Basic Mathematics

Matrix Multiplication

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The aim of this document is to provide a short, self assessment programme for students who wish to learn how to multiply matrices.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Introduction

In the package **Introduction to Matrices** the basic rules of *addition* and *subtraction* of matrices, as well as *scalar multiplication*, were introduced. The rule for the *multiplication of two matrices* is the subject of this package. The first example is the simplest.

Recall that if M is a matrix then the transpose of M, written M^T, is the matrix obtained from M by writing the rows of M as the columns of M^T.

If $A = (a_1\, a_2\, \ldots\, a_n)$ is a $1 \times n$ (row) matrix and $B = (b_1\, b_2\, \ldots\, b_n)^T$ is a $n \times 1$ (column) matrix then the product AB is defined as

$$AB = (a_1\, a_2\, \ldots\, a_n) \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

This general rule is sometimes called the *inner product*.

N.B. The *row matrix* is on the left and the *column matrix* is on the right.

Example 1 In each of the following cases, find the product AB.

(a) $A = (1\ 2), \quad B = (4\ 3)^T$. **(b)** $A = (1\ 1\ 1), \quad B = (2\ 3\ 4)^T$.

(c) $A = (1\ -1\ 2\ 3), \quad B = (1\ 1\ -3\ 2)^T$.

Solution

(a) $AB = (1\ 2)\begin{pmatrix} 4 \\ 3 \end{pmatrix} = 1 \times 4 + 2 \times 3 = 4 + 6 = 10.$

(b) $AB = (1\ 1\ 1)\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 1 \times 2 + 1 \times 3 + 1 \times 4 = 2 + 3 + 4 = 9.$

(c) $$AB = (1\ -1\ 2\ 3)\begin{pmatrix} 1 \\ 1 \\ -3 \\ 2 \end{pmatrix} = \begin{aligned} & 1 \times 1 + 1 \times (-1) + 2 \times (-3) + 3 \times 2 \\ & = 1 + (-1) + (-6) + 6 = 0\,. \end{aligned}$$

Exercise 1. For each of the cases below, calculate AB. (Click on the green letters for solutions.)

(a) $A = (-2\ 4)$, $\quad B = (3\ 2)^T$,
(b) $A = (5\ 3\ -2)$, $\quad B = (3\ -4\ 2)^T$,
(c) $A = (4\ 4\ -2\ -3)$, $\quad B = (5\ -4\ 3\ 2)^T$.

The following observations are worth noting.

- The row matrix is on the left, the column matrix is on the right.
- The row and column have the same number of elements.
- The inner product AB is a 1×1 matrix, i.e. a *number*.
- Nothing has yet been said about a matrix product BA.

Quiz If $A = (x\ x\ 1)$ and $B = (x\ 6\ 9)^T$, which of the following values of x will result in $AB = 0$?

(a) $x = 1$, (b) $x = 3$, (c) $x = -3$, (d) $x = -2$.

2. Matrix Multiplication 1

The previous section gave the rule for the multiplication of a row vector A with a column vector B, the *inner product* AB. This section will extend this idea to more general matrices.

Suppose that $A = \begin{pmatrix} a_1 & a_2 & \dots & a_n \\ c_1 & c_2 & \dots & c_n \end{pmatrix}$ and $B = (b_1\ b_2\ \dots\ b_n)^T$.

Then

$$AB = \begin{pmatrix} a_1 & a_2 & \dots & a_n \\ c_1 & c_2 & \dots & c_n \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a_1b_1 + a_2b_2 + \dots + a_nb_n \\ c_1b_1 + c_2b_2 + \dots + c_nb_n \end{pmatrix}$$

Example 2 Find AB for each of the following cases.

(a) $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$, $\qquad B = (4\ 3)^T$.

(b) $A = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & -3 \end{pmatrix}$, $\quad B = (2\ 3\ 4)^T$.

Solution

(a) $AB = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \times 4 + 2 \times 3 \\ 3 \times 4 + (-1) \times 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 9 \end{pmatrix}$

(b) $AB = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & -3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \times 2 + 1 \times 3 + 1 \times 4 \\ (-2) \times 2 + 1 \times 3 + (-3) \times 4 \end{pmatrix} = \begin{pmatrix} 9 \\ -13 \end{pmatrix}$

The following observations on AB are worth noting.

- The element in the *first row* of AB is the *inner product* of the *first row* of A with the column matrix B.
- The element in the *second row* of AB is the *inner product* of *the second row* of A with the column matrix B.
- The number of *columns* of A must be equal to the number of *rows* of B.
- If A is $2 \times n$ and B is $n \times 1$ then AB is 2×1.

This rule for multiplication may be extended to matrices, A, which have more than two rows. For example, if A had 3 rows then the resulting matrix, AB, would have a third row; the value of this element would be the *inner product* of the *third row* of A with the column matrix B.

EXERCISE 2. For each of the cases below, calculate AB. (Click on the green letters for solutions.)

(a) $A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$, $B = (4\ 3)^T$.

(b) $A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix}$, $B = (2\ 3\ 4)^T$.

(c) $A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}$, $B = (4\ 3)^T$.

(d) $A = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix}$, $B = (5\ -4\ \ 3\ \ 2)^T$.

3. Matrix Multiplication 2

The extension of the concept of matrix multiplication to matrices, A, B, in which A has more than one row and B has more than one column is now possible. The product matrix AB will have the same number of columns as B and each column is obtained by taking the product of A with each column of B, in turn, as shown below.

Let $A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ and let b_1, b_2 be the first and second columns of B respectively. Then

$$Ab_1 = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 13 \\ 4 \end{pmatrix} \quad \text{and} \quad Ab_2 = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}.$$

Thus

$$AB = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 11 & -2 \\ 13 & 4 \\ 4 & 5 \end{pmatrix}.$$

EXERCISE 3. For each of the cases below, calculate AB. (Click on the green letters for solutions.)

(a) $A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$, $\qquad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$.

(b) $A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$, $\qquad B = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$.

(c) $A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}$, $\qquad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$.

(d) $A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix}$, $\qquad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}$.

NB The rules for finding the product of two matrices are summarised on the next page.

- If A is $m \times n$ and B is $n \times r$ then the product AB exists.
- The resulting matrix is $m \times r$. $((m \times \underbrace{n)(n}\times r) = m \times r)$
- The element in the i th row, j th column of the matrix AB is the *inner product* of the i th row of A with the j th column of B.

Example 3 Find the element in the $2nd$ row $3rd$ column of AB if

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 4 & -2 \\ 3 & -1 & 2 \end{pmatrix}.$$

Solution Since A is 2×2 and B is 2×3, the product AB exists and is a 2×3 matrix. The required element is the inner product of the *second row* of A with the *third column* of B, i.e.

$$(-1) \times (-2) + 3 \times 2 = 2 + 6 = 8\,.$$

Exercise 4. If

$$A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$$

find the ij element, *i.e.* the element in the i th row j th column, of AB for the following cases. (Click on the green letters for solutions.)

(a) $i = 3,\ j = 2$, (b) $i = 2,\ j = 3$, (c) $i = 1,\ j = 2$,

(d) $i = 2,\ j = 1$, (e) $i = 3,\ j = 1$, (f) $i = 1,\ j = 3$,

Quiz Which of the following is the element in the 3 rd row, 3 rd column, of the matrix AB in the above exercise?

(a) 26, (b) -26, (c) -12, (d) 12.

4. The Identity Matrix

If A and B are two matrices, the product AB can be found if the number of *columns* of A equals the number of *rows* of B. If A is 2×3 and B is 3×5 then AB can be calculated but BA does not exist. The *order* in which matrices are multiplied together matters. Even when AB and BA both exist it is usually the case that $AB \neq BA$.

There is one particular matrix, the *identity matrix*, which has very special multiplication properties. The $n \times n$ *identity matrix* is the $n \times n$ matrix with 1s and 0s *as shown below*.

Example 4 The 2×2, 3×3 and 4×4 identity matrices are

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The most important property of the identity matrix is revealed in the following exercise.

EXERCISE 5. If the *identity matrix* is denoted by I and the matrix M is

$$M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix},$$

use the appropriate identity matrix to calculate the following matrix products. (Click on the green letters for solutions.)

(a) IM, where I is the 2×2 identity matrix,

(b) MI, where I is the 3×3 identity matrix.

In matrix multiplication the identity matrix, I, behaves exactly like the number 1 in ordinary multiplication. This was seen in the previous exercise. For part (a), the matrix I is the 2×2 identity matrix; in part (b), I was 3×3; they satisfy the equation $IM = M = MI$.

Example 5 The matrices A, B are

$$A = \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix}.$$

Calculate AB and BA.

Solution Using the rules of matrix multiplication,

$$AB = \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I\,.$$

$$BA = \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I\,.$$

The matrix B is the *inverse* of the matrix A, and this is usually written as A^{-1}. Equally, the matrix A is the *inverse* of the matrix B. The equation $AA^{-1} = A^{-1}A = I$ is always true.

5. Quiz on Matrix Multiplication

Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -2 & 1 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{pmatrix}$.

Choose the correct option from the following.

Begin Quiz

1. The 2×3 element of AB is

(a) -1, (b) 1, (c) 0, (d) 2.

2. The 3×1 element of CB is

(a) 3, (b) -1, (c) 4, (d) -6.

3. The 2×2 element of CA is

(a) 4, (b) -3, (c) 0, (d) 2.

4. (a) $B = C^{-1}$, (b) $A = B^{-1}$, (c) $C = A^{-1}$

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)

If the row matrix $A = (-2\ 4)$ and the column matrix

$$B = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

are multiplied, the resulting inner product is

$$\begin{aligned} AB = (-2\ \ 4)\begin{pmatrix} 3 \\ 2 \end{pmatrix} &= -2 \times 3 + 4 \times 2 \\ &= -6 + 8 \\ &= 2\,. \end{aligned}$$

Click on the green square to return □

Exercise 1(b)
If the row matrix $A = (5\ \ 3\ \ -2\,)$ and the column matrix

$$B = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$$

are multiplied, the resulting inner product is

$$\begin{aligned} AB = (5\ 3\ \ -2\,)\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} &= 5\times 3 + 3\times(-4) + (-2)\times 2 \\ &= 15 - 12 - 4 = -1\,. \end{aligned}$$

Click on the green square to return □

Exercise 1(c)

If the row matrix $A = (4\,4\ -2\ -3\,)$ and the column matrix

$$B = \begin{pmatrix} 5 \\ -4 \\ 3 \\ -2 \end{pmatrix}$$

are multiplied, their inner product AB is

$$\begin{aligned} (4\,4-2-3)\begin{pmatrix} 5 \\ -4 \\ 3 \\ -2 \end{pmatrix} &= 4\times 5+4\times(-4)+(-2)\times 3+(-3)\times(-2) \\ &= 20-16-6+6=4\,. \end{aligned}$$

Click on the green square to return □

Exercise 2(a)
For the 2×2 matrix

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$$

and the column $B = (4\ 3)^T$, the product AB is

$$AB = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} (-2) \times 4 + 4 \times 3 \\ 5 \times 4 + 3 \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 29 \end{pmatrix}.$$

Click on the green square to return □

Exercise 2(b)

If the 2×3 matrix

$$A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix}$$

and the column matrix $B = (2\ 3\ 4)^T$ are multiplied together, then the resulting product AB is

$$AB = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \times 2 + 3 \times 3 + 2 \times 4 \\ 4 \times 2(-1) \times 3 + (-1) \times 4 \end{pmatrix}$$

$$= \begin{pmatrix} 27 \\ 1 \end{pmatrix}.$$

Click on the green square to return □

Exercise 2(c)
If the 3×2 matrix is

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}$$

and the column matrix is $B = (4\ 3)^T$, then the product AB is

$$AB = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} (-2) \times 4 + 4 \times 3 \\ 5 \times 4 + 3 \times 3 \\ 4 \times 4 + (-1) \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 29 \\ 13 \end{pmatrix}$$

Click on the green square to return □

Exercise 2(d)
If the 2×4 matrix

$$A = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix}$$

is multiplied with the column matrix $B = (5\ -4\ \ 3\ \ 2)^T$, the resulting product, AB, is

$$AB = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ -4 \\ 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \times 5 + 4 \times (-4) + (-2) \times 3 + (-3) \times 2 \\ 3 \times 5 + (-1) \times (-4) + (-1) \times 3 + 2 \times 2 \end{pmatrix} = \begin{pmatrix} -8 \\ 20 \end{pmatrix} .$$

Click on the green square to return ☐

Exercise 3(a)
Let A and B be the 2×2 matrices:

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}.$$

The matrix AB is

$$AB = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} -2 \times (-2) + 4 \times 5 & -2 \times 4 + 4 \times 3 \\ 5 \times (-2) + 3 \times 5 & 5 \times 4 + 3 \times 3 \end{pmatrix}$$

$$= \begin{pmatrix} 24 & 4 \\ 5 & 29 \end{pmatrix}.$$

Click on the green square to return ☐

Exercise 3(b)
If A and B are the 2×2 matrices:

$$A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix},$$

then the matrix product AB is

$$AB = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 3 \times 5 + 2 \times (-7) & 3 \times (-2) + 2 \times 3 \\ 7 \times 5 + 5 \times (-7) & 7 \times (-2) + 5 \times 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

This is called the 2×2 **identity matrix**.

Click on the green square to return □

Exercise 3(c)
If A and B are the matrices

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$$

then the matrix product AB is

$$AB = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} (-2)\times(-2)+4\times 5 & (-2)\times 4+4\times 3 \\ 5\times(-2)+3\times 5 & 5\times 4+3\times 3 \\ 4\times(-2)+(-1)\times 5 & 4\times 4+(-1)\times 3 \end{pmatrix}$$

$$= \begin{pmatrix} 24 & 4 \\ 5 & 29 \\ -13 & 13 \end{pmatrix} .$$

Click on the green square to return $\square$

Exercise 3(d)
If A is 2×3 and B be is 3×2 given by the following

$$A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix},$$

then the matrix product AB is

$$AB = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \times (-2) + 3 \times 5 + 2 \times 4 & 5 \times 4 + 3 \times 3 + 2 \times (-1) \\ 4 \times (-2) + (-1) \times 5 + (-1) \times 4 & 4 \times 4 + (-1) \times 3 + (-1) \times (-1) \end{pmatrix}$$

$$= \begin{pmatrix} 13 & 27 \\ -17 & 14 \end{pmatrix}.$$

Click on the green square to return □

Exercise 4(a)

If $A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$,

the (32) element in the matrix AB, $(AB)_{32}$, is the inner product of the *third row* of A with the *second column* of B, i.e.

$$\begin{aligned} (AB)_{32} &= 2 \times 1 + 3 \times (-2) + (-1) \times (-3) + (-2) \times 0 \\ &= 2 - 6 + 3 + 0 = -1\,. \end{aligned}$$

Click on the green square to return □

Exercise 4(b)

If $A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$,

the (23) element of AB, $(AB)_{23}$, is the inner product of the *second row* of A with the *third column* of B, i.e.

$$\begin{aligned}(AB)_{23} &= 7 \times (-3) + (-8) \times (-5) + (-6) \times (-7) + 2 \times 6 \\ &= -21 + 40 + 42 + 12 = 73\,.\end{aligned}$$

Click on the green square to return $\square$

Exercise 4(c)

If $A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$,

the (12) element in the matrix AB, $(AB)_{12}$, is the inner product of the *first row* of A with the *second column* of B, i.e.

$$\begin{aligned} (AB)_{12} &= 1 \times 1 + (-2) \times (-2) + 4 \times (-3) + 5 \times 0 \\ &= 1 + 4 - 12 + 0 \\ &= -7\,. \end{aligned}$$

Click on the green square to return $\square$

Exercise 4(d)

If $A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$,

the (21) element in the matrix AB, $(AB)_{21}$, is the inner product of the *second row* of A with the *first column* of B, i.e.

$$\begin{aligned} (AB)_{21} &= 7 \times (-2) + (-8) \times 0 + (-6) \times 4 + 2 \times 0 \\ &= -14 + 0 - 24 + 0 = -38\,. \end{aligned}$$

Click on the green square to return □

Exercise 4(e)

If $A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$,

the (31) element in the matrix AB, $(AB)_{31}$, is the inner product of the *third row* of A with the *first column* of B, i.e.

$$\begin{aligned} (AB)_{31} &= 2 \times (-2) + 3 \times 0 + (-1) \times 4 + (-2) \times 0 \\ &= -4 + 0 - 4 + 0 = -8\,. \end{aligned}$$

Click on the green square to return □

Exercise 4(f)

If $A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$,

the (13) element in the matrix AB, $(AB)_{13}$, is the inner product of the *first row* of A with the *third column* of B, i.e.

$$\begin{aligned} (AB)_{13} &= 1 \times (-3) + (-2) \times (-5) + 4 \times (-7) + 5 \times 6 \\ &= -3 + 10 - 28 + 30 = 9\,. \end{aligned}$$

Click on the green square to return □

Exercise 5(a)

For the 2×3 matrix $$M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix},$$
the left identity matrix (multiplying M on the left to obtain IM) is the 2×2 matrix I:
$$I = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right).$$

The product IM is then
$$IM = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 1 + 0 \times 7 & 1 \times 2 + 0 \times 8 & 1 \times 4 + 0 \times 6 \\ 0 \times 1 + 1 \times 7 & 0 \times 2 + 1 \times 8 & 0 \times 4 + 1 \times 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} = M.$$

Click on the green square to return □

Exercise 5(b)
For the 2×3 matrix $M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix}$, the right identity matrix (multiplying M on the right to obtain MI) is the 3×3 matrix I:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

The product MI is thus

$$MI = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 1 + 2 \times 0 + 4 \times 0 & 1 \times 0 + 2 \times 1 + 4 \times 0 & 1 \times 0 + 2 \times 0 + 4 \times 1 \\ 7 \times 1 + 8 \times 0 + 6 \times 0 & 7 \times 0 + 8 \times 1 + 6 \times 0 & 7 \times 0 + 8 \times 0 + 6 \times 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} = M .$$

Click on the green square to return □

Solutions to Quizzes

Solution to Quiz:

Multiplying the row matrix $A = (x\ x\ 1)$ with the column matrix

$$B = \begin{pmatrix} x \\ 6 \\ 9 \end{pmatrix}$$

from the left we have

$$\begin{aligned} AB = (x\ x\ 1)\begin{pmatrix} x \\ 6 \\ 9 \end{pmatrix} &= x \times x + x \times 6 + 1 \times 9 \\ &= x^2 + 6x + 9 = (x+3)^2\,. \end{aligned}$$

Therefore the inner product $AB = 0$, if $x = -3$. End Quiz

Solution to Quiz:
The matrices A and B from Exercise 4 are

$$A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}.$$

The (33) element in the matrix of AB, $(AB)_{33}$, is the inner product of the *third row* of A with the *third column* of B, i.e.

$$\begin{aligned} (AB)_{33} &= 2 \times (-3) + 3 \times (-5) + (-1) \times (-7) + (-2) \times 6 \\ &= -6 - 15 + 7 - 12 = -26\,. \end{aligned}$$

End Quiz