MATHS QUEST 10+10A

FOR VICTORIA

Australian Curriculum edition

TI-NSPIRE CAS
CALCULATOR COMPANION

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Contents

Introduction

This booklet is designed as a companion to *Maths Quest 10 + 10A for Victoria Australian Curriculum edition.*

It contains worked examples from the student text that have been reworked using the T1-Nspire CX CAS calculator with the most up-to-date Operating System (November 2014).

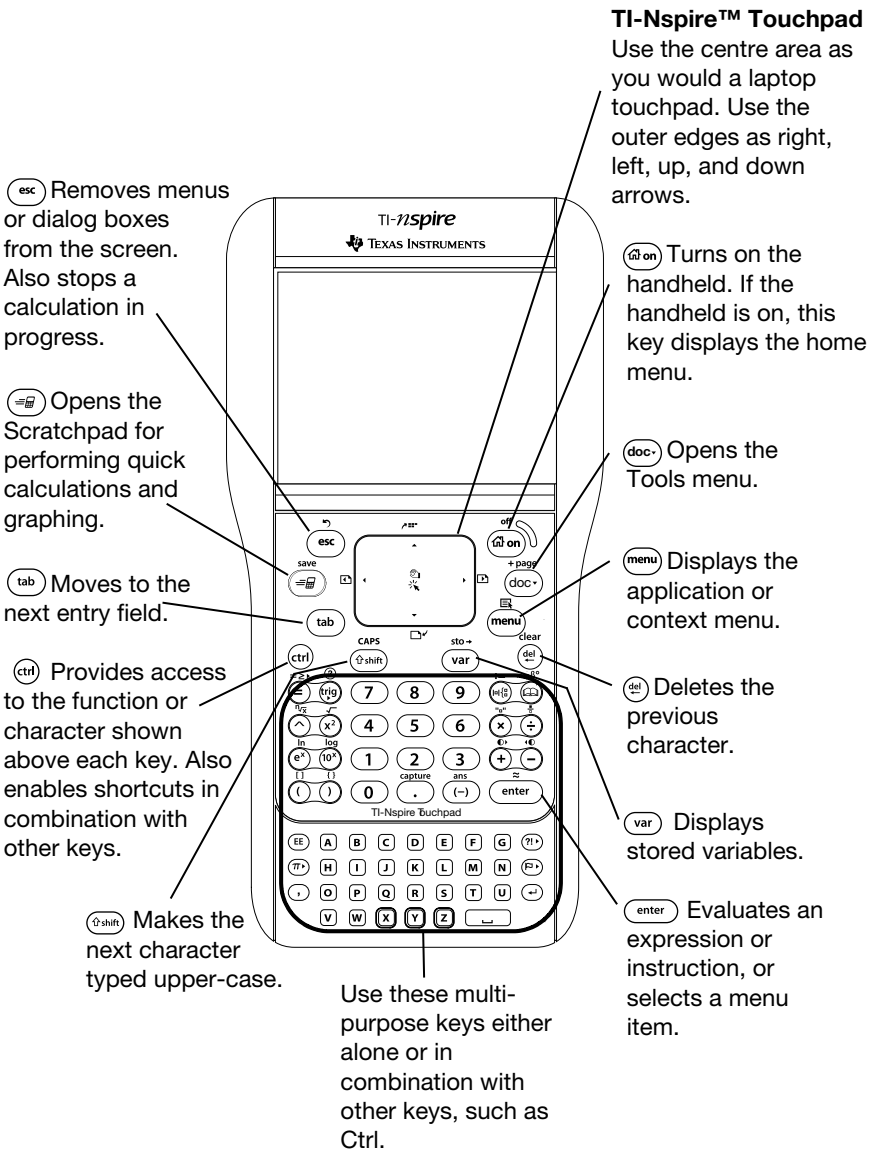
The content of this booklet will be updated online as new operating systems are released by Texas Instruments.

The companion is designed to assist students and teachers in making decisions about the judicious use of CAS technology in answering mathematical questions.

The calculator companion booklet is also available as a PDF file on the eBookPLUS under the preliminary section of *Maths Quest 10 + 10A for Victoria Australian Curriculum edition*.

Navigating the TI-Nspire

To begin using the TI-Nspire, look carefully at the diagram below and note the important features. The features highlighted are the most commonly used features and will be referred to throughout this manual.



Note: The operating system used in this manual is version 3.9. The keystrokes described in this book are those on the TI-Nspire CX TouchPad, and all instructions are given for default settings.

1.1 How to change Document Settings

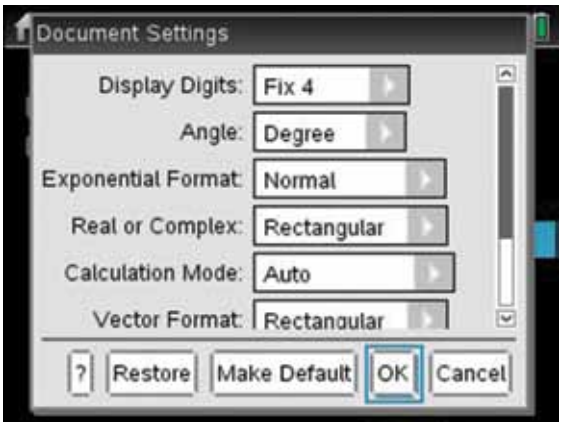
When the TI-Nspire is first turned on, it starts with the Home screen as shown. You can return to this screen by pressing the key with the house icon [on]. The Settings contain tools that allow the user to change the settings on the calculator.

Press:

- **5: Settings** [5]
- **2: Document Settings** [2]

or use the arrow keys on the Touchpad to move to select Document Settings.

Use TAB [tab] to move down through the settings. To move back to a previous setting, use the shift key [shift] followed by TAB [tab]. To change a setting, press on the arrow and select the required setting. To exit, TAB [tab] to OK and press ENTER [enter].



1.2 Pages in a TI-Nspire Document

A TI-Nspire document can consist of many pages. Each page can be one of seven different types. The most commonly used pages are the Calculator and Graphs pages. However, we can also add a Geometry page, a Lists & Spreadsheet page, a Data & Statistics page, a Notes page or a Vernier DataQuest to a document.

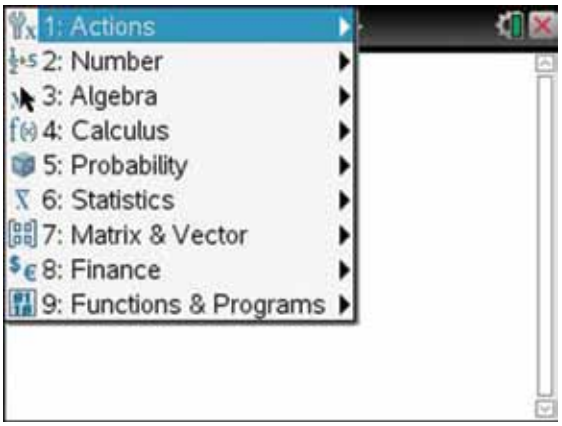
A Calculator page is where we perform basic calculations and algebraic manipulations. A Graphs page is where we can draw the graphs of functions and some relations. A Geometry page can be used to draw geometric shapes and measure side lengths and angles of geometrical figures. A Lists & Spreadsheet page has functions similar to a spreadsheet such as Excel and can be used to create columns of data. A Data & Statistics page is one where we can draw statistical plots of data created in a Lists & Spreadsheet page.

A Notes page can be used to create interactive mathematical summaries, questions and text. The Vernier DataQuest is used in conjunction with various types of sensors that can be plugged into the handheld or a computer running the TI-Nspire software to collect, tabulate and analyse data over a period of time. The most commonly used types of sensors are ones that measure temperature, motion, light and electrical quantities.

In this document we will describe only the basic methods for a Calculator page and a Graphs page. We could also use the Scratchpad, which contains only a Calculator page and a Graphs page; however, the ScratchPad is used for only simple calculations and when we do not want to save a document.

There are many ways to insert pages into a document. One method is to press the house icon [on] and use the arrow keys to highlight and select one of the seven different types of pages from the selections as shown. For example, to insert a Calculator page, select it and press ENTER [enter].

The menu system is context sensitive; that is, when we press [menu], the menus that appear are different when we are on different pages. For example, when we are on a Calculator page and press [menu], the screenshot on the right is shown.

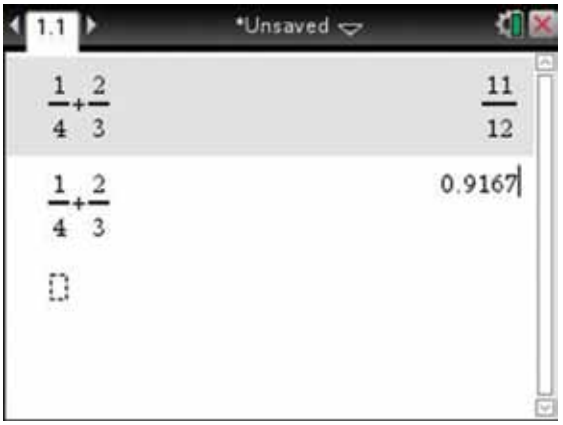


1.3 How to perform basic calculations

The calculator is a CAS calculator, that is, it can perform numerical operations and it also has the ability to perform Computer Algebra Software (CAS). Many of the mathematical operations required can be performed by choosing from the menus or, if you know the syntax of a command, then you can simply type it. The following are some typical examples for simplifying fractions, factorising and expanding algebraic expressions and for solving equations.

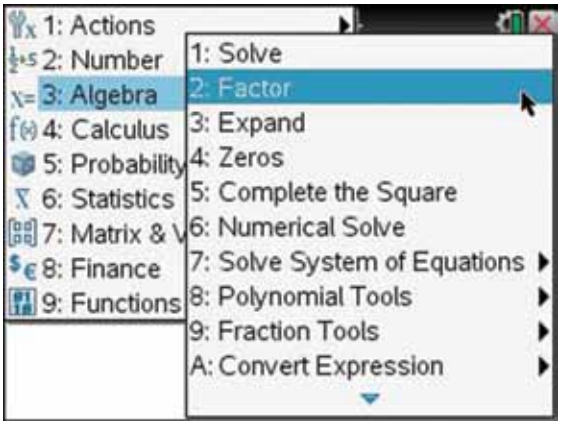
On the Calculator page we can perform basic mathematical calculations. For example, to simplify, $\frac{1}{4}+\frac{2}{3}$ use the fraction template, which is accessed by pressing CTRL [ctrl] divide [÷]. Then type 1, press the down arrow ▼, type 4, press the right arrow ▶, press [+], type 2, then press the down arrow ▼, type 3, then Press ENTER [enter].

If the calculator is in the Exact or Auto mode for the Calculation mode, the result will be shown as an exact fraction. To get a decimal answer, press CTRL [ctrl] ENTER [enter]. This answer will be given to the required number of decimal places as shown in the Display Digits.



For example, to factorise an expression on the Calculator page, press:

- **MENU** [menu]
- **3: Algebra** [3]
- **2: Factor** [2].

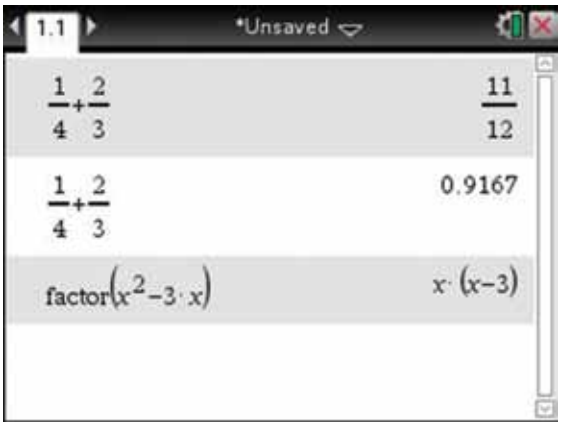


Complete the entry line as:

factor $(x^2 - 3x)$

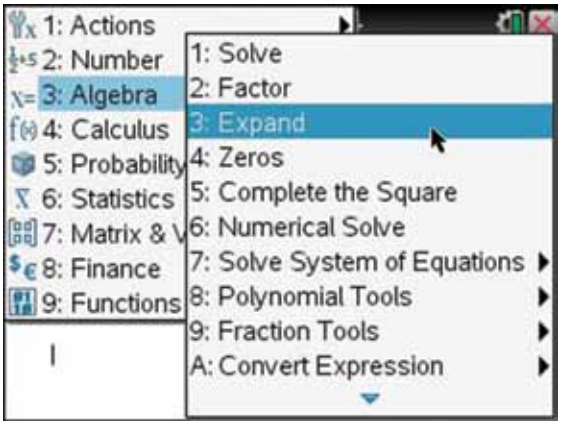
and press ENTER [enter]. The result is shown.

$x^2 - 3x = x(x - 3)$



To expand an expression on a Calculator page, press:

- **MENU** [menu]
- **3: Algebra** [3]
- **3: Expand** [3].



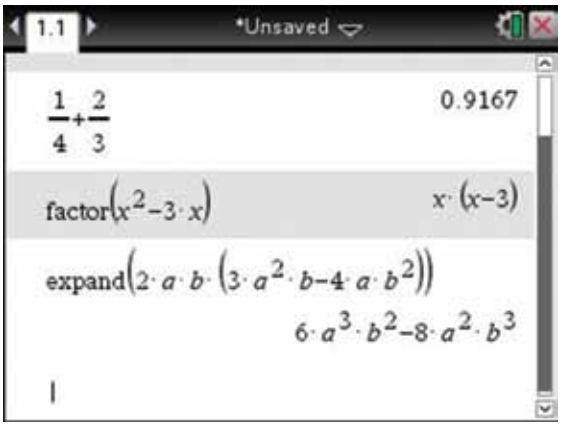
Complete the entry line as:

expand $(2a.b\ (3a^2b - 4a.b^2))$

and press ENTER [enter]. The result is shown.

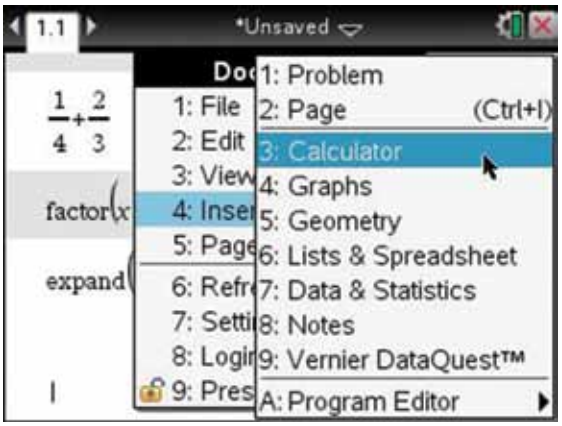
$2a.b\ (3a^2b - 4a.b^2) = 6a^3b^2 - 8a^2b^3$

Note: You must include the multiplication sign between the brackets and between the *a* and *b*.



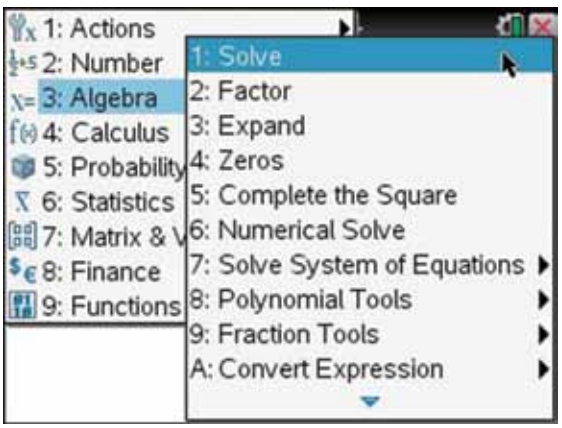
Open a new calculator page. Another way to do this is to press:

- **DOC** [doc]
- **4: Insert** [4]
- **3: Calculator** [3].



For example, to solve an equation on a Calculator page, press:

- **MENU** [menu]
- **3: Algebra** [3]
- **1: Solve** [1].

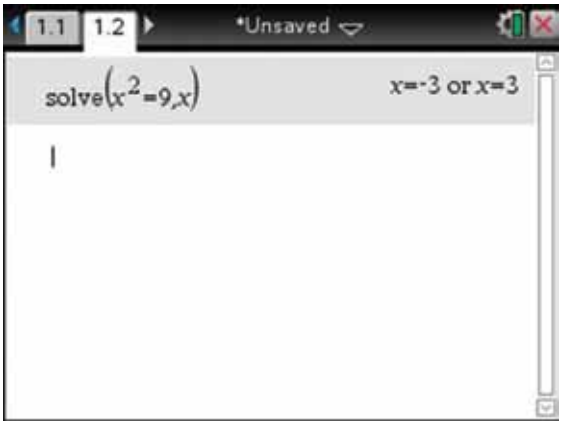


Complete the entry line as:

solve $(x^2 = 9, x)$

and press ENTER enter. Note that we must include the comma x, to instruct the calculator to solve the equation for x. The result is shown.

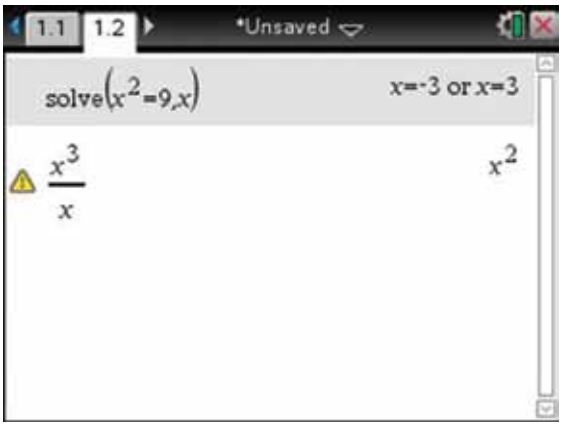
$x^2 = 0 \Rightarrow x = \pm 3$



As another example, to simplify $\frac{x^3}{x}$ press the fraction template. Type x, then use the power hat key ^ and type 3 to make it x^3, then press the down arrow ▼ twice, once to get out of the power and then to get to the denominator. Now type x again, then press ENTER enter. The answer appears.

$\frac{x^3}{x} = x^2$

However, notice the yellow warning sign that is shown! This sign will always be shown when the expression may not always be true. That is, the expression $\frac{x^3}{x} \neq x^2$ when $x = 0$. A warning sign often appears when cancelling expressions, or when an expression is only true on a restricted domain.

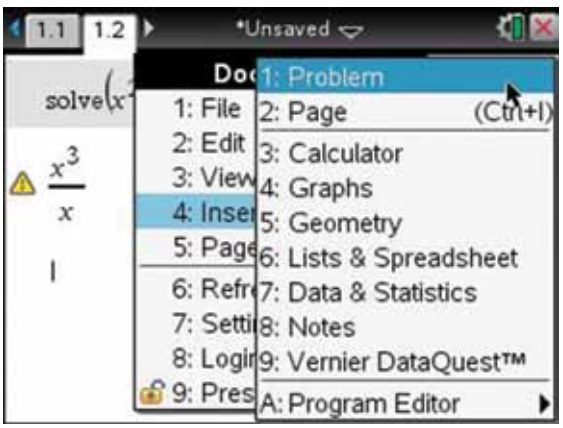


1.4 How to graph functions

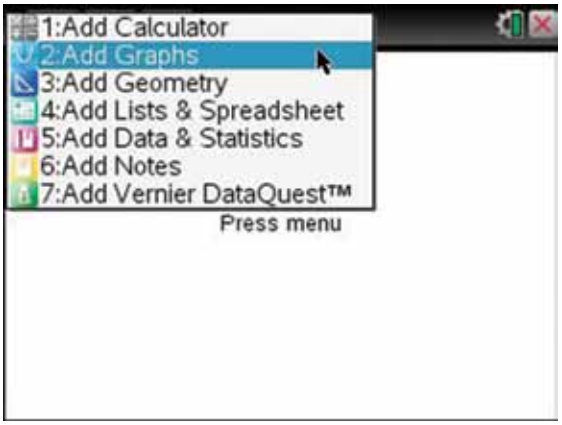
The calculator can also graph functions and some relations and inequalities. These graphs are done on a Graphs page. On the Graphs page we can also find critical points on the graph, such as axial intercepts and turning points. Later instructions will show how to perform these tasks. We can also sketch many graphs on the one Graphs page and find points of intersection between the graphs.

Open a new problem. To do this, press:

- **DOC** doc
- **4: Insert** 4
- **1: Problem** 1.

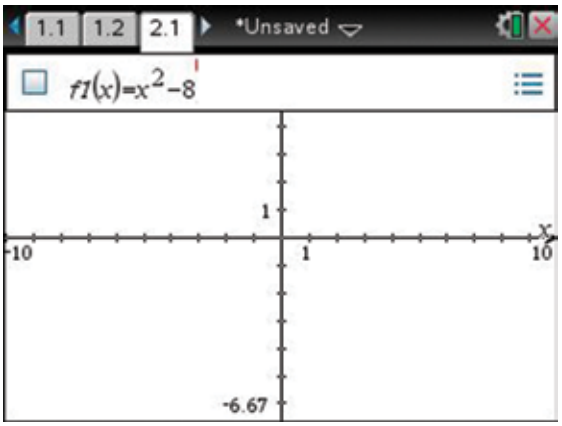


Insert a Graphs page. To do this, select 2 and press ENTER [enter].

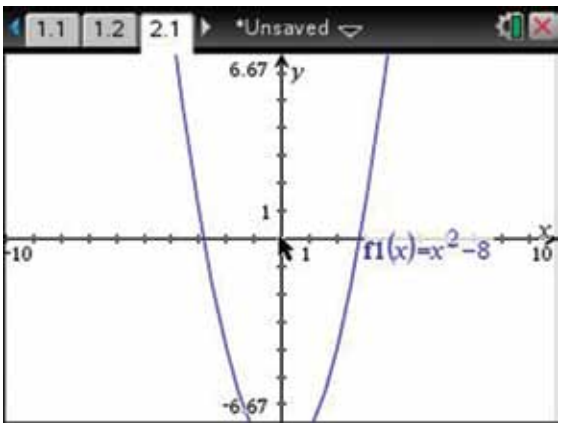


At the function entry line, type in $x^2 - 8$ as shown.

$f1(x) = x^2 - 8$

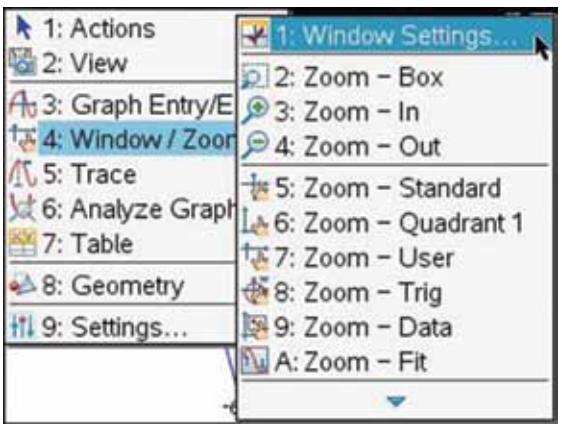


Press ENTER [enter] and the graph is displayed.

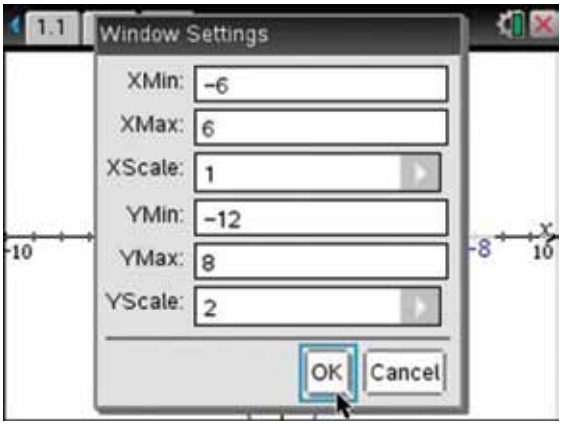


Although the graph is shown, we can adjust the viewing window to get a better picture of the graph. To do this press:

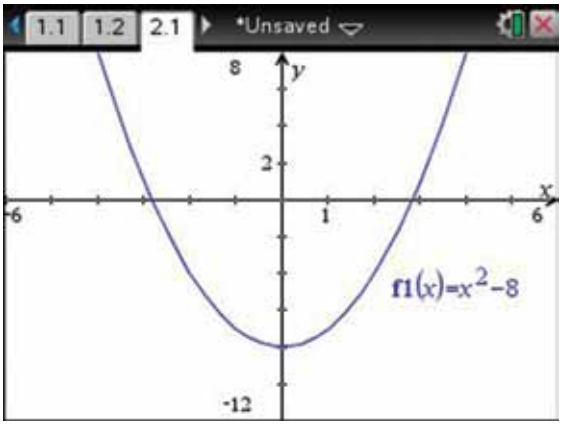
- **MENU** [menu]
- **4: Window/Zoom** [4]
- **1: Window Settings...** [1].



Complete the entry fields as shown using TAB [tab] to move between the fields.

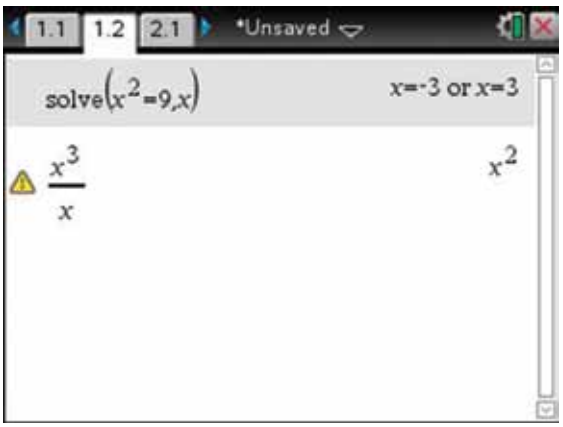


If we TAB [tab] to OK and press ENTER [enter], we see the graph in the new viewing window.



1.5 How to navigate between documents and pages

Notice the 1.1,1.2 and 2.1 in the tabs in the top left corner of the screen. These refer to pages 1 and 2 of problem 1 and page 1 of problem 2. The current page is indicated by the light background; this is page 2 in problem 1 of the current Unsaved Document. To toggle between these pages, press:



- **CTRL** [ctrl]
- **left arrow** ◀

or

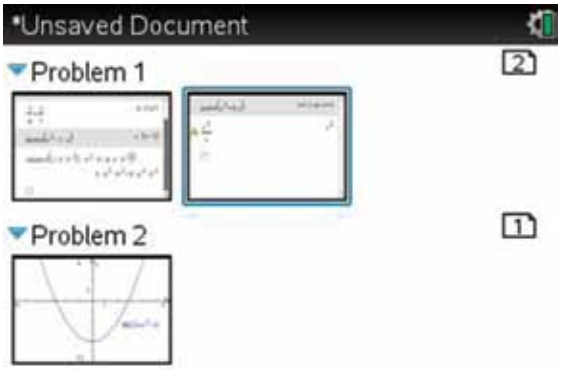
- **CTRL** [ctrl]
- **right arrow** ▶

or simply click on the blue arrows next to the tabs.

Press:

- **CTRL** ctrl
- **up arrow ▲.**

This brings up the page sorter view. From here we can see the current page with a thicker blue border around it. We could delete the page from the document using del, or we could copy the page or reorder the pages. This can be very useful if a document has many pages. When a page is selected, pressing ENTER enter makes that page active.

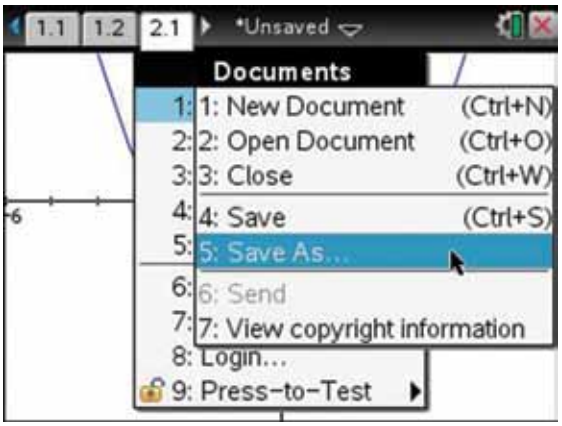


Each document can have at least one and up to 30 problems. Each problem can have at least one and up to 50 pages. Each page can have up to 4 different work sections, which can be grouped as any combination of the different applications.

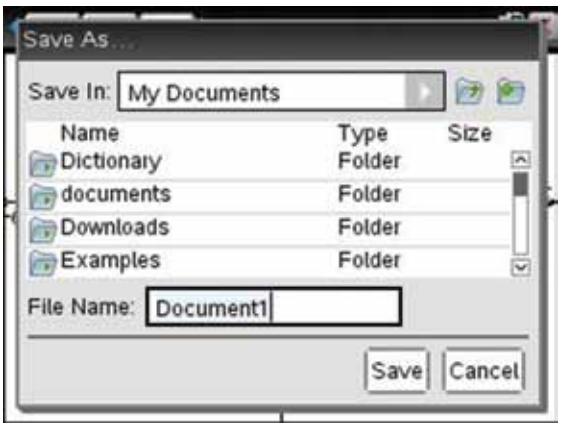
1.6 How to save and delete TNS files

We have not as yet saved our document, as indicated by the *Unsaved at the top of the screen. To save a document, press:

- **DOC** doc
- **1: File** 1
- **5: Save As …** 5.

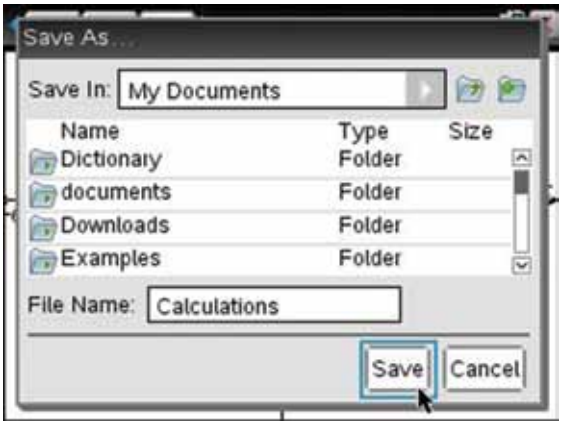


The following screen appears. You can press TAB tab to move to a folder in which to save your document. There may be some saved files or folders already there, so name your document something you will remember easily.



We will name this file Calculations. Type 'Calculations' in the File Name dialog box, TAB [tab] to Save and press ENTER [enter]. 'Calculations' is now saved as a TNS file. We can see the file name at the top of the screen. Also, there is no * next to the file name. If you make any changes to the document, the * will appear in front of the file name. This indicates that you have made changes but not resaved your document. To resave the document at any time, press:

- **CTRL** [ctrl]
- **S:** [S].



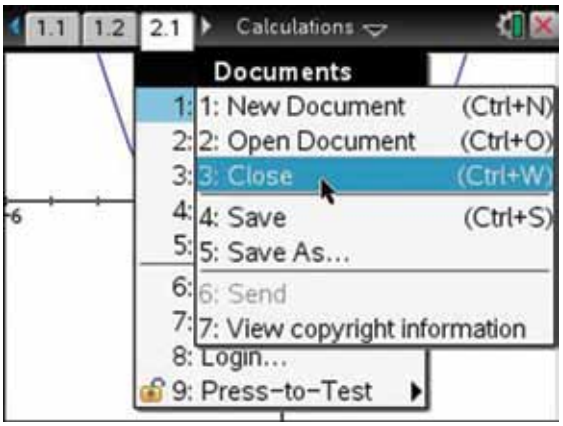
To close the file, press:

- **DOC** [doc]
- **1: File** [1]
- **3: Close** [3]

or simply click on the red X close box in the top right-hand corner of the screen, or press

- **CTRL** [ctrl]
- **W:** [W].

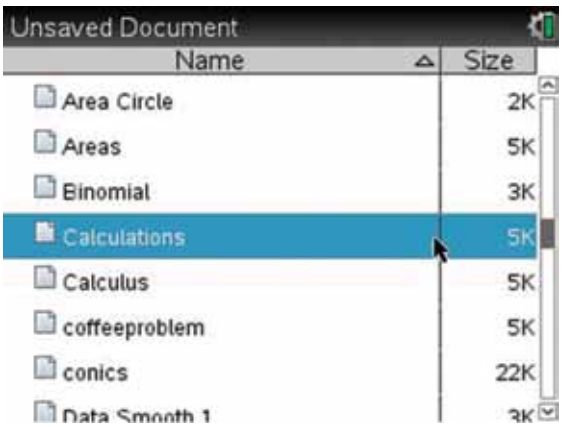
Note that these shortcut key commands, along with many others, are similar to those used in Microsoft Office.



To find this document at a later stage, press:

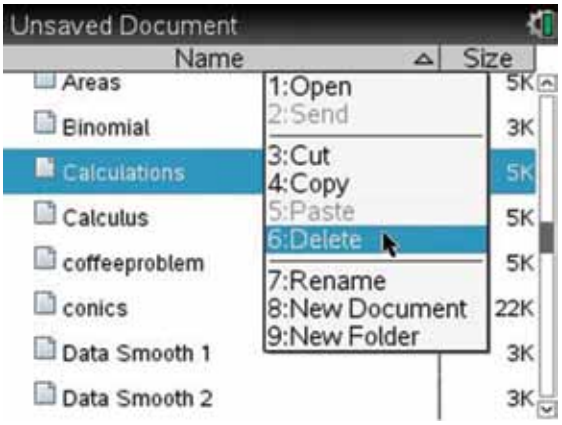
- **HOME** [⌂]
- **2: My Docs** [2].

Scroll down to your saved document, Calculations. Press ENTER [enter] to recover and reload the document.

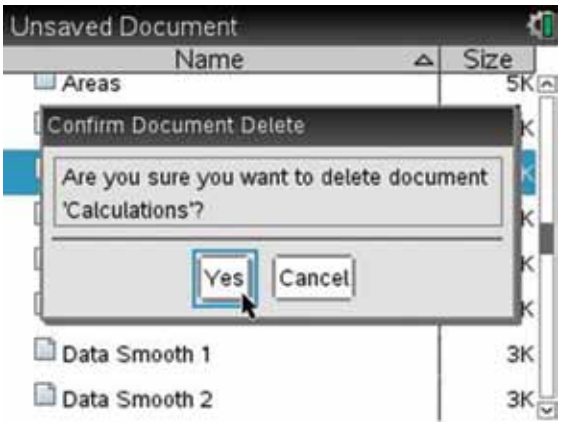


To delete an unneeded document or to free up memory, select a file and press CTRL ctrl ENTER enter. The screen shown appears. Press:

- **6: Delete 6.**



You will be prompted to confirm the deletion of the file. If you select YES and press ENTER enter, the selected file will be deleted and the file name will disappear from the file listing. This action is final and the file cannot be recovered.



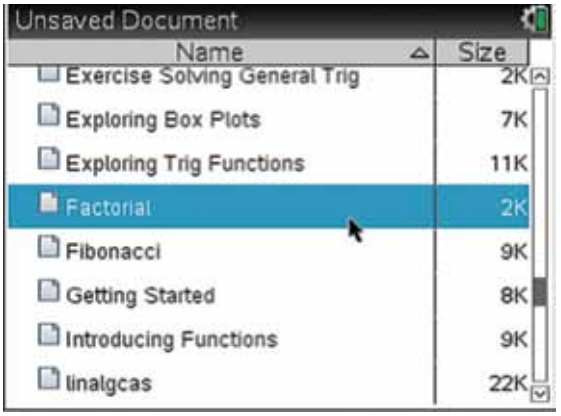
1.7 How to transfer files between calculators

You can send documents and operating system (OS) files to another calculator. If a file with the same name is already stored in the receiving calculator, the file will be renamed automatically. For example, if the file was FileName and the receiving calculator already has a file called FileName, then the incoming file will be renamed FileName (2).

To send a file between calculators, first you must connect the calculators with the connector cable. To locate the file to send, open My Docs by pressing:

- **HOME** home on
- **2: My Docs 2.**

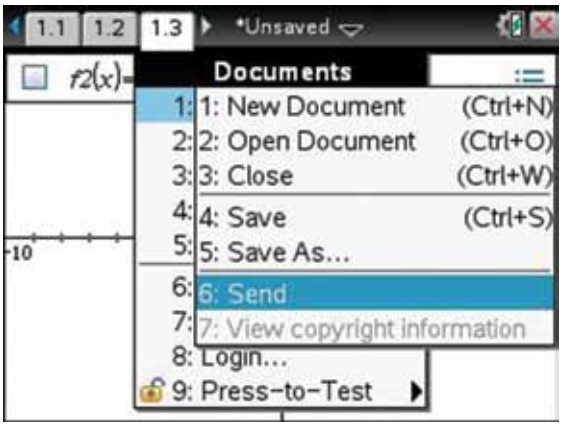
Select the file (or folder) you wish to send by using the arrows on the Touchpad.



To send a file from My Docs, press:

- **DOC** doc
- **1: File** 1
- **6: Send** 6**.**

While the file is being transferred, a progress bar will be displayed. A message will appear when the transfer is complete. No action is required on the receiving calculator.



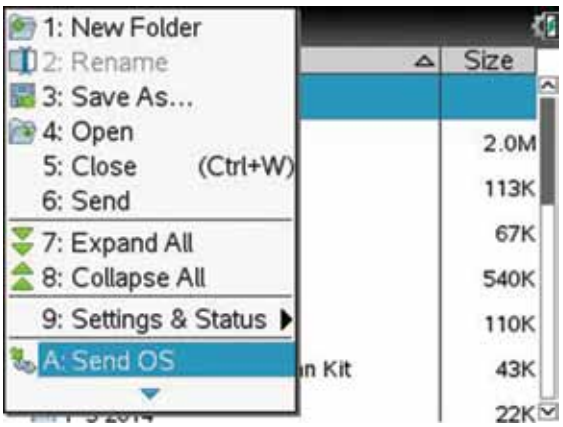
To see the current version of the operating system on your handheld calculator, along with battery levels and available memory space, press:

- **HOME**
- **5: Settings** 5
- **4: Status** 4**.**

To send the operating system from one handheld to another (to update the operating system — no action is required on the receiving handheld), press:

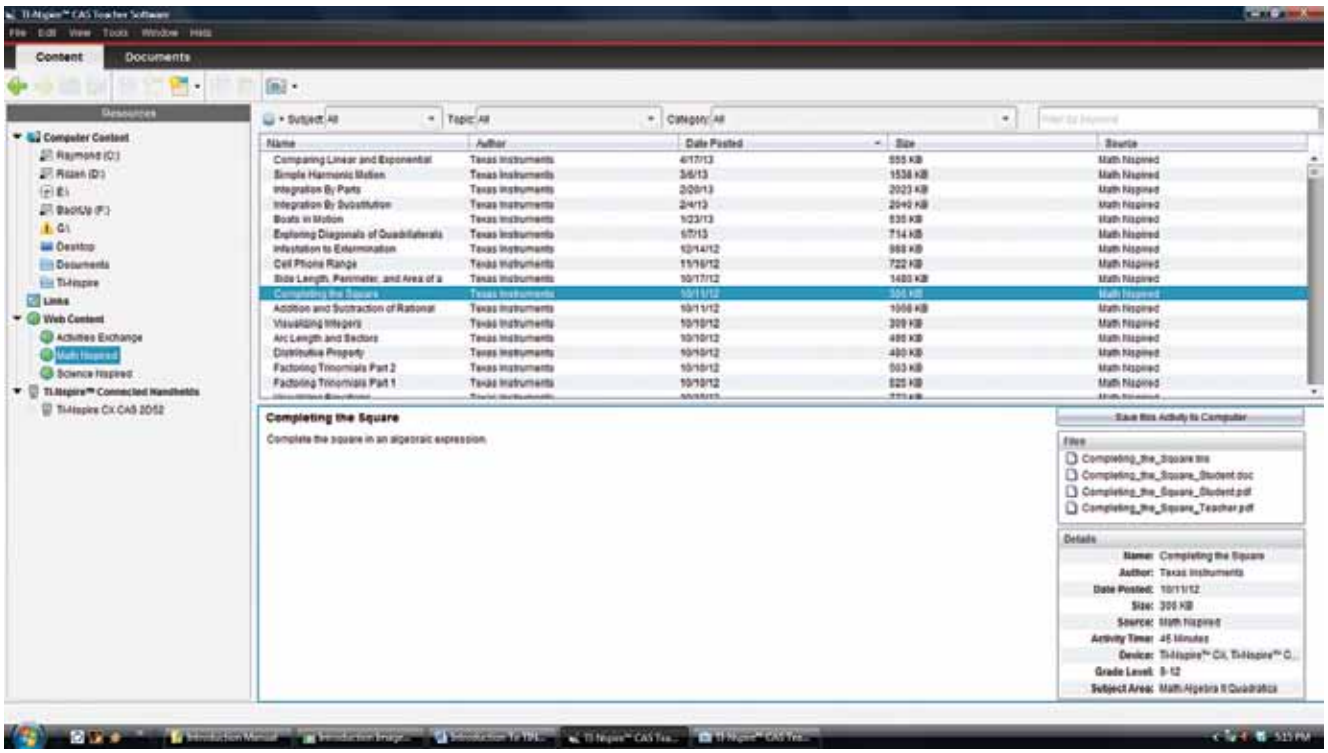
- **HOME**
- **2: My Docs** 2
- **MENU** menu
- **A: Send OS** A**.**

This may take a few minutes. Do not disconnect the two handheld calculators until prompted to. The files on the receiving handheld calculator will remain intact. Make sure you send operating systems between compatible handhelds, for example from a CAS CX to a CAS CX. You cannot send a CAS operating system to a non-CAS handheld.



1.8 How to transfer files between calculators

Many useful TNS Mathematical files are located at http://education.ti.com/en/timathnspired/us/home and can be downloaded to a handheld calculator free of charge. These files can be transferred between computers and calculators. To send a file between a computer and a calculator, connect the handheld calculator to the computer with the USB cable. Open the TI-Nspire Teacher or Student Software, and select your handheld calculator. Now select the Content tab on the screen. While connected to the internet, browse to the TIMath Nspired Web Content Lessons and open the folder in which the desired TNS files are saved. Drag this file or folder to the TI-Nspire HandHeld. This will copy the file(s) onto the handheld calculator. A new folder with the name 'Transfers' along with the current date will be in the My Documents folder along with the transferred files.



You can also update a handheld operating system by using

- **Tools**
- **Install OS**

on the Document TAB.

TOPIC 1

Indices

WORKED EXAMPLE 1

Simplify each of the following.

a $m^4n^3p \times m^2n^5p^3$ **b** $2a^2b^3 \times 3ab^4$ **c** $\dfrac{2x^5y^4}{10x^2y^3}$

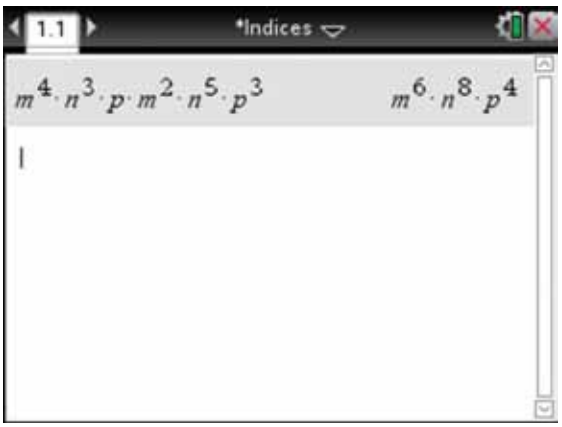
THINK

a In a new document on a calculator page, complete the entry line as:
$m^4n^3p \times m^2n^5p^3$
Be sure to include the implied multiplication signs, and use the hat key [^] to type in the index. Note that when you use the hat key [^] to raise the number to a power, you must press TAB [tab] after the power to bring the cursor into line with your next entry.
To type squared values, you can press [x²].
Then press ENTER [enter].

b On a Calculator page, complete the entry line as:
$2a^2b^3 \times 3ab^4$
Then press ENTER [enter].

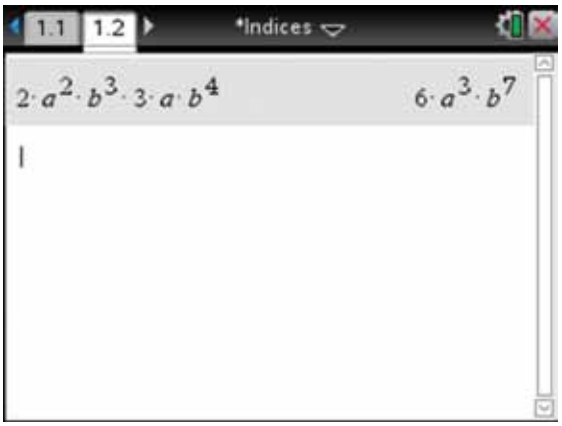
WRITE

a



$m^4n^3p \times m^2n^5p^3 = m^6n^8p^4$

b



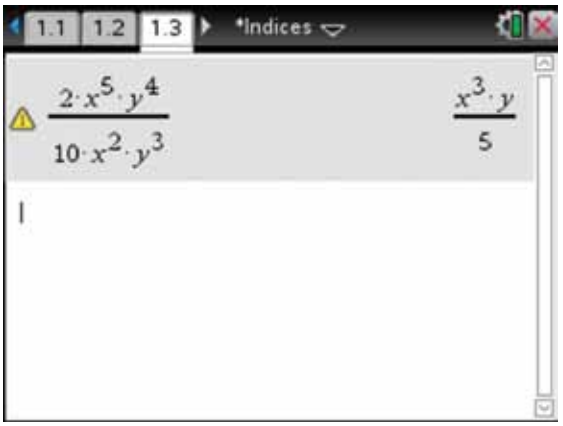
$2a^2b^3 \times 3ab^4 = 6a^3b^7$

c On a Calculator page, to divide the expressions, press CTRL [ctrl] and [÷] to get the fraction template, and then type the indices directly onto the screen as shown.

$$\frac{2x^5y^4}{10x^2y^3}$$

Then press ENTER [enter].

c



$$\frac{2x^5y^4}{10x^2y^3} = \frac{x^3y}{5}$$

WORKED EXAMPLE 3

Simplify each of the following.

a $(2n^4)^3$ **b** $(3a^2b^7)^3$ **c** $\left(\frac{2x^3}{y^4}\right)^4$ **d** $(-4)^3$

THINK

a–d On a Calculator page, use the brackets and complete the entry lines as:

$(2n^4)^3$

$(3a^2b^7)^3$

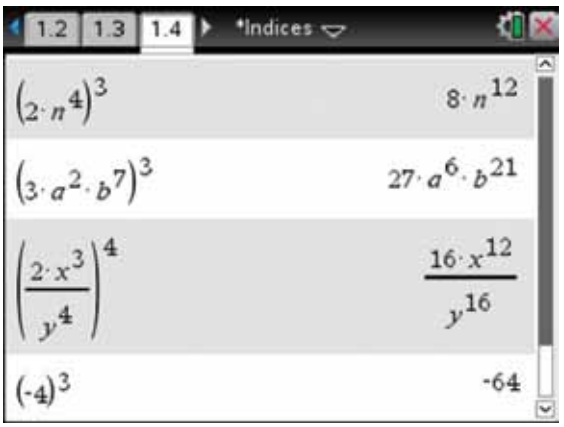
$\left(\frac{2x^3}{y^4}\right)^4$

$(-4)^3$

Press ENTER [enter] after each entry.

WRITE

a–d



$(2n^4)^3 = 8n^{12}$

$(3a^2b^7)^3 = 27a^6b^{21}$

$\left(\frac{2x^3}{y^4}\right)^4 = \frac{16x^{12}}{y^{16}}$

$(-4)^3 = -64$

WORKED EXAMPLE 5

Simplify each of the following, expressing the answers with positive indices.

a $a^2b^{-3} \times a^{-5}b$ **b** $\dfrac{2x^4y^2}{3xy^5}$ **c** $\left(\dfrac{2m^3}{n^{-2}}\right)^{-2}$

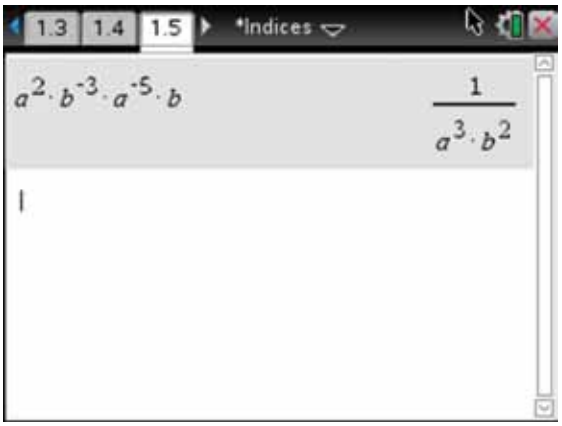
THINK

a On a Calculator page, complete the entry line as: $a^2b^{-3} \times a^{-5}b$
Then press ENTER enter.
Note that the CAS calculator has automatically expressed the answers with positive indices.

b On a Calculator page, complete the entry line as: $\dfrac{2x^4y^2}{3xy^5}$
Then press ENTER enter.

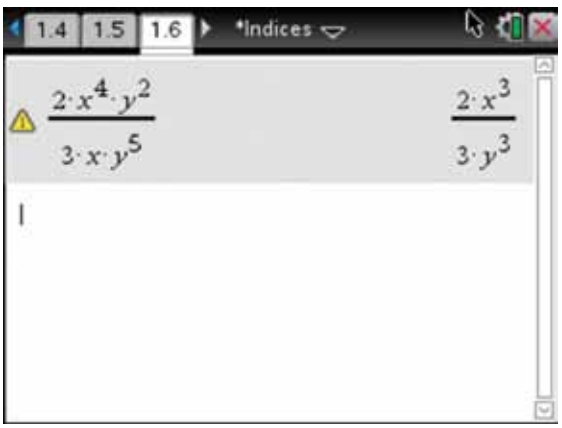
WRITE

a



$a^2b^{-3} \times a^{-5}b = \dfrac{1}{a^3b^2}$

b



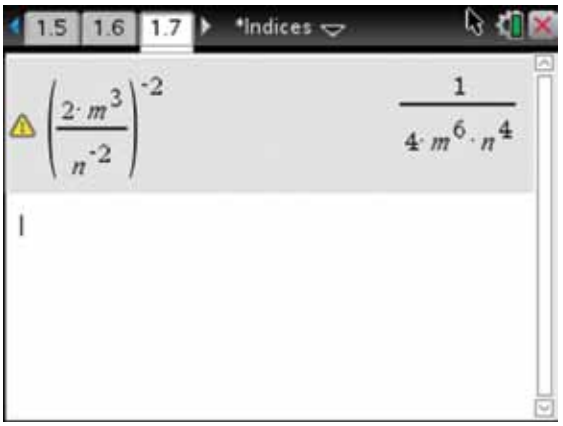
$\dfrac{2x^4y^2}{3xy^5} = \dfrac{2x^3}{3y^3}$

c On a Calculator page, complete the entry line as:

$\left(\frac{2m^3}{n^{-2}}\right)^{-2}$

Then press ENTER enter.

c



$\left(\frac{2m^3}{n^{-2}}\right)^{-2} = \frac{1}{4m^6n^4}$

WORKED EXAMPLE 8

Simplify each of the following.

a $m^{\frac{1}{5}} \times m^{\frac{2}{5}}$ **b** $(a^2b^3)^{\frac{1}{6}}$ **c** $\left(\frac{x^{\frac{2}{3}}}{y^{\frac{3}{4}}}\right)^{\frac{1}{2}}$

THINK

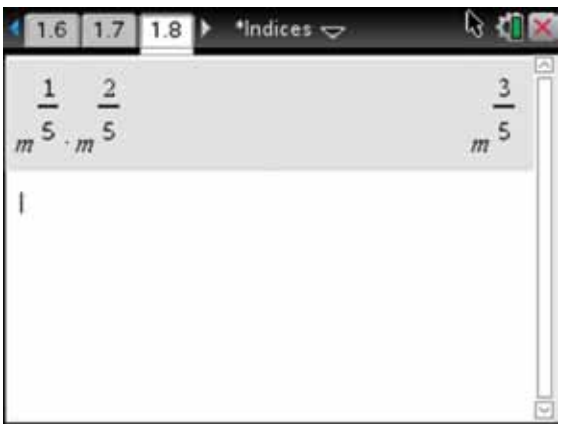
a On a Calculator page, complete the entry line as:

$m^{\frac{1}{5}} \times m^{\frac{2}{5}}$

Then press ENTER enter.

WRITE

a



$m^{\frac{1}{5}} \times m^{\frac{2}{5}} = m^{\frac{3}{5}}$

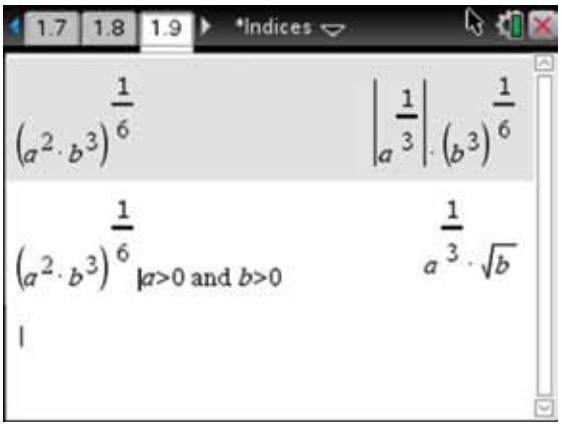
b On a Calculator page, complete the entry line as:

$\left(a^2b^3\right)^{\frac{1}{6}}$

Then press ENTER enter.

Note that the answer of $a^{\frac{1}{3}}\sqrt{b}$ will only be given if a and b are both positive real numbers; that is, if $a > 0$ and $b > 0$. To get this answer, use the symbol |. Press CTRL ctrl and then = to bring up the palette; use the Touchpad to select the | symbol; then insert a space and type 'and'. Alternatively, 'and' can be found in the catalog 📖. Complete as shown, then press ENTER enter.

b



$\left(a^2b^3\right)^{\frac{1}{6}} = a^{\frac{1}{3}}b^{\frac{1}{2}} = a^{\frac{1}{3}}\sqrt{b}$ if $a > 0$ and $b > 0$

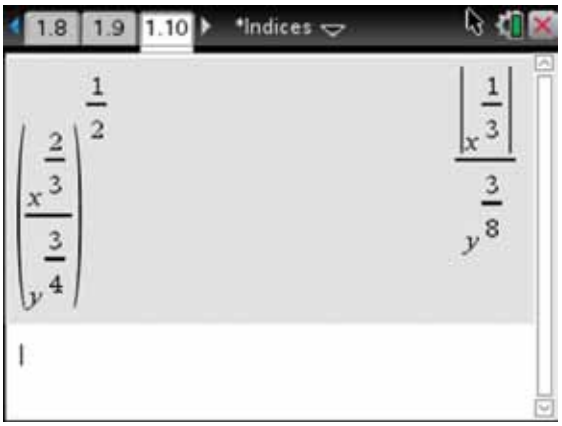
c On a Calculator page, complete the entry line as:

$$\left(\frac{x^{\frac{2}{3}}}{y^{\frac{3}{4}}}\right)^{\frac{1}{2}}$$

Then press ENTER enter.

Note again that $|x|$ will appear, unless you restrict $x > 0$.

c



$$\left(\frac{x^{\frac{2}{3}}}{y^{\frac{3}{4}}}\right)^{\frac{1}{2}} = \frac{x^{\frac{1}{3}}}{y^{\frac{3}{8}}}$$

WORKED EXAMPLE 11

Simplify each of the following.

a $\dfrac{(5a^2b^3)^2}{a^{10}} \times \dfrac{a^2b^5}{(a^3b)^7}$ **b** $\dfrac{8m^3n^{-4}}{(6mn^2)^3} \div \dfrac{4m^{-2}n^{-4}}{6m^{-5}n}$

THINK

a On a Calculator page, complete the entry line as:

$$\frac{\left(5a^2b^3\right)^2}{a^{10}} \times \frac{a^2b^5}{\left(a^3 \times b\right)^7}$$

Then press ENTER enter.

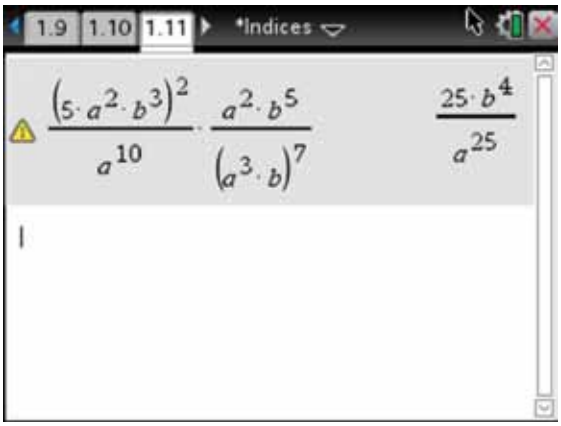
b On a Calculator page, use the fraction template twice to complete the entry line as:

$$\frac{8m^3n^{-4}}{\left(6m \times n^2\right)^3} \div \frac{4m^{-2}n^{-4}}{6m^{-5} \times n}$$

When you press ENTER enter, the answer will display as shown.

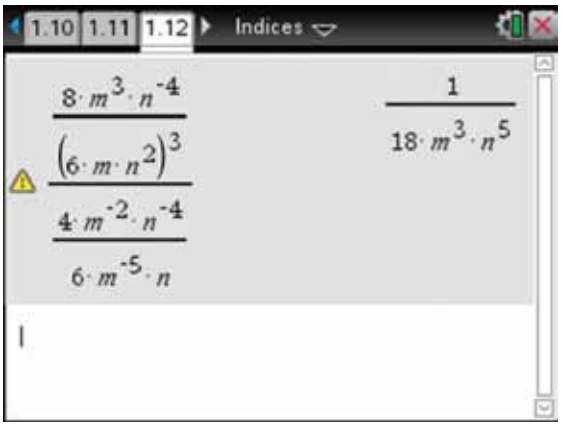
WRITE

a



$$\frac{\left(5a^2b^3\right)^2}{a^{10}} \times \frac{a^2b^5}{\left(a^3b\right)^7} = \frac{25b^4}{a^{25}}$$

b



$$\frac{8m^3n^{-4}}{\left(6mn^2\right)^3} \div \frac{4m^{-2}n^{-4}}{6m^{-5}n} = \frac{1}{18m^3n^5}$$

TOPIC 2

Algebra and equations

WORKED EXAMPLE 2

If $c = \sqrt{a^2 + b^2}$, calculate c if $a = 12$ and $b = -5$.

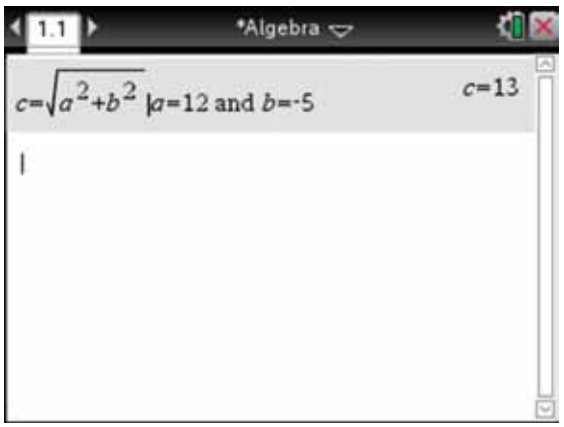
THINK

In a new document, open a calculator page.
To substitute values, use the symbol |. Press CTRL [ctrl] and then [=] to bring up the palette; use the Touchpad to select the | symbol. Then type 'and' or find it in the catalog [📖].
Complete the entry line as:

$c = \sqrt{a^2 + b^2} \mid a = 12$ and $b = -5$

Then press ENTER [enter].

WRITE



If $a = 12$ and $b = -5$, then $c = \sqrt{a^2 + b^2} = 13$.

WORKED EXAMPLE 4

Simplify the following expressions.

a $\dfrac{2x}{3} - \dfrac{x}{2}$ **b** $\dfrac{x+1}{6} + \dfrac{x+4}{4}$

THINK

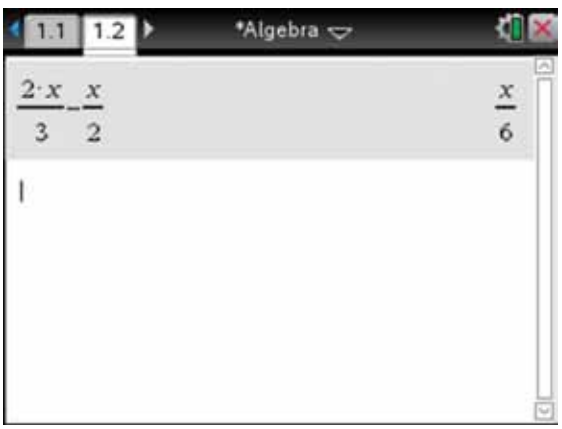
a On a Calculator page, press CTRL [ctrl] and [÷] to get the fraction template, then type the expressions directly as:

$\dfrac{2x}{3} - \dfrac{x}{2}$

Then press ENTER [enter].

WRITE

a



$\dfrac{2x}{3} - \dfrac{x}{2} = \dfrac{x}{6}$

b On a Calculator page, complete the entry line as:

$$\frac{x+1}{6}+\frac{x+4}{4}$$

Then press ENTER [enter].

To add these algebraic fractions, it is necessary to find a common denominator. To do this, press:

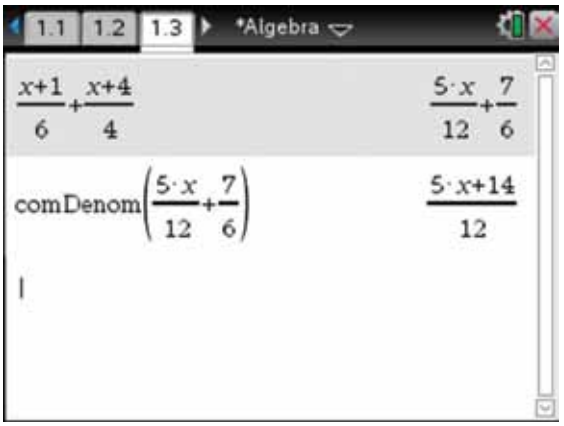
- MENU [menu]
- 3: Algebra [3]
- 9: Fraction Tools [9]
- 4: Common Denominator [4].

Then complete the entry line as:

$$\text{combine}\left(\frac{5x}{12}+\frac{7}{6}\right)$$

Then press ENTER [enter].

b



$$\frac{x+1}{6}+\frac{x+4}{4}=\frac{5x+14}{12}$$

WORKED EXAMPLE 7

Simplify $\frac{x+2}{x-3}+\frac{x-1}{(x-3)^2}$ by writing it first as a single fraction.

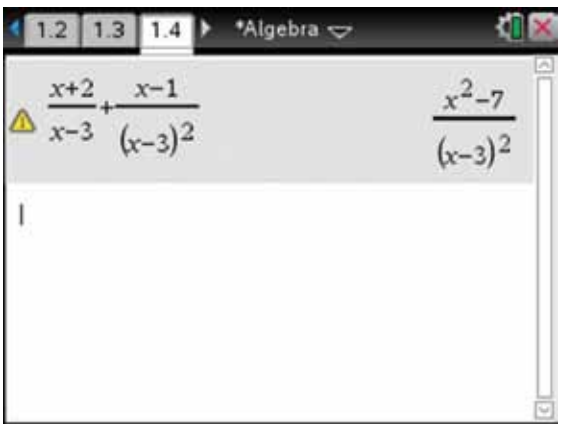
THINK

On a Calculator page, press CTRL [ctrl] and [÷] to get the fraction template, and then complete the entry line as:

$$\frac{x+2}{x-3}+\frac{x-1}{(x-3)^2}$$

Then press ENTER [enter].

WRITE



$$\frac{x+2}{x-3}+\frac{x-1}{(x-3)^2}=\frac{x^2-7}{(x-3)^2}$$

WORKED EXAMPLE 9

Simplify the following expressions.

a $\dfrac{3xy}{2} \div \dfrac{4x}{9y}$

b $\dfrac{4}{(x+1)(3x-5)} \div \dfrac{x-7}{x+1}$

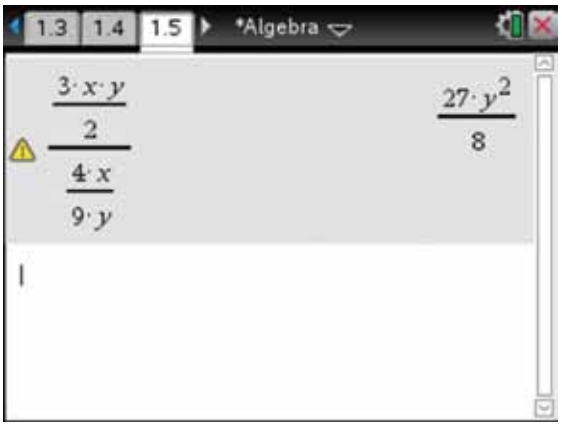
THINK

a On a Calculator page, use the fraction template twice to complete the entry line as:
$\dfrac{3xy}{2} \div \dfrac{4x}{9y}$
When you press ENTER enter, the answer will display as shown.

b On a Calculator page, use the fraction template twice to complete the entry line as:
$\dfrac{4}{(x+1)(3x-5)} \div \dfrac{x-7}{x+1}$
When you press ENTER enter, the answer will display as shown.

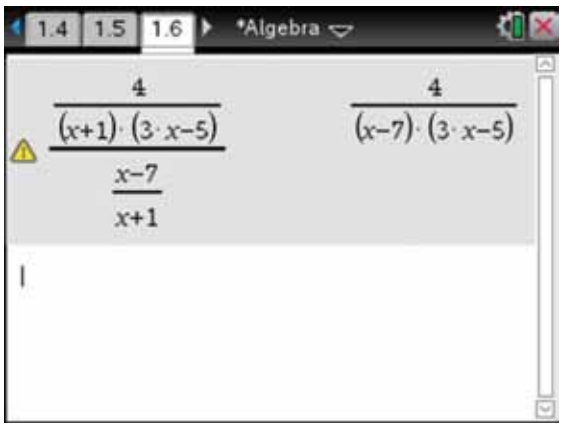
WRITE

a



$\dfrac{3xy}{2} \div \dfrac{4x}{9y} = \dfrac{27y^2}{8}$

b



$\dfrac{4}{(x+1)(3x-5)} \div \dfrac{x-7}{x+1} = \dfrac{4}{(x-7)(3x-5)}$

WORKED EXAMPLE 11

Solve the following equations.

a $5y - 6 = 79$ **b** $\frac{4x}{9} = 5$

THINK

a On a Calculator page, to solve equations press:
- MENU [menu]
- 3: Algebra [3]
- 1: Solve [1].

Then complete the line as:
$\text{solve}(5y - 6 = 79, y)$

The 'comma y' $(, y)$ instructs the calculator to solve for the variable y.
Then press ENTER [enter].

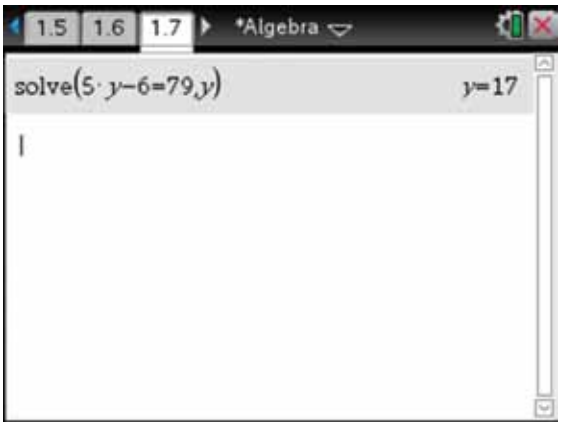
b On a Calculator page, complete the entry line as:
$\text{solve}\left(\frac{4x}{9} = 5, x\right)$

The result is given as an improper fraction.
To change to a proper fraction, press:
- MENU [menu]
- 2: Number [2]
- 7: Fraction Tools [7]
- 1: Proper Fraction [1].

Then complete as shown and press ENTER [enter].

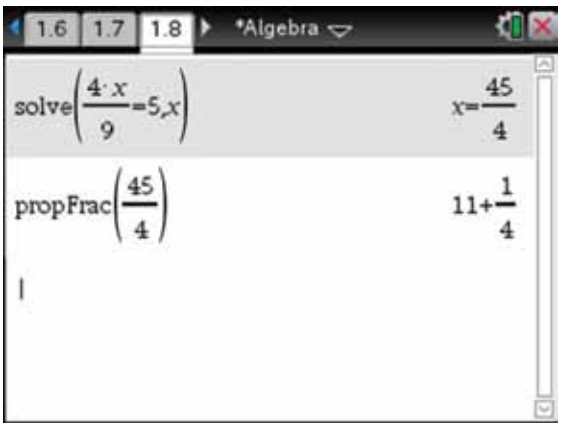
WRITE

a



$5y - 6 = 79$
$\Rightarrow y = 17$

b



$\frac{4x}{9} = 5$
$\Rightarrow x = 11\frac{1}{4}$

WORKED EXAMPLE 13

Solve each of the following linear equations.

a $6(x+1)-4(x-2)=0$

b $7(5-x)=3(x+1)-10$

THINK

a–b On a Calculator page, complete the entry lines as:

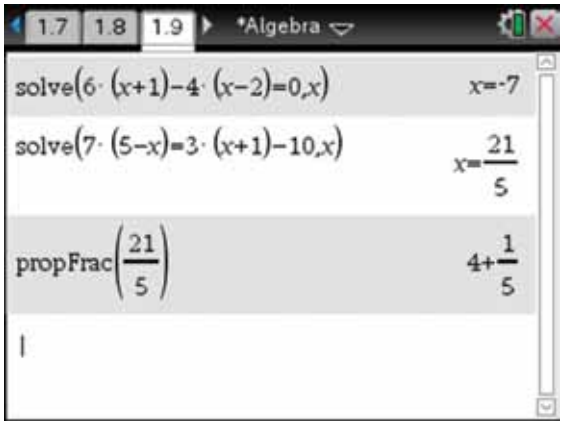
solve$\left(6(x+1)-4(x-2)=0,\ x\right)$

solve$\left(7(5-x)=3(x+1)-10,\ x\right)$

Press ENTER [enter] after each entry.

WRITE

a–b



$6(x+1)-4(x-2)=0$

$\Rightarrow x=-7$

$7(5-x)=3(x+1)-10$

$\Rightarrow x=4\frac{1}{5}$

WORKED EXAMPLE 15

Solve each of the following equations.

a $\dfrac{5(x+3)}{6}=4+\dfrac{3(x-1)}{5}$ **b** $\dfrac{4}{3(x-1)}=\dfrac{1}{x+1}$

THINK

a–b On a Calculator page, complete the entry lines as:

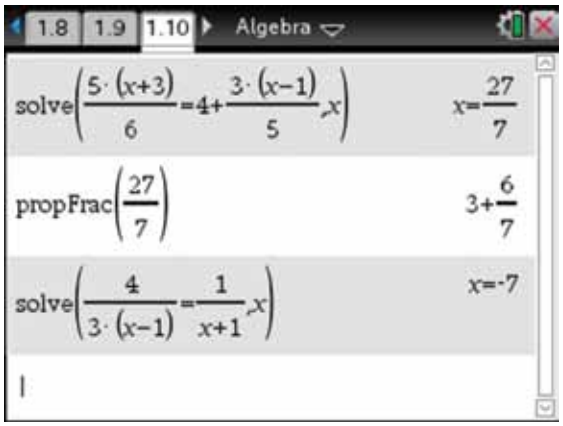
solve$\left(\dfrac{5(x+3)}{6}=4+\dfrac{3(x-1)}{5},\ x\right)$

solve$\left(\dfrac{4}{3(x-1)}=\dfrac{1}{x+1},\ x\right)$

Then Press ENTER [enter] after each entry.

WRITE

a–b



$\dfrac{5(x+3)}{6}=4+\dfrac{3(x-1)}{5}$

$\Rightarrow x=3\dfrac{6}{7}$

$\dfrac{4}{3(x-1)}=\dfrac{1}{x+1}$

$\Rightarrow x=-7$

TOPIC 3

Coordinate geometry

WORKED EXAMPLE 1

Plot the linear graph defined by the rule $y = 2x - 5$ for the x-values $-3, -2, -1, 0, 1, 2$ and 3.

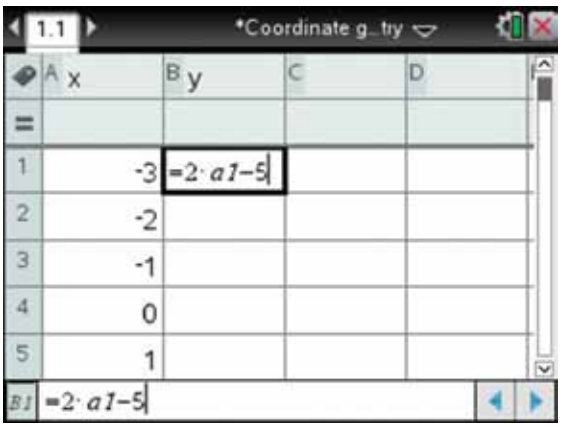
THINK

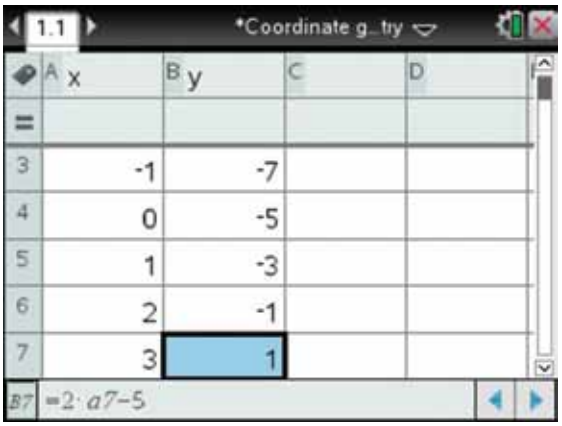
1. In a new document, on a Lists & Spreadsheet page, label column A as x and label column B as y.
   Enter the x-values into column A.
   Then in cell B1, complete the entry line as:
   $= 2a1 - 5$
   Then press ENTER [enter].

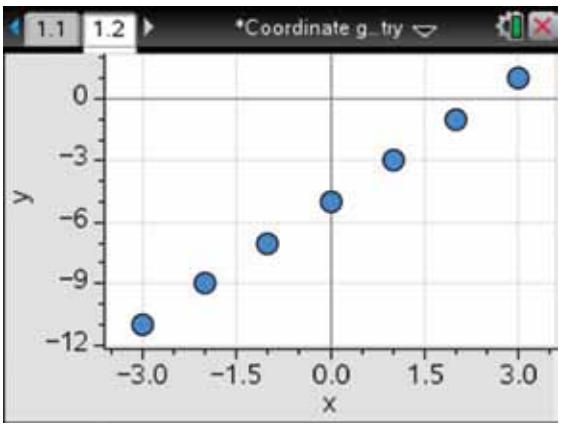
2. Hold down the SHIFT key and the down arrow to fill down the y-values.

3. Open a Data & Statistics page.
   Press TAB [tab] to locate the label of the horizontal axis and select the variable x.
   Press TAB [tab] again to locate the label of the vertical axis and select the variable y. The graph will be plotted as shown.

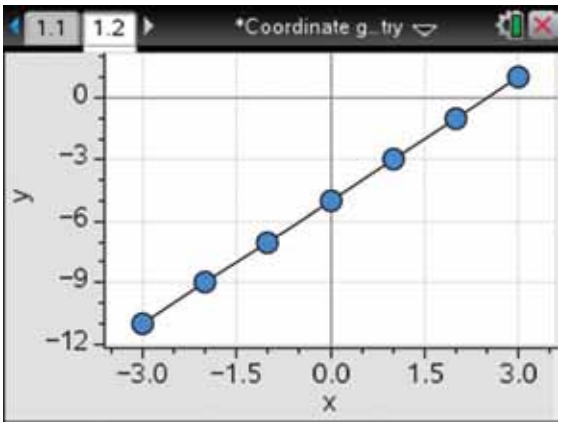
WRITE







4 To join the dots with a line, press:
- MENU [menu]
- 2: Plot Properties [2]
- 1: Connect Data Points [1].



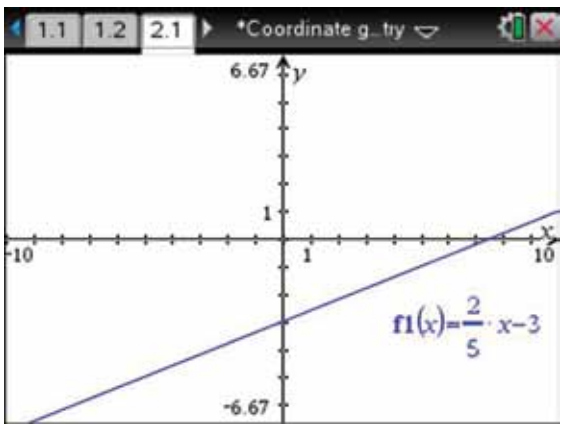
WORKED EXAMPLE 3

Plot the graph of $y = \frac{2}{5}x - 3$ using the gradient–intercept method.

THINK

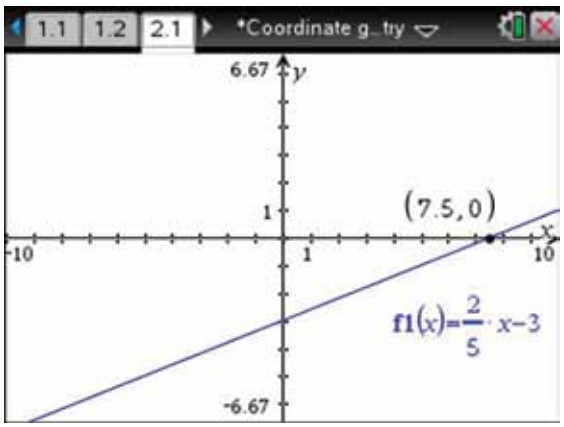
1 Insert a new problem and open a Graphs page.
Complete the function entry line as:
$f1(x) = \frac{2}{5}x - 3$
Then press ENTER [enter].
The line appears.

WRITE/DRAW



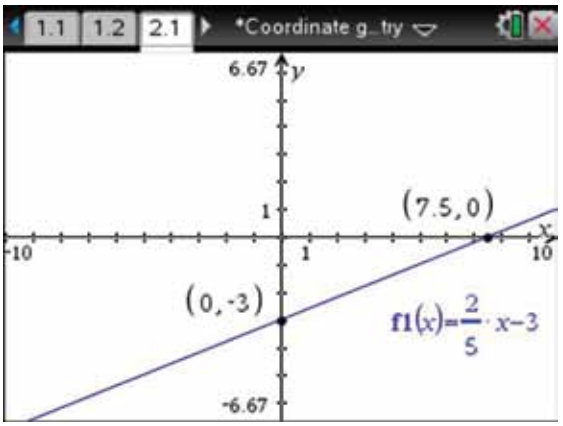
2 To find the x-intercept, press:
- MENU [menu]
- 6: Analyze Graph [6]
- 1: Zero [1].

Move the cursor to the left of the x-intercept and press ENTER [enter], then move the cursor to the right of the x-intercept and press ENTER [enter]. The x-intercept is displayed as (7.5, 0).



3 To find the *y*-intercept, press:
- MENU [menu]
- 5: Trace [5]
- 1: Graph Trace [1].

The cursor will be over the *y*-intercept. Press ENTER [enter], then press ESCAPE [esc].
The *y*-intercept is displayed as (0, –3).



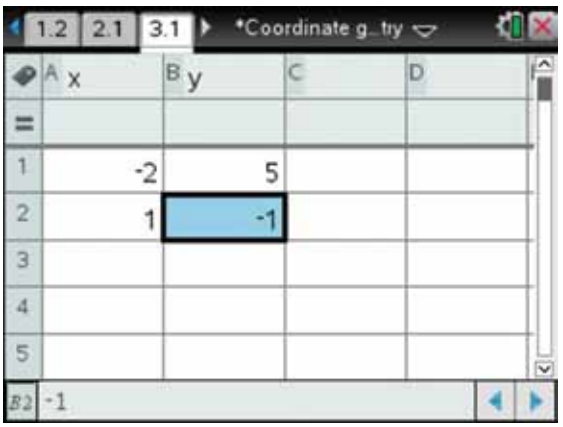
WORKED EXAMPLE 8

Find the equation of the straight line passing through (–2, 5) and (1, –1).

THINK

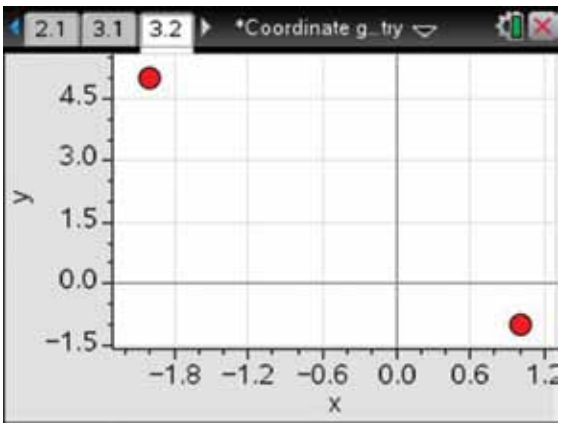
1 The CAS calculator can be used to determine the equation of the line joining the two points as follows. In a new problem on a Lists & Spreadsheet page, complete the entries as shown.

WRITE



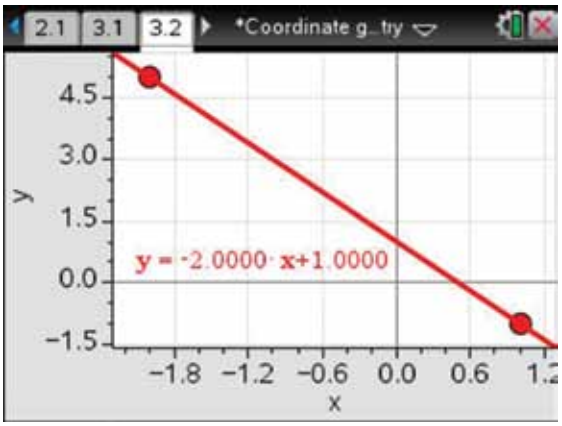
2 Open a Data & Statistics page and press TAB [tab] to locate the label for the horizontal axis and select the variable *x*.
Press TAB [tab] again to locate the label for the vertical axis and select the variable *y*.
The points will be plotted.
To change the colour, press:
- CTRL [ctrl]
- MENU [menu]
- 3: Colour [3]

and select a colour from the palette.



3 To get the equation of the line, press:
- MENU [menu]
- 4: Analyze [4]
- 6: Regression [6]
- 1: Show Linear (mx+c) [1].

The equation will appear on the graph and the points will be joined by a line.



The equation of the line passing through the points (–2, 5) and (1, –1) is $y = -2x + 1$.

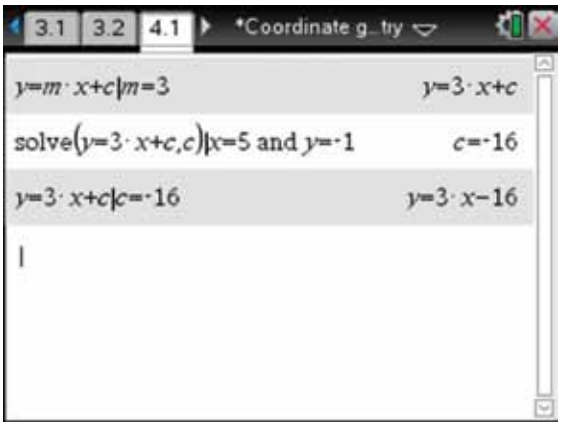
WORKED EXAMPLE 10

Find the equation of the straight line passing through the point (5, –1) with a gradient of 3.

THINK

The equation can be found using a CAS calculator as follows.
In a new problem on a Calculator page, complete the entry lines as:
$y = m \times x + c \,|\, m = 3$
$\text{solve}\left(y = 3x + c,\ c\right) \,|\, x = 5 \text{ and } y = -1$
$y = 3x + c \,|\, c = -16$
Press ENTER [enter] after each line.

WRITE



The equation is $y = 3x - 16$.

WORKED EXAMPLE 13

Find the distance between the points P (–1, 5) and Q (3, –2).

THINK

On a Calculator page, complete the entry lines as:

$x1 := -1$

$y1 := 5$

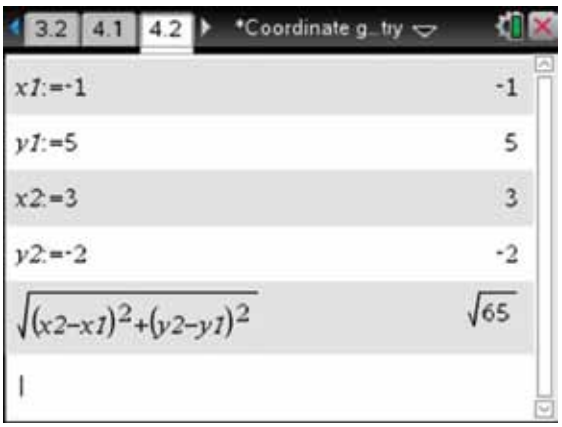
$x2 := 3$

$y2 := -2$

$\sqrt{(x2-x1)^2+(y2-y1)^2}$

Press ENTER enter after each entry.

WRITE



The distance between the two points is $\sqrt{65}$.

WORKED EXAMPLE 15

Find the coordinates of the midpoint of the line segment joining (–2, 5) and (7, 1).

THINK

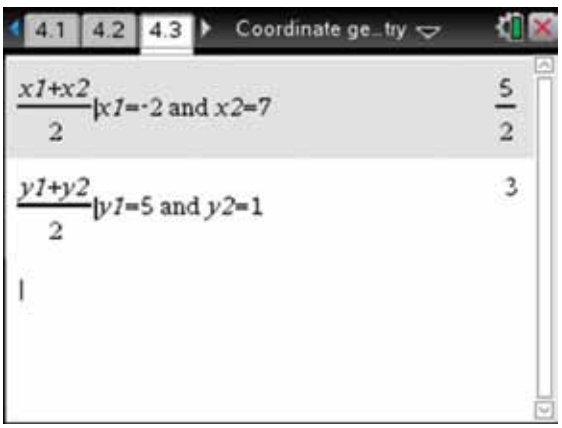
On a Calculator page, complete the entry lines as:

$\frac{x1+x2}{2} \mid x1 = -2$ and $x2 = 7$

$\frac{y1+y2}{2} \mid y1 = 5$ and $y2 = 1$

Press ENTER enter after each entry.

WRITE



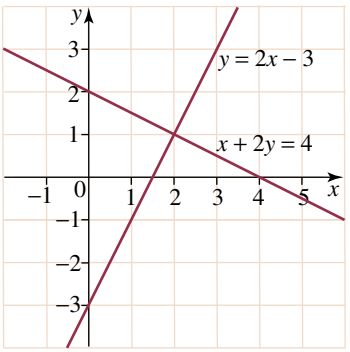
The midpoint is $\left(2\frac{1}{2}, 3\right)$.

TOPIC 4
Simultaneous linear equations and inequations

WORKED EXAMPLE 1

Use the graph of the given simultaneous equations to determine the point of intersection and, hence, the solution of the simultaneous equations.

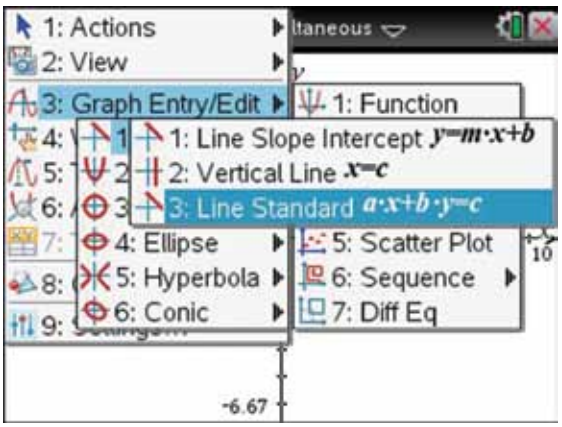
$x + 2y = 4$
$y = 2x - 3$



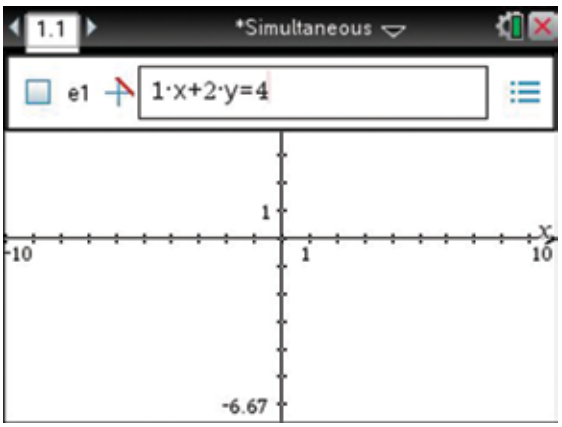
THINK

1. To graph $x + 2y = 4$ in a new document on a Graphs page, press:
   - MENU [menu]
   - 3: Graph Entry/Edit [3]
   - 2: Equation [2]
   - 1: Line [1]
   - 3: Line Standard $ax + by = c$ [3].

WRITE

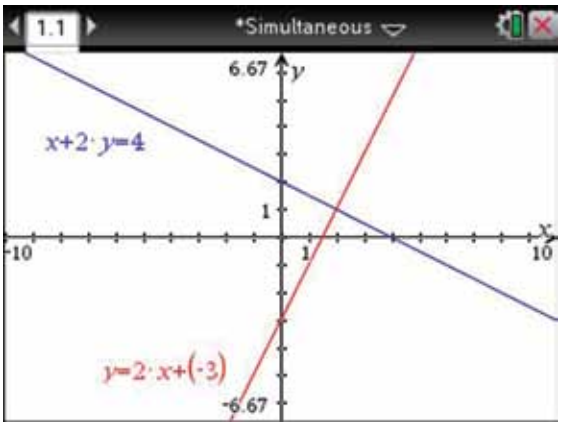


2. Complete the entry line as:
   $1.x + 2.y = 4$
   Press TAB [tab] to move between the fields.
   Press ENTER [enter].
   The graph of the straight line will be shown.



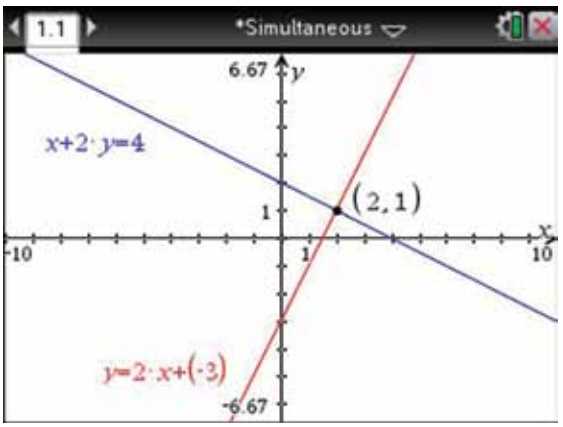
3 To graph $y = 2x - 3$, press:
- MENU [menu]
- 3: Graph Entry/Edit [3]
- 2: Equation [2]
- 1: Line [1]
- 1: Line Slope Intercept $y = mx + b$ [1].

Complete the entry as described above.
Press ENTER [enter].
The graph of the straight line will be shown.



4 To find the point of intersection between the two lines, press:
- MENU [menu]
- 6: Analyze Graph [6]
- 4: Intersection [4].

Move the cursor to the left of the intersection point, press ENTER [enter], then move the cursor to the right of the intersection point and press ENTER [enter]. The intersection point is displayed.



The point of intersection is (2, 1)

WORKED EXAMPLE 2

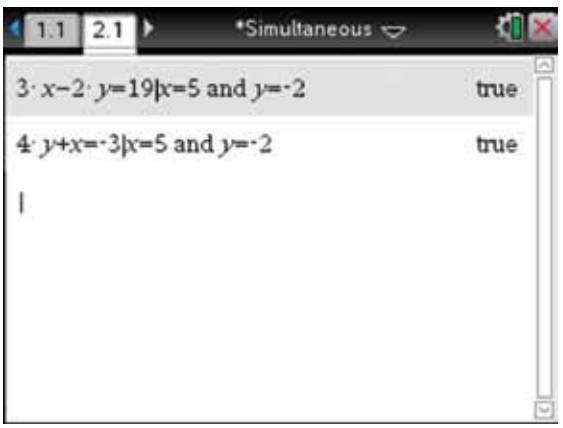
Check whether the given pair of coordinates, (5, –2), is the solution to the following pair of simultaneous equations.

$$3x - 2y = 19 \quad [1]$$
$$4y + x = -3 \quad [2]$$

THINK

In a new problem, on a Calculator page, complete the entry lines as:
$3x - 2y = 19 \,|\, x = 5$ and $y = -2$
$4y + x = -3 \,|\, x = 5$ and $y = -2$
Press ENTER [enter] after each entry.

WRITE



The point (5, –2) is a solution to the pair of simultaneous equations.

WORKED EXAMPLE 4

Solve the simultaneous equations $y = 2x - 1$ and $3x + 4y = 29$ using the substitution method.

THINK

In a new problem, on a Calculator page, complete the entry lines as:

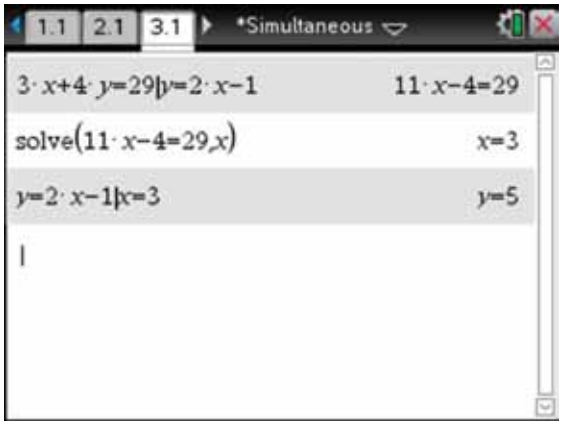
$3x+4y=29 \mid y=2x-1$

solve$(11x-4=29, x)$

$y=2x-1 \mid x=3$

Press ENTER enter after each entry.

WRITE



The point (3, 5) is a solution to the pair of simultaneous equations.

WORKED EXAMPLE 12

Find the point(s) of intersection between $y = x + 5$ and $y = \frac{6}{x}$:

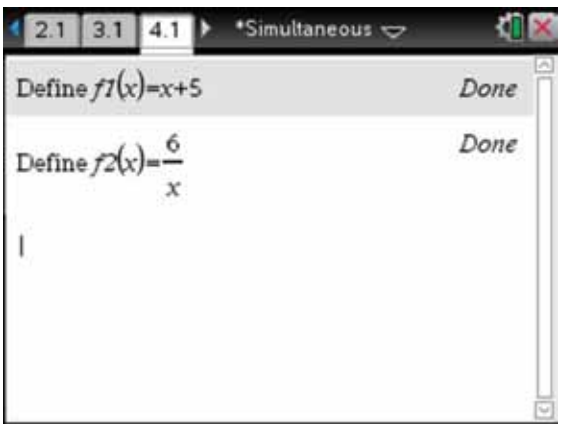
a algebraically
b graphically.

THINK

a 1 In a new problem, on a Calculator page, press:
- MENU menu
- 1: Actions 1
- 1: Define 1.

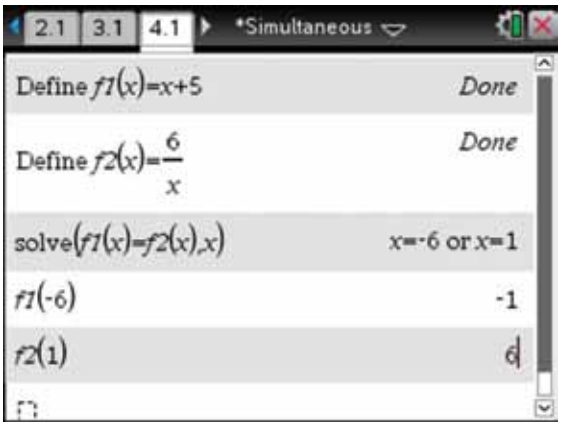
Complete the entry line as:
Define $f1(x) = x + 5$
Repeat for the second function:
Define $f2(x) = \frac{6}{x}$
Press ENTER enter after each entry.

WRITE/DRAW



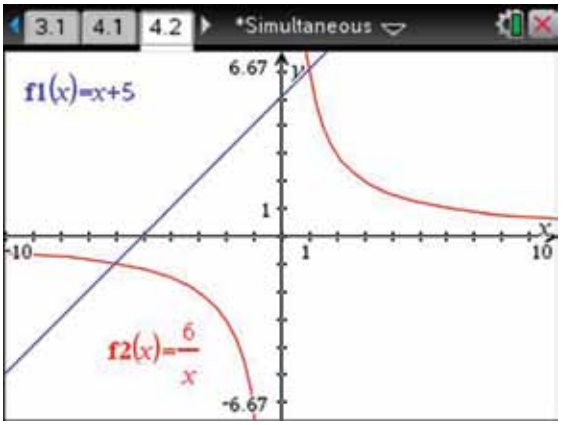
2 To find the intersection points algebraically, press:
- MENU [menu]
- 3: Algebra [3]
- 1: Solve [1].

Complete the entry line as:
solve ($f1(x) = f2(x)$, x)



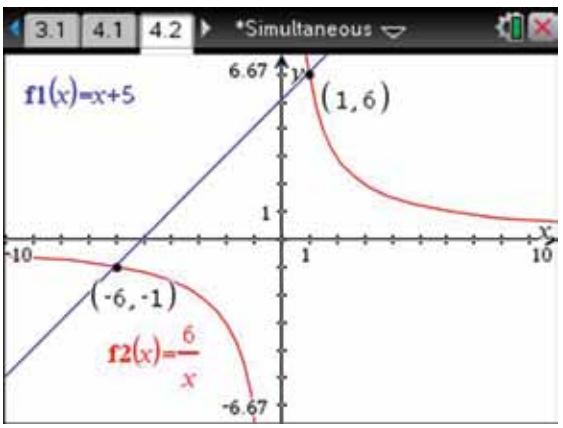
The points (–6, –1) and (1, 6) are the points of intersection.

b **1** On a Calculator page, press the up arrow ▲ to select the function $f2(x)$, then press ENTER [enter]. The graph will be displayed. Now press TAB [tab], select the function $f1(x)$ and press ENTER [enter] to draw the function.



2 To find the points of intersection between the two graphs, press:
- MENU [menu]
- 6: Analyze Graph [6]
- 4: Intersection [4].

Move the cursor to the left of one of the intersection points, press ENTER [enter], then move the cursor to the right of this intersection point and press ENTER [enter]. The intersection point is displayed. Repeat for the other point of intersection.



The points (–6, –1) and (1, 6) are the points of intersection.

WORKED EXAMPLE 14

Solve each of the following linear inequalities.
a $-3m + 5 < -7$ **b** $5(x - 2) \geq 7(x + 3)$

THINK

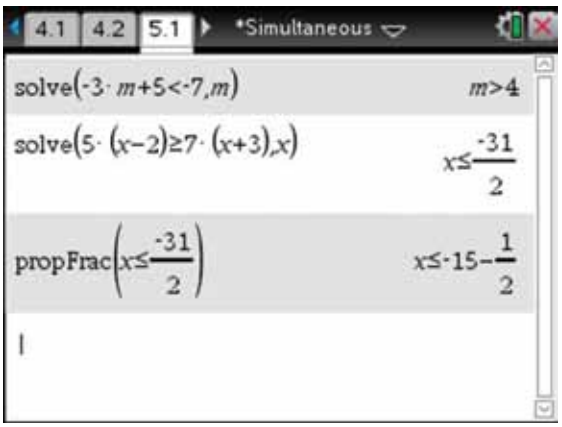
a–b In a new problem, on a Calculator page, complete the entry lines as:
$\text{solve}(-3m+5<-7,m)$
$\text{solve}(5(x-2)\geq 7(x+3),x)$

$\text{propFrac}\left(x \leq \frac{-31}{2}\right)$

Press ENTER [enter] after each entry.

WRITE

a–b



The solution to the first inequation is $m > 4$.
The solution to the second inequation is $x \leq -15\frac{1}{2}$.

WORKED EXAMPLE 15

Sketch a graph of each of the following regions.
a $x \geq -1$ **b** $y < 3$

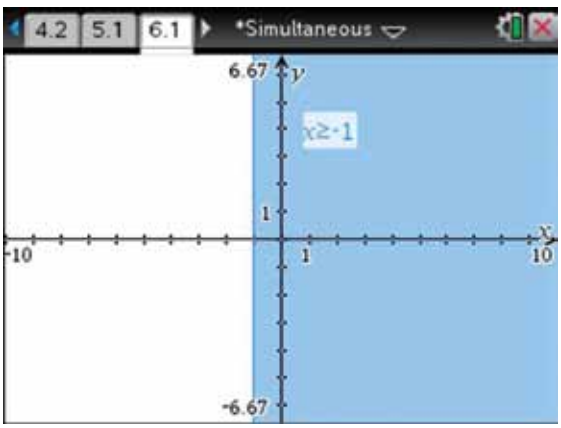
THINK

a In a new problem, on a Graphs page, press:
- MENU [menu]
- 1: Actions [1]
- 7: Text [7].

Click anywhere on the screen. In the text box that appears, type in $x \geq -1$, then press ENTER [enter]. Now press ESC [esc] to get the pointer back, move the pointer until it is over the text box, then press CLICK. When the pointer turns into a closed hand, move it by dragging the text box over either axis. The shaded region defined by the inequality will be displayed.
This method will allow only inequalities such as $x < g(y)$, $x > g(y)$ and $x \leq g(y)$, $x \geq g(y)$ to be graphed.

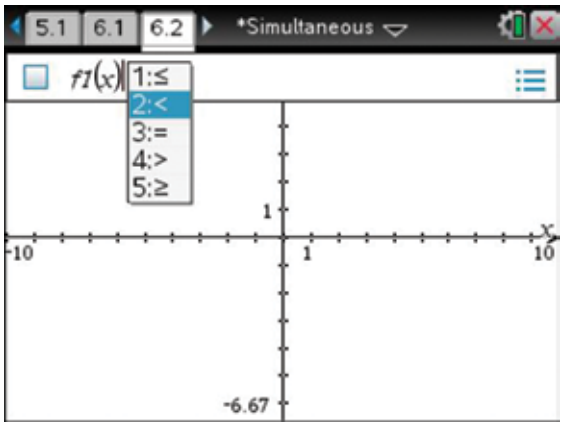
DRAW

a

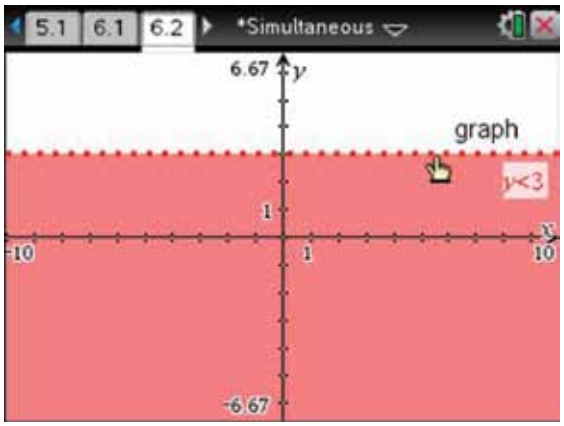


The shaded region corresponding to $x \geq -1$ is displayed.

b **1** On a Graphs page, at the function entry line, press the delete key [del] to delete the equal sign = , choose the option to insert the less than sign, and then type 3.



2 As soon as an inequality symbol is used, you are no longer graphing a function, so your rule appears as $y < 3$. Press ENTER [enter]. The shaded region is displayed. Note that the line $y = 3$ is dotted, indicating that this line is not part of the required region.



The shaded region corresponding to $y < 3$ is displayed.

WORKED EXAMPLE 17

Sketch a graph of the region $2x + 3y < 6$.

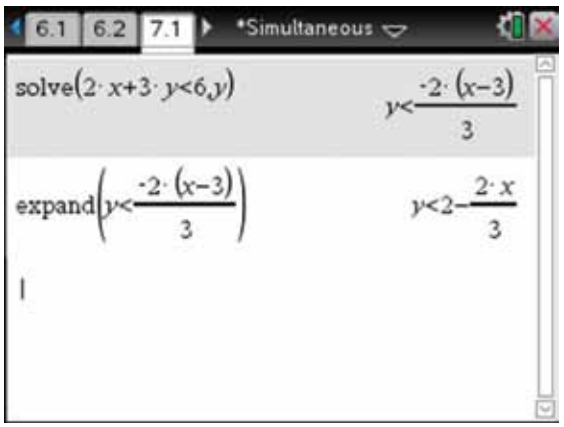
THINK

1 In a new problem, on a Calculator page, complete the entry lines as:

solve$(2x + 3y < 6, y)$

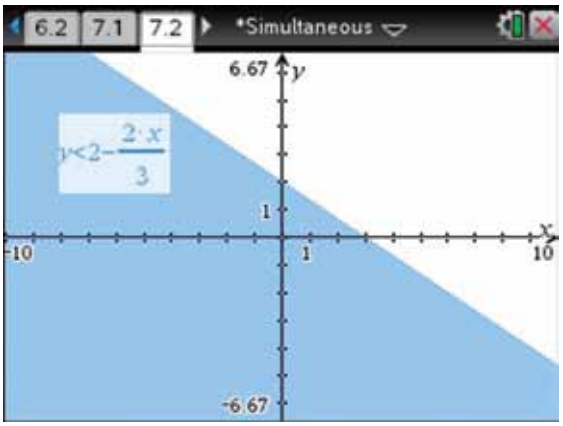
expand$\left(y < \dfrac{-2(x-3)}{3}\right)$

WRITE/DRAW



2 On a Graphs page, at the function entry line, press the delete key [del] to delete the equals sign =, choose the option to insert the less than sign, and complete as shown. As soon as an inequality symbol is used, you are no longer graphing a function, so your rule appears as

$y < 2 - \frac{2x}{3}$.



The shaded region corresponding to $2x + 3y < 6$ is displayed.

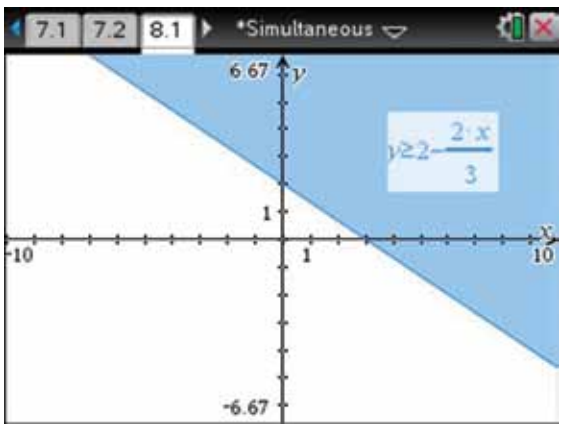
WORKED EXAMPLE 18

Identify the required region in the following pair of linear inequalities.
$2x + 3y \geq 6$, $y < 2x - 3$

THINK

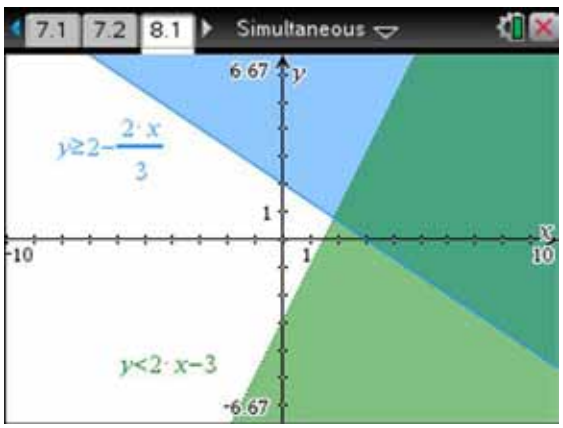
1 In a new problem, on a Graphs page at the function entry line, press the delete key [del] to delete the equals sign =. Complete the entry line as $y \geq 2 - \frac{2x}{3}$. Then press ENTER [enter].

WRITE/DRAW



The graph region corresponding to $2x + 3y \geq 6$ is displayed.

2 Press TAB [tab]. At the function entry line, press the delete key [del] to delete the equals sign =, then complete the entry line as $y < 2x - 3$.
Then press ENTER [enter].
You may need to change the Line Colour and Fill Colour of this inequality to green to see the shaded region in dark green as shown.



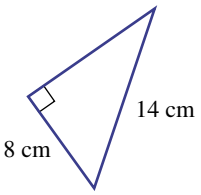
The shaded region indicated is the area corresponding to $2x + 3y \geq 6$ and $y < 2x - 3$.

TOPIC 5

Trigonometry I

WORKED EXAMPLE 2

Calculate the length, correct to 1 decimal place, of the unmarked side of the triangle at right.



THINK

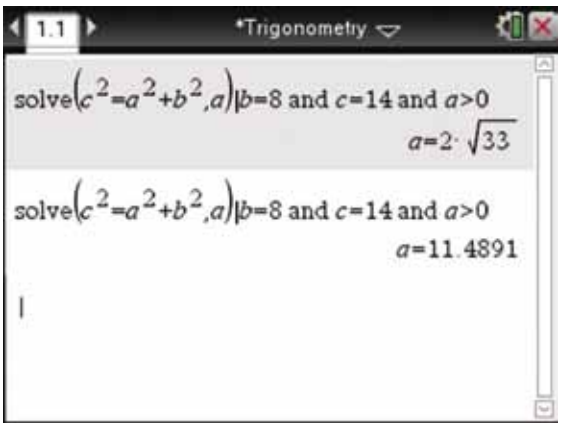
In a new document, on a Calculator page, to solve equations press:
- MENU [menu]
- 3: Algebra [3]
- 1: Solve [1].

Complete the entry line as:
$\text{solve}(c^2 = a^2 + b^2, a) \mid b = 8 \text{ and } c = 14 \text{ and } a > 0$

To substitute values, use the symbol |. Press the ctrl key [ctrl] and then [=] to bring up the palette. Use the Touchpad to select the | symbol, and then type 'and' or find it in the catalog [book]. Complete as shown, and then press ENTER [enter].

Press CTRL [ctrl] ENTER [enter] to get a decimal approximation.

WRITE/DRAW



The length of the unmarked side is $a = 2\sqrt{33} = 11.5$ cm to 1 decimal place.

WORKED EXAMPLE 7

Calculate the value of each of the following, correct to 4 decimal places, using a calculator.

a **cos 65°57′**

b **tan 56°45′30″**

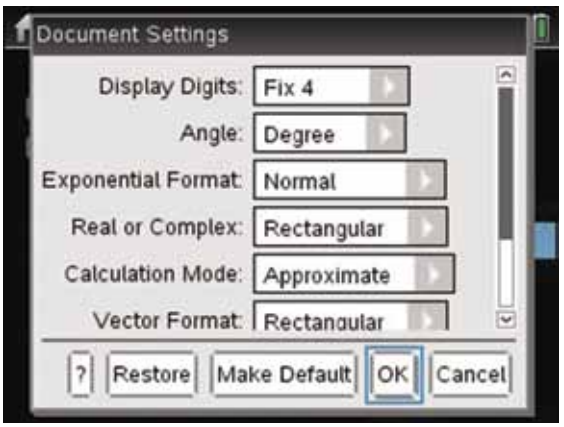
THINK

1 To ensure your calculator is set to degree and approximate mode, press:
- HOME [on]
- 5: Settings [5]
- 2: Document Settings [2].

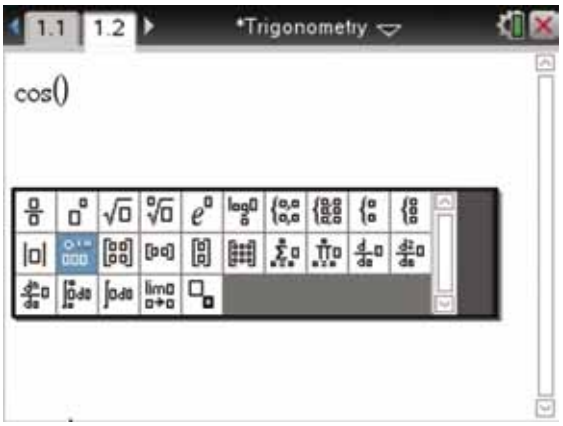
In the Display Digits, select Fix 4. Tab to Angle and select Degree; tab to Calculation Mode and select Approximate.

Tab to OK and press ENTER [enter].

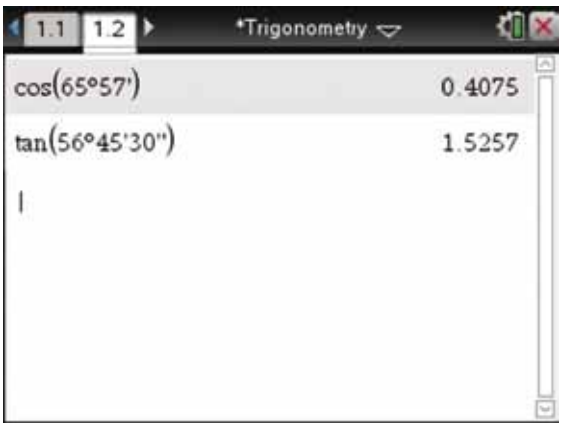
WRITE



2 On a Calculator page, press TRIG [trig] to access and select the appropriate trigonometric ratio. Then press [▫/▫] and choose the template for degrees, minutes and seconds as shown.



3 Complete the entry lines as:
$\cos(65°57')$
$\tan(56°45'30'')$
Press ENTER [enter] after each entry. Since the Calculation Mode is set to Approximate and Fix 4, the answers are shown correct to four decimal places.



$\cos(65°57') = 0.4075$
$\tan(56°45'30'') = 1.5257$

WORKED EXAMPLE 9

Calculate the value of θ:
a correct to the nearest minute, given that $\cos\theta = 0.2547$
b correct to the nearest second, given that $\tan\theta = 2.364$.

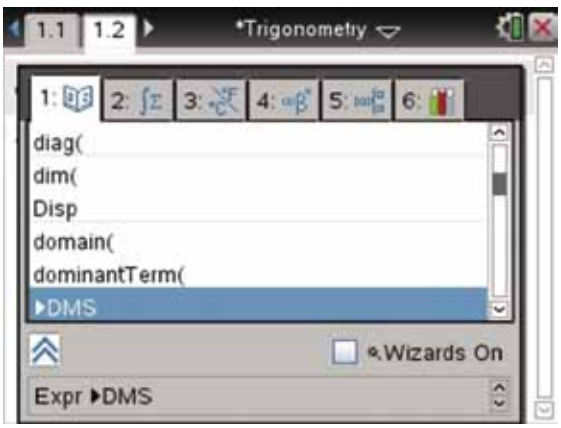
THINK

a On a Calculator page, press TRIG [trig] to access and select the appropriate trigonometric ratio, in this case $\cos^{-1}$.
Complete the entry line as:
$\cos^{-1}(0.2547)$
To convert the decimal degree into degrees, minutes and seconds, press:
- CATALOG [⌨]
- 1: [1]
- d: [D].

Scroll and select ▸DMS.

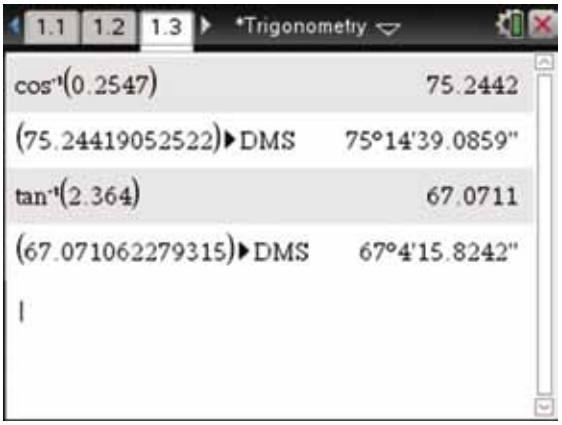
WRITE

a



b Complete the entry line as:
$\tan^{-1}(2.364)$
Convert to degrees, minutes and seconds as above.

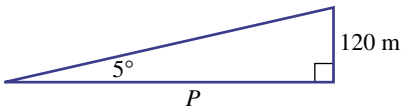
b



$\cos^{-1}(0.2547) = 75°15'$ rounding to the nearest minute.
$\tan^{-1}(2.364) = 64°4'16''$ rounding to the nearest second.

WORKED EXAMPLE 13

Find the value of the pronumeral in the triangle shown. Give the answer correct to 2 decimal places.



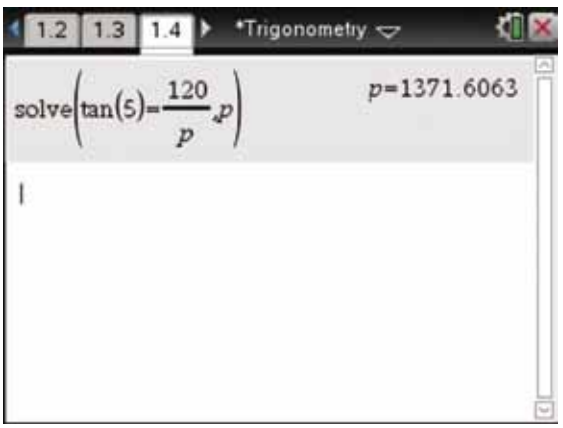
THINK

Ensure your calculator is set to degree mode.
On a Calculator page, complete the entry lines as:

$$\text{solve}\left(\tan(5)=\frac{120}{p}, p\right)$$

Then press ENTER enter.

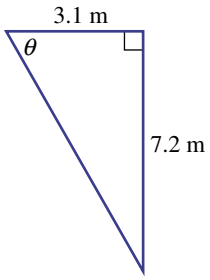
WRITE/DRAW



$P = 1371.61$ m correct to 2 decimal places.

WORKED EXAMPLE 15

Find the size of angle θ:
a correct to the nearest second
b correct to the nearest minute.



THINK

a On a Calculator page, complete the entry line as:

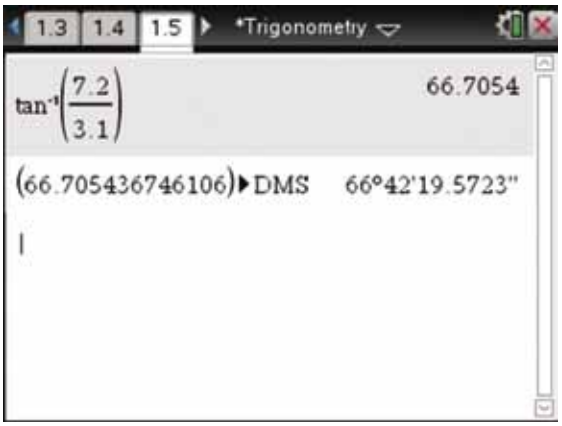
$\tan^{-1}\left(\frac{7.2}{3.1}\right)$

Then convert the decimal degrees to degrees, minutes and seconds as described in Worked Example 9. Then press ENTER enter.

b Using the same screen, round to the nearest minute.

WRITE/DRAW

a



$\theta = 66°42'20''$ correct to the nearest second.

b $\theta = 66°42'$ correct to the nearest minute.

WORKED EXAMPLE 19

A ladder of length 3 m makes an angle of 32° with the wall.
a How far is the foot of the ladder from the wall?
b How far up the wall does the ladder reach?
c What angle does the ladder make with the ground?

THINK

Sketch a diagram and label the sides of the right-angled triangle with respect to the given angle.

On a Calculator page, complete the entry lines as:

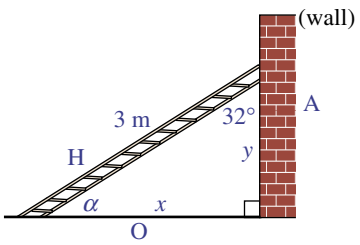
$\text{solve}\left(\sin(32)=\frac{x}{3}, x\right)$

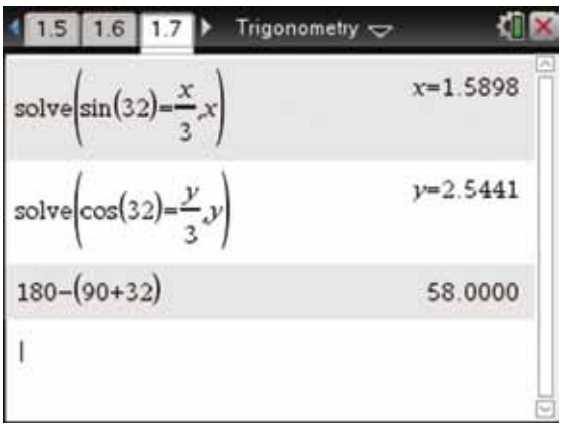
$\text{solve}\left(\cos(32)=\frac{y}{3}, y\right)$

$180-(90+32)$

Press ENTER enter after each entry.

WRITE/DRAW





$x = 1.59$ m correct to 2 decimal places.
$y = 2.54$ m correct to 2 decimal places.
$\alpha = 58°$

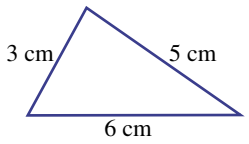
TOPIC 6

Surface area and volume

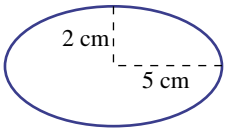
WORKED EXAMPLE 1

Find the areas of the following plane figures, correct to 2 decimal places.

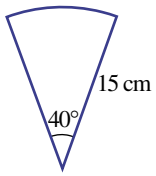
a



b



c



THINK

a In a new document, open a Calculator page. Store the values of a, b and c, and then compute the values of s and A. Complete the entry lines as:

$a := 3$

$b := 5$

$c := 6$

$s := \dfrac{a+b+c}{2}$

$\sqrt{s(s-a)(s-b)(s-c)}$

Press ENTER [enter] after each entry. Remember to include the implied multiplication sign between the expressions.

b–c On a Calculator page, complete the entry lines as:

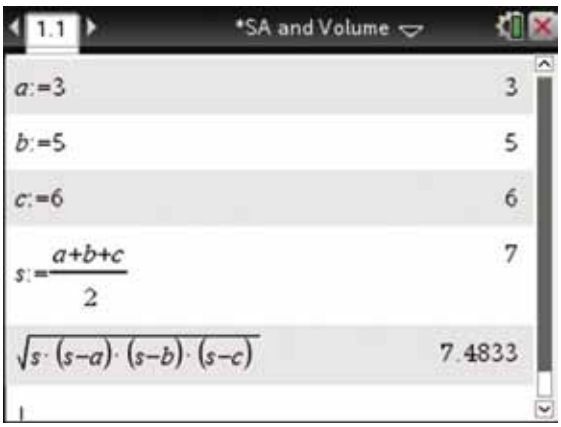
$\pi ab \mid a = 5$ and $b = 2$

$\dfrac{\theta}{360}\ \pi r^2 \mid \theta = 40$ and $r = 15$

Press ENTER [enter] and then CTRL [ctrl] ENTER [enter] after each entry, to get a decimal approximation.

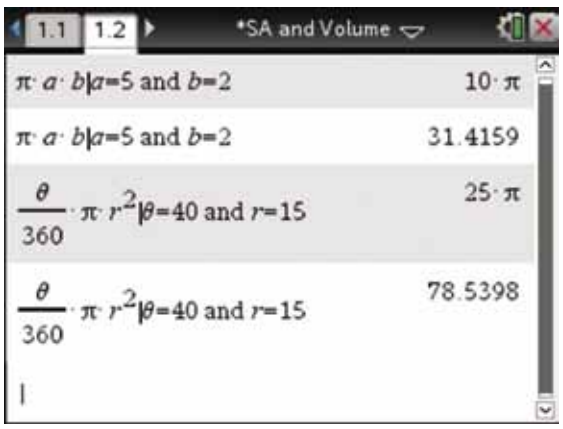
WRITE

a



The area is 7.48 cm^2 correct to 2 decimal places.

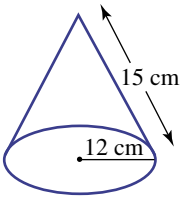
b–c



The area of the ellipse is 31.42 cm^2 correct to 2 decimal places.
The area of the sector is 78.54 cm^2 correct to 2 decimal places.

WORKED EXAMPLE 4

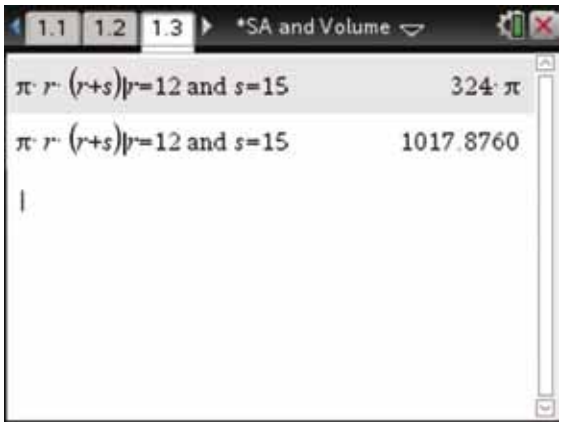
Find the total surface area of the cone shown.



THINK

On the Calculator page, complete the entry line as:
$\pi(r+s) \mid r = 12 \ \ s = 15$
Press CTRL ctrl ENTER enter to get a decimal approximation.

WRITE

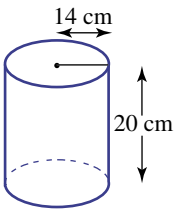


The total surface area of the cone is $1017.9\,\text{cm}^2$ correct to 1 decimal place.

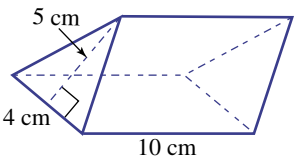
WORKED EXAMPLE 8

Find the volumes of the following shapes.

a



b

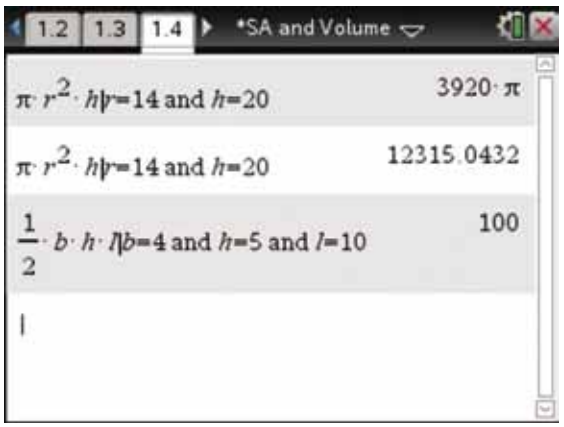


THINK

a–b On a Calculator page, complete the entry lines as:
$\pi r^2 h \mid r = 14$ and $h = 20$
Press CTRL ctrl ENTER enter to get a decimal approximation.
$\frac{1}{2}bh \times l \mid b = 4$ and $h = 5$ and $l = 10$
Press ENTER enter after each entry.

WRITE

a–b

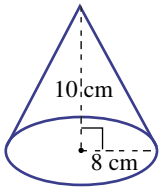


The volume of the cylinder is $12\,315.04\,\text{cm}^3$ correct to 2 decimal places.
The volume of the prism is $100\,\text{cm}^3$.

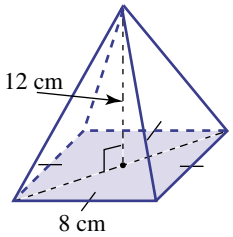
WORKED EXAMPLE 11

Find the volume of each of the following solids.

a



b



THINK

a–b On a Calculator page, complete the entry lines as:

$\frac{1}{3}\pi r^2 h \mid r = 8$ and $h = 10$

Press CTRL ctrl ENTER enter to get a decimal approximation.

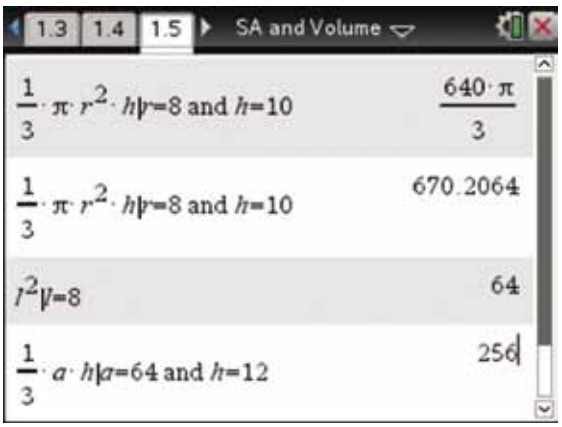
$l^2 \mid l = 8$

$\frac{1}{3}ah \mid a = 64$ and $h = 12$

Press ENTER enter after each entry.

WRITE

a–b



The volume of the cone is 670.21 cm^3 correct to 2 decimal places.
The volume of the pyramid is 256 cm^3.

TOPIC 7

Quadratic expressions

WORKED EXAMPLE 2

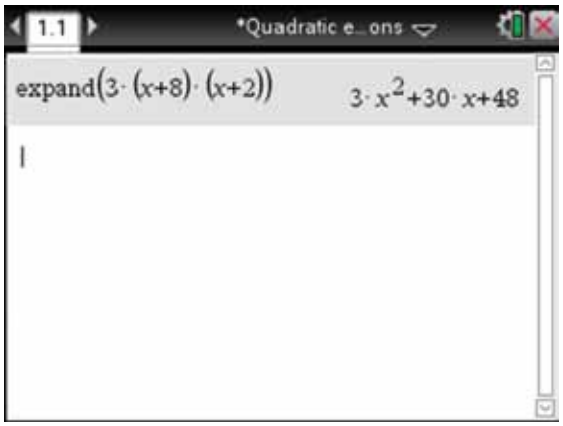
Expand $3(x + 8)(x + 2)$.

THINK

In a new problem, on a Calculator page, press:
- MENU menu
- 3: Algebra 3
- 3: Expand 3.

Complete the entry line as:
expand $(3(x+8)(x+2))$
Then press ENTER enter.

WRITE



$3(x+8)(x+2) = 3x^2 + 30x + 48$

WORKED EXAMPLE 4

Expand and simplify each of the following.

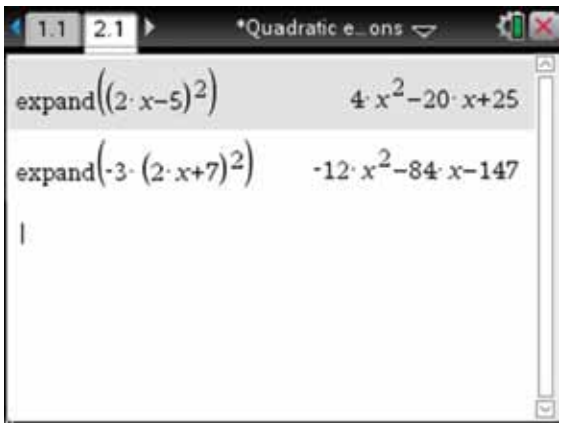
a $(2x - 5)^2$ **b** $-3(2x + 7)^2$

THINK

a–b In a new problem, on a Calculator page, complete the entry lines as:
expand$(2x-5)^2$
expand$\left(-3(2x+7)^2\right)$
Press ENTER enter after each entry.

WRITE

a–b



$(2x-5)^2 = 4x^2 - 20x + 25$
$-3(2x+7)^2 = -12x^2 - 84x - 147$

WORKED EXAMPLE 4

Expand and simplify each of the following.

a $(3x + 1)(3x - 1)$

b $4(2x - 7)(2x + 7)$

THINK

a–b In a new problem, on a Calculator page, complete the entry lines as:

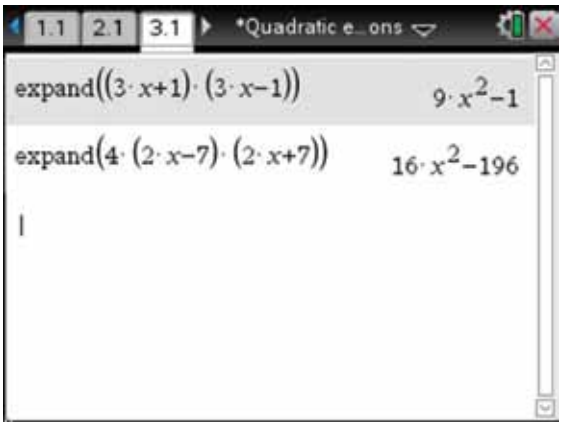
expand$(3x+1)(3x-1)$

expand$(4(2x-7)(2x+7))$

Press ENTER enter after each entry.

WRITE

a–b



$(3x+1)(3x-1) = 9x^2 - 1$

$4(2x-7)(2x+7) = 16x^2 - 196$

WORKED EXAMPLE 6

Factorise $6x^2 - 11x - 10$.

THINK

In a new problem, on a Calculator page, press:

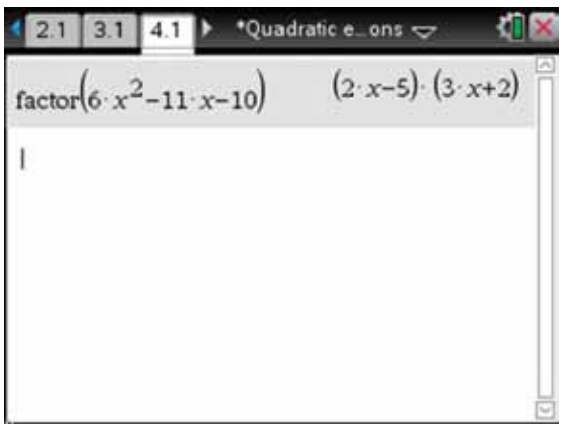
- MENU menu
- 3: Algebra 3
- 2: Factor 2.

Complete the entry line as:

factor$\left(6x^2 - 11x - 10\right)$

Then press ENTER enter.

WRITE



$6x^2 - 11x - 10 = (2x-5)(3x+2)$

WORKED EXAMPLE 7

Factorise the following.

a $12k^2 + 18$

b $16a^2 - 25b^4$

THINK

a–b In a new problem, on a Calculator page, complete the entry lines as:

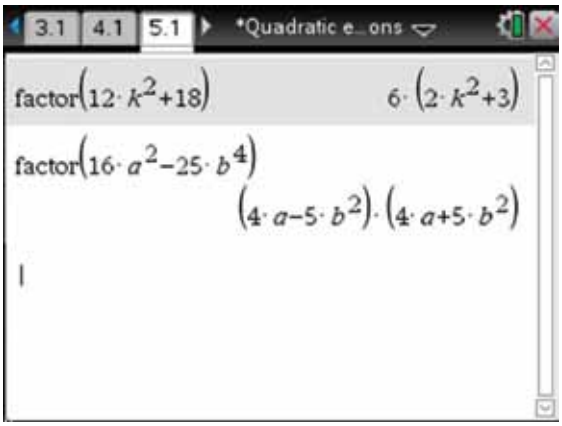
factor$\left(12k^2 + 18\right)$

factor$\left(16a^2 - 25b^4\right)$

Press ENTER enter after each entry.

WRITE

a–b



$12k^2 + 18 = 6\left(2k^2 + 3\right)$

$16a^2 - 25b^4 = \left(4a - 5b^2\right)\left(4a + 5b^2\right)$

WORKED EXAMPLE 9

Factorise the following expression: $x^2 + 12x + 36 - y^2$.

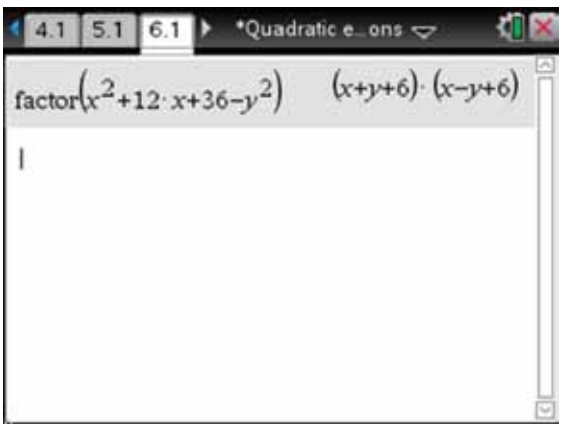
THINK

In a new problem, on a Calculator page, complete the entry lines as:

factor$\left(x^2 + 12x + 36 - y^2\right)$

Press ENTER enter.

WRITE



$x^2 + 12x + 36 - y^2 = \left(x + y + 6\right)\left(x - y + 6\right)$

WORKED EXAMPLE 11

Factorise the following by completing the square.

a $x^2 + 4x + 2$

b $x^2 - 9x + 1$

THINK

a **1** In a new problem, on a Calculator page, to express a quadratic in the completing the square form, press:
- CATALOG
- 1: 1
- C: C

then scroll down, until completeSquare(is highlighted, then press ENTER enter. Using the catalog is one method to show the syntax required for various commands.

2 Complete the entry lines as:

completeSquare$\left(x^2+4x+2, x\right)$

factor$\left(x^2+4x+2, x\right)$

Press ENTER enter after each entry.

b Press:
- MENU menu
- 3: Algebra 3
- 5: Complete the Square 5.

Complete the entry lines as:

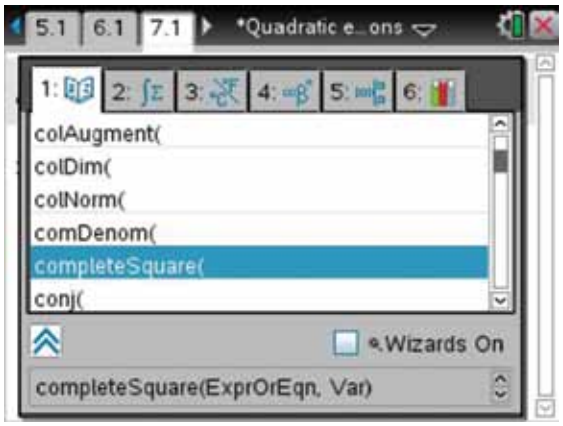
completeSquare$\left(x^2-9x+1, x\right)$

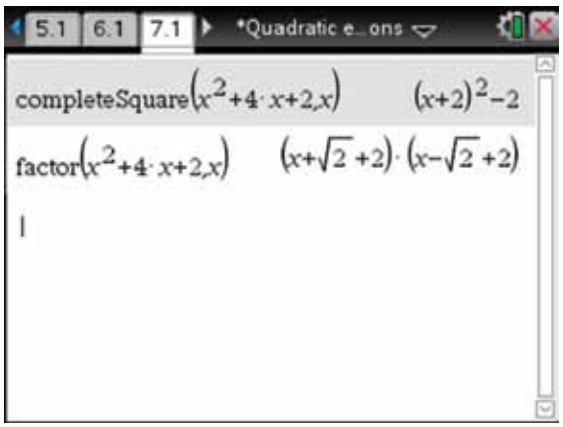
factor$\left(x^2-9x+1, x\right)$

Press ENTER enter after each entry.

WRITE

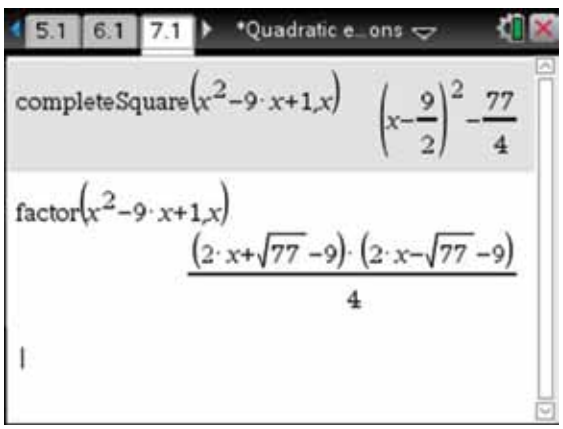
a





$$x^2+4x+2=(x+2)^2-2$$
$$=\left(x+\sqrt{2}+2\right)\left(x-\sqrt{2}+2\right)$$

b



$$x^2-9x+1=\left(x-\frac{9}{2}\right)^2-\frac{77}{4}$$
$$=\frac{\left(2x+\sqrt{77}-9\right)\left(2x-\sqrt{77}-9\right)}{4}$$

TOPIC 8

Quadratic equations

WORKED EXAMPLE 3

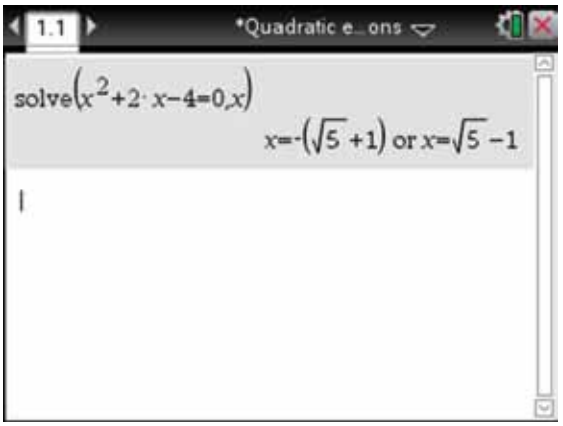
Find the solutions to the equation $x^2 + 2x - 4 = 0$. Give exact answers.

THINK

In a new document, on a Calculator page, press:
- Menu [menu]
- 3: Algebra [3]
- 1: Solve [1].

Complete the entry line as:
solve $(x^2 + 2x - 4 = 0, x)$
Then press ENTER [enter].

WRITE



$x^2 + 2x - 4 = 0$
$\Rightarrow x = -1 + \sqrt{5}$ or $-1 - \sqrt{5}$

WORKED EXAMPLE 6

Use the quadratic formula to solve each of the following equations.

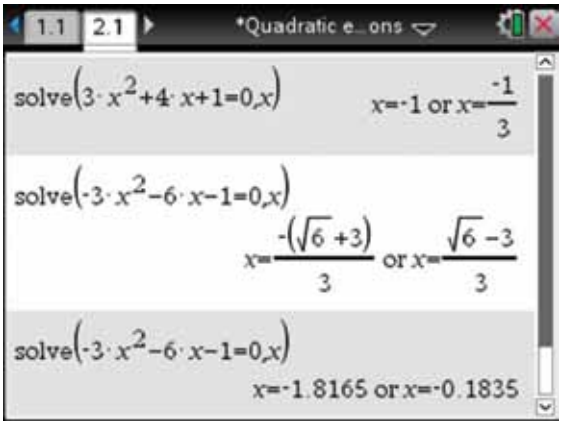
a $3x^2 + 4x + 1 = 0$ **(exact answer)** **b** $-3x^2 - 6x - 1 = 0$ **(round to 2 decimal places)**

THINK

a–b In a new problem, on a Calculator page, complete the entry lines as:
$\text{solve}(3x^2+4x+1=0, x)$
$\text{solve}(-3x^2-6x-1=0, x)$
Then press ENTER [enter] after each entry. Press CTRL [ctrl] ENTER [enter] to get a decimal approximation for **b**.

WRITE

a–b



$3x^2 + 4x + 1 = 0$

$\Rightarrow x = -1 \text{ or } -\frac{1}{3}$

$-3x^2 - 6x - 1 = 0$

$\Rightarrow x = \frac{-(\sqrt{6}+3)}{3} \text{ or } \frac{\sqrt{6}-3}{3}$

$x \approx -1.82 \text{ or } -0.18$ rounding to 2 decimal places.

WORKED EXAMPLE 7

Determine the solution of each of the following quadratic equations by inspecting their corresponding graphs. Give answers to 1 decimal place where appropriate.

a $x^2 + x - 2 = 0$ **b** $2x^2 - 4x - 5 = 0$

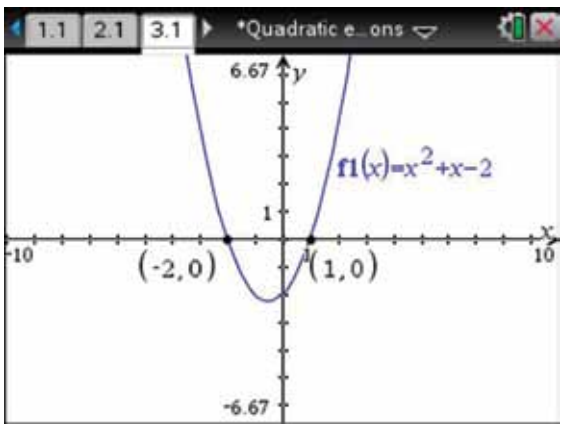
THINK

a In a new problem, on a Graphs page, complete the function entry line as:
$f1(x) = x^2 + x - 2$
Then press ENTER [enter]. The graph will be displayed.
To find the x-intercepts, press:
- MENU [menu]
- 6: Analyze Graph [1]
- 1: Zero [1].

Move the cursor to the left of the zero, press ENTER [enter], then move the cursor to the right of the zero and press ENTER [enter]. The coordinates of the x-intercept are displayed. Press ENTER [enter] to fix the coordinates on the graph. Repeat for the other x-intercept.

WRITE/DRAW

a



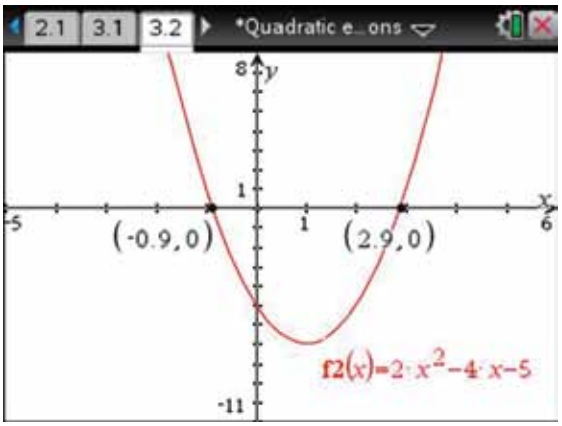
$x^2 + x - 2 = 0$

$\Rightarrow x = 1 \text{ or } -2$

b On a Graphs page, complete the function entry line as:
$f1(x) = 2x^2 - 4x - 5$
Then press ENTER [enter]. The graph will be displayed. To find the x-intercepts, press:
- MENU [menu]
- 6: Analyze Graph [1]
- 1: Zero [1].

Move the cursor to the left of the zero, press ENTER [enter], then move the cursor to the right of the zero and press ENTER [enter]. The coordinates of the x-intercept are displayed. Press ENTER [enter] to fix the coordinates on the graph. Repeat for the other x-intercept.

b



$2x^2 - 4x - 5 = 0$
$\Rightarrow x \approx -0.9$ or 2.9 correct to 1 decimal place.

WORKED EXAMPLE 11

By using the discriminant, determine whether the following equations have:
- **i two rational solutions**
- **ii two irrational solutions**
- **iii one rational solution (two equal solutions)**
- **iv no real solutions.**

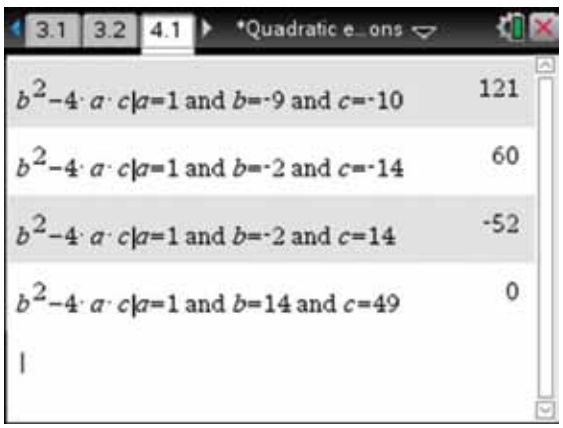
a $x^2 - 9x - 10 = 0$
b $x^2 - 2x - 14 = 0$
c $x^2 - 2x + 14 = 0$
d $x^2 + 14x = -49$

THINK

a–d On a Calculator page, complete the entry lines as:
$b^2 - 4ac \mid a = 1$ and $b = -9$ and $c = -10$
$b^2 - 4ac \mid a = 1$ and $b = -2$ and $c = -14$
$b^2 - 4ac \mid a = 1$ and $b = -2$ and $c = 14$
$b^2 - 4ac \mid a = 1$ and $b = 14$ and $c = 49$
Press ENTER [enter] after each entry.

WRITE

a–d



$x^2 - 9x - 10 = 0$
$\Rightarrow \Delta = 121$. The equation has two rational solutions.
$x^2 - 2x - 14 = 0$
$\Rightarrow \Delta = 60$. The equation has two irrational solutions.
$x^2 - 2x + 14 = 0$
$\Rightarrow \Delta = -52$. The equation has no real solutions.
$x^2 + 14x = -49$
$\Rightarrow \Delta = 0$. The equation has one rational solution.

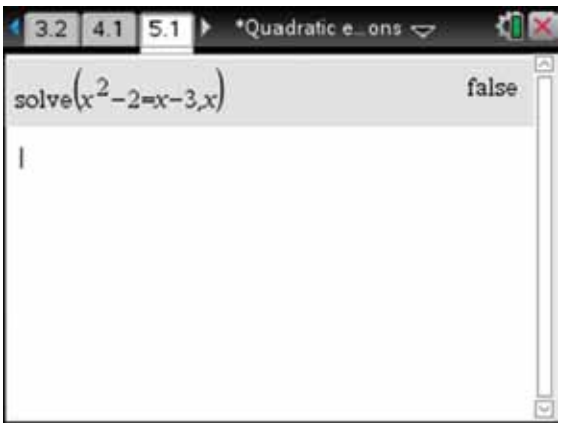
WORKED EXAMPLE 12

Determine whether the parabola $y = x^2 - 2$ and the line $y = x - 3$ intersect.

THINK

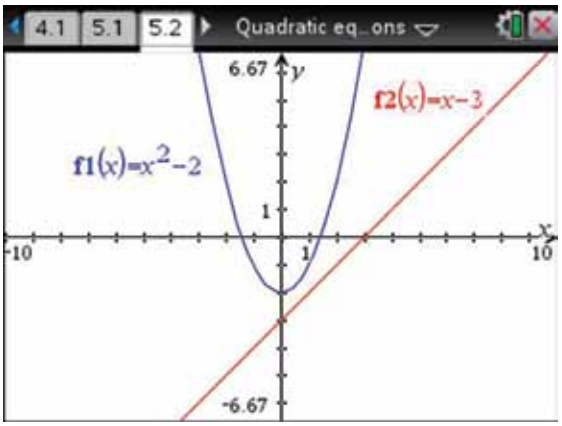
In a new problem, on a Calculator page, complete the entry line as:
solve $(x^2 - 2 = x - 3, x)$
Then press ENTER [enter].

WRITE



There is no point of intersection.

On a Graphs page, to draw the parabola, complete the function entry line as:
$f1(x) = x^2 - 2$
Then press ENTER [enter]. The graph will be displayed.
To draw the straight line, complete the function entry line as:
$f2(x) = x - 3$
Then press ENTER [enter].The graph will be displayed.



The graphs do not intersect.

TOPIC 9

Non-linear relationships

WORKED EXAMPLE 2

Plot the graph of each of the following equations. In each case, use the values of x shown as the values in your table. State the equation of the axis of symmetry, the coordinates of the turning point and the y-intercept for each one.

a $y = x^2 + 2$ **for** $-3 \leq x \leq 3$
b $y = (x + 3)^2$ **for** $-6 \leq x \leq 0$
c $y = -x^2$ **for** $-3 \leq x \leq 3$

THINK

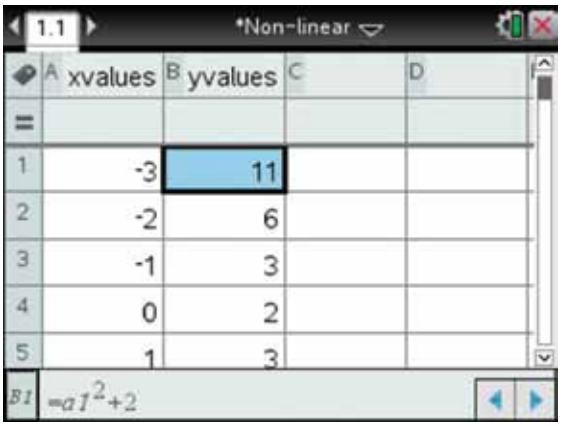
a **1** In a new document, on a Lists & Spreadsheet page, label column A as 'xvalues' and label column B as 'yvalues'. Enter the x-values from -3 to 3 into column A. Then in cell B1, complete the entry line as:
$= a1^2 + 2$
Then press ENTER [enter].
Hold down the SHIFT [⇧shift] key and the down arrow ▼ to fill down the y-values.

2 Open a Data & Statistics page.
Press TAB [tab] to locate the label of the horizontal axis and select the variable 'xvalues'. Press TAB [tab] again to locate the label of the vertical axis and select the variable 'yvalues'.
The points will be plotted. To sketch the graph, press:
- MENU [menu]
- 4: Analyze [4]
- 4: Plot Functions [4].

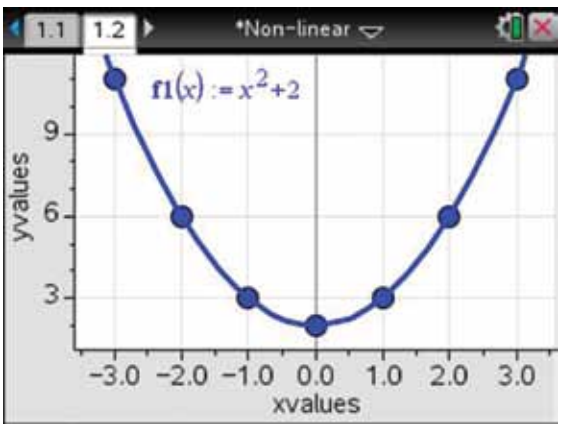
Complete the entry line as
$f1(x) = x^2 + 2$

WRITE

a



The table of values is shown.



The graph is shown. The axis of symmetry is $x = 0$, the turning point is (0, 2) and the y-intercept is 2.

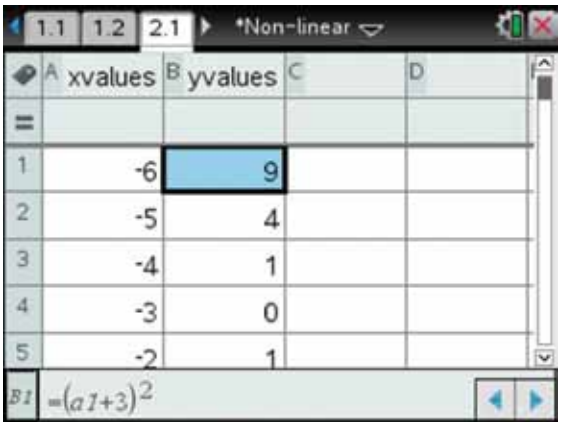
b 1 In a new problem, on a Lists & Spreadsheet page, label column A as 'xvalues' and label column B as 'yvalues'. Enter the x-values from -6 to 0 into column A. Then in cell B1, complete the entry line as:

$= (a1 + 3)^2$

Then press ENTER [enter].

Hold down the SHIFT [shift] key and the down arrow ▼ to fill down the y-values.

b



The table of values is shown.

2 Open a Data & Statistics page.

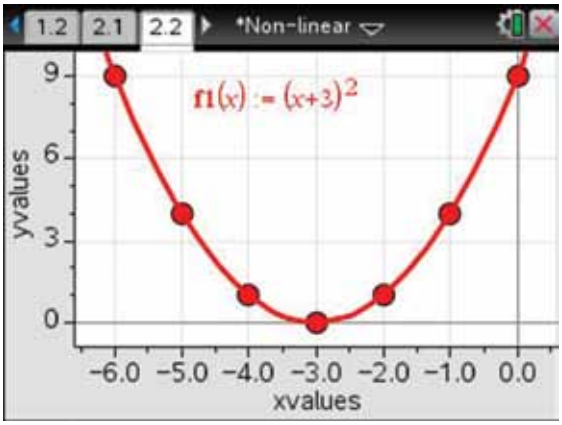
Press TAB [tab] to locate the label of the horizontal axis and select the variable 'xvalues'. Press TAB [tab] again to locate the label of the vertical axis and select the variable 'yvalues'.

The points will be plotted. To sketch the graph, press:

- MENU [menu]
- 4: Analyze [4]
- 4: Plot Functions [4].

Complete the entry line as

$f1(x) = (x + 3)^2$



The graph is shown. The axis of symmetry is $x = -3$, the turning point is $(-3, 0)$ and the y-intercept is 9.

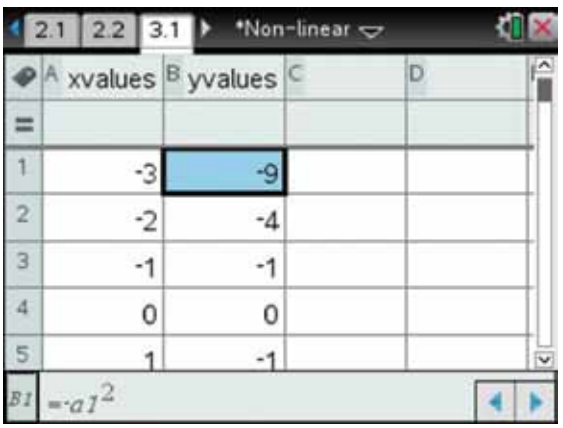
c 1 In a new problem, on a Lists & Spreadsheet page, label column A as 'xvalues' and label column B as 'yvalues'. Enter the x-values from -3 to 3 into column A. Then in cell B1, complete the entry line as:

$= -a1^2$

Then press ENTER [enter].

Hold down the SHIFT [shift] key and the down arrow ▼ to fill down the y-values.

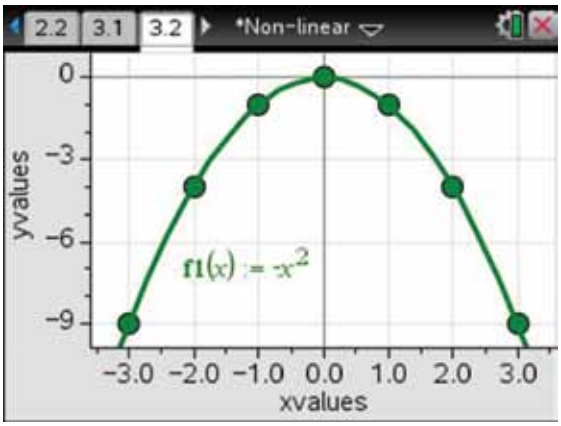
c



The table of values is shown.

2 Open a Data & Statistics page.
Press TAB [tab] to locate the label of the horizontal axis and select the variable 'xvalues'. Press TAB [tab] again to locate the label of the vertical axis and select the variable 'yvalues'.
The points will be plotted. To sketch the graph, press:
- MENU [menu]
- 4: Analyze [4]
- 4: Plot Functions [4].

Complete the entry line as
$f1(x) = -x^2$



The graph is shown. The axis of symmetry is $x = 0$, the turning point is (0, 0) and the y-intercept is 0.

WORKED EXAMPLE 3

State whether each of the following graphs is wider or narrower than the graph of $y = x^2$ and state the coordinates of the turning point of each one.

a $y = \frac{1}{5}x^2$ **b** $y = 4x^2$

THINK

a–b In a new problem, on a Graphs page, complete the function entry lines as:
$f1(x) = x^2$

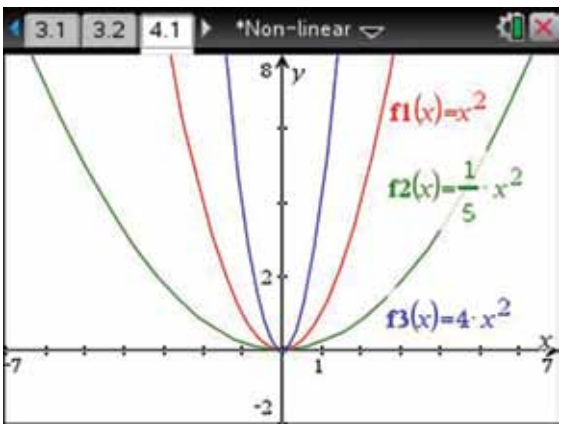
$$f2(x) = \frac{1}{5}x^2$$

$f3(x) = 4x^2$

Press ENTER [enter] after each entry.
Note that the viewing window has been changed to display the features of the graphs.

WRITE

a–b



The graph of $f2(x) = \frac{1}{5}x^2$ is wider than the graph of $f1(x) = x^2$, and the graph of $f3(x) = 4x^2$ is narrower than the graph of $f1(x) = x^2$. The turning point of all graphs is at the origin (0, 0).

WORKED EXAMPLE 6

For each of the following graphs, give the coordinates of the turning point and state whether it is a maximum or a minimum.

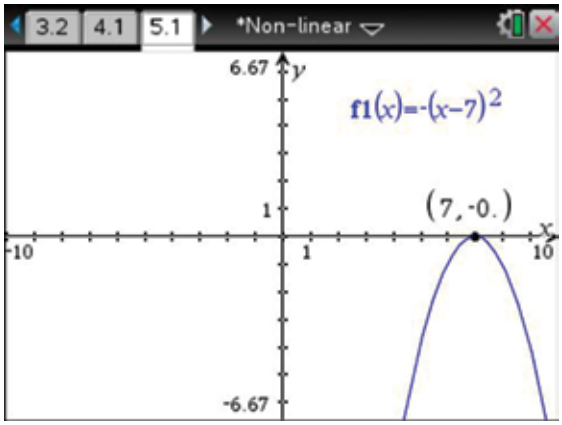
a $y = -(x - 7)^2$ **b** $y = 5 - x^2$

THINK

a In a new problem, on a Graphs page, complete the function entry line as:
$f1(x) = -(x - 7)^2$
Then press ENTER enter.

WRITE

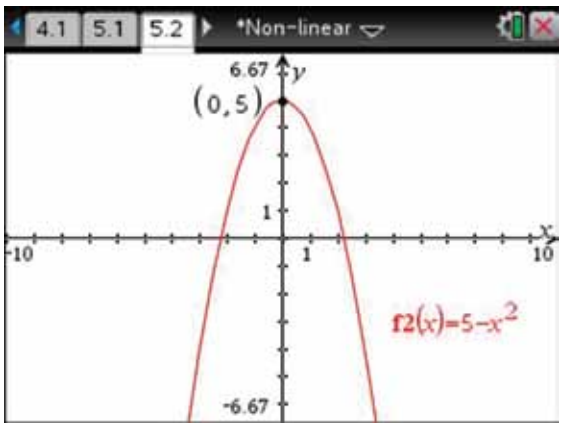
a



The turning point (7, 0) is a maximum.

b In a new problem, on a new Graphs page, complete the function entry line as:
$f1(x) = 5 - x^2$
Then press ENTER enter.

b



The turning point (0, 5) is a maximum.

WORKED EXAMPLE 9

Determine i the y-intercept and ii the x-intercepts (where they exist) for the parabolas with equations:

a $y = (x + 3)^2 - 4$ **b** $y = 2(x - 1)^2$ **c** $y = -(x + 2)^2 - 1$.

THINK

a In a new problem, on a Calculator page, press:
- MENU menu
- 3: Algebra 3
- 1: Solve 1.

Complete the entry lines as:

$\text{solve}\left(y = (x+3)^2 - 4,\ y\right) \mid x = 0$

$\text{solve}\left(y = (x+3)^2 - 4, x\right) \mid y = 0$

Press ENTER enter after each entry.

b On a new Calculator page, press:
- MENU menu
- 3: Algebra 3
- 1: Solve 1.

Complete the entry lines as:

$\text{solve}\left(y = 2(x-1)^2,\ y\right) \mid x = 0$

$\text{solve}\left(y = 2(x-1)^2,\ x\right) \mid y = 0$

Press ENTER enter after each entry.

c On a new Calculator page, press:
- MENU menu
- 3: Algebra 3
- 1: Solve 1.

Complete the entry lines as:

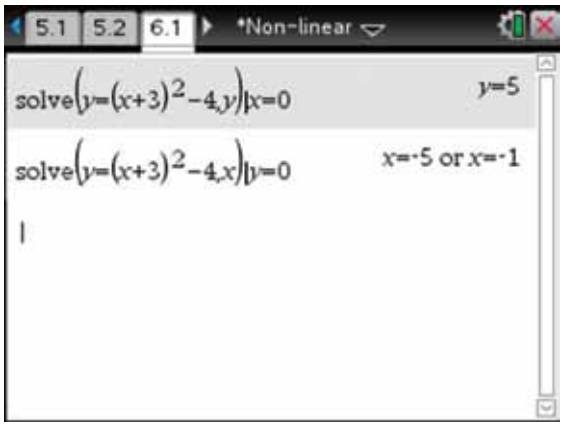
$\text{solve}\left(y = -(x+2)^2 - 1,\ y\right) \mid x = 0$

$\text{solve}\left(y = -(x+2)^2 - 1, x\right) \mid y = 0$

Press ENTER enter after each entry.

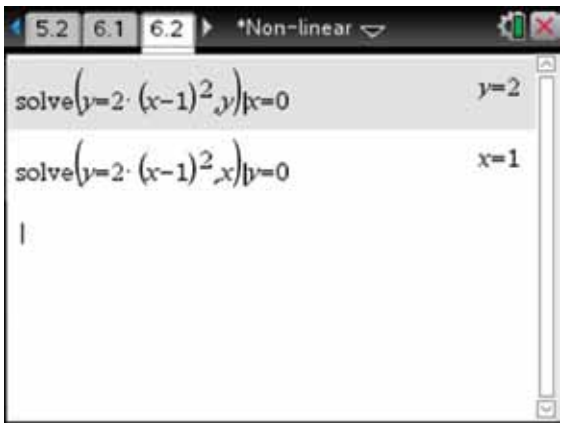
WRITE

a



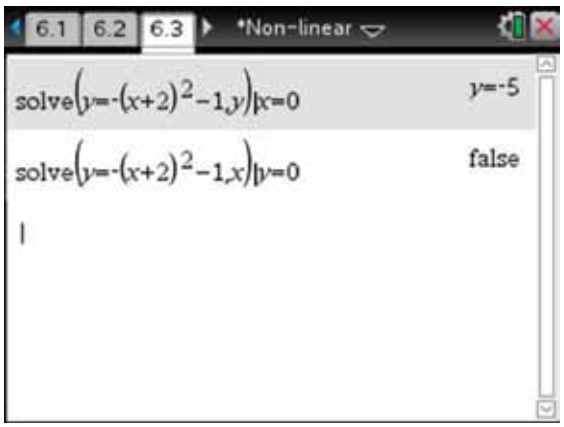
The y-intercept is 5, and the x-intercepts are $x = -5$ and $x = -1$.

b



The y-intercept is 2, and the x-intercept is $x = 1$.

c



The y-intercept is –5, and there are no x-intercepts.

WORKED EXAMPLE 12

Sketch the graph of $y = 2x^2 - 6x - 6$.

THINK

1 In a new problem, on a Calculator page, complete the entry lines as:
solve $(y = 2x^2 - 6x - 6, y) \mid x = 0$
solve $(y = 2x^2 - 6x - 6, x) \mid y = 0$
Press ENTER enter, after each entry. Press CTRL / ENTER ctrl enter to get a decimal approximation.

2 Press:
- MENU menu
- 1: Actions 1
- 1: Define 1.

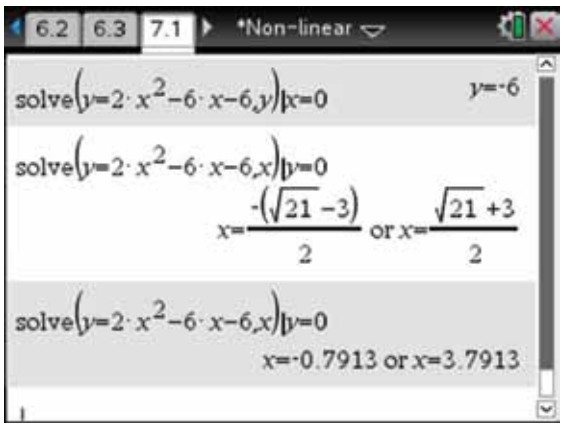
Complete the entry line as:
Define $f1(x) = 2x^2 - 6x - 6$
- MENU menu
- 3: Algebra 3
- 5: Complete the Square 5.

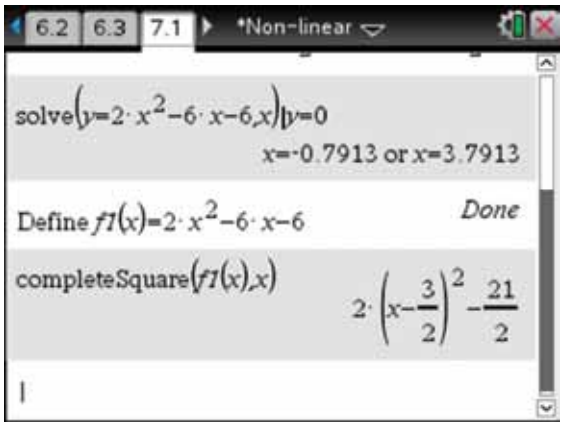
Complete the entry line as:
completeSquare $(f1(x), x)$
Press ENTER enter after each entry.

3 On a new Graphs page, press the up arrow ▲ to select the function entry line as:
$f1(x) = 2x^2 - 6x - 6$
Then press ENTER enter.
Note that the viewing window has been changed to display the features of the graphs.
To find the y-intercept, press:
- MENU menu
- 5: Trace 5
- 1: Graph Trace 1.

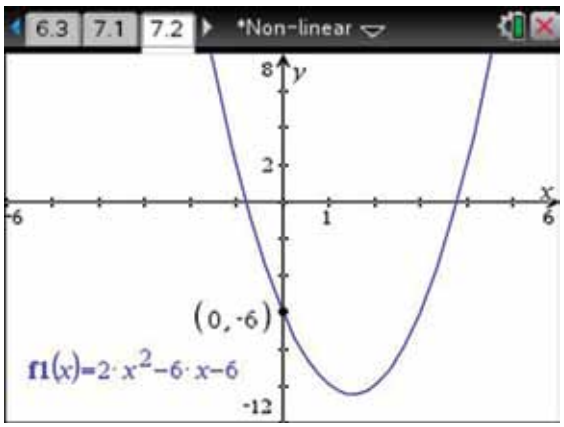
The y-intercept will be displayed. Press ESCAPE esc.

WRITE/DRAW





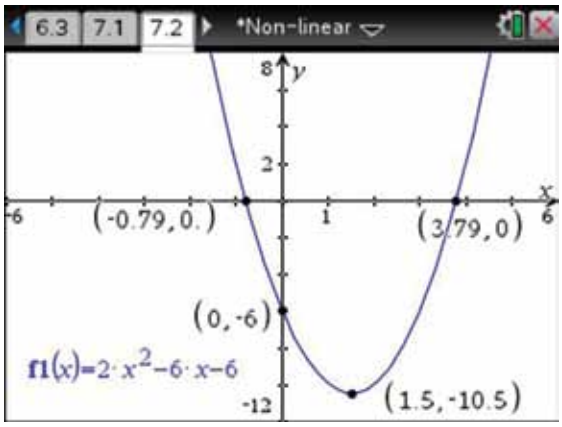
The y-intercept is -6, and the x-intercepts are $x = \frac{3+\sqrt{21}}{2}$ or $x = \frac{3-\sqrt{21}}{2}$, and the turning point is a minimum at $\left(\frac{3}{2}, -10\frac{1}{2}\right)$.



4 To find the x-intercepts, press:
- MENU menu
- 6: Analyze Graph 6
- 1: Zero 1.

Move the cursor to the left of the zero, press ENTER enter, then move the cursor to the right of the zero and press ENTER enter. The coordinates of the x-intercept are displayed. Press ENTER enter to fix the coordinates on the graph. Repeat for the other x-intercept.
To find the minimum turning point, press:
- MENU menu
- 6: Analyze Graph 6
- 2: Minimum 2.

Move the cursor to the left of the turning point, press ENTER enter, then move the cursor to the right of the turning point and press ENTER enter.



The graph is shown, along with the critical points.

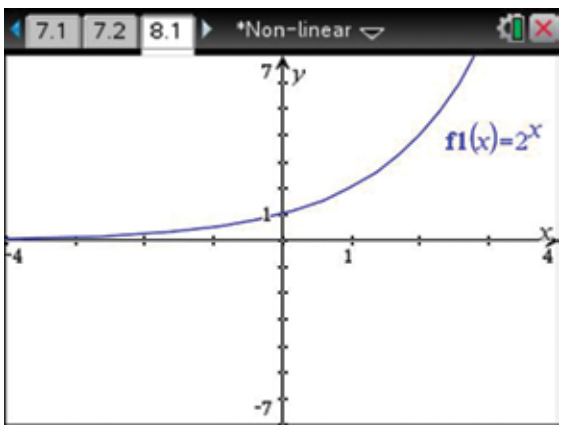
WORKED EXAMPLE 14

By considering transformations to the graph of $y = 2^x$, sketch the graph of $y = -2^x + 1$.

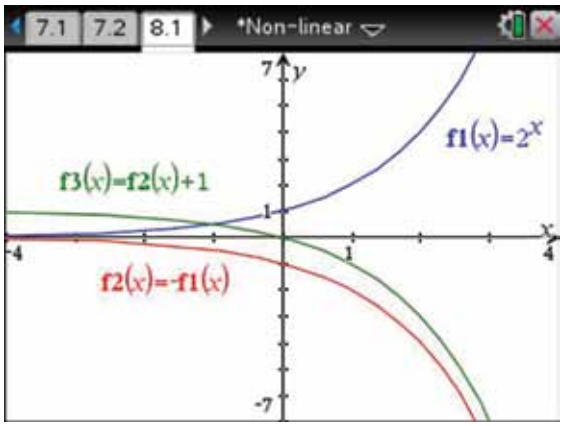
THINK

1 In a new problem, on a Graphs page, complete the function entry line as:
$f1(x) = 2^x$
Then press ENTER enter.

DRAW



2 Press TAB [tab] and complete the function entry line as:
$f2(x) = -f1(x)$
Then press ENTER [enter].
Press TAB [tab] and complete the function entry line as:
$f3(x) = f2(x) + 1$
Then press ENTER [enter].



The graph of $y = -2^x$ is the reflection of the graph of $y = 2^x$ in the x-axis.
The graph of $y = -2^x + 1$ is the graph of $y = -2^x$ translated upwards by 1 unit.
The graph of $y = -2^x + 1$ passes through the origin and has a horizontal asymptote at $y = 1$.

WORKED EXAMPLE 17

Plot the graph of $y = \frac{-3}{x}$ for $-3 \leq x \leq 3$.

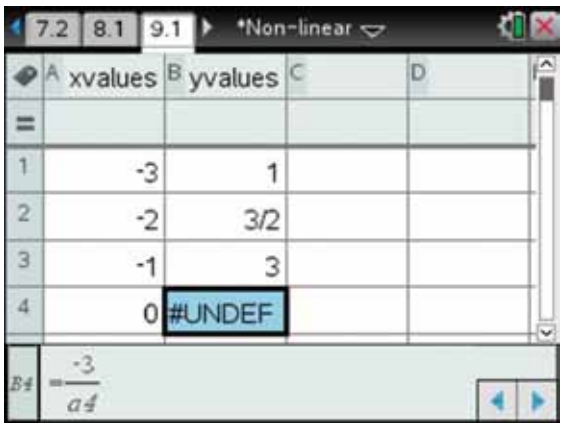
THINK

1 In a new problem, on a Lists & Spreadsheet page, label column A as 'xvalues' and label column B as 'yvalues'. Enter the x-values from –3 to 3 into column A. Then in cell B1, complete the entry line as:

$= \frac{-3}{a1}$

Then press ENTER [enter].
Hold down the SHIFT [shift] key and the down arrow ▼ to fill down the y-values.

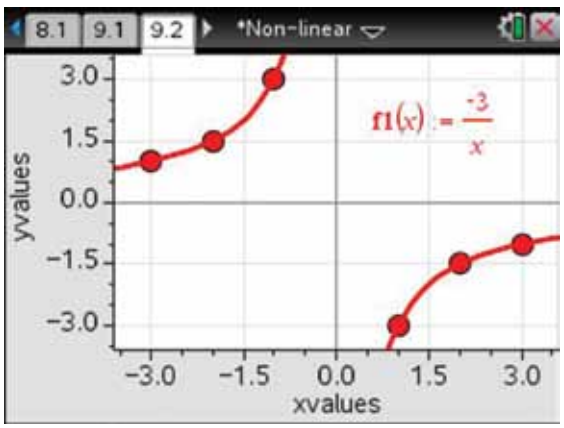
WRITE/DRAW



The table of values is shown.

2 Open a Data & Statistics page.
Press TAB [tab] to locate the label of the horizontal axis and select the variable 'xvalues'. Press TAB [tab] again to locate the label of the vertical axis and select the variable 'yvalues'. The points will be plotted. To sketch the graph, press:
- MENU [menu]
- 4: Analyze [4]
- 4: Plot Functions [4].

Complete the entry line as
$f1(x) = \frac{-3}{x}$



The graph is shown.

WORKED EXAMPLE 20

Sketch the graph of the circle $x^2 + 2x + y^2 - 6y + 6 = 0$.

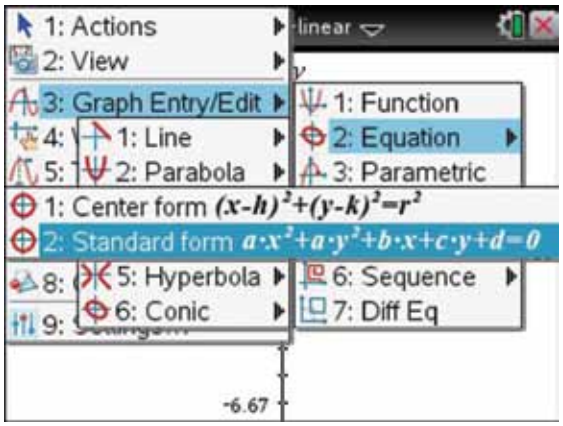
THINK

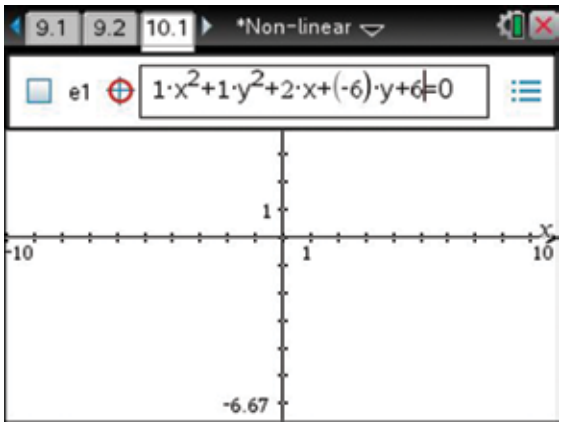
1. In a new problem, on a Graphs page, press:
   - MENU [menu]
   - 3: Graph Entry/Edit [3]
   - 2: Equation [2]
   - 3: Circle [3]
   - 2: Standard form [2].

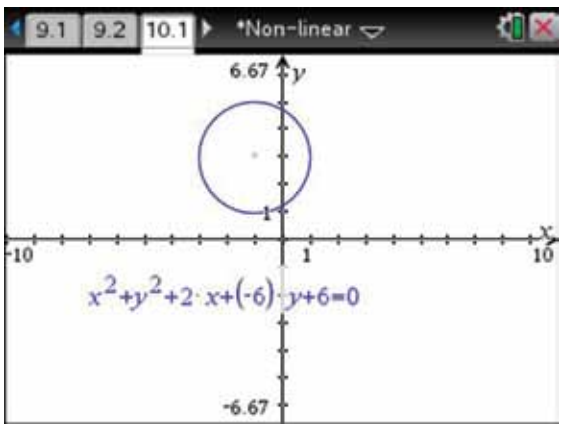
2. Complete the function entry line as shown, press TAB [tab] to move between the fields, and press ENTER [enter].

3. The graph of the circle is shown.

WRITE/DRAW



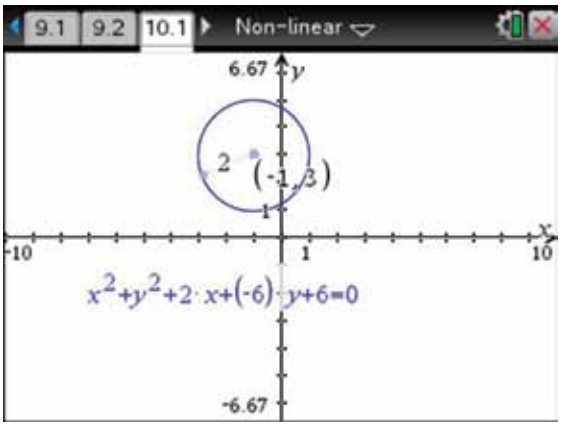




4 To find the centre of the circle, press:
- MENU menu
- 6: Analyze Graph 6
- 8: Analyze Conics 8
- 1: Centre 1.

Press ENTER enter and click on the circle.
The coordinates of the centre are displayed.
To find the radius of the circle, press:
- MENU menu
- 6: Analyze Graph 6
- 8: Analyze Conics 8
- 7: Radius 7.

Press ENTER enter and click on the circle.
The radius of the circle is displayed.



The circle $x^2 + 2x + y^2 - 6y + 6 = 0$ has its centre at $(-1, 3)$ and has a radius of 2.

TOPIC 12

Univariate data

WORKED EXAMPLE 1

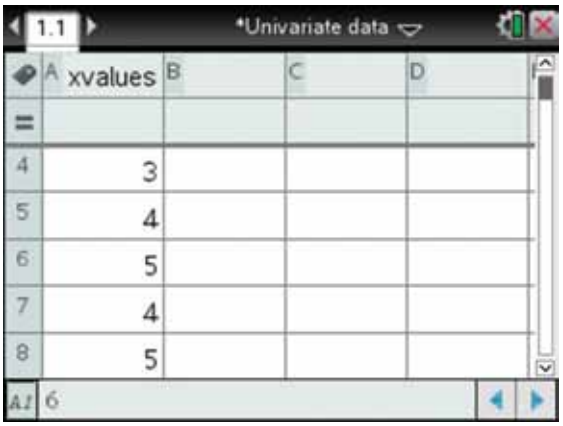
For the data set 6, 2, 4, 3, 4, 5, 4, 5, find the:

a mean
b median
c mode.

THINK

1 In a new document, on a Lists & Spreadsheet page, label column A as 'xvalues', and enter the values in the data set. Press ENTER [enter] after entering each value.

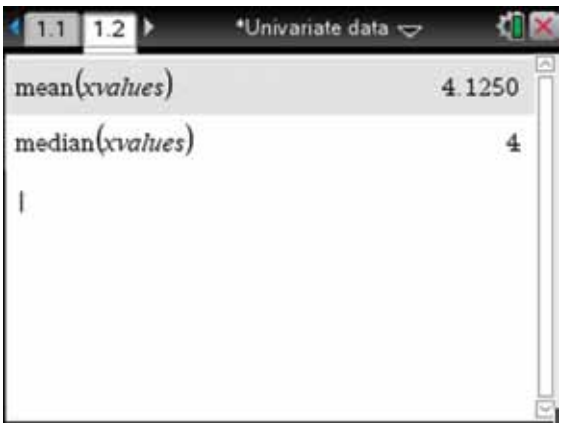
WRITE



2 Although you can find many summary statistics, to find the mean only, open a Calculator page and press:
- MENU [menu]
- 6: Statistics [6]
- 3: List Math [3]
- 3: Mean [3].

Press VAR [var] and select 'xvalues', then press ENTER [enter].
To find the median only, press:
- MENU [menu]
- 6: Statistics [6]
- 3: List Math [3]
- 4: Median [4].

Press VAR [var] and select 'xvalues', then press ENTER [enter].



The mean is 4.125 and the median is 4.
The mode is 4.

WORKED EXAMPLE 2

For the table at right, find the:

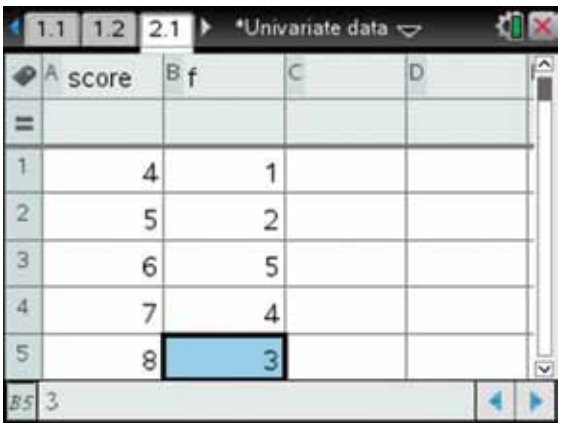
a mean
b median
c mode.

Score (x)	Frequency (f)
4	1
5	2
6	5
7	4
8	3
Total	15

THINK

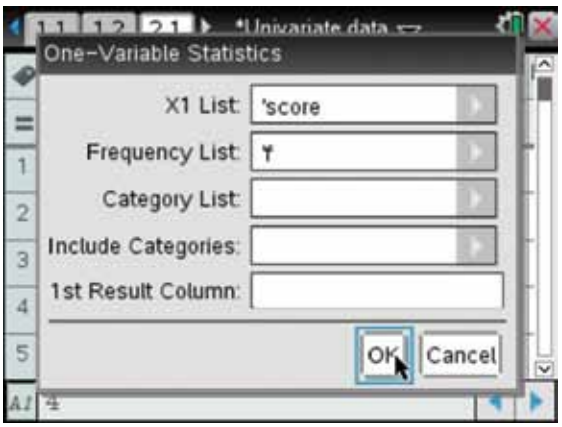
1 In a new problem, on a Lists & Spreadsheet page, label column A as 'score' and column B as 'f'. Enter the values as shown in the table and press ENTER [enter] after entering each value.

WRITE

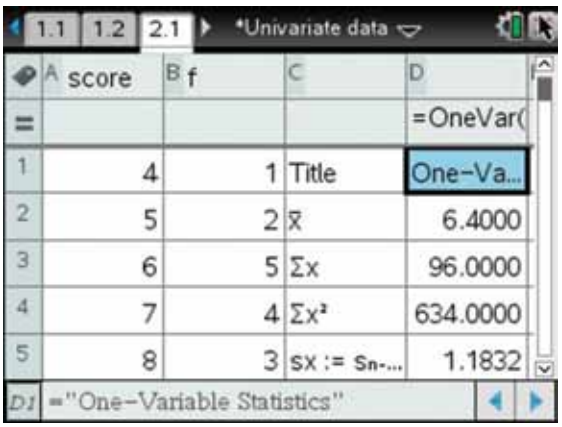


2 To find the summary statistics, open a Calculator page and press:
- MENU [menu]
- 6: Statistics [6]
- 1: Stat Calculations [1]
- 1: One-Variable Statistics [1].

Select 1 as the number of lists. Then on the One-Variable Statistics page (shown opposite) select 'score' as the X1 List and 'f' as the Frequency List. Leave the next fields empty, TAB [tab] to OK and press ENTER [enter].



3 The results are displayed.
The mean $\overline{x} = 6.4$ and the median is 6.
The mode is the data set with the highest frequency value, which in this case is 6.



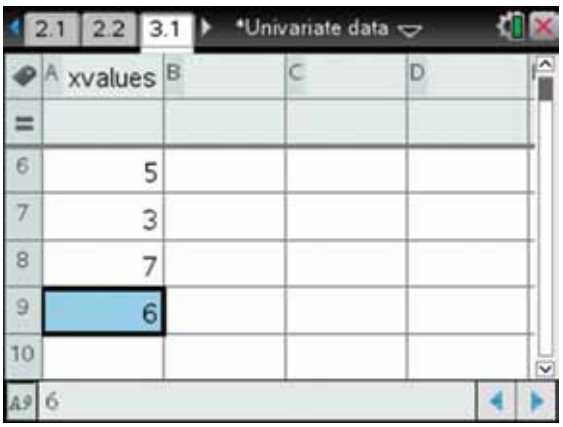
WORKED EXAMPLE 5

Calculate the interquartile range (IQR) of the following set of data: 3, 2, 8, 6, 1, 5, 3, 7, 6.

THINK

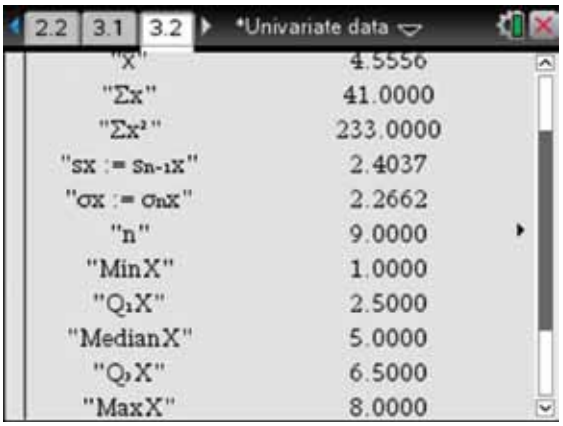
1 In a new problem, on a Lists & Spreadsheet page, label column A as 'xvalues'. Enter the values from the data set. Press ENTER [enter] after entering each value.

WRITE



2 To find the summary statistics, open a Calculator page and press:
- MENU [menu]
- 6: Statistics [6]
- 1: Stat Calculations [1]
- 1: One-Variable Statistics [1].

Select 1 as the number of lists. Then on the One-Variable Statistics page select 'xvalues' as the X1 List and leave the Frequency as 1. Leave the remaining fields empty, TAB [tab] to OK, and then press ENTER [enter].
The summary statistics are shown.



The IQR = $Q_3 - Q_1 = 6.5 - 2.5 = 4$

WORKED EXAMPLE 8

The following stem-and-leaf plot gives the speed of 25 cars caught by a roadside speed camera.

Key: 8 | 2 = 82 km/h, 8* | 6 = 86 km/h

Stem	*Leaf*
8	2 2 4 4 4 4
8*	5 5 6 6 7 9 9 9
9	0 1 1 2 4
9*	5 6 9
10	0 2
10*	
11	4

a Prepare a five-number summary of the data.
b Draw a box-and-whisker plot of the data. (Identify any extreme values.)
c Describe the distribution of the data.

THINK

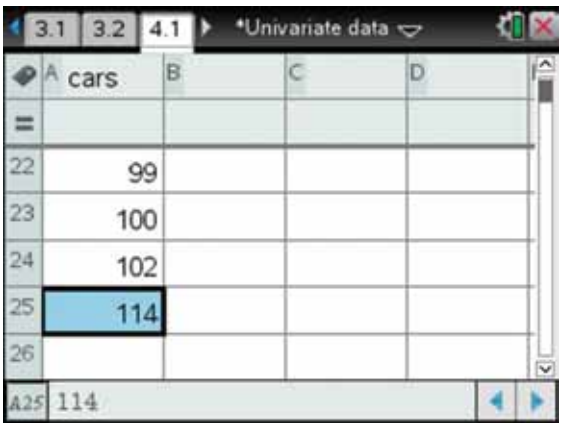
a **1** In a new problem, on a Lists & Spreadsheet page, label column A as 'cars' and enter the data set as:
82, 82, 84, 84, 84, 84, 85, 85, 86, 86, 87, 89, 89, 89, 90, 91, 91, 92, 94, 95, 96, 99, 100, 102, 114.
Press ENTER [enter] after each value.

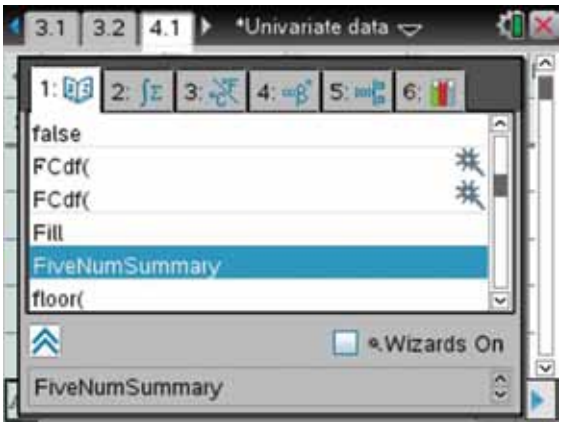
2 To find a five-point summary of the data, on a Calculator page press:
- CATALOG [doc]
- 1: [1]
- F: [F].

Then use the down arrow ▼ to scroll down to FiveNumSummary.

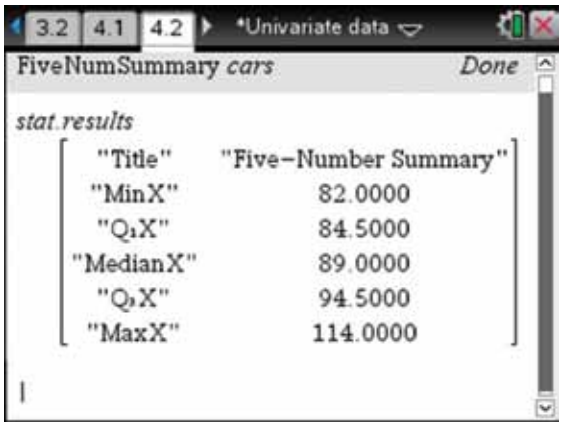
WRITE

a





3 Press VAR [var] and select 'cars'.
Complete the entry line as:
FiveNumSummary *cars*
Then press VAR [var] and select 'stat.results' and press ENTER [enter].



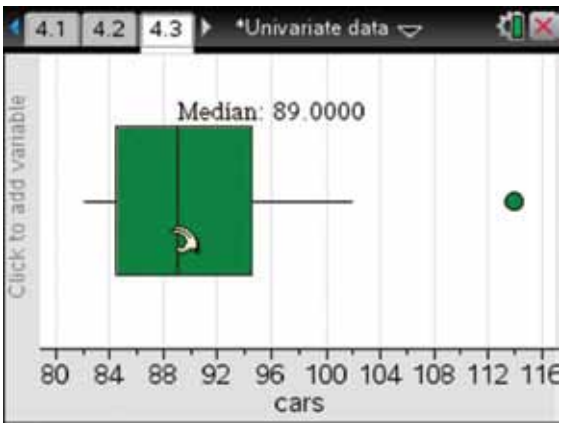
The five-point summary statistics are shown.

b To construct the box-and-whisker plot, open a Data & Statistics page.
Press TAB [tab] to locate the label of the horizontal axis and select the variable 'cars'. Then press:
- MENU [menu]
- 1: Plot Type [1]
- 2: Box Plot [2].

To change the colour, press CTRL [ctrl] MENU [menu]. Then press:
- 6: Color [6]
- 2: Fill Color [2].

Select whichever colour you like from the palette. Press ENTER [enter].

b



The box-and-whisker plot is displayed. As you scroll over the box-and-whisker plot, the values of the five-number summary statistics are displayed.
The data are skewed (positively).

WORKED EXAMPLE 9

The number of lollies in each of 8 packets is 11, 12, 13, 14, 16, 17, 18, 19. Calculate the mean and standard deviation correct to 2 decimal places.

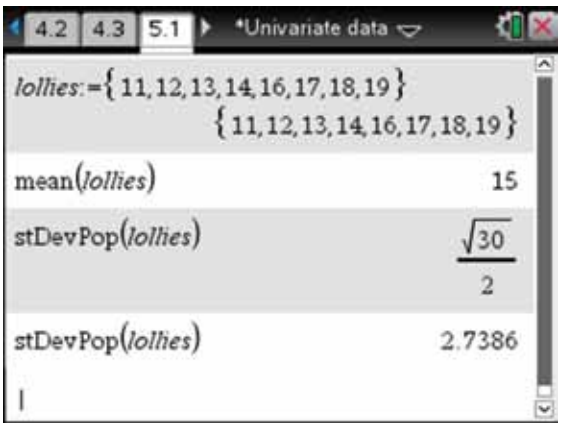
THINK

In a new problem, on a Calculator page, complete the entry lines as shown. This stores the data values to the variable *'lollies'*.
lollies := {11, 12, 13, 14, 15, 16, 17, 18, 19}
Although we can find many summary statistics, to find the mean only, open a Calculator page and press:
- MENU [menu]
- 6: Statistics [6]
- 3: List Math [3]
- 3: Mean [3].

Press var [var] and select *'lollies'*, then press ENTER [enter].
To find the population standard deviation only, press:
- MENU [menu]
- 6: Statistics [6]
- 3: List Math [3]
- 9: Population standard deviation [9].

Press var [var] and select *'lollies'*, then press ENTER [enter]. Press CTRL / ENTER [ctrl] [enter] to get a decimal approximation.

WRITE

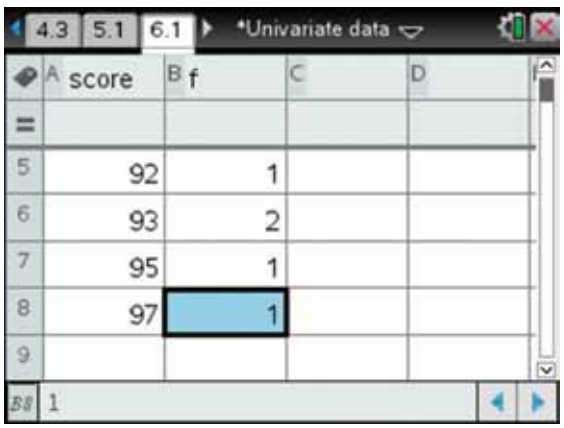


The mean number of lollies is 15 and the population standard deviation is $\sigma = 2.74$.

WORKED EXAMPLE 10

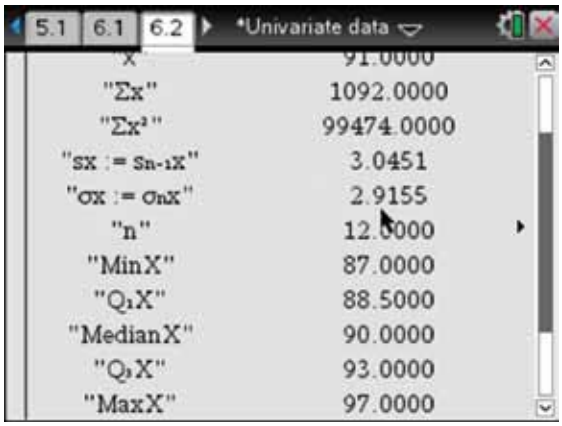
Lucy's scores in her last 12 games of golf were 87, 88, 88, 89, 90, 90, 90, 92, 93, 93, 95 and 97. Calculate the mean score and the standard deviation correct to 2 decimal places.

1. In a new problem, on a Lists & Spreadsheet page, label column A as 'score' and label column B as 'f'. Enter the values and the frequency corresponding to each score as shown in the table. Press ENTER [enter], after each value.



2 To find all the summary statistics, open a Calculator page and press:
- MENU [menu]
- 6: Statistics [6]
- 1: Stat Calculations [1]
- 1: One Variable Statistics [1].

Select 1 as the number of lists, then on the One-Variable Statistics page, select 'score' as the X1 List and 'f' as the Frequency List. Leave the next two fields empty and TAB [tab] to OK, then press ENTER [enter].



The mean is $\bar{x} = 91$ and the population standard deviation is $\sigma = 2.92$ correct to two decimal places.

WORKED EXAMPLE 14

Below are the scores for two students in eight Mathematics tests throughout the year:
John: 45, 62, 64, 55, 58, 51, 59, 62
Penny: 84, 37, 45, 80, 74, 44, 46, 50

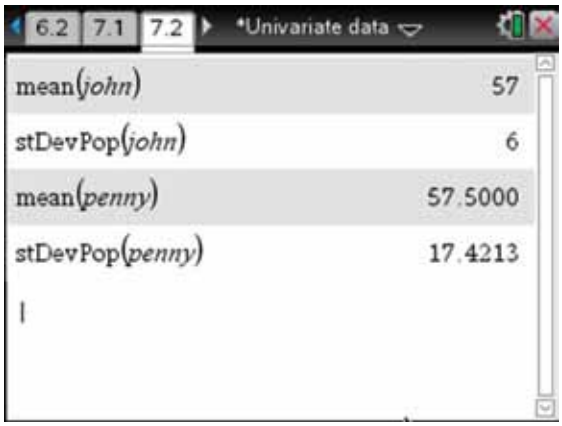
a Use the statistics function on a calculator to find the mean and standard deviation for each student.
b Which student had the better overall performance on the eight tests?
c Which student was more consistent over the eight tests?

THINK

a 1 In a new problem, on a Lists & Spreadsheet page, label column A as 'john' and column B as 'penny'. Enter the data sets from the question. Press ENTER [enter] after each value.

WRITE

2 To find only the mean and standard deviation of each data set, open a Calculator page and complete the entry lines as:
mean(*john*)
stDevPop(*john*)
mean(*penny*)
stDevPop(*penny*)
Press CTRL ctrl ENTER enter after each entry to get a decimal approximation.



John: $\bar{x} = 57$, $\sigma = 6$
Penny: $\bar{x} = 57.5$, $\sigma = 17.42$ correct to 2 decimal places.

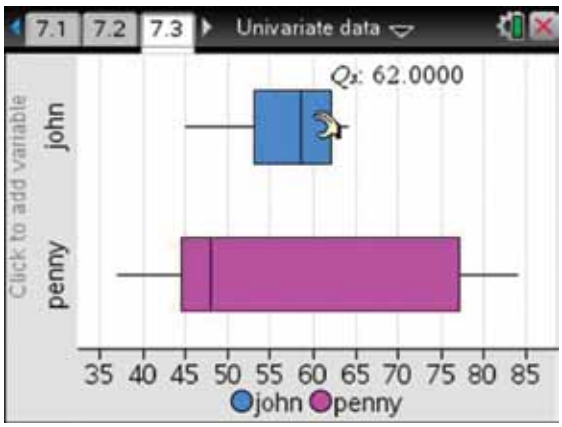
b–c To draw the two boxplots on the same Data & Statistics page, press TAB tab to locate the label of the horizontal axis and select the variable 'john'. Then press:
- MENU menu
- 1: Plot Type 1
- 2: Box Plot 2.

Then press:
- MENU menu
- 2: Plot Type 2
- 5: Add X-variable 5

and select 'penny'.
To change the colour of each boxplot, press CTRL ctrl MENU menu. Then press:
- 6: Color 6
- 2: Fill Color 2.

Select whichever colour you like from the palette for each of the boxplots.

b–c



Penny performed slightly better overall as her mean mark was higher than John's; however, John was more consistent as his standard deviation was lower than Penny's.

TOPIC 13

Bivariate data

WORKED EXAMPLE 3

Mary sells business shirts in a department store. She always records the number of different styles of shirt sold during the day. The table below shows her sales over one week.

Price ($)	14	18	20	21	24	25	28	30	32	35
Number of shirts sold	21	22	18	19	17	17	15	16	14	11

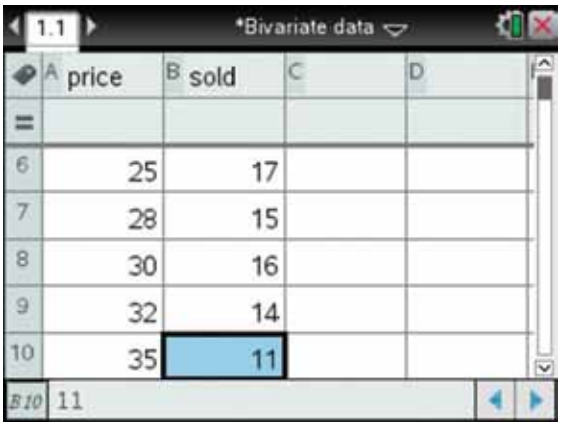
a Construct a scatterplot of the data.
b State the type of correlation between the two variables and, hence, draw a corresponding conclusion.

THINK

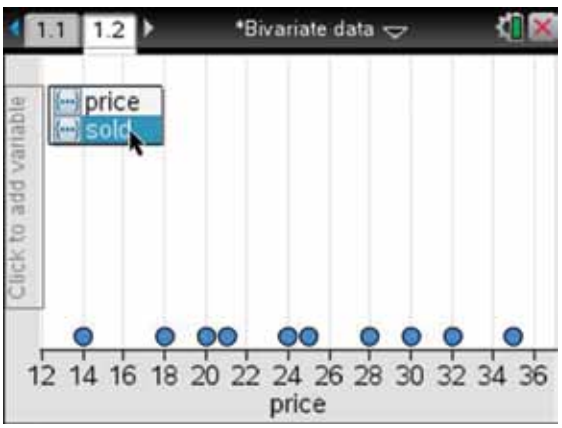
a–b 1 In a new document, on a Lists & Spreadsheet page, label column A as 'price' and label column B as 'sold'. Enter the data as shown in the table.

WRITE/DRAW

a–b

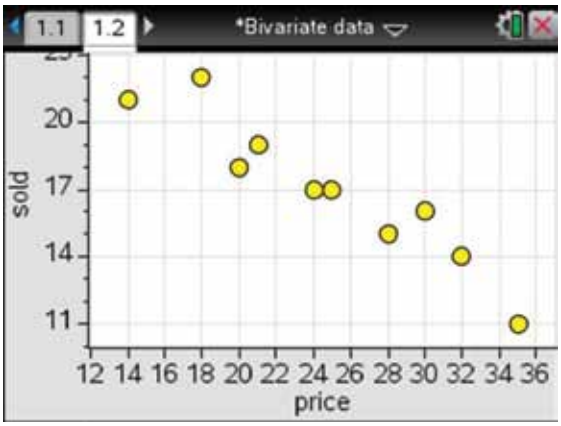


2 Open a Data & Statistics page. Press TAB [tab] to locate the label of the horizontal axis and select the variable 'price'. Press TAB [tab] again to locate the label of the vertical axis and select the variable 'sold'.



3 To change the colour of the scatterplot, place the pointer over one of the data points. Then press CTRL [ctrl] MENU [menu]. Press:
- 3: Colour [3]
- 2: Fill Colour [2].

Select a colour from the palette for the scatterplot. Press ENTER [enter].



The scatterplot is shown, using a suitable scale for both axes. The points are close to forming a straight line. There is a strong negative, linear correlation between the two variables. The trend indicates that the price of a shirt appears to affect the number sold; that is, the more expensive the shirt, the fewer are sold.

WORKED EXAMPLE 7

The percentages from two tests (English and Maths) for a group of 5 students are as shown.

a Calculate the correlation coefficient between the two sets of results.

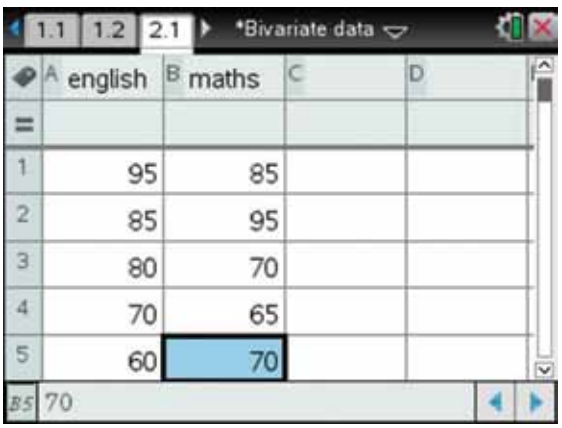
b Based on this value, describe the relationship between the English and Maths results for this group of students.

Student	English (%)	Maths (%)
1	95	85
2	85	95
3	80	70
4	70	65
5	60	70

THINK

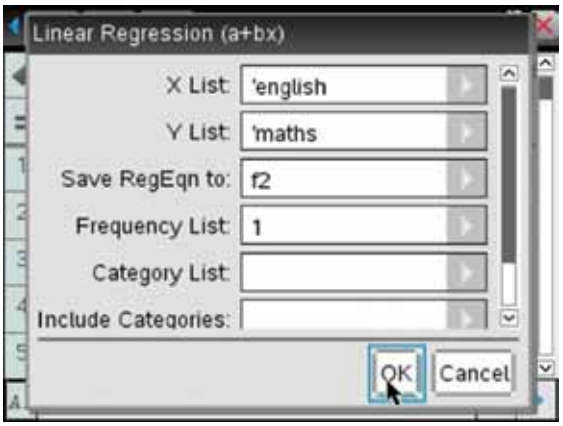
a 1 In a new problem, on a Lists & Spreadsheet page, label column A as 'english' and label column B as 'maths'. Enter the data as in the table.

WRITE



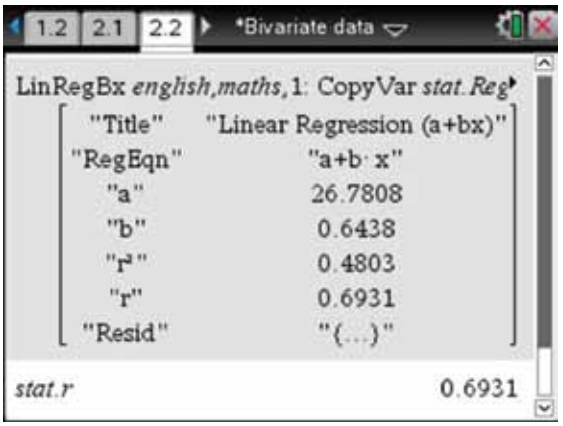
2 Open a Calculator page. Press:
- MENU [menu]
- 6: Statistics [6]
- 1: Stat Calculations [1]
- 4: Linear Regression ($a + bx$) [4].

Select 'english' as the X List and 'maths' as the Y List, and leave the next fields as shown. TAB [tab] to OK and press ENTER [enter].



b The value of the correlation is shown as r, and its value is stored in the variable *stat.r*.

b



A correlation coefficient of 0.69 indicates the relationship between English and Maths marks for this group of students is only moderate. This seems to indicate that students who are good at English are not necessarily good at Maths, and vice versa.

WORKED EXAMPLE 11

Data were recorded about the number of families who moved from Sydney to Newcastle over the past 10 years.

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Number moved	97	118	125	106	144	155	162	140	158	170

a Use technology to construct a time series graph, with a line of best fit, that represents the data.
b Describe the trend.
c Measure the correlation.
d Comment on the results.

THINK

a–b **1** In a new problem on a Lists & Spreadsheet page, label column A as 'year' and label column B as 'number'. Enter the data as in the table.

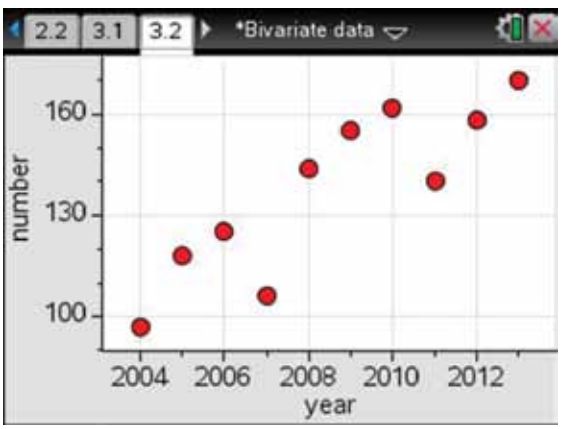
2 Open a Data & Statistics page. Press TAB [tab] to locate the label of the horizontal axis and select the variable 'year'. Press TAB [tab] again to locate the label of the vertical axis and select the variable 'number'.
To change the colour of the scatterplot, place the pointer over one of the data points. Then press CTRL [ctrl] MENU [menu]. Press:
- 3: Colour [3]
- 2: Fill Colour [2].

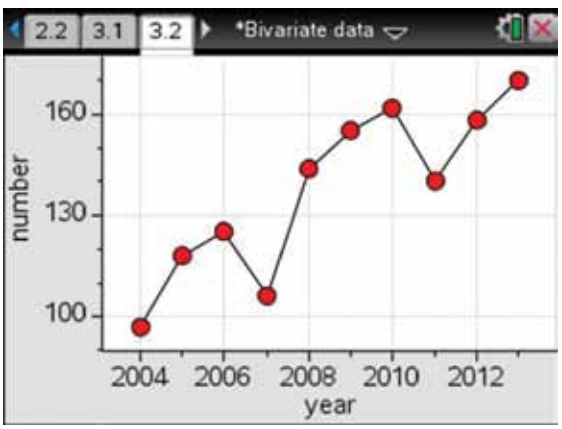
Select a colour from the palette. Press ENTER [enter].

3 Press:
- MENU [menu]
- 2: Plot Properties [2]
- 1: Connect Data Points [1].

WRITE

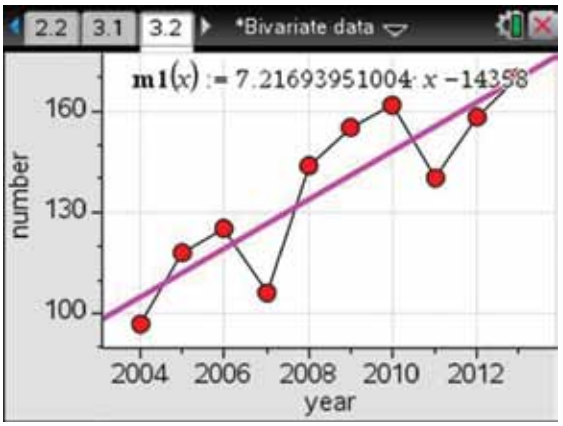
a–b





4 Press:
- MENU [menu]
- 4: Analyze [4]
- 2: Add Moveable Line [2].

A line and its equation appear automatically on the graph as shown. Repositioning the line is done in two steps by moving the position of the *y*-intercept and altering the gradient. To change the *y*-intercept, move the pointer until it rests somewhere around the middle of the line. Press the Click key [ctrl], then use the Touchpad to move the line parallel to itself until the *y*-intercept is in an appropriate position. To change the gradient, move the pointer until it rests somewhere near one end of the line. Press the Click key [ctrl], then use the Touchpad to rotate the line and change the gradient. Continue to use these tools until you are satisfied with the line of best fit by eye.

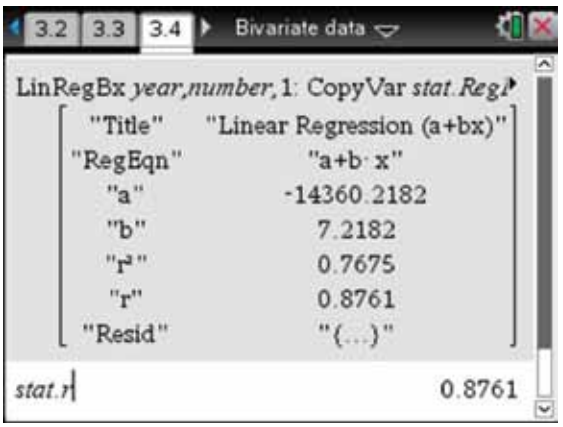


The scatterplot is shown, using a suitable scale for both axes. There appears to be an upward trend over 10 years.

c Open a Calculator page. Press:
- MENU [menu]
- 6: Statistics [6]
- 1: Stat Calculations [1]
- 4: Linear Regression ($a + bx$) [4].

Select 'year' as the X List and 'number' as the Y List, leave the next fields blank, TAB [tab] to OK, then press ENTER [enter]. Type:
stat.r
then press ENTER [enter].

c



d Interpret the results.

d Over the last 10 years, an increasing number of families have decided to move from Sydney to Newcastle. The correlation is strong and positive, $r = 0.8761$, making it possible that this trend is likely to continue.

TOPIC 14

Statistics in the media

WORKED EXAMPLE 2

A die was rolled 50 times and the following results were obtained.
6 5 3 1 6 2 3 6 2 5 3 4 1 3 2 6 4 5 5 4 3 1 2 1 6 4 5 2
3 6 1 5 3 3 2 4 1 4 2 3 2 6 3 4 6 2 1 2 4 2.

a Determine the mean of the population (to 1 decimal place).

b A suitable sample size for this population would be 7 ($\sqrt{50} \approx 7.1$).
- **i Select a random sample of 7 scores, and determine the mean of these scores.**
- **ii Select a second random sample of 7 scores, and determine the mean of these.**
- **iii Select a third random sample of 20 scores, and determine the mean of these.**

c Comment on your answers to parts a and b.

THINK

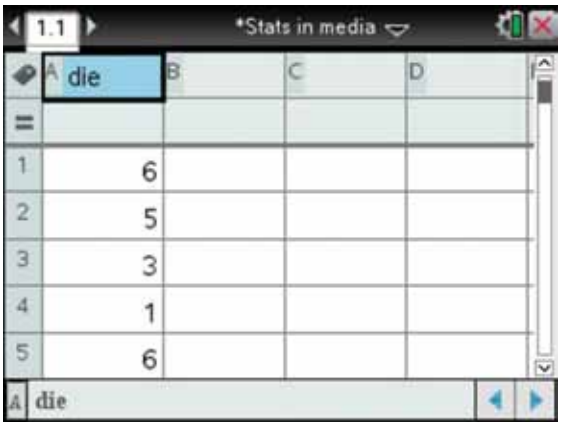
a 1 In a new document, on a Lists & Spreadsheet page, label column A as 'die'. Enter the data from the question.

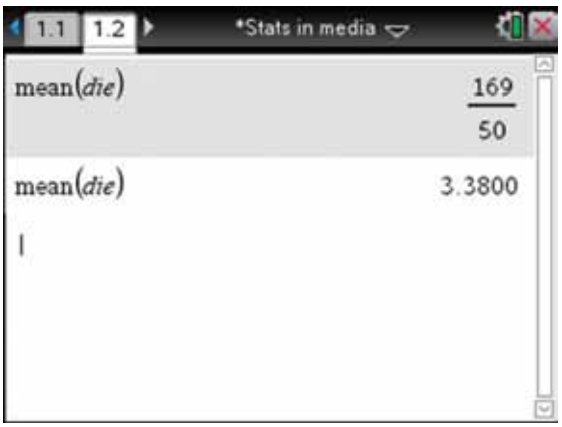
2 Although you can find many summary statistics, to find the mean only, open a Calculator page and press:
- MENU [menu]
- 6: Statistics [6]
- 3: List Math [3]
- 3: Mean [3].

Press VAR [var] and select 'die', then press CTRL [ctrl] ENTER [enter] to get a decimal approximation.

WRITE

a





The mean of the 50 die rolls is 3.38.

b i-ii To select a random sample of 7 scores, on the Calculator page, press:

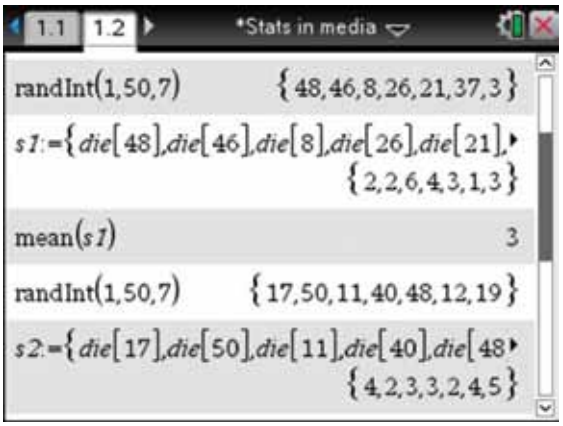
- MENU menu
- 5: Probability 5
- 4: Random 4
- 2: Integer 2.

randInt() will appear on the screen. To generate 7 random numbers within the range 1 to 50, complete the entry line as: randInt(1,50,7)
Then press ENTER enter.
Let the list *s1* represent the 7 randomly chosen values, by the index from these random numbers.
To find the mean only, open a Calculator page and press:

- MENU menu
- 6: Statistics 6
- 3: List Math 3
- 3: Mean 3.

Press VAR var and select 's1'.
Repeat the procedure and select another 7 randomly chosen values as list *s2*, and find the mean.

b i-ii



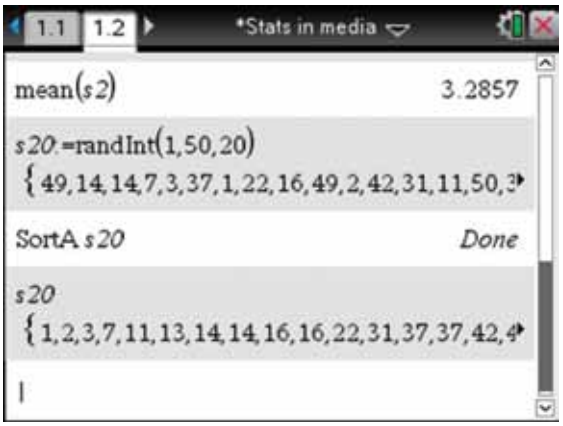
The mean of the first sample of 7 rolls is 3, and the mean of the second sample of 7 rolls is 3.2857.

b iii **1** To repeat the above procedure with 20 randomly selected values, type:
*s*20 := randInt(1,50,20)
Then press ENTER enter.
This will store 20 random numbers between 1 and 50 into the variable s20. Note that some of the values may be repeated, and the numbers at the end of the list cannot be seen. Press:

- MENU menu
- 6: Statistics 6
- 4: List Operations 4
- 1: Sort Ascending 1.

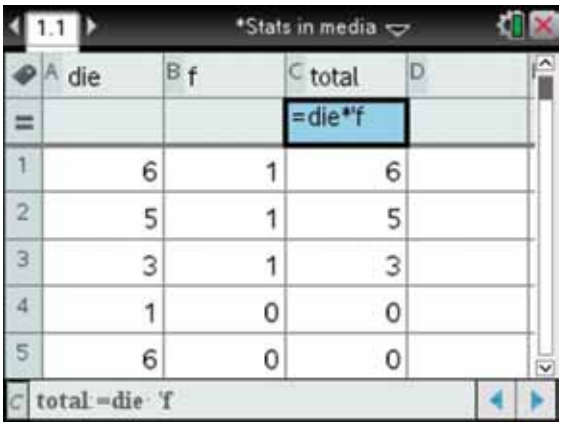
Type *s*20 then press ENTER enter.
Then type *s*20 and ENTER enter again to see the sorted list.

b iii

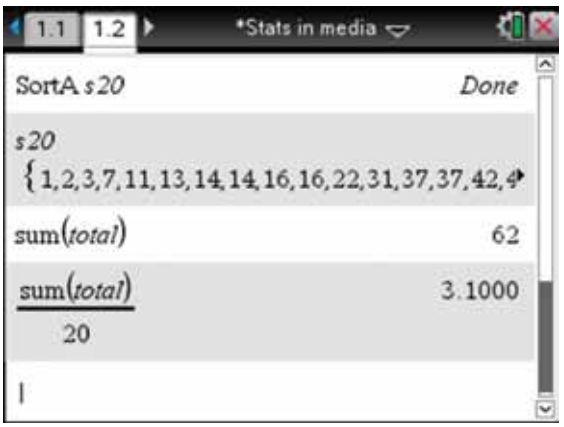


20 randomly selected values are displayed; there may be some values that are repeated.

2 Go back to the Lists & Spreadsheet page, and label column B as 'f'.
Place a 0 in this column if the random number is not in the list. Place a 1 in this column if the random number is in the list and appears only once, place a 2 in this column if the random number is in the list and appears twice, and place a 3 in this column if any number in the list appears three times.
Find the product of the columns A and B by placing the formula
=die.f
in the grey cell under the name for column C, which in this case has been labelled 'total'.



3 Go back to the Calculator page and type:
sum(*total*)
then:
$$\frac{\text{sum}(total)}{20}$$
Then press CTRL ctrl ENTER enter to get a decimal approximation.



c

c The mean of the third sample of 20 rolls is 3.1. This indicates that the results obtained from a bigger sample are more accurate than those from smaller samples.

WORKED EXAMPLE 11

The Australian women's national basketball team, the Opals, competed at the 2008 Olympic Games in Beijing, winning a silver medal. These are the heights (in metres) of the 12 team members:
1.73, 1.65, 1.8, 1.83, 1.96, 1.88, 1.63, 1.88, 1.83, 1.88, 1.8, 1.96
Provide calculations and explanations as evidence to verify or refute the following statements.

a The mean height of the team is greater than their median height.
b The range of the heights of the 12 players is almost 3 times their interquartile range.
c Only 5 players are on the court at any one time. A team of 5 players can be chosen such that their mean, median and modal heights are all the same.

THINK

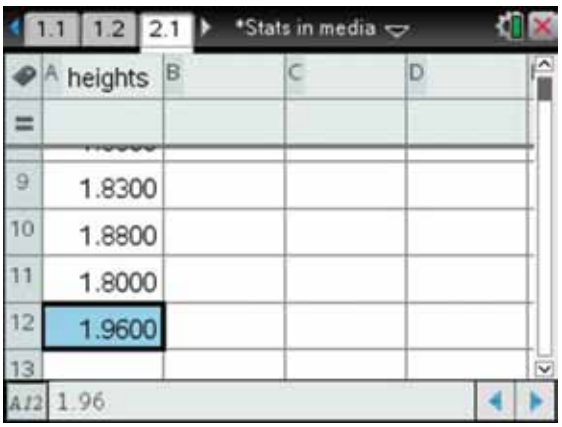
a

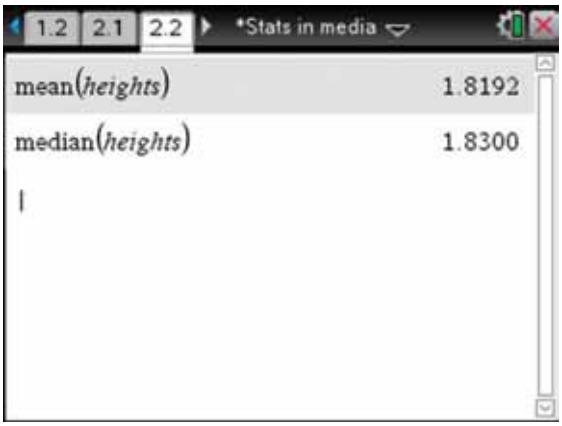
1. In a new problem, on a Lists & Spreadsheet page, label column A as 'heights'. Enter the data from the question.

2. Open a Calculator page and complete the entry lines as:
mean(*heights*)
median(*heights*)
Press ENTER [enter] after each entry.

WRITE

a



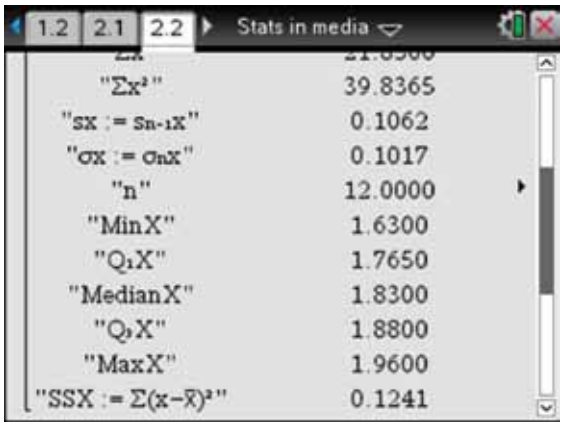


The mean heights are less than the median heights, so the statement is false.

b To find all the summary statistics, open the Calculator page and press:
- MENU [menu]
- 6: Statistics [6]
- 1: Stat Calculations [1]
- 1: One-Variable Statistics [1].

Select 1 as the number of lists. Then on the One-Variable Statistics page, select 'heights' as the X1 List and leave the frequency as 1. Leave the next two fields empty and TAB [tab] to OK and then press ENTER [enter].

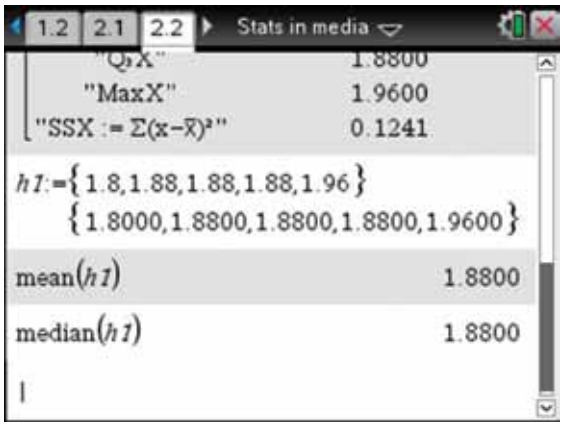
b



The range is max − min = 1.96 − 1.63 = 0.33
$Q1 = 1.765$ and $Q3 = 1.88$
$IQR = Q3 - Q1 = 1.88 - 1.765 = 0.115$.
Now 2.9IQR ≈ range so the statement is true.

c Note that the mode is 1.88 as three players have this height.
If you chose one player shorter and one player taller than the mode by the same amount, then the 5 heights chosen from this sample will have their mode, median and mean all equal.
To verify this, open a Calculator page and complete the entry lines as:
$h1 := \{1.8, 1.88, 1.88, 1.88, 1.96\}$
$\text{mean}(h1)$
$\text{median}(h1)$
Press ENTER [enter] after each entry.

c



The mode, median and mean of the sample chosen are all equal.

TOPIC 15

Financial mathematics

WORKED EXAMPLE 2

The ticketed price of a mobile phone is $600. Andrew decides to purchase the phone using his credit card. At the end of 1 month the credit card company charges interest at a rate of 15% p.a. Calculate the amount of interest that Andrew must pay on his credit card after 1 month.

THINK

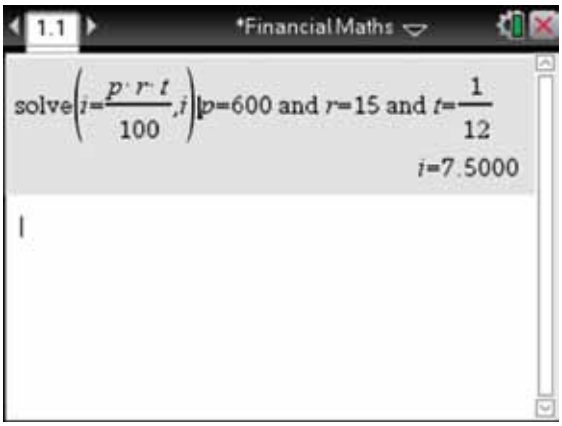
In a new document, on a Calculator page, press:
- MENU [menu]
- 3: Algebra [3]
- 1: Solve [1].

Complete the entry line as:

$$\text{solve}\left(i = \frac{prt}{100}, i\right) \mid p = 600 \text{ and } r = 15 \text{ and } t = \frac{1}{12}$$

Press CTRL [ctrl] ENTER [enter] to get a decimal approximation.
To substitute values, use the symbol |. Press CTRL [ctrl] and then = to bring up the palette. Use the Touchpad to select the | symbol, then type 'and' or find it in the catalog [book].

WRITE



The interest is $7.50.

WORKED EXAMPLE 5

A furniture store offers a discount of 15% during a sale. A further 5% discount is then offered to customers who pay cash.

a Find the price paid by Lily, who pays cash for a bedroom suite priced at $2500.

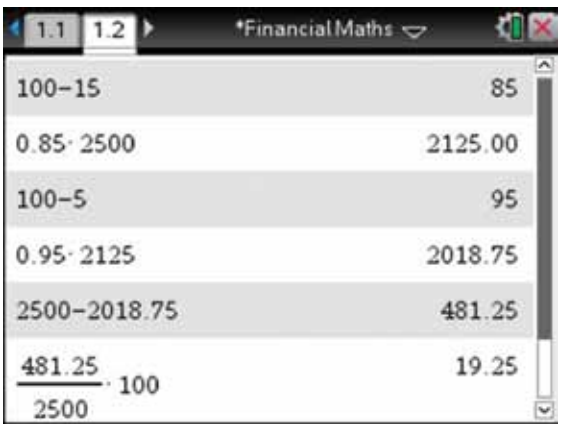
b What single percentage discount does Lily receive on the price of the bedroom suite?

THINK

a–b On a Calculator page, complete the entry lines as shown in the screenshot.
Press ENTER [enter] after each entry.

WRITE

a–b



The price paid is $2018.75.
This is a 19.25% discount.

WORKED EXAMPLE 8

William has $14 000 to invest. He invests the money at 9% p.a. for 5 years with interest compounded annually.

a Use the formula $A = P(1 + R)^n$ to calculate the amount to which this investment will grow.

b Calculate the compound interest earned on the investment.

THINK

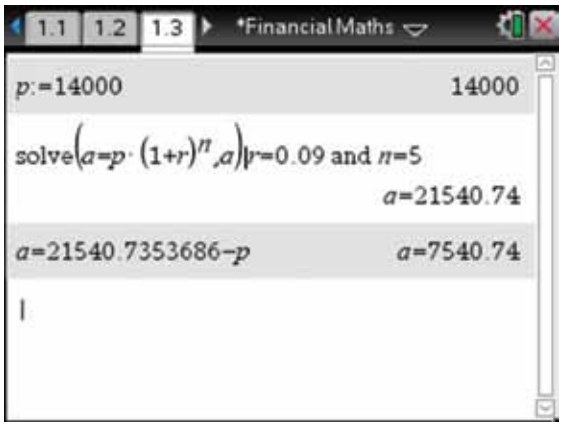
On a Calculator page, store the value of p. To do this, complete the entry line as:
$p := 14000$

Then press:
- MENU [menu]
- 3: Algebra [3]
- 1: Solve [1].

Complete the entry line as:
$\text{solve}\left(a = p(1+r)^n,\ a\right) | r = 0.09 \text{ and } n = 5$

Then press ENTER [enter] and complete as shown.

WRITE



The investment grows to $21 540.74.
The compound interest earned is $7540.74.

WORKED EXAMPLE 9

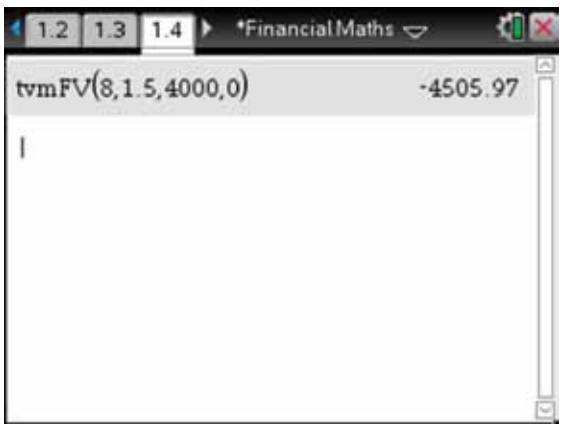
Calculate the future value of an investment of $4000 at 6% p.a. for 2 years with interest compounded quarterly.

THINK

Use the finance functions available on the calculator for this question.
On a Calculator page, press:
- MENU [menu]
- 8: Finance [8]
- 2: TVM Functions [2]
- 5: Future Value [5].

Complete the entry line as:
tvmFV(8,1.5,4000,0)
Press ENTER [enter].
Note that the number of compounding periods is 8, that is 4 times a year for 2 years, and the interest is $\frac{6}{4} = 1.5\%$ quarterly.

WRITE



The future value is $4505.97.

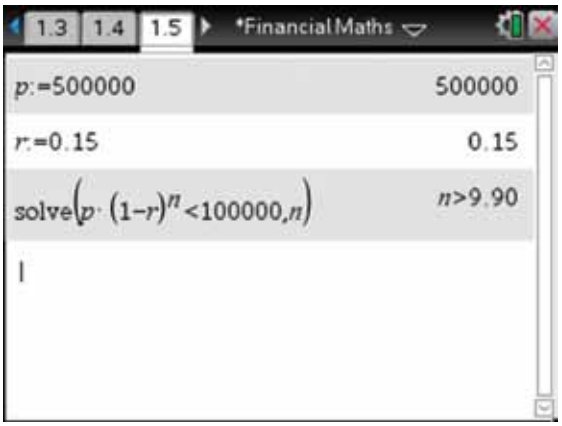
WORKED EXAMPLE 11

A truck driver buys a new prime mover for $500 000. The prime mover depreciates at the rate of 15% p.a. and is written off when its value falls below $100 000. How long will it take for the prime mover to be written off?

THINK

On a Calculator page, store the principal and interest.
Complete the entry lines as:
$p := 500000$
$r := 0.15$
Press ENTER [enter] after each entry.
Complete the entry line as:
$\text{solve}\left(p(1-r)^n < 100000, n\right)$
Then press ENTER [enter].

WRITE



The prime mover will be written off in 10 years.

WORKED EXAMPLE 12

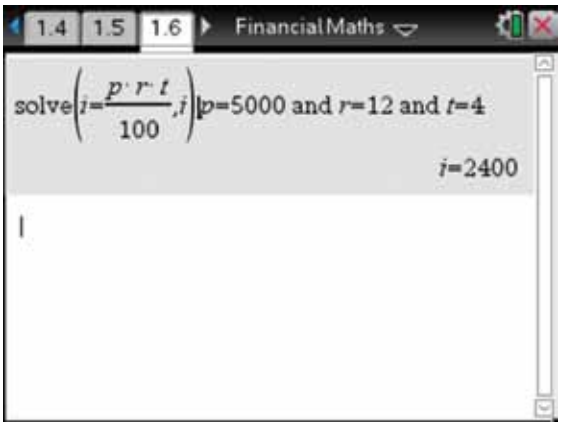
Calculate the interest payable on a loan of $5000 to be repaid at 12% p.a. flat interest over 4 years.

THINK

On a Calculator page, press:
- MENU [menu]
- 3: Algebra [3]
- 1: Solve [1].

Complete the entry line as:
$\text{solve}\left(i = \frac{prt}{100}, i\right) | p = 5000 \text{ and } r = 12 \text{ and } t = 4$
Then press ENTER [enter].

WRITE



The interest payable is $2400.

TOPIC 16

Real numbers

WORKED EXAMPLE 5

Simplify each of the following expressions containing surds. Assume that a and b are positive real numbers.

a $3\sqrt{6}+17\sqrt{6}-2\sqrt{6}$ **b** $5\sqrt{3}+2\sqrt{12}-5\sqrt{2}+3\sqrt{8}$ **c** $\frac{1}{2}\sqrt{100a^3b^2}+ab\sqrt{36a}-5\sqrt{4a^2b}$

THINK

a–c In a new document, on a Calculator page, complete the entry lines as:

$3\sqrt{6}+17\sqrt{6}-2\sqrt{6}$

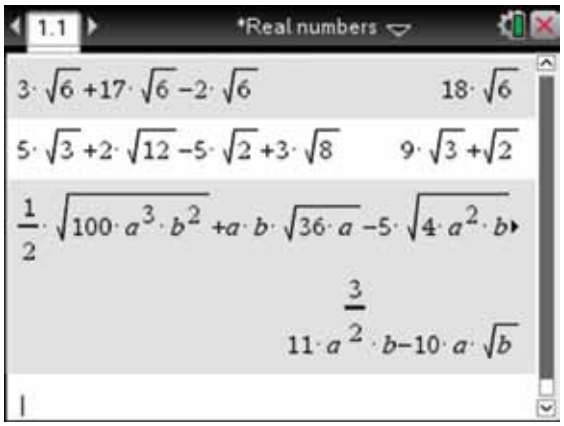
$5\sqrt{3}+2\sqrt{12}-5\sqrt{2}+3\sqrt{8}$

$\frac{1}{2}\sqrt{100a^3b^2}+ab\sqrt{36a}-5\sqrt{4a^2b}\mid a>0$ and $b>0$

Then Press ENTER enter after each entry.

WRITE

a–c



$3\sqrt{6}+17\sqrt{6}-2\sqrt{6}=18\sqrt{6}$

$5\sqrt{3}+2\sqrt{12}-5\sqrt{2}+3\sqrt{8}=9\sqrt{3}+\sqrt{2}$

$\frac{1}{2}\sqrt{100a^3b^2}+ab\sqrt{36a}-5\sqrt{4a^2b}$

$=11a^{\frac{3}{2}}b-10a\sqrt{b}$

WORKED EXAMPLE 6

Multiply the following surds, expressing answers in the simplest form. Assume that x and y are positive real numbers.

a $\sqrt{11} \times \sqrt{7}$ **b** $5\sqrt{3} \times 8\sqrt{5}$ **c** $6\sqrt{12} \times 2\sqrt{6}$ **d** $\sqrt{15x^5y^2} \times \sqrt{12x^2y}$

THINK

a–d On a Calculator page, complete the entry lines as:

$\sqrt{11} \times \sqrt{7}$

$5\sqrt{3} \times 8\sqrt{5}$

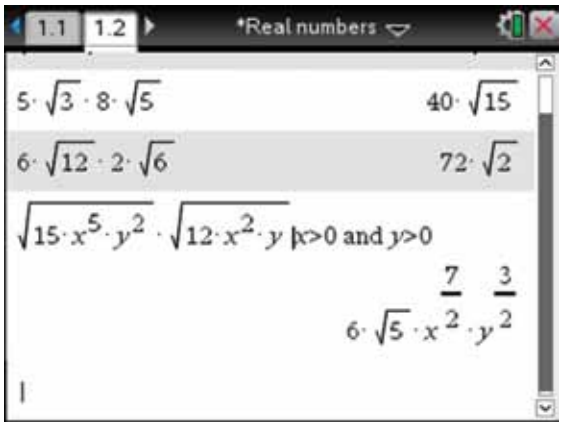
$6\sqrt{12} \times 2\sqrt{6}$

$\sqrt{15x^5y^2} \times \sqrt{12x^2y} \mid x > 0 \text{ and } y > 0$

Then Press ENTER [enter] after each entry.

WRITE

a–d



$\sqrt{11} \times \sqrt{7} = \sqrt{77}$

$5\sqrt{3} \times 8\sqrt{5} = 40\sqrt{15}$

$6\sqrt{12} \times 2\sqrt{6} = 72\sqrt{2}$

$\sqrt{15x^5y^2} \times \sqrt{12x^2y} = 6\sqrt{5}\, x^{\frac{7}{2}} y^{\frac{3}{2}}$

WORKED EXAMPLE 8

Divide the following surds, expressing answers in the simplest form. Assume that x and y are positive real numbers.

a $\dfrac{\sqrt{55}}{\sqrt{5}}$ **b** $\dfrac{\sqrt{48}}{\sqrt{3}}$

c $\dfrac{9\sqrt{88}}{6\sqrt{99}}$ **d** $\dfrac{\sqrt{36xy}}{\sqrt{25x^9y^{11}}}$

THINK

a–b On a Calculator page, use the fraction template to complete the entry lines as:

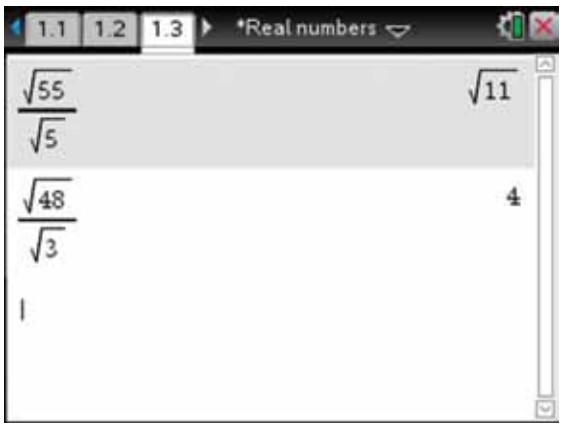
$\dfrac{\sqrt{55}}{5}$

$\dfrac{\sqrt{48}}{\sqrt{3}}$

Then press ENTER enter after each entry.

WRITE

a–b



$\dfrac{\sqrt{55}}{5} = \sqrt{11}$

$\dfrac{\sqrt{48}}{\sqrt{3}} = 4$

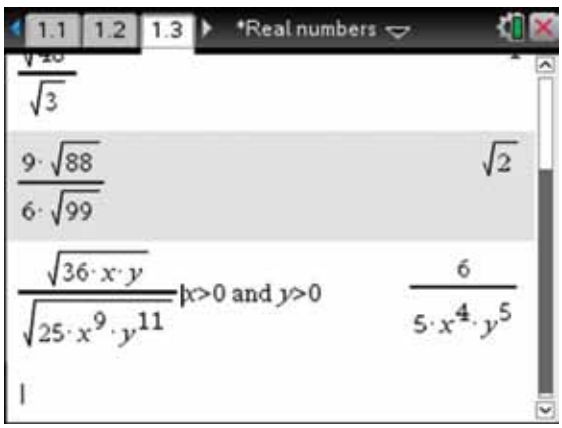
c–d On a Calculator page, use the fraction template to complete the entry lines as:

$\dfrac{9\sqrt{88}}{6\sqrt{99}}$

$\dfrac{\sqrt{36xy}}{\sqrt{25x^9y^{11}}} \mid x > 0 \text{ and } y > 0$

Then press ENTER enter after each entry. Remember to include the implied multiplication sign between x and y in the numerator.

c–d



$\dfrac{9\sqrt{88}}{6\sqrt{99}} = \sqrt{2}$

$\dfrac{\sqrt{36xy}}{\sqrt{25x^9y^{11}}} = \dfrac{6}{5x^4y^5}$

WORKED EXAMPLE 9

Express the following in their simplest form with a rational denominator.

a $\dfrac{\sqrt{6}}{\sqrt{13}}$ **b** $\dfrac{2\sqrt{12}}{3\sqrt{54}}$ **c** $\dfrac{\sqrt{17}-3\sqrt{14}}{\sqrt{7}}$

THINK

a–c On a Calculator page, complete the entry lines as:

$\dfrac{\sqrt{6}}{\sqrt{13}}$

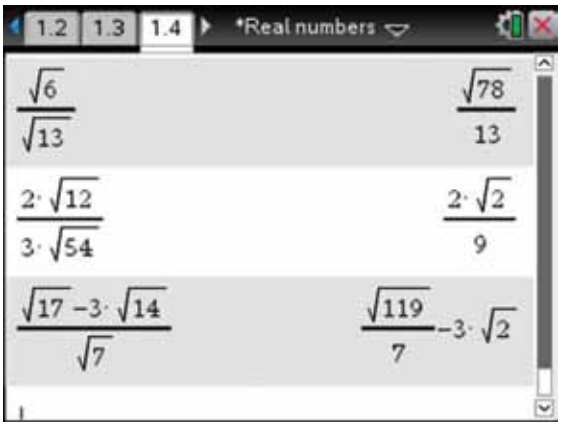
$\dfrac{2\sqrt{12}}{3\sqrt{54}}$

$\dfrac{\sqrt{17}-3\sqrt{14}}{\sqrt{7}}$

Then press ENTER enter after each entry. Notice that the CAS calculator automatically rationalises the denominator.

WRITE

a–c



$\dfrac{\sqrt{6}}{\sqrt{13}} = \dfrac{\sqrt{78}}{13}$

$\dfrac{2\sqrt{12}}{3\sqrt{54}} = \dfrac{2\sqrt{2}}{9}$

$\dfrac{\sqrt{17}-3\sqrt{14}}{\sqrt{7}} = \dfrac{\sqrt{119}}{7} - 3\sqrt{2}$

WORKED EXAMPLE 10

Rationalise the denominator and simplify the following.

a $\dfrac{1}{4-\sqrt{3}}$

b $\dfrac{\sqrt{6}+3\sqrt{2}}{3+\sqrt{3}}$

THINK

a–b On a Calculator page, complete the entry lines as:

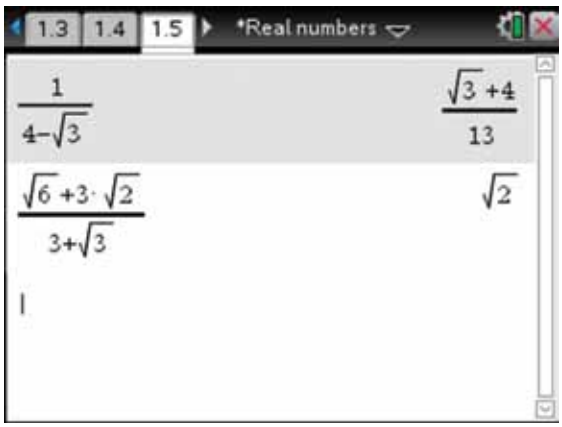
$\dfrac{1}{4-\sqrt{3}}$

$\dfrac{\sqrt{6}+3\sqrt{2}}{3+\sqrt{3}}$

Then press ENTER enter after each entry.

WRITE

a–b



$\dfrac{1}{4-\sqrt{3}} = \dfrac{4+\sqrt{3}}{13}$

$\dfrac{\sqrt{6}+3\sqrt{2}}{3+\sqrt{3}} = \sqrt{2}$

WORKED EXAMPLE 12

Use a calculator to find the value of the following, correct to 1 decimal place.

a $10^{\frac{1}{4}}$ **b** $200^{\frac{1}{5}}$

THINK

a–b On a Calculator page, complete the entry lines as:

$10^{\frac{1}{4}}$

$200^{\frac{1}{5}}$

Then press CTRL ctrl ENTER enter after each entry to get a decimal approximation.

WRITE

a–b

$10^{\frac{1}{4}} \approx 1.8$

$200^{\frac{1}{5}} \approx 2.9$

Both answers are correct to 1 decimal place.

WORKED EXAMPLE 16

Evaluate each of the following.

a 4^{-1} **b** 2^{-4}

THINK

a–b On a Calculator page, complete the entry lines as:

4^{-1}

2^{-4}

Press ENTER enter. Then press CTRL ctrl ENTER enter to get a decimal approximation for each entry.

WRITE

a–b

$4^{-1} = \frac{1}{4} = 0.25$

$2^{-4} = \frac{1}{16} = 0.0625$

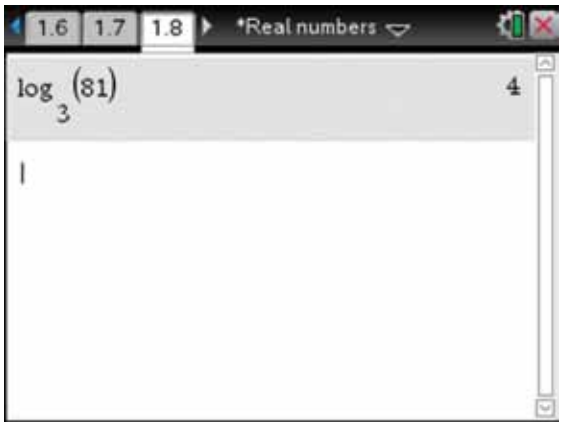
WORKED EXAMPLE 20

Evaluate $\log_3 81$.

THINK

On a Calculator page, press CTRL ctrl log 10× and complete the entry line as:
$\log_3 81$
Then press ENTER enter.

WRITE



$\log_3 81 = 4$

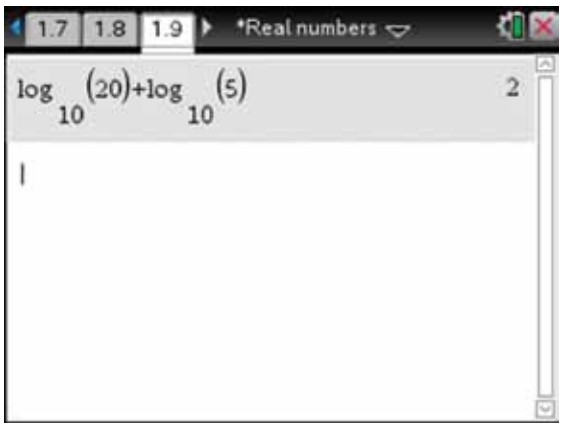
WORKED EXAMPLE 21

Evaluate $\log_{10} 20 + \log_{10} 5$.

THINK

On a Calculator page, press CTRL ctrl log 10× and complete the entry line as:
$\log_{10} 20 + \log_{10} 5$
Then press ENTER enter.

WRITE



$\log_{10} 20 + \log_{10} 5 = 2$

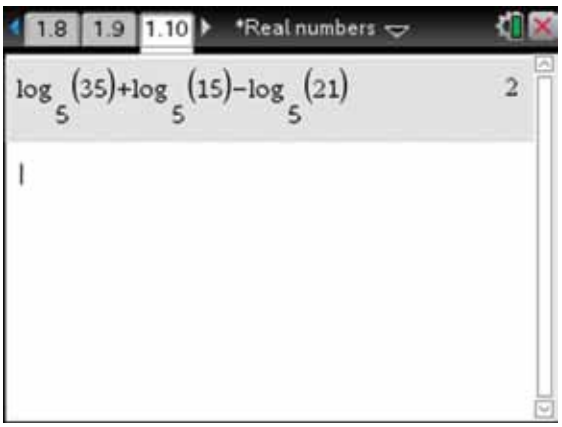
WORKED EXAMPLE 23

Evaluate $\log_5 35 + \log_5 15 - \log_5 21$.

THINK

On a Calculator page, press CTRL ctrl log 10^x and complete the entry line as:
$\log_5 35 + \log_5 15 - \log_5 21$
Then press ENTER enter.

WRITE



$\log_5 35 + \log_5 15 - \log_5 21 = 2$

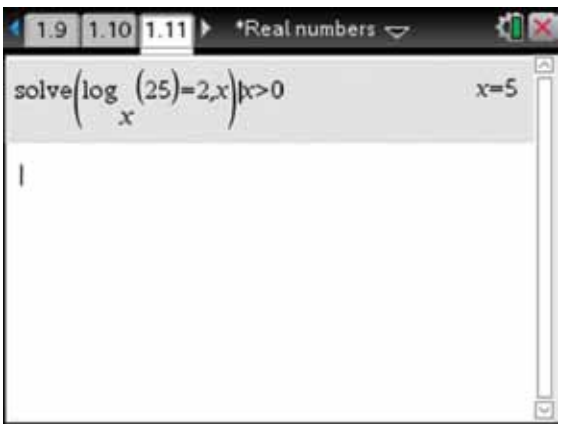
WORKED EXAMPLE 26

Solve for x in $\log_x 25 = 2$, given that $x > 0$.

THINK

On a Calculator page, complete the entry line as:
$\text{solve}\left(\log_x 25 = 2, x\right) \mid x > 0$
Then press ENTER enter.

WRITE



$\log_x 25 = 2$
$\Rightarrow \; x = 5$

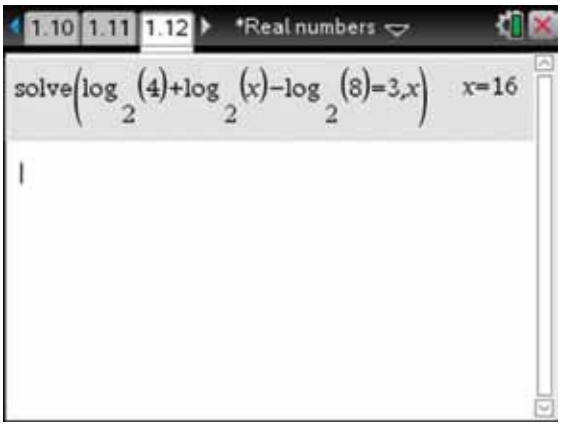
WORKED EXAMPLE 28

Solve for x in the equation $\log_2 4 + \log_2 x - \log_2 8 = 3$.

THINK

On a Calculator page, complete the entry line as:
solve$\left(\log_2 4 + \log_2 x - \log_2 8 = 3, x\right)$
Then press ENTER enter.

WRITE



$\log_2 4 + \log_2 x - \log_2 8 = 3$
$\Rightarrow x = 16$

WORKED EXAMPLE 29

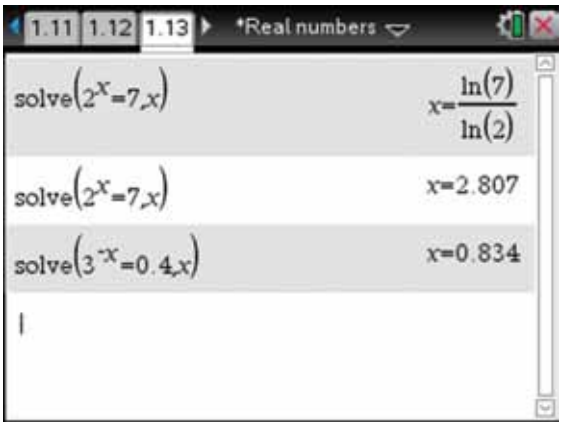
Solve for x, correct to 3 decimal places, if:
a $2^x = 7$ **b** $3^{-x} = 0.4$.

THINK

a–b For **a**, on a Calculator page, complete the entry line as:
solve$\left(2^x = 7, x\right)$
Then press ENTER enter. Press CTRL ctrl ENTER enter to get a decimal approximation.
For **b**, complete the entry line as:
solve$\left(3^{-x} = 0.4, x\right)$
Then press ENTER enter

WRITE

a–b



$2^x = 7$
$\Rightarrow x \approx 2.807$
$3^{-x} = 0.4$
$\Rightarrow x \approx 0.834$
Both answers are correct to 3 decimal places.

TOPIC 17

Polynomials

WORKED EXAMPLE 1

Simplify each of the following.

a $(5x^3 + 3x^2 - 2x - 1) + (x^4 + 5x^2 - 4)$

b $(5x^3 + 3x^2 - 2x - 1) - (x^4 + 5x^2 - 4)$

THINK

a–b In a new document, on a Calculator page, press:
- MENU [menu]
- 1: Actions [1]
- 1: Define [1].

Complete the entry lines as:

Define $p1(x) = 5x^3 + 3x^2 - 2x - 1$

Define $p2(x) = x^4 + 5x^2 - 4$

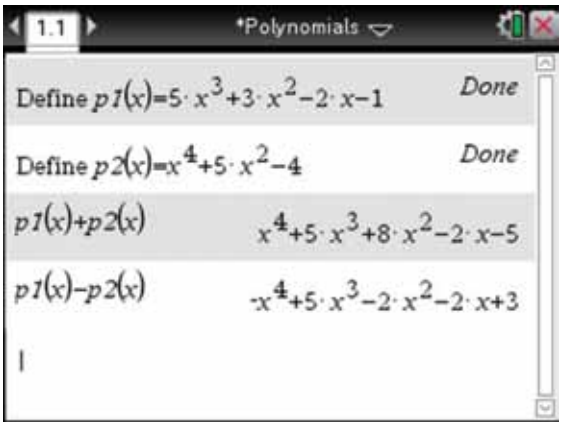
$p1(x) + p2(x)$

$p1(x) - p2(x)$

Press ENTER [enter] after each entry.

WRITE

a–b



$p1(x) + p2(x) = x^4 + 5x^3 + 8x^2 - 2x - 5$

$p1(x) - p2(x) = -x^4 + 5x^3 - 2x^2 - 2x + 3$

WORKED EXAMPLE 2

Expand:

a $x(x + 2)(x - 3)$ **b** $(x - 1)(x + 5)(x + 2)$.

THINK

a–b On a Calculator page, press:
- MENU [menu]
- 3: Algebra [3]
- 3: Expand [3].

Complete the entry lines as:

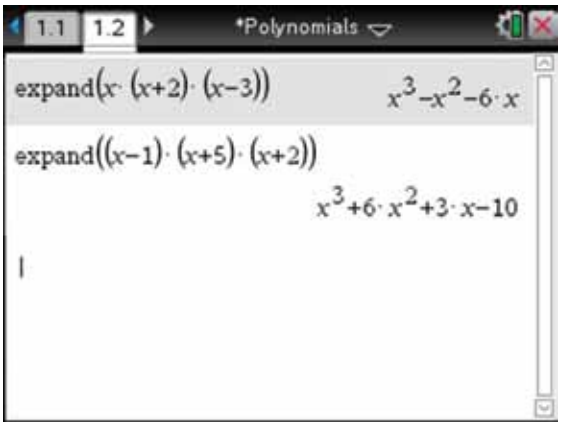
$\text{expand}(x(x+2)(x-3))$

$\text{expand}((x-1)(x+5)(x+2))$

Press ENTER [enter] after each entry.
Remember to include the multiplication sign between the brackets.

WRITE

a–b



$x(x+2)(x-3) = x^3 - x^2 - 6x$

$(x-1)(x+5)(x+2) = x^3 + 6x^2 + 3x - 10$

WORKED EXAMPLE 4

State the quotient and remainder for $(x^3 - 7x + 1) \div (x + 5)$.

THINK

On a Calculator page, press:
- MENU menu
- 3: Algebra 3
- 8: Polynomial Tool 8
- 5: Quotient of Polynomial 5.

Complete the entry line as:
polyQuotient$\left(x^3 - 7x + 1,\ x + 5\right)$

Press ENTER enter.
Then press:
- MENU menu
- 3: Algebra 3
- 8: Polynomial Tool 8
- 4: Remainder of Polynomial 4.

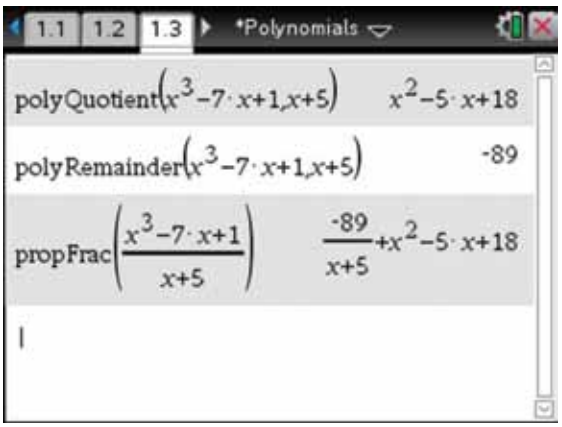
Complete the entry line as:
polyRemainder$\left(x^3 - 7x + 1,\ x + 5\right)$

Then press ENTER enter.
Alternatively, on a Calculator page, press:
- MENU menu
- 3: Algebra 3
- 9: Fractional Tools 9
- 1: Proper Fraction 1.

Complete the entry line as:
$\text{propFrac}\left(\dfrac{x^3 - 7x + 1}{x + 5}\right)$

Press ENTER enter.

WRITE



$$\frac{x^3 - 7x + 1}{x + 5} = \frac{-89}{x + 5} + x^2 - 5x + 18$$

The quotient is $x^2 - 5x + 18$ and the remainder is -89.

WORKED EXAMPLE 6

If $P(x) = 2x^3 + x^2 - 3x - 4$, find:
a $P(1)$ **b** $P(-2)$ **c** $P(a)$ **d** $P(2b)$ **e** $P(x+1)$.

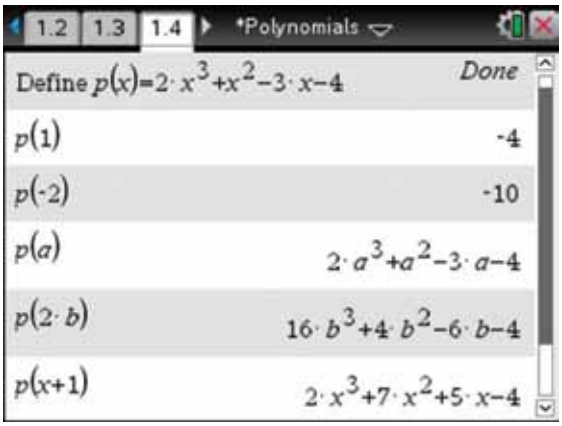
THINK

a–e On a Calculator page, press:
- MENU menu
- 1: Actions 1
- 1: Define 1.

Complete the entry lines as:
Define $p(x) = 2x^3 + x^2 - 3x - 4$
$p(1)$
$p(-2)$
$p(a)$
$p(2b)$
$p(x+1)$
Press ENTER enter after each entry.

WRITE

a–e



$P(1) = -4$
$P(-2) = -10$
$P(a) = 2a^3 + a^2 - 3a - 4$
$P(2b) = 16b^3 + 4b^2 - 6b - 4$
$P(x+1) = 2x^3 + 7x^2 + 5x - 4$

WORKED EXAMPLE 8

$(x - 2)$ is a factor of $x^3 + kx^2 + x - 2$.
Find the value of k.

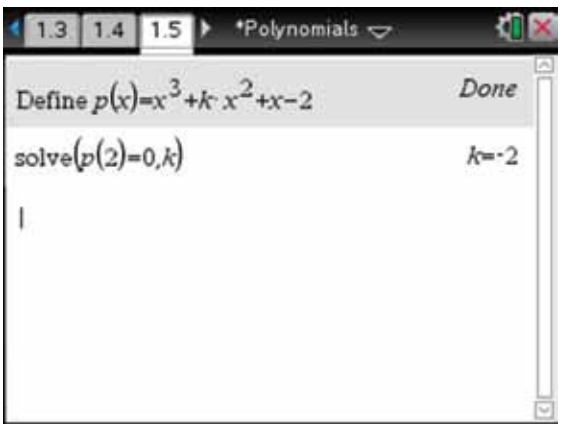
THINK

On a Calculator page, press:
- MENU menu
- 1: Actions 1
- 1: Define 1.

Complete the entry line as:
Define $p(x) = x^3 + kx^2 + x - 2$
Then press ENTER enter.
Then press:
- MENU menu
- 3: Algebra 3
- 1: Solve 1.

Complete the entry line as:
solve$\left(p(2) = 0, k\right)$
Then press ENTER enter.

WRITE



$k = -2$

WORKED EXAMPLE 10

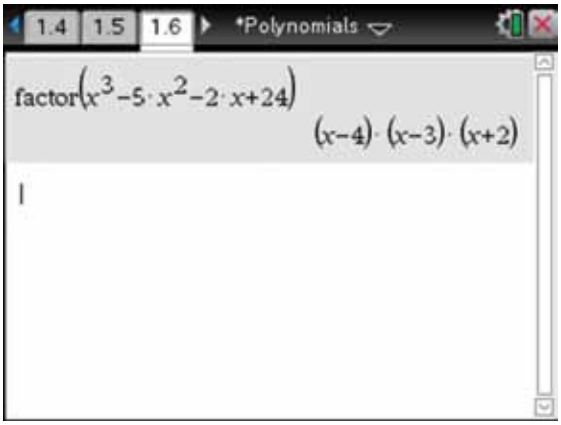
Use short division to factorise $x^3 - 5x^2 - 2x + 24$.

THINK

On a Calculator page, press:
- MENU menu
- 3: Algebra 3
- 2: Factor 2.

Complete the entry line as:
factor$\left(x^3 - 5x^2 - 2x + 24\right)$
Then press ENTER enter.

WRITE



$x^3 - 5x^2 - 2x + 24 = (x - 4)(x - 3)(x + 2)$

WORKED EXAMPLE 11

Solve:
a $x^3 = 9x$
b $-2x^3 + 4x^2 + 70x = 0$
c $2x^3 - 11x^2 + 18x - 9 = 0$

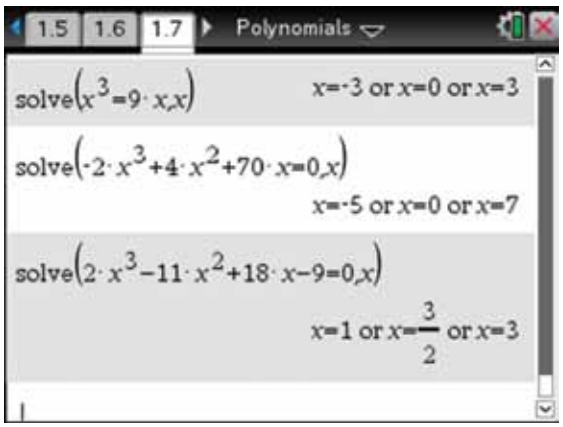
THINK

a–c On a Calculator page, press:
- MENU menu
- 3: Algebra 3
- 1: Solve 1.

Complete the entry lines as:
solve$\left(x^3 = 9x,\ x\right)$
solve$\left(-2x^3 + 4x^2 + 70x = 0,\ x\right)$
solve$\left(2x^3 - 11x^2 + 18x - 9 = 0,\ x\right)$
Press ENTER enter after each entry.

WRITE

a–c



$x^3 = 9x$
$\Rightarrow x = -3$ or $x = 0$ or $x = 3$
$-2x^3 + 4x^2 + 70x = 0$
$\Rightarrow x = -5$ or $x = 0$ or $x = 7$
$2x^3 - 11x^2 + 18x - 9 = 0$
$\Rightarrow x = 1$ or $x = \frac{3}{2}$ or $x = 3$

TOPIC 18

Functions and relations

WORKED EXAMPLE 3

If $f(x) = x^2 - 3$, find:

a $f(1)$
b $f(a)$
c $3f(2a)$
d $f(a) + f(b)$
e $f(a + b)$.

THINK

a–d In a new document, on a Calculator page, press:

- MENU menu
- 1: Actions 1
- 1: Define 1.

Complete the entry lines as:

Define $f(x) = x^2 - 3$

$f(1)$

$f(a)$

$3f(2a)$

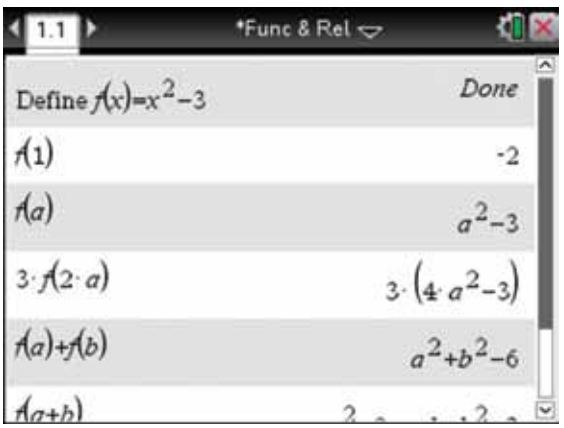
$f(a) + f(b)$

$f(a + b)$

Press ENTER enter after each entry.

WRITE

a–d



$f(1) = -2$

$f(a) = a^2 - 3$

$3f(2a) = 3(4a^2 - 3)$

$f(a) + f(b) = a^2 + b^2 - 6$

$f(a + b) = a^2 + 2ab + b^2 - 3$

WORKED EXAMPLE 5

Find any points of intersection between $f(x) = 2x + 1$ and $g(x) = \frac{1}{x}$.

THINK

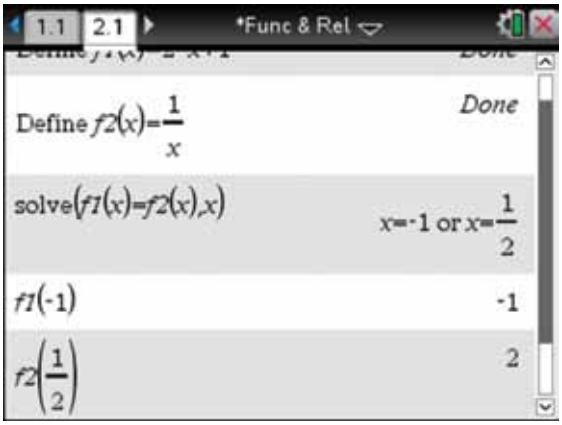
1 In a new problem, on a Calculator page, press:
- MENU menu
- 1: Actions 1
- 1: Define 1.

Complete the entry lines as:
Define $f1(x) = 2x + 1$
Define $f2(x) = \frac{1}{x}$
Press ENTER enter after each entry.
Then press:
- MENU menu
- 3: Algebra 3
- 1: Solve 1.

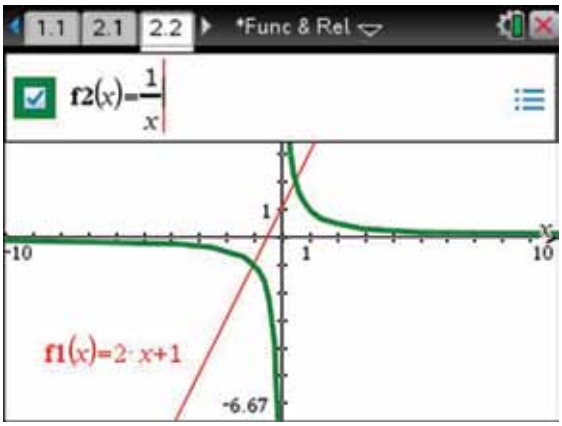
Complete the entry lines as:
$\text{solve}(f1(x) = f2(x), x)$
$f1(-1)$
$f2\left(\frac{1}{2}\right)$
Press ENTER enter after each entry.

2 Alternatively, open a Graphs page in the current document. Since the functions have already been entered, just select the functions and press ENTER enter. The graphs will be displayed.

WRITE

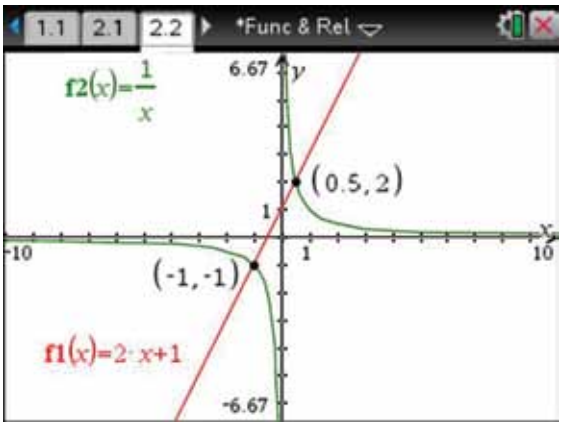


The points of intersection are $(-1, -1)$ and $\left(\frac{1}{2}, 2\right)$.



3 The viewing window needs to be altered to see the graphs more clearly.
To find the points of intersection between the two functions, press:
- MENU menu
- 6: Analyze Graph 6
- 4: Intersection 4.

Move the cursor to the left of the intersection point, and press ENTER enter. Then move the cursor to the right of the intersection point and press ENTER enter. The intersection point is displayed.
Repeat for the other intersection point.



The points of intersection are (−1, −1) and (0.5, 2).

WORKED EXAMPLE 7

The number of bacteria, N, in a Petri dish after x hours is given by the equation $N = 50 \times 2^x$.
a Determine the initial number of bacteria in the Petri dish.
b Determine the number of bacteria in the Petri dish after 3 hours.
c Draw the graph of the function of N against x.
d Use the graph to estimate the length of time it will take for the initial number of bacteria to treble.

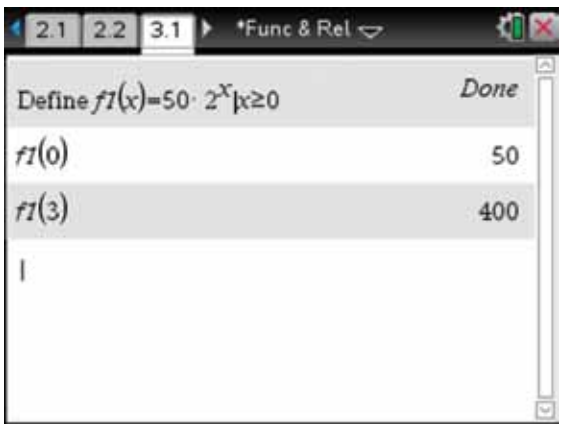
THINK

a–b In a new problem, on a Calculator page, press:
- MENU menu
- 1: Actions 1
- 1: Define 1.

Complete the function entry line as:
Define $f1(x) = 50 \times 2^x \mid x \geq 0$
Then press ENTER enter.
Note that the $x \geq 0$ needs to be included as the graph is only sketched for $x \geq 0$.
To determine the initial number of bacteria, complete the entry line as:
$f1(0)$
To determine the number of bacteria after 3 hours, complete the entry line as:
$f1(3)$
Press ENTER enter after each entry.

WRITE

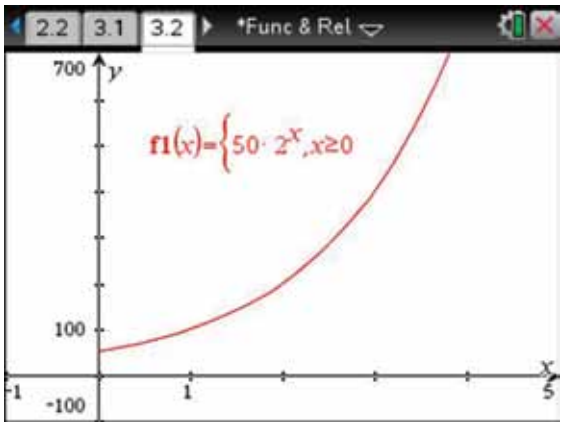
a–b



Initially there are 50 bacteria present, and after 3 hours there are 400 bacteria present.

c Open a Graphs page in the current document. Since the function has already been entered, just select the function and press ENTER [enter], and the graph will be displayed. However, reset the viewing window to a more appropriate scale as shown.

c



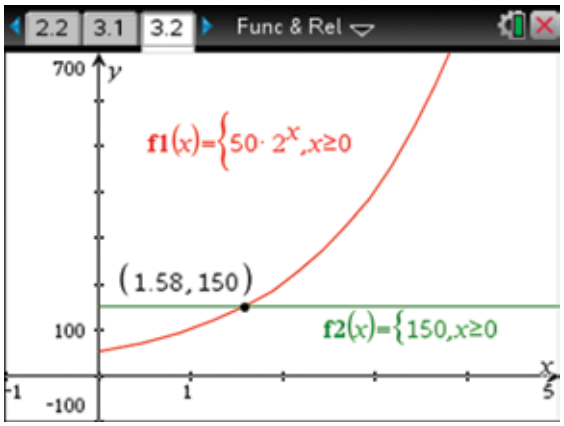
d Now enter the function as:
$f2(x) = 150 \mid x \geq 0$

Then press ENTER [enter]. The graph will be displayed. To find the point of intersection between the two graphs, press:

- MENU [menu]
- 6: Analyze Graph [6]
- 4: Intersection [4].

Move the cursor to the left of the intersection point, and press ENTER [enter]. Then move the cursor to the right of the intersection point and press ENTER [enter]. The intersection point is displayed.

d



The point of intersection is at (1.58, 150). The initial number of bacteria will treble after 1.58 hours.

WORKED EXAMPLE 12

Sketch the following, showing all intercepts.

a $y = (x - 2)(x - 3)(x + 5)$
b $y = (x - 6)^2(4 - x)$
c $y = (x - 2)^3$

THINK

a In a new problem, on a Calculator page, complete the entry lines as:
Define $f1(x) = (x - 2)(x - 3)(x + 5)$

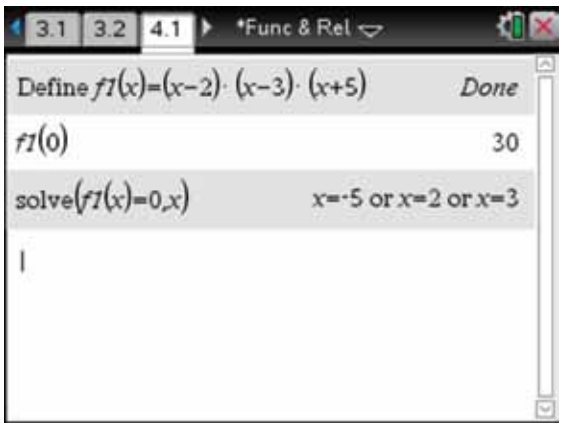
$f1(0)$

$\text{solve}(f1(x) = 0, x)$

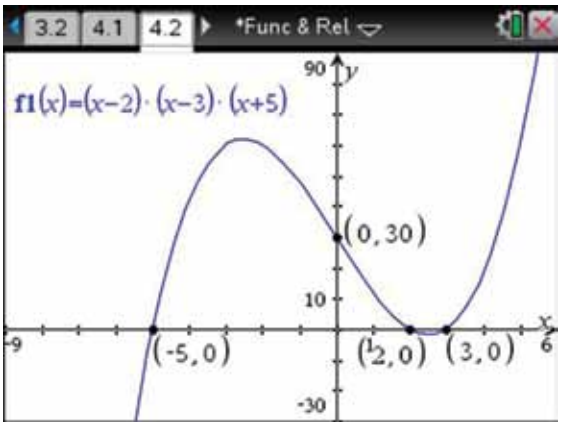
Press ENTER [enter] after each entry.
Remember to include the implied multiplication sign between the brackets.

WRITE

a



Open a Graphs page in the current document. Since the function has already been entered, just select the function and press ENTER enter, and the graph will be displayed. Reset the viewing window to a more appropriate scale as shown. This graph does cross the x-axis at three distinct points. Find all the axial intercepts as described earlier.



The y-intercept is (0, 30) and the x-intercepts are (–5, 0), (2, 0) and (3, 0).

b On a Calculator page, complete the entry lines as:

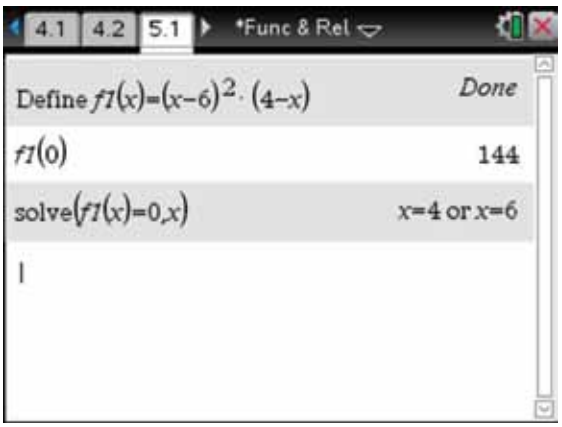
Define $f1(x) = (x-6)^2(4-x)$

$f1(0)$

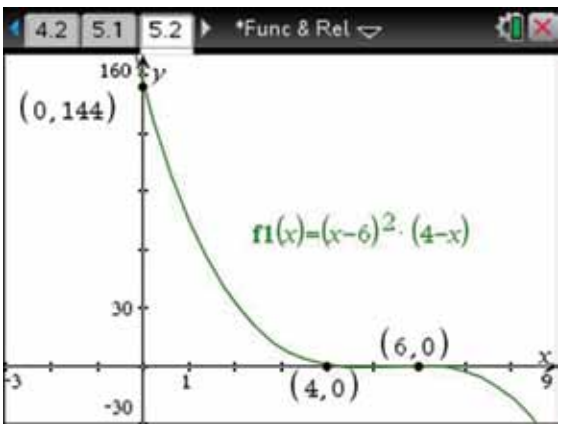
solve$(f1(x) = 0, x)$

Then press ENTER enter.
Remember to include the implied multiplication sign between the brackets.

b



Open a Graphs page in the current document. Since the function has already been entered, just select the function and press ENTER enter, and the graph will be displayed. Reset the viewing window to a more appropriate scale as shown. This graph does cross the x-axis at two distinct points; however, this is not clear from the graph shown. Find all the axial intercepts as described earlier.



The y-intercept is (0, 144) and the x-intercepts are (4, 0) and (6, 0).

c In a new problem, on a Calculator page, complete the entry lines as:

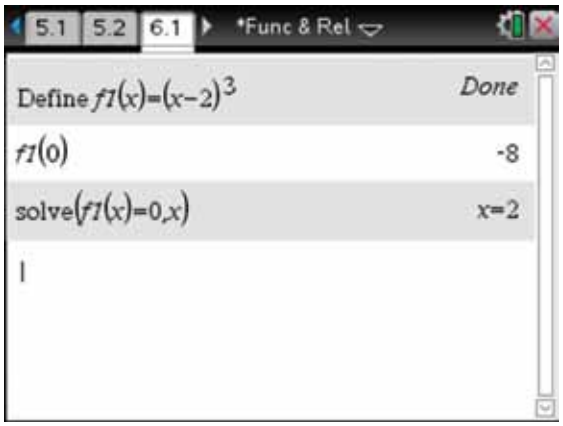
Define $f1(x) = (x-2)^3$

$f1(0)$

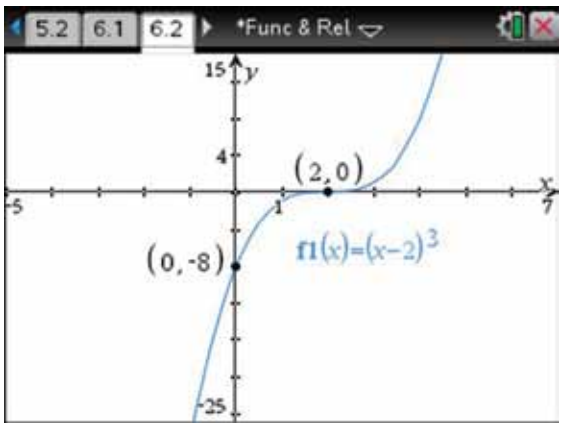
solve$\left(f1(x) = 0, x\right)$

Press ENTER enter after each entry.

c



Open a Graphs page in the current document. Since the function has already been entered, just select the function and press ENTER enter, and the graph will be displayed. Reset the viewing window to a more appropriate scale as shown. Find all the axial intercepts as described earlier.



The y-intercept is (0, –8) and the x-intercept is (2, 0). For this example there is only one x-intercept as it is a triple factor; this point is called a point of inflexion.

WORKED EXAMPLE 13

Sketch the graph of $y = x^4 - 2x^3 - 7x^2 + 8x + 12$, showing all intercepts.

THINK

In a new problem, on a Calculator page, complete the entry lines as:

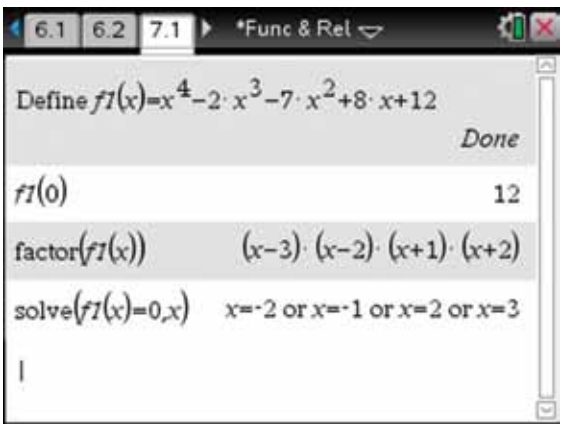
Define $f1(x) = x^4 - 2x^3 - 7x^2 + 8x + 12$

$f1(0)$

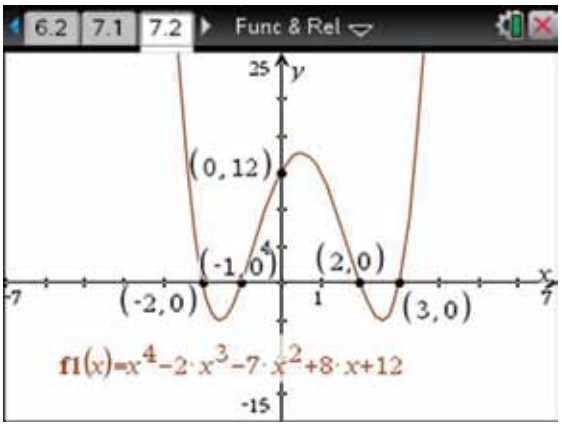
solve$\left(f1(x) = 0, x\right)$

Press ENTER enter after each entry.

WRITE



Open a Graphs page in the current document. Since the function has already been entered, just select the function and press ENTER [enter], and the graph will be displayed. Reset the viewing window to a more appropriate scale as shown. This graph does cross the x-axis at four distinct points. Find all the axial intercepts as described earlier.



The y-intercept is (0, 12) and the x-intercepts are (−2, 0), (−1, 0), (2, 0) and (3, 0).

TOPIC 20

Trigonometry II

WORKED EXAMPLE 1

In the triangle ABC, $a = 4$ m, $b = 7$ m and $B = 80°$. Find A, C and c.

THINK

Open a new document and a Calculator page. Ensure your calculator is set to the degree and approximate mode, as shown for the next set of examples.
To do this, press:
- HOME [on]
- 5: Settings [5]
- 2: Document Settings [2].

TAB [tab] to Angle and select 'Degree'. TAB [tab] to Calculation Mode and select 'Approximate'. TAB [tab] to OK and press ENTER [enter].

1 On a Calculator page, press TRIG [trig] to access and select the appropriate trigonometric ratio, in this case $\sin^{-1}$. Then complete the entry line as:

$$\sin^{-1}\left(\frac{4\sin(80)}{7}\right)$$

To convert the decimal degree into degrees, minutes and seconds, press:
- CATALOG [book]
- 1: [1]
- d: [D].

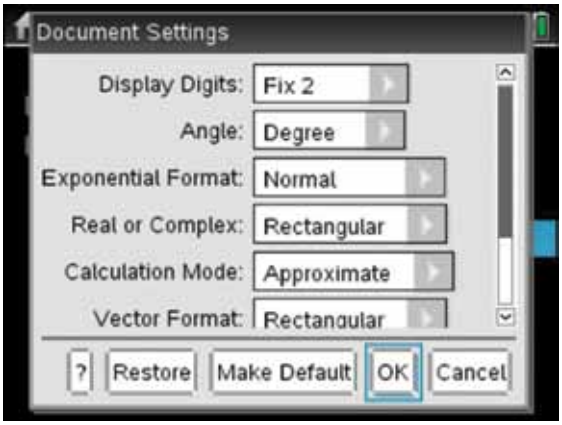
Scroll to and select ▶DMS. Press ENTER [enter] after each entry as shown.

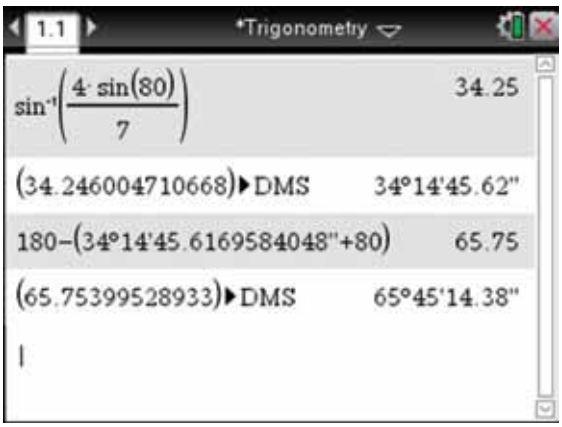
2 To find the value of c, complete the entry line as:

$$\frac{7\sin(65°45')}{\sin(80)}$$

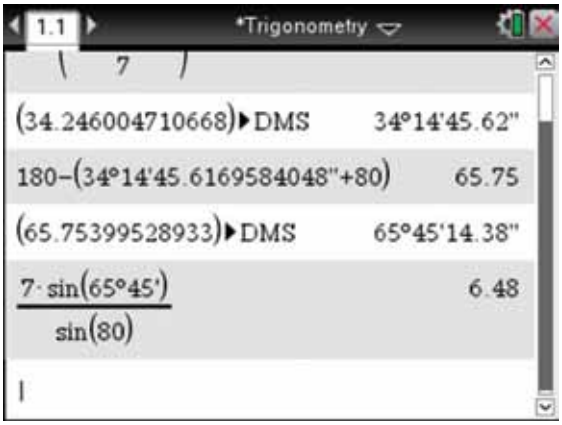
Then press ENTER [enter].

WRITE





$A = 34°15'$ and $C = 65°45'$



$c = 6.48$ m

WORKED EXAMPLE 2

In the triangle ABC, $a = 10$ m, $c = 6$ m and $C = 30°$. Find two possible values of A, and hence two possible values of B and b.

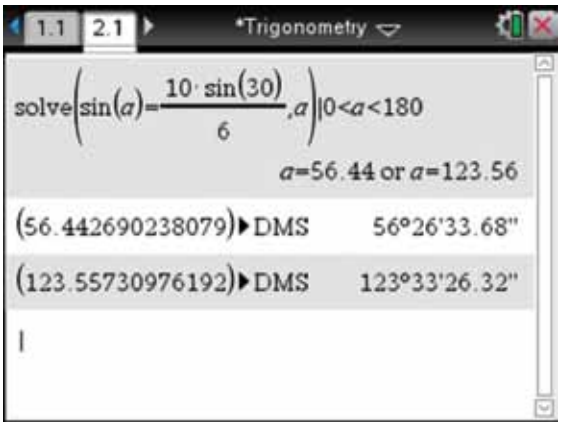
THINK

1 In a new problem, on a Calculator page, complete the entry line as:

$$\text{solve}\left(\sin(a)=\frac{10\sin(30)}{6}\middle|\, 0<a<180\right)$$

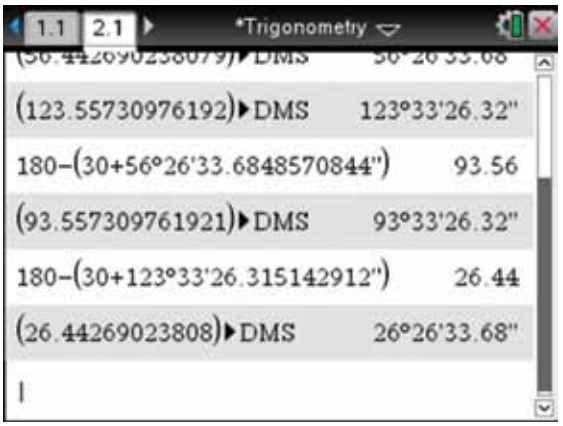
Convert these angles to degrees and minutes as shown in Worked Example 1. Press ENTER [enter] after each entry.

WRITE/DRAW



$A = 56°26'$ or $123°34'$

2 Solve for the two values of B as shown in the screenshot. Press ENTER [enter] after each entry. Convert these angles to degrees and minutes.



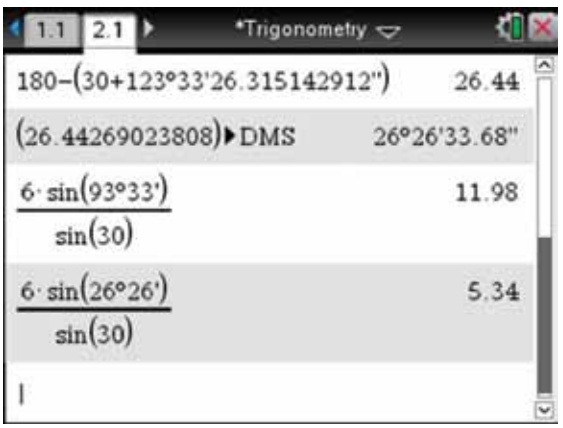
$B = 93°34'$ or $26°26'$

3 To solve for the two values of b, complete the entry lines as:

$$\frac{6\sin(93°34')}{\sin(30)}$$

$$\frac{6\sin(26°26')}{\sin(30)}$$

Press ENTER [enter] after each entry.



$b = 11.98$ m or 5.34 m

WORKED EXAMPLE 5

Find the smallest angle in the triangle with sides 4 cm, 7 cm and 9 cm.

THINK

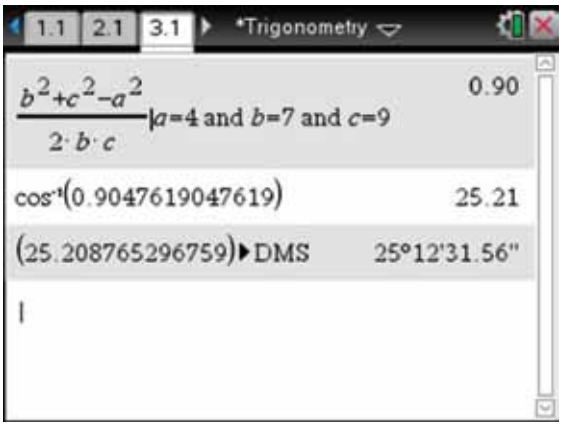
In a new problem, on a Calculator page, complete the entry lines as:

$\frac{b^2+c^2-a^2}{2bc} \mid a=4 \text{ and } b=7 \text{ and } c=9$

$\cos^{-1}(0.90476)$

Convert the angle to DMS as shown previously.
Press ENTER enter after each entry.

WRITE



The smallest angle is 25°13′ rounded up to the nearest minute.

WORKED EXAMPLE 8

A triangle has known dimensions of a = 5 cm, b = 7 cm and B = 52°. Find A and C and hence the area.

THINK

In a new problem, open a Calculator page. To find the angle A, press TRIG trig to access and select the appropriate trigonometric ratio ($\sin^{-1}$). Then complete the entry line as:

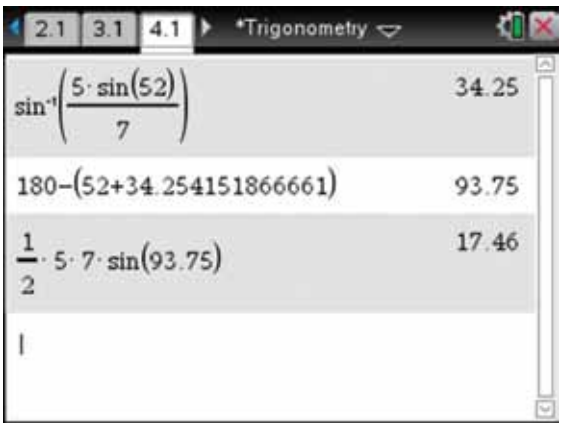
$\sin^{-1}\left(\frac{5\sin(52)}{7}\right)$

Note that you can leave the angle in Decimal Degrees and work with this value.
Find the value of C as shown in the screenshot.
Then find the area by completing the entry line as:

$\frac{1}{2} \times 5 \times 7 \sin(93.75)$

Then press ENTER enter.
Remember to include the implied multiplication signs.

WRITE



$A = 34.25°$
$C = 93.75°$
The area of the triangle is 17.46 cm^2.

WORKED EXAMPLE 11

Find the approximate value of each of the following.
a sin 200° **b cos 200°** **c tan 200°**

THINK

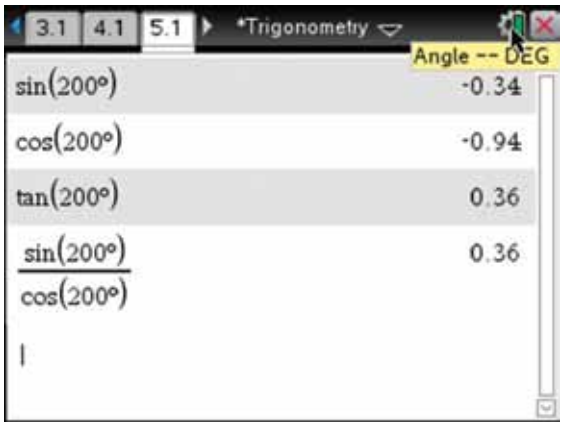
a–c In a new problem, on a Calculator page, press TRIG [trig] to access and select the appropriate trigonometric ratio. Then complete the entry lines as:

$\sin(200)$
$\cos(200)$
$\tan(200)$

Press ENTER [enter] after each entry.
Note that if you hover over the settings icon, as shown in the screenshot, the Angle settings are shown to be in Degrees.
The values are shown correct to two decimal places.

WRITE

a–c



$\sin(200) = -0.34$
$\cos(200) = -0.94$
$\tan(200) = 0.36$

WORKED EXAMPLE 12

Sketch the graphs of a $y = 2\sin x$ and b $y = \cos 2x$ for $0° \leq x \leq 360°$.

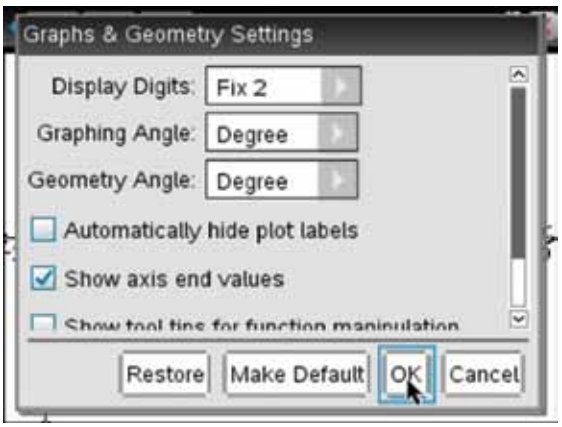
THINK

a 1 In a new problem, on a Graphs page, ensure the Graphs & Geometry Settings are set to the degrees mode, as shown in the screenshot.
To do this, press:
- MENU [menu]
- 9: Settings [9].

TAB [tab] to Graphing Angle and select 'Degree'. TAB [tab] to OK and press ENTER [enter].

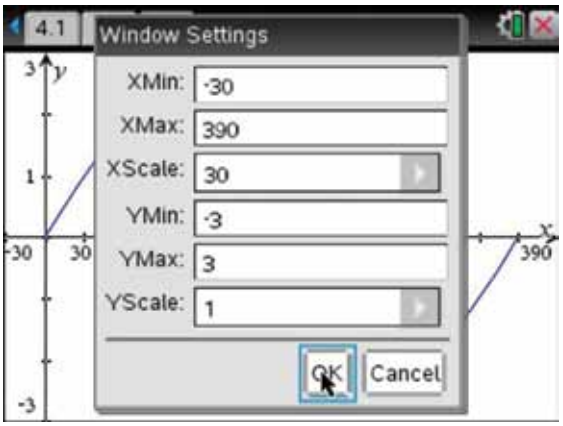
WRITE

a

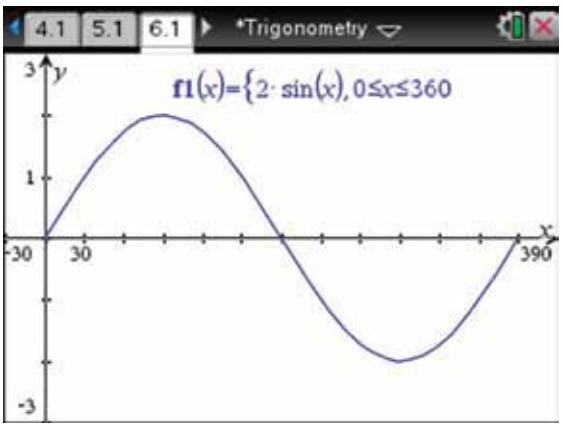


2 To set an appropriate viewing window, press:
- MENU [menu]
- 4: Window/Zoom [4]
- 1: Window Settings [1].

Select the values as shown in the screenshot. TAB [tab] to OK and press ENTER [enter].

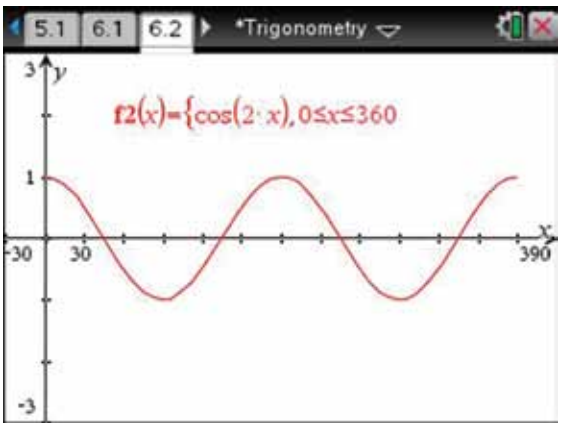


3 Complete the function entry line as:
$f1(x) = 2\sin(x) \mid 0 \le x \le 360$
Press ENTER [enter]. The graph is displayed as required for $0 \le x \le 360$.



b Complete the function entry line as:
$f1(x) = \cos(2x) \mid 0 \le x \le 360$
Press ENTER [enter], and the graph is displayed, as required only for $0 \le x \le 360$.

b



WORKED EXAMPLE 14

Solve the following equations.

a $\sin x = \frac{\sqrt{3}}{2}, x \in [0°, 360°]$

b $\cos 2x = -\frac{1}{\sqrt{2}}, x \in [0, 360°]$

THINK

a In a new problem, on a Calculator page, complete the entry line as:

$$\text{solve}\left(\sin(x) = \frac{\sqrt{3}}{2}, x\right) \mid 0 \le x \le 360$$

Then press ENTER [enter].
Note that the calculator is set to the degrees mode.

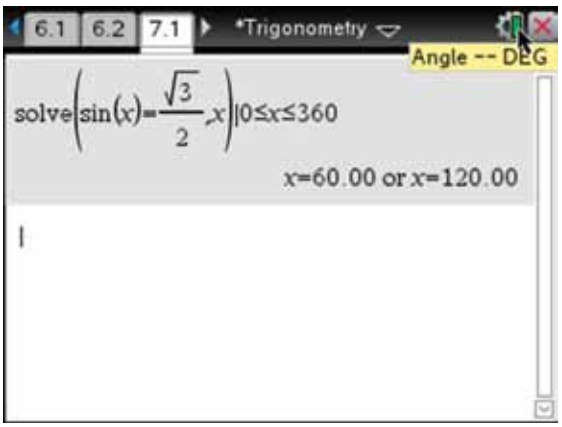
b On a Calculator page, complete the entry line as:

$$\text{solve}\left(\cos(2x) = -\frac{1}{\sqrt{2}}, x\right) \mid 0 \le x \le 360$$

Then press ENTER [enter].
Note that the calculator is set to the degrees mode.

WRITE

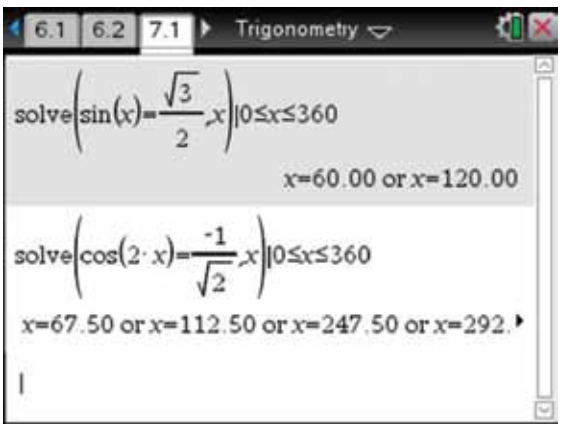
a



$\sin(x) = \frac{\sqrt{3}}{2}$ for $x \in [0°, 360°]$

$\Rightarrow x = 60°$ or $120°$

b



$\cos(2x) = -\frac{1}{\sqrt{2}}$ for $x \in [0°, 360°]$

$\Rightarrow x = 67.5°, 112.5°, 247.5°$ or $292.5°$