MATHEMATICS WORKSHEET

XI Grade (Semester 1)

Chapter 1

STATISTICS

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PREFACE

Mathematics Module “STATISTICS” is written for students of St. Albert Senior High School at XI Grade Semester 1.

The contents are arranged under some worksheets which the students can fill to learn Statistics. Each worksheet is expanded in detail and step by step manner for easy understanding. It starts with a brief introduction and explanation follwed by filled examples and numerous simple exercises to build up the student’s technical skills and to reinforce his or her understanding.

It is hoped that this approach will enable the individual student working on his or her own to make effective use of the module besides enabling the teacher to use them with mixed ability groups.

Finally, we would like to thank to all those involved in the production of this module.

Ignatia Maria Midawati

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TERM in MATHEMATICS
STATISTICS

NO	Mathematics Expressions	How to read in English
1.	$5 + 2 = 7$	Five plus two is equal to seven
2.	$9 - 3 = 6$	Nine minus three is equal to six
3.	$7 \times 9 = 63$	Seven times nine is equal to sixty three
4.	$12 : 3 = 4$	Twelve devided by three is equal to four
5.	$\frac{1}{2}$, $\frac{1}{4}$	a half , a quarter
6.	$\frac{2}{3}$, $\frac{3}{5}$, $2\frac{1}{3}$	Two thirds , three fifths , two and one third Two over three, three over five, two and one over three
6.	$\frac{2}{3}$, $\frac{3}{5}$, $2\frac{1}{3}$	Two over three, three over five, two and one over three
7.	$a \neq b$	a is not equal to b
8	$a > b$	a is greater than b
9.	$a \geq b$	a is greater than or equal to b
10	$a < b$	a is less than b
11.	$a \leq b$	a is less than or equal to b
12.	34,528	Thirty four thousands five hundred twenty eight
13.	34.528	Thirty four point five two eight
14.	45^0	Forty five degrees
15.	20 – 30	Interval twenty until thirty
16.	$20 < x \leq 30$	Twenty is less than x and x is less than or equal to thirty
17.	$20 < x < 30$	x is between twenty and thirty
18.	$20 < x < 30$	Twenty is less than x and x is less than thirty

Worksheet 1st
Topic : Collecting & Organizing Data, Graphical representation
TIME : 4 X 45 minutes

STANDARD COMPETENCY :

1. To use the rules of statistics, the rules of counting, and the properties of probability in problem solving.

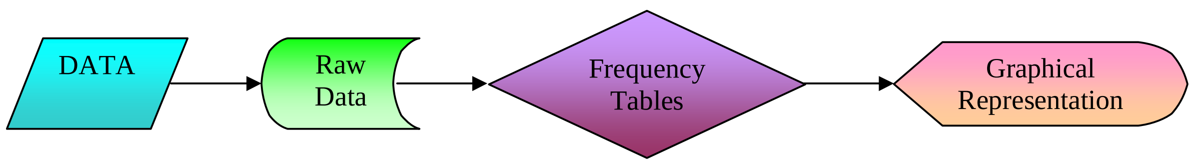
BASIC COMPETENCY:

1.1 To read and present the data in a frequency table and bar chart, line chart, pie chart of singular data with its interpretation.

In this chapter, you will learn :
- How to collect and organize data
- Graphical representation

Statistics is the branch of mathematics in which facts and information are collected, sorted, displayed, and analyzed. Statistics are used to make decisions and predict what may happen in the future.

A. Collecting and organizing data



Data that have been collected but not organized in any way are called ***raw data***. Raw data are difficult to interpret, so it can be arranged in a ***frequency table***. A frequency table shows the number of times (frequency) each value occurs.

Tallying is a system of recording and counting results using diagonal lines grouped in fives. Each time five is reached, a horizontal line is drawn through the tally marks to make a group of five. The next line starts a new group.

B. Graphical representation

1. Pictograms
   A pictogram is a simple way of representing data. Frequency is indicated by identical pictures (called symbols or motifs) arranged rows or columns.

2. Bar graphs
   In bar charts or bar graphs, data are represented in a series of bars that are equally wide. The width itself is not significant, but all the bars should be the same width.

3. Pie charts (circle graph)
   A pie chart is a circle graph in which the angles of the sectors represent the frequency.

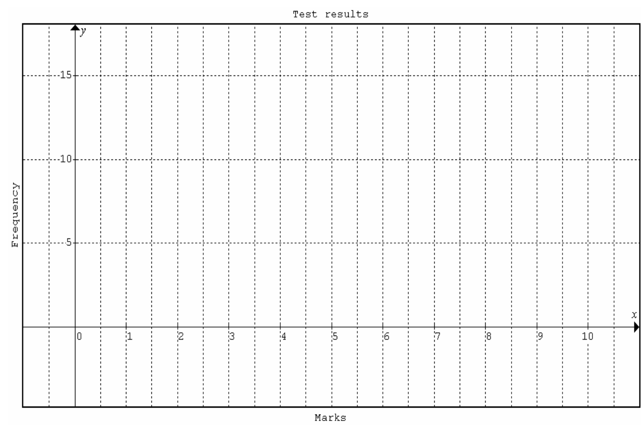
Example 1

The marks obtained by 50 students in a class test are given on the below. Make a frequency table for the given marks. Draw a bar graph to represent the data.

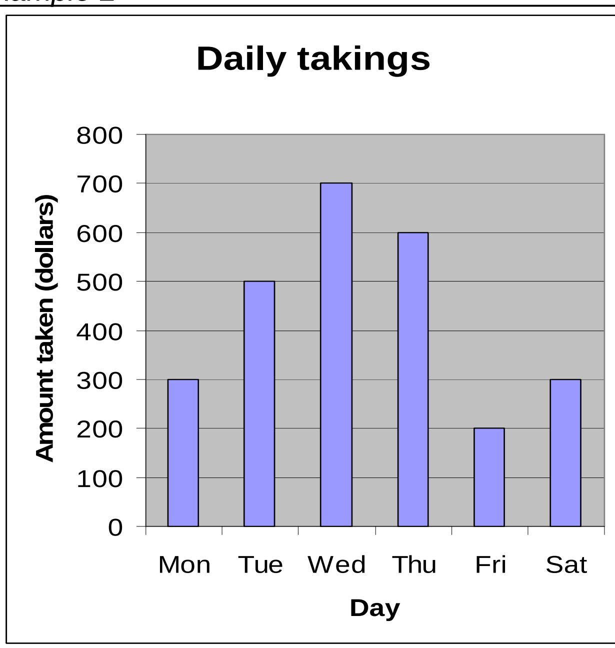
Raw data				
10	3	6	4	7
7	4	5	6	9
4	8	6	7	5
5	6	7	5	4
6	5	6	9	1
8	2	3	4	1
7	5	4	6	7
6	4	5	6	8
7	5	6	1	6
5	4	6	7	7

Solution

Marks	Tally	Frequency
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total number		



Example 2



This bar graph shows the takings of a small business.

a. On which day was the smallest amount of money collected?

b. On which day was the largest amount of money collected?

c. Find the total money collected for the week.

Solution

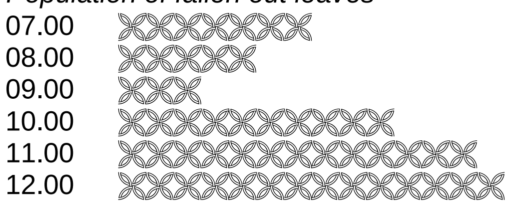
a. ……………… – it has the shortest bar.

b. ……………… – it has the longest bar.

c. The total money collected for the week = ………………………………………………..

Example 3

Population of fallen out leaves



Look at the left pictogram.

a. How many leaves are fallen out at 10.00?

b. How many leaves are the most fallen? When?

Solution

a.

b.

Example 4

The table below shows how a student spends her day.

Activity	*School*	*Sleeping*	*Homework*	*Eating*	*Other*
No. of hours	*8*	*8*	*3*	*1*	*4*

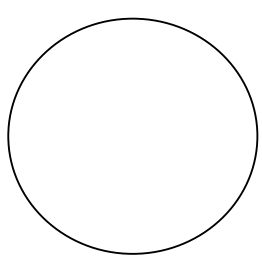
Show this on a pie chart!

Solution

Total no. of hours = ……
School : ……..
Sleeping : ……..
Homework : ……..
Eating : ……..
Other : ……..

Activity	School	Sleeping	Homework	Eating	Other
Angle					

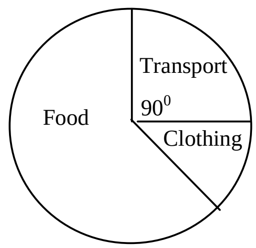
Draw the pie chart! Label the graph and give it a title!



Example 5

A person's expenditure each month is $1200, split as shown in the pie graph on the below.

a. If the angle at the centre in the transport sector is 90^0, what amount of money is spent on transport?
b. If this person spends $700 on food, find the angle at the centre of this sector.
c. What fraction of this person's expenditure is on clothing?



Solution

Exercise 1

1. A survey recorded the number of people living in each of 40 houses. The numbers were as follows:

3	4	2	4	3	2	2	5	4	3
4	1	2	6	3	5	5	2	4	1
4	3	4	2	4	4	6	2	4	3
2	5	4	5	6	4	2	3	2	4

   a. Make a frequency table.
   b. Draw a bar graph to illustrate your results.
   c. What is the total number of people living in these 40 houses?

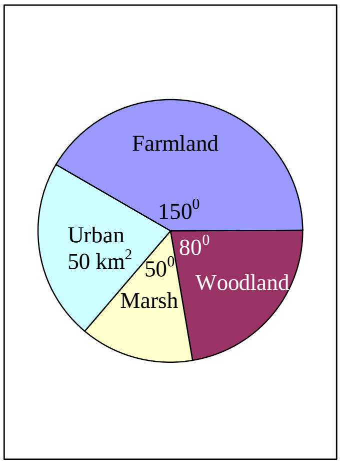
2. The table below shows how an income of $400 was spent. Show these data on a bar graph and a pie chart.

	Food	Rent	Clothing	Transport	Savings
Amount	$120	$80	$40	$110	$50

3.

The pie chart on the right, which is not drawn to scale, shows the distribution of various types of land in a district.
Calculate:

a. the area of woodland as a fraction of the total area shown,
b. the angle of the urban sector,
c. the total area of the district.

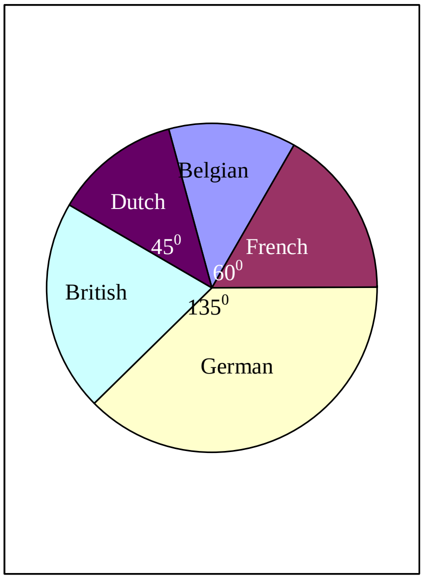


4. A number of students were asked to name their favorite sport. $\frac{1}{4}$ of the students said tennis, $\frac{1}{8}$ said rugby, $\frac{1}{3}$ said football and the rest said swimming.
   a. What fraction said swimming?
   b. Calculate the value of x, if x is the angle of the sector representing rugby in the pie chart.
   c. If 32 students chose football, how many said tennis?

5.

The pie chart on the right, which is accurately drawn, shows the nationalities of people staying in a holiday hotel.

a. Which of these five nationalities had the smallest number of people in the hotel?
b. What fraction of the people in the hotel were French? Give the answer in its lowest terms.
c. Write the answer to b). as a percentage correct to the nearest whole number.
d. Write the ratio $\dfrac{number\ \ of\ \ Germans}{total\ \ number\ \ of\ \ people}$ *as a decimal.*
e. If there were 288 people in the hotel altogether, how many of them were Dutch?



6. Make a simple research of any data that you can collect in your daily life. Collect the data and make the frequency table and the charts.

7. Find a chart from any newspapers, make a conclusion from that chart.

Worksheet 2nd
Topic : Mean, Median, Quartile and Mode of singular data
TIME : 3 X 45 minutes

STANDARD COMPETENCY :

1. To use the rules of statistics, the rules of counting, and the properties of probability in problem solving.

BASIC COMPETENCY:

1.2. To calculate the centre of measurement, the location of measurement, the dispersion of measurement from the singular data, altogether with its interpretations.

In this chapter, you will learn:

- How to determine the centre of measurement: averages – mean, median, mode, quartile from singular data.
- How to determine the location of measurement: quartile from singular data.

C. Averages – mean, median, and mode

The Mean

The mean is the most common type of average. It is the number obtained by dividing the sum of all the items by the number of items.

$$\text{Mean} = \bar{x} = \frac{\sum x_i}{n} \quad \text{or} \quad \text{Mean} = \bar{x} = \frac{\sum f_i . x_i}{\sum f_i}$$

Example 6

Find the mean of 8,9,7,6,8, and 10.

Solution

$$\text{Mean} = \bar{x} = \frac{\ldots + \ldots + \ldots + \ldots + \ldots + \ldots}{\ldots} = \frac{\ldots}{\ldots} = \ldots$$

Example 7

Calculate the mean for the following frequency distribution.

Test marks	1	2	3	4	5	6	7	8	9	10
Frequency	2	3	4	10	5	3	2	1	0	1

Solution

Test marks (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	2	3	4	10	5	3	2	1	0	1
fx										

$$\bar{x} = \frac{\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots}{\ldots} = \frac{\ldots}{\ldots} = \ldots$$

*) The ***Mean*** can be determined by ***Assumed Mean***

$$\text{Mean} = \bar{x} = A + \frac{\sum fu}{\sum f}$$, A = assumed mean

Test marks (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	2	3	4	10	5	3	2	1	0	1
u										
fu										

$$\bar{x} = A + \frac{\sum fu}{\sum f} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Example 8

Tickets for a circus have been sold at the following prices: 180@$6.50, 215@$8, 124@$10.

a. What is total amount of money received for the tickets?

b. What is the mean price of tickets sold (to 3 significant figures)?

Solution

a. Total amount of money is = ……………………………..

b. $\bar{x} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots$

Example 9

The average height of 50 students is 162.3 cm. If the teacher is included, then the average height becomes 162.34 cm. Find the height of the teacher.

Solution

For example: the height of the teacher is x cm. If $\bar{x} = \frac{\sum f_i . x_i}{\sum f_i}$, so

$$\ldots\ldots = \frac{50\times\ldots\ldots + x}{\ldots\ldots}$$

$$\ldots\ldots\times\ldots\ldots = \ldots\ldots\ldots + x$$

$x = \ldots\ldots\ldots$. So the height of the teacher is ……… cm

The Median

In order to find the median of a set of data, these must be arranged in ascending or descending order. ***The median*** is the central or middle figure.

For an odd number of items, the median is the value of the item that is in the middle.
For an even number of items, the median is the mean value of the two middle items.

Example 10

Find the median of the following scores:

20 70 50 30 35 45 75 15 90

Solution

Arrange the data in ascending or descending order. In ascending order,

……, ……, ……, ……, ……, ……, ……, ……, …….

There are ……. scores, so the …… is the middle one. Thus the median is ……

In descending order,

……, ……, ……, ……, ……, ……, ……, ……, …….

There are … … scores, so the … … is the middle one. Thus the median is … …

Example 11

Find the median of the following scores:

6 5 3 8 4 2

Solution

Arrange in order:

……, ……, ……, ……, ……, …….

There is an even number, so the median is the mean value of the …… and ……scores.
Thus the median is ……

Example 12

The distribution of marks obtained by the students in a class is shown in the table below.

Mark obtained	0	1	2	3	4	5	6	7	8	9	10
Number of students	1	0	3	2	2	4	3	4	6	3	2

Find the median of this distribution.

Solution

The total number of students = ... + … + … + … + … + … + … + … + … + … + … = …

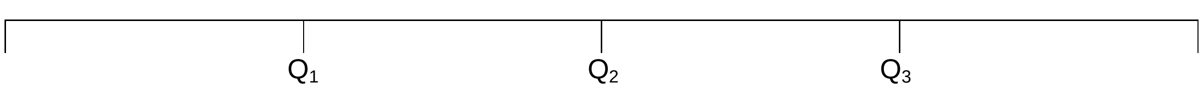
The median is the mean of the …… and …… marks.
Thus the median is ……

The Quartile

This is a measure of the middle half of data, so it is more representative. A distribution is divided into four subgroups by three ***quartiles***.
In ascending order,

- ✓ *The first or lower quartile (Q_1)* is the point below which 25% of the items lie and above which 75% of the items lie.
- ✓ *The second quartile (Q_2)* is the point below which 50% of the items lie and above which 50% of the items lie. You will realize that the second quartile is the same as the median.
- ✓ *The third or upper quartile (Q_3)* is the point below which 75% of the items lie and above which 25% of the items lie.
- ✓ If there are n values, in ascending order, then the lower quartile Q_1 is the $\frac{(n+1)}{4}$ th value, and the upper quartile Q_3 is the $\frac{3(n+1)}{4}$ th value.

Now, what do you think about descending order ?



Example 13

Find the Q_1, Q_2, Q_3 of the following scores:

20 70 50 30 35 45 75 15 90

Solution

Arrange the data in ascending or descending order. Ascending order:

……, ……, ……, ……, ……, ……, ……, ……, …….

Q_1 =

Q_2 =

Q_3 =

Example 14

Find the Q_1, Q_2, Q_3 of the following scores:

20 15 12 21 23 25 14 15

Solution

Arrange the data in ascending or descending order. Ascending order:

……, ……, ……, ……, ……, ……, ……, ……

Q_1 = ……………… ; Q_2 = ………………… ; Q_3 = ………………

The Mode

The ***mode*** of a set data is the value with the highest frequency. A distribution that has two modes is called bimodal. If there are more than two values that appear most frequently, than there is no mode such a distribution is non-modal.
The mode requires no calculation, only counting.

Example 15

Find the mode for the following distribution:

70 80 50 95 80 73 90 85

Solution

…… appears twice, so the mode is ……

Example 16

Find the mode for the following data:

3 10 6 3 5 3 8 1 8 5 4 2 8

Solution

Arrange the data in an array:
……………………………………………………………………………………………

…… and …… appear 3 times each. This is a bimodal distribution.

Example 17

Find the mode of the given distribution:

Marks	0	1	2	3	4	5	6	7	8	9	10
Students	2	1	1	2	6	10	7	6	3	1	1

Solution

The highest frequency is ……. However, remember that the mode refers to the actual data, so the modal value is …….

Exercise 2

1. Construct a frequency table for the following data and calculate the mean.

3	4	5	1	2	8	9	6	5	3
2	1	6	4	7	8	1	1	5	5
2	3	4	5	7	8	3	4	2	5
1	9	4	5	6	7	8	9	2	1
5	4	3	4	5	6	1	4	4	8

2. Find the median value of:
   a. 8, 1, 6, 7, 5, 2, 3
   b. 100, 75, 85, 95, 43, 99, 70, 60
   c. 2, 3, 1, 5, 6, 4
   d. 31, 28, 25, 21, 22, 20
   e. 41, 47, 42, 41, 47, 43, 45, 41

3. Find the mode of each of the following sets of numbers:
   a. 4, 5, 5, 1, 2, 9, 5, 6, 4, 5, 7, 5, 5
   b. 1, 8, 19, 12, 3, 4, 6, 9
   c. 2, 2, 3, 5, 8, 2, 5, 6, 6, 5
   d. 41, 47, 43, 41, 42, 45, 42.

4. A man kept count of the number of letters he received each day over a period of 60 days. The results are shown in the table below.

Number of letters per day	0	1	2	3	4	5
Frequency	28	21	6	3	1	1

   For this distribution, find:
   a. the mode
   b. the median
   c. the mean

5. A class contains 27 men and 23 women. If men's average height is 166 cm and women's average height is 157 cm, then find the average height of students in that class.

6. The average height of 50 students is 165.2 cm. If a new student is included, then the average height becomes 165.28 cm. Find the height of the new student.

7. Find Q_1, Q_2, and Q_3
   a. 34, 31, 20, 18, 50, 45, 31, 30
   b. 7, 6, 1, 3, 4, 5, 2
   c. 10, 11, 12, 13, 14, 15
   d. 80, 100, 60, 50, 70

8. Find Q_1, Q_2, and Q_3 for the distribution table.
   a.

Age	43	44	45	46	47	48	49	50
f	3	7	9	13	15	18	24	11

   b.

Test marks	1	2	3	4	5	6	7	8	9	10
Frequency	2	3	4	10	5	3	2	1	0	1

Worksheet 3rd	
Topic :	THE DISPERSION OF MEASUREMENT of singular data
TIME :	3 X 45 minutes

STANDARD COMPETENCY :

1. To use the rules of statistics, the rules of counting, and the properties of probability in problem solving.

BASIC COMPETENCY:

1.3. To calculate the centre of measurement, the location of measurement, the dispersion of measurement from the singular data, altogether with its interpretations.

In this chapter, you will learn :

- How to determine the statistic connected five
- How to calculate the dispersion of measurement: range, inter quartile, quartile deviation, mean-deviation, variance, standard deviation, from singular data.

D. Range, Inter quartile, Quartile deviation, Mean-deviation, Variance, Standard deviation

The Statistic Connected Five (=Statistik Lima Serangkai)

If $x_1, x_2, x_3, x_4, \ldots, x_{n-1}, x_n$ is a group of statistic data, x_1 = the lowest value, x_n = the highest value, Q_1 = the lower quartile, Q_2 = median, Q_3 = the upper quartile, then the Statistic Connected Five are x_1, x_n, Q_1, Q_2, Q_3

Example 18

Find the statistic connected five of the data

28	29	31	31	36	37	37	39	39
40	41	41	43	44	46	56	57	68

Solution

x_1 = the lowest value = ……

x_n = the highest value = ……

Q_1 = the lower quartile = ……

Q_2 = median = ……

Q_3 = the upper quartile = ……

Thus The Statistic Connected Five is

	Q_2 = ……
Q_1 = ……	Q_3 = ……
x_1 = ……	x_n = ……

Dispersion

The average (mean, median or mode) gives a general idea of the size of the data, but two sets of numbers can have the same mean while being very different in other ways.
The other main statistic we need to find is a measure of *dispersion* or *spread*. There are several ways of measuring dispersion.

Range (=R)

The range is the easiest measure of dispersion to calculate. It is defined as **the difference between the highest value and the lowest value**. The range is a crude measure of dispersion since it makes no use of the intermediate values and it can be distorted by one or two extreme values.

Range = R = the highest value – the lowest value

Inter-quartile range

The inter-quartile range = $Q_3 - Q_1$

Quartile Deviation(=QD)

The Quartile deviation = QD = ½ ($Q_3 - Q_1$)

Example 19

A student's marks in ten subjects in two sets of tests are given below.

Test 1	22	28	20	19	20	24	23	20	24	20
Test 2	13	15	36	11	18	30	23	8	32	34

Find, for each set of tests:

a. the range

b. the inter-quartile range

c. the quartile deviation

Solution

Mean-deviation(=MD)

$$\boxed{MD = \frac{\sum \left| x_i - \bar{x} \right|}{n}} \quad \textbf{or} \quad \boxed{MD = \frac{\sum f_i \cdot \left| x_i - \bar{x} \right|}{\sum f_i}} \quad , \ \bar{x} = mean$$

Example 20

Find the mean-deviation of 8,9,7,6,8, and 10.

Solution

Mean = $\bar{x} = \dfrac{\ldots + \ldots + \ldots + \ldots + \ldots + \ldots}{\ldots} = \dfrac{\ldots}{\ldots} = \ldots$

$$MD = \frac{|\ldots - \ldots| + |\ldots - \ldots| + |\ldots - \ldots| + |\ldots - \ldots| + |\ldots - \ldots| + |\ldots - \ldots|}{\ldots\ldots} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots$$

Example 21

Calculate the mean-deviation for the following frequency distribution.

Test marks	1	2	3	4	5	6	7	8	9	10
Frequency	2	3	4	10	5	3	2	1	0	1

Solution

Test marks (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	2	3	4	10	5	3	2	1	0	1
fx										
$\left\|x_i - \bar{x}\right\|$										
$f_i.\left\|x_i - \bar{x}\right\|$										

Mean = $\bar{x} = \dfrac{\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots}{\ldots} = \dfrac{\ldots}{\ldots} = \ldots$

$$MD = \frac{\sum f_i.\left|x_i - \bar{x}\right|}{\sum f_i} = \frac{\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots}{\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots}$$

$$MD = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots$$

Variance(=Var)

$$\boxed{Var = \frac{\sum f_i.(x_i - \bar{x})^2}{\sum f_i}} \quad , \ \bar{x} = mean$$

Example 22

Find the variance of 2,1,5, 7 and 10.

Solution

Mean = $\bar{x} = \dfrac{\ldots+\ldots+\ldots+\ldots+\ldots}{\ldots} = \dfrac{\ldots}{\ldots} = \ldots$

$$Var = \frac{(\ldots-\ldots)^2 + (\ldots-\ldots)^2 + (\ldots-\ldots)^2 + (\ldots-\ldots)^2 + (\ldots-\ldots)^2 +}{\ldots\ldots} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots$$

Example 23

Calculate the variance for the following frequency distribution.

Test marks	1	2	3	4	5	6	7	8	9	10
Frequency	2	3	4	10	5	3	2	1	0	1

Solution

Test marks (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	2	3	4	10	5	3	2	1	0	1
fx										
$(x_i - \bar{x})$										
$(x_i - \bar{x})^2$										
$f_i.(x_i - \bar{x})^2$										

Mean = $\bar{x} = \dfrac{\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots+\ldots}{\ldots} = \dfrac{\ldots}{\ldots} = \ldots$

$$Var = \frac{\sum f_i.(x_i - \bar{x})^2}{\sum f_i} = \frac{\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots}{\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots+\ldots\ldots}$$

$$Var = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots$$

Standard deviation (=σ)

$$\boxed{\sigma = \sqrt{Var} = \sqrt{\frac{\sum f_i.(x_i - \bar{x})^2}{\sum f_i}}}\ , \ \bar{x} = mean$$

Example 24

Find the standard deviation of 2,1,5, 7 and 10.

Solution

$Var = \ldots\ldots$, $\sigma = \ldots\ldots$

Example 25

Calculate the standard deviation for the following frequency distribution.

Test marks	1	2	3	4	5	6	7	8	9	10
Frequency	2	3	4	10	5	3	2	1	0	1

Solution

$Var = \ldots\ldots,\ \sigma = \ldots\ldots$

Exercise 3

1. A survey recorded the weight of students in a class. The data were as follows:

   41 44 45 46 47 47 48 49 41 43 54
   44 42 45 46 47 47 48 49 50 52

   Find the Statistic Connected Five of the data.

2. Find the Statistic Connected Five of the following data:
   a. 7, 5, 10, 20, 13, 8, 2
   b. 27, 60, 25, 43, 38, 26, 44, 23, 31, 46
   c. 92, 87, 86, 77, 83, 70, 80, 66, 64, 96, 100, 99, 94, 68

3. Find the range, median, inter-quartile range, and deviation quartile of the following data:
   d. 88, 67, 64, 76, 86, 85, 82, 81, 68
   e. 51, 38, 34, 37, 25, 45, 22, 41, 75, 49

4. Find the mean deviation of the data: 3, 2, 1, 2, 2, 1, 4, 5

5. Find the variance and the standard deviation of the data: 4, 5, 6, 7, 8, 6

6. The table below shows the number of fire incidents per day in a city over a period 60 days. Find the mean deviation.

Number of fire incidents	0	1	2	3	4	5	6	7
Number of days	16	12	11	10	6	2	1	2

7. The table below shows the number of hours 60 workers work per week. Find the variance and the standard deviation.

Number of hours	37	38	39	40	41	42	43	44	45
Frequency	3	4	9	18	10	6	3	5	2

Worksheet 4th
Topic : FREQUENCY DISTRIBUTION of grouped data
TIME : 4 X 45 minutes

STANDARD COMPETENCY :

1. To use the rules of statistics, the rules of counting, and the properties of probability in problem solving.

BASIC COMPETENCY:

To present the grouped data into a distribution frequency table.

In this chapter, you will learn:

- How to construct a grouped frequency table.
- How to construct a histogram representing a grouped frequency table.
- How to construct a frequency polygon representing a grouped frequency table.

E. Grouped Frequency Distribution

Grouping a set of data into class intervals requires the following steps :

1. Range = R
2. The number of class intervals = k

$$\boxed{k = 1 + 3.3 \log n}$$ n = the number of data

3. The class length or class width = C

$$\boxed{C = \frac{R}{k}}$$

4. All of data must have a class, include the lowest value and the highest value.
5. Each class of a data is only one.

Example 26

The following set of raw data shows the lengths, in millimeters, measured to the nearest mm, of 40 leaves taken from plants of a certain species. Make the table of frequency distribution.

40	54	25	50	58	45	47	49	30	28
52	31	52	41	47	44	46	39	51	59
49	38	43	48	43	43	40	51	40	56
31	53	44	37	35	37	33	38	46	36

Solution

a. x_1 = the lowest value = …… , x_n = the highest value = …….
So R = …… - …… = ……

b. k = 1 + 3.3 log(……) = 1 + 3.3 x …… = 1 + …… = …… ≈ ……

c. $C = \frac{R}{k} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots \approx \ldots\ldots$

d. We get $C = \frac{34}{6} = 5.6$, so we can choose one of 5 or 6.

e. If we take $C = \frac{34}{6} = 5.6 \approx 6$, we have 25 – 30 , 31 – 36 , 37 – 42, 43 – 48 , 49 – 54 , 55 – 60 as our class interval.
We take $C = \frac{34}{6} = 5.6 \approx 5$, so we have 25 – 29 , 30 – 34, 35 – 39 , 40 – 44 , 45 – 49 , 50 – 54 , 55 – 59 as our class interval.

Thus the table below shows the frequency distribution of the lengths of the 40 leaves.

Lengths (mm)	Tally	Frequency
25 – 29		
…… – ……		
…… – ……		
…… – ……		
…… – ……		
…… – ……		
…… – ……		
Total frequency =		

For the first class, we have:
- the lower class limit = 25
- the upper class limit = 29
- the lower class boundary = 24.5
- the upper class boundary = 29.5

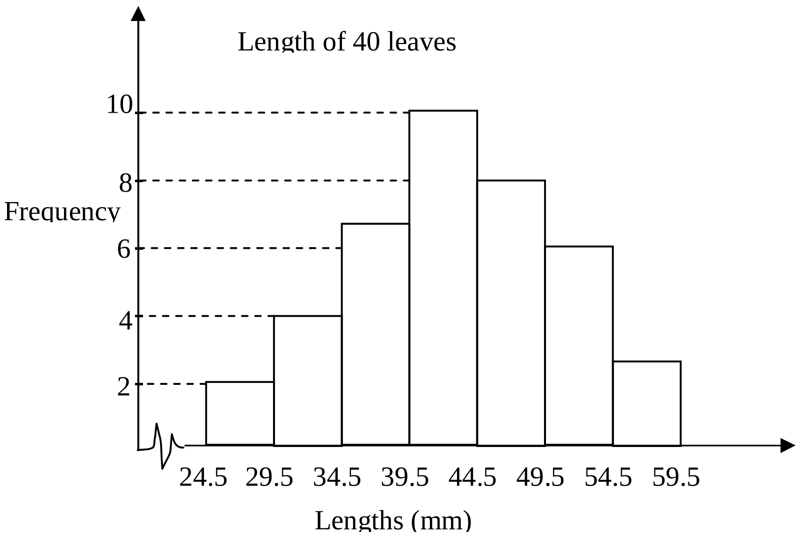
For the second class, we have:
- the lower class limit = ……
- the upper class limit = ……
- the lower class boundary = ……
- the upper class boundary = ……

So the upper class boundary of the class interval 25 – 29 = the ………… class boundary of the class interval …… – ……

For the fourth class, we have:
- the lower class limit = ……
- the upper class limit = ……
- the lower class boundary = ……
- the upper class boundary = ……

Histogram

The diagram below shows *the histogram* representing the frequency distribution of the lengths of 40 leaves.



If we take $C = \dfrac{34}{6} = 5.6 \approx 6$, so we have 25 – 30 , 31 – 36 , 37 – 42 , 43 – 48 , 49 – 54 , 55 – 60 as our class interval.

Thus the table shows the frequency distribution of the lengths of the 40 leaves.

Lengths (mm)	Class boundaries	Tally	Frequency
…… – ……			
…… – ……			
…… – ……			
…… – ……			
…… – ……			
…… – ……			
Total Frequency = ……			

For the first class, we have:
- the lower class limit = ……
- the upper class limit = ……
- the lower class boundary = ……
- the upper class boundary = ……

For the last class, we have:
- the lower class limit = ……
- the upper class limit = ……
- the lower class boundary = ……
- the upper class boundary = ……

The histogram:

Example 27

The fluoride levels, measured in parts per million (PPM), of drinking water treated in a certain water treatment plant were monitored for 30 days. The measurement are given correct to 2 decimal places. The results are given below:

0.76	0.75	0.84	0.98	0.88	0.71	0.87	0.79	0.91	0.82
0.87	0.91	0.83	0.84	0.88	0.99	0.84	0.83	0.83	0.90
0.93	0.85	0.78	0.77	0.81	0.92	1.04	0.92	0.79	0.87

Construct a frequency table and draw a histogram representing it.

Solution

Frequency Polygons

The value mid-way between the class boundaries of a class is called the **class mark**, or the **mid-value**, of the class.
The mid-value of a class is given by

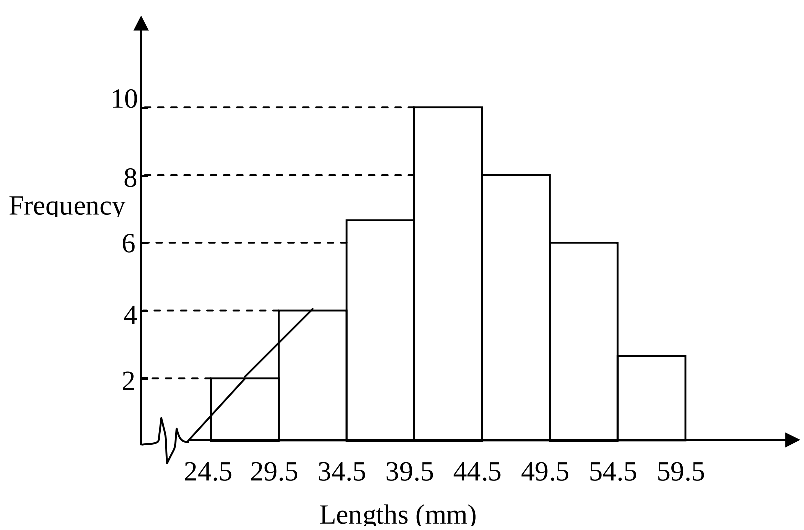
½ (lower class limit + upper class limit) or
½ (lower class boundary + upper class boundary)

The table in example 26 is reproduced as shown below. It shows, in addition, the class mark for each class interval in the frequency distribution.

Lengths (mm)	Class boundaries	Class mark	Tally	Frequency
25 – 29				
…… – ……				
…… – ……				
…… – ……				
…… – ……				
…… – ……				
…… – ……				
				Total =

A frequency polygon is drawn by joining all the mid-points at the top of each rectangle. The mid-points at both ends are joined to the horizontal axis to accommodate the end points of the polygon. This will make the graph neater with the end points falling off to zero on the horizontal axis.
A frequency polygon is often useful when we wish to observe trends. It is also useful when we wish to compare two distributions.

The frequency polygon



We can use another interval like: $25 < x \leq 30$, $30 < x \leq 35$, $35 < x \leq 40$, $40 < x \leq 45$, $45 < x \leq 50$. The class width is **5**. We don't need the lower and upper class boundary, because it is same with the lower and upper class limit respectively.

For the first class, we have:

- the lower class limit = the lower class boundary = 25
- the upper class limit = the upper class boundary = 30
- the mid-point = 27.5

For the second class, we have:

- the lower class limit = the lower class boundary = ….
- the upper class limit = the upper class boundary = ….
- the mid point = ….

Example 28

The weight, in kg, of 50 boys were recorded as shown in the table below:

Weight (x kg)	Number of boys
$40 < x \leq 45$	*4*
$45 < x \leq 50$	*5*
$50 < x \leq 55$	*10*
$55 < x \leq 60$	*14*
$60 < x \leq 65$	*8*
$65 < x \leq 70$	*6*
$70 < x \leq 75$	*3*

Draw the histogram and the frequency polygon of the distribution.

Solution

Exercise 4

1. The following table shows the distribution of marks of some students who took part in science quiz.

Marks	Tally	Lower class boundary	Upper class boundary	Frequency
56 – 60	𝍸 //			
61 – 65	𝍸 //			
66 – 70	𝍸			
71 – 75	𝍸 𝍸			
76 – 80	𝍸			
81 – 85	𝍸			
86 – 90	//			
91 – 95	///			
96 – 100	///			

a. Copy and complete the table
b. To which classes do the marks 90.9 , 66.2, 81.5 belong?
c. Draw a histogram to represent this distribution.

2. The length, in mm, of 48 rubber tree leaves are given below.

137	152	127	147	141	157	132	153	166	147	136	134
146	142	162	169	149	135	166	157	141	146	147	148
163	133	148	150	136	127	162	152	143	138	142	153
145	154	144	126	139	126	158	147	136	144	159	161

Copy and complete the following table:

Lengths (x mm)	Tally	Frequency
$125 < x \le 130$		
$130 < x \le 135$		
$135 < x \le 140$		
$140 < x \le 145$		
$145 < x \le 150$		
$150 < x \le 155$		
$155 < x \le 160$		
$160 < x \le 165$		
$165 < x \le 170$		

a. Determine the class width of the second class.
b. Draw a histogram to illustrate the frequency distribution.

3. The waiting times, x minutes, for 60 patients at a certain clinic are as follows:

25	12	53	8	26	5	19	73	67	18	87	42
6	21	14	19	12	15	13	36	36	16	72	36
13	37	11	51	39	32	30	47	6	22	68	25
98	23	45	22	7	9	26	35	27	48	58	56
29	20	32	62	80	41	58	17	54	15	14	74

a. Construct a frequency table using class intervals $0 < x \le 10$, $10 < x \le 20$, $20 < x \le 30$ and so on.
b. Draw a histogram and a frequency polygon for the distribution.

4. The daily wages of 50 workers, in dollars, are given below. Construct a frequency table with class intervals 10 – 14, 15 – 19, 20 – 24, and so on. Draw a histogram to represent the data. Also draw the frequency polygon.

12	21	13	17	29	33	26	47	10	17
36	31	32	27	25	16	36	29	22	24
21	25	45	18	37	42	35	28	20	44
34	43	22	36	34	20	15	26	17	21
25	30	27	32	26	28	30	38	19	26

5.

The weights, in kg, of 80 members of a sports club were measured and recorded as shown in the table.
a. Draw a histogram for the frequency distribution.
b. Using a separate diagram, draw a frequency polygon to represent the data.

Weight (x kg)	Number of members
$40 < x \le 50$	7
$50 < x \le 60$	10
$60 < x \le 70$	14
$70 < x \le 80$	27
$80 < x \le 90$	12
$90 < x \le 100$	6
$100 < x \le 110$	4

6. Given that the median of five different integers 4, 9, 13, x, and (2x – 3) is 9, find the value of x.

7. The table shows the number of passangers in each of 100 taxis London Airport.

No. of passangers	1	2	3	4
No. of taxis	x	40	Y	26

a. Find the value of x+y
b. If the mean of passangers per taxi is 2.66, show that x+3y = 82
c. Find the value of x & y.

Worksheet 5th	
Topic :	**Measures of Central Tendency of grouped data**
TIME :	4 X 45 minutes

STANDARD COMPETENCY :

1. To use the rules of statistics, the rules of counting, and the properties of probability in problem solving.

BASIC COMPETENCY:

1.4 To calculate the centre of measurement, the location of measurement, and the dispersion of measurement from the grouped data, altogether with their interpretations.

In this chapter, you will learn:

- How to calculate the mean of a grouped frequency distribution.
- How to calculate the mean of a grouped frequency distribution using an “assumed mean” method.
- How to calculate the mode of a grouped frequency distribution.
- How to calculate the mode of a grouped frequency distribution using histogram.

F. Mean and Mode of grouped data

The Mean of grouped data

1. In order to calculate the mean of grouped data, you need to:
   - Find the mid-point of each interval (x_i)
   - Multiply the frequency of each interval by its mid-point ($f_i.x_i$)
   - Find the sum of all the products $f_i.x_i$
   - Find the sum of all the frequencies
   - Divide the sum of the products $f_i.x_i$ by the sum of the frequencies.

$$\text{Mean} = \bar{x} = \frac{\sum f_i.x_i}{\sum f_i}$$

Example 29

The following set of raw data shows the lengths, in millimeters, measured to the nearest mm, of 40 leaves taken from plants of a certain species. This is the table of frequency distribution. Calculate the mean.

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

Solution

Lengths (mm)	Frequency (f_i)	Mid-point (x_i)	$f_i.x_i$
25 – 29	2		
30 – 34	4		
35 – 39	7		
40 – 44	10		
45 – 49	8		
50 – 54	6		
55 – 59	3		
	$\sum f_i =$		$\sum f_i.x_i =$

$$\bar{x} = \frac{\sum f_i.x_i}{\sum f_i} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots$$

8. By Assumed Mean

In order to calculate the mean of grouped data by deviation, you need to:

- Find the mid-point of each interval (x_i)
- Find the assumed mean = A
- Find the difference between A with x_i, we call the deviation (= d_i)
- Multiply the frequency of each interval by its deviation ($f_i.d_i$)
- Find the sum of all the products $f_i.d_i$
- Find the sum of all the frequencies
- Divide the sum of the products $f_i.d_i$ by the sum of the frequencies, then add it to A.

Mean = $\bar{x} = A + \dfrac{\sum f_i.d_i}{\sum f_i}$

Example 30

The following set of raw data shows the lengths, in millimeters, measured to the nearest mm, of 40 leaves taken from plants of a certain species. This is the table of frequency distribution.

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

Solution

Lengths (mm)	Frequency (f_i)	Mid-point (x_i)	Deviation (d_i) $= x_i - A$	$f_i.d_i$
25 – 29	2			
30 – 34	4			
35 – 39	7			
40 – 44	10			
45 – 49	8			
50 – 54	6			
55 – 59	3			
	$\sum f_i =$			$\sum f_i.d_i =$

$$\bar{x} = A + \frac{\sum f_i.d_i}{\sum f_i} = \ldots\ldots + \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots + \ldots\ldots = \ldots\ldots$$

9. By Coding Method

In order to calculate the mean of grouped data by Coding Method, you need to:

- Find the mid-point of each interval (x_i)
- Find the assumed mean = A
- Fill the u_i with zero (=0) in the class of A, then fill the u_i with -1, -2, -3, …to the upper, 1, 2, 3, … to the below of the class of A.
- Multiply he frequency of each interval by its deviation ($f_i.u_i$)
- Find the sum of all the products $f_i.u_i$
- Find the sum of all the frequencies
- Divide the sum of the products $f_i.u_i$ by the sum of the frequencies, multiply it with C, then add it to A.

Mean = $\bar{x} = A + \frac{\sum f_i.u_i}{\sum f_i}.C$

Example 31

The following set of raw data shows the lengths, in millimeters, measured to the nearest mm, of 40 leaves taken from plants of a certain species. This is the table of frequency distribution.

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

Solution

Lengths (mm)	Frequency (f_i)	Mid-point (x_i)	Deviation (u_i)	$f_i.u_i$
25 – 29	2			
30 – 34	4			
35 – 39	7			
40 – 44	10			
45 – 49	8			
50 – 54	6			
55 – 59	3			
	$\sum f_i =$			$\sum f_i.u_i =$

$$\bar{x} = A + \frac{\sum f_i.u_i}{\sum f_i}.C = \ldots\ldots + \frac{\ldots\ldots}{\ldots\ldots}\ldots\ldots = \ldots\ldots + \ldots\ldots = \ldots\ldots$$

Example 32

The table below shows the length of 50 pieces of wire used in a physics laboratory. Lengths have been measured to the nearest centimetre. Find the mean by usual method and Coding Method.

Lengths (mm)	Frequency (f_i)
26 – 30	4
31 – 35	10
36 – 40	12
41 – 45	18
46 – 50	6

Solution

The Mode of grouped frequency distribution

In order to calculate the mode of grouped data, you need to:

- Find the modal class. The modal class is the class interval that has the largest frequency.
- Find the lower class boundary of the modal class ($= Lb$)
- Find the difference of frequency between the modal class to its upper class ($= a$).
- Find the difference of frequency between the modal class to its lower class ($= b$).
- Add the Lb to products $\frac{a}{a+b}$ by C, then add it to A.

Mode = $Mo = Lb_{Mo} + \frac{a}{a+b}.C$

Example 33

The following set of raw data shows the lengths, in millimeters, measured to the nearest mm, of 40 leaves taken from plants of a certain species. This is the table of frequency distribution.

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

Solution

The modal class is 40 – 44, so $Lb_{Mo} = \ldots\ldots$

$a = \ldots\ldots - \ldots\ldots = \ldots\ldots$ and $b = \ldots\ldots - \ldots\ldots = \ldots\ldots$

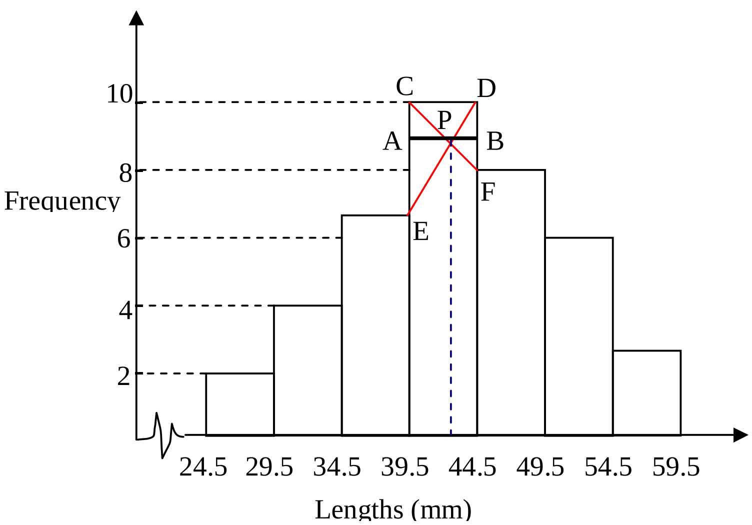
Thus $Mo = Lb_{Mo} + \dfrac{a}{a+b}.C$

$$Mo = \ldots\ldots + \frac{\ldots\ldots}{\ldots\ldots + \ldots\ldots}.\ldots\ldots$$

$$Mo = \ldots\ldots + \ldots\ldots$$

$$Mo = \ldots\ldots\ldots$$

By histogram



CE = …… - …… = ……

DF = …… - …… = ……

AP : PB = CE : DF

AP : PB = …… : …….

AP : AB = …… : (…. + ….)

AP : …… = …… : …….

AP = $\dfrac{\ldots\ldots \times \ldots\ldots}{\ldots\ldots}$

AP = …….

∴ Mo = 39.5 + AP

Mo = 39.5 + …… = ……

Example 34

The weight, in kg, of 50 boys were recorded as shown in the table below:

Weight (x kg)	Number of boys
$40 < x \le 45$	*4*
$45 < x \le 50$	*5*
$50 < x \le 55$	*10*
$55 < x \le 60$	*14*
$60 < x \le 65$	*8*
$65 < x \le 70$	*6*
$70 < x \le 75$	*3*

Find the Mode.

Solution

Exercise 5

1. The following table shows the distribution of marks of some students who took part in science quiz.

Marks	Tally	Lower class boundary	Upper class boundary	Frequency
56 – 60	~~////~~ //			
61 – 65	~~////~~ //			
66 – 70	~~////~~			
71 – 75	~~////~~ ~~////~~			
76 – 80	~~////~~			
81 – 85	~~////~~			
86 – 90	//			
91 – 95	///			
96 – 100	///			

   a. Copy and complete the table
   b. Calculate the mean and the mode.

2. The length, in mm, of 48 rubber tree leaves are given below.

137	152	127	147	141	157	132	153	166	147	136	134
146	142	162	169	149	135	166	157	141	146	147	148
163	133	148	150	136	127	162	152	143	138	142	153
145	154	144	126	139	126	158	147	136	144	159	161

   Copy and complete the following table:

Lengths (x mm)	Tally	Frequency
$125 < x \leq 130$		
$130 < x \leq 135$		
$135 < x \leq 140$		
$140 < x \leq 145$		
$145 < x \leq 150$		
$150 < x \leq 155$		
$155 < x \leq 160$		
$160 < x \leq 165$		
$165 < x \leq 170$		

a. Calculate the mean and the mode.
b. Use the histogram in exercise 4) to calculate the mode.

3. The waiting times, x minutes, for 60 patients at a certain clinic are as follows:

25	12	53	8	26	5	19	73	67	18	87	42
6	21	14	19	12	15	13	36	36	16	72	36
13	37	11	51	39	32	30	47	6	22	68	25
98	23	45	22	7	9	26	35	27	48	58	56
29	20	32	62	80	41	58	17	54	15	14	74

a. Using the frequency table in exercise 4), calculate the mean.
b. Using the histogram in exercise 4), calculate the mode.

4.

The weights, in kg, of 80 members of a sports club were measured and recorded as shown in the table.
a. Calculate the mean.
b. Calculate the mode.

Weight (x kg)	Number of members
$40 < x \le 50$	7
$50 < x \le 60$	10
$60 < x \le 70$	14
$70 < x \le 80$	27
$80 < x \le 90$	12
$90 < x \le 100$	6
$100 < x \le 110$	4

5. The marks scored in a test by 500 children are given in the following table:

Marks (x)	Number of children
$60 < x \le 80$	81
$80 < x \le 100$	103
$100 < x \le 120$	127
$120 < x \le 140$	99
$140 < x \le 160$	90

a. Using an assumed mean of 110, calculate the mean mark.
b. Calculate the mode.

6. Thirty bulbs were life-tested and their lifespan to the nearest hour are as follows:

167	171	179	167	171	165	175	179	169	171
177	169	171	177	173	165	175	167	174	177
172	164	175	179	179	174	174	168	171	168

a. Find the mean of lifespan by dividing their sum by 30.
b. Find the mean of lifespan by grouping the lifespan using class intervals 164 – 166, 167 – 169, and so on.
c. Find the mode of lifespan by looking the data.
d. Find the mode of lifespan by grouping data at b).

7. In an examination taken by 400 students, the scores were as shown in the following distribution table:

Marks	Frequency
1 – 10	8
11 – 20	14
21 – 30	32
31 – 40	56
41 – 50	102
51 – 60	80
61 – 70	54
71 – 80	30
81 – 90	16
91 – 100	8

Find :

a. The mode

b. The mean

Worksheet 6th
Topic : Cumulative Frequency Distribution
TIME : 3 X 45 minutes

STANDARD COMPETENCY :

1. To use the rules of statistics, the rules of counting, and the properties of probability in problem solving.

BASIC COMPETENCY:

1.4 To calculate the centre of measurement, the location of measurement, and the dispersion of measurement from the grouped data, altogether with their interpretations.

In this chapter, you will learn:

- How to construct a cumulative frequency table.
- How to draw a cumulative frequency curve.

G. Cumulative Frequency Distribution

Cumulative Frequency Distribution

So far, we have learnt some different ways of presenting data. Another way of presenting a set of data is by using **the table of cumulative frequencies**. A cumulative frequency table can be represented by a cumulative frequency curve which is also known as an **ogive**.

Example 35

The length of 40 insects of a certain species were measured correct to the nearest millimeter. The frequency distribution is given below:

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

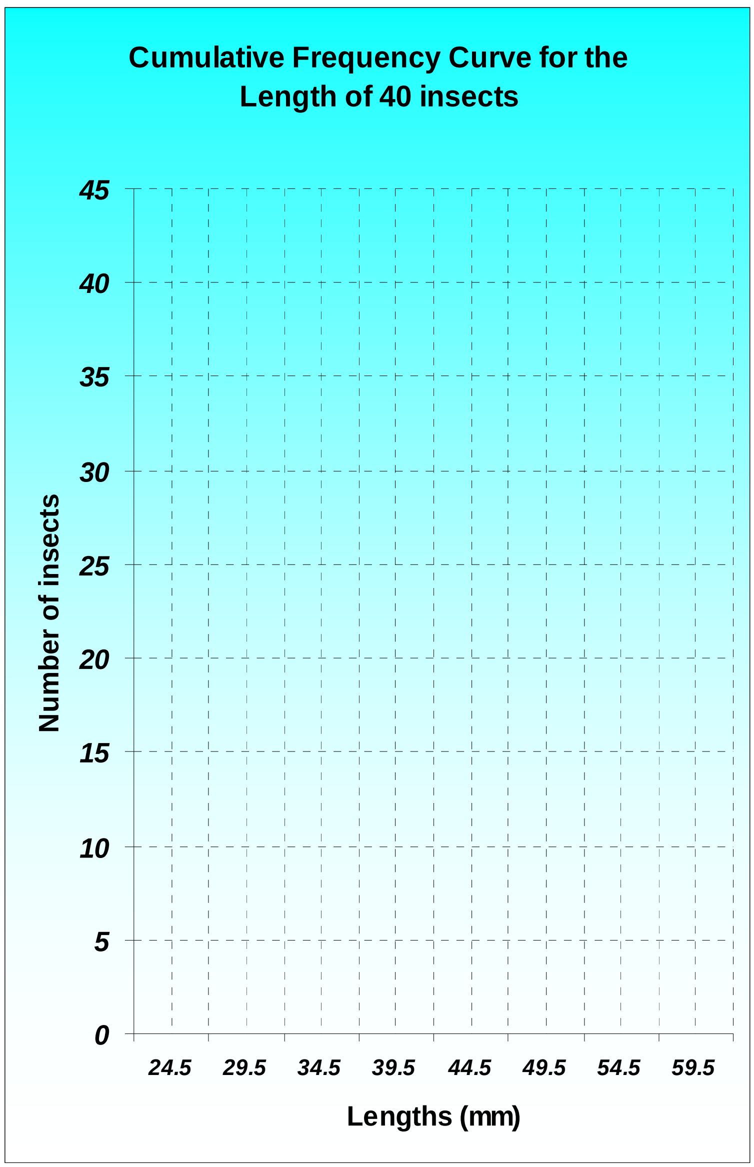
a. Construct a cumulative frequency table for the given data.
b. Draw a cumulative frequency curve for the data.
c. Estimate from the curve
   i. the number of insects that were less than 43.5 mm long,
   (ii) the percentage of insects that were of length 37.5 mm or more,
   (iii) the value of k, if 75% of the insects were less than k mm long.

Solution

a. The cumulative frequency table is constructed below. The table shows the cumulative frequency distribution of the length of 40 insects.

Lengths (mm)	Upper class boundaries	Frequency (f_i)	Length less than	Cumulative frequency
25 – 29		2		
30 – 34		4		
35 – 39		7		
40 – 44		10		
45 – 49		8		
50 – 54		6		
55 – 59		3		

b. The cumulative frequency curve is drawn by plotting the cumulative frequencies against the upper class boundaries, i.e. plotting the points corresponding to the ordered pairs (29.5 , 2), (……, ……) , (……, ……) , (……, ……) , (……, ……) , (……, ……) , (……, ……) .



c. To estimate
   (i) the number of insects that were less than 43.5 mm long, locate the length 43.5 mm on the horizontal axis. Draw a vertical line to meet the curve followed by a horizontal line to meet the vertical axis or cumulative frequency axis as shown in the diagram. *From the graph*, the number of insects that were less than 43.5 mm long is …….

   1. the percentage of insects that were of length 37.5 mm or more, find 37.5 on the horizontal axis and draw a vertical line to meet the curve and then draw a horizontal line to meet the vertical axis. *From the graph,* …… insects were less than 37.5 mm long.
   $\therefore$ the number of insects that were of length 37.5 mm or more = …… - …… = ……
   The percentage of insects that were of length 37.5 mm or more is

$$\frac{\ldots\ldots}{\ldots\ldots}\times 100\% = \ldots\ldots\%$$

   2. the value of *k*, if 75% of the insects were less than *k* mm long.

   75% of …… = $\frac{\ldots\ldots}{\ldots\ldots}\times\ldots\ldots=\ldots\ldots$
   $\therefore$ …… insects were less than *k* mm long.
   From …… on the vertical axis, draw a horizontal line to meet the curve followed by a vertical line to meet the horizontal axis. *From the graph,* …… insects were less than …… mm long.
   $\therefore$ *k* = ……

Example 36

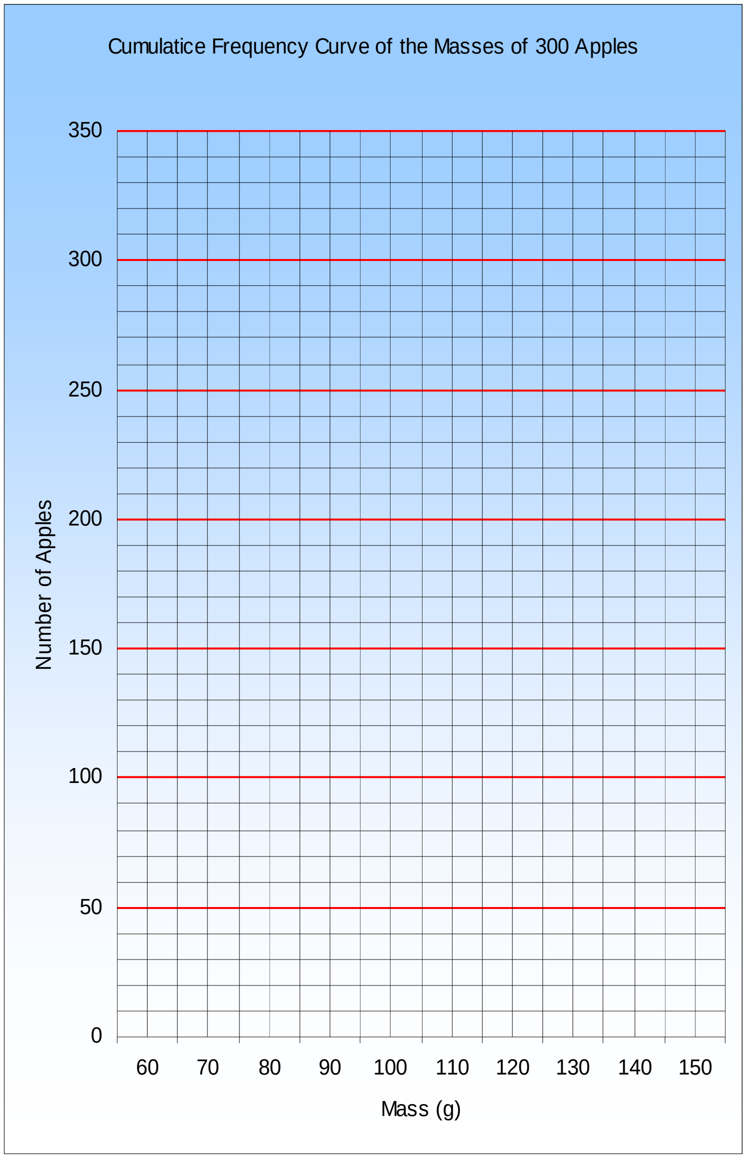
The mass of 300 apples were measured. The table gives the cumulative frequency distribution of the masses.

a. Draw a cumulative frequency curve.
b. Estimate from the curve
   1. the number of apples having masses 98 g or less,
   2. the value of m given that 20% of the apples had masses more than m g.
c. Taking class interval $60 < x \le 70$, $70 < x \le 80$, $80 < x \le 90$, $90 < x \le 95$, … , *construct a frequency distribution and draw a histogram.*

Mass (x g)	Number of apples
$x \le 60$	*0*
$x \le 70$	*8*
$x \le 80$	*19*
$x \le 90$	*57*
$x \le 100$	*89*
$x \le 110$	*141*
$x \le 120$	*216*
$x \le 130$	*266*
$x \le 140$	*290*
$x \le 150$	*300*

Solution

a. The graph shows the cumulative frequency curve.



b. Estimate from the curve

(i) *From the curve*, we estimate that the number of apples having mass 98 g or less are …… apples.

(ii) 20% of …… = $\frac{\ldots\ldots}{\ldots\ldots}\times\ldots\ldots=\ldots\ldots$

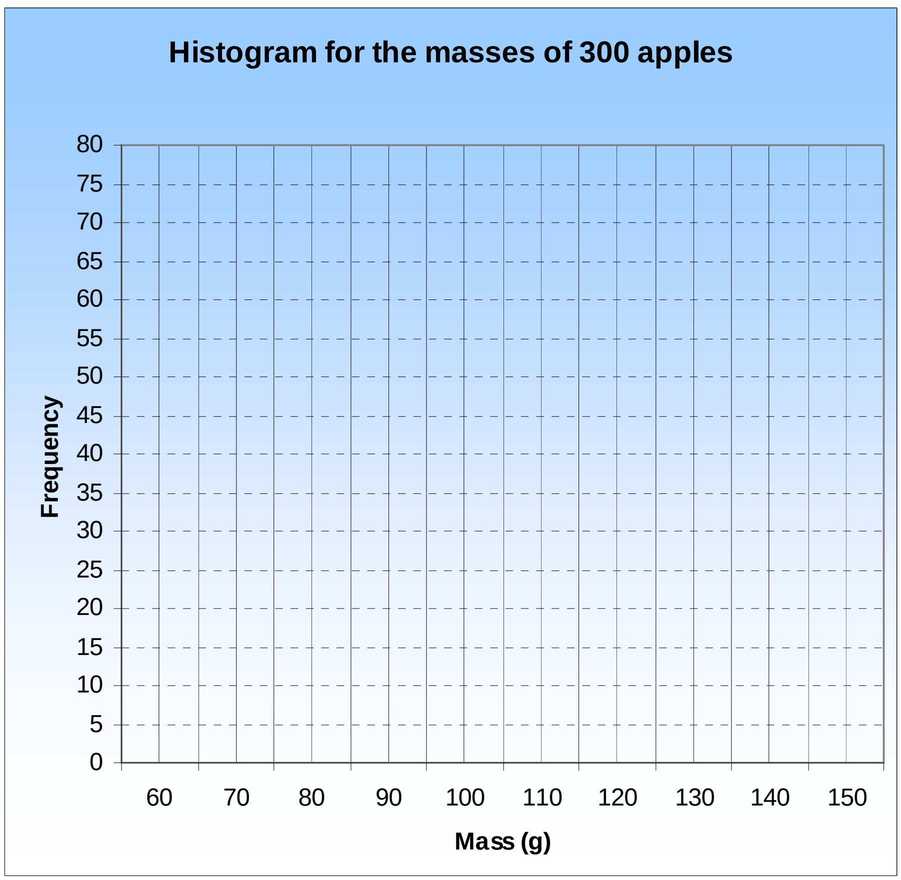
$\therefore$ …… apples have masses more than m g, i.e. …… - …… = …… apples have masses m g or less. *From the curve*, …… apples have masses …… g or less.

$\therefore$ m = ……

c. The frequency distribution is constructed as shown in the following table:

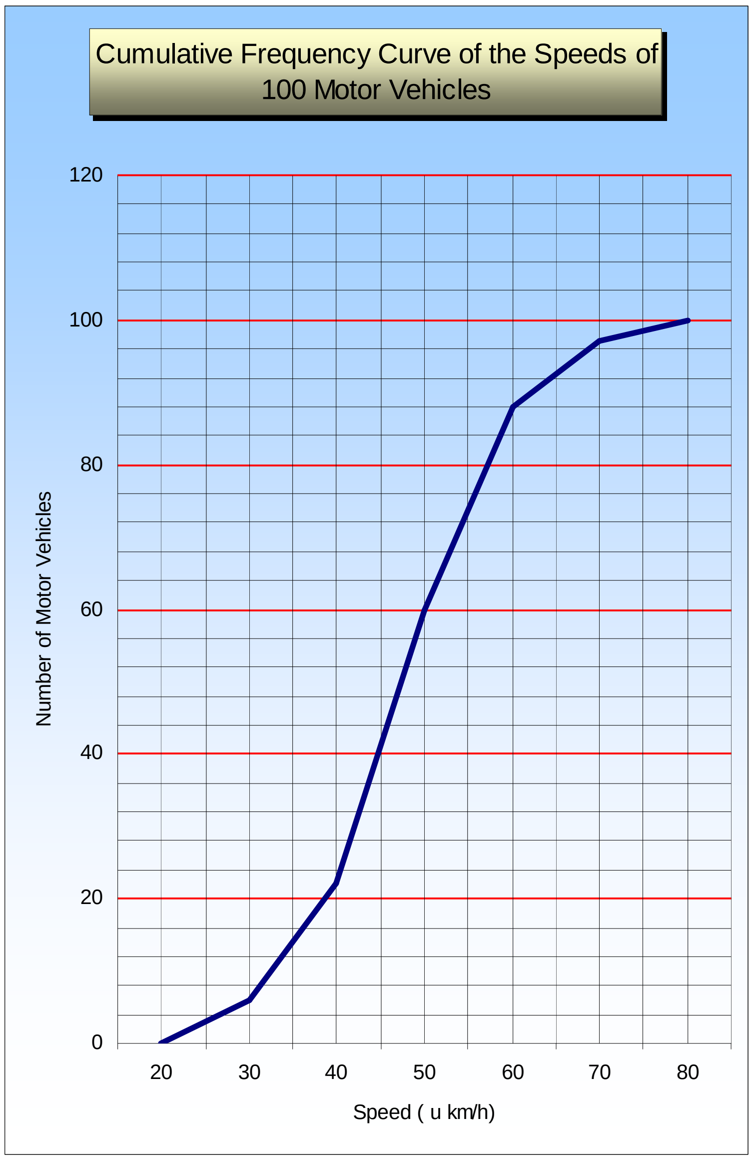
Mass (x g)	Cumulative frequency	Mass (x g)	Frequency
$x \leq 70$	*8*	$60 < x \leq 70$	*8*
$x \leq 80$	*19*		
$x \leq 90$	*57*		
$x \leq 100$	*89*		
$x \leq 110$	*141*		
$x \leq 120$	*216*		
$x \leq 130$	*266*		
$x \leq 140$	*290*		
$x \leq 150$	*300*		

The histogram



Exercise 6

1. The speeds of 100 motor vehicles passing a certain point in a busy street are recorded. The cumulative are frequency curve shows the speed, *u* km/h and the number of vehicles, whose speeds are less than *u* km/h. (As an example, 74 vehicles have speeds less than 53 km/h). Use the curve to estimate
   a. the number of vehicles whose speeds are less than 34 km/h,
   b. the fraction of the total number of vehicles whose speeds are greater than or equal to 59 km/h,
   c. the value of *v*, if 40% of the vehicles have a speed less than *v* km/h.



2. The results of a music examination taken by 160 pupils are shown in the cumulative frequency table below:

Mark	< 10	< 20	< 30	< 40	< 50	< 60	< 70	< 80
Number of pupils	0	8	21	55	103	135	150	160

a. Using a horizontal scale of 2 cm to represent 10 marks and a vertical scale of 1 cm to represent 10 pupils, draw a cumulative frequency curve for the results.
b. Use your graph to estimate
   (i) the number of pupils who scored less than 45 marks,
   (ii) the fraction of the total number of pupils who failed the music examination given that 34 is the lowest mark to pass the examination,
   (iii) the value of x if 22.5% of the pupils obtained at least x marks in the music examination.

3. 500 earthworms were collected from a sample of soil. Their lengths were recorded and the results are given in the following table:

Length (mm)	Number of worms
$10 < x \le 20$	10
$20 < x \le 30$	20
$30 < x \le 40$	50
$40 < x \le 50$	90
$50 < x \le 60$	150
$60 < x \le 70$	100
$70 < x \le 80$	50
$80 < x \le 90$	20
$90 < x \le 100$	10

a. Copy and complete the following cumulative frequency table:

Lengths (mm)	Number of worms
≤ 10	0
≤ 20	10
≤ 30	
≤ 40	
≤ 50	
≤ 60	
≤ 70	
≤ 80	
≤ 90	
≤ 100	500

b. Draw a cumulative frequency curve to represent the results by using 2 cm to represent 100 worms on the vertical axis and taking values of the cumulative frequency from 0 to 500. On the horizontal axis, take values of the length from 10 mm to 100 mm and use a scale if 1 cm to represent 10 mm.

c. Use your graph to estimate
   (i) the number of earthworms whose lengths are less than or equal to 58 mm,
   (ii) the percentage of earthworms whose lengths are greater than 76 mm,
   (iii) the value of x if 18% of the earthworms are of length x mm or less.

4. The lengths of 600 leaves from a tree are measured. The following table gives the cumulative frequency distribution of these lengths:

Length (x mm)	$x \le 20$	$x \le 25$	$x \le 30$	$x \le 35$	$x \le 40$	$x \le 45$	$x \le 50$
Number of leaves	0	20	80	260	500	580	600

a. Draw a cumulative frequency to represent these results using the following scales: On the horizontal axis, take values of the length from 20 mm to 50 mm and use a scale of 2 cm to represent 5 mm.
On the vertical axis, take values of the cumulative frequency from 0 to 600 and use a scale of 2 cm to represent 100 leaves.

b. Use your graph to estimate
(i) the number of leaves whose lengths are less than or equal to 41.5 mm,
(ii) the percentage of leaves whose lengths are greater than 33 mm.

c. Copy and complete the following frequency distribution table:

Length (x mm)	Number of worms
$20 < x \le 25$	20
$25 < x \le 30$	60
$30 < x \le 35$	
$35 < x \le 40$	
$40 < x \le 45$	
$45 < x \le 50$	

d. Draw a histogram to represent the frequency distribution in ©.

5. The table below shows the amount of milk (in kg) obtained from each of the 70 cows of a dairy farm on a particular day:

Amount of milk (x kg)	Number of cows
$0 \le x < 4$	7
$4 \le x < 6$	11
$6 \le x < 8$	17
$8 \le x < 10$	20
$10 \le x < 12$	10
$12 \le x < 14$	5

a. Construct a cumulative frequency table and draw a cumulative frequency curve.
b. Use your curve to estimate
(i) the number of cows that give less than 9.4 kg of milk,
(ii) the fraction of the 70 cows that give at least 7.4 kg of milk,
(iii) the value of x if 70% of the cows give at least x kg of milk.

Worksheet 7th
Topic : Median, Quartiles, and Percentiles of grouped data
TIME : 3 X 45 minutes

STANDARD COMPETENCY :

1. To use the rules of statistics, the rules of counting, and the properties of probability in problem solving.

BASIC COMPETENCY:

1.4 To calculate the centre of measurement, the location of measurement, and the dispersion of measurement from the grouped data, altogether with their interpretations.

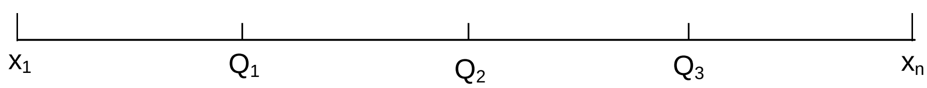
In this chapter, you will learn:

- How to calculate the median and quartiles of grouped data.
- How to estimate the median and quartiles from the cumulative frequency curve.

H. The Median, Quartile, and Percentile of grouped data

By Ogive

We recall that the median (=Q_2) is the middle value when a set of data is arranged in order of increasing magnitude.



Q_1 = the lower quartile
Q_2 = median
Q_3 = the upper quartile

We can estimate Q_1, Q_2, Q_3 from the cumulative frequency and calculate them with the formula.

Median corresponds to the 50th percentile, Q_1 corresponds to the 25th percentile, Q_3 corresponds to the 75th, i.e. $Q_2 = P_{50}$, $Q_1 = P_{25}$, $Q_3 = P_{75}$.

By Formula

The Median

$$Q_2 = Lb_{Q_2} + \frac{\frac{1}{2}n - f_c}{f_{Q_2}}.C$$

Q_2 = median

Lb_{Q_2} = the lower boundary of median

n = the sum of data

f_c = the cumulative frequency before the median class

f_{Q_2} = the frequency of the median class

C = the width of interval class

The Lower Quartile

$$Q_1 = Lb_{Q_1} + \frac{\frac{1}{4}n - f_c}{f_{Q_1}}.C$$

Q_1 = the lower quartile

Lb_{Q_1} = the lower boundary of the lower quartile

n = the sum of data

f_c = the cumulative frequency before the lower quartile class

f_{Q_1} = the frequency of the lower quartile class

C = the width of interval class

The Upper Quartile

$$Q_3 = Lb_{Q_3} + \frac{\ldots n - f_c}{f_{Q_3}}.C$$

Q_3 = the upper quartile

Lb_{Q_3} = the lower boundary of the upper quartile

n = the sum of data

f_c = the cumulative frequency before the upper quartile class

f_{Q_1} = the frequency of the upper quartile class

C = the width of interval class

The Percentile

$$P_x = Lb_{P_x} + \frac{\frac{x}{100}n - f_c}{f_{P_x}}.C$$

P_x = the x^{th} percentile

Lb_{P_x} = the lower boundary of the x^{th} percentile

n = the sum of data

f_c = the cumulative frequency before the x^{th} percentile class

f_{P_x} = the frequency of the x^{th} percentile class

C = the width of interval class

The Limit of X

$$X_i = Lb_{X_i} + \frac{i.n - f_c}{f_{X_i}}.C$$

Example 37

The length of 40 insects of a certain species were measured correct to the nearest millimeter.

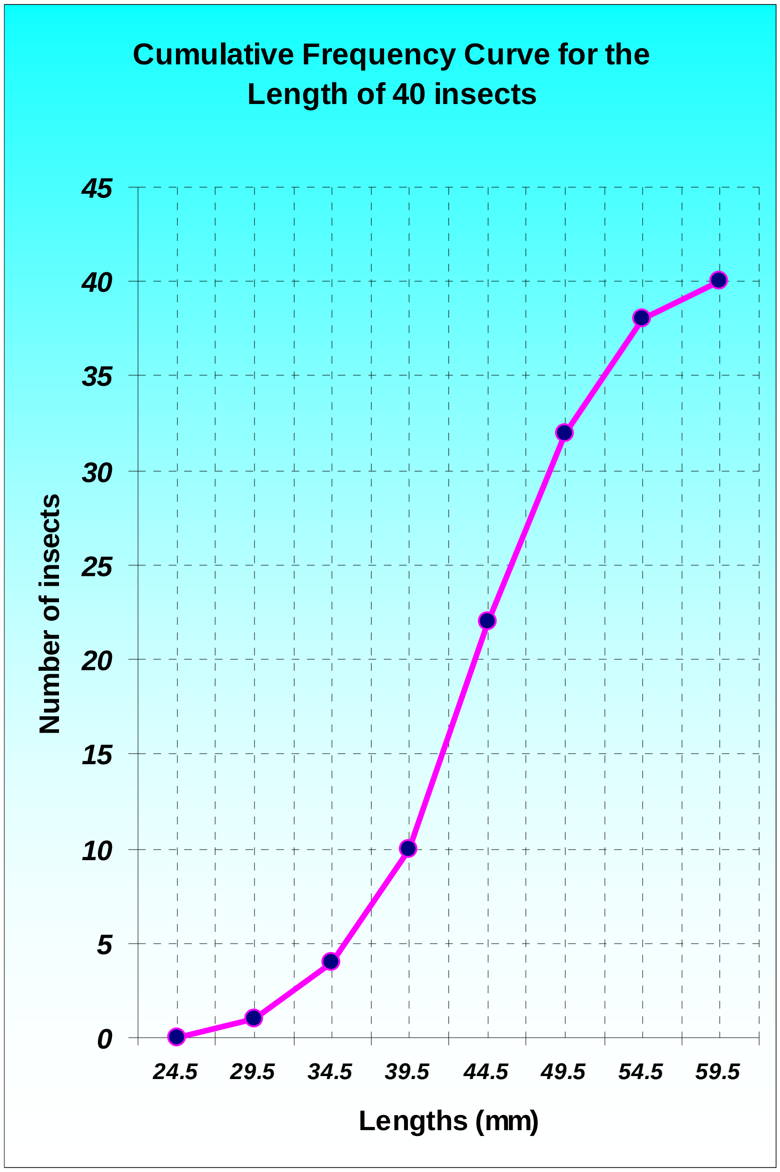
Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

Use the cumulative frequency curve (ogive) to estimate:

a. the median length
b. the upper quartile
c. the lower quartile

Solution

The cumulative frequency table is constructed below. The table shows the cumulative frequency distribution of the length of 40 insects.



a. the median length, 50% of the total frequency = $\frac{50}{100} \times 40 = \ldots\ldots$

From the curve, the median length = ……

b. the upper quartile, 75% of the total frequency = $\frac{75}{100} \times 40 = \ldots\ldots$

From the curve, the upper quartile = ……

c. the lower quartile, 25% of the total frequency = $\frac{25}{100} \times 40 = \ldots\ldots$

From the curve, the lower quartile = ……

By formula:

Lengths (mm)	Frequency (f_i)	The cumulative frequency
25 – 29	2	2
30 – 34	4	
35 – 39	7	
40 – 44	10	
45 – 49	8	
50 – 54	6	
55 – 59	3	

a. ½ *n* = ½ x 40 = 20, 20 in the class 40 – 44 .

$$Q_2 = Lb_{Q_2} + \frac{\frac{1}{2}n - f_c}{f_{Q_2}}.C$$

$$Q_2 = 39.5 + \frac{\frac{1}{2}.40 - 13}{10}.5$$

$$Q_2 = 39.5 + \frac{20 - 13}{10}.5$$

$$Q_2 = 39.5 + \frac{7}{2} = 39.5 + 3.5 = 43$$

b. ¾ *n* = ¾ x …… = ……, …… in the class …… – ……

$$Q_3 = Lb_{Q_3} + \frac{\frac{3}{4}n - f_c}{f_{Q_3}}.C$$

$$Q_3 = \ldots\ldots + \frac{\frac{3}{4}\times \ldots\ldots - \ldots\ldots}{\ldots\ldots}.\ldots\ldots$$

$$Q_3 = \ldots\ldots + \frac{\ldots\ldots - \ldots\ldots}{\ldots\ldots}.\ldots\ldots$$

$$Q_3 = \ldots\ldots + \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots + \ldots\ldots = \ldots\ldots$$

c. ¼ *n* = ¼ x …… = ……, …… in the class …… – ……

$$Q_1 = Lb_{Q_1} + \frac{\frac{1}{4}n - f_c}{f_{Q_1}}.C$$

$$Q_1 = \ldots\ldots + \frac{\frac{1}{4}\times \ldots\ldots - \ldots\ldots}{\ldots\ldots}.\ldots\ldots$$

$$Q_1 = \ldots\ldots + \frac{\ldots\ldots - \ldots\ldots}{\ldots\ldots}.\ldots\ldots$$

$$Q_1 = \ldots\ldots + \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots + \ldots\ldots = \ldots\ldots$$

Example 38

The examination marks of 100 pupils are given in the table:

a. Construct a cumulative frequency table, using the classes "≤ 10","≤ 20", and so on.

b. Draw the cumulative frequency curve for the result obtained.

c. Use your curve to estimate and the formula to calculate

(i) the median mark

(ii) the upper quartile

(iii) the lower quartile

(iv) *the minimum mark required to gain a distinction if the top 5% of the pupils are awarded a distinction*

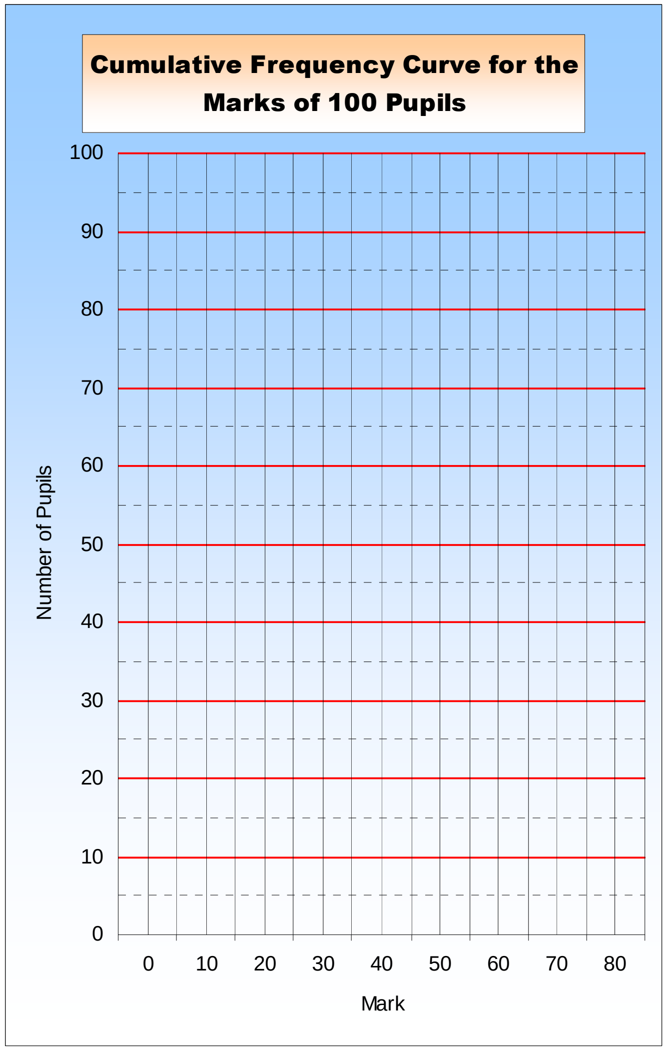
Mark	Number of pupils
$x \leq 10$	2
$10 < x \leq 20$	12
$20 < x \leq 30$	25
$30 < x \leq 40$	29
$40 < x \leq 50$	15
$50 < x \leq 60$	10
$60 < x \leq 70$	4
$70 < x \leq 80$	3

Solution

a. The table below shows the cumulative frequency table.

Mark	Number of pupils
$x \leq 10$	
$x \leq 20$	
$x \leq 30$	
$x \leq 40$	
$x \leq 50$	
$x \leq 60$	
$x \leq 70$	
$x \leq 80$	

b. The graph below shows the cumulative frequency curve for the results:



c. **By formula**

(i) $Q_2 =$

(ii) $Q_3 =$

(iii) $Q_1 =$

(iv) $X_{95} =$

Exercise 7

1. In an agricultural experiment, the lengths of 124 ears of barley were measured. The data obtained is expressed in the following table:

Length (mm)	Number of ears of barley
$x \le 20$	1
$20 < x \le 30$	8
$30 < x \le 40$	35
$40 < x \le 50$	50
$50 < x \le 60$	25
$60 < x \le 70$	5

   a. Construct a cumulative frequency table, using the classes “20 or less”, “30 or less”, and so on
   b. Draw the cumulative frequency curve for the results.
   c. Use your graph to estimate the median.
   d. Use the formula to calculate the median.
   e. From the graph find the number of ears of barley with lengths
      (i) greater that 55 mm,
      (ii) either not greater than 25 mm or greater than 64 mm.
   f. It was discovered later than all the lengths were wrongly recorded such that all lengths should be 5 mm more. Find the correct value of the median.

2. The table below shows the distribution of the marks scored by 600 pupils in an examination:

Marks	10	20	30	40	50	60	70	80	90	100
Number of pupils scoring less than this mark	9	27	88	180	308	415	497	568	590	600

   Using a vertical scale of 1 cm for 50 pupils and a horizontal scale of 1 cm for 10 marks, plot these values and draw a smooth curve through your plotted points.
   a. Use your graph to estimate
      (i) the median mark,
      (ii) the pass mark such that 60% of the pupils will pass the examination.
   b. Indicate clearly the upper and the lower quartile on your graph.
   c. Use the formula to calculate the upper and the lower quartile.

3. 64 adults were asked to indicate the weekly number of hours they spend watching television. The table below shows the information obtained:

Length (mm)	Number of adults
$x \le 5$	2
$5 < x \le 10$	8
$10 < x \le 15$	22
$15 < x \le 20$	16
$20 < x \le 25$	10
$25 < x \le 30$	4
$30 < x \le 35$	2

a. Copy and complete the cumulative frequency table below:

Time (h)	$x \le 5$	$x \le 10$	$x \le 15$	$x \le 20$	$x \le 25$	$x \le 30$	$x \le 35$
Number of adults							

b. Using a vertical scale of 2 cm to represent 10 adults and a horizontal scale of 2 cm to represent 5 hours, draw a cumulative frequency curve to display the information.

c. Use your graph to estimate
(i) the median,
(ii) the upper and the lower quartile,
(iii) the number of adults who spend more than 25 hours per week watching television.

4. The results of 56 students in an examination are tabulated below:

Mark (x)	Frequency
$0 \le x < 10$	1
$10 \le x < 20$	3
$20 \le x < 30$	4
$30 \le x < 40$	5
$40 \le x < 50$	7
$50 \le x < 60$	8
$60 \le x < 70$	11
$70 \le x < 80$	9
$80 \le x < 90$	6
$90 \le x < 100$	2

a. Using the formula, calculate the median, the lower quartile, and the upper quartile.
b. Calculate the percentage of students who scored a mark
(i) greater than or equal to 65,
(ii) less than 34

5. The table below shows the distribution of marks scored by 500 cadets in a physical test:

a. Calculate the mean work.
b. Construct the cumulative frequency table.
c. Draw a cumulative frequency curve representing the distribution.
d. Estimate from the graph and calculate by the formula:
(i) the median,
(ii) the 70^{th} percentile,
(iii) the upper and the lower quartile,
(iv) the number of cadets who scored less than 43 marks,
(v) the pass mark given that 60% of the cadets passed the physical test.

Mark (x)	Number of cadets
$0 \le x < 10$	9
$10 \le x < 20$	17
$20 \le x < 30$	63
$30 \le x < 40$	65
$40 \le x < 50$	86
$50 \le x < 60$	112
$60 \le x < 70$	68
$70 \le x < 80$	55
$80 \le x < 90$	17
$90 \le x < 100$	8

Worksheet 8^{th}
Topic : THE DISPERSION OF MEASUREMENT of grouped data
TIME : 3 X 45 minutes

STANDARD COMPETENCY :

1. To use the rules of statistics, the rules of counting, and the properties of probability in problem solving.

BASIC COMPETENCY:

1.4 To calculate the centre of measurement, the location of measurement, and the dispersion of measurement from the grouped data, altogether with their interpretations.

In this chapter, you will learn :

- How to calculate the dispersion of measurement: range, inter quartile-range, quartile deviation, mean-deviation, variance, standard deviation, from grouped data.

I. Inter quartile, Quartile deviation, Mean-deviation, Variance, Standard deviation

Dispersion

The average (mean, median or mode) gives a general idea of the size of the data, but two sets of numbers can have the same mean while being very different in other ways.
Another main statistic we need to find is a measure of *dispersion* or *spread*. There are several ways of measuring dispersion.

Range

Range = upper boundary of the highest class – lower boundary of the lowest class

Inter-quartile range

The inter-quartile range = $Q_3 - Q_1$

Quartile Deviation(=QD)

The Quartile deviation = QD = ½ (Q_3 – Q_1)

Example 39

The length of 40 insects of a certain species were measured correct to the nearest mm
.

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

By the formula, calculate:

a. the inter-quartile range
b. the quartile deviation

Solution

Lengths (mm)	Frequency (f_i)	The cumulative frequency
25 – 29	2	2
30 – 34	4	
35 – 39	7	
40 – 44	10	
45 – 49	8	
50 – 54	6	
55 – 59	3	

a. The Inter-quartile range = …… - ……

The class of Q_3 : ¾ *n* = ¾ x …… = ……, …… in the class …… – ……

$$Q_3 = Lb_{Q_3} + \frac{\frac{3}{4}n - f_c}{f_{Q_3}}.C$$

$$Q_3 = \ldots\ldots + \frac{\frac{3}{4}\times\ldots\ldots - \ldots\ldots}{\ldots\ldots}.\ldots\ldots$$

$$Q_3 = \ldots\ldots + \frac{\ldots\ldots - \ldots\ldots}{\ldots\ldots}.\ldots\ldots$$

$$Q_3 = \ldots\ldots + \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots + \ldots\ldots = \ldots\ldots$$

The class of Q_1 : ¼ *n* = ¼ x …… = ……, …… in the class …… – ……

$$Q_1 = Lb_{Q_1} + \frac{\frac{1}{4}n - f_c}{f_{Q_1}}.C$$

$$Q_1 = \ldots\ldots + \frac{\frac{1}{4}\times\ldots\ldots - \ldots\ldots}{\ldots\ldots}.\ldots\ldots$$

$$Q_1 = \ldots\ldots + \frac{\ldots\ldots - \ldots\ldots}{\ldots\ldots}.\ldots\ldots$$

$$Q_1 = \ldots\ldots + \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots + \ldots\ldots = \ldots\ldots$$

The Inter-quartile range = …… - …… = ……

b. The Quartile Deviation = …… (…… - ……) = ……. x ……. = ……

Mean-deviation(=MD)

$$\boxed{MD = \frac{\sum f_i.\left|x_i - \overline{x}\right|}{\sum f_i}} \quad , \ \overline{x} = mean$$

Example 40

The length of 40 insects of a certain species were measured correct to the nearest mm.

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

By the formula, calculate:

a. the mean
b. the mean deviation

Solution

Lengths (mm)	Frequency (f_i)	Mid-point (x_i)	$f_i.x_i$	$\left|x_i - \overline{x}\right|$	$f_i.\left|x_i - \overline{x}\right|$
25 – 29	2				
30 – 34	4				
35 – 39	7				

40 – 44	10				
45 – 49	8				
50 – 54	6				
55 – 59	3				
	$\sum f_i =$		$\sum f_i.x_i =$		$\sum f_i.\left\|x_i - \bar{x}\right\| =$

$$\bar{x} = \frac{\sum f_i.x_i}{\sum f_i} = \frac{\cdots\cdots}{\cdots\cdots} = \cdots\cdots$$

$$MD = \frac{\sum f_i.\left|x_i - \bar{x}\right|}{\sum f_i} = \frac{\cdots\cdots}{\cdots\cdots} = \cdots\cdots$$

Variance(=Var)

$$\boxed{Var = \frac{\sum f_i.\left(x_i - \bar{x}\right)^2}{\sum f_i}} \quad , \; \bar{x} = mean$$

Example 40

The length of 40 insects of a certain species were measured correct to the nearest mm
.

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

By the formula, calculate the variance.
You can use your calculator, simplify until
3 significant figures.

Solution

Lengths (mm)	(f_i)	Mid-point (x_i)	$\left(x_i - \bar{x}\right)$	$\left(x_i - \bar{x}\right)^2$	$f_i.\left(x_i - \bar{x}\right)^2$
25 – 29	2				
30 – 34	4				
35 – 39	7				

40 – 44	10				
45 – 49	8				
50 – 54	6				
55 – 59	3				
	$\sum f_i =$				$\sum f_i.\left(x_i - \bar{x}\right)^2 =$

$\bar{x} = \ldots\ldots$

$$Var = \frac{\sum f_i.\left(x_i - \bar{x}\right)^2}{\sum f_i} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots$$

Standard deviation (=σ)

$$\boxed{\sigma = \sqrt{Var} = \sqrt{\frac{\sum f_i.\left(x_i - \bar{x}\right)^2}{\sum f_i}}}\ ,\ \bar{x} = mean$$

Example 41

By the formula, calculate the standard deviation of the problem of example 40.

Solution

$Var = \ldots\ldots\ ,\ \sigma = \ldots\ldots$

An alternative formula of standard deviation : $\boxed{\sigma = \sqrt{Var} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}}$

Calculating standard deviation by Coding method

The coding method can also be used to simplify the calculation of standard deviation as used in the calculation of mean.

$$\sigma = \sqrt{Var} = C.\sqrt{\frac{\sum fu^2}{\sum f} - \left(\frac{\sum fu}{\sum f}\right)^2}$$

Example 42

The length of 40 insects of a certain species were measured correct to the nearest mm .

Lengths (mm)	Frequency (f_i)
25 – 29	2
30 – 34	4
35 – 39	7
40 – 44	10
45 – 49	8
50 – 54	6
55 – 59	3

By the coding method, calculate the standard deviation.

Solution

Lengths (mm)	(f_i)	u	fu	u^2	$f.u^2$
25 – 29	2				
30 – 34	4				
35 – 39	7				
40 – 44	10				
45 – 49	8				
50 – 54	6				
55 – 59	3				
	$\sum f_i =$		$\sum fu =$		$\sum f.u^2 =$

$$\sigma = \sqrt{Var} = C.\sqrt{\frac{\sum fu^2}{\sum f} - \left(\frac{\sum fu}{\sum f}\right)^2}$$

$\sigma =$.....

Exercise 8

1. The table below shows the length of 50 pieces of wire used in a physics laboratory. Lengths have been measured to the nearest centimetre.
   a. Calculate the mean by Coding Method.
   b. Calculate the mean deviation.
   c. Calculate the standard deviation.

Lengths (mm)	Frequency (f_i)
26 – 30	4
31 – 35	10
36 – 40	12
41 – 45	18
46 – 50	6

2. 160 electric light bulbs of brand A were tested to find the life span of each bulb (i.e., the time it lasted before it failed). The results are given in the table below:

Life span (t hours)	Number of bulbs
$t \le 500$	2
$500 < t \le 1000$	4
$1000 < t \le 1500$	13
$1500 < t \le 2000$	68
$2000 < t \le 2500$	51
$2500 < t \le 3000$	18
$3000 < t \le 3500$	3
$3500 < t \le 4000$	1

a. Copy and complete the following cumulative frequency table:

Life in hours	Number of bulbs
≤ 500	2
≤ 1000	6
≤ 1500	
≤ 2000	
≤ 2500	
≤ 3000	
≤ 3500	
≤ 4000	

b. Using a horizontal scale of 2 cm to represent 500 hours and a vertical scale of 2 cm to represent 20 bulbs, draw a smooth cumulative frequency curve for these results.

c. Showing your method clearly, use your graph to estimate
   (i) the median
   (ii) the 10^{th} percentile of the distribution.

160 brand B bulbs were also tested and a report on the test gave the following information:

4 bulbs had a life span ≤ 500.
None lasted beyond 3200 hours.
The median life span was 2300 hours.
The upper quartile of the distribution was 2600 hours.
The inter-quartile range of the distribution was 600 hours.

d. Use this information to draw, on the same axes, a smooth cumulative frequency curve for the brands B bulbs.

e. Use your graph to estimate the number of bulbs with a life span 2750 hours or less
   (i) for brand A,

(ii) for brand B.

f. Both brands cost the same price. Which do you think is a better buy? Give a reason for your choice.

3. The table shows the heights in cm, of 56 plants grown under experimental conditions:

Height (x cm)	Number of plants
$0 < x \le 20$	3
$20 < x \le 30$	4
$30 < x \le 40$	6
$40 < x \le 50$	15
$50 < x \le 60$	20
$60 < x \le 70$	8

a. Draw a histogram to illustrate the information.
b. Calculate the mean height.
c. Construct the cumulative frequency table for the distribution and draw the cumulative frequency curve.
d. Use your curve to estimate
   (i) the median,
   (ii) the upper quartile,
   (iii) the lower quartile,
   (iv) the number of plants having heights greater than 57 cm,
   (v) the value of x if 37.5% of the 56 plants have a height of less than or equal to x cm.

4. The tables gives the frequency distribution of marks obtained by 80 candidates in the Mathematics and English examination:

Mark	Mathematics	English
$0 < x \le 20$	8	2
$20 < x \le 40$	12	10
$40 < x \le 60$	18	33
$60 < x \le 80$	25	31
$80 < x \le 100$	17	4

a. Copy and complete the table on the right showing the cumulative frequency distribution in each subject.

	Number of candidates with this mark or less	
Mark	Mathematics	English
20	8	
40	20	
60		
80		
100	80	

b. Using a scale of 2 cm to represent 20 marks on the horizontal axis and 2 cm to represent 20 candidates on the vertical axis, draw separate cumulative frequency diagrams for each of the subjects, Mathematics and English.
Showing your method clearly, use your graphs to estimate
   (i) the median mark in Mathematics,
   (ii) the inter-quartile range in English,
   (iii) the number of candidates who will obtain a distinction in English, if the minimum mark for a distinction is 76,
   (iv) how many more candidates will fail to achieve a credit in Mathematics than in English if the minimum mark for a credit is 60 in each subject.

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