McGraw-Hill Ryerson

Mathematics of Data Management

Solutions CD-ROM

Roland W. Meisel, B.Sc., B.Ed., M.Sc.
Port Colborne, Ontario

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CONTENTS

Using *McGraw-Hill Ryerson Mathematics of Data Management, Solutions*

McGraw-Hill Ryerson Mathematics of Data Management, *Solutions* provides complete model solutions to all the even-numbered questions for:
- each Review of Prerequisite Skills
- each numbered section, except for Achievement Check questions
- each Technology Extension
- each Review of Key Concepts
- each Chapter Test
- each Cumulative Review
- the Course Review

Teachers will find the *McGraw-Hill Ryerson Mathematics of Data Management*, *Solutions* helpful in planning students' assignments. Depending on their level of ability, the time available, and local curriculum constraints, students will probably only be able to work on a selection of the questions presented in the *McGraw-Hill Ryerson Mathematics of Data Management* Student text. A review of the solutions provides a valuable tool in deciding which problems might be better choices in any particular situation. The solutions will also be helpful in determining which questions might be suitable for extra practice of a particular skill.

In mathematics, for all but the most routine of practice questions, multiple solutions exist. The methods used in *McGraw-Hill Ryerson Mathematics of Data Management*, *Solutions* are generally modelled after the examples presented in the student text. Although only one solution is presented per question, teachers and students are encouraged to develop as many different solutions as possible. Discussion and comparison of different methods can be highly rewarding. This leads to a deeper understanding of the concepts involved in solving the problem and to a greater appreciation of the many connections among topics. Occasionally different approaches are used. This is done deliberately to enrich and extend the reader's insight or to emphasize a particular concept. In such cases, the foundations of the approach are supplied.

There are many complex numerical expressions that are evaluated in a single step. The solutions are developed with the understanding that the reader has access to a scientific calculator, and that one has been used to achieve the result. Despite access to calculators, some problems offer irresistible challenges to develop solutions in a manner that avoids the need for the calculator. Such challenges should be encouraged.

There are a number of situations, particularly in the solutions to the Practise questions, where the reader may sense a repetition in the style of presentation. These solutions were developed with an understanding that a solution may, from time to time, be viewed in isolation and as such might require the full treatment.

The entire body of *McGraw-Hill Ryerson Mathematics of Data Management, Solutions* was created on a home computer in Microsoft® Word 2000. Graphics for the solutions were created with the help of spreadsheets (Microsoft® Excel 2000 and Corel® Quattro® Pro 8), Fathom™ 1.1, and TI-83 Plus graphing calculator screens captured to the computer. Some of the traditional elements of the accompanying graphic support are missing in favour of the rapid capabilities provided by the electronic tools. Since many students will be working with such tools in their future careers, discussion of the features and interpretation of these various graphs and tables are encouraged, and will provide a very worthwhile learning experience.

CHAPTER 1   Tools for Data Management

Review of Prerequisite Skills

a) $f(2) = 3(2)^2 - 5(2) + 2$
$= 12 - 10 + 2$
$= 4$

b) $g(2) = 2(2) - 1$
$= 4 - 1$
$= 3$

c) $f(g(-1)) = f(2(-1) - 1)$
$= f(-2 - 1)$
$= f(-3)$
$= 3(-3)^2 - 5(-3) + 2$
$= 27 + 15 + 2$
$= 44$

d) $f(g(1)) = f(2(1) - 1)$
$= f(1)$
$= 3(1)^2 - 5(1) + 2$
$= 3 - 5 + 2$
$= 0$

e) $f(f(2)) = f(4)$
$= 3(4)^2 - 5(4) + 2$
$= 48 - 20 + 2$
$= 30$

f) $g(f(2)) = g(4)$
$= 2(4) - 1$
$= 8 - 1$
$= 7$

Review of Prerequisite Skills   Page 4   Question 4

The angle required for Type O is calculated by finding 46% of 360° or 166°. Similarly the angle for Type A is 155°, the angle for Type B is 29°, and the angle for Type AB is 11°. The circle graph at the right was generated using a spreadsheet. See Appendix B of the student text for instructions on using spreadsheets.
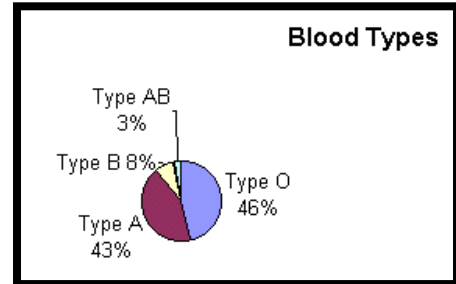
Review of Prerequisite Skills   Page 4   Question 6

The double-line graph below was generated using a spreadsheet. See Appendix B of the student text for instructions on using spreadsheets. You can also generate the graph using pencil and paper.

Review of Prerequisite Skills   Page 5   Question 8
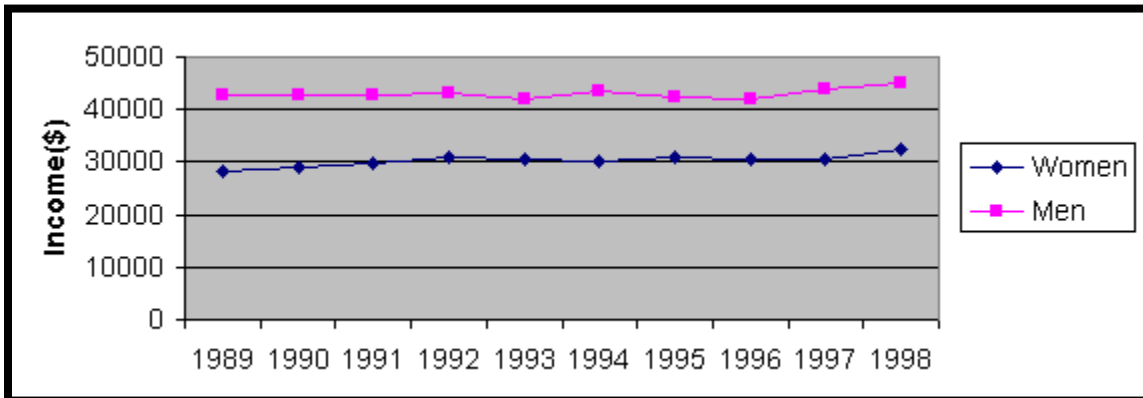
For similarity, all corresponding pairs of sides of two triangles must be in the same ratio.

The ratio $\dfrac{DF}{CA} = \dfrac{4}{2} = 2.$ The ratio $\dfrac{DE}{CB} = \dfrac{7}{4}.$ Therefore, ΔDFE is not similar to ΔCAB.

The ratio $\dfrac{GH}{DF} = \dfrac{6}{4} = \dfrac{3}{2}.$ The ratio $\dfrac{GJ}{DE} = \dfrac{12}{7}.$ Therefore, ΔDFE is not similar to ΔGHJ.

The ratio $\dfrac{GH}{CA} = \dfrac{6}{2} = 3.$ The ratio $\dfrac{HJ}{AB} = \dfrac{9}{3} = 3.$ The ratio $\dfrac{GJ}{CB} = \dfrac{12}{4} = 3.$ Therefore, ΔGHJ is similar to ΔCAB.

Review of Prerequisite Skills   Page 5   Question 10

a) The area of square ABCD at the right is
1 unit × 1 unit = 1 unit². The area of square
EFGH at the right is 2 units × 2 units = 4 units².
Therefore, the ratio of the areas is 4:1.

b) This ratio can be confirmed by counting the
number of grid units in each square, as shown in
the diagram at the right.

c) The diagram at the right was constructed using
The Geometer's Sketchpad®.

Review of Prerequisite Skills   Page 5   Question 12
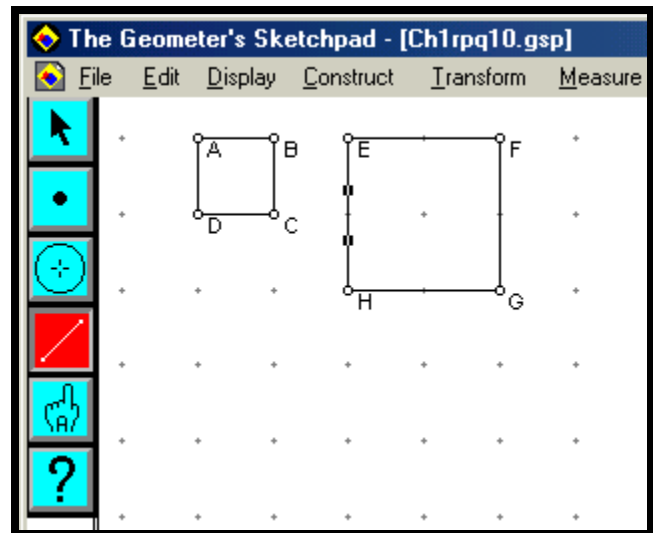
To change a fraction into a decimal, divide the numerator by the denominator. To change a percent to a decimal, divide by 100. A shortcut to dividing by 100 is to move the decimal point two places to the left. When the result is rounded, use the approximately equal to sign $\approx$.

a) $\dfrac{5}{20} = 0.25$ **b)** $\dfrac{23}{50} = 0.46$ **c)** $\dfrac{2}{3} \approx 0.67$

d) $\dfrac{138}{12} = 11.5$ **e)** $\dfrac{6}{7} \approx 0.857$ **f)** 73% = 0.73

CHAPTER 1   Tools for Data Management

Section 1.1   The Iterative Process

Practise

Section 1.1   Page 10   Question 2

The gains and losses of carbon dioxide in the atmosphere are shown using arrows in the following diagram. For example, the eruption of a volcano at the right will result in a gain of carbon dioxide in the atmosphere. On the other hand, photosynthesis in green plants will remove carbon dioxide from the atmosphere, as shown in the centre of the diagram.
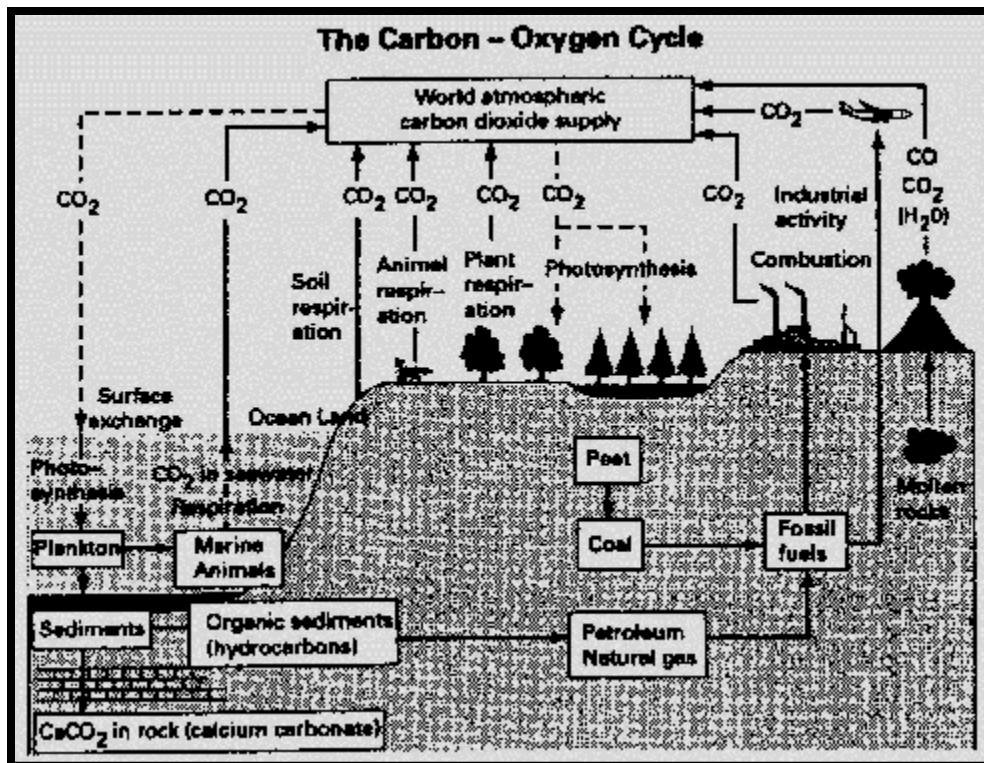
Section 1.1   Page 11   Question 4

Answers will vary.

Section 1.1   Page 11   Question 6

a) Begin with the first birthday. Compare it to the second birthday. If the second is higher than the first, interchange the two. Compare the first to the third. If the third is higher than the first, interchange the two. Continue until the first has been compared to all the birthdays in the list. Next, start with the second birthday compared to the third. If the third is higher than the second, interchange the two. Continue to the bottom of the list. Continue selecting each item and comparing it with the rest of the list until the list has been sorted from highest to lowest.

b) Compare the first birthday to the second. If the second is higher, interchange the two. Compare the second birthday to the third. If the third is higher, interchange the two. Continue down the list to the bottom. Then, start the process again from the top. When you can go through the list with no interchanges, you are finished. This method of sorting is called a "bubble sort."

Section 1.1   **Page 11**   **Question 8**

Pencil-and-Paper Solution:
Sketch each side of your triangle using a length that can be divided by three multiple times, such as 8.1 cm or 24.3 cm. Use this convenient measurement to trisect each side of the triangle, as shown in the diagram at the right. Continue to construct the required second iteration equilateral triangle, using a compass. Continue this process for the third iteration. If you can, perform a fourth iteration. Then, erase the internal lines to obtain the final fractal, as shown in the series of diagrams below.
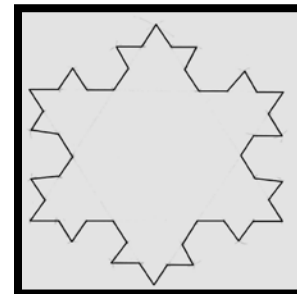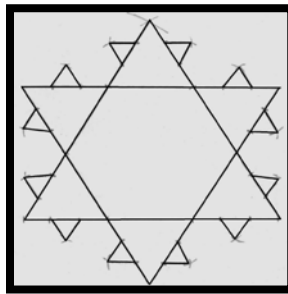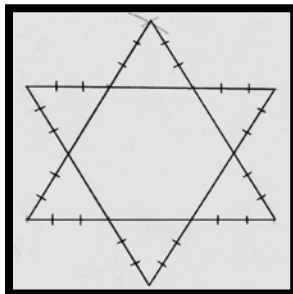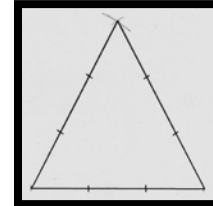
a) The original triangle has three sides. After the second iteration, this has increased to 12, a factor of 4. After the third iteration, it has increased to 48, another factor of 4.

b) After the fourth iteration, you may expect another factor of 4, which gives you 192 sides.

c) The pattern appears to work, as shown in the table at the right. Therefore, after the nth iteration, the number of sides is given by the expression $3 \times 4^{n-1}$.

Iteration	Number of Sides
1	3
2	$3 \times 4 = 12$
3	$3 \times 4 \times 4 = 48$
4	$3 \times 4 \times 4 \times 4 = 192$

Section 1.1   **Page 12**   **Question 10**

a) i) Begin with a vertical line segment of length one unit, as shown in the diagram at the right. Branch off the top at an angle of 90° and construct lengths of one-half unit. Repeat the process from the end of each new branch, making each succeeding set of branches one half the length of the previous set, until the desired fractal tree is drawn.
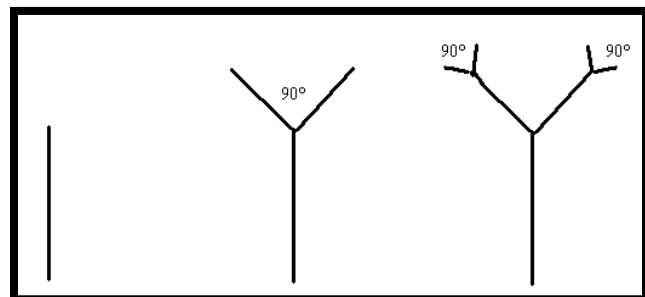
a) ii) Begin with a vertical line segment of length one unit, as shown in the diagram at the right. Branch off the top at an angle of 60° on each side and construct lengths of one-third unit. Extend the first branch upward by one-third unit. Repeat the process from the end of each new branch, making each succeeding set of branches one third the length of the previous set, until the desired fractal tree is drawn.
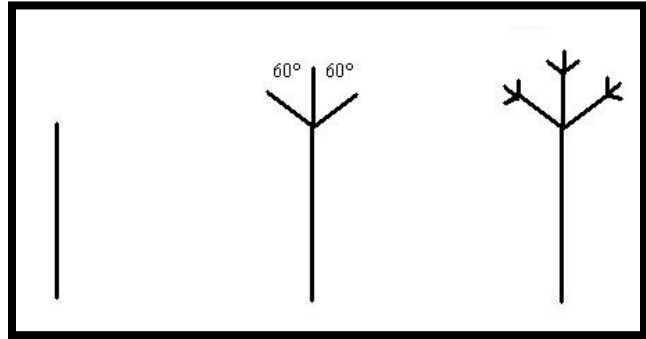
a) iii) Begin with a vertical line segment of length one unit, as shown in the diagram at the right. Branch off the top left at an angle of 120° and construct a branch of length one unit. Extend the first branch upward by one unit. Branch off the top right at an angle of 120° and construct a branch of length one unit.
Iteration: go up one, branch left, and repeat everything one step below. Go up one, and branch left one. Go up one and branch right, repeating what is on the left two steps below. Repeat the iteration.
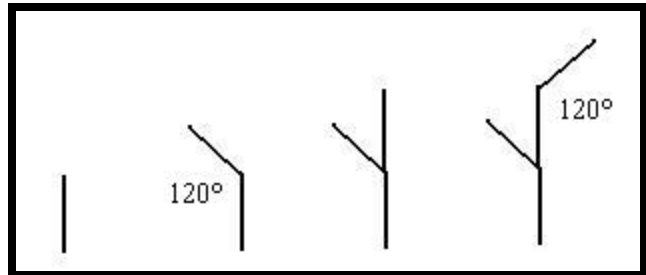
b) i) The number of branches at each stage of length can be counted and summarized in a table, as shown at the right. Adding up the last column gives a total length of 5 units.

Length	Number of Branches	Total
1.0	1	1
0.5	2	1
0.25	4	1
0.125	8	1
0.0625	16	1
	Total Length	5

b) ii) The number of branches at each stage of length can be counted and summarized in a table, as shown at the right. Adding up the last column gives a total length of 3 units.

b) iii) Counting up each segment of length one unit yields a total length of 64 units.

Length	Number of Branches	Total
1	1	1
$\dfrac{1}{3}$	3	1
$\dfrac{1}{9}$	9	1
	Total Length	3

c) No. If you trace branches in part iii), for example, the spirals will not be similar due to the different shape of each branch.

d) Answers may vary.

a) $t_1 = 0$ $t_2 = 2^{-0}$ $t_3 = 2^{-1}$ $t_4 = 2^{-\frac{1}{2}}$ $t_5 = 2^{-\frac{1}{\sqrt{2}}}$

$= 1$ $= \dfrac{1}{2}$ $= \dfrac{1}{\sqrt{2}}$ $= \dfrac{1}{\sqrt[\sqrt{2}]{2}}$

Succeeding terms will be multiple roots of 2 as shown at the right. $\dfrac{1}{\sqrt[\sqrt{2}\cdot\sqrt[\sqrt{2}]{2}]{2}}$

b) $t_1 = 256$ $t_2 = \sqrt{256}$ $t_3 = \sqrt{16}$ $t_4 = \sqrt{4}$

$= 16$ $= 4$ $= 2$

$t_5 = \sqrt{2}$ $t_6 = \sqrt{\sqrt{2}}$ $t_7 = \sqrt{\sqrt{\sqrt{2}}}$ $t_8 = \sqrt{\sqrt{\sqrt{\sqrt{2}}}}$

Succeeding terms are multiple roots of 2.

c) $t_1 = 2$ $t_2 = \dfrac{1}{2}$ $t_3 = \dfrac{1}{\frac{1}{2}}$ $t_4 = \dfrac{1}{2}$

$= 2$

This series alternates between a value of 2 for odd terms and a value of $\dfrac{1}{2}$ for even terms.

Answers will vary.

Answers may vary.

CHAPTER 1   Tools for Data Management

Section 1.2   Data Management Software

Practise

Microsoft® Excel:

a) SUM(A1:A9)   **b)** MAX(F3:K3)   **c)** MIN(A1:K4)   **d)** SUM(A2,B5,C7,D9)

e) AVERAGE(F5:M5), MEDIAN(F5:M5), MODE(F5:M5)   **f)** SQRT(A3)

g) B6^3   **h)** ROUND(D2,4)   **i)** COUNTA(D3:M9)

j) PRODUCT(A1,B3,C5:C10)   **k)** PI

Corel® Quattro® Pro:

a) @SUM(A1..A9)   **b)** @MAX(F3..K3)   **c)** @MIN(A1..K4)

d) @SUM(A2,B5,C7,D9)   **e)** @AVG(F5..M5, @MEDIAN(F5..M5), @MODE(F5..M5)

f) @SQRT(A3)   **g)** B6^3   **h)** @ROUND(D2,4)   **i)** @COUNT(D3..M9)

j) @MULT(A1,B3,C5..C10)   **i)** @PI

Section 1.2   **Page 22**   **Question 4**

If you need instruction on sheet referencing, see pages 20–21 of the student text. If you need general information on spreadsheets, see Appendix B of the student text. This solution is done using Microsoft® Excel. If you are using Corel® Quattro® Pro and need information on equivalent procedures, refer to Appendix B in the student text. Enter the branch data on Sheet1. Enter the price data on Sheet2. Enter the formula for total revenue on Sheet3, as shown in the screen shot below.
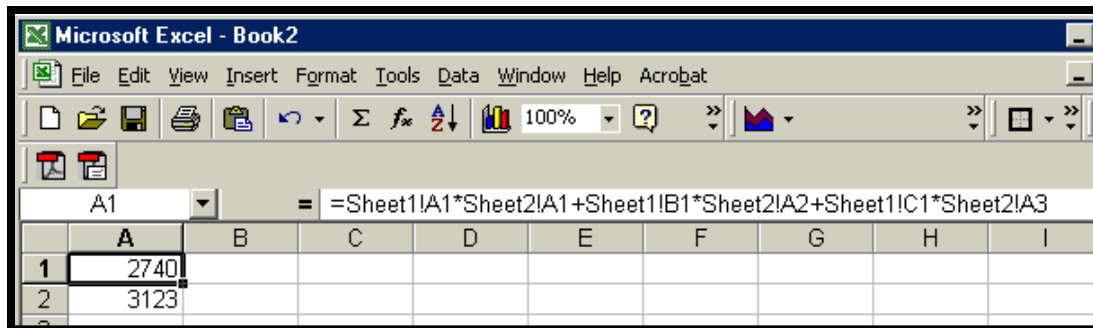
If you are using Corel® Quattro® Pro, the sheets are labelled alphabetically rather than numerically.

This solution is done using Corel® Quattro® Pro. If you are using Microsoft® Excel and need information on equivalent procedures, see Appendix B in the student text.

b) You can calculate each entry in the TOTALS column using the formula: @SUM(B4..G4). You can calculate each entry in the AVERAGES row using the formula: @AVG(B4..B17).

c) You can calculate each entry in the MAXIMUM row using the formula: @MAX(B4..B17). You can calculate each

	A	B	C	D	E	F	G	H
1	Nutritional Content of 14 Breakfast Cereals							
2	(amounts in grams)							
3	Name	Protein	Fat	Sugars	Starch	Fibre	Other	TOTALS
4	Alphabits	2.4	1.1	12.0	12.0	0.9	1.6	30.0
5	Bran Flakes	4.4	1.2	6.3	4.7	11.0	2.4	30.0
6	Cheerios	4.0	2.3	0.8	18.7	2.2	2.0	30.0
7	Crispix	2.2	0.3	3.2	22.0	0.5	1.8	30.0
8	Froot Loops	1.3	0.8	14.0	12.0	0.5	1.4	30.0
9	Frosted Flakes	1.4	0.2	12.0	15.0	0.5	0.9	30.0
10	Just Right	2.2	0.8	6.6	17.0	1.4	2.0	30.0
11	Lucky charms	2.1	1.0	13.0	11.0	1.4	1.5	30.0
12	Nuts 'n Crunch	2.3	1.6	7.1	16.5	0.7	1.8	30.0
13	Rice Krispies	2.1	0.4	2.9	22.0	0.3	2.3	30.0
14	Shreddies	2.9	0.6	5.0	16.0	3.5	2.0	30.0
15	Special K	5.1	0.4	2.5	20.0	0.4	1.6	30.0
16	Sugar Crisp	2.0	0.7	14.0	11.0	1.1	1.2	30.0
17	Trix	0.9	1.6	13.0	12.0	1.1	1.4	30.0
18	AVERAGES	2.52	0.93	8.03	14.99	1.82	1.71	
19	MAXIMUM	5.1	2.3	14.0	22.0	11.0	2.4	
20	MINIMUM	0.9	0.2	0.8	4.7	0.3	0.9	

entry in the MINIMUM row using the formula: @MIN(B4..B17). Your final spreadsheet will look like the screen shot above.

d) Set up your sort parameters, as shown in the screen shot at the right.
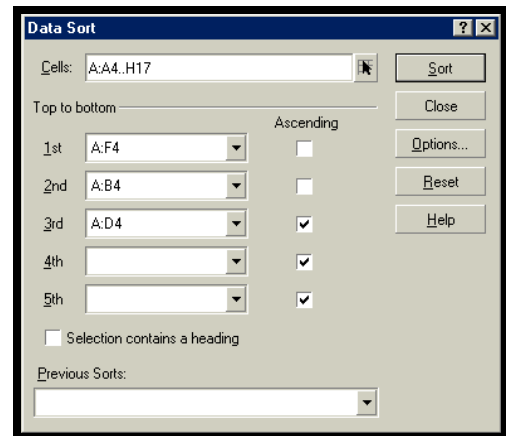
Your sorted data will look like the screen shot at the right.

	A	B	C	D	E	F	G	H
1	Nutritional Content of 14 Breakfast Cereals							
2	(amounts in grams)							
3	Name	Protein	Fat	Sugars	Starch	Fibre	Other	TOTALS
4	Bran Flakes	4.4	1.2	6.3	4.7	11.0	2.4	30.0
5	Shreddies	2.9	0.6	5.0	16.0	3.5	2.0	30.0
6	Cheerios	4.0	2.3	0.8	18.7	2.2	2.0	30.0
7	Just Right	2.2	0.8	6.6	17.0	1.4	2.0	30.0
8	Lucky charms	2.1	1.0	13.0	11.0	1.4	1.5	30.0
9	Sugar Crisp	2.0	0.7	14.0	11.0	1.1	1.2	30.0
10	Trix	0.9	1.6	13.0	12.0	1.1	1.4	30.0
11	Alphabits	2.4	1.1	12.0	12.0	0.9	1.6	30.0
12	Nuts 'n Crunch	2.3	1.6	7.1	16.5	0.7	1.8	30.0
13	Crispix	2.2	0.3	3.2	22.0	0.5	1.8	30.0
14	Frosted Flakes	1.4	0.2	12.0	15.0	0.5	0.9	30.0
15	Froot Loops	1.3	0.8	14.0	12.0	0.5	1.4	30.0
16	Special K	5.1	0.4	2.5	20.0	0.4	1.6	30.0
17	Rice Krispies	2.1	0.4	2.9	22.0	0.3	2.3	30.0
18	AVERAGES	2.52	0.93	8.03	14.99	1.82	1.71	
19	MAXIMUM	5.1	2.3	14.0	22.0	11.0	2.4	
20	MINIMUM	0.9	0.2	0.8	4.7	0.3	0.9	

e) The three pie charts will appear as shown below:

Averages

Other (5.69%)
Fibre (6.07%)
Protein (8.40%)
Fat (3.10%)
Sugars (26.76%)
Starch (49.98%)

Bran Flakes

Other (8.00%)
Protein (14.67%)
Fat (4.00%)
Sugars (21.00%)
Fibre (36.67%)
Starch (15.67%)

Rice Krispies

Other (7.67%)
Fibre (1.00%)
Protein (7.00%)
Fat (1.33%)
Sugars (9.67%)
Starch (73.33%)

f) Sheet B will look like the screen shot at the right.

Corel Quattro Pro - C:\My Documents\My Office
File Edit View Insert Format Tools Window

Arial   10   B I U Nor

B:F1

	A	B	C
1	Name	Fat	Fibre
2	Bran Flakes	1.2	11.0
3	Shreddies	0.6	3.5
4	Cheerios	2.3	2.2
5	Just Right	0.8	1.4
6	Lucky charms	1.0	1.4
7	Sugar Crisp	0.7	1.1
8	Trix	1.6	1.1
9	Alphabits	1.1	0.9
10	Nuts 'n Crunch	1.6	0.7
11	Crispix	0.3	0.5
12	Frosted Flakes	0.2	0.5
13	Froot Loops	0.8	0.5
14	Special K	0.4	0.4
15	Rice Krispies	0.4	0.3
16	AVERAGES	0.93	1.82
17	MAXIMUM	2.3	11.0
18	MINIMUM	0.2	0.3

Turn on **QuickFilter** under the **Tools** menu and set your filters, as shown at the right.

Custom QuickFilter

Fat

less than   1   And
                   And

OK
Cancel
Help

Custom QuickFilter

Fibre

greater than   1.5   And
                        And

OK
Cancel
Help
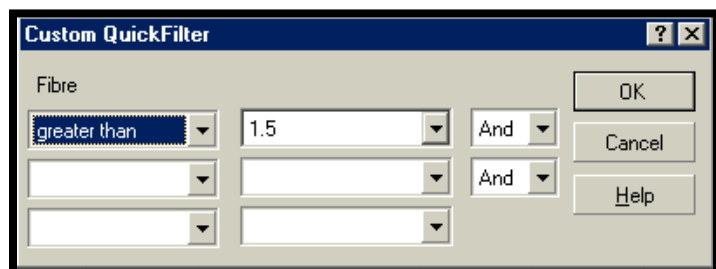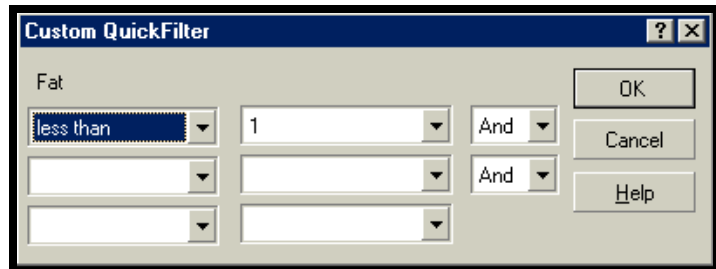
Your final sheet and bar graph will look like the screen shots below.
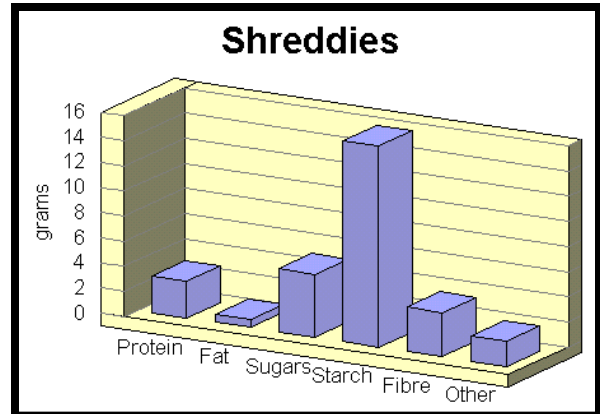
Section 1.2   **Page 23**   **Question 8**

If you are using Microsoft® Excel, select **Freeze Panes** under the **Window** menu. If you are using Corel® Quattro® Pro, select **Locked Titles** under the **View** menu.

CHAPTER 1   Tools for Data Management

Technology Extension   **Page 24**   **Introduction to Fathom™**

Note: The Student e-book contains a Microsoft® PowerPoint presentation showing the solutions for this section. Appendix B of the student text contains detailed information on using Fathom™.

Solution for Example 1

Start Fathom™ and drag the **case table** icon from the menu bar to the work area. Click on the attribute **<new>**, type the heading **Team**, and press **Enter**. Similarly, create attribute columns for **Wins**, **Losses**, **Ties**, **GF**, **GA**, and **Points**. Enter the data into each attribute column. When you are finished, your **case table** will look like the screen shot shown below.

Collection 1	Team	Wins	Losses	Ties	GF	GA	Points
1	Blades	12	3	5	70	56	29
2	Nice Guys	10	5	2	64	42	22
3	Leaves	7	5	8	55	51	22
4	Rinkrats	6	7	3	58	63	15
5	Ice Hogs	6	8	3	46	46	15
6	Slapshots	2	15	1	24	71	5

Note that your collection has been given the generic name **Collection 1**. You can double-click on the collection box and change the name to something more descriptive, like Hockey Stats.
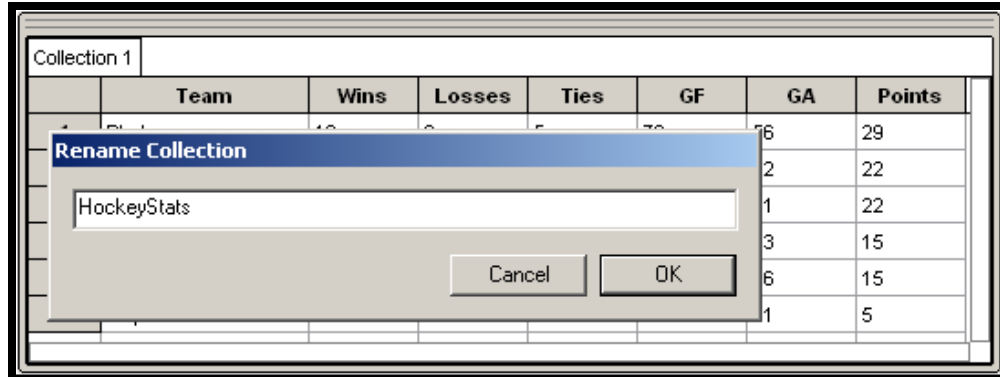
Drag the graph icon [icon] onto the work area. Drag the **Team** attribute from the **case table** to the horizontal axis of the graph, and the **Points** attribute to the vertical axis of the graph. Your graph will look like the screen shot at the right.
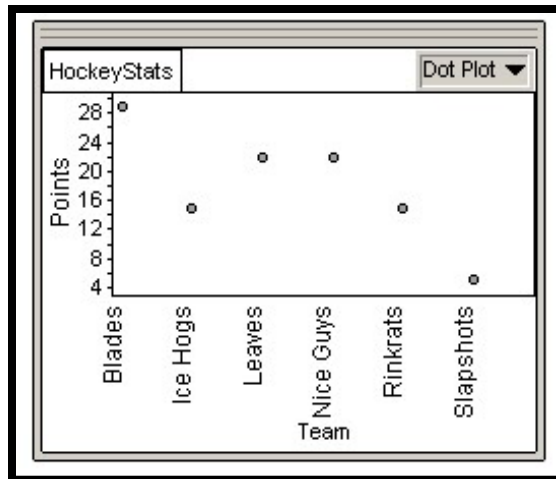
Solution for Example 2

a) Right-click on the **Points** attribute, and select **Sort Descending**. This will set the primary sort to use the **Points** attribute, and sort the case table in descending order of points. Set the secondary sort by right clicking on the **Wins** attribute, and selecting **Sort Descending**. Set the tertiary sort by right clicking on the **Losses** attribute, and selecting **Sort Ascending**. Your final sort will result in a case table that looks like the following screen shot.
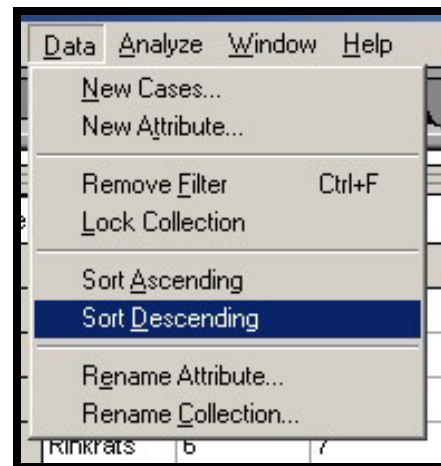
HockeyStats							
	Team	**Wins**	**Losses**	**Ties**	**GF**	**GA**	**Points**
1	Blades	12	3	5	70	56	29
2	Nice Guys	10	5	2	64	42	22
3	Leaves	7	5	8	55	51	22
4	Rinkrats	6	7	3	58	63	15
5	Ice Hogs	6	8	3	46	46	15
6	Slapshots	2	15	1	24	71	5

b) Select **Add Filter** from the **Data** menu. Click on the plus sign beside **Attributes**, as shown in the screen shot at the right. Double-click on the **Points** attribute, select the less-than button, and type 16. Click the **Apply** button, and then **OK** button.
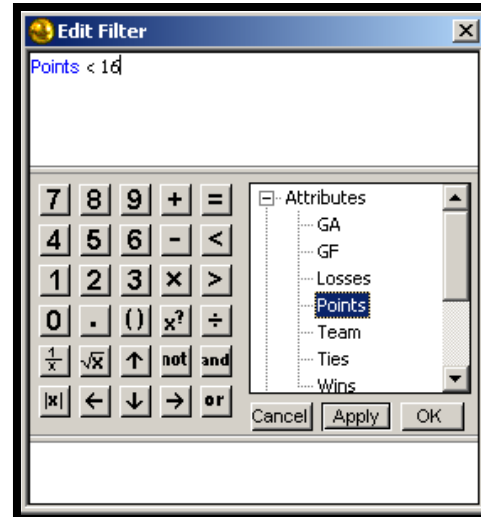
The results will look like the screen shot below:

HockeyStats							
	Team	**Wins**	**Losses**	**Ties**	**GF**	**GA**	**Points**
1	Rinkrats	6	7	3	58	63	15
2	Ice Hogs	6	8	3	46	46	15
3	Slapshots	2	15	1	24	71	5

Points < 16

Note that the **Filter** is listed at the bottom as **Points** < 16.

c) Click on the HockeyStats collection box, and drag a new **case table** onto the work area. Click on the **Wins** attribute. Select **Hide Attribute** from the **Display** menu. Use the same method to hide the **Losses**, **Ties**, and **Points** attributes. Right-click on the **GF** attribute, and use **Sort Descending** to rank the teams. Your result will look like the screen shot at the right.

HockeyStats			
	Team	**GF**	**GA**
1	Blades	70	56
2	Nice Guys	64	42
3	Leaves	55	51
4	Rinkrats	58	63
5	Ice Hogs	46	46
6	Slapshots	24	71

a) Double-click on the HockeyStats collection box to open the **Inspector**. Select the **Measures** tab. Double-click on **<new>** and rename it **MeanGA**. Right-click on the box under **Formula**, and select **Edit Formula**. Select **Functions/Statistical/One Attribute**. Double-click on **Mean**. Move up to **Attributes**, and double click on **GA**. Your final formula will look like the screen shot at the left below.
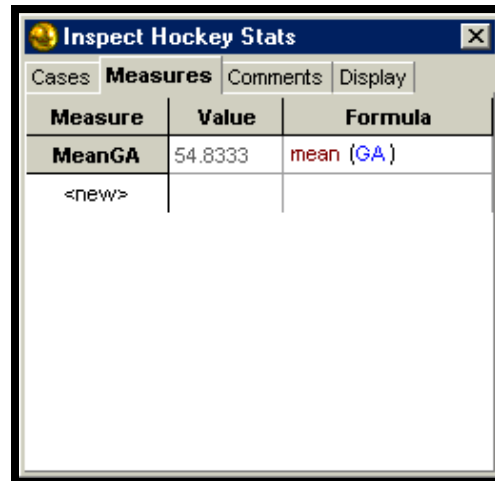
Click **Apply** and **OK**. The **Inspector** will look like the screen shot at the right above. Note that the mean of goals against has been calculated under **Value**.

b) In a manner similar to part a), add the largest value of goals for **GF** to the **Inspector** using the **max** function under **Functions/Statistical/One Attribute**.

c) Add the smallest value of goals for **GF** to the **Inspector** using the **min** function under **Functions/Statistical/One Attribute**.

d) Add the sum of goals against **GA** to the **Inspector** using the **sum** function under **Functions/Statistical/One Attribute**. The final result will look like the screen shot at the right.
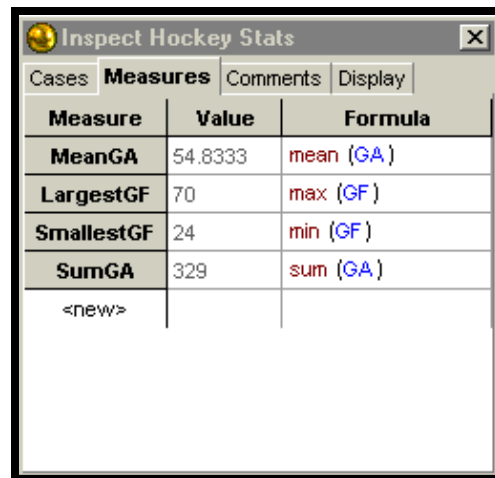
e) Add an attribute called **GAplusGF**. Edit the formula to GA + GF, as shown in the screen shot below.

	Team	Wins	Losses	Ties	GF	GA	Points	GAplusGF	<ne
=								GA + GF	
1	Blades	12	3	5	70	56	29	126	
2	Nice Guys	10	5	2	64	42	22	106	
3	Leaves	7	5	8	55	51	22	106	
4	Rinkrats	6	7	3	58	63	15	121	
5	Ice Hogs	6	8	3	46	46	15	92	
6	Slapshots	2	15	1	24	71	5	95	

Technology Extension   **Page 27**   **Question 2**

a) Select **New** from the **File** menu, drag a **case table** to the work area, rename the **<new>** attribute **StudentMarks**, and enter the data.

b) Right-click on the **StudentMarks** attribute, and select **Sort Ascending**. Your result will look like the screen shot at the right.

c) Double-click on the Student Marks collection box to open the **Inspector**. Select the **Measures** tab. Double-click on **<new>** and rename it **Mean**. Right-click on the box under **Formula**, and select **Edit Formula**. Select **Functions/Statistical/One Attribute**. Double-click on **Mean**. Move up to **Attributes**, and double-click on **StudentMarks**. Click on **Apply** and then **OK**. The mean of 70.2 will appear in the **Value** column.

d) In a similar manner to part c), add the median to the **Inspector** using the **median** function under **Functions/Statistical/One Attribute**. The value is 74, as shown in the screen shot at the right.
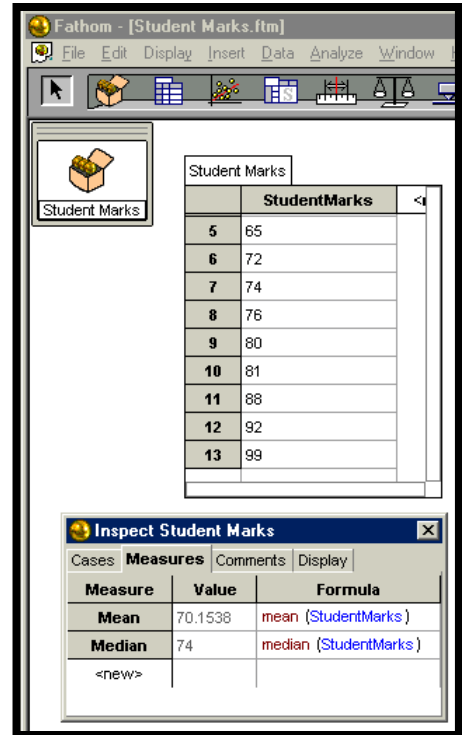
Technology Extension   **Page 27**   **Question 3**

Answers may vary.

Technology Extension   **Page 27**   **Question 4**

Answers may vary.

CHAPTER 1   Tools for Data Management

Section 1.3   **Databases**

Practise

Section 1.3   **Page 31**   **Question 2**

Answers may vary.

Section 1.3   **Page 32**   **Question 4**

a) Answers may vary.

c) i) Your final edited spreadsheet will look like the screen shot shown below.

	A	B	C	D	E	F	G
1	Numbers with Employment Income	All	Males	Females	%Males	%Females	Difference
2	Alberta	1519200	820740	698455	54.0	46.0	8.0
3	British Columbia	1988110	1062600	925515	53.4	46.6	6.9
4	Manitoba	580840	311050	269790	53.6	46.4	7.1
5	New Brunswick	373485	202085	171400	54.1	45.9	8.2
6	Newfoundland	249930	136965	112970	54.8	45.2	9.6
7	Northwest Territories	34225	18700	15520	54.6	45.3	9.3
8	Nova Scotia	448975	243005	205970	54.1	45.9	8.2
9	Ontario	5653475	2998725	2654750	53.0	47.0	6.1
10	Prince Edward Island	73610	39195	34415	53.2	46.8	6.5
11	Quebec	3533955	1926820	1607135	54.5	45.5	9.0
12	Saskatchewan	520915	281780	239140	54.1	45.9	8.2
13	Yukon Territory	19390	10240	9150	52.8	47.2	5.6

c) ii) Your bar graph will look like the one shown at the right.
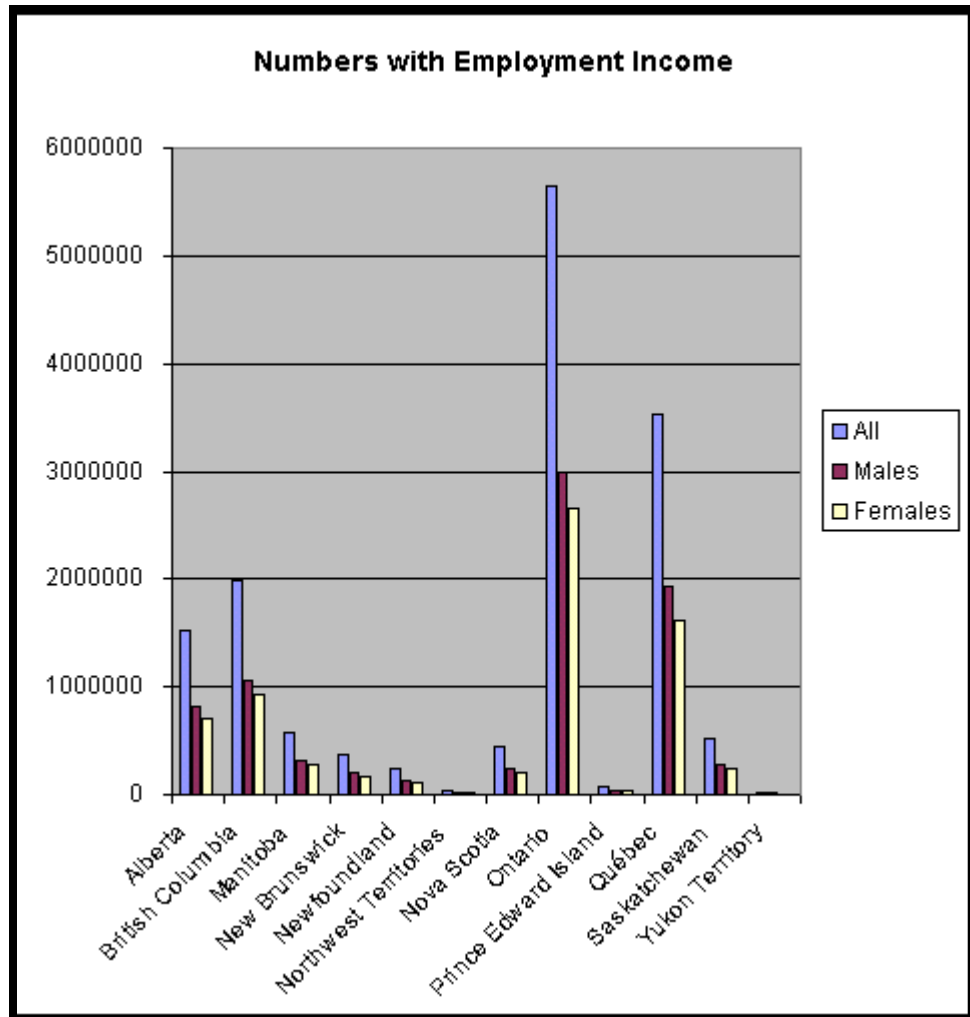
Section 1.3   **Page 32**   **Question 6**

Answers may vary.

Section 1.3   **Page 32**   **Question 8**

Answers may vary.

CHAPTER 1   Tools for Data Management

Section 1.4   Simulations

Practise

Section 1.4   **Page 40**   **Question 2**

a) Microsoft® Excel: 1 + 24*RAND().

Corel® Quattro® Pro: 1 + 24*@RAND.

Copy this formula to fill 100 cells.

b) Microsoft® Excel: 1 + ROUND(24*RAND(),0).

Corel® Quattro® Pro: @RANDBETWEEN(1,25).

Copy this formula to fill 100 cells.

c) Microsoft® Excel: -40 + ROUND(80*RAND(),0).

Corel® Quattro® Pro: @RANDBETWEEN(-40,40).

Copy this formula to fill 16 cells.

d) Microsoft® Excel: COUNTIF(C10:V40,42.5).

Corel® Quattro® Pro: @COUNTIF(C10..V40,42.5).

Refer to Appendix B in the student text if you need information on spreadsheets.

Section 1.4   **Page 40**   **Question 4**

Answers may vary.

Section 1.4   **Page 40**   **Question 6**

Answers may vary.

Solutions to Achievement Checks are described in the Teacher's Resource.

CHAPTER 1   Tools for Data Management

Section 1.5   Graph Theory

Practise

Section 1.5   **Page 49**   **Question 2**

Label each region of the map with a letter. Use each letter as a vertex. Draw an edge between all pairs of regions that share a border.

1d)

1e)

Section 1.5　　　　　　**Page 49**　　　　　　**Question 4**
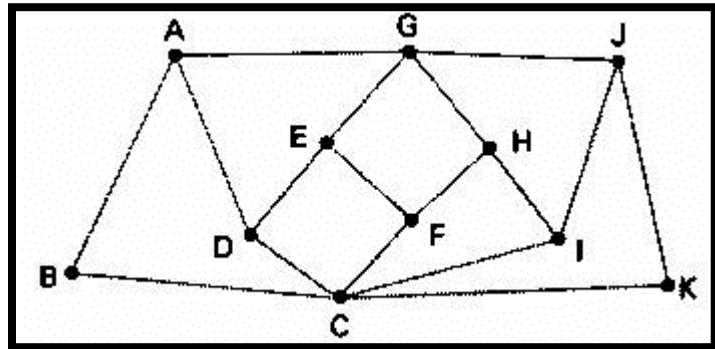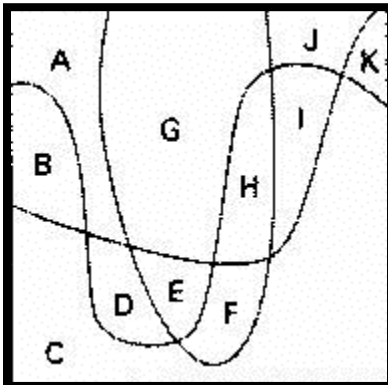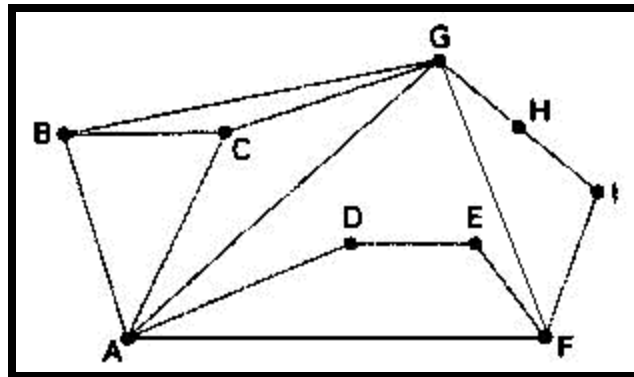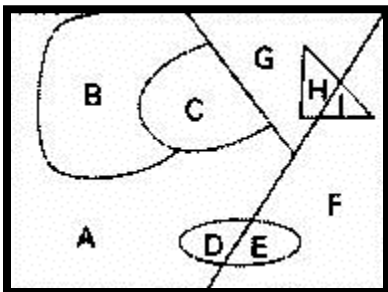
The structure of this map allows it to be coloured with just two colours, as shown in the diagram at the right.

Section 1.5　　　　　　**Page 49**　　　　　　**Question 6**
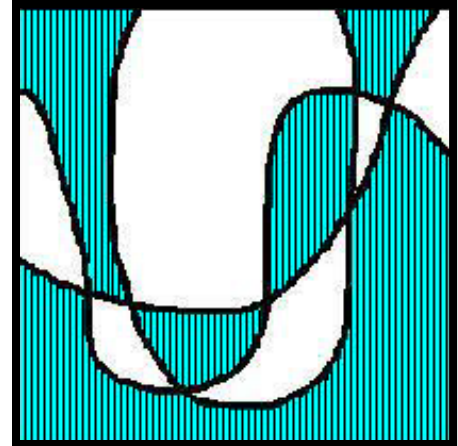
a) The network for the overlap of these subjects is shown at the left below.
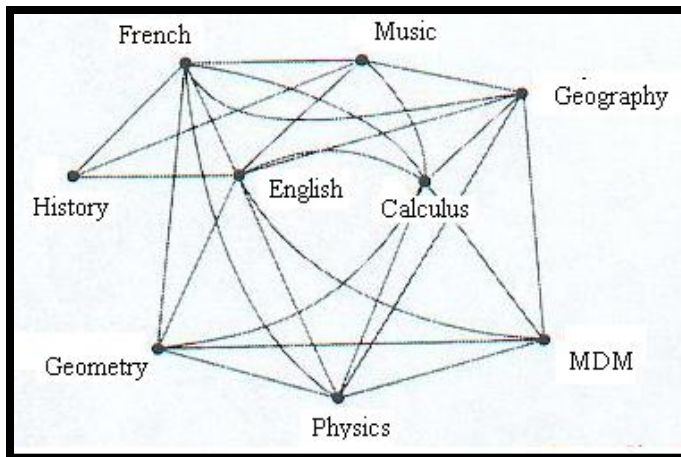
Time Slot	Subject 1	Subject 2
1	English	
2	Geography	Geometry
3	Calculus	History
4	Physics	Music
5	French	MDM

b) A possible examination schedule could look like the one shown at the right above. There are others that will work.

Section 1.5　　　　　　**Page 50**　　　　　　**Question 8**

a) There are four edges emanating from vertex A. Therefore, the degree of A is four. Similarly, the degree of B is two, C is three, and D is three.

b) The sum of the degrees is $4 + 2 + 3 + 3 = 12$.

c) The number of edges is 6.

d) Investigation of other networks in this chapter indicates that the number of edges is half the sum of the degrees.

a) House A can be connected to all three utilities. House B can then be connected to any two. The third connection from house B must cross one of the previous ones. This network is not planar.
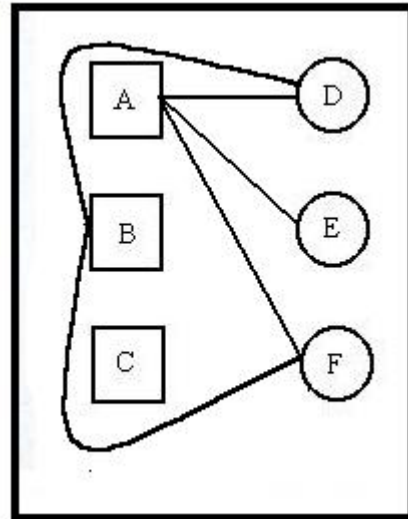
b) If two houses A and B are to be connected to two utilities, this can be done, as shown in the first diagram at the right. If a third utility is added, the lines from B can be drawn, as shown in the second diagram at the right. As additional utilities are added, it is always possible to draw lines from B below those from A, such that lines never cross. Hence, any number *n* utilities can be added without making the network non-planar.

Section 1.5 **Page 50** **Question 12**
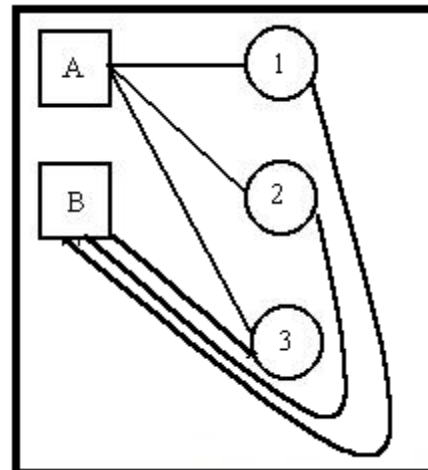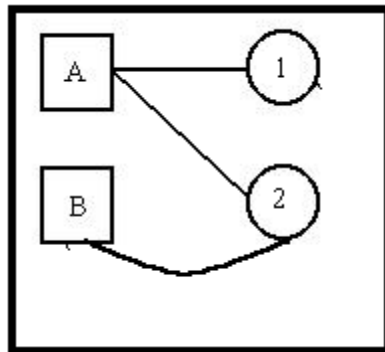
To get from A to B, you must cross four lines. If A is inside the closed figure, then in crossing the lines you go out, then in, then out, and then in again. Hence, B must be inside the closed figure. One possible arrangement is shown in the diagram at the right.

Section 1.5   **Page 51**   **Question 14**
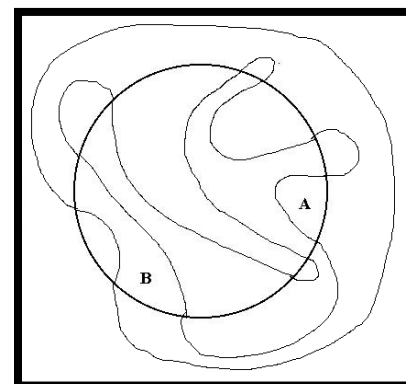
a) Yes. There are many possible routes. One such is Orangeville–Hamilton–Cambridge–Brantford–Woodstock–Stratford–Waterloo–Guelph.

b) It is possible to begin and end in the same city. For example, if you start in Stratford, you can visit Waterloo, Guelph, Orangeville, Hamilton, Brantford, Cambridge, Woodstock and then return to Stratford. Other routes are possible.

c) The longest edge is Orangeville to Hamilton at 116 km. You should avoid this route. The shortest route to and from Cambridge is Guelph to Cambridge to Waterloo or the reverse. Using this strategy, travel the route Orangeville–Guelph–Cambridge–Waterloo–Stratford–Woodstock–Brantford–Hamilton. This adds up to: 60 km + 23 km + 23 km + 55 km + 41 km + 38 km + 45 km = 285 km. Alternatively, you can reverse the route, beginning in Hamilton and finishing in Orangeville.

Section 1.5   **Page 51**   **Question 16**

The least expensive route will visit each city only once. Hence, Vancouver can only be visited on the trip out from Toronto or on the trip back to Toronto from Calgary. A similar argument is true for Windsor. Using this strategy, you should take the route Toronto–Vancouver–Calgary–Thunder Bay–Sudbury–Halifax–Montréal–Windsor–Toronto. This results in a cost of:

$378 + $111 + $319 + $150 + $349 + $321 + $109 + $119 = $1856.

Alternatively, you can take the trip in reverse.

Section 1.5   **Page 52**   **Question 18**

a) Answers may vary. One possible model is shown at the right. If any more edges are drawn, such as from A to E, then a group of three people who know each other is created. Note that B, D, and F form a triple who do not know each other.
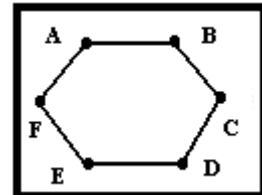
b) Answers may vary. One possible model is shown at the right. As in part a), there are no triples who know each other. There are also no triples who are mutual strangers.

Section 1.5   **Page 52**   **Question 20**
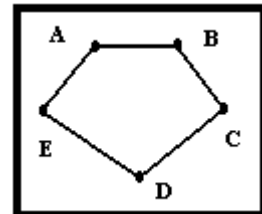
a) Yes. The graph shown for question 18a) is both connected and planar.

b) No. To make the graph referred to in part a) complete, you must join each pair of vertices with at least one edge. You can connect A to D, E and F. You can then connect B to D, E and F without crossing, as shown in the diagram at the right. However, C cannot be connected to E without crossing one of the previous edges.
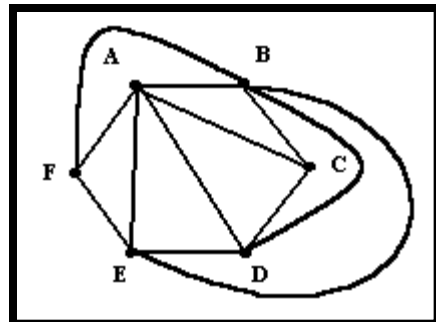
No. If only one vertex is odd, then the sum of the degrees must be odd. Since each edge must end in two vertices, the sum of the degrees must be even. Hence, it is not possible to have only one odd vertex.

CHAPTER 1   Tools for Data Management

Section 1.6   Modelling With Matrices

Practise

Section 1.6   **Page 60**   **Question 2**

a) i) Row 2, column 1 contains a 6
  ii) Row 4, column 3 contains a −4.
 iii) Row 1, column 3 contains a 2.

b) i)   4 is found in row 3, column 1, or element a_{31}.
  ii) −3 is found in row 3, column 3, or element a_{33}.
 iii)   1 is found in row 4, column 2, or element a_{42}.

Section 1.6   **Page 60**   **Question 4**

a) Answers may vary. Two row matrices are [2 −1] and [1 2 0].

Two column matrices are $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$.

b) The dimensions of the matrices in part a), respectively, are 1×2, 1×3, 2×1, and 3×1.

Section 1.6   **Page 60**   **Question 6**

a) $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 3 \\ 4 & 5 & 6 & 7 \end{bmatrix}$ **b)** $\begin{bmatrix} 3 & 2 & 3 & 4 \\ 2 & 3 & 6 & 8 \\ 3 & 6 & 3 & 12 \\ 4 & 8 & 12 & 3 \end{bmatrix}$

a)

$$A + B = \begin{bmatrix} 2 & -1 \\ 3 & 9 \\ 5 & 0 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -6 & 1 \\ 8 & 2 \\ -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ -3 & 10 \\ 13 & 2 \\ -5 & -4 \end{bmatrix}$$

b)

$$B + A = \begin{bmatrix} 3 & 4 \\ -6 & 1 \\ 8 & 2 \\ -1 & -5 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 9 \\ 5 & 0 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ -3 & 10 \\ 13 & 2 \\ -5 & -4 \end{bmatrix}$$

c) $B - C$ is not possible. The two matrices have different dimensions.

d)

$$3A = 3\begin{bmatrix} 2 & -1 \\ 3 & 9 \\ 5 & 0 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 \\ 9 & 27 \\ 15 & 0 \\ -12 & 3 \end{bmatrix}$$

e)

$$-\frac{1}{2}B = -\frac{1}{2}\begin{bmatrix} 3 & 4 \\ -6 & 1 \\ 8 & 2 \\ -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{3}{2} & -2 \\ 3 & -\dfrac{1}{2} \\ -4 & -1 \\ \dfrac{1}{2} & \dfrac{5}{2} \end{bmatrix}$$

f)

$$2(B - A) = 2\left(\begin{bmatrix} 3 & 4 \\ -6 & 1 \\ 8 & 2 \\ -1 & -5 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 9 \\ 5 & 0 \\ -4 & 1 \end{bmatrix}\right)$$

$$= 2\begin{bmatrix} 1 & 5 \\ -9 & -8 \\ 3 & 2 \\ 3 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 10 \\ -18 & -16 \\ 6 & 4 \\ 6 & -12 \end{bmatrix}$$

g)

$$3A - 2B = 3\begin{bmatrix} 2 & -1 \\ 3 & 9 \\ 5 & 0 \\ -4 & 1 \end{bmatrix} - 2\begin{bmatrix} 3 & 4 \\ -6 & 1 \\ 8 & 2 \\ -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 \\ 9 & 27 \\ 15 & 0 \\ -12 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 8 \\ -12 & 2 \\ 16 & 4 \\ -2 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -11 \\ 21 & 25 \\ -1 & -4 \\ -10 & 13 \end{bmatrix}$$

Section 1.6   **Page 61**   **Question 10**

Using matrix addition rules, you can generate the following equations:

$7 + 4 = w$
$x + 4 = 12$
$-1 + 2y = 5$
$3 + 2z = 21$

Solve each equation to obtain: $w = 11$, $x = 8$, $y = 3$, and $z = 9$.

Section 1.6   **Page 61**   **Question 12**

a)

	Thunder Bay	Sault Ste. Marie	North Bay	Ottawa	Toronto	
	0	710	1135	1500	1365	Thunder Bay
	710	0	425	790	655	Sault Ste. Marie
	1135	425	0	365	350	North Bay
	1500	790	365	0	400	Ottawa
	1365	655	350	400	0	Toronto

b) Since the matrix is symmetric about the diagonal, the transpose matrix is the same as the matrix.

c) a_{23} in matrix A and a_{32} in matrix A^t are equal.

Section 1.6 **Page 61** **Question 14**

This solution was done using Microsoft® Excel. A similar solution can be performed using Corel® Quattro® Pro or a graphing calculator.

a) Enter the raw data in columns B, C, D, and E.

b) To find the total number of degrees earned by males in 1997 and 1998, use the formula B2 + D2 in cell F2, and copy it down to cell F10.

c) To find the total number of degrees earned by females in 1997 and 1998, use the formula C2 + E2 in cell G2, and copy it down to cell G10.

d) To find the average number of degrees earned by females in 1997 and 1998, use the formula (C2 + E2)/2 in cell H2, and copy it down to cell H10. Your final matrix will look like the following screen shot.

	A	B	C	D	E	F	G	H
1	Field of Study	1997 Males	1997 Females	1998 Males	1998 Females	Males	Females	Average Females
2	Social Sciences	28421	38244	27993	39026	56414	77270	38635
3	Education	8036	19771	7565	18391	15601	38162	19081
4	Humanities	8034	13339	7589	13227	15623	26566	13283
5	Health	3460	9613	3514	9144	6974	18757	9378.5
6	Engineering	10125	2643	10121	2709	20246	5352	2676
7	Agriculture	4780	6995	4779	7430	9559	14425	7212.5
8	Mathematics	6749	2989	6876	3116	13625	6105	3052.5
9	Fine & Applied Arts	1706	3500	1735	3521	3441	7021	3510.5
10	Arts & Sciences	1730	3802	1777	3563	3507	7365	3682.5

Section 1.6 **Page 62** **Question 16**

a) The matrix which shows connections for the VIA Rail routes shown on page 3 of the student text is displayed at the right.

b) The zero in row 4, column 3 means that there is no direct connection between Niagara Falls and Montréal.

c) The zero in row 3, column 4 means that there is no direct connection between Montréal and Niagara Falls.

$$
\begin{array}{cccccccc}
K & L & M & NF & O & S & T & W \\
\end{array}
$$

$$
\begin{bmatrix}
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{array}{l}
\text{Kingston} \\
\text{London} \\
\text{Montréal} \\
\text{Niagara Falls} \\
\text{Ottawa} \\
\text{Sudbury} \\
\text{Toronto} \\
\text{Windsor} \\
\end{array}
$$

d) A direct connection between Montréal and Niagara Falls would produce a 1 in both row 4, column 3, and row 3, column 4.

e) The sum of the entries in the first row indicates the number of direct connections to Kingston.

f) The sum of the entries in the first column indicates the number of direct connections from Kingston.

g) The answers to parts e) and f) are the same because direct connections work both ways.

Section 1.6   **Page 62**   **Question 18**

For information on how to do matrix calculations with spreadsheets and graphing calculators, see Appendix B of the student text.

CHAPTER 1   Tools for Data Management

Section 1.7   Problem Solving With Matrices

Practise

Section 1.7   **Page 74**   **Question 2**

$$A^2 + 2B^3 = \begin{bmatrix} 4 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 0 & -1 \end{bmatrix} + 2\begin{bmatrix} 0 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 6 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 6 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & -18 \\ 12 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -30 \\ 24 & 1 \end{bmatrix}$$

Section 1.7   **Page 75**   **Question 4**

$$\textbf{a) } A(B + C) = \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix}\left(\begin{bmatrix} 3 & 4 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -2 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 5 \\ 10 & -5 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 20 \\ 8 & 8 \end{bmatrix} + \begin{bmatrix} 5 & -15 \\ 2 & -13 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 5 \\ 10 & -5 \end{bmatrix}$$

Therefore, $A(B + C) = AB + AC$.

b) $(AB)C = \left(\begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -2 & 0 \end{bmatrix} \right) \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 15 & 20 \\ 8 & 8 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 95 \\ 8 & 32 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix} \left(\begin{bmatrix} 3 & 4 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & 19 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 95 \\ 8 & 32 \end{bmatrix}$$

Therefore, $(AB)C = A(BC)$.

c) $AB = \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -2 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 15 & 20 \\ 8 & 8 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 4 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & -4 \\ -10 & 0 \end{bmatrix}$$

Therefore, $AB = BA$.

Section 1.7   **Page 75**   **Question 6**

The following inverse matrices were found using a graphing calculator. If you need instructions on how to do this, refer to Appendix B of the student text.

a)

```
[A]⁻¹
  [[.6 -.2 -2]
   [0  0  -1]
   [.4 .2  -1]]
```

b)

```
[B]⁻¹                    [B]⁻¹
[[.0851  .4255 ..    .4255  .1064]
 [-.0638 -.3191:..  -.3191 .1702]
 [.2340  .1702 ..    .1702 .0426]]
```

c)

```
[C]⁻¹                       [C]⁻¹
[[.2000  0.0000...  -.2000  .2000 ]
 [-.4000 0.0000:.   -.6000 -.4000]
 [.2000  0.0000:.   -.2000 -.8000]
 [.2000  .5000 :..   .3000  .2000 ]]
```

Section 1.7   **Page 75**   **Question 8**

	BU	SC	GR					
Downtown	12	40	30	$40	Price BU		$6180	Downtown Sales
Northern	8	30	21	$30	Price SC	=	$4370	Northern Sales
Southern	10	25	23	$150	Price GR		$4600	Southern Sales

Enter the raw data into a spreadsheet, as shown in the screen shot below. Enter the formula C2/100*11937000 in cell E2 and copy down to cell E11. Enter the formula D2/100*12323000 in cell F2 and copy down to cell F11. In terms of matrices, you are multiplying each element of the column matrix in cells C2 to C11 by 119 370 and storing the results in cells E2 to E11. Then, you are multiplying each element of the column matrix in cells D2 to D11 by 123 230 and storing the results in cells F2 to F11.

Microsoft Excel - Section 1.7 Question 10.xls

File Edit View Insert Format Tools Data Window Help Acrobat

K12 =

	A	B	C	D	E	F
1	Sport	Total(%)	Male(%)	Female(%)	Male	Female
2	Golf	7.4	11.1	3.9	1325007	480597
3	Ice Hockey	6.2	12.0	0.5	1432440	61615
4	Baseball	5.5	8.0	3.1	954960	382013
5	Swimming	4.6	3.6	5.6	429732	690088
6	Basketball	3.2	4.6	1.9	549102	234137
7	Volleyball	3.1	3.3	2.8	393921	345044
8	Soccer	3.0	4.6	1.5	549102	184845
9	Tennis	2.7	3.6	1.8	429732	221814
10	Downhill/Alpine Skiing	2.7	2.9	2.6	346173	320398
11	Cycling	2.5	3.0	2.0	358110	246460

a) Encode the message using 2×2 matrices as shown below. Use a Z to fill in any missing entries.

$$\begin{bmatrix} B & I \\ R & T \end{bmatrix}, \begin{bmatrix} H & D \\ A & Y \end{bmatrix}, \begin{bmatrix} P & A \\ R & T \end{bmatrix}, \begin{bmatrix} Y & F \\ R & I \end{bmatrix}, \begin{bmatrix} D & A \\ Y & Z \end{bmatrix}$$

Replace each letter by its corresponding number in the alphabet.

$$\begin{bmatrix} 2 & 9 \\ 18 & 20 \end{bmatrix}, \begin{bmatrix} 8 & 4 \\ 1 & 25 \end{bmatrix}, \begin{bmatrix} 16 & 1 \\ 18 & 20 \end{bmatrix}, \begin{bmatrix} 25 & 6 \\ 18 & 9 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 25 & 26 \end{bmatrix}$$

Multiply each matrix by the encoding matrix $\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$.

$$\begin{bmatrix} -50 & -42 \\ 86 & 82 \end{bmatrix}, \begin{bmatrix} 13 & -67 \\ -11 & 117 \end{bmatrix}, \begin{bmatrix} -22 & -58 \\ 58 & 98 \end{bmatrix}, \begin{bmatrix} -4 & -15 \\ 40 & 33 \end{bmatrix}, \begin{bmatrix} -67 & -76 \\ 117 & 128 \end{bmatrix}$$

Write the numbers linearly:

−50, −42, 86, 82, 13, −67, −11, 117, −22, −58, 58, 98, −4, −15, 40, 33, −67, −76, 117, 128.

b) Encode the message using 2×2 matrices as shown below. Use a Z to fill in any missing entries.

$$\begin{bmatrix} S & E \\ E & Y \end{bmatrix}, \begin{bmatrix} O & U \\ S & A \end{bmatrix}, \begin{bmatrix} T & U \\ R & D \end{bmatrix}, \begin{bmatrix} A & Y \\ N & I \end{bmatrix}, \begin{bmatrix} G & H \\ T & Z \end{bmatrix}$$

Replace each letter by its corresponding number in the alphabet.

$$\begin{bmatrix} 19 & 5 \\ 5 & 25 \end{bmatrix}, \begin{bmatrix} 15 & 21 \\ 19 & 1 \end{bmatrix}, \begin{bmatrix} 20 & 21 \\ 18 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 25 \\ 14 & 9 \end{bmatrix}, \begin{bmatrix} 7 & 8 \\ 20 & 26 \end{bmatrix}$$

Multiply each matrix by the encoding matrix $\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$.

$$\begin{bmatrix} 23 & -65 \\ -13 & 115 \end{bmatrix}, \begin{bmatrix} -27 & 39 \\ 65 & -37 \end{bmatrix}, \begin{bmatrix} -14 & 30 \\ 50 & -22 \end{bmatrix}, \begin{bmatrix} -40 & 23 \\ 68 & -5 \end{bmatrix}, \begin{bmatrix} -46 & -62 \\ 86 & 114 \end{bmatrix}$$

Write the numbers linearly:

23, −65, −13, 115, −27, 39, 65, −37, −14, 30, 50, −22, −40, 23, 68, −5, −46, −62, 86, 114.

Section 1.7   **Page 76**   **Question 14**

Answers may vary.

Section 1.7   **Page 77**   **Question 16**

a) Consider the four vertices as the north bank, south bank, west island, and east island. The network and corresponding matrix for the bridges of Koenigsberg look like the diagrams shown at the right. To get from Honey Bridge to Connecting Bridge by crossing only one of the other bridges, there are two routes: Honey Bridge–Blacksmith Bridge–Connecting Bridge and Honey Bridge–Green Bridge–Connecting Bridge.
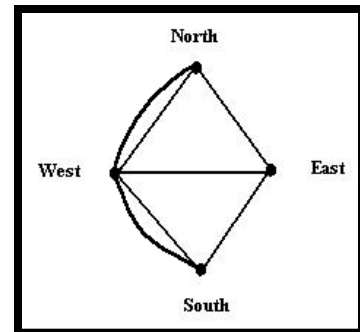
b) To get from Blacksmith Bridge to Connecting Bridge without crossing more than one of the other bridges, there are three routes: Blacksmith Bridge–Connecting Bridge, Blacksmith Bridge–Honey Bridge–Connecting Bridge and Blacksmith Bridge–Green Bridge–Connecting Bridge.

c) To get from Wooden Bridge to Green Bridge without crossing at least two other bridges, take the route Wooden Bridge–Merchants Bridge–Green Bridge.

Section 1.7   **Page 77**   **Question 18**

Answers may vary.

Section 1.7   **Page 77**   **Question 20**

Answers may vary.

Section 1.7   **Page 77**   **Question 22**

a) If A is an $m \times n$ matrix, then it consists of m rows and n columns. Call the rows ar1 to arm. If B is a $n \times p$ matrix, then it consists of n rows and p columns. Call the columns bc1 to bcp. When the matrices are multiplied, the rows of A are multiplied by the columns of B. The product matrix AB has dimensions $m \times p$ and the general form as shown at the right.

$$AB = \begin{bmatrix} ar1 \times bc1 & ar1 \times bc2 & ... & ar1 \times bcp \\ ar2 \times bc1 & ar2 \times bc2 & ... & ar2 \times bcp \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ arm \times bc1 & arm \times bc2 & ... & arm \times bcp \end{bmatrix}$$

When matrix A is transposed, the rows become columns and vice-versa. The same occurs for matrix B. When B^t is multiplied by A^t, the product matrix takes on the form shown at the right, where the rows and columns refer to the original rows of matrices A and B.

$$B^t A^t = \begin{bmatrix} ar1 \times bc1 & ar2 \times bc1 & ... & arm \times bc1 \\ ar1 \times bc2 & ar2 \times bc2 & ... & arm \times bc2 \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ ar1 \times bcp & ar2 \times bcp & ... & arm \times bcp \end{bmatrix}$$

This is clearly a matrix of dimensions $p \times m$, and is the transpose of the product matrix AB.

b) In the result above, let $A = C$ and $B = C^{-1}$.
Therefore, $(CC^{-1})^t = (C^{-1})^t C^t$.
However, $(CC^{-1})^t = I^t = I$.
Therefore, $(C^{-1})^t C^t = I$ and hence $(C^t)^{-1} = (C^{-1})^t$.

CHAPTER 1   Tools for Data Management

Review of Key Concepts

Review of Key Concepts   **Page 78**   **Question 2**

a) Begin with the three-word MATH triangle at the top. Place one copy on the next line such that the left copy centres the T under the M in the previous line, and the right copy centres the M under the T in the previous line. Repeat for subsequent lines.

b) Eight more rows will produce the pattern shown at the right.

c) A pyramid has been formed.
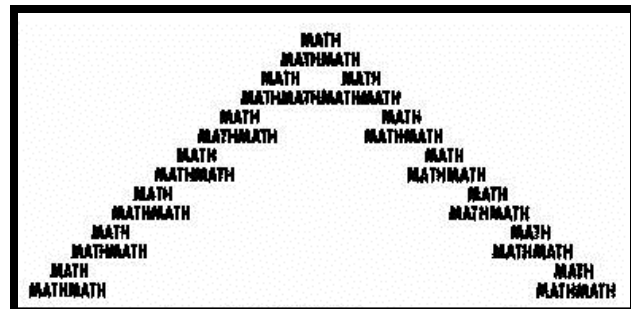
Review of Key Concepts   **Page 78**   **Question 4**

Answers may vary.

Review of Key Concepts   **Page 78**   **Question 6**
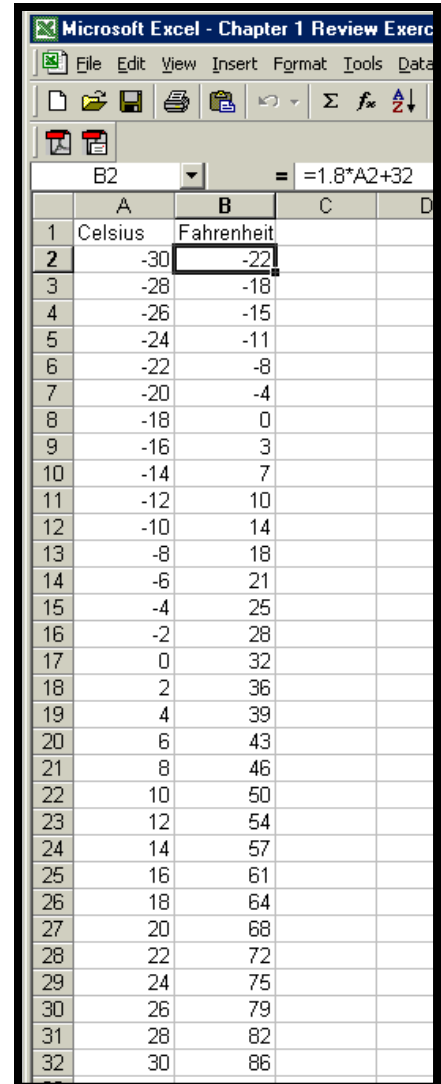
a) $\text{F2} + \text{G7} - \text{A12} = 5 + (-9) - [5 + (-9)]$
$$= 0$$

b) $\text{PROD(D3,F9)} = 6 \times 5$
$$= 30$$

c) $\text{SQRT(B1)} = \sqrt{144}$
$$= 12$$

Review of Key Concepts   **Page 78**   **Question 8**

The spreadsheet that converts Celsius temperatures to Fahrenheit temperatures is shown to the right. In order to space the Celsius column at two-degree spacing, you can use the **Fill...Series** command under the **Edit** menu and set the increment to two. Alternatively, you can calculate each degree, and then apply a filter to select only even degrees.

Microsoft Excel - Chapter 1 Review Exerc
File Edit View Insert Format Tools Data

B2   = =1.8*A2+32

	A	B	C	D
1	Celsius	Fahrenheit		
2	-30	-22		
3	-28	-18		
4	-26	-15		
5	-24	-11		
6	-22	-8		
7	-20	-4		
8	-18	0		
9	-16	3		
10	-14	7		
11	-12	10		
12	-10	14		
13	-8	18		
14	-6	21		
15	-4	25		
16	-2	28		
17	0	32		
18	2	36		
19	4	39		
20	6	43		
21	8	46		
22	10	50		
23	12	54		
24	14	57		
25	16	61		
26	18	64		
27	20	68		
28	22	72		
29	24	75		
30	26	79		
31	28	82		
32	30	86		

Review of Key Concepts   Page 78   Question 10

Answers will vary.

Review of Key Concepts   Page 79   Question 12

Answers may vary.

Review of Key Concepts   Page 79   Question 14

a) randInt(18,65)

b) Microsoft® Excel: 18 + ROUND(47*RAND(),0).

   Corel® Quattro® Pro: @RANDBETWEEN(18,65)

Review of Key Concepts   Page 79   Question 16

Use Fathom™, Microsoft® Excel, Corel® Quattro® Pro or a graphing calculator to generate five random integers from 1 to 50. If two or more are the same, count it as a success. Repeat the random number generation 500 times. Refer to Appendix B of the student text if you need information on how to generate random numbers.
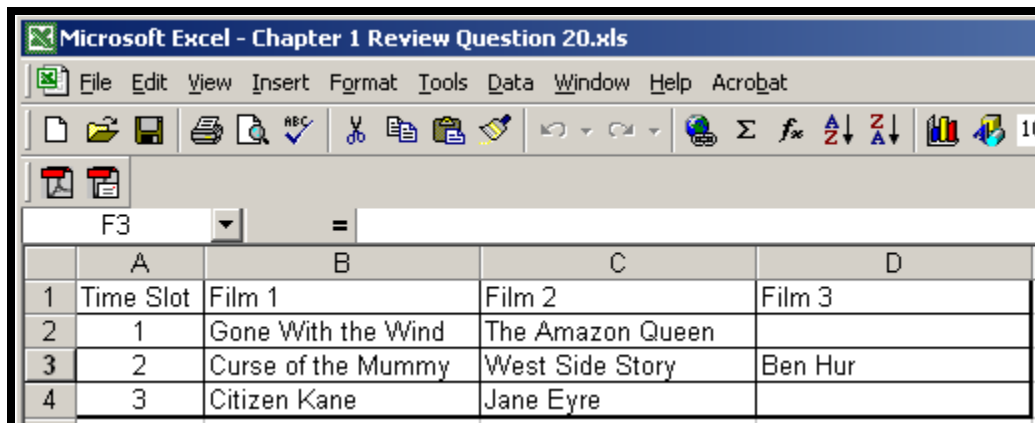
Review of Key Concepts   Page 79   Question 18

a) Since there is a path between any two vertices, the network is connected. Since there are exactly two odd vertices (A and B), the network is traceable. Since none of the edges cross, the network is planar.

b) Since there is a path between any two vertices, the network is connected. Since all vertices are odd, the network is non-traceable. Since many of the edges cross, the network is non-planar.

c) Since there is a path between any two vertices, the network is connected. Since there are four odd vertices (K, L, M, and N), the network is non-traceable. Since two of the edges cross, the network is non-planar.

Review of Key Concepts   Page 79   Question 20

Review of Key Concepts   **Page 80**   **Question 22**

a) Matrix A has dimensions of 4×3.

b) i) $a_{32} = -8$ **ii)** $a_{13} = 5$ **iii)** $a_{41} = -2$

c) i) $a_{23} = 3$ **ii)** $a_{42} = 9$ **iii)** $a_{12} = -1$

Review of Key Concepts   **Page 80**   **Question 24**

a) $A + C = \begin{bmatrix} 3 & 2 & -1 \\ -7 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 1 & -4 \\ -5 & 9 & 0 \end{bmatrix}$

$\quad = \begin{bmatrix} 9 & 3 & -5 \\ -12 & 9 & 5 \end{bmatrix}$

b) Since C and B have different dimensions, subtraction is not possible.

c) Since A and B have different dimensions, addition is not possible.

d) $3D = 3 \begin{bmatrix} 4 & 3 \\ -1 & 7 \\ 6 & 2 \end{bmatrix}$

$\quad = \begin{bmatrix} 12 & 9 \\ -3 & 21 \\ 18 & 6 \end{bmatrix}$

e) $-\dfrac{1}{2}C = -\dfrac{1}{2}\begin{bmatrix} 6 & 1 & -4 \\ -5 & 9 & 0 \end{bmatrix}$

$\quad = \begin{bmatrix} 3 & \dfrac{1}{2} & -2 \\ -\dfrac{5}{2} & \dfrac{9}{2} & 0 \end{bmatrix}$

f) $3(B + D) = 3\left(\begin{bmatrix} 8 & -2 \\ 3 & 4 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 7 \\ 6 & 2 \end{bmatrix} \right)$

$\quad = 3 \begin{bmatrix} 12 & 1 \\ 2 & 11 \\ 8 & 7 \end{bmatrix}$

$\quad = \begin{bmatrix} 36 & 3 \\ 6 & 33 \\ 24 & 21 \end{bmatrix}$

g) $A^{t} + B = \begin{bmatrix} 3 & -7 \\ 2 & 0 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} 8 & -2 \\ 3 & 4 \\ 2 & 5 \end{bmatrix}$

$\quad = \begin{bmatrix} 11 & -9 \\ 5 & 4 \\ 1 & 10 \end{bmatrix}$

h) Since *B* and *C* have different dimensions, then so do their transposes. Hence, addition of the transposes is not possible.

Review of Key Concepts   **Page 80**   **Question 26**

a) Ensure that the matrices are of the same dimensions. Subtract corresponding elements.

b) Enter the matrices into the calculator. Subtract them. If you need information on entering and working with matrices, refer to Appendix B of the student text.

c) Enter the matrices into the spreadsheet. Enter the subtraction formula in the cell where you would like the solution and copy to all other relevant cells. If you need information on entering and working with matrices, refer to Appendix B of the student text.

Review of Key Concepts   **Page 81**   **Question 28**

a) $A^t = \begin{bmatrix} 1 & 8 \\ 5 & -2 \end{bmatrix}$ $B^t = \begin{bmatrix} 0 & 6 \\ 4 & -1 \end{bmatrix}$

b) $(AB)^t = \left(\begin{bmatrix} 1 & 5 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ 6 & -1 \end{bmatrix} \right)^t$

$$= \begin{bmatrix} 30 & -1 \\ -12 & 34 \end{bmatrix}^t$$

$$= \begin{bmatrix} 30 & -12 \\ -1 & 34 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 0 & 6 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 8 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & -12 \\ -1 & 34 \end{bmatrix}$$

Therefore, $(AB)^t = B^t A^t$.

Review of Key Concepts   **Page 81**   **Question 30**

Answers may vary.

a)   B LF P SF S

```
[A]
  [[0 0 0 0 0]
   [1 0 0 0 1]
   [0 1 0 1 1]
   [1 1 0 0 1]
   [1 0 0 0 0]]
```
Bacteria
Large Fish
Plants
Small Fish
Snails

b)   B LF P SF S

```
[A]*[A]
  [[0 0 0 0 0]
   [1 0 0 0 0]
   [3 1 0 0 2]
   [2 0 0 0 1]
   [0 0 0 0 0]]
```
Bacteria
Large Fish
Plants
Small Fish
Snails

c) Row 3, column 5 indicates two indirect links from plants to snails with one intermediate step.

d)   B LF P SF S

```
[A]+[A]²
  [[0 0 0 0 0]
   [2 0 0 0 1]
   [3 2 0 1 3]
   [3 1 0 0 2]
   [1 0 0 0 0]]
```
Bacteria
Large Fish
Plants
Small Fish
Snails

The entries in $A + A^2$ indicate the number of links with either no intermediate step or exactly one intermediate step.

e)   B LF P SF S

```
[A]^3
  [[0 0 0 0 0]
   [0 0 0 0 0]
   [3 0 0 0 1]
   [1 0 0 0 0]
   [0 0 0 0 0]]
```
Bacteria
Large Fish
Plants
Small Fish
Snails

f) Row 3, column 1 indicates that there are three links from plants to bacteria with two intermediate steps. They are:

Plants–Small Fish–Large Fish–Bacteria
Plants–Large Fish–Snails–Bacteria
Plants–Small Fish–Snails–Bacteria

CHAPTER 1   Tools for Data Management

Chapter Test

$$t_1 = 0 \qquad t_2 = \frac{1}{0+2} \qquad t_3 = \frac{1}{\frac{1}{2}+2} \qquad t_4 = \frac{1}{\frac{2}{5}+2} \qquad t_5 = \frac{1}{\frac{5}{12}+2}$$

$$= \frac{1}{2} \qquad\qquad = \frac{2}{5} \qquad\qquad = \frac{5}{12} \qquad\qquad = \frac{12}{29}$$

Starting with the second term, the numerator of each term is the same as the denominator of the previous term. The denominator of each term is the sum of the numerator and denominator of the previous term plus the numerator of the term.

Chapter Test   Page 82   Question 4

a) Microsoft® Excel: SUM(C1:C8)   **Corel® Quattro® Pro**: @SUM(C1..C8)

b) Microsoft® Excel: MIN(A5:G5)   **Corel® Quattro® Pro**: @MIN(A5..G5)

c) Microsoft® Excel: $(5 - \text{SQRT}(6)/(10 + 15))$ **Corel® Quattro® Pro**: $(5 - @\text{SQRT}(6)/(10 + 15))$

Chapter Test   Page 82   Question 6

Answers may vary.

Chapter Test   Page 82   Question 8

a) Label the regions on the map, as shown in the diagram at the right. Then, generate the corresponding network as shown.
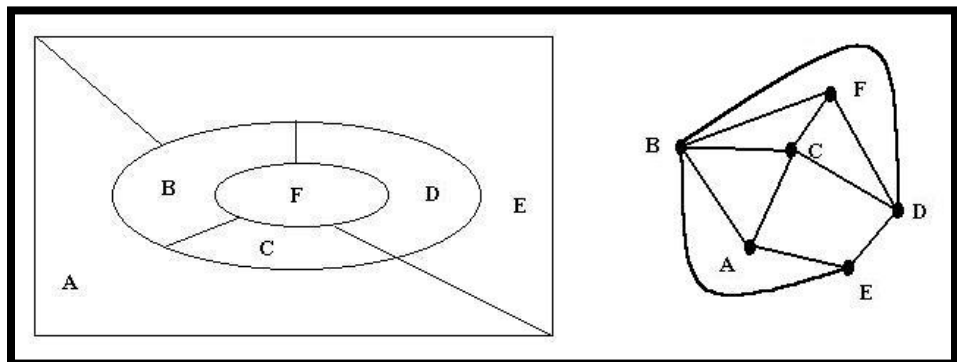
b) Region A is not connected to region F. Hence, they can have the same colour. Region C is not connected to region E. Hence, they can have the same colour. Therefore, you need at least four colours.

Chapter Test   **Page 83**   **Question 10**

Label the banks and islands, as shown in the diagram at the right. The corresponding network is also shown in the diagram. Since there are exactly two vertices of odd degree, A and B, the network is traceable. Hence, it is possible to take a continuous path that traverses each bridge exactly once.

Chapter Test   **Page 83**   **Question 12**
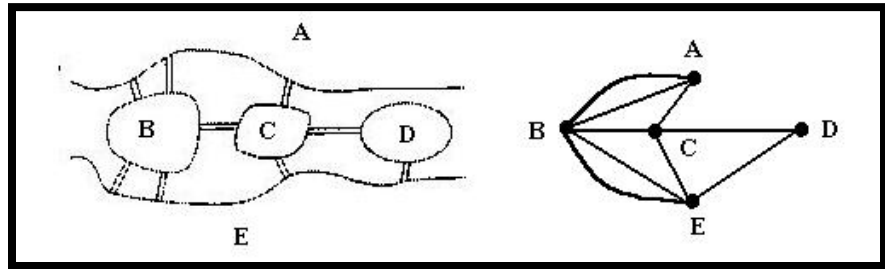
a) $2C + D = 2\begin{bmatrix} 4 & 8 \\ 5 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -7 \\ 9 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 16 \\ 10 & -6 \end{bmatrix} + \begin{bmatrix} 2 & -7 \\ 9 & 1 \end{bmatrix}$

$= \begin{bmatrix} 10 & 9 \\ 19 & -5 \end{bmatrix}$

b) Since A and B have different dimensions, addition is not possible.

c) D would need to be a $1 \times n$ matrix to allow matrix multiplication with A. Hence, it is not possible.

d) $EC = \begin{bmatrix} 8 & -2 \\ 5 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ 5 & -3 \end{bmatrix}$ **e)** $E^t = \begin{bmatrix} 8 & 5 & -4 \\ -2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 22 & 70 \\ 20 & 40 \\ -11 & -35 \end{bmatrix}$

Chapter Test   **Page 83**   **Question 14**

Solutions to Achievement Checks are described in the Teacher's Resource.

CHAPTER 2   Statistics of One Variable

Review of Prerequisite Skills

Review of Prerequisite Skills   **Page 90**   **Question 2**

a) Josh's new hourly rate will be

$$108\% \text{ of } \$12.50 = 1.08 \times \$12.50$$
$$= \$13.50/\text{h}$$

b) Josh's net hourly pay before the raise is

$$83\% \text{ of } \$12.50 = 0.83 \times \$12.50$$
$$= \$10.38/\text{h}$$

His net hourly pay after the raise is

$$83\% \text{ of } \$13.50 = 0.83 \times \$13.50$$
$$= \$11.21/\text{h}$$

Hence, the increase in his take-home pay is $\$11.21 - \$10.38 = \$0.83$.

Review of Prerequisite Skills   **Page 90**   **Question 4**

The discounted price before taxes is

$$75\% \text{ of } \$219.00 = 0.75 \times \$219.00$$
$$= \$164.25$$

The total sales taxes in Ontario amount to 15%. The cost after taxes is

$$115\% \text{ of } \$164.25 = 1.15 \times \$164.25$$
$$= \$188.89$$

Review of Prerequisite Skills   **Page 90**   **Question 6**

a) Move to 1996 on the horizontal axis. Follow the vertical gridline to the graph. Move left to the vertical axis. The price of thingamajigs was approximately $1.44 in 1996.

b) Find $1.50 on the vertical axis. Move horizontally to the graph. This occurred during the year 1997.

c) The overall trend over the time period shown is increasing price.

d) The price in 2001 was approximately $1.59. This is an increase of $0.15 over the price in 1996.

The percent increase was

$$\frac{\$0.15}{\$1.44} \times 100\% = 10.4\%.$$

e) The domain is the extent of the data on the horizontal axis, which is from 1996 to 2001. The range is the extent of the data on the vertical axis, which is from $1.44 to $1.59.

CHAPTER 2   Statistics of One Variable

Section 2.1 Data Analysis With Graphs

Practise

Section 2.1   **Page 101**   **Question 2**

a) The numbers from a roll of a die do not form a continuous range of values. Choose a bar graph.

b) The distances that a shot-put is thrown form a continuous range of values. Choose a histogram.

c) The ages of players do not form a continuous range of values. Choose a bar graph.

d) The heights of players form a continuous range of values. Choose a histogram.

Section 2.1   **Page 101**   **Question 4**

Answers will vary.

Section 2.1   **Page 102**   **Question 6**

a) Since each number on a die has the same chance of coming up on a roll, you may expect the bar graph to have the same height of five for each number, as shown in the graph at the right.
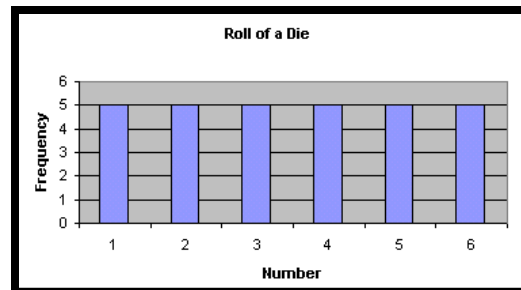
b) Use a random number function such as 1+ROUND(5*RAND(),0) for **Microsoft® Excel** or @RANDBETWEEN(1,6) for **Corel® Quattro® Pro** to simulate the roll of a die. Copy the function to a total of 30 cells. Use a counting function such as COUNTIF($A$16:$A$45,1) in **Microsoft® Excel** or @COUNTIF($A$16..$A$45,1) in **Corel® Quattro® Pro**. Copy the function to a total of six cells, changing the number to match the numbers on the die. Your result will look like the following screen shot.
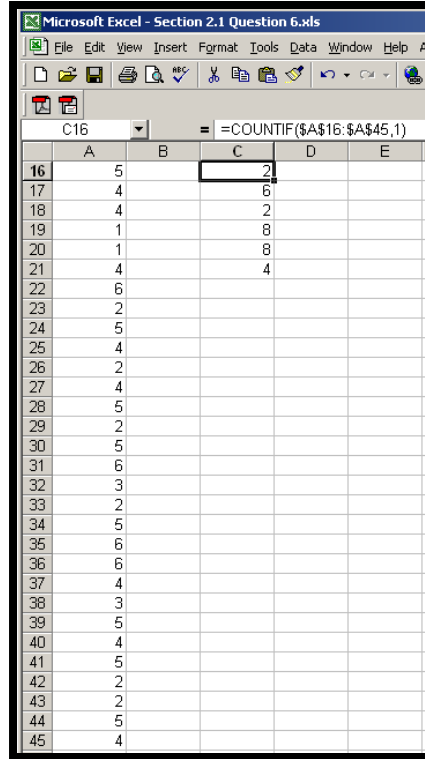
c) Use the charting feature to construct the required bar graph, as shown below.
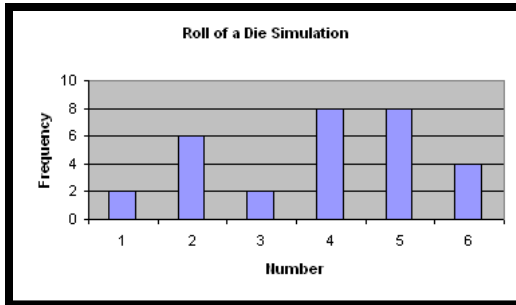
d) The bar graphs from parts a) and c) are not the same. Since the frequency of each number coming up depends on random chance, the frequencies are not the same across the graph. If the simulation were run again, the graph would likely change.

Section 2.1   Page 102   Question 8

a) You should present these data in a grouped format. The numbers are from a continuous distribution.

b) The frequency table is shown at the right. The frequency diagram is shown at the left below. The diagram was generated using Fathom™.

c) The cumulative-frequency diagram is shown at the right below. This diagram was generated using Fathom™.

Purchase Amount($)	Tally	Frequency
5.00-14.99	11	2
15.00-24.99	111	3
25.00-34.99	11111	5
35.00-44.99	1111	4
45.00-54.99	1111	4
55.00-64.99	111	3
65.00-74.99	111	3
75.00-84.99	11	2
85.00-94.99	11	2
95.00-104.99		0
105.00-114.99	11	2

d) Answers may vary.

Section 2.1   **Page 102**   **Question 10**
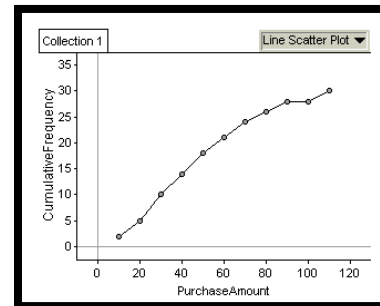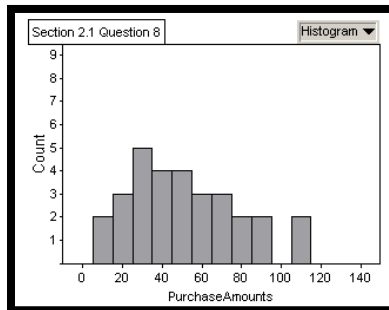
a) The frequency table is shown at the left below.

Salary	Tally	Frequency
0 - 499 999	11	2
500 000 - 999 999	⊥⊥⊥⊤ ⊥⊥⊥⊤ ⊥⊥⊥⊤ 11	17
1 000 000 - 1 499 999	11	2
1 500 000 - 1 999 999	1	1
2 000 000 - 2 499 999		0
2 500 000 - 2 999 999		0
3 000 000 - 3 499 999	1	1
3 500 000 - 3 999 999		0
4 000 000 - 4 499 999	1	1
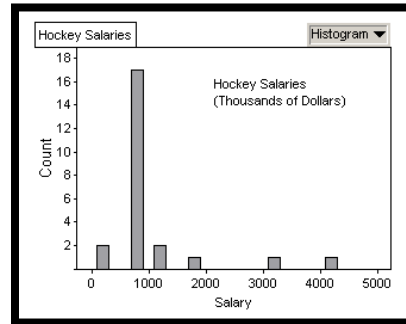
b) The histogram is shown at the right above. This histogram was generated using Fathom™.

c) This distribution is skewed to the left. Few players are paid the highest salaries.

Section 2.1   **Page 103**   **Question 12**

Score	Frequency
29.5-39.5	1
39.5-49.5	2
49.5-59.5	4
59.5-69.5	8
69.5-79.5	5
79.5-89.5	3
89.5-99.5	2

To obtain the frequencies, multiply the relative frequency by 25. For example, the relative frequency for a score of 35 is 0.04. Therefore, the frequency is $0.04 \times 25 = 1$. The resulting frequency distribution table is shown at the right.

Section 2.1   **Page 103**   **Question 14**

a) The sum of the dice may take on values of 2, 4, 6, 7, 8, 9, 10, 11, 12, 13, and 14.

A	B	C	D	E	F	G
	1	1	3	5	6	7
1	2	2	4	6	7	8
1	2	2	4	6	7	8
3	4	4	6	8	9	10
5	6	6	8	10	11	12
6	7	7	9	11	12	13
7	8	8	10	12	13	14
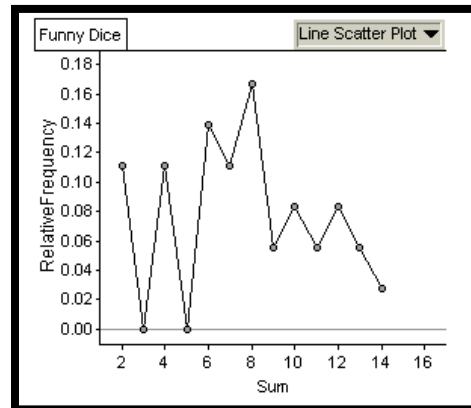
b) There are 36 possible outcomes. These are shown in the screen shot at the above left. To calculate the expected relative frequency, divide the frequency of each outcome by 36. For example, there are four ways to obtain a sum of 2. For example, the relative frequency for a sum of 2 is

$\dfrac{4}{36} = 0.11$. The resulting relative-frequency diagram is shown at the above right.

c) See part b).

d) The solution shown below was generated using Fathom™. If you need information on using Fathom™, refer to Appendix B of the student text. Open Fathom™ and drag a new case table to the work area. Create 100 **New Cases**. Create attributes **Die1**, **Die2,** and **Sum**. For the first two, edit the formula to

randomPick(1,1,3,5,6,7).

Edit the formula for Sum to **Die1 + Die2**. Create a histogram for the **Sum**. Use this histogram to construct a new collection of **Frequency** versus the sum on the dice. Create a **RelativeFrequency** attribute by dividing the **Frequency** attribute by 100. Use the **RelativeFrequency** attribute to create a relative-frequency diagram, as shown below.
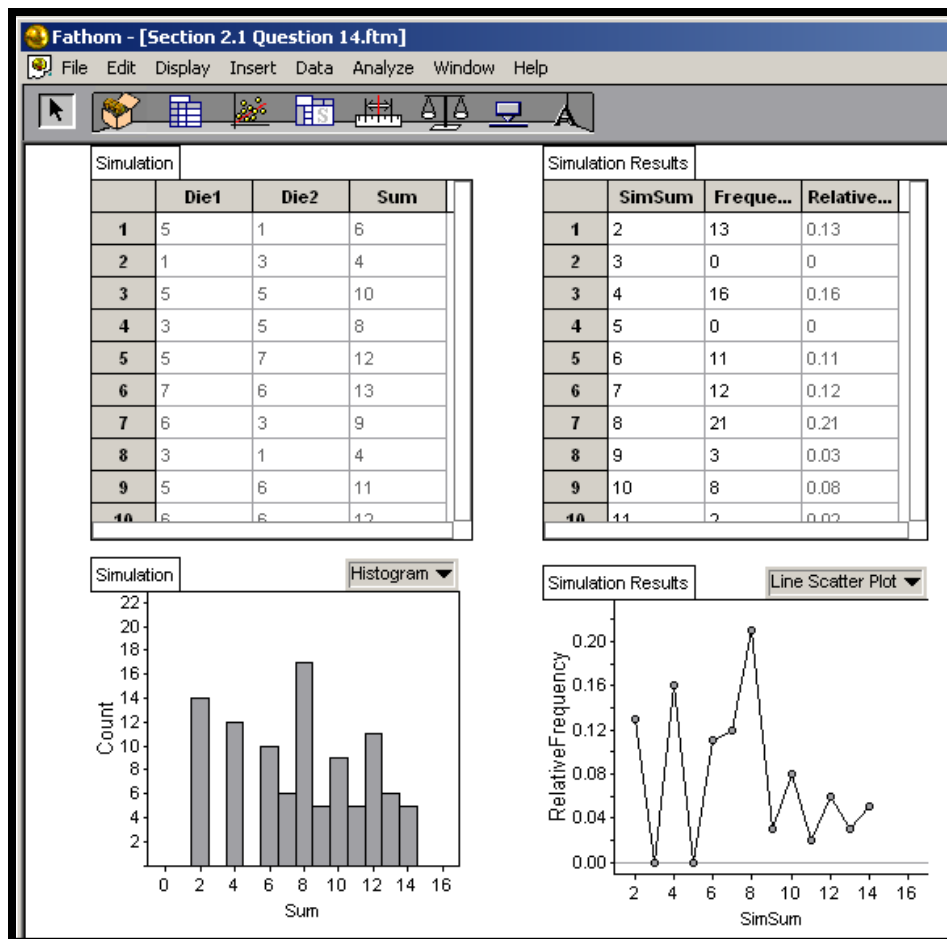
e) The diagrams in parts b) and d) are not the same. There is an element of chance in the simulation.

Section 2.1 **Page 103** **Question 16**

Answers may vary.

CHAPTER 2   Statistics of One Variable

Section 2.2   Indices

Practise

Section 2.2   Page 109   Question 2

a) Canadians purchase a wide variety of items. In order to represent their purchases with reliability, a wide variety of items must be in the CPI basket.

b) Each item will have its own percent increase, with no necessary relation between various items. The CPI uses the individual percent increases to calculate an average percent increase.

Section 2.2   Page 109   Question 4

a) Inflation refers to an increase in prices with a consequent decrease in the purchasing power of the currency.

b) The consumer price index and the new housing price index provide a measure of the increase in prices with time.

c) Since new houses are built with other items, you may expect that an increase in one of these indices will reflect a similar increase in the other.

d) The cost of housing is a significant expenditure. It will have a noticeable effect on the consumer price index.

Section 2.2   Page 110   Question 6

a) This graph begins in 1997 rather than in 1996. Hence, the mean annual change will be lower.

b) Energy costs have increased more rapidly than the costs of other items.

c) Removing the energy component of the CPI brings to light the more rapid increase of energy costs.

d) The average increase in the energy-adjusted CPI over the five years shown is about 1.6%. Hence, the increase in the CPI over five years is a factor of $1.016^5 = 1.08$, or 8%.

e) Part b) of Example 2 estimates an average annual increase of 2%. This translates to a factor of $1.02^5 = 1.10$, or 10%.

Section 2.2   Page 110   Question 8

a) The index for each year after 1988-89 is calculated by dividing the number of wolves per 100 km^2 by 4.9, the base number, and then multiplying by 100. The graph of this index is shown at the right.

Wolf Index
(1988-89 = 100)
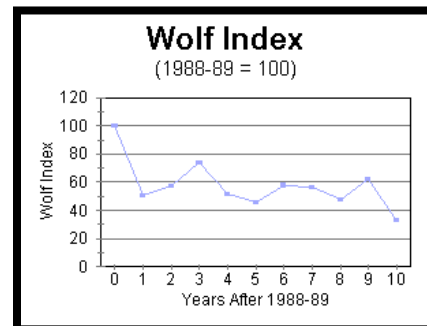
b) The overall trend appears to be downward. In several instances, a year in which there was a decrease was followed by a year in which there was an increase.

Section 2.2   **Page 111**   **Question 10**

Answers may vary.

Section 2.2   **Page 111**   **Question 12**

Answers may vary.

Section 2.2   **Page 112**   **Question 14**

Solutions to Achievement Checks are described in the Teacher's Resource.

Section 2.2   **Page 112**   **Question 16**

Answers will vary.

CHAPTER 2   Statistics of One Variable

Section 2.3   Sampling Techniques

Practise

Section 2.3   **Page 117**   **Question 2**

a) Since callers choose whether to call or not, this is a voluntary-response sample.

b) Since each country of origin has the same proportion in the sample as in the population, this is a stratified sample.

c) Since people on the street are easily approached, this is a convenience sample.

d) Since students are chosen at specific intervals from a roster, this is a systematic sample.

e) Since there are several steps to the sampling process, this is a multi-stage sample.

f) Since each school is a likely representative of the population as a whole, the random selection of 25 schools is a cluster sample.

Section 2.3   **Page 117**   **Question 4**

The total population expected at the picnic is 37. To find the number of each category of attendee to survey, divide the number in the population by 37 and multiply by 12. For example, the number of children to sample is given by

$\dfrac{13}{37} \times 12 = 4$ (when rounded). Results are shown in the screen shot at the right.

	A	B	C	D
1		Population	Sample	
2	Children	13	4	
3	Teens	8	3	
4	Adults	16	5	
5		37	12	

Section 2.3   **Page 118**   **Question 6**

Answers may vary.

Section 2.3   **Page 118**   **Question 8**

a) Since students are easy to stop as they enter the cafeteria, this is a convenience sample.

b) i) This is appropriate for surveying students' opinions on the new mural in the cafeteria, since students who do not use the cafeteria will not see the mural.

ii) This is not appropriate for surveying students' opinions on the location for the graduation prom, since students who do not use the cafeteria may still wish to attend the prom.

c) A random sample of all of the graduating students would be more appropriate for part b) ii).

Section 2.3   **Page 118**   **Question 10**

Answers will vary.

Section 2.3   **Page 118**   **Question 12**

Answers will vary.

CHAPTER 2   Statistics of One Variable

Section 2.4   **Bias in Surveys**

Practise

Section 2.4   **Page 123**   **Question 2**

a) Golf club members have an interest in preserving their golf course. On the other hand, the highway interchange may benefit all residents. Remove bias by taking a random sample of all area residents.

b) Councillors may be hesitant to admit to participating in an illegal protest lest it should affect their political careers. If the responses can be given anonymously, the bias is removed.

c) The preamble to the question lauds the benefits from the construction of a casino, but neglects to mention negative effects. It should be removed. The adjective "forward-thinking" creates bias by implying that the initiative is inherently desirable, and should be removed.

d) The elements of the cluster may not each be representative of the population as a whole. The sample should be randomly taken from the population as a whole.

Section 2.4   Page 123   Question 4

Answers may vary.

Section 2.4   Page 124   Question 6

This claim is not statistically valid. Since listeners choose whether to call or not, those who feel strongly are more likely to call, creating a response bias. Furthermore, listeners of a particular station are those who prefer that station's programming and hence may be largely conservative in their thinking. The sample is not representative of the population as a whole, resulting in sampling bias.

Section 2.4   Page 124   Question 8

Answers may vary.

CHAPTER 2   Statistics of One Variable

Section 2.5   Measures of Central Tendency

Practise

Section 2.5   Page 133   Question 2

a) Consider the data set 1, 2, 3, 4, 5, 6, 7, 8. Since no value occurs more than once, there is no mode. Answers may vary.

b) Consider the data set 1, 2, 3, 4, 6, 7, 8, 9. The median is the mean of the middle two values, which is 5. This is not one of the values in the list. Answers may vary.

c) Consider the data set 1, 1, 1, 2, 3, 4, 4, 4. Both 1 and 4 occur three times, resulting in two modes. Answers may vary.

d) Consider the data set 1, 2, 3, 4, 4, 5, 6, 7. The median is the mean of the middle two values, which is 4. This value is in the list. Answers may vary.

Section 2.5   Page 134   Question 4

a) The mode is the most appropriate measure of central tendency for baseball cap sizes. This would result in a number that is a cap size.

b) Either the mean or the median would be appropriate for standardized test scores for 2000 students. The mode could occur anywhere, and not necessarily near the centre of the data. The large size of the sample means that outliers would have little effect on the mean or the median.

c) The median would be appropriate for the final grades of a class of 18 students. The mode could occur anywhere, and not necessarily near the centre of the data. The small size of the sample means that outliers would have a greater effect on the mean than on the median.

d) The lifetimes for manufactured items form a continuous distribution. The mean would be appropriate for this set of data.

Section 2.5   **Page 134**   **Question 6**

The mean for all the students is 68 with the outliers and 71 without the outliers. The mean mark for Class B is 64 with the outliers and 71 without the outliers. The effect of removing the outliers is greater on Class B than on all the students because there are fewer students in Class B than in the entire set of students.

Section 2.5   **Page 134**   **Question 8**

a) The mean score for each student is shown in the screen shot below.

b) The weighted score for each student is shown in the screen shot above. If you are using **Corel® Quattro® Pro** rather than **Microsoft® Excel**, the equivalent formula is

@SUM(H2..H5)/SUM($B$2..$B$5).

c) According to the weighted scores, Jamie should win the award.

Section 2.5   **Page 134**   **Question 10**

a) The total number of goals scored by the team is the product of the mean score and the number of players, or 90.

b) The mean is calculated by adding up the goals scored by each player and dividing by the number of players, as shown in the equation below. Solving for the total of the goals yields the answer from part a).

$$\frac{\text{Total}}{15} = 6$$

Section 2.5   **Page 135**   **Question 12**

a) Enter the 30 ages into list 1 of a graphing calculator. Select **1-Var Stats** from the **STAT CALC** menu. The mean is 34.5 and the median is 32.5, as shown in the two screen shots at the right.
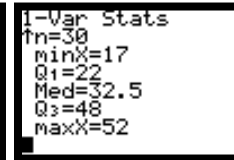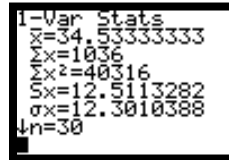
Select the **SortA(** function from the **LIST OPS** menu. Inspect the list to find the mode, which is 48, as shown in the screen shot at the right. If you need information on using a graphing calculator, refer to Appendix B on page 508 of the student text.
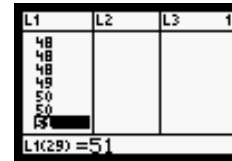
Section 2.5   **Page 135**   **Question 14**

a) Outliers have no effect on the median of a distribution. Consider the set of data below.

  8, 8, 9, 9, 10, 10, 10, 11, 11, 12, 12

The median is 10. If the two 8s were changed to 1s, the median would still be 10.

b) Using the same data as part a), the mode is 10. If the two 8s were changed to 1s, the mode would still be 10. Hence, the outliers have no effect on the mode.

Section 2.5   **Page 135**   **Question 16**

a) The geometric mean of the four pay raises can be calculated as shown below.

$$\sqrt[4]{0.04 \times 0.02 \times 0.02 \times 0.02} = 0.0238$$
$$= 2.38\%$$

b) The geometric mean is useful for calculations involving geometric series.

CHAPTER 2   Statistics of One Variable

Section 2.6   Measures of Spread

Practise

a) Enter the number of home runs into a list on your graphing calculator. Select **1-Var Stats** from the **STAT CALC** menu. The results are shown in the screen shot at the right. The median is 6, Q_1 is 4.5, Q_3 is 10, the interquartile range is the difference between Q_1 and Q_3, or 5.5, and the semi-interquartile range is half of the interquartile range, or 2.75.

```
1-Var Stats
↑n=9
minX=3
Q₁=4.5
Med=6
Q₃=10
maxX=15
```
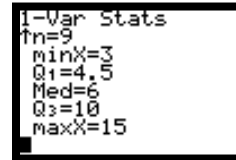
b) Enter the final grades into a list on your graphing calculator. Select **1-Var Stats** from the **STAT CALC** menu. The results are shown in the screen shot at the right. The median is 71.5, Q_1 is 60.5, Q_3 is 79.5, the interquartile range is the difference between Q_1 and Q_3, or 19 and the semi-interquartile range is half of the interquartile range, or 9.5.

```
1-Var Stats
↑n=16
minX=43
Q₁=60.5
Med=71.5
Q₃=79.5
maxX=90
```
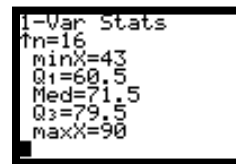
Section 2.6   Page 148   Question 4

a) Since there are 100 percentiles, the first quartile corresponds to the 25th percentile.

b) The median corresponds to the 50th percentile.

c) The third quartile corresponds to the 75th percentile.

Section 2.6   Page 148   Question 6

a) Enter the distances into a list on your graphing calculator. Select **1-Var Stats** from the **STAT CALC** menu. The results are shown in the screen shot at the right. The mean is 79.4, and the standard deviation is 57.53. Square the standard deviation to obtain the variance, which is 3309.7.

```
1-Var Stats
x̄=79.4
Σx=1191
Σx²=140897
Sx=57.52738478
σx=55.57673374
↓n=15
```
```
1-Var Stats
↑n=15
minX=18
Q₁=44
Med=68
Q₃=96
maxX=260
```
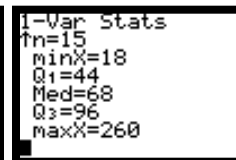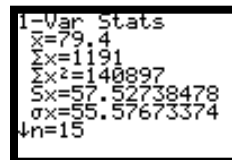
b) The median is 68, the interquartile range is 96 – 44 = 52, and the semi-interquartile range is 26.

c) The box-and-whisker plot for these data is shown in the screen shot at the right.

```
18 68              260
```
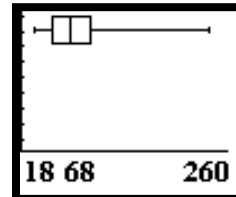
d) The data point at 260 appears to be an outlier.

Section 2.6   Page 148   Question 8

a) If the outlier were removed, the standard deviation would decrease, and the left whisker in the box-and-whisker plot would become shorter.

```
1-Var Stats
x̄=35.80769231
Σx=931
Σx²=34211
Sx=5.912828296
σx=5.798005166
↓n=26
```
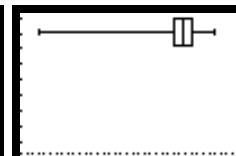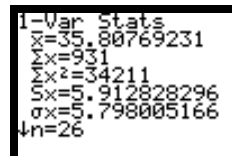
b) Enter the hours per week for each employee. Enter 12 once, 32 five times, and so on. Select **1-Var Stats** from the **STAT CALC** menu. The results are shown in the screen shot at the right. The mean is 35.8 and the standard deviation is 5.9. Square the standard deviation to obtain the variance, which is 34.8. The associated box-and-whisker plot is shown beside the statistics.

Remove the outlier and recalculate the statistics and the box-and-whisker plot. The results are shown at the right, agreeing with the predictions made in part a). The new standard deviation is 3.4.

```
1-Var Stats
x̄=36.76
Σx=919
Σx²=34067
Sx=3.443351081
σx=3.373781261
↓n=25
```

Section 2.6   Page 149   Question 10

Chi-Yan has a smaller standard deviation that indicates she has better control of her drives. As a result, she is more likely to achieve a better score.

Section 2.6   Page 149   Question 12

a) Consider the data set 7, 8, 8, 8, 8, 9. In this case, $Q_1 = Q_3 = 8$. This kind of data set arises when the standard deviation is very small. It could be the sizes of six pairs of shoes sold to female grade 9 students.

b) In this kind of distribution, the median would also be equal to Q_1 and Q_3. Since the data are arranged in ascending order, the median can be nothing else.

Section 2.6   Page 149   Question 14

a) i) The 50th percentile will occur between the 25th and 26th data point. Since both are 68, a score of 68 represents the 50th percentile.
a) ii) The 95th percentile will occur between the 47th and 48th data point. The average of 89 and 91 is 90. Therefore, a score of 90 represents the 50th percentile.

b) The 6th and 7th data points have a score of 50. This corresponds to a range from the 12th to the 14th percentile.

c) The 75th percentile implies that 37.5 scores lie below and 12.5 scores lie above. This occurs at a score of 78. The 90th percentile occurs between the 45th and 46th data points. The average of these two is 86.5.

Section 2.6   Page 150   Question 16

The summation rules can be used to expand the given sum as shown below.

$$\sum (x - \bar{x}) = \sum x - \sum \bar{x}$$

The first sum adds up to the total, as does the second. See Section 2.5, question 10b) for a demonstration of this fact. Therefore, the difference will always be zero.

a) The lowest value is 19 and the highest is 112. Therefore, the midrange is 65.5. The interquartile range can be found using a graphing calculator as 27.5. See the screen shot at the right.

```
1-Var Stats
↑n=20
 minX=19
 Q₁=29.5
 Med=41.5
 Q₃=57
 maxX=112
▁
```

b) Both of these measure half of a range of data. Outliers affect the midrange, but not the interquartile range.

CHAPTER 2   Statistics of One Variable

Review of Key Concepts

Review of Key Concepts   Page 151   Question 2

a) The grouped data are shown in the table at the right.

b) A histogram is shown in the screen shot below.

Mass(g)	Tally	Frequency
230.5-240.5	11	2
240.5-250.5	1	1
250.5-260.5	1	1
260.5-270.5	111	3
270.5-280.5	‖‖‖	5
280.5-290.5	11	2
290.5-300.5	111	3
300.5-310.5	11	2
310.5-320.5	111	3
320.5-330.5	1	1
330.5-340.5	1	1
340.5-350.5	1	1
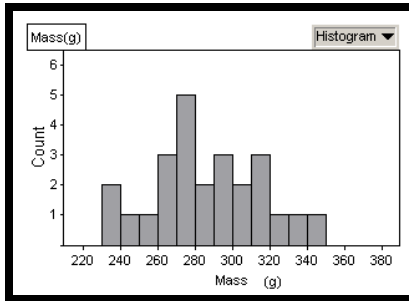
c) and **d)** The frequency polygon and relative-frequency diagram are shown below.
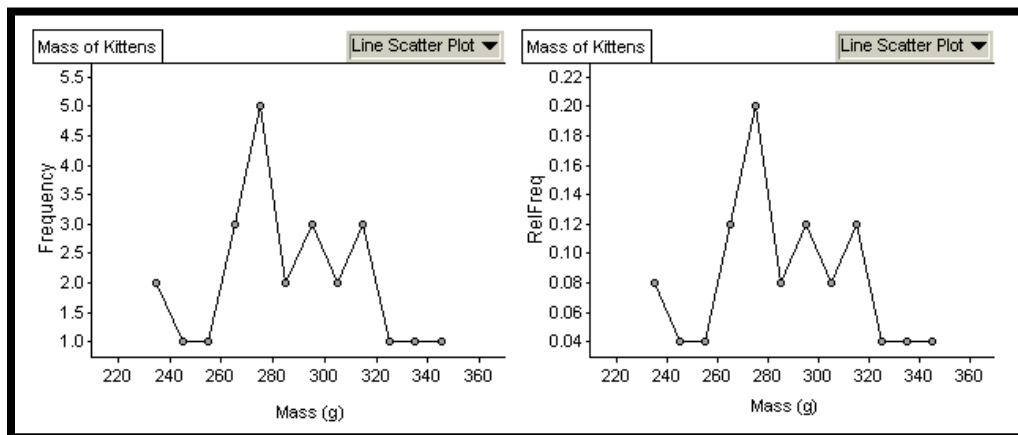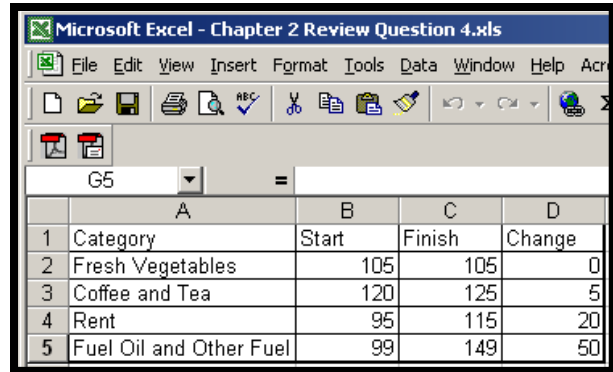
Review of Key Concepts   Page 151   Question 4

a) This graph shows a time series for an index.

b) The changes in the indices are shown in the table at the right. Fuel Oil and Other Fuel had the largest increase.

c) All four graphs intersect at 1992 because that year was chosen as the base year.

d) Coffee and tea was the most volatile category. Rent was the least volatile.

Microsoft Excel - Chapter 2 Review Question 4.xls

	A	B	C	D
1	Category	Start	Finish	Change
2	Fresh Vegetables	105	105	0
3	Coffee and Tea	120	125	5
4	Rent	95	115	20
5	Fuel Oil and Other Fuel	99	149	50

e) Coffee and tea prices depend on the supply, which in turn depends on uncontrollable factors like the weather. Rent is controlled in many places, preventing wild fluctuations.

Review of Key Concepts   Page 152   Question 6

a) In a systematic sample, members are selected from the population at regular intervals defined by some sort of formula. In a stratified sample, the population is divided into groups according to characteristics, and the sample contains proportional representation from each characteristic group.

b) A convenience sample is appropriate when easily accessible members of the population also form a representative group. An example is selecting members of a class to decide on the location of a class field trip.

c) A voluntary-response sample is convenient and inexpensive, since the respondents are responsible for submitting their responses. On the other hand, the respondents may not be representative of the population as a whole. As an example, a survey as to whether cats should be licensed is more likely to generate responses from cat owners.

Review of Key Concepts   Page 152   Question 8

Answers will vary.

Review of Key Concepts   Page 152   Question 10

a) Rather than ask an existing group, select a systematic sample of children from the population as a whole. Replace the term "junk food", which has a negative connotation, with a neutral term such as "snack food."

b) Rather than raise hands, students should indicate responses anonymously such as a secret ballot.

c) Family members and friends may be hesitant to offer realistic criticism. The song should be performed in front of a random audience selected from those with an appreciation of music such as the patrons at a club where similar music is regularly performed.

d) The adjectives "brilliant" and "critically-acclaimed" should be removed, since they imply that the book has already been judged to be excellent. Respondents should be selected from a group, which is already known to have read this book, and others in the same category that the prize is given for.

Review of Key Concepts   Page 152   Question 12

a) From the grouped data, the mean appears to be about 285 and the median appears to be about 285.

b) The mean is 288.6 and the median is 285.

c) The results in part a) are only estimates, and not expected to be completely accurate.

Review of Key Concepts   Page 153   Question 14

a) The table used to calculate the weighted average is shown at the right. Angela's weighted average is 90.2.

b) From the same table, Angela's unweighted average is 88.9.

c) Students in an engineering program must be proficient in the sciences and maths. Therefore, these subjects are given higher weightings. The ability to write is also important. English is given a higher weighting than physical education.

Microsoft Excel - Chapter 2 Review Question 14.xls

File Edit View Insert Format Tools Data Window Help

	A	B	C	D
1	Subject	Mark	Weight	Weighted Mark
2	Calculus	95	3	285
3	English	89	2	178
4	Geometry	94	3	282
5	Phys Ed	80	1	80
6	Computers	84	2	168
7	Chemistry	90	3	270
8	MDM	87	2	174
9	Physics	92	3	276
10				
11	Average	88.9		
12				
13	Weighted	90.2		
14	Average			

Review of Key Concepts   Page 153   Question 16

a) Enter the data from question 1 into a list on your graphing calculator. Select **1-Var Stats** from the **STAT CALC** menu. The results are shown in the screen shots at the right. The standard deviation is 1.5, the interquartile range is the difference between Q_1 and Q_3, or 2. The semi-interquartile range is half of the interquartile range, or 1.

b) The box-and-whisker plot is shown in the screen shot at the right.
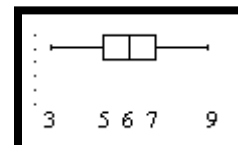
c) The distribution appears to be symmetric. Therefore, there are no outliers in the data.

Review of Key Concepts   **Page 153**   **Question 18**

a) Since there are 25 data, the first quartile occurs between the 6th and 7th data points, which average to 267.5. The third quartile occurs between the 19th and 20th data points, which average to 310.

b) The 10th percentile occurs between the 2nd and 3rd data points, which average to 242.5. The 25th percentile occurs between the 6th and 7th data points, which average to 267.5. The 75th percentile occurs between the 19th and 20th data points, which average to 310. The 90th percentile occurs between the 23rd and 24th data points, which average to 332.5.

c) The 25th percentile is the same as the first quartile. The 75th percentile is the same as the third quartile.

Review of Key Concepts   **Page 153**   **Question 20**

a) Enter the data into a list on your graphing calculator. Select **1-Var Stats** from the **STAT CALC** menu. The results are shown in the screen shots at the right. The mean is 68.1 and the median is 68. These are almost equal.
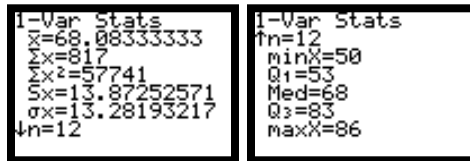
b) The standard deviation is 13.9. The interquartile range is the difference between Q_1 and Q_3, or 30. The interquartile range is larger than the spread implied by the standard deviation.

c) Because of the small number of students in the class, the standard deviation is more suitable for describing this data set.

CHAPTER 2   Statistics of One Variable

Chapter Test

Chapter Test   **Page 154**   **Question 2**

a) Enter the data into a list on your graphing calculator. Select **1-Var Stats** from the **STAT CALC** menu. The results are shown in the screen shots at the right. The mean is 66.05 and the median is 63.5. Select the **SortA(** function from the **LIST OPS** menu. Inspect the list to find the mode, which is 70.

b) The sample standard deviation is 14.514 8744. Square the sample standard deviation to obtain the sample variance, which is 210.681 5788.

c) The interquartile range is the difference between Q_1 and Q_3, or 13. The semi-interquartile range is half of the interquartile range, or 6.5.

Chapter Test   **Page 154**   **Question 4**

Since this distribution is skewed towards higher scores, the mean and mode are too high. The median is the most appropriate measure of central tendency.

Chapter Test   **Page 154**   **Question 6**

The calculation of the weighted scores is shown in the screen shot below. The committee should hire Steven, who has the highest weighted score at 4.

	A	B	C	D	E	F	G	H
	Criterion	Weight	Clarise	Clarise W	Pina	PinaW	Steven	Steven W
1								
2	Education	2	3	6	3	6	4	8
3	Experience	2	4	8	5	10	3	6
4	Interpersonal Skills	3	3	9	3	9	5	15
5	First Interview	1	5	5	4	4	3	3
6								
7	Weighted Score			3.5		3.625		4

H7 = =SUM(H2:H5)/SUM($B$2:$B$5)

Chapter Test   **Page 154**   **Question 8**

Make a list of the 52 children at the centre. Randomly select one of the first six names on the list. Then, choose every 6th name after that one.

Chapter Test   **Page 155**   **Question 10**

The company is stating that all of its funds have returns below the first quartile of comparable funds.

CHAPTER 3   Statistics of Two Variables

Review of Prerequisite Skills

Review of Prerequisite Skills   **Page 158**   **Question 2**

a) i) Enter the data into the lists of a graphing
calculator. Select the **LinReg(ax+b)** function under the
STAT CALC menu to find the line of best fit. If you
need instructions on how to use a graphing calculator,
refer to Appendix B of the student text. The equation

and the graph are shown in the screen shots above.
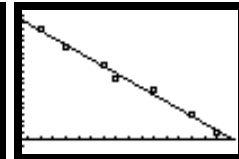
ii) Turn off the **STATPLOT** and then use the **TRACE** key on the line of
best fit to find the intercepts, as shown for the x-intercept in the screen shot
at the right. The x-intercept is about 18.2. The y-intercept is about 21.2.

iii) Use the **TRACE** key again to estimate the value of y when x is 7. The
answer is about 13.1, as shown in the screen shot at the right.

b) i) Enter the data into the lists of a graphing
calculator, and proceed as in part a) above. The results
are shown in the screen shots at the right.
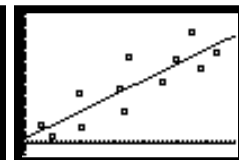
ii) Using the same method as part a), the x-intercept is
about -1.6. The y-intercept is about 1.2.

iii) Using the same method as part a), the value of y, when x is 7, is about 6.3.

Review of Prerequisite Skills   **Page 158**   **Question 4**

a) Enter the function into a graphing calculator, as
shown in the screen shot at the right. If you need
instructions on how to use a graphing calculator, refer
to Appendix B of the student text. The graph is shown
in the screen shot at the right. You can use the **TRACE**
key to show that both the x-intercept and the y-intercept
are zero.
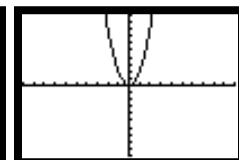
b) Enter the function into a graphing calculator, as
shown in the screen shot at the right. The graph is
shown in the screen shot at the right. The x-intercepts
are at -6 and 1, and the y-intercept is at -6.
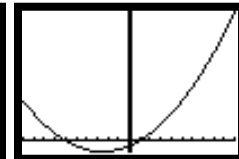
c) Enter the function into a graphing calculator, as
shown in the screen shot at the right. The graph is
shown in the screen shot at the right. The x-intercepts
are at -0.7 and 1, and the y-intercept is at 2.

Review of Prerequisite Skills   Page 158   Question 6
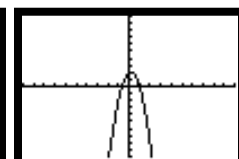
a) $\displaystyle\sum_{i=1}^{8} i = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$

$\qquad = 36$

b) $\displaystyle\sum_{i=1}^{5} i^2 = 1 + 4 + 9 + 16 + 25$

$\qquad = 55$

Review of Prerequisite Skills   Page 158   Question 8

a) A spreadsheet solution for this question is shown at the right. If you need instructions on how to use a spreadsheet, refer to Appendix B of the student text.

b) The raw numbers for 6a), 6b), and 7a) were entered directly. The numbers for 7b) were derived from the first four numbers of 7a) by entering the formula C2^2 in cell D2 and copying it down three more cells. The sums were calculated by using the **SUM** function, as shown in the screen shot at the right. The equivalent expression for **Corel® Quattro® Pro** is @SUM(D2..D9).

Review of Prerequisite Skills   Page 158   Question 10

a) Measurement bias refers to a measurement technique or sampling method that systematically decreases or increases the variable that it is measuring.

b) If a survey method includes sensitive questions that cannot be answered anonymously, then respondents may not answer truthfully. This is an example of unintentional measurement bias.

c) If a survey includes leading questions, then the measurement bias is intentional.

d) Answers may vary.

CHAPTER 3   Statistics of Two Variables

Section 3.1   Scatter Plots and Linear Correlation

Practise

Section 3.1   Page 168   Question 2

a) Since heart disease depends on cholesterol level, cholesterol level is the independent variable and heart disease is the dependent variable.

b) Since free-throw success level depends on hours of basketball practice, hours of basketball practice is the independent variable and free-throw success level is the dependent variable.

c) Since height of plant depends on amount of fertilizer, amount of fertilizer is the independent variable and height of plant is the dependent variable.

d) Since income depends on level of education, level of education is the independent variable and income is the dependent variable.

e) Since pulse rate depends on running speed, running speed is the independent variable and pulse rate is the dependent variable.

Section 3.1   Page 169   Question 4

This question lends itself conveniently to a solution using Fathom™. If you need information on how to use Fathom™, refer to Appendix B of the student text. Enter the data into a **Case Table** as shown at the right. Drag and drop four graphs into the workspace. Drag and drop attributes to generate the graphs shown. Add a least-squares line to each graph from the **Graph** menu.
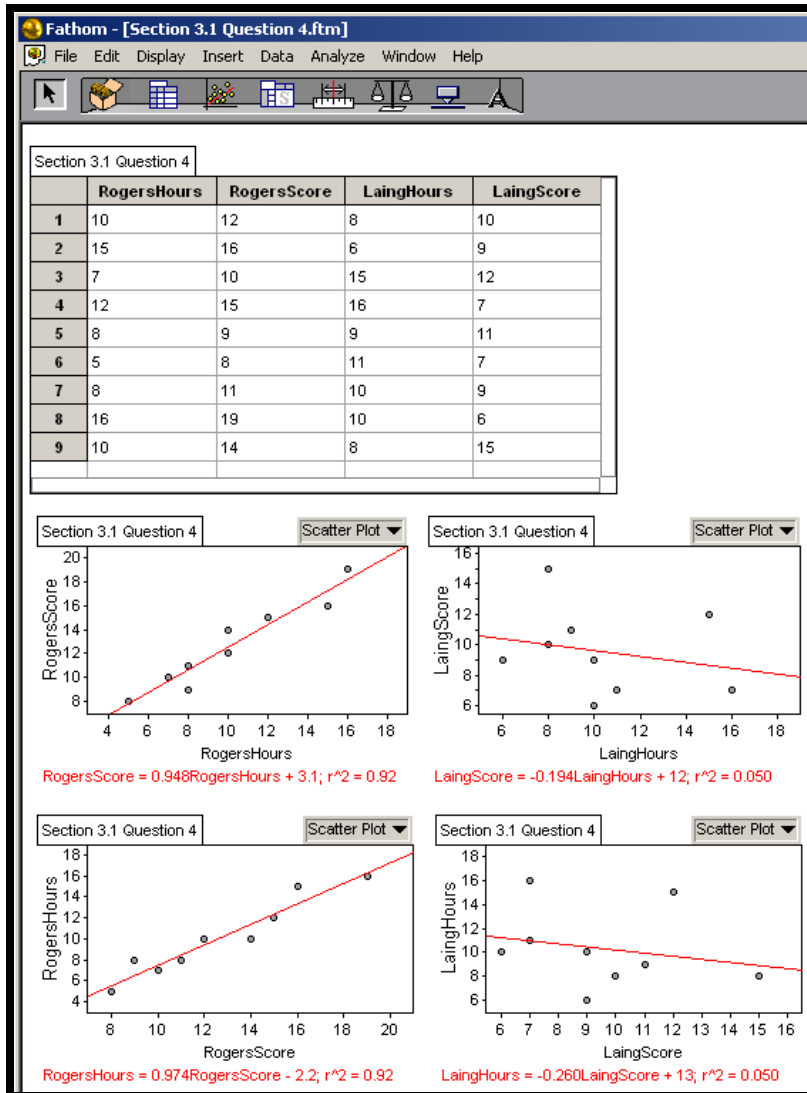
a) The Rogers method shows a strong positive linear correlation with a correlation coefficient of 0.96. The Laing method shows a weak negative linear correlation of −0.22.

b) If the dependent and independent variables are interchanged, the scatter plot is reflected in the line $y = x$. The correlation coefficients will not change.

c) The graphs at the right bear out the predictions in part b).

Section 3.1   **Page 169**   **Question 6**

a) and **b)** Enter the data into the lists of a graphing calculator. If you need information on the use of graphing calculators, refer to Appendix B of the student text. Be sure to turn **Diagnostics On** so that you can see the correlation coefficient. The scatter plot is shown at the above left. There is a very weak negative linear correlation with a correlation coefficient of −0.05.
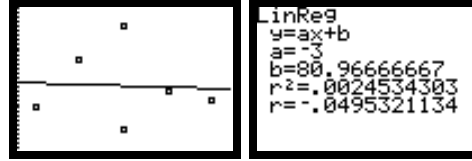
c) The students may conclude that it is unlikely that there is any relationship between arm span and score on a mathematics test.

Section 3.1   **Page 169**   **Question 8**

Answers may vary.

Section 3.1   **Page 170**   **Question 10**

a) You would likely expect to see a negative correlation between the temperature and the number of people using the track for temperatures above the comfort zone, which is around 20ºC to 24ºC. You would expect to see a positive correlation between the temperature and the number of people using the track for temperatures below the comfort zone.

b) A typical scatter plot might look like the one shown at the right.

c) The peak of the graph occurs at the middle of the "comfort zone." As the temperature either rises or drops from here, the number of people using the track will decrease.
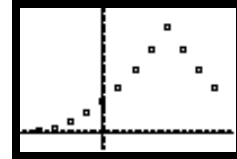
Section 3.1   **Page 170**   **Question 12**

The formula for covariance is $S_{xy} = \dfrac{1}{n-1}\sum(x-\bar{x})(y-\bar{y})$.

Interchanging the dependent and independent variables has the same effect as interchanging the two expressions within the brackets. Since multiplication is commutative, the value of the covariance remains unchanged.

The correlation coefficient is calculated from $r = \dfrac{S_{xy}}{S_x \times S_y}$. Since none of these values are changed by interchanging the dependent and independent variables, the value of the correlation coefficient is also unchanged.

a) There are many possible answers. One possibility is shown in the screen shot at the right.

b) The correlation is zero because the least-squares line is horizontal.

c) The screen shot at the right was generated using Fathom™.
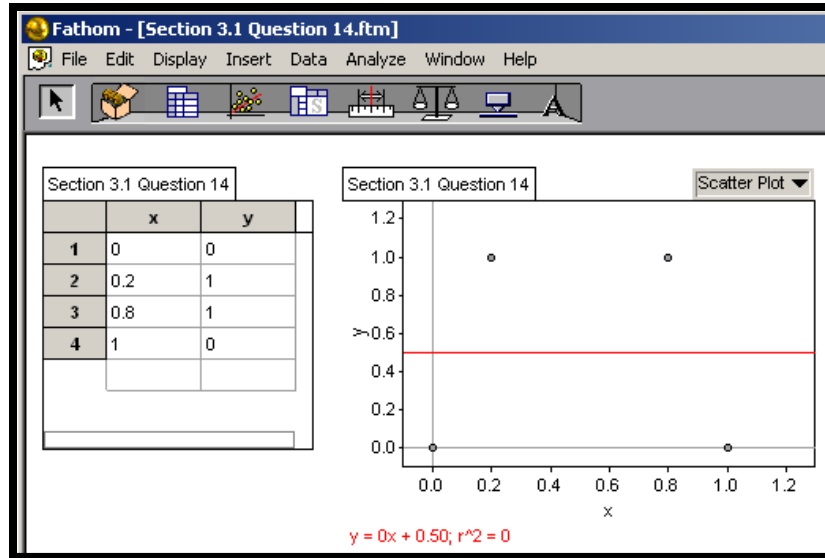
CHAPTER 3   Statistics of Two Variables

Section 3.2   Linear Regression

Practise

Section 3.2   Page 180   Question 2

a) Enter the data from question 1a) into lists L1 and L2 of your graphing calculator. Perform a linear regression as described in Appendix B of the student text. You will create a graph as shown in the screen shot at the right. The equation of the line of best fit is approximately $y = 0.61x + 17.2$.
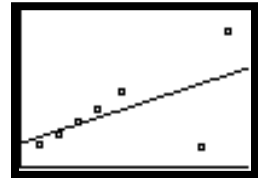
Repeat the process for the data in question 1b). You may wish to use lists L3 and L4 for part b). Then, use Plot 2 and place the equation in Y2 rather than Y1. You will create a graph as in the screen shot at the right. The equation for the line of best fit is approximately $y = -39.9x + 392.9$
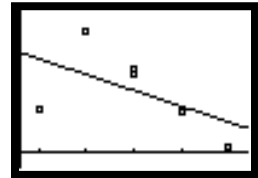
b) In question 1a), the point (92, 18) appears to be an outlier. Remove this point from your lists, and perform the linear regression again. You will create a graph as shown in the screen shot at the right. The equation for the line of best fit is approximately $y = 1.12x + 3.16$.
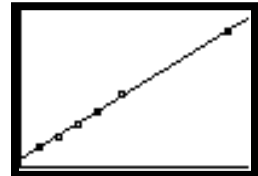
In question 1b), the point (4, 106) appears to be an outlier. Remove this point from your lists, and perform the linear regression again. You will create a graph as shown in the screen shot at the right. The equation for the line of best fit is approximately $y = -98.7x + 794.4$.
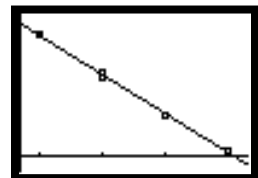
c) The lines of best fit from question 1a) are:

$$y = 0.61x + 17.2 \text{ (with outlier)}$$
$$y = 1.12x + 3.16 \text{ (with outlier removed)}$$

Note the large difference in both the slope and the intercept. Note also how much better the line of best fit matches the remaining points when the outlier is removed.

The lines of best fit from question 1b) are:

$$y = -39.9x + 392.9 \text{ (with outlier)}$$
$$y = -98.7x + 794.4 \text{ (with outlier removed)}$$

Note the large difference in both the slope and the intercept. Note also how much better the line of best fit matches the remaining points when the outlier is removed.

Section 3.2   Page 180   Question 4

A graphing calculator was used in question 2 to verify the answers to question 1. In this question, a spreadsheet will be used. If you need instructions on using spreadsheets, refer to Appendix B of the student text. The results are shown in the screen shot below.
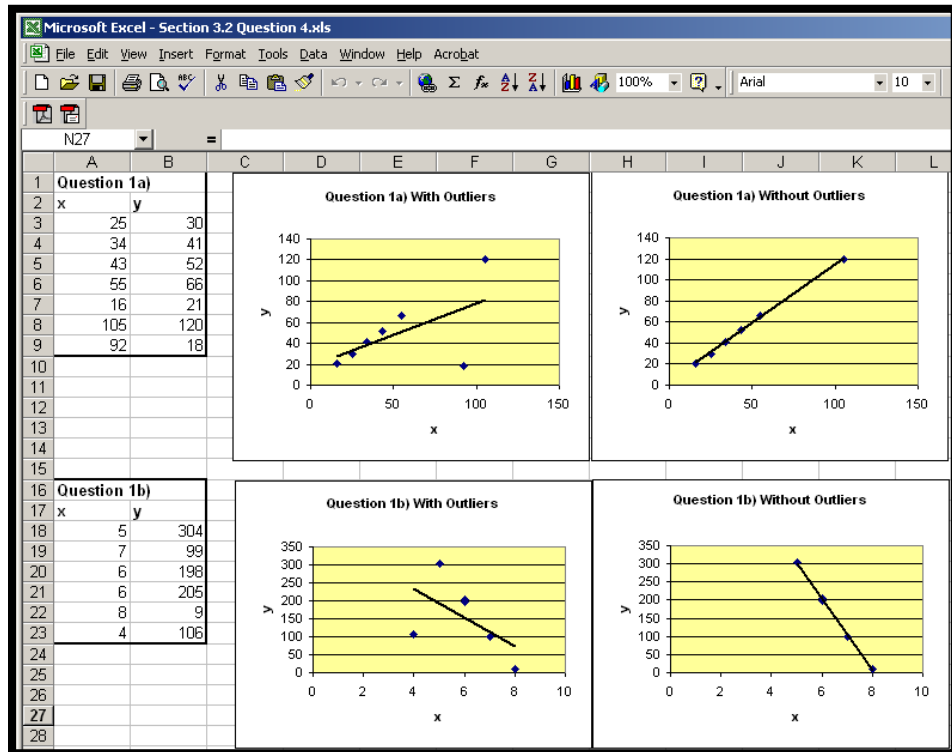
Section 3.2   Page 180   Question 6

a) The scatter plot for these data is shown at the right. There appears to be a moderate positive linear correlation.
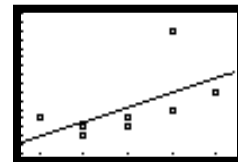
b) The correlation coefficient is about 0.568, as shown in the screen shot at the right. The equation of the line of best fit is approximately $y = 1.67x - 20.42$.

```
LinReg
y=ax+b
a=1.666666667
b=-20.41666667
r²=.3220611916
r=.5675043538
```

c) The outlier appears to be (17, 15).

d) If the outlier is removed, the new graph looks like the screen shot at the right. The new correlation coefficient is 0.759 and the new equation of the line of best fit is $y = 0.92x - 9.61$.

```
LinReg
y=ax+b
a=.9210526316
b=-9.605263158
r²=.5756578947
r=.7587212233
```
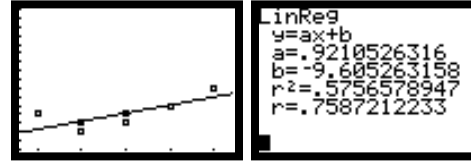
e) Removing the outlier has improved the linear model. The correlation coefficient has increased.

f) The model may be improved by surveying a larger group of students, and by using a stratified sampling technique to ensure that all ages are represented equally.

g) Since there is a positive correlation, you may conclude that the number of books a student reads depends on the student's age.

Section 3.2   Page 181   Question 8

a) The solution for this question was generated using Fathom™. If you need information on using Fathom™, refer to Appendix B of the student text. The scatter plot for the productions in 2001 is shown below left. There appears to be an outlier at (4.2, 1.5).

b) The correlation coefficient is 0.45 and the equation of the line of best fit is $y = 1.71x + 2.6$.

Section 3.2 Question 8 — Scatter Plot
Sales2001 = 1.71Cost2001 + 2.6; r^2 = 0.20

Section 3.2 Question 8 — Scatter Plot
Sales2001NoOutlier = 2.34Cost2001NoOutlier + 2.4; r^2 = 0.99
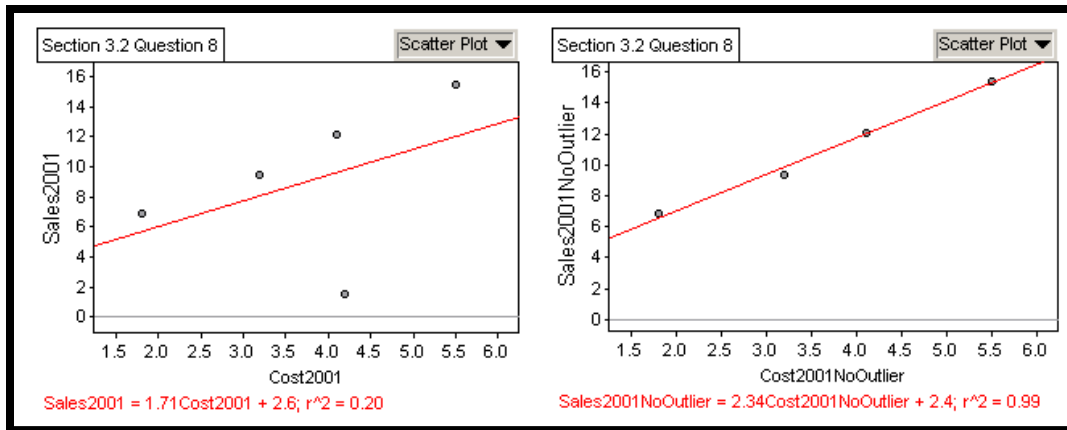
c) With the outlier removed, the scatter plot and regression analysis appear as shown above right. The correlation coefficient is 0.99 and the equation of the line of best fit is $y = 2.34x + 2.4$.

d) The results for 2002 are shown below left. There appear to be no outliers. The correlation coefficient is 0.70 and the equation of the line of best fit is $y = 1.55x - 0.97$.
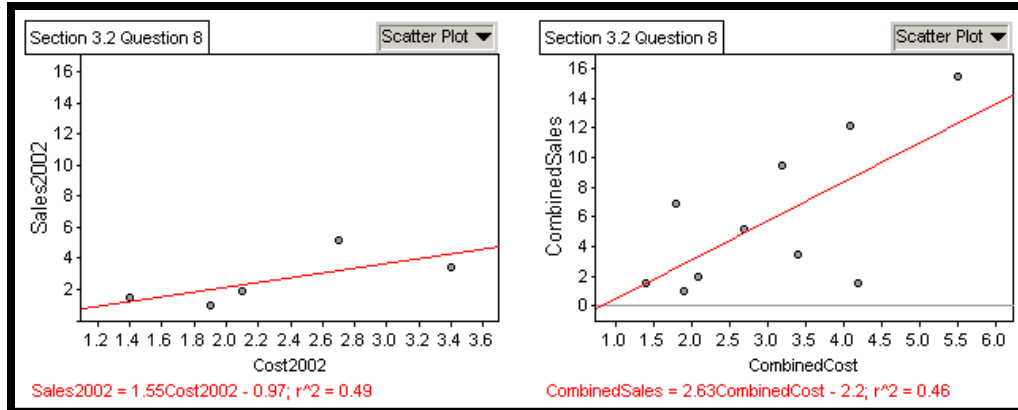
e) The results from combining the data are shown above right. There appear to be no outliers. The correlation coefficient is 0.68 and the equation of the line of best fit is $y = 2.63x - 2.2$.

f) Answers will vary.

g) The equation from part d) is based on more recent data than the equation from part b). It is more reliable as an extrapolation tool.

Section 3.2   Page 182   Question 10

a) The wolf population versus the rabbit population is shown in the left graph below. The rabbit population versus the wolf population is shown in the right graph below. The data points of the second graph are the inverse of those in the first graph.
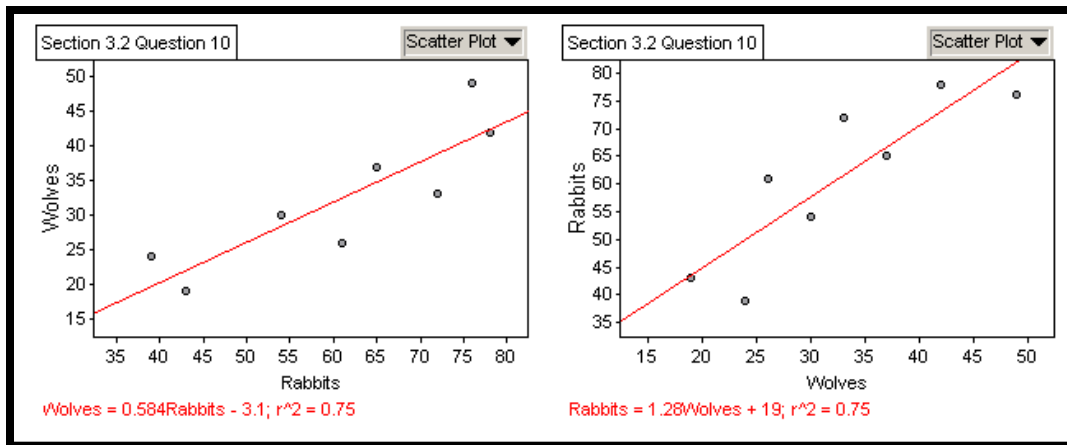
b) The correlation coefficient is 0.87 and the equation of the line of best fit is $y = 1.28x + 19.3$.

c) The line from the first graph has the equation $y = 0.58x - 3.1$. The inverse of this equation is $x = 0.58y - 3.1$. Solving for y, you can show that $y = 1.72x + 5.34$. This is not the same as the line in part b). The reason is that the correlation is not equal to 1.

d) A plot of both populations as a time series is shown at the right. The two populations seem to increase and decrease in step.

e) Since the wolf population seems to lag slightly behind the rabbit population, you may infer that the rabbit population is the independent variable and the wolf population is the dependent variable.
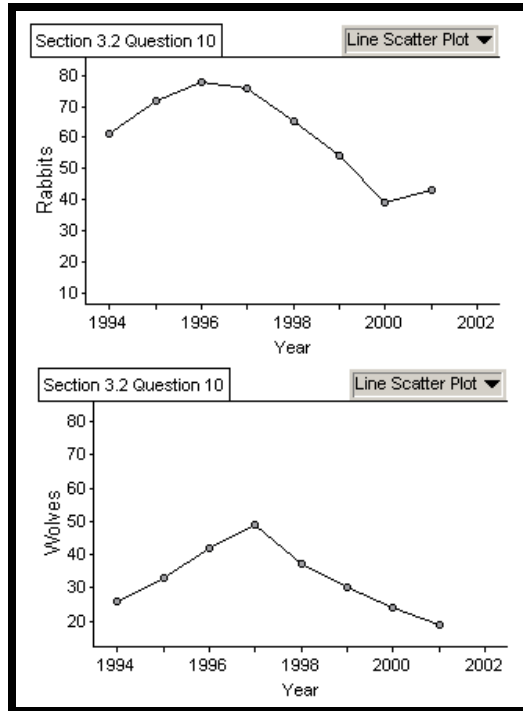
The solution for this question was generated using Fathom™. If you need information on using Fathom™, refer to Appendix B of the student text.

Section 3.2   Page 182   Question 12

a) The data in this question were obtained from E-STAT. The graph and line of best fit are shown at the right. A straight line does not appear to be a good model for this data.

b) The slope of the line of best fit tells you that the average rate of inflation from 1914 to 2000, assuming a linear model, was about 13%. Since a linear model is not appropriate for this data, this information cannot be used for extrapolation purposes.

c) The slope of the line of best fit for the last 20 years is 2.78. The slope for the last 5 years is 1.81.
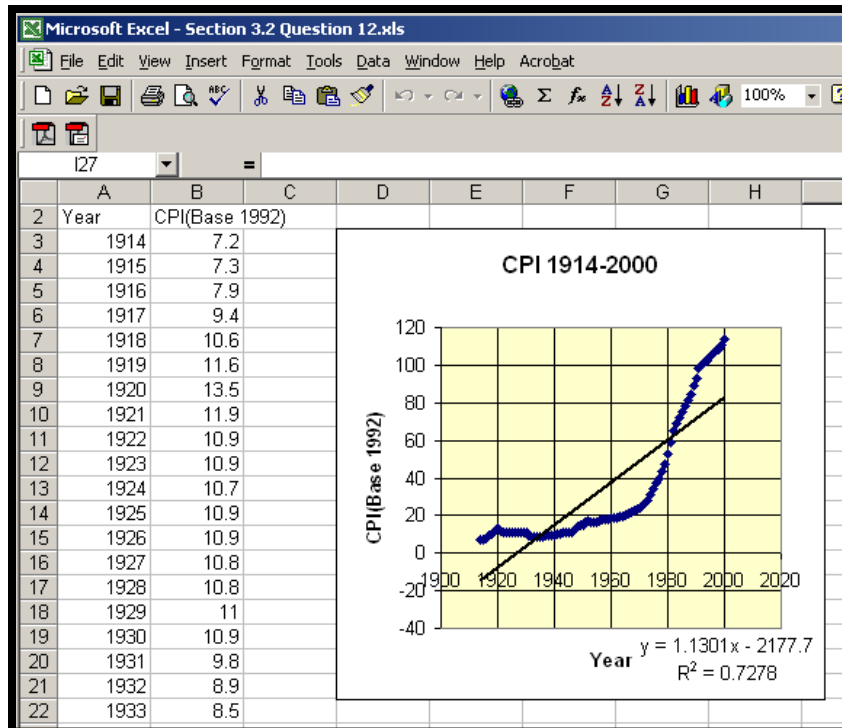
d) The three slopes indicate that the rate of inflation increased since 1914, but has decreased over the last 5 years.

Section 3.2   **Page 183**   **Question 14**

Consider the points (1, 2), (2, 3), (3, 5), (4, 5), (5, 6), (6, 6), (7, 8), and (8, 9). These points all lie on the line $y = x + 1$ except for (3, 5), which is one unit above the line, and (6, 6), which is one unit below the line. The results of a scatter plot and linear regression are shown at the right. Note that the correlation is not perfect.


```
LinReg
y=ax+b
a=.9285714286
b=1.321428571
r²=.9530075188
r=.9762210399
```
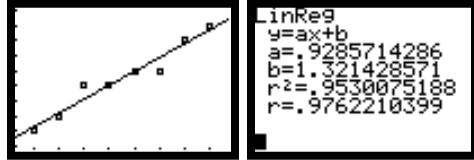
Section 3.2   **Page 183**   **Question 16**

a) The solution for this question was generated using Fathom™. If you need information on using Fathom™, refer to Appendix B of the student text. Enter the **Hours** and **Score** data into a **Case Table**. Make a scatter plot of **Score** versus **Hours**. Add a least-squares line, as shown below. Add another attribute called **Residual**. Edit the formula for this attribute to **Score** − (−1.33**Hours** + 80). This will calculate the difference between the real **Score** and the score predicted by the least squares line.
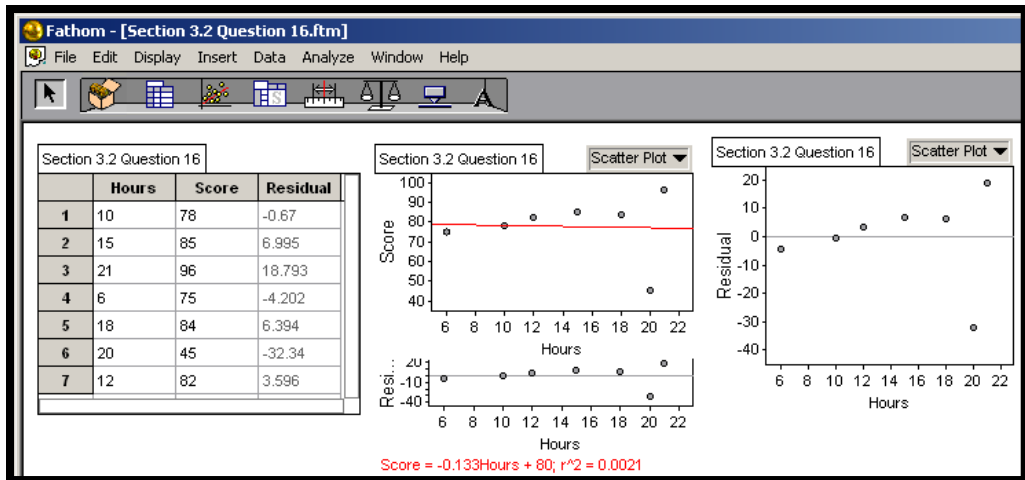
Create a new graph of **Residual** versus **Hours**. Note: Fathom™ can also plot residuals automatically by choosing **Make Residual Plot** under the **Graph** menu.

CHAPTER 3   Statistics of Two Variables

Section 3.3   Non-Linear Regression

Practise

Section 3.3   **Page 191**   **Question 2**

a) Enter the data into a graphing calculator and create a scatter plot, as shown at the right. It appears to be quadratic. Perform a quadratic regression. The results are shown at the right. The equation is $y = -1.43x^2 - 2.46x + 4.64$. The coefficient of determination is 0.966, indicating a good fit.

b) Enter the data into a graphing calculator and create a scatter plot, as shown at the right. It appears to be cubic. Perform a cubic regression. The results are shown at the right. The equation is $y = 0.89x^3 + 3.68x^2 + 3.29x + 1.01$. The coefficient of determination is 0.959, indicating a good fit.
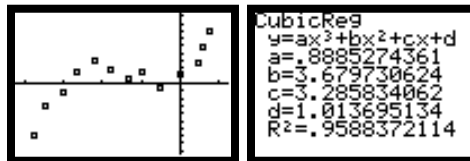
c) Enter the data into a graphing calculator and create a scatter plot, as shown at the right. It appears to be exponential. Perform an exponential regression. The results are shown at the right. The equation is $y = 1.60(1.77)^x$. The coefficient of determination is 0.979, indicating a good fit.
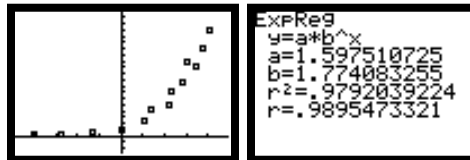
Section 3.3   **Page 192**   **Question 4**

a) Enter the data into Microsoft® Excel and create a scatter plot as described in Appendix B of the student text.

b) From the **Chart** menu, select **Add Trendline** and select **Power** as the type of regression to be performed. On the **Options** tab, check the boxes for "**Display equation on chart**" and "**Display R-squared value on chart**". Your final screen will look like the one in the following screen shot. According to the regression, the exponent is 0.7513. Using whole number exponents, this equation can be written as $y = 1.0506x^{\frac{3}{4}}$.

c) Note that the coefficient of determination is $R^2 = 0.9991$, which indicates that this power equation is a good mathematical model for the relationship between an animal's mass and its basal metabolic rate.
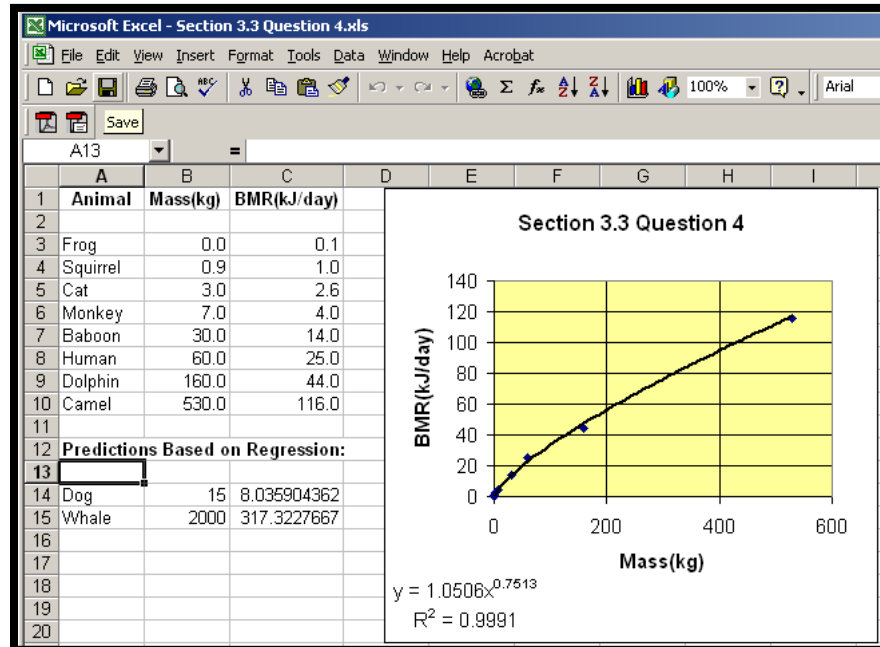
d) Substituting for the mass in the equation on the chart, we find that a 15-kg dog should have a BMR of 8.0 kJ/day and a 2-tonne (2000-kg) whale should have a BMR of 317.3 kJ/day.

Section 3.3   Page 193   Question 6

a) The time-series graph for the mean starting salary of the graduates who find jobs is shown at the right. It might be cubic, or a long-term quadratic with a short-term anomaly.
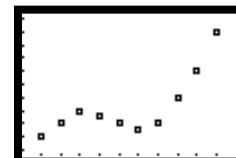
b) A quadratic regression produces the results shown at the right. The equation of the curve is $y = 0.157x^2 - 1.10x + 28.3$. The coefficient of determination is 0.83. The results of a cubic regression are also shown at the right. The equation is $y = 0.05x^3 - 0.70x^2 + 2.83x + 23.88$. The coefficient of determination is 0.98.
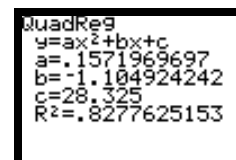
c) The cubic model shows a better coefficient of determination than the quadratic model.
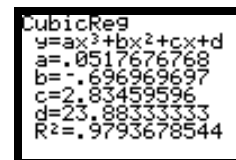
Section 3.3   Page 193   Question 8

a) The logistic regression function is selection B: under the **STAT CALC** menu. The results of a logistic regression of the bacteria population versus time are shown at the right. The equation is $y = \dfrac{1647}{1 + 46.9e^{-0.39x}}$.
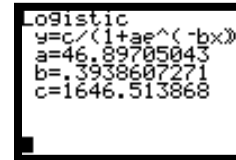
b) The graph of this curve superimposed over the scatter plot is shown at the right.
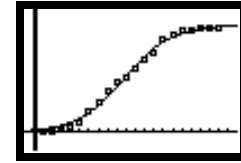
c) The curve appears to fit the data set well over the entire range of values. It has a thin "s" shape.

d) Bacterial growth is quick at first, since there is adequate food and space available. As the population approaches the carrying capacity of the environment, the growth rate levels off to zero.

Section 3.3 **Page 194** **Question 10**

A spreadsheet solution for this problem is shown at the right. If you need instructions on using spreadsheets, refer to Appendix B of the student text. This solution was done using Microsoft® Excel. Equivalent formulas for Corel® Quattro® Pro are shown in brackets after those for Microsoft® Excel. Enter the Age and Annual Income data from Section 3.2, Example 1, as shown at the right. Use the CORREL function, (@CORREL), to calculate Pearson's correlation coefficient. Use the AVERAGE function, (@MEAN), to calculate the mean of the Annual Income. For the line of best fit, you can copy the slope and y-intercept from the trendline on the graphs from section 3.2. Alternatively, you can use the LINEST function, (@LINEST), to generate the slope and y-intercept shown in cells B15 and C15. Use this line of best fit to calculate y_{est} in column C. The formula for the first value is $B$15*A4+$C$15.

Add a column for $(y_{est} - \bar{y})^2$ and

$(y - \bar{y})^2$. Use the following formula to calculate the coefficient of determination.

Microsoft® Excel
SUM(D4:D11)/SUM(E4:E11)

Corel® Quattro® Pro
(@SUM(D4..D11)/@SUM(E4..E11))

Repeat the above steps for Example 2 and Example 3 of section 3.2.

Microsoft Excel - Section 3.3 Question 10.xls

	A	B	C	D	E	F
1	Section 3.2 Example 1					
2						
3	Age(years)	Annual Income($000)	yest	(yest-y-bar)^2	(y-y-bar)^2	
4	33	33	37.098672	16.21131936	66.015625	
5	25	31	27.895636	175.0160805	102.515625	
6	19	18	20.993359	405.2829841	534.765625	
7	44	52	49.752846	74.43973177	118.265625	
8	50	56	56.655123	241.1847309	221.265625	
9	54	60	61.256641	405.2829841	356.265625	
10	38	44	42.850569	2.977589271	8.265625	
11	29	35	32.497154	74.43973177	37.515625	
12						
13	r	0.982531094		Pearson's correlation coefficient		
14	y-bar	41.125				
15	Trendline	1.150379507	-0.86385			
16	r-squared	0.965367351		Coefficient of determination		
17						
18	Section 3.2 Example 2					
19						
20	Rabbits	Wolves	yest	(yest-y-bar)^2	(y-y-bar)^2	
21	61	26	32.5	0	42.25	
22	72	33	38.919098	41.20482097	0.25	
23	78	42	42.420424	98.41482034	90.25	
24	76	49	41.253316	76.62053487	272.25	
25	65	37	34.834218	5.448571368	20.25	
26	54	30	28.415119	16.68624982	6.25	
27	39	24	19.661804	164.8192839	72.25	
28	43	19	21.996021	110.3335702	182.25	
29						
30	r	0.865206839		Pearson's correlation coefficient		
31	y-bar	32.5				
32	Trendline	0.583554377	-3.09682			
33	r-squared	0.748582874		Coefficient of determination		
34						
35						
36						
37						
38						
39	Section 3.2 Example 3					
40						
41	Hours	Score	yest	(yest-y-bar)^2	(y-y-bar)^2	
42	10	78	78.465008	0.36949976	0.020408163	
43	15	85	77.800156	0.003247556	51.02040816	
44	21	96	77.002333	0.730700208	329.1632653	
45	6	75	78.99689	1.299022592	8.163265306	
46	18	84	77.401244	0.207843615	37.73469388	
47	20	45	77.135303	0.521052395	1079.591837	
48	12	82	78.199067	0.116912033	17.16326531	
49						
50						
51	r	-0.046184583		Pearson's correlation coefficient		
52	y-bar	77.85714286				
53	Trendline	-0.132970451	79.79471			
54	r-squared	0.002133016		Coefficient of determination		

In all three examples from section 3.2, the square of Pearson's correlation coefficient is equal to the coefficient of determination for linear regression.

CHAPTER 3   Statistics of Two Variables

Section 3.4   Cause and Effect

Practise

Section 3.4   **Page 200**   **Question 2**

a) The common cause is overall athletic ability.

b) The common cause is overall disregard of safe driving practices.

c) The common cause is a large income.

Section 3.4   **Page 200**   **Question 4**

a) The classes may be taught by teachers of varying ability and training in the methods being used.

b) There is not enough evidence to reject the new method.

c) Repeat the trial several times, using the same teacher for both classes. Ensure that the teacher is well-trained in both methods.

Section 3.4   **Page 200**   **Question 6**

a) The relationship is likely either accidental or there is a common-cause factor at work.

b) Answers may vary.

Section 3.4   **Page 200**   **Question 8**

Answers may vary.

Section 3.4   **Page 200**   **Question 10**

a) This correlation does not imply causality. The number of graduates hired depends on economic factors, and not on how many there are, unless a shortage of graduates exists. In that case, increasing the number of graduates will increase the number hired until the shortage is alleviated.

b) The data shows that, during the recession, there were fewer graduates hired.

Section 3.4   **Page 201**   **Question 12**

Answers may vary.

Section 3.4   Page 201   Question 14

a) Gender may be removed as an extraneous variable by using groups made up of the same gender.

b) The effect of gender may be determined by comparing the results of same-gender groups and mixed-gender groups.

CHAPTER 3   Statistics of Two Variables

Section 3.5   Critical Analysis

Practise

Section 3.5   Page 209   Question 2

A hidden variable is income. As students get older, their income level increases.

Section 3.5   Page 210   Question 4

The level of customer satisfaction rises because dissatisfied customers will go elsewhere while satisfied customers will return.

Section 3.5   Page 210   Question 6

a) The predicted starting salary for the graduates of 2005 can be calculated as follows:

$$\text{Salary(thousands of dollars)} = 0.0518(2005)^3 - 310(2005)^2 + 618\,412(2005) - 411\,344\,091$$
$$= -120\,004.53.$$

Therefore, the predicted starting salary is approximately $-\$120$ million.

b) This is not a realistic projection. Salaries cannot be negative.

c) The cubic model appears attractive due to the dip caused by the recession of the mid-1990s. In fact, it does not model the increase in salaries correctly, and gives erroneous results when extrapolated.

d) A more accurate model would result from taking data over a longer period of time and ignoring short, temporary anomalies. Non-linear models work better if the time is stated as years elapsed as a chosen base year rather than a date.

Section 3.5   Page 210   Question 8

a) A scatter plot of production versus number of employees is shown at the right. The correlation appears to be positive and piecewise-linear with a jump at 120 employees.
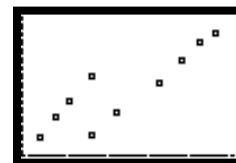
b) The number of employees starts to decrease in 1995. That is when the layoffs appear to start.

c) Answers may vary. One possibility is that a general recession caused demand for the product to drop, resulting in both layoffs and a drop in production.

d) A scatter plot of productivity versus time is shown at the right. Time is entered in number of years elapsed since 1992.
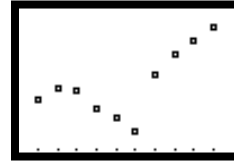
e) The linear regression and line of best fit plot are shown at the right. The equation of the line is $y = 0.011x + 0.447$.
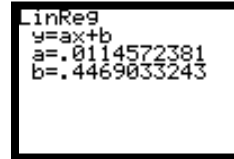
f) The decrease in productivity stops in 1998 and begins to increase. This is likely when a better management system was adopted.
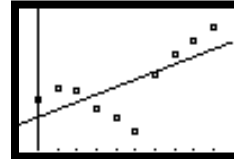
Section 3.5   Page 211   Question 10

Answers may vary.

CHAPTER 3   Statistics of Two Variables

Review of Key Concepts

Review of Key Concepts   Page 212   Question 2

a) The scatter plot for these data is shown at the right. There appears to be a weak negative correlation.
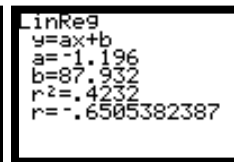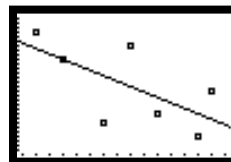
b) The results of the linear regression are shown at the right. The correlation coefficient is −0.651.

c) Answers may vary.

Review of Key Concepts   Page 212   Question 4

a) The scatter plot for these data is shown at the right.
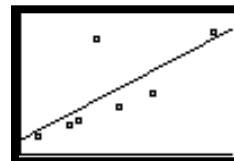
b) The results of the linear regression are shown at the right. The line of best fit has the equation $y = 0.87x + 36.15$. The correlation coefficient is 0.74.
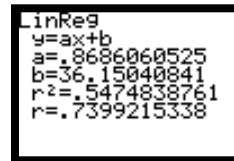
c) The point (177, 235) appears to be an outlier.

d) The results with the outlier removed are shown at the right. The line of best fit has the equation $y = 0.94x + 15.57$. The correlation coefficient is 0.97.

```
LinReg
y=ax+b
a=.9397068096
b=15.57384931
r²=.9334825487
r=.9661690063
```
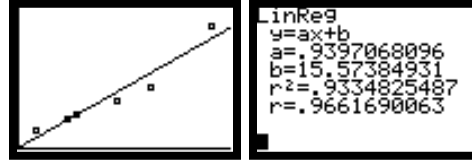
e) A player who scores 250 in the first game would have a predicted score of 254 using the first model and a score of 251 using the second model. This is a difference of 3.

Review of Key Concepts   Page 213   Question 6

a) The scatter plot suggests a logistic fit, as shown at the right. The equation is

$$y = \frac{452.7}{1 + 34.6e^{-0.26x}}$$
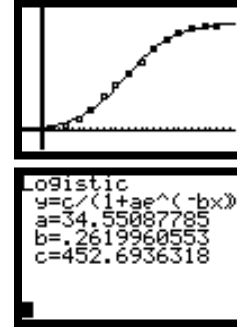
b) Answers may vary.

c) Between 0 and 28 s, the driver accelerated, held speed for a short time, and then braked to a stop.

```
Logistic
y=c/(1+ae^(-bx))
a=34.55087785
b=.2619960553
c=452.6936318
```

Review of Key Concepts   Page 213   Question 8

a) In a control group, all variables are held constant. In an experimental group, one variable is changed. The results are compared. This method minimizes the effect of extraneous variables.

b) A control group is often told that a variable has been changed, when in fact it has not. This is called a "placebo." For example, in the case of testing a new drug, the control group is given sugar pills while the experimental group is given the drug, although both think they have been given the drug. Sometimes an effect is produced psychologically even though no drug has been given. This is called the "placebo effect."

Review of Key Concepts   Page 213   Question 10

The price of eggs and wages are both subject to common-cause factors. For example, in times of inflation, there is upward pressure on the price of eggs as well as on wages. The increase in one does not cause the increase in the other.

Review of Key Concepts   Page 213   Question 12

Answers may vary. Possible factors to consider are that the gifted class is not representative of the student body as a whole or that poor spellers may not enjoy playing the game.

CHAPTER 3   Statistics of Two Variables

Chapter Test

Chapter Test   **Page 214**   **Question 2**

a) strong negative linear: −0.8   **b)** direct: 1

c) weak positive linear: 0.3   **d)** moderate positive linear: 0.6

e) perfect negative linear: −1

Chapter Test   **Page 214**   **Question 4**

a) The scatter plot for Jerome's data is shown at the right. There is a moderate positive linear correlation.
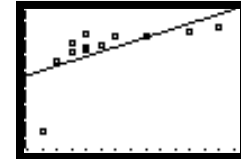
b) The results of the regression analysis are shown at the right. The equation of the line of best fit is $y = 1.79x + 57.86$. The correlation coefficient is 0.618.
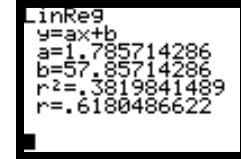
c) The point (2, 38) appears to be an outlier.

d) With the outlier removed, the scatter plot and results of the linear regression are shown at the right. The equation of the line of best fit is $y = 0.89x + 65.99$. The correlation coefficient is 0.819.
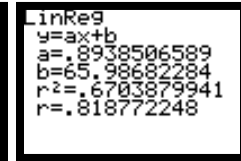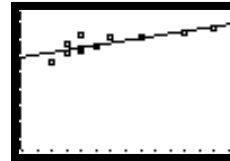
Chapter Test   **Page 215**   **Question 6**

Answers may vary.

Chapter Test   **Page 215**   **Question 8**

Establishing control and experimental groups can minimize the effects of extraneous variables in a correlation study. All variables are held constant for the control groups. One selected variable is changed for the experimental groups while all others are held constant.

Chapter Test   **Page 215**   **Question 10**

Solutions for Achievement Checks are described in the Teacher's Resource.

Cumulative Review: Chapters 1 to 3

Cumulative Review

Cumulative Review   **Page 218**   **Question 2**

a) Start with a 1 in the centre of the grid. Move one cell to the right and place a 2. Continue in a counter-clockwise manner around the 1, increasing the number by 1 for each cell. When you reach 9, move one cell to the right, and then proceed in a counter-clockwise manner again.

b) Filling all of the cells results in a grid, as shown at the right.

37	36	35	34	33	32	31
38	17	16	15	14	13	30
39	18	5	4	3	12	29
40	19	6	1	2	11	28
41	20	7	8	9	10	27
42	21	22	23	24	25	26
43	44	45	46	47	48	49

Cumulative Review   **Page 218**   **Question 4**

a) Since listeners choose whether to call in or not, this is a voluntary-response sample.

b) Since this is a poll, the sample will likely be randomly selected.

c) Since science is involved, the sample will be an observational study.

Cumulative Review   **Page 218**   **Question 6**

Answers may vary. The following are possible answers.

a) Grade point average and starting salary upon graduation are likely related by a common cause such as ability.

b) Grade in chemistry and grade in physics are likely related by a common cause such as scientific aptitude.

c) Sales of symphony tickets and carrot harvest would be an accidental relationship.

d) Increased monthly rainfall likely causes increased monthly umbrella sales.

Cumulative Review   **Page 218**   **Question 8**

a) Since there is a route from any one vertex to any other, the network is connected. Since there are exactly two odd vertices, the network is traceable. Since none of the edges cross, it is planar.

b) Since there is a route from any one vertex to any other, the network is connected. Since there are no odd vertices, the network is traceable. Since it cannot be drawn such that none of the edges cross, it is non-planar.

Cumulative Review   **Page 219**   **Question 10**

a) Buffalo has two routes with exactly one stopover. The musician may travel from Toronto–Detroit–Chicago–Cleveland–Pittsburgh–Washington–Philadelphia (through Buffalo but not stopping over)–New York–Buffalo–Toronto or the musician may travel from Toronto–Detroit–Chicago–Cleveland–Pittsburgh–Washington–Buffalo–Philadelphia–New York–Toronto (through Buffalo but not stopping over).

b) Buffalo is the only city with two possible stopovers. The musician may travel from Toronto–Detroit–Chicago–Cleveland–Pittsburgh–Washington–Buffalo–Philadelphia–New York–Buffalo–Toronto.

Cumulative Review   **Page 219**   **Question 12**

a) The frequency table for the grouped data is shown at the right.

Class Interval	Tally	Frequency
15.9- 25.9	11	2
25.9- 35.9	⊮	5
35.9- 45.9	⊮ 11	7
45.9- 55.9	⊮	5
55.9- 65.9	⊮ 11	7
65.9- 75.9	1111	4
75.9- 85.9	111	3
85.9- 95.9	11	2
95.9-105.9		0
105.9-115.9	111	3
115.9-125.9	1	1
125.9-135.9	1	1

b) The histogram for these data is shown at the left below. The cumulative-frequency diagram is shown at the right below.
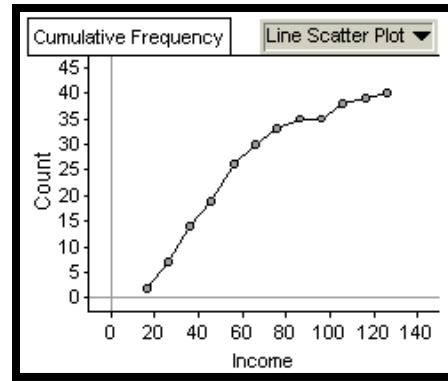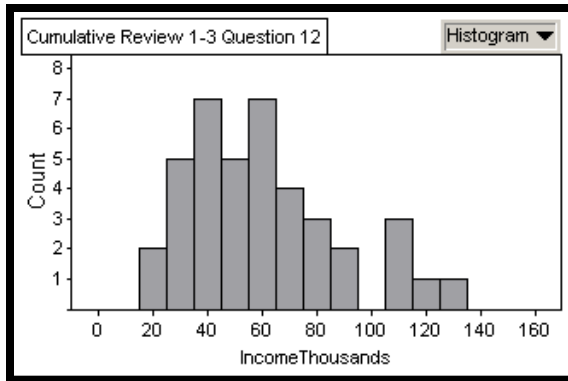
c) Twenty-four of the families earn \$60 000 or less. This is $\dfrac{24}{40} \times 100\% = 60\%$.

Cumulative Review   **Page 219**   **Question 14**

a) Enter the data into a graphing calculator. Select
1-Var Stats from the **STAT CALC** menu. The mean is 198.6 and the
median is 193.5, as shown in the screen shots at the right. Select the
SortA(function from the **LIST OPS** menu. Inspect the list to find the
mode, which is 260. The mean or median is most descriptive of this
distribution. The mode occurs, by chance, near the high end of the data.

b) The standard deviation is 40.4, assuming that the set of bowling scores
is a population and not a sample. Q_1 is 170 and Q_3 is 222, which gives an
interquartile range of 52.

c) The standard deviation measures the spread of the data about the mean. Twenty-five percent of
the scores are below the first quartile of 170 or above the third quartile of 222. The interquartile
range contains the middle 50% of the data, which lies between 170 and 222.

d) The 50th percentile lies between the 15th and 16th data points. Hence, is it the average of 192
and 195 or 193.5.

e) A player who scored 222 lies at the 23rd data point. This is about the 77th percentile, well
below the 80th.

Cumulative Review   **Page 220**   **Question 16**

Answers will vary.

Cumulative Review   **Page 220**   **Question 18**

a) The base for this index is 100. The index was half of this base, or 50, around 1980.

b) The index went from about 25 in 1971 to 100 in 1992. This indicates that the average price of
goods doubled twice from 1971 to 1992.

c) From the graph, you can estimate that the index was 25 in 1971, 55 in 1981, 95 in 1991, and
115 in 2001. The largest change occurred from 1981 to 1991. This indicates that the highest rate
of inflation was during the decade of the 1980s.

d) From 1971 to 2001 the index changed from 25 to 115. The rate of inflation can be estimated
from a geometric series as shown below.

$$25(1+i)^{30} = 115$$

Make an estimate for *i*, and evaluate the expression on the left of the equals sign. Note the
answer, and refine your estimate until you achieve the desired answer on the right. The rate of
inflation is 0.052 or 5.2%.

CHAPTER 4   Permutations and Organized Counting

Review of Prerequisite Skills

Review of Prerequisite Skills   Page 224   Question 2

a) The tree diagram illustrating the possible outcomes of tossing a coin and rolling a six-sided die is shown at the right.

b) There are a total of 12 possible outcomes.
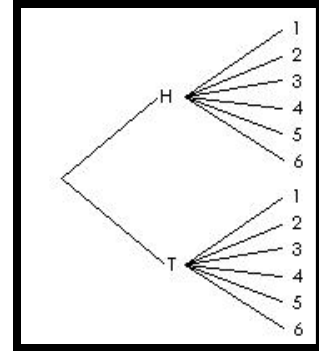
Review of Prerequisite Skills   Page 224   Question 4

a) Start with rectangles of one square. Five of these can be drawn, as shown at the right. There are four rectangles of two squares, three of three squares, two of four squares, and one of five squares, making a total of 15. Similar reasoning is used for the following answers.

b) 45   **c)** 90   **d)** 150

Review of Prerequisite Skills   Page 224   Question 6

a)

$$5(4) + (-1)^3(3)^2$$
$$= 20 - 9$$
$$= 11$$

b)

$$\frac{(10-2)^2(10-3)^2}{(10-2)^2 - (10-3)^2}$$
$$= \frac{64 \times 49}{64 - 49}$$
$$= 209.1$$

c)

$$\frac{6(6-1)(6-2)(6-3)(6-4)(6-5)}{3(3-1)(3-2)}$$
$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$
$$= \frac{720}{6}$$
$$= 120$$

d)

$$\frac{50(50-1)(50-2)\ldots(50-49)}{48(48-1)(48-2)\ldots(48-47)}$$
$$= \frac{50 \times 49 \times 48 \ldots 1}{48 \times 47 \times 46 \ldots 1}$$
$$= 50 \times 49$$
$$= 2450$$

e)

$$\frac{12 \times 11 \times 10 \times 9}{6^2} + \frac{10 \times 9 \times 8 \times 7}{2^4} - \frac{8 \times 7 \times 6 \times 5}{42}$$
$$= \frac{11880}{36} + \frac{5040}{16} - \frac{1680}{42}$$
$$= 330 + 315 - 40$$
$$= 605$$

CHAPTER 4   Permutations and Organized Counting

Section 4.1   Organized Counting

Practise

Section 4.1   **Page 229**   **Question 2**

A sum of 4 may be rolled as 1 + 3, 2 + 2, or 3 + 1. A sum of 11 may be rolled as 5 + 6 or 6 + 5, giving a total of 5 ways to roll a sum of 4 or 11.

Section 4.1   **Page 229**   **Question 4**

Since there are four 10s and four queens, there are a total of eight cards that can be drawn that are either a 10 or a queen.

Section 4.1   **Page 229**   **Question 6**

For each of the ten books, there are four pens that can be selected. Hence, there are $10 \times 4 = 40$ different ways of making your selection. Use the product rule.

Section 4.1   **Page 229**   **Question 8**

Since there are six possible outcomes every time the die is rolled, the total number of outcomes is $6^5 = 7776$.

Section 4.1   **Page 229**   **Question 10**

Assume that a student must choose either True or False. For each question, there are two possible outcomes. Hence, there are $2^6 = 64$ possible ways to answer the questions on the test.

Section 4.1   **Page 230**   **Question 12**

For each window there are two choices, either open or closed. The total number of arrangements is $2^5 = 32$.

Section 4.1   Page 230   Question 14

Since the first digit must be a number from 2 to 9, there are eight choices. Since the second digit must be a 0 or 1, there are two choices. Since there is no restriction on the last digit, there are ten choices. The total number of possible area codes is $8 \times 2 \times 10 = 160$.

Section 4.1   Page 230   Question 16

a) There are 4 ways to select the first five. For each of these, there are 3 ways to select the second five. Therefore, there are $4 \times 3 = 12$ ways to select the two fives.

b) There are 2 ways to select the red five. For each of these, there are 13 ways to select the spade. Therefore, there are $2 \times 13 = 26$ ways to select a red five followed by a spade.

c) There are two red fives and thirteen spades in the deck. Therefore, you may choose a red five or a spade in 15 ways.

d) There are two red fives in the deck and thirteen hearts. However, one of the red fives is also a heart. Therefore, you may choose a red heart or a five in $2 + 13 - 1 = 14$ ways.

Section 4.1   Page 230   Question 18

a) There are 10 ways of selecting each of the three digits, and 26 ways of selecting each of the three letters. Therefore, there are $10 \times 10 \times 10 \times 26 \times 26 \times 26 = 17\,576\,000$ ways of making the licence plates.

b) The extra letter increases the answer to part a) by an additional factor of 26 for a total of 456 976 000 licence plates.

c) Answers may vary.

Section 4.1   Page 230   Question 20

a) There are two possibilities with one pulse. There are 2 × 2 = 4 possibilities with two pulses. There are 2 × 2 × 2 = 8 possibilities with three pulses. There are 2 × 2 × 2 × 2 = 16 possibilities with four pulses. The total number of possibilities is 2 + 4 + 8 + 16 = 30.

b) Five pulses would add another 32 to the answer in part a) for a total of 62. Therefore, in order to obtain 72 possibilities, at least six pulses would be required.

Section 4.1   Page 231   Question 22

Each blank may be cut in 2 × 3 × 3 × 3 × 3 = 162 ways. Since there are ten blanks, the total number of keys is 10 × 162 = 1620.

a) There are 40 choices for the first number. Since the second number cannot be the same as the first, there are 39 choices for the second number. The third number cannot be the same as the second, but it can be the same as the first. Therefore, there are 39 choices for the third number. The total number of combinations is $40 \times 39 \times 39 = 60\,840$.

b) Answers may vary.

c) If the first number must be dialled clockwise from 0, then the choices for the first number are reduced to 39. The choices for the second and third numbers are still 39 each. Therefore, the number of possible combinations is $39 \times 39 \times 39 = 59\,319$.

d) If the first number can be dialled clockwise or counter-clockwise from 0, then there are 78 choices for the first number. The choices for the second and third numbers are still 39 each, assuming that they must be dialled clockwise. Therefore, the number of possible combinations is $78 \times 39 \times 39 = 118\,638$.

e) If another number is added, then the answer for part a) is increased by another factor of 39 to 2 372 760.

CHAPTER 4   Permutations and Organized Counting

Section 4.2   Factorials and Permutations

Practise

Section 4.2   **Page 239**   **Question 2**

a)
$$\frac{7!}{4!} = 7 \times 6 \times 5$$
$$= 210$$

b)
$$\frac{11!}{9!} = 11 \times 10$$
$$= 110$$

c)
$$\frac{8!}{5!\,2!} = \frac{8 \times 7 \times 6}{2 \times 1}$$
$$= 168$$

d)
$$\frac{15!}{3!\,8!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{3 \times 2 \times 1}$$
$$= 168$$

e)
$$\frac{85!}{82!} = 85 \times 84 \times 83$$
$$= 592\,620$$

f)
$$\frac{14!}{4!\,5!} = \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}$$
$$= 30\,270\,240$$

a)

$$P(10, 4) = \frac{10!}{(10-4)!}$$
$$= \frac{10!}{6!}$$
$$= 10 \times 9 \times 8 \times 7$$
$$= 5040$$

b)

$$P(16, 4) = \frac{16!}{(16-4)!}$$
$$= \frac{16!}{12!}$$
$$= 16 \times 15 \times 14 \times 13$$
$$= 43\,680$$

c)

$$_5P_2 = \frac{5!}{3!}$$
$$= 5 \times 4$$
$$= 20$$

d)

$$_9P_4 = \frac{9!}{5!}$$
$$= 9 \times 8 \times 7 \times 6$$
$$= 3024$$

e)

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 5040$$

Section 4.2   **Page 239**   **Question 6**

a) Since each letter is different, there are six choices for the first letter, five for the second, four for the third, three for the fourth, two for the fifth, and one for the sixth. Therefore, the total number of arrangements is 6! = 720.

b) There are four choices for the first textbook, three for the second, two for the third, and one for the fourth. Therefore, the total number of arrangements is 4! = 24.

c) There are 12 choices for the first region, 11 for the second, 10 for the third, and 9 for the fourth. Therefore, the total number of choices is $12 \times 11 \times 10 \times 9 = 11\,880$.

Section 4.2   **Page 239**   **Question 8**

You can define n! as an iterative process as follows. Start with 1. Multiply by 2. Continue multiplying, increasing the multiplier by 1 each time, until the multiplier reaches a value of n.

Section 4.2   **Page 239**   **Question 10**

The number of arrangements with no restrictions is 12! = 479 001 600. Since there is only one correct arrangement, the number of incorrect arrangements is 479 001 599.

Section 4.2   **Page 239**   **Question 12**

Tanya is already seated. Therefore, there are 9! = 362 880 ways of seating the remaining people with no restrictions. If you put Henry and Wilson together, and count them as one entity, there are $2 \times 8! = 80\ 640$ ways of seating them together. The factor of 2 arises because Henry and Wilson can be interchanged. By the same reasoning, there are 80 640 ways of seating Henry next to Nancy. Subtract these unwanted arrangements from the number with no restrictions to obtain the final answer of 201 600 arrangements.

Section 4.2   **Page 239**   **Question 14**

There are 20 ways to choose the valedictorian, 19 ways to choose the class poet, and 18 ways to choose the presenter of the class gift. Multiply these together to obtain the total number of choices, which is 6840.

Section 4.2   **Page 240**   **Question 16**

a) Since digits must not be repeated, an *n*-digit code will have *n*! arrangements. Evaluation on a calculator shows that, to obtain 60 000 different codes, the value of *n* must be 9.

b) If the first three digits of a ten-digit code must be 1, 3, or 6, then there are 3! = 6 ways of choosing the first three digits. The remaining seven digits can be chosen in 7! = 5040 ways. Therefore, the total number of codes is $6 \times 5040 = 30\ 240$.

Section 4.2   **Page 240**   **Question 18**

a) There are four positions to be filled from a choice of 10 candidates. The number of ways this can be done is *P*(10, 4) = 5040.

b) If the president and vice-president must be grade 12 students, there are *P*(5, 2) = 20 ways of doing this. Once these are chosen, there are 3 ways of selecting the grade 12 representative, 3 for the grade 11 representative, and one each for the grade 9 and 10 representatives. The remaining four positions can be filled in 4! = 24 ways from the remaining students. The total number of possible arrangements is the product of these: $20 \times 3 \times 3 \times 1 \times 1 \times 24 = 4320$.

Section 4.2   **Page 240**   **Question 20**

Solutions for Achievements Checks are described in the Teacher's Resource.

Section 4.2   **Page 240**   **Question 22**

Normally, the number of arrangements of six people is 6! = 720. However, since the table is round, shifting each person by one seat does not change the arrangement. There are six ways of shifting people around the table. Therefore, you should divide 720 by 6 to obtain 120 possible seating arrangements. Arrangements like this are called "circular permutations."

There are 10 ways to select the student whose name is repeated. There are three choices of rank for the first teacher. For each of these, there are two choices for the second teacher and one for the third teacher. The total number of ways of choosing the repeated student is $10 \times 3! = 60$. For each of these, the remaining six slots must be chosen from the nine students that are left. There are $P(9, 6) = 60\,480$ ways of doing this. Hence, the total number of ways that the teachers could have made their lists is given by $60 \times 60\,480 = 3\,628\,800$.

CHAPTER 4   Permutations and Organized Counting

Section 4.3   Permutations With Some Identical Items

Practise

Section 4.3   **Page 245**   **Question 2**

a) Since there are two rs, the number of permutations is $\dfrac{8!}{2!} = 20\,160$.

b) Since there are two es and two ls, the number of permutations is $\dfrac{10!}{2!\,2!} = 907\,200$.

c) Since there are two ts and three as, the number of permutations is $\dfrac{7!}{2!\,3!} = 420$.

d) Since there are four es and three ns, the number of permutations is $\dfrac{15!}{4!\,3!} = 9\,081\,072\,000$.

Section 4.3   **Page 245**   **Question 4**

a) Since all digits are different, the number of numbers is $6! = 720$.

b) Since there are three 1s, the number of numbers is $\dfrac{6!}{3!} = 120$.

c) Since there are two 3s and two 4s, the number of numbers is $\dfrac{6!}{2!\,2!} = 180$.

d) Since there are four 6s, the number of numbers is $\dfrac{6!}{4!} = 30$.

Section 4.3   **Page 245**   **Question 6**

Since the number is even, the last digit must be a 2 or a 6. Since the number must be less than 3 000 000, the first digit must be a 1 or a 2. Since there are two 2s available, you must consider four cases:

Case 1: The first digit is a 1 and the last digit is a 2. The remaining digits contain two 5s. The number of numbers is, therefore, $\dfrac{5!}{2!} = 60$.

Case 2: The first digit is a 1 and the last digit is a 6. The remaining digits contain two 2s and two 5s. The number of numbers is, therefore, $\dfrac{5!}{2!\,2!} = 30$.

Case 3: The first digit is a 2 and the last digit is a 2. The remaining digits contain two 5s. The number of numbers is, therefore, $\dfrac{5!}{2!} = 60$.

Case 4: The first digit is a 2 and the last digit is a 6. The remaining digits contain two 5s. The number of numbers is, therefore, $\dfrac{5!}{2!} = 60$.

The total number of numbers is $60 + 30 + 60 + 60 = 210$.

Section 4.3   **Page 245**   **Question 8**

a) SUYFS has two *S*s. Therefore, the number of permutations is $\dfrac{5!}{2!} = 60$.

YATTS has two *T*s. Therefore, the number of permutations is $\dfrac{5!}{2!} = 60$.

SPEEXO has two *E*s. Therefore, the number of permutations is $\dfrac{6!}{2!} = 360$.

HAREMM has two *M*s. Therefore, the number of permutations is $\dfrac{6!}{2!} = 360$.

b) Answers may vary.

Section 4.3   **Page 245**   **Question 10**

a) If there are six different doughnuts, the number of ways of sharing the doughnuts is $6! = 720$.

b) If there are three each of two varieties, the number of ways of sharing the doughnuts is $\dfrac{6!}{3!\,3!} = 20$.

c) If there are two each of three varieties, the number of ways of sharing the doughnuts is

$\dfrac{6!}{2!\,2!\,2!} = 90$.

Section 4.3 **Page 246** **Question 12**

Since there are four green flags, three red flags, and two blue flags for a total of nine flags, the

number of arrangements is $\dfrac{9!}{4!\,3!\,2!} = 1260$.

Section 4.3 **Page 246** **Question 14**

Solutions for Achievement Checks are described in the Teacher's Resource.

Section 4.3 **Page 246** **Question 16**

Since there are three *A*s, the number of arrangements without restriction is $\dfrac{6!}{3!} = 120$. If the

consonants are not to appear out of order, you must divide by the number of ways of arranging
the three consonants among themselves, which is 3! = 6. Therefore, the number of possible
arrangements with this restriction is 20.

Section 4.3 **Page 246** **Question 18**

Place the 8 brown bottles in a row. There are now 9 places to put green bottles, 7 between the
brown bottles and 1 at either end. Select 1 of the 9 spaces for the pair of green bottles. There are 9
ways of doing this. For each of these 9, you must select 5 of the remaining 8 spaces to hold one
green bottle each. This is given by $_8P_5$. However, since the 5 green bottles are identical, you
must divide by 5!. Therefore, the number of ways of placing the bottles so that exactly one pair of

green bottles is together is given by $9 \times \dfrac{_8P_5}{5!} = 504$.

CHAPTER 4 Permutations and Organized Counting

Section 4.4 Pascal's Triangle

Practise

Section 4.4 **Page 251** **Question 2**

a) $t_{7,2} + t_{7,3} = t_{8,3}$ **b)** $t_{51,40} + t_{51,41} = t_{52,41}$

c) $t_{18,12} - t_{17,12} = t_{17,11}$ **d)** $t_{n,r} + t_{n-1,r} = t_{n-1,r-1}$

Section 4.4   Page 251   Question 4

Each row sum is of the form 2^n, where n is the row number.

a) Sum $= 256 \Rightarrow n = 8$.

b) Sum $= 2048 \Rightarrow n = 11$.

c) Sum $= 16\ 384 \Rightarrow n = 14$.

d) Sum $= 65\ 536 \Rightarrow n = 16$.

Section 4.4   Page 251   Question 6

a) The sum of the squares of the terms in the rows of Pascal's triangle is shown in the first two columns of the table at the right. From the pattern produced, you can predict that the sum of the squares of the *nth* row is given

Row	Sum of Squares	Alternately Add and Subtract
0	1	1
1	2	0
2	6	–2
3	20	0
4	70	6

by the term $t_{2n,n}$. For example, the sum of the squares of the terms in row 4 is 70. This is $t_{8,4}$.

b) If you alternately add and subtract the squares of the terms, the result is shown in the third column of the table. If the row number is odd, the result is 0. If the row number is even, the result is $(-1)^{0.5n}t_{n,0.5n}$. For example, if $n = 4$, then the result is $(-1)^{0.5\times4}t_{4,2}$, which is 6.

Section 4.4   Page 252   Question 8

a) The number of diagonals is shown in the diagram at the right.

Quadrilateral   Pentagon   Hexagon
Two diagonals   Five diagonals   Nine Diagonals
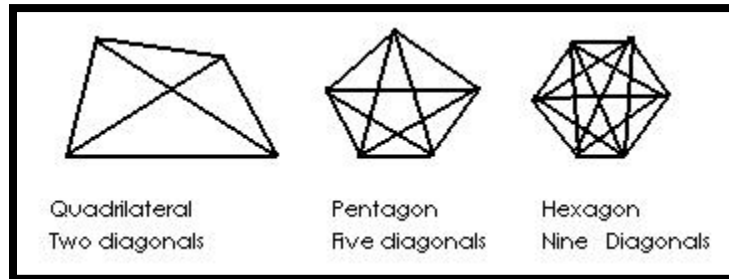
b) The number of diagonals of an *n*-sided polygon is given by $t_{n-3,1} + t_{n-2,2}$. For example, the number of diagonals for a hexagon is given by $t_{3,1} + t_{4,2} = 3 + 6 = 9$.

c) The number of diagonals in a heptagon is $t_{4,1} + t_{5,2} = 4 + 10 = 14$. The number of diagonals in an octagon is $t_{5,1} + t_{6,2} = 5 + 15 = 20$. The diagrams are shown at the right.

Heptagon   Octagon
14 diagonals   20 diagonals
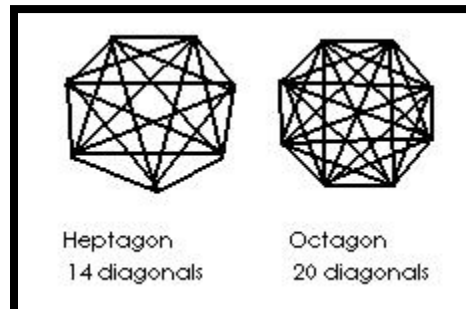
Section 4.4 **Page 252** **Question 10**

a) The rows 1, 3, 7, 15, etc. contain only odd numbers. These are related to the row number n as 2^{n-1}.

b) Since each row begins and ends with a 1, there are no rows with only even numbers.

c) Inspection of the first 12 rows of Pascal's triangle indicates more odd than even entries.

Section 4.4 **Page 252** **Question 12**

a) The sums of the squares of the first n integers are shown in the table at the right. The sum is equal to $t_{n+1,n-2} + t_{n+2,n-1}$. For example, when $n = 5$, the sum is 55. This is equivalent to

$t_{6,3} + t_{7,4} = 20 + 35$
$\phantom{t_{6,3} + t_{7,4}} = 55$

b) When $n = 10$, the sum is 385. This is equivalent to

$t_{11,8} + t_{12,9} = 165 + 220$
$\phantom{t_{11,8} + t_{12,9}} = 385$

Integer	Square	Sum
1	1	1
2	4	5
3	9	14
4	16	30
5	25	55
6	36	91
7	49	140
8	64	204
9	81	285
10	100	385

Section 4.4 **Page 252** **Question 14**

One way to use a spreadsheet to calculate Pascal's triangle is shown in the screen shot at the right. Enter a 1 into cells A1, A2, and B2. Enter the formula A2+B2 into cell B3. Copy the formula down the desired number of rows. Copy the formula right one cell. Copy this new formula down the required number of cells. Continue until you have constructed Pascal's triangle to the desired number of rows. Note: you may get error messages for referencing empty cells in your formulas. You may safely ignore these error messages.
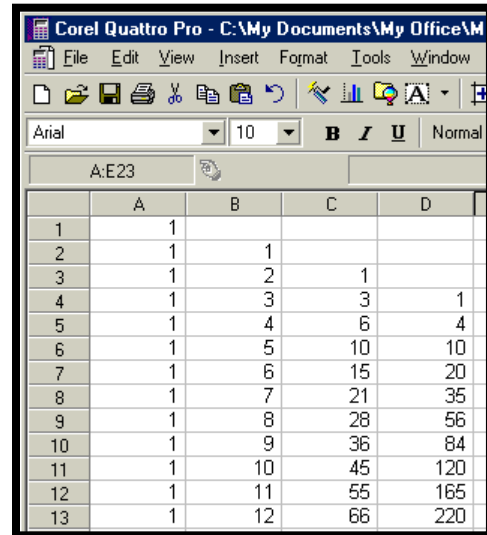
Section 4.4 **Page 253** **Question 16**

a) The number of toothpicks in the nth triangle is related to Pascal's triangle as $3t_{n+1,n-1}$. For example, if $n = 3$, then the number of toothpicks is $3t_{4,2} = 3 \times 6 = 18$.

b) The tenth triangle would contain $3t_{11,9} = 3 \times 55 = 165$ toothpicks.

a) and **b)** The first 20 rows of Pascal's triangle, as well as the entries which are divisible by 2, are shown in bold italics in the table below.

1																			
1	1																		
1	***2***	1																	
1	3	3	1																
1	***4***	***6***	***4***	1															
1	5	***10***	***10***	5	1														
1	***6***	15	***20***	15	***6***	1													
1	7	21	35	35	21	7	1												
1	***8***	***28***	***56***	***70***	***56***	***28***	***8***	1											
1	9	***36***	***84***	***126***	***126***	***84***	***36***	9	1										
1	***10***	45	***120***	***210***	***252***	***210***	***120***	45	***10***	1									
1	11	55	165	***330***	***462***	***462***	***330***	165	55	11	1								
1	***12***	***66***	***220***	495	***792***	***924***	***792***	495	***220***	***66***	***12***	1							
1	13	***78***	***286***	715	1287	***1716***	***1716***	1287	715	***286***	***78***	13	1						
1	***14***	91	***364***	1001	***2002***	3003	***3432***	3003	***2002***	1001	***364***	91	***14***	1					
1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1				
1	***16***	***120***	***560***	***1820***	***4368***	***8008***	***11440***	***12870***	***11440***	***8008***	***4368***	***1820***	***560***	***120***	***16***	1			
1	17	***136***	***680***	***2380***	***6188***	***12376***	***19448***	***24310***	***24310***	***19448***	***12376***	***6188***	***2380***	***680***	***136***	17	1		
1	***18***	153	***816***	***3060***	***8568***	***18564***	***31824***	***43758***	***48620***	***43758***	***31824***	***18564***	***8568***	***3060***	***816***	153	***18***	1	
1	19	171	969	***3876***	***11628***	***27132***	***50388***	***75582***	***92378***	***92378***	***75582***	***50388***	***27132***	***11628***	***3876***	969	171	19	1

c) Answers will vary.

d) Many answers are possible. The result for numbers divisible by 5 is shown in bold italics below.

1																			
1	1																		
1	2	1																	
1	3	3	1																
1	4	6	4	1															
1	***5***	***10***	***10***	***5***	1														
1	6	***15***	***20***	***15***	6	1													
1	7	21	***35***	***35***	21	7	1												
1	8	28	56	***70***	56	28	8	1											
1	9	36	84	126	126	84	36	9	1										
1	***10***	***45***	***120***	***210***	252	***210***	***120***	***45***	***10***	1									
1	11	***55***	***165***	***330***	462	462	***330***	***165***	***55***	11	1								
1	12	66	***220***	***495***	792	924	792	***495***	***220***	66	12	1							
1	13	78	286	***715***	1287	1716	1716	1287	***715***	286	78	13	1						
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1					
1	***15***	***105***	***455***	***1365***	3003	***5005***	***6435***	***6435***	***5005***	3003	***1365***	***455***	***105***	***15***	1				
1	16	***120***	***560***	***1820***	4368	8008	***11440***	***12870***	***11440***	8008	4368	***1820***	***560***	***120***	16	1			
1	17	136	***680***	***2380***	6188	12376	19448	***24310***	***24310***	19448	12376	6188	***2380***	***680***	136	17	1		
1	18	153	816	***3060***	8568	18564	31824	43758	***48620***	43758	31824	18564	8568	***3060***	816	153	18	1	
1	19	171	969	3876	11628	27132	50388	75582	92378	92378	75582	50388	27132	11628	3876	969	171	19	1

CHAPTER 4   Permutations and Organized Counting

Section 4.5   Applying Pascal's Method

Practise

Section 4.5   Page 256   Question 2

a) The different paths form the rows of Pascal's triangle. Since the final row is the 7th, the sum of the entries in this row is $2^7 = 128$.

b) This arrangement of letters generates the number pattern shown below.

						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		6		4		1		
	1		5		10		10		5		1	
1		6		15		20		15		6		1
	7		21		35		35		21		7	
		28		56		70		56		28		
			84		126		126		84			
				210		252		210				

The sum of the last row gives the number of paths as 672.

c) This arrangement of letters generates the number pattern shown at the right. The sum of the last row gives the number of paths as 80.

			1			
		1		1		
	1		2		1	
1		3		3		1
	4		6		4	
		10		10		
	10		20		10	
10		30		30		10

Section 4.5   Page 257   Question 4

a) The number of possible routes from A to B is 126, as shown in the table at the right.

1	1	1	1	1
1	2	3	4	5
1	3	6	10	15
1	4	10	20	35
1	5	15	35	70
1	6	21	56	126

b) The number of possible routes from A to B is 270, as shown in the table at the right.

1	1	1	1	1		
1	2	3	4	5		
1	3	6	10	15	15	15
1	4	10	20	35	50	65
				35	85	150
				35	120	270

c) The number of possible routes from A to B is 136, as shown in the table at the right.

1	1	1	1	1	1
1	2	3	4	1	2
1	3	6	10	11	13
1	4	10	20	31	44
1	5	5	25	56	100
1	6	11	36	36	136

There are 126 different routes to school, as shown in the table at the right. Therefore, Ryan cannot take a different route on each of the 194 school days.

1	1	1	1	1	1
1	2	3	4	5	6
1	3	6	10	15	21
1	4	10	20	35	56
1	5	15	35	70	126

Section 4.5   **Page 257**   **Question 8**

a) The routes across the checkerboard are shown in the diagram at the right. The total number of routes to the other side is given by $5 + 9 + 5 + 1 = 20$.

5		9		5		1
	5		4		1	
2		3		1		
	2		1			
1		1				
	1					
1						

   1   **2**   **3**   **4**

b) The routes beginning from various squares are shown in the three diagrams below.

Diagram 1:

9		19		15		6
	9		10		5		1
3		6		4		1
	3		3		1	
1		2		1		
	1		1			
	1					

   1   **2**   **3**   **4**

Diagram 2:

5		15		20		14
	5		10		10		4
1		4		6		4
	1		3		3		1
		1		2		1
			1		1	
			1			

   1   **2**   **3**   **4**

Diagram 3:

1		6		14		14
	1		5		9		5
		1		4		5
			1		3		2
				1		2
					1		1
					1	

   1   **2**   **3**   **4**

The number of routes beginning at position **2** is 49. The number of routes beginning at position **3** is 54. The number of routes beginning at position **4** is 35. Therefore, position **3** yields the greatest number of routes.

Section 4.5   **Page 258**   **Question 10**

The number of possible routes to Grandma's House is 25, as shown in the diagram at the right.

a) This new triangle is shown below.

						2						2
					2		2					4
				2		4		2				8
			2		6		6		2			16
		2		8		12		8		2		32
	2		10		20		20		10		2	64
2		12		30		40		30		12		2	128
2		14		42		70		70		42		14		2	256

b) Each term of this triangle is double the corresponding term in Pascal's triangle.

c) The sum of the terms in each row is shown to the right of the table. Each number is of the form 2^{n+1}, where n is the row number.

Solutions for Achievement Checks are described in the Teacher's Resource.

The possible routes across the chessboard are shown at the right, for a total of 269.

22	26	43	55	44	33	22	24	269
12	18	18	14	16	20	12	6	
4	4	8	11	9	3	4	3	
3	4	4	1	3	3	3		
1		2	2	2			1	
1	1	1			1			
			1					
	1							

The pyramid of buckets can be arranged such that the buckets on the bottom row fill evenly. One such arrangement is shown at the right.
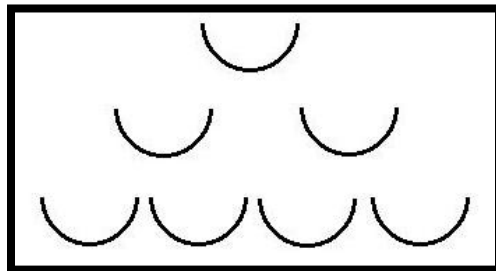
Answers may vary.

CHAPTER 4   Permutations and Organized Counting

Review of Key Concepts

Review of Key Concepts   Page 260   Question 2

There are eight ways to select the first play. For each of these there are seven ways to select the second, and six to select the third. The total number of choices is $8 \times 7 \times 6 = 336$.

Review of Key Concepts   Page 260   Question 4

There are two choices for the vehicle over the rear axle, the station wagon or the minivan. For each of these, there are four choices for the next vehicle, three for the next, then two, and finally one. The total number of choices is $2 \times 4 \times 3 \times 2 \times 1 = 48$.

Review of Key Concepts   Page 260   Question 6

There are five choices for the first song, four for the second, and three for the third. The total number of choices is $5 \times 4 \times 3 = 60$.

Review of Key Concepts   Page 260   Question 8

Since some of the digits are identical, the total number of choices is given by

$$\frac{10!}{4!\,3!\,3!} = 4200.$$

Review of Key Concepts   Page 260   Question 10

Since there are 16 tiles to select, the number of choices with no repetition would be 16!. However, since some of the tiles are identical, the number of choices is given by

$$\frac{16!}{8!\,4!\,4!} = 900\,900.$$

Review of Key Concepts   Page 260   Question 12

The sum of the entries in the seventh row of Pascal's triangle is given by $2^7 = 128$.

Review of Key Concepts   Page 260   Question 14

Pascal's method is an iterative process. Each term is defined in terms of the previous terms.

CHAPTER 4 Permutations and Organized Counting

Chapter Test

Chapter Test   **Page 261**   **Question 2**

a)
$$_{15}P_6 = \frac{15!}{9!}$$
$$= 3\,603\,600$$

b)
$$P(6, 2) = \frac{6!}{4!}$$
$$= 30$$

c)
$$_7P_3 = \frac{7!}{4!}$$
$$= 210$$

d)
$$_9P_9 = \frac{9!}{0!}$$
$$= 362\,880$$

e)
$$P(7, 0) = \frac{7!}{7!}$$
$$= 1$$

Chapter Test   **Page 261**   **Question 4**

a) Since no digit is repeated, the number of arrangements is $P(7, 4) = 840$.

b) Odd numbers must end in 1, 3, 5, and 7. Therefore, the number of arrangements is $4 \times 6 \times 5 \times 4 = 480$.

c) Even numbers must end in 2, 4, or 6. Therefore, the number of arrangements is $3 \times 6 \times 5 \times 4 = 360$.

Chapter Test   **Page 261**   **Question 6**

a) Since there are two as, the number of permutations is $\dfrac{4!}{2!} = 12$.

b) Since there are two as, two ms, two ns and two es, the number of permutations is

$$\frac{10!}{2!\,2!\,2!\,2!} = 226\,800.$$

c) Since no letters are repeated, the number of permutations is $9! = 362\,880$.

Chapter Test   **Page 261**   **Question 8**

Achievement Checks are described in the Teacher's Resource.

CHAPTER 5   Combinations and the Binomial Theorem

Review of Prerequisite Skills

a)
$$_5P_5 = \frac{5!}{(5-5)!}$$
$$= 120$$

b)
$$_{10}P_2 = \frac{10!}{(10-2)!}$$
$$= 90$$

c)
$$_{12}P_1 = \frac{12!}{(12-1)!}$$
$$= 12$$

d)
$$_7P_3 = \frac{7!}{(7-3)!}$$
$$= 210$$

Use the ***nPr*** function under the **MATH PRB** menu of your graphing calculator to evaluate the following. If you need instructions on using this function, refer to Appendix B of the student text.

a) $_{50}P_{25} = 1.96 \times 10^{39}$

b) $P(37, 16) = 2.69 \times 10^{23}$

c) $_{29}P_{29} = 8.84 \times 10^{30}$

d) $_{46}P_{23} = 2.13 \times 10^{35}$

a) Since there are no repeated letters, the number of arrangements is given by $_5P_3 = 60$.

b) Since there are two *m*s, two *a*s, two *n*s, and two *e*s, the number of arrangements is given by

$$\frac{10!}{2!\,2!\,2!\,2!} = 226\,800.$$

c) There are 12 choices for the first prize. For each of these, there are 11 choices for the second prize, and then 10 choices for the third prize. Hence, the total number of ways that the prizes could be awarded is given by $12 \times 11 \times 10 = 1320$.

a)
$$(x-5)^2 = x^2 - 5x - 5x + 25$$
$$= x^2 - 10x + 25$$

b)
$$(5x-y)^2 = 25x^2 - 5xy - 5xy + y^2$$
$$= 25x^2 - 10xy + y^2$$

c)

$$(x^2 + 5)^2 = x^4 + 5x^2 + 5x^2 + 25$$
$$= x^4 + 10x^2 + 25$$

d)

$$(x+3)(x-5)^2$$
$$= (x+3)(x^2 - 10x + 25)$$
$$= x^3 - 10x^2 + 25x + 3x^2 - 30x + 75$$
$$= x^3 - 7x^2 - 5x + 75$$

e)

$$(x^2 - y)^2 = x^4 - x^2 y - x^2 y + y^2$$
$$= x^4 - 2x^2 y + y^2$$

f)

$$(2x+3)^2 = 4x^2 + 6x + 6x + 9$$
$$= 4x^2 + 12x + 9$$

g)

$$(x-4)^2(x-2)$$
$$= (x-2)(x^2 - 8x + 16)$$
$$= x^3 - 8x^2 + 16x - 2x^2 + 16x - 32$$
$$= x^3 - 10x^2 + 32x - 32$$

h)

$$(2x^2 + 3y)^2$$
$$= 4x^4 + 6x^2 y + 6x^2 y + 9y^2$$
$$= 4x^4 + 12x^2 y + 9y^2$$

i)

$$(2x+1)^2(x-2)$$
$$= (x-2)(4x^2 + 4x + 1)$$
$$= 4x^3 + 4x^2 + x - 8x^2 - 8x - 2$$
$$= 4x^3 - 4x^2 - 7x - 2$$

j)

$$(x+y)(x-2y)^2$$
$$= (x+y)(x^2 - 4xy + 4y^2)$$
$$= x^3 - 4x^2 y + 4xy^2 + x^2 y - 4xy^2 + 4y^3$$
$$= x^3 - 3x^2 y + 4y^3$$

Review of Prerequisite Skills   **Page 264**   **Question 10**

a)

$$\sum_{n=2}^{5} 2n = 4 + 6 + 8 + 10$$
$$= 28$$

b)

$$\sum_{n=1}^{4} \frac{x^n}{n!} = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

c)

$$\sum_{n=1}^{5} (2^n + n^2) = 2^1 + 1^2 + 2^2 + 2^2 + 2^3 + 3^2 + 2^4 + 4^2 + 2^5 + 5^2$$
$$= 117$$

CHAPTER 5   Combinations and the Binomial Theorem

Section 5.1   Organized Counting With Venn Diagrams

Practise

Section 5.1   **Page 270**   **Question 2**

a) Add up the numbers in each segment, including the 5 who do not play any of these sports, to obtain the answer: $27 + 8 + 6 + 3 + 10 + 4 + 19 + 5 = 82$.

b) Add up the intersection between soccer and baseball to obtain the answer: $6 + 4 = 10$.

c) Add up those values outside the intersections to obtain the answer: $27 + 10 + 19 = 56$.

d) Add up the intersection between football and soccer to obtain the answer: $3 + 6 = 9$.

e) The intersection of all three circles indicates that 6 play all three sports.

f) Add up the numbers outside the soccer circle to obtain the answer: $27 + 8 + 10 + 5 = 50$.

Section 5.1   **Page 271**   **Question 4**

Model this data using a Venn diagram, as shown at the right.

a) The percents add up to 95%. Therefore, 95% of the viewers watch at least one of these programs.

b) The remaining 5% watch none of these news broadcasts.

c) The diagram shows that 50% of the viewers only watch the news at 23 00. Hence, 45% must watch the news only at 12 00 and 18 00.
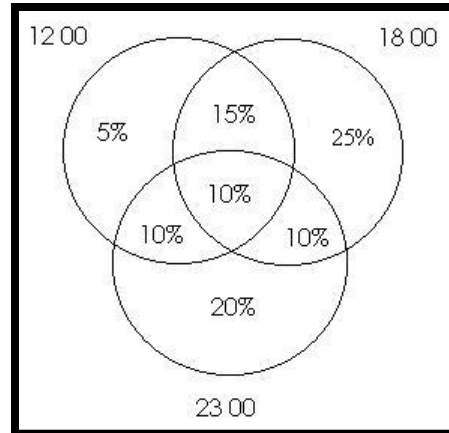
d) Add up the percents outside the intersections to obtain the percent of viewers who watch only one of these shows: $5\% + 25\% + 20\% = 50\%$.

e) Add up the percents in the intersections but not in the intersection of all three to obtain what percent view exactly two of these shows: $10\% + 10\% + 15\% = 35\%$.

Section 5.1   **Page 271**   **Question 6**

Model this data using a Venn diagram, as shown at the right.

Married males who are at least 35 years old fit in none of these categories. Since there are 900 employees, and 890 are accounted for in the Venn diagram, there must be 10 who are married males at least 35 years old.
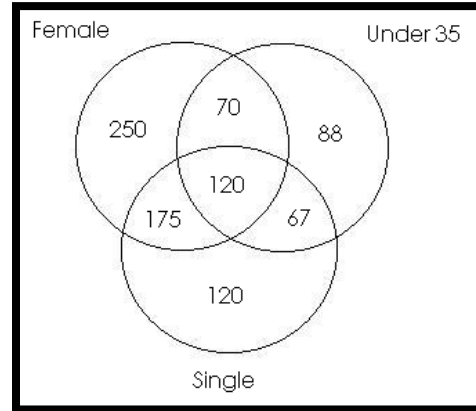

Section 5.1   **Page 272**   **Question 8**

a) Model this data using a Venn diagram, as shown at the right.

Let x represent the number in the intersection of all three sets. The numbers in other segments of the Venn diagram, in terms of x, are shown at the right. Since there are a total of 200 selections, all of the segments add up to 200. Hence,

$$35 + x + 65 + x + 6 + x + 45 - x + 20 - x + 24 - x + x = 200$$

$$x = 5$$
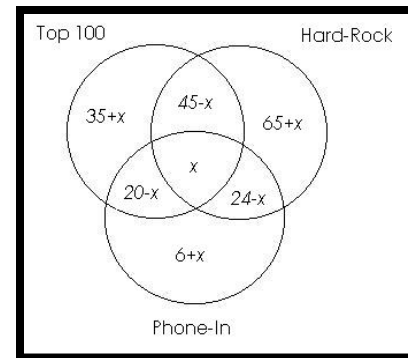
b) The number of songs in the top 100 which were neither hard-rock nor phone-in requests is given by $35 + x = 40$.

Section 5.1   **Page 272**   **Question 10**

a) Refer to the Venn diagram for three sets in question 8. To find the formula for three sets, consider that adding up the number in the three sets adds in all of the intersections twice. Hence, you must subtract the intersections. However, this subtracts the intersection of all three sets three times. Hence, you must add it back in again. Following this reasoning, the formula for three sets is:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

b) You can extend the formula to four sets by extending the reasoning from part a). Add the cardinality of the four sets, subtract each intersection of two sets, add each intersection of three sets, and subtract the intersection of all four sets, as shown below:

$$n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D) - n(A \cap B) - n(A \cap C) - n(A \cap D)$$
$$- n(B \cap C) - n(B \cap D) - n(C \cap D) + n(A \cap B \cap C) + n(A \cap B \cap D)$$
$$+ n(B \cap C \cap D) + n(A \cap C \cap D) - n(A \cap B \cap C \cap D)$$

c) The formula may be extended to any arbitrary number of sets following the same reasoning. For n sets, the formula can be written:

$$(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} A_i - \sum_{\substack{i,j \\ i<j}}^{n} n(A_i \cap A_{j)} + \sum_{\substack{i,j,k \\ i<j<k}}^{n} n(A_i \cap A_j \cap A_k) - \ldots$$

$$+ (-1)^{n+1} n(A_1 \cap A_2 \cap \ldots \cap A_n)$$

CHAPTER 5   Combinations and the Binomial Theorem

Section 5.2   Combinations

Practise

Section 5.2   **Page 279**   **Question 2**

a)

$$_{11}C_1 = \frac{11!}{10!\,1!} \qquad\qquad _{11}C_{10} = \frac{11!}{1!\,10!}$$
$$= 11 \qquad\qquad\qquad\qquad = 11$$

b)

$$_{11}C_2 = \frac{11!}{9!\,2!} \qquad\qquad _{11}C_9 = \frac{11!}{2!\,9!}$$
$$= \frac{11 \times 10}{2} \qquad\qquad = \frac{11 \times 10}{2}$$
$$= 55 \qquad\qquad\qquad = 55$$

c)

$$_{11}C_3 = \frac{11!}{8!\,3!} \qquad\qquad _{11}C_8 = \frac{11!}{3!\,8!}$$
$$= \frac{11 \times 10 \times 9}{6} \qquad\qquad = \frac{11 \times 10 \times 9}{6}$$
$$= 165 \qquad\qquad\qquad = 165$$

You may conclude that $_nC_r = {_n}C_{n-r}$.

Section 5.2   **Page 279**   **Question 4**

There are $_{52}C_4 = 270\ 725$ ways of choosing 4 cards from a deck of 52 if the order does not matter.

Section 5.2　　　　　　**Page 279**　　　**Question 6**

There are $_{22}C_6 = 74\,613$ ways of choosing 6 questions from a list of 22 questions.

Section 5.2　　　　　　**Page 279**　　　**Question 8**

a) There are $_{27}C_8 = 2\,220\,075$ ways of choosing 8 different posters from 27 posters.

b) Answers will vary.

Section 5.2　　　　　　**Page 280**　　　**Question 10**

This solution was prepared using Fathom™. If you need instructions on using Fathom™, refer to Appendix B of the student text.

Start Fathom™ and drag a case table to the work area. Create the attributes **Cards** and **Combinations**. Enter the choices from 0 to 52 for the number of cards that Fritz may choose. Edit the formula for the **Combinations** attribute to **combinations**(52,**Cards**), using the **combinations()** function under the **Functions**/**Arithmetic** menu. Drag a graph to the work area. Drag the **Cards** attribute to the horizontal axis and the **Combinations** attribute to the vertical axis. Your final screen will look like the one shown below.
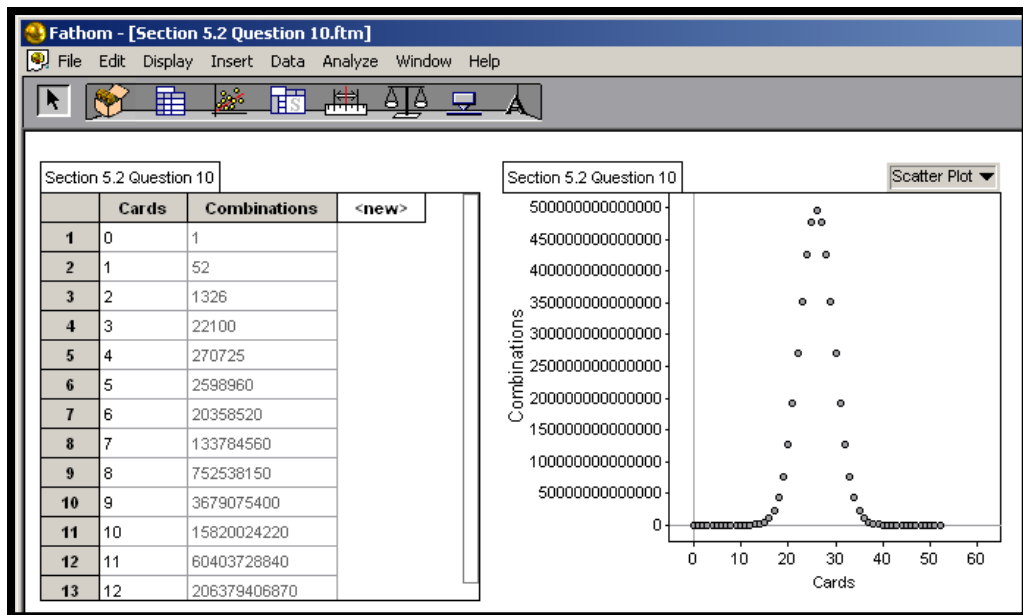
Section 5.2　　　　　　**Page 280**　　　**Question 12**

There are $_{10}C_6 = 210$ ways of selecting the men, and $_{15}C_6 = 5005$ ways of choosing the women. Hence, there are $210 \times 5005 = 1\,051\,050$ ways of selecting the jury.

Section 5.2   **Page 280**   **Question 14**

a) There are $_{20}C_{12} = 125\,970$ ways to select 12 songs from the top 20.

b) There are $_{10}C_8 = 45$ ways to select 8 songs from the top 10. There are $_{10}C_4 = 210$ ways of selecting 4 songs from songs 11 through 20. Hence, there are $45 \times 210 = 9450$ ways of selecting the 12 songs.

c) There are $_{20}C_{12} = 125\,970$ ways to choose 12 songs and $_{20}C_{13} = 77\,520$ ways to choose 13 songs. Hence, there are $125\,970 + 77\,520 = 203\,490$ ways of selecting either 12 or 13 songs for the first hour of Jeffrey's shift.

d) The following solution was prepared using a spreadsheet. If you need instructions on using spreadsheets, refer to Appendix B of the student text.

Enter the number of songs desired in the first column. In the second column, enter a formula for calculating combinations. This will be COMBIN(20,A3) for Microsoft® Excel and @COMB(A3,20) for Corel® Quattro® Pro. Copy the formula down three more cells. To find the total, select cell B8. Use the formula SUM(B3:B6) for Microsoft® Excel and @SUM(B3..B6) for Corel® Quattro® Pro.

	A	B
1	Songs	Choices
2		
3	12	125970
4	13	77520
5	14	38760
6	15	15504
7		
8	Total	257754

(Microsoft Excel - Section, File Edit View Insert, A10)

Section 5.2   **Page 280**   **Question 16**

a) There are $_{11}C_4 = 330$ choices for the first trip.

b) Since there are now only 7 students left to transport, there are $_7C_4 = 35$ choices for the second trip.

Section 5.2   **Page 281**   **Question 18**

There are $_{20}C_2 = 190$ ways of selecting two people to shake hands.

Section 5.2   **Page 281**   **Question 20**

a) The number of possible bridge hands is $_{52}C_{13} = \dfrac{52!}{39!\,13!}$. The order in which the cards are received is not important. The number of ways that a bridge hand can be dealt is $_{52}P_{13} = \dfrac{52!}{39!}$. The order in which the cards are received is important. Hence, there are 13! more ways of dealing a bridge hand than there are possible bridge hands.

b) The number of possible bridge hands with exactly five clubs, two spades, three diamonds, and three hearts is given by $_{13}C_5 \times _{13}C_2 \times _{13}C_3 \times _{13}C_3$.

c) The number of possible bridge hands with exactly five hearts is given by $_{13}C_5 \times _{39}C_8$.

d) Use the ***nCr*** function under the **MATH PRB** menu.

The expression in part b) evaluates to 8 211 173 256.

The expression in part c) evaluates to 79 181 063 676.

Section 5.2   **Page 281**   **Question 22**

a) If they may be any type of ball, there are $_{11}C_4 = 330$ ways of selecting four balls.

b) If there must be two volleyballs and two basketballs, there are $_5C_2 \times _6C_2 = 150$ ways of selecting four balls.

c) If all four must be volleyballs, there are $_5C_4 = 5$ ways of selecting four balls.

d) If none may be volleyballs, there are $_6C_4 = 15$ ways of selecting four balls.

CHAPTER 5   Combinations and the Binomial Theorem

Section 5.3   **Problem Solving With Combinations**

Practise

Section 5.3   **Page 286**   **Question 2**

Since there is one $5 bill, you may either include it or not include it in the sum, giving you two choices. For the two $10 bills, you may include both, just one, or none, giving you three choices. Similarly, there are two choices for the $50 bill. This gives you $2 \times 3 \times 2 = 12$ choices. If you wish to exclude $0 as a possible sum, then there are only 11 choices.

Section 5.3   **Page 286**   **Question 4**

Each member on the committee may be included or excluded from the subcommittee, giving you two choices for each member. Since you must have at least one member on the subcommittee, there are $2^8 - 1 = 255$ ways of forming the subcommittee.

Section 5.3   **Page 287**   **Question 6**

a) If there are no restrictions, there are $_{15}C_5 = 3003$ ways of selecting the five people who will attend the trade show.

b) There are $_7C_5 = 21$ ways of sending all managers and $_8C_5 = 56$ ways of sending all sales representatives. If at least one of each is required, there are $3003 - 21 - 56 = 2926$ ways of selecting the five people who will attend the trade show.

Section 5.3   **Page 287**   **Question 8**

a) There are $_{30}C_6 = 573\ 775$ different ways of selecting the team.

b) If Pierre, Gregory, and Miguel must be on the team, there are $_{27}C_3 = 2925$ ways of selecting the rest of the team.

c) The number of teams without these three is given by $573\ 775 - 2925 = 590\ 850$.

Section 5.3   **Page 287**   **Question 10**

a) If a 4-card hand must have at least one card from each suit, there are 13 choices for each of the cards. Therefore, there are $13^4 = 28\ 561$ ways of doing this.

b) The number of possible 5-card hands with no restrictions is $_{52}C_5 = 2\ 598\ 960$. The number of 5-card hands with no spades is $_{39}C_5 = 575\ 757$. Therefore, the number of 5-card hands with at least one spade is $2\ 598\ 960 - 575\ 757 = 2\ 023\ 203$.

c) Since there are 12 face cards, the number of possible 5-card hands with no face cards is $_{40}C_5 = 658\ 008$. The number of hands with exactly one face card is $_{12}C_1 \times _{40}C_4 = 1\ 096\ 680$. Therefore, the number of hands with at least two face cards is

$2\ 598\ 960 - 1\ 096\ 680 - 658\ 008 = 844\ 272$.

Section 5.3   **Page 287**   **Question 12**

The number of different combinations is $7 \times 4 \times 9 = 252$.

The flow chart is shown below.

Section 5.3   **Page 287**   **Question 16**
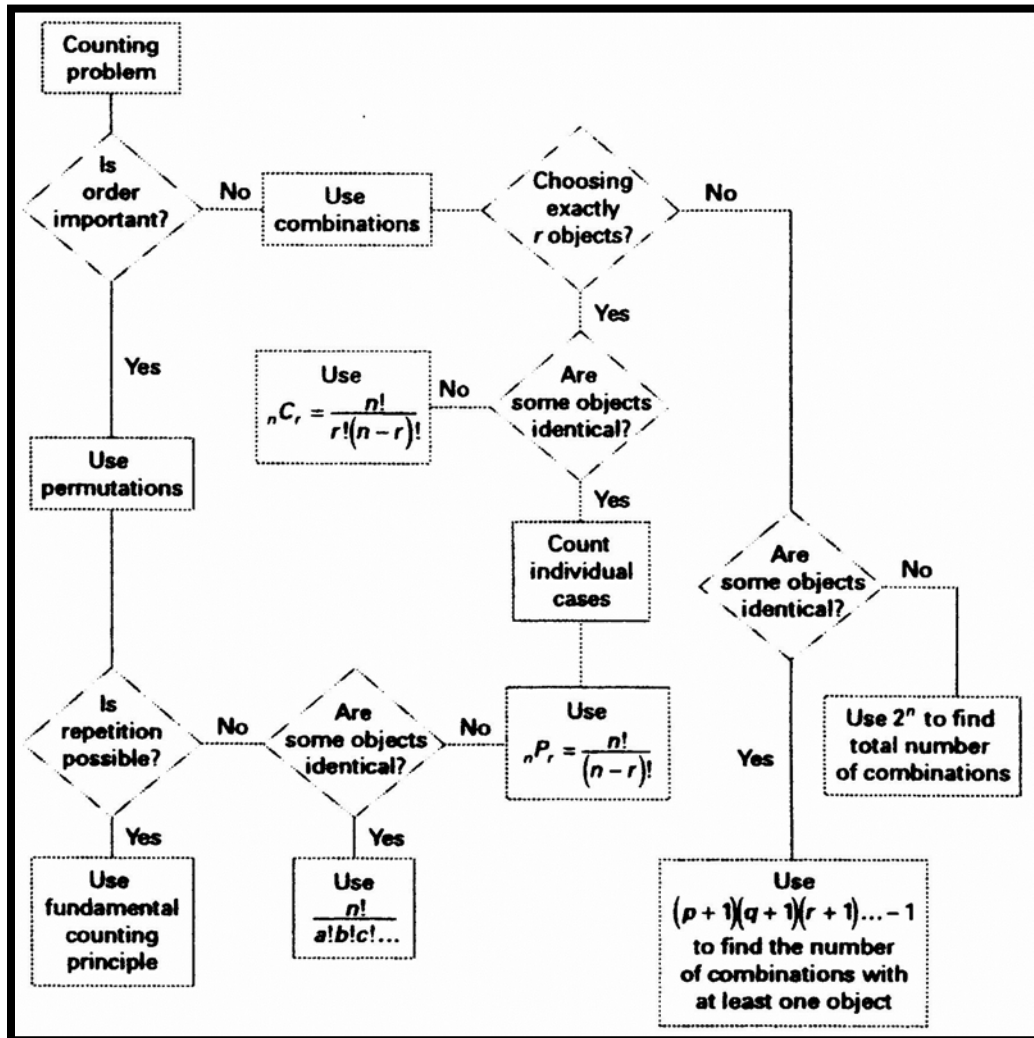
a) i) A diagonal is made by joining two vertices. A pentagon has five vertices. Therefore, there are $_5C_2 = 10$ ways of selecting vertices. However, this includes the outside edges, of which there are 5. Hence, the number of diagonals is $10 - 5 = 5$.

ii) Using reasoning as in part i), the number of diagonals on a hexagon is $_6C_2 - 6 = 9$.

b) The sketches that verify part a) are shown at the right.

Pentagon
Five diagonals

Hexagon
Nine Diagonals
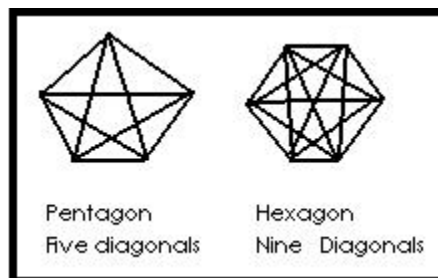
Section 5.3   **Page 288**   **Question 18**

The number of ways with no restrictions is $_{12}C_2 \times _{10}C_4 \times _6C_6 = 13\,860$. The number of ways with Joe and Jim at the door is $_2C_2 \times _{10}C_4 \times _6C_6 = 210$. The number of ways with Joe and Jim on the floor is $_{10}C_2 \times _8C_2 \times _6C_6 = 1260$. The number of ways with Joe and Jim as floaters is $_{10}C_2 \times _8C_4 \times _4C_4 = 3150$. Therefore, the number of ways with Joe and Jim separate is $13\,860 - 210 - 1260 - 3150 = 9240$.

Section 5.3   **Page 288**   **Question 20**

Solutions for Achievement Checks are described in the Teacher's Resource.

Section 5.3   **Page 288**   **Question 22**

The word *parallelogram* contains three *a*s, three *l*s, two *r*s, and five single letters. Consider the following cases.

Case 1: Three letters the same, one different. There are two ways to select the three letters. For each of these, there are seven ways to select the remaining letter, giving a total of 14 ways.

Case 2: Two pairs of two letters the same. There are $_3C_2 = 3$ ways to select two pairs of two letters the same.

Case 3: One pair of letters the same, two different. There are three ways to select the pair of letters the same. For each of these, there are $_7C_2 = 21$ ways to select the two remaining letters, giving a total of 63.

Case 4: All four letters different. There are $_8C_4 = 70$ ways to select four different letters.

All of these add up to 150.

Section 5.3   **Page 288**   **Question 24**

Two of the $5 bills will add up to $10. An equivalent system is one $2, one $5, three $10, and one $20. Note: this only works if at least one $5 is still left. Otherwise, you are eliminating some possible sums. Since there are now three $10 bills, we can eliminate two of them, and add another $20 bill, giving an equivalent system of one $2, one $5, one $10, and two $20. The first three can be treated in two ways each, either including or excluding them from the amount. The $20 bills can be treated in three ways: no $20, one $20, or two $20. Therefore, the total number of sums possible is $2 \times 2 \times 2 \times 3 = 24$. If you do not want to include $0 as a possible sum, then you must subtract 1 to give an answer of 23.

CHAPTER 5   Combinations and the Binomial Theorem

Section 5.4   The Binomial Theorem

Practise

Section 5.4　　　　　　**Page 293**　　　**Question 2**

The sum of the exponents in each case must be 10.

a) $k = 4$　　　　**b)** $k = 2$　　　**c)** $k = 5$

Section 5.4　　　　　　**Page 293**　　　**Question 4**

The exponent on the variable a determines the value of r.

a) $_{11}C_2 = 55$　　　　　**b)** $_{11}C_{11} = 1$　　　　　**c)** $_{11}C_6 = 462$

Section 5.4　　　　　　**Page 294**　　　**Question 6**

The sum of the terms in the nth row of Pascal's triangle is given by 2^n. If the sum is 16 384, then you can use a graphing calculator to show that $n = 14$.

Section 5.4　　　　　　**Page 294**　　　**Question 8**

Since the lowest value of r is 0 and the highest is 8, there will be 9 terms in the expansion of $(x^2 + x)^8$.

Section 5.4　　　　　　**Page 294**　　　**Question 10**

a)

$$(3x + y)^{10} = {}_{10}C_0 (3x)^{10} + {}_{10}C_1 (3x)^9 y + {}_{10}C_2 (3x)^8 y^2 + {}_{10}C_3 (3x)^7 y^3 + {}_{10}C_4 (3x)^6 y^4 + \ldots$$
$$= 59\ 049x^{10} + 196\ 830x^9 y + 295\ 245x^8 y^2 + 262\ 440x^7 y^3 + 153\ 090x^6 y^4 + \ldots$$

b) The terms in $(3x - y)^{10}$ will be the same, except that they will alternate from positive to negative, due to the coefficient of -1 in front of the y.

$$(3x - y)^{10} = 59\ 049x^{10} - 196\ 830x^9 y + 295\ 245x^8 y^2 - 262\ 440x^7 y^3 + 153\ 090x^6 y^4 + \ldots$$

c) See part b).

Section 5.4　　　　　　**Page 294**　　　**Question 12**

a) The number of terms is 7. Therefore, the value of n is 6. The coefficients are the entries in the 6th row of Pascal's triangle. Therefore, the required form is $(x + y)^6$.

b) The number of terms is 5, and the exponent of y is decreasing by 3 on each successive term. Therefore, the value of n is 4. The last term is 16, which is 2^4. Therefore, the required form is $(y^3 + 2)^4$.

c) The number of terms is 6. Therefore, the value of n is 5. The first coefficient is 243, which is 3^5. The last coefficient is -1, and the terms alternate from positive to negative. Therefore, the required form is $(3a - b)^5$.

Section 5.4   **Page 294**   **Question 14**

a)

$$\left(x + \frac{2}{x}\right)^4 = {_4C_0}x^4 + {_4C_1}x^3\left(\frac{2}{x}\right) + {_4C_2}x^2\left(\frac{2}{x}\right)^2 + {_4C_3}x\left(\frac{2}{x}\right)^3 + {_4C_4}\left(\frac{2}{x}\right)^4$$
$$= x^4 + 8x^2 + 24 + 32x^{-2} + 16x^{-4}$$

$$\frac{1}{x^4}\left(x^2 + 2\right)^4 = \frac{1}{x^4}\left({_4C_0}x^8 + {_4C_1}x^6(2) + {_4C_2}x^4(2)^2 + {_4C_3}x^2(2)^3 + {_4C_4}(2)^4\right)$$
$$= x^4 + 8x^2 + 24 + 32x^{-2} + 16x^{-4}$$

b) The results are the same. In the first expression, a factor of $\frac{1}{x^4}$ may be taken out of the brackets, resulting in the second expression.

Section 5.4   **Page 295**   **Question 16**

a) A spreadsheet solution for this problem is shown at the right. Use the formula COMBIN(9,A3)*0.2^A3 in the **Value** column for Microsoft® Excel or @COMB(A3,9)*0.2^A3 for Corel® Quattro® Pro. Use the formula B3+C2 for the **Sum** column.

b) Two decimal place accuracy is achieved after 7 terms have been added.

	A	B	C
	Microsoft Excel - Section 5.4 Exerci		
	File Edit View Insert Format Tools		
	F14	=	
1	Term	Value	Sum
2			
3	0	1	1
4	1	1.8	2.8
5	2	1.44	4.24
6	3	0.672	4.912
7	4	0.2016	5.1136
8	5	0.04032	5.15392
9	6	0.005376	5.159296
10	7	0.000461	5.159757
11	8	2.3E-05	5.15978
12	9	5.12E-07	5.15978

Section 5.4   **Page 295**   **Question 18**

Solutions for Achievement Checks are described in the Teacher's Resource.

Section 5.4   **Page 295**   **Question 20**

a)

$$\left[x + (y + z)\right]^2 = x^2 + 2x(y + z) + (y + z)^2$$
$$= x^2 + 2xy + 2xz + y^2 + 2yz + z^2$$

b)

$$\left[x+(y+z)\right]^3 = x^3 + 3x^2(y+z) + 3x(y+z)^2 + (y+z)^3$$
$$= x^3 + 3x^2y + 3x^2z + 3xy^2 + 6xyz + 3xz^2 + y^3 + 3y^2z + 3yz^2 + z^3$$

c) Prediction:

$$\left[x+y+z\right]^4 = x^4 + 4x^3y + 4x^3z + 6x^2y^2 + 12x^2yz + 6x^2z^2 + 4xy^3 + 12xy^2z + 12xyz^2$$
$$+ 4xz^3 + y^4 + 4y^3z + 6y^2z^2 + 4yz^3 + z^4$$

Calculation:

$$\left[x+(y+z)\right]^4 = x^4 + 4x^3(y+z) + 6x^2(y+z)^2 + 4x(y+z)^3 + (y+z)^4$$
$$= x^4 + 4x^3y + 4x^3z + 6x^2y^2 + 12x^2yz + 6x^2z^2 + 4xy^3 + 12xy^2z + 12xyz^2$$
$$+ 4xz^3 + y^4 + 4y^3z + 6y^2z^2 + 4yz^3 + z^4$$

The prediction and the calculation give the same result.

d) Each term in the expansion of $(x + y + z)^n$ will be of the form $x^i y^j z^k$ such that $i + j + k = n$. Hence, you must choose i xs, j ys, and k zs. The number of ways that this can be done is given by

$$_nC_i \times {}_{(n-i)}C_j \times {}_{(n-i-j)}C_k = \frac{n!}{(n-i)!\,i!} \times \frac{(n-i)!}{(n-i-j)!\,j!} \times \frac{(n-i-j)!}{(n-i-j-k)!\,k!}$$
$$= \frac{n!}{i!\,j!\,k!}$$

Add up all of these terms to obtain a general formula as shown below:

$$(x+y+z)^n = \sum_{\substack{i,j,k=0 \\ i+j+k=n}} \frac{n!}{i!\,j!\,k!} x^i y^j z^k$$

e)

$$[x + y + z]^5 = \frac{5!}{5!\,0!\,0!}x^5 y^0 z^0 + \frac{5!}{4!\,1!\,0!}x^4 y^1 z^0 + \frac{5!}{4!\,0!\,1!}x^4 y^0 z^1 + \frac{5!}{3!\,2!\,0!}x^3 y^2 z^0 +$$

$$\frac{5!}{3!\,1!\,1!}x^3 y^1 z^1 + \frac{5!}{3!\,0!\,2!}x^3 y^0 z^2 + \frac{5!}{2!\,3!\,0!}x^2 y^3 z^0 + \frac{5!}{2!\,2!\,1!}x^2 y^2 z^1 +$$

$$\frac{5!}{2!\,1!\,2!}x^2 y^1 z^2 + \frac{5!}{2!\,0!\,3!}x^2 y^0 z^3 + \frac{5!}{1!\,4!\,0!}x^1 y^4 z^0 + \frac{5!}{1!\,3!\,1!}x^1 y^3 z^1 +$$

$$\frac{5!}{1!\,2!\,2!}x^1 y^2 z^2 + \frac{5!}{1!\,1!\,3!}x^1 y^1 z^3 + \frac{5!}{1!\,0!\,4!}x^1 y^0 z^4 + \frac{5!}{0!\,5!\,0!}x^0 y^5 z^0 +$$

$$\frac{5!}{0!\,4!\,1!}x^0 y^4 z^1 + \frac{5!}{0!\,3!\,2!}x^0 y^3 z^2 + \frac{5!}{0!\,2!\,3!}x^0 y^2 z^3 + \frac{5!}{0!\,1!\,4!}x^0 y^1 z^4 +$$

$$\frac{5!}{0!\,0!\,5!}x^0 y^0 z^5$$

$$= x^5 + 5x^4 y + 5x^4 z + 10x^3 y^2 + 20x^3 yz + 10x^3 z^2 + 10x^2 y^3 + 30x^2 y^2 z + 30x^2 yz^2$$

$$+ 10x^2 z^3 + 5xy^4 + 20xy^3 z + 30xy^2 z^2 + 20xyz^3 + 5xz^4 + y^5 + 5y^4 z + 10y^3 z^2$$

$$+ 10y^2 z^3 + 5yz^4 + z^5$$

Section 5.4   **Page 295**   **Question 22**

a) Since there are two choices for each symbol, there are $2^5 = 32$ possible letters.

b) If exactly two dots are required, there will be $_5C_2 = 10$ possible letters.

c) There is only one code that will contain no dashes. Therefore, there will be 31 codes with at least one dash.

CHAPTER 5   Combinations and the Binomial Theorem

Review of Key Concepts

Review of Key Concepts   **Page 296**   **Question 2**

a) The number of elements in either of two sets A and B is given by:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

b) $n(A) + n(B)$ counts the elements in the intersection of the sets twice. Therefore, it is necessary to subtract $n(A \cap B)$.

c) Answers will vary. One example follows.

$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5, 6, 7\}$

$$n(A \cup B) = 5 + 4 - 2$$
$$= 7$$

Review of Key Concepts   Page 296   Question 4

a) $_{41}C_8 = 95\,548\,245$

b) $_{33}C_{15} = 1\,037\,158\,320$

c) $_{25}C_{17} = 1\,081\,575$

d) $_{50}C_{10} = 10\,272\,278\,170$

e) $_{10}C_8 = 45$

f) $_{15}C_{13} = 105$

g) $_5C_4 = 5$

h) $_{25}C_{24} = 25$

i) $_{15}C_{11} = 1365$

j) $_{25}C_{20} = 53\,130$

k) $_{16}C_8 = 12\,870$

l) $_{30}C_{26} = 27\,405$

Combinations with the form $_nC_{n-1}$ will give a result of n, as in part g) and part h) above.

Combinations with the form $_nC_{n-2}$ will give a result of $\dfrac{n(n-1)}{2}$, as in part e) and part f) above.

Review of Key Concepts   Page 296   Question 6

Select three of the four suits. Since you want a different number of cards from each suit, order is important. Therefore, there are $_4P_3 = 24$ ways of selecting three of the four suits. For each of these, there are $_{13}C_5 = 1287$ ways of selecting five cards from the first suit, $_{13}C_6 = 1716$ ways of selecting six cards from the second suit, and $_{13}C_2 = 78$ ways of selecting two cards from the third suit. Therefore, the total number of possible combinations is

$24 \times 1287 \times 1716 \times 78 = 4\ 134\ 297\ 024.$

Review of Key Concepts   Page 296   Question 8

There are 9 choices of toppings. Each one may be treated in 2 ways: take it or leave it. Therefore, there are $2^9 = 512$ possible "dressings", including no topping at all.

Review of Key Concepts   Page 297   Question 10

Since a student may choose either 4 or 5 from the first five questions, you must consider two cases.

Case 1: Select 4 from the first 5.

There are $_5C_4 = 5$ ways of selecting 4 questions from the first 5. For each of these, there are $_7C_4 = 35$ ways of selecting 4 more questions from the remaining 7. Therefore, the number of combinations is $5 \times 35 = 175$.

Case 2: Select 5 from the first 5.

There is one way of selecting all 5 questions from the first 5. There are $_7C_3 = 35$ ways of selecting 3 more questions from the remaining 7. Therefore, there are 35 choices for this case.

Hence, the total number of combinations is $175 + 35 = 210$.

Review of Key Concepts   Page 297   Question 12

Each person will receive five books. The total number of combinations is given by:

$$_{15}C_5 \times_{10} C_5 \times_5 C_5 = 3003 \times 252 \times 1$$
$$= 756\ 756$$

Review of Key Concepts   Page 297   Question 14

a) Since $n = 5$, there are 6 terms in the expansion.

b) The sum of the exponents must be 5. Therefore, $k = 3$.

Review of Key Concepts   Page 297   Question 16

a)
$$(x + y)^6 = {_6C_0}\, x^6 + {_6C_1}\, x^5 y + {_6C_2}\, x^4 y^2 + {_6C_3}\, x^3 y^3 + {_6C_4}\, x^2 y^4 + {_6C_5}\, xy^5 + {_6C_6}\, y^6$$
$$= x^6 + 6x^5 y + 15x^4 y^2 + 20x^3 y^3 + 15x^2 y^4 + 6xy^5 + y^6$$

b)
$$(6x - 5y)^4 = {_4C_0}\,(6x)^4 + {_4C_1}\,(6x)^3(-5y) + {_4C_2}\,(6x)^2(-5y)^2 + {_4C_3}\,(6x)(-5y)^3 + {_4C_4}\,(-5y)^4$$
$$= 1296x^4 - 4320x^3 y + 5400x^2 y^2 - 3000xy^3 + 625y^4$$

c)
$$(5x + 2y)^5 = {_5C_0}\,(5x)^5 + {_5C_1}\,(5x)^4(2y) + {_5C_2}\,(5x)^3(2y)^2 + {_5C_3}\,(5x)^2(2y)^3 + {_5C_4}\,(5x)(2y)^4 + {_5C_5}\,(2y)^5$$
$$= 3125x^5 + 6250x^4 y + 5000x^3 y^2 + 2000x^2 y^3 + 400xy^4 + 32y^5$$

d)

$$(3x-2)^6 = {}_6C_0\,(3x)^6 + {}_6C_1\,(3x)^5(-2) + {}_6C_2\,(3x)^4(-2)^2 + {}_6C_3\,(3x)^3(-2)^3 + {}_6C_4\,(3x)^2(-2)^4$$
$$+ {}_6C_5\,(3x)(-2)^5 + {}_6C_6\,(-2)^6$$
$$= 729x^6 - 2916x^5 + 4860x^4 - 4320x^3 + 2160x^2 - 576x + 64$$

Review of Key Concepts   Page 297   Question 18

There are 7 terms in the expansion of $(2x-3y)^6$. The $(r+1)$st term is given by:

$$t_{r+1} = {}_6C_r(2x)^{n-r}(-3y)^r$$

Evaluate each term from $r = 0$ to $r = 6$, and add them, as shown below:

$$(2x-3y)^6 = {}_6C_0\,(2x)^6 + {}_6C_1\,(2x)^5(-3y) + {}_6C_2\,(2x)^4(-3y)^2 + {}_6C_3\,(2x)^3(-3y)^3 + {}_6C_4\,(2x)^2(-3y)^4$$
$$+ {}_6C_5\,(2x)(-3y)^5 + {}_6C_6\,(-2y)^6$$
$$= 64x^6 - 576x^5y + 2160x^4y^2 - 4320x^3y^3 + 4860x^2y^4 - 2916xy^5 + 729y^6$$

Review of Key Concepts   Page 297   Question 20

Since $n = 8$, there are nine terms. The middle term is the fifth term, where $r = 4$.

$$t_5 = {}_8C_4(\sqrt{x})^4\left(\frac{5}{\sqrt{x}}\right)^4$$
$$= 70 \times x^2 \times \frac{625}{x^2}$$
$$= 43\ 750$$

Review of Key Concepts   Page 297   Question 22

$$(y^2-2)^6(y^2+2)^6 = (y^4-4)^6$$
$$= {}_6C_0\,y^{24} + {}_6C_1\,y^{20}(-4) + {}_6C_2\,y^{16}(-4)^2 + {}_6C_3\,y^{12}(-4)^3 + {}_6C_4\,y^8(-4)^4$$
$$+ {}_6C_5\,y^4(-4)^5 + {}_6C_6\,(-4)^6$$
$$= y^{24} - 24y^{20} + 240y^{16} - 1280y^{12} + 3480y^8 - 6144y^4 + 4096$$

CHAPTER 5   Combinations and the Binomial Theorem

Chapter Test

Use Pascal's formula to deduce the answers.

a) $_{10}C_7 + {}_{10}C_8 = {}_{11}C_8$

b) $_{23}C_{15} - {}_{22}C_{14} = {}_{22}C_{15}$

a)
$$(8x-3)^5 = {}_5C_0 (8x)^5 + {}_5C_1 (8x)^4(-3) + {}_5C_2 (8x)^3(-3)^2 + {}_5C_3 (8x)^2(-3)^3 + {}_5C_4 (8x)(-3)^4 + {}_5C_5 (-3)^5$$
$$= 32\,768x^5 - 61\,440x^4 + 46\,080x^3 - 17\,280x^2 + 3240x - 243$$

b)
$$(2x-5y)^6 = {}_6C_0 (2x)^6 + {}_6C_1 (2x)^5(-5y) + {}_6C_2 (2x)^4(-5y)^2 + {}_6C_3 (2x)^3(-5y)^3 + {}_6C_4 (2x)^2(-5y)^4$$
$$+ {}_6C_5 (2x)(-5y)^5 + {}_6C_6 (-5y)^6$$
$$= 64x^6 - 960x^5 y + 6000x^4 y^2 - 20\,000x^3 y^3 + 37\,500x^2 y^4 - 37\,500xy^5 + 15\,625y^6$$

a) Since you want 3 members chosen from 20, the number of ways is $_{20}C_3 = 1140$.

b) Since order is now important, the number of ways of choosing a starter, a marshal, and a timer is $_{20}P_3 = 6840$.

c) The answers to parts a) and b) should not be the same. In part a), order is not important. In part b), order is important.

a) The number of arrangements of four wins and three losses is given by $_7C_4 \times {}_3C_3 = 35$.

b) There is only one way of having two wins not in a row: WLWLWLW. Hence, there are 34 ways of having at least two wins in a row.

Chapter Test   **Page 299**   **Question 10**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots$$

$$nx = -0.9 \qquad\qquad 1)$$

$$\frac{n(n-1)}{2}x^2 = 0.36 \quad 2)$$

Solving equation 1) for x and substituting into equation 2) gives you:

$$\frac{n(n-1)}{2} \times \frac{0.81}{n^2} = 0.36$$
$$0.81n - 0.81 = 0.72n$$
$$0.09n = 0.81$$
$$n = 9$$

Therefore,

$$x = \frac{-0.9}{9}$$
$$= -0.1$$

Chapter Test   **Page 299**   **Question 12**

a) Since the occupant of a given seat is not important, there is $_8C_8 = 1$ way of filling the 8 window seats. For each of these, there are $_{12}C_2 = 66$ ways of filling 2 of the 12 aisle seats. Therefore, there are $1 \times 66 = 66$ ways of seating the 10 passengers.

b) Since order is now important, there are $_{10}P_8 = 1\,814\,400$ ways of arranging the 10 passengers in the window seats. For each of these, there are $_{12}P_2 = 132$ ways of arranging the remaining 2 passengers in the aisle seats. Therefore, there are $1\,814\,400 \times 132 = 239\,500\,800$ ways of arranging the passengers.

CHAPTER 6   Introduction to Probability

Review of Prerequisite Skills

Review of Prerequisite Skills   **Page 302**   **Question 2**

To express a percent as a decimal, divide by 100. This is equivalent to moving the decimal point two places to the left.

a) 15% = 0.15

b) 3% = 0.03

c) 85% = 0.85

d) 6.5% = 0.065

e) 26.5% = 0.265

f) 75.2% = 0.752

Review of Prerequisite Skills   **Page 302**   **Question 4**

To change a fraction to a percent, first change it to a decimal by dividing the numerator by the denominator. Then, multiply by 100%, which is equivalent to moving the decimal point two places to the right.

a)
$$\frac{1}{4} = 0.25$$
$$= 25\%$$

b)
$$\frac{13}{15} = 0.867$$
$$= 86.7\%$$

c)
$$\frac{11}{14} = 0.786$$
$$= 78.6\%$$

d)
$$\frac{7}{10} = 0.70$$
$$= 70\%$$

e)
$$\frac{4}{9} = 0.444$$
$$= 44.4\%$$

f)
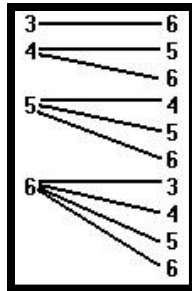$$\frac{13}{20} = 0.65$$
$$= 65\%$$
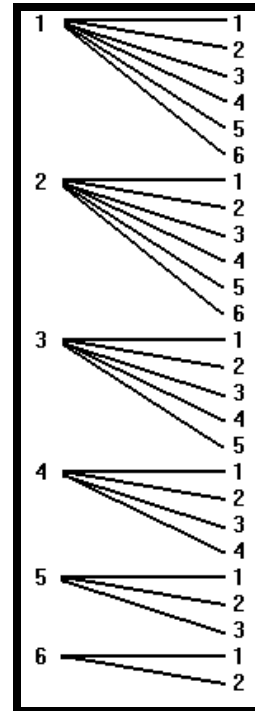
Review of Prerequisite Skills   **Page 302**   **Question 6**

The required tree diagrams for the game of backgammon are shown below and to the right.

a)

b)

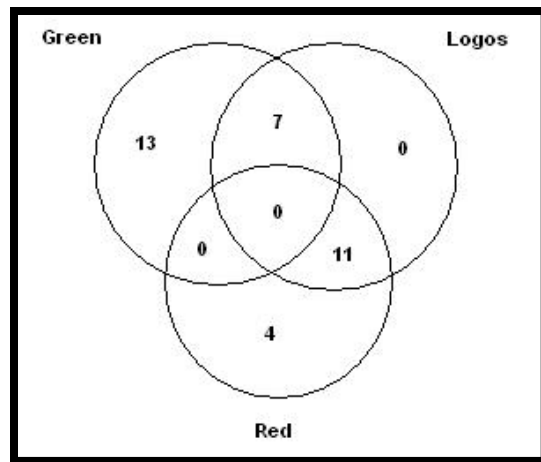**Review of Prerequisite Skills**   **Page 302**   **Question 8**

The number of bridge hands with seven hearts is $_{13}C_7 \times _{39}C_6 = 5.59866 \times 10^9$. The number with eight diamonds is $_{13}C_8 \times _{39}C_5 = 7.40999 \times 10^8$. The total number of hands with either seven hearts or eight diamonds is $5.59866 \times 10^9 + 7.40999 \times 10^8 = 6.33966 \times 10^9$.

Review of Prerequisite Skills   **Page 303**   **Question 10**

a) In order to count the elements in a set, you must be careful to count each element only once. If your counting method counts some elements more than once, they must be subtracted an appropriate number of times. For example, if you have two sets A and B, and add $n(A) + n(B)$, then you have counted each of the elements in the intersection of the sets twice. Therefore, you must subtract the number of elements in the intersection once. The formula is shown below:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

b) Model this problem with a Venn diagram, as shown at the right. Since there are 18 hats with logos, and 11 of them are red, the intersection of the sets Green and Logos contains 7 hats. Since there are 20 green hats, there are 13 green hats without logos. This accounts for 31 hats. Since there are 35 hats in the store, there are 4 red hats without logos. Therefore, the number of hats that are red, have logos, or both is given by
$7 + 11 + 4 = 22$.

Review of Prerequisite Skills   **Page 303**   **Question 12**

a) $_5P_3 = 60$ **b)** $_7P_1 = 7$ **c)** $P(6, 2) = 30$

d) $_9P_9 = 362\,880$ **e)** $_{100}P_1 = 100$ **f)** $P(100, 2) = 9900$

Review of Prerequisite Skills   **Page 303**   **Question 14**

There are 8 choices for the first digit, and 10 choices for each of the other two digits. Therefore, there are $8 \times 10 \times 10 = 800$ possible area codes.

Review of Prerequisite Skills   **Page 303**   **Question 16**

Since repetitions are not allowed, the number of 3-topping pizzas possible is $_9C_3 = 84$.

Review of Prerequisite Skills   **Page 303**   **Question 18**

Matrices a) and d) are square. Matrix b) is a row matrix. Matrix f) is a column matrix.

CHAPTER 6   Introduction to Probability

Section 6.1   Basic Probability Concepts

Practise

Section 6.1   **Page 312**   **Question 2**

a) Since the physics behind the sun rising are well known, you may estimate the probability that it will rise tomorrow as 100%.

b) Since the water cycle is well understood, you may estimate the probability that it will never rain again as 0%.

c) Answers may vary.

d) Answers may vary.

Section 6.1   **Page 312**   **Question 4**

a) The probability that a resident owns a home is $\dfrac{10\ 000}{16\ 000} = 0.625$.

b) The probability that a resident rents and has lived at the present address for less than two years is $\dfrac{4500}{16\ 000} = 0.281\ 25$.

c) We know that we are dealing with a homeowner in this part. Therefore, the number of possible outcomes is reduced to 10 000. The probability that a homeowner has lived at the present address for more than two years is $\dfrac{8000}{10\ 000} = 0.8$.

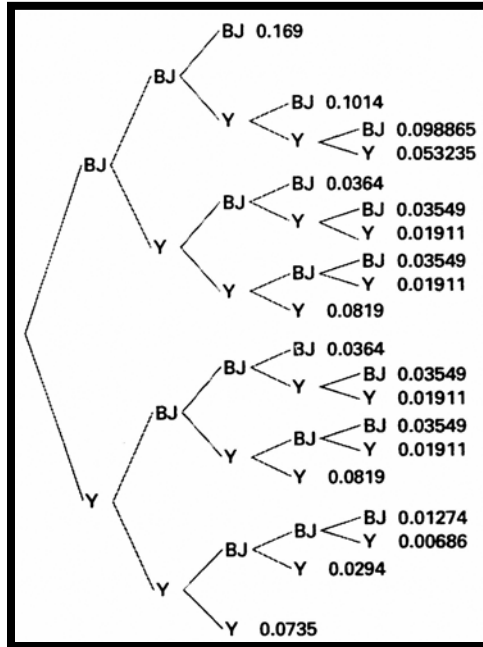**Section 6.1**   **Page 312**   **Question 6**

There are four prime numbers in the first ten natural numbers: 2, 3, 5, and 7. Therefore, the probability that a random number chosen from these digits will be prime is given by $\dfrac{4}{10} = \dfrac{2}{5}$.

Section 6.1   Page 313   Question 8

a) There are 48 deer in the randomly tagged sample, and 26 of these are does. Therefore, the empirical probability that a deer caught at random will be a doe is $\dfrac{26}{48} = \dfrac{13}{24}$.

b) Of the 48 deer in the randomly tagged sample, 22 were bucks. In a group of 10 deer captured at random, the number expected to be bucks is given by $\dfrac{22}{48} \times 10 = 4.56$. Therefore, about 5 are expected to be bucks.

Section 6.1   Page 313   Question 10

a) The tree diagram for all possible outcomes is shown at the right.

b) The probability of Toronto winning three straight games is given by $0.65 \times 0.65 \times 0.40 = 0.169$.

c) The probability of each outcome is shown at the right.

d) The answers to parts b) and c) are calculated by multiplying the branch probabilities leading to that outcome.

Section 6.1   Page 313   Question 12

a) Answers may vary. For example, choosing answer C always will guarantee a 30% success rate. Answer B is the second-most attractive.

b) Answers may vary. It would be unwise to choose answer C all of the time, since if an answer is indeed C, then the next answer is unlikely to be C. You can alternate answers B and C, and hope that you do not choose wrongly on the first one.

CHAPTER 6   Introduction to Probability

Section 6.2   Odds

Practise

Section 6.2   Page 318   Question 2

Since the odds are 19:1, the number of outcomes is 20. Therefore, the probability that the Argonauts will win the Grey Cup is $\dfrac{1}{20}$.

Section 6.2   Page 318   Question 4

a) There is one way that New Year's Day can fall on a Friday for six ways that it will not. Therefore, the odds in favour of New Year's Day falling on a Friday are 1:6.

b) There are $2 \times 2 \times 2 = 8$ possible outcomes when 3 coins are tossed. One of these is favourable while 7 are not. Therefore, the odds in favour of throwing 3 tails with 3 coins are 1:7.

c) There are 3 ways of tossing exactly 2 heads with 3 coins, and 5 ways of not tossing exactly 2 heads. Therefore, the odds in favour of not tossing exactly 2 heads are 5:3.

d) There are 2 ways of drawing a black 6 from a deck of 52 cards, and 50 ways of not drawing a black 6. Therefore, the odds in favour of drawing a black 6 are 2:50 or 1:25.

e) There are four even numbers and five odd numbers from 1 to 9 inclusive. Therefore, the odds in favour of selecting an even number are 4:5.

Section 6.2   Page 318   Question 6

The odds in favour of Boris winning are 5:4. Therefore, the probability of Boris winning is $\frac{5}{9}$, and the probability of Elena winning is $\frac{4}{9}$. The ways that Elena can win, along with probabilities, are summarized in the table at the right. The probabilities are calculated by multiplying the branch probabilities, as shown in the spreadsheet for the sequence WWLLW. The sum of these probabilities is approximately 0.397 which can be expressed in fractional form as $\frac{7808}{19\,683}$.

Microsoft Excel - Section 6.2 Question 6.xls

File Edit View Insert Format Tools Data Windo

B5   =   =4/9*4/9*5/9*5/9*4/9

	A	B	C
1	Sequence	Probability	
2	WWW	0.087791495	
3	WWLW	0.048773053	
4	WLWW	0.048773053	
5	WWLLW	0.02709614	
6	WLWLW	0.02709614	
7	WLLWW	0.02709614	
8	LWWW	0.048773053	
9	LWWLW	0.02709614	
10	LWLWW	0.02709614	
11	LLWWW	0.02709614	
12			
13	Total	0.396687497	

The table in this solution was prepared using Microsoft® Excel. If you need information on using spreadsheets, refer to Appendix B of the student text.

Section 6.2   Page 319   Question 8

Let the total number of events be n. Let the number of ways that event A can occur be m. Therefore, the number of ways that A' can occur is $n - m$. The odds in favour of A are $\frac{n}{n-m}$. The odds against A are equivalent to the odds in favour of A'. The odds in favour of A' are $\frac{n-m}{n}$. Hence, the odds against A are the reciprocal of the odds in favour of A.

Section 6.2   **Page 319**   **Question 10**

a) There are 12 ways of drawing a face card from a deck of 52, and 40 ways of drawing a non-face card. The odds against drawing a face card are 40:12 or 10:3.

b) There are $_{12}C_2 = 66$ ways of selecting two face cards and $_{52}C_2 = 1326$ ways of selecting any two cards. Therefore, there are $1326 - 66 = 1260$ ways of selecting two non-face cards. The odds against selecting two face cards are 1260:66 or 210:11.

Section 6.2   **Page 319**   **Question 12**

a) If George waters the lawn, there is a 30% chance of rain and a 70% chance of no rain. Therefore, the odds in favour of rain tomorrow if he waters the lawn are 30:70 or 3:7.

b) If George washes the car, there is a 40% chance of rain and a 60% chance of no rain. Therefore, the odds in favour of rain tomorrow if he washes the car are 40:60 or 2:3.

c) If George plans a trip to the beach, there is a 50% chance of rain and a 50% chance of no rain. Therefore, the odds in favour of rain tomorrow if he plans a trip to the beach are 50:50 or 1:1.

Section 6.2   **Page 319**   **Question 14**

On each trial, Aki has a probability of $\frac{1}{3}$ of getting a hard question and then a probability of $\frac{1}{2}$ of answering it correctly, giving a probability of $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ of getting it right. Similarly, he has a $\frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$ chance of getting an easy question right. He has a $\frac{2}{3} \times \frac{1}{10} = \frac{1}{15}$ chance of getting an easy question wrong, and a $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ chance of getting a hard question wrong. Since he must answer at least two questions correctly to win, the possible routes to winning are shown in the spreadsheet at the right. The probability for each route is calculated by multiplying the branch probabilities, as shown for the route ER–HR–HW at the right. The sum of these probabilities is about 0.86. Therefore, the odds in favour of Aki winning the contest are about 86:14 or 43:7.

Microsoft Excel - Section 6.2 Question 14.xls
File Edit View Insert Format Tools Data Windo

D18   = =$B$1*$B$3*$B$4

|	A	B	C	D
1	ER	0.6		
2	EW	0.066667		
3	HR	0.166667		
4	HW	0.166667		
5				Probability
6	ER	ER	ER	0.216
7			HR	0.06
8		HR	ER	0.06
9			HR	0.0166667
10	HR	ER	ER	0.06
11			HR	0.0166667
12		HR	ER	0.0166667
13			HR	0.0046296
14				
15	ER	ER	EW	0.024
16			HW	0.06
17		HR	EW	0.0066667
18			HW	0.0166667
19	HR	ER	EW	0.0066667
20			HW	0.0166667
21		HR	EW	0.0018519
22			HW	0.0046296
23	ER	EW	ER	0.024
24			HR	0.0066667
25		HW	ER	0.06
26			HR	0.0166667
27	HR	EW	ER	0.0066667
28			HR	0.0018519
29		HW	ER	0.0166667
30			HR	0.0046296
31	EW	ER	ER	0.024
32			HR	0.0066667
33		HR	ER	0.0066667
34			HR	0.0018519
35	HW	ER	ER	0.06
36			HR	0.0166667
37		HR	ER	0.0166667
38			HR	0.0046296
39			Total	0.8620741

CHAPTER 6   Introduction to Probability

Section 6.3   Probabilities Using Counting Techniques

Practise

Section 6.3   Page 324   Question 2

Follow the reasoning in Example 3 of this section. Since there are only 8 people in this sample, the lower value is changed from 342 to 358. The probability that at least two of the eight friends will have the same birthday is approximately 0.074. The calculation on a graphing calculator is shown at the right. If you need instructions on using a graphing calculator, refer to Appendix B of the student text.

```
1-prod(seq(X/365
,X,358,364))
          .0743352924
```
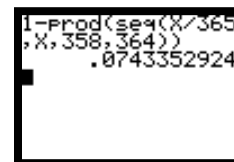
Section 6.3   Page 324   Question 4

In Example 1, it was shown that the total number of outcomes for this problem was 42. Since there are 7 lanes, there are 6 ways to place the two brothers. However, they may be placed in either order, making a total of 12 ways. The probability that they will be next to each other in any pair of lanes is $\dfrac{12}{42} = \dfrac{2}{7}$.

Section 6.3   Page 325   Question 6

There are 16 socks in the drawer. Therefore, there are $_{16}C_2 = 120$ ways to select two socks. Of these, $_8C_2 = 28$ will yield two white socks. Therefore, the probability of choosing two white socks is $\dfrac{28}{120} = \dfrac{7}{30}$.

Section 6.3   Page 325   Question 8

a) There are $_5P_2 = 20$ ways of picking the first two people to arrive. There are $_3P_2 = 6$ ways that two of the three girls can arrive first. Therefore, the probability that two of the girls will arrive first is $\dfrac{6}{20} = \dfrac{3}{10}$.

b) There are 5! = 120 ways that the friends can arrive at the party. Of these, there is only one way that is in ascending order of age. Therefore, the probability that they will arrive in ascending order of age is $\dfrac{1}{120}$.

c) In parts a) and b) it is assumed that the friends arrive one at a time. In part b), it is assumed that all have different ages.

Section 6.3   **Page 325**   **Question 10**

a) i) Since there are 5 400 000 tickets and 10 grand prizes, you have a probability of
$\dfrac{10}{5\ 400\ 000} = \dfrac{1}{540\ 000}$ of winning a grand prize.

ii) If you buy 10 tickets, your probability increases to $\dfrac{100}{5\ 400\ 000} = \dfrac{1}{54\ 000}$.

iii) If you buy 100 tickets, your probability increases to $\dfrac{1000}{5\ 400\ 000} = \dfrac{1}{5400}$.

b) In order to have a 5% chance of winning a grand prize, you need to buy 0.5% of the tickets, or $0.005 \times 5\ 400\ 000 = 27\ 000$ tickets. This is not a very sensible strategy due to the high cost of purchasing that many tickets.

c) In order to have a 50% chance of winning, you must multiply the answer to part b) by 10, resulting in 270 000 tickets.

Section 6.3   **Page 326**   **Question 12**

a) There are $_{22}C_4 = 7315$ ways of selecting four bucks from the 22 captured.

i) There are $_7C_4 = 35$ ways of selecting four bucks from the seven with cross-hatched antlers.

Therefore, the probability that all four bucks will have cross-hatched antlers is $\dfrac{35}{7315} = \dfrac{1}{209}$.

ii) There are $_{15}C_4 = 1365$ ways of choosing four bucks with no cross-hatching. Therefore, there are $7315 - 1365 = 5950$ ways of selecting at least one buck with cross-hatching. The probability of selecting at least one buck with cross-hatching is $\dfrac{5950}{7315} = \dfrac{170}{209}$.

b) Since order is important, there are $_7P_2 = 42$ ways of selecting two bucks from the seven. Of these, there is only one way to choose the eldest followed by the second eldest. Therefore, the probability that they will be chosen in this order is $\dfrac{1}{42}$.

Section 6.3   **Page 326**   **Question 14**

a) i) There are 50 fish in the pond, and the ratio of catfish:trout:bass is 5:2:3. Therefore,

$$5x + 2x + 3x = 50$$
$$10x = 50$$
$$x = 5$$

You may conclude that there are 25 catfish, 10 trout, and 15 bass in the pond.

There are $_{50}C_3 = 19\,600$ ways of catching three fish. Of these, there are $_{10}C_3 = 120$ ways of catching three trout. Therefore, the probability of catching three trout is $\dfrac{120}{19\,600} = \dfrac{3}{490}$.

ii) If order is important, there are $_{50}P_3 = 117\,600$ ways of catching three fish. There are $15 \times 25 \times 10 = 3750$ ways of catching three fish in alphabetical order. Therefore, the probability that the fish are caught in alphabetical order is $\dfrac{3750}{117\,600} = \dfrac{25}{784}$.

b) Answers may vary. Due to the random chance inherent in the empirical investigation, results are not likely to exactly match those arrived at theoretically.

c) Answers may vary. You could use the **randInt(**1,50,3**)** function under the **MATH PRB** menu to generate three random numbers from 1 to 50, as shown in the screen shot at the right. You could assign numbers 1 to 25 to be catfish, numbers 26 to 35 to be trout, and numbers 36 to 50 to be bass. You would have to discard any trials in which the same number occurred more than once.
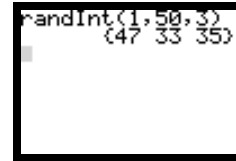
d) Answers may vary. You can use the random number functions in Fathom™ or a spreadsheet to generate three random numbers, and create cases or copy formulas for any desired number of trials. You can assign fish types in a manner similar to part c) above.

e) Answers may vary. Fish are usually not randomly distributed in a pond. Some fish tend to stay in schools. Others may be solitary, but will congregate where food is more abundant. Since the fish are thrown back before the next trial, the calculated probabilities assume that a fish that has been caught is just as likely to be caught again. In fact, a fish that has been caught and thrown back may retreat for at least a short time, and be wary of hooks bearing food.

Section 6.3 **Page 326** **Question 16**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text.

	A	B	C	D	E
1	Committee Size	Total Choices	Nurses Only	Mixed	P(Mixed)
2	1	10	8	2	0.2
3	2	45	28	17	0.377777778
4	3	120	56	64	0.533333333
5	4	210	70	140	0.666666667
6	5	252	56	196	0.777777778
7	6	210	28	182	0.866666667
8	7	120	8	112	0.933333333
9	8	45	1	44	0.977777778

Let the number of people on the committee be r. Insert values of r ranging from 1 to 8 in column A of the spreadsheet. There are $_{10}C_r$ ways of selecting each committee. Enter this formula in cell B2 and copy it down to cell B9. Use the **COMBIN** function in Microsoft® Excel, or the **@COMB** function in Corel® Quattro® Pro. There are $_8C_r$ ways of selecting only nurses for the committee. Enter this formula in cell C2 and copy it down to cell C9. Enter $B2 - C2$ in cell D2 and copy it down to cell D9. Enter $D2/B2$ in cell E2 and copy it down to cell E9. The probabilities of having a mixed committee are now in column E. The committee size must be at least 7 to ensure a 90% or better probability of having a mixed committee.

CHAPTER 6 Introduction to Probability

Section 6.4 Dependent and Independent Events

Practise

Section 6.4 **Page 334** **Question 2**

The probability of being cut from the basketball team is 30%. The probability of failing the geometry quiz is 20%. Since you may take these as independent events, the probability of having both happen on the same day is $0.3 \times 0.2 = 0.06$.

Section 6.4 **Page 334** **Question 4**

a) The probability that both tests give the correct result is $0.95 \times 0.89 = 0.8455$ or 84.55%.

b) The probability that neither test gives the correct result is $0.05 \times 0.11 = 0.0055$ or 0.55%.

c) The probability that at least one test gives the correct result is $1 - 0.0055 = 0.9945$ or 99.45%.

Section 6.4   **Page 334**   **Question 6**
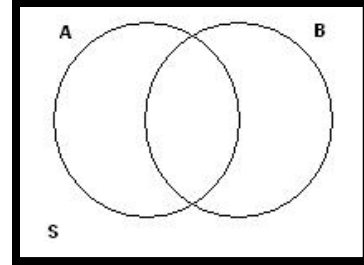
The probability that Tenzin will ask Mikala to go to the dance, and that Mikala will agree, is $0.60 \times 0.85 = 0.51$.

Section 6.4   **Page 334**   **Question 8**

Refer to the Venn diagram at the right. The $P(A|B)$ implies that event B has already occurred. Therefore, the number of possible outcomes is restricted to $n(B)$. The number of these outcomes in which event A occurs must be $n(A \cap B)$. Therefore,

$$P(A \mid B) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{\dfrac{n(A \cap B)}{n(S)}}{\dfrac{n(B)}{n(S)}}$$

$$= \frac{P(A \cap B)}{P(B)}$$

Section 6.4   **Page 335**   **Question 10**

a) If the odds in favour of passing the data management examination are 17:3, then the probability of passing is $\dfrac{17}{20}$. If the odds against passing the biology examination are 3:7, then the probability of passing is $\dfrac{7}{10}$. The probability that Pinder will pass both examinations is

$\dfrac{17}{20} \times \dfrac{7}{10} = \dfrac{119}{200}$ or 59.5%.

b) The probability of failing both examinations is $\dfrac{3}{20} \times \dfrac{3}{10} = \dfrac{9}{200}$ or 4.5%. Therefore, the odds in favour of failing are 9:191.

c) These two events may be dependent if the amount of study time is limited. Adding time studying for one subject may subtract from the time allotted for the other, thus changing the probabilities of success.

Section 6.4   **Page 335**   **Question 12**

Of the 48 deer captured, 26 were does. Of these, 30% had the bald patch. Therefore, the probability that a captured deer has a bald patch is $\dfrac{26}{48} \times 0.30 = 0.1625$, or $\dfrac{13}{80}$ in fractional form.

The desired probability that any one relay fails within five years is 10% or less. Since the relays are independent, each relay must have a probability of failure of $\dfrac{10\%}{8} = 1.25\%$ or less.

There are six ways of throwing a 7 and six ways of throwing doubles, for a total of 12 out of the 36 possible outcomes with a pair of dice. The probability of not throwing a 7 or doubles in six throws is $\left(\dfrac{24}{36}\right)^6 = \dfrac{64}{729}$.

You are told that the Tigers have won the first game. Therefore, the possible five-game outcomes are restricted to those shown in the spreadsheet at the right. The probability for each branch that favours a series win by the Storm is calculated in column F, as shown for the sequence T–T–S–S–S at the right. The sum of the probabilities is shown in cell F13 as 0.2187.

	A	B	C	D	E	F
						Probability
1						
2	T	T	T			
3	T	T	S	T		
4	T	T	S	S	T	
5	T	T	S	S	S	0.0486
6	T	S	T	T		
7	T	S	T	S	T	
8	T	S	T	S	S	0.0396
9	T	S	S	T	T	
10	T	S	S	T	S	0.0405
11	T	S	S	S		0.09
12						
13					Total	0.2187

F5 = =0.6*0.4*0.45*0.45

Microsoft Excel - Section 6.4 Question 18.xls
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CHAPTER 6   Introduction to Probability

Section 6.5   Mutually Exclusive Events

Practise

a) There are nine positions available. Therefore, the probability that he will be assigned to play catcher is $\dfrac{1}{9}$.

b) Since there are three outfielders, the probability that he will be assigned to play outfielder is $\dfrac{3}{9} = \dfrac{1}{3}$.

c) There are four positions that Troy does not like. Therefore, the probability that he will be assigned to a position he does not like is $\dfrac{4}{9}$.

Section 6.5   Page 341   Question 4

a) i) There are 365 possible day-passes. The four vacation months have 31 days each, which adds up to 124 days. The probability that the day-pass occurs during a vacation month is $\dfrac{124}{365}$.

ii) The months of June, July, or August add up to 92 days. The probability that the day-pass occurs during these months is $\dfrac{92}{365}$.

**b)** The Venn diagram for the events in part a) is shown at the right.
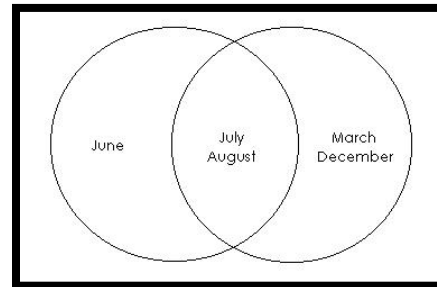
Section 6.5   Page 341   Question 6

This problem can be modelled using a Venn diagram, as shown at the right.

a) The probability that the viewer was an adult who did not watch the program is $\dfrac{550}{1000} = 0.55$.

b) The probability that the viewer was a child who watched the program is $\dfrac{148}{1000} = 0.148$.

c) The probability that the viewer was either an adult or someone who watched the program is $\dfrac{800 + 148}{1000} = 0.948$.
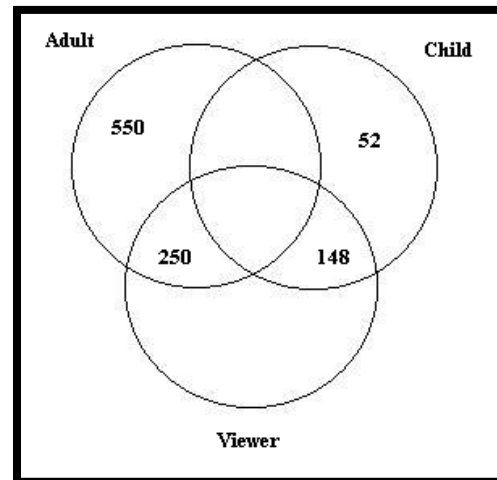
Section 6.5   **Page 341**   **Question 8**

a) From the principle of inclusion and exclusion, you know that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Divide each term by $n(S)$.

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

Therefore,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

b) If $P(A \cap B) = 0$, the events A and B are mutually exclusive.

**Section 6.5**   **Page 341**   **Question 10**

Cross-hatched antlers are only found among bucks and bald patches are only found among does, so the events are mutually exclusive. Out of 48 deer captured, there were 7 bucks with cross-hatched antlers. Since the rate of bald patches is 30% of does, the expected number of does with bald patches among the 26 does captured is 7.8. Therefore, the probability that a captured deer has either cross-hatched antlers or bald patches is $\dfrac{7 + 7.8}{48} = 0.308\overline{3}$, or $\dfrac{37}{120}$ in fractional form.

Section 6.5   **Page 342**   **Question 12**

Solutions for Achievement Checks are described in the Teacher's Resource.

Section 6.5   **Page 342**   **Question 14**

Model this problem using a Venn diagram, as shown at the right. You know that $x + x + 0.1 = 0.25$. Therefore, $x = 0.075$. The probability that a kitten will have blue eyes is given by $x + 0.1 = 0.175$. The odds in favour of a kitten having blue eyes are 0.175:0.825 or 7:33.

Section 6.5   **Page 343**   **Question 16**
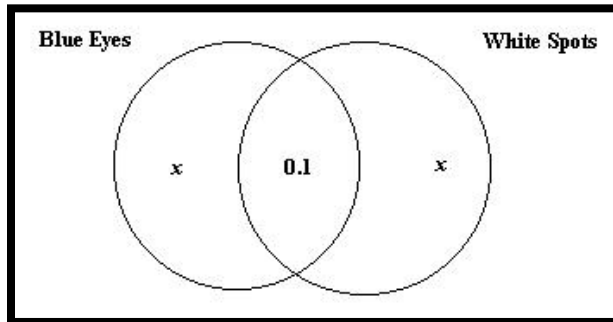
a) i) The probability of a 1998 graduate being a male is $\dfrac{71\,949}{172\,076} = 0.418$.

ii) The probability of a 1998 graduate being in mathematics and physical sciences is $\dfrac{9992}{172\,076} = 0.058$.

iii) The probability of a 1998 graduate being a male in mathematics and physical sciences is $\dfrac{6876}{172\,076} = 0.040$.

iv) The probability of a 1998 graduate being not a male in mathematics and physical sciences is $1 - 0.040 = 0.960$.

v) The probability of a 1998 graduate being a male or a graduate in mathematics and physical sciences is $0.418 + 0.058 - 0.040 = 0.436$.

b) The probability that a male graduate from 1996 graduated in mathematics and physical sciences is $\dfrac{6726}{75\,106} = 0.090$.

c) If a mathematics and physical sciences graduate is selected at random from the period 1994 to 1996, the probability that the graduate is a male is $\dfrac{6697 + 6941 + 6726}{9551 + 9879 + 9786} = 0.697$.

d) Being a male and graduating in mathematics and physical sciences are not independent events. There are societal pressures in favour of males and against females operating in these fields of study.

CHAPTER 6   Introduction to Probability

Section 6.6   Applying Matrices to Probability Problems

Practise

Section 6.6   **Page 353**   **Question 2**

a) This cannot be a transition matrix since the entries in the third row do not add up to 1.

b) This is a transition matrix. All entries are positive. It is square. Each row adds up to 1.

c) This is not a transition matrix since it is not square.

a) Answers may vary.

b) Let the steady-state vector be $\begin{bmatrix} a & b \end{bmatrix}$ such that $\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} a & b \end{bmatrix}$. This leads to the

equations:

$0.6a + 0.5b = a$ 1)
$0.4a + 0.5b = b$ 2)
$a + b = 1$ 3)

These equations simplify to:

$-0.4a + 0.5b = 0$ 4)
$0.4a - 0.5b = 0$ 5)
$a + b = 1$ 6)

Since equations 4) and 5) are identical, solve for b and substitute into equation 6):

$$b = \frac{4}{5}a$$

$$a + \frac{4}{5}a = 1$$

$$\frac{9}{5}a = 1$$

$$a = \frac{5}{9}$$

$$\therefore b = \frac{4}{9}$$

$$\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} \dfrac{5}{9} & \dfrac{4}{9} \end{bmatrix}$$

c) E-pics increased its market share from 0.33 or about $\dfrac{3}{9}$ to $\dfrac{4}{9}$.

d) Although ZapShot has a 0.6 chance of getting customers back compared to 0.5 for E-pics, the initial probability vector of 0.67 was too high to sustain over the long term.

Section 6.6   **Page 353**   **Question 6**

$$\begin{array}{cc} & \text{W} \quad \text{L} \end{array}$$

a) The transition matrix for this situation is $\begin{array}{c} \text{W} \\ \text{L} \end{array}\begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$.

b) Store the transition matrix in your graphing calculator, and square it to obtain the matrix shown in the screen shot at the right. If you need instructions on using a graphing calculator, refer to Appendix B of the student text. Hence, the probability that Gina's team will win two games later is given by the element in row 2, column 1, which is 0.52.
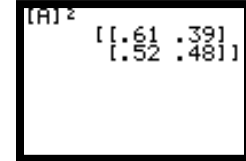
c) Let the steady-state vector be $\begin{bmatrix} a & b \end{bmatrix}$ such that $\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} a & b \end{bmatrix}$. This leads to the equations:

$$0.7a + 0.4b = a \qquad 1)$$
$$0.3a + 0.6b = b \qquad 2)$$
$$a + b = 1 \qquad\qquad 3)$$

These equations simplify to:

$$-0.3a + 0.4b = 0 \qquad 4)$$
$$0.3a - 0.4b = 0 \qquad 5)$$
$$a + b = 1 \qquad\qquad 6)$$

Since equations 4) and 5) are identical, solve for b and substitute into equation 6):

$$b = \frac{3}{4}a$$
$$a + \frac{3}{4}a = 1$$
$$\frac{7}{4}a = 1$$
$$a = \frac{4}{7}$$
$$\therefore b = \frac{3}{7} \quad \text{and} \quad \begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} \dfrac{4}{7} & \dfrac{3}{7} \end{bmatrix}$$

Hence, Gina's team will win $\dfrac{4}{7}$ of their games.

Section 6.6   **Page 354**   **Question 8**

S   C   R

a) The transition matrix is
$$
\begin{array}{c}
S \\ C \\ R
\end{array}
\begin{bmatrix}
0.50 & 0.30 & 0.20 \\
0.25 & 0.35 & 0.40 \\
0.20 & 0.35 & 0.45
\end{bmatrix}.
$$

b) If it is cloudy on Wednesday, then the forecast for Saturday requires the transition matrix to be cubed. The results from a graphing calculator are shown in the screen shots at the right. The probability that it will be sunny on Saturday is given by the element in row 2, column 1, which is about 0.305.
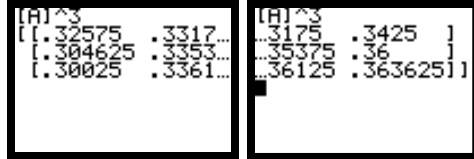
c) To find the probability that it will be sunny four months from now, raise the transition matrix to a large power. If your power is high enough, all of the elements in the first column will be the same, as shown in the screen shot at the right. The probability that it will be sunny no longer depends on the initial probability vector, which reaches a steady-state at 0.310.
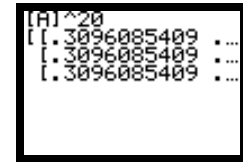
d) In part c) you must assume that the weather patterns do not change throughout the year.

Section 6.6   **Page 354**   **Question 10**

a) Since 30% of the does currently have bald patches, the initial probability vector is $\begin{bmatrix} 0.7 & 0.3 \end{bmatrix}$.

D   B

b) The transition matrix is
$$
\begin{array}{c}
D \\ B
\end{array}
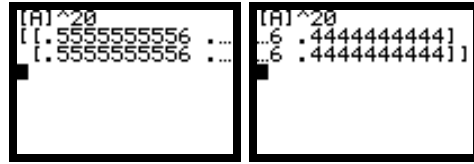\begin{bmatrix}
0.80 & 0.20 \\
0.35 & 0.65
\end{bmatrix}.
$$

c) Enter the transition matrix into your graphing calculator, and raise it to a high power, such that the elements in each column are the same, as shown in the screen shots at the right. The long-term probability of a doe being born with bald patches it about 0.364.
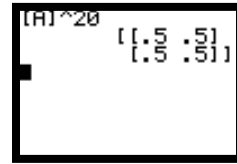
d) Answers may vary.

Section 6.6   **Page 355**   **Question 12**
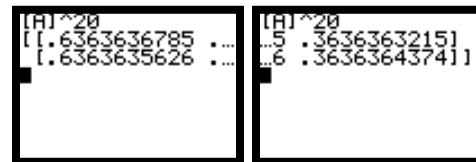
a) The new transition matrix is $\begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$. The steady-state vector probability of calm will

decrease while that of stormy weather will increase.

b) The effect of the long term on this matrix is shown in the screen shots at the right. The new steady-state vector is about $\begin{bmatrix} 0.56 & 0.44 \end{bmatrix}$.
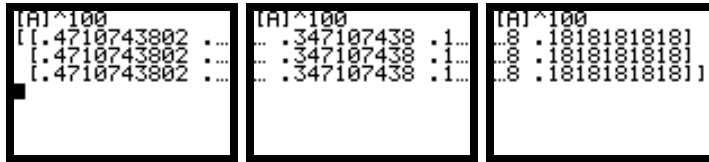

          C   S

c) The new transition matrix is $\begin{array}{c} C \\ S \end{array} \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$. The long-term effect of this

change is shown in the screen shot at the right. The new steady-state vector is $\begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$.

d) Answers may vary.

Section 6.6   **Page 355**   **Question 14**

a) P^{100} is shown in the screen shots at the right.

b)

$$S^{(0)}P^{100} = \begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} 0.471 & 0.347 & 0.182 \\ 0.471 & 0.347 & 0.182 \\ 0.471 & 0.347 & 0.182 \end{bmatrix}$$
$$= \begin{bmatrix} 0.471(a+b+c) & 0.347(a+b+c) & 0.182(a+b+c) \end{bmatrix}$$
$$= \begin{bmatrix} 0.471 & 0.347 & 0.182 \end{bmatrix}$$

since $(a+b+c) = 1$. This expression does not depend on $S^{(0)}$, but it does depend on P.

c) The answer to part b) illustrates the property that the steady-state vector for a regular Markov chain does not depend on the choice of the initial probability vector.

CHAPTER 6   Introduction to Probability

Review of Key Concepts

a) The empirical probability of rolling a 4 with a die based on the 20 trials is $\dfrac{5}{20} = \dfrac{1}{4}$.

b) There is one way of rolling a 4 with a die with 6 sides. Therefore, the theoretical probability of rolling a 4 with a die is $\dfrac{1}{6}$.

c) Random chance accounts for the variation between parts a) and b).

When three coins are flipped, there are $2 \times 2 \times 2 = 8$ possible outcomes. Of these, only 1 is 3 heads. Therefore, the probability of flipping 3 heads is $\dfrac{1}{8}$, and the odds in favour of flipping 3 heads are 1:7.

a) The tree diagram illustrating all of the possible outcomes is shown at the right.

b) Since probabilities vary among branches, not all of the outcomes are equally likely.

c) There are $_8C_3 = 56$ ways of selecting 3 cat's-eyes. There are $_{20}C_3 = 1140$ ways of selecting 3 marbles. Therefore, the probability of selecting 3 cat's-eyes is $\dfrac{56}{1140} = \dfrac{14}{285}$.

d) There are $_{12}C_3 = 220$ ways of not choosing cat's-eyes. Therefore, the probability that no cat's-eyes are chosen is $\dfrac{220}{1140} = \dfrac{11}{57}$.
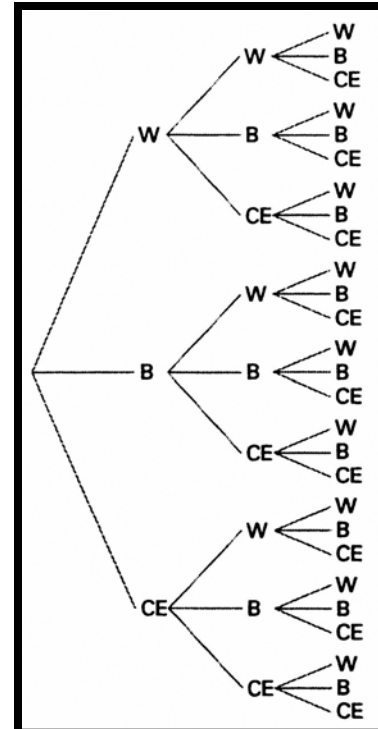
Review of Key Concepts **Page 357** **Question 8**

a) Since order is important, there are $_{10}P_3 = 720$ ways of selecting the committee. Of these, $_5P_3 = 60$ are composed of runners only. Therefore, the probability that the committee is composed of all runners is $\dfrac{60}{720} = \dfrac{1}{12}$.

b) There are $3 \times 2 \times 1 = 6$ ways of forming the committee using the three eldest runners. Therefore, the probability that the committee is comprised of the three eldest runners is $\dfrac{6}{720} = \dfrac{1}{120}$.

c) There is only one way of forming a committee such that the eldest runner is president, the second-eldest runner is secretary, and the third-eldest runner is treasurer. The probability of forming this committee is $\dfrac{1}{720}$.

Review of Key Concepts **Page 358** **Question 10**

a) The probability that Bruno will receive both job offers is $0.4 \times 0.75 = 0.3$.

b) Since Bruno is making a personal estimate of the probabilities, he is applying the concept of subjective probability.

Review of Key Concepts **Page 358** **Question 12**

The probability of being called at all is 20%. The window spans 4 h. You are eating dinner for 0.5 h during this window. The probability that you will be called during dinner is $0.2 \times \dfrac{0.5}{4} = \dfrac{1}{40}$ or 0.025.

Review of Key Concepts **Page 358** **Question 14**

a) There are four 5s in the deck, and six black face cards. The probability of drawing either a 5 or a black face card is $\dfrac{4+6}{52} = \dfrac{5}{26}$.

b) The Venn diagram illustrating this situation is shown at the right.

Review of Key Concepts   Page 359   Question 16
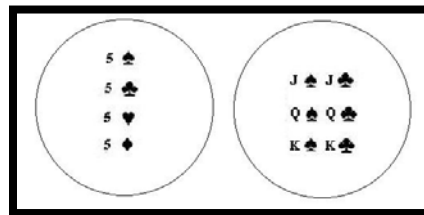
a) There are 110 students who like pop songs and 40 who do not. Therefore, the odds in favour of a randomly selected student liking a pop song are 11:4.

b) Since 110 students like pop songs, 70 like heavy-metal songs and 50 like both, the number who like either one is given by $110 + 70 - 50 = 130.$ Since there are 150 students at the dance, there are 20 who do not like either. Therefore, the odds in favour of a student liking neither are 20:130 or 2:13.

c) Answers may vary.

Review of Key Concepts   Page 359   Question 18

a) Since you know that she has bowled a strike, the initial probability vector is $\begin{bmatrix} 1 & 0 \end{bmatrix}$.

b) The transition matrix is $\begin{array}{c} \\ S \\ N \end{array} \begin{bmatrix} \overset{S}{0.50} & \overset{N}{0.50} \\ 0.35 & 0.65 \end{bmatrix}$.

c) The probability that Alysia will bowl a strike in the second frame is given by the entry in row 1, column 1 of the transition matrix, which is 0.50.

The matrix for the third frame is obtained by squaring the transition matrix. The matrix for the tenth frame is obtained by raising the transition matrix to the ninth power. The results for the third and tenth frames are shown in the screen shots at the right. Therefore, the probability of a strike in the third frame is 0.425, and in the tenth frame is 0.4118.
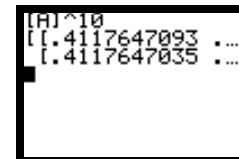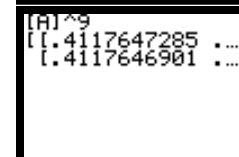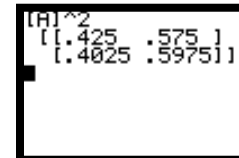
d) To obtain the matrix for the first frame of the second game, you must raise the transition matrix to the tenth power. The result is shown in the screen shot at the right. The probability of rolling a strike is about 0.4118.

e) It appears that the long-term probability that Alysia will bowl a strike is 0.4118.

f) Answers may vary.

CHAPTER 6   Introduction to Probability

Chapter Test

Chapter Test   **Page 360**   **Question 2**

a) Of the nine writing instruments in the drawer, six are not pencils. Therefore, the probability of selecting one other than a pencil is $\frac{6}{9} = \frac{2}{3}$.

b) There are four pens in the drawer, and five other instruments. The odds in favour of pulling out a pen are 4:5.

Chapter Test   **Page 360**   **Question 4**

The probability that Juanita will bump into Troy and be late for class is $0.20 \times 0.25 = 0.05$.

Chapter Test   **Page 360**   **Question 6**

The child may inherit genes in four ways: AA, AO, OA, and OO. Only one of these results in type O blood. Therefore, the odds in favour of the child having type O blood are 1:3.

Chapter Test   **Page 360**   **Question 8**

a) There are 5! = 120 ways of seating the five siblings at the table. There are 3! = 6 ways of seating the three girls in the middle. For each of these, there are 2! = 2 ways of seating the two boys, giving $6 \times 2 = 12$ ways. The probability that the children are seated with the three girls in the middle is $\frac{12}{120} = \frac{1}{10}$.

b) The children may be seated in order of age ascending or descending, giving 2 ways. The probability that they are seated in order of age is $\frac{2}{120} = \frac{1}{60}$.

Chapter Test   **Page 361**   **Question 10**

a) i) Since Ursula has just found a defective SCR, the initial probability vector is $\begin{bmatrix} 1 & 0 \end{bmatrix}$.

ii) The transition matrix is $\begin{array}{c} \\ D \\ ND \end{array} \begin{array}{cc} D & ND \\ \begin{bmatrix} 0.070 & 0.930 \\ 0.004 & 0.996 \end{bmatrix} \end{array}$.

iii) The probability that the next SCR will be defective is 0.07. Given that it is defective, the probability that the one after that will be defective is 0.07. The probability that both will be defective is $0.07 \times 0.07 = 0.0049$.

b) This Markov chain is regular. All entries in the transition matrix are positive.

c) The long-term probability of an SCR failing inspection will reach a steady-state. To find the steady-state, you must raise the transition matrix to a large power, as shown in the screen shot at the right. The long-term probability of failure appears to be about 0.00428.

```
[A]^20
[[.0042826552  :…
 [.0042826552  :…
```
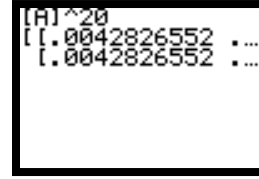
CHAPTER 6   Introduction to Probability

Cumulative Review: Chapters 4 to 6

Cumulative Review: Chapters 4 to 6   Page 364   Question 2

a)

$$(3x-2y)^5 = \ _5C_0\,(3x)^5 - _5C_1\,(3x)^4(2y) + _5C_2\,(3x)^3(2y)^2 - _5C_3\,(3x)^2(2y)^3 + _5C_4\,(3x)(2y)^4 - _5C_5\,(2y)^5$$
$$= 243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$$

b)

$$2a^4 - 8a^3b + 12a^2b^2 - 8ab^3 + 2b^4 = 2(a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4)$$
$$= 2(a-b)^4$$

Cumulative Review: Chapters 4 to 6   Page 364   Question 4

There are $12! = 479\,001\,600$ ways of arranging the cars in the front row. If the red car and orange car are placed together, there are $2 \times 11! = 79\,833\,600$ ways of parking the cars. Therefore, the number of ways of parking the cars with the red car and the orange car not next to each other is $479\,001\,600 - 79\,833\,600 = 399\,168\,000$.

Cumulative Review: Chapters 4 to 6   Page 364   Question 6

The odds in favour of the chess grand master defeating the computer are 4:5. Therefore, the probability that the chess grand master will defeat the computer is $\dfrac{4}{9}$.

Cumulative Review: Chapters 4 to 6   Page 364   Question 8

a) There are $_8C_4 = 70$ ways of selecting the first team. Of these, there are $_5C_4 = 5$ ways of selecting a team with only right-handed players. Therefore, the probability that the team consists of only right-handed players is $\dfrac{5}{70} = \dfrac{1}{14}$.

b) There are $_5C_2 \times _3C_2 = 30$ ways of selecting two right-handed and two left-handed players. Therefore, the probability of choosing such a team is $\dfrac{30}{70} = \dfrac{3}{7}$.

Cumulative Review: Chapters 4 to 6   Page 364   Question 10

There are 208 cards in four decks. Therefore, there are $_{208}P_{13}$ ways of dealing 13 cards. There are $_{200}C_{11} \times {_8}C_2 \times 13!$ ways of dealing a hand with exactly two black 3s in it. Therefore, the probability of dealing a hand with exactly two black 3s in it is
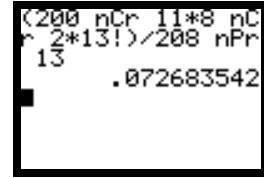
$$\frac{_{200}C_{11} \times {_8}C_2 \times 13!}{_{208}P_{13}} = 0.073$$

The screen shot for this calculation is shown at the right.

Cumulative Review: Chapters 4 to 6   Page 364   Question 12

The probability of rain is 12%, and the probability of snow is 24%, giving a total of 36%. Therefore, the probability that it will neither rain nor snow is $100\% - 36\% = 64\%$.

CHAPTER 7 Probability Distributions

Review of Prerequisite Skills

Review of Prerequisite Skills   **Page 368**   **Question 2**

a) $\displaystyle\sum_{i=1}^{12} t_i$ **b)** $\displaystyle\sum_{i=0}^{9} i_9 C_i$ **c)** $\displaystyle\sum_{i=1}^{6} \frac{i+1}{i+2}$ **d)** $\displaystyle\frac{1}{6}\sum_{i=0}^{5} a_i$

Review of Prerequisite Skills   **Page 368**   **Question 4**

Use your copy of Pascal's triangle for the numerical coefficients in this question.

a) $(x+y)^6 = x^6 + 6x^5 y + 15x^4 y^2 + 20x^3 y^3 + 15x^2 y^4 + 6xy^5 + y^6$

b) $(0.4+0.6)^4 = (0.4)^4 + 4(0.4)^3(0.6) + 6(0.4)^2(0.6)^2 + 4(0.4)(0.6)^3 + (0.6)^4$

$$= 1$$

c) $\left(\dfrac{1}{3}+\dfrac{2}{3}\right)^5 = \left(\dfrac{1}{3}\right)^5 + 5\left(\dfrac{1}{3}\right)^4\left(\dfrac{2}{3}\right) + 10\left(\dfrac{1}{3}\right)^3\left(\dfrac{2}{3}\right)^2 + 10\left(\dfrac{1}{3}\right)^2\left(\dfrac{2}{3}\right)^3 + 5\left(\dfrac{1}{3}\right)\left(\dfrac{2}{3}\right)^4 + \left(\dfrac{2}{3}\right)^5$

$$= 1$$

d) $(p+q)^n = p^n + {}_nC_1 p^{n-1}q + {}_nC_2 p^{n-2}q^2 + \ldots + {}_nC_n q^n$

$$= \sum_{r=0}^{n} {}_nC_r p^{n-r}q^r$$

Review of Prerequisite Skills   **Page 368**   **Question 6**

There are $2^4 = 16$ possible outcomes for a family of four children. Of these, only one results in four girls. Therefore, the probability of having four girls in the family is $\dfrac{1}{16}$.

CHAPTER 7 Probability Distributions

Section 7.1 Probability Distributions

Practise

Section 7.1   **Page 374**   **Question 2**

a) Since there is an equal probability of each number winning, this is a uniform distribution.

b) Different people might have a different probability of being selected, depending on their roles and the topic of the conference. This is likely not a uniform distribution.

c) Since there is an equal probability of throwing heads or tails, this is a uniform distribution.

d) Since there is an equal probability of each random number being selected, this is a uniform distribution.

e) Since the value of the guess will be influenced by factors such as appearance, this is not a uniform distribution.

f) Since there is an equal probability of selecting each card, this is a uniform distribution.

g) Since there are different probabilities for different sums, this is not a uniform distribution.

Section 7.1   **Page 375**   **Question 4**

a) There are four prime numbers, 2, 3, 5, and 7, on the spinner. Therefore, the probability of a prime number coming up is $\dfrac{4}{8} = \dfrac{1}{2}$.

b) A spreadsheet solution for this problem is shown at the right, giving the answer $E(x) = 4.5$. If you need information on how to use spreadsheets, refer to Appendix B of the student text.

	A	B	C
	x	p(x)	xp(x)
1			
2	1	0.125	0.125
3	2	0.125	0.25
4	3	0.125	0.375
5	4	0.125	0.5
6	5	0.125	0.625
7	6	0.125	0.75
8	7	0.125	0.875
9	8	0.125	1
10			
11		E(x)	4.5

Section 7.1   **Page 375**   **Question 6**

a) and **b)** A spreadsheet solution for this problem is shown at the right, giving the answer $E(x) = 7$. If you need information on how to use spreadsheets, refer to Appendix B of the student text.

	A	B	C
	x	p(x)	xp(x)
1			
2	2	1/36	1/18
3	3	1/18	1/6
4	4	1/12	1/3
5	5	1/9	5/9
6	6	5/36	5/6
7	7	1/6	1 1/6
8	8	5/36	1 1/9
9	9	1/9	1
10	10	1/12	5/6
11	11	1/18	11/18
12	12	1/36	1/3
13			
14		E(x)	7

c) A spreadsheet solution for this problem is shown at the right, giving the answer $E(x) = 10.5$.

	A	B	C
17	x	p(x)	xp(x)
18	3	1/216	1/72
19	4	1/72	1/18
20	5	1/36	5/36
21	6	5/108	5/18
22	7	5/72	35/72
23	8	7/72	7/9
24	9	25/216	1 1/24
25	10	1/8	1 1/4
26	11	1/8	1 3/8
27	12	25/216	1 7/18
28	13	7/72	1 19/72
29	14	5/72	35/36
30	15	5/108	25/36
31	16	1/36	4/9
32	17	1/72	17/72
33	18	1/216	1/12
34			
35		E(x)	10.5

Section 7.1   Page 375   Question 8

a) Since there are 7 prizes and 2 000 000 tickets, the probability of winning a prize is $\dfrac{7}{2\,000\,000}$.

b) A spreadsheet solution for this problem is shown at the right, giving an expected prize value of $0.52 per ticket. Therefore, the profit per ticket is $2.00 − $0.52 = $1.48. If you need information on how to use spreadsheets, refer to Appendix B of the student text.

	A	B	C
1	x	p(x)	xp(x)
2	1000000	1/2000000	1/2
3	25000	1/2000000	1/80
4	1000	1/2000000	1/2000
5	1000	1/2000000	1/2000
6	1000	1/2000000	1/2000
7	1000	1/2000000	1/2000
8	1000	1/2000000	1/2000
9			
10		E(x)	0.515000
11			
12			

Section 7.1   Page 375   Question 10

A spreadsheet solution for this problem is shown at the right, giving an expected prize value of $0.63 per ticket. If the operators wish to make a 40% profit, they will need to charge $\dfrac{\$0.63}{0.6} = \$1.05$ per ticket. If you need information on how to use spreadsheets, refer to Appendix B of the student text.

	A	B	C
1	x	p(x)	xp(x)
2	1000000	1/2000000	1/2
3	50000	1/400000	1/8
4	1000	1/200000	1/200
5	50	1/40000	1/800
6			
7		E(x)	0.631250

Section 7.1   **Page 376**   **Question 12**

A spreadsheet solution for this problem is shown at the right, giving an expected area of 26 cm^2. If you need information on how to use spreadsheets, refer to Appendix B of the student text.

Section 7.1   **Page 376**   **Question 14**

a) Answers may vary.

b) Since Cheryl's oldest child is a boy, there are two possible outcomes for her family: BB or BG. Only one of these results in two sons. Therefore, the probability that Cheryl has two sons is $\frac{1}{2}$. Fatima has at least one son. The possible outcomes for her family are BG, GB, or BB. One of these results in having two sons. The probability that Fatima has two sons is $\frac{1}{3}$.

Section 7.1   **Page 376**   **Question 16**

Solutions for Achievement Checks are described in the Teacher's Resource.

Section 7.1   **Page 377**   **Question 18**

a) i) The probability that a given number comes up is $\frac{1}{38}$, resulting in odds against winning of 37:1. If the game is to be fair, the house should pay 37:1. If the house pays 35:1, then the house is paying two less than it should. Therefore, the house advantage is $\frac{2}{38} \times 100\% = 5.3\%$.

ii) The probability of a red number winning is $\frac{18}{38}$, resulting in odds against winning of 20:18. If the game is to be fair, the house should pay 20:18. If the house pays 1:1, or 18:18, then the house is paying two less than it should. Therefore, the house advantage is $\frac{2}{38} \times 100\% = 5.3\%$.

iii) The probability of an odd number winning is $\dfrac{18}{38}$, resulting in odds against winning of 20:18. If the game is to be fair, the house should pay 20:18. If the house pays 1:1, or 18:18, then the house is paying two less than it should. Therefore, the house advantage is $\dfrac{2}{38} \times 100\% = 5.3\%$.

iv) The probability of a 6-number group winning is $\dfrac{6}{38}$, resulting in odds against winning of 32:6. If the game is to be fair, the house should pay 32:6. If the house pays 5:1, or 30:6, then the house is paying two less than it should. Therefore, the house advantage is $\dfrac{2}{38} \times 100\% = 5.3\%$.

v) The probability of a 12-number group winning is $\dfrac{12}{38}$, resulting in odds against winning of 26:12. If the game is to be fair, the house should pay 26:12. If the house pays 2:1, or 24:12, then the house is paying two less than it should. Therefore, the house advantage is $\dfrac{2}{38} \times 100\% = 5.3\%$.

b) Answers may vary.

c) If the payout ratios stay the same, having one less number on the wheel would decrease the house advantage to 1 in every 37, or 2.7%. As an example, consider a bet on a specific number coming up. The probability of this occurring is $\dfrac{1}{37}$ on a European roulette wheel. The odds against winning are 36:1, and the house should pay 36:1. If the house still pays 35:1, then the house is paying one less than it should. Therefore, the house advantage is $\dfrac{1}{37} \times 100\% = 2.7\%$.

Section 7.1 **Page 377** **Question 20**

Answers for the design of the device may vary.

a) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The expected value is 2.625.

b) From the spreadsheet at the right, the expected value is 1.75.

	A	B	C
		Corel Quattro Pro - C:\My Documents\	
		File Edit View Insert Format Tool	
	Arial	10 B I	
	A:E17		
	A	**B**	**C**
1	x	p(x)	xp(x)
2	0	1/4	0
3	1	1/6	1/6
4	2	1/8	1/4
5	3	1/8	3/8
6	4	1/12	1/3
7	5	1/12	5/12
8	6	1/12	1/2
9	7	1/12	7/12
10			
11		E(x)	2.625
12			
13			
14	x	p(x)	xp(x)
15	0	1/6	0
16	1	1/4	1/4
17	2	1/4	1/2
18	3	1/3	1
19			
20		E(x)	1.75

CHAPTER 7   Probability Distributions

Section 7.2   Binomial Distributions

Practise

Section 7.2   Page 385   Question 2

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text.
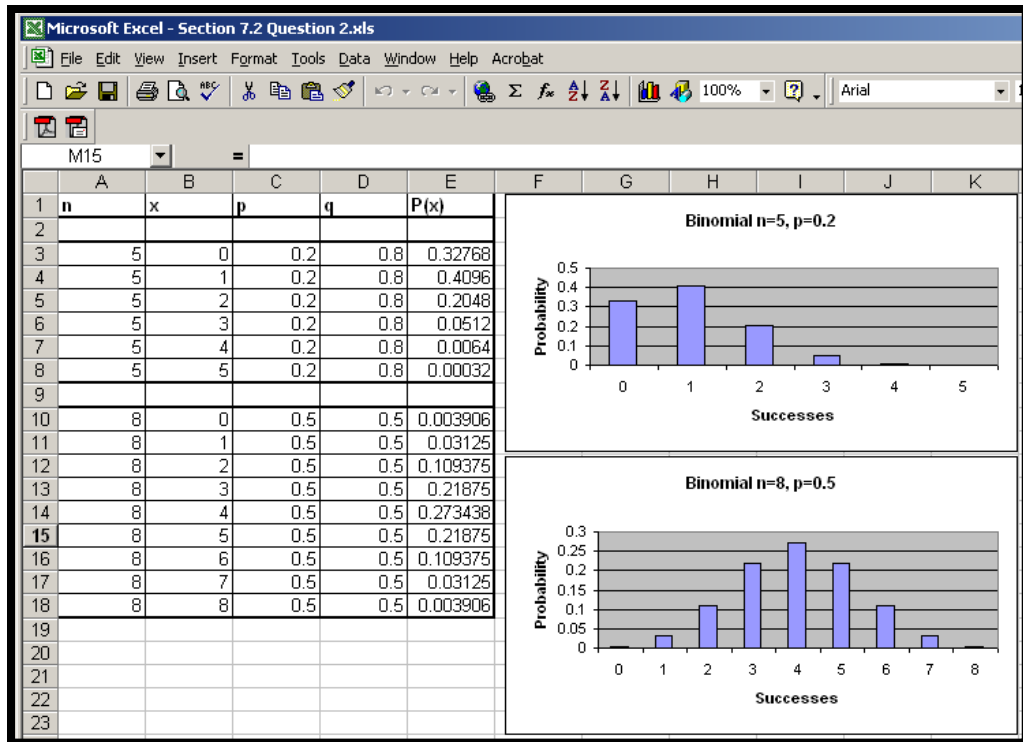The formula for $P(x)$ for $x = 0$ is:

Microsoft® Excel
COMBIN(A3,B3)*D3^(A3-B3)*C3^B3
Corel® Quattro® Pro
@COMB(B3,A3)*D3^(A3-B3)*C3^B3

Section 7.2   Page 385   Question 4

Answers may vary.

Section 7.2 **Page 385** **Question 6**

a) Answers may vary.

b) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the entry in the probability column for $x = 0$ is:
Microsoft® Excel
COMBIN(A3,B3)*D3^(A3-B3)*C3^B3
Corel® Quattro® Pro
@COMB(B3,A3)*D3^(A3-B3)*C3^B3

	A	B	C	D	E
	Microsoft Excel - Section 7.2 Question 6.xls				
	File Edit View Insert Format Tools Data Window Help				
	F11		=		
	A	B	C	D	E
1	n	x	p	q	P(x)
2					
3	100	0	0.015	0.985	0.220609
4	100	1	0.015	0.985	0.335953
5	100	2	0.015	0.985	0.253243
6	100	3	0.015	0.985	0.125979
7					
8				Sum	0.935784

The probability of fewer than 4 failures in 100 launches is 0.9358.

c) The expected number of failures in 100 launches of the rocket is given by

$$np = 100 \times 0.015$$
$$= 1.5$$

Section 7.2 **Page 386** **Question 8**

a) In this problem, $n = 20$, $p = 0.10$, and $x = 5$. Therefore,

$$P(x) = {}_nC_x p^x q^{n-x}$$
$$P(5) = {}_{20}C_5(0.10)^5(0.90)^{15}$$
$$= 0.0319$$

b) The expected number of left-handed people in a group of 20 is given by

$$E(x) = np$$
$$= 20 \times 0.10$$
$$= 2$$

c) Answers may vary.

Section 7.2 **Page 386** **Question 10**

Since the number of passengers overbooked is equal to the expected number of passengers who will not show up for their flights, the expected payout is $0.

Section 7.2   **Page 386**   **Question 12**

Use your copy of Pascal's triangle to determine the numerical coefficient of each term in the expansions below.

a) i) $(p+q)^6 = p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$

ii) $(0.2+0.8)^5 = (0.2)^5 + 5(0.2)^4(0.8) + 10(0.2)^3(0.8)^2 + 10(0.2)^2(0.8)^3 + 5(0.2)(0.8)^4$
$$+ (0.8)^5$$

b) Each term in the above expansions results in the probability of getting a certain number of successes for a binomial random variable.

Section 7.2   **Page 386**   **Question 14**

a) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Poisson distribution predicts a probability of about 1.2276×10^{-8}, while the binomial distribution predicts a probability of about 9.7936×10^{-9}.

The formula for the Poisson distribution is:
Microsoft® Excel
(EXP(-A3*C3)*(A3*C3)^B3)/FACT(B3)
Corel® Quattro® Pro
(@EXP(-A3*C3)*(A3*C3)^B3)/@FACT(B3)

The formula for the binomial distribution is:
Microsoft® Excel
COMBIN(A3,B3)*C3^B3*(1-C3)^(A3-B3)
Corel® Quattro® Pro
@COMB(B3,A3)*C3^B3*(1-C3)^(A3-B3)

b) If fewer than 5 claims are required, you must calculate the probabilities for 0, 1, 2, 3, and 4 claims. The previous screen shot shows these probabilities calculated for both the binomial and Poisson distributions, using formulas similar to those in part a). Add up the probabilities to obtain the final answers. The Poisson distribution predicts a probability of about 5.3559×10^{-15}, while the binomial distribution predicts a probability of about 4.4357×10^{-15}.

c) If fewer than 8 votes are required, you must calculate the probabilities for 0, 1, 2, 3, 4, 5, 6, and 7 votes. The previous screen shot shows these probabilities calculated for both the binomial and Poisson distributions, using formulas similar to those in part a). Add up the probabilities to obtain the final answers. The Poisson distribution predicts a probability of about 5.2337×10^{-7}, while the binomial distribution predicts a probability of about 3.9805×10^{-7}.

Section 7.2   **Page 387**   **Question 16**

a) Let the number of successes of the outcome with probability p be n_1. Let the number of successes of the outcome with probability q be n_2. Let the number of successes of the outcome with probability r be n_3. Therefore, $n_1 + n_2 + n_3 = n$. The number of ways of selecting n_1 successes for the outcome with probability p, followed by n_2 successes of the outcome with probability q, followed by n_3 successes of the outcome with probability r, is given by

$$_nC_{n_1} \times {}_{n-n_1}C_{n_2} \times {}_{n-n_1-n_2}C_{n_3} = \frac{n!}{(n-n_1)!n_1!} \times \frac{(n-n_1)!}{(n-n_1-n_2)!n_2!} \times \frac{(n-n_1-n_2)!}{(n-n_1-n_2-n_3)!n_3!}$$

$$= \frac{n!}{n_1!n_2!n_3!}$$

Therefore, the probability of this "trinomial" distribution is given by

$$\frac{n!}{n_1!n_2!n_3!}p^{n_1}q^{n_2}r^{n_3}$$

b) The probability of rolling a 3 is $\frac{1}{6}$. The probability of rolling a 5 is $\frac{1}{6}$. The probability of rolling anything else is $\frac{2}{3}$. Using the formula from part a), the probability of rolling a 3 twice, a 5 four times, and neither on the other four rolls is given by

$$\frac{10!}{2!\,4!\,4!}\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^4\left(\frac{2}{3}\right)^4 = 0.0133$$
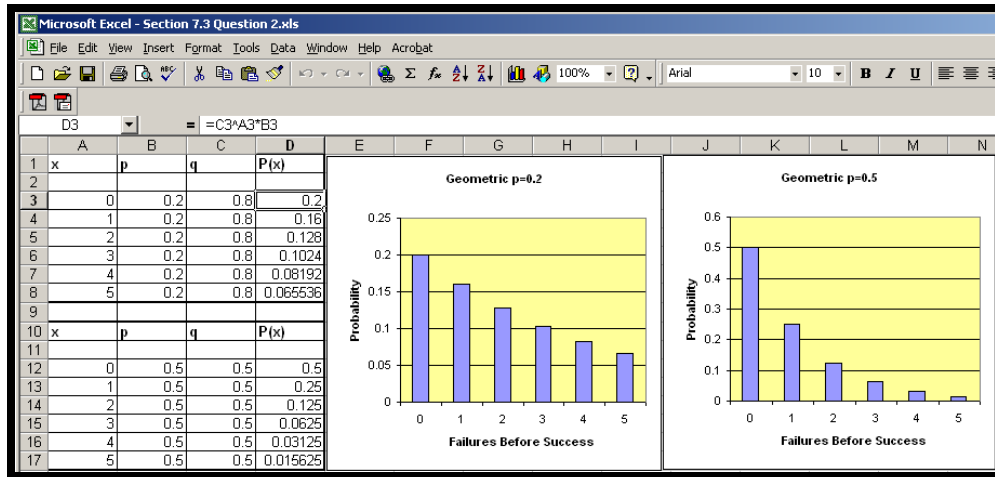
CHAPTER 7   Probability Distributions

Section 7.3   Geometric Distributions

Practise

Section 7.3   **Page 395**   **Question 2**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the geometric distribution for $x = 0$ and $p = 0.2$ is C3^A3*B3.

Section 7.3   **Page 395**   **Question 4**

a) The probability that a flight will be on time is $\frac{3}{4}$. The probability of this happening eight times in a row is $\left(\frac{3}{4}\right)^8 = 0.1001$.

b) The expected waiting time before a delay is $\dfrac{\left(\frac{3}{4}\right)}{\left(\frac{1}{4}\right)} = 3$ flights.

Section 7.3   **Page 395**   **Question 6**

The probability of selecting the wrong door is $\frac{2}{3}$. The probability that this will happen four times in a row is $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$.

Section 7.3   **Page 395**   **Question 8**

a) The probability of a success is $\dfrac{1}{10}$. Therefore, the probability of 11 failures before the first

success is $\left(\dfrac{9}{10}\right)^{11}\left(\dfrac{1}{10}\right) = 0.0314$.

b) The expected number of hangups before Minh can do a sales pitch is $\dfrac{\left(\dfrac{9}{10}\right)}{\left(\dfrac{1}{10}\right)} = 9$.

Section 7.3   **Page 395**   **Question 10**

a) The probability of a success is 0.34. Therefore, the probability that the pollster would have to interview five failures before a success is $(0.66)^5(0.34) = 0.0426$.

b) The expected waiting time before the pollster interviews someone who agrees with the policy is $\dfrac{0.66}{0.34} = 1.9$ people.

Section 7.3   **Page 395**   **Question 12**

a) The probability of scoring a goal is 0.93. Therefore, the probability of scoring 19 times before missing on the 20th try is $(0.93)^{19}(0.07) = 0.0176$.

b) The expected number of shots before he misses is $\dfrac{0.93}{0.07} = 13.3$.

Section 7.3   **Page 396**   **Question 14**

a) Answers may vary.

b) The waiting time for getting the first brand is 0. However, the first round is used up getting the first brand. Therefore, the waiting time for the second brand does not begin until the second round. Since each ball is returned to the bag before the following round, the probability of choosing a different brand from the first on subsequent trials is $\dfrac{2}{3}$. Therefore, the expected

waiting time before getting a ball of a second brand is $\dfrac{\left(\dfrac{1}{3}\right)}{\left(\dfrac{2}{3}\right)} = 0.5$. The waiting time for the third

brand does not begin until the round after the second brand has been obtained.

Once the two brands have been chosen, the probability of getting the third brand is $\frac{1}{3}$. The

expected waiting time for the third brand is $\dfrac{\left(\dfrac{2}{3}\right)}{\left(\dfrac{1}{3}\right)} = 2$. Therefore, the expected waiting time

before trying all three brands is the sum of the three waiting times and the two rounds in which the balls were obtained. This works out as follows: $0 + 1 + 0.5 + 1 + 2 = 4.5$ rounds.

Section 7.3　　　　　**Page 396**　　　　**Question 16**

Let x be the number of the trial on which the first success occurs. The number of the trial with the first success is one more than the number of failures before a success. Therefore, the probability that the first success will occur on the xth trial is $q^{x-1}p$.

The expected value for x will be one more than the expected number of failures before a success. Therefore,

$$E(x) = \frac{q}{p} + 1$$
$$= \frac{q+p}{p}$$
$$= \frac{1}{p}$$

Section 7.3　　　　　**Page 396**　　　　**Question 18**

If the second success is to occur on the fifth trial, then the following patterns are possible: SFFFS, FSFFS, FFSFS, and FFFSS. All of these have the probability p^2q^3. Therefore, the probability that the second success occurs on the fifth trial is $4p^2q^3$.

CHAPTER 7   Probability Distributions

Section 7.4   Hypergeometric Distributions

Practise

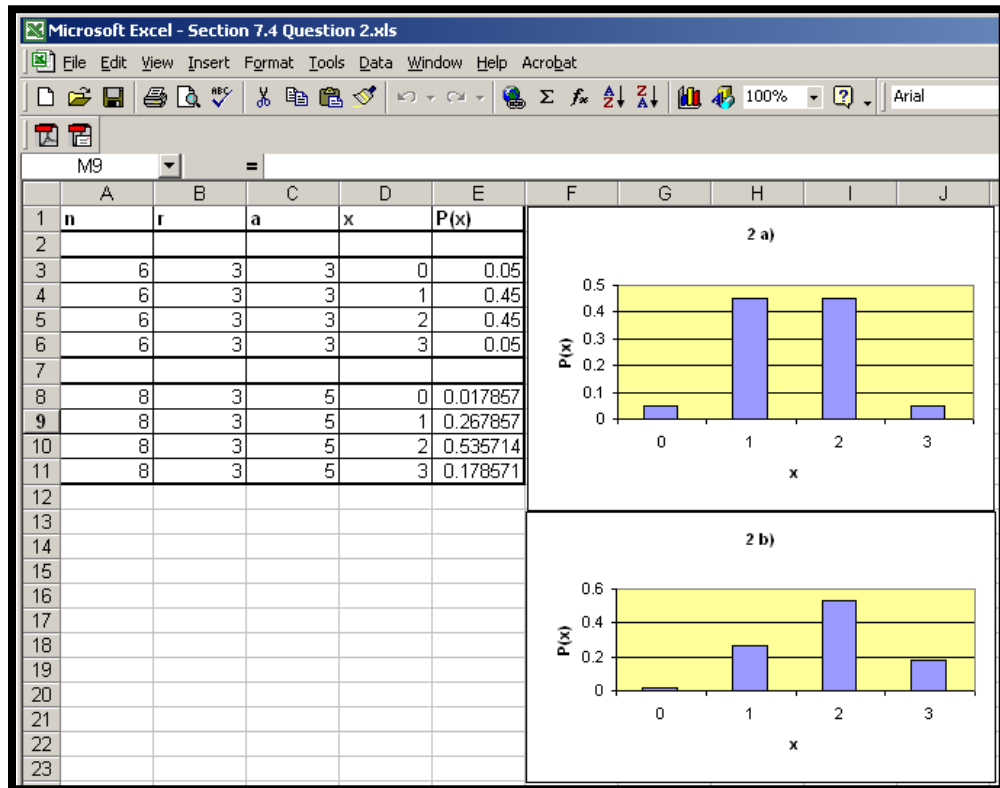**Section 7.4   Page 404   Question 2**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the hypergeometric distribution for the first line in part a) is:

Microsoft® Excel
COMBIN(C3,D3)*COMBIN(A3-C3,B3-D3)/COMBIN(A3,B3)
Corel® Quattro® Pro
@COMB(D3,C3)*@COMB(B3-D3,A3-C3)/@COMB(B3,A3)

Section 7.4   **Page 404**   **Question 4**

a) In this question, $a = 520$, $r = 125$, and $E(x) = 30$. Therefore,

$$E(x) = \frac{ra}{n}$$
$$30 = \frac{125 \times 520}{n}$$
$$30n = 125 \times 520$$
$$n = 2167$$

b) This may not be accurate due to chance variation in the number of tagged seals caught.

Section 7.4   **Page 404**   **Question 6**

a) Answers may vary.

b) In this question, $a = 200$, $r = 50$, and $n = 1200$. Therefore,

$$E(x) = \frac{ra}{n}$$
$$= \frac{50 \times 200}{1200}$$
$$= 8.3$$

Section 7.4   **Page 404**   **Question 8**

a) In this question, $a = 5$, $r = 5$, $x = 2$, and $n = 16$. Therefore,

$$P(x) = \frac{{}_aC_x \times {}_{n-a}C_{r-x}}{{}_nC_r}$$
$$P(2) = \frac{{}_5C_2 \times {}_{11}C_3}{{}_{16}C_5}$$
$$= 0.3777$$

b) $E(x) = \frac{ra}{n}$
$$= \frac{5 \times 5}{16}$$
$$= 1.6$$

a) In this question, $a = 11$, $r = 12$, $x = 6$, and $n = 25$. Therefore,

$$P(x) = \frac{{}_aC_x \times {}_{n-a}C_{r-x}}{{}_nC_r}$$

$$P(6) = \frac{{}_{11}C_6 \times {}_{14}C_6}{{}_{25}C_{12}}$$

$$= 0.2668$$

b) The probability that 0, 1, or 2 women will be jurors is calculated in the spreadsheet shown below. The probability formula for $x = 0$ is:

Microsoft® Excel
COMBIN(C3,D3)*COMBIN(A3-C3,B3-D3)/COMBIN(A3,B3)
Corel® Quattro® Pro
@COMB(D3,C3)*@COMB(B3-D3,A3-C3)/@COMB(B3,A3)

Therefore, the probability that there will be at least three women on the jury is

$$1 - 0.0114 = 0.9886.$$

c)

$$E(x) = \frac{ra}{n}$$

$$= \frac{12 \times 11}{25}$$

$$= 5.3$$

Section 7.4   **Page 405**   **Question 12**

a) In this question, $a = 5$, $r = 4$, $x = 4$, and $n = 11$. Therefore,

$$P(x) = \frac{{}_aC_x \times {}_{n-a}C_{r-x}}{{}_nC_r}$$

$$P(4) = \frac{{}_5C_4 \times {}_6C_0}{{}_{11}C_4}$$

$$= 0.0152$$

b) In this question, $a = 4$, $r = 4$, $x = 2$, and $n = 11$. Therefore,

$$P(x) = \frac{{}_aC_x \times {}_{n-a}C_{r-x}}{{}_nC_r}$$

$$P(2) = \frac{{}_4C_2 \times {}_7C_2}{{}_{11}C_4}$$

$$= 0.3818$$

c) Since no red marbles may be chosen, the second pair of marbles may only be chosen from the five black marbles.

$$P(2) = \frac{{}_4C_2 \times {}_5C_2}{{}_{11}C_4}$$

$$= 0.1818$$

d) Red: Black: Green:

$$E(x) = \frac{4 \times 2}{11} \qquad E(x) = \frac{4 \times 5}{11} \qquad E(x) = \frac{4 \times 4}{11}$$

$$= 0.73 \qquad\qquad\quad = 1.82 \qquad\qquad\quad = 1.45$$

Section 7.4   **Page 405**   **Question 14**

a) There are ${}_{40}C_5 = 658\,008$ ways of selecting the first five numbers. For each of these, there are ${}_{35}C_1 = 35$ ways of selecting the bonus number. Therefore, the probability of winning the first prize is $\dfrac{1}{658\,008 \times 35} = 4.34 \times 10^{-8}$.

b) In order to win second prize, only 4 of the regular 5 numbers chosen must match. There are ${}_5C_4 = 5$ ways of doing this. Therefore, the probability of winning the second prize is

$$\frac{5}{658\,008 \times 35} = 2.17 \times 10^{-7}.$$

c) If your first three regular numbers match winning numbers, the number of outcomes is reduced. There are $_{37}C_2 = 666$ ways of selecting the remaining two regular numbers, and $_{35}C_1 = 35$ ways of selecting the bonus number. Of these, $_{35}C_2 = 595$ will not match the winning numbers, and $_{33}C_1 = 33$ will not match the bonus number. Therefore, the probability of not winning a prize is $\dfrac{595 \times 33}{666 \times 35} = 0.842$.

Section 7.4   **Page 405**   **Question 16**

The number of ways of travelling from your location to your friend's location is shown in the table at the right. Each of you must walk 10 blocks to reach the other's starting point. Since your speeds are the same, any meetings will occur after each of you has walked five blocks. These places occur along the diagonal of the table. There are $1 + 5 + 10 + 10 + 5 + 1 = 32$ ways of reaching

1	6	21	56	126	252
1	5	15	35	70	126
1	4	10	20	35	56
1	3	6	10	15	21
1	2	3	4	5	6
You	1	1	1	1	1

these meeting points. The probability that you will meet in the northwest corner of the table is the product of the probabilities of each of you selecting that path, which is $\dfrac{1}{32} \times \dfrac{1}{32}$. Therefore, the probability that the two of you will meet is the sum of the probabilities of meeting in any of the six possible locations. This is given by

$$\left(\frac{1}{32}\right)^2 + \left(\frac{5}{32}\right)^2 + \left(\frac{10}{32}\right)^2 + \left(\frac{10}{32}\right)^2 + \left(\frac{5}{32}\right)^2 + \left(\frac{1}{32}\right)^2 = \frac{252}{1024}$$
$$= \frac{63}{256}$$

CHAPTER 7 Probability Distributions

Review of Key Concepts

Review of Key Concepts   **Page 406**   **Question 2**

a) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The expected move per spin is 3.5 squares forwards.

b) Since the expected value is not zero, this is not a fair game. The remainder of this answer will vary.

Microsoft Excel - Chapter 7 Review Question 2.x

File Edit View Insert Format Tools Data Win

G8 =

	A	B	C	D
1	Number	x	P(x)	xP(x)
2	1	2	0.1	0.2
3	2	-1	0.1	-0.1
4	3	6	0.1	0.6
5	4	-2	0.1	-0.2
6	5	10	0.1	1
7	6	-3	0.1	-0.3
8	7	14	0.1	1.4
9	8	-4	0.1	-0.4
10	9	18	0.1	1.8
11	10	-5	0.1	-0.5
12				
13			E(x)	3.5

Review of Key Concepts   **Page 406**   **Question 4**

A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The expected area is 57 m^2.

Microsoft Excel - Chapter 7 Review Question 4.xls

File Edit View Insert Format Tools Data Window Help A

G5 =

	A	B	C	D	E
1	L	W	A	P(A)	AP(A)
2	1	17	17	1/17	1
3	2	16	32	1/17	1 15/17
4	3	15	45	1/17	2 11/17
5	4	14	56	1/17	3 5/17
6	5	13	65	1/17	3 14/17
7	6	12	72	1/17	4 4/17
8	7	11	77	1/17	4 9/17
9	8	10	80	1/17	4 12/17
10	9	9	81	1/17	4 13/17
11	10	8	80	1/17	4 12/17
12	11	7	77	1/17	4 9/17
13	12	6	72	1/17	4 4/17
14	13	5	65	1/17	3 14/17
15	14	4	56	1/17	3 5/17
16	15	3	45	1/17	2 11/17
17	16	2	32	1/17	1 15/17
18	17	1	17	1/17	1
19					
20				E(A)	57

Review of Key Concepts   Page 406   Question 6

A binomial distribution occurs when there are a number of trials in which each trial can have only two possible outcomes. The probabilities of these outcomes must remain constant for each trial. One example is the number of times a 1 is rolled in 10 rolls of a fair die.
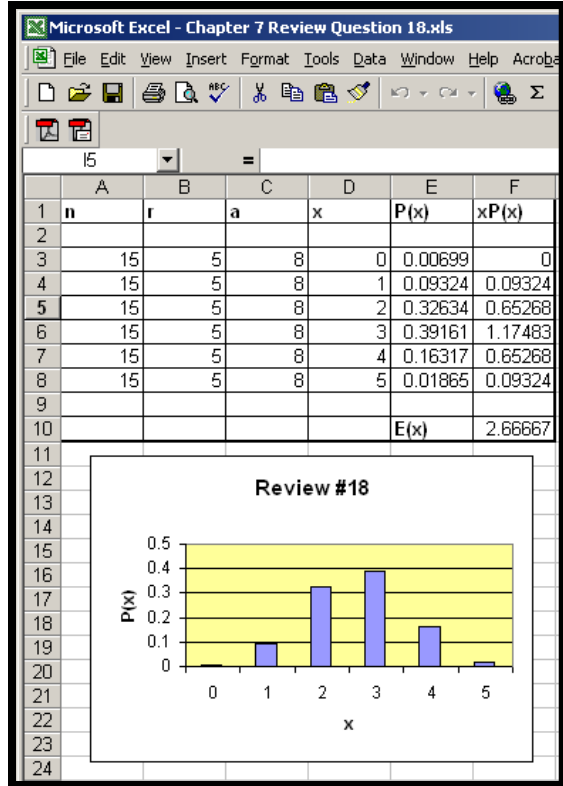
Review of Key Concepts   Page 406   Question 8

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for $P(x)$ when $x = 0$ is COMBIN(5,A2)*B2^A2*(1-B2)^(5-A2) for Microsoft® Excel. If you are using Corel® Quattro® Pro, the formula is @COMB(A2,5)*B2^A2*(1-B2)^(5-A2).
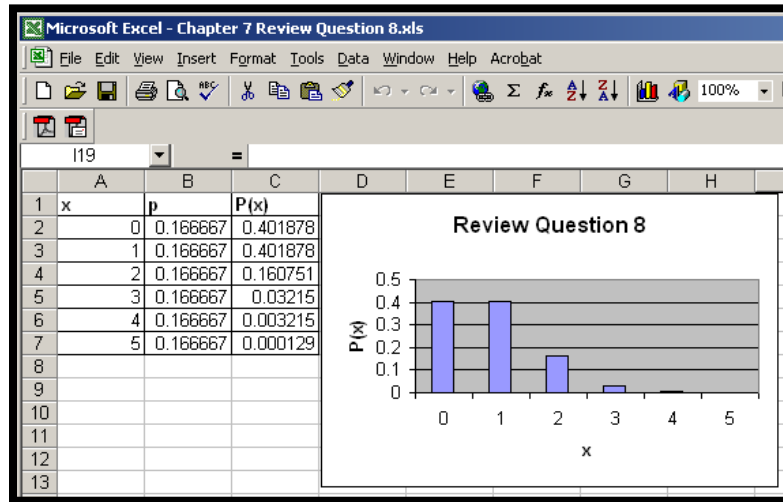
Review of Key Concepts   Page 406   Question 10

a) In this question, $p = \dfrac{1}{20}$, $n = 10$, and $x = 5$. Therefore,

$$P(x) = {}_nC_x p^x q^{n-x}$$

$$P(5) = {}_{10}C_5 \left(\frac{1}{20}\right)^5 \left(\frac{19}{20}\right)^5$$

$$= 0.000\ 0609$$

b) The expected number of times that the dart would land in a given sector is

$$E(x) = np$$

$$= 10 \times \frac{1}{20}$$

$$= 0.5$$

Review of Key Concepts   Page 407   Question 12

A geometric distribution is one in which there are independent trials that have two possible outcomes. The probabilities of these outcomes are constant in each trial. The independent variable is the number of failures before the first success. An example is the number of rolls of a fair die before a 1 is rolled.

Review of Key Concepts   Page 407   Question 14

a) The probability of getting five good boards followed by a defective board is $(0.976)^5(0.024) = 0.0213$.

b) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The probability of the first defective circuit board being among the first six is 0.1356. The formula for $P(x)$ when $x = 0$ is (1-B2)^A2*B2.

Microsoft Excel - Chapter 7 Review
File Edit View Insert Format Tools

	A	B	C
1	x	p	P(x)
2	0	0.024	0.024
3	1	0.024	0.023424
4	2	0.024	0.022862
5	3	0.024	0.022313
6	4	0.024	0.021778
7	5	0.024	0.021255
8			
9		Sum	0.135632

c) Use the expectation formula for a geometric distribution.

$$E(x) = \frac{q}{p}$$
$$= \frac{0.976}{0.024}$$
$$= 40.7$$

Therefore, the expected waiting time until the first defective circuit board is found is 40.7 boards.

Review of Key Concepts   Page 407   Question 16

A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. Note that x in this problem represents the number of rolls required to win. Therefore, the formula for $P(x)$ must be modified. The formula for $P(x)$ for $x = 1$ is (1-B2)^(A2-1)*B2.

Microsoft Excel - Book1
File Edit View Insert Format Tools Data Wind

C2   = =(1-B2)^(A2-1)*B2

	A	B	C	D
1	x	p	P(x)	xP(x)
2	1	0.027778	0.027778	0.000772
3	2	0.027778	0.027006	0.00075
4	3	0.027778	0.026256	0.000729
5	4	0.027778	0.025527	0.000709
6	5	0.027778	0.024818	0.000689
7	6	0.027778	0.024128	0.00067
8	7	0.027778	0.023458	0.000652
9	8	0.027778	0.022806	0.000634
10	9	0.027778	0.022173	0.000616
11	10	0.027778	0.021557	0.000599

Review of Key Concepts   Page 407   Question 18

a) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text.

b) The expected number of solutions in the display that were prepared by mathematics of data management students is 2.7.

	A	B	C	D	E	F
1	n	r	a	x	P(x)	xP(x)
2						
3	15	5	8	0	0.00699	0
4	15	5	8	1	0.09324	0.09324
5	15	5	8	2	0.32634	0.65268
6	15	5	8	3	0.39161	1.17483
7	15	5	8	4	0.16317	0.65268
8	15	5	8	5	0.01865	0.09324
9						
10					E(x)	2.66667

Review #18

Review of Key Concepts   Page 407   Question 20

In this question, $a = 98$, $r = 50$, and $x = 32$. Therefore,

$$E(x) = \frac{ra}{n}$$
$$32 = \frac{50 \times 98}{n}$$
$$32n = 50 \times 98$$
$$n = 153$$

The size of the beaver population is estimated to be 153.

CHAPTER 7   Probability Distributions

Chapter Test

Chapter Test   **Page 408**   **Question 2**

A spreadsheet solution for this problem is shown at the right. If
you need information on how to use spreadsheets, refer to
Appendix B of the student text. The probabilities in column B
are calculated by taking the number of prizes and dividing by
the number of tickets sold. The expected return on each ticket is
$0.14. Since a ticket costs $2.00, each ticket is valued at
$-$1.86.

	A	B	C
1	x	P(x)	xP(x)
2	500000	2.22222E-07	0.111111
3	50000	4.44444E-07	0.022222
4	5000	1.11111E-06	0.005556
5	500	4.44444E-06	0.002222
6	50	2.22222E-05	0.001111
7			
8		E(x)	0.142222

Chapter Test   **Page 408**   **Question 4**

A spreadsheet solution for this problem is shown at the right.
If you need information on how to use spreadsheets, refer to
Appendix B of the student text. The expected area of the
circle is 34.56 cm^2.

	A	B	C	D
1	n	A	P(A)	AP(A)
2		1	3.141593	0.2	0.628319
3		2	12.56637	0.2	2.513274
4		3	28.27433	0.2	5.654867
5		4	50.26548	0.2	10.0531
6		5	78.53982	0.2	15.70796
7				
8			E(A)	34.55752

Chapter Test   **Page 408**   **Question 6**

a) A spreadsheet solution for this problem is shown at the
right. If you need information on how to use spreadsheets,
refer to Appendix B of the student text. The probability that
a triple occurs on any given roll of three dice is $\frac{6}{216} = \frac{1}{36}$.
This is a geometric distribution. The formula for $P(x)$ when
$x = 0$ is C3^A3*B3. The probability that a triple will occur
within the first five rolls is 0.1314, as shown at the right.

b) Use the expectation formula for a geometric distribution.

$$E(x) = \frac{\frac{35}{36}}{\frac{1}{36}} = 35$$

The expected waiting time before a triple occurs is 35 rolls.

	A	B	C	D
1	x	p	q	P(x)
2				
3	0	0.027778	0.972222	0.027778
4	1	0.027778	0.972222	0.027006
5	2	0.027778	0.972222	0.026256
6	3	0.027778	0.972222	0.025527
7	4	0.027778	0.972222	0.024818
8				
9			Sum	0.131384

Chapter Test   **Page 409**   **Question 8**

a) This is a binomial distribution with $n = 10$. The probability of getting an answer right on a guess is $p = 0.25$. The probability of getting 1 or 2 guesses correct is given by

$$P(1) + P(2) = {}_{10}C_1(0.25)(0.75)^9 + {}_{10}C_2(0.25)^2(0.75)^8$$
$$= 0.469$$

b) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for $P(x)$ when $x = 6$ is:
Microsoft® Excel
COMBIN(10,A2)*0.25^A2*0.75^(10-A2).
Corel® Quattro® Pro
@COMB(A2,10)*0.25^A2*0.75^(10-A2).

	A	B
1	x	P(x)
2	6	0.016222
3	7	0.00309
4	8	0.000386
5	9	2.86E-05
6	10	9.54E-07
7		
8	Sum	0.019728

c) The expected number of correct guesses is

$$E(x) = np$$
$$= 10 \times 0.25$$
$$= 2.5$$

Chapter Test   **Page 409**   **Question 10**

Answers may vary. Fathom™ lends itself very well to this kind of simulation. If you need instructions on how to use Fathom™, refer to Appendix B of the student text.

1) Open Fathom™ and drag a case table to the work area. Name an attribute **Toys**, and create three cases. Edit the formula for **Toys** to **randomInteger**(1,3), as shown at the right.

2) Right-click on the collection box and open the **Inspector** for the collection. Select the **Measures** tab. Create a measure called **HowMany**, and edit the formula to **uniqueValues(Toys)**, as shown at the right.

3) Ensure that the collection box is selected, and select **Collect Measures** from the **Analyse** menu. Note that a new collection has been created. Open the **Inspector** for this new collection. Select the **Collect Measures** tab. Turn off the **Animation**, and set the number of measures to 1000. Click on the **Collect More Measures** button.
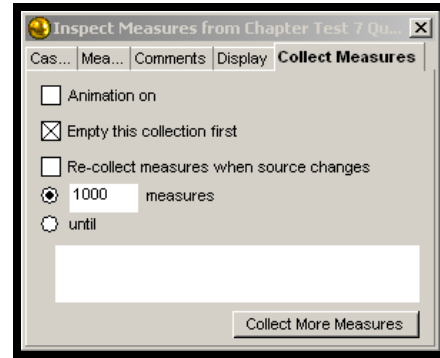
4) Select the **Measures** collection box, and drag a new case table to the work area. This will contain one thousand measures from the original collection, as shown at the right.

5) To see the empirical probability that you obtained three different toys, select the **Measures** collection box, and click on the **Measures** tab in the **Inspector**. Create a new measure called **Probability**, and edit the formula to

$$\frac{\text{count}(\text{HowMany} = 3)}{1000}.$$

It appears that the probability of getting three different toys in three purchases of a burger is about 0.429. This gives odds in favour of success of 429:571 or roughly 2:3.

6) At this point you can add new cases to the original collection and collect more measures. You can try 4 purchases, then 5, and so on, until you achieve the desired odds in favour of success of 3:1.

Chapter Test   **Page 409**   **Question 12**

Solutions for Achievement Checks are described in the Teacher's Resource.
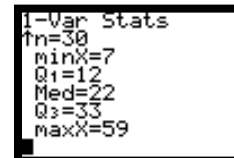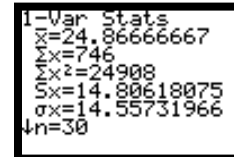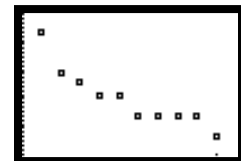
CHAPTER 8   The Normal Distribution

Review of Prerequisite Skills

a) Enter the data into a list on your graphing calculator. If you need instructions on using a graphing calculator, refer to Appendix B of the student text. Set your **STAT PLOT** for a scatter plot. If necessary, use **ZoomStat** under the **ZOOM** menu to adjust the window.
A histogram of these data is shown in the screen shot at the right.
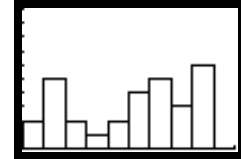
b) Use the **1-Var Stats** command under the **STAT CALC** menu to determine the mean of 150.5 and the standard deviation of 27.2, as shown in the screen shot at the right.
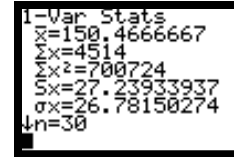
c) A box-and-whisker plot of these data is shown at the right. The minimum is 102, the first quartile is at 122, the median is 157, the third quartile is at 171, and the maximum is 187, as shown in the continuing **1-Var Stats** screen at the right.
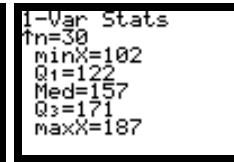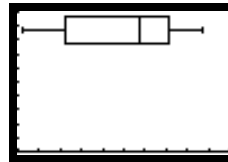
Enter the data into a list on your graphing calculator. If you need instructions on using a graphing calculator, refer to Appendix B of the student text. Use the **SortA(** function under the **STAT EDIT** menu to sort the data. Scroll through the data to find the modes at 12, 14, and 22. Use the **1-Var Stats** command under the **STAT CALC** menu to determine the mean of 24.9, the median of 22, and the standard deviation of 14.8, as shown in the screen shots at the right.

Enter the data into two lists on your graphing calculator. If you need instructions on using a graphing calculator, refer to Appendix B of the student text. Set your **STAT PLOT** for a scatter plot. If necessary, use **ZoomStat** under the **ZOOM** menu to adjust the window. The scatter plot is shown in the screen shot at the right.

a) The *z*-score of a 25-year-old viewer is given by

$$\frac{x - \mu}{\sigma} = \frac{25 - 38.3}{12.71}$$
$$= -1.05$$

b) The *z*-score of a 70-year-old viewer is given by

$$\frac{x-\mu}{\sigma} = \frac{70-38.3}{12.71}$$
$$= 2.49$$

c) The *z*-score of a 40-year-old viewer is given by

$$\frac{x-\mu}{\sigma} = \frac{40-38.3}{12.71}$$
$$= 0.13$$

d) The age range within 1 standard deviation of the mean is 25.59 to 51.01.

e) The age range within 2 standard deviations of the mean is 12.88 to 63.72.

f) The age range within 1.3 standard deviations of the mean is 21.78 to 54.82.

Review of Prerequisite Skills   Page 413   Question 10

a) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the probability that *x* = 0 is:

Microsoft® Excel
COMBIN(12,A2)*0.85^A2*0.15^(12-A2)

Corel® Quattro® Pro
@COMB(A2,12)*0.85^A2*0.15^(12-A2)

The probability that exactly 8 of the 12 people own a VCR is 0.0683, as shown in cell B10 at the right.

b) The probability that no more than 8 people own a VCR is calculated by summing the probabilities from *x* = 0 to *x* = 8. The answer is 0.0922, as shown in cell B16 of the spreadsheet.

c) The probability that 8 people or more own a VCR is calculated by summing the probabilities from *x* = 8 to *x* = 12. The answer is 0.9761, as shown in cell B17 of the spreadsheet.

	A	B
1	x	P(x)
2	0	1.29746E-10
3	1	8.82275E-09
4	2	2.74976E-07
5	3	5.19399E-06
6	4	6.62233E-05
7	5	0.000600425
8	6	0.003969475
9	7	0.019280308
10	8	0.068284423
11	9	0.171975583
12	10	0.29235849
13	11	0.301217839
14	12	0.142241757
15		
16	Sum 0-8	0.092206331
17	Sum 8-12	0.976078091

d) The expected number of VCR owners is

$$E(x) = np$$
$$= 12 \times 0.85$$
$$= 10.2$$

CHAPTER 8   The Normal Distribution

Section 8.1   Continuous Probability Distributions

Practise
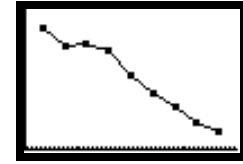
Section 8.1   Page 419   Question 2

a) About 76.

b) Answers may vary.

Section 8.1   Page 420   Question 4

a) The scatter plot for these data is shown at the right.

**b)** The distribution appears to be positively skewed.

c) The decrease in accidents with age may be a result of fewer older workers than younger workers, or it may be that older workers are more experienced and better able to avoid accidents.

d) Answers will vary. The mean appears to be about 420. The mean tells you the average number of accidents per age interval.

Section 8.1   Page 420   Question 6

a) The graph of this distribution is shown at the right. The area under the graph from 18 to 30 is 1. Therefore, the height of the graph must be

$$\frac{1}{30-18} = 0.083.$$

This graph was generated using Fathom™. If you need instructions on using Fathom™, refer to Appendix B of the student text.
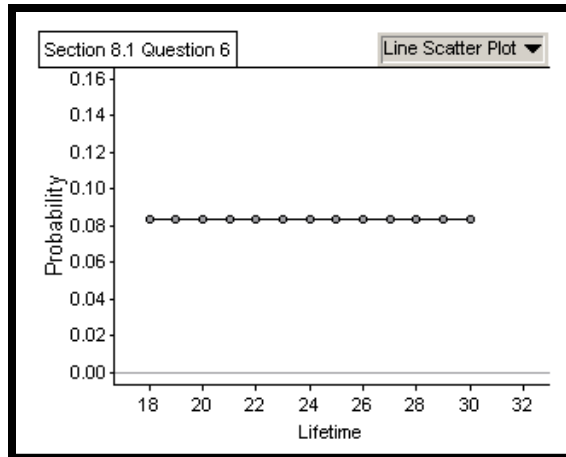
b) The probability that a fly selected at random will live at least 28 h is calculated by finding the area under the graph from 18 h to 28 h. This is $\frac{10}{12} = \frac{5}{6}$ of the total area. Therefore, the required probability is $\frac{5}{6}$.

c) The probability that a fly selected at random will live for less than 24 h is calculated by finding the area under the graph from 18 h to 24 h. This is $\frac{6}{12} = \frac{1}{2}$ of the total area. Therefore, the required probability is $\frac{1}{2}$.

Section 8.1   **Page 421**   **Question 8**

a) The maximum value of $P(X \le x)$ will occur when e^{-kx} is as small as possible. This occurs when x is an arbitrarily large number, as shown in the table at the right. An arbitrarily large number is usually given the symbol ∞. As $x \to \infty$, $e^{-kx} \to 0$. Therefore, the maximum value of $P(X \le x)$ is given by

$$1 - e^{-kx} = 1 - 0$$
$$= 1$$

Microsoft Excel - Section 8.1 Question 8.xls

File   Edit   View   Insert   Format   Tools   Data   W

B2   =   =1-EXP(-0.5*A2)

	A	B	C	D
1	x	P		
2	1	0.393469		
3	5	0.917915		
4	10	0.993262		
5	50	1		
6	100	1		
7	1000	1		

b) The estimates for $P(X \le 3)$ in Example 2 are both 0.8. Since $\mu = 2$,

$$k = \frac{1}{\mu}$$
$$= \frac{1}{2}$$
$$= 0.5$$

Putting $x = 3$ and $k = 0.5$ into the formula in this question gives

$$P(X \le x) = 1 - e^{-kx}$$
$$P(X \le 3) = 1 - e^{-0.5(3)}$$
$$= 0.78$$

c) Since the simulation uses random numbers, the estimate from the simulation may not yield the same result as the method using calculus. Estimating the area under the curve using the midpoints also introduces some error.

d) A longer simulation would reduce errors introduced by random chance. Smaller intervals would decrease the errors in estimating the area under the curve.

Section 8.1   Page 421   Question 10

a) The bar graph for the earthquake data is shown in the screen shot at the right. This graph was prepared using Microsoft® Excel. If you need instructions on how to create graphs using spreadsheets, refer to Appendix B of the student text.

Earthquake Data
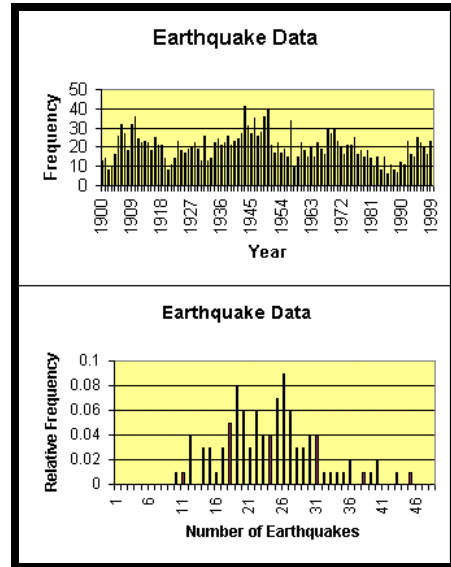
b) The bar graph for the relative frequency is shown in the screen shot at the right. This graph was prepared using Microsoft® Excel.

Earthquake Data

c) The mean is 20.05, and the standard deviation is 7.23. These were calculated using the appropriate functions built into Microsoft® Excel.

Section 8.1   Page 421   Question 12

Answers may vary.

CHAPTER 8   The Normal Distribution

Section 8.2   Properties of the Normal Distribution

Practise

Section 8.2   Page 430   Question 2

Use the **normalcdf(** function on a graphing calculator for this question, as shown in the screen shot at the right. If you need information on using graphing calculators, refer to Appendix B of the student text. In this case, the lower bound is –1E99, the upperbound is 190 cm, the mean is 165 cm, and the standard deviation is 20 cm, giving a probability of 0.8944 that Michael's best friend is shorter than he is.

Section 8.2   Page 430   Question 4

a) If Joey scored 78%, then the probability that others scored below him is 88%, as shown in the screen shot at the right. Use the **normalcdf(** function on a graphing calculator to determine this. If you need information on using graphing calculators, refer to Appendix B of the student text. Therefore, Joey scored at the 88th percentile.
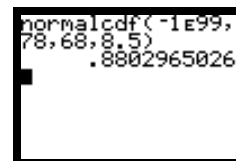
b) If Shaheed obtained 55%, then he scored at the 6th percentile, as shown at the right.
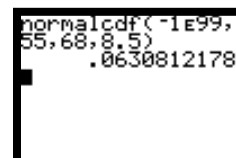
c) If Michelle obtained 89%, then she scored at the 99th percentile, as shown at the right

```
normalcdf(-1E99,
89,68,8.5)
         .9932554398
```

Section 8.2   **Page 431**   **Question 6**

The probability that a crankshaft will have a diameter of less than 223.92 mm is 0.00383, as shown in the screen shot at the right. The probability that a crankshaft will have a diameter of more than 224.08 mm is 0.00383. Since a crankshaft will cause damage if it lies in either of these two regions, the probability that a crankshaft will cause damage is 0.00383 + 0.00383 = 0.00766 or 0.766%.

```
normalcdf(-1E99,
223.92,224,.03)
        .0038304251
```
```
normalcdf(224.08
,1E99,224,.03)
        .0038304251
```

Section 8.2   **Page 431**   **Question 8**

a) The probability that there will be fewer than 40 shots on goal is 0.0221 or 2.21%, as shown in the screen shot at the right.

```
normalcdf(-1E99,
40,56.3,8.1)
        .0220916856
```

b) The probability that there will be between 50 and 60 shots on goal is 0.4577 or 45.77%, as shown in the screen shot at the right.

```
normalcdf(50,60,
56.3,8.1)
        .4577390843
```

c) The probability that there will be more than 66 shots on goal is about 0.116, or 11.6%, as shown in the screen shot at the right.

```
normalcdf(66,1E9
9,56.3,8.1)
        .1155499136
```

Section 8.2   **Page 431**   **Question 10**

a) The probability that a teenage girl's height is greater than 170 cm is 0.0316, as shown in the screen shot at the right.

```
normalcdf(170,1E
99,157,7)
        .0316453536
```

b) Since you know the desired probability, and want to find the height that delineates that probability, you must use the **invNorm(** function. The probability that a teenage girl's height falls in the lowest 5% of the population is 0.05. Using the **invNorm(** function, this gives a lower height limit of 145.49 cm. Similarly, the probability that a teenage girl's height falls in the lower 95% of the population is 0.95. This gives an upper height limit of 168.51 cm, as shown in the screen shots above. Therefore, the range of heights between 145.49 cm and 168.51 cm would occur 90% of the time.

```
invNorm(.05,157,
7)
        145.4860246
```
```
invNorm(.95,157,
7)
        168.5139754
```

Section 8.2   Page 431   Question 12

Since you want the middle 75% of the seedlings, you need the maximum height for the lowest 12.5% and the maximum height for the lowest 87.5%. The maximum height for the lowest 12.5% can be calculated using the **invNorm(** function on a graphing calculator, as shown in the screen shot at the right. This height is 20 cm. The maximum height for the lowest 87.5% is 43 cm, as shown in the screen shot at the right. Therefore, the range of heights between which 75% of the seedlings will grow is 20 cm to 43 cm.

```
invNorm(.125,31.
5,10)
         19.99650621
```

```
invNorm(.875,31.
5,10)
         43.00349379
```

CHAPTER 8   The Normal Distribution

Section 8.3   Normal Sampling and Modelling

Practise

Section 8.3   Page 439   Question 2

This exercise will be solved using Fathom™. If you need information on how to use Fathom™, you may wish to review Introduction to Fathom™ in Chapter 1 of the student text, the solution to Introduction to Fathom™ included in Chapter 1 of the Solutions, or the presentation Introduction to Fathom™ on the Student e-book CD-ROM.

a) Start Fathom™ and create a new **case table**. Rename the <new> attribute **Salaries**. Input the data from the exercise. Rename the collection **Section 8.3 Question 2**. Select a **Summary Table** from the **Insert** menu. Drag the **Salaries** attribute to the **Down arrow**. Note that the mean is automatically calculated. The mean is $32 461.24. Select **Add Formula** from the **Summary** menu. Navigate to **Functions/Statistical/One-Attribute** and select **stdDev**. Note that the standard deviation is added to your summary table. The standard deviation is $3375.77.

b) Select **Add Formula** from the **Summary** menu. Navigate to **Functions/Statistical/One-Attribute** and select **count()**. Inside the brackets type **Salaries < 30000**. Outside the brackets add **/50**. Your final screen will look like the screen shot below. The probability that a graduate will have an entry-level salary below $30 000 is 0.26.

Section 8.3   Page 439   Question 4

a) A Fathom™ solution for this problem is shown below. If you need information on how to use Fathom™, refer to Appendix B of the student text. Download the required data from the Large Data Sets on the Student e-book, or directly from **E-STAT**. Drag a blank graph to the workspace. Drag the Canada attribute to the horizontal axis. Change the type of graph to Histogram.
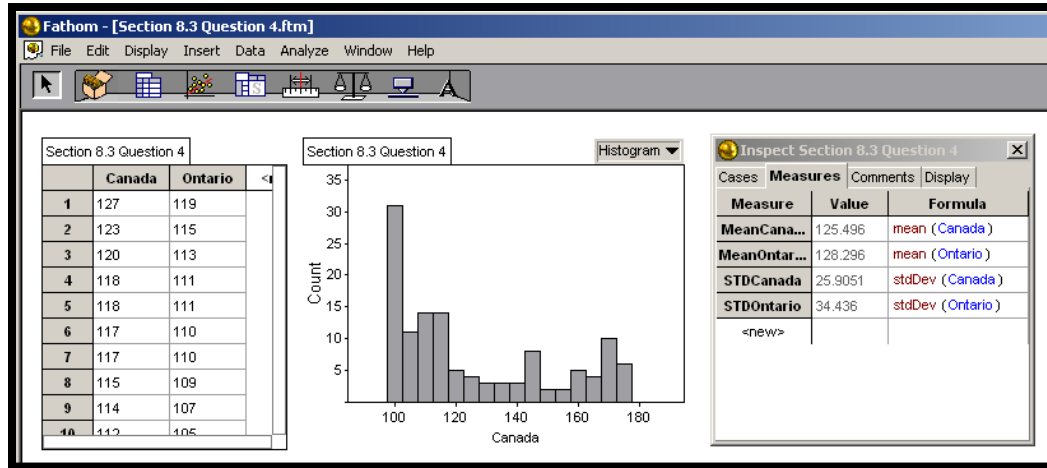
b) According to the histogram, the data do not appear to be normally distributed.

c) Open the **Inspector** by double-clicking on the collection. Use the built-in functions to calculate the means and standard deviations as shown. For Canada, the mean and standard deviation are 125.5 and 25.9. For Ontario, the mean and standard deviation are 128.3 and 34.4.

d) Since the mean for Ontario is higher than the mean for Canada, you may infer that it was easier to find a job in Ontario than in the rest of Canada during this period.

Section 8.3   Page 440   Question 6

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the mean in Microsoft® Excel is AVERAGE(B2:B13). The formula for the standard deviation is STDEV(B2:B13). If you are using Corel® Quattro® Pro, the equivalent formulas are @MEAN(B2..B13) and @STDS(B2..B13). **NOTE**: When you download data from E-STAT, you may get some cells containing #N/A. You must change this to NA, or you will get an error when trying to use the cell in a function.

	A	B	C	D	E	F	G	H
1	Thousands of Tonnes	1990	1991	1992	1993	1994	1995	1996
2	Alberta	166000	167000	172000	179000	186000	194000	199000
3	British Columbia	51200	51400	51300	54500	56300	60500	62400
4	Manitoba	20700	20400	20700	20200	20500	21400	22300
5	New Brunswick	16000	15500	16100	15300	16300	17700	16800
6	Newfoundland	9360	8510	8510	7180	8250	8250	8360
7	Northwest Territorie	2230	2230	1530	1690	2010	1960	2100
8	Nova Scotia	19400	19300	20000	2000	19400	19100	19400
9	Ontario	182000	181000	185000	178000	179000	183000	191000
10	Prince Edward Island	1950	1930	1940	1950	1870	1870	1990
11	Quebec	85500	81100	81800	84800	86300	86400	87000
12	Saskatchewan	46500	46500	50700	51600	55200	57300	59300
13	Yukon Territory	NA	NA	683	641	536	973	638
14								
15	Mean	54622	54079	50855	49738	52639	54371	55857
16	Standard Deviation	64118.8	64008.91	64550.22	65758.98	66188.06	68271.74	70617.6

**Section 8.3   Page 440   Question 8**

a) Enter the data into a list on your graphing calculator. If you need information on using graphing calculators, refer to Appendix B of the student text. Select the **1-Var Stats** command under the **STAT CALC** menu to obtain the screen shot at the right. The mean is 43.9, and the standard deviation is 10.866. In this question, it is assumed that the data represent the population and not a sample.

```
1-Var Stats
x̄=43.93333333
Σx=659
Σx²=30723
Sx=11.24701018
σx=10.86564412
↓n=15
```

b) The probability of hitting more than 46 home runs is estimated by using the **normalcdf(** function under the **DISTR** menu, as shown in the screen shot at the right. Since you are using the normal distribution with discrete data, the lower limit is 46.5. Since this estimate is a sample, use the sample standard deviation of 11.2. The probability of hitting more than 46 home runs is 0.408.

```
normalcdf(46.5,1
E99,43.9,11.2)
          .4082135719
```

Section 8.3 **Page 440** **Question 10**

a) A Fathom™ solution for this problem is shown below. If you need information on how to use Fathom™, refer to Appendix B of the student text. Group the density data into suitable intervals, as shown in the screen shot below. Create a frequency graph for density as shown. These data do not appear to be normally distributed.
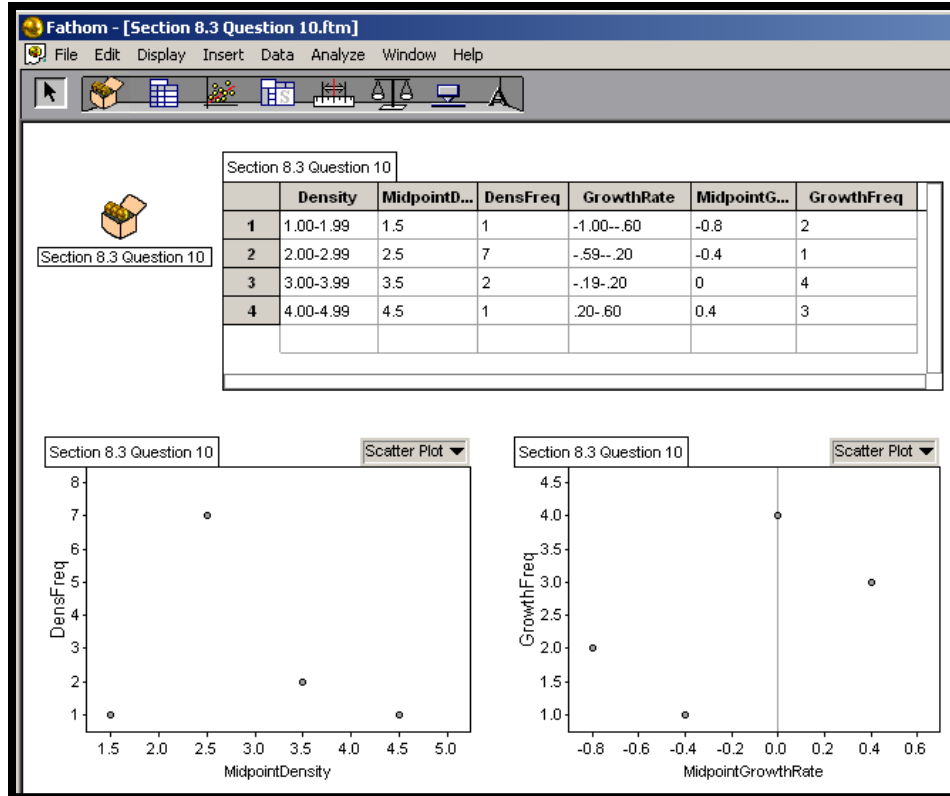
b) Extend the **Case Table** to include intervals for the population growth rate, as shown above. Create a frequency graph for the population growth rate. These data do not appear to be normally distributed.

c) This data set is very small, and the effects of random chance are large. If you had a larger set of data, the distribution might appear closer to a normal distribution.

Section 8.3 **Page 441** **Question 12**

Solutions for Achievement Checks are described in the Teacher's Resource.

CHAPTER 8 The Normal Distribution

Technology Extension **Page 442** **Normal Probability Plots**

1. Enter the data into a list on your graphing calculator. Use the **1-Var Stats** command under the **STAT CALC** menu to calculate the mean as 20 min and the standard deviation as 5.71 min, as shown in the screen shot at the right.

2. Select **Plot 1** under the **STAT PLOT** menu, and set up your parameters as shown in the screen shot at the right. Press the GRAPH key to obtain the normal probability plot, as shown in the screen shot at the right. Since the plot is straight, the assembly times appear to be normally distributed.
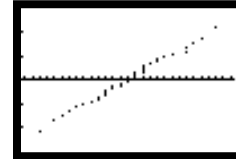
3. a) Since the mean is 20 min, the probability that an adult can complete this assembly in 20 min or less is 0.5. This can be verified by using the **normalcdf(** function under the **DISTR** menu, as shown in the screen shot at the right.
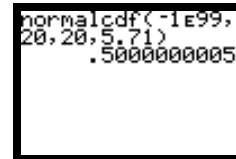
b) The proportion of adults that should complete this assembly within 15 min to 30 min is 76.9% , as shown in the screen shot at the right.
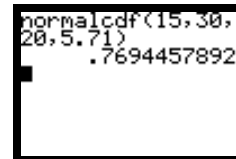
4. a) This part of the extension will use Fathom™. If you need instructions on how to use Fathom™, refer to Appendix B of the student text. Open Fathom™, and open a new document, if necessary. Drag a new **collection** box to the workspace. Rename the collection **Assembly Times**, and create 36 new cases.

b) Drag a new **case table** to the workspace. Name the first column **Times**, the second column **zTimes**, and the third column **Quantiles**.

c) i) Enter the time data in the first column. **Sort** it into ascending order.

ii) Edit the formula in the second column to **zScore(Times)**. This will calculate the z-scores for the data.

iii) Edit the formula in the third column to **normalQuantile((uniqueRank(Times)**-0.5)/36/0/1. The 0.5 in the formula ensures that the normal quantile is calculated for the midpoint of each of the 36 segments of the distribution. This formula will calculate the expected z-score if the data were indeed distributed normally. The **uniqueRank()** function returns the "row number" of the sorted data. Note that most of the quantile z-scores in the following screen shot are different from the z-scores for the corresponding data.

Fathom - [Technology Extension Question 4.ftm]

File Edit Display Insert Data Analyze Window Help

Assembly Times

	Times	zTimes	Quantiles	
=		zScore (Times)	normalQuantile ($\frac{uniqueRank\ (Times)\ -\ 0.5}{36}$, 0, 1)	
1	8	-2.10079	-2.20041	
2	10	-1.75066	-1.73166	
3	11	-1.57559	-1.47994	
4	12	-1.40053	-1.29754	
5	13	-1.22546	-1.15035	
6	14	-1.05039	-1.02459	
7	15	-0.875328	-0.91325	
8	16	-0.700263	-0.718868	
9	16	-0.700263	-0.812218	
10	17	-0.525197	-0.548522	
11	17	-0.525197	-0.469253	
12	17	-0.525197	-0.631402	

Assembly Times   Scatter Plot ▼

Quantiles = 0.174Times - 3.48; r^2 = 0.99

Inspect Assembly Times

Cases **Measures** Comments Display

Measure	Value	Formula
Mean	20	mean (Times)
StDev	5.71214	stdDev (Times)
P20orless	0.5	normalCumulative (20, Mean, StDev)
P15to30	0.7693	normalCumulative (30, Mean, StDev) − normalCumulative (15, Mean, StDev)
<new>		

d) Drag a new graph to the workspace. Drag the **Times** attribute to the horizontal axis, and the **Quantiles** attribute to the vertical axis to generate a normal probability plot. Select **Least Squares Fit** from the **Graph** menu. The linear correlation coefficient for **Times** and **Quantiles** is 0.995, indicating that the data are likely normally distributed.

e) Double-click on the **collection** to open the **Inspector**. Choose the **Measures** tab. Create four measures: **Mean**, **StdDev**, **P20orLess**, and **P15to30**. Use the **mean**, **standard deviation**, and **normalCumulative** functions to calculate the mean, the standard deviation, and the answers to question 3, as shown in the previous screen shot.
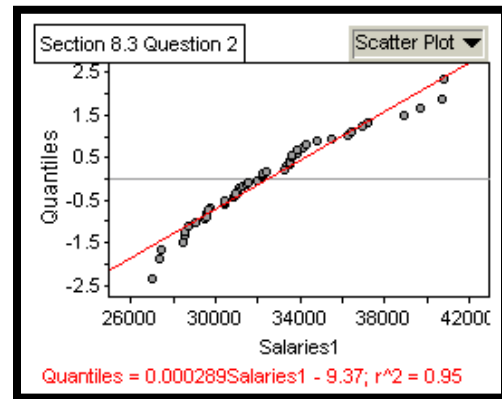
5. The Fathom™ file generated in question 4 can be used as a template to generate the normal probability plots for the questions in Section 8.3, as shown below.
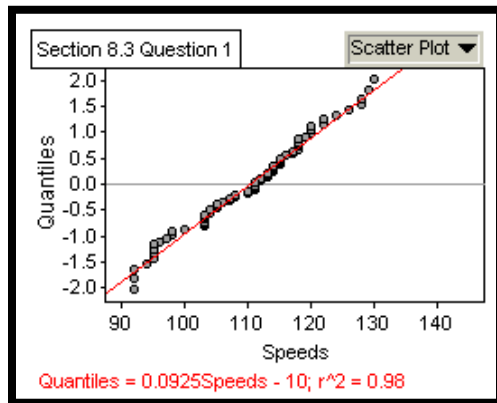
Section 8.3, Question 1.   Section 8.3, Question 2.

      Data appear normally distributed.   Data appear normally distributed.

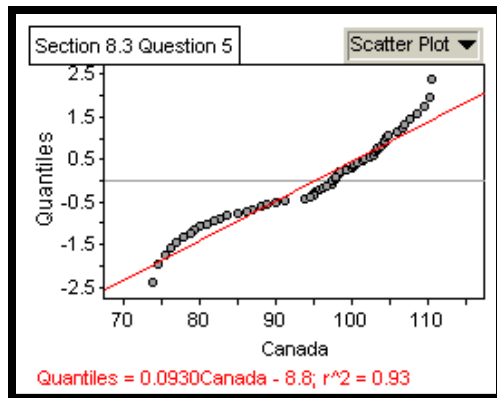Section 8.3, Question 3.   Section 8.3, Question 4.
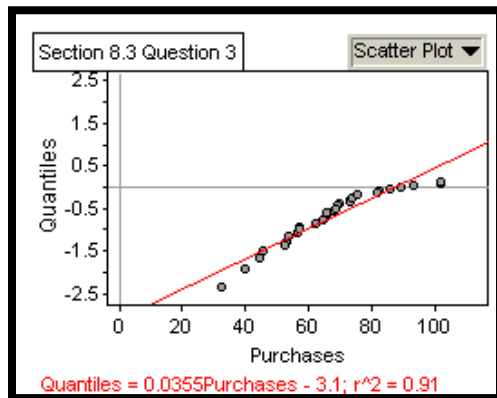
    Data do not appear normally distributed.   Data do not appear normally distributed.
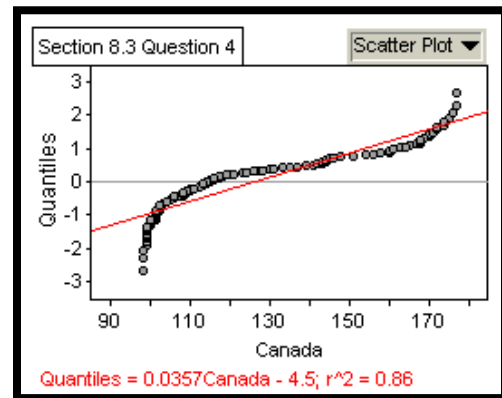
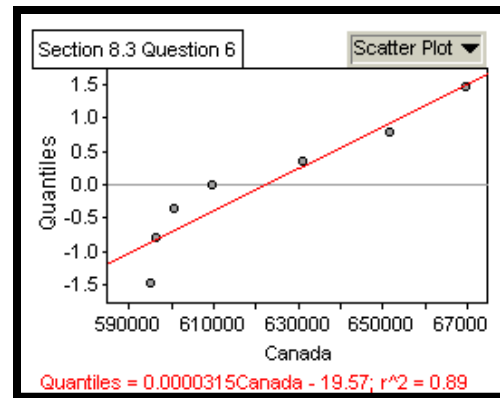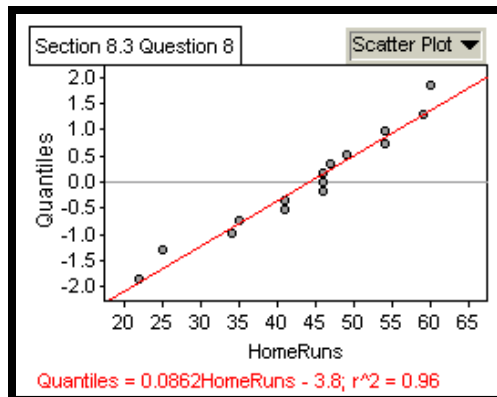Section 8.3, Question 5.   Section 8.3, Question 6.

Data do not appear normally distributed.   Data do not appear normally distributed.
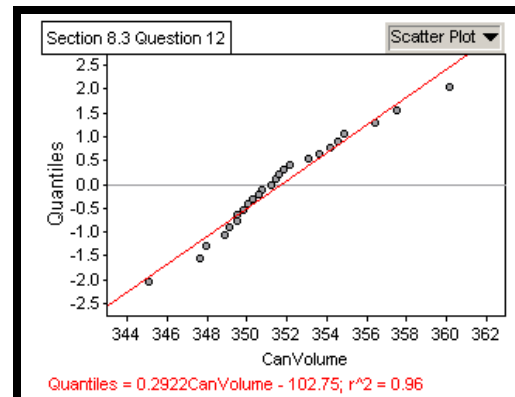
Section 8.3, Question 8.   Section 8.3, Question 12.

Data appear normally distributed.   Data appear normally distributed.

6. Answers will vary.

CHAPTER 8   The Normal Distribution

Section 8.4   Normal Approximation to the Binomial Distribution

Practise

**Section 8.4**   **Page 449**   **Question 2**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Microsoft® Excel formula for the probability in cell F3 is NORMDIST(21.5,C3,D3,TRUE). If you are using Corel® Quattro® Pro, the equivalent formula is @NORMDIST(21.5,C3,D3,1).

	A	B	C	D	E	F
1	Sample Size	Probability of Success	Mean	Standard Deviation	Probability	Probability
2	n	p	μ	σ	Desired	
3	60	0.4	24	3.794733192	P(X<22)	0.25500952
4	200	0.7	140	6.480740698	P(X<160)	0.99999734
5	75	0.6	45	4.242640687	P(X>50)	0.09742539
6	250	0.2	50	6.32455532	P(X>48)	0.59373782
7	1000	0.8	800	12.64911064	P(780<X<840)	0.93751991
8	90	0.65	58.5	4.524930939	P(52<X<62)	0.65391068
9	100	0.36	36	4.8	P(X=40)	0.05869909
10	3000	0.52	1560	27.3642102	P(X=1650)	6.5318E-05

Section 8.4   **Page 449**   **Question 4**

Toni makes $5 \times 8 = 40$ h of calls per week. Since each call takes 90 s or 0.025 h, she makes $\dfrac{40}{0.025} = 1600$ calls per week. The probability of a success on any given call is 0.015. Therefore,

$$\mu = np$$
$$= 1600 \times 0.015$$
$$= 24$$

$$\sigma = \sqrt{npq}$$
$$= \sqrt{1600 \times 0.015 \times 0.985}$$
$$= 4.862$$
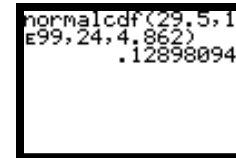
Since 30 or more successes are required, you want $P(X > 29.5) = 0.129$, as shown in the screen shot at the right. The probability that Toni gets at least 30 new customers for the week is 0.129.

Section 8.4   **Page 449**   **Question 6**

Using the normal approximation, you have

$\mu = np$ $\sigma = \sqrt{npq}$

$= 200 \times 0.18$ $= \sqrt{200 \times 0.18 \times 0.82}$

$= 36$ $= 5.433$

You want $P(39.5 < X < 40.5) = 0.0560$, as shown in the screen shot at the right.

Using the binomial distribution, the calculation is

$_{200}C_{40}(0.18)^{40}(0.82)^{160} = 0.0541$

Section 8.4   **Page 450**   **Question 8**

a) i) Using the normal approximation,

$\mu = np$ $\sigma = \sqrt{npq}$

$= 223 \times 0.68$ $= \sqrt{223 \times 0.68 \times 0.32}$

$= 151.64$ $= 6.966$

You want $P(X > 141.5) = 0.9273$, as shown in the screen shot at the right.

ii) Using the normal approximation,

$\mu = np$ $\sigma = \sqrt{npq}$

$= 223 \times 0.97$ $= \sqrt{223 \times 0.97 \times 0.03}$

$= 216.31$ $= 2.547$

You want $P(X < 219.5) = 0.8948$, as shown in the screen shot at the right.

b) The expected number of customers who paid Serena with cash is $223 \times 0.03 = 6.69$, or about 7.

Section 8.4   Page 450   Question 10

a) The probability of getting a sum of 5 is $\dfrac{4}{36} = \dfrac{1}{9}$. Therefore,

$$\mu = np \qquad\qquad \sigma = \sqrt{npq}$$
$$= 200 \times \frac{1}{9} \qquad\qquad = \sqrt{200 \times \frac{1}{9} \times \frac{8}{9}}$$
$$= 22.222 \qquad\qquad = 4.444$$

normalcdf(30.5,1
E99,22.222,4.444
)
        .0312496267

You want $P(X > 30.5) = 0.0312$, as shown in the screen shot at the right.

b) For this part, you want $P(29.5 < X < 40.5) = 0.0507$, as shown in the screen shot at the right.

normalcdf(29.5,4
0.5,22.222,4.444
)
        .0507211531

Section 8.4   Page 450   Question 12

Solutions for Achievement Checks are described in the Teacher's Resource.

CHAPTER 8   The Normal Distribution

Section 8.5   Repeated Sampling and Hypothesis Testing

Practise

Section 8.5   Page 457   Question 2

A spreadsheet solution for this problem follows. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Microsoft® Excel formula for the probability in cell I2 is 1-NORMDIST(22.5,G2,H2,1). If you are using Corel® Quattro® Pro, the equivalent formula is 1-@NORMDIST(22.5,G2,H2,1). Note that the formula uses 22.5 rather than 23. This is necessary, since you are using a normal distribution to approximate a binomial distribution, as explained in Section 8.4.
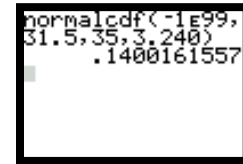
Note that the probability calculated in row 2 is about 23.5%. Therefore, the probability that there were 23 or more successes in 50 trials by random chance is about 23.5%. Since $\alpha = 10\%$, you will only accept H_i if this probability is less than 10%. Therefore, you accept H_o for the first case.

Section 8.5   **Page 457**   **Question 4**

In this problem,

$$\mu = np \qquad\qquad \sigma = \sqrt{npq}$$
$$= 50 \times 0.70 \qquad = \sqrt{50 \times 0.70 \times 0.30}$$
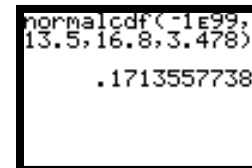$$= 35 \qquad\qquad = 3.240$$

$P(X < 31.5) = 0.140$, as shown in the screen shot at the right. Since this is greater than the 10% allowed by a confidence level of 90%, the result of the survey is not significant. Therefore, the newspaper does not need to print a correction.

Section 8.5   **Page 457**   **Question 6**

In this question,

$$\mu = np \qquad\qquad \sigma = \sqrt{npq}$$
$$= 60 \times 0.28 \qquad = \sqrt{60 \times 0.28 \times 0.72}$$
$$= 16.8 \qquad\qquad = 3.478$$

$P(X < 13.5) = 0.171$, as shown in the screen shot at the right. The probability that the result was as low as 13 in this survey is 17.1%. This is too large for a reasonable confidence level of 90%. There is not enough evidence to reject the claim that the product holds 28% of the market.

Section 8.5   **Page 458**   **Question 8**

Dana needs to know the standard deviation for the loans. A significance level of 1% would be appropriate, since the sample size is small.

Section 8.5 **Page 458** **Question 10**

In this question,

$$\mu = np$$
$$= 200 \times 0.08$$
$$= 16$$

$$\sigma = \sqrt{npq}$$
$$= \sqrt{200 \times 0.08 \times 0.92}$$
$$= 3.837$$

$P(X > 18.5) = 0.257$, as shown in the screen shot at the right. The probability that the result was 18 or more in this survey is 25.7%. This is too large for a reasonable confidence level of 90%. There is not enough evidence to accept the claim that the advertising was effective.

```
normalcdf(18.5,1
E99,16,3.837)
        .2573454653
```

Section 8.5 **Page 458** **Question 12**

Solutions for Achievement Checks are described in the Teacher's Resource.

CHAPTER 8   The Normal Distribution

Section 8.6   Confidence Intervals

Practise

Section 8.6 **Page 464** **Question 2**

a) There is a $\dfrac{19}{20} \times 100\% = 95\%$ probability that between $42\% - 3\% = 39\%$ and

$42\% + 3\% = 45\%$ of high school graduates expect to earn over \$100 000 per year by the time they retire.

b) There is a $\dfrac{9}{10} \times 100\% = 90\%$ probability that between $48\% - 5\% = 43\%$ and

$48\% + 5\% = 53\%$ of decided voters will vote for the incumbent in the next election.

c) There is a $\dfrac{11}{15} \times 100\% \approx 73\%$ probability that between $28\% - 4\% = 24\%$ and

$28\% + 4\% = 32\%$ of teenagers will purchase the latest CD by the rock band Drench.

Section 8.6 **Page 465** **Question 4**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Microsoft® Excel formula for $z_{\frac{\alpha}{2}}$ is

NORMINV(1-A2/200,0,1), as shown in the screen shot below. If you are using Corel® Quattro® Pro, the equivalent formula is @NORMINV(1-A2/200,0,1). The formula for the Lower limit in cell F2 is B2-E2*C2/SQRT(D2). The formula for the Upper limit in cell G2 is B2+E2*C2/SQRT(D2). The desired confidence interval for the mean is 15.8 to 19.4 months.

Section 8.6 **Page 465** **Question 6**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Microsoft® Excel formula for $z_{\frac{\alpha}{2}}$ is

NORMINV(1-A2/200,0,1), as shown in the screen shot below. If you are using Corel® Quattro® Pro, the equivalent formula is @NORMINV(1-A2/200,0,1). The formula for the Lower limit in cell F2 is B2-E2*SQRT(B2*C2)/SQRT(D2). The formula for the Upper limit in cell G2 is B2+E2*SQRT(B2*C2)/SQRT(D2). The desired confidence interval for the proportion of boats with the name "Serenity" is 0.119 to 0.121.

Section 8.6   Page 465   Question 8

a) A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. Enter the percent data for the 33 towns into a spreadsheet. Use spreadsheet functions to calculate the mean and standard deviation to be used by the confidence interval calculation. The desired confidence interval for the mean percent of waste recycled in towns with under 5000 households is 13.7% to 18.3%. See the solution for question 4 above for examples of the formulas.

b) Answers will vary.

Section 8.6   Page 466   Question 10

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the sample size in cell E2 is $(2 \cdot D2 \cdot B2/C2)^2$. The sample size must be about 68 for the desired maximum error and confidence level.

Section 8.6   Page 466   Question 12

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the sample size in cell F2 is $4 \cdot B2 \cdot C2 \cdot (E2/D2)^2$. The sample size must be about 936 for the desired maximum error and confidence level.

Section 8.6   **Page 466**   **Question 14**

Answers may vary.

Section 8.6   **Page 466**   **Question 16**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the sample size in cell E2 is $(2*D2*B2/C2)^2$. The sample size must be about 106 for the desired maximum error and confidence level.

Section 8.6   **Page 467**   **Question 18**

Solutions for Achievement Checks are described in the Teacher's Resource.

Section 8.6   **Page 467**   **Question 20**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the sample size in cell F2 is $4*B2*C2*(E2/D2)^2$. The required sample size for the desired maximum error and confidence interval has been calculated for various probabilities. It appears that a sample size of 1690 would cover all possibilities for people's opinions.
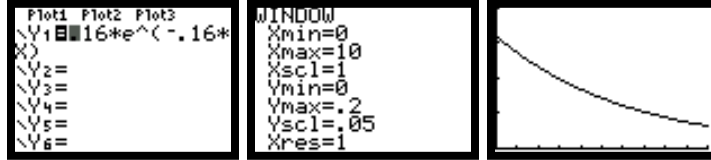
CHAPTER 8   The Normal Distribution
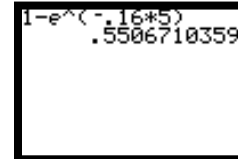
Review of Key Concepts

Review of Key Concepts   Page 468   Question 2

a) Enter the formula for the exponential distribution into your graphing calculator, as shown in the first screen shot below. Set your WINDOW parameters as shown in the middle screen shot below. Your graph will look like that shown in the third screen shot below. If you need instructions on using graphing calculators, refer to Appendix B of the student text.

```
Plot1 Plot2 Plot3
\Y1◼16*e^(-.16*
X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```
```
WINDOW
Xmin=0
Xmax=10
Xscl=1
Ymin=0
Ymax=.2
Yscl=.05
Xres=1
```

b) Use the formula $P(X \le x) = 1 - e^{-kx}$ from Section 8.1, question 8 for this problem. The required probability is about 0.55, as shown in the screen shot at the right.

```
1-e^(-.16*5)
       .5506710359
```

Review of Key Concepts   Page 468   Question 4

a) The probability that a resistor selected randomly has a resistance less than 13 Ω is 0.0856, as shown in the screen shot at the right.

```
normalcdf(-1E99,
13,15.08,1.52)
       .0855902166
```
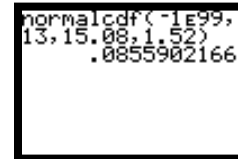
b) The probability that a resistor selected randomly has a resistance greater than 14.5 Ω is 0.6486, as shown in the screen shot at the right.

```
normalcdf(14.5,1
E99,15.08,1.52)
       .6486130972
```

c) The probability that a resistor selected randomly has a resistance between 13.8 Ω and 16.2 Ω is 0.5695, as shown in the screen shot at the right.

```
normalcdf(13.8,1
6.2,15.08,1.52)
       .5695264227
```
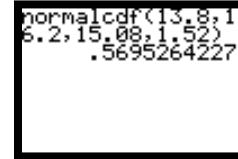
Review of Key Concepts   Page 468   Question 6

a) The probability that a particular light will last more than 250 h is 0.2119, as shown in the screen shot at the right.

```
normalcdf(250,1E
99,210,50)
       .2118553337
```
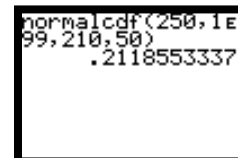
b) The percent of lights that will need to be replaced within 235 h is 69.15%, as shown in the screen shot at the right.

```
normalcdf(-1E99,
235,210,50)
       .6914624678
```

c) Out of 2000 lights, the number which will have a lifetime between 200 h and 400 h is $0.5792 \times 2000 = 1158$. The probability calculation is shown in the screen shot at the right.

```
normalcdf(200,40
0,210,50)
        .5791873153
```

Review of Key Concepts   Page 469   Question 8

a) Enter the data into a list on your graphing calculator. Use the **1-Var Stats** command under the **STAT CALC** menu to find the mean of 2.304 and the standard deviation of 0.06893.

```
1-Var Stats
x̄=2.304
Σx=46.08
Σx²=106.2586
Sx=.0689316977
σx=.0671863081
↓n=20
```

b) The probability that a bolt is below the minimum diameter of 2.25 cm is 0.2167, as shown in the screen shot at the right.

```
normalcdf(-1E99,
2.25,2.304,.0689
3)
        .2166950463
```

Review of Key Concepts   Page 469   Question 10

The mean of 20.05 and standard deviation of 7.23 for the earthquake data in this chapter were calculated in Section 8.1, question 10. Use these two statistics in your graphing calculator to find the probability of 0.503 that in a given year there will be more than 20 major earthquakes, as shown in the screen shot at the right.

```
normalcdf(20,1E9
9,20.05,7.23)
        .5027589318
```

Review of Key Concepts   Page 470   Question 12

In this problem,

$$\mu = np \qquad\qquad \sigma = \sqrt{npq}$$
$$= 500 \times 0.42 \qquad\qquad = \sqrt{500 \times 0.42 \times 0.58}$$
$$= 210 \qquad\qquad = 11.036$$

```
normalcdf(199.5,
250.5,210,11.036
)
        .829185972
```

$P(199.5 < X < 250.5) = 0.8291$, as shown in the screen shot at the right.

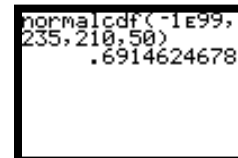**Review of Key Concepts   Page 470   Question 14**

Enter the coach's data into a list on your graphing calculator. Use the **1-Var Stats** command under the **STAT CALC** menu to find the mean of 73.86 and standard deviation of 22.34 for the coach's data, as shown in the first screen shot at the right. The probability that the mean price is $80 or less is 0.608, as shown in the second screen shot at the right. This is well within a 95% confidence level.

```
1-Var Stats
x̄=73.86111111
Σx=2659
Σx²=213871
Sx=22.34425817
σx=22.03173567
↓n=36
```

```
normalcdf(-1E99,
80,73.86,22.34)
        .6082816368
```

Review of Key Concepts   **Page 470**   **Question 16**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Microsoft® Excel formula for $z_{\frac{\alpha}{2}}$ is NORMINV(1-A2/200,0,1), as shown in the screen shot below. If you are using Corel® Quattro® Pro, the equivalent formula is @NORMINV(1-A2/200,0,1). The formula for the Lower limit in cell F2 is B2-E2*C2/SQRT(D2). The formula for the Upper limit in cell G2 is B2+E2*C2/SQRT(D2). The desired confidence interval for the mean time to find a job is 5.2 to 5.6 months.

Review of Key Concepts   **Page 471**   **Question 18**

a) A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. Enter the employee data for the 40 corporations into the spreadsheet. Use spreadsheet functions to calculate the mean and standard deviation for the confidence interval formula. The Microsoft® Excel formula

for $z_{\frac{\alpha}{2}}$ is NORMINV(1-A2/200,0,1), as shown in the screen shot above. If you are using Corel® Quattro® Pro, the equivalent formula is @NORMINV(1-A2/200,0,1). The formula for the Lower limit in cell F2 is B2-E2*C2/SQRT(D2). The formula for the Upper limit in cell G2 is B2+E2*C2/SQRT(D2). The confidence interval for the desired significance level is 1515 to 3185 employees.

b) The confidence interval includes less than half of the data because the data are not normally distributed.

Review of Key Concepts   **Page 471**   **Question 20**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The formula for the sample size in cell F2 is 4*B2*C2*(E2/D2)^2. The sample size must be about 358 for the desired maximum error and confidence level.
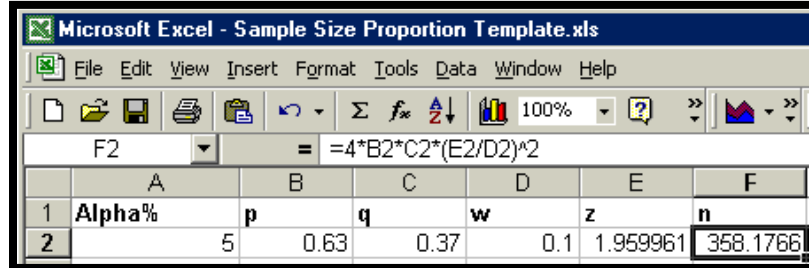
Review of Key Concepts   **Page 471**   **Question 22**

a) A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. According to the table, 26 of 50 students rated the city's support as good or better. This gives an estimate of $p = 0.52$. The Microsoft® Excel formula for $z_{\frac{\alpha}{2}}$ is NORMINV(1-A2/200,0,1), as shown in the screen shot below. If you are using Corel® Quattro® Pro, the equivalent formula is @NORMINV(1-A2/200,0,1). The formula for the Lower limit in cell F2 is B2-E2*SQRT(B2*C2)/SQRT(D2). The formula for the Upper limit in cell G2 is B2+E2*SQRT(B2*C2)/SQRT(D2). The confidence interval for the desired significance level is 0.382 to 0.658, as shown in the screen shot below.

b) Answers may vary.
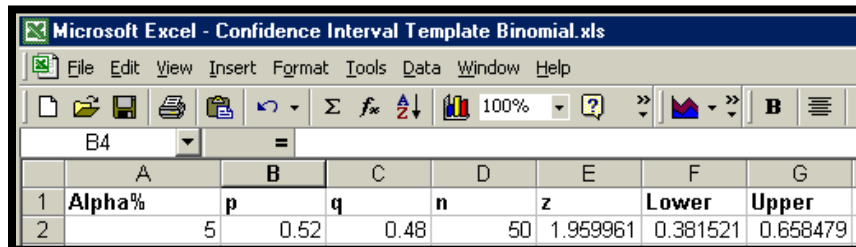
CHAPTER 8   The Normal Distribution

Chapter Test

Chapter Test   **Page 472**   **Question 2**

a) The probability that a container chosen at random has a volume of juice between 1.88 L and 2.15 L is 0.588, as shown in the screen shot at the right.

b) If containers with less than 1.75 L are rejected, then the proportion of juice containers that would be rejected is 9.12%, as shown in the screen shot at the right.

```
normalcdf(-1E99,
1.75,1.95,.15)
          .0912112819
```

c) The probability that a container has a volume of juice greater than 2.2 L is 0.0478, as shown in the screen shot at the right. Out of 500 containers, you would expect $0.0478 \times 500 = 24$ to have a volume greater than 2.2 L.

```
normalcdf(2,2,1E
99,1.95,.15)
          .0477903304
```

Chapter Test   **Page 472**   **Question 4**

Enter the farmer's data into a list on your graphing calculator. Use the **1-Var Stats** command under the **STAT CALC** menu to find the mean of 9.88 and standard deviation of 3.48 for the farmer's data, as shown in the first screen shot at the right. The probability that a tree selected randomly from this orchard has a yield greater than 10 kg is 0.486, as shown in the second screen shot at the right.

```
1-Var Stats
x̄=9.8795
Σx=197.59
Σx²=2181.7273
Sx=3.476514399
σx=3.388487089
↓n=20
```

```
normalcdf(10,1E9
9,9.8795,3.4765)
          .4861748529
```

Chapter Test   **Page 473**   **Question 6**

In this problem,

$$\mu = np \qquad\qquad \sigma = \sqrt{npq}$$
$$\quad = 80 \times 0.85 \qquad\qquad = \sqrt{80 \times 0.85 \times 0.15}$$
$$\quad = 68 \qquad\qquad\qquad = 3.194$$

```
normalcdf(-1E99,
69.5,68,3.194)
          .6806904955
```

If you want the probability that 10 or more applicants need to retake the test, you need $P(X < 69.5) = 0.6807$, as shown in the screen shot at the right.

Chapter Test   **Page 473**   **Question 8**

A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Microsoft® Excel formula for $z_{\frac{\alpha}{2}}$ is NORMINV(1-A2/200,0,1), as shown in the screen shot below. If you are using Corel® Quattro® Pro, the equivalent formula is @NORMINV(1-A2/200,0,1). The formula for the Lower limit in cell F2 is B2-E2*SQRT(B2*C2)/SQRT(D2). The formula for the Upper limit in cell G2 is B2+E2*SQRT(B2*C2)/SQRT(D2).The desired confidence interval for the percent of voters who are likely to vote "Yes" is 0.352 to 0.527, as shown in the screen shot below.
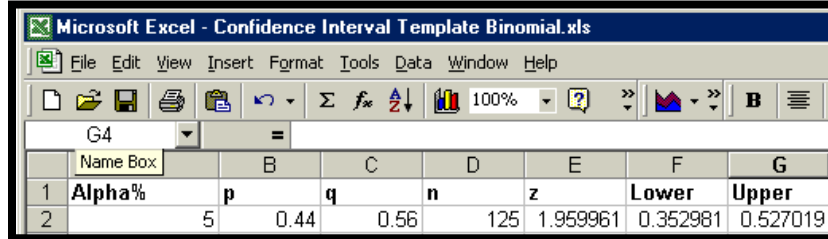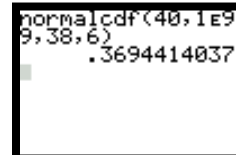

Chapter Test   **Page 473**   **Question 10**

Solutions for Achievements Checks are described in the Teacher's Resource.

Cumulative Review: Chapters 7 and 8

Cumulative Review: Chapters 7 and 8   **Page 476**   **Question 2**

a) The percent of cars passing the school that are speeding is 37%, as shown in the screen shot at the right.

b) A driver will receive a ticket if the speed of the car is 44 km/h or more. The probability that this will occur is 15.9%, as shown in the screen shot at the right.

Cumulative Review: Chapters 7 and 8   **Page 476**   **Question 4**
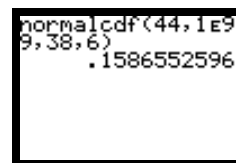
a) The probability that exactly five students use a computer is

$$P(X = 5) = {}_{10}C_5(0.825)^5(0.175)^5$$
$$= 0.0158$$

b) Here,

$$nq = 10 \times 0.175$$
$$= 1.75$$

This is not greater than 5. Therefore, you may not use the normal approximation. A spreadsheet solution for this problem is shown at the right. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Microsoft® Excel formula for the probability that $x = 6$ is COMBIN(10,A2)*(0.825)^A2*(0.175)^(10-A2). If you are using Corel® Quattro® Pro, the equivalent formula is @COMB(A2,10)*(0.825)^A2*(0.175)^(10-A2). Copy this formula to calculate the probabilities for $x = 7$, 8, 9, and 10. The probability that at least six students use a personal computer is 0.981.

	A	B
1	x	P(x)
2	6	0.062101
3	7	0.167291
4	8	0.295747
5	9	0.30983
6	10	0.146063
7		
8	Sum	0.981031

c) The expected number of students who use a personal computer is

$$E(x) = np$$
$$= 10 \times 0.825$$
$$= 8.25$$

Cumulative Review: Chapters 7 and 8   **Page 476**   **Question 6**

a) Answers will vary.

b) The theoretical probability of exactly five red bulbs is

$$\frac{{}_{15}C_5 \times {}_{29}C_{10}}{{}_{44}C_{15}} = 0.2616$$

c) The expected number of blue bulbs is

$$\frac{13}{44} \times 15 = 5.45$$

d) The simulation from part a) may not produce the same probability as in part b) due to random chance. This discrepancy can be minimized by using a large number of trials.

a) A spreadsheet solution for this problem is shown below. If you need information on how to use spreadsheets, refer to Appendix B of the student text. The Microsoft® Excel formula for $z_{\frac{\alpha}{2}}$ is

NORMINV(1-A2/200,0,1), as shown in the screen shot below. If you are using Corel® Quattro® Pro, the equivalent formula is @NORMINV(1-A2/200,0,1). The formula for the Lower limit in cell F2 is B2-E2*C2/SQRT(D2). The formula for the Upper limit in cell G2 is B2+E2*C2/SQRT(D2). The confidence interval for the desired significance level is 1.58 to 1.76 wolves per 100 km^2.

b) Part a) assumes that wolves are normally distributed.

	A	B	C	D	E	F	G
1	Alpha%	Mean	StDev	n	z	Lower	Upper
2	5	1.67	0.32	50	1.959961	1.581302	1.758698

Course Review

Course Review   **Page 477** **Question 2**

Answers may vary.

Course Review   **Page 477** **Question 4**

There are 6 side orders available. Therefore, the number of combinations of 2 side orders is

$$
\begin{aligned}
_6C_2 &= \frac{6!}{(6-2)!\,2!} \\
&= \frac{6!}{4!\,2!} \\
&= 15
\end{aligned}
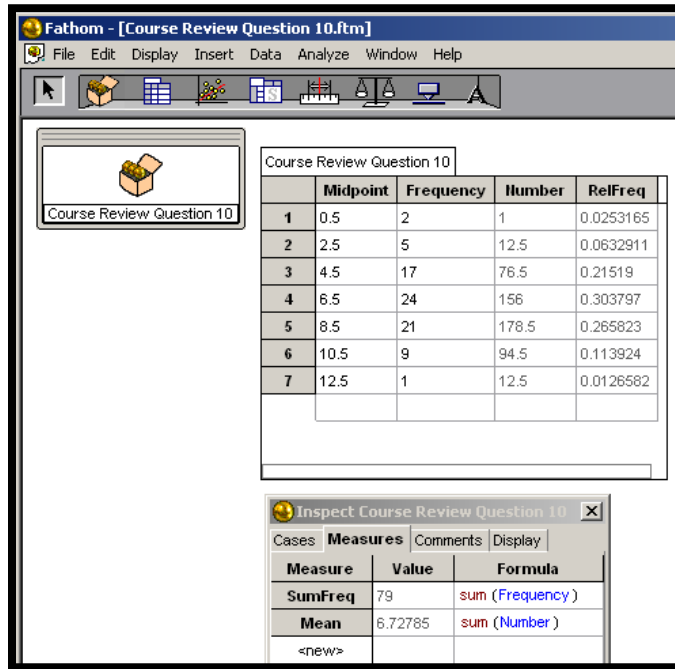$$

Course Review   **Page 477** **Question 6**

a) A Bernoulli trial has only two possible outcomes.

b) Trials in a geometric distribution are independent. Trials in a hypergeometric distribution are dependent. An example of the former is selection of a card from a deck, and then replacement of the card before the next trial. An example of the latter is selection of a card from a deck, and no replacement of the card before the next trial.

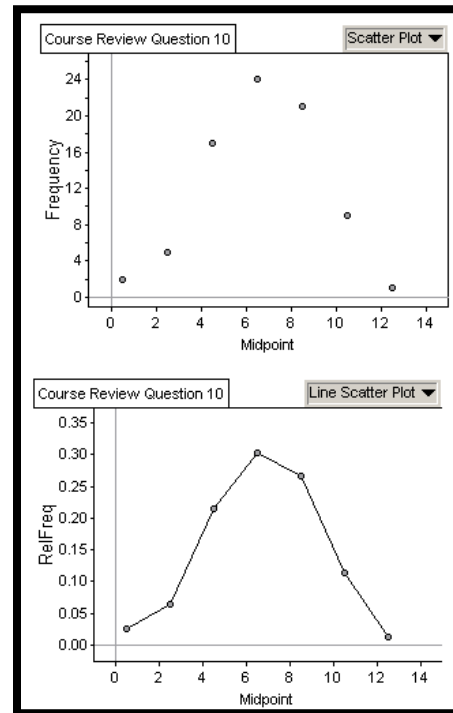**Course Review**   **Page 477** **Question 8**

Answers may vary.

a) An example of a stratified sample is a sampling of students from each of the grades 9, 10, 11, and 12 in numbers according to the proportion each grade comprises of the total. This ensures representation from all grades in the proper proportions.

b) An example of a simple random sample is sampling persons from randomly-chosen households in Ontario for their provincial political preference. Each person has an equal opportunity to be a part of the sample.

c) An example of a systematic sample is sampling 6 students from a class of 30 according to the following procedure: choose a random number from 1 to 5, select this student from the class list, and every sixth student thereafter. This makes it easy to obtain the required percent of respondents.

Course Review   **Page 477**   **Question 10**

a) A Fathom™ solution for this problem is shown at the right. If you need instructions on using Fathom™, refer to Appendix B of the student text. Enter the midpoints and frequencies in the first two attribute columns. Calculate the approximate number of videos rented for each interval by multiplying the midpoint by the frequency. Find the mean by adding the numbers, and dividing by the sum of the frequencies. The mean is 6.7. Since the sum of the frequencies is 79, the median is data point 40. This occurs in the bin with midpoint 6.5. Therefore, the median is estimated as 6.5.

Fathom - [Course Review Question 10.ftm]

File   Edit   Display   Insert   Data   Analyze   Window   Help

Course Review Question 10

	Midpoint	Frequency	Number	RelFreq
1	0.5	2	1	0.0253165
2	2.5	5	12.5	0.0632911
3	4.5	17	76.5	0.21519
4	6.5	24	156	0.303797
5	8.5	21	178.5	0.265823
6	10.5	9	94.5	0.113924
7	12.5	1	12.5	0.0126582

Inspect Course Review Question 10

Cases   **Measures**   Comments   Display

Measure	Value	Formula
SumFreq	79	sum (Frequency)
Mean	6.72785	sum (Number)
<new>		

b) Diagrams for frequency and relative frequency are shown at the right.

c) These calculations may not be accurate. The data are grouped, and calculations were made using the midpoint of the group.

Course Review Question 10   Scatter Plot ▼

Course Review Question 10   Line Scatter Plot ▼

Course Review   **Page 478**   **Question 12**

Since butter and cheese are both dairy products, there is likely a common cause forcing both prices up. One is not causing the other.

Course Review   **Page 478**   **Question 14**

a) Since there are two selections for each jumper, the number of ways a set of six can be set is
$2^6 = 64.$

b) Since there are three selections for each jumper, the number of ways a set of six can be set is
$3^6 = 729.$

Course Review   **Page 478**   **Question 16**

The number of choices is $_{78}C_6 = 256\,851\,595.$

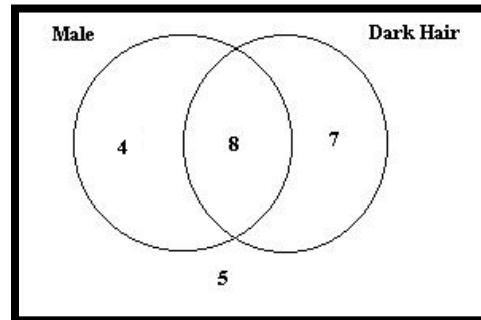**Course Review**   **Page 478**   **Question 18**

The number of ways that 18 chocolates can be evenly distributed among three people is
$_{18}C_6 \times _{12}C_6 \times _6C_6 = 17\,153\,136.$

Course Review   **Page 478**   **Question 20**

Since there are 30 teams, and 6 of them are original NHL teams, Naomi has a probability of
$\dfrac{6}{30} = \dfrac{1}{5}$ of getting one of the original 6 NHL teams in a given box of cereal.

Course Review   **Page 479**   **Question 22**

Set up this problem using a Venn diagram, as shown at the right. Since 15 guests have dark hair, and 7 of them are females, there are 8 males with dark hair. Since there are 12 males, there are 4 males without dark hair. These add up to 19. Since there are 24 people at the party, there are 5 females without dark hair. The probability that the first guest to arrive will have either dark hair or be a male is $\dfrac{19}{24}.$

Course Review   **Page 479**   **Question 24**

a) You want four failures before the first success. This situation can be modelled by a geometric distribution. Therefore, the required probability is
$$P(X = x) = q^x p$$
$$P(X = 4) = (0.835)^4(0.165)$$
$$= 0.080$$

b) The first red car sold could be the first, second, third, fourth, or fifth car sold. This generates values of x from 0 to 4. A spreadsheet solution is shown at the right. The probability that the first red car will be among the first five cars sold is 0.594.

c) The expected waiting time before the first red car is sold is

$$E(x) = \frac{q}{p}$$
$$= \frac{0.835}{0.165}$$
$$= 5.06$$

Course Review   **Page 479**   **Question 26**

a) This situation can be modelled by a geometric distribution. The probability that the first defect found will be in the 20th battery is

$$P(X = x) = q^x p$$
$$P(X = 19) = (0.979)^{19}(0.021)$$
$$= 0.014$$

b) The probability that there are no defects in the first 6 batteries tested is $(0.979)^6 = 0.88$.

c) The expected waiting time until a defective battery is tested is

$$E(x) = \frac{q}{p}$$
$$= \frac{0.979}{0.021}$$
$$= 46.6 \text{ batteries}$$

Course Review   **Page 479**   **Question 28**

a) This situation represents a binomial distribution. The probability that exactly 10 of 32 patients had symptoms of the common cold or flu is

$$P(x) = {_n}C_x p^x q^{n-x}$$
$$P(10) = {_{32}}C_{10}(0.42)^{10}(0.58)^{22}$$
$$= 0.069$$

b) The expected number of patients who have symptoms of the common cold or flu is

$$E(x) = np$$
$$= 32 \times (0.42)$$
$$= 13.4$$