Small Steps Guidance -Addition \& Subtraction

## Year <br> (7)

\#MathsEveryoneCan
White

|  | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Algebraic Thinking |  |  |  |  |  | Place Value and Proportion |  |  |  |  |  |
|  |  | Understanding and using algebraic notation |  |  | Equality and equivalence |  | Place value and ordering integers and decimals |  |  | Fraction, decimal and percentage equivalence |  |  |
| $\stackrel{\text { no }}{\text { 들 }}$ | Applications of Number |  |  |  |  |  | Directed Number |  |  | Fractional Thinking |  |  |
|  | Solving problems with addition \& subtraction |  | Solving problems with multiplication and division |  |  |  | Four operations with directed number |  |  | Addition and subtraction of fractions |  |  |
|  | Lines and Angles |  |  |  |  |  | Reasoning with Number |  |  |  |  |  |
|  | Co <br> meas geom | structi ring and etric no | g, using ation | Deve | ing geo reasoning | metric | Deve num se | ping <br> ber <br> se |  | and <br> bility |  | ne <br> rs and of |

## Spring 1: Application of Number

## Weeks 1 \& 2: Solving problems with addition \& subtraction

The focus for this two weeks is building on the formal methods of addition and subtraction students have developed at Key Stage 2. All students will look at this in the context of interpreting and solving problems, for those for whom these skills are secure, there will be even more emphasis on this. Problems will be drawn from the contexts of perimeter, money, interpreting bar charts and tables and looking at frequency trees; we believe all these are better studied alongside addition and subtraction rather than separately. Calculators should be used to check and/or support calculations, with significant figures and equations explicitly revisited.
National curriculum content covered:

- use formal written methods, applied to positive integers and decimals
- recognise and use relationships between operations including inverse operations
- derive and apply formulae to calculate and solve problems involving: perimeter
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts and pictograms for categorical data, and vertical line (or bar) charts for ungrouped numerical data


## Weeks 3 to 6: Solving problems with multiplication \& division

The rest of the term is dedicated to the study of multiplication and division, so allowing for the study of forming and solving of two-step equations both with and without a calculator. Unit conversions will be the main context for multiplication by 10,100 and 1000 and simple finding fractions and percentages of an amount will be explored. As well as distinguishing between multiples and factors, substitution and simplification can also be revised and extended. Again the emphasis will be on solving problems, particularly involving area of common shapes and the mean. Choosing the correct operation to solve a problem will also be a focus. There will also be some exploration of the order of operations, which will be reinforced alongside much of this content next term when studying directed number.
National curriculum content covered:

- use formal written methods, applied to positive integers and decimals
- select and use appropriate calculation strategies to solve increasingly complex problems
- recognise and use relationships between operations including inverse operations
- use the concepts and vocabulary factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple
- change freely between related standard units [time, length, area, volume/capacity, mass]
- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, and trapezia ( H )
- substitute numerical values into formulae and expressions, including scientific formulae
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- describe, interpret and compare observed distributions of a single variable through: the mean


## Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. It is not the intention that each small step should last a lesson - some will be a short step within a lesson, some will take longer than a lesson. We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

## What We Provide

- Some brief guidance notes to help identify key teaching and learning points
- A list of key vocabulary that we would expect teachers to draw to students' attention when teaching the small step,
- A series of key questions to incorporate in lessons to aid mathematical thinking.
- A set of questions to help exemplify the small step concept that needs to be focussed on.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are

- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in bold. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible - if you feel your class can access any particular small step, then please include it in your planning.

## Key Representations



| Hundreds | Tens | Ones |  |
| :---: | :---: | :---: | :---: |
| $?$ |  | $?$ | $?$ |
| 100 |  |  |  |

Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Number lines are particularly useful for both addition and subtraction and provide a good model of mental methods.

The column methods are sometimes not understood by students and are therefore prone to error. Linking these formal methods to the use of place value counters and/or base 10 blocks illustrating exchanges is very useful.

## Year 7 | Spring Term 1|Application of Number

## Addition and Subtraction

## Small Steps

- Properties of addition and subtraction
- Mental strategies for addition and subtraction
- Use formal methods for addition of integers
- Use formal methods for addition of decimals
- Use formal methods for subtraction of integers
- Use formal methods for subtraction of decimals
- Choose the most appropriate method: mental strategies, formal written or calculator
- Solve problems in the context of perimeter
- Solve financial maths problems


## Addition and Subtraction

## Small Steps

- Solve problems involving tables and timetables
- Solve problems with frequency trees
- Solve problems with bar charts and line charts

Add and subtract numbers given in standard form
(H) denotes higher strand and not necessarily content for Higher Tier GCSE

## Properties of addition \& subtraction

## Notes and guidance

Students will know from earlier study that addition and subtraction are inverses, and that addition is commutative but subtraction is not. This step reinforces these concepts and the associated language and encourages multiple representations of calculations to deepen understanding. It is useful to extend this to algebraic expressions and also to use the associative law to simplify calculations.

## Key vocabulary

| Total Sum | Difference | Number Line |
| :--- | :--- | :--- | :--- |
| Commutative | Associative | Inverse |

## Key questions

If we know $x=y+z$, what other addition facts do we know? What subtraction facts do we know?
What's the easiest way to add a list of numbers like this: $6+8+4+7+2+3$ ?
How could a number line help us to find the difference between, say, 186 and 354 ?

## Exemplar Questions

List all the additions and subtractions that these diagrams show.
What other models could you use?


This bar model illustrates that

$$
7+3=3+7
$$

We say addition is commutative.
Generalise the above example using this second bar model. Is subtraction commutative? Why, or why not?

$$
\begin{array}{rlrl}
17+26+14 & =17+26+14 & & \text { This example shows that } \\
(17+26)+14 & =17+(26+14) & & \text { addition is associative. } \\
43+14 & =17+40 & & \text { Which is the easiest way to find } \\
57 & =57 & & \text { the sum of the three numbers? } \\
& & \text { Why? }
\end{array}
$$

Marcel says that addition and subtraction are inverse operations.
Use examples and diagrams to explain what this means.

## Year 7 | Spring Term 1|Application of Number

## Mental strategies

## Notes and guidance

This small step looks at ways students can develop their flexibility and efficiency in mental addition and subtraction calculations. Increased flexibility in their choice of strategy is developed through regular discussion and comparison of different approaches. The use of part-whole models and number lines to illustrate methods will help students' understanding.

## Key vocabulary

| Bridging | Compensation | Partition |
| :--- | :--- | :--- |
| Difference | Count On | Number bonds |

## Key questions

Make up an example where number bonds to 10 and 100 are useful to perform mental calculations.
How does adding the same number to both parts of a subtraction affect the difference?
Find three ways to mentally calculate 700-438

## Exemplar Questions

Here are some ways of working out $78+96$


Which strategies do you prefer and why?
How would you work out each of these mentally?

| $386+99$ | $57+64$ | $89+254$ | $694+238$ |
| :--- | :--- | :--- | :--- |

How does this number line show that $93-37=90-34$ ?


What strategies would you use to work out these subtractions mentally?

$$
786-299 \quad 852-131 \quad 81-54 \quad 2000-1864
$$

## Formal methods: adding integers

## Notes and guidance

For students who are confident with the formal method of addition, this small step will provide practice and revision. Students who find this more challenging should have the opportunity to revisit with concrete materials alongside the formal method to develop their understanding.

## Key vocabulary

| Column Method | Place Value | Carrying |
| :--- | :--- | :--- |
| Exchange | Placeholder |  |

## Key questions

Why do we start column addition with the column on the right?
When and why do we exchange in column addition? Is the column method always the best way to solve an addition problem?

## Exemplar Questions



What addition calculation is illustrated here? What exchange needs to be done to complete the calculation?
Compare this to the formal written method for adding two integers.

Complete these calculations.


|  | H | T | O |
| :---: | :---: | :---: | :---: |
|  | 3 | 8 | 6 |
| + | 2 | 1 | 5 |
|  |  |  |  |

What are the similarities and differences between the calculations? Estimate the answers to these calculations and then use the column method of addition to find the actual answers.


## Formal methods: adding decimals

## Notes and guidance

Here students will build on the previous small steps on addition, making use of estimation and the column method paying particular attention to alignment and the use of placeholders. It is also a good opportunity to revisit the meanings of tenths and hundredths and to build on last term's work of decimal and fraction equivalence and earlier work on algebraic substitution.

## Key vocabulary

| Place value | Decimal point | Equivalence |
| :--- | :--- | :--- |
| Place holder | Estimating | Partition |

## Key questions

How do we line up decimal addition if one of the numbers is an integer?

What does placeholder mean? Why do we use placeholders?

## Exemplar Questions

Write the above representation as an addition using the column method.
Repeat the calculation if represents 1 instead of 100
What is the same and what is different about your calculations?
 Here are 4 ways of using the column method to set up $4.38+7.9$
Which ones are suitable, and which are not? Why?


Work out the answers to these calculations.
$5.43+\frac{8}{10} 5.43+\frac{59}{100} \quad 5.43+\frac{3}{4} \quad 5.43+\frac{3}{5}$

Given that $a=12.6, b=0.74, \mathrm{c}=20$ and $d=1.08$, evaluate.
$a+b \quad a+b \quad a+b+c+d \quad a$

The first term of a linear sequence is 11.3 , and the common difference between terms is 4.2
How often will the sequence produce integers?

## Formal methods: subtracting integers

## Exemplar Questions

## Notes and guidance

Following on from pervious steps, the use of the formal method of subtraction needs a good understanding of how and when to exchange e.g. one ten for ten ones. Linking back to concrete and pictorial representations may be necessary for some students. Setting questions in the context of equations and checking by addition will reinforce the concept of inverse operations.

## Key vocabulary

| Exchange | Difference | Equation |
| :--- | :--- | :--- |
| Placeholder | Subtraction | Inverse |

## Key questions

Why isn't subtraction commutative?
How can we check the answer to a subtraction? When do we need to perform exchanges when doing a written subtraction?

How would you use place value counters to illustrate $63-21$ ? How does this compare to the written column method? Compare the place value and column methods for 63-25

Complete these calculations.


What are the similarities and differences between the calculations?
Use the column method of subtraction to solve these equations. Check your answers using the column method of addition.

$$
a+3500=8267 \quad 85172=b+27000
$$

$$
c=715000-67800
$$

$$
56302=28275+c
$$

## Formal methods: subtracting decimals

## Notes and guidance

The clear links to the formal method of subtraction of integers and to the addition of decimals need to be emphasised. In particular, the use of zeros as placeholders is essential. Although the emphasis is on the formal method, it is worth discussing whether alternative methods could or should be used e.g. counting on for change.

## Key vocabulary

| Place value | Digit | Equation |
| :--- | :--- | :--- |
| Placeholder | Difference | Exchange |

## Key questions

When would it be appropriate to include a hundredths column in a number that is given in tenths? For what types of subtraction is the formal method most/least useful?

## Exemplar Questions

What mistakes have been made in these calculations?


| 8 | . | 1 | 6 |  |
| ---: | ---: | ---: | ---: | ---: |
| - | 3 | . | 5 | 4 |
| 5 | . | 4 | 2 |  |


| 7 | . | 6 |  |  |
| ---: | ---: | ---: | ---: | ---: |
| - | 6 | . | 5 | 4 |
| 1 | . | 1 | 4 |  |



Work out the correct answers to the calculations.
Solve these equations without using a calculator.

$$
a+13.7=28.6
$$

$$
b-13.7=28.6
$$

$$
324=c+47.2
$$

$$
6.1=d-26.97
$$

Work out the range of the four values $a, b, c$ and $d$.
Joachim says that to work out $£ 10-£ 3.27$, you could work out $£ 9.99$ - $£ 3.26$ instead.
Work out both calculations to show that he is correct.
Why does his method work?
Work out the answers to these calculations.

| $407-126$ | $407-12.6$ | $407-1.26$ |
| :--- | :--- | :--- |
| $6.7-\frac{1}{5}$ | $6.7-\frac{3}{5}$ | $6.7-\frac{1}{4}$ |

## Choosing the appropriate method

## Notes and guidance

As well as flexibility in applying methods, students should be encouraged to choose which method to apply in which situation - mental, jottings, formal written, or calculator. The discussion as to which method can draw out, or lead to, understanding of the methods themselves and this is sometimes as powerful as the practice itself.

## Key vocabulary

| Formal method | Estimate | Mental |
| :--- | :--- | :--- |
| Written | Jottings | Calculator |

## Key questions

How do you decide which method to use to perform a calculation?

Give an example of when a calculator isn't the quickest way to work out an answer.

## Exemplar Questions

Estimate the answers to these calculations, and then check your answers using an appropriate method.


Decide whether a mental, written or calculator method would be best for each of the calculations.

- Bashir earned $£ 942.18$ one month. He spent $£ 787.40$ on rent and bills. How much money did he have left?
- A film starts at 1855 and finishes at 2040 How long did the film last?
- Mary had £2500 in her savings. She withdrew $£ 850$ How much was left in the bank?
- In 2018, the population of England is 54.79 million. 8,136 million people live in London. How many people live in the rest of England?

Explain why a mental method would be best for these calculations.


## Year 7| Spring Term 1|Application of Number

## Solve problems with perimeter

## Notes and guidance

Students will be familiar with perimeter from primary school. This small step is an opportunity to revisit the concept and solve addition and subtraction problems in context. This is also an opportunity to revise forming and solving one-step equations and/or simplifying and substituting into expressions.

## Key vocabulary

| Length | Path | Distance |
| :--- | :--- | :--- |
| Units | Edges | Polygon |

## Key questions

Why is the number of sides on a shape the same as the number of terms in a perimeter addition?

If all the sides of a rectangle are increased by 2 units, how could we know how much the perimeter has increased by?

## Exemplar Questions

The perimeter of this shape is 14.2 cm . What is the length of the missing side? How does the bar model help?


A rectangle has perimeter 20 cm .
If the side lengths are integers, what might the dimensions be?
How many triangles with integer side-lengths and a perimeter of 20 cm can be made?
Why is $14 \mathrm{~cm}, 4 \mathrm{~cm}, 2 \mathrm{~cm}$ not a possible combination?
Two sides of an isosceles triangle are 8.7 cm long. If the perimeter of the triangle is 29.2 cm , calculate the length of the third side.

Write an expression for the
 perimeter of this pentagon. If the perimeter is 26.4 cm , form and solve an equation to find the value of $x$.
If instead $x=4.1$, find the perimeter of the pentagon.

## Solve financial problems

## Notes and guidance

This small step uses addition and subtraction, particularly in a familiar context whilst also introducing potentially new vocabulary. Students may practise calculator or noncalculator skills as appropriate following previous learning. Estimation and checking answers on a calculator will support entering values some of which are in pounds and some in pence, and interpreting displays such as "14.4"

## Key vocabulary

| Profit | Loss | Balance | Credit |
| :--- | :--- | :--- | :--- |
| Debit | Statement | Change | Bill |

## Key questions

What is the difference between the words credit and debit on a bank statement.?

How do you calculate profit?
Why does a calculator display $£ 12.50$ as 12.5 ?

## Exemplar Questions

A bracelet costs $£ 3.99$ and a bobble costs $£ 1.29$
How much change should there be from $£ 10$ If I buy both items?
John spends $£ 112.50$ on ingredients and $£ 17.80$ on advertising for a cake sale.
He sells all the cakes for a total of £145.12
Does he make a profit or a loss?
How much profit or loss does he make?
Complete the bank statement.

| Date | Description | Credit <br> $(£)$ | Debit <br> (£) | Balance <br> $(£)$ |
| :---: | :---: | :---: | :---: | :---: |
| Mar 1 | Opening balance |  |  | 93.68 |
| Mar 3 | Gas bill |  | 84.17 |  |
| Mar 7 | Wages | 312.72 |  |  |
| Mar 9 | Rent |  | 145.10 |  |


| Previous Reading | Current Reading | Unit price |
| :---: | :---: | :---: |
| 16851 | 18123 | 12.7 p |

The table shows part of an electricity bill.
How many units have been used?
If there is a standing charge of $£ 23.56$, work out the total bill.

## Tables and timetables

## Notes and guidance

Reading tables is a key life skill and provides a good context for practising addition and subtraction skills. Calculations with time can create difficulties as students are not used to working with non-decimal contexts. Number lines can be a very valuable support here.

## Key vocabulary

| Row | Column | Entry | Total |
| :--- | :--- | :--- | :--- |
| Hours | Minutes | Difference |  |

## Key questions

Does the column method for subtraction work when dealing with time? Why or why not?
Explain how we could use a number line (or time line) to help us with calculations for time.
Is it true that sum of all the row totals in a table equal to the sum of all the column totals? Why or why not?

## Exemplar Questions

## London

| 211 | Cardiff |  |
| :---: | :---: | :---: |
| 556 | 493 | Glasgow |
| 518 | 392 | 177 |

This table shows the distance by air between some UK cities.

## Belfast

Hoda flies from London to Belfast and then from Belfast to Cardiff. How far does she fly in total? How much longer is her journey than flying directly from London to Cardiff?

The table below shows part of the results of a survey in a school with 900 students.

|  | Left-handed | Right-handed | Total |
| :---: | :---: | :---: | :---: |
| Girls | 34 |  | 361 |
| Boys |  | 463 |  |
| Total |  |  |  |

Work out the missing numbers in the table. To the nearest whole number, what percentage of the boys are left-handed?

| Harton | 1005 | 1045 | 1130 |
| :---: | :---: | :---: | :---: |
| Bridge | 1024 | 1106 | 1147 |
| Aville | 1051 | 1133 | 1205 |
| Ware | 1117 | 1202 | 1233 |

Investigate which of the buses shown in this timetable is the quickest or slowest between each pair of towns.

## Frequency trees

## Notes and guidance

Frequency trees provide a good opportunity for students to practise addition and subtraction in a different context. Although this may be unfamiliar, links can be made to using tables in the previous step, and to the part-whole model. Students can be challenged to create their own frequency tree questions and to investigate the minimum amount of information needed to complete a frequency tree.

## Key vocabulary

| Frequency | Frequency Tree |
| :--- | :--- |
| Total | Part-whole |

## Key questions

Explain the relationships between the numbers in a frequency tree.

When might we have a frequency tree with more than two branches?

## Exemplar Questions

There are 32 students in a class. 17 of the students are boys.
Complete the frequency tree.


80 people took their driving test one week.
45 of the people were men.
28 of the men passed their test. 27 of the women passed their test. Complete the frequency tree.

How many more men than women did not pass their test?


In a year group of 140 students, 74 are girls.
42 of the students wore glasses, including 23 boys.
Show this information on a frequency tree.

|  | Men | Women |
| :---: | :---: | :---: |
| Suitcase | 63 |  |
| Rucksack | 45 | 32 |

The table shows information about luggage carried by 203 passengers on a flight. Complete the table and represent as a frequency tree.

## Bar \& line charts

## Notes and guidance

Students are very familiar with the construction of bar and line charts, so the foci of this small step should be the interpretation of ready-drawn diagrams and linking different forms of charts to tables. As well as opportunities to solve addition and subtraction problems, the notation of scaled axes can be discussed making links with intervals on number lines studied last term.

## Key vocabulary

| Frequency | Axis | Scale |
| :--- | :--- | :--- |
| Difference | Dual | Multiple |

## Key questions

What difference does it make if a bar chart is drawn horizontally rather than vertically?

## Exemplar Questions



The bar chart shows the number of children in a class of 30 pupils who have various types of pet.
What is the total of the frequencies?
Why is this not total not 30? What is the difference between the number of pupils with a dog and the number with a guinea pig? What other questions can you ask?


Wildlife Population


The table shows the ages of girls and boys in a youth group.
Complete the table and represent the information as a multiple bar chart.

The chart shows the expected changes in animal populations in a wildlife park. Describe the changes, using calculations to justify your findings.

## Add \& subtract in standard form $H$ Exemplar Questions

## Notes and guidance

In this small step, students will have the opportunity to revisit standard form notation through exploring addition and subtraction, noticing that adding powers is an incorrect approach. It might also be a good opportunity to consolidate knowledge of working with billions and rounding to one significant figure.

## Key vocabulary

| Standard form | Power | Exponent |
| :--- | :--- | :--- |
| Significant figure | Billion | Million |

## Key questions

Why do you not add/subtract the powers when adding/subtracting numbers written in standard form?

Explain the difference between $10^{-3}$ and $10^{3}$

Round the populations of the three countries shown to 1 significant figure., giving your answers in standard form.

| Country | Population | Population rounded to 1sf |
| :--- | :---: | :--- |
| New Zealand | 4.7 million |  |
| Slovenia | 2100000 |  |
| Djibouti | 960000 |  |
| Total |  |  |

What is the difference between the totals of the two columns?
Work out each calculation, giving your answer as an ordinary numbers.

$$
3 \times 10^{5}+4 \times 10^{4} \quad 2 \times 10^{7}-6 \times 10^{5}
$$

$$
7 \times 10^{5}-6 \times 10^{4}+8 \times 10^{3}
$$

$$
\begin{aligned}
& 4 \times 10^{-2}+5 \times 10^{-3} \\
& 5 \times 10^{3}+4 \times 10^{3}=9 \times 10^{6} \\
& 3 \times 10^{5}+7 \times 10^{5}=10 \times 10^{5}
\end{aligned}
$$

$4 \times 10^{-2}-5 \times 10^{-3}$

For homework, Jon had to add up pairs of numbers and give the answers in standard form. Explain why he has got both these questions wrong.

