MATLAB® Laboratory Manual

with SIMULINK® examples

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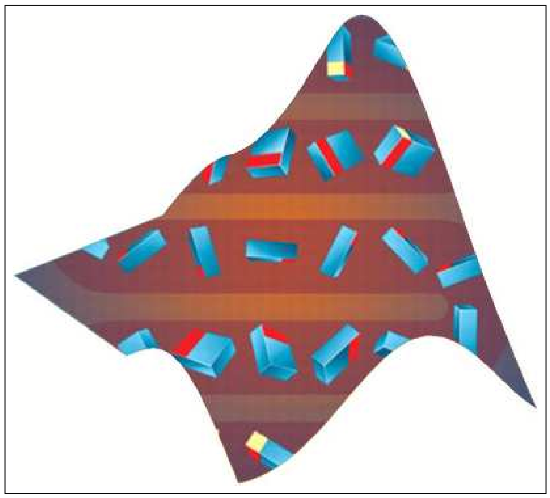

to accompany

FUNDAMENTALS OF
DIFFERENTIAL EQUATIONS
SIXTH EDITION

and

FUNDAMENTALS OF
DIFFERENTIAL EQUATIONS
AND
BOUNDARY VALUE PROBLEMS
FOURTH EDITION

Nagle Saff Snider

Contents

Preface

This manual is designed to accompany the new edition of *Fundamentals of Differential Equations (and Boundary Value Problems)* by Nagle, Saff and Snider and its format was mostly inspired by the companion MAPLE Technology Manual written by Kenneth Pothoven. The usefulness of this manual should however extend well beyond this specific association.

MATLAB is a “high-performance language for technical computing which now integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation” (excerpt from The MathWorks website). MATLAB stands for MATrix LABoratory and was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK projects.

MATLAB has become “a standard instructional tool for introductory and advanced courses in Mathematics, Engineering, and Science. In industry, MATLAB is now the tool of choice for high-productivity research, development, and analysis.” This manual explores the use of MATLAB in solving differential equations and visualizing and interpreting their solutions. The main objectives are:

1. familiarize students with basic MATLAB programming to solve differential equations of varying complexity,
2. provide a platform for testing and experimenting with fundamental aspects of numerical computations of solutions of differential equations, and
3. present simple ways to visualize the numerical solutions.

The use of the MATLAB Symbolic Toolbox was limited to short examples with the functions `laplace` and `ilaplace` in Laboratory 14. Custom functions which do not use this toolbox were used to illustrate the numerical evaluation of these transforms. An introduction to SIMULINK is given in Laboratory 13. SIMULINK provides a tool for building and testing models using the power of MATLAB without requiring the specific knowledge of the MATLAB language necessary to implement these models.

The versions of MATLAB and SIMULINK used are MATLAB 6.0 and SIMULINK 4, respectively (Release 12). Changes made in Release 13 (the last available as of June 2003) have no bearing on the content of this manual.

This manual includes two introductory chapters on MATLAB: the first one shows how to start MATLAB and how to use and organize basic commands. It explains in particular how to save groups of commands in a file, which is important in creating programs that implement a whole problem. The second chapter is more specific to differential equations. It goes over how to implement initial value problems and how to visualize the solution(s), using complete examples. Students are encouraged to read these two chapters and execute the listed commands before moving to the laboratory section.

Following is the main part of this manual which comprises 14 laboratory sessions covering general numerical implementation and execution issues when solving differential equations. This is done in the context of specific applications, many of which are similar to the ones presented in the MAPLE companion manual. As noted there, these laboratory sessions vary in length and difficulty, and are included to demonstrate the use of MATLAB as well as to involve students through exploratory exercises. Instructors should feel free to modify the sessions at their own discretion.

Additional project ideas are included in the last chapter. These projects represent different kinds of applications from various disciplines including chemical, civil, electrical, and mechanical engineering, including most of the projects ideas (sometimes radically changed though) from the Maple companion book by K. Pothoven. A project idea coming from an actual project carried out in a Differential Equations with MATLAB course I have been teaching is also included. Each project is briefly described at the beginning of the chapter.

As always when using a computer software the user should be careful to respect the correct syntax needed for proper execution. In my experience this represents the most significant hurdle for students without any background in computer programming. This manual is designed to help in this regard, by including simple examples and templates that students can conveniently use as a starting point for their own applications. Ultimately, some effort must however be made by the student in order to assimilate the proper ways of using MATLAB by practicing with the examples included in this manual and adapting them to new problems.

The manuscript was prepared using the LaTeX document preparation system. PostScript figures were created using MATLAB or converted from screen capture in Jpeg format using `jpeg2ps 1.9` by Thomas Herz. The cover picture was created from the original book cover of the new (sixth) edition of the Nagle, Saff and Snider text with a mask based on the MATLAB logo using PhotoShop. The MATLAB graphical user interfaces `dfield6.m` and `pplane6.m` by John Polking and David Arnold were briefly used in this manual. A detailed description is available in their book *Ordinary Differential Equations Using MATLAB*. Many of the MATLAB and SIMULINK programs used in this manual are available online at `http://math.asu.edu/~bdw/PUBLIC`. Partial funding for this work was provided by a grant to improve undergraduate education from the College of Liberal Arts and Sciences at Arizona State University.

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Chapter 1

Introduction to MATLAB

MATLAB is a computer software commonly used in both education and industry to solve a wide range of problems.

This chapter provides a brief introduction to MATLAB, and the tools and functions that help you to work with MATLAB variables and files.

The MATLAB Environment

★ To start MATLAB double-click on the MATLAB shortcut icon 

or type `matlab &` at the prompt (Unix). The MATLAB desktop opens.

On the right side of the desktop you find the Command Window, where commands are entered at the prompt `>>`.

On the left side you will generally find the Launch Pad and Workspace windows, and the Command History and Current Directory windows. For all practical purposes we have in mind I recommend closing the Launch Pad, Workspace, and Command History windows, if opened. It is convenient to keep the Current Directory window opened to check for files you create and use in the Command Window.

Note that windows within the MATLAB desktop can be resized by dragging the separator bar(s). A typical MATLAB desktop is shown in Fig. 1.1.

★ To exit MATLAB do one of the following:

- Click on the close box  at the top right of the MATLAB Desktop.
- Select **File** > **Exit** from the desktop File menu.
- Type `quit` or `exit` at the Command Window prompt `>>`.

Basics And Help

Commands are entered in the Command Window.

★ Basic operations are `+`, `-`, `*`, and `/`. The sequence

```
>> a=2; b=3; a+b, a*b,

ans =

     5


ans =
```

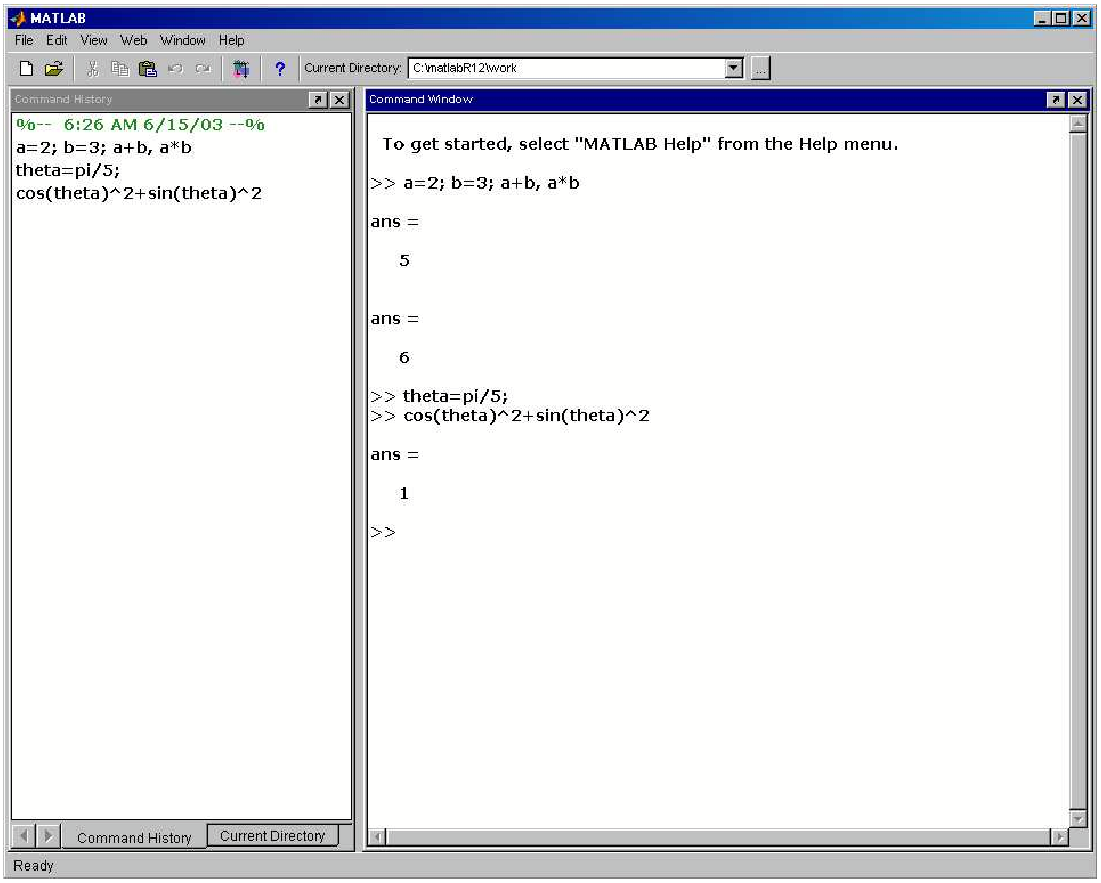

Figure 1.1: A typical MATLAB desktop

```
     6
```

defines variables a and b and assigns values 2 and 3, respectively, then computes the sum $a+b$ and product ab. Each command ends with `,` (output is visible) or `;` (output is suppressed). The last command on a line does not require a `,`.

★ Standard functions can be invoked using their usual mathematical notations. For example

```
>> theta=pi/5;
>> cos(theta)^2+sin(theta)^2

ans =

     1
```

verifies the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ for $\theta = \frac{\pi}{5}$. A list of elementary math functions can be obtained by typing `help elfun` in the Command window:

```
>> help elfun

  Elementary math functions.
```

```
Trigonometric.
  sin         - Sine.
  sinh        - Hyperbolic sine.
  asin        - Inverse sine.
  asinh       - Inverse hyperbolic sine.
  cos         - Cosine.
  cosh        - Hyperbolic cosine.
  acos        - Inverse cosine.
  acosh       - Inverse hyperbolic cosine.
  tan         - Tangent.
  tanh        - Hyperbolic tangent.
  atan        - Inverse tangent.
  atan2       - Four quadrant inverse tangent.
  atanh       - Inverse hyperbolic tangent.
  sec         - Secant.
  sech        - Hyperbolic secant.
  asec        - Inverse secant.
  asech       - Inverse hyperbolic secant.
  csc         - Cosecant.
  csch        - Hyperbolic cosecant.
  acsc        - Inverse cosecant.
  acsch       - Inverse hyperbolic cosecant.
  cot         - Cotangent.
  coth        - Hyperbolic cotangent.
  acot        - Inverse cotangent.
  acoth       - Inverse hyperbolic cotangent.

Exponential.
  exp         - Exponential.
  log         - Natural logarithm.
  log10       - Common (base 10) logarithm.
  log2        - Base 2 logarithm and dissect floating point number.
  pow2        - Base 2 power and scale floating point number.
  sqrt        - Square root.
  nextpow2    - Next higher power of 2.

Complex.
  abs         - Absolute value.
  angle       - Phase angle.
  complex     - Construct complex data from real and imaginary parts.
  conj        - Complex conjugate.
  imag        - Complex imaginary part.
  real        - Complex real part.
  unwrap      - Unwrap phase angle.
  isreal      - True for real array.
  cplxpair    - Sort numbers into complex conjugate pairs.

Rounding and remainder.
  fix         - Round towards zero.
  floor       - Round towards minus infinity.
  ceil        - Round towards plus infinity.
  round       - Round towards nearest integer.
  mod         - Modulus (signed remainder after division).
```

```
      rem         - Remainder after division.
      sign        - Signum.
```

★ To obtain a description of the use of a particular function type help followed by the name of the function. For example

```
>> help cosh

 COSH   Hyperbolic cosine.
    COSH(X) is the hyperbolic cosine of the elements of X.
```

★ To get a list of other groups of MATLAB programs already available enter `help`:

```
>> help

HELP topics:

matlab\general       -  General purpose commands.
matlab\ops           -  Operators and special characters.
matlab\lang          -  Programming language constructs.
matlab\elmat         -  Elementary matrices and matrix manipulation.
matlab\elfun         -  Elementary math functions.
matlab\specfun       -  Specialized math functions.
matlab\matfun        -  Matrix functions - numerical linear algebra.
matlab\datafun       -  Data analysis and Fourier transforms.
matlab\audio         -  Audio support.
matlab\polyfun       -  Interpolation and polynomials.
matlab\funfun        -  Function functions and ODE solvers.
matlab\sparfun       -  Sparse matrices.
matlab\graph2d       -  Two dimensional graphs.
matlab\graph3d       -  Three dimensional graphs.
matlab\specgraph     -  Specialized graphs.
matlab\graphics      -  Handle Graphics.
matlab\uitools       -  Graphical user interface tools.
matlab\strfun        -  Character strings.
matlab\iofun         -  File input/output.
matlab\timefun       -  Time and dates.
matlab\datatypes     -  Data types and structures.
matlab\verctrl       -  Version control.
matlab\winfun        -  Windows Operating System Interface Files (DDE/ActiveX)
matlab\demos         -  Examples and demonstrations.
toolbox\local        -  Preferences.
matlabR12\work       -  (No table of contents file)

For more help on directory/topic, type "help topic".
```

★ Another way to obtain help is through the desktop Help menu, **Help** > **MATLAB Help**, or by connecting to the Mathworks web site at www.mathworks.com.
★ MATLAB is case-sensitive. For example

```
>> theta=1e-3, Theta=2e-5; ratio=theta/Theta

theta =
```

```
    0.0010

Theta =

  2.0000e-005

ratio =

    50
```

★ The quantities `Inf` (∞) and `NaN` (Not a Number) also appear frequently. Compare

```
>> c=1/0
Warning: Divide by zero.

c =

   Inf
```

with

```
>> d=0/0
Warning: Divide by zero.

d =

   NaN
```

Plotting with MATLAB

★ To plot a function you have to create two arrays (vectors): one containing the abscissae, the other the corresponding function values. Both arrays should have the same length. For example, consider plotting the function

$$y = f(x) = \frac{x^2 - \sin(\pi x) + e^x}{x - 1}$$

for $0 \leq x \leq 2$. First choose a sample of x values in this interval:

```
>> x=[0,.1,.2,.3,.4,.5,.6,.7,.8,.9,1, ...
1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2]

x =

  Columns 1 through 4

         0    0.1000    0.2000    0.3000

  Columns 5 through 8

    0.4000    0.5000    0.6000    0.7000

  Columns 9 through 12
```

```
    0.8000    0.9000    1.0000    1.1000

  Columns 13 through 16

    1.2000    1.3000    1.4000    1.5000

  Columns 17 through 20

    1.6000    1.7000    1.8000    1.9000

  Column 21

    2.0000
```

or simply

```
>> x=0:.1:2

x =

  Columns 1 through 4

         0    0.1000    0.2000    0.3000

  Columns 5 through 8

    0.4000    0.5000    0.6000    0.7000

  Columns 9 through 12

    0.8000    0.9000    1.0000    1.1000

  Columns 13 through 16

    1.2000    1.3000    1.4000    1.5000

  Columns 17 through 20

    1.6000    1.7000    1.8000    1.9000

  Column 21

    2.0000
```

Try also

```
>> x=linspace(0,2,21)

x =

  Columns 1 through 4

         0    0.1000    0.2000    0.3000
```

```
  Columns 5 through 8

    0.4000    0.5000    0.6000    0.7000

  Columns 9 through 12

    0.8000    0.9000    1.0000    1.1000

  Columns 13 through 16

    1.2000    1.3000    1.4000    1.5000

  Columns 17 through 20

    1.6000    1.7000    1.8000    1.9000

  Column 21

    2.0000
```

★ Note that an ellipsis `...` was used to continue a command too long to fit in a single line.
★ The output for x can be suppressed (by adding `;` at the end of the command) or condensed by entering `format compact`:

```
>> format compact
>> x
  Columns 1 through 4
         0    0.1000    0.2000    0.3000
  Columns 5 through 8
    0.4000    0.5000    0.6000    0.7000
  Columns 9 through 12
    0.8000    0.9000    1.0000    1.1000
  Columns 13 through 16
    1.2000    1.3000    1.4000    1.5000
  Columns 17 through 20
    1.6000    1.7000    1.8000    1.9000
  Column 21
    2.0000
```

From now on we shall use such format for all output.

To evaluate the function f simultaneously at all the values contained in x, type

```
>> y=(x.^2-sin(pi.*x)+exp(x))./(x-1)
Warning: Divide by zero.
y =
  Columns 1 through 4
   -1.0000   -0.8957   -0.8420   -0.9012
  Columns 5 through 8
   -1.1679   -1.7974   -3.0777   -5.6491
  Columns 9 through 12
  -11.3888  -29.6059       Inf   45.2318
  Columns 13 through 16
   26.7395   20.5610   17.4156   15.4634
  Columns 17 through 20
```

```
  14.1068   13.1042   12.3468   11.7832
 Column 21
  11.3891
```

Note that the function becomes infinite at $x = 1$ (vertical asymptote). The array y inherits the dimension of x, namely 1 (row) by 21 (columns). Note also the use of parentheses.

IMPORTANT REMARK

In the above example `*`, `/` and `^` are preceded by a dot . in order for the expression to be evaluated for each component (entry) of x. This is necessary to prevent MATLAB from interpreting these symbols as standard linear algebra symbols operating on arrays. Because the standard `+` and `-` operations on arrays already work componentwise, a dot is not necessary for `+` and `-`.

The command

```
>> plot(x,y)
```

creates a Figure window and shows the function, see Fig. 1.2. The figure can be edited and manipulated using the Figure window menus and buttons. Alternately, properties of the figure can also be defined directly at the command line:

```
>> x=0:.01:2;
>> y=(x.^2-sin(pi.*x)+exp(x))./(x-1);
>> plot(x,y,'r-','LineWidth',2);
>> axis([0,2,-10,20]); grid on;
>> title('f(x)=(x^2-sin(\pi x)+e^x)/(x-1)');
>> xlabel('x'); ylabel('y');
```

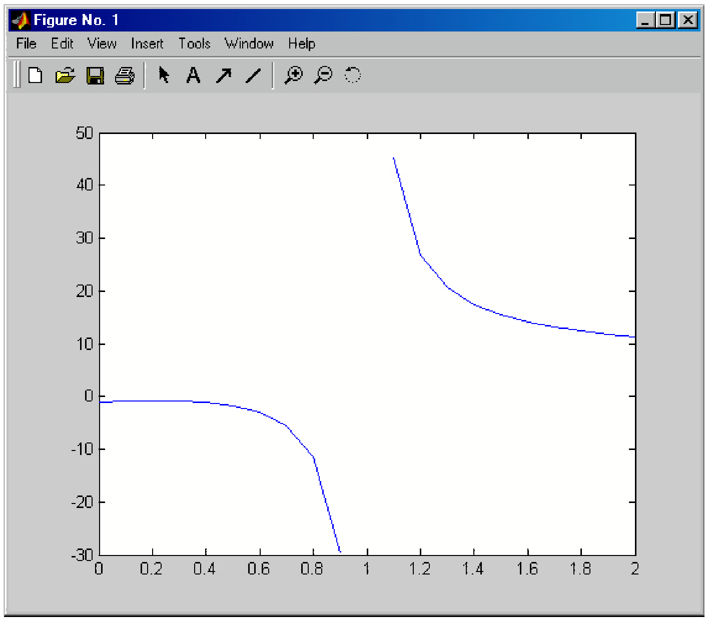

Figure 1.2: A Figure window

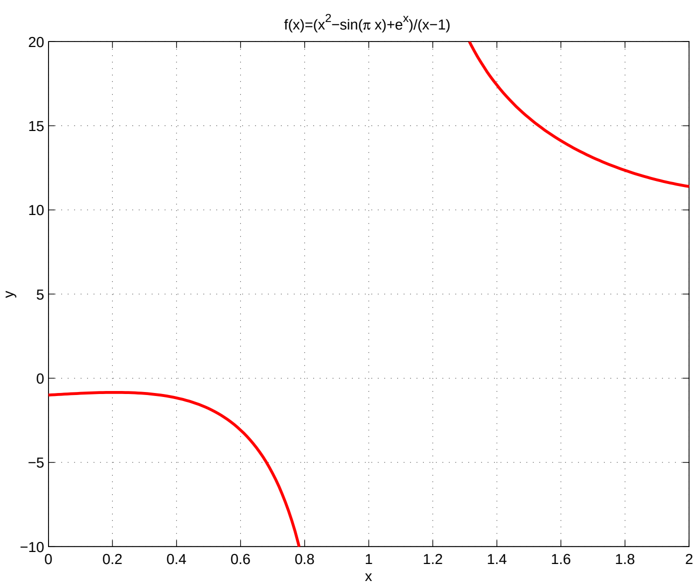

Figure 1.3: The function $y = f(x) = \frac{x^2 - \sin(\pi x) + e^x}{x-1}$.

The number of x-values has been increased for a smoother curve (what is the new size of x?). The curve now appears wider and in red. The range of x and y values has been reset (always a good idea in the presence of vertical asymptotes). A title and labels have been added. The resulting new plot is shown in Fig. 1.3. For more options type `help plot` in the Command Window.

Scripts and Functions

★ Files containing MATLAB commands are called `m`-files and have a `.m` extension. They are two types:

1. A *script* is simply a collection of MATLAB commands gathered in a single file. The value of the data created in a script is still available in the Command Window after execution.

2. A function is similar to a script, but can accept and return arguments. Unless otherwise specified any variable inside a function is local to the function and not available in the Workspace. A function invariably starts with the command

   ```
   function output = function_name(input)
   ```

   and should contain one or several commands defining the output.

Use a function when a group of commands needs to be evaluated multiple times.
★ To create a new script or function select the MATLAB desktop File menu **File** > **New** > **M-file**. In the MATLAB text editor window enter the commands as you would in the Command window. To save the file use the menu **File** > **Save** or **File** > **Save As...**, or the shortcut Save button.

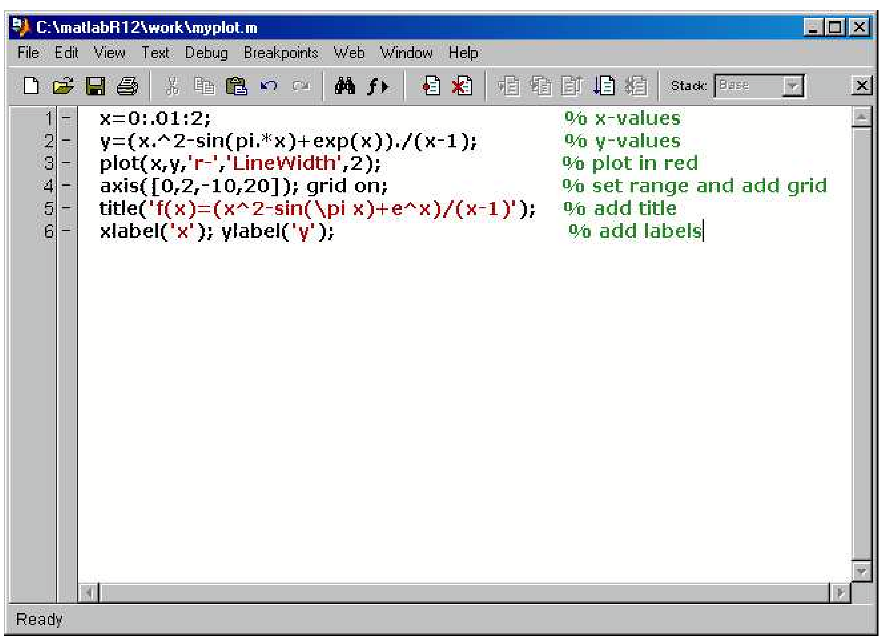

Figure 1.4: The script `myplot.m` in the MATLAB Editor window.

★ Examples of script/function:

1. **script**

   `myplot.m` `myplot.m` (see Fig. 1.4).

```
x=0:.01:2;                                     % x-values
y=(x.^2-sin(pi.*x)+exp(x))./(x-1);             % y-values
plot(x,y,'r-','LineWidth',2);                  % plot in red
axis([0,2,-10,20]); grid on;                   % set range and add grid
title('f(x)=(x^2-sin(\pi x)+e^x)/(x-1)');      % add title
xlabel('x'); ylabel('y');                      % add labels
```

2. **script+function** (two separate files)

   `myplot2.m` (driver script)

```
x=0:.01:2;                                     % x-values
y=feval(@myfunction,x);                        % evaluate myfunction at x
plot(x,y,'r-','LineWidth',2);                  % plot in red
axis([0,2,-10,20]); grid on;                   % set range and add grid
title('f(x)=(x^2-sin(\pi x)+e^x)/(x-1)');      % add title
xlabel('x'); ylabel('y');                      % add labels
```

   `myfunction.m` (function)

```
function y=myfunction(x)                       % defines function
y=(x.^2-sin(pi.*x)+exp(x))./(x-1);             % y-values
```

3. **script+function** (one single file)

   `myplot1.m` (driver script converted to function + function)

```
function myplot1
x=0:.01:2;                                     % x-values
y=feval(@myfunction,x);                        % evaluate myfunction at x
plot(x,y,'r-','LineWidth',2);                  % plot in red
axis([0,2,-10,20]); grid on;                   % set range and add grid
title('f(x)=(x^2-sin(\pi x)+e^x)/(x-1)');      % add title
xlabel('x'); ylabel('y');                      % add labels
%-----------------------------------------
function y=myfunction(x)                       % defines function
y=(x.^2-sin(pi.*x)+exp(x))./(x-1);             % y-values
```

In case 2 `myfunction.m` can be used in any other `m`-file (just as other predefined MATLAB functions). In case 3 `myfunction.m` can be used by any other function in the same `m`-file (`myplot1.m`) only. Use 3 when dealing with a single project and 2 when a function is used by several projects.
★ It is convenient to add descriptive comments into the script file. Anything appearing after `%` on any given line is understood as a comment (in green in the MATLAB text editor).
★ To execute a script simply enter its name (without the .m extension) in the Command Window, e.g.,

```
>> myplot;
```

in case 1,

```
>> myplot2;
```

in case 2 and

```
>> myplot1;
```

in case 3 above. The function `myfunction` can also be used independently if implemented in a separate file `myfunction.m`:

```
>> x=2; y=myfunction(x)
y =
  11.3891
```

A script can be called from another script or function (in which case it is local to that function).

If any modification is made, the script or function can be re-executed by simply retyping the script or function name in the Command Window (or use the up-arrow on the keyboard to browse through past commands).

IMPORTANT REMARK
By default MATLAB saves files in the Current Directory (folder). When entering MATLAB the Current Directory is the Work Directory (e.g., `C:\matlabR12\work`). Make sure the file is saved where you want it. To change directory use the Current Directory window or the Current Directory box 

on top of the MATLAB desktop.

★ A function file can contain a lot more than a simple evaluation of a function $f(x)$ or $f(t, y)$. But in simple cases $f(x)$ or $f(t, y)$ can simply be defined using the `inline` syntax. Compare

```
>> ...
>> slope = feval(@f,2,1)   % note use of @. Try also slope=f(2,1)
slope =
   3
```

where f.m is the file containing

```
function dydt = f(t,y)
dydt = t^2-y;
```

to

```
>> ...
>> f = inline('t^2-y','t','y')
f =
   Inline function:
   f(t,y) = t^2-y
>> slope = feval(f,2,1)    % note no @
slope =
   3
```

However, an inline function is only available where it is used and not to other functions. It is not recommended when the function implemented is too complicated or involves too many statements.

Matrices and Linear Algebra

We have used one-dimensional 1×21 arrays x and y in previous examples. MATLAB can handle higher dimensional arrays. Two-dimensional arrays (matrices) are commonly used in many situations.
★ Matrices can be constructed in MATLAB in different ways. For example the 3×3 matrix $A = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$ can be entered as

```
>> A=[8,1,6;3,5,7;4,9,2]
A =
   8     1     6
   3     5     7
   4     9     2
```

or

```
>> A=[8,1,6;
3,5,7;
4,9,2]
A =
   8     1     6
   3     5     7
   4     9     2
```

or defined as the concatenation of 3 rows

```
>> row1=[8,1,6]; row2=[3,5,7]; row3=[4,9,2]; A=[row1;row2;row3]
A =
   8     1     6
   3     5     7
   4     9     2
```

or 3 columns

```
>> col1=[8;3;4]; col2=[1;5;9]; col3=[6;7;2]; A=[col1,col2,col2]
A =
   8     1     6
   3     5     7
   4     9     2
```

Note the use of `,` and `;`. Concatenated rows/columns must have the same length. Larger matrices can be created from smaller ones in the same way:

```
>> C=[A,A]  % Same as C=[A A]
C =
   8     1     6     8     1     6
   3     5     7     3     5     7
   4     9     2     4     9     2
```

The matrix C has dimension 3×6 ("3 by 6"). On the other hand smaller matrices (submatrices) can be extracted from any given matrix:

```
>> A(2,3)  % coefficient of A in 2nd row, 3rd column
ans =
   7
>> A(1,:)  % 1st row of A
ans =
   8     1     6
>> A(:,3)  % 3rd column of A
ans =
   6
   7
   2
>> A([1,3],[2,3]) % keep coefficients in rows 1 & 3 and columns 2 & 3
ans =
   1     6
   9     2
```

★ Some matrices are already predefined in MATLAB:

```
>> I=eye(3)  % the Identity matrix
I =
   1     0     0
   0     1     0
```

```
   0     0     1
>> magic(3)
ans =
   8     1     6
   3     5     7
   4     9     2
```

(what is magic about this matrix?)
★ Matrices can be manipulated very easily in MATLAB (unlike MAPLE). Here are sample commands to exercise with:

```
>> A=magic(3);
>> B=A'    % transpose of A, i.e, rows of B are columns of A
B =
   8     3     4
   1     5     9
   6     7     2
>> A+B     % sum of A and B
ans =
   16     4    10
    4    10    16
   10    16     4
>> A*B     % standard linear algebra matrix multiplication
ans =
   101    71    53
    71    83    71
    53    71   101
>> A.*B    % coefficientwise multiplication
ans =
    64     3    24
     3    25    63
    24    63     4
```

Try `A*A`, `A^2`, `A.^2`.
★ Two MATLAB commands are especially relevant when studying the solution of linear systems of differentials equations:

1. `x=A\b` determines the solution $x = A^{-1}b$ of the linear system $Ax = b$. The array `b` must have as many rows as the matrix `A`.
2. `[S,D]=eig(A)` determines the eigenvectors of A (columns of `S`) and associated eigenvalues (diagonal coefficients of `D`) (note that the `eig` function has one input and two output arguments).

As an example consider the matrix A =magic(3) again and $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$:

```
>> A=magic(3)
A =
   8     1     6
   3     5     7
   4     9     2
>> x=[1,2,3]'   % same as x=[1;2;3]
x =
   1
   2
   3
```

```
>> b=A*x
b =
  28
  34
  28
>> A\b   % this is x!
ans =
   1
   2
   3
>> inv(A)*b % less efficient and accurate
ans =
  1.0000
  2.0000
  3.0000
>> [S,D]=eig(A)
S =
 -0.5774   -0.8131   -0.3416
 -0.5774    0.4714   -0.4714
 -0.5774    0.3416    0.8131
D =
 15.0000         0         0
       0    4.8990         0
       0         0   -4.8990
>> A*S(:,1)-D(1,1)*S(:,1) % test 1st eigenvector-eigenvalue pair
ans =
1.0e-014 *

 -0.1776
  0.3553
 -0.3553
```

Note the multiplicative factor 10^{-14} in the last computation. MATLAB performs all operations using standard IEEE double precision.

MATLAB Programming and Debugging

Several constructs are used in MATLAB:

1. repetitive loops (fixed number of times)

    ```
    for <expression>
        <list of commands>
    end
    ```

2. repetitive loops (indefinite number of times)

    ```
    while <expression>
        <list of commands>
    end
    ```

3. conditional branching

    ```
    if expression
    ```

```
        <list of commands>
    elseif expression
        <list of commands>
        :
    else
        <list of commands>
    end
```

or

```
    switch expression
    case <expression>
        <list of commands>
        :
    case <expression>
        <list of commands>
    otherwise
        <list of commands>
    end
```

The following examples illustrate the use of each construct:

1. `for` loop: determine the sum of the squares of integers from 1 to 10

```
S = 0;  % initialize running sum
for k = 1:10
    S = S+k^2;
end
```

   What is S? Verify with MATLAB.

2. `while` loop: determine the sum of the inverses of squares of integers from 1 until the inverse of the integer square is less than 10^{-5}

```
S = 0;  % initialize running sum
k = 1;  % initialize current integer
incr = 1; % initialize test value
while incr>=1e-10
    S = S+incr;
    k = k+1;
    incr = 1/k^2;
end
```

   What is the value of S returned by this script? Compare to $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$.

   Can `k` and `incr` be both initialized by 0?

3. `if` statement: evaluate $y = \frac{1}{x-2}$ for a given (but unknown) scalar x

```
function y=f(x)
if x==2
    disp('y is undefined at x = 2')
else
    y=1/(x-2);
end
```

or with `switch` statement:

```
function y=f(x)
switch x
case 2
    disp('y is undefined at x = 2')
otherwise
    y=1/(x-2);
end
```

Try `f(1)`, `f(2)`. Modify the example to allow for arrays as input.

★ Whenever possible all these construct should be avoided and available MATLAB functions used to improve efficiency. In particular lengthy `do` loops introduce a substantial overhead. Compare

```
>> tic; S=0; for k=1:1000000; S=S+1/k^2; end; toc; S
elapsed_time =
   1.8830
S =
   1.6449
```

and

```
>> tic; S=sum(1./(1:1000000).^2); toc; S
elapsed_time =
   0.0800
S =
   1.6449
```

★ Programming with MATLAB is fairly easy. Still a script or function may not execute properly due to some programming error. In this case MATLAB returns an error indicating where it stopped and why. Most of the time this is sufficient to find out the error and correct it, especially when you get used to it. But keep in mind that even non-fatal mistakes may eventually force a program to later crash. Debugging tools are available in the MATLAB editor to force the execution to stop at specific places within a script or function(s) and access the current state of available variables.

Write a script or function with the MATLAB editor and position the cursor on a selected line. Then try the buttons  and  in the editor window before executing the file in the Command Window. Observe what happens. Check the value of variables. To continue and eventually exit the debugger press return.

Exercises

Now that you have been through the essential elements of MATLAB relevant in this text, a good exercise is to go through the commands and change values, functions, and problems to familiarize yourself with the MATLAB syntax and commands introduced in this chapter.