MathLinks

6-1
TEACHER PACKET

MATHLINKS: GRADE 6
TEACHER PACKET 1
WHOLE NUMBER MULTIPLICATION AND DIVISION

Table of Contents

GENERAL INFORMATION

PACING PLAN SUGGESTIONS

TRADITIONAL MATH SCHEDULE				
Days-Modified	Days-Basic	Day-Enriched	Lesson	Review/Practice
2	1	1	[1.1] Pages 0, 1-9	Pages 23-25
2	1	1	[1.2] Pages 0, 10-16	Pages 26-28
2	1	1	[1.3] Pages 0, 17-22	Pages 29-35
3	2	2	Catch up, Tasks, Assessment	

BLOCK SCHEDULE				
Days-Modified	Days-Basic	Days-Enriched	Lesson	Review/Practice
2	0.5	0.5	[1.1] Pages 0, 1-9	Pages 23-25
1	1	1	[1.2] Pages 0, 10-16	Pages 26-28
1	0.5	0.5	[1.3] Pages 0, 17-22	Pages 29-35
2	1	1	Catch up, Tasks, Assessment	

- Lesson pages are not intended to be used only as classwork, and review/practice pages are not intended to be used only as homework. How they are used is up to the teacher.
- The number of days estimated for each lesson will vary depending on school schedules, instruction styles, and student proficiency.
- Although they are listed at the end of the tables, use catch up days when needed.
- Tasks may be assigned at any time after students have completed the prerequisite content work.
- Multiple assessment measures are encouraged, including (but not limited to) quizzes, tasks, proficiency challenges, strategically selected student pages, skill builders, selected response page, knowledge check, etc.
- Consider requiring a math journal, to be collected and checked periodically, or collecting an “exit slip” at the end of selected class periods. Journals and exits slips may include short skills review, explanations of concepts, or anything else the instructor may want to assess.
- This packet includes review of concepts from previous grades. Use Quiz 1 to pre-assess student fluency if desired.

COMMON CORE STATE STANDARDS – MATHEMATICS

STANDARDS FOR MATHEMATICAL CONTENT

4.NBT.B*	**Use place value understanding and properties of operations to perform multi-digit arithmetic.[1]**
4.NBT.5*	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculations by using equations, rectangular arrays, and/or area models.
4.NBT.6*	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculations by using equations, rectangular arrays, and/or area models.
5.NBT.A*	**Understand the place value system.[1]**
5.NBT.2*	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. ~~Use whole-number exponents to denote powers of 10.~~
5.NBT.5*	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NBT.6*	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.2	Fluently divide multi-digit numbers using the standard algorithm.
6.EE.A	**Apply and extend previous understandings of arithmetic to algebraic expressions.[1]**
6.EE.2b	Write, read, and evaluate expressions in which letters stand for numbers: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

* Review of content essential for success in 6th grade.
[1] A major cluster for the grade level.

STANDARDS FOR MATHEMATICAL PRACTICE

MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP7	Look for and make use of structure.
MP8	Look for and express regularity in repeated reasoning.

PACKET PLANNING INFORMATION

Assessments*, Reproducibles, and Tasks****

Quiz 1A, 1B
Proficiency Challenge 1
Test Part 1 (See Assessment Tab, page iv)

Reproducible 1:	Base-10 Blocks (optional) [1.1, 1.2, 1.3]
Reproducible 2:	Division Template 1 (for one problem) [1.2, 1.3]
Reproducible 3:	Division Template 2 (for two problems) [1.2, 1.3]
Reproducible 4:	Grid Paper [1.1, 1.3]
Task, Page 1:	Multiplication Patterns [1.1]
Task, Page 2:	Interpreting the Division Algorithm [1.3]

*Located in the assessment envelope and on the secure website
**Located in the back of the Teacher Guide.

Materials

- Square tiles (20-30 tiles, optional) [1.1]
- Base-10 blocks (1 small set, optional) [1.1, 1.2, 1.3]
- Grid paper (1-2/student, optional) [1.1, 1.3]

***MathLinks*: Grade 6 Resource Guide (Part 1)**

Key vocabulary in the Word Bank:

- algorithm
- difference
- dividend
- divisor
- factor of a number
- product
- quotient
- remainder
- sum

Explanations and examples:

- Mathematical Symbols and Language
- Mathematical Properties
- Whole Numbers: Multiplication and Division

Prepare Ahead

Go to www.mathandteaching.org:
- For PowerPoints designed to deepen teacher understanding of the mathematics and provide ideas for instruction
- To gain access to additional resources and updates

Lesson 1.1:
- Become familiar with area models for explaining multiplication.

Lesson 1.2:
- Become familiar with the division algorithm variation and with the area model division examples before demonstrating in class.
- Make extra copies of the division templates for the students to use in this lesson and in future lessons.

Technology Resources

Permission is granted to project the student pages from the Grade 6 Program CD.

A good source for virtual manipulatives is the National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

Options for a Substitute

After 1.1:	Pages 9, 23-25
After 1.2:	Pages 26-28
After 1.3:	Pages 22, 29-35

TEACHER CONTENT INFORMATION

MATH NOTES

MN1: Some Properties of Arithmetic [1.1]

The associative property of addition states that $a + (b + c) = (a + b) + c$ for any three numbers a, b, and c. In other words, changing the grouping of addends does not change the sum.

Example: $9 + (1 + 14) = (9 + 1) + 14 = 9 + 1 + 14$

The associative property of multiplication states that $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for any three numbers a, b, and c. In other words, changing the grouping of the factors does not change the product.

Example: $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5) = 3 \cdot 4 \cdot 5$

The commutative property of addition states that $a + b = b + a$ for any two numbers a and b. In other words, changing the order of the addends does not change the sum.

Example: $14 + 6 = 6 + 14$

The commutative property of multiplication states that $a \cdot b = b \cdot a$ for any two numbers a and b. In other words, changing the order of the factors does not change the product.

Example: $3 \cdot 5 = 5 \cdot 3$

The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. This property relates two operations (multiplication and addition). It is called the "distributive property" because it distributes the factor outside the parentheses over the two terms within the parentheses.

Example: $3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$

TEACHING NOTES

TN1: Select Standards for Mathematical Practice Examples [1.1, 1.2, 1.3]

Here are a few examples of how the Standards for Mathematical Practice are applied in these lessons.

MP2 Reason abstractly and quantitatively. [1.2] Students must interpret the meaning of the quotient and remainder in context.

MP3 Construct viable arguments and critique the reasoning of others. [1.4] Students analyze Lauren's multiplication work in a skill builder.

MP7 Look for and make use of structure. [1.1] Students describe and apply strategies for multiplying by multiples of 10.

MP8 Look for and express regularity in repeated reasoning. [1.3] The division algorithm makes use of repeated reasoning.

TN2: Strategies for English Learners [1.1, 1.2, 1.3]

Teaching Note 2 in each packet is devoted to lesson-specific strategies for English learners. For a research-based list of general strategies for English learners, please refer to the front matter of the teacher guide.

Building Background

(Emphasize key vocabulary) [1.1] Formal definitions of mathematical properties may be difficult for some students. Start with examples of the properties in action and work from there to create a class definition. After this knowledge base is established, move to the formal definition.

(Link concepts to past learning) [1.3] Some English learners may have learned other algorithms for multiplication or division in their country. Encourage them to share their strategies.

Comprehensible Input

(Use appropriate speech and body language) [1.2] Act out word problems and discuss vocabulary to be sure students understand the problems.

TEACHING NOTES (Continued)

TN3: Strategies for Special Learners [1.1, 1.2, 1.3]

Teaching Note 3 in each packet is devoted to lesson-specific strategies for struggling students and special learners. For a research-based list of strategies for these learners, please refer to the front matter of the teacher guide.

Know your learner

(Assess student knowledge of prerequisite mathematics content.) [1.1, 1.2, 1.3] Use pre-assessment tools to determine student mathematical strengths and needs.

Create a positive classroom culture

(Maintain high standards for all students.) [1.2] All students can learn to divide. The chunking algorithm is an excellent alternative for students who have difficulty remembering all of their multiplication facts.

Increase communication and participation

(Encourage students to demonstrate what they have learned to their peers.) [1.1] All students benefit from explaining their strategies in small groups and to the whole class.

(Allow alternative methods to express mathematical ideas.) [1.1] Making connections explicit from the blocks in an area model to a pictorial model and then to an abstract multiplication algorithm are important for all special learners.

Differentiate instructional strategies

(Create special templates to isolate information or remove distractions.) [1.3] Provide a Division Template so students have extra room to write. Encourage them to create a toolkit of multiplication facts prior to beginning the division process.

TN4: Strategies for Enrichment [1.1, 1.2, 1.3]

Teaching Note 4 in each packet is devoted to lesson-specific strategies enrichment. For more research-based information about enrichment, please refer to the front matter of the teacher guide.

[1.1] Encourage students to devise efficient ways to use the area model to multiply by chunking numbers in strategic ways.

[1.2] Encourage students to represent chunking algorithm partial quotients in an area model diagram for multiplication, and explain how they connected the two representations.

[1.3] Advanced learners often devise “tricks” for remembering facts and making smart estimates when dividing. Encourage them to share their strategies with others.

TEACHING NOTES (Continued)

TN5: Creating the "Itch" [1.1, 1.2, 1.3]

"You have to create the itch before you scratch it," states Malcolm Swan, a Mathematics Education Professor at the University of Nottingham and the Shell Centre for Mathematics Education. His research suggests that students learn when teachers put them in situations that foster reflection and allow them to construct their own mathematical arguments by connecting what they're about to learn to what they already know. He suggests starting math lessons with an "itch" that justifies for them the learning of the "scratch" that is contained in the lesson.

TN5 in every packet contains suggestions about how to create the "itch." The "itch" will be referenced briefly as a useful reminder in the lesson pages. The primary purpose of the "itch" is to get students engaged by helping them access what they already know or to prime them for why the mathematics they are about to learn is useful.

[1.1] ***What are some examples in your life when the order in which you do something does not affect the outcome?*** Brushing your teeth and combing your hair, for example. ***What are some examples when the order you do things does affect the outcome?*** Taking a shower and getting dressed, for example. The goal is help students see that math contains rules/operations where the order matters sometimes and they need to learn these rules so that they don't end up "getting dressed and then taking a shower."

[1.2] ***When in life do we actually divide things into groups?*** One possibility, when organizing supplies for groups for an art project in class. ***When and why might estimation be more useful for dividing than "doing the math"?*** For the art project, if there are four groups, each group might get about ¼ of the pipe cleaners. It is unnecessary to count them.

[1.3] ***In a general sense, an algorithm is a sequence of actions that leads to a desired result. Tying your shoelaces can be a difficult challenge for young children to learn because it requires a lot of finger coordination and it's a really complex sequence of steps…an algorithm. What are some other algorithms that you do in your life successfully on a daily basis?*** One possibility: Make a peanut butter sandwich. ***What are some algorithms you'd like to learn?*** One possibility: Learn to parallel park a car.

TN6: Writing Computation Sentences Horizontally [1.1, 1.2, 1.3]

In early years, students typically line up whole number computation problems vertically. This is especially useful for addition and subtraction when adding digits with the same place value. However, as students mature mathematically, they will transition to writing mathematical sentences horizontally. This is so that equations can be written on one line, and equivalent equations can be lined up, one under the other.

TN7: Digit Name vs. Digit Value [1.1]

Stress place value in multiplication by distinguishing between the name of the digit and the value for which it stands. The 2 in 24 stands for $2 \times 10 = 20$, not 2. Base-10 blocks and area model diagrams emphasize the value that each digit stands for because they use expanded notation to build the answer.

TEACHING NOTES (Continued)

TN8: Cooperative Learning – Getting Started [1.1, 1.2, 1.3]

Cooperative learning is advantageous in many ways, but it takes time, patience, and practice for both the teacher and students alike. Simply putting kids in the same physical proximity with a task does not necessarily promote cooperation.

It is imperative that we help students to reach not only their academic content goals, but their academic personal and social goals as well. Some contributions of cooperative learning are:

- Higher achievement
- More effective problem-solving skills
- Increased use of higher-level cognition
- More positive attitude toward mathematics
- Increased motivation
- Greater acceptance of other's differences (race, religion, etc.)
- Promotion of equity of educational opportunities
- Increased use of academic language
- Greater language acquisition (for non-native speakers)
- Greater sense of belonging
- Integration of students with special needs

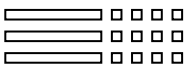
Much of learning is acquired through talking. Students in groups talk more than students receiving direct instruction. Teachers that facilitate cooperative learning tend to know their students better through observing these conversations, which in turn helps in formative assessment and lesson planning.

Some guidelines:

1. Create a climate and expectation for mutual respect and cooperation.
2. Ask students to make lists of "dos and don'ts" at the start of the school year. Post a chart of these in the classroom.
3. Teach students to follow these rules and to "police" themselves.
4. Arrange seats to facilitate partner work. It is recommended that seats are always in partner-ready positions unless individual assessment is required.
5. If students are not accustomed to much group work, start slowly. Consider pair work before larger group work. Plan initial group tasks (not all of which need be mathematics-centered) to help students learn how to be good partners.
6. Try to keep groups to a maximum of four, or five if necessary.
7. Monitor group interaction, intervene as necessary, and provide guidance and feedback. Encourage effort, and praise cooperative behaviors.
8. Create roles with responsibilities (leader, scribe, spokesperson, taskmaster, etc.) and change roles periodically (say once per week), then change groups (say once per month).
9. Consider making a point system and assign points based upon following roles, staying on task, etc. Carrying a clipboard for taking notes and assigning points may be helpful.
10. Teach students to self-evaluate and peer-evaluate. This may include a "quiet time" writing task to be turned in to the teacher assessing one's own progress and that of one's group mates.

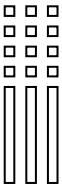
TEACHING NOTES (Continued)

TN9: Does 3 • 14 Have the Same Value as 14 • 3? [1.1]



$$\begin{array}{r} 3 \\ \times\ 14 \\ \hline \end{array}$$

or



$$\begin{array}{r} 14 \\ \times\ 3 \\ \hline \end{array}$$

The commutative property of multiplication asserts that the product does not depend on the order of the factors. Each of the products 3×14 and 14×3 is equal to 42. Nonetheless, for some problems context is important. Although both actions require 42 marbles, the filling of 3 bags with 14 marbles each will require different supplies than the filling of 14 bags with 3 marbles each. A piece of beachfront property (100 meters $\times$ 30 meters) will be worth more money if the beachfront is 100 meters long rather than 30 meters long.

TN10: Drawing Rectangles for an Area Model [1.1]

An area model is an alternative and efficient way to multiply. Encourage students to draw rectangles, even though the rectangles may not be drawn to scale. If students need to use base-10 blocks as a transitional step, change the numbers in the problems to match the quantity of blocks that are available.

TN11: Using an Area Model to Record Multiplication [1.1]

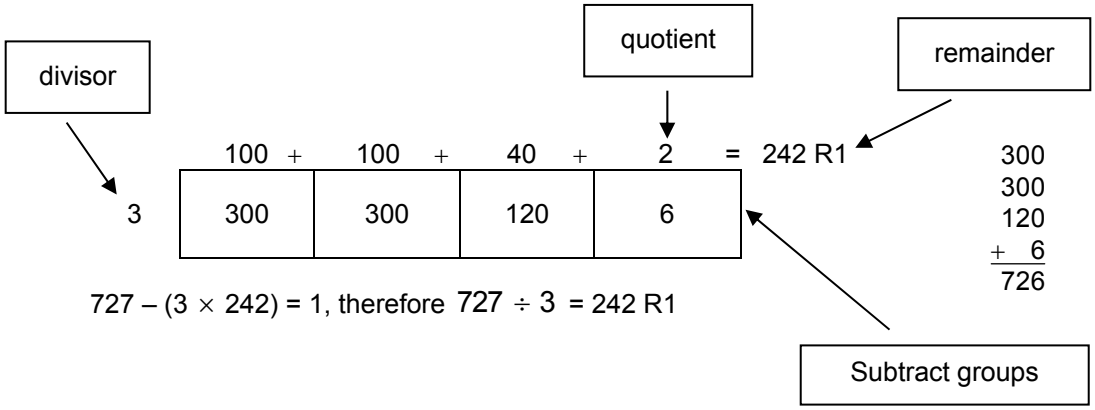
Is it acceptable to permit students to use an area model as a recording method for multiplication? Yes. An area model not only helps to explain why the standard algorithm commonly taught in the United States for multiplication works, it is an efficient recording alternative. Some students (especially visual learners and those who have difficulty keeping numbers lined up in multiplication problems) may prefer it. Furthermore, this method has certain benefits. It illuminates important mathematical concepts (such as the distributive property), allows for computational flexibility (expanded notations allow students to use derived facts), and reinforces the concept of area. Later in this course, students will explore the area model for determining the plausibility of the multiplication rule for rational numbers. Finally, when students take algebra, they can extend the area model to make sense of multiplication and factoring of polynomials.

TEACHING NOTES (Continued)

TN12: Using an Area Model to Record Division [1.2]

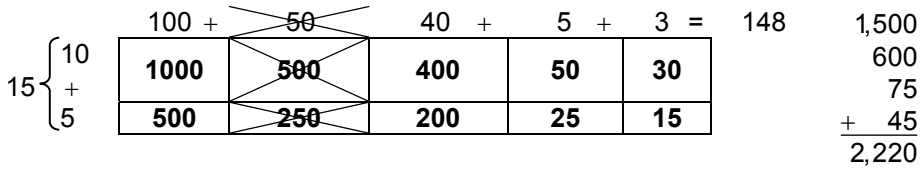
The area model can be used to solve division problems.

Example 1: $727 \div 3$



$727 - (3 \times 242) = 1$, therefore $727 \div 3 = 242$ R1

Example 2: Divide: $2{,}220 \div 15$ (Note that the guess of 50 is too big, so it was crossed out.)



Therefore, $2{,}220 = 15 \times 148$.

Rectangles are not drawn to scale.

TEACHING NOTES (Continued)

TN13: A Chunking Division Procedure [1.2]

This chunking division procedure keeps the dividend intact as we "close in" on the quotient. Students who are not fluent with all their multiplication facts may find this procedure more accessible than the standard division algorithm because they subtract out groups of the divisor more flexibly, but still arrive at the correct quotient. If the largest amount possible is chosen to subtract at each step, this procedure is very efficient.

Divide 761 highlighters into 3 boxes.

Step 1. Rewrite problem

$3\overline{)761}$

Step 2**:** Make a toolkit of multiplication facts that may be useful for this problem.

$3 \times 1 = 3$	$3 \times 10 = 30$	$3 \times 100 = 300$
$3 \times 2 = 6$	$3 \times 20 = 60$	$3 \times 200 = 600$
$3 \times 3 = 9$	$3 \times 30 = 90$	$3 \times 300 = 900$
$3 \times 4 = 12$	$3 \times 40 = 120$	$3 \times 400 = 1{,}200$

Step 3: Select a fact from the toolkit that is less than or equal to the dividend, and record.

761	
− 600	200
161	

Repeat Step 3: Continue the routine until the remainder is less than the divisor.

761		→	761		→			240 R 2	
−600	200		−600	200		761			
161			161			−600	200		
−120	40		−120	40		161			
41			41			−120	40		
			−30	10		41			
			11			−30	10		
						11			
						−9	3		
						2			
							253		

The last calculation shows that the quotient is (200 + 40 + 10 + 3) = 253, and the remainder is 2.

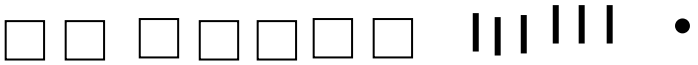
TEACHING NOTES (Continued)

TN14: Using Base-10 Blocks to Model The Division Algorithm [1.3]

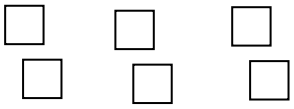
Base-10 block diagrams may be used to illustrate the standard division algorithm. We use squares, sticks, and dots  to represent 100, 10, and 1. Consider the problem:

Divide 761 highlighters into 3 boxes.

Step 1: Begin with 761, the initial number of highlighters to be shared. Think about how they can be divided into 3 groups.

Step 2: Toolkit (does not apply to diagram) 

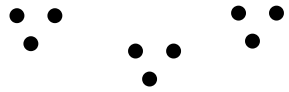
Step 3: "Divvie up" into 3 groups. Continue until the remaining units are less than 3 (the divisor).

THE MOST WE CAN TAKE OUT		REMAINING	
Words	Pictures	Words	Pictures
3 groups of 200 (or 600)		161 highlighters	

(trade out one 100 for ten 10's)  so 161 remain as sticks and dots

THE MOST WE CAN TAKE OUT		REMAINING	
Words	Pictures	Words	Pictures
3 groups of 50 (or 150)		11 highlighters	

(trade out one 10 for ten 1's)  so 11 remain.

THE MOST WE CAN TAKE OUT		REMAINING	
Words	Pictures	Words	Pictures
3 groups of 3 (or 9)		2 highlighters	

TOTALS

TAKE OUT		REMAINING	
3 groups of 253 (or 759)		2 highlighters remain	

$761 = 3 \cdot 253 + 2$

APPLYING PROPERTIES OF ARITHMETIC

Summary

Students learn and use properties of multiplication. Students learn strategies for recalling multiplication facts. Students use an area model to multiply numbers.

4.NBT.B, 5.NBT.A, 6.EE.A

Goals

- Learn strategies for deriving multiplication facts from simpler facts.
- Use associative and commutative properties.
- Use the distributive property.
- Use expanded notation.
- Multiply using an area model.

PREVIEW / WARMUP

Whole Class

Page 0
Word Bank

Page 1
Applying Properties of Arithmetic

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.

Create an "itch" here. See Teaching Note 5.

- Review the commutative and associative properties of addition and connect them to real life examples. Students identify the properties illustrated by numerical examples.
- Practice with addition and subtraction of whole numbers is located in the Skill Builders of the Student Packet.

INTRODUCE 1

Whole Class / Partners

Page 2
Properties of Multiplication

Page 3
Multiplication Facts

- Students read the properties of multiplication aloud and do the problems. Discuss as needed.
- Introduce the strategy of remembering math facts by using "derived facts."

Last year a student said she had difficulty remembering 6 • 7, but she did know 3 • 7 = 21. She added 21 + 21 = 42 in her head, and that is how she derived the multiplication fact 6 • 7 = 42. Why does her method work? By the distributive property, 6 • 7 = (3 + 3)7 = 3 • 7 + 3 • 7 = 21 + 21 = 42. This can also be related to doubling, namely 2 • (3 • 7).

What is another simpler multiplication fact you might use to derive the product for 6 • 7? Some student may find it easier to multiply 5 • 7 and then add another 7.

Page 4
Multiplying by Multiples of 10

If we know that 6 • 7 = 42, how can we quickly find the product for 6 • 70? 60 • 70? 60 • 700? Students may recall that they can count the number of zeroes in each factor and "write zeros at the end." Avoid using the ambiguous expression "add zeros," since adding zero to a number does not change the number. This is a good opportunity to discuss the addition property of zero, and the fact that we call zero the additive identity.

EXPLORE 1

Partners

Page 3
Multiplication Facts

Page 4
Multiplying by Multiples of 10

- Students select one of the multiplication facts that are typically difficult and record ways to remember it. Encourage students to share derived fact strategies with each other.
- Students find products of numbers that are multiples of 10.

SUMMARIZE 1

Whole Class

Page 3
Multiplication Facts

Page 4
Multiplying by Multiples of 10

- Record derived fact strategies for difficult facts on the board or on chart paper. Organize them according to the type of strategy used. Some examples are listed here.

Derived Fact Strategies				
Skip Count for 4×4	Double For 6×7	Halve for 6×5	Add On for $7 \times 3 = 21$	Take Away for $9 \times 3 = 27$*
4 8 12 16 So $4 \times 4 = 16$	$3 \times 7 = 21$, $21 + 21 = 42$, So $6 \times 7 = 42$ $7 \times 6 = 42$	$6 \times 10 = 60$, So $6 \times 5 = 30$ $5 \times 6 = 30$	$6 \times 3 = 18$ Think 19, 20, 21, So $7 \times 3 = 21$ $3 \times 7 = 21$	$10 \times 3 = 30$, $30 - 3 = 27$, So $9 \times 3 = 27$ $3 \times 9 = 27$

*For multistep strategies such as this, be careful not to misuse the equal sign. Avoid writing statements such as "$10 \times 3 = 30 - 3 = 27$" as these statements are false: 10×3 is *not* equal to $30 - 3$.

- Review strategies for multiplying by multiples of 10. Combine with fact recall strategies if needed.

What do you think about when you multiply 60×8? Students may say, "multiply 6×8 and add a zero." This technique works, but offer an alternative to this sloppy, imprecise language because it may interfere with learning. For example, think how this might get in the way of understanding the following number statements: $6 \times 8 + 0 = 48$ and $14.6 \times 10 \neq 14.60$. A better way to state this is, "We must move the (implied) decimal point one place to the right."

Why does this work? Write 60 as the product of 6×10. Use the associative and commutative properties: $(6 \times 10) \times 8 = (6 \times 8) \times 10 = 48 \times 10 = 480$.

INTRODUCE 2

Whole Class

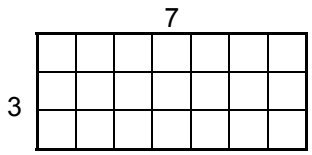
Page 5-6
Multiplication Using an Area Model

Materials
- Square tiles
- Base-10 Block
- Grid paper

Reproducible 1
Base-10 Blocks

Reproducible 4
Grid Paper

- Remind students that the area of a figure is a measure of size expressed in square units, and the area of a rectangle is the product of its length and width.
- Use square tiles or draw a diagram to create a rectangle that is 3×7.

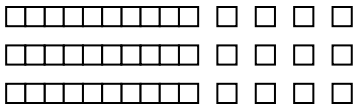


What does the first rectangle illustrate? 3 rows of 7 = 21, $3 \times 7 = 21$; 7 columns of 3 = 21, $7 \times 3 = 21$. This is an example of the commutative property of multiplication.

- Break the rectangle into two parts (for example $3 \times 7 = 3(2 + 5)$). Write in the partial products.

Suppose we break the rectangle into two parts. What does this rectangle illustrate? $3(2 + 5) = 21$, $(3 \times 2) + (3 \times 5) = 21$. This is an example of the distributive property.

- Use base-10 blocks or grid paper to illustrate problem 7. Demonstrate 3×14 as 3 rows of 14.



What is the total value of this collection? 42.

How can we find the value without counting one by one? A possible strategy: (3 groups of 10) + (3 groups of 4) = 30 + 12 = 42.

What property is illustrated here? The distributive property.

How does this picture connect to the distributive property? $3(10 + 4) = 3(10) + 3(4)$.

EXPLORE 2

Partners

Page 7
Multiplication Using an Area Model

- Students use area models to find the remaining products. Share and discuss.

SUMMARIZE 2

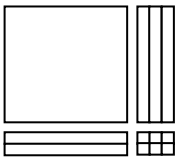
Whole Class

Page 7
Multiplication Using an Area Model

- Ask students to record their methods for problems 13 and 14 for the class, including the alternative methods asked for in problem 14. Encourage students to share routine and non-routine strategies (for example: 10 groups of 12 is 120, and 3 groups of 12 is 36, so 120 + 36 = 156). Accept the traditional algorithm, but require the students to use the area model. The traditional algorithm will be explored in an upcoming lesson, as well as an alternative, transitional algorithm below.

Rectangle 12×13

$100 + 20 + 30 + 6 = 156$



	10	3
10	100	30
2	20	6

Rectangle not drawn to scale.

Alternative Algorithm

$$\begin{array}{r} 12 \\ \times 13 \\ \hline 6 \\ 30 \\ 20 \\ +100 \\ \hline 156 \end{array} \quad \begin{array}{l} \\ \\ = 2 \times 3 \\ = 10 \times 3 \\ = 10 \times 2 \\ = 10 \times 10 \\ \\ \end{array}$$

Traditional Algorithm

$$\begin{array}{r} 12 \\ \times 13 \\ \hline 36 \\ +120 \\ \hline 156 \end{array}$$

Where do the partial products come from? These are easily identified in an area model (and the algorithms), so require the students to explain their methods clearly to the class.

In problem 14, 18 × 74, the rectangle was divided in two different ways. Did you get the same answer each time? Yes. ***Why?*** The area of a rectangle is the number of unit squares inside of it. The sum of the areas of the smaller rectangles is the same as the area of the big rectangle.

Which area model did you prefer? Some students may prefer the first model because there is less multiplying needed. Some students may prefer the second model because the expanded form uses easier multiplication facts. Some may have offered other approaches that they prefer.

Did anyone get different answers for the same problem when using different strategies? What does that mean? If so, then at least one of the procedures was incorrect. It is possible that students found their own computational errors when doing problems in two different ways.

PRACTICE

Individuals

Pages 8-9
Area Model Practice

Task, Page 1
Multiplication Patterns

- Students complete the problems for more practice. Time permitting, invite students to share and discuss solutions. Emphasize the flexibility of an area model. Recall that later this model will be a springboard to help make sense of both an alternative and the traditional algorithm. These problems are appropriate for class work or homework.
- This problem requires students to observe a pattern and make predictions.

CLOSURE

Whole Class

Page 0
Word Bank

Page 1
Applying Properties of Arithmetic

- Review the goals, standards, and vocabulary for the lesson.

DIVISION WITH REMAINDER

Summary

Students learn a division algorithm and use it to find quotients of whole numbers. Students solve problems and interpret the meaning of the remainder in the context of the problem.

4.NBT.B, 5.NBT.A, 6.NS.B, 6.EE.A

Goals

- Learn strategies for finding quotients of whole numbers.
- Estimate quotients.
- Use division vocabulary.
- Use a variation of the standard division algorithm.
- Interpret the meaning of remainders in division problems.

PREVIEW / WARMUP

Whole Class

Page 0
Word Bank

Page 10
Division with Remainder

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.

Create an "itch" here. See Teaching Note 5.

- Students complete warmup.

Can you give an example that illustrates why subtracting 6 from 24 until nothing remains illustrates a meaning for division? Measurement division could be modeled as repeated subtraction. For example, there are 24 cookies and each person gets 6.

INTRODUCE

Whole Class

Pages 11-12
A Division Algorithm

Materials
- Base-10 Blocks

Reproducible 1
Base-10 Blocks

- Present students with a variation of the traditional division algorithm. This algorithm keeps the dividend intact and uses a chunking technique to "close in" on the quotient. Students who are not fluent with all their multiplication facts may find this procedure more accessible. For students fluent in the traditional algorithm (which is introduced in the next lesson), this procedure may provide added insight into the meaning of division as subtracting groups of the divisor. Use base-10 blocks or block diagrams to illustrate each step if desired.

- **Step 1:** Write the problem.

In the context of this problem, what does 850 represent?
Total number of colored pencils.

In this context, what does 24 represent? Number of pencils in each group.

What will our answer represent? The number of groups.

INTRODUCE (Continued)

Whole Class

Pages 11-12
A Division Algorithm

Materials
- Base-10 Blocks

Reproducible 1
Base-10 Blocks

Reproducible 2
Division Template 1

Reproducible 3
Division Template 2

- **Step 2:** Make a toolkit of multiplication facts that may be useful for this problem. Explain to students that they need **not** write down every fact. They can add facts later, and they will be able to divide, even if every fact is not included. Discuss some techniques for easily making a tool kit such as the ones shown below.

  If I know 24 × 1 and 24 × 2, how can I get 24 × 3? Add 24 × 1 + 24 × 2.

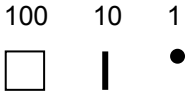
  How can I get 24 × 4? Add 24 × 1 + 24 × 3; or double 24 × 2. ***How might I get 24 × 8?*** Double 24 × 4.

  If I know 24 × 1, how can I get 24 × 10? Move (implied) decimal one place larger.

  How do I know that I do not need any facts larger than 24 × 40? Because 24 × 40 = 960. This is already larger than the dividend.

  For special learners:

  Use Base-10 blocks and a base-10 diagram to support this procedure.

- **Step 3:** Show how to select a fact from the toolkit and record. Emphasize that with this procedure, we will subtract groups of 24 from the dividend in chunks to determine the quotient. That is, with each step, we are finding groups of 24 colored pencils, determining the number of colored pencils used up so far and then subtracting to determine how many colored pencils are still left.

  Why does the first step show that 850 = 24 × 30 + 130? Show where various components appear in the recording.

  Why do we need to continue the division process? There are still enough pencils to form more groups of 24.

- **Repeat Step 3:** Continue the routine until the remainder is less than the divisor. Discuss the meaning of each step in the process. Compare to the traditional algorithm if appropriate, or wait until the next lesson.

  How can you tell what the quotient is? Add the numbers on the right. ***How can you tell what the remainder is?*** It is the final difference.

  Consider re-doing the same problem, and suggesting that students instead try a first step of dividing out 10 or 20 groups of pencils. Then compare to the initial process. Let students know that they do not have to begin with the largest/closest fact in their toolkit if they are more comfortable removing a smaller amount.

- Try another example with a multi-digit divisor (such as 2,220 ÷ 15 or 7,792 ÷ 35). Ask students to compare underestimating quotients using friendly numbers (such as 15 × 100 = 1,500 or 35 × 200 = 7000) to "close in" on the quotient as well as using the closest fact to get to the dividend.

EXPLORE

Partners

Pages 13-14
Division Problems

- Students practice the division algorithm for problems 1 and 2. Have students check their work using an area model to display multiplication.

  What happens to the remainder when checking a division problem? Multiplying (using an area model for practice) will give the product of the quotient and divisor. The remainder must then be added to get the original dividend.

- Students use this division algorithm to solve each story problem. Be sure students pay particular attention to the meaning of the remainder for problem 4.

  For English learners and special learners:

  As students read, take time to emphasize some of the study skills needed to read a mathematics text.
  - Read the text more than once. First scan for the big idea, and then focus on the details.
  - Read slowly. Stop to make sure each phrase is understood.
  - Read with pencil and paper. Take good notes or try examples.

SUMMARIZE

Whole Class

Pages 13-14
Division Problems

- Invite students to share computational strategies and solutions to problems 1 and 2. Try to have more than one approach demonstrated for each, time permitting.

- Discuss results for problems 3 and 4, and strategies for problem 5.

  For problem 3, what is the meaning of the quotient? Mr. Garcia was able to drive 22 miles per gallon, and there is no remainder to interpret.

  For problem 4, what is the meaning of the quotient and the remainder? Is it more appropriate to round down or round up? We can fill 38 buses, but then 48 students remain. Rounding up is necessary, so the answer to the question is that 39 buses are needed to take all of the students on the field trip.

  What was necessary for finding the values at A and B? A common way is to notice that each pair of adjacent hatch marks must have a difference of (164 – 20) ÷ 6. Then students can count up or back by that amount from the given values. Look for other creative students strategies to share with the class.

PRACTICE

Whole Class

Pages 15-16
Division Practice

- These pages are appropriate for class work or homework.

CLOSURE

Whole Class

Page 0
Word Bank

Page 10
Division with Remainder

- Review the goals, standards, and vocabulary for the lesson.

MULTIPLICATION AND DIVISION: STANDARD ALGORITHMS

Summary

Students review the standard algorithm for multiplication of whole numbers. Then students review the standard algorithm for division of whole numbers.

4.NBT.B, 5.NBT.A, 6.NS.B

Goals

- Multiply multi-digit numbers using the standard algorithm.
- Divide multi-digit numbers using the standard algorithm.

PREVIEW / WARMUP

Whole Class

Page 0
Word Bank

Page 17
Multiplication and Division: Standard Algorithms

- Introduce the goals and standards for the lesson. Discuss important vocabulary as relevant.

Create an "itch" here. See Teaching Note 5.

- Review the alternative algorithms presented in the previous lessons. Point out to students that when they write problems 3 and 4 horizontally, they are using a notation that is useful for algebra. In this course, they will see that notation often.

INTRODUCE 1

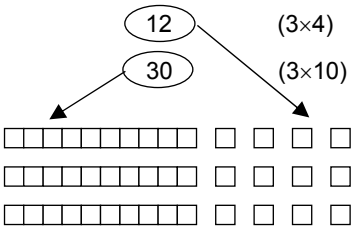
Whole Class

Materials
- Base-10 Blocks or
- Grid paper

Reproducible 1
Base-10 Blocks

Reproducible 4
Grid Paper

- Begin with a simple multiplication problem such as 3×14. Connect representations to each area model, a longer alternative algorithm, and finally the standard algorithm). Be sure students see the relationship between the partial products in the first three methods (illustrated below) and the standard algorithm, even though they are obscured by the efficient recording.



	10	+	4
3	30		12

$$\begin{array}{r} 14 \\ \times\ 3 \\ \hline 12 \\ +\ 30 \\ \hline 42 \end{array}$$

In the standard algorithm, how is the product of 12 represented? We write only the 2 in the ones place and "carry" the 1 in the tens place.

In the standard algorithm, how is the product of 30 represented? After multiplying the 3 times the 1, we add the carried 1, resulting in 40, and write the 4 in the tens place to the left of the 2 in the ones place.

EXPLORE 1 / SUMMARIZE 1

Partners / Individuals

Page 18
Multiplication: Three Methods

- Students work through problem 1, and make connections between the representations.

  Why is expanded form used to create the lengths of the rectangle? This breakdown will lead to partial products that match standard procedures.

  What does the "zero" mean in the standard algorithm? We may say "$2 \times 4 = 8$ to begin the second partial product line. But we mean "$20 \times 4 = 80$. The zero is the "0" in 80.

- Discuss problem 2 as needed.

- Students use any two methods to find each product. Encourage them to try each of them at least once or twice. Have several students demonstrate their work by writing and talking it through, and ask follow up question to check for understanding.

PRACTICE 1

Partners / Individuals

Page 19
Multiplication Practice

- This page is appropriate for class work or homework.

INTRODUCE 2 / EXPLORE 2 / SUMMARIZE 2

Whole Class / Partners / Individuals

Reproducible 2
Division Template 1

Reproducible 3
Division Template 2

- Review basic division terminology. Explain to students that we are going to review a more efficient process to divide whole numbers. We will "take out" as much as possible at each step, which differs from the algorithm in the previous lesson that allowed for student choice at each step.

- Begin with $850 \div 24$. First compute using the chunking division procedure from the previous lesson. Then show the standard algorithm next to it, using language commonly associated with the standard algorithm, while recording on the board.

  How are the two procedures different? The chunking procedure might take more steps. We work with the dividend as a whole number, rather than as individual digits.

  How are the two procedures the same? Both lead to same result.

Page 20
Division: The Standard Algorithm

- Repeat the process for $963 \div 14$. Ask questions as needed to check for understanding.

PRACTICE 2

Partners / Individuals

Page 21
Practice: The Standard Division Algorithm

- Both of these pages are appropriate for class work or homework.

- Students practice the standard division algorithm. If done in class, monitor student progress, and have several students share their work. Make sure that students understand the meanings of the quotients and remainders for problems 4 and 5. This can be done the next day if done for homework.

For Special Learners

Repeat this problem with different start numbers throughout the year for multiplication and division fluency practice.

Page 22
Multiplication and Division Fluency Challenges

- Begin with an example, such as starting with 2. Multiply it by 3, 4, and 5. Then divide by 2, 3, 4, 5 so that students understand the task.

EXTEND	
Whole Class / Partners / Individuals Task, Page 2 Interpreting the Division Algorithm	• Use this page as group work with discussion, a performance task, or as a problem of the week.

CLOSURE	
Whole Class Page 0 Word Bank Page 17 Multiplication and Division: Standard Algorithms	• Review the goals, standards, and vocabulary for the lesson.