Matthew Schwartz

Lecture 18: Antennas and interference

1 Visualizing radiation patterns

In the Lecture 16, we showed that an accelerating charge produces a field which decays with distance differently in different directions. In the plane transverse to the acceleration, the field dies like $E_\theta \sim \frac{1}{r}$ while in the direction parallel to the acceleration it decays much faster like $E_r \sim \frac{1}{r^2}$. In this lecture, we will be considering more and more elaborate arrangements of sources placed near the origin ($r=0$) and looking at how large the field is very far away from those sources. We call this the **far-field limit**. In the far-field limit, we only care about the transverse component of the electric field E_θ since it is parametrically larger than the parallel component.

We will be calculating expressions where this field varies in space and time

$$E_\theta = E_0 e^{i(kr-\omega t+\delta)} \tag{1}$$

We will try to write this complex representation of the electric field in a form where E_0 is real, so that the actual electric field is $\mathrm{Re}(E_\theta) = E_0 \cos(kr-\omega t)$. The corresponding intensity averaged over time is then $I = \epsilon_0 \mathrm{Re}(E_\theta)^2 = \frac{1}{2}\epsilon_0 E_0^2$.

A useful observation is that when you add two fields, the time-averaged intensity only depends on their amplitudes and phase difference, not the phases separately. There are many ways to see this, but the most effective might simply be direct calculation. Say we have a field

$$E = E_1 e^{i(kx-\omega t+\phi_1)} + E_2 e^{i(kx-\omega t+\phi_2)} \tag{2}$$

Then averaging the intensity over a period $T = \frac{2\pi}{\omega}$ we get

$$\langle I \rangle = \frac{1}{T}\int_0^T dt \frac{1}{2}\varepsilon_0 (\mathrm{Re}[E])^2 = \frac{\varepsilon_0}{2}\left\{ \frac{(E_1+E_2)^2}{2}\cos^2\frac{\Delta\phi}{2} + \frac{(E_1-E_2)^2}{2}\sin^2\frac{\Delta\phi}{2} \right\} \tag{3}$$

where $\Delta\phi = \phi_1 - \phi_2$. The reason the separate phases drop out is that we can always write the sum of two waves as having an overall, average phase, and a phase difference. The average phase combines with the time oscillation and gets averaged out, but the phase difference does not. Thus, in the following we will concentrate on phase differences among sets of waves. We will go back and forth between talking about fields and intensities, with the time-averaging of the intensities always being implicit.

To begin, say we just have a group of charges exactly at the origin moving up and down in the z direction. Such an arrangement is called a **monopole antenna**. Now, consider how the amplitude of the electric field produced looks in the x-y plane. It will be $E_\theta \sim \frac{1}{r}\cos(kr-\omega t)$. We can draw this pattern as follows

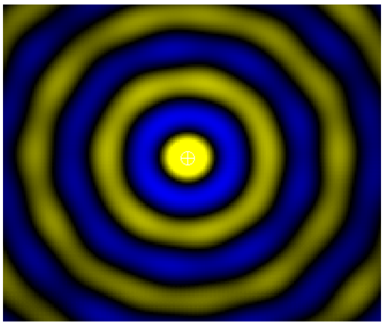


Figure 1. A monopole antenna produces the radiation pattern as generated in Falstad's ripple program.

In this visualization, the blue means $E_\theta > 0$ the yellow means $E_\theta < 0$ and the black is $E_\theta = 0$. The brightness of the color represents $|E_\theta|$.

Now, pictures like Fig. 1 show how the amplitude of the electric field varies in two dimensions. To understand how much power can be received from a transmitter, we instead want to know how large the intensity is at different points in three dimensions. So let's imagine a spherical shell or radius R surrounding the antenna with the antenna direction pointing from the north pole to the south pole and consider how large the generated field is around the shell. The amplitude and intensity are given by functions of the latitude and longitude angles θ and ϕ around the sphere. We can then plot a surface where the radial distance from the origin in the plot is the intensity $I(\theta, \phi)$.

For example, for the monopole antenna, the surface looks like

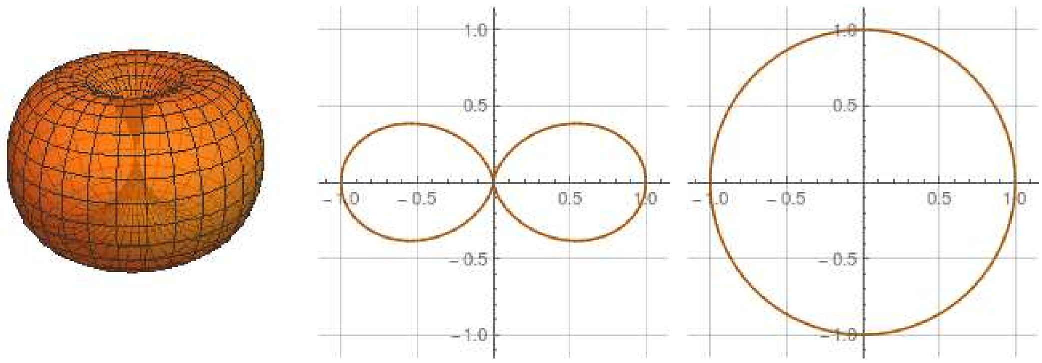


Figure 2. Radiation pattern for a monopole antenna. Left is the 3D pattern. Middle and right show vertical and equatorial slices, respectively.

This is known as a **3D radiation pattern**.

To make sure this is clear, let's review what is plotted. In Lecture 16 we showed that an accelerating charge produces a field whose components are $E_r = \frac{q}{4\pi\varepsilon_0 R^2}$ and $E_\theta = \frac{q a}{4\pi\varepsilon_0 c^2 R}\sin\theta$. For an antenna, the net charge is neutral so $E_r = 0$ and only E_θ matters. At a fixed R very far away from the source, $E_\theta \sim \sin\theta$, where θ is the angle to the direction of the accelerating charges. Thus the 3D pattern above is a spherical plot of the surface $R = \sin^2\theta$; the middle pattern above is a polar plot of the contour $R = \sin^2\theta$ which is a vertical slice through the 3D plot; the right pattern is a polar plot along the equator $\theta = \frac{\pi}{2}$, so $R = 1$. There is a notebook Interference.nb on the canvas site which generates these patterns.

Since the field dies off so fast in the vertical direction, we are often only interested in the 2D pattern in the plane of the equator, as in the right panel of Figure 2. In this case, since the intensity is constant along the equator at a distance r from the antenna, the equatorial cross section of the 3D radiation pattern is just a circle. When we put more antennas together, the resulting interference patterns will be more interesting, as we will now see. So we will work with two different 2D visualizations: the Falstad ripple type, as in Fig. 1 which attempts to show the phase of the amplitude, and the 2D projections as in the right panel of Fig. 2 which shows the relative intensity as a function of angle.

2 Two sources

What happens if we put two sources with the same amplitude and in phase a distance d apart?

Let's start with a very small separation. If the distance $d \ll \lambda$, where λ is the wavelength, then the waves will be entirely in phase everywhere. This radiation pattern will look just like the single source but more intense. Say the field from one source is E_0; the intensity from one source is therefore $I_0 = \frac{1}{2}\epsilon_0 E_0^2$. Two sources at the same spot will produces a field $2E_0$ and an intensity $4I_0$. Since power is conserved, one factor of 2 comes from having two sources instead of 1, and the other factor of two comes from source loading, as we discussed in the context of sound.

Now consider moving the sources farther apart. For a general d, the field at a distant point is given by the sum of the fields produced from the two charges. The picture is like this

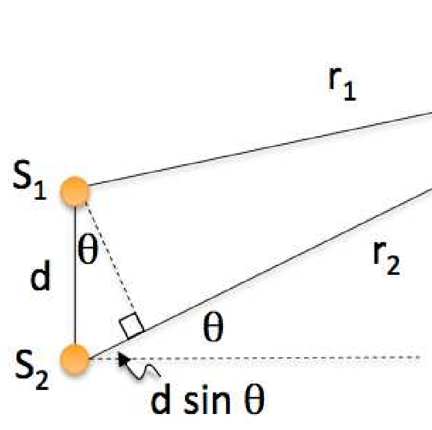


Figure 3. Two antennas spaced d apart produce fields which go to P.

Let us denote by r_1 and r_2 the distances from S_1 and S_2 to the point P where we want to evaluate the field, as shown in the picture. Then the field at P produced from the two sources are

$$E_1 = E_0 e^{i(kr_1 - \omega t)}, \quad E_2 = E_0 e^{i\,(kr_2 - \omega t)}, \tag{4}$$

So, by linearity, the total field at P is

$$E_P = E_1 + E_2 = E_0 e^{-i\omega t}(e^{ikr_1} + e^{ikr_2}) \tag{5}$$

Even though the sources are in phase, since the distance to the far point is different from the two sources, the fields may or may not add coherently.

Let us call θ the angle between the line connecting S_2 to P and the x axis. In terms of θ, the difference in the distance the light travels from S_2 and S_1 to get to P is (see Fig. 3):

$$\Delta r = r_2 - r_1 = d\sin\theta \tag{6}$$

The corresponding phase difference is therefore

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} = 2\pi\frac{d}{\lambda}\sin\theta \tag{7}$$

To see the effect on the amplitude, define

$$r = \frac{r_1 + r_2}{2} \tag{8}$$

Then

$$\begin{aligned}
E_P &= E_0 e^{-i\omega t}\left[e^{ik\left(r + \frac{\Delta r}{2}\right)} + e^{ik\left(r - \frac{\Delta r}{2}\right)}\right] \\
&= E_0 e^{-i\omega t} e^{ikr}\left[e^{i\frac{k\Delta r}{2}} + e^{-i\frac{k\Delta r}{2}}\right] \\
&= 2E_0 e^{-i\omega t} e^{ikr}\cos\left(k\frac{\Delta r}{2}\right) \\
&= 2\,E_0 e^{-i\omega t} e^{ikr}\cos\left(\pi\frac{d}{\lambda}\sin\theta\right) \\
&= 2\,E_0 e^{-i\omega t} e^{ikr}\cos\left(\frac{\Delta\phi}{2}\right)
\end{aligned}$$

So the intensity (averaged over time) is

$$I = \frac{1}{2}\epsilon_0 \mathrm{Re}(E_P)^2 = 4 I_0 \cos^2\left(\pi \frac{d}{\lambda}\sin\theta\right) = 4 I_0 \cos^2\left(\frac{\Delta\phi}{2}\right) \tag{9}$$

where $I_0 = \frac{1}{2}\epsilon_0 E_0^2$ is the intensity from a single source, as above. Taking the limit $d \ll \lambda$ we find that $I = 4I_0$, which is in agreement with total coherence, as discussed above.

In limit $d \gg \lambda$, the radiation pattern has a bunch of maxima and minima

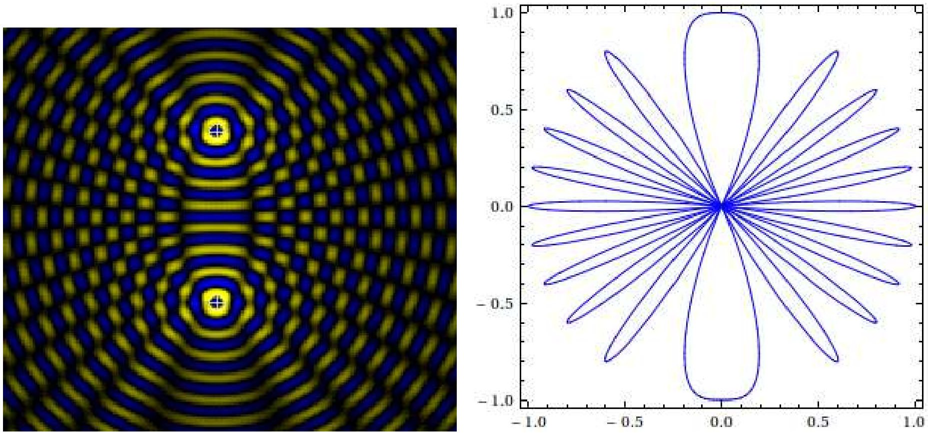


Figure 4. Two coherent sources separated by $d \gg \lambda$. Left shows the amplitude from Ripple, and the right shows the radiation pattern.

In the $d \gg \lambda$ case, the argument of the cosine, $\pi\frac{d}{\lambda}\sin\theta$ goes around the circle many times. The average intensity along a circle is

$$\langle I \rangle = \frac{1}{2\pi}\int_0^{2\pi} 4 I_0 \cos^2\left(\pi\frac{d}{\lambda}\sin\theta\right) \approx 2 I_0 \quad (d \gg \lambda) \tag{10}$$

The integral has been evaluated by replacing $\cos^2\theta$ by its average $\frac{1}{2}$ (if you don't trust this approximation, go ahead and can check it numerically yourself). Thus when the sources are farther than λ apart, all the constructive interference effects cancel on average – the total power emitted is then just the sum of the power begin emitted by the sources.

Ok, so neither $d \ll \lambda$ or $d \gg \lambda$ are particularly interesting. What happens if the distance between the sources is half a wavelength: $d = \frac{\lambda}{2}$? In this case, when the wave from one source gets to the other, there will be complete destructive interference. Thus, along the direction of the line between the sources, even far away from the sources, the intensity will be zero. On the other hand, on the line which goes perpendicular to the sources, there must be constructive interference. So we get $I = 4I_0$ along that line. We can confirm these assessments with a direct evaluation of Eq. (9) with $\lambda = \frac{d}{2}$:

$$I = 4 I_0 \cos^2\left(\frac{\pi}{2}\sin\theta\right) = I_0 \times \begin{cases} 4, & \theta = 0 \\ 0, & \theta = \frac{\pi}{2} \\ 4, & \theta = \pi \\ 0, & \theta = \frac{3\pi}{2} \end{cases} \tag{11}$$

So it has maxima at $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ and minima at 0 and π. The pattern looks like this

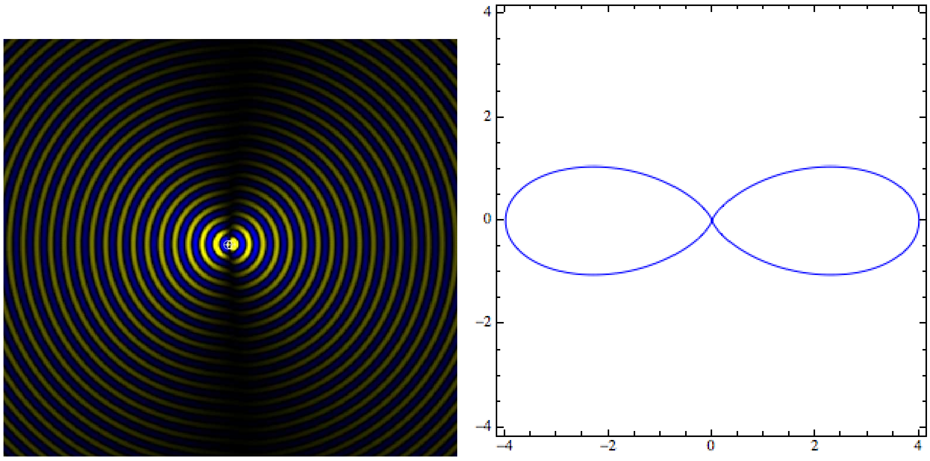


Figure 5. Amplitude and radiation pattern for two antennas separated in the y direction by $d = \frac{\lambda}{2}$.

So the field is focuses more in the x direction than the y direction.

This is useful: if we have a broadcasting antenna, we can arrange it to broadcast only in the East and West directions, with little power going North and South. That is, we can use interference to direct our transmission.

Unlike the $d \ll \lambda$ case, the power does not go out uniformly. Recall that in the $d \ll \lambda$ case the intensity along a circle was constant and the average intensity was just $\langle I \rangle = 4 I_0$. In this case, the average intensity around the circle is

$$\langle I \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta 4 I_0 \cos^2\left(\frac{\pi}{2} \sin\theta\right) \approx 1.4 I_0 \tag{12}$$

So we are able to achieve a local intensity of $4 I_0$ in the $\pm x$ direction using only 1.4 times the power of a single source.

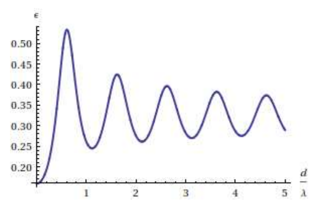
Let us define the transmission efficiency as amount of power going into a particular direction, say $\theta = 0$, divided by the power averaged over the whole circle. That is

$$\varepsilon = \frac{I(\theta = 0)}{\langle I \rangle} \tag{13}$$

For example, if we had a set of transmitting antennas at the origin and another receiving set far away at $\theta = 0$, this efficiency would tell us how much of the emitted power would get to the receiver. With one source $\varepsilon = 1$. With two sources with $d \ll \lambda$, $\varepsilon = 1$ as well. In this case, because the gain in intensity is due to source loading, although more power is being converted into radiation, the transfer of power is not actually more efficient than for a single source. For $d = \frac{\lambda}{2}$ we found $\varepsilon = \frac{4}{1.4} = 2.9$. So an antenna array with two sources set a half wavelength apart is nearly 3 times as efficient as a single source for broadcasting in a particular direction.

The next obvious question is how well can we do? That is, how can we maximize $\frac{I_0(\theta = 0)}{\langle I \rangle}$? With two sources spaced d apart, we want to maximize

$$\varepsilon = \frac{\cos^2\left(\pi \frac{d}{\lambda} \sin(\theta = 0)\right)}{\frac{1}{2\pi} \int_0^{2\pi} d\theta' \cos^2\left(\pi \frac{d}{\lambda} \sin\theta'\right)} = \frac{2\pi}{\int_0^{2\pi} d\theta' \cos^2\left(\pi \frac{d}{\lambda} \sin\theta'\right)} = \tag{14}$$



Plotting this function it's not hard to see that the maximum is at $d = \frac{\lambda}{2}$. The efficiency at that point is $\varepsilon = 2.9$, as above.

Can we do better?

3 Phased arrays

We found that if we have two sources, the field at a point P is determined by the phase shift $\Delta\phi = 2\pi\frac{d}{\lambda}\sin\theta$ from the different distance light has to go to get to P from the two sources. Then,

$$E_P = 2E_0 e^{-i\omega t} e^{ikr} \cos\left(\frac{\Delta\phi}{2}\right) \tag{15}$$

How does the pattern change if we decide to produce the sources out of phase. Say the sources differ by a phase δ.

This phase shift then add to the phase shift $\Delta\phi$ from the path lengths at the point P and we get

$$E_p = 2E_0 e^{-i\omega t} e^{ikr} \cos\left(\frac{\Delta\phi + \delta}{2}\right) \tag{16}$$

So that

$$I = 4I_0 \cos^2\left(\frac{\Delta\phi + \delta}{2}\right) = 4I_0 \cos^2\left(\pi\frac{d}{\lambda}\sin\theta + \frac{\delta}{2}\right) \tag{17}$$

How does δ affect the radiation pattern?

For example, consider again the case with two antennas spaced $d = \frac{\lambda}{2}$ apart in the y direction, but now have them be broadcasting exactly out of phase $\delta = \pi$. The radiation pattern from Eq. (17) looks like

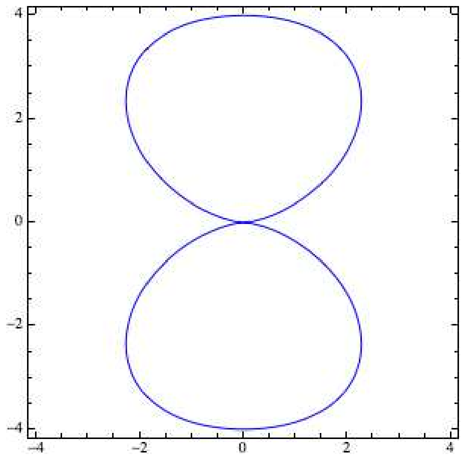


Figure 6. Radiation pattern with $d = \frac{\lambda}{2}$ and the two sources exactly out of phase ($\delta = \pi$).

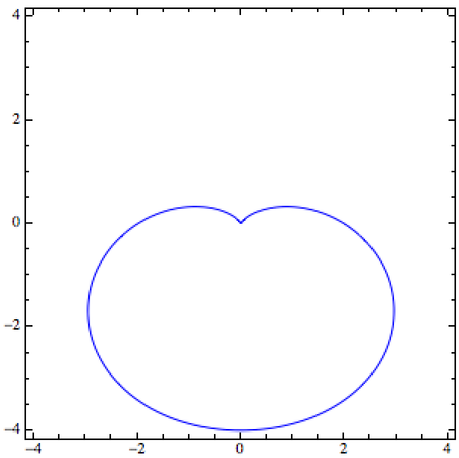
This looks just like Fig. 5, but rotated 90°. We can check that this makes sense. When a wave leaves the bottom antenna and goes towards the top one, it will have rotated by π since the two antennas are exactly half a wavelength apart. This phase shift of π adds to the phase shift $\delta = \pi$ we put in by hand so that the wave going upward from the bottom antenna is now exactly in phase with the wave from the second antenna. Thus they will continue to be in phase as we move upward, which explains the upward lobe (and similarly the downward lobe).

This result is very practical: if we want to broadcast North-South instead of East-West we don't have to go up on the roof and reorient the antenna. Instead, we can simply insert a delay into the current driving one antenna to change its phase by π. If you play with the plots in the Interference.nb Mathematica notebook, you can see how the pattern changes as δ goes from 0 to 2π.

Now consider the case with $d = \frac{\lambda}{4}$ and $\delta = \frac{\pi}{2}$. Then,

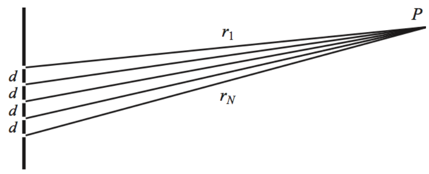
$$I(\theta) = 4I_0 \cos^2\left(\frac{\pi}{4}\sin\theta + \frac{\pi}{4}\right) = I_0 \times \begin{cases} 2, & \theta = 0 \\ 0, & \theta = \frac{\pi}{2} \\ 2, & \theta = \pi \\ 4, & \theta = \frac{3\pi}{2} \end{cases} \tag{18}$$

Which looks like this



So now the radiation points mostly in a single direction. This is clearly a much more efficient way of transmitting a signal from point to point. Now our antenna array can radiate South only, with little going North, East or West.

To do better, we can put a bunch of sources in a row. Say we have N sources in a line, each separated from the previous by a distance d with a phase shift of δ. This is known as a **phased array**:



Then far away, at an angle θ to the array, each one has a phase shift of

$$\Delta = 2\pi\frac{d}{\lambda}\sin\theta + \delta \tag{19}$$

from the previous one. If E_0 is the field from one source, then the net field at P is

$$E_P = E_0 e^{-i\omega t} e^{ikr}\left(1 + e^{i\Delta} + e^{2i\Delta} + \cdots + e^{(N-1)\,i\Delta}\right) \tag{20}$$

To sum this, we use the mathematical formula

$$\sum_{n=0}^{N-1} x^n = \frac{x^N - 1}{x - 1} \tag{21}$$

So that

$$\begin{aligned} E_P &= E_0 e^{-i\omega t} e^{ikr} \sum_{n=0}^{N-1} (e^{i\Delta})^n \\ &= E_0 e^{-i\omega t} e^{ikr} \frac{e^{iN\Delta} - 1}{e^{i\Delta} - 1} \\ &= E_0 e^{-i\omega t} e^{ikr} e^{iN\frac{\Delta}{2}} e^{-i\frac{\Delta}{2}} \left[\frac{\sin\left(N\frac{\Delta}{2}\right)}{\sin\left(\frac{\Delta}{2}\right)}\right] \end{aligned}$$

Averaging over time the phases drop out and the intensity is then

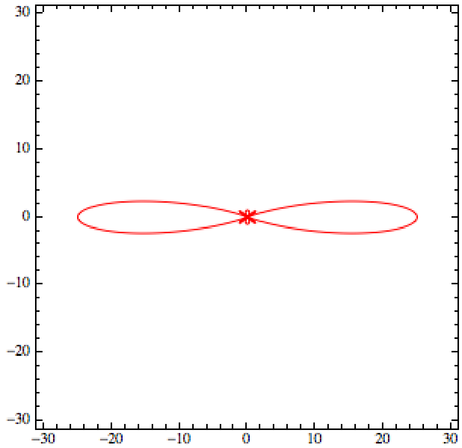
$$I = I_0 \frac{\sin^2\left(N\frac{\Delta}{2}\right)}{\sin^2\left(\frac{\Delta}{2}\right)} \tag{22}$$

We should check this against our previous results. For $N=1$ we find $I=I_0$. For $N=2$ we can use $\frac{\sin(2x)}{\sin(x)}=2\cos(x)$ to see that $I=4I_0\cos^2\frac{\Delta}{2}$ as in Eq. (17).

Since $\Delta=2\pi\frac{d}{\lambda}\sin\theta+\delta$, our general formula for N antennas with a phase shift of δ between each pair is:

$$I=I_0\frac{\sin^2\left(N\left(\pi\frac{d}{\lambda}\sin\theta+\frac{\delta}{2}\right)\right)}{\sin^2\left(\pi\frac{d}{\lambda}\sin\theta+\frac{\delta}{2}\right)} \tag{23}$$

For $N=5$, $d=\frac{\lambda}{2}$ and $\delta=0$ this looks like



So the power is more narrowly focused in the x direction. The efficiency at $\theta=0$ is now

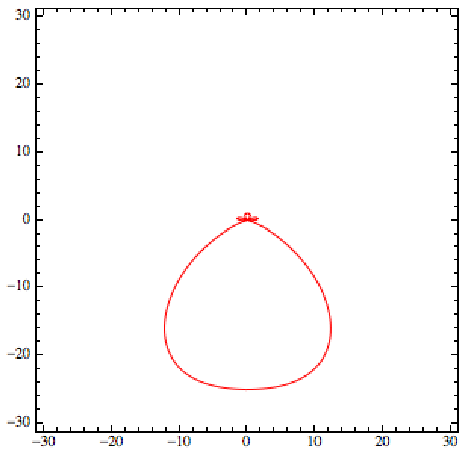
$$\varepsilon=\frac{I(0)}{\langle I\rangle}=\frac{25}{3.5}=8.18 \tag{24}$$

More generally, the intensity at $\theta=0$ with $\delta=0$ is

$$I=I_0\lim_{\theta\to 0}\frac{\sin^2\left(N\left(\pi\frac{d}{\lambda}\sin\theta\right)\right)}{\sin^2\left(\pi\left(\frac{d}{\lambda}\sin\theta\right)\right)}=I_0\lim_{\theta\to 0}\frac{\left(N\pi\frac{d}{\lambda}\theta\right)^2}{\left(\pi\frac{d}{\lambda}\theta\right)^2}=N^2I_0 \tag{25}$$

On the other hand $\langle I\rangle$ grows linearly with N, since when one averages over all directions there is as much destructive as constructive broadcast. Thus, for $\delta=0$, the efficiency $\varepsilon\sim N$ which grows linearly with the number of antenna.

We can also adjust the phases so that the antenna points in one direction. With $N=5$ $d=\frac{\lambda}{4}$ and $\delta=\frac{\pi}{2}$ we find



What does a phased array of N evenly spaced sources look like? It looks like

Figure 7. Yagi antenna array.

This is called a **Yagi antenna array**.

Phased arrays are very common for radar in the military, since radar can scan very fast in different directions by changing phases electronically, rather than rotating a radar dish. Ever heard of a patriot missile? Did you know that PATRIOT shands for "Phased Array Tracking Radar to Intercept On Target". These missiles are guided by ground-based phased-array radar. Ever wonder what's in the nose of a fighter jet?

Here's a MIG-31 with the nose cone removed:

Figure 8. That orange grid under the nose cone of a fighter jet is a phased array.

Fighter jets, and many modern aircraft are equipped with phased array radar. The orange lines you see under the cone are antennas whose phases can be adjusted electronically to point the radar in any direction. There are horizontal antennas in front and vertical ones in back, so the antenna can point in any direction in three-dimensions.

4 Other antennas

There are a number of other types of antennas you might want to know about.

A car antenna is a called a **whip antenna** or **monopole antenna**. It is not direction specific, since the care is driving around, there would be no reason to point it in a particular direction

Figure 9. Whip antenna on a car

Next, there are dipole antennas, which have two arms

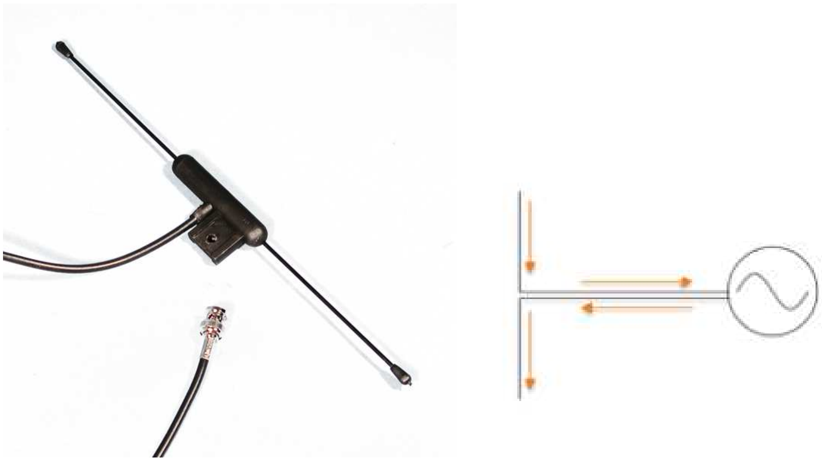


Figure 10. In a dipole antenna charge moves up and down in the two arms coherently. This produces waves which are mostly confined to the plane transverse to the antenna.

Walkie-talkies have antennas that are much shorter. They curl around to pick up the $\vec{B}$ field rather than the $\vec{E}$ field. The are also non-directional

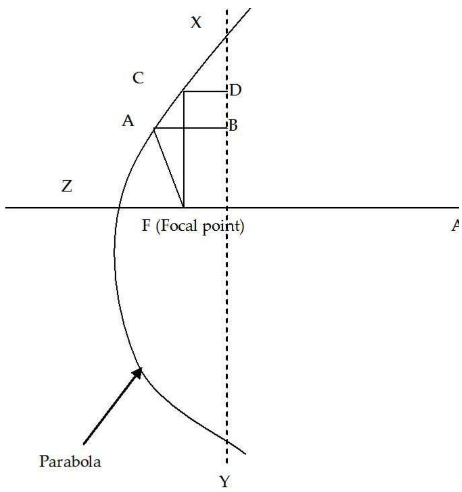
Figure 11. Rubber ducky antenna

Finally, there are the good old rabbit ears

Figure 12. Rabbit ear antenna.

Fiddling with the straight antennas (the whips) can pick up greater signal. When signals come into a house they can bounce off walls and the polarizations can change. Thus changing the angle of the whips can help pick up particular polarizations. The middle round ring is for picking up magnetic fields. Usually it can rotate to pick up different polarizations as well. The whips pick up VHF (very-high frequency) stations, with frequencies 3 MHz - 300 MHz (100 m-1m). The ring picks up UHF (ultra-high frequency) stations which have frequencies 300 MHz – 1 MHz or wavelengths of 1m to 10cm.

As you probably already know, dish antennas are usually parabolic. The parabolic shape focuses incoming plane waves to a point, the focus of a parabola. The picture looks like this

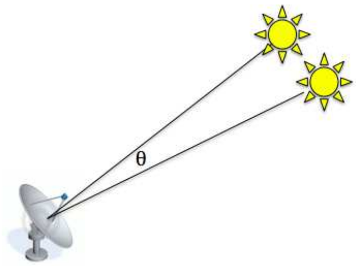


A plane wave has the same phase at D and B. Using Snell's law and some calculus you can then show that the shape of the dish must be a parabola for the reflected waves all to be in phase when they reach a single point (the focal point). There is a nice simulation of this in Ripple.

5 Interferometry

Sometimes we want to use an antenna to do more than receive a signal as intensely as possible. For example, we might be interested in angular resolution. Suppose two stars are separated by an angle θ in the sky. When θ is very small, what kind of antenna would be able to make out that there are two stars instead of just one?

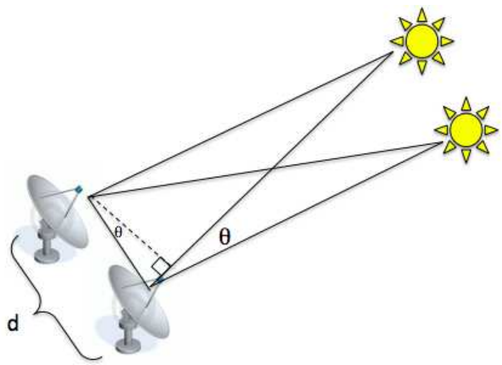
Suppose we only had one antenna to receive the signal.



This antenna would measure the sum of the amplitudes of the waves from the two stars. If the antenna focuses the signal to just a point, then it just receives some amplitude $A(t) = A_0 \cos(\omega t)$. One can get the frequency and intensity from such an amplitude, but there is nothing more. A single star or two nearby stars are indistinguishable. A bigger dish collects more light and hence is more **sensitive**. But it's angular **resolution** is no better than a simple monopole antenna. Since the wave is still measured at just a single point and has no angular information.

Of course, you can point the dish in a different direction to get some angular resolution. But there's actually a much smarter and more powerful way to increase the angular resolution.

Consider an array of antennas now, each separated from the next by a distance d



(26)

To an excellent approximation, the light coming from both stars will be plane waves in the direction between the stars and the earth. This is true no matter what produces the light in the stars, or how far they are from us. Let us write

$$A_{\text{dish 1}} = A_{\text{star 1}} + A_{\text{star 2}} \tag{27}$$

where $A_{\text{star 1}}$ and $A_{\text{star 2}}$ are the amplitudes for the two waves, including their phases, when they hit dish 1.

For simplicity, let us first consider the case where the antennas are in a line perpendicular to the direction to star 1. Then the light from star 1 will hit all the antennas with exactly the same phase. Since star 2 is offset from star 1 by an angle θ, its light will not have the same phase at both antennas. The phase difference will between the two waves at dish 2 is the usual$\Delta\phi = 2\pi\frac{d}{\lambda}\sin\theta$. So

$$A_{\text{dish 2}} = A_{\text{star 1}} + A_{\text{star 2}}\cos(\Delta\phi) \tag{28}$$

Similarly if we for a third dish equally spaced from the other two, its amplitude is

$$A_{\text{dish 3}} = A_{\text{star 1}} + A_{\text{star 2}}\cos(2\Delta\phi) \tag{29}$$

Now we have three equations. Assuming that these amplitudes are measured perfectly, we can solve them for the three unknowns $A_{\text{star 1}}$, $A_{\text{star 2}}$ and $\Delta\phi$.

Note that if we only had the intensity instead of the amplitude at each antenna, the phase difference would simply get time-averaged away and each dish would measure the same intensity. This way of resolving small angles is an example of **interferometry**. It requires amplitude-level phase information.

What is the angular resolution of an antenna array? Well, if the phase difference is too small, the signal will get washed out by noise. The biggest the phase difference can possibly be is $\Delta\phi = \pi$. Basically, if $\Delta\phi \ll \pi$ we are not going to see much interference. For small angles θ, $\Delta\phi = 2\pi\frac{d}{\lambda}\sin\theta \approx 2\pi\frac{d}{\lambda}\theta$. Thus setting $\Delta\phi = \pi$ we find

$$\theta_{\text{resolvable}} = \frac{\lambda}{2d} \tag{30}$$

If you can resolve phase shifts smaller than π, you might do a little better than this. The important point is the scaling of the resolvable angles with wavelength and the distance d between the the antennas. The bigger d is the smaller the angles are that you can resolve. For this reason, you want very large interferometers. Also, the smaller the wavelengths, the smaller angles you can resolve.

There is an interferometer in New Mexico called the VLA (Very Large Array) which looks like this:

Figure 13. VLA array in New Mexico has 27 antennas each 25 meters in diameter. The antennas can be moved up to a maximum baseline of 22 miles

Here is an example of the improvement in angular resolution which you can get with an interferometer.

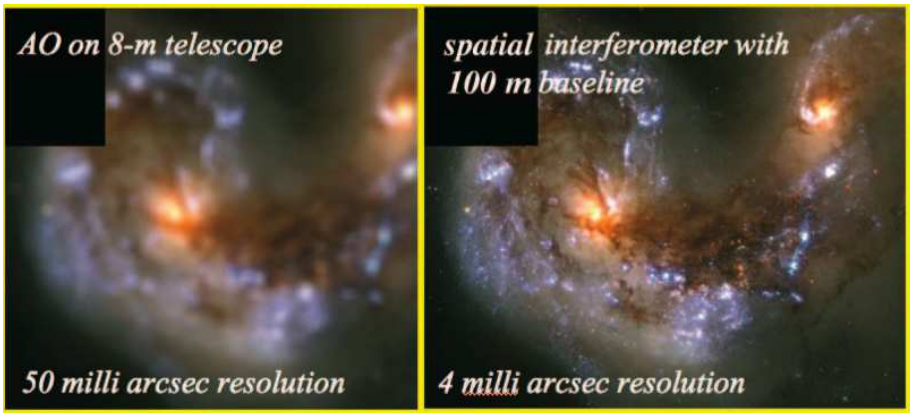


Figure 14. The left is how well one can see with a single 8m diameter telescope. The second shows what you can see with 2 such telescopes separated by 100 m.