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Features

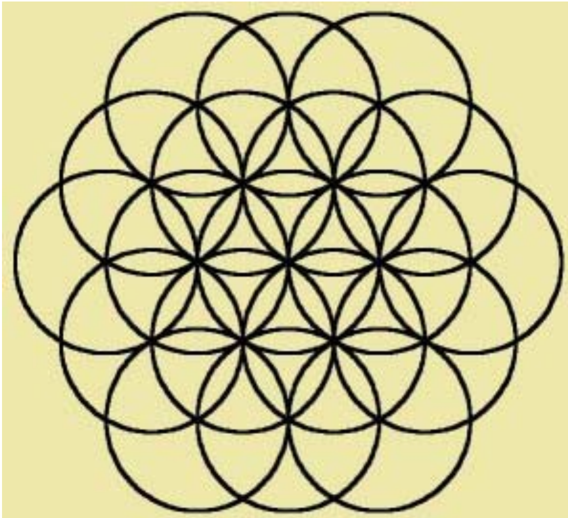
# Maths and art: the whistlestop tour

## by Lewis Dartnell

The world around us is full of relationships, rhythms, correlations, patterns. And mathematics underlies all of these, and can be used to predict future outcomes. Our brains have evolved to survive in this world: to analyse the information it receives through our senses and spot patterns in the complexity around us. In fact, it's thought that the mathematical structure embedded in the rhythm and melody of music is what our brains latch on to, and that this is why we enjoy listening to it. It is perhaps not surprising then that there is a great deal of overlap between mathematics and the art that our brain finds so pleasing to look at.

This article is a whistle−stop tour of some of the types of art with a strong mathematical component, or conversely where a mathematical visualisation has an astonishing beauty.

## Geometric patterns



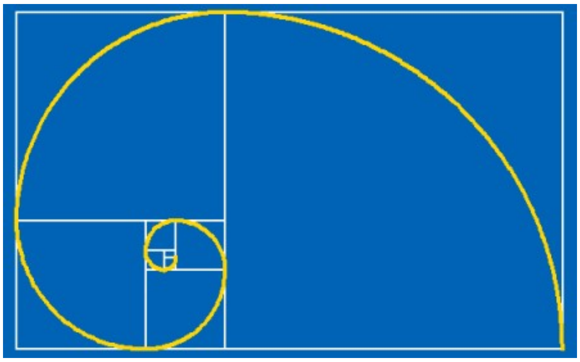
The "Flower of Life" pattern

Simple arrangements of mathematical figures, like circles and triangles, have been extensively used in decoration throughout history. For example, the "Flower of Life" can be seen on the Temple of Osiris at Abydos, Egypt, which dates back about 5,000 years. This pattern, as seen to the right, consists of an array of circles positioned in rows, each one centred on the circumference of circles in the neighbouring rows. (This arrangement provides an interesting geometric problem to solve. If each circle has a radius of 1 unit, can you show that the rows are $^3/_2$ apart? Hint: think about the type of triangle formed by the centre−points of three neighbouring circles, and your sine and cosine rules about triangle sides).

Mosques throughout the world are decorated with extremely elaborate geometrical patterns of simple shapes. This is because Islamic art (in contrast to that of other faiths, with their murals, paintings and stained glass windows) traditionally avoids the depiction of people and animals, and instead involves repeating geometric patterns. These regular arrangements are said to symbolise the divine order of the Universe. However, the artist often includes a deliberate mistake in acknowledgement of the belief that only Allah is truly perfect.

## The Golden Ratio

The ancient Greeks did some of the finest maths in history − we owe much of our understanding to them. One of the numbers that particularly interested them is known as the *Golden Ratio*. A rectangle constructed with sides in proportion to the ratio of 1 to 1.618 to 3 decimal places (to be precise, $\frac{1+\sqrt{5}}{2}$) has a very interesting property. Removing the largest square contained in the rectangle leaves a smaller rectangle, with exactly the same proportions as the original. This means that you can keep on removing squares to leave smaller and smaller rectangles. Joining up the corners of these rectangles produces a spiral that has been named the Golden Curve due to its graceful aesthetic look.



The Golden Curve

The value 1.618 is itself very curious. It was discovered by the Greeks as the positive solution to the basic quadratic equation $x^2 - x - 1 = 0$. The ratio of successive terms of the Fibonacci sequence, where each new term is the sum of the two previous (i.e. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,...), also tends towards this value of 1.618.... This series turns up throughout nature, from the number of petals in a flower, to the pattern of seeds in Sunflowers to the swirl in a Nautilus shell.

The Greeks appreciated the special nature of the Golden Ratio, and thought that objects in this proportion were particularly pleasing to the eye. It is said that they used it to ensure the beauty of statues and architecture, for example in the dimensions of the Parthenon. This practice was apparently passed down through history, so that even the figures in Leonardo Da Vinci's paintings, or Michaelangelo's David are proportioned according to this ratio. You can find out more about the use of the Golden Ratio in art in The golden ratio and aesthetics from a previous issue of *Plus*.

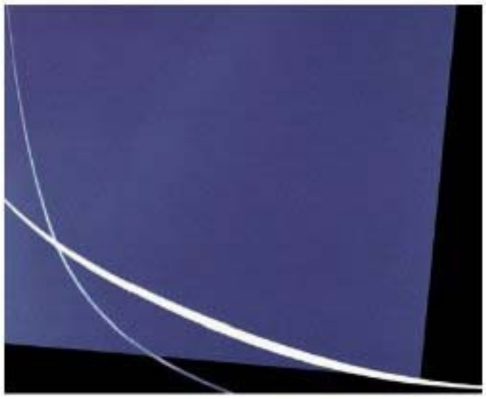
However, although there is strong support for the presence of this Fibonacci "Golden" ratio in nature, whether Ancient or Renaissance artists really did use it in their work or whether it is only a numerological coincidence is much more contentious. In fact, recent psychological experiments have failed even to show that people have any preference for this proportion, compared to either "thinner" or "squatter" rectangles. There is a very interesting article by Keith Devlin on the facts and fictions of the Golden Ratio in art on the website of the Mathematical Association of America.

## Geometric abstractionism

Piet Mondrian was one of the founding members in 1917 of a Dutch art movement called *De Stijl* (meaning "The Style") or neo−Plasticism. Mondrian's style of painting involved the use of strictly horizontal or vertical black lines to create a grid of rectangles, some of which were filled in with black or white, or vivid red, blue or yellow. You can see an example of Mondrian's work at the website of the Museum of Modern Art in New York.

Mondrian believed that, despite their complexity and variability, natural scenes are composed of basic components and regularity. So he tried to create faithful reproductions by painting only these fundamentals and their relationship with each other, using only the prime colours and elements of geometric mathematics − straight lines, right angles and quadrangles − to create his reflection of reality.

## Abstract paintings from graphs



Crossing at the Left Border, by Margaret Leiteritz

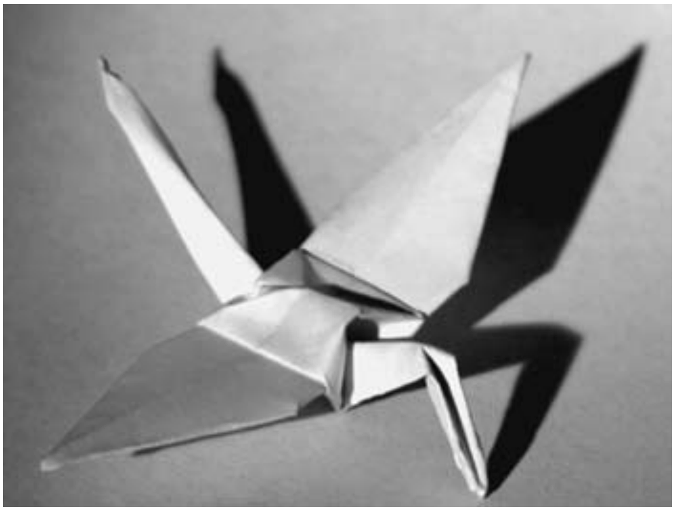
Margaret Leiteritz was always intrigued by the graphs in her scientific journals and books about chemical engineering. She was fascinated by the way that a simple line on a graph could represent something real about the energies and motions of the universe, such as combustion or viscosity of a liquid, as well as the innate aesthetic appeal of many of the curves. Between 1961 and 1974 Leiteritz produced a series of "painted diagrams". One piece from this period is named *Crossing at the Left Border*, which she painted from a graph labelled in the paper only as "Figure 13: Separation as a function of flow rate with plate spacing as parameter". It's amazing that something with such a dry and unimaginative name inspired such a colourful and dynamic piece of art.

## Tessellations

M.C. Escher's "Regular division of the plane".


Maurits Cornelis Escher (1898−1972) is well known for his graphic art. He enjoyed playing with aspects of surfaces and reflection (including spherical mirrors), perspective, symmetry, and impossible figures such as the paradoxical staircase depicted in *Ascending and Descending*. He sometimes included polyhedra in the background of his drawings, and was particularly interested in mathematical curiosities such as the Möbius Strip, a surface with only one side. He also produced a great number of tessellating patterns composed of irregular shapes. You may have learned at school that only certain regular polygons can be repeated indefinitely and all fit together without any gaps − triangles, squares and hexagons can make tiles, but pentagons cannot. Escher made an art form out of colourful patterns of tessellating reptiles, birds, fish and even crabs and sea−horses.

# Origami



Origami is the Japanese craft of creating wonderful three−dimensional shapes and models solely by folding paper − usually a single square sheet (*ori* meaning folded and *kami* meaning paper). The links between this ancient art and mathematics are profound. If you unfold a finished model you will see a complex geometrical pattern of creases made up of triangles and squares, many of which will be congruent (because they were produced by the same fold). The construction step known as a "push fold" is mathematically equivalent to bisecting an angle.

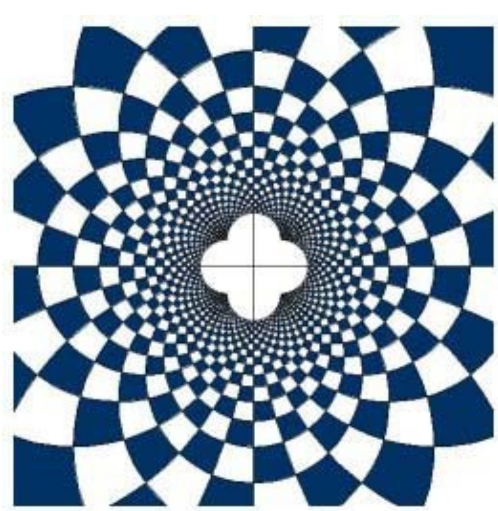
The regular polyhedra. Image used by permission of Polycell

Maths classrooms are often decorated with models of the nine "regular polyhedra". These are the three−dimensional solids that have every face the same. Five of them are *convex*, that is, the straight line joining any two points lies entirely inside the solid. These are often known as the *Platonic solids*: the tetrahedron, cube, octahedron, dodecahedron and icosahedron. The other four were not known to the Greeks, and are sometimes known as the *stellated* (starlike) regular polyhedra. The classroom models are usually made from card with scissors and glue, but they can also be made by origami. In fact, mathematicians have been able to prove not only that any polyhedron can be constructed by origami, but also that any outline drawn with straight lines (such as a star symbol *, or arrow ) can be cut out from a sheet of paper using a single scissor cut, once the sheet has been folded in the appropriate way.

## Anamorphic Art

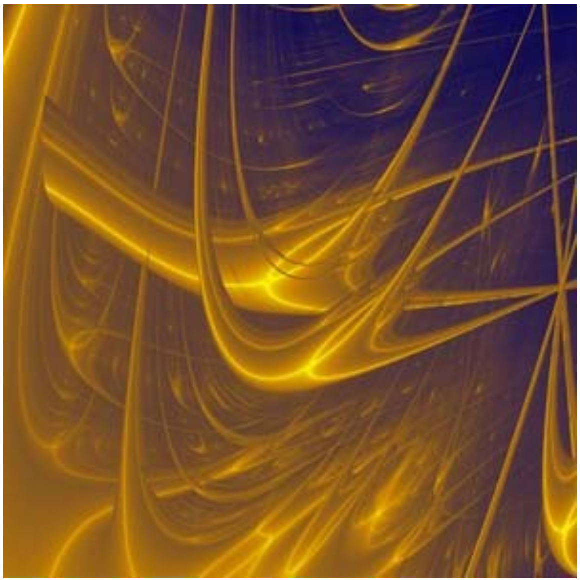
The Ambassadors (1533), Holbein

"Anamorphis" is a form of art that was first experimented with during the Renaissance and became particularly popular during the Victorian era. It involves distorting an image so that it is unrecognisable unless viewed in the right way. For example, Hans Holbein's *The Ambassadors* (1533), at the National Gallery in London, contains a skull (symbolising the transience of life) that has been grossly stretched. It only becomes recognisable when viewed from a very oblique angle, by standing practically alongside the left−hand edge of the painting.



Anamorphic art is a good demonstration of the mathematics of transformations. Translations, rotations and reflections don't tend to distort an object in interesting ways, but stretching along one axis (like in *The Ambassadors*) is a popular method. However, another very striking effect is produced when an image is transformed through an "inverse curve". If the curve used is a circle then the image only becomes recognisable in the reflection of a cylindrical mirror. One good example is "The Well", drawn by István Orosz (1998), which contains a portrait of Escher (you can see a copy of this image and the hidden portrait at planetperplex). The image shown to the right is of an inverted chessboard that only appears normally if a cylindrical mirror is placed in the gap in the centre.

## Fractals



A detail from the "Lyapunov Exponent"
by Andy Burbanks

A fractal is usually defined as a mathematical structure that exhibits some sort of "self−similarity", meaning that if you zoom in on one, the same type of structure will keep appearing. For example, a fern leaf is built up of leaflets and fronds with the same reiterative structure. They are also often defined as shapes that have fine detail at all scales, and it is this combination of repetition and intricacy that makes them so aesthetically interesting.

When the sensitivity of a complex system to small changes is mapped on the plane, the result is often a fractal. When this map is colour−coded, the result can be strikingly beautiful. The "Lyapunov Exponent" fractal on the left is one example, and you can find out more, and see some more beautiful images, in Extracting beauty from chaos from issue 9 of *Plus*.

## Minimal energy surfaces

A rope hung between two points, like a clothes drying line, describes a particular mathematical curve. It is sagging under its own weight, but is also constrained because the rope cannot stretch much. So the curve produced is a compromise − it sags as low to the ground as it can get. The rope is said to have found a "minimum energy solution" (in this case it is potential energy that is being minimised).

Minimisation problems can also be solved in three dimensions. The spherical shape of a soap bubble is due to all the molecules in the surface attracting each other and pulling the thin film into the smallest possible area. A raindrop on a leaf is under further constraints and the resultant "minimal energy surface" is distorted into a flattened droplet shape.

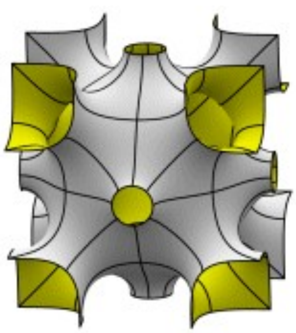


Image by Ken Brakke

Mathematicians have used computers to work through the same equations that are solved by nature, but given them much more complicated constraints. The picture to the left shows the surface that would be adopted by a soap bubble formed within a cube connected to its neighbours, if it never popped and wasn't subject to gravity.

Antipot, by Bathsheba Grossman

Some artists have appreciated the beauty of such forms and incorporating them into their work. For example, Bathsheba Grossman, who has a maths degree from Yale and a Masters degree in sculpture, describes herself as an explorer of the region between art and mathematics. Her "Antipot" sculpture is an example. You can find out more about Grossman and her work at her website.

This article can give only a brief glimpse of the enormous region of overlap between mathematical patterns and processes, and the world of art and aesthetics. A web search reveals many more examples, and it would be well worth keeping your maths hat on when you visit a gallery....

## About this article

Lewis Dartnell read Biological Sciences at Queen's College, Oxford. He is in his second year of a four−year combined MRes−PhD program in Modelling Biological Complexity at University College London's Centre for multidisciplinary science, CoMPLEX. In 2003 he came second in the THES/OUP science writing competition, and in 2004 he came second in the Daily Telegraph/BASF Young Science Writer Awards. You can read more of Lewis's work at www.ucl.ac.uk/~ucbplrd.

*Plus* is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.