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Mathematics Syllabus – S4-S5[1]
6 Periods (6P)

Approved by the Joint Teaching Committee at its meeting of 7 and 8 February 2019 in Brussels

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[1] The document "Mathematics Syllabus – S5 - 6 Periods" (ref. 2019-01-D-49-en-3) was approved by the Joint Teaching Committee at its meeting of 13 and 14 February 2020 in Brussels

Europeans Schools - Mathematics Syllabus

Year S4-S5 – 6 P

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1. General Objectives

The European Schools have the two objectives of providing formal education and of encouraging pupils' personal development in a wider social and cultural context. Formal education involves the acquisition of competences (knowledge, skills and attitudes) across a range of domains. Personal development takes place in a variety of spiritual, moral, social and cultural contexts. It involves an awareness of appropriate behaviour, an understanding of the environment in which pupils live, and a development of their individual identity.

These two objectives are nurtured in the context of an enhanced awareness of the richness of European culture. Awareness and experience of a shared European life should lead pupils towards a greater respect for the traditions of each individual country and region in Europe, while developing and preserving their own national identities.

The pupils of the European Schools are future citizens of Europe and the world. As such, they need a range of competences if they are to meet the challenges of a rapidly-changing world. In 2006 the European Council and European Parliament adopted a European Framework for Key Competences for Lifelong Learning. It identifies eight key competences which all individuals need for personal fulfilment and development, for active citizenship, for social inclusion and for employment:

1. Literacy competence
2. Multilingual competence
3. Mathematical competence and competence in science, technology and engineering
4. Digital competence
5. Personal, social and learning to learn competence
6. Civic competence
7. Entrepreneurship competence
8. Cultural awareness and expression competence

The European Schools' syllabuses seek to develop all of these key competences in the pupils.

Key competences are that general, that we do not mention them all the time in the Science and Mathematics syllabuses.

2. Didactical Principles

General

In the description of the learning objectives, competences, connected to content, play an important role. This position in the learning objectives reflects the importance of competences acquisition in actual education. Exploratory activities by pupils support this acquisition of competences, such as in experimenting, designing, searching for explanations and discussing with peers and teachers. In science education, a teaching approach is recommended that helps pupils to get acquainted with concepts by having them observe, investigate and explain phenomena, followed by the step to have them make abstractions and models. In mathematics education, investigations, making abstractions and modelling are equally important. In these approaches, it is essential that a maximum of activity by pupils themselves is stimulated – not to be confused with an absent teacher: teacher guidance is an essential contribution to targeted stimulation of pupils' activities.

The concept of *inquiry-based learning* (IBL) refers to these approaches. An overview of useful literature on this can be found in the *PRIMAS guide for professional development providers*.

http://primas-project.eu/wp-content/uploads/sites/323/2017/10/PRIMAS_Guide-for-Professional-Development-Providers-IBL_110510.pdf

Mathematics

Careful thought has been given to the content and the structure to where topics are first met in a pupil's time learning mathematics in secondary education. It is believed that this is a journey and if too much content is met at one point, there is a risk that it will not be adequately understood and thus a general mathematical concept will not be fully appreciated. By limiting the content of this syllabus (found in section 4.2.) each year more time can be used to develop core mathematical concepts that may have been met before or new mathematical concepts introduced are given ample time for extension. It must be noted that extension activities are conducted at the discretion of the teacher, however, it is suggested that rather than look at a vertical approach to extension a horizontal approach is used, thus giving the pupil a deeper understanding of the mathematical concept (in section 4 the word 'limitation' is used to ensure the extension does not go too far).

Furthermore, to this point it is believed that with a focus on competences this syllabus can encourage pupils to have a greater enjoyment of mathematics, as they not only understand the content better but understand the historical context (where it is expected a history of mathematics can be told over the cycles) and how the mathematics can be applied in other subjects, cross cutting (these can be seen in the fourth column in section 4.2.). As such the syllabuses have specifically been designed with reflection to the key competences (section 1.) and the subject specific competences (section 3.1.). In some cases, the key competences are clear for example the numerous history suggested activities (shown by the icon ) that maps to key competency 8 (Cultural awareness and expression). In other areas the link may not be so apparent.

One of the tasks in the pupil's learning process is developing inference skills, analytical skills and strategic thinking, which are linked to both the key and subject specific competences. This is the ability to plan further steps in order to succeed solving a problem as well as dividing the process of solving more complex problems into smaller steps. A goal of teaching mathematics is to develop pupil's intuitions in mathematics appropriate for their age. The ability to understand and use mathematical concepts (e.g. angle, length, area, formulae and equations) is much more important than memorising formal definitions.

This syllabus has also been written so that it can be accessible by teachers, parents and pupils. This is one reason why icons have been used (listed in section 4.2.). These icons represent different areas of mathematics and are not necessarily connected to just one competency but can cover a number of competences.

To ensure pupils have a good understanding of the mathematics the courses from S1 to S7 have been developed linearly with each year the work from the previous year is used as a foundation to build onto. Thus, it is essential before commencing a year the preceding course must have been covered or a course that is similar. The teacher is in the best position to understand the specific needs of the class and before beginning a particular topic it is expected that pupils have the pre-required knowledge. A refresh is always a good idea when meeting a concept for the first time in a while. It should be noted that revision is not included in the syllabus, however, as mentioned earlier about limiting new content, there is time to do this when needed.

The use of technology and digital tools plays an important role in both theoretical and applied mathematics, which is reflected in this syllabus. The pupils should get the opportunity to work and solve problems with different tools such as spreadsheets, computer algebra system (CAS) software, dynamic geometric software (DGS), programming software or other software that are available in the respective schools. Technology and digital tools should be used to support and promote pupils' understanding, for example by visualising difficult concepts and providing interactive and personalised learning opportunities, rather than as a substitute for understanding. Their use will also lead to improved digital competence.

Teachers have full discretion with how to teach this course, materials to use and even the sequence the content is taught in. The content and the competencies (indicated in the tables in section 4.2., columns 2 and 3) to be covered is, however, mandatory.

The S4 6 Period Course

The S4 4 Period course has been developed alongside the 6 Period course where the core work is done in the 4 Period course, and the 6 Period course will explore the content in more depth. With this approach, changing between the courses is possible, with the understanding that pupils having studied the 6 Period course will often have a greater depth of understanding.

Students opting for the 6 Period course should have already gained confidence in handling the basic requirements in algebra, arithmetic and plane geometry from past years. Though a few teaching periods are allocated to building upon the ground work of the previous years, the major part of the teaching time addresses new concepts such as functions and vectors or deepens understanding of statistics and probability. The students embarking on the 6 Period course should be aware that this course is demanding and that they will have to dedicate a considerable part of their working time to it, especially because no other course has so many teaching periods in total.

The S5 6 Period Course

This course has been specifically written for those who will be studying fields where mathematics plays a significant role. This includes all scientific studies, but also some fields of economics, finance and social studies, keeping in mind that the list is not exhaustive. In this course, together with acquiring skills that are essential for their broader studies, students will also develop an understanding of the culture and value of mathematics for its own enjoyment. Though there is no noticeable gap in difficulty between the 6 Period course in S4 and the 6 Period course in S5, the students should be aware that without a strong foundation from S4, they will struggle in S5.

Pupils must note that the 4 Period and 6 Period courses in S5 are different. Thus, pupils wishing to study the 5 Period course in S6 will need to be aware of this before embarking on the 4 Period course in not just S5 but S4 too.

3. Learning Objectives

3.1. Competences

The following are the list of subject specific competences for mathematics. Here the key vocabulary is listed so that when it comes to reading the tables in section 4.2. the competency being assessed can be quickly seen. Please note that the list of key vocabulary is not exhaustive, and the same word can apply to more than one competency depending on the context.

Further information about assessing the level of competences can be found in section 5.1. Attainment Descriptors. The key concepts here are those needed to attain a sufficient mark.

	Competency	Key concepts (attain 5.0-5.9)	Key vocabulary
1.	**Knowledge and comprehension**	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and principles	Apply, classify, compare, convert, define, determine, distinguish, expand, express, factorise, identify, know, manipulate, name, order, prove, recall, recognise, round, simplify, understand, verify
2.	**Methods**	Carries out mathematical processes in straightforward contexts, but with some errors	Apply, calculate, construct, convert, draw, manipulate model, organise, plot, show, simplify sketch solve, use, verify
3.	**Problem solving**	Translates routine problems into mathematical symbols and attempts to reason to a result	Analyse, classify, compare, create, develop, display, estimate, generate, interpret, investigate, measure, model, represent, round, simplify, solve
4.	**Interpretation**	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Calculate, conduct, create, develop, discover, display, generate, interpret, investigate, model
5.	**Communication**	Generally presents reasoning and results adequately; using some mathematical terminology and notation	Calculate, conduct, create, discover, display, interpret, investigate, model, present
6.	**Digital competence**[2]	Uses technology satisfactorily in straightforward situations	Calculate, construct, create, display, draw, model, plot, present, solve

[2] This competence is part of the European Digital Competence Framework (https://ex.europa.eu/jrc/en/digcomp)

3.2. Cross-cutting concepts

Cross cutting concepts will be carried by the joint competences. The list of cross cutting concepts that will be composed will be shared by all science and mathematics syllabuses. The tentative list to be taught is based on the next generation science standards in the United states (National Research Council, 2013):

	Concept	Description
1.	**Patterns**	Observed patterns of forms and events guide organisation and classification, and they prompt questions about relationships and the factors that influence them.
2.	**Cause and effect**	Mechanism and explanation. Events have causes, sometimes simple, sometimes multifaceted. A major activity of science is investigating and explaining causal relationships and the mechanisms by which they are mediated. Such mechanisms can then be tested across given contexts and used to predict and explain events in new contexts.
3.	**Scale, proportion and quantity**	In considering phenomena, it is critical to recognise what is relevant at different measures of size, time, and energy and to recognise how changes in scale, proportion, or quantity affect a system's structure or performance.
4.	**Systems and system models**	Defining the system under study—specifying its boundaries and making explicit a model of that system—provides tools for understanding the world. Often, systems can be divided into subsystems and systems can be combined into larger systems depending on the question of interest
5.	**Flows, cycles and conservation**	Tracking fluxes of energy and matter into, out of, and within systems helps one understand the systems' possibilities and limitations.
6.	**Structure and function**	The way in which an object or living thing is shaped and its substructure determine many of its properties and functions and vice versa.
7.	**Stability and change**	For natural and built systems alike, conditions of stability and determinants of rates of change or evolution of a system are critical for its behaviour and therefore worth studying.
8.	**Nature of Science**	All science relies on a number of basic concepts, like the necessity of empirical proof and the process of peer review.
9.	**Value thinking**	Values thinking involves concepts of justice, equity, social–ecological integrity and ethics within the application of scientific knowledge.

In the mathematics syllabuses, the concepts 5 and 8 will be addressed only to a limited extent. The lists of competences and cross cutting concepts will serve as a main cross-curricular binding mechanism. The subtopics within the individual syllabuses will refer to these two aspects by linking to them in the learning goals.

http://ngss.nsta.org/Professional-Learning.aspx

4. Content

4.1. Topics

This section contains the tables with the learning objectives and the mandatory content for the strand Mathematics in S4 (6 periods per week).

4.2. Tables

How to read the tables on the following pages

The learning objectives are the curriculum goals. They are described in the third column. These include the key vocabulary, highlighted in bold, that are linked to the specific mathematics competences found in section 3.1. of this document.

These goals are related to content and to competences. The mandatory content is described in the second column. The final column is used for suggested activities, key contexts and phenomena. The teacher is free to use these suggestions or use their own providing that the learning objective and competencies have been met.

Please note that the word ‘limitation’ is used to ensure that when extension is planned it is planned with the idea of horizontal extension rather than vertical extension as mentioned in section 2. of this document.

Use of icons

Furthermore, there are six different icons which indicate the areas met in the final column:

	Activity
	Cross-cutting concepts
	Digital competence
	Extension
	History
	Phenomenon

Each of these icons highlight a different area and are used to make the syllabus easier to read. These areas are based on the key competences mentioned in section 1 of this document.

S4 – 6 Period (6P)

YEAR 4 (6P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Basic calculations *This chapter is a prerequisite and gives the opportunity to revise the basics calculations while investigating more challenging problems; All items of the chapter do not have to be taught separately but only revised if need arises*	Basic calculations over the set of $\mathbb{Q}$	**Apply** basic calculations (+, –, x, /, ≤) over the set of $\mathbb{Q}$		Investigate the division-by-zero fallacy
	Calculation rules and properties	**Apply** calculation rules and properties established in years 1 to 3 and **use** them in simple algebraic and numerical expressions		
	Prime numbers	**Use** prime numbers factorisation in simple cases and applications: LCM/HCF **Investigate** factors, multiples, LCM/HCF, prime, numbers factorisation with and without a technological tool		"Mathematicians stand on each other's shoulders". Prime numbers mystery: Mersenne's primes, Bertrand's Postulate, Hardy and Littlewood conjecture F, Ulam's prime spiral
	Notation Δ and Σ for difference and summation	**Understand** the meaning of Δ and Σ in various elementary examples		• Give examples from mathematics, physics and chemistry, just for interpretation • Calculations involving Δ
	Rational numbers	**Know** that any rational number *q* can be written as: $q = \frac{a}{b}\ (a \in \mathbb{Z}, b \in \mathbb{N}^*)$		
	Decimals and fractions	**Convert** terminating and recurring decimals to fractions and vice versa		Recurring decimals and other periodic phenomena
Radicals and surds	Square numbers, square roots and surds	**Recall** the first 20 square numbers **Understand** that squaring and square rooting are inverse operations		

YEAR 4 (6P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	    Properties of radicals Rationalise a denominator	**Know, prove and understand** that $\sqrt{2} \notin \mathbb{Q}$ and recognise other surds **Know** the distinction between exact and approximate calculations **Apply** the following properties of radicals: • $\sqrt{a}\sqrt{b} = \sqrt{ab}$ for $a, b \in \mathbb{R}^+ \cup \{0\}$ • $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ for $a \in \mathbb{R}^+ \cup \{0\}, b \in \mathbb{R}^+$ • $\sqrt{a^2 b} = a\sqrt{b}$ for $a, b \in \mathbb{N}$ • $\sqrt{a^2} = \|a\|$ for $a \in \mathbb{R}$ Rationalise a denominator: • division by $\sqrt{a}$ • division by $b \pm \sqrt{a}$ • division by $\sqrt{a} \pm \sqrt{b}$ for $a, b \in \mathbb{N}$		In the ISO paper size system, the height-to-width ratio of all pages is equal to the square root of two (with proof) • Pythagoras and irrational numbers • Socrates: Menon
Real numbers	Definition Number line Arithmetic rules in $\mathbb{R}$	**Know** that rational and irrational numbers define all the real numbers **Understand** that each point of a number line corresponds to one and only one real number **Distinguish** between approximate and exact values **Know** that all arithmetic rules in $\mathbb{Q}$		Number Φ and golden ratio

YEAR 4 (6P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		apply in $\mathbb{R}$		
Powers and algebraic expressions	Definitions Formulae Scientific notation Simplifying expressions using index laws	**Know** the definitions and the formulae concerning powers where the indices are integers **Convert** a number to and from scientific notation, including rounding **Calculate** using scientific notation	 	Examples of algebraic formulae from natural and social sciences Round the answer to a certain number of significant figures
Proportionality	Direct proportion Inverse proportion Representations of direct and inverse proportions	**Investigate** phenomena which can be modelled with direct proportion: $y = k \cdot x$ **Investigate** phenomena which can be modelled with inverse proportion: $y = \frac{k}{x}$ **Use** tables of values **Investigate** relationships between two measurable quantities to identify whether they are proportional or not **Plot** direct and inverse proportions *Limitation: Just rewriting formulae in three forms and simple substitution of numbers*		Use phenomena from different subjects with focus on the similar underlying structure, e.g. $s = v \cdot t$
Linear models	Relations and functions	**Define** a relation in establishing that one variable (quantity) is dependent on another variable (quantity)		Examples of linear relations from natural and social sciences e.g. taxi rates, calling rates

YEAR 4 (6P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Understand the difference between relations and functions (e.g. using a vertical line test)		Modelling: Make students familiar with various relevant linear processes and formulae from other fields
		Use function notation ($y = f(x)$) and vocabulary with and without technological tool		
	Variables and parameters	**Understand** the difference between variables and parameters		
	Linear equations	**Solve** linear equations (including equations involving algebraic fractions)		
	Equation of a line: $y = mx + p$	**Determine** the equation of a line given two points, one point and m, using only the graph		
	Graph of linear dependency	**Draw** the graph of a linear function (also using a technological tool)		
		Investigate the graphical meaning of *m* and *p*		
		Determine the intercepts with horizontal and vertical lines as well as axes intercepts		
	Linear function: $x \to mx + p$ $f(x) = mx + p$	**Determine** algebraically and geometrically zero (root) of a linear function		
		Recognise and **solve** problems using linear models		

YEAR 4 (6P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Equation of a line: $ax + by + c = 0$	**Verify** results by use of a technological tool **Understand** and **apply** the following equations: $ax + by + c = 0$ and $y = mx + p$ and **convert** from the first form to the second and vice versa **Use** a technological tool to manipulate parameters and see the effect on linear graphs **Investigate** parallel and perpendicular lines **Use** a technological tool to show by dynamical procedure that points having coordinates satisfying an equation: $ax + by + c = 0$ are colinear		
Simultaneous equations of the type: $\begin{cases} ax + by = c \\ dx + fy = e \end{cases}$	Simultaneous equations: algebraic methods, graphical representation	**Solve** simultaneous equations geometrically (paper and technology) **Solve** simultaneous equations by substitution and elimination methods **Create** and **solve** simultaneous equations from real-life problems	 	Real problems which lead to simultaneous equations Discuss limitations of each method
Polynomials	Simplify, factorize and order polynomial expressions	**Recognise** polynomial expressions and calculate their value **Simplify** and **order** polynomial expressions **Recognise** equivalent expressions		Design a polynomial Roller Coaster

YEAR 4 (6P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Add and **multiply** polynomial expressions **Factorise** simple polynomial expressions **Apply** special identities: • $(a \pm b)^2 = a^2 \pm 2ab + b^2$ • $(a + b)(a - b) = a^2 - b^2$		
	Pascal's triangle	**Investigate** the expansion of $y = (a + b)^n$ and relate it to Pascal's triangle		History of Pascal's triangle using examples from India, Persia/Iran, China, Germany and Italy
	Power function $y = ax^n, n = 1, 2, 3, \ldots$	**Investigate** models that describe a power relation **Investigate** their graphs and tables of values using graphing software		Use examples of formulae with powers from natural and social sciences, e.g. accelerated motion
	Simple quadratic equations	**Solve** simple quadratic equations: • $ax^2 = b$, for $a, b \in \mathbb{Q}$ • $ax^2 + bx = 0$, for $a, b \in \mathbb{Q}$ **Solve** quadratic equations graphically or with a technological tool		

YEAR 4 (6P)	TOPIC: GEOMETRY		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**

Right-angled triangles	Properties of right-angled triangles	**Prove** the properties of a right-angled triangle:
		• a right-angled triangle is half of a rectangle; its hypotenuse is a diagonal of the rectangle; its non-right angles are complementary
		• the length of the median line extended from the right-angle is equal to half the hypotenuse
		• the midpoint of the hypotenuse is the centre of the circumscribed circle of the triangle
		• the hypotenuse is the diameter of the circumscribed circle
		Use a technological tool to verify these properties using construction and measurements
			• “Big problems” for space study:
			  ○ Measuring lengths for reproduction of angles, and back
			  ○ Drawing plans and maps from measures of length
			• Use right triangles to construct an angle congruent to a given one

	Pythagoras’ theorem and the converse of Pythagoras’ theorem	**Know** Pythagoras’ theorem	Pythagorean tree, the spiral of Theodorus, lunes of Hippocrates
		Know the converse of Pythagoras’ theorem	Find different “proofs” of Pythagoras’ theorem and compare them
		Model real problems using Pythagoras' theorem	Investigate Pythagorean triples
	Trigonometric ratios: sin, cos, tan	**Define, recognise** and **apply** the trigonometric ratios: sine, cosine and tangent of an acute angle in a right-angled triangle	

YEAR 4 (6P)	TOPIC: GEOMETRY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Understand the property $\tan p = \frac{\sin p}{\cos p}$		
Properties of the circle and connections with right-angled triangles	Circle, centre, diameter, radius Sector, arc, chord Circumference of a circle, area of a circle Length of an arc sector of a circle Relative position of a straight line and a circle Right-angled triangles inscribed in a circle	**Define** a circle from the compass construction **Draw** a circle, an arc, a chord, a sector, an arc and the related chord **Know** the formulae to calculate the circumference of a circle and the area of a circle **Calculate** the length of an arc and the area of a sector **Investigate** tangent and secant lines **Define** properties of right-angled triangles inscribed in a circle		The locus of the midpoint of a ladder sliding down a wall is a quarter of a circle Pythagoras theorem is used to approximate the length of a circle Find out by dynamical procedure how to inscribe a triangle in a circle
Enlargement	Enlargement and reduction of a figure Scale factor of an enlargement: similar figures Effect of enlargement on angles, areas and volumes	**Understand** enlargement and reduction as proportional transformation of lengths: scale factor **Model** geometrical enlargement **Calculate** the scale factor of an enlargement **Investigate** the effect of enlargement on angles, lengths, areas and volumes **Investigate** invariants of		Suggestion for a project: slides and screens, pictures, plans and maps

YEAR 4 (6P)	**TOPIC: GEOMETRY**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		enlargements **Use** a technological tool to find out the scale factor using variables and sliders		Geometric figures are scale free: enlargements do not change angles and shapes of objects. Geometric figures may describe completely any object or relationship in space
Congruent and similar triangles	Congruent and similar triangles Intercept theorem	**Investigate, define** and **construct** congruent and similar triangles *Note: all circles are similar* **Know, recognise** and **apply** the Intercept theorem	 	Give examples from real-life where intercept theorem is used Fractals, e.g. Sierpinski's triangle
Numbers and points on a plane: coordinate and vectors	Numbers and points on planes Position vectors Numbers and transformations on planes Free vectors	**Understand** and **know** how an ordered pair of numbers denote a point in a plane, given three points (O, I, J) non-collinear as a coordinate system, $\overrightarrow{OM} = x \cdot \overrightarrow{OI} + y \cdot \overrightarrow{OJ}$ **Use** vectors as translation operators: addition and subtraction, zero vector, opposite vector, parallelogram **Multiply** a vector by a scalar **Use** linear combination of position vectors in geometric problems **Prove** and **apply** the parallelogram formula $\overrightarrow{AB} = \overrightarrow{CD} \Leftrightarrow \overrightarrow{AC} = \overrightarrow{BD}$ **Investigate** collinear vectors and parallel lines	 	• From line to plane (one-dimensional objects to two dimensional objects), numbers and geometry • Continuing modelling and computing for proving geometric properties Problems from physics: composition of forces and movements in a plane

YEAR 4 (6P)	TOPIC: GEOMETRY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Vectors and trans-formations	Operators in the plane	**Discover** that parallel projections, translations, enlargements and rotations of 180° all map equal vectors onto equal vectors		Isometric transformations of the plane, displacements

YEAR 4 (6P)	TOPIC: STATISTICS AND PROBABILITY		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Data collection	Types of data	**Analyse** different types of data in a concrete situation, being either nominal, ordinal (both categorical), interval or ratio (both numerical)	• Classify statistical data, as they appear in news media according to their measurement levels • Make students find misleading statistical representations, e.g. Simpson's paradox Use data from scientific sources
	Interpreting data	**Develop** a critical attitude towards interpreting data and the (mis)use of statistics	
Organise data	Absolute, relative, cumulative frequencies	**Understand** the meaning of absolute, relative, and cumulative frequencies **Calculate** these different types of frequencies (for relative frequencies both in terms of proportions and percentages) using raw data, both by hand and a spreadsheet or other appropriate digital technology **Use** relative frequencies to **compare** different data sets *Limitation: do not use too many data, and make students aware of the limitations of this representation*	Use data from scientific sources
	Frequency table	**Organise** data in a frequency table, by hand and through spreadsheet software or other appropriate digital technology, including all of the above frequency types	Make students change frequency tables according to specific purposes

YEAR 4 (6P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Stem-and-leaf diagram	**Organise** a small set ($N \leq 25$) of numerical data in a stem-and-leaf diagram		Make stem-and-leaf plots using data collected on your students e.g. the number of books read last year
Data set characteristics	Measures of central tendency	**Understand** the meaning of different measures of central tendency **Know** when to apply which measure appropriately **Know** how to interpret its value		Use data from scientific sources
	Mode	**Identify** and **interpret** the mode in an appropriate data set, also in case the data are presented in a frequency table		Make students compare the impact of adding or deleting one extreme value on the three measures *Limitation: only use these measures for non-categorical data*
	Mean	**Calculate** and **interpret** the (arithmetic) mean of a set of data at interval or ratio measurement level, also in case the data are presented in a frequency table		
	Median	**Determine** and **interpret** the median of a set of data, also in case the data are presented in a frequency table		
	Quartiles	**Understand** the meaning and **determine** the quartiles of a set of data		
	Measures of spread	**Understand** the meaning of different measures of spread and **know** when to **apply** which measure appropriately and how to interpret its value *Note: this is just meant to make students*		

YEAR 4 (6P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		experience the calculation procedure		
	Range	**Understand** the meaning of the range and **calculate** it for a set of data		
	Interquartile range (IQR)	**Understand** the meaning of the interquartile range and **calculate** it for a set of data		
Graphical re-presentations	Interpret graphical representations	**Interpret** graphical representations and **use** them to find estimations of central tendency and spread *By hand and while using spreadsheet software or other appropriate digital technology.*		Make students choose between different possible representations of the same data set from a scientific or daily life context
	Bar chart	**Understand** and **interpret** a given bar chart and **create** it for an appropriate data set		
	Cumulative frequency polygon	**Understand** and **interpret** a given cumulative frequency polygon and **create** it for an appropriate data set		
	Histogram	**Understand and interpret** a given histogram and **create** it for a set of grouped data at interval or ratio measurement level provided in a table with equal class width *Note: make students aware of the difference between a histogram and a bar chart and prepare them for continuous probability distributions in years 6 and 7*		
	Density in histograms	**Understand** the notion of density in		

YEAR 4 (6P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		histograms and **create** a histogram for a set of grouped data at interval or ratio measurement level provided in a table with unequal class width		
	Boxplot	**Understand and interpret** a given boxplot and **create** it for a set of data at interval or ratio measurement level		
Datasets	Frequency distributions	**Compare** and **interpret** frequency distributions with respect to their: • means and medians • interquartile ranges • histograms and box plots		
	Histograms and boxplots	**Show simultaneously** and **compare** histograms and box plots for two different distributions		
Probability	Sample space	**Define** the sample space in a random experiment *Note: Make students aware that outcome sets may be infinite*		
	Event	**Understand** that an event is a subset of the set of all possible outcomes		
	Venn diagram	**Use** a Venn diagram to represent the set of possible outcomes and events		Make a Human Venn diagram.
	Tree diagram	**Represent** events using a tree diagram *Limitation: no more than three sets of branches in the tree diagram*		

YEAR 4 (6P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Concept of complementary, independent, mutually exclusive, and exhaustive events	**Understand** the idea of probability leading on from relative frequency **Determine** probabilities using Venn diagrams and tree diagrams		Make students distinguish between experimental and theoretical probability, e.g. tossing a coin, rolling a dice, ...

S5 – 6 Period (6P)

YEAR 5 (6P)	**TOPIC: ALGEBRA**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Powers and algebraic expressions	Negative and rational indices	**Calculate** powers with negative and rational indices **Understand** the relationship between rational indices $\left(\frac{1}{n}, \frac{m}{n}\right)$ and radicals **Use** negative and rational indices to rewrite scientific formulae, e.g. $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\left(\frac{l}{g}\right)^{\frac{1}{2}}$ or $l = g\left(\frac{T}{2\pi}\right)^2$ **Convert** between scientific notation with positive and negative indices and standard form using a technological tool		Relation with all kind of formulae from physics, chemistry and biology, e.g. Avogadro, metric system, … Formulae can be expressed in many ways using negative and rational indices instead of divisions and radicals
Quadratic models and formulae	Quadratic equations (the three forms of a quadratic function and its graph: $y = ax^2 + bx + c$ $y = a(x - r)(x - s)$ $y = a(x - p)^2 + q$)	**Use** factorisation to solve a quadratic equation **Solve** a quadratic equation by completing the square **Apply** the formula for the general solution of a quadratic equation **Understand** the relation between the coefficients and the nature of the solutions of the general equation by using the discriminant		François Viète, the Father of Modern Algebra, Viète's formula for quadratics Proof that completing the square method leads to the quadratic formula Parabolas in art and in architecture $f(x) = ax + b$, $g(x) = cx + d$ and $h(x) = f(x) \cdot g(x)$; in a graphing software environment, manipulate a, b, c and d to generate specific effects on the graph of h, such as: • opened upwards

YEAR 5 (6P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
				• touching the horizontal axis • vertex coincides intersection points of the two lines
		Solve problems leading to a quadratic equation including worded problem		Use quadratic models in physical contexts, such as fall movements or projectile trajectories
		Determine the axis of symmetry and the coordinates of the vertex of a parabola, graphically and algebraically **Calculate** the zeros of a quadratic function and interpret them geometrically		
		Use graphs to **find** the values of x when $f(x) < 0$ and when $f(x) > 0$		When there is a root a , then $f(x) = (x - a) \cdot g(x)$
		Determine algebraically and geometrically the intersection of a straight line and a parabola		
Exponential (growth) models and formulae	$y = C \cdot a^x$	**Investigate** models that describe exponential growth and decay: • multiplication per (time) unit • growth factor: $a > 1$ and $0 < a < 1$ • standard model $y = C \cdot a^x$ **Use** technology to investigate the graphs of exponential growth **Solve** equations and inequalities from practical situations using a technological tool		Exponential models from: • physics, chemistry and biology, e.g. radioactive decay, cell division, … • economics: interest, inflation, …

YEAR 5 (6P)	TOPIC: ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Introduction of logarithms	Logarithms with a positive integer base	**Understand** the concept of logarithm with a positive integer base *Note: logarithms should be introduced from powers* **Use** the basic properties of indices and logarithms with a positive integer base **Solve** simple exponential and logarithmic equations of the type: • $2^x = 4^{2x+1}$ • $\log(3x) = \log(2x+9)$ • $\log_3 \mathrm{x} + \log_3 7 = \log_3 49$ **Check the answers** in the original equations.	• Compound interest rate • Growth of computer and biological virus
Periodic models	Trigonometric functions	**Investigate** trigonometric functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ and their expansion to domain $\mathbb{R}$	Use and show periodic models from physics (waves) and biology (prey/predator) Investigate graphs of $y = a \cdot \sin\big(b(x+c)\big) + d$ for different values of a, , b , c and d and explore the impact of each parameter
Algorithm and program notions	Simple programming	**Use** a software to break down a problem into sub-problems, and write, test and execute a simple program **Create** a flow chart for basic algorithm **Know** how to assign labels to	Estimate a root of a quadratic: Heron algorithm.

YEAR 5 (6P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		variables in a program **Understand** and **apply** different types of conditional instructions **Understand** and **apply** different types of computer loops and notions		

YEAR 5 (6P)	TOPIC: GEOMETRY		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Measure of angles	Degrees Radians	**Investigate** two unit-systems to measure angles: degrees and radians **Know** and **understand** the trigonometric ratios for a set of standard angles in degrees and radians: • $0°$, $30°$, $45°$, $60°$ and $90°$ • 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$ radians	
Unit circle	Trigonometric formulae	**Define** an oriented angle and **represent** it in the unit circle **Calculate** the trigonometric ratios of an oriented angle, and of angles associated with it, both in degrees and radians **Compare** the trigonometric ratios of an angle with those of its: • complementary angles: $\cos q = \sin(90 - q) = \sin\left(\frac{\pi}{2} - q\right)$ • supplementary angles: $\sin q = sin(180 - q\ ) = \sin(\pi - q)$ • the Pythagorean identity: $\cos^2(p) + \sin^2(p) = 1$ **Solve** equations of the type: • $\sin q = a$, $\cos q = a$ and $\tan q = a$	Use geometric models used in physics, chemistry and biology • Measuring the earth • Astronomical measures

YEAR 5 (6P)	TOPIC: GEOMETRY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		• $\cos\left(q+\frac{\pi}{6}\right)=\frac{1}{2}$ • $3\cos^2(q)-\sin q-1=0$ **Know** and **use** (only with numerical values of angles) formulae of the type • $\cos(a \pm b) =$ $\cos a \cos b \mp \sin a \sin b$ • $\sin(a \pm b) =$ $\sin a \cos b \pm \cos a \sin b$ • $\sin(2a) = 2\sin a \cos a$ • $\cos(2a) = \cos^2 a - \sin^2 a$ **Use and prove** the following formulae for any triangle: • $a^2 = b^2 + c^2 - 2\, b \cdot c \cdot \cos\alpha$ • $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ • Area = $\frac{1}{2}\, b \cdot \mathrm{c} \cdot \sin\alpha$ **Apply** the formulae to determine angles and sides of triangles, including real life problems		
Vectors in two dimensions	Basis Coordinate system	**Recognise** linearly dependent and independent vectors, a basis, a coordinate system and the dimension of a vector space **Define** a basis and a coordinate system **Express** a vector as a linear		Explore the mathematical properties of a new mathematical object: vectors

YEAR 5 (6P)	TOPIC: GEOMETRY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Linear combination of two vectors	combination of two given vectors that form a basis		
	Parallelism of two vectors	**Show** the bijection which exists between the set of vectors and the set of ordered pairs of real numbers		
	Scalar product of two vectors	**Define** and calculate the scalar product of two vectors		
	Orthogonality of two vectors	*Note: This concept allows to make calculations in plane and solid geometry, especially on parallelism and orthogonality*		
		Investigate geometrical properties of the scalar product		Vectors in computer games development
		Define the scalar product of a vector with itself		
		Define the magnitude of a vector		
		Define the orthogonality of two vectors and use the properties of the scalar product		
		Express the scalar product of two vectors in terms of their magnitudes and the cosine of the angle between them		
		Use the scalar product to verify orthogonality		
	Scalar product in an orthonormal basis	**Express** a scalar product in an orthonormal basis		

YEAR 5 (6P)	TOPIC: GEOMETRY			
Subtopic	**Content**	**Learning objectives**		**Key contexts, phenomena and activities**
		Calculate the distance between two points, $AB^2 = \left\|\overrightarrow{AB}\right\|^2 = \overrightarrow{AB}^2$		
Lengths and distances in 3D objects	Plane sections of solids	**Apply** Pythagoras' Theorem and the intercept theorem to plane sections of solids		Chemical models, Physical models
	Internal diagonal Edges Height of a cone	**Calculate** the internal diagonal of a cube or cuboid, the edges of a pyramid or the height of a cone with particular angles **Recognise** and **solve** real problems which can be modelled with regular solids		Use examples from real life which can be modelled with regular solids (church towers, houses, containers, …)

YEAR 5 (6P)	**TOPIC: STATISTICS AND PROBABILITY**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Probability	Probability laws	**Know** the following probability rules and apply them to solve problems: • $0 \leq \mathrm{P}(A) \leq 1$ • $\mathrm{P}(\bar{A}) = 1 - \mathrm{P}(A)$ • $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ • $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ for mutually exclusive events		Chevalier de Mere's probability problem in dice game
	Conditional probability	**Understand** the concept of conditional probability and the notations $\mathrm{P}_B(A)$ and $\mathrm{P}(A\|B)$ **Use** information from Venn diagrams, tree diagrams, contingency tables and the formula $\mathrm{P_B}(A) = \mathrm{P}(A\|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ to calculate conditional probability		Monty Hall problem
	Independent events	**Understand** the concept of independent events **Use** the formulae $\mathrm{P}(A \cap B) = \mathrm{P}(A) \cdot \mathrm{P}(B)$ and $\mathrm{P}_B(A) = \mathrm{P}(A\|B) = \mathrm{P}(A)$ to check if two events are independent		
Sampling	Population Random sample Sample variation Simulate sampling	**Recognise** populations and random samples in everyday life situations and explain the difference between the two **Know** that different samples will show variation **Use** a digital tool to simulate statistical		Provide examples from scientific contexts; make students really experience sample variation through sampling or simulation

YEAR 5 (6P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		sampling and interpret the results		
Data set characteristics	Measures of spread Standard deviation Calculation of standard deviation Linear transformation of data Comparing data sets	**Understand** the meaning of the standard deviation, **interpret** it and **calculate** it using a technological tool **Calculate** for a small sample size ($n \leq 6$) variance and standard deviation by hand, using one of the following formulas • $\sigma = \sqrt{\sum_{i=1}^{i=n} \frac{(x_i - \bar{x})^2}{n}}$ • $\sigma = \sqrt{\sum_{i=1}^{i=n} \frac{x_i^2}{n} - \bar{x}^2}$ **Know** and **use** the impact of a linear transformation of data on measures of central tendency and spread **Compare** and **interpret** frequency distributions with respect to their: • means and standard deviations • medians and interquartile ranges **Compare** histograms and box plots for two different distributions		Use data from scientific sources
Inference	Statistical inference	**Know** that statistical inference concerns making claims about a population based on a sample and **know** that we can never be sure about these types of inferences		Use data from scientific contexts, as well as daily life contexts

YEAR 5 (6P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Generalise from sample to population	**Generalise** findings from a sample to an underlying population, considering the uncertainty of these generalisations		Examples: biased and unbiased samples when carrying out surveys, e.g. cinema going survey, human height survey, …
	Statistical enquiry cycle	**Know** the main elements of the statistical enquiry cycle, i.e., problem definition, data collection plan, data collection, data analysis, conclusion		Carry out a small statistical investigation in which the different phases of the statistical enquiry cycle are addressed

5. Assessment

For each level there are attainment descriptors written by the competences, which give an idea of the level that pupils have to reach. They also give an idea of the kind of assessments that can be done.

With the competences are verbs that give an idea of what kind of assessment can be used to assess that goal. In the table with learning objectives these verbs are used and put bold, so there is a direct link between the competences and the learning objectives.

Assessing content knowledge can be done by written questions where the pupil has to respond on. Partly that can be done by multiple choice but competences as constructing explanations and engaging in argument as well as the key competences as communication and mathematical competence need open questions or other ways of assessing.

An assignment where pupils have to use their factual knowledge to make an article or poster about a (broader) subject can be used to also judge the ability to critically analyse data and use concepts in unfamiliar situations and communicate logically and concisely about the subject.

In Europe (and America) pupils must have some competence in designing and/or engineering (STEM education). So there has to be an assessment that shows the ability in designing and communicating. A design assessment can also show the ability in teamwork.

Pupils have to be able to do an (experimental) inquiry. An (open) inquiry should be part of the assessments. Assessing designing and inquiry can be combined with other subjects or done by one subject, so pupils are not obliged to do too many designing or open inquiry just for assessment at the end of a year.

Digital competence can be assessed by working with spreadsheets, gathering information from internet, measuring data with measuring programs and hardware, modelling theory on the computer and comparing the outcomes of a model with measured data. Do combine this with other assessments where this competence is needed.

Assessment is formative when either formal or informal procedures are used to gather evidence of learning during the learning process and are used to adapt teaching to meet student needs. The process permits teachers and students to collect information about student progress and to suggest adjustments to the teacher’s approach to instruction and the student’s approach to learning.

Assessment is summative when it is used to evaluate student learning at the end of the instructional process or of a period of learning. The purpose is to summarise the students’ achievements and to determine whether, and to what degree, the students have demonstrated understanding of that learning.

For all assessment, the marking scale of the European schools shall be used, as described in “*Marking system of the European schools: Guidelines for use*” (Ref.: 2017-05-D-29-en-7).

5.1. Attainment Descriptors

	A	B	C	D	E	F	FX
	(9,0 - 10 Excellent)	(8,0 - 8,9 Very good)	(7,0 - 7,9 Good)	(6,0 - 6,9 Satisfactory)	(5,0 - 5,9 Sufficient)	(3,0 - 4,9 Failed / Weak)	(0 - 2,9 Failed / Very weak)
Knowledge and comprehension	Demonstrates comprehensive knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows broad knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows satisfactory knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows satisfactory knowledge and understanding of mathematical terms, symbols and principles in most areas of the programme	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and principles	Shows partial knowledge and limited understanding of mathematical terms, symbols, and principles	Shows very little knowledge and understanding of mathematical terms, symbols and principles
Methods	Successfully carries out mathematical processes in all areas of the syllabus	Successfully carries out mathematical processes in most areas of the syllabus	Successfully carries out mathematical processes in a variety of contexts	Successfully carries out mathematical processes in straightforward contexts	Carries out mathematical processes in straightforward contexts, but with some errors	Carries out mathematical processes in straightforward contexts, but makes frequent errors	Does not carry out appropriate processes
Problem solving	Translates complex non-routine problems into mathematical symbols and reasons to a correct result; makes and uses connections between different parts of the programme	Translates non-routine problems into mathematical symbols and reasons to a correct result; makes some connections between different parts of the programme	Translates routine problems into mathematical symbols and reasons to a correct result	Translates routine problems into mathematical symbols and reasons to a result	Translates routine problems into mathematical symbols and attempts to reason to a result	Translates routine problems into mathematical symbols and attempts to reason to a result only with help	Does not translate routine problems into mathematical symbols nor attempts to reason to a result

Interpretation	Draws full and relevant conclusions from information; evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information, evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information and attempts to evaluate reasonableness of results	Attempts to draw conclusions from information given, shows some understanding of the reasonableness of results	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Makes little attempt to interpret information	Does not interpret information
Communication	Consistently presents reasoning and results in a clear, effective and concise manner, using mathematical terminology and notation correctly	Consistently presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results adequately using mathematical terminology and notation	Generally presents reasoning and results adequately; using some mathematical terminology and notation	Attempts to present reasoning and results using mathematical terms	Displays insufficient reasoning and use of mathematical terms
Digital competence	Uses technology appropriately and creatively in a wide range of situations	Uses technology appropriately in a wide range of situations	Uses technology appropriately most of the time	Uses technology satisfactorily most of the time	Uses technology satisfactorily in straightforward situations	Uses technology to a limited extent	Does not use technology satisfactorily

Annex 1: Suggested time frame

For this cycle, the following topics are described with only an estimated number of weeks to be reviewed by the teacher depending on the class.

Note: The designated weeks include assessments, time needed for practice and rehearsal, mathematics projects, school projects, etcetera.

Course	**S4P6**	**S5P6**
Topic	**Weeks**	
Algebra	13	11
Geometry	10	11
Statistics and Probability	9	8
Total	**32**	**30**

Annex 2: Modelling

Physicists know very well, that it requires mathematics to describe the world. But also, many other scientific subjects like chemistry, economics or physical education use mathematical knowledge to explain phenomena or predict results. Above this, we live in the digital age and artificial intelligence will become a very important part of our lives.

When we talk about applied mathematics, we will find many examples, where problems are solved by using a modelling circle (see Figure 1). The weather forecast is such an example that comes up in our daily lives.

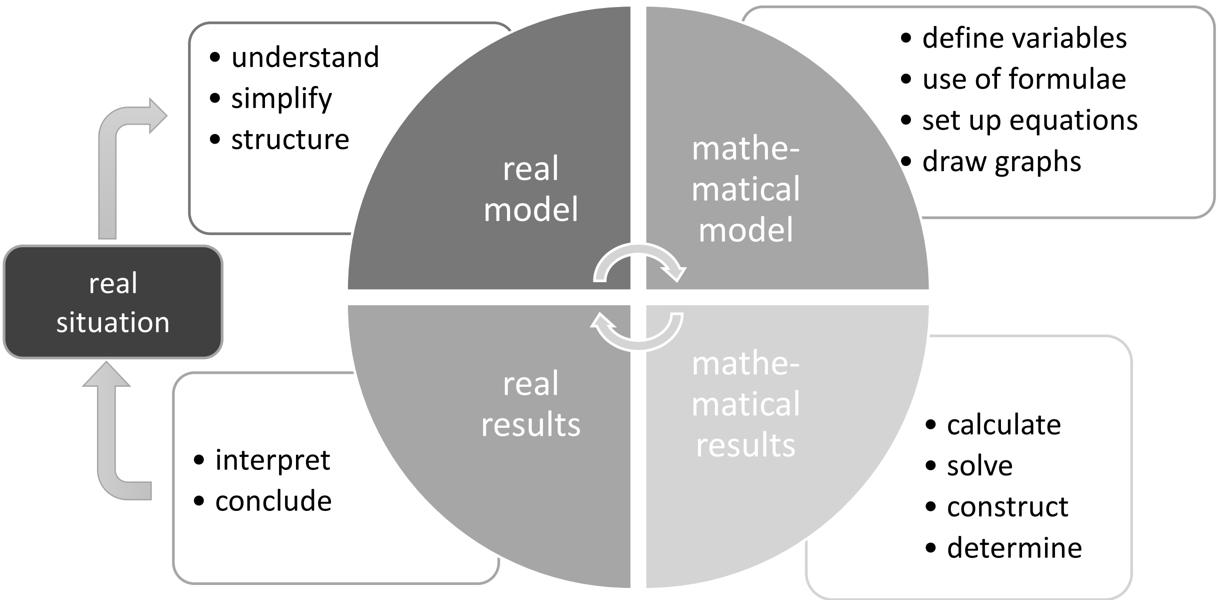

Figure 1: Modelling Circle (Borromeo-Ferri, 2006)

In this case, the *real situation* consists of data about the present weather and its development within the days before.

To create a *real model* for the weather, we need to understand for example, how humidity is related to the temperature. The weather is a very complex phenomenon, so we need to simplify and structure it before we can move on to the next step.

The *mathematical model* is based on many different equations that contain for example derivatives, Integrals and random variables.

To find *mathematical results* we need very fast computers that provide lots of data.

The *real results* can be found by interpreting the data and draw conclusions.

After going through the first circle the same procedure will be done again with slightly different results. So, in the end we can say for example, that the probability of rain for the next day is 20% because in 2 out of 10 trials this was our result.

One day later the results can be compared with the real situation to improve our model for the future. That the prediction is not always right shows, that we are dealing with a model. But when we look back for 10 years it is obvious, that the model has been optimised constantly.

When artificial intelligence is learning, the modelling circle plays an important role. In the 5 Period and the 6 Period courses we have pupils who might study informatics and develop new devices in the future. So, it is necessary that they are trained well in the complete circle. In the 3 Period and 4 Period courses we have pupils who use these devices. Therefore, it is important for them to understand, how these devices work in general. They should be able to question these devices and be critical towards the results given to them.

The modelling circle is quite complex itself, so it is recommended, that pupils are trained on single steps before putting them together or even undertake more than one stage. For the teacher it is possible to create tasks and exercises. The following example is to demonstrate how an existing question can be extended so that it even allows differentiation within the course.

Example of real situation: a sprinkler is used to water the garden.

1. To understand how the sprinkler works, the pupils can do some research on the internet, try it themselves and describe the way of the water in their own words. For simplification they might reduce the number of dimensions and consider the initial speed and the angle of the water jet as constant.
2. To mathematise this model, the pupils might see that the water follows a parabola. They could measure or estimate the maximum height and length of the water jet. This data can also be given to them. However, they should be able to develop a quadratic equation that describes the water jet. That means, that they must adjust certain parameters. Finally, they need to ask questions like: What is the maximum height/width of the water jet? The pupils must find solution approaches like $f(x) = 0$.
3. To find mathematical results, they need to solve quadratic equations with or without the calculator or find out the maximum or the intersection point with the x-axis.
4. To get reals results, the solutions must be interpreted, answers to the questions must be given in a text and numbers must relate to units.
5. The last step is to look at the real situation again. Do the results match with the measurements? Is the model good enough? What can be optimised? Are there other questions that can be asked? Can we reduce the simplifications that were made in the beginning?

This example shows, how many possibilities can be given to a mathematics lesson by a simple situation from daily live. Single steps can be trained or be connected with others. The amount of information that is given, can be adjusted to bear in mind the individual skills of the pupils. Some pupils can work faster and go further in their investigations. All pupils can explore and see, that mathematics is connected to their daily lives. By using the modelling circle, it is possible to teach and train all key competences that are stated in the syllabus.