1-3 Measuring and Constructing Angles

Warm Up

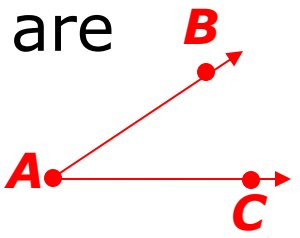
Lesson Presentation

Lesson Quiz

Warm Up

1. Draw $\overrightarrow{AB}$ and $\overrightarrow{AC}$, where A, B, and C are noncollinear.

Possible answer:



2. Draw opposite rays $\overrightarrow{DE}$ and $\overrightarrow{DF}$.

Solve each equation.

3. $2x + 3 + x - 4 + 3x - 5 = 180$ 31

4. $5x + 2 = 8x - 10$ 4

Objectives

Name and classify angles.

Measure and construct angles and angle bisectors.

Vocabulary

angle
vertex
interior of an angle
exterior of an angle
measure
degree
acute angle
right angle
obtuse angle
straight angle
congruent angles
angle bisector

1-3 Measuring and Constructing Angles

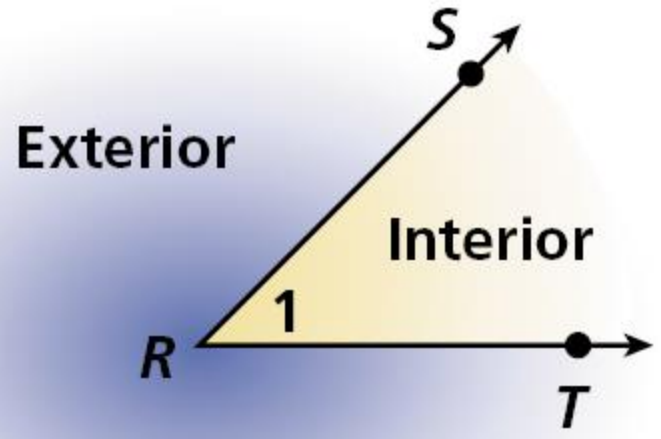
A transit is a tool for measuring angles. It consists of a telescope that swivels horizontally and vertically. Using a transit, a survey or can measure the *angle* formed by his or her location and two distant points.

An **angle** is a figure formed by two rays, or sides, with a common endpoint called the **vertex** (plural: *vertices*). You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the **interior of an angle**. The **exterior of an angle** is the set of all points outside the angle.

Angle Name

$\angle R$, $\angle SRT$, $\angle TRS$, or $\angle 1$



You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.

Example 1: Naming Angles

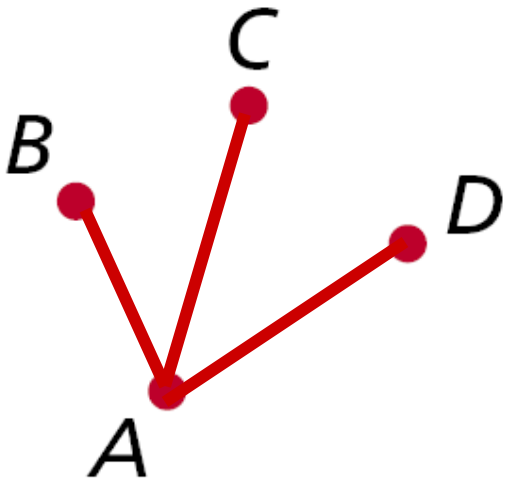
A surveyor recorded the angles formed by a transit (point *A*) and three distant points, *B*, *C*, and *D*. Name three of the angles.

Possible answer:

$\angle BAC$

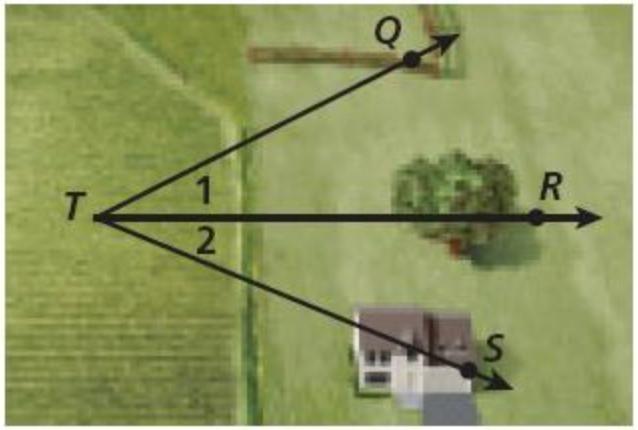
$\angle CAD$

$\angle BAD$



Check It Out! Example 1

Write the different ways you can name the angles in the diagram.



∠RTQ, ∠T, ∠STR, ∠1, ∠2

1-3 Measuring and Constructing Angles

The **measure** of an angle is usually given in degrees. Since there are 360° in a circle, one **degree** is $\frac{1}{360}$ of a circle. When you use a protractor to measure angles, you are applying the following postulate.

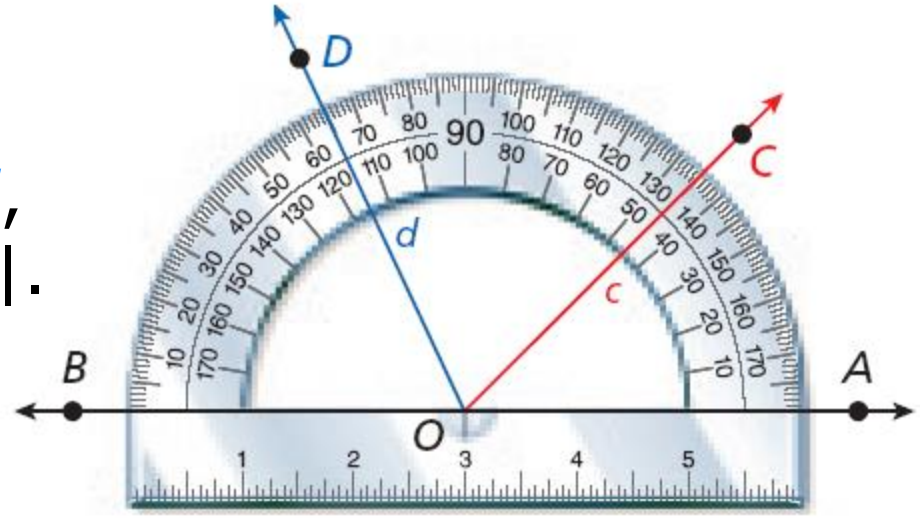
Postulate 1-3-1 Protractor Postulate

Given $\overleftrightarrow{AB}$ and a point O on $\overleftrightarrow{AB}$, all rays that can be drawn from O can be put into a one-to-one correspondence with the real numbers from 0 to 180.

1-3 Measuring and Constructing Angles

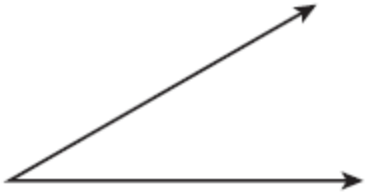
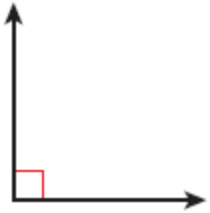
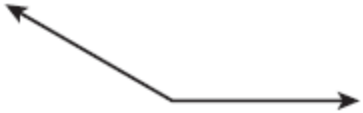
You can use the Protractor Postulate to help you classify angles by their measure. The measure of an angle is the absolute value of the difference of the real numbers that the rays correspond with on a protractor.

If $\overrightarrow{OC}$ corresponds with c and $\overrightarrow{OD}$ corresponds with d, $m\angle DOC = |d - c|$ or $|c - d|$.



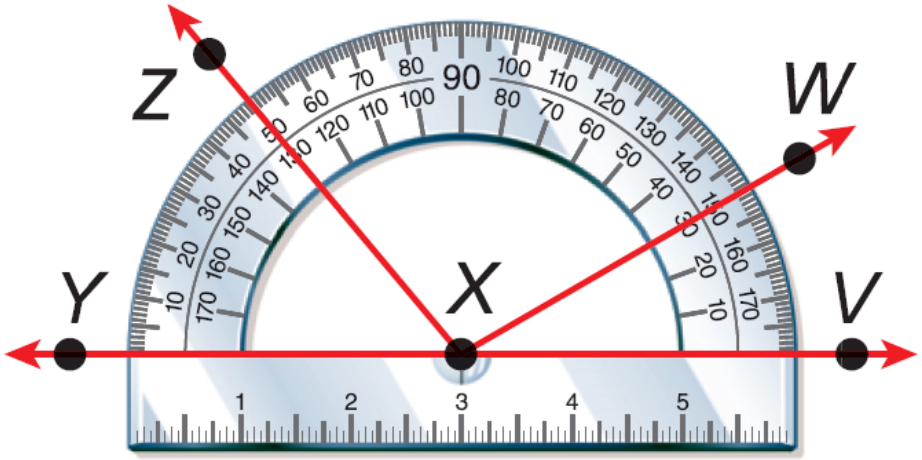
1-3 Measuring and Constructing Angles

Types of Angles

Acute Angle	Right Angle	Obtuse Angle	Straight Angle
			
Measures greater than 0° and less than 90°	Measures 90°	Measures greater than 90° and less than 180°	Formed by two opposite rays and meaures 180°

Example 2: Measuring and Classifying Angles

Find the measure of each angle. Then classify each as acute, right, or obtuse.



A. $\angle WXV$

$m\angle WXV = 30°$

$\angle WXV$ is acute.

B. $\angle ZXW$

$m\angle ZXW = |130° - 30°| = 100°$

$\angle ZXW$ = is obtuse.

Check It Out! Example 2

Use the diagram to find the measure of each angle. Then classify each as acute, right, or obtuse.

a. $\angle BOA$

$m\angle BOA = 40°$

$\angle BOA$ is acute.

b. $\angle DOB$

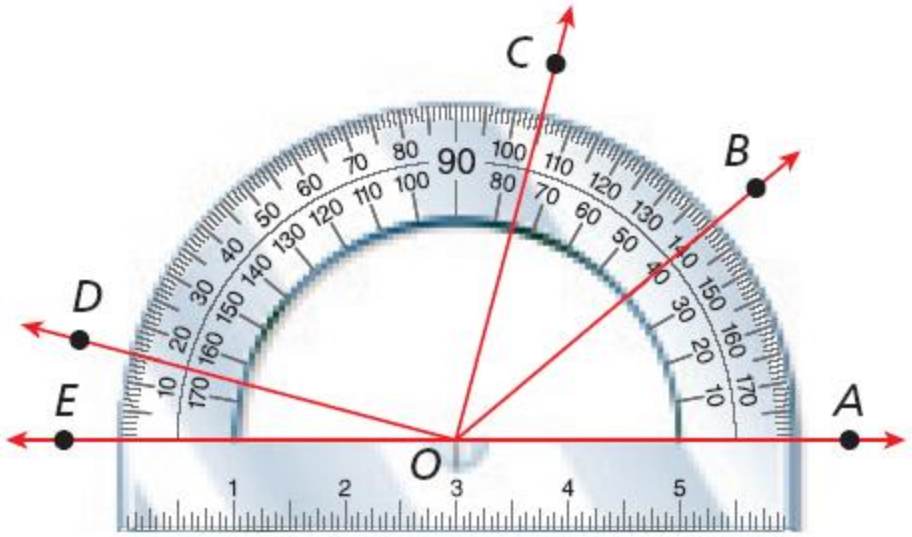
$m\angle DOB = 125°$

$\angle DOB$ is obtuse.

c. $\angle EOC$

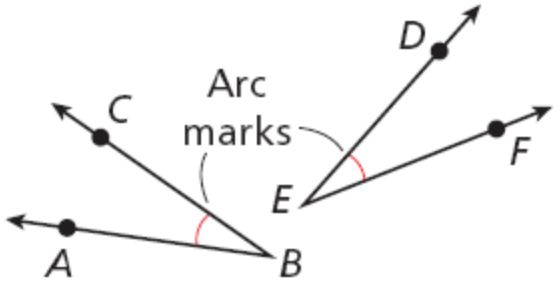
$m\angle EOC = 105°$

$\angle EOC$ is obtuse.



Congruent angles are angles that have the same measure. In the diagram, m∠*ABC* = m∠*DEF*, so you can write ∠*ABC* ≅ ∠*DEF*. This is read as "angle ABC is congruent to angle *DEF*." *Arc marks* are used to show that the two angles are congruent.

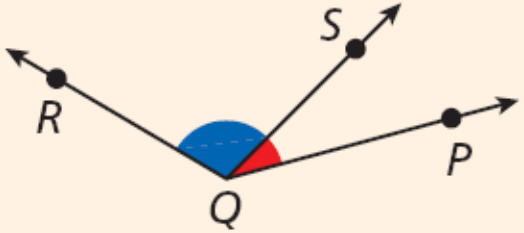
The Angle Addition Postulate is very similar to the Segment Addition Postulate that you learned in the previous lesson.



Postulate 1-3-2 **Angle Addition Postulate**

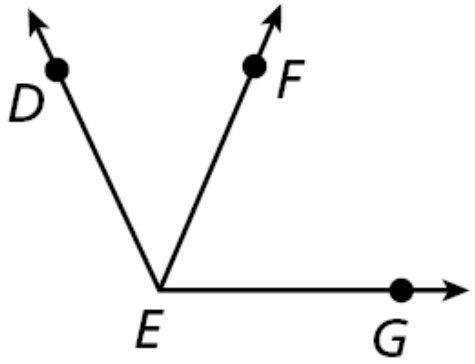
If S is in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$.

($\angle$ Add. Post.)



Example 3: Using the Angle Addition Postulate

m∠*DEG* = 115°, and m∠*DEF* = 48°. Find m∠*FEG*



m∠*DEG* = m∠*DEF* + m∠*FEG* ∠ *Add. Post.*

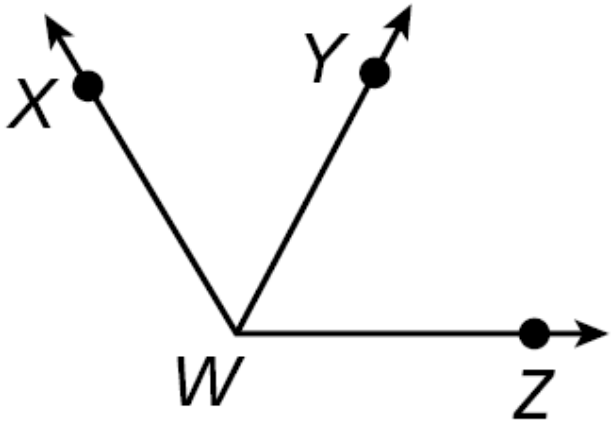
115° = 48° + m∠*FEG* *Substitute the given values.*

−48° −48° *Subtract 48 from both sides.*

67° = m∠*FEG* *Simplify.*

Check It Out! Example 3

m∠*XWZ* = 121° and m∠*XWY* = 59°. Find m∠*YWZ*.



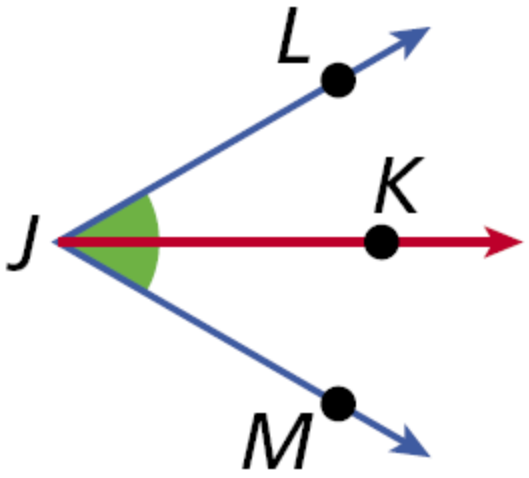
m∠*YWZ* = m∠*XWZ* − m∠*XWY* *∠ Add. Post.*

m∠*YWZ* = 121° − 59° *Substitute the given values.*

m∠*YWZ* = 62° *Subtract.*

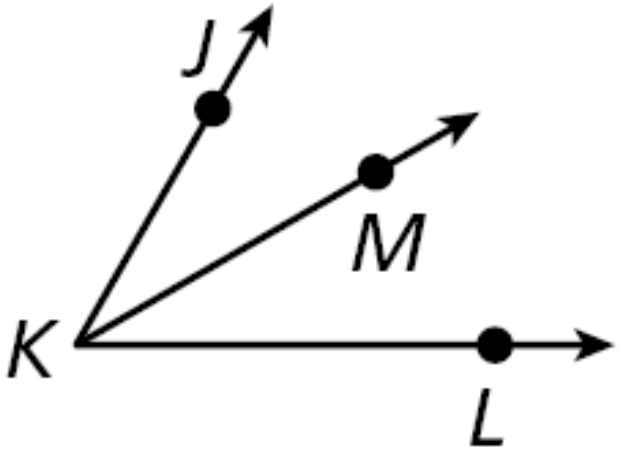
An **angle bisector** is a ray that divides an angle into two congruent angles.

$\overrightarrow{JK}$ bisects $\angle LJM$; thus $\angle LJK \cong \angle KJM$.



Example 4: Finding the Measure of an Angle

$\overrightarrow{KM}$ bisects $\angle JKL$, $m\angle JKM = (4x + 6)°$, and $m\angle MKL = (7x - 12)°$. Find $m\angle JKM$.



Example 4 Continued

Step 1 Find x.

$m\angle JKM = m\angle MKL$	*Def. of ∠ bisector*
$(4x + 6)° = (7x - 12)°$	*Substitute the given values.*
$+12 \quad +12$	*Add 12 to both sides.*
$4x + 18 = 7x$	*Simplify.*
$-4x \quad -4x$	*Subtract 4x from both sides.*
$18 = 3x$	*Divide both sides by 3.*
$6 = x$	*Simplify.*

Example 4 Continued

Step 2 Find m∠*JKM*.

m∠*JKM* = 4*x* + 6

= 4(6) + 6 *Substitute 6 for x.*

= 30° *Simplify.*

Check It Out! Example 4a

Find the measure of each angle.

$\overrightarrow{QS}$ bisects $\angle PQR$, $m\angle PQS = (5y - 1)°$, and $m\angle PQR = (8y + 12)°$. Find $m\angle PQS$.

Step 1 Find *y*.

$\angle PQS = \frac{1}{2}\angle PQR$ *Def. of ∠ bisector*

$(5y - 1)° = \frac{1}{2}(8y + 12)°$ *Substitute the given values.*

$5y - 1 = 4y + 6$ *Simplify.*

$y - 1 = 6$ *Subtract 4y from both sides.*

$y = 7$ *Add 1 to both sides.*

Check It Out! Example 4a Continued

Step 2 Find m∠*PQS*.

$m\angle PQS = 5y - 1$

$= 5(7) - 1$ *Substitute 7 for y.*

$= 34°$ *Simplify.*

Check It Out! Example 4b

Find the measure of each angle.

$\overrightarrow{JK}$ bisects $\angle LJM$, $m\angle LJK = (-10x + 3)°$, and $m\angle KJM = (-x + 21)°$. Find $m\angle LJM$.

Step 1 Find x.

$\angle LJK = \angle KJM$	*Def. of ∠ bisector*
$(-10x + 3)° = (-x + 21)°$	*Substitute the given values.*
$+x \quad +x$	*Add x to both sides.*
$-9x + 3 = 21$	*Simplify.*
$-3 \quad -3$	*Subtract 3 from both sides.*
$-9x = 18$	*Divide both sides by –9.*
$x = -2$	*Simplify.*

Check It Out! Example 4b Continued

Step 2 Find m∠*LJM*.

$m\angle LJM = m\angle LJK + m\angle KJM$

$= (-10x + 3)° + (-x + 21)°$

$= -10(-2) + 3 - (-2) + 21$ *Substitute –2 for x.*

$= 20 + 3 + 2 + 21$ *Simplify.*

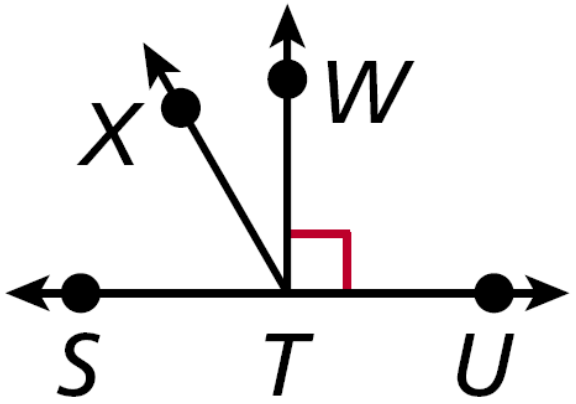
$= 46°$

Lesson Quiz: Part I

Classify each angle as acute, right, or obtuse.

1. $\angle XTS$ acute

2. $\angle WTU$ right



3. K is in the interior of $\angle LMN$, $m\angle LMK = 52°$, and $m\angle KMN = 12°$. Find $m\angle LMN$.

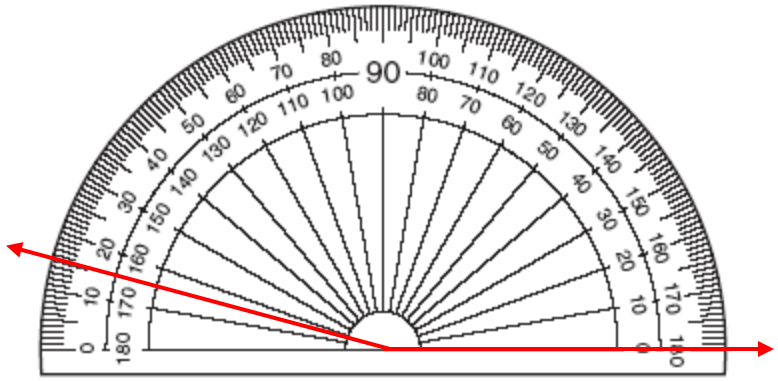
64°

Lesson Quiz: Part II

4. $\overrightarrow{BD}$ bisects $\angle ABC$, $m\angle ABD = \left(\frac{1}{2}y + 10\right)°$, and $m\angle DBC = (y + 4)°$. Find $m\angle ABC$.

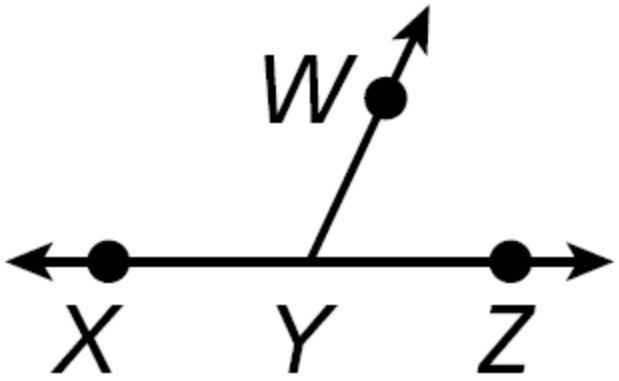
32°

5. Use a protractor to draw an angle with a measure of 165°.



Lesson Quiz: Part III

6. $m\angle WYZ = (2x - 5)°$ and $m\angle XYW = (3x + 10)°$. Find the value of x.



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