हमारा विश्वास...
हर एक विद्यार्थी है ख़ास

PAPER WITH SOLUTION

JEE Advanced 2019

MATHEMATICS PAPER - 1

हमारा विश्वास... हर एक विद्यार्थी है ख़ास

MOTION™
Nurturing potential through education

CRITERIA FOR DIRECT ADMISSION IN STAR BATCHES

V STAR BATCH
XII Pass (JEE M+A)

ELIGIBILITY

JEE Main'19 %tile > 98%tile

JEE Advanced'19 Rank (Gen.) < 15,000

J STAR BATCH
XII Pass (NEET/AIIMS)

ELIGIBILITY

NEET'19 Score > 450 Marks

AIIMS'19 %tile > 98%tile

P STAR BATCH
XI Moving (JEE M+A)

ELIGIBILITY

NTSE Stage-1 Qualified or NTSE Score > 160

100 marks in Science or Maths in Board Exam

H STAR BATCH
XI Moving (NEET/AIIMS)

ELIGIBILITY

NTSE Stage-1 Qualified or NTSE Score > 160

100 marks in Science or Maths in Board Exam

Scholarship Criteria

JEE Main Percentile	SCHOLARSHIP+ STIPEND
98 - 99	100%
Above 99	100% + ₹ 5000/ month

JEE Advanced Rank	SCHOLARSHIP+ STIPEND
10000-20000	100%
Under 10000	100% + ₹ 5000/ month

NEET 2019 Marks	SCHOLARSHIP+ STIPEND
450	100%
530-550	100% + ₹ 2000/ month
550-560	100% + ₹ 4000/month
560	100% + ₹ 5000/month

NTSE STAGE-1 2019 Marks	SCHOLARSHIP+ STIPEND
160-170	100% + ₹ 2000/ month
171-180	100% + ₹ 4000/month
180+	100% + ₹ 5000/month

FEATURES :

- Batch will be taught by NV Sir & HOD's Only.
- Weekly Quizes apart from regular test.
- Under direct guidance of NV Sir.
- Residential campus facility available.
- 20 CBT (Computer Based Test) for better practice.
- Permanent academic coordinator for personal academic requirement.
- Small batch with only selected student.
- All the top brands material will be discussed.

SECTION -1 (Maximum Marks : 12)

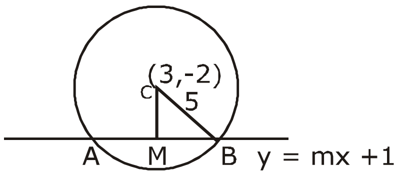
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +3 If ONLY the correct option is chosen.
Zero Marks : 0 If none of the options is choosen (i.e. the question is unanswered)
Negative marks : −1 In all other cases

1. A line $y = mx + 1$ intersects the circle $(x - 3)^2 + (y + 2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x - coordinate $\frac{-3}{5}$, then which one of the following options is correct ?

(1) $-3 \le m < -1$ (2) $6 \le m < 8$ (3) $4 \le m < 6$ (4) $2 \le m < 4$

Sol. **4**



$m_{AB} \cdot m_{cm} = -1$

$\Rightarrow m \cdot \left(\frac{1-\frac{3}{5}m+2}{-\frac{3}{5}-3}\right) = -1$

$\Rightarrow m\left(\frac{15-3m}{-18}\right) = -1$

$\Rightarrow 15m - 3m^2 - 18 = 0$

$m^2 - 5m + 6 = 0$

$m = 2, m = 3 \Rightarrow 2 \le m < 4$

2. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If
α^* is the minimum of set $\{\alpha(\theta): \theta \in [0,2\pi)\}$ and
β^* is the minimum of the set $\{\beta(\theta): \theta \in [0,2\pi)\}$
then the value of $\alpha^* + \beta^*$ is

(1) $\frac{-29}{16}$ (2) $-\frac{37}{16}$ (3) $-\frac{17}{16}$ (4) $-\frac{31}{16}$

Sol. **1**

$M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

$M = \alpha I + \beta M^{-1}$

$M^2 = \alpha M + \beta I$

$$\begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix}\begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \beta\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \alpha\begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix}$$

$\sin^8\theta - 1 - \sin^2\theta - \cos^2\theta - \cos^2\theta\sin^2\theta = \beta + \alpha\sin^4\theta$

$\sin^8\theta - 2 - \cos^2\theta\sin^2\theta = \beta + \alpha\sin^4\theta$ .....(1)

$\sin^2\theta + \cos^2\theta\sin^4\theta + \cos^4\theta + \cos^6\theta = \alpha(1 + \cos^2\theta)$

$$\alpha = \frac{\sin^4\theta(1+\cos^2\theta) + \cos^4\theta(1+\cos^2\theta)}{(1+\cos^2\theta)}$$

$\alpha = \sin^4\theta + \cos^4\theta = = 1 - \frac{1}{2}\sin^2 2\theta$

$\alpha_{min} = 1 - \frac{1}{2} = -\frac{1}{2}$

for equation (1)

$\sin^8\theta - 2 - \cos^2\theta\sin^2\theta - \alpha\sin^4\theta = \beta$

$\beta = \sin^2\theta - 2 - \sin^2\theta\cos^2\theta - \sin^4\theta(\sin^4\theta + \cos^4\theta)$

$\beta = -2 - \sin^2\theta\cos^2\theta - \sin^4\theta\cos^4\theta$

$\beta = -2 - \frac{1}{4}\sin^2 2\theta - \frac{1}{16}(\sin 2\theta)^4$

$\beta = -2 - \frac{1}{16}\left\{(\sin 2\theta)^4 + 4(\sin^2 2\theta) + 4\right\} + \frac{1}{4}$

$\beta = -\frac{7}{4} - \frac{1}{16}\{\sin 2\theta + 2\}^2$

$\beta = -\frac{7}{4} - \frac{1}{16}.9 = \frac{-7}{4} - \frac{9}{16} = \frac{-28-9}{16} = -\frac{37}{16}$

$\alpha^*_{min} + \beta^*_{min} = \frac{-37+8}{16} = \frac{-29}{16}$

3. Let S be the set of all complex numbers z satsfying $|z - 2 + i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) $-\frac{\pi}{2}$

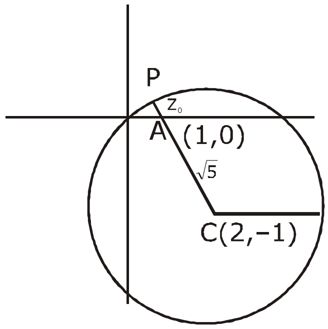
Sol. **4**

$|z - 2 + i| \geq \sqrt{5}$ for max of $\frac{1}{|z_0 - 1|}$

$\Rightarrow \min|z_0 - 1|$

$m_{CA} = \tan\theta = = \frac{1}{-1} = -1$

Now use parametric coordinate $\theta = 135°$



$$P(z_0) = \left\{\left(2+\sqrt{5}.\left(\frac{-1}{\sqrt{2}}\right)\right), \left(-1+\sqrt{5}\left(\frac{1}{\sqrt{2}}\right)\right)\right\}$$

$$\Rightarrow z_0 = \left(2-\sqrt{\frac{5}{2}}, -1+\sqrt{\frac{5}{2}}\right)$$

$$\Rightarrow \arg\left(\frac{4-(z_0+\bar{z}_0)}{(z_0-\bar{z}_0)+2i}\right) \quad \Rightarrow \arg\left(\frac{4-\left(2\left\{2-\sqrt{\frac{5}{2}}\right\}\right)}{2i+2\left(-1+\sqrt{\frac{5}{2}}\right)i}\right)$$

$$\Rightarrow \arg\left(\frac{\sqrt{10}}{i\sqrt{10}}\right) \quad \Rightarrow \arg\left(\frac{1}{i}\right)$$

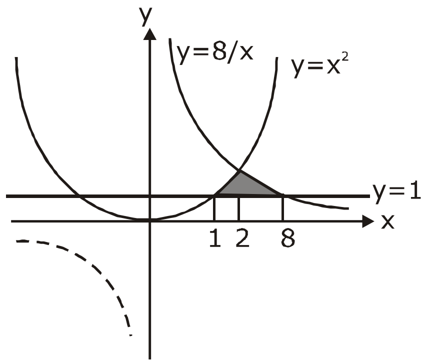
$$\Rightarrow \arg(-i) = \frac{-\pi}{2}$$

4. The area of region $\{(x,y) : xy \le 8,\ 1 \le y \le x^2\}$ is

(1) $16\log_e 2 - \frac{14}{3}$ (2) $8\log_e 2 - \frac{7}{3}$ (3) $8\log_e 2 - \frac{14}{3}$ (4) $16\log_e 2 - 6$

Sol. **1**

$xy \le 8$ & $1 \le y \le x^2$



$$A = \int_1^2 (x^2 - 1)\,dx + \int_2^8 \left(\frac{8}{x} - 1\right)dx$$

$$A = \left.\frac{x^3}{3}\right|_1^2 + 8\ln x\Big|_2^8 - 1 - 6$$

$$A = \left(\frac{8}{3} - \frac{1}{3}\right) + 8(\ln 8 - \ln 2) - 7$$

$$A = \frac{7}{3} - 7 + 16 \ln 2$$

$$A = 16\ln 2 - \frac{14}{3}$$

SECTION -2 (Maximum Marks : 12)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.
  Full marks : +4 If only (all) the correct option(s) is (are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen and both of which are correct
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If two or more options is chosen (i.e. the question is unanswered)
  Negative Marks : –1 in all other cases
- For example, in a question, if (A),(B) and (D) are the ONLY three options corresponding to correct answer, then
  choosing ONLY (A), (B) and (D) will get +4 marks
  choosing ONLY (A) and (B) will get +2 marks

choosing ONLY (A) and (D) will get +2 marks
choosing ONLY (B) and (D) will get +2 marks
choosing ONLY (A) will get +1 mark
choosing ONLY (B) will get +1 mark
choosing ONLY (D) will get +1 mark
choosing no option (i.e., the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –1 mark

1. Let Γ denotes a curve y = y(x) which is in the first quadrant and let the point (1,0) lie on it. Let the tangent to Γ at a point P intersect the y - axis at Y_P. If PY_P has length 1 for each point P on Γ, then Which of the following options is/are correct ?

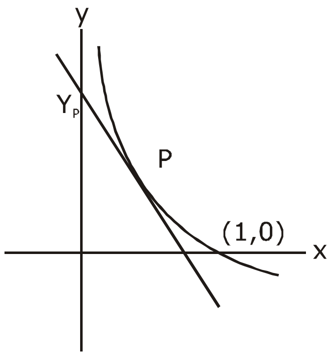
(1) $xy' - \sqrt{1-x^2} = 0$

(2) $y = -\log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(3) $xy' + \sqrt{1-x^2} = 0$

(4) $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

Sol. 3,4



Equation of Tangent at P

$Y - y = \frac{dy}{dx}(X - x)$

For $Y_P \Rightarrow$ (X = 0)

$Y_P = y - x\frac{dy}{dx}$

distance $Y_pP = 1$

$x^2 + \left(y - y + x\frac{dy}{dx}\right)^2 = 1$

$$x^2\left(1+\left(\frac{dy}{dx}\right)^2\right) = 1$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{x^2}-1$$

$$\frac{dy}{dx} = \pm\frac{\sqrt{1-x^2}}{x} \rightarrow \text{option 1 and 3}$$

$$\int dy = \pm\int\frac{\sqrt{1-x^2}}{x}dx$$

$x = \sin\theta$

$$y = \pm\int\frac{\cos\theta}{\sin\theta}\cos\theta d\theta$$

$$y = \pm\int\frac{1-\sin^2\theta}{\sin\theta}d\theta$$

$$y = \pm\int(\operatorname{cosec}\theta-\sin\theta)d\theta$$

$$y = \pm(\ln|\operatorname{cosec}\theta+\cot\theta|+\cos\theta) + C$$

$$y = \pm\left(\ln\left|\frac{1}{x}-\frac{\sqrt{1-x^2}}{x}\right|+\sqrt{1-x^2}\right) + C$$

P as (1,0) $\Rightarrow$ c = 0

$$y = \pm\left(\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)+\sqrt{1-x^2}\right) \rightarrow \text{option (2), (4)}$$

2. Define the collections $\{E_1, E_2, E_3 \ldots\ldots\}$ of ellipse and $\{R_1, R_2, R_3 \ldots\ldots\}$ of rectangles as follows :

$E_1 : \frac{x^2}{9}+\frac{y^2}{4}=1$;

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1;

E_n : ellipse $\frac{x^2}{a_n^2}+\frac{y^2}{b_n^2}$ = 1 of largest area inscribed in R_{n-1}, n > 1;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n , n > 1

Then which of the following options is/are correct?

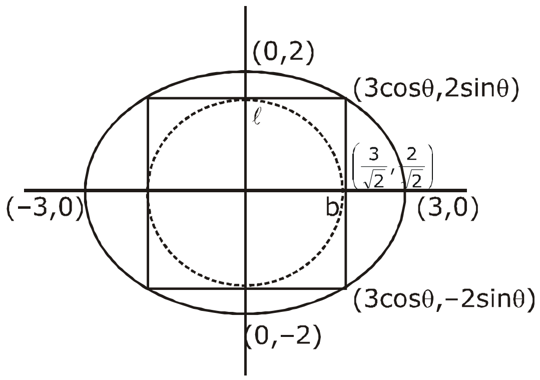
(1)The eccentricities of E_{18} and E_{19} are NOT equal

(2) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

(3) $\sum_{n=1}^{N}(\text{area of } R_n) < 24$, for each positive integer N

(4) The length of latus rectum of E_9 is $\frac{1}{6}$

2. **(3),(4)**



$E_1 \Rightarrow \frac{x^2}{9}+\frac{y^2}{4} = 1$

$l = 6\cos\theta$

$b = 4\sin\theta$

Area $= 12 \times \sin 2\theta$

$A_{max} = 12$

$\sin 2\theta = 1$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

E_2 : $a = \frac{3}{\sqrt{2}}$; $b = \frac{2}{\sqrt{2}}$; $a = 3$; $r = \frac{1}{\sqrt{2}}$; $b = 2$; $r = \frac{1}{\sqrt{2}}$

(i) $e^2 = 1-\frac{b^2}{a^2}$ eccentricities of all ellipse will be equal

(ii) for E_9 ; $e = \frac{\sqrt{5}}{3}$ and $a = 3\times\left(\frac{1}{\sqrt{2}}\right)^8$

$\therefore$ distance of focus from centre

$= ae = \frac{3}{16} \times \frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{16}$

(iii) sum of area of rectangles = 12 + 6 + 3 + ...

$$A = \frac{12}{1-\frac{1}{2}} = 24$$

$$\text{(iv) L.R.} = \frac{2b^2}{a} = \frac{2\times\left(2\times\frac{1}{16}\right)^2}{2.\frac{1}{16}} = \frac{2\times\frac{1}{64}}{3\times\frac{1}{16}} = \frac{1}{6}$$

3. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct ?

(1) det $(\text{adj}M^2) = 81$

(2) $a + b = 3$

(3) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

(4) if $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

Sol. **2,3,4**

$M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

$$\Rightarrow \text{adj } M = \begin{bmatrix} 2-3b & ab-1 & -1 \\ 8 & -6 & 2 \\ b-6 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$2 - 3b = -1$; $ab - 1 = 1$

$b - 6 = -5$; $a = 2$

$b = 1$

Now $M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$|M| = 8 - 10 = -2$

$\Rightarrow a + b = 3$ option (2)

$|\text{adj}(M^2)| = |M^2|^2$

$= |M|^4 = 16$

(3) $(\text{adj}M)^{-1} + \text{adj}(M^{-1})$ option(3)

$= adj(M^{-1}) + adj(M^{-1})$
$= 2adj(M^{-1})$
$= 2(|M^{-1}|M)$

$= 2\left(\frac{1}{-2}M\right)$

$= -M$

(4) $M\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$

$$\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$\beta + 2\gamma = 1$
$\alpha + 2\beta + 3\gamma = 2$
$3\alpha + \beta + \gamma = 1$
$\alpha \quad 1$
$\beta = -1$
$\gamma = 1$
$\alpha - \beta + \gamma = 3$ option (4)

4. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integer n, define

$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 1$

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$

Then which of the following options is/are correct ?

(1) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \geq 1$

(2) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

(3) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(4) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

Sol. **1,2,4**

$x^2 - x - 1 = 0$

$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$ (2) $b_1 = 1$ $b_n = a_{n-1} + a_{n+1}$

$\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$

$b_n = \dfrac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} + \dfrac{\alpha^{n+1} + \beta^{n+1}}{\alpha - \beta}$

$= \dfrac{\alpha^{n-1}\left(1+\alpha^2\right) - \beta^{n-1}\left(1+\beta^2\right)}{\alpha - \beta}$

$= \dfrac{\alpha^{n-1}(\alpha+2) - \beta^{n-1}(\beta+2)}{\alpha - \beta}$

$= \dfrac{\alpha^{n-1}\left(\dfrac{5+\sqrt{5}}{2}\right) - \beta^{n-1}\left(\dfrac{5-\sqrt{5}}{2}\right)}{\alpha - \beta}$

$= \dfrac{\sqrt{5}\alpha^n + \sqrt{5}\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$

(i) $a_1 + a_2 + a_3 + \ldots. + a_n$

$= \dfrac{\left(\alpha + \alpha^2 + \ldots. + \alpha^n\right) - \left(\beta + \beta^2 + \ldots.\beta^n\right)}{\alpha - \beta}$

$= \dfrac{\dfrac{\alpha\left(1-\alpha^n\right)}{1-\alpha} - \dfrac{\beta\left(1-\beta^n\right)}{1-\beta}}{\alpha - \beta}$

$\alpha^2 - \alpha - 1 = 0$

$\alpha^2 - 1 = \alpha$

$\alpha + 1 = \dfrac{\alpha}{\alpha - 1}$

$= \dfrac{-\alpha^2\left(1-\alpha^n\right) + \beta^2\left(1-\beta^n\right)}{\alpha - \beta}$

$= \dfrac{-\alpha^2 + \alpha^{n+2} + \beta^2 - \beta^{n+2}}{(\alpha - \beta)}$

$= \dfrac{\alpha^{n+2} - \beta^{n+2}}{\alpha - \beta} - (\alpha + \beta)$

$= a_{n+2} - 1$

(3) $\sum \frac{b_n}{10^n} = \sum\left(\frac{\alpha^n}{10^n}+\frac{\beta^n}{10^n}\right)$

$$= \left(\frac{\alpha}{10}+\frac{\alpha^2}{10^2}+\ldots.\right)$$

$$= \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}}+\frac{\beta}{1-\frac{\beta}{10}}$$

$$= \frac{\alpha}{10-\alpha}+\frac{\beta}{10-\beta}$$

$$= \frac{10(\alpha+\beta)-2\alpha\beta}{100-10(\alpha+\beta)+\alpha\beta}$$

$$= \frac{10+2}{100-10-1} = \frac{12}{89}$$

(4) $\sum \frac{a^n}{10^n} = \frac{1}{\alpha-\beta}\left\{\frac{\alpha}{10-\alpha}-\frac{\beta}{10-\beta}\right\}$

$$= \frac{1}{\alpha-\beta}\left\{\frac{10(\alpha-\beta)}{89}\right\} = \frac{10}{89}$$

5. Let f : R $\to$ R be given by

$$f(x) = \begin{cases} x^5+5x^4+10x^3+10x^2+3x+1, & x<0 \\ x^2-x+1, & 0\le x<1; \\ \frac{2}{3}x^3-4x^2+7x-\frac{8}{3}, & 1\le x<3 \\ (x-2)\log_e(x-2)-x+\frac{10}{3}, & x\ge 3 \end{cases}$$

Then which of the following options is /are correct ?

(1) f is increasing on $(-\infty,0)$

(2) f is onto

(3) f' has a local maximum at x = 1

(4) f' is NOT differentiable at x = 1

Sol. **2,3,4**

$$f(x)=\begin{cases} x^5+5x^4+10x^3+10x^2+3x+1 & x<0 \\ x^2-x+1 & 0\le x<1 \\ \dfrac{2}{3}x^3-4x^2+7x-\dfrac{8}{3} & 1\le x<3 \\ (x-2)\ell n(x-2)-x+\dfrac{10}{3} & x\ge 3 \end{cases}$$

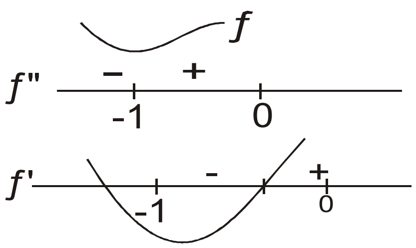
f is onto $\because$ Range = R (ℓn (x–2) contains all real values)

$$f'(x)=\begin{cases} 5x^4+20x^3+30x^2+20x+3 & x<0 \\ 2x-1 & 0\le x<1 \\ 2x^2-8x+7 & 1\le x<3 \\ 1+\ell n(x-2)-1 & x\ge 3 \end{cases}$$

Check diff of f' at x = 1 < RHD=-4, LHD=2 $\quad f$ is not diff.

$$f''(x)=\begin{cases} 20x^3+60x^2+60x+20 & x<0 \\ 2 & 0\le x<1 \\ 4x-8 & 1\le x<3 \\ \dfrac{1}{x-2} & x\ge 3 \end{cases}$$

$$f''(x)=\begin{cases} 20(1+x)^3 & x<0 \\ 2 & 0\le x<1 \\ 4x-8 & 1\le x<3 \\ \dfrac{1}{x-2} & x\ge 3 \end{cases}$$



$\Rightarrow$ Not always one

6. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ?

(1) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(2) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(3) Probability that the selected bag is B_3, given that chosen ball is green, equals $\frac{5}{13}$

(4) Probability that the chosen ball is green equals $\frac{39}{80}$

Sol. **1, 4**

5R + 5G	3R + 5G	5R + 3G
B_1	B_2	B_3

$P(B_1) = \frac{3}{10} \Big| P(B_2) = \frac{3}{10} \Big| P(B_3) = \frac{4}{10}$

1. $P(G_1|B_3) = \frac{3}{8} = \boxed{\frac{3}{8}}$

2. $P(B_3|G) = \boxed{\frac{4}{13}}$

3. $P(B_3|G) = \frac{12}{39} = \boxed{\frac{4}{13}}$

4. $P(G) = \frac{3}{10}\cdot\frac{5}{10} + \frac{3}{10}\cdot\frac{5}{8} + \frac{4}{10}\cdot\frac{3}{8} = \frac{12+15+12}{80} = \boxed{\frac{39}{80}}$

7. In a non-right angled triangle ΔPQR, let p,q,r denote the lengths of the sides opposite to the angles at P,Q,R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, q=1, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

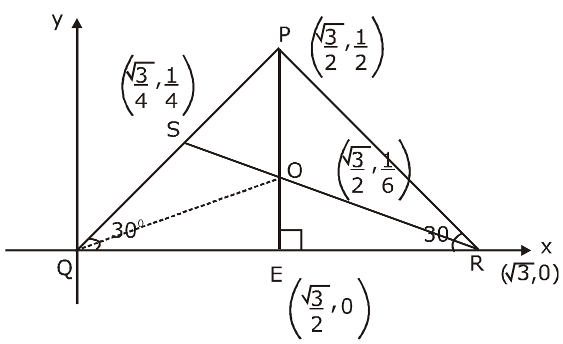
(1) Length of RS $= \frac{\sqrt{7}}{2}$

(2) Length of OE $= \frac{1}{6}$

(3) Radius of incircle ΔPQR $= \frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

(4) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

Sol. **1,2,3**



sin Law

$$\frac{QP}{\sin P} = \frac{PR}{\sin\theta} = 2R$$

$$\frac{\sqrt{3}}{\sin P} = \frac{1}{\sin\theta} = 2$$

$$\sin P = \frac{\sqrt{3}}{2} \left\langle \begin{matrix} P = 60 \\ P = 120 \end{matrix} \right.$$

$$\sin\theta = \frac{1}{2} \left\langle \begin{matrix} \theta = 30 \\ \theta = 150 \end{matrix} \right.$$

$\angle P = 120^0$, $\theta = 30^0$, $\angle R = 30^0$

(1) $RS = \frac{1}{2}\sqrt{2\left(\sqrt{3}\right)^2 + 2(1)^2 - 1} = \frac{\sqrt{7}}{2}$ Ans 1

(2) Eq. of RS : $(y - 0) = \frac{\frac{1}{4} - 0}{\frac{\sqrt{3}}{4} - \sqrt{3}}\left(x - \sqrt{3}\right) \Rightarrow y = -\frac{1}{3\sqrt{3}}\left(x - \sqrt{3}\right)$

Hence coordinate of O : $\left(\frac{\sqrt{3}}{2}, \frac{1}{6}\right)$

$\Rightarrow$ $\boxed{OE = \frac{1}{6}}$

(3) $r = \frac{\Delta}{S} = \frac{\frac{1}{2}.\sqrt{3}.\frac{1}{2}}{\frac{\sqrt{3}+1+1}{2}} = \frac{\sqrt{3}}{2(2+\sqrt{3})}$

$$\frac{\sqrt{3}}{2}(2-\sqrt{3})$$

(4) $\Delta = \frac{1}{2}\begin{vmatrix} \frac{\sqrt{3}}{2} & 0 & 1 \\ \frac{\sqrt{3}}{2} & \frac{1}{6} & 1 \\ \frac{\sqrt{3}}{4} & \frac{1}{4} & 1 \end{vmatrix}$

$$= \left| \frac{\sqrt{3}}{4} \begin{vmatrix} 1 & 0 & 1 \\ 1 & \frac{1}{6} & 1 \\ \frac{1}{2} & \frac{1}{4} & 1 \end{vmatrix} \right| \qquad = \left| \frac{\sqrt{3}}{4} \left\{ 1\left(\frac{1}{6} - \frac{1}{4}\right) + 1\left(\frac{1}{4} - \frac{1}{12}\right) \right\} \right|$$

$$= \left| \frac{\sqrt{3}}{4} \left\{ \frac{-2}{24} + \frac{2}{12} \right\} \right| \qquad = \left| \frac{\sqrt{3}}{4} \cdot \frac{2}{24} \right| = \frac{\sqrt{3}}{48}$$

8. Let L_1 and L_2 denote the lines

$$\vec{r} = \hat{i} + \lambda(-\hat{i} + +2\hat{j} + 2\hat{k}), \lambda \in R \text{ and}$$

$$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$$

respectively, If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

(1) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}),\ t \in R$ (2) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}),\ t \in R$

(3) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}),\ t \in R$ (4) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k})\ t \in R$

Sol. **1,2**

$$L_1 \rightarrow \frac{x-1}{-1} = \frac{y-0}{2} = \frac{z-0}{2}$$

$$L_2 \rightarrow \frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$$

$$L_3 \rightarrow \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$

$L_3 \perp L_1$ & L_2

$L_3 \parallel (L_1 \times L_2)$

$\therefore L_3 \parallel (6\hat{i} + 6\hat{j} - 3\hat{k})$

Let any point on L_1 is $\equiv (-\lambda + 1, 2\lambda, 2\lambda)$

Let any point on L_2 is $B \equiv (2\mu, -\mu, 2\mu)$

DR(s) of AB will be

$2\mu + \lambda - 1, -\mu - 2\lambda, 2\mu - 2\lambda$

But D.R. of AB are

6, 6, −3 or 2, 2, −1

$$\therefore \frac{2\mu + \lambda - 1}{2} = \frac{-\mu - 2\lambda}{2} = \frac{2\mu - 2\lambda}{-1} = k(\text{let})$$

$\therefore\ 2\mu + \lambda - 1 = 2k$ ....(1)
$-\mu - 2\lambda = 2k$ ....(2)
$2\mu - 2\lambda = -k$ ....(3)
Solve (1) & (3)

$$\lambda = \frac{3k+1}{3}$$

Put $\lambda = \frac{3k+1}{3}$ in equation (2)

$$\mu = \frac{12k+2}{(-3)}$$

Put λ & μ in eq. (3)

$$2\left(\frac{12k+2}{-3}\right) - 2\left(\frac{3k+1}{3}\right) + k = 0$$

$$k = -\frac{2}{9}$$

$$\therefore \lambda = \frac{3\left(-\frac{2}{9}\right)+1}{3} = \frac{-\frac{2}{3}+1}{3} = \frac{1}{9}$$

$$\mu = \frac{12\left(\frac{-2}{9}\right)+2}{-3} = \frac{\frac{-8}{3}+2}{-3} = \frac{2}{9}$$

$$\therefore A \equiv (-\lambda + 1, 2\lambda, 2\lambda) \Rightarrow \left(\frac{-1}{9}+1, \frac{2}{9}, \frac{2}{9}\right)$$

$$A \equiv \left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$$

$$\therefore B \equiv (2\mu, -\mu, 2\mu) \Rightarrow B \equiv \left(\frac{4}{9}, \frac{-2}{9}, \frac{4}{9}\right)$$

$\therefore$ Equation of L_3 can be

$$L_3 \to \vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$$

$$\text{or } L_3 \to \vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$$

Section - 3

- This section contains SIX (06) qeustions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/roundoff the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme;

Full Marks : +3 If ONLY the correct numerical value is entered
Zero Marks : 0 in all other cases.

1. Three lines are given by

$$\vec{r} = \lambda\hat{i},\ \lambda \in R$$

$$\vec{r} = \mu(\hat{i} + \hat{j}),\ \mu \in R$$

$$\vec{r} = v(\hat{i} + \hat{j} + \hat{k}),\ v \in R$$

Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then value of $(6\Delta)^2$ equals ____________.

Sol. **0.75**

$\vec{r} = \lambda\hat{i}$ $\quad \vec{r} = \mu\left(\hat{i} + \hat{j}\right)$ $\quad \vec{r} = v\left(\hat{i} + \hat{j} + \hat{k}\right)$

x + y + z = 1

Ist line

$x = \lambda,\quad y = 0,\quad z = 0$

$\therefore \boxed{\lambda = 1}$ A(1,0,0)

For 2^{nd} Line

$x = \mu,\ y = \mu,\ z = 0$

$\therefore 2\mu = 1 \qquad B\left(\frac{1}{2}, \frac{1}{2}, 0\right)$

Similarly $C\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$$\therefore \text{Area of } \Delta = \frac{1}{2}\left|\overline{AB} \times \overline{AC}\right|$$

$$= \frac{1}{2}\left|\left(-\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}\right) \times \left(-\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + +\frac{1}{3}\hat{k}\right)\right|$$

$$= \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix} = \frac{1}{2}\left\{\hat{i}\left(\frac{1}{6}\right) - \hat{j}\left(\frac{-1}{6}\right) + \hat{k}\left(\frac{1}{6}\right)\right\}$$

$$= \frac{1}{2}\left|\frac{\hat{i}}{6} + \frac{\hat{j}}{5} + \frac{\hat{k}}{6}\right| = \frac{1}{2}\sqrt{\frac{3}{36}}\ ; \quad \Delta = \frac{\sqrt{3}}{12}$$

$$\therefore (6\Delta)^2 = \frac{3}{4} = .75$$

2. Let S be the sample space of all 3 × 3 matrices with entries from the set {0,1}, Let the events E_1 and E_2 be given by

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{sum of entries of A is 7}\}$

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals

2. **1/2**

Sample space = 2^9

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

E_2 : sum of entries 7

$\therefore$ '7' one and '2' zero

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} \quad \text{total } E_2 = \frac{9!}{7!2!} = \frac{8 \times 9}{2} = 36$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$ for |A| to be zero both zeros should by in same row or column

$\therefore$ (3×3)2 = 18

$$\therefore P(E_1/E_2) = \frac{18}{36} = \frac{1}{2}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1(1) - 1(-1)$$

3. Let $\omega \neq 1$ be a cube root of unity . Then the minimum of the set $\{|a + b\omega + c\omega^2|^2 : a, b, c$ distinct non-zero integers$\}$ equals ____________.

Sol. **3**

$$\left|a + b\omega + c\omega^2\right|^2$$

$$= \left(a + b\omega + c\omega^2\right)\left(a + b\omega^2 + c\omega\right)$$

$$= \left\{a^2 + b^2 + c^2 - ab - bc - ca\right\}$$

$$= \frac{1}{2}\left\{(a-b)^2 + (b-c)^2 + (c-a)^2\right\}$$

$$= \frac{1}{2}\{1+1+4\} = 3$$

4. Let AP(a; d) denote the set of all the terms of an infinite arithmetic progression with first term α and common difference d > 0, If

$AP(1;3) \cap AP(2;5) \cap AP(3;7) = AP(a;d)$ then a + d equals ____________.

Sol. 157

First AP

a = 1, common diff. = 3

Second AP

a = 2, common diff. = 5

Third AP

a = 3, common diff. = 7

Now on AP whose first term and common diff. is common of all three

$\therefore$ 1+(n–1)3 = 2+(m–1)5 = 3+(k–1)7

(i) $\frac{3n+1}{5} = m$ and $\frac{3n+2}{7} = k$

m and k are integer

So at n = 18 m = 11 and k = 8

first term of AP $\Rightarrow$ 1+(18–1)3 = 52

Common diff. = LCM (3,5,7) = 105

$\therefore$ $\boxed{a + d = 157}$

5. If $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then $27I^2$ equals ____________.

Sol. 4

$$I = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$$

Apply King x $\to$ –x

$$I = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{e^{\sin x}}{(1+e^{\sin x})(2-\cos 2x)} \; ; \; 2I = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{2-\cos 2x}$$

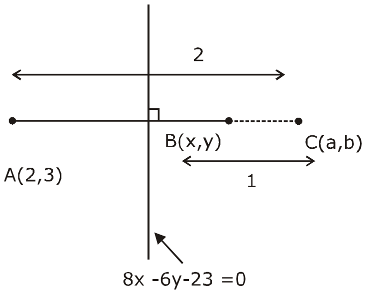
$$\therefore I = \frac{2}{\pi}\int_{0}^{\frac{\pi}{4}} \frac{dx}{1+2\sin^2 x} = \frac{2}{\pi}\int_{0}^{\frac{\pi}{4}} \frac{\sec^2 x dx}{1-\tan^2 x + 2\tan^2 x}, \tan x = t$$

$$= \frac{2}{\pi}\int_{0}^{1} \frac{dt}{1+3t^2} = \frac{2}{3\pi} = \frac{2}{\sqrt{3}\pi}\tan^{-1}\left(\sqrt{3}t\right)_0^1 = \frac{2}{\sqrt{3}\pi}\left(\frac{\pi}{3}\right) = \frac{2}{3\sqrt{3}} = I$$

$$\therefore 27 \times \frac{4}{27} = 4$$

6. Let the point B be the reflection of the point A(2,3) with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B resepectively.Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is ___________.

6. **10**



For B

$$\frac{x-2}{8} = \frac{y-3}{-6} = \frac{-2(16-18-23)}{64+36}$$

$$\frac{x-2}{8} = \frac{y-3}{6} = \frac{-2(-25)}{100}$$

$$\frac{x-2}{8} = \frac{y-3}{6} = \frac{1}{2} \qquad \therefore x = 6 \text{ and } y = 6$$

B (6, 6)

Now for 'C' external division in ratio $r_1 : r_2$

$$a = \frac{2.6-1.2}{2-1} \quad b = \frac{2.6-1.3}{2-1}$$

a = 10, b = 9

$$\therefore AC = \sqrt{8^2+6^2}$$

$$= \sqrt{64+36}$$

$$= \sqrt{100} = 10$$

हमारा विश्वास... हर एक विद्यार्थी है ख़ास

Motion™
Nurturing potential through education

Based on JEE Advanced'19

MARKS	FEE (After Scholarship)
140 above	Drona Residential Program Free
120 to 139	₹ 0
100 to 120	₹ 14,500
90 to 99	₹ 29,000
80 to 89	₹ 43,500
69 to 79	₹ 58,000
40 to 69	₹ 87,000

*Scholarship Applicable at Kota Center Only

Based on JEE Main'19

JEE Main Percentile	English	Hindi
	Fees (After Scholarship)	
99 & Above	Drona Residential Program Free	
97.5 To 99	₹ 0	₹ 0
97 To 97.5	₹ 14,500	₹ 14,500
96.5 To 97	₹ 29,000	₹ 29,000
96 To 96.5	₹ 58,000	₹ 58,000
95.5 To 96	₹ 65,250	₹ 65,250
95 To 95.5	₹ 72,500	₹ 72,500
93 To 95	₹ 87,000	₹ 87,000
90 To 93	₹ 1,01,500	₹ 94,250
85 To 90	₹ 1,08,750	₹ 1,01,500
80 To 85	₹ 1,16,000	₹ 1,08,750
75 To 80	₹ 1,30,500	₹ 1,23,250

JEE MAIN Special Batch
for Class 14th Repeaters

Flat 50% Scholarship

(Fee after Scholarship) **Only ₹ 46,750**