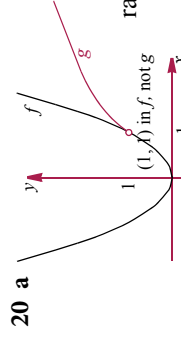
**b**  $f \circ g$  does not exist:  $r_g = \mathbb{R} \not\subseteq d_f = ]0, \infty[$

$g \circ f$  exists as  $r_f = ]e, \infty[ \subseteq d_g = ]0, \infty[$

**c**  $g \circ f(x) = |x|$ ,  $x \in \mathbb{R}$ , where  $g \circ f(x) = (x+1) + \ln 2$

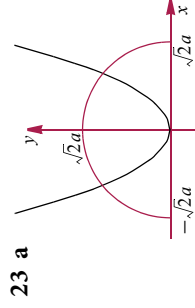
**18**  $(f \circ g)(x) = |x|$ ,  $x \in \mathbb{R}$ ; **range** =  $[0, \infty[$

**19 a** **y** **c**



**b**  $g \circ f: ]1, \infty[ \mapsto \mathbb{R}$ , where  $g \circ f(x) = x$   
**d**  $f \circ g^*: ]1, \infty[ \mapsto \mathbb{R}$ , where  $g \circ f(x) = x$

$d_f = \mathbb{R} \setminus \left\{ \frac{a}{c} \right\}$ ,  $r_f = \mathbb{R} \setminus \left\{ \frac{a}{c} \right\}$ ,  $r_f \subseteq d_f$ ,  $f \circ f(x) = x$



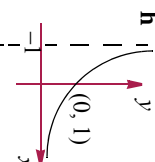
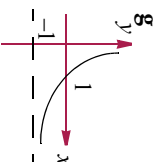
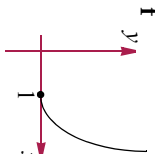
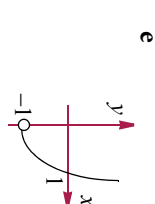
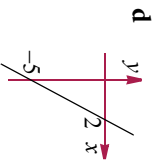
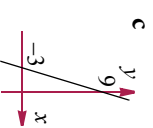
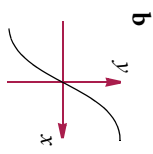
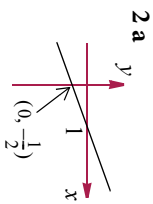
**b**  $d_{f \circ g} = [-\sqrt{2}a, \sqrt{2}a]$ ,  $f \circ g = 2a - \frac{x^2}{a}$

**c**  $d_{g \circ f} = [-2^{1/4}a, 2^{1/4}a]$ ,  $f \circ g = \frac{1}{a} \sqrt{2a^4 - x^4}$ ,  
**range** =  $[0, \sqrt{2}a]$

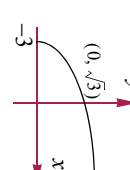
**Exercise 5.4.2**

1 a  $\frac{1}{2}(x-1), x \in \mathbb{R}$     b  $\sqrt[3]{x}, x \in \mathbb{R}$     c  $3(x+3), x \in \mathbb{R}$     d  $\frac{5}{2}(x-2), x \in \mathbb{R}$

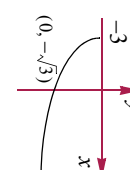
$e x^2 - 1, x > 0$     f  $(x-1)^2, x \geq 1$     g  $\frac{1}{x} - 1, x > 0$     h  $\frac{1}{(x+1)^2}, x > -1$



3a  $\sqrt{x+3}, x \geq -3$



b  $-\sqrt{x+3}, x \geq -3$



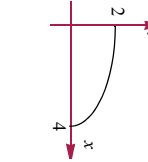
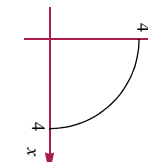
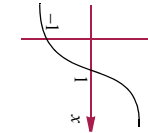
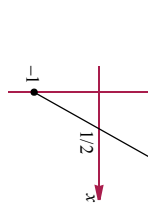
4  $\frac{\pm|x|}{\sqrt{1-x^2}}, -1 < x < 1$

5a

b

c

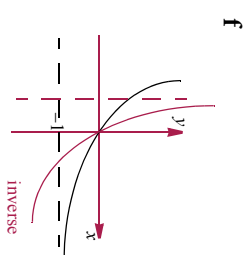
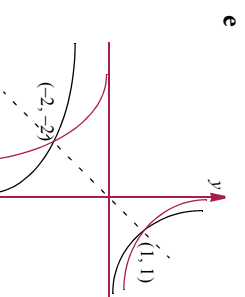
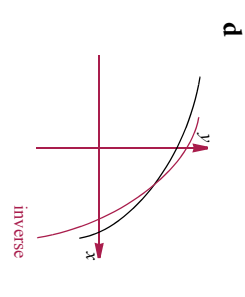
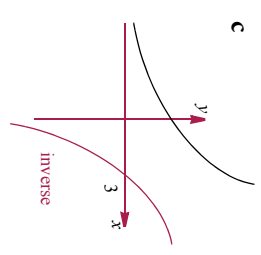
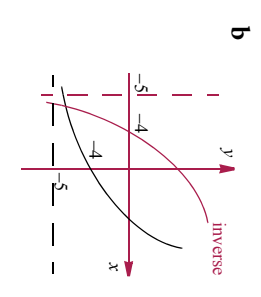
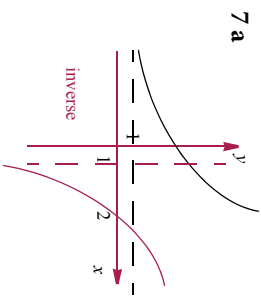
d



6 a  $f^{-1}(x) = \log_3(x-1), x > 1$     b  $f^{-1}(x) = \log_2(x+5), x > -5$

c  $f^{-1}(x) = \frac{1}{2}(\log_3 x - 1), x > 0$     d  $g^{-1}(x) = 1 + \log_{10}(3-x), x < 3$

e  $h^{-1}(x) = \log_3\left(1 + \frac{2}{x}\right), x \in \mathbb{R} \setminus [-2, 0]$     f  $g^{-1}(x) = \log_2\left(\frac{1}{x+1}\right), x > -1$

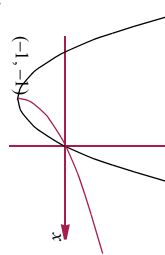


8 a  $f^{-1}(x) = 2x - 1, x \in \mathbb{R}$     b  $f^{-1}(x) = \frac{1}{2} \cdot 10^x, x \in \mathbb{R}$

c  $f^{-1}(x) = 21 - x, x \in \mathbb{R}$     d  $f^{-1}(x) = 3x + 1 + 1, x \in \mathbb{R}$

e  $f^{-1}(x) = 5x/2 + 5, x \in \mathbb{R}$     f  $f^{-1}(x) = 1 - 10^3(2-x), x \in \mathbb{R}$

9  $f^{-1}(x) = -1 + \sqrt{x+1}, x > -1$

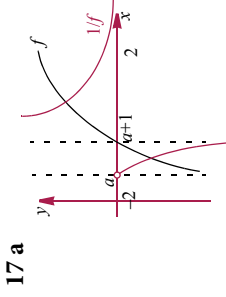


dom =  $[-1, \infty[$ , ran =  $[-1, \infty[$

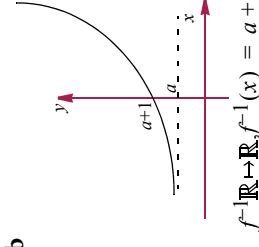
10a  $f^{-1}(x) = a - x$     b  $f^{-1}(x) = \frac{2}{x-a} + a$     c  $f^{-1}(x) = \sqrt{a^2 - x^2}$



11  $f^{-1}(x) = \sqrt[3]{2-x}$



17 a



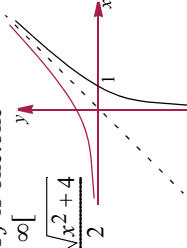
12  $[2, \infty[$

13  $\mathbb{R}^+ \setminus \{1.5\}$

14 a Inverse exists as  $f$  is one:one

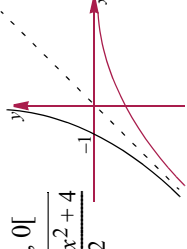
b Case 1:  $S = ]0, \infty[$

$$g^{-1}(x) = \frac{x + \sqrt{x^2 + 4}}{2}$$

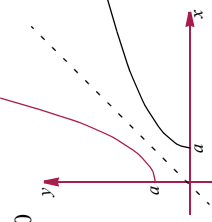


Case 2:  $S = ]-\infty, 0[$

$$g^{-1}(x) = \frac{x - \sqrt{x^2 + 4}}{2}$$

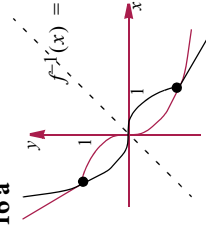


15  $f^{-1}(x) = a(x^2 + 1), x \geq 0$



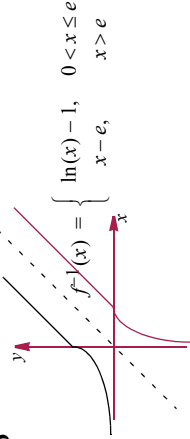
$\{x: f(x) = f^{-1}(x)\} = \emptyset$

16 a



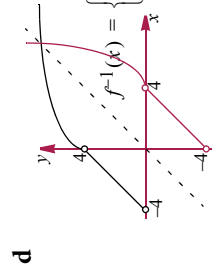
b

$$f^{-1}(x) = \begin{cases} -(2x+1), & x < -1 \\ -3\sqrt{x}, & x \geq -1 \end{cases}$$



c

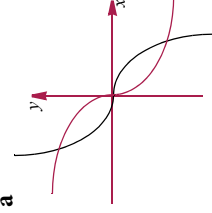
$$f^{-1}(x) = \begin{cases} 1 + e^{-x}, & x < 0 \\ 2 - x, & x \geq 0 \end{cases}$$



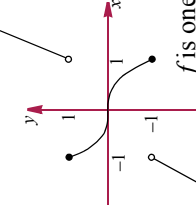
d

$$f^{-1}(x) = \begin{cases} (x-4)^2, & x > 4 \\ x-4, & 0 < x < 4 \end{cases}$$

19 a



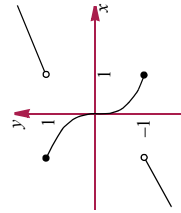
b i



ii

$$f(x) = \begin{cases} \frac{1}{2}(x-1) & x < -1 \\ -3\sqrt{x} & -1 \leq x \leq 1 \\ \frac{1}{2}(x+1) & x > 1 \end{cases}$$

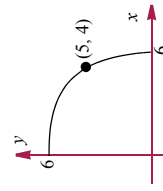
iii



iv  $\{-1, 0, 1\}$

$f$  is one:one

18  $\text{gof}$  exists as  $r_f \subseteq d_g$ . It is one:one so the inverse exists:



19 a i  $\text{tom}(x) = e^{\sqrt{x}}, x \geq 0$  ii  $\text{mot}(x) = \sqrt{e^x}, x \in \mathbb{R}$

b i  $(\text{tom})^{-1}(x) = (\ln(x))^2, x > 0$

ii  $(\text{mot})^{-1}(x) = \ln x^2, x > 0$

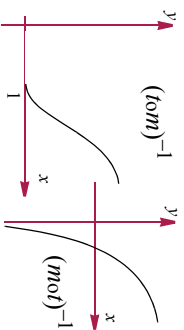
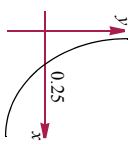
c i & ii neither exist

d Adjusting domains so that the functions in part c exist, we have:

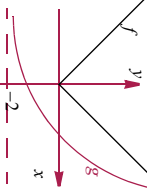
$r^{-1} \circ m^{-1}(x) = (\text{mot})^{-1}(x)$  and  $m^{-1} \circ r^{-1}(x) = (\text{tom})^{-1}(x)$

e Yes as rules of composition OK.

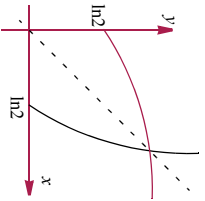
20 a 1 b  $y^y$  0.206



21 a  $f$   $y$   $y$   $g$   $f \circ g$   $g \circ f$  b  $f \circ g$  exists but is not one-one



c i  $B = [\ln 2, \infty[$  ii  $(f \circ g)^{-1}: [0, \infty[ \mapsto \mathbb{R}$  where,  $(f \circ g)^{-1}(x) = \ln(x+2)$

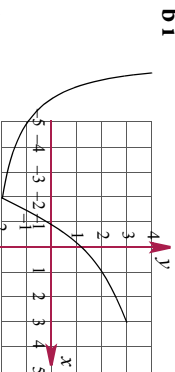
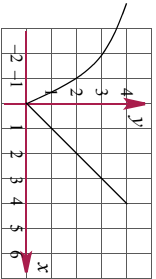


Chapter 6

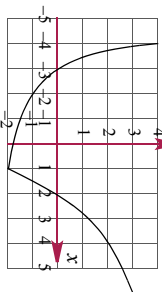
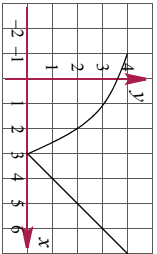
Exercise 6.1

- 1 a  $y = (x-4)^2$  b  $y = (x+2)^2$  c  $y = x^2 + 5$  d  $(x-2)^2 + y = 2$   
 e  $x^2 + y = 4$  f  $x^2 + y = 0$  g  $y = \frac{8}{x-4}, x \neq 4$  h  $y = \frac{8}{x-1}, x \neq 1$   
 i  $(x+1)^2 + y^2 = 4$  j  $y^2 = \frac{9}{x-3}, x \neq 3$  k  $(y+3)^2 = \frac{9}{x}, x \neq 0$  l  $x + y^2 = 8$

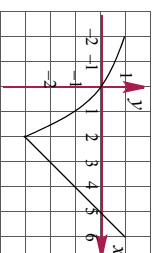
2 a i b i



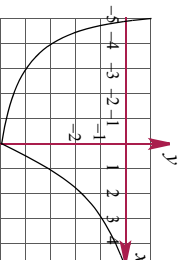
a ii b ii



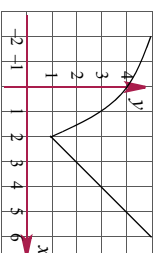
a iii



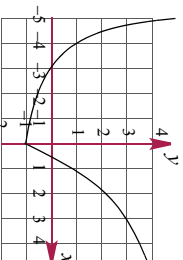
b iii



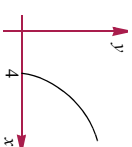
a iv



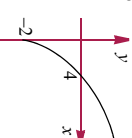
b iv



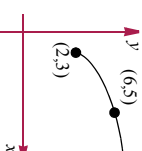
3 a



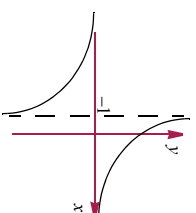
b



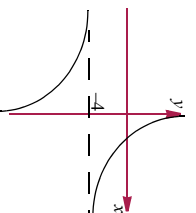
c



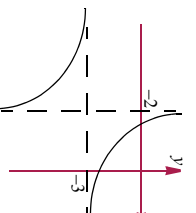
4 a



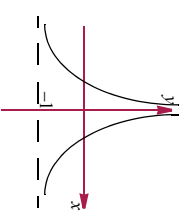
b



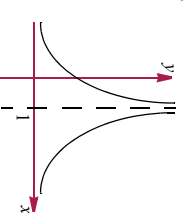
c



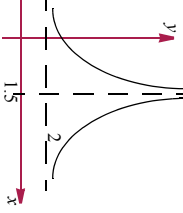
5 a



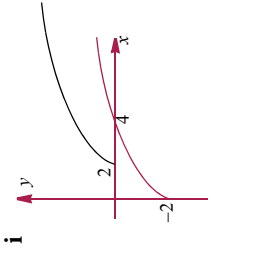
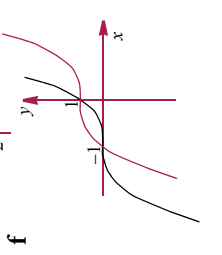
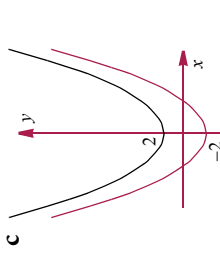
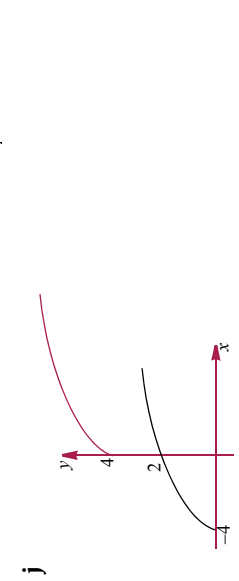
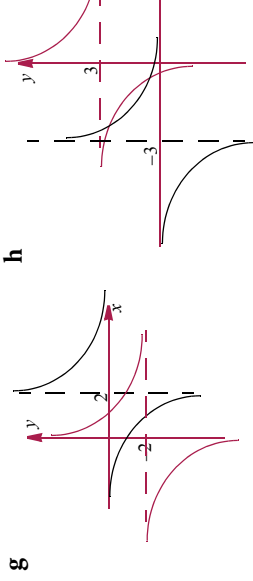
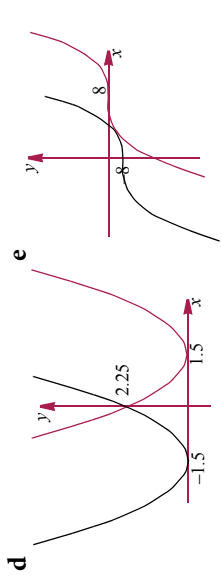
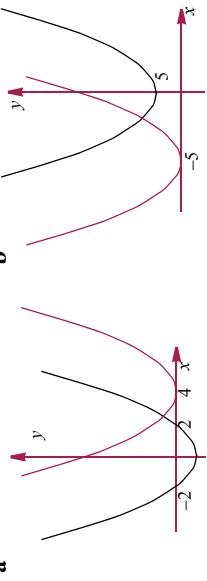
b



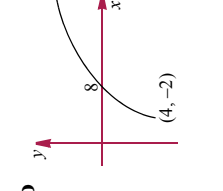
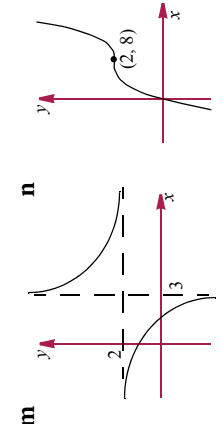
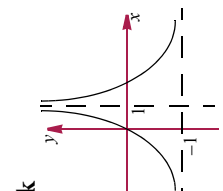
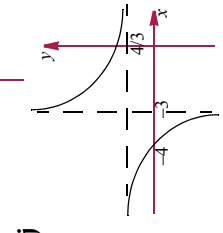
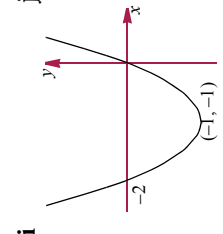
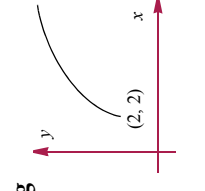
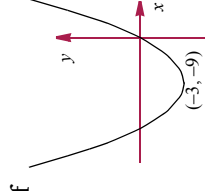
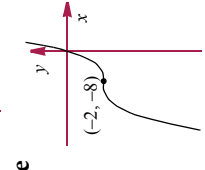
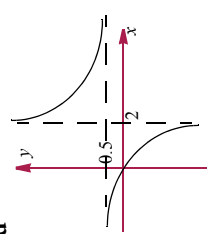
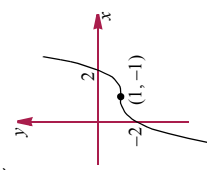
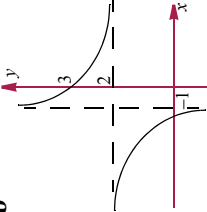
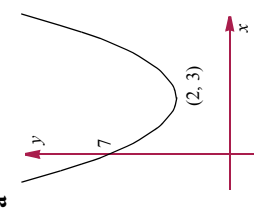
c



6 First function in black, second function in maroon



7 Note: coordinates were asked for. We have labelled most of these with single numbers.

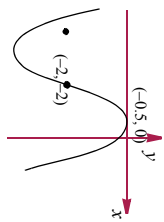


- 8  $a \begin{pmatrix} 0 \\ 4 \end{pmatrix}$   $b \begin{pmatrix} 0 \\ -2 \end{pmatrix}$   $c \begin{pmatrix} -1 \\ 0 \end{pmatrix}$   $d \begin{pmatrix} 2 \\ 0 \end{pmatrix}$   $e \begin{pmatrix} -2 \\ 0 \end{pmatrix}$   $f \begin{pmatrix} 0 \\ -4 \end{pmatrix}$   $g \begin{pmatrix} 2 \\ -2 \end{pmatrix}$   
 $h \begin{pmatrix} -2 \\ 3 \end{pmatrix}$   $i \begin{pmatrix} 4 \\ 2 \end{pmatrix}$   $j \begin{pmatrix} 2 \\ 3 \end{pmatrix}$   $k \begin{pmatrix} 3 \\ -1 \end{pmatrix}$   $l \begin{pmatrix} -k \\ h \end{pmatrix}$   $m \begin{pmatrix} -2 \\ 4 \end{pmatrix}$   $n \begin{pmatrix} 1 \\ -1 \end{pmatrix}$   
 $o \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

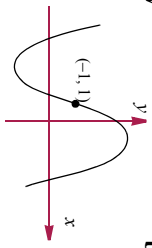
9  $a g(x) = f(x-1)+1$   $b g(x) = f(x+2)-4$   $c g(x) = f(x-2)$

d  $g(x) = f(x-1) + 1$     e  $g(x) = f(x-1) + 3$

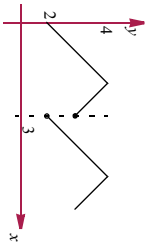
10 a i



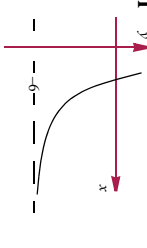
iv



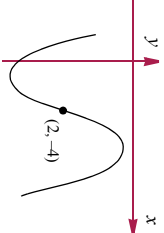
iii



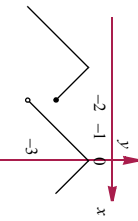
ii



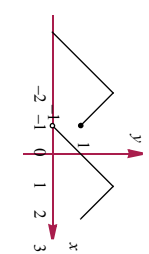
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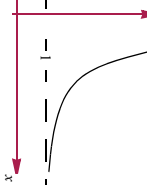
b i



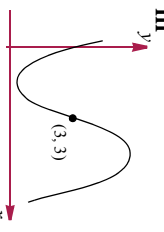
iv



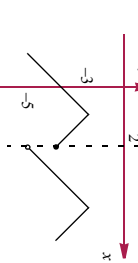
iii



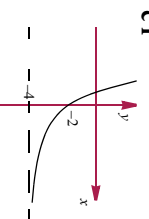
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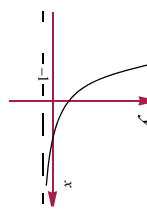
ii



c i

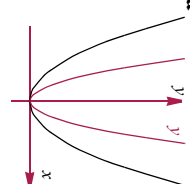


iv

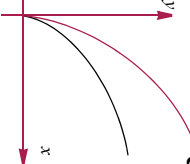


Exercise 6.2

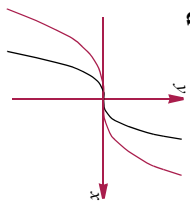
1 a



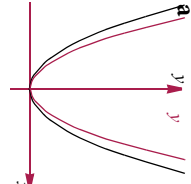
b



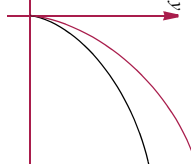
d



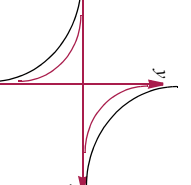
2 a



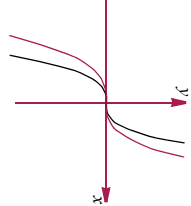
b



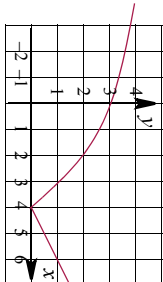
c



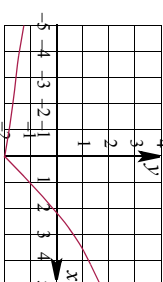
d



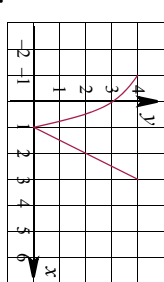
3 a i



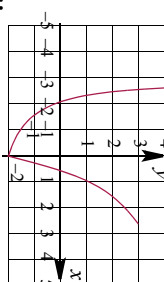
b i



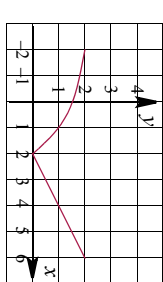
ii



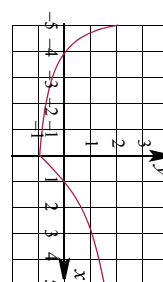
ii



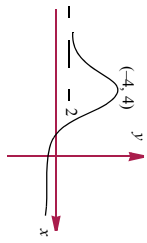
iii



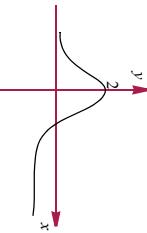
iii



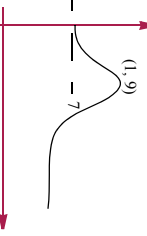
d i



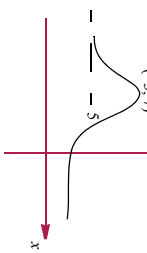
ii



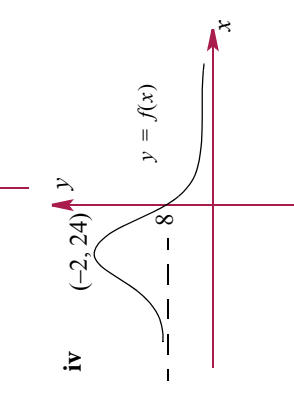
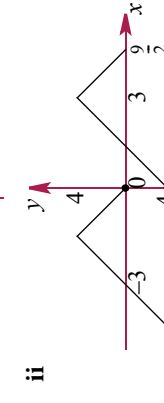
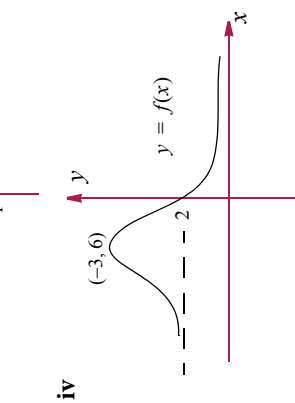
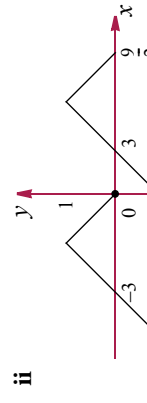
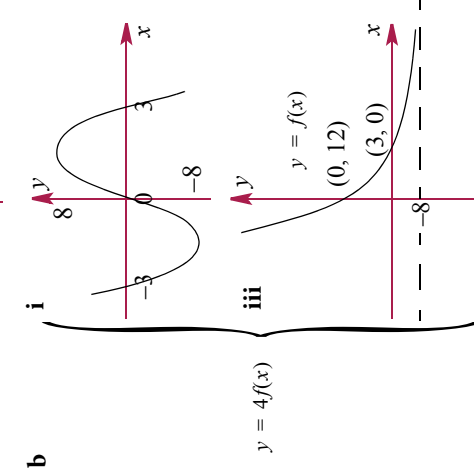
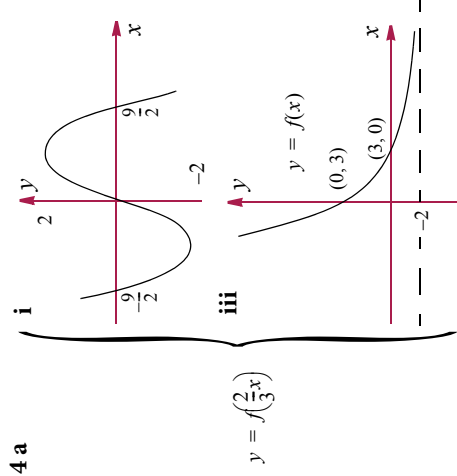
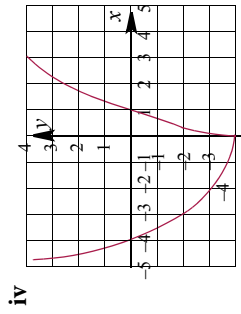
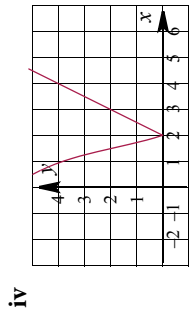
iii



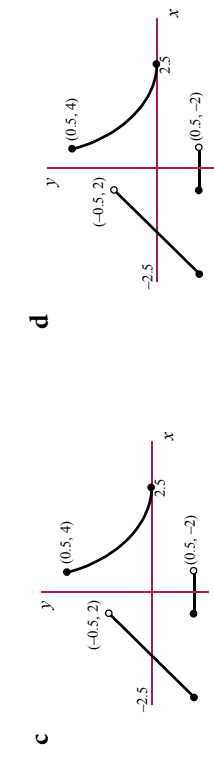
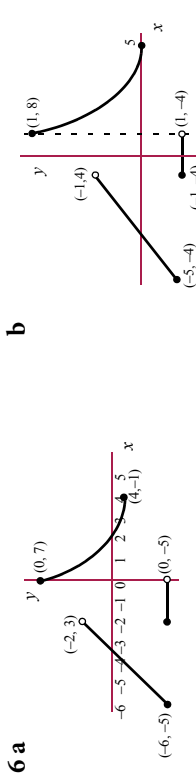
iv



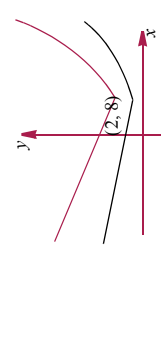
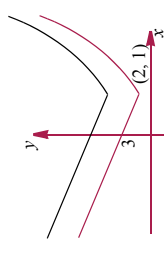
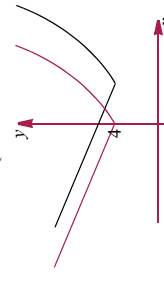
11  $y = \begin{cases} f(x+2) + 2, & -3 \leq x \leq -1 \\ f(x+4) + 2, & -5 \leq x \leq -3 \end{cases}$



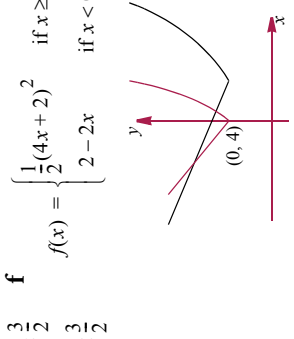
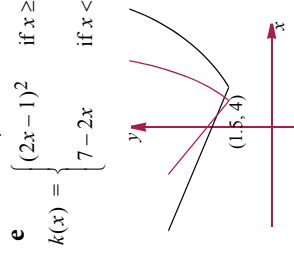
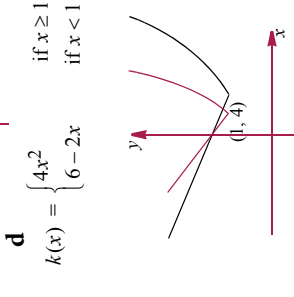
5 a  $f(x) = |x|$     $y = f(2x) + 1$    **b**  $f(x) = x^2$     $y = \frac{1}{2}f(x-2) - 3$   
 c  $f(x) = \frac{1}{x}$     $y = \frac{1}{2}f(x - \frac{1}{2})$    **d**  $f(x) = x^3$     $y = 27f(x - \frac{2}{3})$   
 e  $f(x) = x^4$     $y = 128f(x - \frac{1}{2}) - 2$    **f**  $f(x) = \sqrt{x}$     $y = \sqrt{2}f(x) + 2$

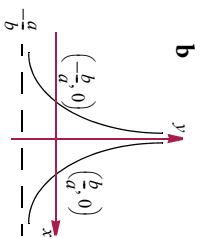
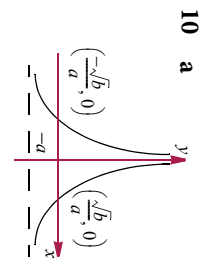
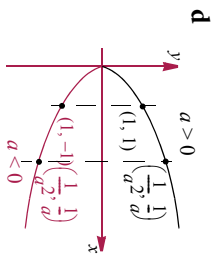
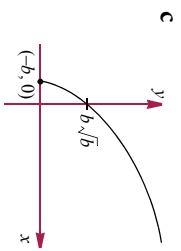
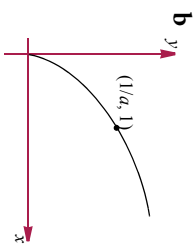
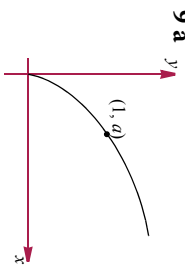
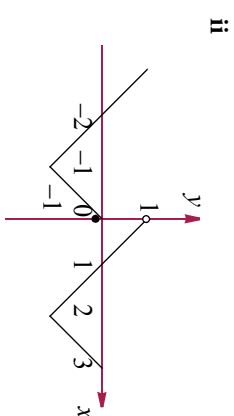
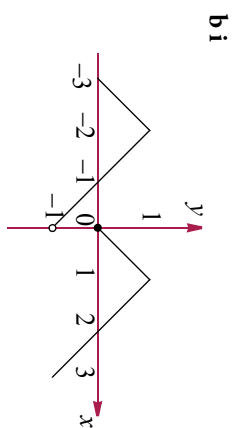
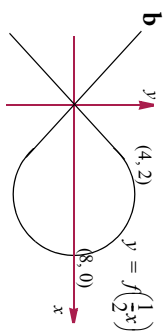
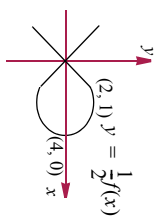
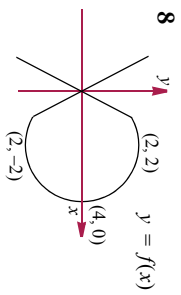


7 a  $f(x) = \begin{cases} (x+2)^2 & \text{if } x \geq 0 \\ 4-x & \text{if } x < 0 \end{cases}$    **b**  $h(x) = \begin{cases} x^2-3 & \text{if } x \geq 2 \\ 3-x & \text{if } x < 2 \end{cases}$    **c**  $h(x) = \begin{cases} 2x^2 & \text{if } x \geq 2 \\ 12-2x & \text{if } x < 2 \end{cases}$

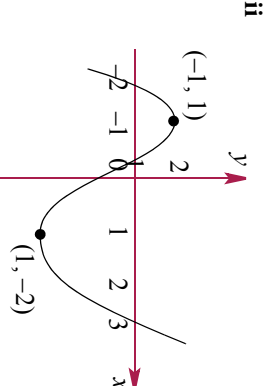
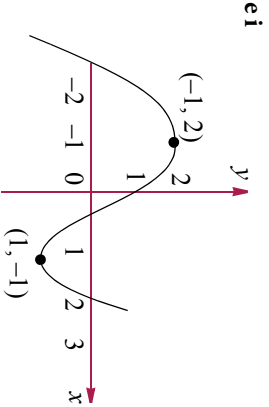
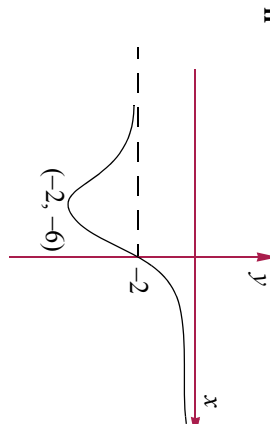
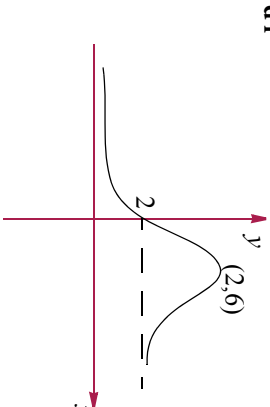
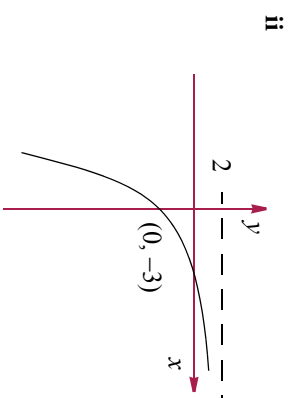
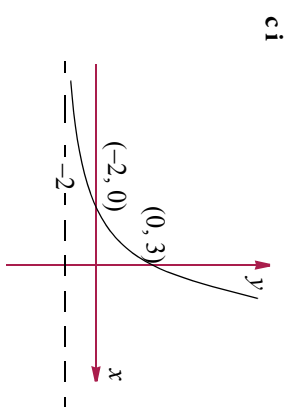
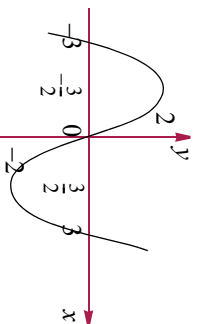
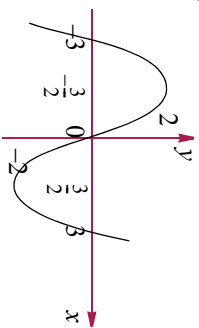


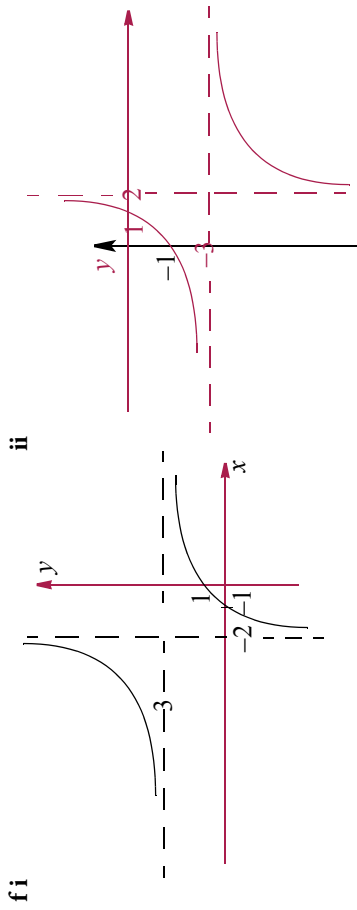
**d**  $k(x) = \begin{cases} 4x^2 & \text{if } x \geq 1 \\ 6-2x & \text{if } x < 1 \end{cases}$    **e**  $k(x) = \begin{cases} (2x-1)^2 & \text{if } x \geq \frac{3}{2} \\ 7-2x & \text{if } x < \frac{3}{2} \end{cases}$    **f**  $f(x) = \begin{cases} \frac{1}{2}(4x+2)^2 & \text{if } x \geq 0 \\ 2-2x & \text{if } x < 0 \end{cases}$



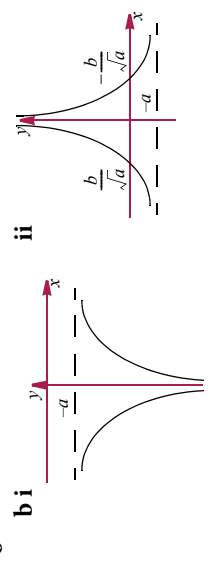
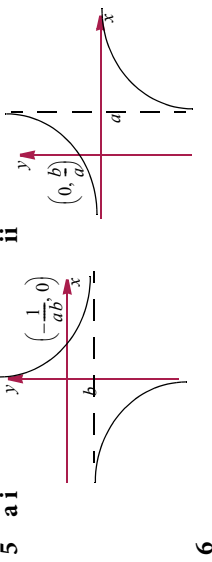
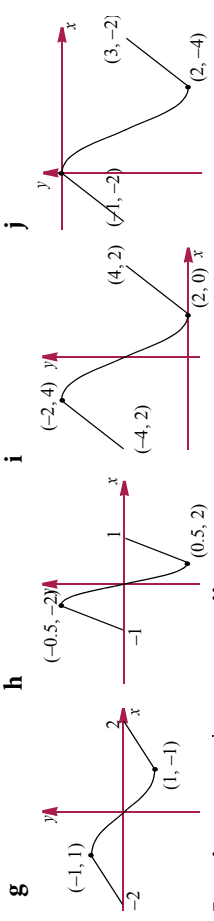
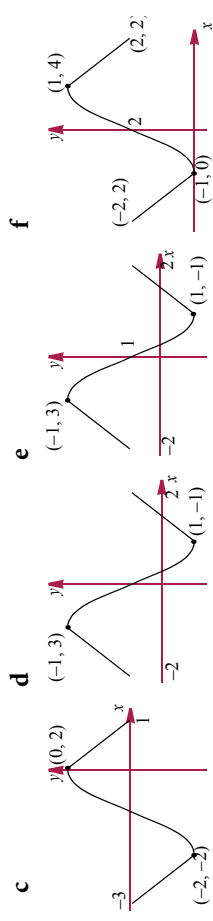
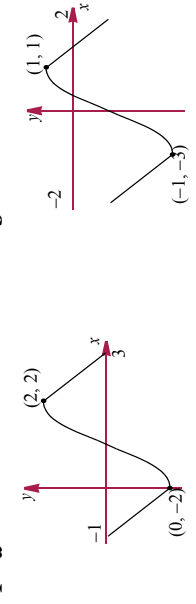
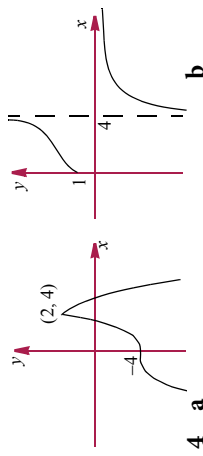
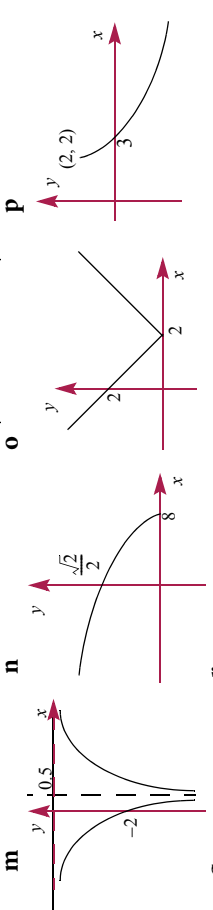
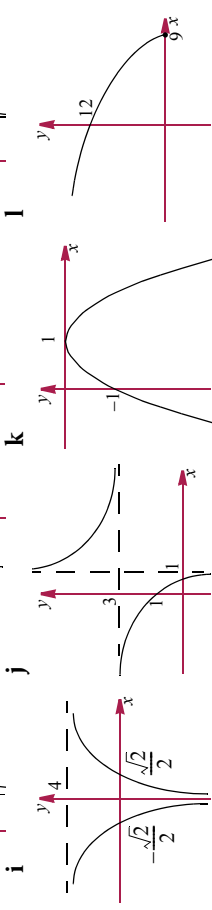
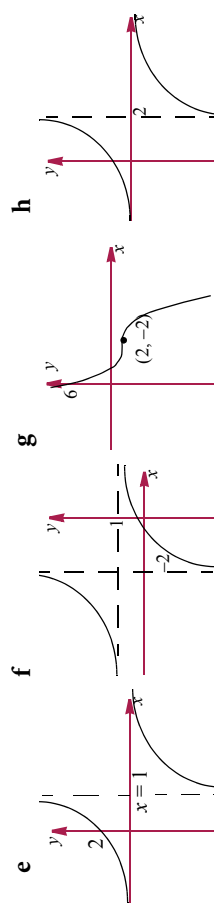
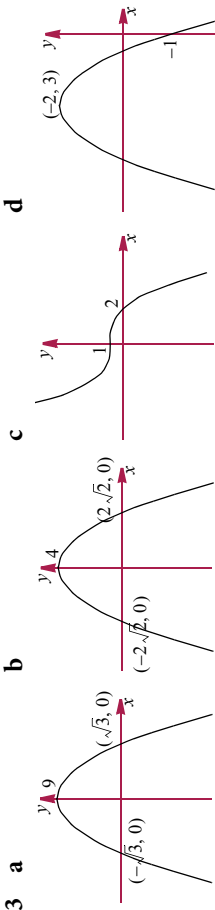


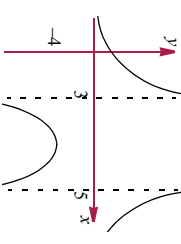
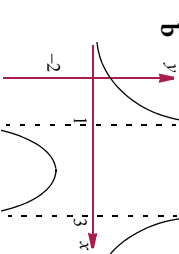
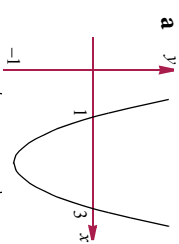
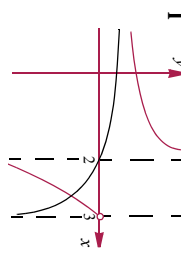
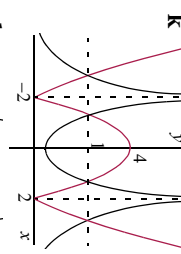
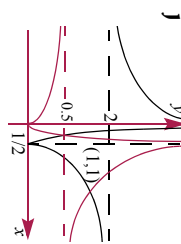
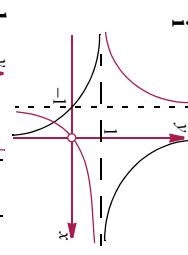
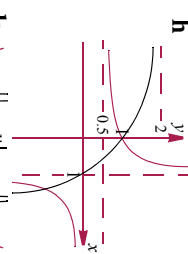
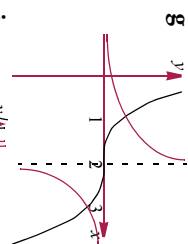
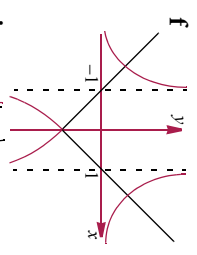
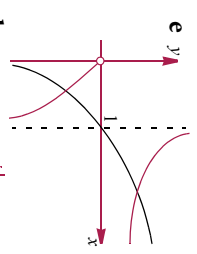
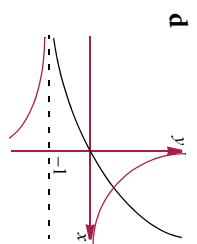
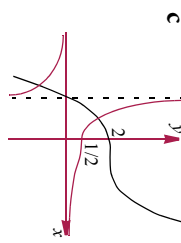
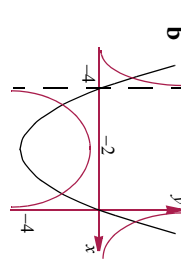
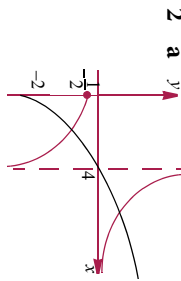
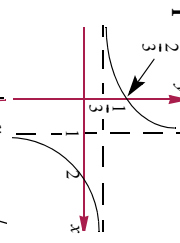
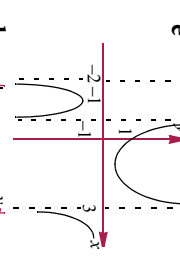
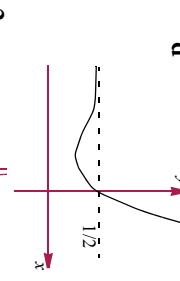
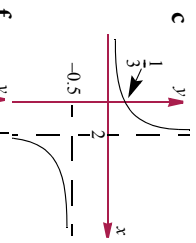
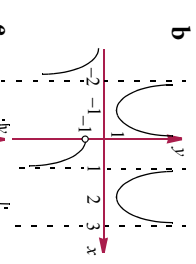
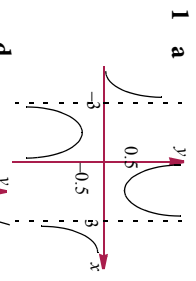
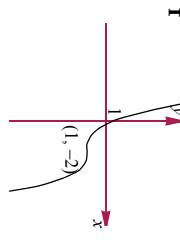
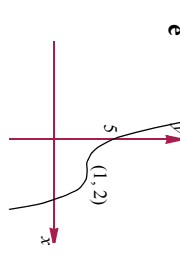
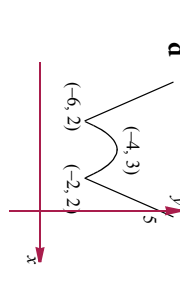
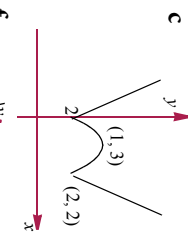
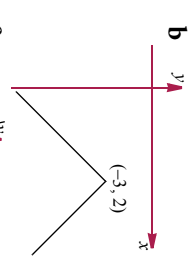
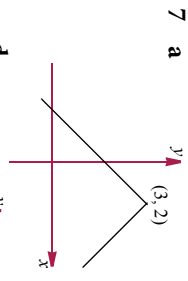
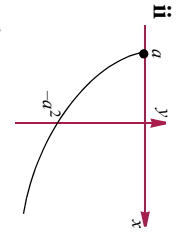
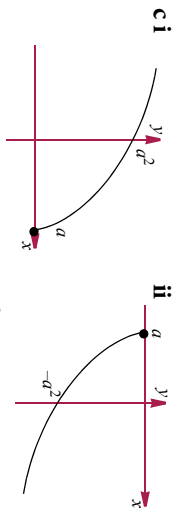
**Exercise 6.3**  
1 a i



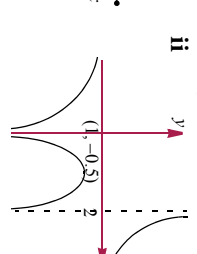
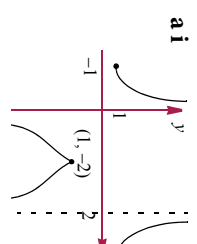


**2 a**  $y = -f(x)$    **b**  $y = f(-x)$    **c**  $y = f(x+1)$    **d**  $y = f(2x)$    **e**  $y = 2f(x)$

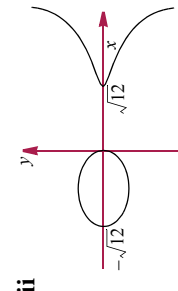
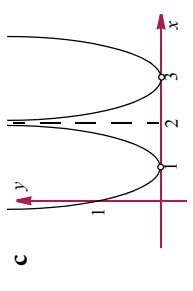
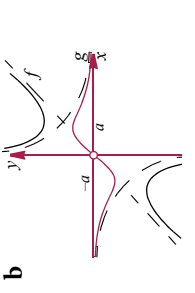
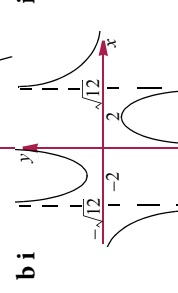
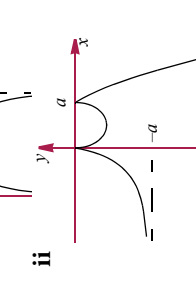
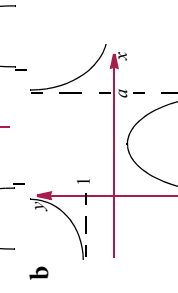
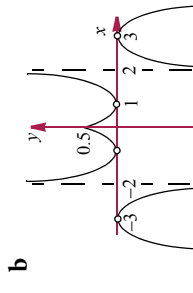
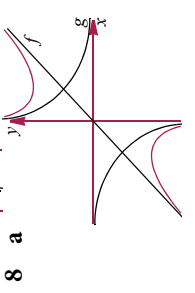
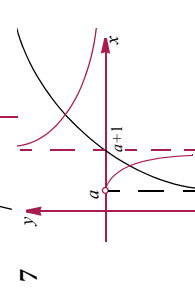
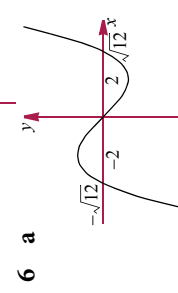
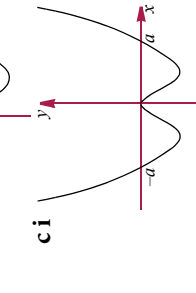
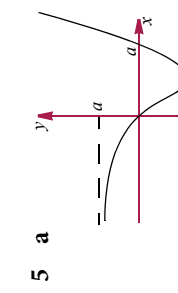
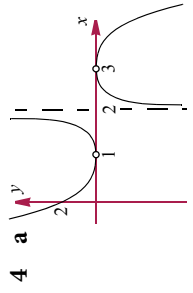
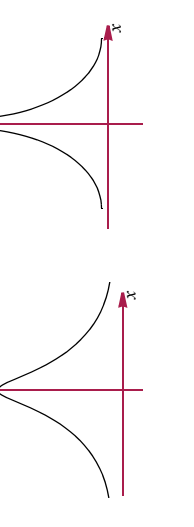
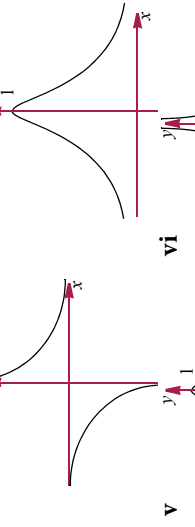
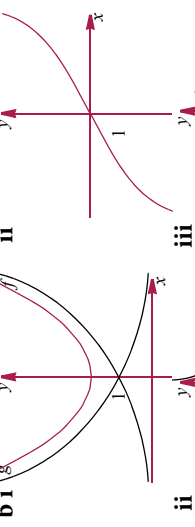
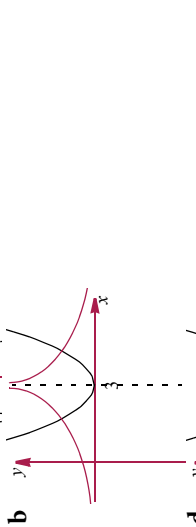
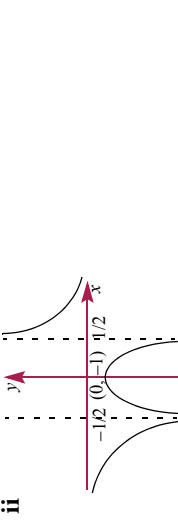
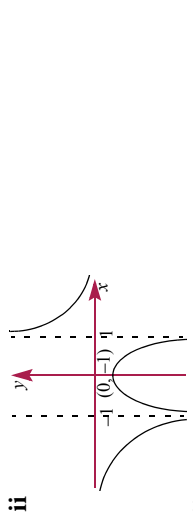
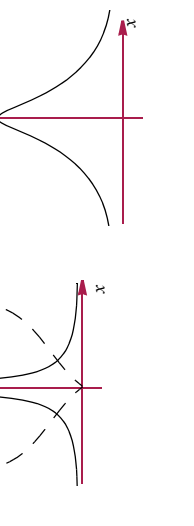
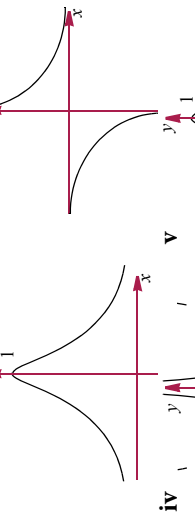
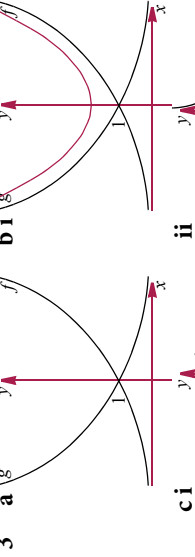
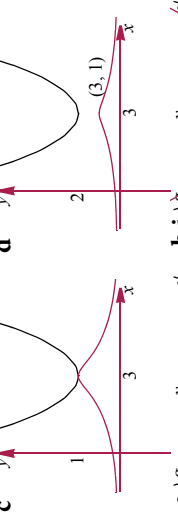
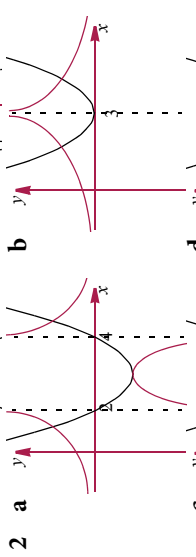
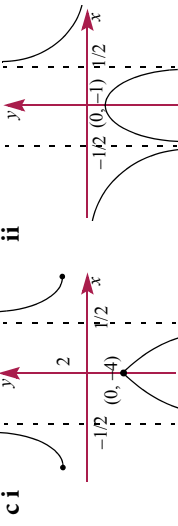
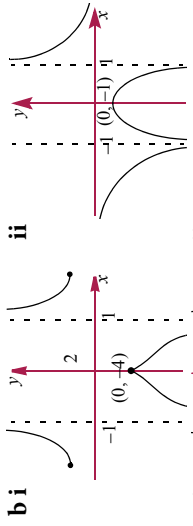




**Exercise 6.5 Miscellaneous questions**







Chapter 7

Exercise 7.1.1

- 1 a  $\frac{27y^{15}}{8x^3}$  b  $\frac{9l}{216a^6}$  c  $2^n + 2$  d  $\frac{8x^{11}}{27y^2}$  e  $\frac{3x^2y^2}{8}$   
 f  $3^n + 1 + 3$  g  $4^{n+1} - 4$  h  $2(4^{n+1} - 4)$  i  $\frac{1-b^6}{16b^4}$
- 2 a 64 b  $\left(\frac{2}{3}\right)^x$  c)  $2^{2y+1}$  d  $\frac{1}{b^{2x}}$  e  $\left(\frac{1}{2}\right)^6$  f  $\left(\frac{9}{2}\right)^n + 2$
- 3 a  $\frac{z^2}{xy}$  b  $3^{7n} - 2$  c  $5^n + 1$  d 9 e  $26^n + 1$  f  $21 - 3^n$  g  $x^2 + 4n - n^2$   
 h  $x^{3n^2+n+1}$  i 27
- 4  $\frac{y^{2m-2}}{x^m}$
- 5 a -81 b  $\frac{9x^8}{8y^4}$  c  $y^{-x}$  d  $\frac{2x+1}{x+1}$  e -1 f -b
- 6 a  $\frac{1}{x^2y^2}$  b  $\frac{1}{x^4}$  c  $\frac{1}{x(\alpha+h)}$  d  $\frac{1}{x-1}$  e  $\frac{1}{(x+1)(\alpha-1)^5}$  f  $\frac{1}{x^2}$
- 7 a  $118 \times 5^{n-2}$  b 1 c  $\frac{b^7}{a^4}$  d  $a^{mn}$  e  $\frac{p+q}{pq}$  f  $\frac{2\sqrt{a}}{a-1}$  g  $\frac{7}{8}$  h  $a^{7/8}$
- 8 a  $x^{11/12}$  b  $2a^{3n-2}b^{2n-2}$  c  $2^n$  d  $\frac{7^{m-n}}{8}$  e  $\frac{6 \times 5^n}{5^{n+5}}$  f  $x+1$

Exercise 7.1.2

- 1 a 2 b -2 c  $\frac{2}{3}$  d 5 e 6 f -2.5 g 2 h 1.25 i  $\frac{1}{3}$
- 2 a -6 b  $-\frac{2}{3}$  c -3 d 1.5 e 0.25 f 0.25 g  $-\frac{1}{8}$  h  $-\frac{11}{4}$  i -1.25

Exercise 7.1.3

- 1 a 3.5 b 3.5 c -3 d 1.5 e 3.5 f 1.5 g 1.8 h  $-\frac{4}{7}$  i 0
- 2 a -0.75 b -1, 4 c 0, 1 d 3, 4 e -1, 4 f 0, 2
- 3 a -1, 1, 2 b -3, 1, 3, 4 c  $\frac{4}{3}, \frac{5}{3}, 2$  d -1, 1, 2 e 3, 7,  $\frac{-1 \pm \sqrt{233}}{2}, \frac{1}{3}$

Exercise 7.1.4

- 1 a i 5.32 ii 9.99 iii 2.58 b i 2.26 ii 3.99 iii 5.66  
 c i 3.32 ii -4.32 iii -6.32 d i -1.43 ii 1.68 iii -2.86
- 2 a 0 b 0.54 c -0.21 d -0.75, 0 e 1.13 f 0, 0.16

Exercise 7.1.5

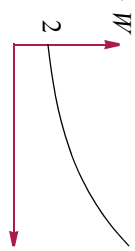
- 1 a 2 b -1 c 0.5 d 0.5  
 2 a 1 b 0.6 c 0

- 3 a 0 b  $\frac{2}{3}$
- 4 a -1, 2 b -2, 3 c -1 d -6, 1 e 0, 1 f 1
- 5 a 1.3863 b 2.1972 c 3.2189 d 0
- 6 a 0.4236 b 0.4055 c 0.3054 d -0.4176
- 7 a 0 b -0.6733 c 0  
 9 9.36

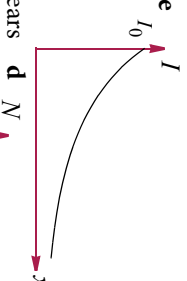
10 a =  $\sqrt{2}e$ , k =  $\ln(\sqrt{2})$

Exercise 7.2

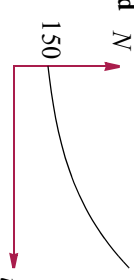
- 1 a 1000 b 1516 c 2000 d 10 days  
 2 a 0.0013 b 2.061 kg c 231.56 years d W



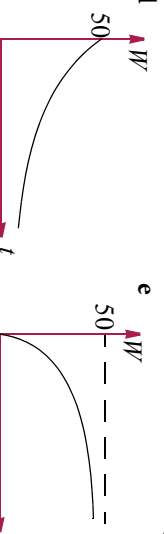
- 3 a 0.01398 b 52.53% c 51.53 m d 21.53 m e  $I_0$



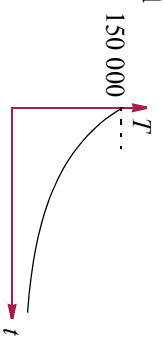
- 4 a i 157 ii 165 iii 191 b 14.2 years c 20.1 years d N



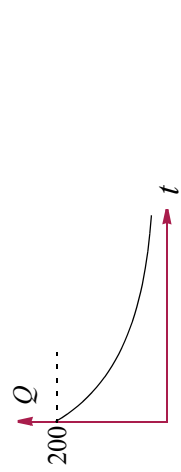
- 5 a 50 b 0.0222 c 17.99 kg d W



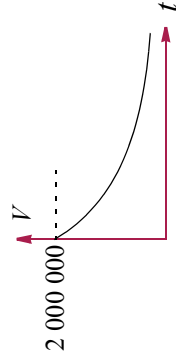
- 6 a 15 000°C b i 11 900°C ii 1500°C c 3.01 million years



- 7 a 0.0151 b 12.50 gm c 20 years d



- 8 a \$2 million b \$1.589 million c 30.1 years d

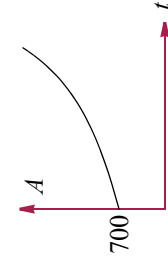


- 9 b 0.01761 c 199 230 d 22.6 years

- 10 a 20 cm<sup>2</sup> b 19.72 cm<sup>2</sup> c 100 days d 332 days

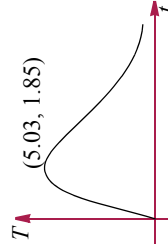
- 11 a i b i 512170 ii 517217 c 54.1 early 2014

- 12 a i \$933.55 ii \$935.50 b 11.95 years c



- 13 a 99 b  $99 \times 20.1394t$  c 684

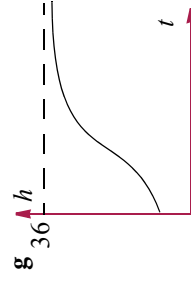
- 14 a  $T$  (5.03, 1.85) b 38.85°C at ~midnight



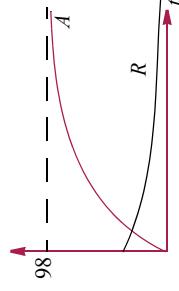
- 15 a 19 b 2.63 c 100

- 16 a 18 cm b 4 cm c 1.28 m d 36 m

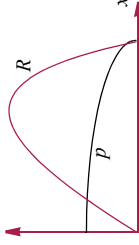
- e i 21.7 years ii 27.6 years iii 34.5 years f 36



- 17 a 5 mg/min b 13.51 min c i 2.1 ii 13.9 iii 68 min d 19.6 mg  
e f No



- 18 a i \$499 ii \$496 iii \$467 c 15537 d i \$499 000 ii \$2.48 million  
iii \$4.67 million f 12358 g \$5.14 million b & e



Exercise 7.3

- 1 a 2 b 2 c 5 d 3 e -3 f -2 g 0 h 0 i -1 j -2 k 0.5 l -2

- 2 a  $\log_{10} 10000 = 4$  b  $\log_{10} 0.001 = -3$  c  $\log_{10}(x+1) = y$  d  $\log_{10} p = 7$

- e  $\log_2(x-1) = y$  f  $\log_2(y-2) = 4x$

- 3 a  $2^9 = x$  b  $b^x = y$  c  $b^{ax} = t$  d  $10^{x^2} = z$  e  $10^{1-x} = y$  f  $2^y = ax - b$

- 4 a 16 b 2 c 2 d 9 e  $\sqrt[4]{2}$  f 125 g 4 h 9 i  $\sqrt[3]{\frac{1}{3}}$  j 21 k 3 l 13

- 5 a 54.5982 b 1.3863 c 1.6487 d 7.3891 e 1.6487 f 0.3679 g 52.5982  
h 4.7183 i 0.6065

Exercise 7.4

- 1 a 5 b 2 c 2 d 1 e 2 f 1

- 2 a  $\log a = \log b + \log c$  b  $\log a = 2\log b + \log c$  c  $\log a = -2\log c$

- d  $\log a = \log b + 0.5\log c$  e  $\log a = 3\log b + 4\log c$  f  $\log a = 2\log b - 0.5\log c$

- 3 a 0.18 b 0.045 c -0.09

- 4 a  $x = yz$  b  $y = x^2$  c  $y = \frac{x+1}{x}$  d  $x = 2^y + 1$  e  $y = \sqrt{x}$  f  $y^2 = (x+1)^3$

- 5 a  $\frac{1}{2}$  b  $\frac{1}{2}$  c  $\frac{17}{15}$  d  $\frac{3}{2}$  e  $\frac{1}{3}$  f no real soln g 3, 7 h  $\frac{\sqrt{33}-1}{2}$  i 4

- j  $\sqrt{10} + 3$  k  $\frac{64}{63}$  l  $\frac{2}{15}$

- 6 a  $\log_3 2^{wx}$  b  $\log_{47} x$  c  $\log_d [x^2(x+1)^3]$  d  $\log_d \left[ \frac{(x^5)(x+1)^3}{\sqrt{2x-3}} \right]$  e  $\log_{10} \left( \frac{y^2}{x} \right)$

- f  $\log_2 \left( \frac{1}{x} \right)$

- 7 a 1 b -2 c 3 d 9 e 2 f 9

- 8 a 1, 4 b 1,  $3 \pm \sqrt{3}$  c 1,  $4 \sqrt[3]{4}$  d 1,  $5 \pm \sqrt[4]{5}$

9 a  $\frac{\log 14}{\log 2} = 3.81$  b  $\frac{\log 8}{\log 10} = 0.90$  c  $\frac{\log 125}{\log 3} = 4.39$

d  $\frac{1}{\log 2} \times \log\left(\frac{11}{3}\right) - 2 = -0.13$  e  $\frac{\log 10 - \log 3}{4 \log 3} = 0.27$  f 5.11 g  $\frac{-\log 2}{2 \log 10} = -0.15$

h 7.37 i 0.93 j no real solution k  $\frac{\log 3}{\log 2} - 2 = -0.42$  l  $\frac{\log 1.5}{\log 3} = 0.37$

10 a 0.5, 4 b 3 c -1, 4 d 10, 10<sup>10</sup> e 5 f 3

11 a (4, log<sub>4</sub> 11) b 100, 10 c 2, 1

12 a  $y = xz$  b  $y = x^3$  c  $x = e^{y-1}$

13 a  $\frac{1}{e^4 - 1}$  b  $\frac{1}{3}$  c  $\frac{\sqrt{5}-1}{2}$  d  $\emptyset$

14 a ln 21 = 3.0445 b ln 10 = 2.3026 c -ln 7 = -1.9459 d ln 2 = 0.6931

e ln 3 = 1.0986 f  $2 \ln\left(\frac{14}{9}\right) = 0.8837$  g  $e^3 = 20.0855$  h  $\frac{1}{3}e^2 = 2.4630$

i  $\pm\sqrt{e^9} = \pm 90.0171$  j  $\emptyset$  k  $e^2 - 4 = 3.3891$  l  $\sqrt[3]{e^9} = 20.0855$

15 a 0, ln 2 b ln 5 c ln 2, ln 3 d 0 e 0, ln 5 f ln 10

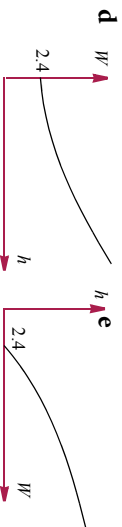
16 a 4.5222 b 0.2643 c 0, 0.2619 d -1, 0.3219 e -1.2925, 0.6610 f 0, 1.8928  
g 0.25, 2 h 1 i 121.5 j 2

17 a -3.1831 b 1.3098 c 0.1422, 0.5574 d 2.6692 e 1.8960 f 1.7162

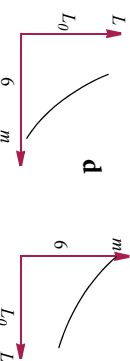
**Exercise 7.5**

1 a 10 b 30 c 40

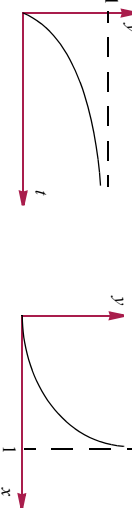
2 a 31.64 kg b 1.65 c  $W = 2.4 \times 10^{0.8h}$



3 a 4.75 b  $L = L_0 \times 10^{\left(\frac{6-m}{2.5}\right)}$  c



4 a [0, 1] b i 2.22 ii 1.11 iii 0.74 years c As c increases, reliability reduces.  
d  $x = 1 - 10^{-ct}$  e 1



5 a  $I = \frac{d}{rk}$

6 a 0.10 b  $\lambda = \lambda_0 \times 10^{-kx}$  c 16.82% d  $k = -\frac{1}{x} \log\left(\frac{\lambda}{\lambda_0}\right)$

**Chapter 8**

**Exercise 8.1.1**

1 i b 4 c  $t_n = 4n - 2$  ii b -3 c  $t_n = -3n + 23$  iii b -5 c  $t_n = -5n + 6$   
iv b 0.5 c  $t_n = 0.5n$  v b 2 c  $t_n = y + 2n - 1$  vi b -2 c  $t_n = x - 2n + 4$

2 -28

3 9, 17

4 -43

5 7

6 7

7 -5

8 0

9 a 41 b 31st

10  $2, \sqrt{3}$

11 a i 2 ii -3 bi 4 iii 11

12  $x - 8y$

13  $t_n = 5 + \frac{10}{3}(n-1)$

14 a -1 b 0

**Exercise 8.1.2**

1 a 145 b 300 c -170

2 a -18 b 690 c 70.4

3 a -105 b 507 c 224

4 a 126 b 3900 c 14th week

5 855

6 a 420 b -210

7  $a = 9, b = 7$

**Exercise 8.1.3 Miscellaneous questions**

1 123

2 -3, -0.5, 2, 4.5, 7, 9.5, 12

3 3.25

4  $a = 3$   $d = -0.05$

5 10 000

6 330

7 -20

8 328

9 \$725, 37 weeks

10 a \$55 b 2750

11 a 8 m ii 40 m b 84 m c Dist =  $2n^2 - 2n = 2n(n-1)$  d 8

e 26 players, 1300 m

12 a 5050 b 10200 c 4233

- 13 a 145 b 390 c -1845  
14 b  $3n - 2$

**Exercise 8.2.1**

- 1 a  $r = 2, u_5 = 48, u_n = 3 \times 2^{n-1}$  b  $r = \frac{1}{3}, u_5 = \frac{1}{27}, u_n = 3 \times \left(\frac{1}{3}\right)^{n-1}$   
c  $r = \frac{1}{5}, u_5 = \frac{2}{625}, u_n = 2 \times \left(\frac{1}{5}\right)^{n-1}$  d  $r = -4, u_5 = -256, u_n = -1 \times (-4)^{n-1}$   
e  $r = \frac{1}{b}, u_5 = \frac{a}{b^3}, u_n = ab \times \left(\frac{1}{b}\right)^{n-1}$  f  $r = \frac{b}{a}, u_5 = \frac{b^4}{a^2}, u_n = a^2 \times \left(\frac{b}{a}\right)^{n-1}$
- 2 a  $\pm 12$  b  $\frac{\pm\sqrt{5}}{2}$
- 3 a  $\pm 96$  b 15th
- 4 a  $u_n = 10 \times \left(\frac{5}{6}\right)^{n-1}$  b  $\frac{15625}{3888} \approx 4.02$  c  $n = 5$  4 times

- 5  $-2, \frac{4}{3}$

- 6 a i \$4096 ii \$2097.15 b 6.2 years

- 7  $\left(u_n = \frac{1000}{169} \times \left(\frac{12}{5}\right)^{n-1}\right), \frac{1990656}{4225} \approx 471.16$

- 8 2.5, 5, 10 or 10, 5, 2.5

- 9 53757

- 10 108 952

- 11 a \$56156 b \$299284

**Exercise 8.2.2**

- 1 a 3 b  $\frac{1}{3}$  c -1 d  $-\frac{1}{3}$  e 1.25 f  $-\frac{2}{3}$

- 2 a 216513 b  $1.6384 \times 10^{-10}$  c  $\frac{256}{729}$  d  $\frac{729}{2401}$  e  $-\frac{81}{1024}$

- 3 a 11; 354292 b 7; 473 c 8; 90.90909 d 8; 172.778 e 5; 2.256  
f 13; 111.1111111111

- 4 a  $\frac{127}{128}$  b  $\frac{63}{8}$  c  $\frac{130}{81}$  d 60 e  $\frac{63}{64}$

- 5 4; 118096

- 6 \$2109.50

- 7 9.28 cm

- 8 a  $V_n = V_0 \times 0.7^n$  b 7

- 9 54

- 10 53.5 gms; 50 weeks.

- 11 7

- 12 9

- 13 -0.5, -0.7797

- 14  $r = 5, 1.8 \times 10^{10}$

- 15 \$8407.35

- 16  $1.8 \times 10^{19}$  or about 200 billion tonnes.

**Exercise 8.2.3**

- 1 Term 9 AP = 180, GP = 256. Sum to 11 terms AP = 1650, GP = 2047.

- 2 18

- 3 12

- 4 7, 12

- 5 8 weeks Ken \$220 and Bo-Youn \$255)

- 6 a week 8 b week 12

- 7 a 1.618 b 121379 (~121400, depends on rounding errors)

**Exercise 8.2.4**

- 1 a  $\frac{81}{2}$  b  $\frac{10}{13}$  c 5000 d  $\frac{30}{11}$

- 2  $23\frac{23}{99}$

- 3 6667 fish. (Note:  $t_{43} < 1$ . If we use  $n = 43$  then ans is 6660 fish); 20 000 fish.

Overfishing means that fewer fish are caught in the long run.

- 4 27

- 5 48, 12, 3 or 16, 12, 9

- 6 a  $\frac{11}{30}$  b  $\frac{37}{99}$  c  $\frac{191}{90}$

- 7 128 cm

- 8  $\frac{121}{9}$

- 9  $2 + \frac{4}{3}\sqrt{3}$

- 10  $\frac{1 - (-t)^n}{1 + t} - \frac{1}{1 + t}$

- 11  $\frac{1 - (-t^2)^n}{1 + t^2} - \frac{1}{1 + t^2}$

**Exercise 8.2.5 Miscellaneous questions**

- 1 3, -0.2

- 2  $\frac{2560}{93}$

- 3  $\frac{10}{3}$

- 4 a  $\frac{43}{18}$  b  $\frac{458}{99}$  c  $\frac{413}{990}$

- 5 9900

- 6 3275

- 7 3

- 8  $t_n = 6n - 14$   
 9 6  
 10  $-\frac{1}{6}$   
 11 a 12 b 26  
 12 9, 12  
 13  $\pm 2$   
 14 (5, -5, 5), (5, -10, 20)  
 15 a 2, 7 b 2, 5, 8 c  $3n - 1$   
 16 a 5 b 2 m

**Exercise 8.3**

- 1 \$2773.08  
 2 \$4377.63  
 3 \$1781.94  
 4 \$12216  
 5 \$35816.95  
 6 \$40349.37  
 7 \$64006.80  
 8 \$276971.93, \$281325.41  
 9 \$63762.25  
 10 \$98.62, \$9467.14, interest \$4467.14. Flat interest = \$6000  
 11 \$134.41, \$3790.44, 0.602%/month (or 7.22% p.a.)

**Chapter 9**

**Exercise 9.1**

	a cm	b cm	c cm	A	B	C
1	3.8	4.1	1.6	67°	67°	90°
a	3.8	4.1	1.6	67°	67°	90°
b	81.5	98.3	55.0	56°	56°	90°
c	32.7	47.1	33.9	44°	44°	90°
d	1.61	30.7	30.7	3°	3°	90°
e	2.3	2.74	1.49	57°	57°	90°
f	48.5	77	59.8	39°	39°	90°
g	44.4	81.6	68.4	33°	33°	90°
h	2.93	13.0	12.7	13°	13°	90°
i	74.4	94.4	58.1	52°	52°	90°
j	71.8	96.5	64.6	48°	48°	90°
k	23.3	34.1	24.9	43°	43°	90°
l	43.1	43.2	2.3	87°	87°	90°
m	71.5	80.2	36.4	63°	63°	90°
n	33.5	34.1	6.5	79°	79°	90°
o	6.1	7.2	3.82	58°	58°	90°
p	29.1	30	7.3	76°	76°	90°
q	29.0	29.1	2.0	86°	86°	90°
r	34.5	88.2	81.2	23°	23°	90°
s	24.0	29.7	17.5	54°	54°	90°
t	41.2	46.2	21.0	63°	63°	90°
u	59.6	72.9	41.8	55°	55°	90°

- v 5.43 6.8 4.09 53° 90° 37°  
 w 13.0 19.8 14.9 49° 90° 49°  
 x 14.0 21.3 16.1 41° 90° 49°  
 y 82.4 88.9 33.3 68° 90° 22°  
 2 a  $2\sqrt{3}$  b  $5(1 + \sqrt{3})$  c 4 d  $2(1 + \sqrt{3})$  e  $\frac{4}{3}(3 + \sqrt{3})$  f  $\sqrt{106} - 5$   
 4 a  $25(1 + \sqrt{3})$  b  $\frac{40\sqrt{3}}{3}$

**Exercise 9.2**

- 1 a i 030°T ii 330°T iii 195°T iv 200°T  
 b i N25°E ii S iii S40°W iv N10°W  
 2 37.49 m  
 3 18.94 m  
 4 37° 18'  
 5  $\frac{26}{9}$  m/s  
 6 N58° 33'W, 37.23 km  
 7 199.82 m  
 8 10.58 m  
 9 72.25 m  
 10 25.39 km  
 11 15.76 m  
 12 a 3.01 km N, 3.99 km E b 2.87 km E, 0.88 km S c 6.86 km E, 2.13 km N  
 d 7.19 km 253°T  
 13 524 m

**Exercise 9.3**

- 1 a 39°48' b 64°46'  
 2 a 12.81 cm b 61.35 cm c 77°57' d 60.83 cm e 80° 32'  
 3 a 21°48' b 42°2' c 26°34'  
 4 a 2274 b 12.7°  
 5 251.29 m  
 6 a 103.5 m b 35.26° c 39.23°  
 7 b 53.43 c 155.16 m d 145.68 m  
 8 b 48.54 m

9 a  $\sqrt{(b-c)^2 + h^2}$  b  $\tan^{-1}\left(\frac{h}{a}\right)$  c  $\tan^{-1}\left(\frac{h}{b-c}\right)$

d  $2(b+c)\sqrt{h^2 + a^2} + 2a\sqrt{(b-c)^2 + h^2}$

- 10 82.80 m  
 11 a 40.61 m b 49.46 m  
 12 a 10.61 cm b 75° 58' c 93° 22'  
 13 a 1.44 m b 73° 13' c 62° 11'

**Exercise 9.4**

- 1 a 1999.2 cm<sup>2</sup> b 756.8 cm<sup>2</sup> c 3854.8 cm<sup>2</sup> d 2704.9 cm<sup>2</sup> e 538.0 cm<sup>2</sup>

- f 417.5 cm<sup>2</sup> g 549.4 cm<sup>2</sup> h 14.2 cm<sup>2</sup> i 516.2 cm<sup>2</sup> j 281.5 cm<sup>2</sup> k 918.8 cm<sup>2</sup>  
 l 387.2 cm<sup>2</sup> m 139.0 cm<sup>2</sup> n 853.7 cm<sup>2</sup> o 314.6 cm<sup>2</sup>  
 2 69345 m<sup>2</sup>  
 3  $100\pi - 6\sqrt{91}$  cm<sup>2</sup>  
 4 17.34 cm  
 5 a 36.77 sq units b 14.70 sq units c 62.53 sq units  
 6 52.16 cm<sup>2</sup>  
 7 7° 2'  
 8  $\frac{(b+a \times \tan \theta)^2}{2 \tan \theta}$

9 Area of  $\triangle ACD = 101.78$  cm<sup>2</sup>, Area of  $\triangle ABC = 61.38$  cm<sup>2</sup>

**Exercise 9.5.1**

l	a cm	b cm	c cm	A	B	C
a	13.3	37.1	48.2	10°	29°	141°
b	2.7	1.2	2.8	74°	25°	81°
c	11.0	0.7	11.3	60°	3°	117°
d	31.9	39.1	51.7	38°	49°	93°
e	18.5	11.4	19.5	68°	35°	77°
f	14.6	15.0	5.3	75°	84°	21°
g	26.0	7.3	26.4	79°	16°	85°
h	21.6	10.1	28.5	39°	17°	124°
i	0.8	0.2	0.8	82°	16°	82°
j	27.7	7.4	33.3	36°	9°	135°
k	16.4	20.7	14.5	52°	84°	44°
l	21.4	45.6	64.3	11°	24°	145°
m	30.9	27.7	22.6	75°	60°	45°
n	29.3	45.6	59.1	29°	49°	102°
o	9.7	9.8	7.9	65°	67°	48°
p	21.5	36.6	54.2	16°	28°	136°
q	14.8	29.3	27.2	30°	83°	67°
r	10.5	0.7	10.9	52°	3°	125°
s	11.2	6.9	17.0	25°	15°	140°
t	25.8	18.5	40.1	30°	21°	129°

**Exercise 9.5.2**

l	a	b	c	A°	B°	C°	c*	B*°	C*°
a	7.40	18.10	21.06	20.00	56.78	103.22	12.95	123.22	36.78
b	13.30	19.50	31.36	14.00	20.77	145.23	6.49	159.23	6.77
c	13.50	17.00	25.90	28.00	36.24	115.76	4.12	143.76	8.24
d	10.20	17.00	25.62	15.00	25.55	139.45	7.22	154.45	10.55
e	7.40	15.20	19.55	20.00	44.63	115.37	9.02	135.37	24.63
f	10.70	14.10	21.41	26.00	35.29	118.71	3.94	144.71	9.29
g	11.50	12.60	22.94	17.00	18.68	144.32	1.16	161.32	1.68
h	8.30	13.70	18.67	24.00	42.17	113.83	6.36	137.83	18.17
i	13.70	17.80	30.28	14.00	18.32	147.68	4.27	161.68	4.32

- j 13.40 17.80 26.19 28.00 113.42 5.24 141.42 10.58  
 k 12.10 16.80 25.63 23.00 124.15 5.30 147.15 9.85  
 l 12.00 14.50 24.35 21.00 133.34 2.72 154.34 4.66  
 m 12.10 19.20 29.34 16.00 138.06 7.57 154.06 9.94  
 n 7.20 13.10 19.01 15.00 136.91 6.30 151.91 13.09  
 o 12.20 17.70 23.73 30.00 103.50 6.93 133.50 16.50  
 p 9.20 20.90 27.97 14.00 132.66 12.59 146.66 19.34  
 q 10.50 13.30 21.96 20.00 134.33 3.03 154.33 5.67  
 r 9.20 19.20 26.29 15.00 132.31 10.80 147.31 17.69  
 s 7.20 13.30 18.33 19.00 124.03 6.82 143.03 17.97  
 t 13.50 20.40 25.96 31.00 97.90 9.01 128.90 20.10  
 2 a–d no triangles exist.

**Exercise 9.5.3**

- 1 30.64 km  
 2 4.57 m  
 3 476.4 m  
 4 201°47'T  
 5 222.9 m  
 6 a 3.40 m b 3.11 m  
 7 b 1.000 m c 1.715 m  
 8 a 51.19 min b 1 hr 15.96 min c 14.08 km  
 9 \$4886  
 10 906 m

**Exercise 9.5.4**

l	a cm	b cm	c cm	A	B	C
a	13.5	9.8	16.7	54°	36°	90°
b	8.9	10.8	15.2	35°	44°	101°
c	22.8	25.6	12.8	63°	87°	30°
d	21.1	4.4	21.0	85°	12°	83°
e	15.9	10.6	15.1	74°	40°	66°
f	8.8	13.6	20.3	20°	32°	128°
g	9.2	9.5	13.2	44°	46°	90°
h	23.4	62.5	58.4	22°	89°	69°
i	10.5	9.6	15.7	41°	37°	102°
j	21.7	36.0	36.2	35°	72°	73°
k	7.6	3.4	9.4	49°	20°	111°
l	9.1	12.5	14.3	28°	83°	69°
m	14.9	11.2	15.8	35°	52°	93°
n	2.0	0.7	16.2	63°	42°	75°
o	7.6	3.7	2.5	38°	13°	129°
p	18.5	9.8	9.0	56°	24°	100°
q	20.7	16.3	24.1	45°	22°	113°
r	14.6	22.4	13.6	87°	52°	41°
s	7.0	6.6	29.9	28°	46°	106°
t	21.8	20.8	9.9	45°	42°	93°
u	1.1	1.7	23.8	58°	54°	68°
			1.3	41°	89°	50°

v	1.2	1.2	0.4	85°	76°	19°
w	23.7	27.2	29.7	49°	60°	71°
x	3.4	4.6	5.2	40°	60°	80°

**Exercise 9.5.5**

- 1 a 10.14 km b 121°T
- 2 7° 33'
- 3 4.12 cm
- 4 57.32 m
- 5 315.5 m
- 6 124.3 km b W28° 47' S

**Exercise 9.5.6 Miscellaneous questions**

- 1 39.60 m 52.84 m
- 2 30.2 m
- 3 54°.42', 84°
- 4 37°
- 5 028°T.
- 6 108.1 cm
- 7 a 135° b 136 cm
- 8 41°.56', 83°
- 9 a 158° left b 43.22 km
- 10 264 m
- 11 53.33 cm
- 12 186 m
- 13 50.12 cm
- 14 5.17 cm
- 15 a 5950 m b 13341 m c 160° d 243°
- 17 a 20.70° b 2.578 m c 1.994 m<sup>3</sup>
- 18 a 4243 m<sup>2</sup> b 86 m c 101 m

**Exercise 9.6**

- 1 5.36 cm
- 2 12.3 m
- 3 24 m
- 4 40.3 m, 48.2°
- 5 16.5 min, 8.9°
- 6 ~10:49 am

$$7 \text{ a i } \frac{d \sin \phi}{\sin(\phi - \theta)} \quad \text{ii } \frac{d \sin \theta}{\sin(\phi - \theta)} \quad \text{b } \frac{d \sin \phi \tan \alpha}{\sin(\phi - \theta)} \text{ or } \frac{d \sin \theta \tan \beta}{\sin(\phi - \theta)} \quad \text{c } d \left( \frac{\sin \phi \cos \theta}{\sin(\phi - \theta)} - 1 \right)$$

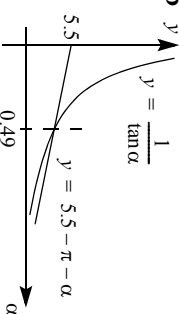
**Exercise 9.7**

- 1 a  $\frac{169\pi}{150}$  cm<sup>2</sup>,  $5.2 + \frac{13\pi}{15}$  cm b  $\frac{529\pi}{32}$  cm<sup>2</sup>,  $23 + \frac{23\pi}{8}$  cm  
c  $242\pi$  cm<sup>2</sup>,  $88 + 11\pi$  cm d  $\frac{1156\pi}{75}$  m<sup>2</sup>,  $13.6 + \frac{68\pi}{15}$  m

- e  $\frac{96\pi}{625}$  cm<sup>2</sup>,  $1.28 + \frac{12\pi}{25}$  cm f  $\frac{361\pi}{15}$  cm<sup>2</sup>,  $15.2 + \frac{19\pi}{3}$  cm
- g  $5248.8\pi$  m<sup>2</sup>,  $648 + 32.4\pi$  cm h  $\frac{12943\pi}{300}$  cm<sup>2</sup>,  $17.2 + \frac{301\pi}{30}$  cm
- i  $\frac{1922\pi}{75}$  cm<sup>2</sup>,  $12.4 + \frac{124\pi}{15}$  cm j  $\frac{15884\pi}{3}$  cm<sup>2</sup>,  $152 + \frac{418\pi}{3}$  cm
- k  $12\pi$  cm<sup>2</sup>,  $24 + 2\pi$  cm l  $\frac{98\pi}{3}$  cm<sup>2</sup>,  $28 + \frac{14\pi}{3}$  cm m  $\frac{196\pi}{75}$  cm<sup>2</sup>,  $5.6 + \frac{28\pi}{15}$  cm
- n  $\frac{11532\pi}{25}$  cm<sup>2</sup>,  $49.6 + \frac{186\pi}{5}$  cm o  $\frac{3\pi}{50}$  cm<sup>2</sup>,  $2.4 + \frac{\pi}{10}$  cm

- 2 0.63° c, 36°
- 3 0.0942 m<sup>3</sup>
- 4 1.64°
- 5 79 cm
- 6 5.25 cm<sup>2</sup>
- 7 a 31.83 m b 406.28 m c 11°
- 8 1.11°
- 9 0.75°

- 10 a 1.85° b 137.09 cm ii 88.57 cm c 370.92 cm<sup>2</sup>
- 11 26.57 cm<sup>2</sup>
- 12 193.5 cm
- 13 a 105.22 cm b 118.83 cm
- 14 a 9 cm b 12 cm c 36°52'
- 15 b  $y'$  c 0.49



**Chapter 10**

**Exercise 10.1**

- 1 a 120° b 108° c 216° d 50°
- 2 a  $\pi^c$  b  $\frac{3\pi^c}{2}$  c  $\frac{7\pi^c}{9}$  d  $\frac{16\pi^c}{9}$  3



- 3 a  $\frac{\sqrt{3}}{2}$  b  $\frac{1}{2}$  c  $-\sqrt{3}$  d  $-2$  e  $-\frac{1}{2}$  f  $-\frac{\sqrt{3}}{2}$  g  $\frac{1}{\sqrt{3}}$  h  $\sqrt{3}$  i  $-\frac{1}{\sqrt{2}}$  j  $-\frac{1}{\sqrt{2}}$   
 k 1 l  $-\sqrt{2}$  m  $-\frac{1}{\sqrt{2}}$  n  $\frac{1}{\sqrt{2}}$  o  $-1$  p  $\sqrt{2}$  q 0 r 1 s 0 t undefined
- 4 a 0 b  $-1$  c 0 d  $-1$  e  $\frac{1}{\sqrt{2}}$  f  $-\frac{1}{\sqrt{2}}$  g  $-1$  h  $\sqrt{2}$  i  $-\frac{1}{2}$  j  $-\frac{\sqrt{3}}{2}$  k  $\frac{1}{\sqrt{3}}$   
 l  $\sqrt{3}$  m  $-\frac{\sqrt{3}}{2}$  n  $\frac{1}{2}$  o  $-\sqrt{3}$  p 2 q  $-\frac{1}{\sqrt{2}}$  r  $\frac{1}{\sqrt{2}}$  s  $-1$  t  $-\sqrt{2}$
- 5 a  $\frac{1}{2}$  b  $\frac{\sqrt{3}}{2}$  c 11 d  $\frac{1}{2}$  e  $-\frac{1}{\sqrt{3}}$  f  $-\frac{1}{2}$  g  $-\sqrt{2}$  h  $-\frac{2}{\sqrt{3}}$
- 6 a  $-\frac{1}{2}$  b  $-\frac{1}{\sqrt{2}}$  c  $\sqrt{3}$  d  $-2$  e 1 f  $\frac{1}{2}$  g  $-\frac{1}{\sqrt{3}}$  h  $-\frac{\sqrt{3}}{2}$  i  $-\frac{2}{\sqrt{3}}$  j  $\frac{1}{\sqrt{3}}$   
 k  $\frac{2}{\sqrt{3}}$  l  $-\frac{\sqrt{3}}{2}$
- 7 a  $(\frac{1}{2}, \frac{\sqrt{3}}{2})$  b  $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$  c  $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$  d  $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$
- 8 a 0 b  $\frac{\sqrt{3}}{2}$  c  $\frac{1}{\sqrt{3}}$  d  $\frac{1+\sqrt{3}}{2\sqrt{2}}$
- 10 a  $\frac{2}{3}$  b  $\frac{2}{3}$  c  $-\frac{2}{3}$
- 11 a  $\frac{2}{5}$  b  $\frac{5}{2}$  c  $\frac{5}{2}$
- 12 a k b  $-\frac{1}{k}$  c  $-k$
- 13 a  $\frac{\sqrt{5}}{3}$  b  $\frac{3}{\sqrt{5}}$  c  $-\frac{\sqrt{5}}{3}$
- 14 a  $-\frac{3}{5}$  b  $\frac{3}{4}$  c  $\frac{4}{5}$
- 15 a  $\frac{4}{5}$  b  $\frac{3}{4}$  c  $-\frac{5}{3}$
- 16 a  $-k$  b  $-\sqrt{1-k^2}$  c  $-\frac{k}{\sqrt{1-k^2}}$
- 17 a  $-\sqrt{1-k^2}$  b  $\frac{k}{\sqrt{1-k^2}}$  c  $-\frac{1}{\sqrt{1-k^2}}$
- 18 a  $\sin \theta$  b  $\cot \theta$  c 1 d 1 e  $\cot \theta$  f  $\tan \theta$

- 19 a  $\frac{\pi}{3}, \frac{2\pi}{3}$  b  $\frac{\pi}{3}, \frac{5\pi}{3}$  c  $\frac{\pi}{3}, \frac{4\pi}{3}$  d  $\frac{5\pi}{6}, \frac{7\pi}{6}$  e  $\frac{5\pi}{6}, \frac{11\pi}{6}$  f  $\frac{7\pi}{6}, \frac{11\pi}{6}$

**Exercise 10.2.1**

- 1 a  $x^2 + y^2 = k^2, -k \leq x \leq k$  b  $\frac{x^2 + y^2}{b^2} = 1, -b \leq x \leq b$   
 c  $(x-1)^2 + (2-y)^2 = 1, 0 \leq x \leq 2$  d  $\frac{(1-x)^2}{b^2} + \frac{(y-2)^2}{a^2} = 1$   
 e  $5x^2 + 5y^2 + 6xy = 16$
- 2 a i  $-\frac{4}{5}$  ii  $-\frac{5}{3}$  b i  $\frac{4}{\sqrt{7}}$  ii  $-\frac{\sqrt{7}}{3}$
- 3 a  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$  b  $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$  c 0,  $\frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi$  d  $\frac{\pi}{2}, \frac{3\pi}{2}$
- 9 a  $\frac{2a}{a^2+1}$  b  $\frac{a^2-1}{a^2+1}$
- 10 a i 1 ii 1 b 1
- 11 a  $\frac{1-\sqrt{x^2-1}}{x}$  b  $\frac{1+\sqrt{x^2-1}}{x}$  c  $\frac{2}{x^2} - 1$
- 12 a i 6 ii  $\frac{5}{2}$  iii  $\frac{9}{8}$  b i 5 ii 1 iii  $-2$
- 13 a  $\pm 2$  b  $\frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$  or  $\frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}$
- 14 a i 25 ii  $\frac{1}{5^4}$  b i 27 ii  $\frac{1}{3}$
- 15 a 1 + 2k b  $(1-k)\sqrt{1+2k}$
- 16 a  $\frac{1-a}{2\sqrt{a}}$  b i  $2 + \sqrt{2a-a^2}$  ii  $-\frac{\sqrt{2a-a^2}}{1-a}$
- 17 a  $\frac{2}{3}$  b 0,  $\pm \frac{2\sqrt{2}}{3}$
- 18 0,  $\frac{\pi}{3}, \frac{2\pi}{3}, \pi$

**Exercise 10.2.2**

- 1 a  $\sin \alpha \cos \phi + \cos \alpha \sin \phi$  b  $\cos 3\alpha \cos 2\beta - \sin 3\alpha \sin 2\beta$  c  $\sin 2x \cos y - \cos 2x \sin y$   
 d  $\cos \phi \cos 2\alpha + \sin \phi \sin 2\alpha$  e  $\frac{\tan 2\theta - \tan \alpha}{1 + \tan 2\theta \tan \alpha}$  f  $\frac{\tan \phi - \tan 3\omega}{1 + \tan \phi \tan 3\omega}$
- 2 a  $\sin(2\alpha - 3\beta)$  b  $\cos(2\alpha + 5\beta)$  c  $\sin(x+2y)$  d  $\cos(x-3y)$   
 e  $\tan(2\alpha - \beta)$  f  $\tan x$  g  $\tan(\frac{\pi}{4} - \phi)$  h  $\sin(\frac{\pi}{4} + \alpha + \beta)$  i  $\sin 2x$

3 a  $-\frac{56}{65}$  b  $\frac{33}{65}$  c  $-\frac{16}{63}$

4 a  $\frac{16}{65}$  b  $\frac{63}{65}$  c  $\frac{56}{33}$

5 a  $-\frac{5\sqrt{11}}{18}$  b  $-\frac{7}{18}$  c  $\frac{5\sqrt{11}}{7}$  d  $\frac{35\sqrt{11}}{162}$

6 a  $-\frac{3}{5}$  b  $-\frac{4}{5}$  c  $\frac{3}{4}$  d  $\frac{24}{7}$

7 a  $\frac{1+\sqrt{3}}{2\sqrt{2}}$  b  $\frac{1+\sqrt{3}}{2\sqrt{2}}$  c  $-\frac{1+\sqrt{3}}{2\sqrt{2}}$  d  $\sqrt{3}-2$

8 a  $\frac{2ab}{a^2+b^2}$  b  $\frac{a^2+b^2}{2ab}$  c  $\frac{a^4-6a^2b^2+b^4}{(a^2+b^2)^2}$  d  $\frac{2ab}{b^2-a^2}$

12  $\sqrt{2}-1$

14 a  $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$  b  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$  c  $0, \pi, 2\pi, \alpha, \pi \pm \alpha, 2\pi - \alpha, \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

15 a  $R = \sqrt{a^2+b^2}$ ,  $\tan \alpha = \frac{b}{a}$  b 10

16 a  $R = \sqrt{a^2+b^2}$ ,  $\tan \alpha = \frac{b}{a}$  b -11

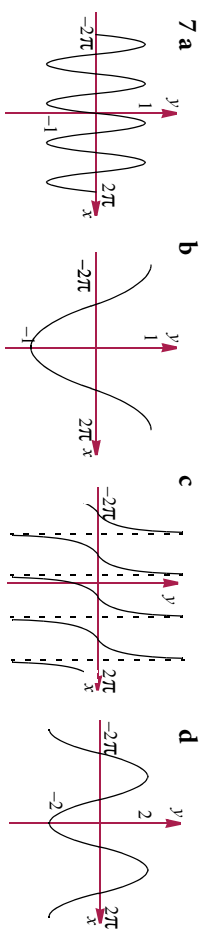
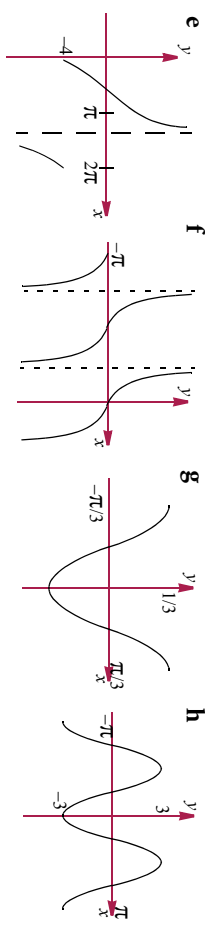
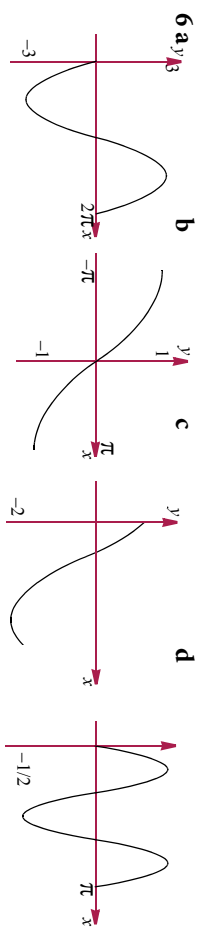
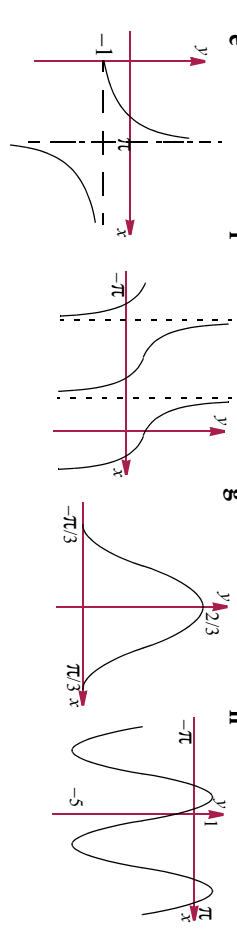
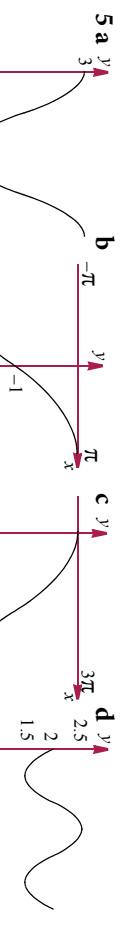
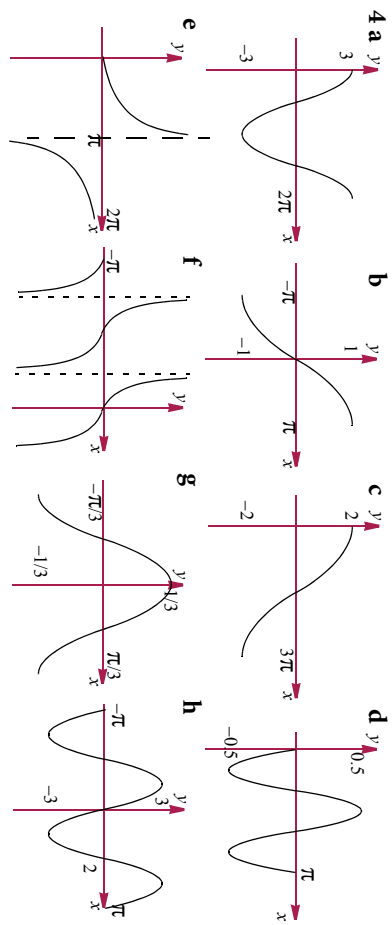
18  $2-\sqrt{3}$

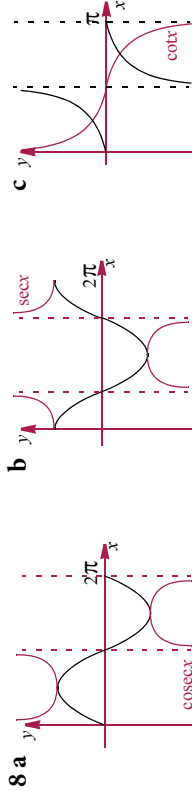
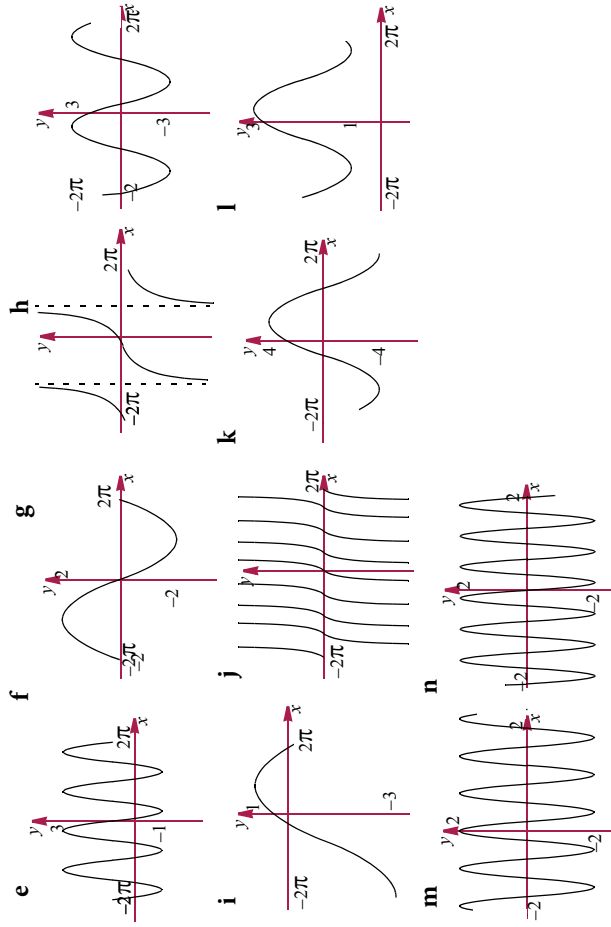
**Exercise 10.3**

1 a  $4\pi$  b  $\frac{2\pi}{3}$  c  $3\pi$  d  $4\pi$  e 2 f  $\frac{\pi}{2}$

2 a 5 b 3 c 5 d 0.5

3 a  $2\pi, 2$  b  $6\pi, 3$  c  $\pi$  d  $\pi$  e  $\pi, 4$  f  $\pi, 3$  g  $6\pi$  h  $\frac{2\pi}{3}, \frac{1}{4}$  i  $3\pi$  j  $\frac{8\pi}{3}, \frac{2}{3}$

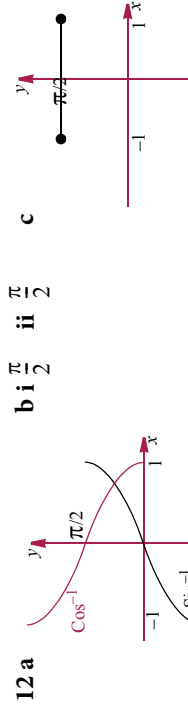
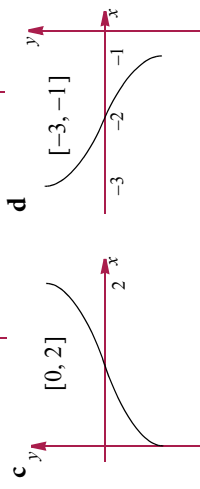
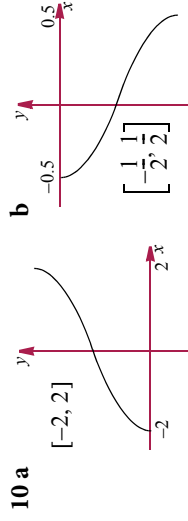




**Exercise 10.4**

- 1 a  $\frac{\pi}{4}$  b  $\frac{\pi}{2}$  c  $\pi$  d  $\frac{\pi}{3}$  e  $\frac{\pi}{4}$  f  $-\frac{\pi}{3}$  g 1.1071° h -0.7754° i 0.0997°  
 j 1.2661° k -0.6435° l 1.3734° m undefined n -1.5375° o 1.0654°  
 2 a -1 b  $\frac{\sqrt{3}}{4}$  c  $-\frac{1}{3\sqrt{2}}$   
 4  $\frac{1}{3}, \frac{2}{3}$   
 5 a  $\frac{2}{3}$  b  $\frac{1}{3}$  c  $\frac{1}{2}$  d  $\frac{3}{4}$  e  $\frac{3\sqrt{2}}{4}$  f -1  
 6 a 1 b  $\frac{7}{25}$  c  $\frac{63}{65}$  d undefined e  $\frac{4\sqrt{5}}{9}$  f  $\frac{3}{5}$  g  $\frac{4}{3}$  h  $\frac{1}{2}$

9 a  $\frac{\sqrt{1-k^2}}{k}$  b  $\frac{1}{\sqrt{1+k^2}}$



13  $\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{n+1}\right)$

**Exercise 10.5**

- 1 a  $\frac{\pi}{4}, \frac{3\pi}{4}$  b  $\frac{7\pi}{6}, \frac{11\pi}{6}$  c  $\frac{\pi}{3}, \frac{2\pi}{3}$  d  $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$   
 e  $\frac{\pi}{3}, \frac{5\pi}{3}$  f  $\frac{5}{4}, \frac{7}{4}, \frac{13}{4}, \frac{15}{4}, \frac{21}{4}, \frac{23}{4}$   
 2 a  $\frac{\pi}{4}, \frac{7\pi}{4}$  b  $\frac{2\pi}{3}, \frac{4\pi}{3}$  c  $\frac{\pi}{6}, \frac{11\pi}{6}$  d  $\pi$  e  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$  f  $\frac{3}{2}, \frac{5}{2}, \frac{11}{2}$   
 3 a  $\frac{\pi}{6}, \frac{7\pi}{6}$  b  $\frac{3\pi}{4}, \frac{7\pi}{4}$  c  $\frac{\pi}{3}, \frac{4\pi}{3}$  d  $4\tan^{-1}2$  e  $\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$  f 3  
 4 a 90°, 330° b 180°, 240° c 90°, 270° d 65°, 335°  
 e  $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$  f 0,  $\pi, 2\pi$  g  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}$  h  $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

- 5 a  $60^\circ, 300^\circ$  b  $\frac{4\pi}{3}, \frac{5\pi}{3}$  c  $\frac{\pi}{6}, \frac{7\pi}{6}$  d  $23^\circ 35', 156^\circ 25'$  e  $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$  f  $\frac{2\pi}{3}, \frac{5\pi}{3}$
- g  $\frac{5\pi}{6}, \frac{9\pi}{6}$  h  $3.35559^\circ, 5.2105^\circ$  i  $\frac{\pi}{3}, \frac{4\pi}{3}$  j  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$  k  $\frac{2\pi}{6}, \frac{7\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$
- 1  $68^\circ 12', 248^\circ 12'$  m  $\frac{\pi}{3}, \frac{5\pi}{3}$  n  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$  o  $\emptyset$
- 6 a  $-\frac{3\pi}{4}, \frac{\pi}{4}$  b  $\pm \frac{\pi}{3}$  c  $-\frac{7\pi}{8}, -\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}$  d  $-\frac{\pi}{2}$  e  $\pm \frac{\pi}{2}$  f  $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$
- g  $\frac{\pi}{2}, \frac{3\pi}{2}$  h  $\frac{\pi}{2}, \frac{3\pi}{2}$
- 7 a  $\frac{3\pi}{4}, \frac{7\pi}{4}, \tan^{-1}\left(\frac{2}{3}\right), \pi + \tan^{-1}\left(\frac{2}{3}\right)$  b  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{4}$
- 8 a  $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$  b  $\frac{2\pi}{3}, \frac{4\pi}{3}$  c  $0, 1, 2, 3, 4, 5, 6$
- 9 a  $\frac{\pi}{3}, \frac{5\pi}{3}, \pi \pm \cos^{-1}\left(\frac{1}{4}\right)$  b  $\frac{3\pi}{4}, \frac{7\pi}{4}, \tan^{-1}(3), \pi + \tan^{-1}(3)$  c  $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$
- d  $\tan^{-1}\left(\frac{3}{2}\right), \pi - \tan^{-1}(2), \pi + \tan^{-1}\left(\frac{3}{2}\right), 2\pi - \tan^{-1}(2)$
- 10 a  $2\sin\left(x + \frac{\pi}{6}\right)$  b  $0, \frac{2\pi}{3}, 2\pi$
- 11 a  $2\sin\left(x - \frac{\pi}{3}\right)$  b  $\frac{\pi}{6}, \frac{3\pi}{2}$
- 12  $\frac{\pi}{3}, \frac{2\pi}{3}$
- 13 a  $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \cup \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$
- b  $\left(\pi + \sin^{-1}\left(\frac{1}{\sqrt{3}}\right), 2\pi - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) \cup \left(3\pi + \sin^{-1}\left(\frac{1}{\sqrt{3}}\right), 4\pi - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$
- 14 a ii  $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$  b ii  $\left[0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{2}, \frac{5\pi}{6}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$
- 16 a i  $\{x|x = k\pi + \alpha(-1)^k, k \in \mathbb{Z}\}$  ii  $\{x|2k\pi + \alpha \leq x \leq (2k+1)\pi - \alpha, k \in \mathbb{Z}\}$
- b  $\left\{x|x = (2k+1)\frac{\pi}{5}\right\} \cup \{x|x = 2k\pi\}, k \in \mathbb{Z}$
- c  $\left\{x|x = \frac{2k\pi + \pi}{5}\right\} \cup \left\{x|x = 2k\pi - \frac{\pi}{2}\right\}, k \in \mathbb{Z}$

17 a  $0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$  b  $\sqrt{2}, \frac{\sqrt{2}}{2}$

18 c  $2\cos\frac{\pi}{9}, 2\cos\frac{5\pi}{9}, 2\cos\frac{7\pi}{9}$

19  $\left\{\pm\frac{\pi}{4}, \pm\frac{2\pi}{3}, \pm\frac{3\pi}{4}\right\}$

21 a  $90^\circ, 199^\circ 28', 340^\circ 32'$  b  $(199^\circ 28', 340^\circ 32')$

24  $\{(x, y)|x = 2k\pi + \frac{\pi}{2}, y = 2k\pi\} \cup \{(x, y)|x = 2k\pi - \frac{\pi}{2}, y = 2k\pi + \pi\}, k \in \mathbb{Z}$

**Exercise 10.6**

1 a 5, 24, 11, 19 b  $T = 5\sin\left(\frac{\pi t}{12} - 3\right) + 19$  c  $23.6^\circ$

2 a 3, 4.2, 2, 7 b  $L = 3\sin\left(\frac{\pi t}{2.1} - 3\right) + 7$

3 a 5, 11, 0, 7 b  $V = 5\sin\left(\frac{2\pi t}{11}\right) + 7$

4 a 1, 11, 1, 12 b  $P = \sin\frac{2\pi}{11}(t-1) + 12$

5 a 2.6, 7, 2, 6 b  $S = 2.6\sin\frac{2\pi}{7}(t-2) + 6$

6 a 0.6, 3.5, 0, 11 b  $P = 0.6\sin\left(\frac{4\pi t}{7}\right) + 11$

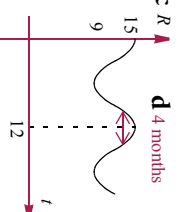
7 a 0.8, 4.6, 2.7, 11 b  $D = 0.8\sin\frac{\pi}{2.3}(t-2.7) + 11$

8 a 3000 b 1000, 5000 c  $\frac{4}{9}$

9 a 6.5 m, 7.5 m b 1.58 sec, 3.42 sec

10 a 750, 1850 b 3.44 c mid-April to end of August

11 a 15000 b 12 months c  $R$  d 4 months

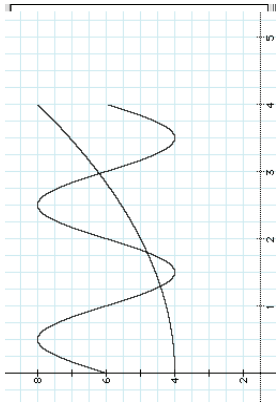


12 a  $\pi, -2, 2$  b  $\frac{1}{3} \text{ m}$  c  $\frac{4}{3} \text{ m}$

13 a

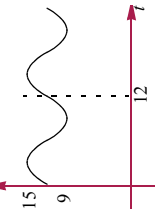
$t$	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(t)$	6	8	6	4	6	8	6	4	6
$G(t)$	4	4.0625	4.25	4.5625	5	5.5625	6.25	7.0625	8

b



c 3 d 38.4%

14 a



b i 7, 11, 19, 23 ii  $[0, 7] \cup [11, 19] \cup [23, 24]$  c 14.9 m

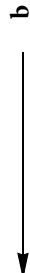
Chapter 11

Exercise 11.1

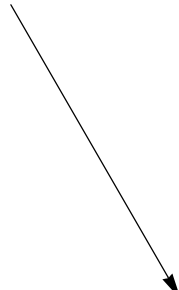
- vector
- scalar
- scalar
- vector
- vector
- vector
- scalar
- scalar

Exercise 11.2

1 a



d

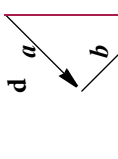
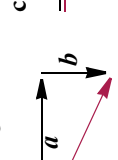
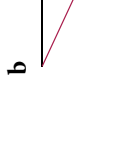
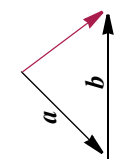


2 a

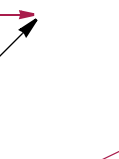
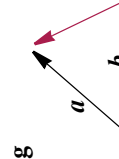
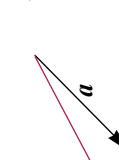
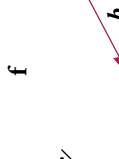
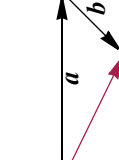


3 a {a,b,e,g,u}; {d,f} b {d,f}; {a,c}; {b,e} c {a,g}; {c,g} d {d,f}; {b,e} e {d,f}, {b,e}, {a,c,g}

4 a



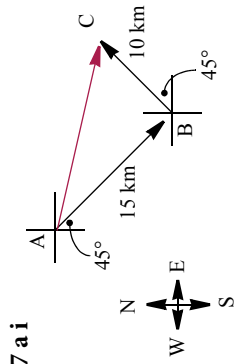
e



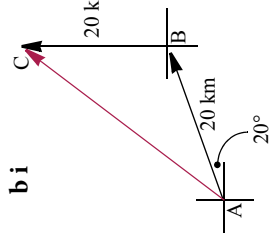
5 a AC b AB c AD d BA e 0

6 a Y b N c Y d Y e N

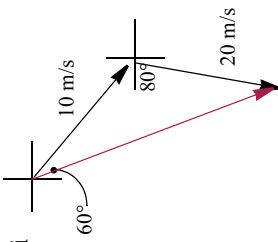
7 a i



b i



c i



ii  $\sqrt{325}$  iv  $20\sqrt{2}(1 - \cos 110^\circ)$  v  $10\sqrt{5 - 4\cos 110^\circ}$

8 72.11 N, E 33°41' N

9 2719 N along river

10 b i 200 kph N ii 213.6 kph, N 7°37' W

11 b i 200 ii 369.32

Exercise 11.3

1 a c - a b b - c c  $\frac{1}{2}(b + a)$

2 a b - a b b - 2a c 2b - 3a d  $\frac{1}{2}(b + 2a)$

3 a 0 b PS c AY d 6OC

4 a  $\frac{1}{2}(b+a)$  b  $\frac{1}{3}(2b+a)$  c  $\frac{1}{4}(a+b+2c)$

7 a  $c-b$  b  $c+a$  c  $a+c-2b$

8 a  $2\sqrt{21}$  b  $2\sqrt{26}$

15  $m = \frac{13}{23}$ ,  $n = \frac{50}{23}$

16  $m = \frac{4}{3}$

**Exercise 11.4**

1 a  $4i+28j-4k$  b  $12i+21j+15k$  c  $-2i+7j-7k$  d  $-6i-12k$

2 a  $3i-4j+2k$  b  $-8i+24j+13k$  c  $18i-32j+k$  d  $-15i+36j+12k$

3 a  $\begin{pmatrix} 11 \\ 0 \\ 8 \end{pmatrix}$  b  $\begin{pmatrix} -27 \\ 1 \\ -22 \end{pmatrix}$  c  $\begin{pmatrix} -3 \\ -6 \\ 12 \end{pmatrix}$  d  $\begin{pmatrix} 16 \\ -1 \\ 14 \end{pmatrix}$

4  $\begin{pmatrix} -5 \\ 3 \end{pmatrix}$

5  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ ,  $(-2, 3)$

6 a  $8i-4j-28k$  b  $-19i-7j-16k$  c  $-17i+j+22k$  d  $40i+4j-20k$

7 a  $\begin{pmatrix} 20 \\ 1 \\ 25 \end{pmatrix}$  b  $\begin{pmatrix} 12 \\ 2 \\ 16 \end{pmatrix}$  c  $\begin{pmatrix} -4 \\ -38 \\ -32 \end{pmatrix}$  d  $\begin{pmatrix} -20 \\ -22 \\ -40 \end{pmatrix}$

8  $A = -4$ ,  $B = -7$

9 a  $(2, -5)$  b  $(-4, 3)$  c  $(-6, -5)$

10 Depends on basis used. Here we used: East as  $i$ , North  $j$  and vertically up  $k$   
 b  $D = 600i - 800j + 60k$ ,  $A = -1200i - 300j + 60k$  c  $1800i - 500j$

**Exercise 11.5**

1 a  $\sqrt{10}$  b  $5\sqrt{2}$  c  $\sqrt{30}$  d 3 e  $\sqrt{53}$  f  $\sqrt{41}$  g  $\sqrt{14}$  h  $\sqrt{17}$

2 a  $\frac{1}{\sqrt{2}}(i+j)$  b  $\frac{1}{\sqrt{41}}(4i+5j)$  c  $\frac{1}{\sqrt{5}}(-i-2j)$  d  $\frac{1}{\sqrt{46}}(i+6j-3k)$

e  $\frac{1}{\sqrt{5}}(i+2k)$  f  $\frac{1}{\sqrt{17}}(2i-2j-3k)$  g  $\frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$  h  $\frac{1}{3\sqrt{3}}\begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix}$

3 a Depends on the basis:  $-3i+4j+k$  or  $-4i-3j+k$  b  $\sqrt{26}$

4 a  $\sqrt{3}(i-j+k)$  b  $\frac{1}{4}(3i-j+\sqrt{2}k)$

5  $\pm\sqrt{11}$

6  $\sqrt{13}$

**Exercise 11.6**

1 a 4 b  $-11.49$  c 25

2 a 12 b  $27^\circ$  c  $-8$  d  $-49$  f 4 g  $-21$  h 6 i  $-4$  j  $-10$

3 a  $79^\circ$  b  $108^\circ$  c  $55^\circ$  d  $50^\circ$  e  $74^\circ$  f  $172^\circ$  g  $80^\circ$  h  $58^\circ$

4 a  $-8$  b  $0.5$

5 a  $-6$  b 2 c Not possible d 5 e Not possible f 0

6 a  $4-2\sqrt{3}$  b  $2\sqrt{3}-4$  c  $14-2\sqrt{3}$  d Not possible

7 1

8  $105.2^\circ$

9  $x = -\frac{16}{7}$ ,  $y = -\frac{44}{7}$

10  $\pm\frac{1}{\sqrt{11}}(-i+j+3k)$

12 a  $\lambda(-16i-10j+k)$  b e.g.  $i+j+\frac{3}{2}k$

14  $a \perp b - c$  if  $b \neq c$  or  $b = c$

15 a  $\begin{pmatrix} 3 \\ 5 \\ 5 \end{pmatrix}$  b  $\begin{pmatrix} \sqrt{2} \\ 2 \\ 2 \end{pmatrix}$ ,  $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

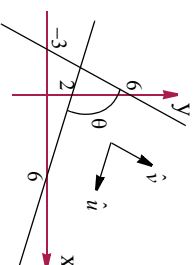
16 a  $\begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$ ,  $\begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$  b  $131.8^\circ$ ,  $48.2^\circ$ ,  $70.5^\circ$

18 a  $\frac{1}{3}$  b  $\frac{1}{\sqrt{3}}$

19 a

b  $i \cdot \hat{u} = \frac{1}{\sqrt{10}}(3i-j)$   $i \cdot \hat{v} = \frac{1}{\sqrt{5}}(i+2j)$

c  $81.87^\circ$



19  $\frac{1}{2}(-i+j+\sqrt{2}k)$

25 a Use  $i$  as a 1 km eastward vector and  $j$  as a 1 km northward vector.

b  $\vec{WD} = 4i+8j$ ,  $\vec{WS} = 13i+j$  and  $\vec{DS} = 9i-7j$  c  $\frac{1}{\sqrt{80}}(4i+8j)$  d  $\frac{d}{\sqrt{80}}(4i+8j)$

e  $3i+6j$

**Exercise 11.7.1**

- 1 **a**  $r = i + 2j$  **ii**  $r = -5i + 11j$  **iii**  $r = 5i - 4j$  **b** line joins (1, 2) and (5, -4)  
 2 **a**  $r = 2i + 5j + \lambda(3i - 4j)$  **b**  $r = -3i + 4j + \lambda(-i + 5j)$  **c**  $r = j + \lambda(7i + 8j)$   
**d**  $r = i - 6j + \lambda(2i + 3j)$  **e**  $r = \begin{pmatrix} -1 \\ -1 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -2 \\ 10 \end{pmatrix}$  or  $r = -i - j + \lambda(-2i + 10j)$   
**f**  $r = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$  or  $r = i + 2j + \lambda(5i + j)$   
 3 **a**  $r = 2i + 3j + \lambda(2i + 5j)$  **b**  $r = i + 5j + \lambda(-3i - 4j)$  **c**  $r = 4i - 3j + \lambda(-5i + j)$   
 4 **a**  $r = 9i + 5j + \lambda(i - 3j)$  **b**  $r = 6i - 6j + i(-4i - 2j)$   
**c**  $r = -i + 3j + \lambda(-4i + 8j)$  **d**  $r = i + 2j + \mu \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} - \frac{1}{3}j$   
 5 **a**  $x = -8 + 2\mu$  **b**  $x = 7 - 3\mu$  **c**  $x = 5 + 2.5\mu$  **d**  $x = 0.5 - 0.1t$   
 $y = 10 + \mu$   $y = 4 - 2\mu$   $y = 3 + 0.5\mu$   $y = 0.4 + 0.2t$   
 6 **a**  $\frac{x-1}{3} = y-3$  **b**  $\frac{x-2}{-7} = \frac{y-4}{-5}$  **c**  $x+2 = \frac{y+4}{8}$  **d**  $x-0.5 = \frac{y-0.2}{-11}$   
**e**  $x = 7$  **f**  $y = 6$   
 7 **a**  $r = 2j + t(3i + j)$  **b**  $r = 5i + t(i + j)$  **c**  $r = -6i + t(2i + j)$   
 8 **a**  $6t + 13j$  **b**  $-\frac{16}{3}i - \frac{28}{3}j$   
 9  $r = 2i + 7j + t(4i + 3j)$   
 11 **a** (4, -2), (-1, 1), (9, -5) **b** -2 **d**  $r = 4i - 2j + \lambda(-5i + 3j)$  **e**  $M \parallel L$  **ii**  $M = L$   
 12  $4x + 3y = 11$   
 13 **a**  $\frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}}$  **b**  $\frac{4}{5}, \frac{3}{5}$   
 14 **b** ii and iii  
 15 (-83, -215)  
 16  $r = \frac{k}{7}(19i + 20j)$   
 17 **a**  $\begin{pmatrix} 92 \\ 11 \\ 11 \end{pmatrix}$  **b**  $\emptyset$  **c** Lines are coincident, all points are common.

**Exercise 11.7.2**

- 1 **a** No **b** 52.5 mins after A  
 2 **a**  $r_A = \begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$  **ii**  $r_B = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$  **b** No **c**  $i \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} + (t-1) \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$  **ii** 11 a.m.

**Exercise 11.7.3**

- 1 **a**  $r = 2i + j + 3k + t(i - 2j + 3k)$  **b**  $r = 2i - 3j - k + t(-2i + k)$   
 2 **a**  $r = 2i + 5k + t(i + 4j + 3k)$  **b**  $r = 3i - 4j + 7k + t(4i + 9j - 5k)$   
**c**  $r = 4i + 4j + 4k + t(7i + 7k)$   
 3 **a**  $\frac{x}{3} = \frac{y-2}{4} = \frac{z-3}{5}$  **b**  $\frac{x+2}{5} = \frac{z+1}{-2}, y = 3$  **c**  $x = y = z$

$x = 5 - 7t$   $\begin{pmatrix} 5 \\ 2 + t \\ 6 \end{pmatrix} + t \begin{pmatrix} -7 \\ -4 \\ -4 \end{pmatrix}$   
 $y = 2 + 2t$   $r = \begin{pmatrix} 5 \\ 2 + t \\ 6 \end{pmatrix} + t \begin{pmatrix} -7 \\ -4 \\ -4 \end{pmatrix}$   
 $z = 6 - 4t$

5  $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$

$x = 2 + 3t$   $x = 1 + 1.5t$   $x = 3 - t$   $x = 1 + 2t$

6 **a**  $y = 5 + t$  **b**  $y = t$  **c**  $y = 2 - 3t$  **d**  $y = 3 + 2t$   
 $z = 4 + 0.5t$   $z = 4 - 2t$   $z = 4 + 2t$   $z = 2 + 0.5t$

7 **a**  $\frac{x-4}{3} = \frac{y-1}{-4} = \frac{z+2}{-2}$  **b**  $x = 2, y = \frac{z-1}{-3}$

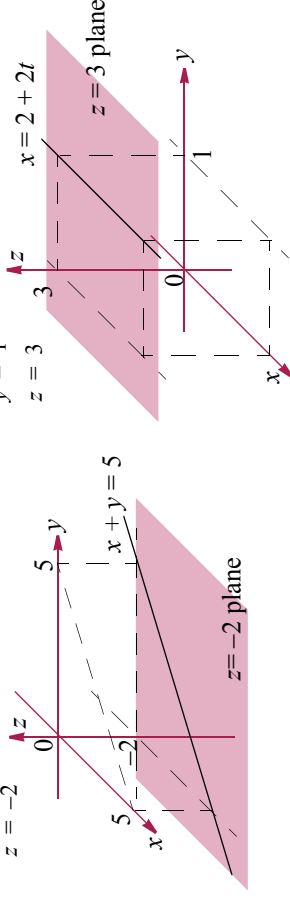
9 **a**  $\frac{x+1}{2} = y-3 = \frac{z-5}{-1}$  **b**  $\frac{x-2}{2} = \frac{z-1}{-2}, y = 1$

10 **a** (1, -1, 0) **b**  $a = 15, b = -11$

11 **a**  $x = 1 + t$

$y = 4 - t$

$z = -2$



12  $r = \begin{pmatrix} 1 \\ 0.5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1.5 \\ 1 \end{pmatrix}$ . Line passes through (1, 0.5, 2) and is parallel to the

vector  $2i - \frac{3}{2}j + k$

13 **a** 54.74° **b** 82.25° **c** 57.69°

14 **a** (4, 10.5, 15) **b** Does not intersect.

15 **a** L:  $x = \frac{y-2}{2} = \frac{z}{5}, M: \frac{x+1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$  **b**  $\emptyset$  **c** 84.92°

**d**  $i(0, 2, 0)$  **ii**  $\left(0, \frac{1}{2}, 0\right)$

18  $\frac{x}{4} = \frac{y}{9} = \frac{z}{3}$

19  $k = -\frac{7}{2}$

20  $64^\circ$

21 3 or -2

22  $12i + 6j - 7k$  (or any multiple thereof)

23 Not parallel. Do not intersect. Lines are skew.

**Chapter 12**

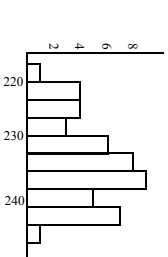
**Exercise 12.1**

- 1 a i 14 500 ii 2 000 b 305 (304.5)  
Population estimate is 5000.
- 2 Sample size is large but may be biased by factors such as the location of the catch.
- 3 a i 1500 ii 120 b 100 c 1 000
- 4 a, c numerical, b, d, e categorical
- 5 a, d discrete, b, c, e continuous

**Exercise 12.2**

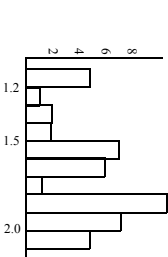
1

218– 220	221– 223	224– 226	227– 229	230– 232	233– 235	236– 238	239– 241	242– 244	245– 247
1	4	4	3	6	8	9	5	7	1



2

1.1– 1.2	1.2– 1.3	1.3– 1.4	1.4– 1.5	1.5– 1.6	1.6– 1.7	1.7– 1.8	1.8– 1.9	1.9– 2.0	2.0– 2.1
5	1	2	2	7	6	1	12	7	5



3 Set A Mode = 29.1 Mean = 27.2 Median = 27.85

Set B Mode = 9 Mean = 26.6 Median = 9. Set B is much more spread out than set A

and although the two sets have a similar mean, they have very different mode and median.

**Exercise 12.3**

- 1 Mode = 236–238 g, Mean = 234 g, Median = 235 g
- 2 Mode = 1.8–1.9 g, Mean = 1.69 g, Median = 1.80 g
- 3 Set A Mode = 29.1, Mean = 27.2, Median = 27.85; Set B Mode = 9, Mean = 26.6, Median = 9.

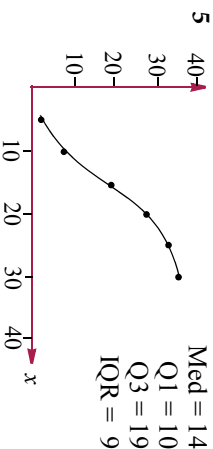
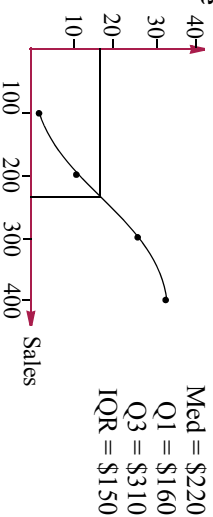
- 4 a \$27 522 b \$21 025 c Median
- 5 a \$233 300 b \$169 000 c Median
- 6 a 14.375 b 14.354

**Exercise 12.4**

- 1 a Sample A Mean = 1.99 kg; Sample B Mean = 2.00 kg  
b Sample A Sample std = 0.0552 kg; Sample B Sample std = 0.1877 kg  
c Sample A Population std = 0.0547 kg; Sample B Population std = 0.1858 kg
- 2 a 16.41 b 6.84
- 3 Mean = 49.97, Std = 1.365

**Exercise 12.5**

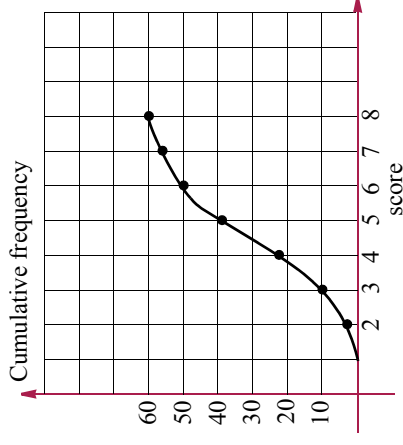
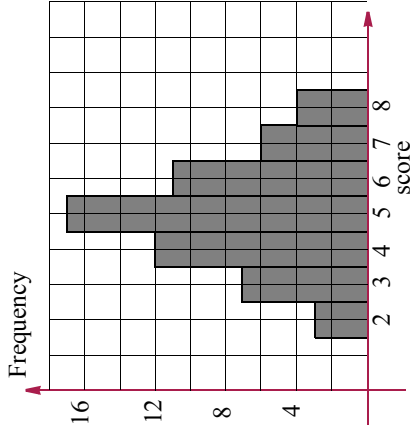
- 1 a Med = 5, Q1 = 2, Q3 = 7, IQR = 5 b Med = 3.3, Q1 = 2.8, Q3 = 5.1, IQR = 2.3  
c Med = 163.5, Q1 = 143, Q3 = 182, IQR = 39  
d Med = 1.055, Q1 = 0.46, Q3 = 1.67, IQR = 1.21  
e Med = 5143.5, Q1 = 2046, Q3 = 6252, IQR = 4206
- 2 a Med = 3, Q1 = 2, Q3 = 4, IQR = 2 b Med = 13, Q1 = 12, Q3 = 13, IQR = 1  
c Med = 2, Q1 = 2, Q3 = 2.5, IQR = 0.5  
d Med = 40, Q1 = 30, Q3 = 50, IQR = 20  
a \$84.67 b \$147.8 c \$11 d Q1 = \$4.50, Q3 = \$65 IQR = \$60.50
- 3 a 2.35 b 1.25 c 2 d Q1 = 1, Q3 = 3, IQR = 2
- 4 a \$232 b \$83 c–e





**Exercise 12.6 Miscellaneous questions**

- 1 a Sample–100 randomly selected patients, population – all suffering from AIDS  
 b Sample–1000 working aged people in N.S.W, population – all working aged people in N.S.W.  
 c Sample – John’s I.B. Higher Maths class, population – all seniors at Nappa Valley High School.
- 2 Discrete: a, b, d; Continuous: c, e, f, g.
- 3 b



- 4 suggested answers only: a 200–224; 225–249; 250–274; ... 575–599  
 b 100–119; 120–139; ... 400–419 c 440–459; 460–479; ... 780–799.
- 5 Make use of your graphics calculator.
- 6 a 16 b graphics calculator c 15.23 d 1.9892
- 7 a 30–34 b graphics calculator c 30.4 d 8.9205
- 8 b 215.5 c 216.2 d 18.80 sec
- 9 48.17, 14.14
- 10 a Q1 ~ 35, Q3 ~ 95 b ~ 105 c 61% d 67.15
- 11 range = 19, s = 5.49
- 12 5.8; 1.50
- 13 17.4;  $s_n = 3.12$   $s_{n-1} = 3.18$
- 14 a 6.15 b 1.61
- 15  $s_n = 18.8$ ,  $s_{n-1} = 19.1$

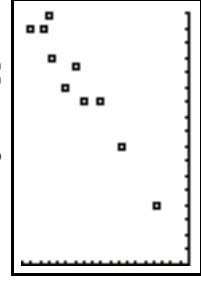
**Chapter 13**

**Exercise 13.2**

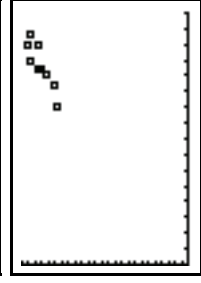
- 1 a i Increasing, positive ii Approx. linear iii Mild (to weak)  
 b i No association ii–iii 0  
 b i Increasing, positive ii Linear iii Very strong  
 d i Increasing, positive ii Square root iii Mild (strength not appropriate as it is a non-linear relationship!)  
 e i Decreasing, negative ii Exponential iii Mild (strength not appropriate as it is a

non-linear relationship!)

- f i Decreasing ii Approx. linear iii Mild

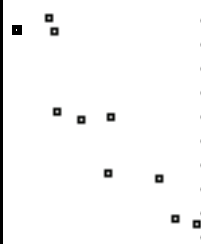


2 a



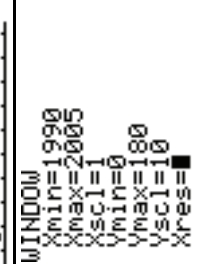
3 a

b Positive association, linear, strength: very strong



4

Data displays a strong positive association. Increase in lead content can be attributed to increase in traffic flow.

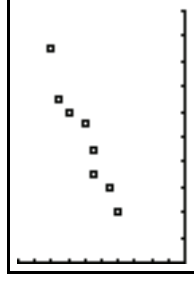


5

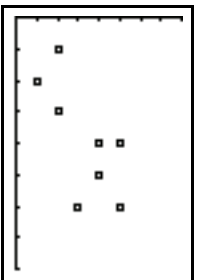
Work safety policy has had desired effect, i.e. number of accidents has decreased. Data displays a strong negative association.

**Exercise 13.3**

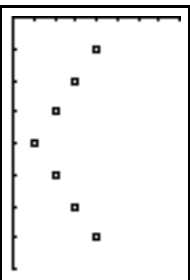
- 1 a  $r = 0.96$  b



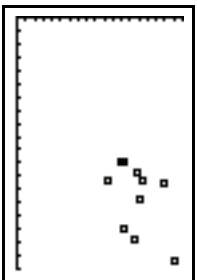
2 a  $r = 0.70$  (assumed linear)



3 a  $r = 0$ . No, not linear!



4 No. The relationship is not linear.  
5 a i 64% ii 81% b i 51% ii 64%



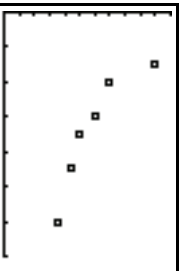
b  $r = 0.45$

6 a  
7 3  
8  $\pm 0.922$

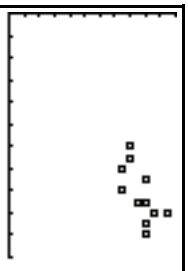
10 b  $r = 0.7715$  There is strong evidence to suggest that a student who does well in Maths will also do well in Biology. c Values of  $r$  are the same.

**Exercise 13.4**

1 a i ii  $y = -1.33x + 21.11$

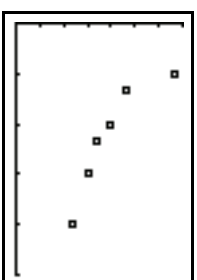


b i ii  $y = 0.64x + 6.94$



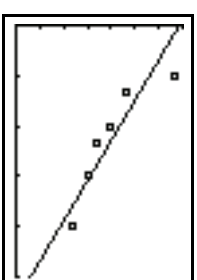
2 a i ii  $y = 20.6 - 1.26x$

iii

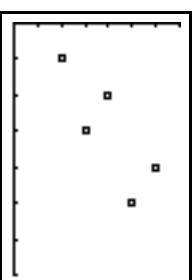


```
LinReg
y=a+bx
a=20.598789419
b=-1.2583586637
r2=.9050526291
r=.

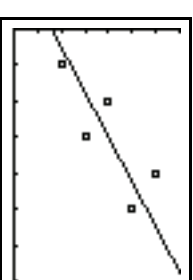
```



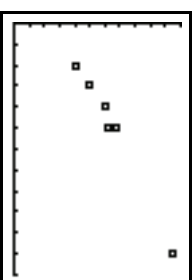
b i ii  $y = 1.6 + 0.8x$



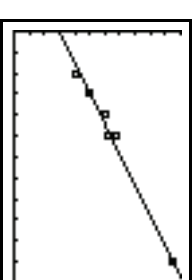
```
LinReg
y=a+bx
a=1.6
b=.8
r2=.64
r=.8
```



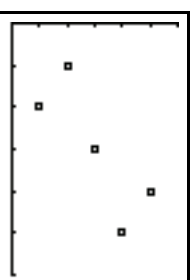
c i ii  $y = 14.8 + 3.44x$



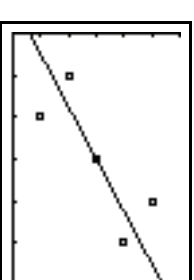
```
LinReg
y=a+bx
a=14.8
b=3.44
r2=.9828571429
r=.9913915185
```



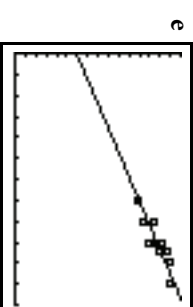
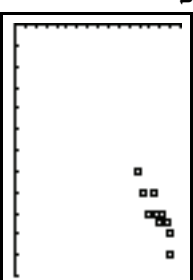
d i ii  $y = 0.6 + 0.8x$



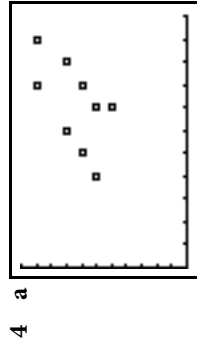
```
LinReg
y=a+bx
a=.6
b=.8
r2=.64
r=.8
```



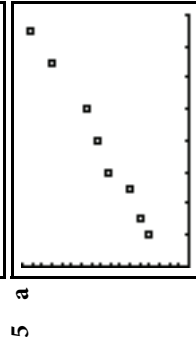
3 a b  $r = 0.891$



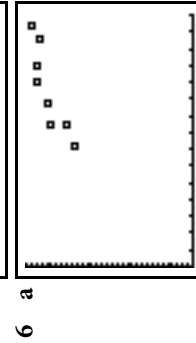
c 79.4%  
d  $y = 29.76 + 2.15x$



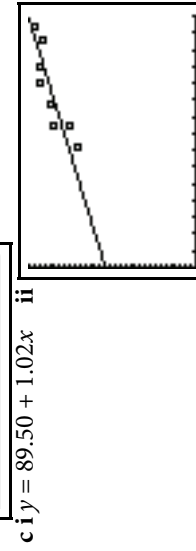
- b  $r = 0.553$   
 c i  $y = 40 + 0.5x$   
 ii  $x = 24.1 + 0.61y$



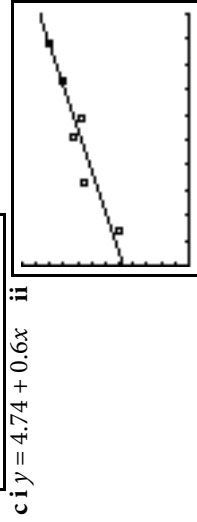
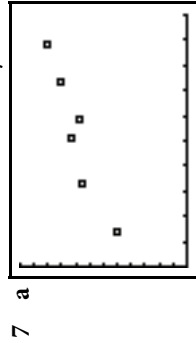
- b Based on the scatter diagram, there is a definite linear relationship. Therefore, owner is justified.  
 c i  $r = 0.99$  ii  $C = 4.19 + 1.82w$   
 d i 20.57, i.e. 21 ii 95.19, i.e. 95  
 iii From ii, serving 95 people per hour is unrealistic.



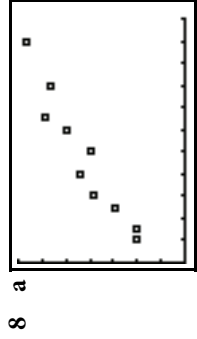
- b Scatter diagram shows a linear relationship. Therefore statistic is appropriate,  $r = 0.877$ .



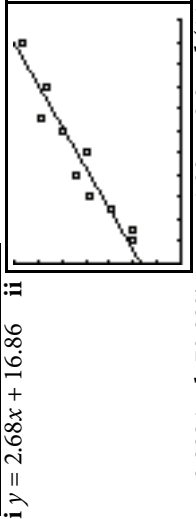
- d i 135.6 ii 176.5  
 iii  $x = 85$  is a fair way out from the set of values used to obtain the regression line  
 b Scatter diagram shows a linear relationship. Therefore statistic is appropriate,  $r = 0.945$



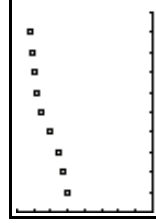
- d i 8.63 ii 10.73



- b Linear trend exists,  $r = 0.96$

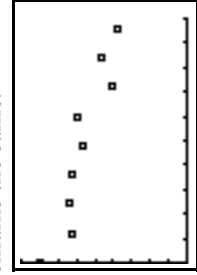


- d i 27.57 ii 57.03

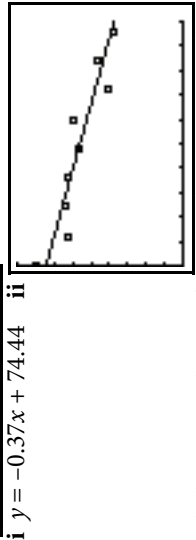


- b i  $r = 0.97$  ii 222 c  $M = 0.2967T + 48.28$

11 Remains the same.

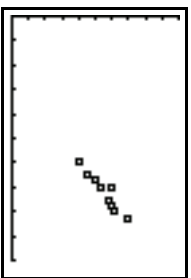


- b -0.93



- d  $y(40) = 59.722$  (interpolation),  $y(120) = 30.278$  (extrapolation)

**Exercise 13.5 Miscellaneous questions**



- 1 a  $y = 0.57x - 26.2$   
 c 0.9388 Because of the strong positive linear association, and the high  $r$  value, we can say that the taller the student the greater their weight.

2 0.057

3 B

4 a 0.8 b Strong positive relationship

5 1.5

6 a 0.78 b i P = 1.07M - 12.91 ii 73% c i M = 0.77E + 27.14 ii 100

ii Extrapolated. Continued linear trend highly likely. Therefore confident

d Find regression equation of E on M, then use M = 90 into this new equation.

7 a Positive b Linear c Very strong

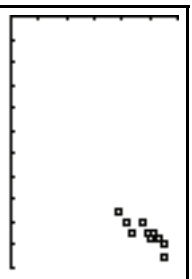
8 a  $\bar{x} = 20.57, \bar{y} = 31.86$  b 0.9645 c  $y = 1.68x - 2.7$

9 a  $y = -1.75x + 64.67$  b 22.67 c -11.12

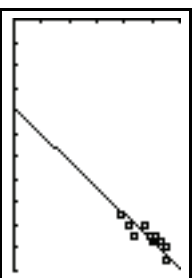
10 a i ii = 0.8908

b  $r^2 = 1.7935$ , that is, 79.35%

c  $y = 2.15x - 33.28$



d e  $x = 37, y = 46.35$ ; Expenditure is \$4635

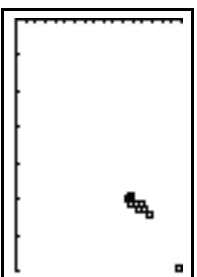


11 a i 4.4; 2.02 ii 14.06; 2.92 b  $b = 0.4895; r = 0.34$

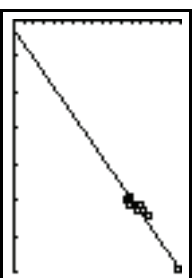
c  $r^2 = (0.3397)^2 = 0.1154$

d Regression equation is  $(y - 14.06) = 0.4895(x - 4.4)$  when  $x = 3.5, y = 13.63$

12 a i ii  $r = 0.9629$  b  $y = 0.635x - 33.815$



d When  $x = 1040, y = 626.59$ . The carcass weighs 626.59 lbs.



**Chapter 14**

**Exercise 14.1**

1 15

2 a 25 b 625

3 a 24 b 256

4 a 24 b 48

5 15

6 270

7 120

8 336

9 60

10 a 362880 b 80640 c 1728

11 20

12 a 10! b  $2 \times 8!$  c  $2 \times 9!$  ii  $8 \times 9!$

13 34650

14 4200

15 4

**Exercise 14.2**

1 792

2 a 1140 b 171

3 1050

4 70

5 2688

6 a 210 b 420

7 24000

8 8

9 155

10 5

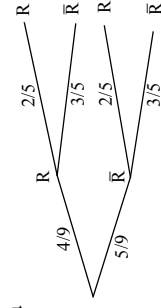
**Exercise 14.3 Miscellaneous questions**

1 a 120 b 325

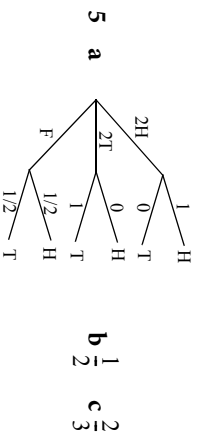
2 5040

- 3 a 144 b 1440  
 4 a 720 b 240  
 5 11760  
 6 7056; 4606  
 7 a 840 b 1680  
 8 190  
 9 10080  
 10 226800  
 11 a 71 b 315 c 665  
 13  ${}^n C_2$   
 14  ${}^n C_4$   
 15 b 92  
 16 252  
 17 a 1287 b 560  
 18 256  
 19 288  
 20 a 10080 b 30240 c 14400  
 21 10080, 1080  
 22 3528000  
 23 720; 240  
 24 103680  
 25 a 12 b 128  
 26 2880  
 27 a 30030 b 37310  
 28 77055  
 29 a 48 b 72
- Chapter 15**  
**Exercise 15.1**  
 1 a  $\frac{2}{5}$  b  $\frac{3}{5}$  c  $\frac{2}{5}$   
 2 a  $\frac{2}{7}$  b  $\frac{5}{7}$   
 3 a  $\frac{5}{26}$  b  $\frac{21}{26}$   
 4 {HH, HT, TH, TT} a  $\frac{1}{4}$  b  $\frac{3}{4}$   
 5 {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT} a  $\frac{3}{8}$  b  $\frac{1}{2}$  c  $\frac{1}{4}$   
 6 a  $\frac{2}{9}$  b  $\frac{2}{9}$  c  $\frac{2}{3}$  d  $\frac{1}{3}$   
 7 a  $\frac{1}{2}$  b  $\frac{3}{10}$  c  $\frac{9}{20}$
- 8 a  $\frac{11}{36}$  b  $\frac{1}{18}$  c  $\frac{1}{6}$  d  $\frac{5}{36}$   
 9 {GGG, GGB, BGB, BGG, BBB, BBG, BGB, GBB} a  $\frac{1}{8}$  b  $\frac{3}{8}$  c  $\frac{1}{2}$   
 10 a  $\frac{1}{2}$  b  $\frac{1}{4}$  c  $\frac{1}{4}$   
 11 a  $\frac{3}{8}$  b  $\frac{1}{4}$  c  $\frac{3}{8}$  d  $\frac{3}{4}$   
 12 a {(1, H), (2, H), (3, H), (4, H), (5, H), (6, H), (1, T), (2, T), (3, T), (4, T), (5, T), (6, T)}  
 b  $\frac{1}{4}$   
 13 a  $\frac{1}{216}$  b  $\frac{1}{8}$  c  $\frac{3}{8}$   
**Exercise 15.2**  
 1 a  $\frac{1}{4}$  b  $\frac{5}{8}$  c  $\frac{3}{4}$   
 2 a  $\frac{1}{13}$  b  $\frac{1}{2}$  c  $\frac{1}{26}$  d  $\frac{7}{13}$   
 3  $\frac{9}{26}$   
 4 a 1.0 b 0.3 c 0.5  
 5 a 0.65 b 0.70 c 0.65  
 6 a 0.95 b 0.05 c 0.80  
 7 a {TTT, TTH, THT, HTT, HHH, HHT, HTH, THH} b i  $\frac{3}{8}$  ii  $\frac{1}{2}$  iii  $\frac{1}{4}$  iv  $\frac{3}{8}$   
 8 a  $\frac{6}{25}$  b  $\frac{6}{25}$  c  $\frac{13}{25}$   
 9 b  $\frac{3}{4}$  c  $\frac{1}{2}$  d  $\frac{1}{6}$  e  $\frac{7}{12}$   
 10 a  $\frac{1}{4}$  b  $\frac{1}{2}$  c  $\frac{8}{13}$  d  $\frac{7}{13}$   
 11 a 0.1399 b i 0.8797 ii 0.6  
 12 b  $\frac{4}{15}$  c  $\frac{4}{15}$  d  $\frac{11}{15}$

- Exercise 15.3**  
 1 a 0.7 b 0.75 c 0.50 d 0.5  
 2 a 0.5 b 0.83 c 0.10 d 0.90  
 3 a

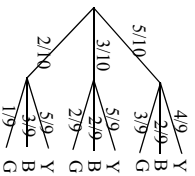


4 a 0.5 b 0.30 c 0.25



6  $\frac{1}{3}$

7 a b  $\frac{31}{45}$  c  $\frac{2}{9}$



8  $\frac{2}{3}$

9 a 0.88 b 0.42 c 0.6 d 0.28

10 a 0.33 b 0.49 c 0.82 d 0.551

11 a 0.22 b 0.985 c 0.8629

12 a 0.44 b 0.733

14 a 0.512 b 0.128 c 0.8571

15 a 0.2625 b 0.75 c 0.4875 d 0.7123

16 a 0.027 b 0.441 c 0.453

**Exercise 15.4**

1 a 0.042 b 0.7143

2 a 0.4667 b 0.3868

3 a  $\frac{5}{7}$  b  $\frac{9}{13}$

4  $\frac{5}{9}$

5 bi  $\frac{1}{40}$  ii 0.2

6 a i  $\frac{2^{N-m}}{2^N}$  ii  $\frac{2^{(N-m)}}{2^{N-m}}$  b  $\frac{m}{m+(N-m)^2}$

7  $\frac{9}{19}$

8 a 0.07 b 0.3429 c 0.30 d 0.0282

9 a 0.8008 b 0.9767 c 0.0003

10 a 0.0464 b 0.5819 c 0.9969

11 0.2 b 0.08 c 0.72

**Exercise 15.5**

1 a  $\frac{5}{126}$  b  $\frac{5}{18}$  c  $\frac{1}{126}$

2 a  $\frac{1}{5}$  b  $\frac{1}{10}$  c  $\frac{2}{5}$  d  $\frac{3}{5}$

3 a  $\frac{72}{5525}$  b  $\frac{1}{5525}$  c  $\frac{1}{1201}$

4  $\frac{2}{5}$

5 a  $\frac{63}{143}$  b  $\frac{133}{143}$

6 a  $\frac{5}{12}$  b  $\frac{5}{33}$  c  $\frac{5}{6}$

7  $\frac{3}{11}$

8 a  $\frac{4}{13}$  b  $\frac{9}{13}$

9 a  $\frac{67}{91}$  b  $\frac{22}{91}$

10 a  $\frac{1}{4}$  b  $\frac{1}{28}$  c  $\frac{5}{14}$

11 a  $\frac{5}{28}$  b  $\frac{1}{28}$

12  $\frac{6}{13}$

13 a  $\frac{1}{6}$  b  $\frac{1}{4}$

14 a  $\frac{1}{210}$  b  $\frac{7}{9}$

15 a  $\frac{7}{1938}$  b 0.6

16  $\frac{11}{21}$

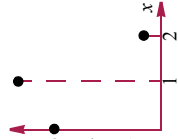
**Chapter 16**

**Exercise 16.1**

1 0.3

2 a 0.1 bi 0.2 ii 0.7

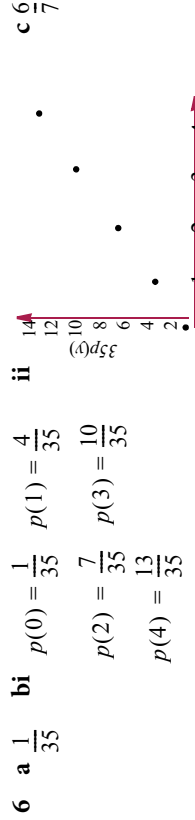
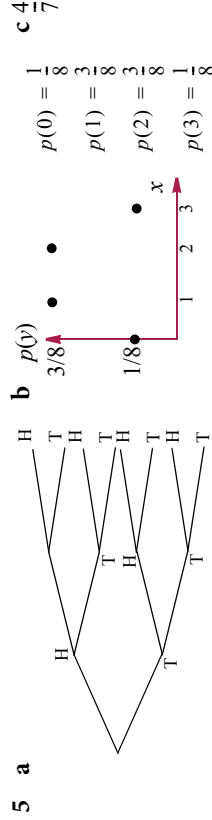
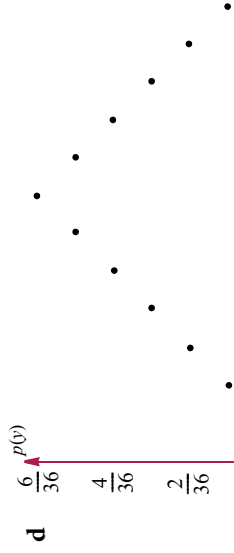
3 a  $p(0) = \frac{6}{15}, p(1) = \frac{8}{15}, p(2) = \frac{1}{15}$  b  $15 \cdot p(x)$  c  $\frac{14}{15}$



4 a {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$x$	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

c  $\frac{5}{36}$



7 a i 0.9048 ii 0.09048 b 0.0002

8 0.3712

9 a  $p(0) = \frac{11}{30}, p(-1) = \frac{1}{2}, p(3) = \frac{2}{15}$  b i  $\frac{11}{30}$  ii  $\frac{13}{15}$

10

$n$	0	1	2
$P(N=n)$	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{1}{15}$

11 a

$n$	1	2	3	4
$P(N=n)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b

$s$	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

12 a 0.81 b 0.2439

Exercise 16.2

1 a 2.8 b 1.86

2 a 3 b i 1 ii 1 ci 6 ii 0.4

3 a i 1.3 ii 2.5 iii -0.1 bi 0.9 ii 7.29 ci  $\frac{31}{60}$  ii 0.3222

4  $\mu = \frac{2}{3}, \sigma^2 = 0.3556$

5 a 7 b 5.8333

6  $np = 3 \times \frac{1}{2} = 1.5$

7 a  $\frac{1}{25}$  b 2.8 c 1.166

8 a 0.1 bi 0.3 ii 1 ci 0 iii 2

9 5.56

10  $p(0) = \frac{35}{120}, p(1) = \frac{63}{120}, p(2) = \frac{21}{120}, p(3) = \frac{1}{120}$  bi 0.9 ii 0.49

c  $W = 3N - 3, E(W) = -0.3$

11 a \$-1.00 b both the same

12 a 50 b 18 c 2

13 a 11 b  $\frac{\sqrt{3}}{3}$  c -4

14 a 0.75 b 0.6339

15 a  $E(X) = 1 - 2p, \text{Var}(X) = 4p(1 - p)$  bi  $n(1 - 2p)$  ii  $4np(1 - p)$

16 a

$n$	0	1	2
$P(N=n)$	$\frac{28}{45}$	$\frac{16}{45}$	$\frac{1}{45}$

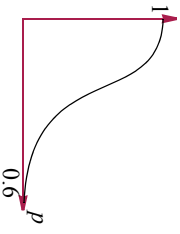
b  $W = 21.43$

17 a  $a = \frac{2}{3}, 0 \leq b \leq 1$     b  $E(X) = \frac{b+1}{3}, Var(X) = \frac{1}{9}(2+7b-b^2)$

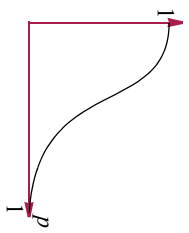
18 a  $E(X) = 4, Var(X) = 20$

**Exercise 16.3**

- 1 a 0.2322    b 0.1737    c 0.5941  
 2 a 0.3292    b 0.8683    c 0.2099    d 0.1317  
 3 a 0.1526    b 0.4812    c 0.5678  
 4 a 0.7738    b  $3.125 \times 10^{-7}$     c 0.9988    d  $3 \times 10^{-5}$   
 5 a 0.2787    b 0.4059  
 6 a 0.2610    b 0.9923  
 7 a 0.2786    b 0.7064    c 0.1061  
 8 a 0.1318    b 0.8484    c 0.054    d 0.326  
 9 a 0.238    b 0.6531    c 0.0027    d 0.726    e 12.86  
 10 a 0.003    b 0.2734    c 0.6367    d 0.648  
 11 a 0.3125    b 0.0156    c 0.3438    d 3  
 12 a 0.2785    b 0.3417    c 120  
 13 a 0.0331    b 0.565  
 14 a 0.4305    b 0.61    c \$720    d 0.2059  
 15 a i 1.4    ii 1    iii 1.058    iv 0.0795    v 0.0047  
     b i 3.04    ii 3    iii 1.373    iv 0.2670    v 0.1390  
 16 38.23  
 19 a i 0.1074    ii  $7.9 \times 10^{-4}$     iii 0.3758    b at least 6  
 20 a  $\frac{4}{3}$     b  $\frac{10}{9}$     c  $\frac{1}{6}$     d  $\frac{5}{288}$   
 21 a 20    b 3.4641  
 22 a 102.6    b 0.000254  
 23 a i 6    ii 2.4    b i 6    ii 3.6  
 24 0.1797  
 25 1.6, 1.472  
 26 a 0.1841    b \$11.93  
 27 a \$8    b \$160  
 28 a 0.0702    c



29 b



30 b 0.8035    c 39.3

**Chapter 17**

**Exercise 17.1**

- 1 a 0.6915    b 0.9671    c 0.9474    d 0.9965    e 0.9756    f 0.0054    g 0.0287  
     h 0.0594    i 0.0073    j 0.8289    k 0.6443    l 0.0823  
 2 a 0.0360    b 0.3759    c 0.0623    d 0.0564    e 0.0111    f 0.2902    g 0.7614  
     h 0.0343    i 0.6014    j 0.1450    k 0.9206    l 0.2668    m 0.7020    n 0.9132  
     o 0.5203    p 0.8160    q 0.9388    r 0.7258

**Exercise 17.2**

- 1 a 0.0228    b 0.9332    c 0.3085    d 0.8849    e 0.0668    f 0.9772  
 2 a 0.9772    b 0.0668    c 0.6915    d 0.1151    e 0.9332    f 0.0228  
 3 a 0.3413    b 0.1359    c 0.0489  
 4 a 0.6827    b 0.1359    c 0.3934  
 5 a 0.8413    b 0.4332    c 0.7734  
 6 a 0.1151    b 0.1039    c 0.1587  
 7 a 0.1587    b 0.6827    c 0.1359  
 8 a 0.1908    b 0.4754    c 16.88  
 9 a 0.1434    b 0.6595  
 10 a 0.2425    b 0.8413    c 0.5050  
 11 a -1.2816    b 0.2533  
 12 a 58.2243    b 41.7757    c 59.80  
 13 39.11  
 14 9.1660  
 15 42%  
 16 0.7021  
 17 a 0.2903    b 0.4583    c 0.2514  
 18 23%  
 19 0.5  
 20 11%  
 21 5%  
 22 14%  
 23 1.8  
 24 252  
 25 0.1517  
 26 0.3821  
 27 0.22  
 28 322  
 29 0.1545  
 30 7  
 31 87  
 32 a i 0.0062    ii 0.0478    iii 0.9460    b 0.0585  
 33 a \$5.11    b \$7.39  
 34 a 0.0062    b i 0.7887    ii 0.0324    c \$1472  
 35 a  $\mu = 66.86, \sigma = 10.25$     b \$0.385  
 36 a  $\mu = 37.2, \sigma = 28.2$     b 20 (19.9)  
 37 a i 0.3446    ii 0.2347    b i 0.3339    ii 0.3852    c 0.9995

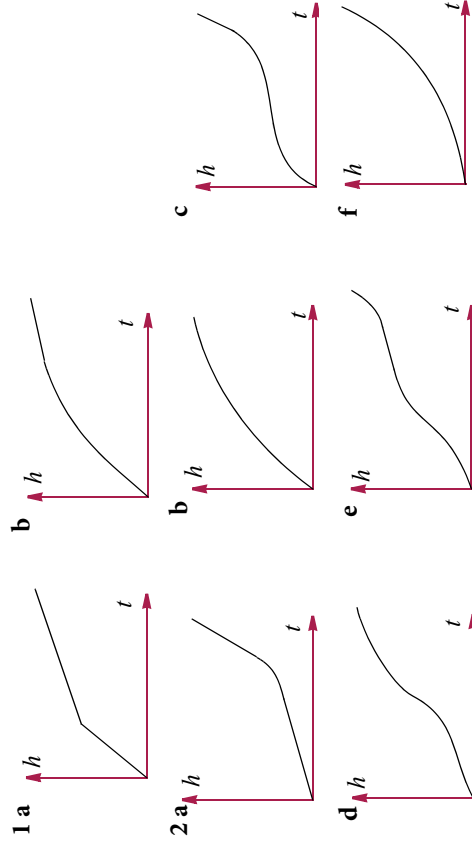


**Chapter 18**

**Exercise 18.1**

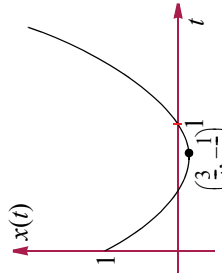
- 1 a  $\frac{3}{4}$  b  $\frac{3a}{4b}$  c -1 d 1 e  $-\frac{15}{8}$  f 0  
 2 a 4 b 0.2 c 0.027 d 0.433 e -0.01 f 6.34 g 6.2 h 0  
 3 a 6 m/s b 30 m/s c 11 + 6h + h<sup>2</sup> m/s  
 4 12 m/s  
 5 8 + 2h  
 6 -3.49°C/sec  
 7 a 127π cm<sup>3</sup>/cm b i 19.6667π cm<sup>3</sup>/cm ii 1.9967π cm<sup>3</sup>/cm iii 0.2000π cm<sup>3</sup>/cm  
 8 1.115  
 9 a -7.5°C/min b t = 2 to t = 6  
 10 a 28 m b 14 m/s c average speed d 49 m e 49 m/s  
 11 a \$1160, \$1345.6, \$1560.90, \$1810.64, \$2100.34 b \$220.07 per year

**Exercise 18.2**

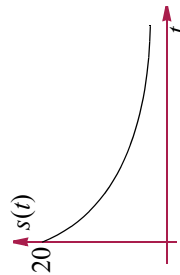


**Exercise 18.3**

- 1 a h + 2 b 4 + h c  $\frac{-1}{1+h}$  d 3 - 3h + h<sup>2</sup>  
 2 a 2 b 4 c -1 d 3  
 3 a 2a + h b -(2a + h) c (2a + 2) + h d 3a<sup>2</sup> + 1 + 3ah + h<sup>2</sup>  
 e -(3a<sup>2</sup> + 3ah + h<sup>2</sup>) f 3a<sup>2</sup> - 2a + (3a - 1)h + h<sup>2</sup> g  $\frac{-2}{a(a+h)}$   
 h  $-\frac{1}{(a-1)(a-1+h)}$  i  $\frac{1}{\sqrt{a+h} + \sqrt{a}}$   
 4 a 1; 1 b 2a + h; 2a c 3a<sup>2</sup> + 3ah + h<sup>2</sup>; 3a<sup>2</sup> d 4a<sup>3</sup> + 6a<sup>2</sup>h + 4ah<sup>2</sup> + h<sup>3</sup>; 4a<sup>3</sup>

- 5 a  b i 3 ms<sup>-1</sup> ii 2 ms<sup>-1</sup> iii 1.2 ms<sup>-1</sup>

d Find (limit) as h → 0 e 4t - 3

- 6 a  b i 20 cm<sup>2</sup> ii 17.41 cm<sup>2</sup> iii 2.59 cm<sup>2</sup>


- iv -1.29 cm<sup>2</sup>/day c 20(1 - 2<sup>-0.1h</sup>) cm<sup>2</sup>/day d i -1.3863 cm<sup>2</sup>/day  
 ii -1.2935 cm<sup>2</sup>/day

**Exercise 18.4**

- 1 a 15 b 8 c 0 d 1 e 0 f 6 g  $\frac{2}{\pi}$  h e i 6 j  $\sqrt{5}$   
 2 a 4 b -3 c 0.5 d 0 e 3  
 3 a 0 b undefined c 1 d 1 e undefined  
 4 a i 0 ii 0 bi -8 ii undefined  
 5 a 2 b 1 c 0.5 d 1 e 3  
 6 a 1 b -1 c undefined  
 7 a  $\frac{1}{3}$  b 3x<sup>2</sup> + 2 c 6 d 12 e  $-\frac{1}{x^2}$   
 8 a 0 b 4 c 4 d 0 e 1  
 9 a 3 b -2 c 2.5 d -1 e 0  
 10 a i 2 ii  $\frac{b}{a}$  iii 2 iv 0

**Exercise 18.5**

- 1 a 3 b 8 c  $-\frac{1}{9}$  d 1.39 e -1 f  $\frac{17}{16}$   
 2 4.9 m b 4.9(h<sup>2</sup> + 2h) m c 9.8 m/s  
 3 a 8x b 10x c 12x<sup>2</sup> d 15x<sup>2</sup> e 16x<sup>3</sup> f 20x<sup>3</sup>  
 4 a 4x b -1 c -1 + 3x<sup>2</sup> d -x<sup>-2</sup> e -2(x + 1)<sup>-2</sup> f 0.5x<sup>-1/2</sup>  
 5 a 1 ms<sup>-1</sup> b (2 - a) ms<sup>-1</sup>

- 6 a  $\uparrow x(t)$   b  $5 \text{ ms}^{-1}$  ii  $4 \text{ ms}^{-1}$  c  $8t - 3t^2 \text{ ms}^{-1}$  d  $\frac{8}{3} \text{ sec}$

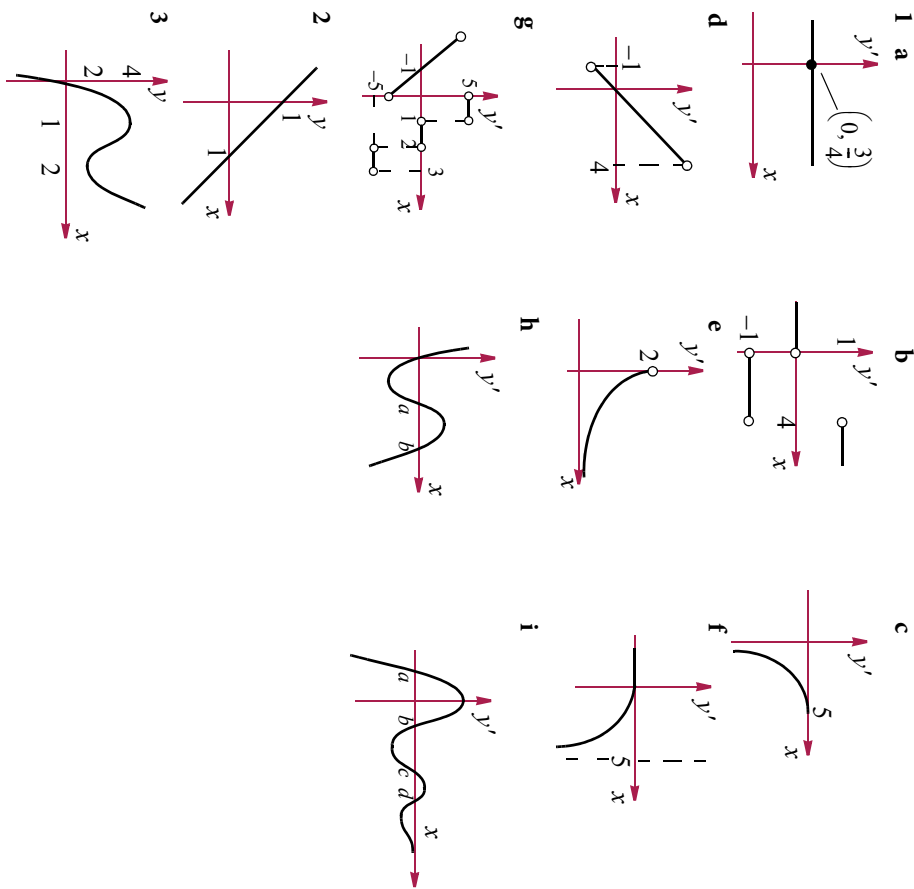
Chapter 19

Exercise 19.1

- 1 a  $5x^4$  b  $9x^8$  c  $25x^{24}$  d  $27x^2$  e  $-28x^6$  f  $2x^7$  g  $2x$  h  $20x^3 + 2$   
 $-15x^4 + 18x^2 - 1$  j  $-\frac{4}{3}x^3 + 10$  k  $9x^2 - 12x$  l  $3 + \frac{2}{5}x + 4x^3$
- 2 a  $-\frac{3}{x^4}$  b  $\frac{3}{2}\sqrt{x}$  c  $\frac{5}{2}\sqrt{x^3}$  d  $\frac{1}{3\sqrt{x^2}}$  e  $\frac{2}{\sqrt{x}}$  f  $9\sqrt{x}$  g  $\frac{1}{\sqrt{x}} + \frac{3}{x^2}$  h  $\frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x^3}}$   
i  $\frac{10}{3\sqrt{x}} - 9$  j  $5 - \frac{1}{2\sqrt{x}} - \frac{8}{5x^3}$  k  $\frac{4}{\sqrt{x}} - \frac{15}{x^6} + \frac{1}{2}$  l  $-\frac{1}{2\sqrt{x^3}} - \frac{1}{\sqrt{x}} + x^2$
- 3 a  $\frac{3}{2}\sqrt{x} + \frac{1}{\sqrt{x}}$  b  $4x^3 + 3x^2 - 1$  c  $3x^2 + 1$  d  $\frac{1}{x^2}$  e  $\frac{1}{\sqrt{x^3}}$  f  $\frac{1}{2} - \frac{1}{4\sqrt{x^3}}$  g  $-7$   
h  $2x - \frac{8}{x^3}$  i  $2x - \frac{2}{x^2} - \frac{4}{x^5}$  j  $\frac{1}{2}\sqrt{\frac{3}{x}} + \frac{1}{6\sqrt{x^3}}$  k  $2x - \frac{12}{5}\sqrt{x} + \frac{2}{5\sqrt{x^3}}$   
l  $-\frac{3}{2\sqrt{x}}\left(\frac{1}{x} + 1\right)\left(\frac{1}{\sqrt{x}} - \sqrt{x}\right)^2$
- Exercise 19.2.1**
- 1  $m_{PQ} = 4 + h$ ;  $\lim_{h \rightarrow 0} m_{PQ} = 4$
- 2  $P(1, 1), Q\left(1+h, \frac{2}{2+h}\right)$ ;  $m_{PQ} = -\frac{1}{2+h}$ ;  $\lim_{h \rightarrow 0} m_{PQ} = -\frac{1}{2}$
- 3  $-12$
- 4 a 3 b  $-\frac{1}{4}$  c 12 d 4 e 4 f  $\frac{7}{6}$  g  $-\frac{1}{12}$  h  $\frac{53}{16}$
- 5  $\pm\sqrt[3]{8}$
- 6 a  $2x - 12$  b  $-18$  c  $(8, -32)$
- 7 a  $-3x^2 + 3$  b 0 c  $(\pm\sqrt{2}, \pm\sqrt{2})$
- 8 a  $\left(\pm\frac{\sqrt{2}}{2}, -\frac{1}{16}\right), (0, 0)$  b  $\left\{x: -\frac{1}{\sqrt{2}} < x < 0\right\} \cup \left\{x: x > \frac{1}{\sqrt{2}}\right\}$
- 9  $x = \frac{1}{3}, -1$

- 10 a  $-2, 6, 3$  b  $-2$   
11 a  $= 1$  b  $= -8$   
12  $f'(a+b) = 2(a+b) = 2a+2b$   
13 a  $4a^2 - 2a, a \geq 0$  b  $4 - \frac{1}{a}, a > 0$   
14  $-56$

Exercise 19.2.2



**Exercise 19.2.3**

- 1 a  $48t^3 - \frac{1}{2\sqrt{t}}$  b  $2n - \frac{2}{n^2} - \frac{4}{n^5}$  c  $\frac{3}{2}\sqrt{r} + \frac{5}{6\sqrt{r}} - \frac{1}{\sqrt{r}}$  d  $20 - \frac{9}{2}\sqrt{\theta} + 3 - \frac{1}{2\sqrt{\theta}}$   
 e  $40 - 3t^2$  f  $-\frac{100}{v^3} - 1$  g  $6t^2 + 5$  h  $2\pi + 8h$  i  $4n^3 - \frac{1}{3\sqrt[3]{n^2}} + \pi$   
 2 a  $\frac{8}{3t^3}$  b  $\frac{2\pi r}{r^2} - \frac{20}{r^2}$  c  $\frac{5}{2}3^{3/2} + \frac{3}{s^2}$  d  $-\frac{6}{t^4} + \frac{2}{t^3} - \frac{1}{t^2}$  e  $-\frac{4}{b^2} + \frac{1}{2b^{3/2}}$  f  $3m^2 - 4m - 4$

**Exercise 19.3**

- 1 a  $3x^2 - 5x^4 + 2x + 2$  b  $6x^5 + 10x^4 + 4x^3 - 3x^2 - 2x$  c  $-\frac{4}{x^5}$  d  $6x^5 + 8x^3 + 2x$   
 2 a  $-\frac{2}{(x-1)^2}$  b  $\frac{1}{(x+1)^2}$  c  $\frac{1-x^2-2x}{(x^2+1)^2}$  d  $\frac{-(x^4+3x^2+2x)}{(x^3-1)^2}$  e  $\frac{2x^2+2x}{(2x+1)^2}$   
 f  $\frac{1}{(1-2x)^2}$

- 3 a  $(\sin x + \cos x)e^x$  b  $\ln x + 1$  c  $e^x(2x^3 + 6x^2 + 4x + 4)$  d  $4x^3 \cos x - x^4 \sin x$   
 e  $-\sin^2 x + \cos^2 x$  f  $2x \tan x + (1+x^2)\sec^2 x$  g  $\frac{4}{x^3}(x \cos x - 2 \sin x)$

h  $e^x(x \cos x + x \sin x + \sin x)$  i  $(\ln x + 1 + x \ln x)e^x$

- 4 a  $\frac{\sin x - x \cos x}{\sin^2 x}$  b  $\frac{-[\sin x(x+1) + \cos x]}{(x+1)^2}$  c  $\frac{e^x}{(e^x+1)^2}$  d  $\frac{2x \cos x - \sin x}{2x\sqrt{x}}$  e  $\frac{\ln x - 1}{(\ln x)^2}$   
 f  $\frac{(x+1) - x \ln x}{x(x+1)^2}$  g  $\frac{xe^x + 1}{(x+1)^2}$  h  $\frac{-2}{(\sin x - \cos x)^2}$  i  $\frac{x^2 - x + 2x \ln x}{(x + \ln x)^2}$

- 5 a  $-5e^{-5x} + 1$  b  $4 \cos 4x + 3 \sin 6x$  c  $-\frac{1}{3}e^{\frac{1}{3}x} - \frac{1}{x} + 18x$  d  $25 \cos 5x + 6e^{2x}$   
 e  $4 \sec^2 4x + 2e^{2x}$  f  $-4 \sin(4x) + 3e^{-3x}$  g  $\frac{4}{4x+1} - 1$  h 0

i  $\frac{1}{2} \cos\left(\frac{x}{2}\right) - 2 \sin 2x$  j  $7 \cos(7x-2)$  k  $\frac{1}{2\sqrt{x}} - \frac{1}{x}$  l  $\frac{1}{x} + 6 \sin 6x$

- 6 a  $2x \cos x^2 + 2 \sin x \cos x$  b  $2 \sec^2 2\theta - \frac{\cos \theta}{\sin^2 \theta}$  c  $\frac{1}{2\sqrt{x}} \cos \sqrt{x}$  d  $\frac{1}{x^2} \sin\left(\frac{1}{x}\right)$   
 e  $-3 \sin \theta \cdot \cos^2 \theta$  f  $e^x \cos(e^x)$  g  $\frac{1}{x} \sec^2(\log_e x)$  h  $\frac{-\sin 2x}{\sqrt{\cos 2x}}$  i  $-\cos \theta \cdot \sin(\sin \theta)$   
 j  $4 \sin \theta \cdot \sec^2 \theta$  k  $-5 \cos 5x \cdot \csc^2(5x)$  l  $-6 \csc^2(2x)$

- 7 a  $2e^{2x+1}$  b  $-6e^{4-3x}$  c  $-12xe^{4-3x^2}$  d  $\frac{1}{2}\sqrt{e^x}$  e  $\frac{1}{2\sqrt{x}}e^{\sqrt{x}}$  f  $e^{2x+4}$   
 g  $2xe^{2x^2+4}$  h  $-\frac{6}{e^{3x+1}}$  i  $(6x-6)e^{3x^2-6x+1}$  j  $\cos(\theta)e^{\sin \theta}$

k  $2 \sin(2\theta)e^{-\cos 2\theta}$  l  $2x$  m  $\frac{2e^{-x}}{(e^{-x}+1)^2}$  n  $3(e^x + e^{-x})(e^x - e^{-x})^2$   
 o  $e^{x+2}$  p  $(-2x+9)e^{-x^2+9x-2}$

8 a  $\frac{2x}{x^2+1}$  b  $\frac{\cos \theta + 1}{\sin \theta + \theta}$  c  $\frac{e^x + e^{-x}}{e^x - e^{-x}}$  d  $-\frac{1}{x+1}$  e  $\frac{3}{x}(\ln x)^2$  f  $\frac{1}{2x\sqrt{\ln x}}$

g  $\frac{1}{2(x-1)}$  h  $\frac{-3x^2}{1-x^3}$  i  $\frac{1}{2(x+2)}$  j  $\frac{-2 \sin x \cos x}{\cos^2 x + 1}$  k  $\frac{1}{x} + \cot x$  l  $\frac{1}{x} + \tan x$

9 a  $\ln(x^3+2) + \frac{3x^3}{x^3+2}$  b  $\frac{\sin^2 x + 2\sqrt{x} \sin x \cos x}{2\sqrt{x}}$  c  $-\frac{1}{\sqrt{\theta}} \sin \sqrt{\theta} \cdot \cos \sqrt{\theta}$

d  $(3x^2 - 4x^4)e^{-2x^2+3}$  e  $-(\ln x + 1) \sin(x \ln x)$  f  $\frac{1}{x \ln x}$

g  $\frac{(2x-4) \cdot \sin(x^2) - 2x \cdot \cos(x^2)(x^2-4x)}{(\sin x)^2}$  h  $\frac{10(\ln(10x+1)-1)}{[\ln(10x+1)]^2}$

i  $(\cos 2x - 2 \sin 2x)e^x - 1$  j  $2x \ln(\sin 4x) + 4x^2 \cot 4x$  k  $(\cos \sqrt{x} - \sin \sqrt{x})\frac{1}{2\sqrt{x}}e^{-\sqrt{x}}$

l  $-(2 \sin x + 2x \cos x) \cdot \sin(2x \sin x)$  m  $\frac{e^{5x+2}(9-20x)}{(1-4x)^2}$  n  $\frac{\cos^2 \theta + \sin^2 \theta \ln(\sin \theta)}{\sin \theta \cos^2 \theta}$

o  $\frac{x+2}{2(x+1)\sqrt{x+1}}$  p  $\frac{2x^2+2}{\sqrt{x^2+2}}$  q  $\frac{10x^3+9x^2+4x+3}{3(x+1)^{2/3}}$  r  $\frac{3x^2(3x^3+1)}{2\sqrt{x^3+1}}$

s  $\frac{2}{x^2+1} - \frac{1}{x^2} \ln(x^2+1)$  t  $\frac{2}{x(x+2)}$  u  $\frac{2-x}{2x^2\sqrt{x-1}}$  v  $\frac{-x^2+x-9}{\sqrt{x^2+9}} \cdot e^{-x}$

w  $\frac{7x^3-12x^2-8}{2\sqrt{2-x}}$  x  $nx^{n-1} \ln(x^n-1) + \frac{nx^{2n-1}}{x^n-1}$

10 x = 1

11 0

12 0

13 1

14 -2e

15 a  $\cos^2 x - \sin^2 x$  b  $\frac{\pi}{180} \cos x^\circ$  c  $-\frac{\pi}{180} \sin x^\circ$

16 b i  $2x \sin x \cos x + x^2 \cos^2 x - x^2 \sin^2 x$

ii  $e^{-x^3}(2 \cos 2x \ln \cos x - 3x^2 \sin 2x \ln \cos x - \sin 2x \tan x)$

17 a  $i - \frac{3}{x}(\ln x)^2$     ii  $-\frac{3x^2}{1-x^3}$     b  $i - 2e^{-2x} \cdot \cos(e^{-2x})$     ii  $-2x \cos x^2 \cdot e^{-\sin x^2}$

18  $-\frac{1}{5}k$

19  $x = a, b, \frac{mb+na}{m+n}$

20  $\{\theta: n \tan \theta^m \cdot \tan \theta^n = m \theta^{m-n}\}$

21 a  $-4 \csc(4x)$     b  $2 \sec(2x) \tan(2x)$     c  $3 \cot(3x) \csc(3x)$     d  $-3 \sin(3x)$   
 e  $\csc^2\left(\frac{\pi}{4} - x\right)$     f  $-2 \sec(2x) \tan(2x)$

22 a  $2x \sec(x^2) \tan(x^2)$     b  $\sec^2 x$     c  $\tan x$     d  $-3 \cot^2 x \csc^2 x$     e  $x \cos x + \sin x$   
 f  $-2 \cot x \csc^2 x$     g  $4x^3 \csc(4x) - 4x^4 \cot(4x) \csc(4x)$   
 h  $2 \cot x \sec^2(2x) - \csc^2 x \tan(2x)$     i  $\frac{\sec x \tan x - \sin x}{2\sqrt{\cos x} + \sec x}$

23 a  $e^{\sec x} \sec x \tan x$     b  $e^x \sec(e^x) \tan(e^x)$     c  $e^x \sec(x) + e^x \sec(x) \tan(x)$   
 d  $\frac{-\csc^2(\log x)}{x}$     e  $-5 \csc(5x) \sec(5x)$     f  $\frac{\cot(x)}{x} - \csc^2(x) \log x$

g  $-\cos x \cot(\sin x) \csc(\sin x)$     h  $-\cos(\csc x) \cot x \csc x$     i 0

**Exercise 19.4**

1 a  $20x^3$     b  $48(1+2x)^2$     c  $\frac{2}{x^3}$     d  $\frac{2}{(1+x)^3}$     e 2    f  $\frac{6}{(x-2)^3}$     g  $\frac{42}{x^8}$     h  $24(1-2x)$

i  $-\frac{1}{x^2}$     j  $\frac{2(x^2+1)}{(1-x^2)^2}$     k  $-16 \sin^4 \theta$     l  $2 \cos x - x \sin x$     m  $6x^2 \cos x + 6x \sin x - x^3 \sin x$

n  $\frac{1}{x}$     o  $\frac{10}{(2x+3)^3}$     p  $6x e^{2x} + 12x^2 e^{2x} + 4x^3 e^{2x}$     q  $\frac{8 \sin 4x - 15 \cos 4x}{e^x}$

r  $2 \cos x^2 - 4x^2 \sin x^2$     s  $\frac{-48(x^2+2x^5)}{(4x^3-1)^3}$     t  $\frac{10}{(x-3)^3}$

2  $\frac{6 \ln x - 5}{x^4}, \frac{n^2 \ln x + n \ln x - 2n - 1}{x^{n+2}}$

3  $f(x) = \left(\frac{x+1}{x-1}\right)^n \Rightarrow f''(x) = \frac{4n(n+x)(x+1)^n}{(x^2-1)^2(x-1)^n}$

4 a  $2 + \frac{1}{8\sqrt{2}}$     b  $\frac{3+\pi}{2}$

5  $-\frac{1}{8}$   
 6  $[0, 1.0768] \cup [3.6436, 2\pi]$

**Chapter 20**

**Exercise 20.1**

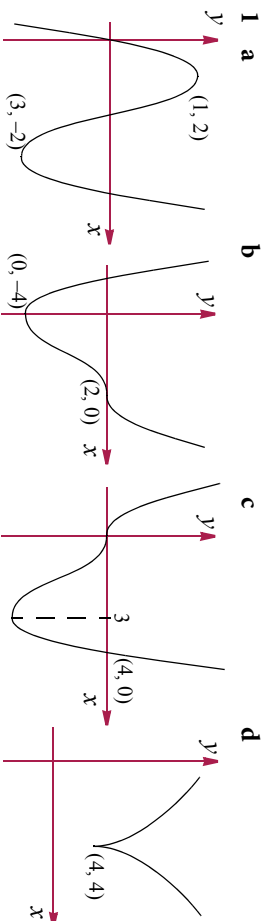
- 1 a  $y = 7x - 10$     b  $y = -4x + 4$     c  $4y = x + 5$     d  $16y = -x + 21$     e  $4y = x + 1$   
 f  $4y = x + 2$     g  $y = 28x - 48$     h  $y = 4$   
 2 a  $7y = -x + 30$     b  $4y = x - 1$     c  $y = -4x + 14$     d  $y = 16x - 79$     e  $2y = 9 - 8x$   
 f  $y = -4x + 9$     g  $28y = -x + 226$     h  $x = 2$   
 3 a  $y = 2ex - e$     b  $y = e$     c  $y = \pi$     d  $y = -x$     e  $y = x$     f  $ey = (2e-1)x - e^2 + 2e - 1$   
 g  $y = ex$     h  $y = 2x + 1$   
 4 a  $2ey = -x + 2e^2 + 1$     b  $x = 1$     c  $x = \pi$     d  $y = x - 2\pi$     e  $y = -x + \pi$   
 f  $(2e-1)y = -ex + 3e^2 - 4e + 1$     g  $ey = -x$     h  $2y = -x + 2$   
 5 A:  $y = 28x - 44$ , B:  $y = -28x - 44$ , Isosceles,  $z \equiv (0, a^2 - 3a^4)$   
 6 2 sq. units,  $y = 2$ ,  $x = 1$   
 7  $4y = 3x$   
 8  $by = \sqrt{a^2 - b^2}x$   
 9  $y = 4x - 9$   
 10  $y = \log_e 4$   
 11  $8y = 4(\pi+2)x - \pi^2$ ;  $4(\pi+2)y = -8x + 4\pi + \pi^2$   
 12 A:  $y = -8x + 32$ , B:  $y = 6x + 25$ ,  $\left(\frac{1}{2}, 28\right)$

13  $y = -x$ , Tangents:  $y = \frac{1}{2}, y = -\frac{1}{2}$ ,  $\left(-\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, -\frac{1}{2}\right)$  tangent and normal meet at  $(0.5, -0.5)$

14 a  $y = 3x - 7$     b  $Q \equiv (2, -1)$

15  $m = -2, n = 5$

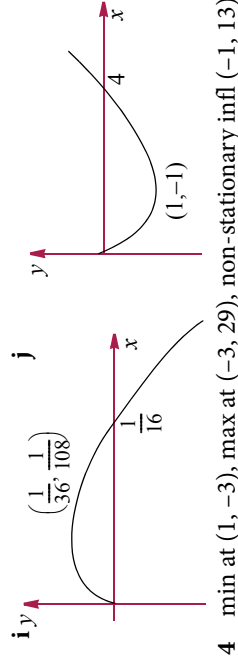
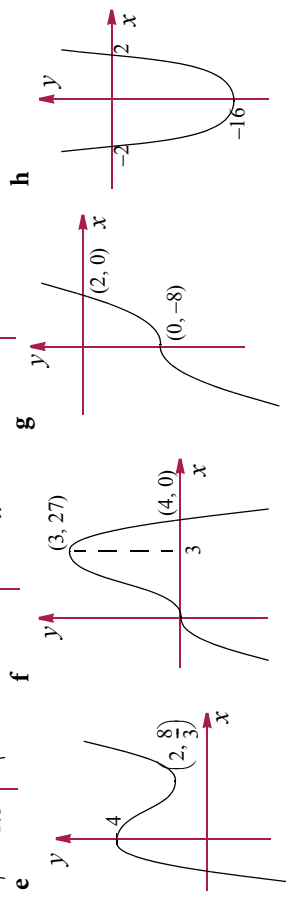
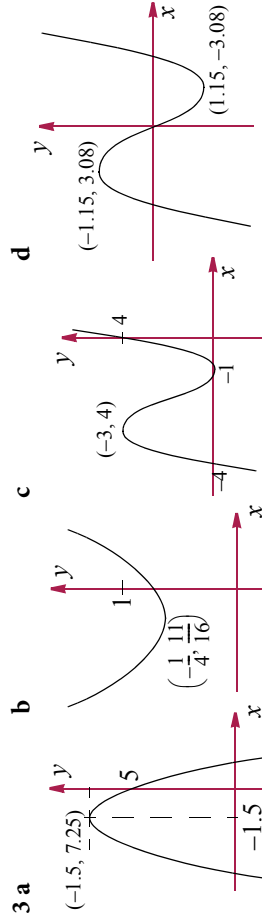
**Exercise 20.2**



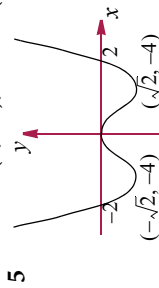
- 2 a max at  $(1, 4)$     b min at  $\left(-\frac{9}{2}, -\frac{81}{4}\right)$     c min at  $(3, -45)$     max  $(-3, 63)$   
 d max at  $(0, 8)$ , min at  $(4, -24)$     e max at  $(1, 8)$ , min at  $(-3, -24)$   
 f min at  $\left(\frac{1+\sqrt{13}}{3}, \frac{70-26\sqrt{13}}{27}\right)$ , max at  $\left(\frac{1-\sqrt{13}}{3}, \frac{70+26\sqrt{13}}{27}\right)$     g min at  $(1, -1)$

**h** max at  $(0, 16)$ , min at  $(2, 0)$    **i** min at  $(1, 0)$  max at  $(-\frac{1}{3}, \frac{32}{27})$

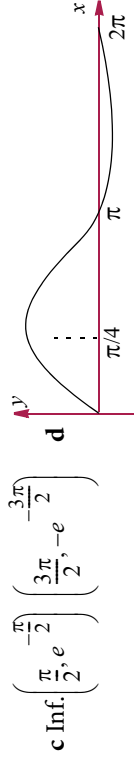
**j** min at  $(\frac{4}{9}, -\frac{4}{27})$    **k** min at  $(2, 4)$ , max at  $(-2, -4)$    **l** min at  $(1, 2)$ , min at  $(-1, 2)$



**4** min at  $(1, -3)$ , max at  $(-3, 29)$ , non-stationary inf  $(-1, 13)$

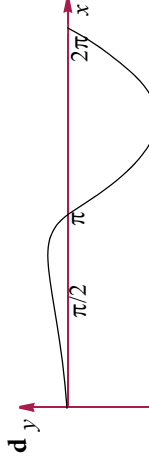


**6 a i**  $(\cos x - \sin x)e^{-x}$    **ii**  $-2\cos x \cdot e^{-x}$    **b i**  $\frac{\pi}{4}, \frac{5\pi}{4}$    **ii**  $\frac{\pi}{2}, \frac{3\pi}{2}$



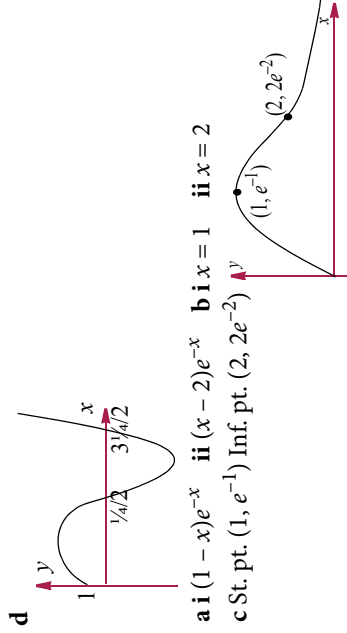
**7 a i**  $e^x(\sin x + \cos x)$    **ii**  $2e^x \cos x$    **b i**  $x = \frac{3\pi}{4}, \frac{7\pi}{4}$    **ii**  $x = \frac{\pi}{2}, \frac{3\pi}{2}$

**c** St. pts.  $(\frac{3\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}})$ ,  $(\frac{7\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{7\pi}{4}})$    Inf. pts.  $(\frac{\pi}{2}, e^{\frac{\pi}{2}})$ ,  $(\frac{3\pi}{2}, -e^{\frac{3\pi}{2}})$

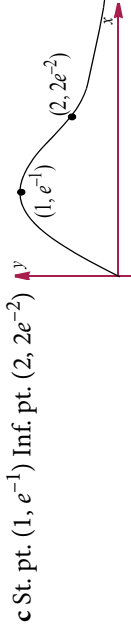


**8 a i**  $e^x(\cos x - \sin x)$    **ii**  $-2\sin x \cdot e^x$    **b i**  $\frac{\pi}{4}, \frac{5\pi}{4}$    **ii**  $0, \pi, 2\pi$

**c** St. pts.  $(\frac{\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{\pi}{4}})$ ,  $(\frac{5\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{5\pi}{4}})$    Inf. pts.  $(0, 1)$ ,  $(\pi, -e^\pi)$ ,  $(2\pi, e^{2\pi})$



**9 a i**  $(1-x)e^{-x}$    **ii**  $(x-2)e^{-x}$    **b i**  $x=1$    **ii**  $x=2$



**10 a 8**   **b 0**   **c 4**   **d**  $27\sqrt[3]{9} \approx 56.16$

**11 a** min value  $-82$    **b** max value  $26$

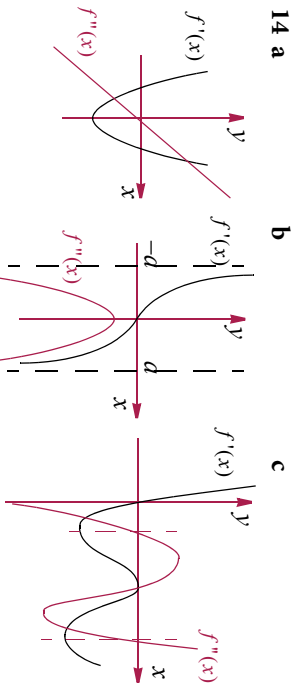
**12 a** pt A: **i** Yes   **ii** non-stationary pt of inflect; pt B: **i** Yes   **ii** Stationary point (local/global min); pt C: **i** Yes   **ii** non-stationary pt of inflect.

**b** pt A: **i** No   **ii** Local/global max; pt B: **i** No   **ii** Local/global min; pt C: **i** Yes   **ii** Stationary point (local max)

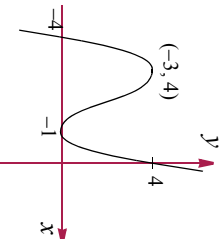
**c** pt A: **i** Yes   **ii** Stationary point (local/global max); pt B: **i** Yes   **ii** Stationary point (local min); pt C: **i** Yes   **ii** non-stationary pt of inflect.

**d** pt A: **i** Yes   **ii** Stationary pt (local/global max); pt B: **i** No   **ii** Local min; pt C: **i** Yes   **ii** Stationary point (local max)

- e pt A: **i** No **ii** Cusp (local min); pt B: **i** Yes **ii** Stationary pt of inflect;  
 pt C: **i** Yes **ii** Stationary point (local max)  
 f pt A: **i** Yes **ii** Stationary point (local/global max); pt B: **i** Yes **ii** Stationary point  
 (local/global min); pt C: **i** No **ii** Tangent parallel to  $y$ -axis.  
 13 a **i** A **ii** B **iii** C **b** **i** C **ii** B **iii** A



15  $y = x^3 + 6x^2 + 9x + 4$

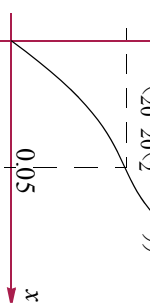


16  $f(x) = \frac{1}{3}x^3 - x^2 - 3x - 6$

17  $f(x) = 3x^5 - 20x^3$

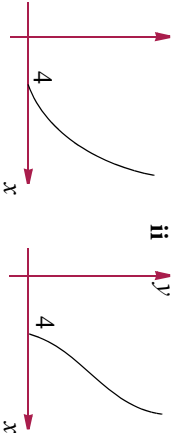


18



19  $m = -0.5, n = 1.5$

20 a **i**



**ii**

**b i**  $\frac{3}{2}\sqrt{x-4}$

**ii**  $\frac{3x-10}{2\sqrt{x-4}}$

21  $a = 2, b = -3, c = 0$

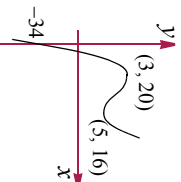
22 Stationary points: local min at  $(-1, 0)$  and local max at  $(1, 4e^{-1})$ .

Inflection pts are:  $(1 + \sqrt{2}, (6 + 4\sqrt{2})e^{-(1 + \sqrt{2})})$  and  $(1 - \sqrt{2}, (6 - 4\sqrt{2})e^{-(1 - \sqrt{2})})$

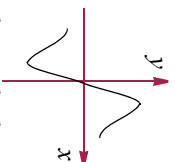
23 Absolute min at  $\sim (-\frac{3 + \sqrt{13}}{2}, -2.1733)$ , local max at  $\sim (-\frac{3 - \sqrt{13}}{2}, 0.2062)$

Inflection pts at  $\sim (-0.4384, -1.4489)$  and  $(-4.5615, 0.1488)$   
 24 – 27 are left as questions for classroom discussion.

28  $a = 1, b = -12, c = 45, d = -34$



29 **b**  $b = 1$  **c**  $a = \frac{1}{\sqrt{2}}$  **d**  $f(x) = \frac{1}{\sqrt{2}}xe^{-x^2}$



30 a 2.7983, 6.1212, 9.3179 **b** Use a graphics calculator to verify your sketch.

**Exercise 20.3**

1 **a** Local min. at  $x = \frac{4}{\sqrt{3}}$ , local max at  $x = -\frac{4}{\sqrt{3}}$

**b** Local max. at  $x = 0$ , local min. at  $x = \pm 1$  **c** Local max. at  $x = 0.25$

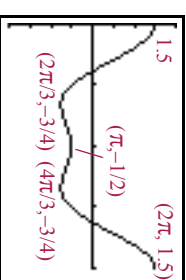
**d** Local max. at  $x = 1$  **e** none **f** Local max. at  $x = 0.5$ , local min. at  $x = 1, 0$

**g** Local max. at  $x = 1$ , local min. at  $x = -1$  **h** none

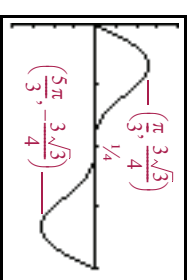
2 **a** max. = 120, min. =  $\frac{128}{3\sqrt{5}}$  **b** max. = 224, min. = -1

**c** max. = 0.5, min. = 0 **d** max. = 1, min. = 0.

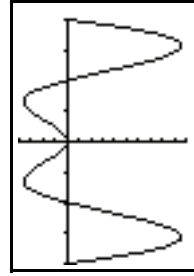
3



4



5

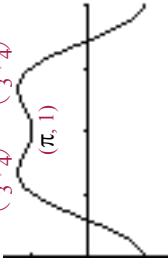


Stationary points occur where  $\tan x = -x$

6 a Local min. at  $(1, 2)$ ; infl. pt. at  $(3, \sqrt{3} + \frac{1}{3}, \sqrt{3})$

b Local min. at  $(1, 2)$ ; local max. at  $(-3, -6)$  c none

7  $(\frac{2\pi}{3}, \frac{5}{4})$   $(\frac{4\pi}{3}, \frac{5}{4})$



8 - 11 Verify your graphs with graphics calculator.

8 a Global min. at  $(0, 0)$ ; local max. at  $(2, 4e^{-2})$  Infl. pts.  $(2 - \sqrt{2}, (6 - 4\sqrt{2})e^{-(2 - \sqrt{2})})$   
 $(2 + \sqrt{2}, (6 + 4\sqrt{2})e^{-(2 + \sqrt{2})})$

b Global max. at  $(0, e^4)$ , infl. pt. at  $(\pm \frac{1}{\sqrt{2}}, e^{3.5})$  c Local max. at  $(-2, -\frac{1}{2}e)$

9 a Global max. at  $(e, e^{-1})$ . Infl. pt. at  $(e^{1.5}, 1.5e^{-1.5})$  b Global min. at  $(\frac{1}{\sqrt{2}}, 2 + \frac{1}{2}\ln 2)$

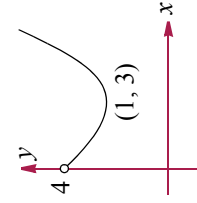
c Global min. at  $(2, 1 + \ln 2)$ ; Infl. pt. at  $(4, 2 + \ln 4)$  d none

10 a  $f(x) = (x-2)^{a-1}(x+2)^{b-1}((a+b)x+2(a-b))$  b i  $f(x) = \frac{x-2}{x+2}$ ; none

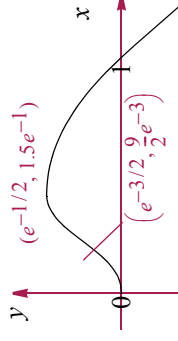
ii  $f(x) = (x-2)^2(x+2)$ ; local max. at  $(-\frac{2}{3}, \frac{256}{27})$ ; local min. at  $(2, 0)$

iii  $f(x) = (x-2)^2(x+2)^2$ ; local min. at  $(\pm 2, 0)$ , local max. at  $(0, 16)$ .

11 a Global min. at  $(1, c-1)$ ;  $c \neq 1$



12



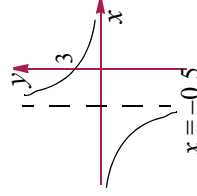
13 Global max. at  $(e^{0.5}, 0.5e^{-1})$ ; infl. pt. at  $(e^{5/6}, \frac{5}{6}e^{-5/3})$ .

**Exercise 20.4**

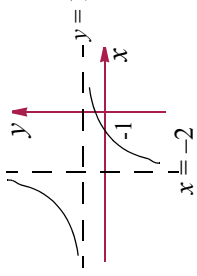
1 a  $y = 2, x = -1$  b  $y = 1, x = -\frac{1}{3}$  c  $y = \frac{1}{2}, x = -\frac{1}{4}$

d  $y = -1, x = -3$  e  $y = 3, x = 0$  f  $y = 5, x = 2$

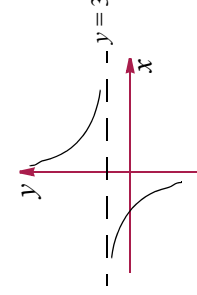
3 a



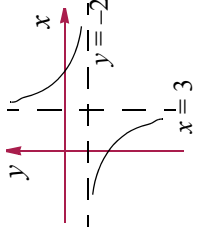
b



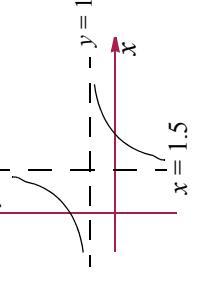
d



e

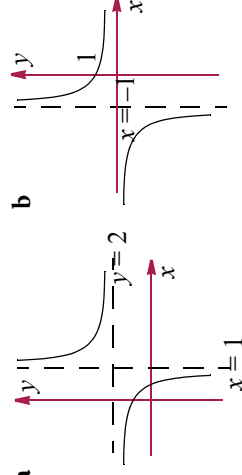


f

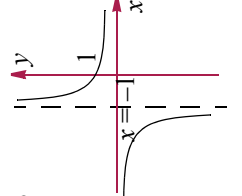


4 a  $2, c = 4$

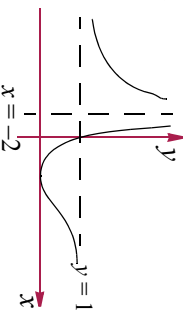
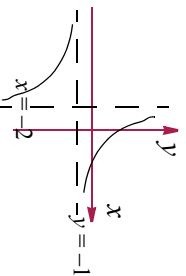
5 a



b



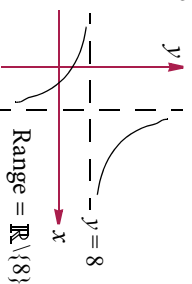
- 6 a i (0, 1), (2, 0) ii  $y = -1, x = -2$  iii iv  $d = \mathbb{R} \setminus \{-2\}$



b  $f^{-1}: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}$ , where  $f^{-1}(x) = \frac{2(1-x)}{(1+x)}$

c

- 7 a  $y = 8, x = 3$  b



Chapter 21

Exercise 21.1

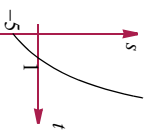
- 1 a i  $x < 0$  ii  $x > 4$  iii  $0 \leq x \leq 4$  b i  $-1 < x < 2$  ii  $x < -1, 2 < x < 5$  iii  $\emptyset$   
 c i  $-1 < x < 1$  ii  $x < -1$  iii  $x \geq 1$  d i  $0 < x < 1$  ii  $2 < x < 3$  iii  $x < 0, 1 \leq x < 2$   
 e i  $\emptyset$  ii  $-2 < x < 4$  iii  $\emptyset$  f i  $-4 < x < -1, 2 < x < 5$  ii  $-1 < x < 2, 5 < x < 8$   
 iii  $\emptyset$

Exercise 21.2

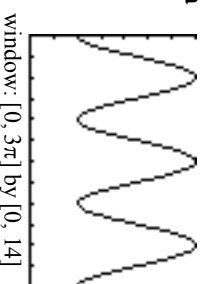
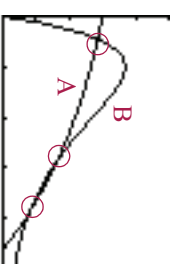
- 1 4.4 (4 deer per year, to nearest integer)  
 2 a 200 cm<sup>3</sup> b 73.5 cm<sup>3</sup>/day  
 3 a 75 b No  
 4 a \$207.66 b \$ 40.79 per year c \$41.54 per year  
 5 a 2.50 b 3.33 c 2.50  
 6 a 1230 <  $x$  < 48770 approx. b i  $0 < x < 25000$  ii 25000 <  $x$  < 50000  
 7 66667 to nearest integer, 1446992 to nearest integer  
 8 b 133.33 d 46.67 e  $0 < x < 5700$   
 9 a  $D'(x) = \frac{-40000(2x+12)}{(x^2+12x+20)^2}$  5  $\leq x \leq 18$  b 22.22 22 items/dollar  
 10 a  $\frac{3000}{(x+32)^2}$  b i  $x \geq 0$  ii  $x \in \emptyset$   
 11 a i 0 mm/s ii  $\sim 90.69$  mm/s b 0.6 sec  
 12 a 8.53 cm/s b never c never  
 13  $-e^{-1}$  ms<sup>-2</sup>

Exercise 21.3

- 1 a i  $v = -\frac{1}{(t-1)^2}, t > 1$  ii  $a = \frac{2}{(t-1)^3}, t > 1$  b i  $v = 2(e^{2t} - e^{-2t}), t \geq 0$   
 ii  $a = 4(e^{2t} + e^{-2t}), t \geq 0$  c i  $v = \frac{2}{\sqrt{4-t^2}}, 0 \leq t < 2$  ii  $a = \frac{2t}{(4-t^2)^{3/2}}, 0 \leq t < 2$   
 d i  $v = \frac{t}{(t+1)\ln 10} + \log_{10}(t+1), t \geq 0$  ii  $a = \frac{1}{\ln 10} \left[ \frac{1}{(t+1)^2} + \frac{1}{t+1} \right], t \geq 0$   
 e i  $v = a - 2bt e^{-t^2}, t \geq 0$  ii  $a = 2be^{-t^2}(2t^2 - 1), t \geq 0$   
 f i  $v = (\ln 2) \times 2^{t+1} - (\ln 3) \times 3^t, t \geq 0$  ii  $v = (\ln 2)^2 \times 2^{t+1} - (\ln 3)^2 \times 3^t, t \geq 0$   
 2 a 8 ms<sup>-1</sup> b never at rest c i 5 m from O in negative direction ii 4 ms<sup>-1</sup>  
 d 40 m e

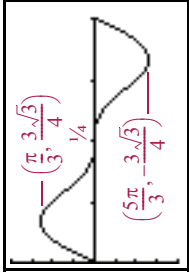


- 3 a 1 ms<sup>-1</sup> b never c  $t = \frac{1}{3}$  or  $t = 1$  d 20 ms<sup>-2</sup>  
 4 a  $v = -6t^2 + 12; a = -12t$  b  $\sim 141$  sec c once d use graphics calculator  
 5 a 3 m in positive direction b i 5 m ii 2 m c 5 ms<sup>-1</sup>  
 e oscillation about origin with amplitude 5 m and period  $2\pi$  seconds  
 7 a 100 m, in negative direction b 3 times c i 80 ms<sup>-1</sup> ii  $-34$  ms<sup>-2</sup> d 1481 m  
 8 a max. = 5 units, min. =  $-1$  unit b  $\frac{\pi}{2}$  s c i  $a = -12 \cos(2t - \pi)$  ii  $a = -4(x-2)$   
 9 a 0318 m above b i  $v = 3.75e^{-0.25t} - 3$  ii  $a = -0.9375e^{-0.25t}$   
 c 0322 m d  $a = -0.25(v+3)$   
 10 a  $0 < t < 05$  or  $t > 1$  b  $t > 05$  c  $t = 1$  or  $168 \leq t \leq 5$   
 11 a This question is best done using a graphics calculator:  
 b From the graph the particles pass each other three times c 0.45 s; 285 s; 387 s  
 d i  $v_A = -0.3e^{0.3t}$  ms<sup>-1</sup> ii  $v_B = 10e^{-t}(1-t)$  ms<sup>-1</sup>  
 e Yes, on two occasions.  
 12 a 2m in positive direction b i 2 s ii never  
 c 0026 ms<sup>-2</sup>  
 13 a b 0295





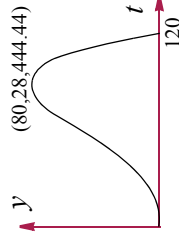
**Exercise 21.4**

- 1 22.6 m  
 2 a 1.5 mh<sup>-1</sup> b \$19,55 per km  
 3 a 400 b \$46,400,000  
 4 \$273.86  
 5 \$0.40  
 6 1.97 m  
 7 0.45 m<sup>3</sup>  
 8 5 m by 5 m  
 9 128  
 10  $r = \frac{50}{4+\pi} \approx 7.00$ , dim of rect.  $\frac{50}{4+\pi} \times \frac{50}{4+\pi}$  i.e. approx 7.00 m by 7.00 m  
 11  $\theta = \frac{\pi}{6}$   
 12 a   
 b  $\frac{3\sqrt{3}}{2}$  units c At infl. pts. when  $\cos x = -\frac{1}{4}$ .

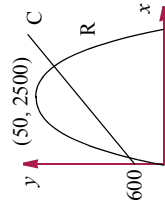
- 13 648 m<sup>2</sup>  
 14 a 10.5 b 5.25  
 15 72

16 a  $y = 100 - 2x$  b A =  $x(100 - 2x)$ ,  $0 < x < 50$  c  $x = 25$ ,  $y = 50$

17 a  $\frac{100}{x} - \frac{1}{2}x$ ,  $0 < x < 10\sqrt{2}$  b  $\frac{2000}{9}\sqrt{6} \approx 544.3$  cm<sup>3</sup>



- 18 a 400 mLs<sup>-1</sup> b 40 s c



- 19 a b 8.38, 71.62 c  $9 \leq x \leq 71$  d  $80x - x^2 - 600$ , \$1000

20  $(\frac{11}{2}, \frac{7}{2})$  &  $(-\frac{11}{2}, \frac{7}{2})$

21  $5\sqrt{2}$  by  $\frac{5}{2}\sqrt{2}$

- 22 4 by  $\frac{8}{3}$   
 23  $348 - 8\sqrt{170} \sim 243.7$  cm<sup>2</sup>  
 24 2  
 25 radius =  $\sqrt{\frac{10}{3}}$  cm, height =  $2\sqrt{\frac{10}{3}}$  cm  
 26  $3\sqrt{\frac{15}{\pi}}$   
 27 5 cm  
 28 a  $h = \frac{24r^2}{r^2 - 144}$  b  $\frac{8\pi r^4}{r^2 - 144}$  c  $r = 12\sqrt{2}$ ,  $h = 48$   
 29  $r : h = 1 : 2$   
 30  $\sim (0.55, 1.31)$   
 31 b 2.5 m  
 32 altitude =  $\frac{1}{3}$  height of cone  
 33  $\sim 1.640$  m wide and 1.040 m high  
 34  $\frac{2\sqrt{2}}{\sqrt{3}}\pi$   
 35 where XP : PY = b : a  
 36 5 km  
 37  $r : h = 1 : 1$   
 38  $\frac{4}{3}$  cm  
 39 2 : 1  
 40  $\frac{10}{\sqrt{3}\pi}$   
 41 0.873 km from P  
 42 b  $r = 3\sqrt{2}$ ,  $h = 6\sqrt{2}$   
 43 b when  $\theta = \arcsin(\frac{2}{6})$ , i.e. approx. 6.030 km from P.  
 44 a  $\tan \theta = \frac{xL}{x^2 + k(l+k)}$  b  $x = \sqrt{k^2 + kl}$   
 45 c if  $k < c$ , swimmer should row directly to Q.  
 46 a i  $\pi r^2 h + \frac{2}{3}\pi r^3$  ii  $3\pi r^2 + 2\pi r h$  c  $r : h = 1 : 1$   
 47  $(a^{2/3} + b^{2/3})^{3/2}$   
 48 b 4 km along the beach c row directly to destination

Chapter 22

Exercise 22.1

- 1 a  $\frac{1}{4}x^4 + c$    b  $\frac{1}{8}x^8 + c$    c  $\frac{1}{6}x^6 + c$    d  $\frac{1}{9}x^9 + c$    e  $\frac{4}{3}x^3 + c$    f  $\frac{7}{6}x^6 + c$    g  $x^9 + c$   
 h  $\frac{1}{8}x^4 + c$
- 2 a  $5x + c$    b  $3x + c$    c  $10x + c$    d  $\frac{2}{3}x + c$    e  $-4x + c$    f  $-6x + c$    g  $-\frac{3}{2}x + c$   
 h  $-x + c$
- 3 a  $x - \frac{1}{2}x^2 + c$    b  $2x + \frac{1}{3}x^3 + c$    c  $\frac{1}{4}x^4 - 9x + c$    d  $\frac{2}{5}x + \frac{1}{9}x^3 + c$    e  $\frac{1}{3}x^{3/2} + \frac{1}{x} + c$   
 f  $x^{5/2} + 4x^2 + c$    g  $\frac{1}{3}x^3 + x^2 + c$    h  $x^3 - x^2 + c$    i  $x - \frac{1}{3}x^3 + c$
- 4 a  $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + c$    b  $\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2 + c$    c  $\frac{1}{4}(x-3)^4 + c$   
 d  $\frac{2}{5}x^5 + \frac{1}{2}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + c$    e  $x + \frac{1}{2}x^2 - \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + c$   
 f  $\frac{2}{7}x^{7/2} + \frac{4}{5}x^{5/2} + \frac{2}{3}x^{3/2} - 2x + c$
- 5 a  $\frac{1}{2}x^2 - 3x + c$    b  $2u^2 + 5u + \frac{1}{u} + c$    c  $-\frac{1}{x} - \frac{2}{x^2} - \frac{4}{3x^3} + c$    d  $\frac{1}{2}x^2 + 3x + c$   
 e  $\frac{1}{2}x^2 - 4x + c$    f  $\frac{1}{3}t^3 + 2t - \frac{1}{t} + c$
- 6 a  $\frac{4}{7}\sqrt{x^7} + 2\sqrt{x} - 5x + c$    b  $\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{4}{7}x^{7/2} - \frac{4}{5}x^{5/2} + c$   
 c  $-\frac{1}{2z^2} + \frac{2}{z} + 2z^2 + z + c$    d  $\frac{1}{2}t^4 + t + c$    e  $\frac{2}{5}\sqrt{t^5} - 2\sqrt{t^3} + c$    f  $\frac{1}{3}u^3 + 2u^2 + 4u + c$
- 8 a  $\frac{1}{8}(2x+3)^4 + c$    b  $3\sqrt{x^2+4} + c$

Exercise 22.2

- 1 a  $x^2 + x + 3$    b  $2x - \frac{1}{3}x^3 + 1$    c  $\frac{8}{3}\sqrt{x^3} - \frac{1}{2}x^2 - \frac{40}{3}$    d  $\frac{1}{2}x^2 + \frac{1}{x} + 2x - \frac{3}{2}$    e  $(x+2)^3$   
 f  $\frac{3}{4}\sqrt{x^4} + \frac{1}{4}x^4 + x$    g  $\frac{1}{3}x^3 + 1$    h  $x^4 - x^3 + 2x + 3$
- 2  $\frac{1}{2}x^2 + \frac{1}{x} + \frac{5}{2}$
- 3 \$3835.03
- 4 9.5
- 5  $\frac{251}{3}\pi \text{ cm}^3$

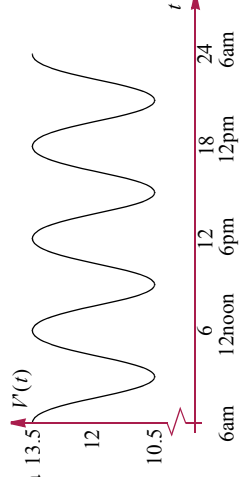
- 6 292
- 7  $\frac{5}{7}\sqrt{x^3} + \frac{23}{7}$
- 8 1, -8
- 9  $P(x) = 25 - 5x + \frac{1}{3}x^2$
- 10  $N = \frac{20000}{201}t^{2.01} + 500, t \geq 0$
- 11 a  $y = -\frac{2}{5}x^2 + 4x$    b  $y = \frac{1}{6}x^3 + \frac{5}{4}x^2 + 2x$
- 12  $y = 2(x^3 + x^2 + x)$
- 13  $f(x) = -\frac{3}{10}x^3 + \frac{49}{10}x - \frac{13}{5}$
- 14 Vol  $\sim 43202 \text{ cm}^3$
- 15  $110 \text{ cm}^2$

Exercise 22.3

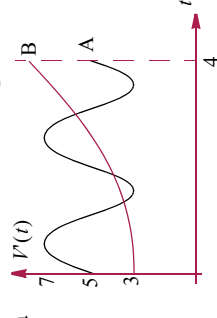
- 1 a  $\frac{1}{5}e^{5x} + c$    b  $\frac{1}{3}e^{3x} + c$    c  $\frac{1}{2}e^{2x} + c$    d  $10e^{0.1x} + c$    e  $-\frac{1}{4}e^{-4x} + c$    f  $e^{-4x} + c$   
 g  $-0.2e^{-0.5x} + c$    h  $-2e^{1-x} + c$    i  $5e^x + 1 + c$    j  $e^{2-2x} + c$   
 k  $3e^{x/3} + c$    l  $2\sqrt{e^x} + c$
- 2 a  $4\log_e x + c, x > 0$    b  $-3\log_e x + c, x > 0$    c  $\frac{2}{5}\log_e x + c, x > 0$   
 d  $\log_e(x+1) + c, x > -1$    e  $\frac{1}{2}\log_e x + c, x > 0$    f  $x - 2\log_e x - \frac{1}{x} + c, x > 0$   
 g  $\frac{1}{2}x^2 - 2x + \log_e x + c, x > 0$    h  $3\ln(x+2) + c$
- 3 a  $-\frac{1}{3}\cos(3x) + c$    b  $\frac{1}{2}\sin(2x) + c$    c  $\frac{1}{5}\tan(5x) + c$    d  $\cos(x) + c$
- 4 a  $-\frac{1}{2}\cos(2x) + \frac{1}{2}x^2 + c$    b  $2x^3 - \frac{1}{4}\sin(4x) + c$    c  $\frac{1}{5}e^{5x} + c$   
 d  $-\frac{4}{3}e^{-3x} - 2\cos\left(\frac{1}{2}x\right) + c$    e  $3\sin\left(\frac{x}{3}\right) + \frac{1}{5}\cos(3x) + c$   
 f  $\frac{1}{2}e^{2x} + 4\log_e x - x + c, x > 0$    g  $\frac{1}{2}e^{2x} + 2e^x + x + c$   
 h  $\frac{5}{4}\cos(4x) + x - \log_e x + c, x > 0$    i  $\frac{1}{3}\tan(3x) - 2\log_e x + 2e^{x/2} + c, x > 0$   
 j  $\frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + c$    k  $\frac{1}{2}e^{2x+3} + c$    l  $1 - \frac{1}{2}\cos(2x+\pi) + c$   
 m  $\sin(x-\pi) + c$

- 5  $n - 4 \cos\left(\frac{1}{4}x + \frac{\pi}{2}\right) + c$    **o**  $2\left(\frac{e^x+2}{\sqrt{e^x}}\right) + c$
- 6 **a**  $\frac{1}{16}(4x-1)^4 + c$    **b**  $\frac{1}{21}(3x+5)^7 + c$    **c**  $-\frac{1}{5}(2-x)^5 + c$    **d**  $\frac{1}{12}(2x+3)^6 + c$   
**e**  $-\frac{1}{27}(7-3x)^9 + c$    **f**  $\frac{1}{5}\left(\frac{1}{2}x-2\right)^{10} + c$    **g**  $-\frac{1}{25}(5x+2)^{-5} + c$    **h**  $\frac{1}{4}(9-4x)^{-1} + c$   
**i**  $-\frac{1}{2}(x+3)^{-2} + c$    **j**  $\ln(x+1) + c, x > -1$    **k**  $\ln(2x+1) + c, x > -\frac{1}{2}$   
**l**  $-2 \ln(3-2x) + c, x < \frac{3}{2}$    **m**  $3 \ln(5-x) + c, x < 5$    **n**  $-\frac{3}{2} \ln(3-6x) + c, x < \frac{1}{2}$   
**o**  $\frac{5}{3} \ln(3x+2) + c, x > -\frac{2}{3}$
- 7 **a**  $\frac{1}{2} \cos(2x-3) - x^2 + c$    **b**  $6 \sin\left(2 + \frac{1}{2}x\right) + 5x + c$    **c**  $\frac{3}{2} \sin\left(\frac{1}{3}x-2\right) + \ln(2x+1) + c$   
**d**  $10 \tan(0.1x-5) - 2x + c$    **e**  $2 \ln(2x+3) + 2e^{-\frac{1}{2}x+2} + c$    **f**  $-\frac{2}{2x+3} - \frac{1}{2}e^{-\frac{1}{2}x+2} + c$   
**g**  $x + \ln(x+1) - 4 \ln(x+2) + c$    **h**  $2x - 3 \ln(x+2) + \frac{1}{2} \ln(2x+1) + c$   
**i**  $-\frac{1}{2x+1} + \ln(2x+1) + c$
- 8 **a**  $f(x) = \frac{1}{6}\sqrt{(4x+5)^3}$    **b**  $f(x) = 2 \ln(4x-3) + 2$   
**c**  $f(x) = \frac{1}{2} \sin(2x+3) + 1$    **d**  $f(x) = 2x + \frac{1}{2}e^{-2x+1} + \frac{1}{2}e$
- 9 14334
- 10 13.19ms<sup>-1</sup> or 1.19ms<sup>-1</sup>
- 11 2.66 cm
- 12  $2e^{x/2} - \frac{1}{2} \sin(2x) - 2$
- 13 **a**  $p = \frac{a}{a^2+b^2}, q = -\frac{b}{a^2+b^2}$    **b**  $\frac{1}{13}e^{2x}(2 \sin 3x - 3 \cos 3x) + c$
- 14 **a** 0.25a   **b**  $a \times \left(\frac{1}{2}\right)^{8/3} \approx 0.11575a$
- 15 **b** 666 g

16 **a** 13.5   **b** 73.23%   **c** ~25.24 litres



17 **a** **b** 7000   **c** 1.16 day   **d** 2 days



**Exercise 22.4.1**

- 1 **a**  $\frac{2}{3}(5x^2+2)^{3/2} + c$    **b**  $-\frac{1}{3(x^3+4)} + c$    **c**  $\frac{3}{8}(1-2x^2)^4 + c$    **d**  $\frac{1}{5}(9+2x^{3/2})^5 + c$   
**e**  $\frac{9}{4}(x^2+4)^{4/3} + c$    **f**  $\frac{-1}{2(x^2+3x+1)^2} + c$    **g**  $4\sqrt{x^2+2} + c$    **h**  $\frac{1}{12(1-x^4)^3} + c$   
**i**  $\frac{2}{3}(1+e^{3x})^{3/2} + c$    **j**  $\frac{-1}{2(x^2+2x-1)} + c$    **k**  $\frac{2}{3}\sqrt{x^3+3x+1} + c$
- 1  $\frac{1}{12}(3+4x^2)^{3/2} + c$    **m**  $2\sqrt{e^x+2} + c$    **n**  $-\frac{1}{4}(1-e^{-2x})^{-2} + c$    **o**  $\frac{2}{3}(x^3+1)^{5/2} + c$   
**p**  $\frac{1}{24}(x^4+8x-3)^6 + c$    **q**  $\frac{1}{5}(x^4+5)^{5/2} + c$    **r**  $-\sqrt{1-\sin 2x} + c$   
**s**  $\frac{2}{9}(4+3 \sin x)^{3/2} + c$    **t**  $-\frac{1}{12(1+3 \tan 4x)} + c$    **u**  $\frac{3}{2}(x+\cos x)^{2/3} + c$   
**v**  $-\frac{1}{2} \cos^4 \frac{x}{2} + c$    **w**  $2\sqrt{1+x \sin x} + c$    **x**  $\frac{4}{3}(x^{1/2}+1)^{3/2} + c$
- 2 **a**  $e^{x^2+1} + c$    **b**  $6e^{\sqrt{x}} + c$    **c**  $\frac{1}{3}e^{\tan 3x} + c$    **d**  $-e^{-(ax^2+bx)} + c$    **e**  $-6e^{\frac{\cos x}{2}} + c$   
**f**  $-4e^{(4+x^{-1})} + c$    **g**  $-\frac{1}{2} \cos(2e^x) + c$    **h**  $\frac{1}{2(1-e^{2x})} + c$    **i**  $-\ln(1+e^{-x}) + c$   
**j**  $\frac{5}{2} \ln(1+2e^x) + c$    **k**  $-\frac{2}{3a}(4+e^{-ax})^{3/2} + c$    **l**  $\frac{(\ln(1+e^{2x}))^2}{4} + c$
- 3 **a**  $-\cos(x^2+1) + c$    **b**  $-10 \cos \sqrt{x} + c$    **c**  $-2 \sin\left(2 + \frac{1}{x}\right) + c$    **d**  $-\frac{2}{3}(\cos x)^{3/2} + c$

$$e^{-\frac{1}{3}}\log(\cos 3x) + c \quad \mathbf{f} \frac{4}{3}\log(1 + \tan 3x) + c \quad \mathbf{g} \frac{-4}{3(\tan(3x) + 1)} + c$$

$$\mathbf{h} 2 \sin(\ln x) + c \quad \mathbf{i} -\frac{1}{6}(1 + \cos 2x)^{3/2} + c \quad \mathbf{j} \sin(e^x) + c \quad \mathbf{k} -e^{-(x^3 + 2)} + c$$

$$\mathbf{l} \left[ \ln\left(\sin\frac{1}{2}x\right) \right]^2 + c \quad \mathbf{m} \sec x + c \quad \mathbf{n} \frac{1}{4}[\ln(1 + 2e^x)]^2 + c \quad \mathbf{o} \tan\left(\frac{1}{3}x^3 - 3x\right) + c$$

$$\mathbf{4} \quad \mathbf{a} \frac{531377}{9} \quad \mathbf{b} -2\sqrt{2} + 2\sqrt{1+e} \quad \mathbf{c} 3\ln 2(2 + \sqrt{2}) \quad \mathbf{d} \sin e - \sin(e^{-1})$$

$$\mathbf{e} \frac{2}{3}\left[1 - \cos\left(\frac{\pi}{5}\right)^{3/2}\right] \quad \mathbf{f} \frac{2}{3} \quad \mathbf{g} e^{-e^{-1}} \quad \mathbf{h} \ln 2 \quad \mathbf{i} \frac{7\sqrt{7}}{3} \quad \mathbf{j} 0 \quad \mathbf{k} \frac{3}{5} \quad \mathbf{l} \frac{1}{64} \quad \mathbf{m} \frac{1}{3} \quad \mathbf{n} -\frac{1}{60}$$

**Exercise 22.4.2**

$$\mathbf{1} \quad \mathbf{a} \frac{2}{3}(x^2 + 1)^{3/2} + c \quad \mathbf{b} \frac{2}{3}(x^3 + 1)^{3/2} + c \quad \mathbf{c} -\frac{1}{3}(4 - x^4)^{1.5} + c \quad \mathbf{d} \ln(x^3 + 1) + c$$

$$\mathbf{e} -\frac{1}{18(3x^2 + 9)^3} + c \quad \mathbf{f} e^{(x^2 + 4)} + c \quad \mathbf{g} \ln(z^2 + 4z - 5) + c \quad \mathbf{h} -\frac{3}{8}(2 - t^2)^{4/3} + c$$

$$\mathbf{i} e^{\sin x} + c \quad \mathbf{j} \ln[e^x + 1] + c \quad \mathbf{k} \frac{1}{5}\sin^5 x + c$$

$$\mathbf{2} \quad \mathbf{a} e^{\tan x} + c \quad \mathbf{b} -\ln(1 - 2x^2) + c \quad \mathbf{c} \frac{1}{1 - 2x^2} + c \quad \mathbf{d} \frac{1}{2}(\ln x)^2 + c$$

$$\mathbf{e} -\ln(1 + e^{-x}) + c \quad \mathbf{f} \ln(\ln x) + c$$

$$\mathbf{3} \quad \mathbf{a} 0 \quad \mathbf{b} \frac{2\ln 2}{3} \quad \mathbf{c} \ln \frac{77}{54} \quad \mathbf{d} \ln 2 \quad \mathbf{e} \frac{1}{3}\ln 2 \quad \mathbf{f} \frac{1}{4}$$

$$\mathbf{4} \quad \mathbf{a} \frac{7\sqrt{7}}{3} - \frac{8}{3} \quad \mathbf{b} \frac{3}{8}(\cos \pi^2 - 1) \quad \mathbf{c} \frac{1042}{5} \quad \mathbf{d} \ln 4 \quad \mathbf{e} 1 \quad \mathbf{f} \frac{5}{4}(e^5 - e^{-1})$$

$$\mathbf{5} \quad \mathbf{a} \frac{1}{4} \quad \mathbf{b} 2 - \frac{2}{3}\sqrt{3} \quad \mathbf{c} \frac{31}{80} \quad \mathbf{d} 4 - 2\sqrt{2}$$

$$\mathbf{6} \quad \mathbf{a} -\frac{2}{5}\sqrt{3} \quad \mathbf{b} \frac{2}{5}\sqrt{3} \quad \mathbf{c} \frac{26}{3} \quad \mathbf{d} -\frac{4}{3} \quad \mathbf{e} 3 + 2\ln 4$$

$$\mathbf{7} \quad \mathbf{a} \frac{\pi}{3} \quad \mathbf{b} 8\sin^{-1}\left(\frac{2}{3}\right) \quad \mathbf{c} \frac{\pi}{4} \quad \mathbf{d} \frac{1}{2}\sin^{-1}(1) \quad \mathbf{e} 2\sqrt{2} - 2 - \frac{\pi}{2} \quad \mathbf{f} \frac{\pi}{4} \quad \mathbf{g} \pi - 2\tan^{-1}\left(\frac{1}{3}\right)$$

**Exercise 22.5**

$$\mathbf{1} \quad \mathbf{a} \frac{15}{2} \quad \mathbf{b} \frac{38}{3} \quad \mathbf{c} \frac{5}{36} \quad \mathbf{d} -8$$

$$\mathbf{2} \quad \mathbf{a} \frac{35}{20} \quad \mathbf{b} \frac{8}{5}\sqrt{2} - 2 \quad \mathbf{c} -2 \quad \mathbf{d} 0 \quad \mathbf{e} \frac{1}{20} \quad \mathbf{f} -\frac{4}{3} \quad \mathbf{g} \frac{7}{6} \quad \mathbf{h} \frac{5}{6} \quad \mathbf{i} \frac{20}{3} \quad \mathbf{j} 0$$

$$\mathbf{k} \frac{20}{3} \quad \mathbf{l} -\frac{\sqrt{2}}{3}$$

$$\mathbf{4} \quad \mathbf{a} e \quad \mathbf{b} 2(e^{-2} - e^{-4}) \quad \mathbf{c} 0 \quad \mathbf{d} 2(e - e^{-1}) \quad \mathbf{e} e^2 + 4 - e^{-2} \quad \mathbf{f} \frac{1}{2}(e - e^5)$$

$$\mathbf{g} 2\sqrt{e} - 3 \quad \mathbf{h} \frac{1}{4}(16e^{1/4} - e^4 - 15) \quad \mathbf{i} \frac{1}{2}(e^{-1} - e^3)$$

$$\mathbf{6} \quad \mathbf{a} 3\ln 2 \quad \mathbf{b} 2\ln 5 \quad \mathbf{c} 4 + 4\ln 3 \quad \mathbf{d} \frac{1717}{4} \quad \mathbf{e} \frac{3}{2}\ln 3 \quad \mathbf{f} 2\ln 2 \quad \mathbf{g} \frac{3}{4} \quad \mathbf{h} 4\ln 2 - 2$$

$$\mathbf{i} \ln 2$$

$$\mathbf{8} \quad \mathbf{a} 1 \quad \mathbf{b} \frac{3\sqrt{3}}{2} \quad \mathbf{c} \frac{\sqrt{3}}{2} \quad \mathbf{d} -2 \quad \mathbf{e} \frac{\pi^2}{32} - 1 \quad \mathbf{f} 0 \quad \mathbf{g} 0 \quad \mathbf{h} \frac{\sqrt{3}}{2} - \frac{1}{2} \quad \mathbf{i} 0$$

$$\mathbf{9} \quad \mathbf{a} \frac{31}{5} \quad \mathbf{b} \frac{7\sqrt{7}}{3} - \sqrt{3} \quad \mathbf{c} 0 \quad \mathbf{d} \frac{5}{72} \quad \mathbf{e} 3\sqrt[3]{2} - \frac{3}{2} \quad \mathbf{f} 1 - \ln 2$$

$$\mathbf{10} \quad \ln\left(\frac{21}{5}\right)$$

$$\mathbf{11} \quad \sin 2x + 2x \cos 2x; 0$$

$$\mathbf{12} \quad \mathbf{a} 2m - n \quad \mathbf{b} m + a - b \quad \mathbf{c} -3n \quad \mathbf{d} m(2a - b) \quad \mathbf{e} na^2$$

$$\mathbf{13} \quad \mathbf{a} e^{0.1x} + 0.1xe^{0.1x}; 10xe^{0.1x} - 100e^{0.1x} + c$$

$$\mathbf{b} \mathbf{i} 99 \text{ accidents} \quad \mathbf{ii} N = 12t + 10te^{0.1t} - 100e^{0.1t} + 978$$

$$\mathbf{14} \quad \mathbf{a} 1612 \text{ subscribers} \quad \mathbf{b} 46220$$

$$\mathbf{15} \quad \mathbf{b} \sim 524 \text{ flies}$$

**Exercise 22.6**

$$\mathbf{1} \quad \mathbf{a} 4 \text{ sq. units} \quad \mathbf{b} \frac{32}{3} \text{ sq. units} \quad \mathbf{c} 4 \text{ sq. units} \quad \mathbf{d} 36 \text{ sq. units} \quad \mathbf{e} \frac{1}{6} \text{ sq. units}$$

$$\mathbf{2} \quad \mathbf{a} e \text{ sq. units} \quad \mathbf{b} \frac{1}{2}(e^4 - 2 - e^2) \text{ sq. units} \quad \mathbf{c} 2(e + e^{-1} - 2) \text{ sq. units}$$

$$\mathbf{d} 2(e^2 - 2 - e) \text{ sq. units}$$

$$\mathbf{3} \quad \mathbf{a} \ln\left(\frac{5}{4}\right) \text{ sq. units} \quad \mathbf{b} 2\ln 5 \text{ sq. units} \quad \mathbf{c} 3\ln 3 \text{ sq. units} \quad \mathbf{d} 0.5 \text{ sq. units}$$

$$\mathbf{4} \quad \mathbf{a} 2 \text{ sq. units} \quad \mathbf{b} \frac{\pi}{2} \text{ sq. units} \quad \mathbf{c} \frac{3}{8}\pi^2 + \sqrt{2} - 2 \text{ sq. units} \quad \mathbf{d} \sqrt{2} \text{ sq. units}$$

$$\mathbf{e} 4\sqrt{3} \text{ sq. units}$$

$$\mathbf{6} \quad 12 \text{ sq. units}$$

$$\mathbf{7} \quad 4\left(\sqrt{3} - \frac{1}{3}\right) \text{ sq. units.}$$

$$\mathbf{8} \quad \ln 2 + 1.5 \text{ sq. units.}$$

$$\mathbf{9} \quad 2 \text{ sq. units.}$$

$$\mathbf{10} \quad \frac{37}{12} \text{ sq. units}$$

- 11 a 0.5 sq. units b 1 sq. unit c  $2(\sqrt{6} - \sqrt{2})$  sq. units  
 12  $\frac{8}{3}$   
 13  $-2\tan 2x; \frac{1}{4}\ln 2$  sq. units  
 14 a  $\frac{9}{2}$  sq. units b 3 sq. units  
 15 a 1 sq. unit b 10 sq. units  
 16 a  $x \ln x - x + c$  b 1 sq. unit  
 17  $\frac{14}{3}$  sq. units  
 18 a  $\frac{7}{6}$  sq. units b  $\frac{9}{2}$  sq. units  
 19 a i  $\frac{15}{4}$  sq. units ii  $\frac{45}{4}$  sq. units  
 20  $\frac{22}{3}$  sq. units  
 21 b i  $e^{-1} + e - 2$  sq. units ii 1 sq. unit iii  $2\ln(2)$  sq. units  
 22 b 3.05 sq. units  
 23 a  $2y = 3ax - a^3$  b  $\frac{1}{15}a^5$  sq. units  
 24 a  $1 - e^{-1}$  sq. units b  $e^{-1}$  sq. units c  $1 - e^{-e^{-1}} - 1 - e^{-1} \sim 0.10066$  sq. units  
 25  $a = 16$

**Exercise 22.7**

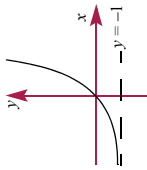
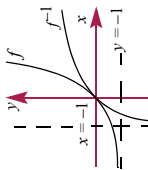
- 1 a  $x = t^3 + 3t + 10, t \geq 0$  b  $x = 4 \sin t + 3 \cos t - 1, t \geq 0$  c  $x = t^2 - 4e^{\frac{-1}{2}t} + 2t + 4, t \geq 0$   
 2 a  $x = t^3 - t^2, t \geq 0$  b 100 c  $100\frac{8}{27}$  m  
 3 a  $x = -\frac{2}{3}(4+t)^{3/2} + 2t + 8$  b 6.92 m  
 4  $\frac{125}{6}$  m  
 5  $\frac{125}{49}$  s; 63.8 m  
 6 a  $\frac{\pi}{6}$  s b  $\frac{\pi}{2} - 1$  m  
 7 80.37 m  
 8 a  $s(t) = \frac{160}{\pi} \left[ 1 - \cos\left(\frac{\pi}{16}t\right) \right], t \geq 0$  b 86.94 m c -6.33 m d 116.78 m  
 9 a  $v = 4 + k - \frac{k}{t^2}, t > 0$  b  $k = 2$  c 52.2 m  
 10 b 0.0893 m

**Exercise 22.8**

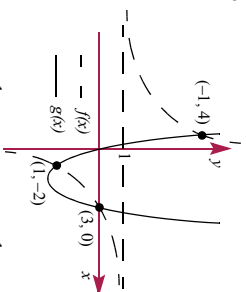
All values are in cubic units.

- 1  $21\pi$   
 2  $\pi \ln 5$   
 3  $\frac{\pi}{2}(e^{10} - e^2)$   
 4  $\pi^2$   
 5  $\frac{\pi}{2}$   
 6  $\pi\left(\frac{8}{3} - 2 \ln 3\right)$   
 9  $\frac{\pi}{2}(5 - 5 \sin 1)$   
 10  $\frac{251}{30}\pi$   
 11  $\frac{242}{5}\pi$   
 12  $\frac{\pi}{4}$   
 13  $\frac{88}{5}\sqrt{3}\pi$   
 14  $\frac{3\pi}{4}$   
 15  $k = 1$   
 16  $4\pi^2 a^2$   
 17  $k = \frac{\pi}{2}$   
 18  $\frac{8\pi}{15\sqrt{1+a^2}} \left( \frac{3a^2+2}{1+a^2} \right)$   
 19 a Two possible solutions: solving  $a^3 - 6a^2 - 36a + 204 = 0, a = 4.95331$ ;  
 solving  $a^3 - 6a^2 - 36a - 28 = 0$ , then  $a = -0.95331$  b  $a = \frac{100}{\pi}$   
 20  $\frac{28}{15}\pi$   
 21  $64\pi$

**Revision Set A**

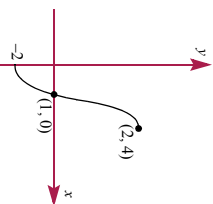
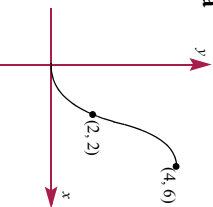
- 1 -84  
 2 a  b i  $]-1, \infty[$  ii  $f^{-1}(x) = \ln(x+1)$  c   
 3 840  
 4 a i 0 ii 2 b  $-2 \leq x \leq 2$  c  $x \geq 0$

- 5 a (1, -2), (-1, 4) and (3, 0) b

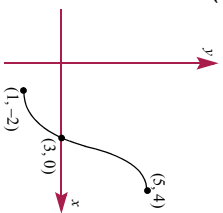


- 6 a 2 b  $S = [0, \infty[$ , range =  $[1, \infty[$  c  $f^{-1}: [1, \infty[ \mapsto \mathbb{R}, f^{-1}(x) = (\ln x)^2$

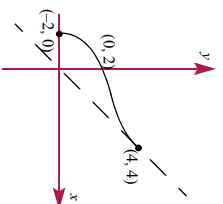
- 7 a



- c



- d



- 8 a i 512 ii 2 b i  $3x^2h + 3xh^2 + h^3$  ii  $3x^2 + 3xh + h^2$   
 9 a i -1 or 6 ii  $\frac{3}{e-1}$  b i  $\mathbb{R} \setminus \{3\}$  ii 0.2 iii 0

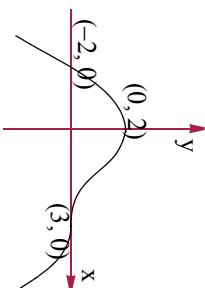
- 10 a i 2 or 6 ii  $\frac{1}{3}(e^2 - 4)$  b i  $0 < x < 1$  ii  $\mathbb{R}$  iii  $\log_e 4 \approx 0.72$  iv  $\frac{e^{0.8}}{1 + e^{0.8}} \approx 0.69$

- 11 a  $g(f(x)) = -\frac{2x}{1-x}, x \in \mathbb{R} \setminus \{\pm 1\}$  b  $P \equiv (2, 4)$

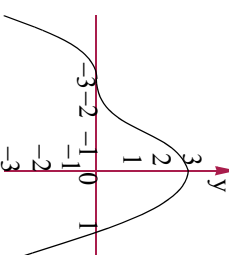
- 12 a i  $x = \frac{\ln 6}{\ln 3}$  ii  $\frac{15}{7}$  b  $1 + \sqrt{3}$  c i  $f(g(x)) = \sqrt{\frac{1}{x^2} - 1}, g(f(x)) = \frac{1}{x-1}$  ii  $[-1, 1] \setminus \{0\}$

- 13 0.5

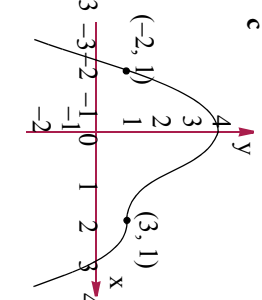
- 14 a



- b



- c



- 15 a  $k = 0$  or 16 b  $(2x-1)(3x+2)(x+3)$  c  $0 < x < 3$

- 16 a  $0 < x < 5$  b 70 c -2,  $-\frac{1}{2}, 1$

- 17 a 9 b -4

- 18  $\pm 3$

- 19 a  $y = -2x$  b  $\frac{x-y}{x+y}$

- 20 b  $x = \frac{4}{9}, y = \frac{1}{9}$

- 21 b ii  $p^5 = 3 + 5p, p^{-5} = 5p - 8$

- 22 a  $\frac{2}{9}$  b 59136

- 23 a  $-\frac{3}{5}, b = -\frac{648}{25}, n = 10$

- 24 a ii  $\{\pm 1\}$  b i  $y = \sqrt{6(x-3)}$  ii  $x = 9, y = 6$

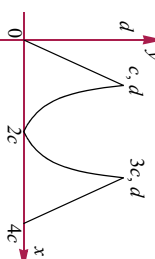
- 25  $1792x^5$

- 26 a  $\frac{5}{2}, \frac{3}{2}$  b  $\frac{3}{2}, \frac{1}{2}$  c  $\frac{17}{2}$

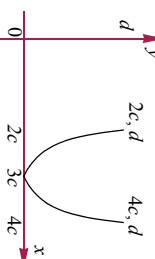
- 27 a  $\mathbb{R}$  b  $]-\infty, 4[$  c  $]-\infty, 4[$

- 28 b  $\frac{5}{8}$  sq units

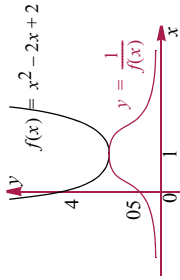
- 29 a



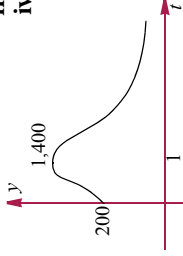
- b



30 a i & ii

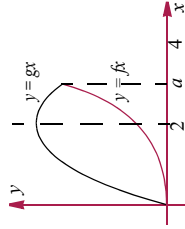


b i

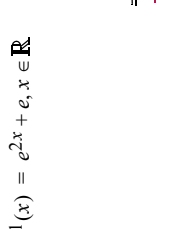


ii \$200 iii \$4  
iv  $t = 0.42, 1.57$

31 a

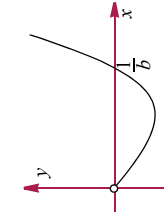


b 3



32  $g^{-1}(x) = e^{2x} + e, x \in \mathbf{R}$

33 a i  $0, \infty[$  ii  $(-\frac{a}{e^b}, \infty[$  c  $(1, a \log_e b)$  d  $x = b^{\frac{1}{1-x}}$

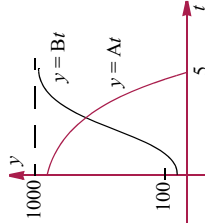


34 a  $a = -36, b = 900$  b

$t$	1	2	35	5
$B(t)$	13104	52702	95723	99778

c 20 d 1000

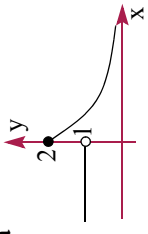
e  $t > 195$  f



35 a 150 cm b 138 cm c 94 hrs d  $[0, 94]$  e  $h^{-1}(x) = \frac{12.5 - \sqrt{f}}{0.13}$

f Use graphics calculator. g 17.3 hrs

36 a b  $]0, 2]$  c No ( $x = 0$ )

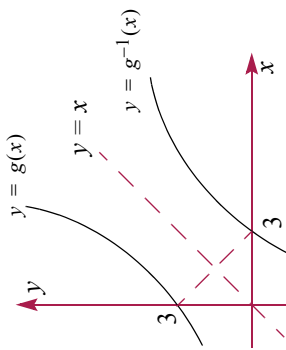


37 78

38 a 0 b  $-\sqrt{2}$  c  $r_f \neq d_g$ , i.e. does not exist

39  $-\frac{63}{8}x^5$

40 a  $g^{-1}(x) = -1 + \sqrt{x-2}, x \geq 2$  b



41 b  $h(x) = 4 - x, x \geq 0$ , range =  $]-\infty, 4]$

42 a Use graphics calculator. b  $f^{-1}(x) = -\log_e(1-x), x < 1$  c Use graphics calculator.

43 -10

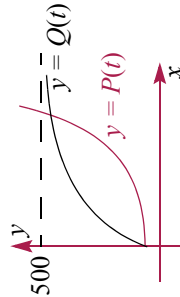
44 a  $r_g \subseteq d_f \Rightarrow f \circ g$  exists;  $r_f \not\subseteq d_g \Rightarrow g \circ f$  doesn't exist. b  $x < -2$  or  $x > 2$

45 a  $f^{-1}(x) = (2-x)^2, x < 2$  b  $r_g \subseteq d_{f^{-1}} \Rightarrow f^{-1} \circ g$  does not exist;

$r_{f^{-1}} \subseteq d_g \Rightarrow g \circ f^{-1}$  exists. c  $F(x) = x-2, x \leq 2$

46 a  $t = 2$  or 3 b  $t = 3$  c  $x = 1 + \lambda, y = 4 - \lambda, z = \lambda, \lambda \in \mathbf{R}$

47 a i 50 ii  $50e \approx 135.9$  c 500 d i 50 ii 334.5



f Increasing at a decreasing rate g  $\sim 460$  wasps h ii  $t = 0$  and  $t = 10 \log_e 9$

Revision Set B

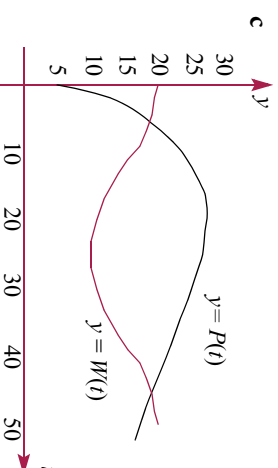
1 a 189 b 99 c -96 d 36

2 b -65

3 b 239 km c  $264^\circ$  d 153 km e 1075

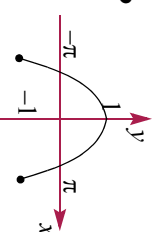
- 4 **a** i A: \$49000; B: \$52400; C: \$19200 **ii** A: \$502400; B: \$506100; C: \$379400  
**b** 46% **c** i 14 months **ii** C never reaches its target
- 5 **a**  $r = 05$  **b** 625 cm
- 6 **b**  $26^\circ 34'$  or  $135^\circ$
- 7 **b**  $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$
- 8 **a** 28
- 9 **a**  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$  **b**  $0, \frac{\pi}{2}, \pi, 2\pi$
- 10 **a** Max. value is  $\frac{17}{2}$  for  $x = \frac{\pi}{2} + 2k\pi$  or  $x = \frac{3\pi}{2} + 2k\pi$ , where  $k$  is an integer;  
 min. value is  $\frac{17}{5}$  for  $x = k\pi$ , where  $k$  is an integer **b**  $\frac{\pi}{3}, \frac{5\pi}{3}$
- 11 **a**  $u_n = 74 - 6n$  **b**  $n = \frac{1}{6}(74 - p)$  **c**  $\frac{1}{12}(74 - p)(68 + p)$ , 420
- 12  $\frac{24(4\sqrt{3} - 3)}{39}$
- 14 **a**  $60^\circ, 109^\circ 28', 250^\circ 32', 300^\circ$  **b** i 2 cosec  $\theta$  **ii**  $\frac{\pi}{3}, \frac{2\pi}{3}$
- 15 **a**  $\sim 342$  **b** 20 terms **c**  $0 < x < 2$  **d** {1, 3, 8, 18, ...} **e**  $u_n = 23 - 3n$   
**f** \$4131.45
- 16 **a**  $-\frac{1}{2}$  **b** 4
- 17 **a**  $120^\circ$  **b**  $14\sqrt{3}$  cm<sup>2</sup>
- 18 **a** i  $0.3\sqrt{3}$  m **ii**  $0.2\sqrt{3}$  m **b**  $\sim 1.15$  m **c**  $73^\circ 13'$
- 19 **a**  $\frac{\pi}{3}, \frac{4\pi}{3}$  **b**  $\left\{x \mid \frac{\pi}{3} < x < \frac{4\pi}{3}\right\}$
- 20 **a** 8 cm **b**  $28^\circ 4'$
- 21 3
- 22 **a**  $\left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}\right\}$  **b**  $\left(\frac{\pi}{3}, 1\right)$
- 23 **a** \$77156.10 **b**  $u_1 = -\sqrt{3}, u_3 = -3\sqrt{3}$
- 24 **a**  $f(x) = 3 \cos(2x)$  **b**  $\left\{\frac{7\pi}{6}\right\}$  **c** 3
- 25 **b** i BP = 660 m, PQ = 688 m
- 26  $216^\circ$
- 27 **b** 906 m
- 28 **a**  $38^\circ 40'$  **b** 0.08004 m<sup>2</sup> **c** \$493.71
- 29 **a**  $\tan \alpha = -\frac{1 + \sqrt{5}}{2}$  **b** range = [3, 3.5] **c** i 3 **ii** 2

- 30 **a** i  $W(4) = 19.38, P(4) = 14.82$  **ii**  $W(20) = 10.95, P(20) = 27.02$   
**iii**  $W(35) = 13.45, P(35) = 23.25$  **b** Amp = 5, period = 50 weeks  
**d** \$27.07 **e** during 7th & 46th weeks



- 31 **a** \$49000, \$47900, \$46690, \$46690 **b** \$34062.58 **c** 18.8 years **d**  $\sim \$248564$
- 32 **a** **ii** 26 cards **b** 26, 40, 57, 77 **c**  $a = 3, b = -$  **d** 155 cards **e**  $t_n = \frac{n}{2}(3n + 1)$
- 33 **a**  $\sim 2.77$  m **b** i 3.0 m **ii** 2.0 m **c** 4.15 pm **d** Use graphics calculator.
- 34 1.262 ha
- 35  $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

36 **a**  $x = -\frac{2\pi}{3}, \frac{2\pi}{3}$  **b**  $c -\frac{2\pi}{3} < x < \frac{2\pi}{3}$



- 37 1623 m
- 38 **a**  $19.5^\circ\text{C}$  **b**  $D(t) = -1 + 2 \cos\left(\frac{\pi}{12}t\right)$  **d** Use graphics calculator.
- 39 1939 m **e** 8 am to midnight
- 40 **a** **ii**  $N_0 = 2000, \alpha = 10$  **b** 2000, 2200, 2420, 2662, 2988.2 **c** 52 hrs **d** 176995
- 41 **a**  $(4 - \pi)$  cm<sup>2</sup> **b**  $\frac{(4 - \pi)}{2}$  cm<sup>2</sup> **c** **ii**  $r = \frac{1}{2}$  **iii**  $A_n = (4 - \pi) \times \left(\frac{1}{2}\right)^{n-1}$ ,  $n = 1, 2, \dots$
- d** i  $\frac{31}{16}(4 - \pi)$  cm<sup>2</sup> **ii**  $2(4 - \pi)$  cm<sup>2</sup> **e** Geometric

**Revision Set C**

- 1 **a**  $-7i + 6j + k$  **b**  $-8$  **c**  $a = \frac{1}{\sqrt{3}}(i + j + k)$
- 2 **a**  $x = \frac{7}{2} + \lambda, y = \lambda, z = \frac{9}{2} + 5\lambda$  **b**  $\frac{x - 3.5}{1} = \frac{y - 4.5}{5}$
- 3 0



- 4  $t = 2, (16, -8, 4)$   
 5 **a**  $3i - j - 2k$  **b**  $100^\circ$  **c**  $4i - 3j - 3k$   
 6 **a**  $|r|_{\min} = 2\sqrt{2}$  **b**  $t = 5, b = \frac{2}{5}$   
 7 **a**  $90^\circ$  **ii**  $\frac{7}{2}\sqrt{26}$  unit<sup>2</sup> **b**  $i s + 3p$  **ii**  $s + 2p$  **iii**  $\frac{1}{2}s + 2p$  **iv**  $-\frac{1}{2}s + 2p$   
 8 **a**  $27^\circ$  **b**  $\frac{1}{2}\sqrt{17}$  unit<sup>2</sup>  
 9 **a**  $\mathbf{i} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 7 \end{pmatrix}, \lambda \in \mathbf{R}$  **ii**  $x = 2 + 3\lambda, y = -3 + 7\lambda, \lambda \in \mathbf{R}$  **iii**  $\frac{x-2}{3} = \frac{y+3}{7}$   
**b**  $-i + 11j$  **c** **no** **ii** lines are skew  
 10  $28^\circ 35'$   
 11 **a**  $a = \frac{3}{2}$  **b**  $b = \frac{3}{2}, c = \frac{1}{3}$   
 12  $\frac{4}{\sqrt{77}} \left( -\frac{5}{4}i + j + \frac{3}{2}k \right)$  or  $-\frac{4}{\sqrt{77}} \left( -\frac{5}{4}i + j + \frac{3}{2}k \right)$   
 13 **a** 5 **b**  $\frac{5}{3}\sqrt{5}$   
 14 **OA** =  $2i - 2j + k$ , **OB** =  $4i - 3k$ ;  $70^\circ 32'$   
 15 Yes  
 16 **a**  $\mathbf{i} (1, -1, 2); \left( \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right)$  **ii**  $(3, 6, 2); \left( \frac{3}{7}, \frac{6}{7}, \frac{2}{7} \right)$  **b** lines do not meet  
 17  $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z+3}{1}$   
 18  $\left( \frac{2}{5}, \frac{23}{5} \right)$   
 19  $\frac{2}{3}$  or 2  
 20 **a**  $r_A = \begin{pmatrix} 0 \\ 80000 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \end{pmatrix}$  **b**  $\begin{pmatrix} 21600 \\ 65600 \end{pmatrix}$  (units in metres) **c** They do not collide.  
 21 **a**  $r = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  **b**  $LP = \begin{pmatrix} -21 \\ -11 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  **c**  $\sim 129.31$  km  
 22 **a**  $\frac{1}{4}$  **b**  $\frac{3}{8}$  **c** 0.3169  
 23 0.0228  
 24 **a** 0.12 **b** 0.6087  
 25 **a** 0.89 **b**  $\frac{21}{40}$  **c**  $\frac{40}{89}$   
 26 **a** 0.46 **b**  $\frac{9}{23}$   
 27 **a** 3326400 **b**  $\frac{2}{11}$  **ii**  $\frac{2}{77}$   
 28 **a** 0.9772 **b** 0.3413  
 29 **a** 0.936 **b** 5  
 30 **a** 792 **b** 35  
 31 **a** 151200 **b** 0.1512  
 32 0.2852  
 33  $\frac{128}{850} \approx 0.1506$   
 34 **a** 0.10 **b** 0.40 **c**  $(x, P(X=x))$  values are: 0, 0.40), 1, 0.50), 2, 0.10)  
**d**  $E(X) = 0.70, \text{var}(X) = 0.41$   
 35 **a** 0.8664 **b** 0.7210 **c** 0.9034 **d** 9.8855 <  $Y < 10.2145$  **e** 79.3350  
 36 **a** 315 **b** 17280  
 37  $\frac{193}{512}$   
 38 **a**  $\frac{2}{3}$  **b**  $\frac{1}{2}$   
 39 **a**  $P(X=x) = \frac{1}{6} \times \left(\frac{5}{6}\right)^x, x = 0, 1, \dots$ , i.e. geometric **b** **i** 0.0670 **ii** 0.4019 **iii**  $\frac{1}{6}$   
 40 **a**  $\frac{13}{44}$  **b**  $\frac{9}{44}$  **b**  $(x, P(X=x))$  values are:  $\left(1, \frac{9}{25}\right), \left(3, \frac{7}{25}\right), \left(5, \frac{5}{25}\right), \left(10, \frac{3}{25}\right), \left(20, \frac{1}{25}\right)$   
**c**  $EX = \frac{105}{25} \approx 4.2, \text{var}X = \frac{11400}{625} \approx 18.24$  **d** 0.00064  
 41 **a** 0.3085 **b** 0.0091 **c** 0.1587  
 42 100  
 43 **a**  $\frac{1}{2}$  **b**  $\frac{1}{7}$  **c**  $\frac{2}{7}$   
 44 **b**  $(x, P(X=x))$  values are: 1, 0.4, 2, 0.3, 3, 0.2, 4, 0.1 **ci** 2 **ii** 5 **iii** 3  
 45 **a** 0.8186 **b** 0.1585  
 46 **a**  $(x, P(X=x))$  values are:  $\left(0, \frac{3}{16}\right), \left(1, \frac{7}{16}\right), \left(2, \frac{5}{16}\right), \left(3, \frac{1}{16}\right)$  **b** **ii**. 0.0064 **iii**. 0.7705  
 47  $\mu = 0.9586, \sigma = 0.0252$   
 48 **a**  $\frac{10}{21}$  **b** **i** 0.3085 **ii** 0.1747  
 49 **a** **i** 0.8 **ii** 0.25 **b** **i** 0.4 **ii**  $E(X) = 0.8, \text{var}(X) = \frac{14}{25}$   
 50 **a**  $\frac{1}{8}$  **ii**  $\frac{47}{72}$  **iii**  $\frac{1}{8}$  **iv**  $\frac{47}{72}$  **v**  $\frac{9}{47}$   
 51  $\frac{189}{8192}$   
 52  $\frac{43}{60} \approx 0.7167$   
 53  $\frac{117}{145} \approx 0.8069$   
 54 **a**  $(x, P(X=x))$  values are:  $\left(0, \frac{1}{6}\right), \left(1, \frac{1}{3}\right), \left(2, \frac{1}{2}\right), \left(2, \frac{1}{2}\right)$ ;  $E(X) = \frac{4}{3}, \text{var}(X) = \frac{5}{9}$  **b**  $\frac{2}{3}$  **c**  $\frac{5}{24}$   
 55 **a** 0.4 **b** 0.096 **c** 0.225 **d** 0.635  
 56 **a**  $\frac{3}{5}$

57 **b** ( $x, P(X=x)$ ) values are:  $(0, \frac{4}{25})$ ,  $(1, \frac{12}{25})$ ,  $(2, \frac{9}{25})$     **c**  $E(X) = 1.2$ ,  $\text{var}(X) = 0.48$

**d**  $\frac{3}{7}$

58 **a**  $i \frac{8}{15}$     **i**  $\frac{7}{15}$     **iii**  $\frac{1}{5}$     **iv**  $\frac{4}{5}$     **v**  $\frac{4}{7}$     **b**  $\frac{x(p-q)+100q}{100}$

59  $\frac{2}{3}$

60 **a** 0.1359    **b** 137.22    **c**  $137\frac{1}{3}$     **d**  $a = 141.21$

61 **a**  $\frac{2}{3}$     **b**  $\frac{2}{9}$     **c** not independent

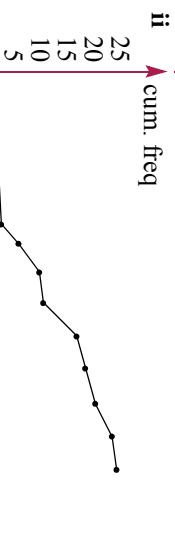
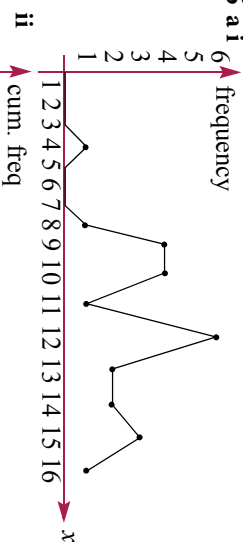
62 **a**  $b+6a$     **b**  $0 \leq b \leq \frac{1}{3}$

63 **a** 0.081    **b**  $\frac{4}{13}$

64 **a** 0.0169    **b**  $i$  0.9342    **ii** 127    **iii** 0.008

65 **a** 0.1587    **b** 0.7745    **c** \$0.23

66 **a** **i**



**b** **i** 11.44    **ii** 2.6695    **c** **i**  $x_{min} = 4$ ,  $x_{max} = 16$ , med = 12,  $Q_3 = 13.5$ ,  $Q_1 = 9.5$



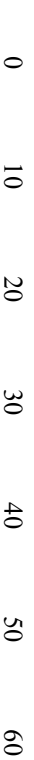
67 **a**  $i$  0.24    **ii** 0.36    **b**  $172+0.96Q$     **c**  $Q > 29.17$

68 **a** use graphics calculator    **b**  $i$   $\bar{x}_A = 37.35$ ,  $\bar{x}_B = 37.31$

**ii** A:  $s_n = 8.801$ , B:  $s_n = 9.025$

**c** **i** Class A:  $x_{min} = 23$ ,  $x_{max} = 56$ , med = 36,  $Q_3 = 43$ ,  $Q_1 = 31$   
Class B:  $x_{min} = 22$ ,  $x_{max} = 57$ , med = 37,  $Q_3 = 44$ ,  $Q_1 = 31$

Class A



Class B



**ii** Class A: med = 36; multimodal – 34, 35, 39, 43, 48

Class B: med = 37; multimodal – 27, 34, 38, 42, 49

**iii** Class A: IQR = 12, Class B: IQR = 13

**d** Results from both classes are very close, however, Class B does slightly better as it has a larger median as well as the larger maximum value.

**Revision Set D**

1 **a**  $\frac{x}{\sqrt{x^2+4}}$     **b**  $2 \cos 2x - 2(2x-1) \sin 2x$

2  $30\frac{1}{3} + \log e^4$

3 **a**  $\frac{\pi}{3}$ ;  $\frac{4\pi}{3}$     **b**  $\left\{ x \mid \frac{\pi}{3} < x < \frac{4\pi}{3} \right\}$     **c** 4 sq. units

4 **a** 19.8°C    **b** 1.6°C per minute    **c** 17.3 min

5 **a**  $x \in [-1, 0] \cup [0, \infty[$     **b**  $x \in ]-\infty, 0[ \cup [2, \infty[$

6 10 m

7 **a**  $\frac{4x}{(x^2+1)^2}$     **b**  $-4 \sin 2x \cos 2x$  OR  $-2 \sin 4x$

8 **a**  $i$  0    **ii** 2    **b**  $x \in [-2, 2]$     **c**  $x \geq 0$     **d**  $-\frac{x}{\sqrt{4-x^2}}$ ,  $-2 < x < 2$

9 **a**  $-\frac{2+h}{(1+h)^2}$ ,  $h \neq 0$     **b** -2

10 **a** 74    **b** 0.69

11 1.455  $\text{ms}^{-1}$

12 **a** Absolute maximum at  $(\pm \frac{1}{\sqrt{2}}, 1)$ ; local min at 0, 0; x-intercept at  $\pm 1$ , 0

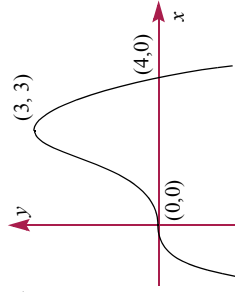
**b** Local min at  $(\pm \frac{1}{\sqrt{2}}, 1)$ ; asymptotes at  $x = \pm 1$ ,  $y = 0$ .

13 **a**  $6 \cos 2x \sin^2 2x$     **b**  $\frac{x+3}{(2x+3)^{3/2}}$

14  $\frac{1}{2}(e^{2x} - 4x + e^{-2x}) + c$

15 720  $\text{m}^3$

16 a



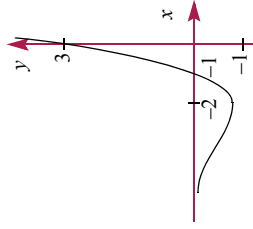
b  $98^\circ$

17 b i 2 ii 72 cm<sup>3</sup>

18 a Area =  $A = \frac{8}{15}h^{3/2}$ , Volume =  $V = 0.48h^{3/2}$  b  $\frac{5}{144}$  m/min

19 a (-1, 4), (1, -2), (3, 0) b use graphics calculator c  $\frac{16}{3} - 3 \log_e 3$  sq. units

20



21 a  $h = \frac{1000}{\pi r^2}$  b radius = 5 cm, height = 12.7 cm

22 a  $-\frac{3x}{\sqrt{1-3x^2}}$  b  $\frac{e^x}{(1+e^x)^2}$

23 a  $3x^2h + 3xh^2 + h^3$  b  $3x^2 + 3xh + h^2$  c  $3x^2$

24 a  $2 - \frac{3}{4}\sqrt{3}$  b  $\log_e 3$

25 a  $p(t) = 0.8(1 - 0.02t)e^{-0.02t}$  b  $\sim 38.3$  million c i decreasing  
ii  $\sim 0.1$  million/year d 50 years time, i.e. 2030; 42.2 million

26 76222 cm<sup>3</sup>

27 a  $A \equiv (-1, 5)$ ,  $B \equiv (1, 3)$ ,  $C \equiv (4, 0)$  b use graphics calculator c  $12 - 4 \log_e 4$  sq. units

28 a  $\frac{1 + \cos x + x \sin x}{(1 + \cos x)^2}$  b  $\frac{x}{x^2 + 1}$

29 a  $12 + 6h + h^2$ ,  $h \neq 0$  b 12

30 a  $A \equiv (2, 2e^{-1})$  b i  $y = x$  ii  $\frac{d}{dx}(xe^{-x}/k) = (1 - \frac{x}{2})e^{-x/2}$  iii  $4 - 2(2 + a)e^{-a/2} - \frac{1}{2}a^2$

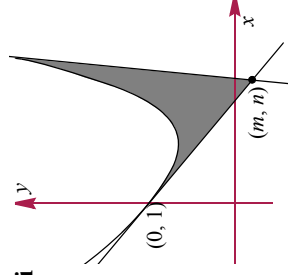
c i  $(2x - x^2)e^{-x}$  ii  $\pi(2 - 10e^{-2})$  cubic units

31 a  $\log_e x$  b  $2(\log_e 2 - 1)$

32 a i 283 sec ii 250 sec c 244 sec

33 a  $A \equiv (\frac{1}{2} \ln 2, 2(1 - \ln 2))$  b  $\frac{1}{2}(e^2 - 5)$  sq. units c i At (0,1):  $y = -2x + 1$

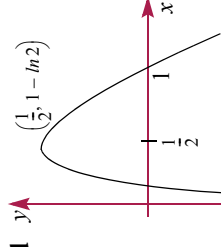
At (1,  $e^2 - 4$ ):  $y = (2e^2 - 4)x - e^2$  ii



iii  $\frac{1}{2}(e^2 - 5)$  sq. units d ii  $\frac{\pi}{12}(3e^4 - 24e^2 + 37)$  cubic units

34 b i  $0 < x < 0.5$  ii  $x = 0.5$  iii  $x < 0$  or  $x > 0.5$  c  $(\frac{1}{2}, 1 - \ln 2)$

d i  $y = -x + 1$  ii  $y = x - 1$



f i  $\frac{3}{8} - \frac{1}{2} \ln 2$  sq. units ii  $\frac{1}{8} + \frac{1}{2} \ln 2$  sq. units

35 a 4.20 b i  $\frac{1}{2} \cos \frac{1}{4}t$  ii -0.40

36  $\pi \log_e 3$  cubic units

37  $a = -1$ ,  $b = 6$ ,  $c = -9$

38 a  $\frac{4\sqrt{3}}{3}$  b i  $-6 \sin 3x \cos 3x$  ii  $\frac{x}{2} + \frac{1}{12} \sin 6x + c$

39  $A = 0$ ,  $B = 0.5$

40 a  $\frac{7}{12}$  sq. units b  $\frac{7}{15}\pi$  cubic units

41 a  $V = \pi r^2 h + \frac{4}{3}\pi r^3$  b  $P = 2\pi k r h + 6\pi k r^2$  c  $P = \frac{2kV}{r} + \frac{10\pi}{3} k r^2$  d  $0 < r < (\frac{3V}{4\pi})^{1/3}$

e  $r = (\frac{3V}{10\pi})^{1/3}$

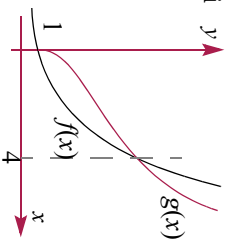
42 b  $[\frac{a}{4}, \frac{a}{2}]$  c  $\frac{\sqrt{3}}{36}a^3$  cubic units

43 a  $2x \log_e x + x$ ;  $2 \log_e 2 - \frac{3}{4}$  c 1,  $e$ ;  $y = ex$  d  $(\frac{1}{2}e - 1)$  sq. units

44  $x_1 = \frac{\pi}{6}, x_2 = \frac{\pi}{4}, x_3 = \frac{\pi}{3}$

45 **a**  $a = 2; f^{-1}(x) = 2 + \sqrt{x}, x \geq 0$     **b**  $\frac{136}{3}\pi$  cubic units

46 **a**  $i \mathbb{R}$     **ii**  $[0, \infty[$     **b** 0.5    **c**  $\{0, 4\}$     **d**  $i$   $y$     **ii**  $\{x \mid x > 4\}$

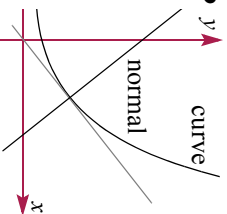


**e**  $i$  4 sq. units    **ii**  $\frac{\pi}{2}(e^4 + 3)$  cubic units

47 **a**  $-6 \sin 2x \cos^2 2x$     **b**  $\frac{1-2x^2}{\sqrt{1-x^2}}$

48 **a**  $\sin x + x \cos x$     **b**  $x \sin x + \cos x$

49 **a**  $y = -e^x + e + e^{-1}$     **b**  $y$     **c**  $\frac{1}{2}e + e^{-2}$  sq. units

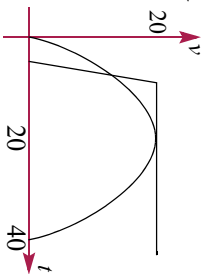


50 **a**  $\frac{4}{3}$  sq. units    **b**  $\frac{64}{15}\pi$  cubic units

51 **a**  $[0, 5]$     **b** use graphics calculator    **c** 0.625    **d**  $a = \frac{1}{2} - \frac{1}{5}t, 0 \leq t \leq 5$

52 **c** Minimum,  $3\pi a^2 \left(\frac{2}{3}\right)^{1/3}$ ; Maximum  $3\pi a^2 \left(\frac{2}{4}\right)^{1/3}$

53 **a**    **b** 30 seconds    **c**  $116\frac{2}{3}$  metres



54 **a**  $-e^{-x}(\cos x + \sin x)$     **b** 1

55 **a**  $-\frac{1}{t^2} + 1$     **b**  $\ln t + 2t + \frac{1}{2}t^2 + c$

56 **a** use graphics calculator    **b**  $f(x) = 2x \cos x, 0 < x < \frac{\pi}{2}$     **d** 1.12 sq. units