Measurement of the Stokes parameters of light

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We describe a measuring system for determining the state of polarization of a beam of light in terms of its Stokes parameters. The technique which can be fully automated incorporates a monochromator and single photon counting detection and can thus be applied over a large wavelength range for very weak optical signals. Fourier transformation of the data by an on-line minicomputer allows immediate calculation of the Stokes parameters. We discuss special applications to light emitted from excited atomic systems with and without cylindrical symmetry.

I. Introduction

Measurements of light polarization are used in many branches of science as probes of excited systems. The light being emitted from the system may be just the scattering of incident light of a particular polarization[1] or the system may have been excited by some other means. The state of polarization of the emitted light in general provides information on various spatial anisotropies of the excited system. In this paper we shall discuss a measurement technique for accurately determining the complete state of polarization of the light. The technique is readily automated by the use of an on-line minicomputer to provide an immediate and complete analysis of the light polarization parameters. In particular, the linear and circular polarization components are measured simultaneously and independently.

Stokes[2] showed that a beam of light (and electromagnetic radiation, in general) can be described completely by four parameters. Light is in general partially elliptically polarized, and the four parameters can be, for example, the size and shape of the ellipse, its orientation with respect to some fixed spatial axes, and the direction of rotation of the ellipse. For actual measurements Stokes found it more convenient to introduce the four parameters which we label as I, M, C, S following the notation of previous authors.[1]

We choose a cartesian coordinate system (x,y,z) so that x and y are perpendicular to the direction of light propagation z. Then if E_x and E_y are the electric field components in the x and y directions, and $I(\varphi)$ is the intensity of light polarized in the direction φ to the x axis,

$$I = I(0°) + I(90°) = \langle |E_x|^2 \rangle + \langle |E_y|^2 \rangle, \qquad (1)$$

$$M = I(0°) - I(90°) = \langle |E_x|^2 \rangle - \langle |E_y|^2 \rangle, \qquad (2)$$

$$C = I(45°) - I(135°) = R_e \langle E_x E_y \rangle, \qquad (3)$$

$$S = I_{\text{RHC}} - I_{\text{LHC}} = I_m \langle E_x E_y \rangle, \qquad (4)$$

where the brackets $\langle \ \rangle$ indicate averaging over a long time (many photons), and I_{RHC} and I_{LHC} are the intensities of right- and left-handed polarized light, respectively.

To measure all four Stokes parameters requires an absolute intensity measurement. However, the polarization state is completely determined by the three ratios M/I, C/I, S/I, which we shall call the relative Stokes parameters. They then have possible values between -1 and $+1$. Clarke and Grainger[3] have discussed the general considerations necessary for measurements of Stokes parameters using various combinations of retarder phase plates and linear polarizers.

II. Polarization Analysis System

The basic polarization elements of any polarization measuring system are a retardation phase plate and a linear polarizer. The retardation plate gives a difference in phase of δ for electric vectors parallel and perpendicular to its fast axis. In Fig. 1 we show the retardation plate reference axis at an angle β to the x axis established for the incident light (which is being propagated along the z axis). The linear polarizer follows the retardation plate, and its transmission axis is at an angle α to the x axis. The transmitted light intensity for an incident light beam with Stokes parameters (I,M,C,S) is[3,4]

$$I_T(\alpha,\beta,\delta) = \frac{1}{2}\{I + (M\cos2\beta + C\sin2\beta)\cos2(\alpha - \beta)$$

$$+ [(C\cos2\beta - M\sin2\beta)\cos\delta + S\sin\delta]\sin2(\alpha - \beta)\}. \quad (5)$$

The transmitted intensity I_T can be analyzed by a monochromator and detected by a photomultiplier
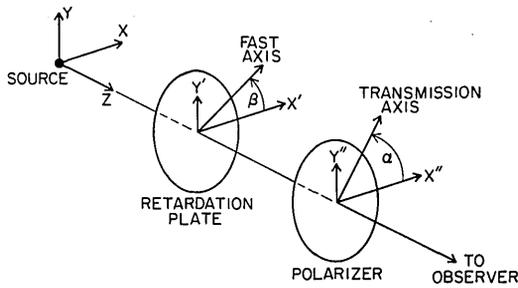
Fig. 1. Geometry of the polarization analysis system.

using pulse counting techniques. The Stokes parameters can be measured by observing the variation of $I_T(\alpha,\beta,\delta)$ as either the linear polarizer (α) or the retardation plate (β) is rotated. Rotation of the final polarizer changes the polarization plane of the transmitted light, and hence the detection system must be made insensitive to polarization. This can be achieved by rotating the detection system with the polarizer or, more conveniently, by introducing a depolarizing element after the polarizer. We have described this technique previously.[5]

An alternative technique is to rotate the retardation plate. This method obviates the need to correct for a polarization dependent detection system and has been discussed by Clarke and Ibbett[6] and by Laloë.[7] The former developed the system to measure linear polarization from astronomical light sources.

For considering rotation of the retardation plate, it is more convenient to rewrite Eq. (5) in the form

$$I_T(\alpha,\beta,\delta) = \frac{1}{2}\left[I + \left(\frac{M}{2}\cos 2\alpha + \frac{C}{2}\sin 2\alpha\right)(1 + \cos\delta)\right]$$
$$+ \frac{1}{2}[S\sin\delta \cdot \sin(2\alpha - 2\beta)] + \frac{1}{4}[(M\cos 2\alpha - C\sin 2\alpha)\cos 4\beta$$
$$+ (M\sin 2\alpha + C\cos 2\alpha)\sin 4\beta](1 - \cos\delta). \quad (6)$$

We consider rotation of the retardation plate angle $\beta = \omega t$, where t is the time and ω is its angular velocity. Then each term on the right-hand side of Eq. (6) has a different frequency dependence. The first term is constant, the second term varies sinusoidally with frequency 2ω, and the third term varies sinusoidally with frequency 4ω. Laloë[7] used a frequency lockin detection scheme to measure the 2ω and 4ω signals which are proportional, respectively, to the circular and linear polarizations of his light source. (His source axes x and y could be chosen so that $C = 0$ in all cases.)

Most generally, the light to be analyzed has two nonzero linear polarization components M and C which are determined not only by the magnitude of the 4ω frequency signal, but also by its phase relative to the 2ω signal phase or the direction of the axes of the retardation plate and polarizer. We shall describe in detail such necessary calibration of the polarization system and two techniques which we have used to obtain the Stokes parameters.

III. Calibration of the Quarterwave Plate

Achromatic retardation plates[8] with a phase $\delta = \pi/2$ over a large wavelength range are obtainable for the visible spectrum, but for accurate work and for measurements in the uv spectrum using zero order or multiple order retardation plates, the phase angle δ can vary significantly from $\pi/2$.

In order to calibrate the phase δ of a retardation plate, one can illuminate the optical system with polarized light of known Stokes parameters. In practice, a simple technique is to remove the retardation plate (see Fig. 1) and introduce a rotatable linear polarizer P_1. Then, if the final polarizer is P_2 at a fixed angle α, maximum transmission will occur when the axis directions of P_1 and P_2 are parallel. If P_1 and P_2 are not perfect polarizing elements—for example—if the Stokes parameters $C = S = 0$ but $M/I \equiv k_{\text{inc}} < 1$ for light transmitted by P_1, complete extinction will not occur when P_1 and P_2 are crossed. If the analyzer P_2 is an ideal polarizer as assumed in the derivation of Eq. (5), the efficiency of P_1 can be measured as a function of wavelength so that

$$\frac{M}{I}(\lambda) \equiv k_{\text{inc}}(\lambda) = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}, \quad (7)$$

where I_{max}, I_{min} are the light intensities observed for the axes P_1 and P_2 parallel and crossed, respectively.

To measure the retardation plate phase shift δ, set the axis of P_1 parallel to the x axis (Stokes parameters I, M, $C = 0$, $S = 0$), set the axis of P_2 at a fixed known angle α, and reinsert the retardation plate. Then the transmitted light as a function of the retardation plate angle β becomes from Eq. (6)

$$I_T(\beta) = 1 + |\eta|\cos(4\beta + \gamma), \quad (8)$$

and the phase shift $\delta(\lambda)$ is given by

$$\cos\delta(\lambda) = \frac{1 - |\eta|\cos 2\alpha - 2|\eta|/k_{\text{inc}}(\lambda)}{1 + |\eta|\cos 2\alpha}. \quad (9)$$

The phase $\delta(\lambda)$ is thus determined from the modulation amplitude (up to a sign) without knowing the location of the quarter waveplate axis. It is convenient to choose $\alpha = \pi/4$ so that $\cos 2\alpha = 0$ in Eq. (9).

For a standard quartz retardation plate, the phase shift can be written

$$\delta(\lambda) = 2\pi(p + \tfrac{1}{4}) = \frac{2\pi d}{\lambda} \cdot [n_e(\lambda) - n_o(\lambda)], \quad (10)$$

where $n_o(\lambda)$ and $n_e(\lambda)$ are the refractive indices of the ordinary and extraordinary ray, respectively, and d is the thickness of the quartz. For wavelengths λ_N, where p is an integer N, the plate is a quarterwave plate of order N, a zero-order plate having $N = 0$. We show in Fig. 2(a) the variation of the phase $\delta(\lambda)$ for a plate with $N \sim 50$. Clearly from Eq. (10), for lower order plates (low N), the variation of $\delta(\lambda)$ with wavelength is less rapid, but even for zero order plates the $1/\lambda$ term is important. Equation (6) shows that the term containing the circular polarization Stokes parameter S is proportional to $\sin\delta$. Hence the system becomes insensitive to S at wavelengths where $\delta(\lambda) = m\pi$ when the retardation plate becomes a half or full waveplate.
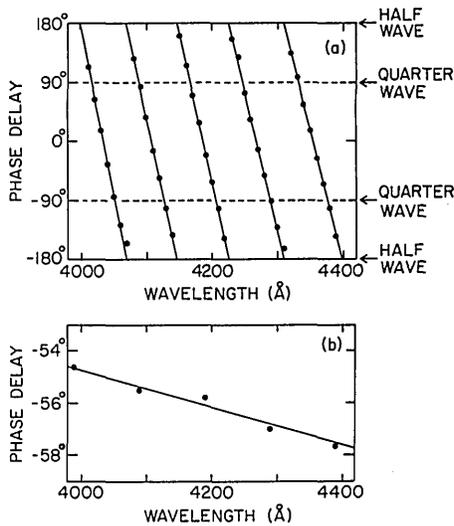
Fig. 2. Wavelength dependence of the phase difference for two retardation plates. Part (a) shows the variation for a quartz retarder of order $N \approx 50$ and a thickness of $\sim$2.5 mm, and (b) shows the wavelength variation of a plastic retarder over the same wavelength region. Note the enlarged scale for the phase delay in (b).

IV. Data Analysis

We consider a data collection system in which the light intensity $I_T(\alpha,\beta,\delta)$ of Eq. (6) is measured for different retardation plate rotation angles β_i. The retardation plate angles β_i can be controlled by a stepping motor drive so that a set of data will consist of N data points obtained at the angular positions $\beta_i (i = 1$ to $N)$, with the step size $\Delta\beta = \beta_{i+1} - \beta_i$ generally being a small fraction of 360°. In the experiments to be described below this step motor is controlled by an on-line minicomputer which also collects and analyzes the data. Immediate reduction of the data to the searched-for Stokes parameters of the light source is very desirable in many experiments and, except for cases of simple light source geometry, requires an on-line minicomputer or its equivalent.

A. Fourier Transform Analysis

The most straightforward technique would be to make a linear least squares fit of the data to Eq. (6), with the four Stokes parameters I, M, C, and S as adjustable parameters. However, we noted that Eq. (6) is a Fourier series

$$I_T(\beta) = C_0 + C_2 \cos2\beta + C_4 \cos4\beta + S_2 \sin2\beta + S_4 \sin4\beta, \quad (11)$$

which can be exactly inverted in a finite and discrete Fourier transform.[9] For an even number of data points $N = 2L$, the Fourier series coefficients are

$$\left. \begin{array}{l} C_{\omega k} = \dfrac{2}{N} \cdot \dfrac{1}{1 + \delta_{k0} + \delta_{kL}} \cdot \displaystyle\sum_{i=1}^{N} I_{T_i} \cos\omega_k \beta i \\[3mm] S_{\omega k} = \dfrac{2}{N} \cdot \dfrac{1}{1 + \delta_{k0} + \delta_{kL}} \cdot \displaystyle\sum_{i=1}^{N} I_{T_i} \sin\omega_k \beta_i \end{array} \right\}, \quad (12)$$

where $k = 0,1\ldots,L$ and δ_{kj} are delta functions, and

$$\omega_k = \frac{2\pi}{N} \cdot \frac{k}{\Delta\beta}, \text{ and } \beta_i = (i-1) \cdot \Delta\beta. \quad (13)$$

I_{Ti} is the measured intensity of the angular position β_i.

These expressions for the Fourier series coefficients are exact. The statistical uncertainty and χ^2 per degree of freedom can be computed in the conventional manner. Alternatively, uncertainties in the parameters can be estimated by computing the average deviation of the point-by-point values of the coefficients from the mean values calculated above. For example,

$$C_0{}^i = I_{T_i} - C_2 \cos2\beta_i - S_2 \sin2\beta_i - C_4 \cos4\beta_i - S_4 \sin4\beta_i, \quad (14)$$

and an uncertainty estimate for the fitting coefficient C_0 is

$$\sigma^2 = \frac{1}{N} \cdot \sum_{i=1}^{N} (C_0{}^i - C_0)^2. \quad (15)$$

All Fourier coefficients $C_{\omega k}$ and $S_{\omega k}$ are defined by Eqs. (12) and (13) for integers $k = 1$ to L and for ideal light polarization data will be zero except for $\omega_k = 0, 2,$ and 4. The relative magnitudes of $C_{\omega k}, S_{\omega k}$ for $\omega_k \neq 0,$ 2, or 4 provide tests of the precision of the data as shown below in some examples. These coefficients are calculated directly in fast Fourier transform routines and can be used with on-line minicomputers.[10]

For the above analysis, it is necessary to choose a step size $\Delta\beta$ such that $\omega_k = 2$ and $\omega_k = 4$ are included in the Fourier series [from Eq. (13)]. Clearly, the step size $\Delta\beta$ must be less than $\pi/4$, but it is also necessary that the retardation plate make a half integral number of revolutions for each set of N measurements. This becomes a natural requirement for automated systems in which the retardation plate is always rotating in the same sense. For an odd number of data points $N = 2L + 1$, Eqs. (12) apply if the δ_{kL} is removed.

Of course, for polarized light it is only necessary to calculate the five coefficients C_0, C_2, C_4, S_2, S_4 from Eq. (12). This can reduce the computation time and required memory storage of an on-line program to a minimum, and the analysis may even be programmed with a hand calculator. The restrictions on the number of data points are no longer applicable, except that $\Delta\beta \leq \pi/4$.

B. Stokes Parameters

The Stokes parameters can be determined from the Fourier series coefficients C_0, S_2, C_2, S_4, C_4 provided the angular position of the retardation plate is known, for example, $\beta = \beta_0$ at the first measurement position, and provided also that the polarizer angle α is known.

Thus, substituting $\beta - \beta_0$ for β in Eq. (6) and com-
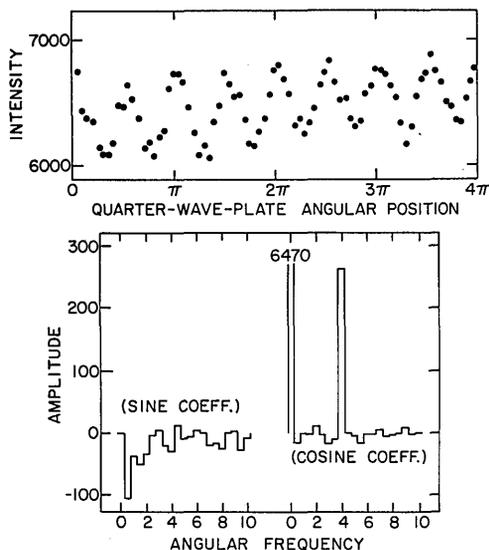
Fig. 3. Data from a linearly polarized source and the resulting Fourier series components C_i, S_i ($i = 0,1,\ldots,10$). The light is the NeII 3230-Å $3s^2D-3p^2D$ transition from 1-MeV Ne$^+$ excited in a thin perpendicular carbon foil. The derived Stokes parameters are $I = 6487 \pm 15$, $M/I = 0.059 \pm 0.003$, $C/I = -0.057 \pm 0.003$, $S/I = -0.006 \pm 0.002$.

paring with the Fourier expansion (11) we obtain expressions for the Stokes parameters

$$M = \frac{2}{1-\cos\delta}[C_4 \cos(2\alpha + 4\beta_0) + S_4 \sin(2\alpha + 4\beta_0)],$$

$$C = \frac{2}{1-\cos\delta}[S_4 \cos(2\alpha + 4\beta_0) - C_4 \sin(2\alpha + 4\beta_0)],$$

$$S = \frac{C_2}{\sin\delta \sin(2\alpha + 4\beta_0)} = \frac{-S_2}{\sin\delta \cos(2\alpha + 4\beta_0)}, \quad (16)$$

$$|S| = (C_2^2 + S_2^2)^{1/2}/\sin^2\delta,$$

$$I = C_0 - \frac{1+\cos\delta}{1-\cos\delta}\cdot[C_4 \cos(4\alpha + 4\beta_0) + S_4 \sin(4\alpha + 4\beta_0)].$$

We have renormalized these expressions so that the Stokes parameter I is near the average number of counts C_0.

Notice that the magnitude of S can be determined without knowledge of the linear polarizer angle α and the first data point position of the phase plate angle β_0. To determine I, M, C, and the sign of S requires an accurate measurement of the angles 2α and $2\beta_0$. These angles are easily obtained by first observing strongly circularly polarized light, and second observing strongly linearly polarized light. In the first case the Fourier coefficients C_2 and S_2 will be well determined, and from Eq. (16) we obtain

$$\tan(2\beta_0 + 2\alpha) = -C_2/S_2. \quad (17)$$

When linearly polarized light is incident such that $C = S = 0$, we obtain from Eq. (16) that

$$\tan(4\beta_0 + 2\alpha) = S_4/C_4. \quad (18)$$

Often it is possible to determine the linear polarizer angle α when the polarization system is being assembled. The angle β_0 can be determined by either above

method. At each measurement of the Stokes parameters, the angles given in Eqs. (17) and (18) can be determined as an additional check on the consistency of the data.

If the polarized light source being analyzed is known to have cylindrical symmetry about the x or y axis in our coordinate system, C and S are identically zero. For this special case, the values of the nonzero Stokes parameters are given by Eq. (16), and

$$|M| = \frac{4}{1-\cos\delta}\cdot(C_4^2 + S_4^2)^{1/2}. \quad (19)$$

The magnitude of the Stokes parameter M can now be measured without knowledge of the angles α and β_0. Accurate determination of the intensity and the sign of M are still only possible when the angles are specified.

The modulation ratio of the observed light is

$$\eta = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{M(1 - \cos\delta)}{2I + M\cos2\alpha(1 + \cos\delta)}. \quad (20)$$

For a phase shift $\delta = \pi/2$, a quarter waveplate, the modulation amplitude is $M/2$, half that of a system with a rotating polarizer. However, the modulation ratio η varies with the polarization angle α used. For 100% linear polarization with $M/I = +1$, η varies from $\frac{1}{3}$ when $\alpha = 0$ to $\eta = 1$ when $\alpha = \pi/2$.

It should be noted that this Fourier transform technique discriminates against long term variations in the source intensity better than a direct nonlinear least squares fitting of the data. In general such long term variations will appear in Fourier series coefficients different from C_0, C_2, S_2C_4, and S_4 which are used for determining the Stokes parameters (e.g., see Fig. 3).

V. Applications to Polarized Light Sources

The state of polarization of the light emitted from excited atoms or molecules provides information about their spatial anisotropies. Relations between the Stokes parameters and such anisotropies were developed by Perrin[1] and Chandrasekhar,[11] while Fano[12] introduced a quantum mechanical description for polarized light. Several recent papers discuss in greater detail the emission of polarized light from anisotropic sources.[13] Examples of typical anisotropic distributions occur in directional excitation experiments such as crossed-beam excitation, beam-gas excitation, Raman spectroscopy, sputtering and ion-surface scattering, channeling, and beam foil spectroscopy. Polarization measurements of light scattered from systems of particles are made in studies of biological specimens,[14] atmospheric pollution, and planetary atmospheres. We discuss here only the Stokes parameters for electric dipole radiation, but the concepts can clearly be generalized to radiation of other multipole moments of the source. We give examples of measurements in beam-foil spectroscopy and beam-surface scattering.

The experimental arrangement is shown in Fig. 4. The polarizer P consists of a Glan-air prism. A lens focuses the source on the monochromator entrance slit
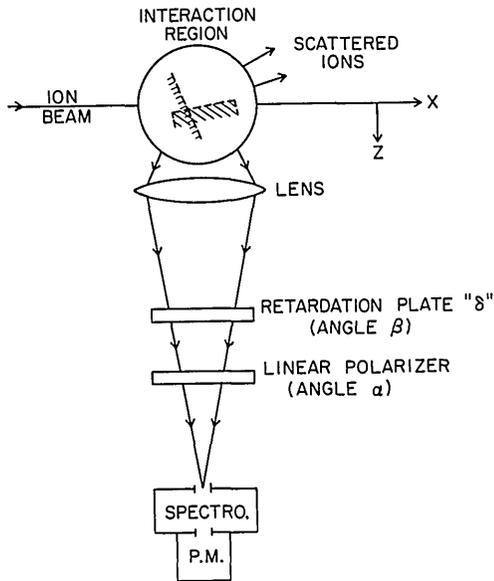
Fig. 4. Experimental arrangement for measuring the Stokes parameters of partially elliptically polarized light from fast ion–solid interactions.
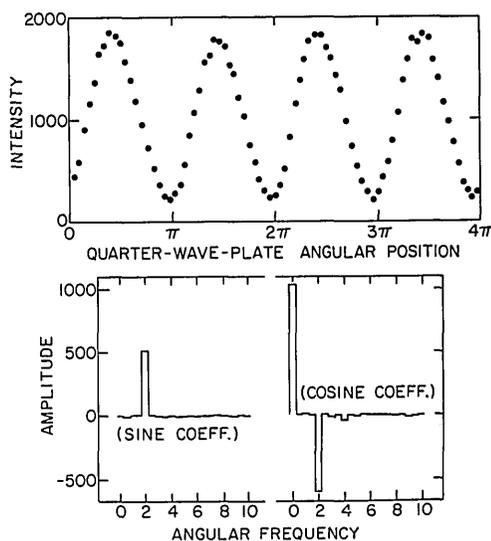
Fig. 5. Data from a strongly circularly polarized source and the Fourier series components C_i, S_i ($i = .. 0,1, .,10$). The light is the ArII 4610-Å $4s'^2 D_{5/2} - 4p'^2 F_{7/2}$ transition from 1-MeV Ar^+ excited by a Cu surface at 3° grazing incidence. The derived Stokes parameters are $I = 1021 \pm 5$, $M/I = -0.063 \pm 0.005$, $C/I = 0.012 \pm 0.005$, $S/I = 0.787 \pm 0.006$.

producing slightly covergent light through the polarization analysis system. Several different retardation plates were used to cover the 2000–6000-Å spectral range.

We consider the following types of observable light polarization:
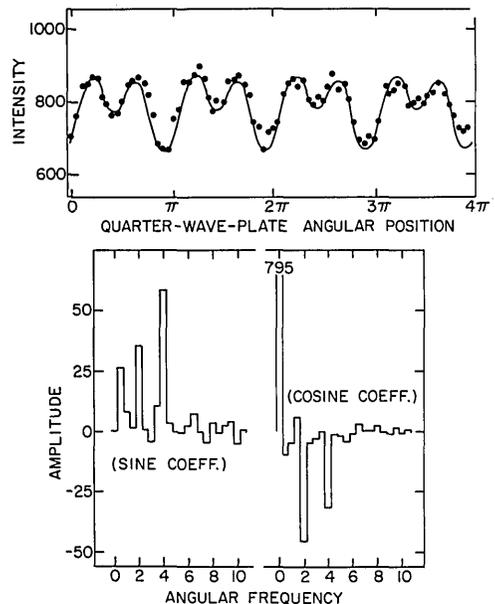
Fig. 6. Data from a source with circular and linear polarization fractions and the Fourier series components C_i, S_i ($i = 0,1,\ldots,10$). The derived Stokes parameters are $I = 780 \pm 3$, $M/I = 0.066 \pm 0.006$, $C/I = -0.086 \pm 0.006$, $S/I = 0.082 \pm 0.007$. The line through the data is the intensity given by Eq. (11) with the coefficients $C_0, C_2 \ldots$ taken from the results shown in the lower part of the figure.

(1) A spherically symmetric source emits unpolarized light of equal intensity in all directions. Thus, the Stokes parameters are zero except for the total intensity I: ($M = C = S = 0$).

(2) A cylindrically symmetric source in general can emit linearly polarized light in a given direction. However, the linear polarization fractions M/I and C/I will be zero for observations along the symmetry axis and reach maxima for observations perpendicular to this axis. For example, for observation along the z axis, if x or y coincides with the symmetry axis, $C/I = 0$. The circular polarization S is zero in all directions, and the transmitted light intensity through the polarization system reduces to Eq. (8).

In Fig. 3 we show an example of linear polarization modulations of frequency 4ω for $\alpha = \pi/4$. The Fourier transform coefficients have three nonzero values for C_0, C_4, and S_4, from which can be calculated the Stokes parameters M and I [Eq. (19)].

(3) Circularly polarized light also has only two nonzero Stokes parameters I and S, with $M = C = 0$, and a single modulation is observed but with a frequency 2ω, the modulation ratio being maximized for a perfect quarter waveplate with $\sin\delta = \pm 1$ and becoming unity for 100% circularly polarized light $I = S = 1$. In Fig. 5 we show an example of strong circular polarization ($S = 0.781$) with very low linear polarization fractions.

(4) Partially elliptically polarized light is a superposition of linear- and circular-polarized and unpolarized fractions and will in general yield modulations of both 2ω and 4ω frequencies. Figure 6 shows such a

complex curve which is readily decomposed into its Fourier series components as shown below. Note that the 2ω and 4ω frequencies are well resolved in the Fourier spectrum. This is an advantage over the rotating polarizer technique where the modulation frequencies are the same for linear and circular polarized fractions which are distinguished only by a phase difference of $\pi/4$.

VI. Conclusions

We have described an optical detection system based on a rotating retardation plate followed by a fixed polarizer. This system can accurately determine the complete polarization state of a beam of light with simultaneous measurement of the three relative Stokes parameters M/I, C/I (the linear polarization fractions), and S/I (the circular polarization fraction) and total intensity I.

We note that the linear polarization signals are well resolved from the circular polarization signal in this rotating retardation plate technique as contrasted with an optical system using a fixed retardation plate followed by a rotating linear polarizer. In the former system, the two types of polarization signal are modulated at different frequencies (4ω and 2ω, respectively), while in the latter system, although the linear polarization signal is doubled, it has the same frequency (2ω) as the circular polarization signal. This clear resolution of the signals is experimentally very important when searching for a small circular polarization signal from a source of large linear polarization or vice versa.

Fourier series analysis of the data can easily be performed by an on-line minicomputer to provide immediate values of the Stokes parameters. This is a useful feature for rapid data collection systems and for systems undergoing rapid variations in time. We have applied this system to measurements of polarized light emitted by excited beam ions.

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Nicholas George, the new director of the Institute of Optics of the University of Rochester.