

Maths: Higher Revision Guide

Name:

Grade C Key topics:

- **Ratio and Proportion**
- **Adding and subtracting fractions**
- **Pie Charts**
- **Drawing graphs e.g. $y=3x+2$**
- **Percentage Increase and Decrease**
- **Stem and Leaf**
- **Pythagoras**
- **Area and circumference of a circle**

Grade B Key topics:

All of the above plus:

- **Simultaneous equations**
- **Compound interest**
- **Cumulative frequency**
- **Factorising and Expanding**
- **Standard form**
- **Trigonometry (SOH CAH TOA)**

Grade A Key topics:

All of the above plus:

- **Circle theorems (angles within a circle)**
- **Using the quadratic equation**
- **Probability**
- **Sine and Cosine rule**
- **Histograms**

Ratio: Key facts

Example

A drink is made up of orange & grapefruit in the ratio 2 : 3. If there is 500ml of drink in a cup, how much is orange and how much is grapefruit.

- | | | |
|--------|--|----------------|
| Step 1 | Add up the ratio numbers | $2 + 3 = 5$ |
| Step 2 | Divide the total by this number | $500 \div 5 =$ |
| 100ml | | |
| Step 3 | Multiply this by the proportion we are interested in | |
| | So orange = $2 \times 100 = 200\text{ml}$ | |
| | grapefruit = $3 \times 100 = 300\text{ml}$ | |

Questions to try

- 1). Divide £90 in the ratio 3 : 7.
- 2). A school collected £180 for charity. It was decided to divide the money between Dr. Barnados and the RSPCA in the ratio 2 : 3. How much did each charity receive ?
- 3). Pocket money is split between Pete, Alan and Helen in the ratio 4 : 5 : 6. Dad pays out £60, how much does each person get ?
- 4). £60 is split between Arnie, Barney and Clancey in the ratio 3 : 4 : 5. How much does each one receive ?
- 5). Mr. Allen, Mr. Book and Mr. Collins own 4, 5, and 6 parts of a business respectively. The business makes £120 profit in a week. How much does each man get ?

Adding and subtracting fractions : Key facts

Try adding up these fractions

$$\frac{1}{2} \overset{\times 3}{\underset{\times 3}{+}} \frac{1}{3} \overset{\times 2}{\underset{\times 2}{+}} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Step 1

Multiply by the bottom number in the other fraction

Step 2

Remember to multiply the top and the bottom by the same number

Questions to try

$$\frac{1}{4} + \frac{1}{3} =$$

$$\frac{1}{7} + \frac{1}{2} =$$

$$\frac{2}{3} + \frac{1}{5} =$$

$$\frac{5}{7} - \frac{1}{4} =$$

$$\frac{5}{6} + \frac{3}{4} =$$

Pie Charts: Key facts

Drawing Pie Charts

Remember:

$$\frac{360^{\circ}}{\text{Total Frequency}} = \text{Total angle per person}$$

Now- Multiply your answer by all the categories

24 people were asked for their favourite football team.
The results are in the table.

Football Team	Frequency	Degrees
Liverpool	3	$3 \times 15 = 45^{\circ}$
Leeds United	7	
Manchester Utd	4	
Arsenal	2	
Manchester City	8	
	24	

Click next to see the
table fill up with
degrees.

Try to work them out
before you click.

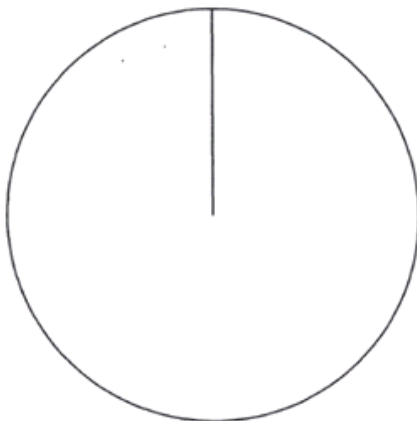
$$\frac{360}{24} = 15$$

Question to try:

- Q1 The members of Four Ways Youth Club each choose an activity for an outing.
Their choices are shown in the table below.

Activity	Frequency
Bowling	8
Theme Park	14
Cinema	12
Other	6
	40

Construct a pie chart for this information.



Drawing graphs: Key facts

Two Operations:

$$y = 3x + 2$$

Create a table

x	-1	0	1	2	3	4
$\times 3$						
$+2$						
y						

Co-
Ordinates

Questions to try (Just Draw the tables)

Easy

$$y = 2x + 2$$

$$y = 3x + 1$$

$$y = 7x + 3$$

Medium

$$y = 5x - 2$$

$$y = 3x - 3$$

$$y = 9x - 7$$

Hard

$$y = 2 - 2x$$

$$y = 3 - 3x$$

$$y = x^2$$

Percentage Increase and Decrease: Key facts

To find 10% divide by 10

To find 1% divide by 100

Remember to add on or subtract at the end of the question.

Depreciate means subtract in this context.

A pair of jeans is in a sale.

The sale offers 20% off all prices.

The jeans originally cost £38.

What was the price of the jeans in the sale?

$$\begin{aligned} \text{find } 20\% \rightarrow & 38 \div 10 = 3.8 = 10\% \\ & 7.6 = 20\% \end{aligned}$$

reducing so subtract

$$38 - 7.60 = \underline{\underline{£30.40}}$$

British Rail have decided to raise ticket prices by 17.5%.

Below is a list of prices before the increase. Work out the new ticket prices please.

To find 17.5% you need to find 10%. Then halve it to find 5%. Then halve it again to find 2.5%. Then add the three parts up.

Liverpool-London Return	£60
Liverpool-Edinburgh Return	£50
Liverpool-Paris Return	£90
Liverpool-Rome Return	£130

Stem and leaf: Key facts

Remember to put on a key!

The stem and Leaf diagram must be ordered



Questions to try

Complete a Stem and Leaf diagram for this party of 30 people.

47	29	41	28	38	51	43	42	65	46
42	32	58	72	59	57	46	41	63	44
61	52	39	71	42	47	53	62	72	55

Find the median, mode and modal group and range for this data.



Area and Circumference of a circle

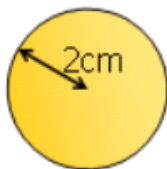
Key facts:

Area of a circle $A = \pi r^2$ or $A = \pi \times r \times r$

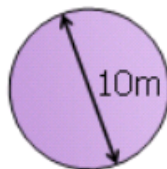
Circumference of a circle $C = \pi \times d$

The area of a circle

Use $\pi = 3.14$ to find the area of the following circles:



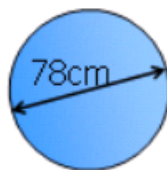
$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 2^2 \\ &= 12.56\text{cm}^2 \end{aligned}$$



$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 5^2 \\ &= 78.5\text{m}^2 \end{aligned}$$



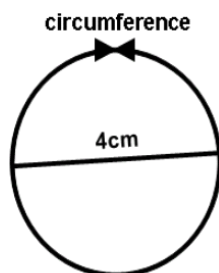
$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 23^2 \\ &= 1661.06\text{mm}^2 \end{aligned}$$



$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 39^2 \\ &= 4775.94\text{cm}^2 \end{aligned}$$

Example 1

Find the circumference of this circle

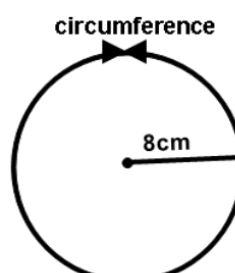


$$\begin{aligned} \text{Circumference} \\ &= \pi \times \text{diameter} \end{aligned}$$

$$\begin{aligned} \text{Circumference} \\ &= \pi \times 4 \\ &= 12.57\text{cm (2 d.p.)} \end{aligned}$$

Example 2

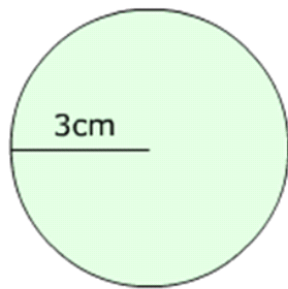
Find the circumference of this circle



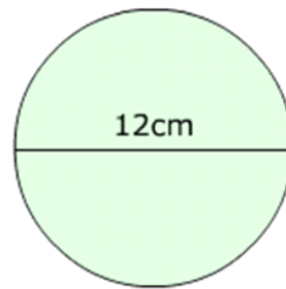
$$\begin{aligned} \text{Circumference} \\ &= \pi \times \text{diameter} \end{aligned}$$

$$\begin{aligned} \text{Circumference} \\ &= \pi \times 16 \\ &= 50.27\text{cm (2 d.p.)} \end{aligned}$$

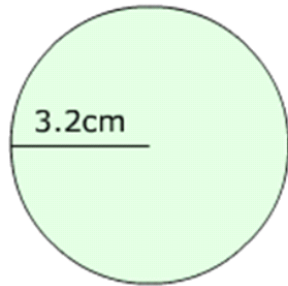
Calculate the area and the circumference of :



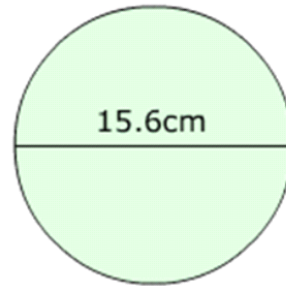
Area = cm^2 [2]



Area = cm^2 [2]



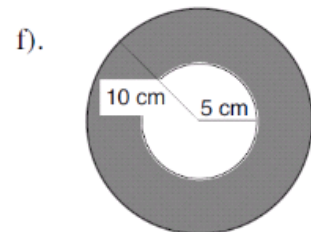
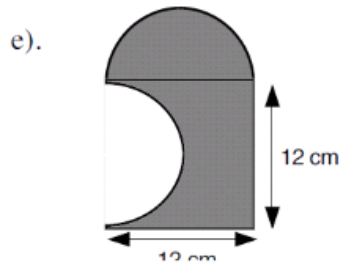
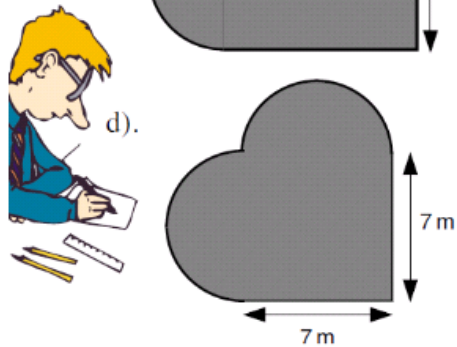
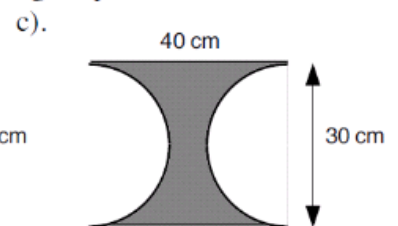
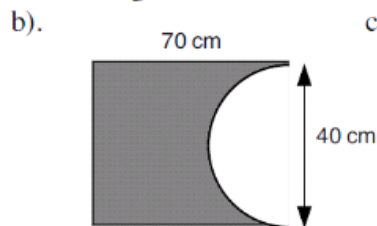
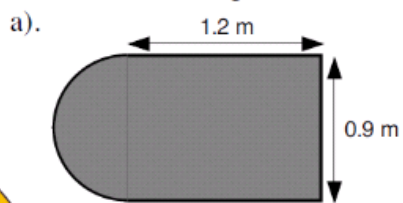
Area = cm^2 [2]



Area = cm^2 [2]

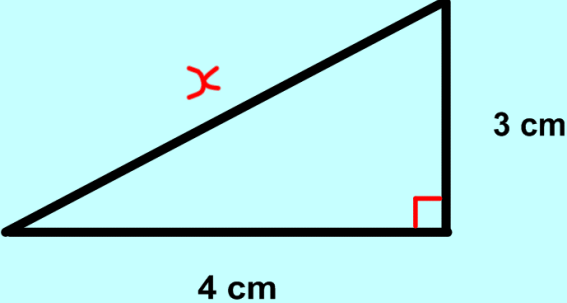
Extension: Tip to find a semicircle half a circle

14). Find the **area** and **perimeter** of the shaded regions in the following shapes.



Pythagoras : Key facts

E.g. 1



3 cm

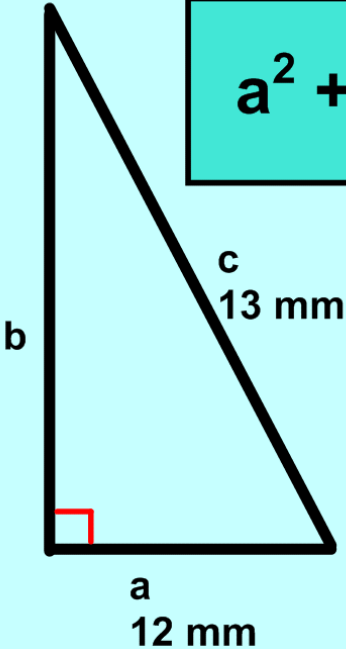
4 cm

So:

$$\begin{aligned} 3^2 + 4^2 &= x^2 \\ 9 + 16 &= x^2 \\ 25 &= x^2 \\ \sqrt{25} &= x \\ \underline{5} &= x \end{aligned}$$

$$a^2 + b^2 = c^2$$

E.g. 2



b

c
13 mm

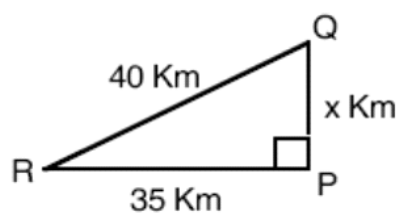
a
12 mm

$$a^2 + b^2 = c^2$$

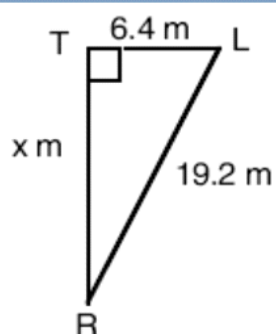
$$\begin{aligned} 13^2 &= 12^2 + b^2 \\ 169 &= 144 + b^2 \\ -144 &-144 \\ \hline 25 &= b^2 \\ \sqrt{25} &= b \\ 5 &= b \end{aligned}$$

Some to try:

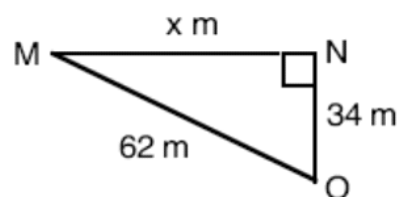
26).



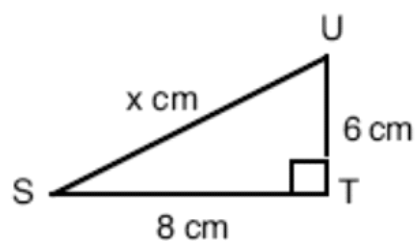
27).



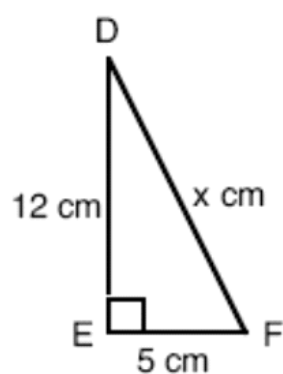
28).



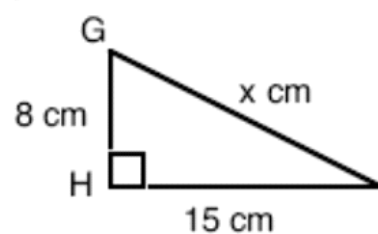
1).



2).



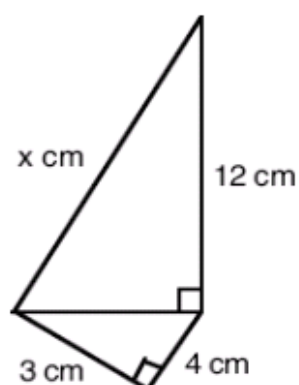
3).



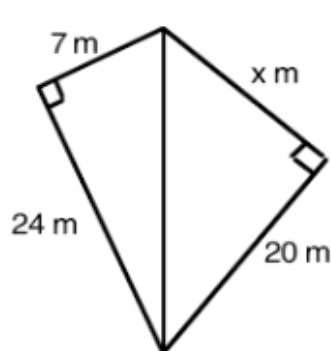
Extension:

D. Find the length of the side marked "x".

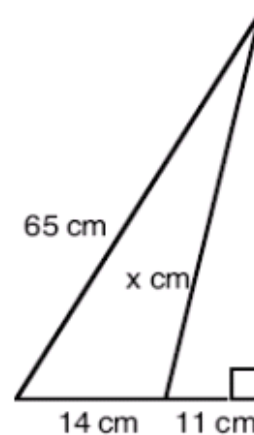
1).



2).



3).



Simultaneous Equations: Key facts

$$\begin{array}{r} \text{x3} \quad 4x - 2y = 7 \\ \text{x4} \quad 3x - 4y = 4 \end{array}$$

Multiply the other equation by the amount of x in the other equation

$$\begin{array}{r} \text{x3} \quad 12x - 6y = 21 \\ \text{x4} \quad 12x - 16y = 16 \end{array}$$

Now Subtract the equations if the signs are the same or add them if it is different

$$\begin{array}{r} -6 - (-16) = 10y \rightarrow 10y = 5 \\ 21 - 16 = 5 \end{array} \quad \begin{array}{l} y = \frac{5}{10} \\ y = 0.5 \end{array}$$

$$\begin{array}{r} 4x - 2 \times 0.5 = 7 \\ 4x - 1 = 7 \\ 4x = 8 \\ x = \frac{8}{4} \quad x = 2 \end{array}$$

Now substitute back in to an original equation

Questions to try:

Solve the simultaneous equations

$$\begin{array}{r} 3x + 7y = 26 \\ 4x + 5y = 13 \end{array}$$

Solve the simultaneous equations.

$$\begin{array}{r} 5a + 3b = 9 \\ 2a - 3b = 12 \end{array}$$

Compound interest: Key facts

Compound Interest

Worked Example 1 Long Method

£2000 is invested at 6% compound interest for 3 years.
Find: (a) the amount in the account at the end of the period.
and (b) the interest accrued.

Amount after 1 year = $2000 + 6\% \text{ of } 2000 = 2000 + 120 = £2120$

Amount after 2 years = $2120 + 6\% \text{ of } 2120 = 2120 + 127.20 = £2247.20$

Amount after 3 years = $2247.20 + 6\% \text{ of } 2247.20 = 2247.20 + 134.83 = £2382.03$

Interest accrued = $£2382.03 - £2000 = £382.03$

Compound Interest

Worked Example 1 Efficient Method

£2000 is invested at 6% compound interest for 3 years.
Find: (a) the amount in the account at the end of the period.

(a) Money at end of 3 years = $2000 \times 1.06^3 = £2382.03$ ✓

Explanation of the Method

Remember that 6% means $\frac{6}{100} = 0.06$

At the end of each year the money grows to 106% of its value at the start of the year = $\frac{106}{100} = 1.06$

After 1 year the money has been multiplied by 1.06 → 2000×1.06

After 2 years the money is again multiplied by 1.06 → $(2000 \times 1.06) \times 1.06$

After 3 years the money is again multiplied by 1.06 → $(2000 \times 1.06 \times 1.06) \times 1.06$

So after 3 years the money will have grown to $£2000 \times 1.06^3$.

If the term had been 7 years and the interest rate 8% then we would simply have calculated 2000×1.08^7 .

Remember that the amount is decreasing in will reduce every year. Per annum means every year. Depreciate means reducing.

Worked Example 1:

£2000 is invested at 6% compound interest for 3 years.

Find: (a) the amount in the account at the end of the period and (b) the interest accrued.

Q1. £600 is invested at 5% compound interest for 3 years.

Find: (a) the amount in the account at the end of the period. and (b) the interest accrued.

Q2. £5000 is invested at 8% compound interest for 4 years.

Find: (a) the amount in the account at the end of the period (nearest £) and (b) the interest accrued (nearest £)

Remember with the question below the decimal multiplier will be 100%-8%.....

A car is bought for £17,000, a nice Toyota Celica.

The car loses 8% of its value every year due to old age and wear and tear (depreciation).

How much will the car be worth after 3 years?



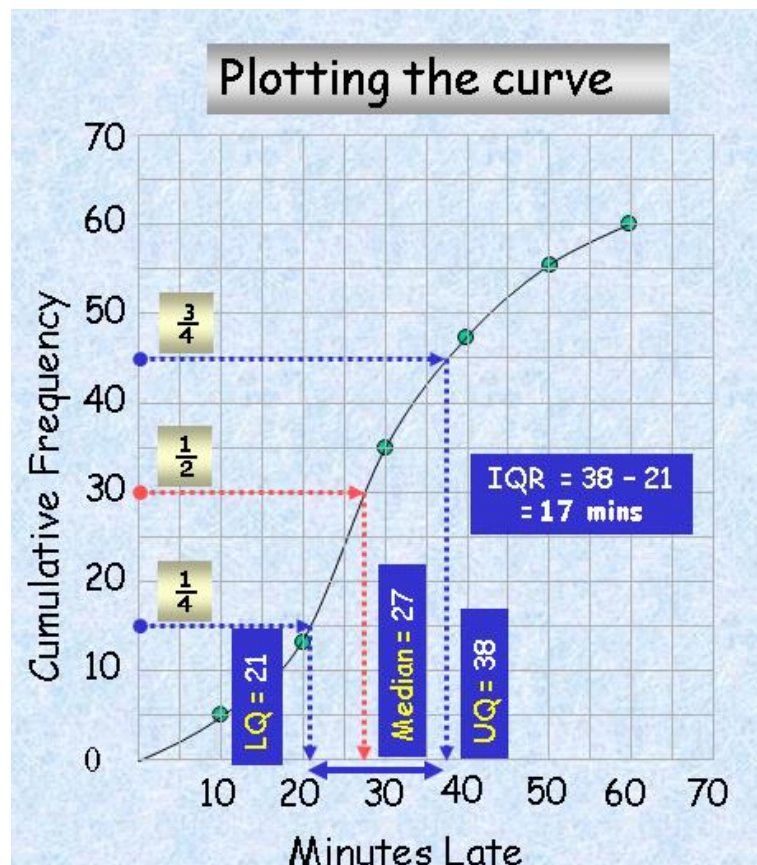
Cumulative Frequency: Key points

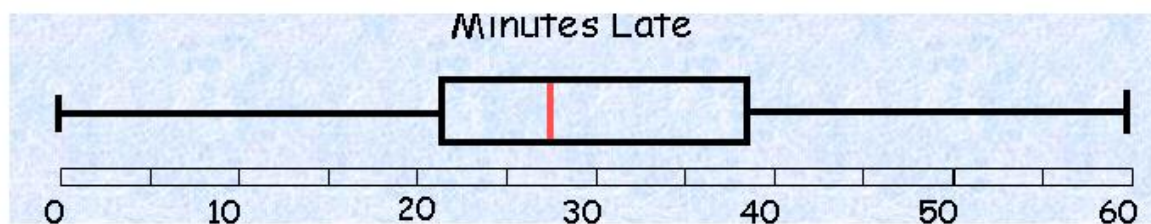
Cumulative frequency table

Minutes Late	Frequency	Upper Limit	Cumulative Frequency
0 - 10	5	< 10	5
10 - 20	8	< 20	13
20 - 30	22	< 30	35
30 - 40	12	< 40	47
40 - 50	8	< 50	55
50 - 60	5	< 60	60

Cumulative frequency just means running total.

Plot the upper limit!

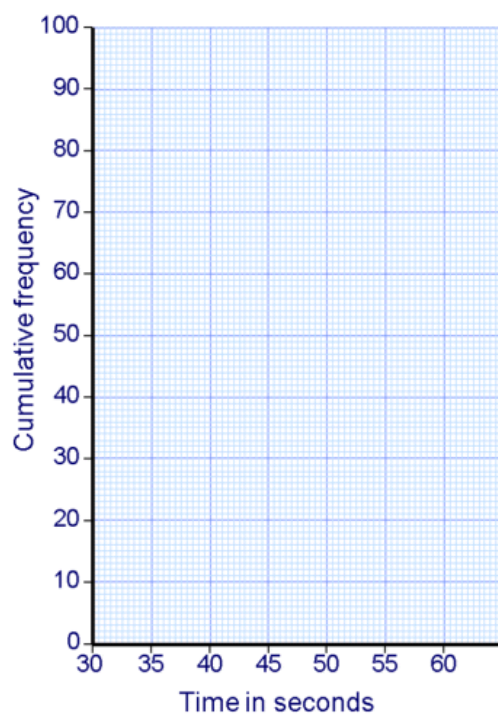




Lowest score **Lower Quartile** **median** **Upper quartile** **Highest score**

Question for you to try:

Time in seconds	Frequency	Cumulative frequency
$30 < t \leq 35$	9	
$35 < t \leq 40$	12	
$40 < t \leq 45$	24	
$45 < t \leq 50$	28	
$50 < t \leq 55$	16	
$55 < t \leq 60$	11	



Factorising Expressions

- Take out a common factor and put the rest in a bracket

eg 1

$$20y - 12 = \boxed{4} (\boxed{5y - 3})$$

$$8x + xy = \boxed{x} (\boxed{8 + y})$$

Tip: remember to factorise fully:

$$15x^2 - 10x$$

$$= 5x (3x - 2)$$

Some to try:

Easy

- | | | | | |
|---------------|---------------|---------------|----------------|-----------------|
| 1). $2x + 6$ | 2). $4x + 12$ | 3). $3t + 9$ | 4). $5a - 20$ | 5). $6y + 9$ |
| 6). $4f - 10$ | 7). $9g + 15$ | 8). $8x + 12$ | 9). $14r - 21$ | 10). $12e - 15$ |

Medium

- | | | | | |
|-----------------|------------------|------------------|------------------|-----------------|
| 11). $xy + 3x$ | 12). $2ab + ad$ | 13). $5t + rt$ | 14). $5ry - rf$ | 15). $3gh - 2g$ |
| 16). $x^2 - 2x$ | 17). $2d^2 - 3d$ | 18). $4p^2 + 5p$ | 19). $6r - 5r^2$ | 20). $9p^2 + p$ |

Hard

- | | | | | |
|-------------------|-------------------|-------------------|-------------------|---------------------|
| 21). $3xy + 6x$ | 22). $8a + 4ab$ | 23). $5fg + 10g$ | 24). $4rt - 6r$ | 25). $6gh - 9h$ |
| 26). $8x^2 - 10x$ | 27). $9x^2 - 12x$ | 28). $6xy - 9x^2$ | 29). $8rt + 6r^2$ | 30). $18c^2 - 12cb$ |

Factorising expressions (2)

$$x^2 - 8x + 12 = 0$$
$$(x - 6)(x - 2)$$

Some to try:

Easy

- | | | | |
|----------------------|----------------------|---------------------|---------------------|
| 1). $x^2 + 4x + 3$ | 2). $x^2 + 6x + 5$ | 3). $x^2 + 6x + 8$ | 4). $x^2 + 7x + 10$ |
| 5). $x^2 + 11x + 10$ | 6). $x^2 - 11x + 10$ | 7). $x^2 + 8x + 15$ | 8). $x^2 - 8x + 15$ |

Medium

- | | | | |
|----------------------|----------------------|-----------------------|-----------------------|
| 9). $x^2 - 12x + 20$ | 10). $x^2 - 9x + 14$ | 11). $x^2 - 11x + 30$ | 12). $x^2 - 7x + 12$ |
| 13). $x^2 + 6x - 7$ | 14). $x^2 + 4x - 5$ | 15). $x^2 + 2x - 8$ | 16). $x^2 + 12x - 28$ |

Hard

- | | | | |
|-----------------------|----------------------|----------------------|-----------------------|
| 17). $x^2 - 2x - 15$ | 18). $x^2 - 4x - 12$ | 19). $x^2 - 6x - 16$ | 20). $x^2 - x - 20$ |
| 21). $x^2 + 11x + 18$ | 22). $x^2 + 6x - 16$ | 23). $x^2 - 8x + 16$ | 24). $x^2 + 10x + 21$ |

Expanding double brackets

Double Brackets: Expanding Quadratics

$$(x+4)(x+6) =$$

\times	x	$+4$
x	x^2	$4x$
$+6$	$6x$	$+24$

$$x^2 + \cancel{4x} + 6x + 24$$
$$x^2 + 10x + 24$$

Some to try:

Easy

- 1). $(x+3)(x+2)$ 2). $(a+4)(a+6)$ 3). $(c+3)(c+4)$ 4). $(r+2)(r+7)$
5). $(y+3)(y+5)$ 6). $(t+8)(t+3)$ 7). $(k+6)(k+2)$ 8). $(d+8)(d+2)$

Medium

- 9). $(f+5)(f-2)$ 10). $(g+2)(g-4)$ 11). $(e+1)(e-2)$ 12). $(b+4)(b-1)$
13). $(h+5)(h-2)$ 14). $(p+5)(p-4)$ 15). $(m+3)(m-4)$ 16). $(h+7)(h-9)$

Hard

- 17). $(w-4)(w+3)$ 18). $(e-6)(e+5)$ 19). $(y-7)(y+2)$ 20). $(f-5)(f+2)$
21). $(p-3)(p+8)$ 22). $(q-6)(q+1)$ 23). $(x-1)(x+4)$ 24). $(b-3)(b+2)$

Trigonometry Key points:

S **O** **H** **C** **A** **H** **T** **O** **A**

$$\text{Sine } A = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin A = \frac{O}{H}$$

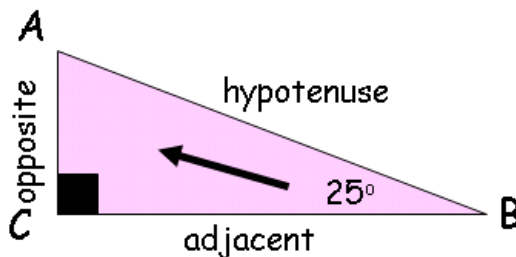
$$\text{Cosine } A = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\cos A = \frac{A}{H}$$

$$\text{Tangent } A = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan A = \frac{O}{A}$$

Convention for naming sides.



• The side opposite the **right-angle** is called the **hypotenuse**

• The side opposite a given angle is called the **opposite side**.

• The sides next to (or adjacent to) a given angle is called **adjacent side**.

The three stage process to find missing sides.

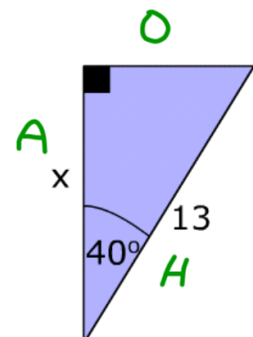
SOHCAHTOA

Step 1 - Which sides do we know?

Opp & Hyp

Adj & Hyp

Opp & Adj



$$\cos A = \frac{A}{H}$$

$$\cos 40 = \frac{x}{13}$$

$$13 \times \cos 40 = x$$

Now put the information into your calculator!

E.g.2

This is the bit no one likes;

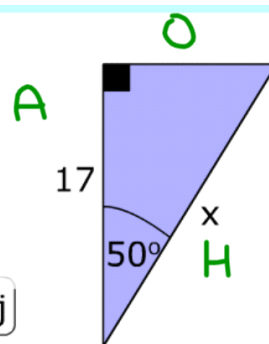
SOHCAHTOA

Step 1 - Which sides do we know?

Opp & Hyp

Adj & Hyp

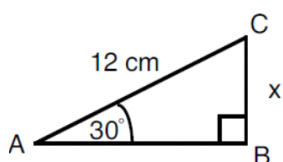
Opp & Adj



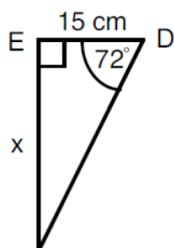
$$\cos A = \frac{A}{H} \quad \cos 50 = \frac{17}{x} \quad x = \frac{17}{\cos 50}$$

Some for you to try:

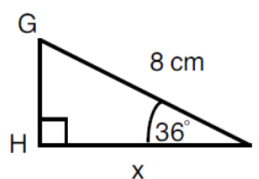
1).



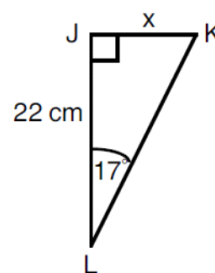
2).



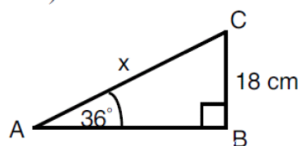
3).



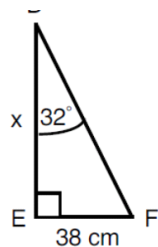
4).



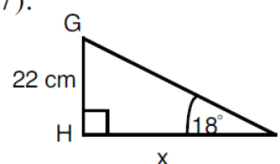
5).



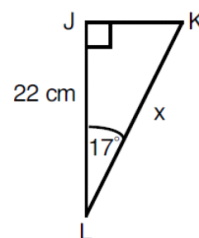
6).



7).

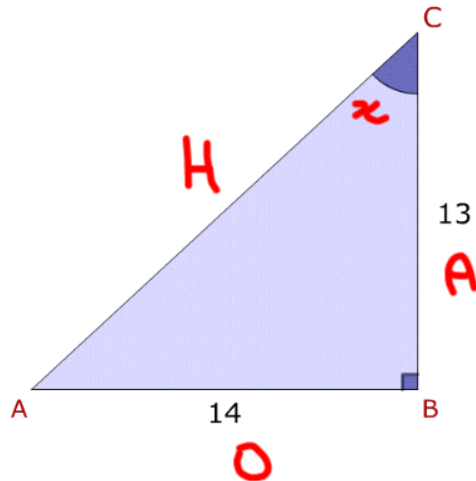


8).



Finding missing angles using trigonometry:

Remember to use the inverse (-1)



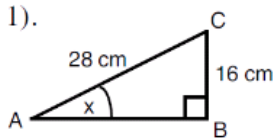
SOH CAH **TOA**

$$\tan = \frac{O}{A}$$

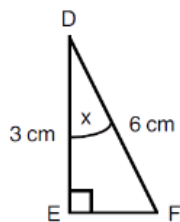
$$\tan^{-1} = \frac{14}{13}$$

Find the angles marked x.

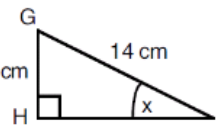
1).



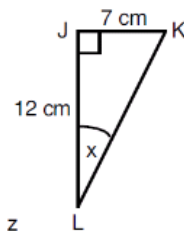
2).



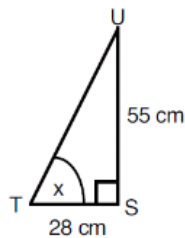
3).



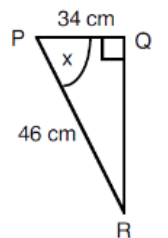
4).



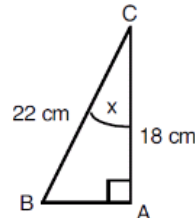
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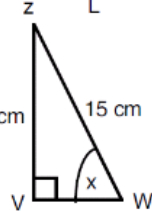
6).



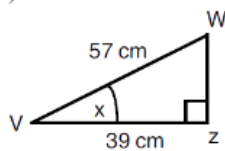
7).



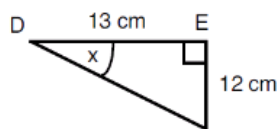
8).



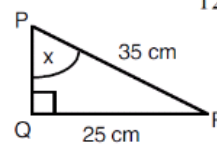
9).



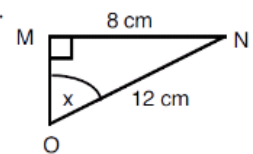
10).



11).



12).



Standard form: key points

What is standard form?

- Anything that ends in $\times 10$ and then a power (+ or -) and has only a unit which is followed by a decimal place

eg 3.6×10^2 4.293×10^{-5}
 4.2×10^{11} 4.2×10^{-11}

Any small number less than 1 can be written in Standard Form. Here are a few examples.

$$0.3 = 3 \times 10^{-1}$$

$$0.041 = 4.1 \times 10^{-2}$$

$$0.002 = 2 \times 10^{-3}$$

$$0.0053 = 5.3 \times 10^{-3}$$

$$0.00027 = 2.7 \times 10^{-4}$$

Any number can be written in Standard Form. Here are a few examples.

$$300 = 3 \times 10^2$$

$$410 = 4.1 \times 10^2$$

$$2000 = 2 \times 10^3$$

$$5300 = 5.3 \times 10^3$$

$$27000 = 2.7 \times 10^4$$

Mixed Questions

1. What is 2300 when written in standard form?

- A 23×10^2 B 2.3×10^{-3} C 2.3×10^3 D 2×10^3 E 230×10^1

2. What is 0.00045 when written in standard form?

- A 4.5×10^{-4} B 4.5×10^{-5} C 4.5×10^4 D 4×10^{-4} E 45×10^{-5}

3. What is 2×10^{-3} when written as a normal number?

- A 2000 B 2003 C 0.0023 D 0.002 E 0.0002

4. What is 67×10^5 when written in standard form?

- A 6.7×10^6 B 0.67×10^7 C 67×10^3 D 0.67×10^3 E 6.7×10^4

5. What is 8.74×10^3 when written as a normal number?

- A 8000 B 0.000874 C 0.00874 D 874000 E 8740

Calculating with standard form remember:

$$a^4 \times a^6 = a^{10}$$

When you multiply powers we add

$$a^{10} \div a^6 = a^4$$

When you divide powers we subtract

$$(3 \times 10^{-1}) \times (9 \times 10^{-9})$$

$$27 \times 10^{-10}$$

Multiply numbers

$$-1 + -9 = -10$$

$$2.7 \times 10^{-9}$$

Not in standard form

Power decreases as the number
has gone from big to small

Questions to try:

7). $(2 \times 10^4) \times (6 \times 10^{-3})$

10). $(7 \times 10^1) \times (4 \times 10^{-4})$

13). $(4 \times 10^{-2}) \times (9 \times 10^4)$

16). $(7 \times 10^{-4}) \times (5 \times 10^3)$

19). $(6 \times 10^{-2}) \times (7 \times 10^{-7})$

22). $(8 \times 10^{-4}) \times (2 \times 10^{-5})$

8). $(5 \times 10^6) \times (4 \times 10^{-4})$

11). $(3 \times 10^6) \times (9 \times 10^{-2})$

14). $(3 \times 10^{-4}) \times (7 \times 10^2)$

17). $(9 \times 10^{-1}) \times (4 \times 10^6)$

20). $(4 \times 10^{-2}) \times (5 \times 10^{-4})$

23). $(3 \times 10^{-1}) \times (9 \times 10^{-9})$

9). $(6 \times 10^2) \times (7 \times 10^{-5})$

12). $(6 \times 10^3) \times (8 \times 10^{-7})$

15). $(9 \times 10^{-4}) \times (6 \times 10^0)$

18). $(3 \times 10^{-2}) \times (5 \times 10^3)$

21). $(3 \times 10^{-1}) \times (7 \times 10^{-3})$

24). $(8 \times 10^{-3}) \times (4 \times 10^{-5})$

Harder exam question:

$$y^2 = \frac{ab}{a+b}$$

$$a = 3 \times 10^8$$

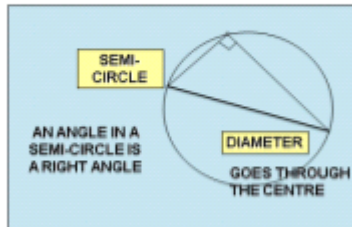
$$b = 2 \times 10^7$$

Find y .

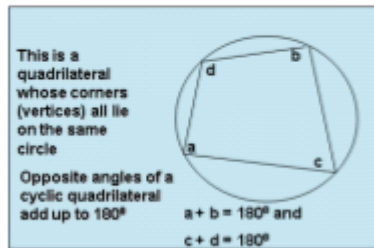
Give your answer in standard form correct to 2 significant figures.

Circle theorems: You need to remember these:

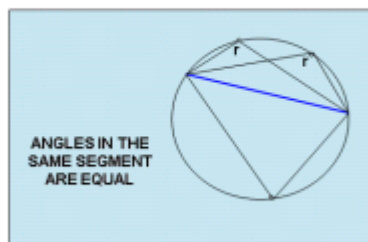
Angles in Circles



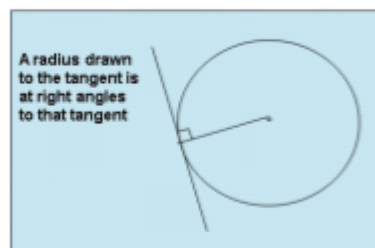
Cyclic quadrilateral



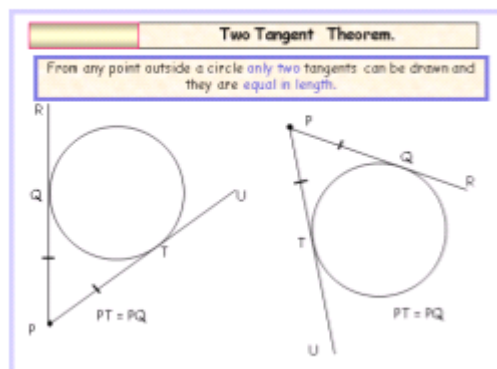
Angles in Segments



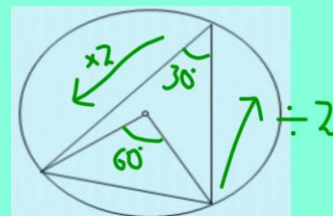
Tangent and Radius



Mr Cottons favourite!



Angles at the centre and the circumference (arrowhead)



Mixed test questions:

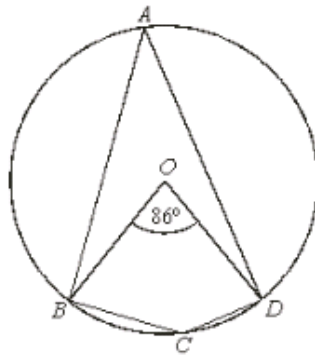


Diagram NOT accurately drawn

A, B, C and D are points on the circle, centre O .
Angle $BOD = 86^\circ$

- (a) (i) Work out the size of angle BAD .

.....^o

- (ii) Give a reason for your answer.

.....
.....

(2)

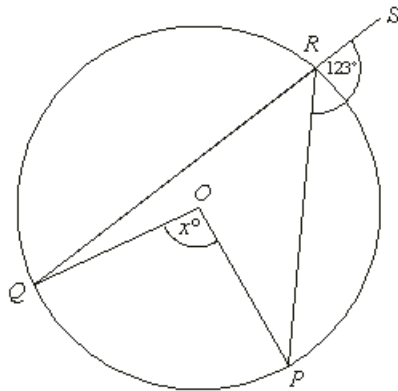
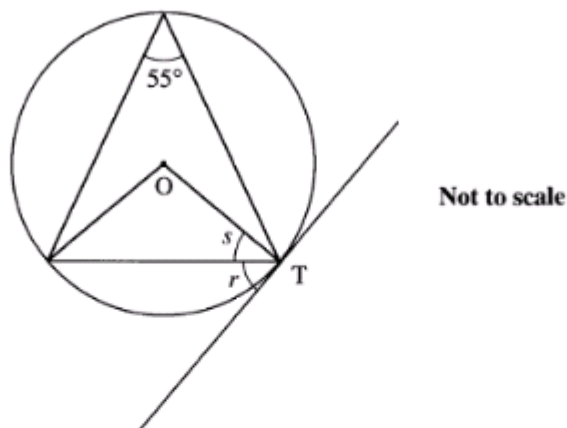


Diagram NOT
accurately drawn

QRS is a straight line.
 QR and PR are chords of a circle, centre O .
Angle $PRS = 123^\circ$.
Angle $QOP = x^\circ$.

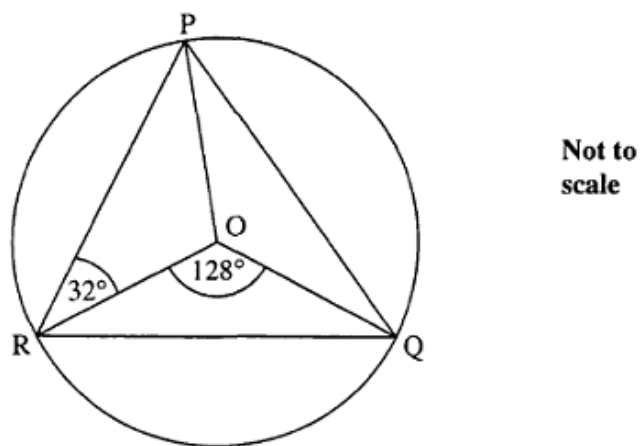
Calculate the size of the angle marked x° .
Give reasons for your answer.



O is the centre of the circle.
T is the point of contact of a tangent to the circle.

Work out angles r and s .
Give a reason for each answer.

$r = \dots\dots\dots^\circ$ because $\dots\dots\dots$
 $s = \dots\dots\dots^\circ$ because $\dots\dots\dots$
 [3]



O is the centre of the circle.
Angle ROQ = 128° .
Angle PRO = 32° .

Prove that PQR is an isosceles triangle.
Give reasons for each step in your argument.

Quadratic Equation: key points

Try to factorise $x^2 + 5x + 2 = 0$
 $(\quad)(\quad) = 0$

The **Quadratic Formula** will help us to find the two values for x that makes the equation equal 0. (roots)

Here is the quadratic formula known as 'The Formula'

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Try to factorise $x^2 + 5x + 2 = 0$
 $(\quad)(\quad) = 0$

$a=1$ $b=5$ $c=2$

Now substitute into the formula:

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{-5 + \sqrt{17}}{2} \quad \text{or}$$

$$x = \frac{-5 - \sqrt{17}}{2}$$

Now put into a calculator to work out, you will have two answers:

So x = _____ or x = _____

Some to try: remember to take the ‘-’ sign if the number is negative!

5 Questions

1) $x^2 + 4x + 1 = 0$

$x =$ $\text{ or } x =$

2) $x^2 - 3x + 1 = 0$

$x =$ $\text{ or } x =$

3) $2x^2 + x - 8 = 0$

$x =$ $\text{ or } x =$

4) $3x^2 - 4x - 9 = 0$

$x =$ $\text{ or } x =$

5) $3x^2 - 3x - 2 = 0$

$x =$ $\text{ or } x =$

Probability

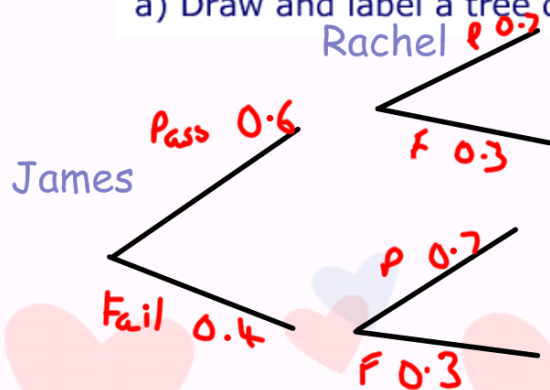
Key points:

Remember to multiply probabilities (do NOT add)

Twins James and Rachel are taking their driving test next week. The probability that James will pass is 0.6; the probability that Rachel will pass is 0.7.

These events are independent.

a) Draw and label a tree diagram to show this information.



Q. What is the probability that they both pass?

James to Pass is 0.6 Rachel to pass is 0.7

Multiply them together so $0.6 \times 0.7 = 0.42$

Top tip if you can't multiply decimals convert them to fractions:

$$\begin{aligned} &0.6 \times 0.7 \\ &= \frac{6}{10} \times \frac{7}{10} = \frac{42}{100} \end{aligned}$$

Q. What is the probability only one of them passes?

James to pass 0.6 Rachel to fail 0.3 $0.6 \times 0.3 = 0.18$

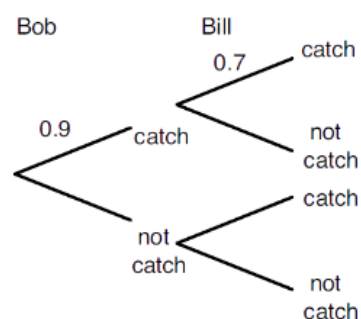
James to fail 0.4 Rachel to pass 0.7 $0.4 \times 0.7 = 0.28$

Now add your two answers together so $0.18 + 0.28 = 0.46$

Questions: Hint draw a tree diagram (probability must add up to one)

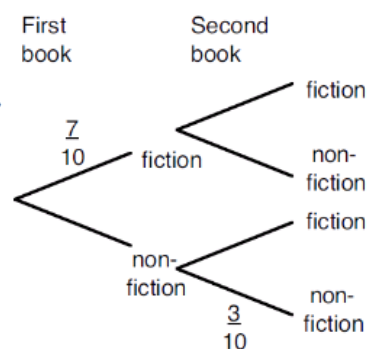
Each morning Bob and Bill catch the same bus. The probability that Bob catches the bus is 0.9 and for Bill it is 0.7. The probabilities are independent of each other.

- a). Copy and **complete** the tree diagram.
- b). Calculate the probability that on a given day :-
 - i). they both catch the bus,
 - ii). Bob catches the bus, but not Bill,
 - iii). neither catch the bus,
 - iv). at least one of them catch the bus.



There are 10 books on a shelf in a library. Seven are fiction and three are nonfiction. A member of the public takes a book at random, looks at it, and then replaces it on the shelf. Another member of the public then takes a book at random from the shelf.

- a). Copy and **complete** the tree diagram.
- b). What is the probability the two books taken are :-
 - i). both nonfiction,
 - ii). both fiction,
 - iii). one of each ?



Extension

In a flower contest the probability that a red rose will win is 0.18, the probability that a yellow rose will win is 0.24. What is the probability that a red **or** yellow rose will win ?

Sine and Cosine rules

These are used only in non-right angled triangles. The formulae will be on your test paper at the front.

Two rules: Different applications

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{Cosine Rule}$$

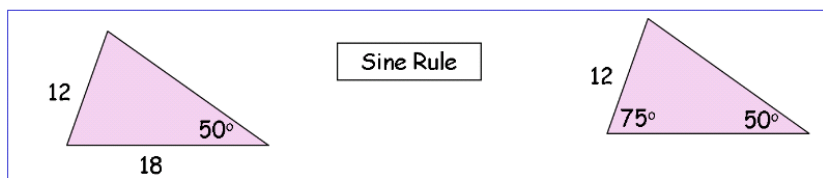
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{Sine rule}$$

Don't Panic, they will be on your formula Sheet!

When to use them:

The **Sine Rule** is used for cases in which the **Cosine Rule** cannot be applied. It is used to find:

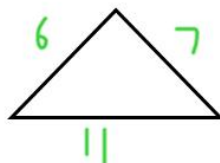
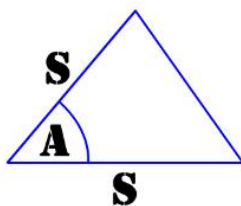
1. An **unknown side**, when we are given **two angles** and a **side**.
2. An **unknown angle** when we are given **two sides** and an **angle** that is **not** included.



The Cosine Rule

The Cosine rule can be used to find:

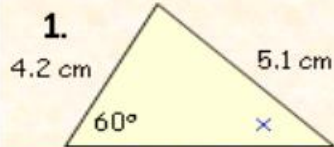
1. An **unknown side** when **two sides** of the triangle and the **included angle** are given.
2. An **unknown angle** when **3 sides** are given.



The Sine Rule

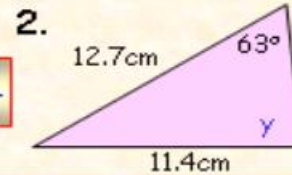
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To find an unknown angle we need 2 sides and an angle not included.



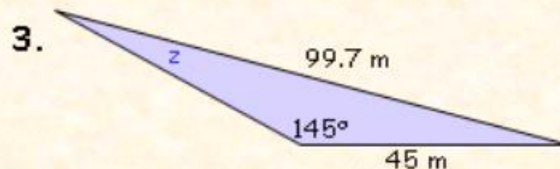
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\begin{aligned}\frac{\sin x}{4.2} &= \frac{\sin 60^\circ}{5.1} \\ \Rightarrow \sin x &= \frac{4.2 \sin 60^\circ}{5.1} \\ \Rightarrow x &= 45.5^\circ (1 \text{ dp})\end{aligned}$$



Not
to
scale

$$\begin{aligned}\frac{\sin y}{12.7} &= \frac{\sin 63^\circ}{11.4} \\ \Rightarrow \sin y &= \frac{12.7 \sin 63^\circ}{11.4} \\ \Rightarrow y &= 83.0^\circ (1 \text{ dp})\end{aligned}$$

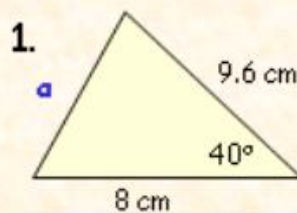


$$\begin{aligned}\frac{\sin z}{45} &= \frac{\sin 145^\circ}{99.7} \\ \Rightarrow \sin z &= \frac{45 \sin 145^\circ}{99.7} \\ \Rightarrow z &= 15^\circ (1 \text{ dp})\end{aligned}$$

The Cosine Rule

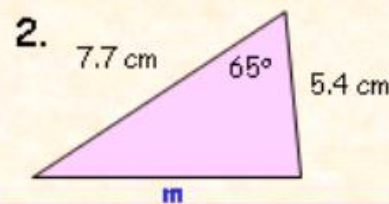
$$a^2 = b^2 + c^2 - 2bc \cos A$$

To find an unknown side we need 2 sides and the included angle.

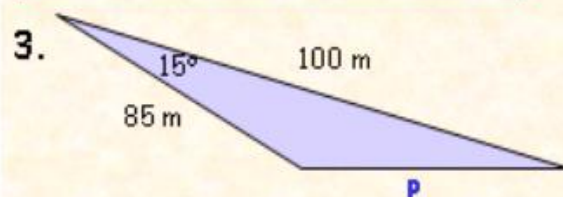


Not
to
scale

$$\begin{aligned}a^2 &= 8^2 + 9.6^2 - 2 \times 8 \times 9.6 \times \cos 40^\circ \\ a &= \sqrt{(8^2 + 9.6^2 - 2 \times 8 \times 9.6 \times \cos 40^\circ)} \\ a &= 6.2 \text{ cm} (1 \text{ dp})\end{aligned}$$

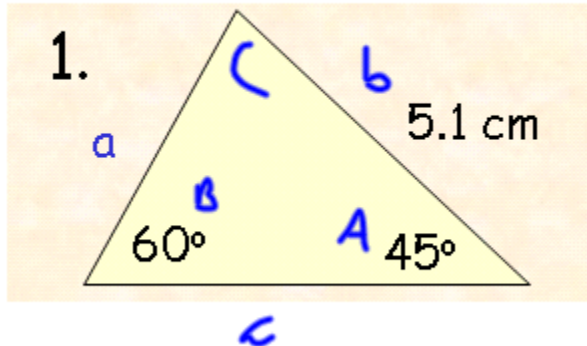


$$\begin{aligned}m^2 &= 5.4^2 + 7.7^2 - 2 \times 5.4 \times 7.7 \times \cos 65^\circ \\ m &= \sqrt{(5.4^2 + 7.7^2 - 2 \times 5.4 \times 7.7 \times \cos 65^\circ)} \\ m &= 7.3 \text{ cm} (1 \text{ dp})\end{aligned}$$

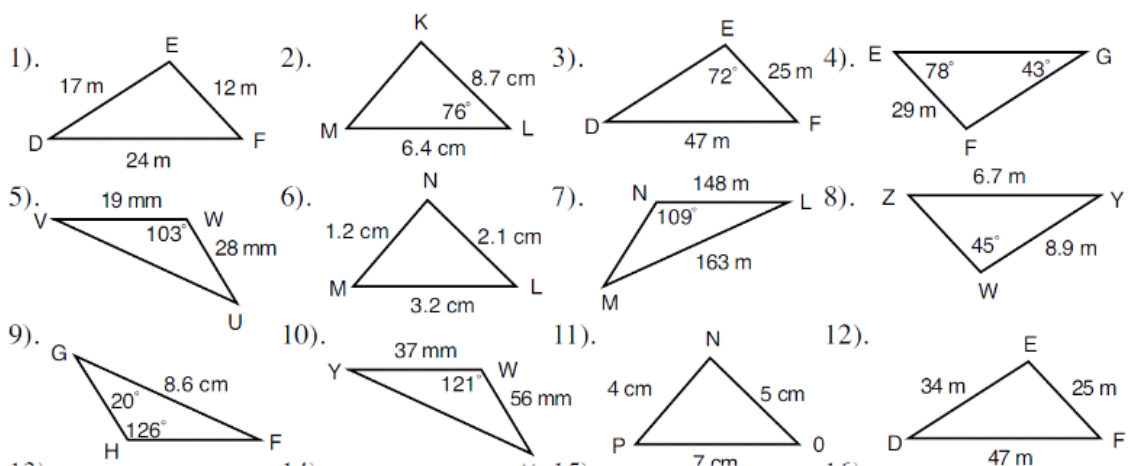


$$\begin{aligned}p^2 &= 85^2 + 100^2 - 2 \times 85 \times 100 \times \cos 15^\circ \\ p &= \sqrt{(85^2 + 100^2 - 2 \times 85 \times 100 \times \cos 15^\circ)} \\ p &= 28.4 \text{ m} (1 \text{ dp})\end{aligned}$$

Tip: It does not matter how you label the sides as long as a side and angle are opposite each other e.g.



Mixed questions. Find which rule to use first. Find all missing measurements/ angles.



Histograms key facts

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Classwidth}}$$

Just remember that: **Frequency = Area**

Here is our table.

Complete the frequency densities.

Mileage m	Freq	Width	Height Freq Den
$240 \leq m < 280$	3	40	0.075
$280 \leq m < 320$	6	<input type="text"/>	<input type="text"/>
$320 \leq m < 340$	8	<input type="text"/>	<input type="text"/>
$340 \leq m < 360$	11	<input type="text"/>	<input type="text"/>
$360 \leq m < 380$	6	<input type="text"/>	<input type="text"/>
$380 \leq m < 420$	4	<input type="text"/>	<input type="text"/>
$420 \leq m < 460$	2	<input type="text"/>	<input type="text"/>

The first one has been done for you.

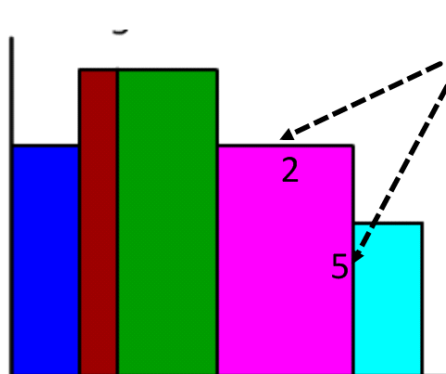
$$\text{Area} = \text{Width} \times \text{Height}$$

So

$$\text{Height frequency density} = \text{Area} \div \text{Width (freq)}$$

$$= \frac{3}{40} \div 40$$

$$= 0.075$$



We multiply the length of the bar by the width of the bar to find out the frequency of a histogram

Questions to try

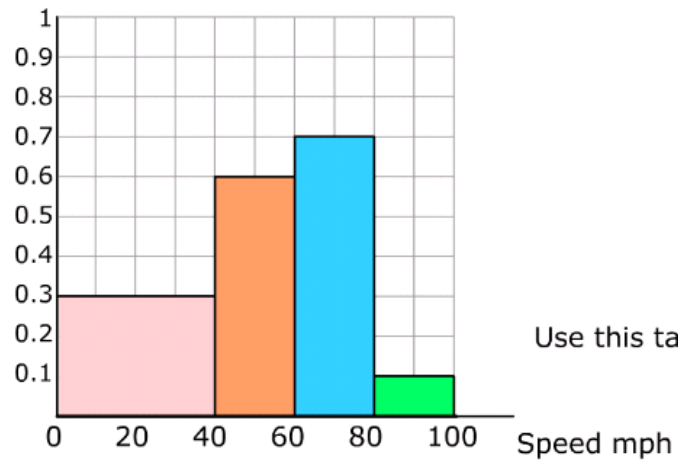
Type 1

Exam Question

The histogram shows the speeds, in miles per hour, of vehicles passing a police check point.

Calculate an estimate of the mean speed of these 40 vehicles.

f.d.



Speed	Frequency
$0 < v \leq 40$	<input type="text"/>
$40 < v \leq 60$	<input type="text"/>
$60 < v \leq 80$	<input type="text"/>
$80 < v \leq 100$	<input type="text"/>

Markit

Use this table to help you find the answer.

Type 2: Remember to calculate frequency density!

One Monday, Victoria measured the time, in seconds, that individual birds spent on her bird table.

She used this information to complete the frequency table.

Time (t seconds)	Frequency
$0 < t \leq 10$	8
$10 < t \leq 20$	16
$20 < t \leq 25$	15
$25 < t \leq 30$	12
$30 < t \leq 50$	6

- (a) Use the table to complete the histogram.

