# Mathematics Standard Level

for the IB Diploma

Exam Preparation Guide

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## INTRODUCTION

## **ABOUT THIS BOOK**

If you are using this book, you're probably getting quite close to your exams. You may have started off as a bright-eyed student keen to explore international perspectives in mathematics and the nature of mathematical knowledge, but now you want to know how to get the best possible grade! This book is designed to revise the entire core material that you need to know, and to provide examples of the most common types of exam questions for you to practise, along with some hints and tips regarding exam technique and common pitfalls.

Any common pitfalls and useful exam hints will be highlighted in these boxes.

This type of box will be used to point out where graphical calculators can be used effectively to simplify a question or speed up your work. Common calculator pitfalls will also be highlighted in such boxes.

If the material in a chapter involves maths outside of that chapter, this kind of box will direct you to the relevant part of the book where you can go and remind yourself of that maths.

The most important ideas and formulae are emphasised in the 'What you need to know' sections at the start of each chapter. When a formula or set of formulae is given in the Formula booklet, there will be a book icon next to it . If formulae are not accompanied by such an icon, they do not appear in the Formula booklet and you may need to learn them or at least know how to derive them.

For Mathematics Standard Level, each of the written papers:

- is worth 40% of the final grade
- is one and a half hours long (plus 5 minutes of reading time)
- has a total of 120 marks available
- contains 9 or 10 short questions and 3 or 4 long questions.

The difference between the two papers is that calculators are **not allowed** for Paper 1, but are required for Paper 2.

In this book questions which are designed to be done without a calculator are accompanied by a non-calculator icon.

Questions which are only sensible to do with a calculator are marked with a calculator icon.

All other questions should be attempted first without a calculator, and then with one.

Between Papers 1 and 2 the majority of the material in the Standard Level course will be assessed.

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## **IMPORTANT EXAM TIPS**

- Grab as many marks as you can.
  - If you cannot do an early part of a question, write down a sensible answer and use that in later parts or, if the part you could not do was a 'show that' task, use the given result. You will still pick up marks.
  - Do not throw away 'easy marks':
    - Give all answers exactly or to three significant figures. Each time you fail to do so you will lose a mark.
    - Do not use rounded intermediate values, as this can result in an inaccurate answer; store all intermediate values in your calculator.
    - Read the questions carefully and make sure you have provided the answer requested. For example, if the question asks for coordinates, give both *x* and *y* values. If the question asks for an equation, make sure that you have something with an equals sign in it.
- The questions are actually worded to help you.
  - Make sure you know what each command term means. (These are explained in the IB syllabus.) For example:
    - 'Write down' means that there does not need to be any working shown. So, for this type of question, if you are writing out lines and lines of algebra, you have missed something.
    - 'Hence' means that you have to use the previous part somehow. You will not get full marks for a correct answer unless you explicitly show how you have used the previous part.
    - 'Hence or otherwise' means that you can use any method you wish, but it's a pretty big hint that the previous part will help in some way.
    - 'Sketch' means that you do not need to do a precise and to-scale drawing; however, you should label all the important points at the very least where the curve crosses any axes.
  - If the question refers to solutions, you should expect to get more than one answer.
  - Look out for links between the parts of a question, particularly in the long questions.
- Use your 5 minutes of reading time effectively.
  - Decide on the order in which you will attempt the questions. It is very rarely most efficient to answer the questions in the order in which they are set; in particular, the last few short questions often involve a lot of work relative to the number of marks available.

Make sure you leave enough time for the long questions, some parts of which can be fairly straightforward.

- For Paper 2, decide which questions can be done easily or checked effectively on the calculator. Do not be surprised if this is the majority of questions.
- Try to classify which section of the course each question is about.

Practise using the reading time when attempting your practice papers.

Most importantly, there is nothing like good preparation to make you feel relaxed and confident going into an exam, which will help you achieve your best possible result.

Good luck! The author team