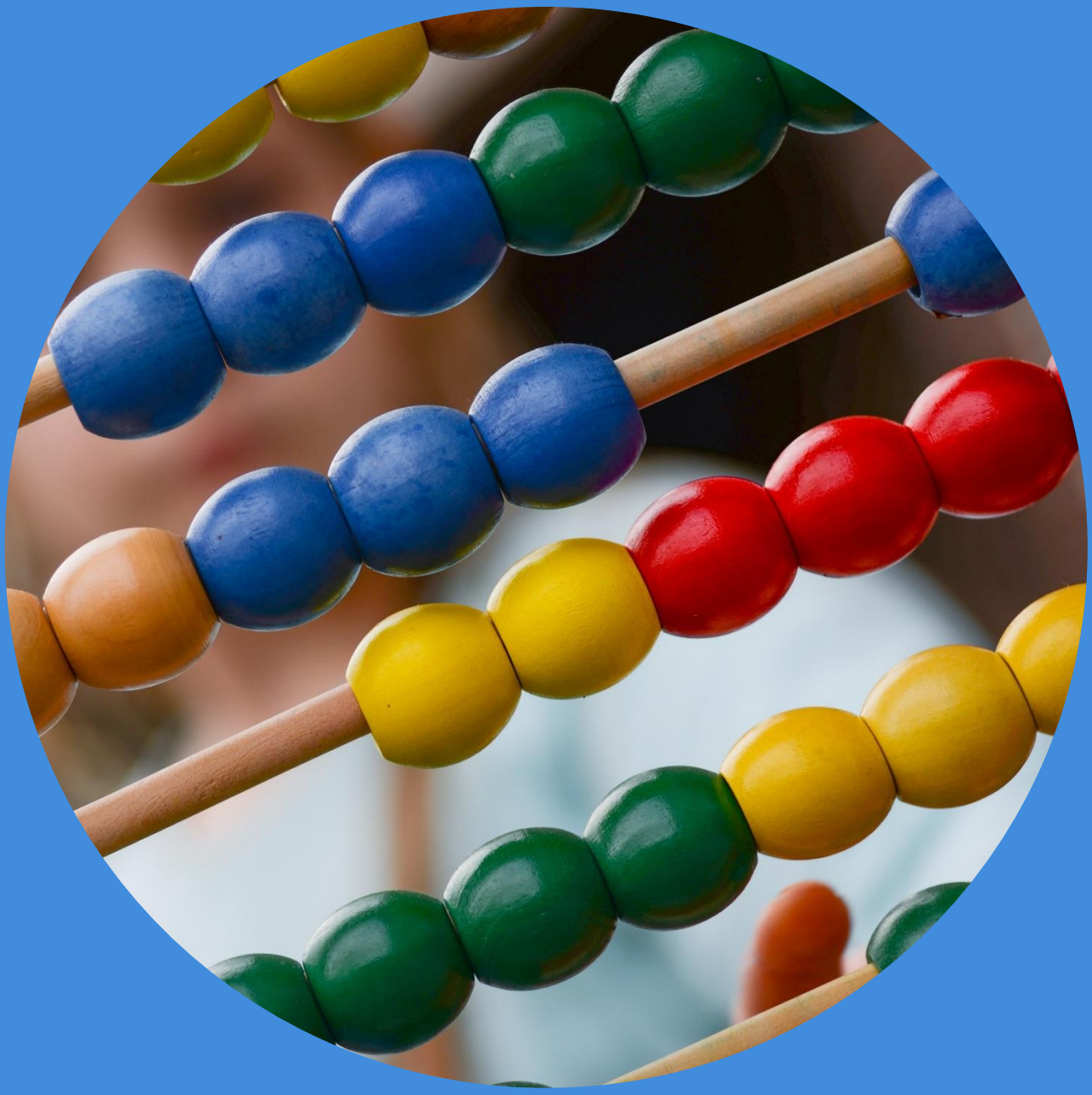

Maths

Key Stage 3 Tutor Guidance

BRILLIANT TUTORING PROGRAMME — T — BRILLIANT TUTORING PROGRAMME

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Module 5 - Angles

This module covers how to find missing angles in various shapes and geometric problems using the angle rules. Review the pupils' answers to the before-module knowledge check to help you identify which areas they seem comfortable with and which areas to focus on and develop.

Tutorial	Topic
Tutorial 1.1	Basic angle rules
Tutorial 1.2	Triangles and quadrilaterals
Tutorial 1.3	Parallel lines
Tutorial 1.4	Polygons

Learning objectives

This module aims to help pupils:

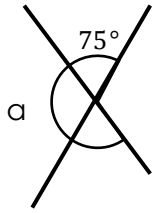
1. Find missing angles using basic angles rules
2. Find missing angles in triangles and quadrilaterals
3. Find missing angles using parallel lines
4. Find the angles of polygons

Knowledge Check #1

Where possible, encourage students to complete this online:

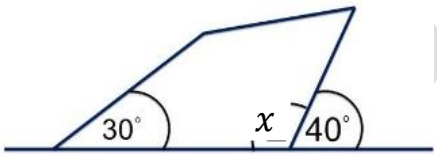
https://forms.office.com/r/HUaiGPUHMe

1 What is the size of angle a?



15° 75° 285° 105°

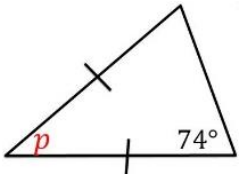
2 What is the value of the angle marked x?



140° 70° 110° 150°

3 This is an isosceles triangle.

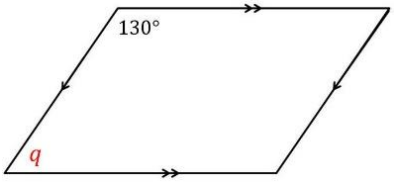
Calculate the size of angle p.



32° 74° 106° Not enough information

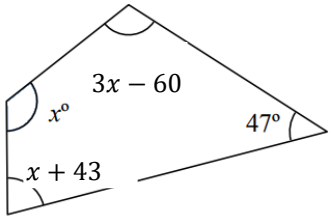
4 This is a parallelogram.

Calculate the size of angle q.



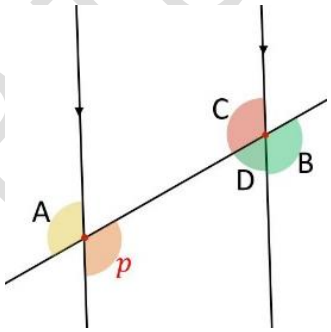
65° | 130° | 50° | Not enough information

5 What is the value of x?



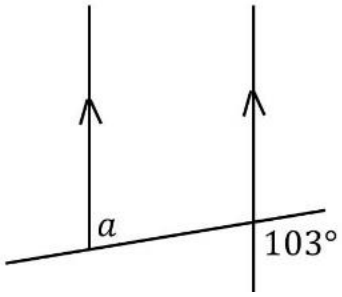
30 | 66 | 90 | 300

6 Which angle is not equal to angle p?



A | B | C | D

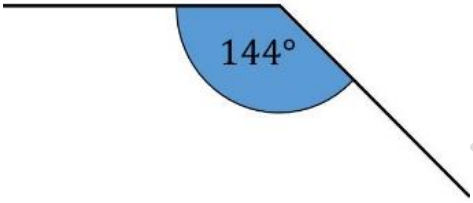
7 Find the missing angle a.



77° | 103° | 257° | Not enough information

8 This is part of a regular polygon.

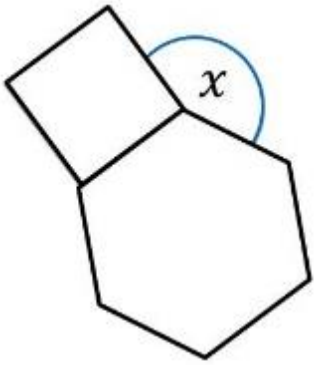
How many sides does it have?



6 | 5 | 10 | Not enough information

9 These are both regular polygons.

Calculate the unknown angle x.



210° | 180° | 225° | 150°

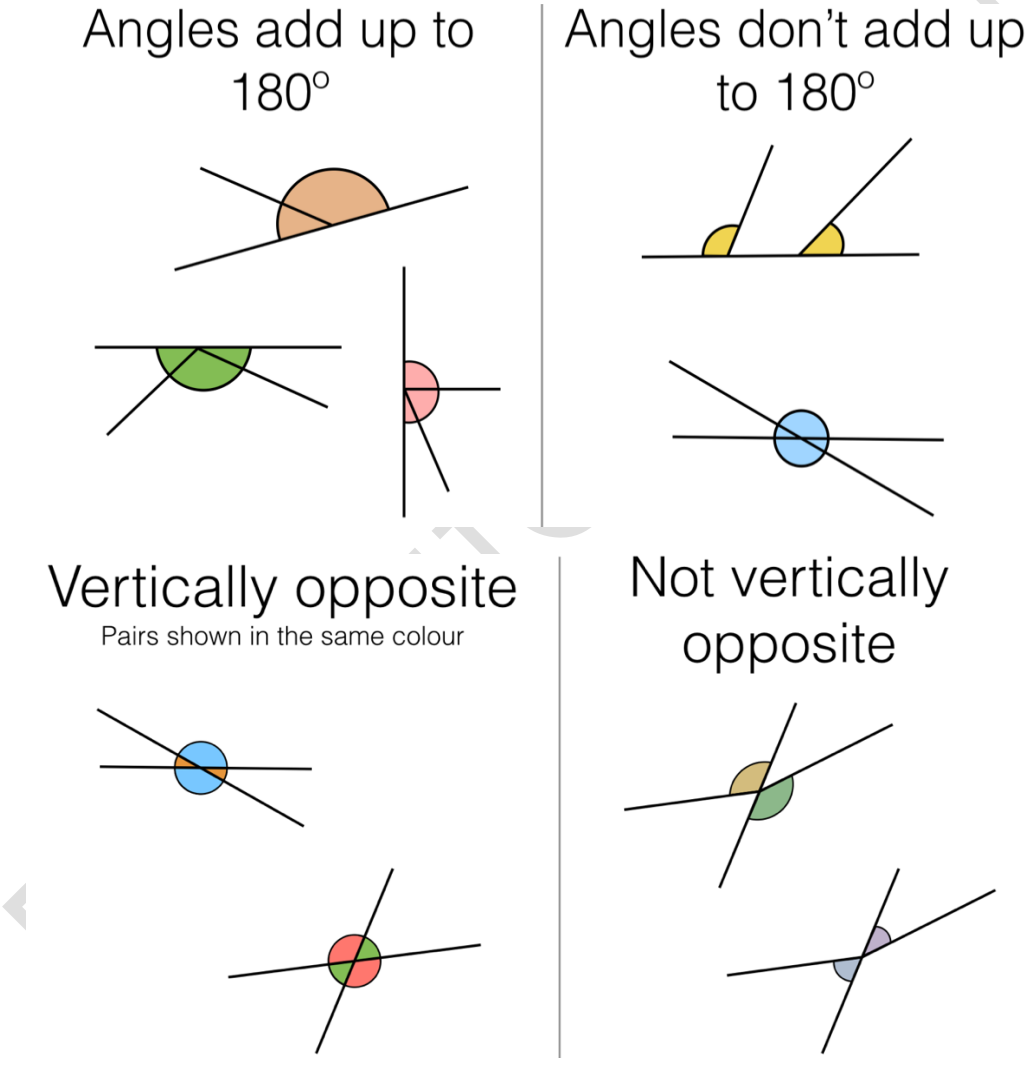
Tutorial 5.1 – Basic angle rules

In this tutorial we will look at:

- Using basic angle rules to find missing angles
- Using algebra to find missing angles

Angles on lines and points

Ensure first that students are comfortable with the individual rules. Beware with the straight line rule that only adjacent angles are considered, and for vertically opposite angles the lines must be straight. The following diagrams may be useful in illustrating these points:



Angle rules with algebra

Ensure that students are happy with the fact that "sum" or "total" are synonyms for addition. If students are struggling to write down the equation, then ask them to explain what they would do if there were no variables and just numbers. Always write down the

full expression before simplifying and encourage students to re-read the question to ensure that they are giving the answer expected.

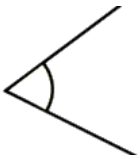
Learning activities

Angles on lines and points

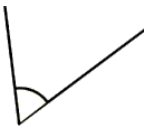
1. Warm up

Estimate the following angles

1. **60°**

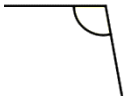


2. **60°**

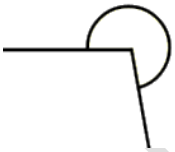


3. **140°**

4. **100°**



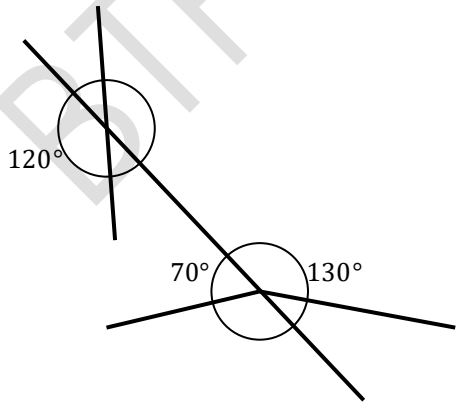
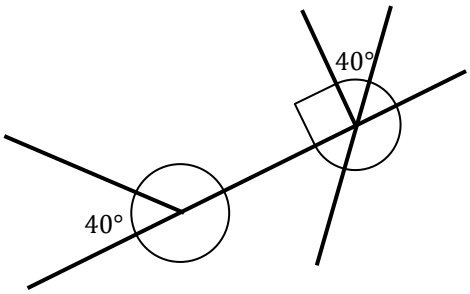
5. **260°**



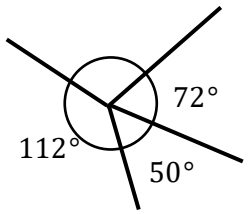
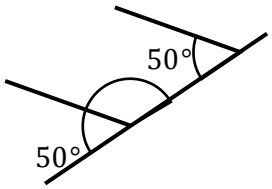
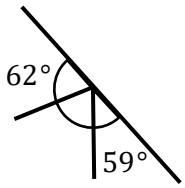
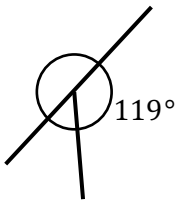
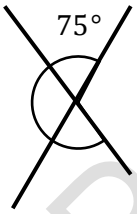
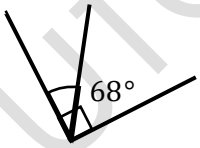
6. **340°**

Accept answers within ±10°

2. Guidance

Worked example	Guided practice
Find the missing angles	Find the missing angles
60°, 120°, 60° **110°, 50°**	**140°, 180°** **50°, 130°, 50°**
	

3. Practice

Fluency	Problem Solving
Find the size of each missing angle	1. Three of these angles together make a straight line. Which three?
126° 	**130°** 
59° 	**180°,61°** 
105°, 75° 	**22°** 
	24° 36° 120° 2. Which of these are impossible? a. acute + acute = right angle **b. obtuse + obtuse = right angle** **c. obtuse + obtuse = straight line** **d. acute + obtuse = full turn** acute + right = obtuse angle

Angle rules with algebra

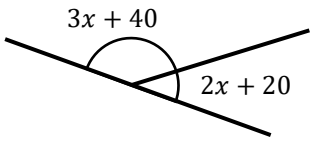
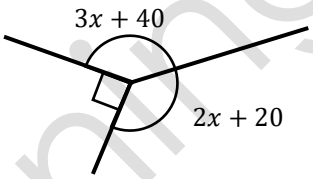
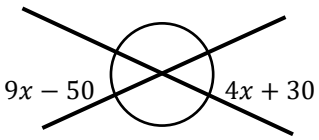
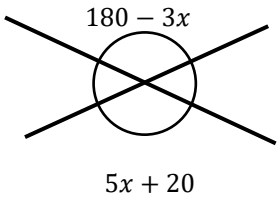
1. Warm up

Solve the following equations

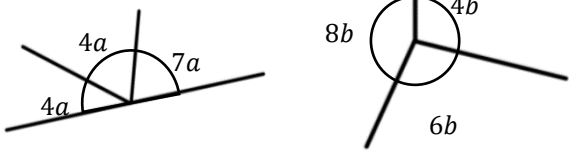
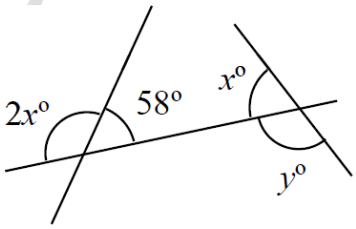
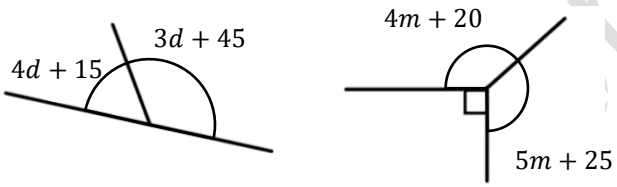
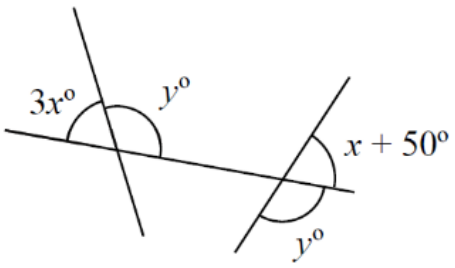
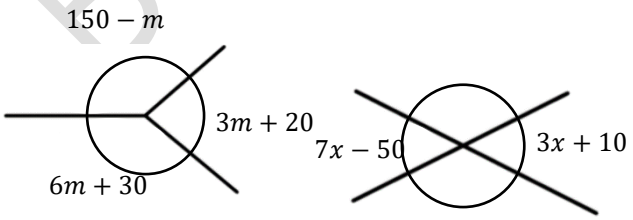
1. $2x = 180, x = 90$
2. $3x = 360, x = 120$
3. $x + 140 = 360, x = 220$
4. $5x + 40 = 180, x = 28$
5. $2x - 30 = x + 40$, x=70
6. $5x - 40 = 2x + 80$

$x = 40$

2. Guidance

Worked example	Guided practice
Find the value of x, then calculate all the missing angles	Find the value of x, then calculate all the missing angles
x=24, angles = 112°, 68°	**x=42, angles = 166°, 104°**
	
x=16, angles = 94°	**x=20, angles = 120°**
	

3. Practice

Fluency	Problem Solving
Work out this value of the letters, then find the missing angles.	1. Three angles fit around a point The second angle is 20° more than the first. The third angle is twice the size of the second. Find the size of all three angles.
a=12, angles = 48°, 48°, 84°	**1st angle = x** **2nd angle = x+20** **3rd angle = 2(x+20)=4x+40** **x = 50, angles = 50°, 70°, 240°**
b=20, angles = 160°, 80°, 120°	
	
d=120/7, angles= 83.57°, 96.43°	2. Find the values of x and y: a.
m=25, angles = 120°, 150°	
	**x=61, y=119**
[bottom left] m = 20, angles = 130°, 150°, 80°	b. **x=25, y=105**
[bottom right] x=15, angles = 55°, 55°, 125°, 125°	
	

Tutorial 5.2 – Triangles and quadrilaterals

In this tutorial we will look at:

- Using angle rules to find missing angles in triangles and quadrilaterals
- Using algebra to find missing angles in triangles and quadrilaterals

Angles in triangles

Common misconceptions include pairing up the incorrect angles in an isosceles triangle and using exterior angles when calculating the interior angle sum of a triangle. This activity will lead students through progressively more complicated examples designed to address these misconceptions. Tutors should encourage students to show full workings for each angle calculation, and to label the diagram as they go.

Angles in quadrilaterals

Many of the same strategies for the previous learning episode apply to this one. Using lines of symmetry or rotational symmetry can help show students which angles must be equal in separate quadrilaterals. As an introduction to Session 4 there could be an exploration of why the angle sum of a quadrilateral is $360°$.

Triangles, quadrilaterals and algebra

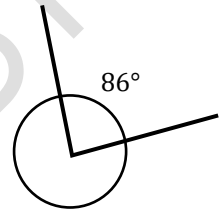
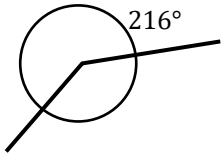
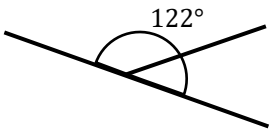
This combines skills from the previous learning episodes along with the end of Session 1. Students should once again write down all expressions before simplifying, taking care with negative signs when finding interior angles given an exterior. As a final check, students can substitute their values for x back into the expressions to see if the interior angle sums are consistent with the rule.

Learning activities

Angles in triangles

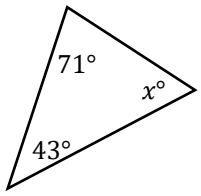
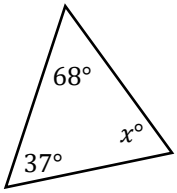
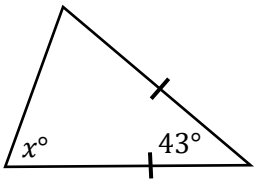
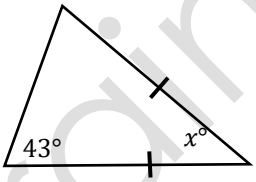
1. Warm up

Find the missing angles

1.	2.	3.
		

1. 274° **2. 144°** **3. 58°**

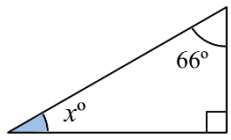
2. Guidance

Worked example	Guided practice
Find the size of angle x in the following	Find the size of angle x in the following
	
x=66°	x=75°
	
x=68.5°	x=94°

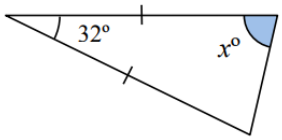
3. Practice

Fluency	Problem Solving

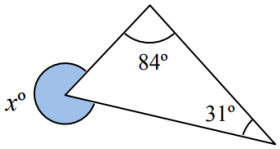
Find the size of angle x in each of the following



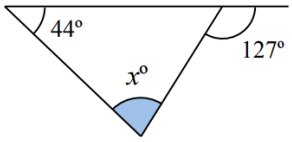
x=24°



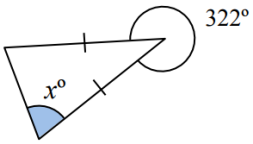
x=74°



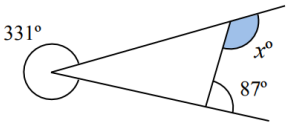
x=295°



x=83°



x=71°



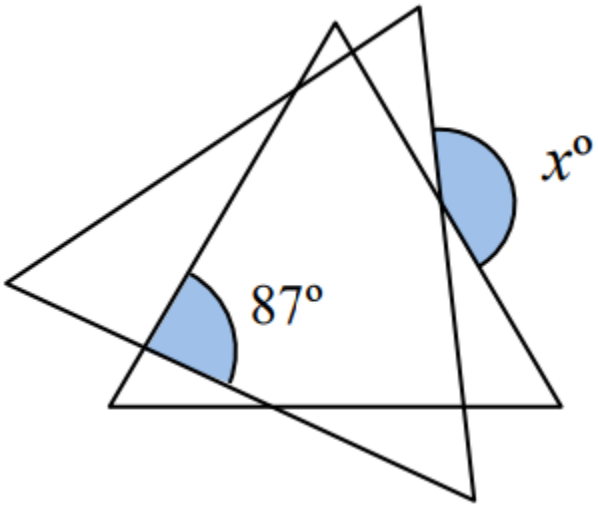
x=122°

These two triangles are equilateral. Find the value of x

Equilateral triangles have angles of 60°

Work round from 87° + y = 180°

x=153°



Angles in quadrilaterals

1. Warm up

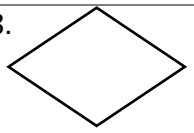
Name each shape and label their equal angles with the same letter.

1. **Square, all angles equal**

2.

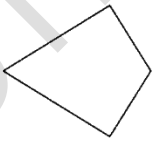
Rectangle, all angles equal

3.



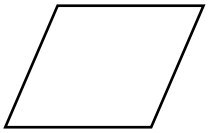
Rhombus, opposing angles equal

4.



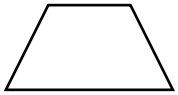
Irregular quadrilateral, no equal angles

5.



Parallelogram, opposing angles equal

6.

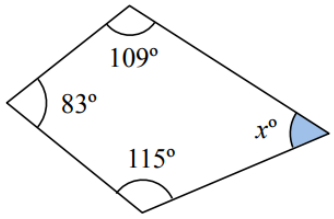


Trapezium, base angles equal, top angles equal

2. Guidance

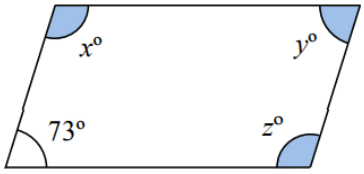
Worked example	Guided practice

Work out the missing angles

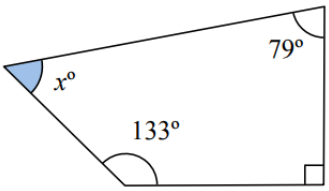


x=53°

x=z=107°, y=73°

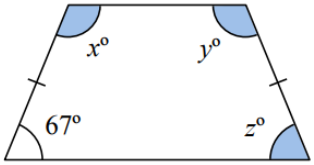


Work out the missing angles



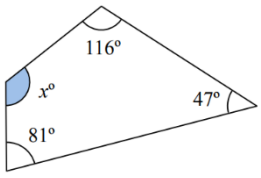
x=58°

x=67°, y=113°

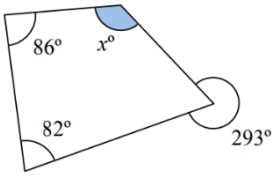


3. Practice

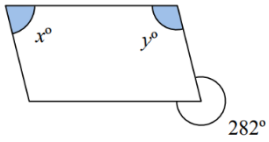
Fluency	Problem Solving
Work out the missing angles	Find the missing angles in the following shapes



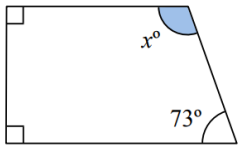
X=116°



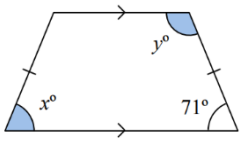
x=125°



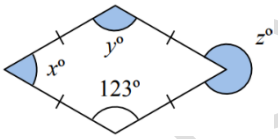
x=78°, y=102°



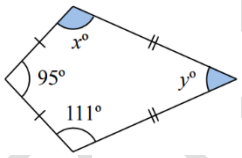
x= 107°



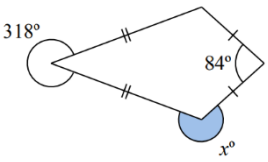
X=71°, y=109°



y=123°, x=57°, z=303°

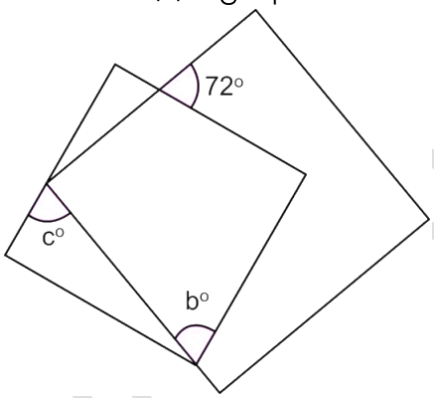


X=111°, y=43°



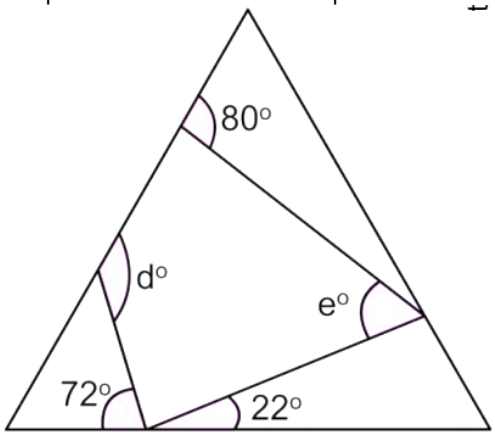
x=243°

1. Two overlapping squares



b=72°, c=72°

2. A quadrilateral in an equilateral triangle



d=132°, e=42°

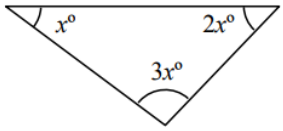
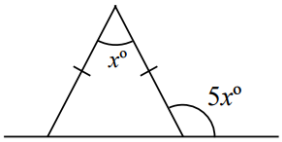
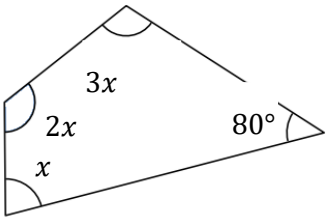
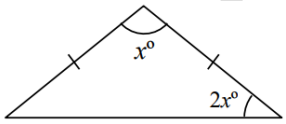
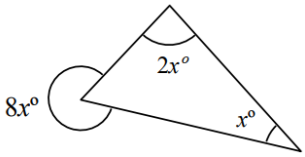
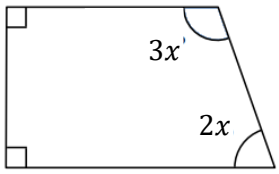
Triangles, quadrilaterals and algebra

1. Warm up

Solve the following equations

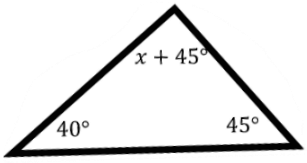
1. $4x = 180, x = 45$	2. $9x = 360,$ x = 40	3. $x + 70 = 180, x = 110$
4. $5x + 40 = 360$ x = 320	5. $3x - 70 = 2x + 80,$ x = 150	6. $5x - 80 = x - 20$ x = 15

2. Guidance

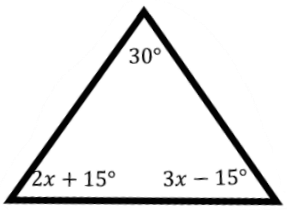
Worked example	Guided practice
Find the value of x  x=30 x=20  x=50  	Find the value of x  x=45  x=36 x=38 

3. Practice

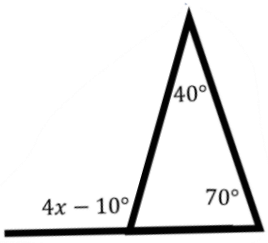
Fluency	Problem Solving
Find the values of x in each of the following	The interior angles of a triangle are $x°$, $2x°$ and $3x°$. Show that this is a right-angled triangle. **x+2x+3x=180 -> x=30, 3x = 90° -> right-angle triangle**



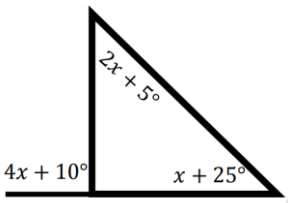
x=50



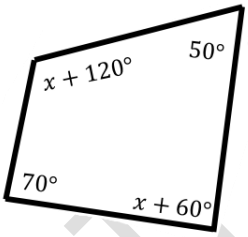
x=30



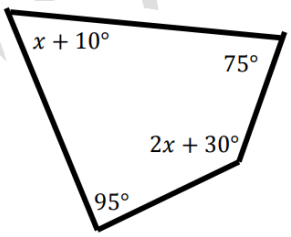
x=30



x=20



x=30



x=50

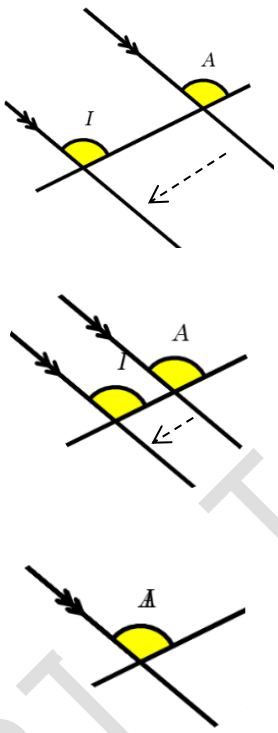
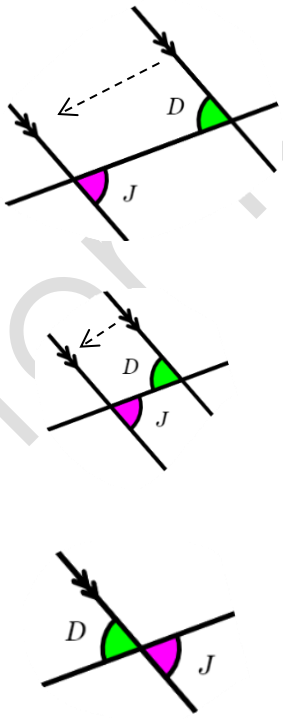
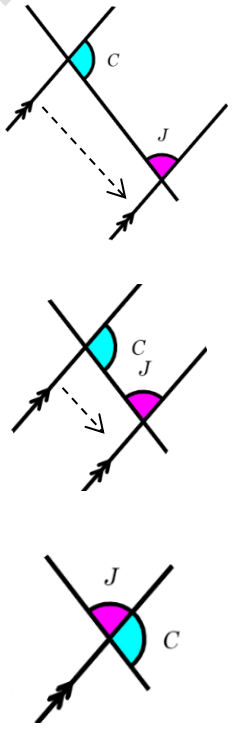
Tutorial 5.3 – Parallel lines

In this tutorial we will look at:

- Using angle rules to find missing angles on parallel lines
- Using algebra to find missing angles on parallel lines

Angles in parallel lines

Ensure that students are using the correct terminology of "alternate", "corresponding" and "co-interior" angles as opposed to "z-, f- and c-angles" as they do not receive marks for these in exams. To distinguish between these, encourage students to imagine sliding one parallel line over the other. Then corresponding angles match up, alternate angles are vertically opposite and co-interior angles are adjacent (see the below diagram):

Corresponding angles	Alternate angles	Co-interior angles
		

Shapes on parallel lines

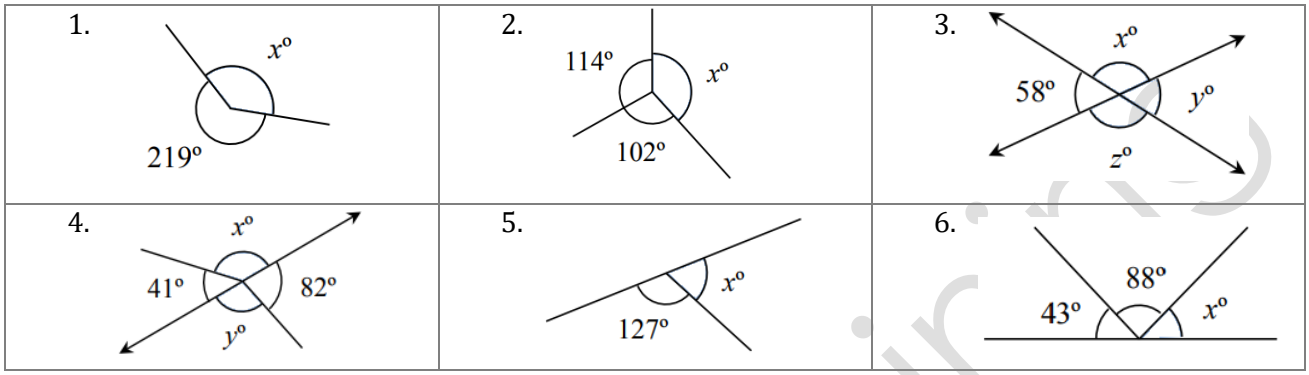
Rather than trying to find the missing angles directly, students should be instructed to find as many missing angles as they can, writing down their reasoning at each stage. Refer students to the previous Session for some strategies on the shapes.

Learning activities

Angles in parallel lines

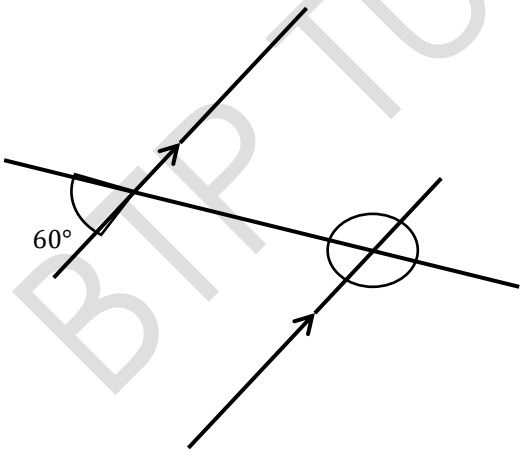
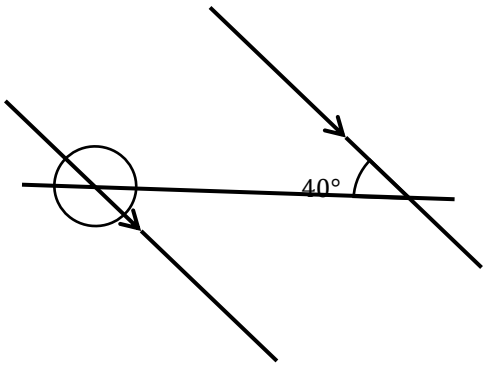
1. Warm up

Find the missing angles



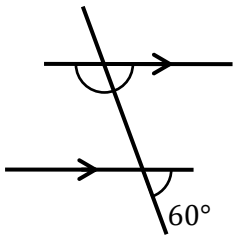
1. x=141° 2. x=144° 3. x=z=122°, y=58° 4. x=139°, y=98° 5. x=53° 6. x=19°

2. Guidance

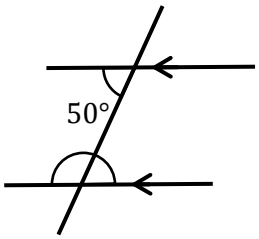
Worked example	Guided practice
Find the missing angles, giving a reason for your answer.	Find the missing angles, giving a reason for your answer.
60°, 120°, 60°, 120°	40°, 140°, 40°, 140°
	

3. Practice

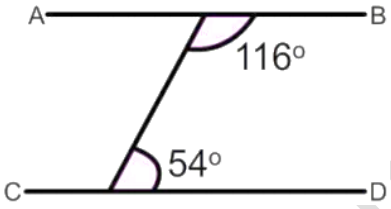
Fluency	Problem Solving
Find the missing angles, giving a reason for your answer.	Are lines AB and CD parallel? Give a reason for your answer.



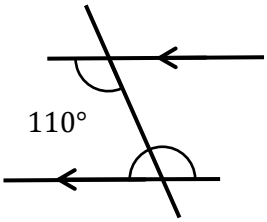
60°, 120°



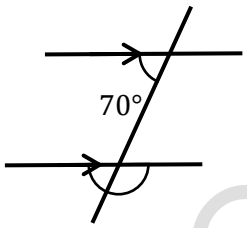
50°, 130°



126°

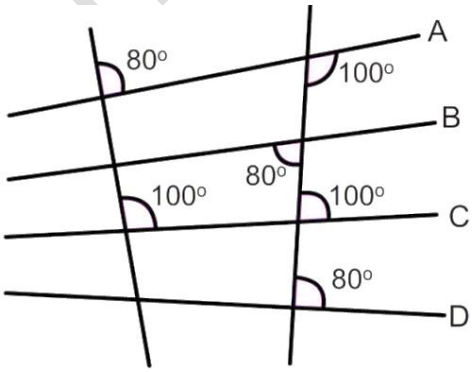


70°, 110°

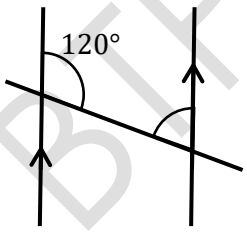


70°, 110°

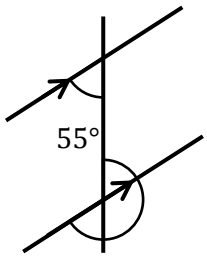
Which of the lines A, B, C and D are parallel to each other?



A, B, D



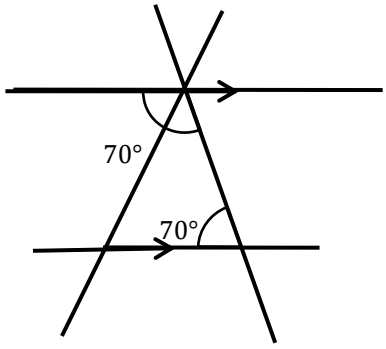
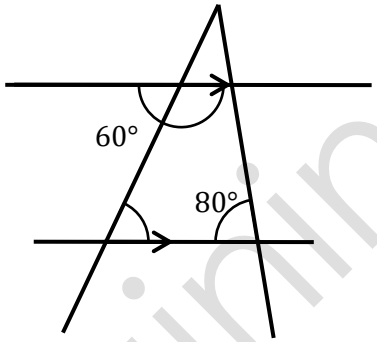
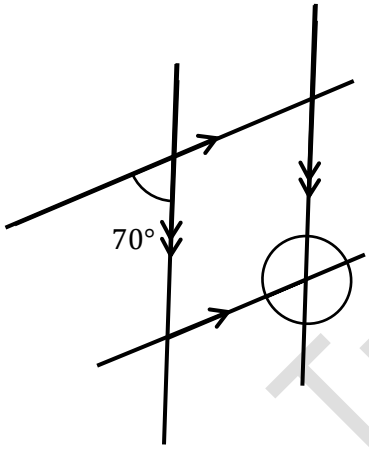
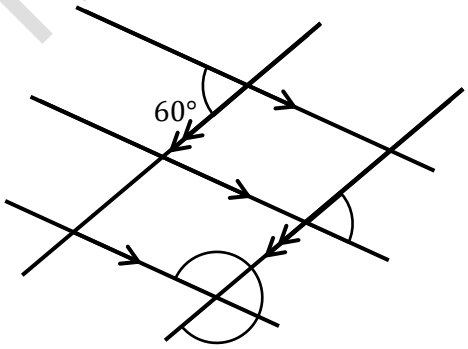
60°, 55°,



145°, 55°

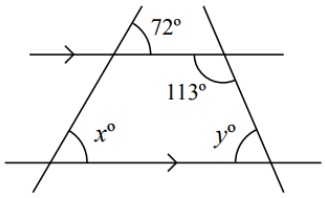
Shapes on parallel lines

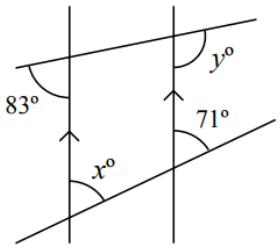
1. Guidance

Worked example	Guided practice
Work out the missing angles. Give a reason for your answers.	Work out the missing angles. Give a reason for your answers.
	
40°	120°, 60°
	
70°, 110°, 70°, 110°	60°, 60°, 120°, 120°

3. Practice

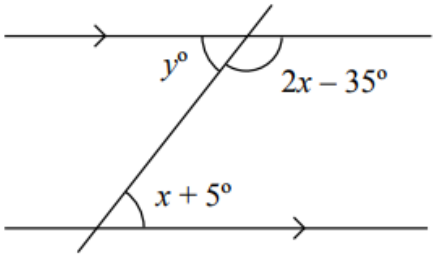
Fluency	Problem Solving
Find the values of the missing angles	Find the values of x and y





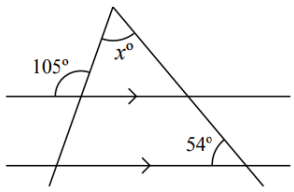
x=72°, y=67°

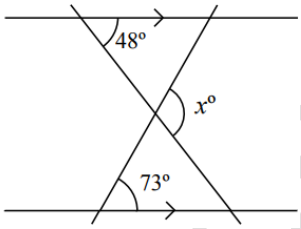
x=71°, y=97°



x=70

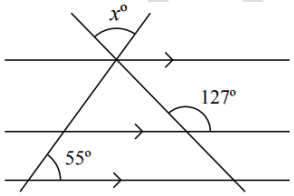
y=75°

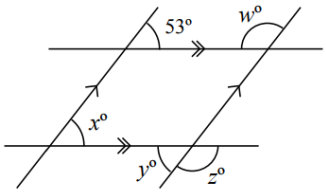




x=51°

x=121°





x=72°
w=z=127°

x=y=53°,

Tutorial 5.4 – Polygons

In this tutorial we will look at:

- Calculating the interior angles of regular polygons
- Calculating the exterior angles of regular polygons

Angle sum of polygons

Demonstrate the angle sum formula by subdividing each polygon into triangles. Encourage students to remember this process as it will be easier to recall rather than the abstract formula. Tutors can also draw attention to the fact that each extra side adds $180°$ to the interior angle sum

Exterior angles and regular polygons

This activity should help students decide on a method for working out interior and exterior angles of regular polygons. Try to emphasise the fact that the sum of exterior angles for *all* polygons is $360°$. A common misconception is that the exterior angle $360°$ subtract the interior angle, rather than $180°$. To avoid this, diagrams should always be drawn when dealing with exterior angles.

Polygon problems

As with the shapes in parallel lines sequence, students should be encouraged to find as many angles they can rather than directly try to find the missing one. All the shapes in this section are regular, and students can be referred to the previous activities this session to aid them in their calculations. Be aware that the angles involving heptagons will introduce decimals, students may think they have made a mistake here.

Learning activities

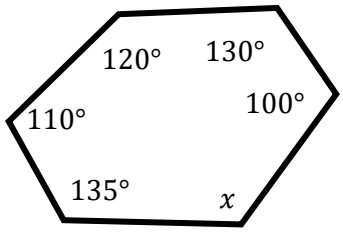
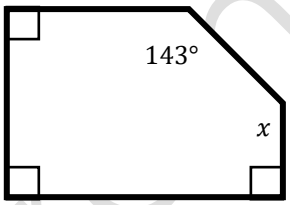
Angle sum of polygons

1. Warm up

Draw a sketch of the following polygons

1. Equilateral triangle	2. Regular pentagon	3. Irregular hexagon

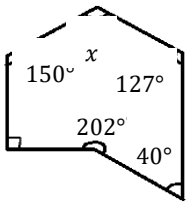
2. Guidance

Worked example	Guided practice
Calculate the interior angle sum of a hexagon. **720°** Find the missing angle in the following polygon.  **125°**	Calculate the interior angle sum of a pentagon. **540°** Find the missing angle in the following polygon. **127°** 

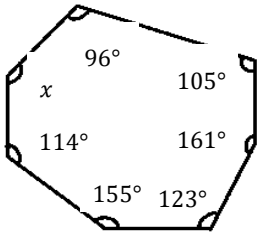
3. Practice

Fluency	Problem Solving

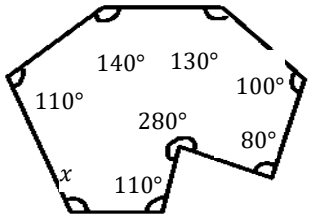
1. Calculate the interior angle sum of:
   a. a heptagon **900°**
   b. an octagon **1080°**
   c. a nonagon **1260°**
   d. a decagon **1440°**
   e. a dodecagon **1800°**
2. Find the missing angles in the following polygons



111°



146°



130°

The sum of the interior angles of a polygon is 2700°. Work out the number of sides the polygon has.

2700 = (x-2) x 180

17 sides

Exterior angles and regular polygons

1. Guidance

Worked example	Guided practice
Work out the exterior angle of a regular hexagon. **240°** Work out the interior angle of a regular hexagon. **120°**	Work out the exterior angle of a regular pentagon. **252°** Work out the interior angle of a regular pentagon. **108°**

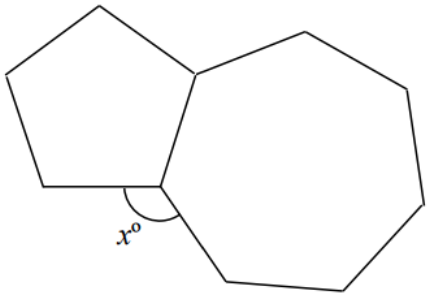
2. Practice

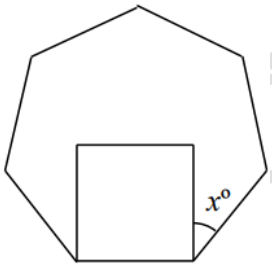
Fluency	Problem Solving
1. For each regular polygon, i. calculate the sum of the interior angles using the triangle method ii. work out the size of one interior angle iii. work out the size of one exterior iv. angle using "angles on a straight line" a. regular octagon b. regular decagon c. regular dodecagon 2. For each regular polygon in Question 1 i. work out the size of one exterior angle using the total of the exterior angles ii. work out the size of one interior angle using "angles on a straight line" iii. Work out the sum of the interior angles using your answer to part ii **As previously calculated**	The size of each interior angle of a regular polygon is $140°$ bigger than the size of each exterior angle. Work out the number of sides the polygon has. Not solvable, skip this question

Polygon problems

1. Guidance

Worked example	Guided practice
Work out the value of x **x=123.4°**	Work out the value of x **x=38.6°**

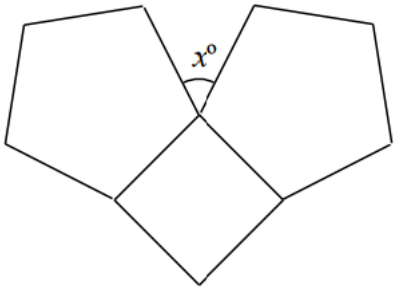


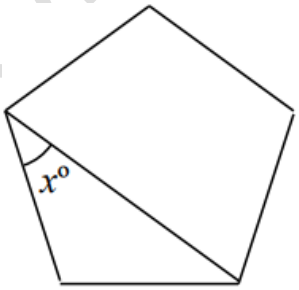


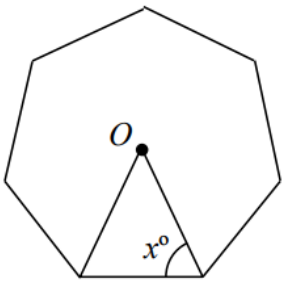
2. Practice

Problem solving

Work out the value of x in each of the following

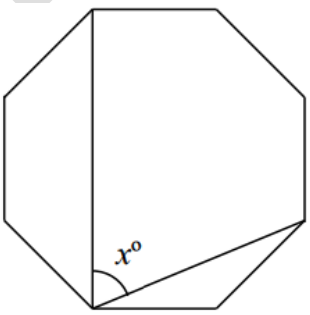


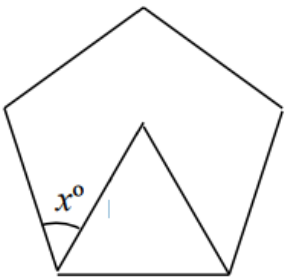


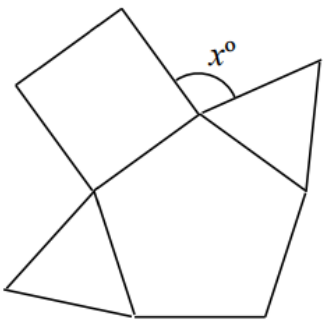


a. 54° **b. 36°** **c. 64.3°**

d.67.5° **e. 48°** **f. 102°**





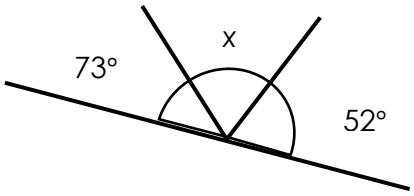


Knowledge Check #2: End of Module

At the end of this tutorial you will guide pupils through a set of confidence and Knowledge Check questions. You will also complete a reflection exercise so that pupils can take time to think about what they found challenging and where they did well – you'll find more details about this on the relevant tutorial slides.

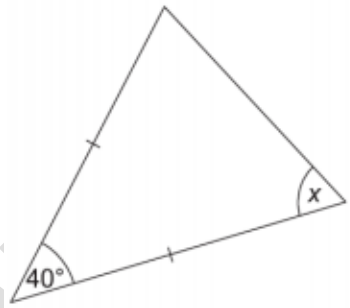
Correct answers for the Knowledge Check are below. Students can complete this online by going to: https://forms.office.com/r/5BHKVGCSNQ

1 What is the size of the angle marked x?



55° | 60° | 235° | 90°

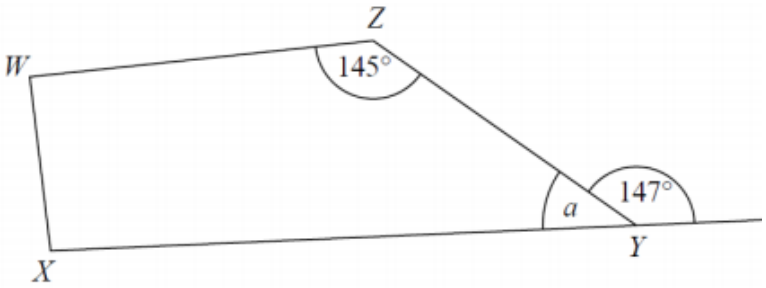
2 This diagram shows a triangle. What is the value of x?



160° | 70° | 40° | Not enough information

3 WXYZ is a quadrilateral.

XYV is a straight line.

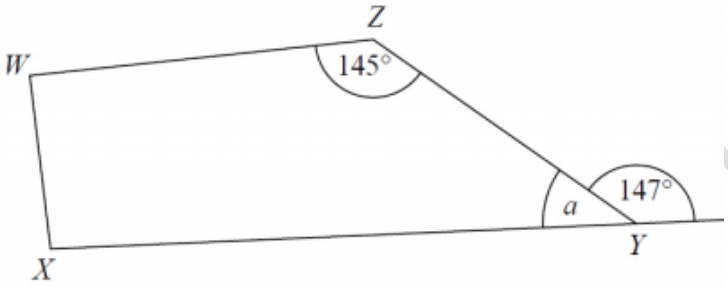


a) What is the size of the angle marked a?

33° 213° 75° 35°

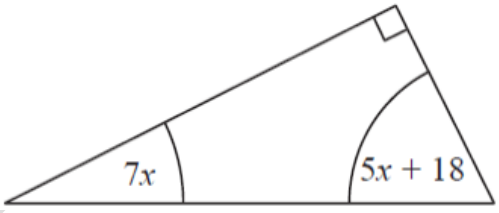
b) Angle x is equal to angle y.

What is the size of angle x?



72° 33° 91° 101°

4 The diagram shows a right-angled triangle. All angles are in degrees.



a) What is the value of x?

3 6 9 21

b) What is the size of the smallest angle?

21° | 42° | 47° | 90°

5 AB and CPD are parallel straight lines.
PQ and PR are straight lines.

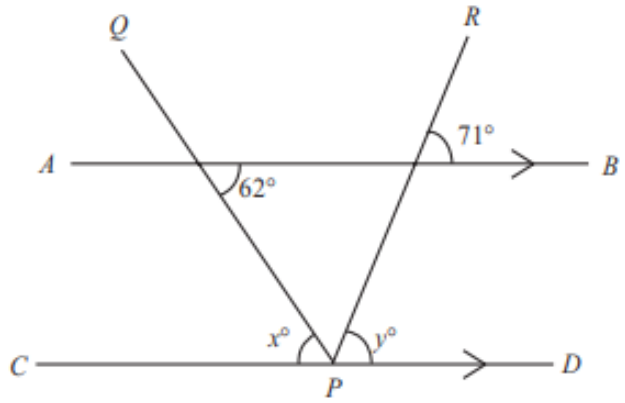


Diagram **NOT** accurately drawn

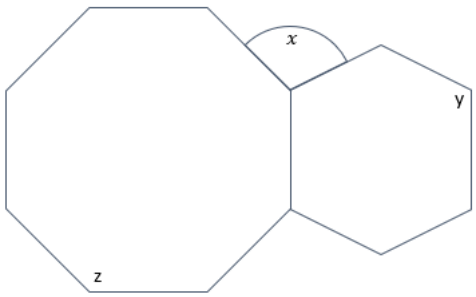
a) What is the size of the angle marked x?

118° | 71° | 109° | 62°

b) What is the size of the angle marked y?

118° | 71° | 109° | 62°

6 The diagram shows a regular octagon and a regular hexagon.



a) What is the size of angle *y*?

60° | 135° | 108° | 120°

b) What is the size of angle *z*?

60° 135° 108° 120°

c) What is the size of angle x?

105° 60° 75° 115°

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