Mathematics Standards Clarification for

Geometry Conceptual Category

High School

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Contents

Congruence

Cluster

Experiment with transformations in the plane

NVACS HSG.CO.A.1 (Major Supporting Work)

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Element	Exemplars
Standards for Mathematical Practice	- MP 6 Students will attend to precision of vocabulary so that students are clear on what each type of transformation describes. - MP 7 Students will use the structure of transformations such as an angle - the second ray is rotated around the vertex
Instructional Strategies	- This is a vocabulary based standard, flash cards and other study devices are appropriate. - Focus on relating simpler vocabulary to more precise vocabulary - Flip becomes reflection - Slide becomes translation - Turn becomes rotation
Prerequisite Skills	- Understand undefined notations such as point, line and plane. - Know that distance is length. - Connect slope to parallel and perpendicular lines. - Understand a transformation changes a shape in some way.
Connections Within and Beyond High School	- Provides vocabulary instruction to be used in other standards. - Distance around a circular arc relates to standards covering circles.
Instructional Examples/Lessons/Tasks	- Polygraph: Transformations (Desmos) - Allows students to practice vocabulary. - Lines (Khan Academy) - Points, lines, planes practice - Points, Lines, Planes (Math Open Reference) - Basic Geometric Terms (mcckc.edu)
Assessment Examples	- Foundations of Geometry Practice Test (RPDP) - Geometry: Parallel and Perpendicular Line Quiz (ProProfs.com)

Congruence

Cluster

Experiment with transformations in the plane.

NVACS HSG.CO.A.2 (Major Supporting Work)

Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Element	Exemplars
Standards for Mathematical Practice	● MP 1 Students will experiment to determine outcomes when applying transformations or a series of transformations. ● MP 5 Students will use a variety of tools such as tracing paper, software, and manipulatives to explore transformations.
Instructional Strategies	● Have students use patty (tracing) paper to experiment with transformations. ● While it is acceptable to mention dilations, they are covered under a different standard (note this is a congruence cluster). ● Have students use dynamic graphing software.
Prerequisite Skills	● Understand properties of congruence. ● Recognize and plot points on a coordinate plane.
Connections Within and Beyond High School	● Will be extended to compare to similarity transformations. ● Used as a method to prove congruent figures. ● Transformations of parent functions is a major part of an advanced algebra course
Instructional Examples/Lessons/Tasks	● The student will be able to explain the connection between the algebraic relationships of functions and one to one functions to the geometric relationships of mapping and transformations. ● The student will be able to use coordinate rules to move and/or alter a pre-image to determine its image or vice versa. ● Geometry: Congruence (Khan Academy) ● Rules for Translations (cK-12.org) ● Rigid Transformations Intro (Khan Academy) ● Transformations Cheat Sheet! ● Transformation Rules ● Des-Patterns (Desmos)
Assessment Examples	● Geometric Transformations (marlboroschools.org) ● Transformations Review Sheet (sklootmath.weebly.com)

Congruence

Cluster
Experiment with transformations in the plane.

NVACS HSG.CO.A.3 (Major Supporting Work)

Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Element	Exemplars
Standards for Mathematical Practice	- MP 3 Students will be able to construct arguments about how a shape will map onto itself. They will also be able to compare their mapping to a different mapping from another student. - MP 5 Students will use tools such as rulers and protractors to determine the transformation that are mapping the figures. - MP 8 Students will begin to see patterns of mapping that can be applied to additional problems.
Instructional Strategies	- Use tracing paper or software to move figures to check for mappings. - Students should be able to follow mapping directions and determine the mapping directions.
Prerequisite Skills	Demonstrate knowledge of lines of symmetry.
Connections Within and Beyond High School	- This standard can be used as a method for proving congruent figures. - Mapping concepts may be used in the development of the unit circle and for exploring reference angles that are greater than 360 degrees.
Instructional Examples/Lessons/Tasks	- Origami Regular Octagon (Illustrative Mathematics) - Rotations & Reflections of Quadrilaterals & Regular Polygons (Study.com) - Transformations and Symmetry (mathematicsvisionproject.org)
Assessment Examples	Mapping a Polygon onto Itself (jmap.org)

Congruence

Cluster

Experiment with transformations in the plane.

NVACS HSG.CO.A.4 (Major Supporting Work)

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Element	Exemplars
Standards for Mathematical Practice	- MP 1 Students will continue to explore transformations until patterns of behavior start to emerge. - MP 3 Students will test each others definitions to see if they are complete and valid. - MP 8 Students will use the patterns they have found from performing transformations to develop specific definitions of each.
Instructional Strategies	Have students in groups create characteristics of each definition on chart paper. The papers will be displayed, and the class will determine what the final definitions will be.
Prerequisite Skills	- Understand and be able to apply the definitions of angles, circles, perpendicular lines, parallel lines, and line segments. - Understand different types of angles and classifications of angles. - Understand how to label figures, angles, and lines using prime notation.
Connections Within and Beyond High School	- These precise terms will have a recurring role throughout the entire course. - Transformation terminology plays a major role with parent functions in higher level algebra courses.

Instructional Examples/Lessons/Tasks	- The student will know the definitions of the isometric transformations (reflect, rotate, & translate). - The student will be able to describe rotations, reflections and translations. - The student will be able to determine and apply the properties of the isometric transformations. - The student will be able to identify which transformation has taken place based on the properties found between the pre-image and image. - The student will be able to identify the orientation relationship between the pre-image and image. - Student questions - What does orientation mean? - If two triangles, ΔABC and ΔA'B'C' have the same orientation and the distances AA' = BB' = CC', what motion would map one onto the other? (**Possible Answer:** A translation.) - If two translations occurred on ΔABC, what would be true about the orientation and the distances AA'', BB'' and CC''? (**Possible Answer:** Orientation would be the same and AA'' = BB'' = CC'', which would mean a translation.) - Geometry Transformation Practice (GeoGebra)
Assessment Examples	Practice Problem **1.** If $R_{O,180°}(H) = T$, which of the below statement is false. A) $\overrightarrow{OH}$ and $\overrightarrow{OT}$ are opposite rays B) m∠TOH = 180° C) T is on $\overrightarrow{OH}$ D) ∠HTO is a straight angle **2.** If $T_{<x,y>}(\triangle ABC) = \triangle DEF$ then which statement is false? A) AD = BE B) $\overline{AD} \parallel \overline{CF}$ C) $\overline{BC} \parallel \overline{EF}$ D) AD = x **3.** ΔA'B'C' is the image of ΔABC under transformation G. Line m is the perpendicular bisector of $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. Which describes the transformation G? Answers: 1) D, 2) D, 3) B

Congruence

Cluster

Experiment with transformations in the plane.

NVACS HSG.CO.A.5 (Major Supporting Work)

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Element	Exemplars
Standards for Mathematical Practice	● MP 3 Students will be able to construct arguments about how a shape will map onto another. They will also be able to compare their mapping to a different mapping from another student. ● MP 5 Students will use tools such as rulers and protractors to determine the transformation that are mapping the figures. ● MP 8 Students will begin to see patterns of mapping that can be applied to additional problems.
Instructional Strategies	● Use tracing paper or software to move figures to check for mappings. ● Students should be able to follow mapping directions and determine the mapping directions.
Prerequisite Skills	● Understand properties of congruence. ● Recognize and plot points on a coordinate plane.
Connections Within and Beyond High School	● This standard can be used as a method for proving congruent figures. ● Mapping concepts may be used in the development of the unit circle and for exploring reference angles that are greater than 360 degrees.
Instructional Examples/Lessons/Tasks	● Transformation Golf: Rigid Motion (Desmos) ● Teaching Geometry Using Transformations (GeoGebra) ● Composition (Sequences of Transformations) (mathbitsnotebook.com) ● Rigid Transformations - Isometries (mathbitsnotebook.com)
Assessment Examples	• Transformations (Copper Trails School)

Congruence

Cluster

Understand congruence in terms of rigid motions.

NVACS HSG.CO.B.6 (Major Supporting Work)

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Element	Exemplars
Standards for Mathematical Practice	● MP 3 Students will discuss and predict outcomes of rigid motions.
Instructional Strategies	● Teacher may want to introduce the term "isometry." Before introducing this new vocabulary term, provide students with pre-image and image examples and have them state whether they are the "same" or not.
Prerequisite Skills	● Students need to understand which transformations are rigid (preserve congruence).
Connections Within and Beyond High School	● Students will use rigid motions to prove congruence of shapes. ● Students must know the difference between transformations that are isometries (rigid) and those that are not isometries. ● Students must differentiate between the various isometries, e.g., reflections, rotations, or translations. ● Students must know that isometries preserve angle measure, betweenness, collinearity, and distance. ● Students must know that a transformation is a function. ● Students must use patty paper, transparencies, or geometry software to demonstrate transformations in a plane. ● Students must show a one-to-one correspondence between the original figure's coordinates (called the pre-image or input values) and the transformed figure's coordinates (called the image or output values). ● Students must create the image of a figure given a transformation. ● In higher Algebra students will use these concepts to relate a graph to an equation.

Instructional Examples/Lessons/Tasks	● Student questions:
	  ◦ Is the statement $\triangle ABC \cong \triangle DEF$ the same as the statement $\triangle ABC \cong \triangle EFD$? (**Possible Answer:** No… the first statement implies that $\overline{DA} \cong \overline{DD}$ but the second statement implies that $\overline{DA} \cong \overline{DE}$.)
	  ◦ If you have two triangles in the plane, what would you attempt to do to prove that they are congruent? (**Possible Answer:** Map one onto the other using only isometric transformations.)
	● Identifying Isometries (CCSD)

Assessment Examples	● What is the difference between a rigid transformation and a non-rigid transformation? What transformations are isometries?

Congruence

Cluster

Understand congruence in terms of rigid motions.

NVACS HSG.CO.B.7 (Major Supporting Work)

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Element	Exemplars
Standards for Mathematical Practice	● MP 1 Students will struggle and be able to persevere when determining which angles correspond, and which sides correspond ● MP 5 Students will utilize construction tools and technology to demonstrate their knowledge of rigid motion triangle congruence.
Instructional Strategies	● Note the "if and only if" part of this standard shows that this needs to be demonstrated both that congruent figures have corresponding congruent parts AND that figures that have corresponding congruent parts are congruent. ● Use patty paper to show that figures that have corresponding parts are congruent. ● Have students find different rigid motions that will move one figure to the other.
Prerequisite Skills	● Understanding of an angle as the space bounded by two rays which share a vertex. ● Understanding of correspondence as it relates to Geometry.
Connections Within and Beyond High School	● Using rigid motions is a method for checking other ways of proven congruence. ● Understanding corresponding parts is necessary for traditional proofs of congruence as well as problems involving similar figures. ● The correct connection of various parts of figures is needed for solving trigonometric problems.
Instructional Examples/Lessons/Tasks	● Investigating Congruence (Richland Parish Schools)
Assessment Examples	● **Activity:** Provide students with sets of triangles and have them determine the sequence of rigid motions that will land one triangle on top of the other. ● Properties of Congruent Triangles (Illustrative Mathematics)

Congruence

Cluster

Understand congruence in terms of rigid motions.

NVACS HSG.CO.B.8 (Major Supporting Work)

Explain how the criteria for triangle congruence (ASA, SAS, and SSS, AAS) follow from the definition of congruence in terms of rigid motions.

Element	Exemplars
Standards for Mathematical Practice	● MP 4 Students will model the concepts of rigid motion that allow us to accept SSS, SAS, ASA, Congruence postulates and AAS congruence theorem ● MP 8 Students will use the ideas from their rigid motion to accept the triangle congruence postulates.
Instructional Strategies	● This standard helps students to understand why we accept these postulates are true, but we are not proving them. ● A common instructional practice in curriculum is to try to "prove" these while introducing the proof process. However, these are better served by having students experiment with trying to perform a rigid motion without creating a different (non-congruent) triangle.
Prerequisite Skills	● Students must know and understand rigid motion transformations.
Connections Within and Beyond High School	● SSS, SAS, AAS, and ASA are the traditional postulates that will be used in developing two-column proofs ● This process is designed to teach students to not just blindly accept things (like postulates) but to also test out their validity.
Instructional Examples/Lessons/Tasks	● Triangle Congruence (Khan Academy) ● Congruent Triangles - A Solidify Understanding Task (mathematicsvisionproject.org) ● Congruent Triangles (Math Open Reference) ● Choose the 'Testing for congruence" for each triangle.
Assessment Examples	● Proving Triangles Congruent (Quizizz) ● (free to register) ● Sketch, then answer. (a) to prove ΔLMN ≅ ΔPQR by SSS you need _____ and _____ and _____. (b) To prove ΔRAT ≅ ΔMED by ASA, given $\overline{RT} \cong \overline{MD}$, you also need _____ and _____.

Congruence

Cluster

Prove geometric theorems.

NVACS HSG.CO.C.9 (Major Supporting Work)

Prove theorems about lines and angles. *Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

Element	Exemplars
Standards for Mathematical Practice	● MP 3 Students work with peers to explain their thinking and listen to the thinking of others. ● MP 5 By making dynamic geometry software available gives students the opportunity to choose tools strategically.
Instructional Strategies	● Have students create conjectures about angle relationships and prove them using what they know about rigid transformations.
Prerequisite Skills	● Be able to make conjectures ● Know definitions of the rigid transformations
Connections Within and Beyond High School	● Prove theorems about triangles ● Prove theorems about parallelograms

Instructional Examples/Lessons/Tasks	● Student questions to further discussion and refine students' arguments
	    • Who can restate _____'s reasoning in a different way?
	    • Did anyone have the same strategy, but would explain it differently?
	    • Do you agree or disagree? Why?
	    • Would these reasons convince someone who didn't think it was true?
	● Prove Geometric Theorems (RPDP)
	● Parallel Lines and Angle Relationships (RPDP)
	● Evidence, Angles, and Proof (Illustrative Math)
	● Transformations, Transversals, and Proof (Illustrative Math)
	● What Went Wrong? (Illustrative Math)

Assessment Examples	● Angle Relationships Test with Transformations (Better Lesson)

Congruence

Cluster

Prove geometric theorems.

NVACS HSG.CO.C.10 (Major Supporting Work)

Prove theorems about triangles. *Theorems include measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

Element	Exemplars
Standards for Mathematical Practice	- MP 3 Students create proofs and compare with classmates. - MP 5 Use tools such as paper folding, patty paper, constructions, etc... to prove geometric theorems. - MP 6 Students use precise mathematical language to express their reasoning.
Instructional Strategies	- Provide mathematical structures and ask students to make conjectures about situations involving triangles. Ask students to critique one another's proofs. - To illustrate the interior angles of a triangle sum - Have students create and cut out a paper triangle - Label angles a, b, c; - Tear triangle into three pieces so the angles can be rearranged to form a line. - Use paper folding, patty paper, or constructions to prove base angles, midsegment, and medians.
Prerequisite Skills	- Relationship between triangle sides and angles (biggest side across from biggest angle) - Knowledge of transformations
Connections Within and Beyond High School	- Prove theorems about lines and angles. - Prove theorems about quadrilaterals. - Define trigonometry ratios
Instructional Examples/Lessons/Tasks	- Triangle Angle Theorems (GeoGebra) - Isosceles Triangle Theorem: Discovery Lab (GeoGebra) - Triangle Midsegment Action! (GeoGebra) - Triangle Medians: Quick Investigation (GeoGebra)
Assessment Examples	Sample Item #3342 (smarterbalanced.org)

Congruence

Cluster

Prove geometric theorems.

NVACS HSG.CO.C.11 (Major Supporting Work)

Prove theorems about parallelograms. *Theorems include opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Element	Exemplars
Standards for Mathematical Practice	● MP 6 Students will express proofs using precise mathematical language, including creating their own diagram with given information and auxiliary lines.
Instructional Strategies	● Encourage different ways of writing proofs including narrative and two-column proofs. ● Encourage students to refine the language used to justify their reasoning.
Prerequisite Skills	● To know and be able to identify different quadrilaterals ● To know basic properties of quadrilaterals ● To recognize and apply Triangle Congruence Theorems
Connections Within and Beyond High School	● Prove theorems about lines and angles. ● Prove theorems about triangles.
Instructional Examples/Lessons/Tasks	● Practicing Proofs (Illustrative Mathematics) ● Proofs about Quadrilaterals (Illustrative Mathematics) ● Proofs about Parallelograms (Illustrative Mathematics) ● Bisect It (Illustrative Mathematics) ● Prove Geometric Theorems (RPDP)
Assessment Examples	● Proof Practice Problems for Quadrilaterals (mathbitsnotebook.com) ● Quadrilaterals Proofs (White Plains Public Schools) • Assessment questions on page 4.

Congruence

Cluster

Make geometric constructions.

NVACS HSG.CO.D.12 (Major Supporting Work)

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Element	Exemplars
Standards for Mathematical Practice	● MP 5 Students will use a compass, straightedge and dynamic software strategically to create precise constructions of geometric figures. ● MP 6 Students attend to precision by constructing congruent geometric figures using appropriate tools.
Instructional Strategies	● Create a semi-permanent display showing the straightedge and compass moves used in constructions. ● Model the appropriate use of construction tools.
Prerequisite Skills	● Students use a straightedge to construct straight lines. ● Students know the definition of congruence.
Connections Within and Beyond High School	● Students will construct an equilateral triangle, square and hexagon inscribed in a circle.
Instructional Examples/Lessons/Tasks	● Build It (Illustrative Mathematics) ● Construction Techniques 1: Perpendicular Bisectors (Illustrative Mathematics) ● Constructions (Math Open Reference)
Assessment Examples	● Geometric Constructions: End of Unit Assessment (Better Lesson) • Assessment continues with G.CO.C.13 ● Basic Constructions Test (soowook.com)

Congruence

Cluster

Make geometric constructions.

NVACS HSG.CO.D.13 (Major Supporting Work)

Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Element	Exemplars
Standards for Mathematical Practice	- MP 5 Students will use a compass, straightedge and dynamic software strategically to create precise constructions of geometric figures. - MP 6 Students attend to precision by constructing congruent geometric figures using appropriate tools.
Instructional Strategies	- Create a semi-permanent display showing the straightedge and compass moves used in constructions. - Model the construction of a hexagon inscribed in a circle and encourage students to discover all the equilaterals within the construction.
Prerequisite Skills	- Students can use construction tools. - Students can construct parallel and perpendicular lines.
Connections Within and Beyond High School	- Copy a segment, angle, bisect a segment, bisect and angle. - Construct perpendicular lines and perpendicular bisectors. - Construct a line parallel to a given line through a point not on the line.
Instructional Examples/Lessons/Tasks	- Construction Techniques 2: Equilateral Triangles (Illustrative Mathematics) - Construction Techniques 5: Squares (Illustrative Mathematics) - Constructions (Math Open Reference)
Assessment Examples	- Geometric Constructions: End of Unit Assessment (Better Lesson) - Assessment continues with G.CO.C.12

Similarity, Right Triangles and Trigonometry

Cluster

Understand similarity in terms of similarity transformations.

NVACS HSG.SRT.A.1 (Major Supporting Work)

Verify experimentally the properties of dilations given by a center and a scale factor.

Element	Exemplars
Standards for Mathematical Practice	● MP 5 Students use the geometric tools of a compass and straight edge when doing dilations. ● MP 6 Repeating the dilations has students attend to the precision and detail of the diagram. ● MP 7 Structure is found in applying the properties of a dilation (congruent angles and proportional sides) as a tool to re-size an object.
Instructional Strategies	● Have students perform dilations on graph paper and then use the slope formula and the distance formula to verify dilation properties (pre-image and image are parallel, scale factor).
Prerequisite Skills	● Students know how to use a compass and straight edge. ● Students know the isometric transformations and that dilations are a non-isometric transformation. ● How to calculate slope using a formula and from a graph ● How to use the distance formula
Connections Within and Beyond High School	● Dilations lead directly to similarity.
Instructional Examples/Lessons/Tasks/ Questions	● The student will be able to dilate when given a center of dilation and a scale factor. ● The student will be able to construct a dilation. ● Student questions • What is it about a dilation that scales the shape? • What is it about a dilation that keeps the shape proportional? • Why does this process work? ● Dilations and Scale Factors (mathworksheetsland.com)
Assessment Examples	● Dilating Monsters (RPDP) ● Dilations (mathworksheetsland.com)

Similarity, Right Triangles and Trigonometry

Cluster

Understand similarity in terms of similarity transformations.

NVACS HSG.SRT.A.1.A (Major Supporting Work)

Verify experimentally the properties of dilations given by a center and a scale factor:
- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

Element	Exemplars
Standards for Mathematical Practice	● MP 7 Working from the concrete of constructions to the abstract of scale factors and notation helps students find structure.
Instructional Strategies	● Introduce this concept using constructions first. The physical constructing helps the student to see the relationship. From there they learn when the center of dilation is not on the line how the scale factor moves the line either closer or farther away. They also see that no new line is formed when the center of dilation is on the line.
Prerequisite Skills	● In G.CO.A.2 we learn what is and isn't an isometry. At that time, we discussed that a dilation WAS NOT isometric because it changed the size of the shape. We knew that at some stage we would look at the properties of dilation up close. ● Parallel lines have equal slopes, opposite rays, rays, angle relationships found with parallel lines & a transversal
Connections Within and Beyond High School	● Scale factor is investigated here. We want to connect the previous number sense ideas of size to a scale factor. That values greater than 1 increase a number and so likewise they will increase a polygon. ● Students will use dilations in lots of places, similarity, parallel line relationships, geometric mean, special right triangles, trigonometry, and partitioning a line segment to name a few. These early concepts and properties found in dilating lay the basis for lots of ratio related mathematics later in the course.

Instructional Examples/Lessons/Tasks	● The student will determine the properties of dilation.
	● The student will be able dilate when given a center of dilation and a scale factor.
	● The student will be able to determine the center of dilation and the scale factor from a diagram.
	● The student will be able to dilate using both positive and negative scale factors.
	● Student questions
	  ● How do you know if it has been a reduction?
	  ● Is there more than one way to know that a reduction has occurred? (**Possible answers:** scale number, notation of image, image is closer to center)
	  ● Is a scale factor of $\frac{4}{3}$ an enlargement or a reduction? (**Possible Answer:** Students think all fractions reduce.)

Assessment Examples	● 8.3 Dilations (Quizziz) ● Practice with Dilations (mathbitsnotebook.com)

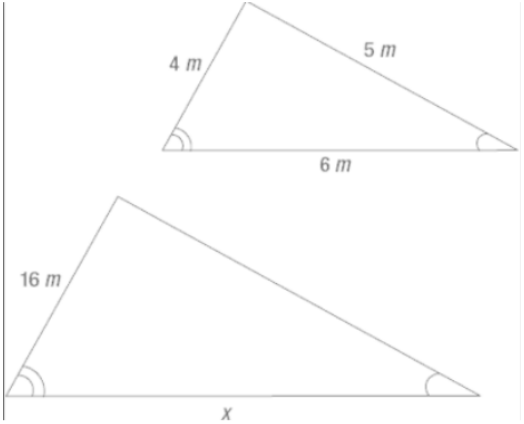# Similarity, Right Triangles and Trigonometry

Cluster

Understand similarity in terms of similarity transformations.

NVACS HSG.SRT.A.1B (Major Supporting Work)

Verify experimentally the properties of dilations given by a center and a scale factor:
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Element	Exemplars
Standards for Mathematical Practice	● MP 2 Accurately interpret the scale factor of given dilations and recognize the corresponding sides' ratio. ● MP 3 Construct viable arguments, make conjectures, and build a logical progression of statements regarding dilations and the ratio given by the scale factor.
Instructional Strategies	● Be sure that students understand the pre-image and image points will lie along the same line. Students should be encouraged to draw these lines in order to verify this property. ● The lines connecting pre-image and image points are concurrent with their point of concurrency at the center of the dilation. Have students use the terms enlargement and reduction with each dilation so they understand the terminology. ● Students can use the slope to determine where the image point is located.
Prerequisite Skills	● Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. ● Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. ● Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. ● Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. ● Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. ● Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Connections Within and Beyond High School	- Students must know the difference between transformations that are isometries (rigid) and those that are not isometries. - Students must differentiate between the various isometries, e.g., reflections, rotations, or translations. - Students must know that isometries preserve angle measure, betweenness, collinearity, and distance. - Students must know that a transformation is a function. - Students must use patty paper, transparencies, or geometry software to demonstrate transformations in a plane. - Students must show a one-to-one correspondence between the original figure's coordinates (called the pre-image or input values) and the transformed figure's coordinates (called the image or output values). - Students must create the image of a figure given a transformation.
Instructional Examples/Lessons/Tasks	- Dilation Practice with Different Scale Factors(CCSD) - Dilation Centers(CCSD)
Assessment Examples	- What is the difference between a rigid transformation and a non-rigid transformation? What transformations are isometries? - If the scale factor is 4, what is the length of x?

Similarity, Right Triangles and Trigonometry

Cluster

Understand similarity in terms of similarity transformations.

NVACS HSG.SRT.A.2 (Major Supporting Work)

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Element	Exemplars
Standards for Mathematical Practice	- MP 1 Students need to determine what information is relevant and then create the correct proportion. - MP 4 Similarity is used in many real-world examples such as scale drawings. - MP 7 Similarity statements rely on structure --writing statements so that the corresponding parts correlate. The dilation properties provide the basis for the congruent angles, the proportional sides and the parallel lines.
Instructional Strategies	- Given a pre-image and an image, students give a sequence of transformations that will carry one onto the other. Have students give two different answers. - Geogebra, a free dynamic geometry software, has many pre-made related activities for students to investigate. - Have students use similarity statements and different colors to help identify corresponding sides.
Prerequisite Skills	- Students can set up and solve a proportion. - Students are familiar with the properties of isometric transformations (rotation, reflection, and translation) and the non-isometric transformation (dilation).
Connections Within and Beyond High School	We need to learn the definition of what makes two shapes similar so that we can establish relationships within the shapes. Knowing that two shapes are similar will open the door to determine lengths using ratios and knowing angles due to their corresponding congruence.

Instructional Examples/Lessons/Tasks	- The student will be able to identify corresponding angles and sides based on similarity statements. - The student will be able to develop and write similarity statements for two polygons. - The student will be able to determine if two triangles are similar based on their corresponding parts. - The student will be able to establish a sequence of similarity transformations between two similar polygons. - Student questioning: - Is there more than one way to map a polygon onto another? Give an example. - What is the difference between isometric transformations and similarity transformations? - Given that ΔABC is similar to ΔFRG, what things can you conclude? (**Possible answer:** congruent angles & proportional sides) - Dilations and Similarity Notes (mathbitsnotebook.com)
Assessment Examples	Dilations and Similarity (Baltimore Polytechnic Institute)

Similarity, Right Triangles and Trigonometry

Cluster

Understand similarity in terms of similarity transformations.

NVACS HSG.SRT.A.3 (Major Supporting Work)

Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Element	Exemplars
Standards for Mathematical Practice	- MP 2 Proofs require a high level of abstraction and reasoning. Students on this worksheet will apply isometric transformation properties and dilation properties to establish the triangles are similar. - MP 3 They will set up logical arguments found in the similarity transformations to show that these relationships can happen.
Instructional Strategies	- This is very abstract. Scale factors don't have numbers, they are represented by sides and that causes students a difficult time. - To help students you will want to walk them through the proof logic before doing any formal writing.
Prerequisite Skills	Establishing the similarity relationships requires a background in proving congruence as well as the good understanding of scale factors.
Connections Within and Beyond High School	- Similarity and the use of similarity properties to solve problems must come first and then comes establishing or proving similarity. - Establishing figures to be similar is a very common theme throughout the rest of the year. Similarity of triangles shows up very often because the only required knowledge is two corresponding congruent angles, and this happens in many different cases.
Instructional Examples/Lessons/Tasks	- The student will be able to prove two triangles to be similar using the minimum requirements of AA, SAS and SSS. - The student will be able to use the properties of similarity transformations to establish the AA, SAS and SSS criterion for two triangles to be similar. - Student questions - What is the goal of this proof? - How do we form a scale factor between two sides that have no measurements? - How can we use congruence to help us establish similarity? - Similarity Postulates and Theorems Notes (Coshocton City Schools)
Assessment Examples	Similar Triangles Proofs

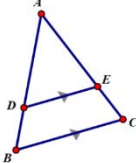# Similarity, Right Triangles and Trigonometry

Cluster

Prove theorems involving similarity.

NVACS HSG.SRT.B.4 (Major Supporting Work)

Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

Element	Exemplars
Standards for Mathematical Practice	● MP 2 Proof is a powerful way to provide students a chance to reason in abstract, to take the postulates, definitions, proven theorems and properties to establish new truths. ● MP 3 Students will establish viable arguments based on the givens and their statements.
Instructional Strategies	● Use the similarity criteria to establish new and important relationships found in similar triangles such as the 'side splitting theorem' and the angle bisector theorem. ● Prove the Pythagorean Theorem using similar triangles and what eventually comes to be known as the geometric mean relationships.
Prerequisite Skills	● The connection to things before this is similarity. Any connections to ratios, scale factors, proportion, etc... all connect to the concept of similarity and the non-isometric transformation, dilation. The side-splitting theorem is the natural progression after being able to prove similarity. Its proof is built on the ability to prove two triangles are similar by AA.
Connections Within and Beyond High School	● Parallel lines, proportional relationships, and congruent angles unlock many problems in the future. These new proven theorems will allow students to go at larger problems and have a logical structure to explain why it works.

Instructional Examples/Lessons/Tasks	- The student will prove (the side-splitting theorem) that a line parallel to one side of a triangle divides the other two proportionally. - The student will prove (the angle bisector theorem) that an angle bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle. - The student will prove the Pythagorean Theorem using similarity and the geometric means. - Student questions - If we are to prove that two sets of sides are proportional, what would be a logical proof structure to accomplish that? (**Possible Answer:** Try to prove similarity of triangles first.) - Why is the proportion found on the sides where they have been split not the same proportion for the parallel sides that were not split? - In some cases, if the triangles are right triangles, what other relationships do you know that might help us to find lengths? (**Possible Answer:** The Pythagorean Theorem) - Triangle Angles (Khan Academy) - Theorems About Triangles Notes(Carnegie Mellon University) - Similarity in Right Triangles (Geometric Mean) Notes (Newark Catholic High School)
Assessment Examples	- Midsegments of Triangles (Holland Patent Central School District) - Geometric Mean in Triangles (Kyrene School District)

Similarity, Right Triangles and Trigonometry

Cluster

Prove theorems involving similarity.

NVACS HSG.SRT.B.5 (Major Supporting Work)

Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Element	Exemplars
Standards for Mathematical Practice	- MP 7 Understanding geometric mean is directly related to structure. The overlapping nature of these three embedded triangles reveals some wonderful relationships all based on similarity. - MP 8 Discovering the patterns that exist in the special right triangles allow students to understand the math behind the rules.
Instructional Strategies	- For geometric mean problems, have students pull apart the three triangles and use similarity to understand the theorems. - Students can discover the properties of 45-45-90 triangles by drawing a diagonal in a square and using the Pythagorean Theorem. - Students can discover the properties of 30-60-90 triangles by drawing an altitude of an equilateral triangle and using the Pythagorean Theorem.
Prerequisite Skills	- Understand how to set up and solve problems using similar triangles. - Use of the Pythagorean theorem. - Simplify expressions involving radicals.
Connections Within and Beyond High School	These relationships all connect to the concepts found in trigonometry. Trigonometry is the relationship between angles and sides in right triangles. The exact values of the special right triangles are early representations of the trigonometric relationships.

Instructional Examples/Lessons/Tasks	- The student will be able to derive the three geometric mean relationships. - The student will be able to use the geometric mean to solve for sides of triangles. - The student will be to construct the special right triangles. - The student will use exact values in solving for sides and angles in the special right triangles. - Student questions - Why do the geometric means exist in this type of triangle? (**Possible answer:** Similarity is found because of AA.) - What other relationship is found in a right triangle that could help us in many of these questions? (**Possible answer:** The Pythagorean Theorem) - The hypotenuse of a 45-45 right triangle is 14. Is that possible? - How do we determine who the short and long legs are in a 30-60 right triangle? (**Possible answer:** The angles tells us about side length. The smallest angle is opposite the shortest side (the short leg). - Similarity in Right Triangles (Geometric Mean) Notes (Newark Catholic High School)
Assessment Examples	- Geometric Mean in Triangles (Kyrene School District) - Geometric Mean and Pythagorean Theorem (Kuta Software) - Maths Special Right Triangles Quiz (ProProfs) - Special Right Triangles (Quizizz) - Special Right Triangles (Rocklin Unified School District)

Similarity, Right Triangles and Trigonometry

Cluster

Define trigonometric ratios and solve problems involving right triangles.

NVACS HSG.SRT.C.6 (Major Supporting Work)

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Element	Exemplars
Standards for Mathematical Practice	- MP 2 Reason quantitatively about the relationships of the side and angle measures in right triangles. - MP6 Attend to precision with exact answers. - MP 8 Look for patterns in explorations, leading to generalizing about the relationships of the sides.
Instructional Strategies	- Have students solve ratios of triangles of varying sizes for comparison. - Build from the concept of similar triangles and that all 30-60-90 triangles have consistent ratios of its sides.
Prerequisite Skills	- Trigonometry is connected heavily to similarity. The entire essence of trigonometry is the fact that the ratios of sine, cosine and tangent are constant for each reference angle and the reason they are constant is because those triangles are all similar by AA. - Know that similar triangles have corresponding sides that are proportional and corresponding angles that are congruent. - Understand that similarity in right triangles, leads to proportional relationships - Ability to solve proportions
Connections Within and Beyond High School	These right triangle relationships allow us to build a formula for area and solve for values of oblique (non-right) triangle.

Instructional Examples/Lessons/Tasks	- The student will be able to label a triangle in relation to the reference angle (opposite, adjacent & hypotenuse). - The student will be able to determine the most appropriate trigonometric ratio (sine, cosine, and tangent) to use for a given problem based on the information provided. - The student will be able to solve for sides and angles of right triangles using trigonometry. - Student questions - How do you know when solving for a side whether the unknown value is in the numerator or denominator? (**Possible answer:** It depends on which side you are solving for and its position in the ratio.) - When would you use special right triangle relationships and when would you use trigonometry? (**Possible answer:** Trigonometry can be used in all cases, but the special right relationships can only be used for 30-60-90 and 45-45-90 triangles.) - How do the ratios of the side lengths of right triangles relate to the angles in the triangle? [Trigonomteric Ratio Discovery](#) (CCSD) [Similarity, Right Triangles, & Trigonometry](#) (CCSD) [Right Triangle Trigonometry: Solving Word Problems](#) (Central Bucks School District)
Assessment Examples	[Right Triangles Test Review](#) (Carlisle County School)

Similarity, Right Triangles and Trigonometry

Cluster

Define trigonometric ratios and solve problems involving right triangles.

NVACS HSG.SRT.C.7 (Major Supporting Work)

Explain and use the relationship between the sine and cosine of complementary angles.

Element	Exemplars
Standards for Mathematical Practice	● MP 7 Student understanding of why this works demonstrates their ability to recognize reference angles and the basic trigonometric ratios.
Instructional Strategies	● Use a trigonometry table to allow students to see matching values in the sine and cosine columns. To help students connect the pattern to the complement relationship use easy angle values like 10 degrees (& 80) or 30 degrees (& 60).
Prerequisite Skills	● The basic trigonometric ratios of sine and cosine.
Connections Within and Beyond High School	● Understanding the concepts underneath the basic trigonometric ratios will help student succeed as we build on trigonometry in future courses.

Instructional Examples/Lessons/Tasks	● The student will be able explain the co-function nature of sine and cosine.
	● The student will be able to calculate values that would make sine and cosine equal.
	● Sine and Cosine Complementary Angles (Khan Academy)
	● Sine and Cosine Complementary Angles (mathbitsnotebook.com)
	● Student questions
	    • Why is $\sin 45° = \cos 45°$? (**Possible answer:** It is a right isosceles triangle.)
	    • Explain why the $\sin 23° = \cos 67°$? (**Possible answer:** In a right triangle the other two angles are complementary. Those two complementary angles can act as reference angles. From one angle the sine ratio will use the same two sides as the cosine from the other reference angle. Thus referring to the same two sides creating equivalent values.)

Assessment Examples	● Sine and Cosine of Complementary Angles PRACTICE (mathbitsnotebook.com) ● Sine and Cosine of Complementary Angles (study.com)

Similarity, Right Triangles and Trigonometry

Cluster

Define trigonometric ratios and solve problems involving right triangles.

NVACS HSG.SRT.C.8 (Major Supporting Work)

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.[*]

Element	Exemplars
Standards for Mathematical Practice	● MP 1 Students utilize their right triangle knowledge to make sense of the problem provided and to plan a solution pathway. ● MP 4 So much of trigonometry is the context of the problem. Trigonometry is used in so many contexts and thus provides a natural modeling context – turning context into mathematical diagrams and equations.
Instructional Strategies	● Progress from basic calculations to more complex word problems. They will gain a greater understanding of the term used to describe locations in the diagram.
Prerequisite Skills	● The past connection is to G.SRT.6 where we discovered that trigonometry was tightly connected to similarity and that it allows us to link angles sizes to side proportions. In this objective we use this knowledge to solve real world problems.
Connections Within and Beyond High School	● This standard elevates the trigonometric relationships to more abstract modeling situations. Students are asked to take the words and break them into mathematics diagrams and equations. This worksheet focuses mostly on the terminology often used in these situations. ● Trigonometry becomes a powerful tool to solve many problems past this point of mathematics.

Instructional Examples/Lessons/Tasks	- The student will be able to interpret the word descriptions into lengths and angles of a right triangle so that they can diagram the relationship. - The student will be able to solve trigonometry and Pythagorean Theorem problems based on written descriptions. - Student questions - Why do you think students put the angle of depression in the wrong location? - Why are the values for angle of elevation and angle of depression equivalent? (**Possible answer:** Alternate interior angles are congruent (**Possible answer:** parallel lines and parallel horizons) - What assumptions do we make when creating a trigonometric problem from these situations? - Why do we make those assumptions? (**Possible answer:** To simplify the problem.) - Trigonometry - Word Problems Notes (mathbitsnotebook.com)
Assessment Examples	- Sine, Cosine, Tangent Applications (Math Warehouse) - Real World Sohcahtoa Worksheet and Answer Key (Math Warehouse) - Sine and Cosine Relationship (Geometry Common Core)

Similarity, Right Triangles and Trigonometry

Cluster

Apply trigonometry to general triangles.

NVACS HSG.SRT.D.9 (Major Supporting Work)

(+) Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

Element	Exemplars
Standards for Mathematical Practice	● MP 1 Students determine the area of triangles given a variety of information. ● MP 2 Students will derive the area formula using sine.
Instructional Strategies	● Have students troubleshoot the issue of determining the height of an oblique triangle. ● Solve for the area of triangles with varied given information.
Prerequisite Skills	● Know how to solve for the area of a triangle. ● Know how to solve triangles using right triangle trigonometry.
Connections Within and Beyond High School	● Use the calculation of the height of an oblique triangle to derive the Law of Sines.
Instructional Examples/Lessons/Tasks	● Derive and Use the Area Formula in Terms of Sine (RPDP)
Assessment Examples	● Derive and Use the Area Formula in Terms of Sine (RPDP)

Similarity, Right Triangles and Trigonometry

Cluster

Apply trigonometry to general triangles.

NVACS HSG.SRT.D.10 (Major Supporting Work)

(+) Prove the Laws of Sines and Cosines and use them to solve problems.

Element	Exemplars
Standards for Mathematical Practice	● MP 3 Students use previous knowledge of triangles and trigonometry to prove the Law of Sines and Cosines
Instructional Strategies	● Use right triangle trigonometry and the Pythagorean theorem to prove the Law of Cosines. ● Use the altitudes of oblique triangles and right triangle trigonometry to prove the Law of Sines. ● Begin by showing students how to determine when to use either the Law of Sines or Cosines. ● Students initially solve for one missing side or angle of an oblique triangle and build to solving the entire triangle.
Prerequisite Skills	● Know how to find trigonometry ratios in right triangles. ● Know how to find the sine/cosine/tangent of given angle measures. ● Know how to solve equations for given variables. ● Know how to solve using the Pythagorean Theorem.
Connections Within and Beyond High School	● Students will continue to use basic trigonometry ratios in geometric proofs. ● Students will find missing parts of triangles in parts of circles.
Instructional Examples/Lessons/Tasks	● Prove and Use the Laws of Sines and Cosines (RPDP) ● Tactile Trigonometry (NCTM) ● The Laws of Sines and Cosines Made Simple! (Utah Education Network) ● Law of Cosines (NCTM)
Assessment Examples	● Prove and Use the Laws of Sines and Cosines (RPDP)

Similarity, Right Triangles and Trigonometry

Cluster

Apply trigonometry to general triangles.

NVACS HSG.SRT.D.11 (Major Supporting Work)

(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Element	Exemplars
Standards for Mathematical Practice	● MP 4 Students recognize and solve oblique triangles in the real world using the Law of Sines and Cosines.
Instructional Strategies	● Give students word problems that must be solved using either the Law of Sines or Law of Cosines ● Have each student create their own real-world word problem that requires the Law of Sines or Law of Cosines to solve, then have students solve each other's problems.
Prerequisite Skills	● Know how to find the sine/cosine/tangent of given angle measures. ● Know how to solve equations for given variables. ● Know how to solve oblique triangles using the Law of Sines and Law of Cosines.
Connections Within and Beyond High School	● Students will continue to use basic trigonometry ratios in geometric proofs. ● Students will find missing parts of triangles in parts of circles.
Instructional Examples/Lessons/Tasks	● Understand and Apply the Laws of Sines and Cosines (RPDP) ● Tactile Trigonometry (NCTM)
Assessment Examples	● Understand and Apply the Laws of Sines and Cosines (RPDP)

Circles

Cluster

Understand and apply theorems about circles.

NVACS HSG.C.A.1 (Major Supporting Work)

Prove that all circles are similar.

Element	Exemplars
Standards for Mathematical Practice	● MP5 Students select and use appropriate tools (pencil paper, geometry software, etc…) to examine the similarity in circles.
Instructional Strategies	● Identifying similarities and differences of circles ● Generating and testing hypotheses regarding circle similarity.
Prerequisite Skills	● Know the definition of and how to apply similar figures. ● Know the properties of dilations and how to dilate figures. ● Know the definition of transformations in order to transform figures. ● Know the definition of isometric transformations and rigid motions and how to transform figures. ● Know the properties of isometric transformations and rigid motions so that they can apply them to transform figures. ● Apply similarity and dilation knowledge to prove all circles are similar through isometric transformations.
Connections Within and Beyond High School	● Understand that the radius of a circle is perpendicular to the tangent where the radius intersects the circle. ● Know that angles inscribed in circles are half of the measure of the intercepted arc and that central angles are the same measure of the intercepted arc. ● Quadrilaterals inscribed in a circle must have opposite angles that are supplementary. ● Understand the proportional relationship between the arc length and the radius of a circle.
Instructional Examples/Lessons/Tasks	● Discuss what transformations might be used to map one circle to another. ● Ask students to draw several examples of two or three circles that are concentric and two or three circles that are not to reinforce their understanding of the concept. ● Prove Circles Similar (mathbitsnotebook.com) ● Similar Circles (Illustrative Mathematics) ● Proving Circles Similar Activity (CCSD) ● Hot Chocolate 3 Act Math Task (tapintoteenminds.com) ● This is an activity for lower grades but could be adapted for this unit by having different cylinders and comparing radii, solving for circumference and finding the scale factor.

Assessment Examples	- Students are given two circles with different radius lengths and are asked to prove that the circles are similar. - What does it mean for two figures to be similar? Why are all circles similar? (**Possible answer:** Two figures are similar when one is a dilation of the other. Dilations preserve angles and proportionality of sides. All circles are similar because they are dilations of each other.)

Circles

Cluster

Understand and apply theorems about circles.

NVACS HSG.C.A.2 (Major Supporting Work)

Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

Element	Exemplars
Standards for Mathematical Practice	- MP1 Students analyze whether their answers make sense using their knowledge of the relationship between central and inscribed angles - MP3 Students recognize relationships between angles and segments in circles and discuss their reasoning with others. - MP4 Students use mathematical models to represent the relationships between angles within circles. - MP5 Students identify appropriate tools to solve circle angle relationship problems.
Instructional Strategies	- Tactile Instruction - Go outside or use the edges of a board to form inscribed angles that help explain a camera angle. - As students step away from the building, have different students move until they can view the edges of the building exactly, no more and no less. - Students can look through an actual camera or a paper tube. - Point out to the students that they form the vertices of the inscribed angles. - Use Geoboards or an interactive computer program, such as Geometer's Sketchpad. The experience of creating the figures with these items will help students understand the relationships between various angles and the other parts of the circle. - Use a mnemonic to help students remember the that the measure of a minor arc is less than the measure of a major arc. For example, many students may be familiar with the use of the word *minor* to mean a young person who is less than the age of full legal responsibility, while someone greater than or equal to the legal age has reached the age of majority. Another hint could be the distinction between a college *minor*, which requires fewer courses, and a college *major*, which requires more.

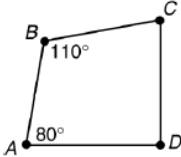|	
Prerequisite Skills	● Understand the relationships among inscribed angles, radii, and chords within circles and utilize this knowledge to perform mathematical tasks. ● Understand the relationship between central, inscribed, and circumscribed angles within circles. ● Know that inscribed angles in a semicircle are right angles. ● Understand that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
Connections Within and Beyond High School	● Construct the inscribed and circumscribed circles of a triangle and know and use the properties of angles for a quadrilateral inscribed in a circle. ● Construct a tangent line from a point outside a given circle to the circle. ● Understand the proportional relationship between the arc length and the radius of a circle.
Instructional Examples/Lessons/Tasks	● Lines, Angles, and Curves (Illustrative Mathematics) ● Inscribed Angles (Illustrative Mathematics) ● Circle Folding Activity (CCSD)
Assessment Examples	● What are the relationships between parts of a circle? Can those relationships be used to find unknown parts of a circle? ● Name the characteristics involved with each of the following parts of a circle: inscribed angles, radii, diameters, and central angles (**Possible answers:** diameter = twice the radius, goes through the center, and endpoints are on the outside edge of the circle). ● Circles Test (RPDP)

Circles

Cluster

Understand and apply theorems about circles.

NVACS HSG.C.A.3 (Major Supporting Work)

Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Element	Exemplars
Standards for Mathematical Practice	- MP 1 Explore a wide range of relationships of the centers of the triangles to the triangles and to various circles related to triangles. - MP 5 Use dynamic geometry software to explore hwo9 the centers of the triangles are related. - MP 6 Be precise in constructions in order to identify possible relationships. - MP 8 Look for patterns across a range of examples to better understand the behavior of the centers of a triangle.
Instructional Strategies	- Have students construct a circle with many inscribed angles, attempting to find the largest possible inscribed angle. After giving them time to make some discoveries, lead them into a discussion about the largest angle and its intercepted arc. - Students should inscribe a quadrilateral inside of a circle (be sure that students are creating different types of quadrilaterals). Have the students measure all four angles and then compare them with the answers of other team members. Students should discover that the opposite angles are supplementary. Lead them in a discussion about why this must be true, creating first an informal and then formal proof. - The constructions of these 3 polygons is an excellent time to review and emphasize their special characteristics and how they relate to the angles and segments of the circumscribed circle. - Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. - Some of these constructions are closely related to other standards and can be introduced in conjunction with them.
Prerequisite Skills	- Know what inscribed angles are and how to find their measures. - Know theorems of quadrilaterals - Recall an inscribed angle with endpoints on the diameter is a right angle. - Properties of equilateral triangles

Connections Within and Beyond High School	- Students must construct a square given a side. - Students must construct an equilateral triangle given a side. - Students must construct an inscribed hexagon given the radius or edge length. - Students must construct a square and then circumscribe a circle about the square. - Students must construct an inscribed equilateral triangle given the radius of a circle. - Students must construct an inscribed square given the radius of a circle.
Instructional Examples/Lessons/Tasks	- Equidistant Arena (whenmathhappens.com) - Quadrilaterals in Circles (Illustrative Mathematics) - Circle Folding Activity (CCSD) - Inscribed Quadrilateral Proof (CCSD) - Constructions (Math Open Reference) - Congruence BLAST(CCSD)
Assessment Examples	What must be the measures of ∠C and ∠D so that a circle may be circumscribed about *ABCD* below? *B* 110° *A* 80° *D* *C* Not drawn to scale

Circles

Cluster

Understand and apply theorems about circles.

NVACS HSG.C.A.4 (Major Supporting Work)

(+) Construct a tangent line from a point outside a given circle to the circle.

Element	Exemplars
Standards for Mathematical Practice	● MP 5 Use of construction tools and dynamic software to ensure accurate constructions. ● MP 6 Attending to precision when marking for construction.

Instructional Strategies	● Have students explore methods for constructing a tangent line to a circle from an external point and to justify why their method works.
	    • First Partner 1 and 2 writes down the process, individually to constructing the tangent line to their circle.
	    • Then partners exchange their papers and follow the written procedure to construct the tangent line to the given circle

Prerequisite Skills	● Define tangent lines
	● Understand the properties of circles and how to apply them.
	● Construct
	    • Angle bisectors
	    • Segment bisectors
	    • Copy an angle
	    • Perpendicular line
	    • Parallel line

Connections Within and Beyond High School	● Students must construct a tangent line to a given circle using an external point. ● Students must construct a tangent line to a given circle using an internal point. ● Students must construct an internal tangent line to two circles. ● Students must construct an external tangent line to two circles.
Instructional Examples/Lessons/Tasks	● Tangents through an external point (Math Open Reference) ● Constructing a Line Tangent to a Circle (Khan Academy) ● How to Construct the Tangent Line to a Circle (Wikihow)
Assessment Examples	● Provide a circle and an external point. Tell students to construct a line that is tangent to the given circle. ● Challenge: Prove two circles. Tell the students to construct an internal tangent line to the two circles.

Circles

Cluster

Find arc lengths and areas of sectors of circles.

NVACS HSG.C.B.5 (Major Supporting Work)

Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Element	Exemplars
Standards for Mathematical Practice	● MP 6 Students are careful to attend to precision in their units, degrees/radians.

Instructional Strategies	● Tactile Strategy
	   1. Have students fold a piece of paper in half.
	   2. Separate each half into two columns.
	   3. On the top half, have students write the terms *sector* and *segment*.
	   4. On the bottom half, have them draw a model of each term, write each term's definition, and give the method for finding the area of each shape.
	   5. Students will define and explain the formula for arc length and area of a sector.
	● Visual Representation
	   1. Have a student stand several feet away from a classroom wall.
	   2. Tie a string loosely to the student's arm. Ask another student to hold the opposite end of the string on the wall so that it is taut.
	   3. Explain that the string represents the radius of a circle.
	   4. Direct the first student to walk in an arc toward the wall.
	   5. Ask the class how they might measure the distance traveled by the student. Explain that they will learn how to find arc lengths in this lesson.
	● Students should understand that arc length is a portion of the circumference and be able to find this length using both degrees and radians.

Prerequisite Skills	● Students must know how to use the formula for circumference or area of a circle. ● Students must understand proportional relationships. ● Students must understand the difference between area and circumference of a circle and the formulas associated with both.

Connections Within and Beyond High School	- Be able to solve using the formula for circumference and area of a circle. - Apply the formula for the volume of a cylinder, cone, pyramid. - Understand cross-sections of three-dimensional shapes and rotations of two-dimensional shapes.
Instructional Examples/Lessons/Tasks	- Virginia Museum 3 Act Task (whenmathhappens.com) - Lava Field 3 Act Task (whenmathhappens.com) - Discovering Pi Activity (CCSD) - Arc Lengths and Areas of Sectors (CCSD) - Circles BLAST Module (CCSD)
Assessment Examples	- What is a radian? What is the advantage of measuring angles using radians instead of degrees? - Convert radian measure into degrees and degree measure into radians. - Calculate the area of a sector for a circle with a radius of 12 cm and a central angle of 35 degrees. - Calculate the area of a sector for a circle with a radius of 11 in and a central angle of 75 degrees.

Expressing Geometric Properties with Equations

Cluster

Translate between the geometric description and the equation for a conic section.

NVACS HSG.GPE.A.1 (Major Supporting Work)

Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Element	Exemplars
Standards for Mathematical Practice	- MP 3 Use of distance formula and the Pythagorean Theorem knowledge to build a logical progression of statements to derive the equation of a circle. - MP 6 Attend to precision when deriving the equation of a circle to ensure that the x and y are placed correctly.
Instructional Strategies	- Show students how to derive the equation of a circle based on the distance formula using the fact that all points on a circle are equidistant from the center. - Derive the general equation of a circle by applying the Pythagorean Theorem. - Determine the standard form of circle by completing the square. - Determine the equation of a circle given the radius and center of the circle. - Determine the radius and center of a circle, given the equation of the circle.
Prerequisite Skills	- Know how to use and apply the Pythagorean Theorem. - Know how the distance formula is derived from the Pythagorean Theorem.
Connections Within and Beyond High School	- Completing the square - Distance formula - Pythagorean Theorem
Instructional Examples/Lessons/Tasks	- Equations of Circles (CCSD) - Circle Equation Review (Khan Academy) - Completing the Square: Circle Equations (purplemath.com)
Assessment Examples	- How do you write the equation of a circle? - Write the equation of a circle with a radius of 4 and a center of (6, -2). - Find the center and radius of the circle having the following equation $$4x^2 + 4y^2 - 16x - 24y + 52 = 0$$

Expressing Geometric Properties with Equations

Cluster

Translate between the geometric description and the equation for a conic section.

NVACS HSG.GPE.A.2 (Major Supporting Work)

Derive the equation of a parabola given a focus and directrix.

Element	Exemplars
Standards for Mathematical Practice	- MP 3 Construct viable arguments, make conjectures, and build a logical progression of statements to derive the equation of a parabola. - MP 7 Relate work with parabolas to quadratic functions. - MP 8 Recognizing the repetition in the equal distance between each point on a parabola from the focus and directrix.
Instructional Strategies	- Lecture with guided notes - Videos - Guided Activities - Patty Paper Activities
Prerequisite Skills	- Know the distance formula and how to apply it - Understand the definition of derive - Know the properties of a parabola - Able to rearrange literals and formulas - Factoring - Multiplying polynomials
Connections Within and Beyond High School	- Deriving the equation of a circle - Deriving the equation of a hyperbola - Deriving the equation of an ellipse

Instructional Examples/Lessons/Tasks	- To give students a tactile understanding of the relationship of the focus and directrix of a parabola, have them follow these instructions: - Draw one point an inch or two from the bottom of a piece of paper. The point should not be on a side edge. - Fold the bottom of the paper such that a point on the bottom edge touches the point drawn in Step 1. Make a crease on the fold. - Repeat Step 2 twenty or more times. When finished, the creases will form a parabola. Have students draw a line from the point on the bottom edge to the point drawn for Step 1. Have students make a conjecture about the distance from the bottom edge to the fold and the distance from the fold to the point in Step 1. - Construct a Parabola Class Activity (CCSD) - Deriving the Equation of a Parabola (CCSD) - String Parabola Activity(CCSD) - Defining Parabolas Geometrically (Illustrative Mathematics) - Expressing Geometric Properties with Equations BLAST Module (CCSD)
Assessment Examples	- Find the focus and the directrix of the following parabola $$y^2 = 16x$$ - Find the directrix of the parabola with the following equation: $$(x-5)^2 = 20y$$ - Write the equation of a parabola with a focus of (8, 0) and directrix of $x = 2$

Expressing Geometric Properties with Equations

Cluster

Translate between the geometric description and the equation for a conic section.

NVACS HSG.GPE.A.3 (Major Supporting Work)

(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Element	Exemplars
Standards for Mathematical Practice	• MP 3 Construct viable arguments, make conjectures, and build a logical progression of statements to derive the equations of ellipses and hyperbolas. • MP 4 Use coordinates to model real world and geometric situations.
Instructional Strategies	• Begin using technology to show what ellipses and hyperbolas look like and the change in foci affects the graph. • Break down the steps to graphing hyperbolas and ellipses as a whole. In partners, have the students explain the steps. • Students work in partners to graph hyperbolas and ellipses. • Students explain to each other, aloud the difference between all conic sections
Prerequisite Skills	• Know the distance formula. • Understand and apply the circle equations. • Derive the circle equation.
Connections Within and Beyond High School	• Distance formula • Deriving the equation of a circle • Deriving the equation of a parabola
Instructional Examples/Lessons/Tasks	• Ellipse Building Background Knowledge (Math Open Reference) • Development of an Elipse from the Definition (richland.edu) • Deriving the Equation of a Hyperbola Centered at the Origin (Lumen Learning) • Ellipses Vs Hyperbolas Similarities and Differences (youtube.com) • How to Graph a Hyperbola (dummies.com) • Proof: Hyperbola Foci (Khan Academy) • Building Conic Sections (Desmos)
Assessment Examples	• Conic Sections Practice Test (murrieta.k12.ca.us) • Conic Sections Exam (study.com)

Expressing Geometric Properties with Equations

Cluster

Use coordinates to prove simple geometric theorems algebraically.

NVACS HSG.GPE.B.4 (Major Supporting Work)

Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

Element	Exemplars
Standards for Mathematical Practice	- MP 3 Students create proofs and compare with classmates. - MP 5 Use tools such as paper folding, patty paper, constructions, etc. to prove geometric theorems. - MP 6 Students use precise mathematical language to express their reasoning.
Instructional Strategies	- Students explore geometric theorems using coordinates by using technology such as Geogebra, Geometer's Sketchpad, or Desmos. - This is a good opportunity to review characteristics of specific quadrilaterals or triangles through an algebraic lens.
Prerequisite Skills	- Graph points - Find a midpoint - Find the distance between two points - Know characteristics of triangles and polygons, especially special quadrilaterals.
Connections Within and Beyond High School	Students use graphing skills in the coordinate plane from elementary school & middle school.
Instructional Examples/Lessons/Tasks	[Connecting Algebra and Geometry](mathematicsvisionproject.org) (mathematicsvisionproject.org)
Assessment Examples	[Mathematics II Resourced for EOC Remediation: Coordinate Geometry Cluster](RPDP) (RPDP)

Expressing Geometric Properties with Equations

Cluster

Use coordinates to prove simple geometric theorems algebraically.

NVACS HSG.GPE.B.5 (Major Supporting Work)

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Element	Exemplars
Standards for Mathematical Practice	● MP 3 Construct viable arguments, make conjectures, and build a logical progression of statements. ● MP 7 Look for and make use of the structure of an equation in point slope form to solve problems with parallel and perpendicular lines.

Instructional Strategies	● Visual Representation
	    ● Have students start by graphing two equations that are parallel so they can SEE that the slopes are what allow the lines to "travel" the same.
	    ● Then have them graph a perpendicular line so they can SEE the same concept.
	    ● Have them measure the angle where the lines intersect.
	● Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in a two-column format, and using diagrams without words.

Prerequisite Skills	● Students will know the slopes of opposite sides of parallelograms are equal. ● Understand that slopes of parallel lines are equal. ● Comprehend that the slopes of perpendicular lines are the opposite reciprocals. ● Students will identify the slope of a line from an equation or graph and differentiate between parallel, perpendicular, or neither. ● Be able to apply the slope of a parallel or perpendicular line to write a new equation in slope-intercept, point-slope, and standard forms.

Connections Within and Beyond High School	- Find the point on a directed line segment between two given points that partitions the segment in a given ratio. - Prove theorems about triangles. Theorems include - Measures of interior angles of a triangle sum to 180° - Base angles of isosceles triangles are congruent - The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length - The medians of a triangle meet at a point - Prove theorems about parallelograms. Theorems include - Opposite sides are congruent - Opposite angles are congruent - The diagonals of a parallelogram bisect each other - Conversely, rectangles are parallelograms with congruent diagonals
Instructional Examples/Lessons/Tasks	- Expressing Geometric Properties with Equations (RPDP) - Identify Parallel, Perpendicular, and Intersecting Lines PRACTICE (RPDP) - Parallel Lines and Angle Relationships (RPDP) - Writing Equations of Perpendicular Lines (Khan Academy)
Assessment Examples	- What is the relationship between slopes of perpendicular and parallel lines? - Given $y = 2x + 7$, write the equation of a line parallel to that line that passes through the point (3, -2). - Given the $y = 2x + 7$, write the equation of a line perpendicular to that line that passes through the point (-4, -3).

Expressing Geometric Properties with Equations

Cluster

Use coordinates to prove simple geometric theorems algebraically.

NVACS HSG.GPE.B.6 (Major Supporting Work)

Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Element	Exemplars
Standards for Mathematical Practice	● MP 1 Students will persevere in finding the solutions that partition line segments.
Instructional Strategies	● The key is to link a dilation by a value between 0 and 1 to be the same as partitioning a line segment. Help students to take a partitioning ratio and convert it to a scale factor (ex. 1:4 partition ratio is a 1:5 scale factor).
Prerequisite Skills	● This standard connects slope and dilation together. The big idea here is simply using skills to solve problems using coordinates and coordinate skills.
Connections Within and Beyond High School	● The student needs to use a reduction dilation to be able to perform this standard, which is an earlier standard. The concept of slope is also a very important foundational understanding here. ● Ultimately, this is about ratios and coordinates which do show up in different shapes and problems later on. This also bridges a student from "directed line segments" to VECTORS.
Instructional Examples/Lessons/Tasks	● The student will be able to partition a line segment based on a provided ratio. ● Student questions 　• How is dilating a point by a value between 0 and 1 like a directed line segment? 　• What is the relationship between a partitioning ratio and a scale factor? ● Divide Line Segments (Khan Academy)
Assessment Examples	● Partitioning a Line Segment PRACTICE (rcboe.org)

Expressing Geometric Properties with Equations

Cluster

Use coordinates to prove simple geometric theorems algebraically.

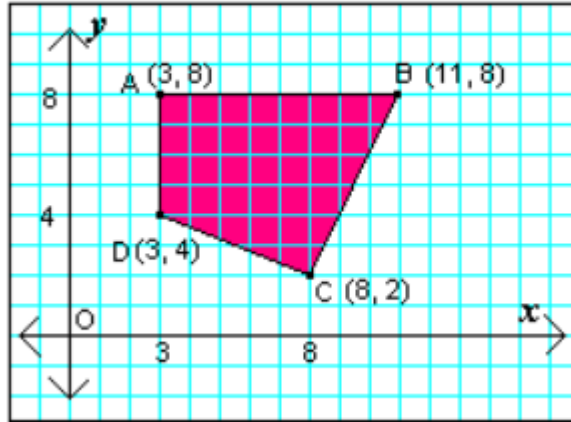**NVACS HSG.GPE.B.7 (Major Supporting Work)**

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.[*]

Element	Exemplars
Standards for Mathematical Practice	- MP 2 Use coordinate geometry to reason quantitatively in terms of areas and perimeters of figures - MP 3 Construct viable arguments, make conjectures, and build a logical progression of statements. - MP 6 Attend to precision with exact answers by finding the perimeter and area of polygons..
Instructional Strategies	- Visual Representation - Show examples in the coordinate grid with right triangles whose legs are vertical and horizontal so students can easily determine the length of the two legs - Use the distance formula to determine the length of the hypotenuse. - Show examples where students must graph four coordinates and use the distance formulas and slope to show properties of the sides and diagonals.
Prerequisite Skills	- Students can plot and identify points on a coordinate plane. - Students can differentiate among the formulas, theorems, and postulates needed for each coordinate proof. - Students will apply geometric formulas, theorems, or postulates needed to justify each step of a coordinate proof. - Students can compare and contrast algebraic proofs using coordinate geometry with other forms of proof. - Students can prove geometric theorems using coordinate geometry.
Connections Within and Beyond High School	- Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). - Find the point on a directed line segment between two given points that partitions the segment in a given ratio. - Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).*(Modeling Standard)
Instructional Examples/Lessons/Tasks	- Coordinate Geometry PRACTICE (hitbullseye.com) - Coordinate Geometry (onlinemathlearning.com) - Coordinate Geometry Distance Formula (youtube.com)

Assessment Examples	- What theorems can be proven using coordinate geometry? - Find the area of the triangle formed by the vertices (4, 5), (10, 12), and (-3, 2). **Find the perimeter of quadrilateral ABCD.**

Geometric Measurement and Dimension

Cluster

Explain volume formulas and use them to solve problems.

NVACS HSG.GMD.A.1 (Major Supporting Work)

Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*.

Element	Exemplars
Standards for Mathematical Practice	● MP 3 Use reasoning to defend the formulas for circumference and area of circles and volumes of solids. The reasoning may look at dissection principles, Cavaleri's Principle or informal limits. ● MP 6 Attend to precision with exact answers by finding the area of the base and leaving the answer in terms of π.
Instructional Strategies	● Some two-dimensional figures may be composed of more than one polygon. Students can determine the area of this type of figure by calculating each polygon separately then adding the total sum of the areas. ● Show students that the surface area of three-dimensional figures is determined by calculating the separate areas of each face and using the sum of these values using nets or by cutting up a cereal box and having students measure up each side and adding them together. The volume is determined by calculating the area of the base and multiplying it by the height.
Prerequisite Skills	● Students need to know how to calculate the area of a triangle and quadrilateral. ● Students must know how to identify a polygon in order to identify the base of the three-dimensional figure. ● Students must know how to determine the appropriate order of operations needed to calculate the area of the base of the three-dimensional shape. ● Students must know how to calculate the surface area of a cube by using the sum of the areas of each face. ● Students must know how to calculate the surface area of a right prism by using the sum of the areas of each face.
Connections Within and Beyond High School	● Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. ● Know the formulas for the volumes of cones, cylinders, and spheres and use the formulas to solve real-world and mathematical problems.
Instructional Examples/Lessons/Tasks	● [Air Mattress 3 Act Task](whenmathhappens.com) (whenmathhappens.com) ● [Mr. Clean 3 Act Task](whenmathhappens.com) (whenmathhappens.com)
Assessment Examples	● How can you use rectangles and triangles to find the area of other polygons? ● How do you determine the volume of a right prism? How would you calculate the surface area? ● How would you find the volume of a cylinder with a circumference of 16π and a height of 15 ft.? ● Find the volume of a prism with a height of 7 in. and a base of a right triangle with side lengths of 3 in., 4 in., and 5 in.

Geometric Measurement and Dimension

Cluster

Explain volume formulas and use them to solve problems.

NVACS HSG.GMD.A.2 (Major Supporting Work)

(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Element	Exemplars
Standards for Mathematical Practice	- MP 3 Develop arguments in favor of using Cavalieri's Principle and critique the reasoning of others. - MP 6 Attend to precision with exact answers by finding the area of the base and leaving the answer in terms of π.
Instructional Strategies	- Illustrate Cavalieri's principle using a stack of CDs or a stack of pennies. - Ben and Jerry's pint slices
Prerequisite Skills	- Students must know the formula for - Circumference of a circle - Area of a circle - Volume of a cylinder - Volume of a pyramid - Volume of a cone - Students must know Cavalieri's principle. - Students can use dissection, Cavalieri's principle, and/or limits to justify an informal argument for the circumference of a circle, the area of a circle, and the volume of a cylinder/pyramid/cone.
Connections Within and Beyond High School	- Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. - Use volume formulas for cylinders, pyramids, cones, and spheres, and the surface area of a sphere to solve problems. *(Modeling Standard)
Instructional Examples/Lessons/Tasks	[Cavalieri's Principle](#) (CCSD) [Geometric Measurement and Dimension BLAST Module](#) (CCSD)
Assessment Examples	- How are the formulas for the circumference and area of a circle derived? - How are the formulas for the volume of cylinders, pyramids, and cones derived? - If the surface area of your globe is 256π squared centimeters, what is the volume?

Geometric Measurement and Dimension

Cluster

Explain volume formulas and use them to solve problems.

NVACS HSG.GMD.A.3 (Major Supporting Work)

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. [*]

Element	Exemplars
Standards for Mathematical Practice	- MP 1 Understand what the variables in formulas are and persevere in solving problems. - MP 4 Solve real world volume problems. - MP 6 Attend to precision by calculating exact answers using appropriate labels (Example: $6\ in^3$).
Instructional Strategies	- Have geometric figures for students to manipulate. - Explain that the volume of all prisms including cylinders use the same formula V=Bh. - Explain that the volume of all pyramids, including cones use the same basic formula V=$\frac{1}{3}$Bh, which is a variation of the prism formula. - Demonstrate why the volume of a cone is $\frac{1}{3}$ the volume of a cylinder with equal height and radius.
Prerequisite Skills	- Know how to calculate the area of triangles, quadrilaterals and circles. - Know properties of triangles, quadrilaterals and circles.
Connections Within and Beyond High School	- Students will be able to use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). - Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Instructional Examples/Lessons/Tasks	- Volume Tutorials (Virtual Nerd) - Volume of Cone Demonstration (youtube.com) - Geometry Problems (analyzemath.com)
Assessment Examples	- Finding Volume PRACTICE (studylib.net) - Volume of Cylinders, Cones, and Spheres (mayfieldschool.org)

Geometric Measurement and Dimension

Cluster

Visualize relationships between two-dimensional and three-dimensional objects.

NVACS HSG.GMD.B.4 (Major Supporting Work)

Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Element	Exemplars
Standards for Mathematical Practice	● MP 2 Students will use abstract reasoning to determine a two-dimensional cross-section of a three dimensional figure ● MP 5 Use the correct tools to make cross sections of solids for study
Instructional Strategies	● Put a right triangle, obtuse triangle, rectangle, semicircle, and a circle on a pencil so that students can see that two-dimensional figures can represent slices or cross-sections of three-dimensional figures. As you rotate the pencil, students can see that three-dimensional figures can be generated by rotating these two-dimensional figures.
Prerequisite Skills	● Understanding of rotational symmetry ● Reflectional symmetry
Connections Within and Beyond High School	● Students will be able to use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ● Students will be able to apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
Instructional Examples/Lessons/Tasks	● [Rotations of Plane Figures Around a Line](#) (CCSD)
Assessment Examples	● What shapes can be cross sections of three-dimensional figures? ● How can rotations of two-dimensional figures be used to generate three-dimensional figures?

Modeling with Geometry

Cluster

Apply geometric concepts in modeling situations.

NVACS HSG.MG.A.1 (Major Supporting Work)

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).[*]

Element	Exemplars
Standards for Mathematical Practice	● MP 4 Model geometric shapes and solids using everyday objects.
Instructional Strategies	● When studying right triangles, focus on situations well modeled by trigonometric ratios for acute angles. ● When studying the area and volume, focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles. ● When studying circles, focus on situations in which the analysis of circles is required.
Prerequisite Skills	● Students must know the relationships between standard shapes in a plane and the faces of actual three-dimensional objects (e.g., a circle to the face of a clock). ● Students must know the relationships between standard solids in a space and actual three-dimensional objects (e.g., a right rectangular prism to a box). ● Students must use the relationships between standard shapes in a plane and the faces of three-dimensional objects to approximate the perimeter, circumference, or area of a real-world object (e.g., use the area of a trapezoid to approximate the area of Nevada). ● Students must use the relationships between standard solids in a space and the three-dimensional objects to approximate the volume of a real-world object.
Connections Within and Beyond High School	● Students will be able to use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ● Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Instructional Examples/Lessons/Tasks	● [Surface Area and Volume Notes](#) (RPDP.net) ● [Geometric Models](#) (youtube.com) ● [Modeling Geometry](#) (mathbitsnotebook.com)

Assessment Examples	● How can one model the real world by using geometric shapes?
	● How deep is the well?
	   ● The winding drum of a well is 30 cm in diameter. If it takes five and a half turns of the handle before the bucket is in the water, how deep is the well (to the nearest 10 cm)?
	   ● If the radius of the handle is 40 cm, how far has your hand travelled by the time the bucket has been brought to the top?
	● A bicycle wheel has a diameter of 60 cm. How far will it travel if it makes 50 revolutions?

Modeling with Geometry

Cluster

Apply geometric concepts in modeling situations.

NVACS HSG.MG.A.2 (Major Supporting Work)

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).[*]

Element	Exemplars
Standards for Mathematical Practice	- MP 4 Use real world examples to model the concept of density. - MP 6 Attend to precision of the units used in a density problem.
Instructional Strategies	- When studying the area and volume, focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles. - Measure the average diameter of a human head to approximate its volume. - Find the change in volume of a liquid, in a known solid, to calculate the volume of an irregular object. (e.g., apply Archimedes' Principle)
Prerequisite Skills	- Students must calculate the density of an area given a quantity and the shape of the region. - Students must calculate the density of a volume given a quantity and the shape of the solid.
Connections Within and Beyond High School	- Students will be able to use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). - Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Instructional Examples/Lessons/Tasks	- Geometry and Density PRACTICE (mathbitsnotebook.com) - Dealing with Density (mathbitsnotebook.com) - Density Word Problems (Khan Academy)
Assessment Examples	- How can area and volume be used with the concept of density? - The average amount of pasture needed to feed mature mules is 1 acre per mule. A farm has three rectangular fenced pasture sections to be used for grazing. 100 ft. by 300 ft. 150 ft. by 500 ft. 250 ft. by 400 ft. *Note: 1 acre = 43,560 square feet* Six mules are being adopted from a rescue program. The farm does not have sufficient pasture space to support 6 mules according to the 1 acre per mule guideline. What is the least amount of additional square footage of grazing land that will need to be fenced to meet this guideline?

Modeling with Geometry

Cluster

Apply geometric concepts in modeling situations.

NVACS HSG.MG.A.3 (Major Supporting Work)

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).[*]

Element	Exemplars
Standards for Mathematical Practice	- MP 1 Make sense of how Geometry might be used to solve the problem. - MP 3 Justify the designs created and how it was answered. - MP 6 Attend to precision with exact answers by finding the area of the base and leaving the answer in terms of π. - MP 8 Attend to how the structure of elements in the problem are related geometrically, and which properties can be used to aid in the design process.
Instructional Strategies	- Measure and average the diameter of a human head to approximate its volume. - Find the change in volume of a liquid, in a known solid, to calculate the volume of an irregular object (i.e., apply Archimedes' Principle).
Prerequisite Skills	Students must apply previously learned knowledge to solve real-world problems, including complex situations.
Connections Within and Beyond High School	- Students will be able to use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). - Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Instructional Examples/Lessons/Tasks	- Using Geometry in Design Problems (mathworksheetsland.com) - Solve Design Problems by Applying Geometric Methods (LearnZillion) - Modeling with Geometry (Khan Academy)
Assessment Examples	- How can I apply geometry to design an object or structure to solve real world problems? - Design a poster that shows how to find area and perimeter. List out step by step how to find the area and perimeter of a figure. Provide an example with your steps. - Choose any item to paint or carpet, etc. Measure that item in centimeters. Name the type of shape the item is. Find the item's area and perimeter. Go online and determine the cost for completing the task.

Acknowledgements

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Jhone Ebert

Superintendent of Public Instruction

Jonathan Moore, Ed.D

Deputy Superintendent for Student Achievement

Dave Brancamp

Director of Standards and Instructional Support

Tracy Gruber

K-12 Mathematics Education Program Professional

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Nevada Ready!
Network
Standards-Based Instruction for
ALL Nevada Students