

Measurement of the Sodium D Emission Lines Using a Michelson Interferometer

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The Sodium D emission lines (commonly called the ‘Sodium Doublet’) are spectral emission lines that have wavelengths within a few angstroms apart. The experiment performed involved calibrating a Michelson interferometer and measuring the wavelengths of both emission lines. This was accomplished by measuring both the average and beat wavelengths of the interference pattern created. From the experiment it was found that the emission lines were given as $603\pm 37\text{nm}$ and $604\pm 37\text{nm}$ for the D1 and D2 lines respectively.

1 Introduction

The purpose of the experiment was to measure the wavelength of the Sodium D emission lines. Sodium has two emission wavelengths that are extremely close in wavelength and without sensitive equipment cannot be distinguished. These lines, designated the D2 and D1 Fraunhofer lines, have wavelengths of 589.6 nm and 589.0nm respectively¹. Using sensitive interferometry, the average wavelength of the sodium lines was determined as well as the difference between the two emission lines.

When a sodium lamp is used in conjunction with an interferometer, the wavelengths of the two emission line will create interference patterns. The distance between fringes within the interference patterns can be measured to calculate the wavelength of the emission source. By measuring the path length difference that causes the movement of the fringe pattern by one cycle, the average wavelength of the source can be determined. However, because of the relative proximity between the sodium D lines, it becomes more difficult to directly observe the two

interference patterns. Also as a consequence of the proximity of the two lines, the interference patterns together will form an observable beat pattern. From the resulting beat pattern the difference in the wavelengths can be determined.

1.1 Setup

For the experiment, a standard Michelson interferometer was used to create interference patterns which allowed measurements of the wavelength of the light sources used. Below in Figure 1 is a diagram of the experimental setup used:

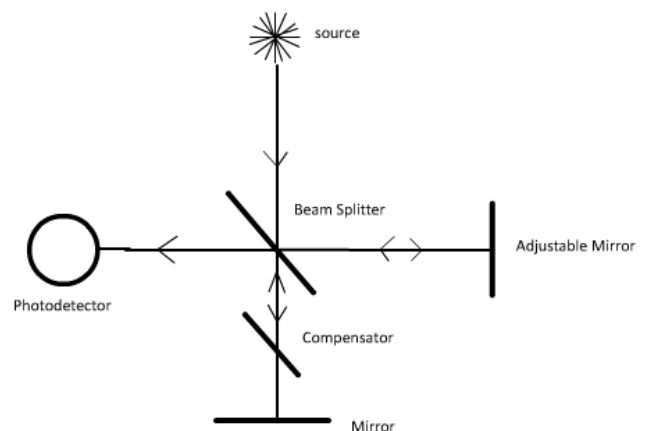


Figure 1 - Experimental Setup

In the experimental setup, a computer controlled motor was used to move the adjustable mirror

¹ Standard D Line values from [University Physics, 11th Edition](#)

small distances on the order of a few nanometers. To measure the interference pattern created from the interferometer, a photodetector was used to collect data. When the photodetector was placed in the path of the outgoing light, the interference pattern would create a sequence of 'high' and 'low' recorded values. The harmonic pattern of the recorded values from the photodetector indicated the passing fringes of the interference pattern.

The computer system used was capable of controlling the position of the adjustable mirror as well as recording the intensity of light upon the photodetector. Thus the system was able to record the intensity of light as a function of position, which was used to experimentally measure the wavelength of the sodium lines. However because the units of distance on the system correlated to 'motor steps', the interferometer was calibrated to find the relationship between the motor step length and nanometers the mirror moved.

1.2 Theory

The working theory behind the experiment is that of wave interference. Consider the interferometer described in Figure 1. To create interference from a monochromatic light source, the following condition must hold:

$$m\lambda = l \quad (1)$$

Where l is the path length difference between the two light sources. The interference is caused by path difference l in the optical path to one mirror and the optical path to the second. In Equation 1, m is the order number and λ is the wavelength of the light source. In the instance of the interferometer, m is a measurement of how many fringes apart the associated distance represents. Note in the case of the Michelson interferometer, if the distance the mirror moves is d , then the total path length difference is $2d$ because of the

fact that the light is reflected and travels back through the same distance twice. Thus it can be said $l=2d$ where d is the distance the mirror is moved. Therefore:

$$m\lambda = 2d \quad (2)$$

So as the mirror is moved a distance d , m fringe patterns will move past a given point on the detector. The wavelength of the light source can be determined if both the number of passed fringes and change and mirror distance are measured. Consequentially, if a light source with a given wavelength is known, the interferometer can be calibrated to find the distance the mirror has moved.

However the previous derivations are only appropriate for monochromatic light sources. Consider in the case of the Sodium D lines two separate wavelengths λ_1, λ_2 separated by a small amount. The resulting interference pattern is the superposition of the two separate fringe systems for each wavelength. Over certain distances of the mirror position the two fringe systems can either constructively or destructively interfere with each other when the fringe patterns overlap. When the two maxima of the patterns are at the same position, a set of dark fringes are produced indicating that the optical path lengths for both wavelengths are equal. Consider the case when both overlap as described. If the mirror distance is increased, the two patterns will overlap again when:

$$2d = m\lambda_2 = (m + 1)\lambda_1 \quad (3)$$

So from Equation 3 and solving for $\Delta\lambda = \lambda_2 - \lambda_1$ then one has a result of:

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{\lambda_1}{m} \quad (4)$$

Using this result in combination with Equation 4 it can be shown that:

$$\Delta\lambda = \frac{\lambda_{gm}^2}{2d} \quad (5)$$

Where λ_{gm} is the geometric mean of the two sodium spectral lines². Using both Equations 2 and 5, the two spectral lines can be determined experimentally by making the appropriate measurements.

2 Experimental Methods

Before measuring the interference patterns of the sodium doublet, the interferometer setup used was calibrated. As stated in Section 1.1, the adjustable mirror on the interferometer was connected to a computer controlled system that moved the mirror in units of 'motor steps'. Using Equation 2, the distance the mirror moves can be directly related to the wavelength of the light source and the number of passing fringes in the interference pattern. To use the relationship, a helium neon laser with a wavelength of $633 \pm 1 \text{ nm}$ was used as a light source in the setup in Figure 1. After fine tuning the placement and angles of the mirrors to focus the interference pattern on a screen, the photodetector was placed in the pattern to measure the intensity of the light. Next the motor attached to the adjustable mirror was activated so that the fringes within the interference pattern passed through ten cycles. The computer system connected to the photodetector was able to plot the intensity of the light crossing the detector versus the number of motor steps. Using Equation 2 and the data collected from the system, the distance between fringes in terms of steps could be related to the calculated distance in nanometer. The process was repeated in order to determine an average value of nanometers the mirror moved per motor step.

With the calibration data for the setup determined, the experimental task of determining

the wavelength and difference of the sodium doublet was prepared. The setup was similar to that in Figure 1 except that a lens was used to focus outgoing light. The photodetector was placed at the focal length of the lens where the contrast in the fringes would be the largest, creating clearly defined maxima and minima of the interference pattern.

Once the experimental setup described produced an observable fringe pattern, the computer system was used to control the mirror motor to vary the mirror distance and observe the intensity pattern created by the passing fringes on the photodetector.

To determine the averaged wavelength of the sodium D lines, the same method used to calibrate the interferometer was used. The mirror was moved via the computer control over approximately 1000 steps of the motor. The computer in conjunction with the photodetector was able to create a plot of the intensity pattern. From the plot, the number of step per the passing of ten fringes was recorded to get an average value for the number of steps per passing fringe. Using Equation 2 and the calibration measurements, the average wavelength of the sodium light was determined. To measure the difference in the two sodium D lines, the mirror was moved over a large number of motor steps so that intensity pattern wavelength and beat pattern could be observed and measured by previous methods.

3 Calibration Data Set

For the calibration data set, the data was taken to determine the distance in nanometers of each motor step on the mirror motor system. The data below in Table 1 displays the values relating the steps per fringe to the nanometers per step.

² C.J. Overback. "Selective Experiments in Science", 1962.

Trial	Steps per 10 Fringes ($\sigma=\pm 5$)	Steps per Fringes ($\sigma=\pm 5$)	Nanometers per Step	Calculated Error (nm)
1	900	90	3.52	0.20
2	800	80	3.96	0.25
3	880	88	3.60	0.20
4	870	87	3.64	0.21
5	825	82.5	3.84	0.23
6	845	84.5	3.75	0.22
7	865	86.5	3.66	0.21
8	850	85	3.72	0.22
9	820	82	3.86	0.24
Average	851	85.1	3.73	0.22
σ	31.8	3.18	0.14	0.02

Table 1 - Calibration Data Set

Note that to measure how many nanometers are in one motor step, Equation 2 was used along with dimensional analysis to provide the following:

$$1 \text{ step} = \frac{316.5\text{nm}}{\# \text{steps}/\text{fringe}} = \frac{\lambda}{F} \quad (6)$$

To find the error in the value, error propagation techniques were used to result in the following relationship:

$$\sigma_x^2 = \sigma_F^2 \left(\frac{1}{F}\right)^2 + \sigma_\lambda^2 \left(\frac{\lambda}{F^2}\right)^2 \quad (7)$$

Where $\lambda=633\pm 1\text{nm}$, F is the number of steps per fringe and σ_x is the error in the nanometers per step calculation.

From the calibration data, it can be said that each step of the motor is approximately given as

$$d = (3.7 \pm 0.2\text{nm})(\# \text{ of Steps}) \quad (8)$$

Where d is movement of the mirror in nanometers. Note that in Table 1, the standard deviation of the distribution of number of steps per fringe (≈ 3) is less than the uncertainty of the measurement (5 steps/fringe) thus the measurement uncertainty was used in the error propagation.

4 Sodium Doublet Data

In order to determine the average wavelength of the sodium D lines, the interferometer mirror was moved approximately the distance that creates 10 passing fringes. The distance was then used to calculate the wavelength of the incoming light creating the interference pattern. The results of the measurements are below in Table 2:

Trial	Steps per 10 Fringes ($\sigma=\pm 5$)	Steps per Fringe ($\sigma=\pm 5$)	Average Wavelength (nm)	Calculated Error (nm)
1	780	78	582	37.5
2	790	79	589	37.5
3	830	83	619	37.5
4	825	82.5	615	37.5
5	790	79	589	37.5
6	800	80	597	37.5
7	820	82	612	37.5
8	825	82.5	615	37.5
9	800	80	597	37.5
10	830	83	619	37.5
Average	809	80.9	604	37.5
σ	19.0	1.90	14.15	0.01

Table 2 - Average Na Wavelength Data

In order to calculate the wavelength of the emission lines, Equation 8 was used to convert the step count into nanometers. To calculate the error in the calculation of the wavelength, the following relationship was derived using error propagation and the relationship in Equation 2:

$$\sigma_x^2 = \sigma_F^2 (2N)^2 + \sigma_N^2 (2F)^2 \quad (9)$$

Where F is the number of steps per fringe, N is the conversion factor between steps to nanometers related in Equation 8 and σ_x is the error in the wavelength calculation. The values for the trials were averaged together to give the final measured wavelength and error. The measured value for the average of the sodium D line is thus given as $604 \pm 37\text{nm}$. Similar to the calibration data, the measurement uncertainty of

the data for the steps per fringe portion is much larger than the variance between the trials. Therefore the measurement uncertainty was used as the error in the measurement instead.

To find the beat pattern of the sodium lines, the mirror motor was set to move over half a million steps so that the entire beat pattern of the sodium doublet interference could be measured. The collected data is displayed below in Figure 2:

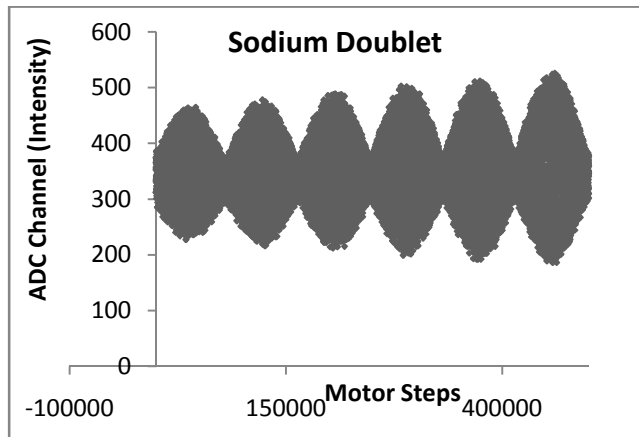


Figure 2 - Sodium D Line Beat Pattern

In order to find the beat pattern, $\Delta\lambda$, the distance between successive beats is required. From the data it is found that the average length of one of the beat patterns over four beats is approximately 82500 ± 2000 steps. Using Equations 8 to find the value and 9 for error it was found that the average beat length was 307000 ± 19000 nm

The value of the difference in the wavelengths of the sodium D lines is given in Equation 5. Using error propagation derivations, the error in the wavelength difference is given as:

$$\sigma_{\Delta}^2 = \sigma_{\lambda}^2 \left(\frac{\lambda}{d}\right)^2 + \sigma_d^2 \left(\frac{\lambda^2}{2d^2}\right)^2 \quad (10)$$

Using Equations 5 and 10 together result in $\Delta\lambda = 0.59 \pm 0.08$ nm. The following is a summary table of the results:

	value	error
Steps per Beat	82500	2000
nm per Beat	307000	19000
$\Delta\lambda$	0.59	0.08

Table 3 - Beat Pattern Summary

Furthermore, using the definitions of beat length and the average wavelength:

$$\lambda_{\text{beat}} = \lambda_2 - \lambda_1 \quad (11)$$

$$\lambda_{\text{avg}} = \frac{\lambda_2 + \lambda_1}{2} \quad (12)$$

The sodium D lines can be given exactly. From the results $\lambda_{\text{beat}} = 0.59$ nm and $\lambda_{\text{avg}} = 604$ nm and solving the linear system, the emission lines are given as 603.7nm and 604.3nm for the D1 and D2 lines respectively. However considering error, the measured values are more fairly given as 603 ± 37 nm and 604 ± 37 nm.

5 Results

From the experiment, it was found that the emission lines for sodium were 603 ± 37 nm and 604 ± 37 nm for the sodium D1 and D2 lines respectively. Although these values differ from the recorded values of 589.6 nm and 589.0nm, it is interesting to note that the relative difference $\Delta\lambda$ for both the experimental and recorded values are very similar. Furthermore the error in $\Delta\lambda$ is less than one angstrom, which is interesting as the uncertainty in the average value is off by approximately 40nm. However, the large amount of error in the average wavelength measurement comes from the measurement uncertainty of the number of motor steps per fringe. As noted earlier the actual deviation of the data points was less than the measurement uncertainty. If the statistical variation was used as the error, the variations in the average wavelength would be reduced to approximately 10 nanometers which would make the value of 603nm for the average slightly out of range of the recorded 590 nm value.