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Measuring income redistribution: beyond the proportionality standard

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Abstract

Traditional analyses of redistributive effects of the tax-benefit system are rooted in the concepts of relative income inequality and proportionality. This observation also applies to decompositions proposed by Kakwani (1977, 1984) and Lambert (1985) that reveal the vertical and horizontal effects of tax-benefit instruments. This paper generalises those decompositions within the frameworks of the alternative inequality concepts suggested by Ebert (2004) and Bosmans et al. (2014). As expected, the results of the empirical analysis indicate that for different views of inequality, different taxes and benefits play significantly different roles in reducing inequality.

JEL: D63, H22, H23

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1 Introduction

Measuring requires making a comparison with some standard. When measuring the redistributive effects of taxes and benefits, the prevalent 'standard' is that of proportionality with pre-tax income. According to this view, a proportional tax neither increases nor decreases income inequality. If a tax is more pro-rich than a counterfactual equal-yield proportional tax, it is inequality-decreasing, 'progressive' or vertically equitable. On the contrary, if a tax is less pro-rich than a counterfactual equal-yield proportional tax, it is inequality-increasing, 'regressive' or vertically inequitable.

According to Pfingsten (1986), an *inequality-preserving tax* is one that leaves income inequality unchanged in the transition from pre-tax to post-tax income. From the proportionality perspective, an inequality-preserving tax is a proportional tax because inequality is defined following the *relative inequality concept* (RIC), in which a proportional change of all peoples' incomes does not change the state of inequality. The RIC is embedded in the widely used tools of income distribution and redistribution analysis, such as the Lorenz curve and the Gini index.

Musgrave and Thin (1948)'s measure of 'effective progression', which became one of the cornerstones of redistribution analysis, was defined using the areas confined by the Lorenz pre-tax and post-tax curves. Similar indices were later proposed by Kakwani (1977) and Reynolds and Smolensky (1977). Fellman (1976), Jakobsson (1976) and Eichhorn et al. (1984) provided theoretical proofs of the relationship between progressivity and inequality reduction in the RIC.

Lambert (1985, 2001) extended the use of Lorenz and concentration curves in analyses of the overall tax-benefit system. By comparing Lorenz (concentration) curves of pre-fiscal income, taxes, benefits and post-fiscal income, conclusions can be reached regarding the vertical and horizontal equity of various tax-benefit instruments and the fiscal system as a whole. The differences between the relevant curves can be aggregated to obtain Gini-like indices of progressivity and vertical and horizontal effects.

However, in the 1980s the proportionality view in the measurement of tax progressivity was challenged by several authors. They employed *alternative inequality concepts*, which were introduced to the modern literature by Kolm (1976) (but envisaged much earlier by Dalton (1920)). According to the *absolute* inequality concept (AIC), income inequality is not changed if all peoples' incomes are changed by the same amount. Later, a range of inequality concepts were conceived that are often referred to as 'intermediate', because they reflect certain combinations of the RIC and AIC transformations; e.g., Bossert and Pfingsten (1990)'s θ-inequality concept (θ-IC), the Ebert (2004)'s d-inequality concept (d-IC) and Bosmans et al. (2014)'s α-inequality concept (α-IC). For other inequality concepts see, e.g., del Río and Ruiz-Castillo (2000), Yoshida (2005) and Zheng (2004).

The conditions for which a tax reduces inequality for the alternative inequality concepts were derived by Moyes (1988) (for the AIC), Pfingsten (1987, 1988) (for the θ-IC) and Ebert (2010) (for the d-IC). Ebert and Moyes (2000) extended the analysis to the case of heterogeneous households. These authors showed that, in order to be inequality-reducing, a tax does not have to be 'progressive' in the RIC sense. In other words, proportionality is only one of the possible measurement 'standards'. Each inequality concept gives rise to its own 'standard' for measuring the redistributive effects of taxes. In the AIC, an inequality-preserving tax is a tax of the same amount for all income units, i.e., a lump-sum tax or head tax. For intermediate inequality concepts, an inequality-preserving tax is a combination of two components: one that is proportional with pre-tax income and the other that is a fixed amount.

Urban (2014a) reviewed many of the methods based on the Lorenz (concentration) curves, which were used to assess the contributions of taxes and benefits to the overall redistributive effects. They are predominantly rooted in the RIC. One exception is the famous Rao (1969) decomposition of disposable income inequality (independently derived by several other authors). It has been demonstrated that Rao (1969)'s model is in fact rooted in the AIC, which explains the significant differences in empirical results obtained by this method *vis-à-vis* RIC-based methods (see, e.g., Fuest et al. (2010)). Furthermore, Urban (2014a) derived the decompositions of vertical and horizontal effects of tax-benefit instruments both in the RIC and AIC.

This paper extends the apparatus for the measurement of tax-benefit redistributive effects by developing *à la* Kakwani-Lambert decompositions within various non-RIC frameworks. Two of the above-mentioned inequality concepts are employed: the α-IC and the d-IC. The former is used because it is intuitive and relatively easy to comprehend; α-inequality-neutral taxes and benefits are created and used to obtain counterfactual post-tax, post-benefit and post-fiscal income variables. By comparing the concentration curves of these counterfactual incomes with the concentration curves of actual incomes, we are generalising the Kakwani-Lambert approach to a multiple inequality view framework. We use d-IC because it provides easy-to-obtain d-IC-Lorenz and concentration curves. The relationship between decompositions based on the α-IC and d-IC has been established: they are fully equivalent for pairs of α and d parameter values.

When alternative inequality concepts are employed along with the RIC, a whole new set of results arises regarding the roles of taxes and benefits in achieving overall redistribution. The relative contributions of some tax-benefit instruments to the overall vertical effect may significantly change as α is varied. Furthermore, even the direction of influence may change; for instance, some instrument that reduces inequality in the RIC may increase inequality in the AIC.

Following the theoretical part, the paper also contains an empirical section that applies the newly derived decompositions to the Croatian tax-benefit system. The investigation is particularly interested in two main instruments—the child benefit and the tax allowance for supported children—that provide support to households with children and are common topics of domestic policy debates.

The paper is organised as follows. Section 2 summarises the basic models: redistributive effects in the RIC and α- and d-inequality concepts. Section 3 develops and discusses new measures of redistributive effects for alternative inequality concepts. Section 4 presents the empirical analysis applying the new method. Section 5 concludes.

2 Background

2.1 Basic variables

The model tax-benefit system consists of one tax and one benefit. The use of such a simple system greatly simplifies the exposition, but all results can be simply extended to real-world cases with several taxes and benefits. Pre-fiscal income is income before the application of taxes and benefits. The matrix $\dot{M}$ shows data 'collected' for the model tax-benefit system. There are n households in the sample; $\dot{X}_i$, $\dot{T}_i$ and $\dot{B}_i$ are the monetary values of pre-fiscal income, the tax and the benefit of household i, respectively. Furthermore, s_i, a_i and c_i are the sampling weight, the number of adults and the number of children, respectively.

$$\dot{M} = \begin{bmatrix} \dot{X}_1 & \dot{T}_1 & \dot{B}_1 & s_1 & a_1 & c_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dot{X}_i & \dot{T}_i & \dot{B}_i & s_i & a_i & c_i \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dot{X}_n & \dot{T}_n & \dot{B}_n & s_n & a_n & c_n \end{bmatrix}$$

Data on household members are used to construct equivalence factors. By applying the equivalence scheme to $\dot{X}_i$, $\dot{T}_i$ and $\dot{B}_i$, we obtain the equivalised values X_i, T_i and B_i, respectively (see section 3.4.3 for details). The income unit becomes the *equivalised* household.

Post-tax income, post-benefit income and post-fiscal income are obtained as $Y_i^T = X_i - T_i$, $Y_i^B = X_i + B_i$ and $Y_i^F = X_i - T_i + B_i$. The matrix M contains all of these newly created variables. Other variables are added in the subsequent analysis.

$$M = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_i & T_i & B_i & Y_i^T & Y_i^B & Y_i^F & s_i & a_i & c_i \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

In the next step the rows of matrix M are sorted in increasing values of pre-fiscal income (the first column) to obtain the matrix M_X. The X in the subscript indicates that pre-fiscal income was used for ordering income units.

$$M_X = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{i:X} & T_{i:X} & B_{i:X} & Y_{i:X}^T & Y_{i:X}^B & Y_{i:X}^F & s_{i:X} & a_{i:X} & c_{i:X} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Other variables can also be used for sorting to obtain, e.g., M_{Y^F}, M_{Y^T} or M_{Y^B}. The *analytical* weight of income unit i, $w_{i:X}$, tells us how many times unit i is counted in calculations of the mean and other related indicators. It is calculated as $w_{i:X} = m_{i:X} s_{i:X}$, where $m_{i:X}$ is the number of members in income unit i (see section 3.4.3 for details).

The size of the total population, mean pre-fiscal income and the quantiles of pre-fiscal income distribution are obtained, respectively, as follows:

$$\begin{aligned} N &= \sum\nolimits_{i=1}^{n} w_{i:X} \\ \mu_X &= \frac{1}{N} \sum\nolimits_{i=1}^{n} w_{i:X} X_{i:X} \\ q_{i:X} &= \frac{1}{N\mu_X} \sum\nolimits_{j=1}^{i} w_{j:X} \end{aligned}$$

The ordinates of the *relative* concentration curves (RCCs) of pre- and post-fiscal income are obtained as follows:

$$\begin{aligned} C(X_{i:X}) &= \frac{1}{N\mu_X} \sum\nolimits_{j=1}^{i} w_{j:X} X_{j:X} \\ C(Y_{i:X}^F) &= \frac{1}{N\mu_{Y^F}} \sum\nolimits_{j=1}^{i} w_{j:X} Y_{i:X}^F \\ C(Y_{i:Y^F}^F) &= \frac{1}{N\mu_{Y^F}} \sum\nolimits_{j=1}^{i} w_{j:Y^F} Y_{i:Y^F}^F \end{aligned}$$

Notice the difference between $C(Y_{i:X}^F)$ and $C(Y_{i:Y^F}^F)$ that results from how income units are sorted. When the *object* variable and the *sorting* variable are the same variable—as in cases of

$C(Y^F_{i:Y^F})$ and $C(X_{i:X})$— we are speaking of the *Lorenz* curve as a special case of concentration curve.

The ordinates of the *absolute* concentration curves (ACCs) of pre- and post-fiscal income are obtained as follows:

$$
\begin{aligned}
\bar{C}^a(X_{i:X}) &= \frac{1}{N}\sum\nolimits_{j=1}^{i} w_{j:X}(X_{j:X}-\mu_X) \\
\bar{C}^a(Y^F_{i:X}) &= \frac{1}{N}\sum\nolimits_{j=1}^{i} w_{j:X}(Y^F_{j:X}-\mu_{Y^F}) \\
\bar{C}^a(Y^F_{i:Y^F}) &= \frac{1}{N}\sum\nolimits_{j=1}^{i} w_{j:Y^F}(Y^F_{j:Y^F}-\mu_{Y^F})
\end{aligned}
$$

For convenience reasons, the ACCs *of all variables* are divided by $-N\mu_X$. Thus, ACC ordinates for pre- and post-fiscal income are obtained as follows:

$$
\begin{aligned}
C^a(X_{i:X}) &= -\frac{1}{N\mu_X}\bar{C}^a(X_{i:X}) \\
C^a(Y^F_{i:X}) &= -\frac{1}{N\mu_X}\bar{C}^a(Y^F_{i:X}) \\
C^a(Y^F_{i:Y^F}) &= -\frac{1}{N\mu_X}\bar{C}^a(Y^F_{i:Y^F})
\end{aligned}
$$

Analogously, the means, RCCs and ACCs of other variables are obtained. Their notation is summarised in Table 1, where $\phi_T = \mu_T/\mu_X$ and $\phi_B = \mu_B/\mu_X$.

Table 1: Basic indicators

Variable	Mean	Share in μ_X	RCC	ACC
X	μ_X	1	$C(X_{i:X})$	$C^a(X_{i:X})$
T	μ_T	ϕ_T	$C(T_{i:X})$	$C^a(T_{i:X})$
B	μ_B	ϕ_B	$C(B_{i:X})$	$C^a(B_{i:X})$
Y^T	μ_{Y^T}	$1-\phi_T$	$C(Y^T_{i:X})$, $C(Y^T_{i:Y^T})$	$C^a(Y^T_{i:X})$, $C^a(Y^T_{i:Y^T})$
Y^B	μ_{Y^B}	$1+\phi_B$	$C(Y^B_{i:X})$, $C(Y^B_{i:Y^B})$	$C^a(Y^B_{i:X})$, $C^a(Y^B_{i:Y^B})$
Y^F	μ_{Y^F}	$1-\phi_T+\phi_B$	$C(Y^F_{i:X})$, $C(Y^F_{i:Y^F})$	$C^a(Y^F_{i:X})$, $C^a(Y^F_{i:Y^F})$

2.2 Inequality, vertical and horizontal effects

$C(Y^F_{i:Y^F})$ is the relative Lorenz curve of post-fiscal income. It is known that $C(Y^F_{i:Y^F})$ never lies above $C(Y^F_{i:X})$ (Lambert, 2001). The two curves will coincide if the fiscal system does not induce the reranking of income units; otherwise, if reranking occurs, $C(Y^F_{i:Y^F}) < C(Y^F_{i:X})$ for some or all i. Based on this property, the following decomposition applies to the fiscal system in the RIC:

$$
\begin{aligned}
C(Y^F_{i:Y^F}) - C(X_{i:X}) &= [C(Y^F_{i:X}) - C(X_{i:X})] - [C(Y^F_{i:X}) - C(Y^F_{i:Y^F})] \\
r_i^{F;r} &= v_i^{F;r} - h_i^{F;r} \qquad (1)
\end{aligned}
$$

The difference on the left-hand side of equation 1, equal to $r_i^{F;r}$, represents the *inequality* effect (IE) of the fiscal system. The differences on the right-hand side, $v_i^{F;r}$ and $h_i^{F;r}$, are the *vertical* effect (VE) and the *horizontal* effect (HE) of the fiscal system, respectively. The letter r in the superscript denotes that the measures belong to the RIC. The decomposition from equation

Table 2: Inequality, vertical and horizontal effects in the RIC

	Fiscal system	Tax	Benefit
IE	$r_i^{F;r} = C(Y^F_{i:Y^F}) - C(X_{i:X})$	$r_i^{T;r} = C(Y^T_{i:Y^T}) - C(X_{i:X})$	$r_i^{B;r} = C(Y^B_{i:Y^B}) - C(X_{i:X})$
VE	$v_i^{F;r} = C(Y^F_{i:X}) - C(X_{i:X})$	$v_i^{T;r} = C(Y^T_{i:X}) - C(X_{i:X})$	$v_i^{B;r} = C(Y^B_{i:X}) - C(X_{i:X})$
HE	$h_i^{F;r} = C(Y^F_{i:X}) - C(Y^F_{i:Y^F})$	$h_i^{T;r} = C(Y^T_{i:X}) - C(Y^T_{i:Y^T})$	$h_i^{B;r} = C(Y^B_{i:X}) - C(Y^B_{i:Y^B})$

1 was first proposed by Kakwani (1984), but for the tax system only, i.e. in the form $r_i^{T;r} = v_i^{T;r} - h_i^{T;r}$. Table 2 summarises different IEs, VEs and HEs in the RIC.

The vertical effect represents the *potential* IE, i.e. one that would be achieved in the absence of horizontal inequity. Because in real-world fiscal systems some horizontal inequity always exists, the actual IE is lower than the potential IE. Lambert (1985, 2001) decomposes the VE of the fiscal system into terms that represent the contributions of the tax and the benefit as follows (proof is given in the Appendix):

$$v_i^{F;r} = \frac{(1-\phi_T)v_i^{T;r} + (1+\phi_B)v_i^{B;r}}{1-\phi_T+\phi_B} \tag{2}$$

Table 3: Inequality, vertical and horizontal effects in the AIC

	Fiscal system	Tax	Benefit
IE	$r_i^{F;a} = C^a(Y^F_{i:Y^F}) - C^a(X_{i:X})$	$r_i^{T;a} = C^a(Y^T_{i:Y^T}) - C^a(X_{i:X})$	$r_i^{B;a} = C^a(Y^B_{i:Y^B}) - C^a(X_{i:X})$
VE	$v_i^{F;a} = C^a(Y^F_{i:X}) - C^a(X_{i:X})$	$v_i^{T;a} = C^a(Y^T_{i:X}) - C^a(X_{i:X})$	$v_i^{B;a} = C^a(Y^B_{i:X}) - C^a(X_{i:X})$
HE	$h_i^{F;a} = C^a(Y^F_{i:X}) - C^a(Y^F_{i:Y^F})$	$h_i^{T;a} = C^a(Y^T_{i:X}) - C^a(Y^T_{i:Y^T})$	$h_i^{B;a} = C^a(Y^B_{i:X}) - C^a(Y^B_{i:Y^B})$

The measures of IE, VE and HE can be analogously derived for the AIC; see Table 3. The AIC counterparts of decompositions from equations 1 and 2 are given as follows:

$$r_i^{F;a} = v_i^{F;a} - h_i^{F;a} \tag{3}$$

$$v_i^{F;a} = v_i^{T;a} + v_i^{B;a} \tag{4}$$

Equation 3 represents the decomposition of IE into VE and HE in the AIC. Equation 4 decomposes VE into contributions of the tax and the benefit in the AIC (proof is given in the Appendix).

2.3 The α-inequality concept

Assume that we want to compare the inequality of pre-fiscal income distribution X and two post-fiscal income distributions, $Y^\star$ and Y, such that $\mu_X = \mu_{Y^\star}$ and $\mu_X \gtrless \mu_Y$. In the first comparison, because X and $Y^\star$ have equal means, the Lorenz dominance criterion is readily applied, as follows:

The fiscal system $X \to Y^\star$ is strictly inequality reducing if
$C(Y^\star_{i:Y^\star}) > C(X_{i:X}), \forall i.$

If $\geq$ holds instead of $>$ for at least some i, then 'strictly' falls out. However, when comparing X and Y the above criterion is no longer applicable because these distributions do not have equal means. Therefore, to make the comparison possible, Bosmans et al. (2014) propose the α-IC that comprises the following procedure. Create a counterfactual distribution, X^{α}, as follows:

$$X_i^{\alpha} = X_i + \alpha \frac{\lambda}{\mu_X} X_i + (1-\alpha)\lambda \tag{5}$$

where $\lambda = \mu_Y - \mu_X$ and $\alpha \in [0,1]$. According to Bosmans et al. (2014), X^{α} has the same α-inequality as X. Equation 5 shows that X^{α} consists of three parts: the original income X, the X-dependent part, and the X-independent part.

Because $\mu_{X^{\alpha}} = \mu_Y$, X^{α} and Y can be compared using the Lorenz dominance criterion. We have the following proposition:

The fiscal system $X \to Y$ is strictly α-inequality reducing if
$C(Y_{i:Y}) > C(X^{\alpha}_{i:X}), \forall i = 1, ..., n.$

For $\alpha = 1$, we have that:

$$X_i^{\alpha=1} = X_i + \frac{\lambda}{\mu_X} X_i = \frac{1+\lambda}{\mu_X} X_i \tag{6}$$

Thus, $X^{\alpha=1}$ is simply obtained from X by multiplying each value of X by the factor $f = (1+\lambda)/\mu_X$. We know that in transition from X to X^{α} inequality does not change. In the RIC, inequality does not change if all incomes are changed by the same percentage. Therefore, the α-IC for $\alpha = 1$ corresponds to the RIC. For $0 < \alpha < 1$ we have a range of intermediate views that can be considered combinations of the RIC and AIC.

For $\alpha = 0$, we have that:

$$X_i^{\alpha=0} = X_i + \lambda \tag{7}$$

In the AIC, inequality does not change if all incomes are increased or decreased by the same absolute amount, and this is exactly what the transition from X to X^{α} describes. Thus, the α-IC for $\alpha = 0$ corresponds to the AIC.

2.4 The d-inequality concept

The Ebert (2004, 2010)'s inequality concept, the d-IC, can be described using the analogy with the α-IC. The counterfactual distribution X^d has the same inequality as the original distribution X, where the former is defined as follows:

$$X_i^d = \begin{cases} X_i + \ln\eta & \text{for } d = -\infty, \\ \eta(X_i - d) + d & \text{for } d \in \mathbb{R}, \end{cases} \tag{8}$$

The parameter d plays the similar role as α, whereas $\eta > 0$ is the counterpart of λ, and also determined by the analyst. Equation 8 can be rewritten as:

$$X_i^d = \begin{cases} X_i + 0X_i + \ln\eta & \text{for } d = -\infty, \\ X_i + (\eta-1)X_i + (\eta-1)(-d) & \text{for } d \in \mathbb{R}, \end{cases} \tag{9}$$

which is compatible with equation 5, demonstrating that X^d consists of X, the X-dependent part and the X-independent part. For $d = -\infty$, we notice that the variable part actually equals zero, and only the fixed part exists, equalling $\ln\eta$; in this case the d-IC resembles the AIC. Conversely, when $d = 0$, $X_i^d = (\eta-1)X_i$, the d-IC corresponds to the RIC.

Equation 9 suggests that the α-IC and d-IC are equivalent; their connection will be discussed in the next subsection. One advantage of the d-IC over α-IC is that that for comparing inequality of distributions in the d-IC, one does not have to create counterfactuals such as X^{α}. Instead, the d-concentration curves are used; the ones for the pre-fiscal and post-fiscal income are defined, respectively, as follows:

$$\begin{aligned} C^d(X_{i:X}) &= \frac{1}{N(\mu_X - d)} \sum_{j=1}^{i} w_{j:X}(X_{j:X} - d) \\ C^d(Y_{i:Y}) &= \frac{1}{N(\mu_Y - d)} \sum_{j=1}^{i} w_{j:Y}(Y_{j:Y} - d) \end{aligned}$$

We have the following proposition:

The fiscal system is strictly d-inequality reducing if
$C^d(Y_{i:Y}) > C^d(X_{i:X}), \forall i = 1, ..., n.$

2.5 The relationship between α- and d-inequality concepts

The previous subsection revealed the connection between the α-IC to d-IC. Recall from equation 5 that the X-dependent and X-independent parts in the α-IC were $\alpha(\lambda/\mu_X)X_i$ and $(1-\alpha)\lambda$, respectively. Let χ^{α} be their ratio:

$$\chi^{\alpha} = \frac{\alpha(\lambda/\mu_X)X_i}{(1-\alpha)\lambda} = \frac{\alpha X_i}{(1-\alpha)\mu_X}$$

Analogous ratios are obtained for the d-IC from equation 9 as follows:

$$\begin{aligned} \chi^{d=-\infty} &= \frac{0}{\ln \eta} = 0 \\ \chi^{d\in\mathbb{R}} &= \frac{(\eta-1)X_i}{(\eta-1)(-d)} = \frac{X_i}{-d} \end{aligned}$$

By equating the ratios for the α-IC to the d-IC, we obtain:

$$\begin{aligned} \chi^{\alpha} = \chi^{d=-\infty} &\Rightarrow \frac{\alpha X_i}{(1-\alpha)\mu_X} = 0 \\ \chi^{\alpha} = \chi^{d\in\mathbb{R}} &\Rightarrow \frac{\alpha X_i}{(1-\alpha)\mu_X} = \frac{X_i}{-d} \end{aligned}$$

It follows that $\alpha = 0$ if $d = -\infty$, which is as expected because we know that for these values both the α-IC and d-IC resemble the AIC. For $d \in \mathbb{R}$ we obtain:

$$d = \frac{-(1-\alpha)\mu_X}{\alpha} \tag{10}$$

$$\alpha = \frac{\mu_X}{\mu_X - d} \tag{11}$$

Recall that Bosmans et al. (2014) set α into the interval $[0,1]$, where the limits of this interval represent the AIC and RIC, respectively. According to equation 10, we have that (a) $\alpha \to 0 \Rightarrow d \to -\infty$, (b) $\alpha \in (0,1) \Rightarrow d \in (-\infty, 0)$, and (c) $\alpha = 1 \Rightarrow d = 0$. However, Ebert (2004, 2010) envisages even the positive values for d, saying that $d > 0$ has the meaning of the 'reference or minimum income'. In this case, equation 11 says that: (d) $d > 0 \Rightarrow \alpha > 1$.

3 Inequality, vertical and horizontal effects in the α-IC and d-IC

3.1 The α-inequality concept

To establish whether the fiscal system reduces inequality, we must compare the inequality of pre-fiscal income distribution, X, with the inequality of post-fiscal income, Y^F. Similarly, to judge the redistributive effects of the tax (benefit) only, X must be compared to post-tax (post-benefit) income distribution Y^T (Y^B).

In the α-IC, X must be transformed into its counterpart, X^α, which has the same α-inequality as X according to equation 5. Depending on what is investigated—the tax only, the benefit only or the whole fiscal system—different counterfactual incomes must be constructed as follows:

$$\begin{aligned}
X_{i:X}^{T;\alpha} &= X_{i:X} - \alpha\phi_T X_{i:X} - (1-\alpha)\phi_T\mu_X \\
X_{i:X}^{B;\alpha} &= X_{i:X} + \alpha\phi_B X_{i:X} + (1-\alpha)\phi_B\mu_X \\
X_{i:X}^{F;\alpha} &= X_{i:X} + \alpha(\phi_B - \phi_T)X_{i:X} + (1-\alpha)(\phi_B - \phi_T)\mu_X
\end{aligned}$$

$X^{T;\alpha}$, $X^{B;\alpha}$, $X^{F;\alpha}$ have the same α-inequality as X. Their concentration curves are $C(X_{i:X}^{T;\alpha})$, $C(X_{i:X}^{B;\alpha})$ and $C(X_{i:X}^{F;\alpha})$. Using these curves, we can construct the measures of IE, VE and HE in the α-IC that are shown in Table 4.

Table 4: Inequality, vertical and horizontal effects in the α-IC

	Fiscal system	Tax	Benefit
IE	$r_i^{F;\alpha} = C(Y_{i:Y^F}^F) - C(X_{i:X}^{F;\alpha})$	$r_i^{T;\alpha} = C(Y_{i:Y^T}^T) - C(X_{i:X}^{T;\alpha})$	$r_i^{B;\alpha} = C(Y_{i:Y^B}^B) - C(X_{i:X}^{B;\alpha})$
VE	$v_i^{F;\alpha} = C(Y_{i:X}^F) - C(X_{i:X}^{F;\alpha})$	$v_i^{T;\alpha} = C(Y_{i:X}^T) - C(X_{i:X}^{T;\alpha})$	$v_i^{B;\alpha} = C(Y_{i:X}^B) - C(X_{i:X}^{B;\alpha})$
HE	$h_i^{F;\alpha} = C(Y_{i:X}^F) - C(Y_{i:Y^F}^F)$	$h_i^{T;\alpha} = C(Y_{i:X}^T) - C(Y_{i:Y^T}^T)$	$h_i^{B;\alpha} = C(Y_{i:X}^B) - C(Y_{i:Y^B}^B)$

IE of the fiscal system in the α-IC is decomposed into VE and HE as follows:

$$r_i^{F;\alpha} = v_i^{F;\alpha} - h_i^{F;\alpha} \tag{12}$$

Furthermore, VE of the fiscal system is decomposed as follows (proof is given in the Appendix):

$$v_i^{F;\alpha} = \frac{(1-\phi_T)v_i^{T;\alpha} + (1+\phi_B)v_i^{B;\alpha}}{1-\phi_T+\phi_B} \tag{13}$$

3.2 The d-inequality concept

The d-concentration curves of Y^T, Y^B and Y^F are defined as follows:

$$\begin{aligned}
C^d(Y_{i:X}^T) &= \frac{1}{N(1-\phi_T-\delta)\mu_X}\sum_{j=1}^{i} w_{j:X}(Y_{j:X}^T - d) \\
C^d(Y_{i:X}^B) &= \frac{1}{N(1+\phi_B-\delta)\mu_X}\sum_{j=1}^{i} w_{j:X}(Y_{j:X}^B - d) \\
C^d(Y_{i:X}^F) &= \frac{1}{N(1-\phi_T+\phi_B-\delta)\mu_X}\sum_{j=1}^{i} w_{j:X}(Y_{j:X}^F - d)
\end{aligned}$$

where $\delta = d/\mu_X$.

Table 5: Inequality, vertical and horizontal effects in the d-IC

	Fiscal system	Tax	Benefit
IE	$r_i^{F;d} = C^d(Y^F_{i:Y^F}) - C^d(X_{i:X})$	$r_i^{T;d} = C^d(Y^T_{i:Y^T}) - C^d(X_{i:X})$	$r_i^{B;d} = C^d(Y^B_{i:Y^B}) - C^d(X_{i:X})$
VE	$v_i^{F;d} = C^d(Y^F_{i:X}) - C^d(X_{i:X})$	$v_i^{T;d} = C^d(Y^T_{i:X}) - C^d(X_{i:X})$	$v_i^{B;d} = C^d(Y^B_{i:X}) - C^d(X_{i:X})$
HE	$h_i^{F;d} = C^d(Y^F_{i:X}) - C^d(Y^F_{i:Y^F})$	$h_i^{T;d} = C^d(Y^T_{i:X}) - C^d(Y^T_{i:Y^T})$	$h_i^{B;d} = C^d(Y^B_{i:X}) - C^d(Y^B_{i:Y^B})$

Using these curves, the measures of IE, VE and HE in the d-IC are obtained; see Table 5. IE of the fiscal system in the d-IC is decomposed into VE and HE as follows:

$$r_i^{F;d} = v_i^{F;d} - h_i^{F;d} \tag{14}$$

VE of the fiscal system is decomposed as follows (proof is given in the Appendix):

$$v_i^{F;d} = \frac{(1-\phi_T-\delta)v_i^{T;d} + (1+\phi_B-\delta)v_i^{B;d}}{1-\phi_T+\phi_B-\delta} \tag{15}$$

3.3 α-inequality neutral taxes and benefits

The counterfactual post-tax and post-benefit income distributions, $X^{T;\alpha}$ and $X^{B;\alpha}$, are created to make Lorenz comparisons in the α-IC. The *fictive* tax and benefit are separated from the pre-fiscal income and represented by the following variables:

$$T^\alpha_{i:X} = X_{i:X} - X^{T;\alpha}_{i:X} = \alpha\phi_T X_{i:X} + (1-\alpha)\phi_T\mu_X \tag{16}$$

$$B^\alpha_{i:X} = X^{B;\alpha}_{i:X} - X_{i:X} = \alpha\phi_B X_{i:X} + (1-\alpha)\phi_B\mu_X \tag{17}$$

T^α and B^α are *α-inequality neutral tax and benefit.* The tax T^α is, in fact, the Pfingsten (1986)'s inequality-preserving tax function derived for the α-IC. The concentration curves of T^α and B^α are identical, as shown by the following equation:

$$C(T^\alpha_{i:X}) = C(B^\alpha_{i:X}) = \alpha C(X_{i:X}) + (1-\alpha)q_{i:X} \tag{18}$$

When $\alpha = 1$, T^α and B^α become the *proportional* tax and benefit. RCCs of T^α and B^α in this case are equivalent to the relative Lorenz curve of pre-fiscal income, $C(X_{i:X})$. When $\alpha = 0$, T^α and B^α are *lump-sum* tax and benefit. In this case, their RCCs coincide with the line of complete equality, whose ordinates are equal to $q_{i:X}$.

Using the concentration curves of T^α and B^α, we can define the following α-progressivity effects:

$$p_i^{T;\alpha} = C(T^\alpha_{i:X}) - C(T_{i:X})$$

$$p_i^{B;\alpha} = C(B_{i:X}) - C(B^\alpha_{i:X})$$

The α-progressivity effects are related to VEs in the α-IC in the following manner:

$$v_i^{T;\alpha} = \frac{\phi_T}{1-\phi_T} p_i^{T;\alpha} \tag{19}$$

$$v_i^{B;\alpha} = \frac{\phi_B}{1+\phi_B} p_i^{B;\alpha} \tag{20}$$

The decomposition of the α-vertical effect of the fiscal system can be rewritten as follows:

$$v_i^{F;\alpha} = \frac{\phi_T p_i^{T;\alpha} + \phi_B p_i^{T;\alpha}}{1-\phi_T+\phi_B} \tag{21}$$

For $\alpha = 1$, i.e., in the RIC, equation 19 is the famous Kakwani (1977) decomposition of the vertical effect of tax, whereas equation 20 is its counterpart for the benefit. Finally, equation 21 is a variant of the Lambert (1985, 2001) decomposition from equation 2.

3.4 Further results

3.4.1 Making equity judgements

In several preceding sections, numerous new measures of IE, VE and HE are presented for several inequality concepts. Their relationship with the old measures, conceived in the RIC, is established. Here we summarise the conditions used to determine both whether the fiscal system and its components are reducing inequality and the intensity of the different effects. These conditions are summarised in Table A1 in the Appendix.

For example, the tax is strictly vertically equitable in the α-IC if $v_i^{T;r} > 0, \forall i = 1, ..., n$. From Table 4 we recall that $v_i^{T;\alpha} = C(Y_{i:X}^T) - C(X_{i:X}^{T;\alpha})$. Therefore, if $C(Y_{i:X}^T)$ lies above $C(X_{i:X}^{T;\alpha})$ for all income units i, it can be said that the tax is *strictly* vertically equitable for the chosen value of α.

Certain regularities can be observed for VEs of the tax and the benefit in the α-IC. They are presented by the following two propositions.

Proposition 1 *For some α and i:*
(a) $p_i^{T;\alpha} > 0 \Rightarrow v_i^{T;\alpha} > 0$, and
(b) $p_i^{B;\alpha} > 0 \Rightarrow v_i^{B;\alpha} > 0$.

Proposition 1 tells us that if the tax or benefit is α-progressive at some quantile $q_{i:X}$, then it is also α-vertically equitable. This follows simply from equations 19 and 20, but is proven by Kakwani (1977) for the RIC, by Pfingsten for θ-IC and by Ebert (2010) for the d-IC.

Proposition 2 *For some α_1 and α_2, such that $\alpha_1 < \alpha_2$:*
(a) $C(T_{i:X}^{\alpha_1}) > C(T_{i:X}^{\alpha_2}), \forall i < n$.
(b) $C(X_{i:X}^{T;\alpha_1}) < C(X_{i:X}^{T;\alpha_2}), \forall i < n$.
(c) $C(B_{i:X}^{\alpha_1}) > C(B_{i:X}^{\alpha_2}), \forall i < n$.
(d) $C(X_{i:X}^{B;\alpha_1}) > C(X_{i:X}^{B;\alpha_2}), \forall i < n$.

(proof is given in the Appendix). Proposition 2(a) says that the lower the value of α, the higher the ordinates of the concentration curve $C(T_{i:X}^{\alpha})$. Thus, as previously mentioned, for $\alpha = 0$, $C(T_{i:X}^{\alpha}) = q_{i:X}$. The consequence is that the α-progressivity of the tax, measured by $p_i^{T;\alpha}$, is increasing in $\alpha, \forall i < n$. At the same time, the VE of the tax, measured by $v_i^{T;\alpha}$, is also increasing in $\alpha, \forall i < n$, as can be deduced from Proposition 2(b). The situation is completely opposite for the benefit. From Proposition 2(c) and Proposition 2(d) it follows that the benefit will be *less* α-progressive and less α-vertically equitable as α increases.

Example 1 A hypothetical fiscal system consists of the tax and the benefit such that $T_{i:X} = B_{i:X}, \forall i$. *For any α, $p_i^{T;\alpha} = -p_i^{B;\alpha}$ and $v_i^{T;\alpha} = -v_i^{B;\alpha}, \forall i$.*

We use Example 1 to illustrate Proposition 2. In a hypothetical situation in which the tax and benefit have identical distributions, their contributions to VE will be judged in symmetrical manner. Notice that VEs of the tax and benefit cancel each other out, leaving $v_i^{F;\alpha} = 0, \forall i$.

3.4.2 The 'reference or minimum income'

T^{α} and B^{α} are linear functions of X and can be represented by simple lines. Assuming that $X_{i:X} = 0$, T^{α} is a line with intercept $(1-\alpha)\phi_T\mu_X$ and a slope $\alpha\phi_B$. When $\alpha = 0$, the slope is zero, i.e., the line is parallel with the x-axis. When $\alpha = 1$, the intercept is zero. For $0 < \alpha < 1$, both the intercept and the slope are positive, and the slope ranges in the interval $(0, \phi_T)$.

However, what happens if $\alpha > 1$? This case was not envisaged by Bosmans et al. (2014), but is implied by the Ebert (2004, 2010)'s concept of 'reference or minimum income', i.e. when $d > 0$. If $X_{i:X} = 0$, the line showing the relationship between T^{α} and X will have a negative intercept and slope larger than ϕ_T. Indeed, T^{α} is negative income tax with a minimum income of $(\alpha-1)\phi_T\mu_X$ and a tax rate of $\alpha\phi_B$. Because typical real-world taxes are non-negative, it follows that for $\alpha > 1$ the progressivity effect, $p_i^{T;\alpha}$ and the VE of the tax, $v_i^{T;\alpha}$, will be negative at the bottom quantiles of the pre-fiscal income distribution.

3.4.3 A note on equivalence scales

The choice of the equivalence scale is one of the key issues in income redistribution measurement and, as the choice of inequality concept, involves ethical judgements. As Ebert and Moyes (2000) note, the 'usual strategy' in research is to assume (a) the relative equivalence scale (RES), (b) single-adult as a reference household type, (c) the RIC as the inequality concept, and (d) $m_i = a_i + c_i$.

However, these assumptions are only one possible choice from a wide range of alternatives, each reflecting a particular ethical stance. Regarding point (c), the current paper extends the analysis to include other inequality concepts. Other points also deserve attention, but their treatment must be left for future consideration. Thus, regarding the points (a), (b) and (d), we stick to the 'usual strategy'.

According to the standard approach, X_i is obtained from $\dot{X}_i$ as $X_i = \dot{X}_i/e_i$, where e_i is the equivalence factor, that depends on the structure of household i. For instance, in the 'modified OECD scale', $e_i = 1 + 0.5(a_i - 1) + 0.3c_i$. This scale is constructed in such a manner that $e_i = 1$ for a single-adult household, that serves as the *reference-type* household.

Alternatively, regarding point (a), the absolute equivalence scales could be used. Concerning point (b), any household type could be selected as the reference-type household. Finally, with regard to point (d), the analyst could use $m_{i:X} = e_i$; thus, instead of using the number of real persons, the number of equivalent persons could be used.

The α-inequality neutral tax and benefit, derived in section 3.3, are expressed in *equivalised* terms. The following short analysis shows their *monetary* values for different types of households when both the relative equivalence scale and the single adult as the reference-type household are employed. We obtain that $\dot{T}_{i:X}^{\alpha} = e_i T_{i:X}^{\alpha}$ and $\dot{B}_{i:X}^{\alpha} = e_i B_{i:X}^{\alpha}$. Imagine four income units, $i = \{r, s, u, v\}$, such that $X_{r:X} = X_{s:X} < X_{u:X} = X_{v:X}$. Units r and u represent single-adult households; units s and v represent multiple-member households of identical structure. Therefore, $e_{r:X} = e_{u:X} = 1$ and $e = e_{s:X} = e_{v:X} > 1$.

Table 6 provides calculations of $\dot{B}_{i:X}^{\alpha}$ for the four hypothetical units in the AIC and RIC and compares these values. Households of the same type and different incomes should obtain the same amounts of benefit in the AIC; observe that $\dot{B}_{u:X}^{\alpha} - \dot{B}_{r:X}^{\alpha} = 0$ and $\dot{B}_{v:X}^{\alpha} - \dot{B}_{s:X}^{\alpha} = 0$. However, in the RIC, single-adult households with higher income obtain a greater benefit because $\phi_B(X_{u:X} - X_{r:X}) > 0$. The same is true for multiple-member households, but the premium is adapted to family size; observe $e\phi_B(X_{u:X} - X_{r:X}) > 0$.

In the AIC we have that $\dot{B}_{s:X}^{\alpha} - \dot{B}_{r:X}^{\alpha} = \dot{B}_{v:X}^{\alpha} - \dot{B}_{u:X}^{\alpha} = (e-1)\phi_B\mu_X$: multiple-member units should receive larger amount of the benefit than the single-adult units and the premium is

Table 6: Monetary values of B^{α} for the relative equivalence scale

Unit	$\alpha = 0$	$\alpha = 1$
$\dot{B}^{\alpha}_{r:X}$	$\phi_B \mu_X$	$\phi_B X_{r:X}$
$\dot{B}^{\alpha}_{s:X}$	$e\phi_B \mu_X$	$e\phi_B X_{r:X}$
$\dot{B}^{\alpha}_{u:X}$	$\phi_B \mu_X$	$\phi_B X_{u:X}$
$\dot{B}^{\alpha}_{v:X}$	$e\phi_B \mu_X$	$e\phi_B X_{u:X}$
$\dot{B}^{\alpha}_{s:X} - \dot{B}^{\alpha}_{r:X}$	$(e-1)\phi_B \mu_X$	$(e-1)\phi_B X_{r:X}$
$\dot{B}^{\alpha}_{v:X} - \dot{B}^{\alpha}_{u:X}$	$(e-1)\phi_B \mu_X$	$(e-1)\phi_B X_{u:X}$
$\dot{B}^{\alpha}_{u:X} - \dot{B}^{\alpha}_{r:X}$	0	$\phi_B (X_{u:X} - X_{r:X})$
$\dot{B}^{\alpha}_{v:X} - \dot{B}^{\alpha}_{s:X}$	0	$e\phi_B (X_{u:X} - X_{r:X})$

identical for all pairs. Conversely, in the RIC, the premium for the larger household increases with income; namely, $(e-1)\phi_B X_{u:X} > (e-1)\phi_B X_{r:X}$ because $X_{u:X} > X_{r:X}$.

3.4.4 Compatible inequality concepts

It has been mentioned on several occasions that the α-IC for $\alpha = 0$ and $\alpha = 1$ corresponds to the AIC and RIC, respectively. Furthermore, the α-IC is connected with the d-IC; see equation 10. Therefore, it should be expected that the conclusions based on the α-IC will be identical to those based on the AIC (for $\alpha = 0$), RIC (for $\alpha = 1$) and the d-IC (for d^*). This is true, but it seems useful to explicitly state some of these identities, which is done in Proposition 2.

Proposition 3 *Let $\alpha^0 = 0$, $\alpha^1 = 1$ and $d^* = -(1-\alpha^*)\mu_X/\alpha^*$ (see equation 10). For α^*, d^* and α^0 we have that:*
(a) $v_i^{T;\alpha^0}/v_i^{F;\alpha^0} = v_i^{T;a}/v_i^{F;a}, \forall i$;
(b) $v_i^{T;\alpha^1}/v_i^{F;\alpha^1} = v_i^{T;r}/v_i^{F;r}, \forall i$;
(c) $v_i^{T;\alpha^*}/v_i^{F;\alpha^*} = v_i^{T;d^*}/v_i^{F;d^*}, \forall i$;
and analogously for the benefit.

Proposition 3 says that, whatever inequality concept we use, for compatible inequality concepts we will obtain the same *relative* results, such as the contributions of the tax and the benefit in VE of the fiscal system.

3.4.5 S-Gini indices of IE, VE and HE

Single-parameter of S-concentration indices aggregate the distances between concentration curves using a weighting scheme that depends on one *ethical* parameter. Following Duclos and Araar (2006) the weighting scheme is defined as follows:

$$\kappa^{\rho}_{i:X} = \rho(\rho-1)(1-q_{i:X})^{\rho-2} \tag{22}$$

where ρ is the ethical parameter that must be greater than 1 for the weights $\kappa^{\rho}_{i:X}$ to be positive everywhere. For $\rho = 2$, $\kappa^{\rho}_{i:X} = 2, \forall i$. For $\rho < 2$ ($\rho < 2$), $\kappa^{\rho}_{i:X}$ is monotonically increasing (decreasing); see Duclos and Araar (2006) for more details.

The S-concentration index of pre-fiscal income in the RIC is obtained as follows:

$$D_{\rho}(X) = \frac{1}{N}\sum\nolimits_{i=1}^{n} w_{i:X} \cdot \kappa^{\rho}_{i:X} \cdot [q_{i:X} - C(X_{i:X})] \tag{23}$$

Note that $D_{\rho=2}(Z)$ is the standard Gini index. Analogously to equation 23 the S-concentration indices are obtained in the α-IC and $d-IC$; e.g., from $C(X_{i:X}^{F;\alpha})$ and $C^d(Y_{i:X}^T)$ the indices $D_\rho(X_{i:X}^{F;\alpha})$ and $D_\rho^d(Y_{i:X}^T)$ are obtained, respectively. Table A2 in the Appendix summarises all of the various IEs, VEs and HEs based on S-Gini indexes.

4 Application: Child benefits in Croatia

4.1 Introduction

The new measurement apparatus is applied to appraise the equity of child contingent benefits (CCBs) in Croatia in 2016. Generally, CCBs are various cash social benefits received by households with children, e.g., child benefits, grants for child birth, maternity and parental benefits, etc. Recent research also includes tax expenditures for children in CCBs.

The Croatian social protection system contains more than one dozen CCBs. The most important CCBs are simulated in EUROMOD (Sutherland and Figari, 2013; Urban and Bezeredi, 2016), including Child benefit, Lump-sum grant for newborn children, Maternity leave benefit, Parental leave benefit and Support during the newborn child care. Personal income tax incorporates Tax allowance for supported children (TASC), which reduces the tax obligations of persons with children.

TASC and Child benefit are obtained each month by families with children during the childhood and schooling years. These instruments create a disposable income wedge between households with and without children, compensating the former type of family for its greater needs. TASC and Child benefit are the most debated of Croatia's CCB instruments. There is a controversy regarding their roles and distributional patterns, particularly with respect to TASC.

EUROMOD will be used to analyse the fiscal subsystem consisting of Personal income tax and Surtax, CCBs and other social benefits. The investigation will reveal the redistributive effects of this subsystem and the relative contributions of various tax-benefit instruments to VE. Particular attention will be devoted to TASC and Child benefit. In addition to the baseline scenario, an alternative scenario will be analysed that assumes a stylised reform of Personal income tax and Child benefit.

4.2 Child benefit and TASC

Before moving to EUROMOD-based analysis, the functioning of the Child benefit and TASC is briefly explained using an analysis of typical families.

Pre-tax wage (PTW), W, is equal to gross wage, W^g, minus employees social insurance contributions. Pre-tax wage is then taxed by the Personal income tax, whose amount is $T^p(W)$. Each person is taxed separately; a basic personal allowance is used, along with TASC, by persons with dependent children. Surtax is equal to $T^s(W) = sT^p(W)$, where s is the surtax rate, determined by the local government unit. PITS is the sum of Personal income tax and Surtax; $T^{ps}(W) = (1+s)T^p(W)$.

For the 1st, 2nd, 3rd and 4th child TASC equals HRK 1,300, 1,820, 2,600 and 3,640 per month, respectively, and the amounts continue to progressively rise for each additional child. The effective value of TASC, however, depends on taxpayer's tax bracket- Personal income tax's rates are 12, 25 and 40%. Henceforth, *TASC benefit* denotes the fictive benefit, representing the values of PITS reduction attributable to the existence of TASC.

Hypothetical families have two spouses and 0, 1, 2, 3 or 4 school children. Both spouses work; their pre-tax wages are W_1 and W_2, such that $W_1 = W_2$ in each family. Spouses are separately taxed, whereby the amount of TASC is equally shared between the two spouses. The surtax rate is $s = 0.12$. PITS equals $T_1^p(W) = T_2^p(W)$ for two spouses. Total PTW of a family with c children is $W_f = W_1 + W_2$ and $T_f^p(W_1, W_2; c)$ is the total PITS paid by this family.

In our hypothetical example, the *TASC benefit* for a family with $c > 0$ children is calculated as $T_f^p(W_1, W_2; 0) - T_f^p(W_1, W_2; c)$. The first step is to calculate PITS for spouses without children and then for spouses with the same PTW, but with children. The north-west panel of Figure 1 shows the schedules of TASC benefit for the four hypothetical family types and a wide range of PTW. TASC benefit begins at the PTW of HRK 5,200, which equals double the amount of the basic personal tax allowance. The amount of TASC is increasing and there are three 'plateaus' representing three tax brackets.

How *rapidly* does the amount of TASC benefit increase? This phenomenon is shown by observing the elasticities of TASC benefit with respect to PTW, shown on the north-east panel of Figure 1. Naturally, on PTW intervals in which TASC is constant the elasticity is zero, but on other income intervals the elasticity always exceeds 1, meaning that TASC benefit *over-proportionately* increases.

[*Figure 1*]

Recall how the α-inequality neutral taxes and benefits are constructed. According to equation 17, for $\alpha = 0$ the amount of benefit should neither rise nor decrease in pre-fiscal income. If $\alpha = 1$ ($0 < \alpha < 1$; $\alpha > 1$), the benefit should increase proportionately (under-proportionately; over-proportionately) in pre-fiscal income. Obviously, the TASC benefit is not inspired by the AIC. It is more likely that the RIC, or even the α-IC with $\alpha > 1$, motivates its design. We can expect that the TASC benefit will be vertically inequitable or 'regressive' for lower values of α.

Child benefit is received by below-median households with children. There are three brackets with monthly amounts of HRK 300, 250 and 200 per child. Brackets are delineated by total net household income per household member, and the third bracket ends at HRK 1,663. The so-called 'pronatality supplement' of HRK 500 is added for the third and the fourth child. The south-west panel of Figure 1 shows the schedules of Child benefit for the same hypothetical family types. There are three brackets; the benefit amount steeply falls at the end of the last bracket, particularly for families with three and four children that receive the 'pronatality supplement'. The south-east panel of Figure 1 represents the sum of the TASC benefit and Child benefit. The prominent features of this combined schedule are the V-letter shaped cracks that appear for all family types. These gaps represent income intervals in which families obtain much lower combined support than neighbouring families with lower or higher PTW.

The amounts of the TASC benefit increase with income and for high-income taxpayers, significantly exceed those of the Child benefit. TASC is therefore criticised for its 'regressiveness' and marked as the 'largest delinquent among various social assistance programs' in Croatia (World Bank, 2014). The total yearly expenditure on TASC benefit is estimated at approximately HRK 1.6 billion, an amount that is similar to the total expenditure on Child benefit. Another objection is that the combined system of TASC benefit and Child benefit excludes some families: those with income high enough to exceed the top bracket in Child benefit, but not enough to obtain satisfactory levels of TASC benefit (Urban, 2014b).

There have been proposals to address the above-mentioned issues. For example, Child benefit could include additional income bracket to assist the families that do not obtain sufficient support, whereas the 'regressive' TASC could be replaced by the wastable tax credit for children (Šućur et al., 2016). Alternatively, the universal child benefit (UCB) that would take the place of the TASC and Child benefit might enable a balanced distribution of the child support across entire income distribution. Despite criticism and these proposals, the TASC is still in place and its position has even been strengthened by the 2016 tax reform, which has increased the nominal allowance amounts.

4.3 Microsimulation data and scenarios

The pre-fiscal income, taxes and benefits are computed using EUROMOD. The starting system is HR2016A, which simulates Croatia's actual 2016 tax-benefit system. The *baseline* scenario takes the variables directly from HR2016A. However, certain adaptations were made to analyse the TASC benefit as a stand-alone instrument. EUROMOD is used to hypothetically separate the TASC benefit from personal income tax and surtax as follows.

An auxiliary system, HR2016B, is created. The new policy, 'TASC-abolished PIT', is obtained from the original policy, 'Personal income tax', by setting all TASC rates to zero. Let the variable *PITS* represent the sum of personal income tax and surtax in HR2016A; the variable *TASC-abolished PITS* contains the sum of values obtained from 'TASC-abolished PIT' and the corresponding surtax amounts. Now, the difference between *TASC-abolished PITS* and *PITS* is the new variable *TASC benefit*; it shows the gain for each person from the use of TASC.

The *reform* scenario assumes a stylised reform of the original policies of 'Personal income tax' and 'Child benefit'. The system HR2016R is created. 'Reformed personal income tax' replaces the three-rate schedule by one rate of 24% while the basic personal allowance is set to HRK 3,500 for all taxpayers. The variable *Reformed PITS* sums up the amounts obtained by 'Reformed personal income tax' and corresponding surtax amounts in HR2016R.

The policy 'Reformed child benefit' abolishes income testing and gives an equal amount of HRK 350 to all eligible children, pursuant to the same eligibility rules as in HR2016A. The 'Pronatality supplement' is abolished, but other supplements remain.

Pre-fiscal income is composed of original incomes and public pensions. All social insurance contributions are deducted. Table 7 presents all the analysed instruments, showing that other cash social benefits are included into the analysis.

Table 8 shows the descriptive statistics for two scenarios. Pre-fiscal income and the majority of social benefits are identical in the baseline and reform scenario (upper section of the table). The total amount of post-fiscal income is equal in the two scenarios because the reform is set to be budget neutral. In the baseline scenario, *Child benefit* (B2) and *TASC benefit* (B1) are the largest benefits and have similar coverage in terms of children and total amounts. The total amount of B1 and B2 is HRK 3.4 billion in the baseline scenario, whereas in the reform scenario the amount of B3 is HRK 3.5 billion. Thus, the reform increases outlays for children by 4%.

4.4 Results based on microsimulation

The measurement apparatus developed in sections 2 and 3 is used to judge the equity characteristics of the current Croatian tax-benefit subsystem and its stylised reform, as defined in section 4. The 'modified OECD scale' is employed.

4.4.1 Baseline scenario

The investigation begins with the quantile analysis of VEs of taxes and benefits in the baseline scenario. The measures $v_i^{T;\alpha}$ and $v_i^{B;\alpha}$ are obtained for $\alpha \in (0, 0.5, 1, 1.5)$ and shown in Figure 2. Starting with $\alpha = 0$, we notice that T1 has the overwhelmingly strongest VE (for convenience reasons, the top sections of T1's curves for $\alpha < 1.5$ are not shown on the graph). However, with greater α values the domination of T1 fades away and VE becomes negative for $\alpha = 1.5$ at the

Table 7: Fiscal subsystem for Croatia, baseline and reform scenarios

<table>
<tr><th>Name</th><th>Variable</th><th>Baseline</th><th>Reform</th></tr>
<tr><td>T1</td><td>TASC-abolished PITS</td><td>Yes</td><td>∅</td></tr>
<tr><td>T2</td><td>Reformed PITS</td><td>∅</td><td>Yes</td></tr>
<tr><td>B1</td><td>TASC benefit</td><td>Yes</td><td>∅</td></tr>
<tr><td>B2</td><td>Child benefit</td><td>Yes</td><td>∅</td></tr>
<tr><td>B3</td><td>Reformed child benefit</td><td>∅</td><td>Yes</td></tr>
<tr><td>B4</td><td>Maternity leave benefit</td><td colspan="2">Yes</td></tr>
<tr><td>B5</td><td>Parental leave benefit</td><td colspan="2">Yes</td></tr>
<tr><td>B6</td><td>Support during the newborn child care</td><td colspan="2">Yes</td></tr>
<tr><td>B7</td><td>Lump-sum grant for newborn children</td><td colspan="2">Yes</td></tr>
<tr><td>B8</td><td>Social assistance benefits
(= Subsistence benefit
+ Other social assistance benefits
+ Housing benefits)</td><td colspan="2">Yes</td></tr>
<tr><td>B9</td><td>Unemployment benefits
(= Unemployment benefit
+ Other unemployment benefits
+ Sickness benefits)</td><td colspan="2">Yes</td></tr>
<tr><td>B10</td><td>Other benefits
(= Disability benefits
+ Survivors benefits
+ Old-age benefits)</td><td colspan="2">Yes</td></tr>
</table>

bottom quantiles. Benefits follow the opposite trend: VE tends to increase with α. At $\alpha = 0.5$ the VE curves cross for T1 and B2. Therefore, an unambiguous judgement of which instrument is more vertically equitable cannot be made. The same ambiguity is present for B8 in relation to T1 and B2. Observe that the VE curve for B8 is very steep at the lowest quantiles because *Social assistance benefits* are primarily consumed at this segment of income distribution.

[*Figure 2*]

Expectations about the 'regressivity' of the *TASC benefit* (B1) were correct: it is vertically inequitable for $\alpha = 0$ and $\alpha = 0.5$; furthermore, VE is negative for $\alpha = 1$ in large section of pre-fiscal distribution. Conversely, the VE is positive for $\alpha = 1.5$. Thus, for observers whose inequality view assumes large values of α, the *TASC benefit* is equitable.

The VEs of instruments shown on the right-hand panels of Figure 2 are notably smaller (compare the sizes of vertical scales on the left- and right-hand scales). B6 and B10 seem to be more important than other benefits. B4 and B5 follow very similar pattern and are inequitable for $\alpha = 0$ at the lowest quantiles.

4.4.2 Baseline vs. reform scenario

Figure 3 shows the quantile analysis of VE in the baseline and reform scenarios. The left and the right panels present the measures $v_i^{F;a}$ and $v_i^{F;d}$ for $\alpha = 1.5$, respectively. In a relative sense, the curves on the two panels are remarkably similar (the same is true of the curves obtained for $\alpha = 0.5$ and $\alpha = 1$ that are not displayed on the graph). In the range from the 50th to 90th percentiles, the reform scenario VE dominates the baseline VE. However, at the bottom

Table 8: Fiscal subsystem for Croatia

Name	Households (in thous.)	Children (in thous.)	Adults (in thous.)	Total yearly amount (in mill. HRK)
		Baseline and reform scenarios		
X	1,489	769	3,393	126,297
B4	30	60	84	531
B5	42	88	112	495
B6	34	86	105	403
B7	29	69	86	68
B8	112	69	218	1,120
B9	102	91	285	1,214
B10	144	56	374	998
		Baseline scenario		
T1	857	550	2,203	9,128
T2	0	0	0	0
B1	358	487	993	1,603
B2	218	427	592	1,769
B3	0	0	0	0
Y	1,517	794	3,438	125,371
		Reform scenario		
T1	0	0	0	0
T2	816	481	2,055	9,265
B1	0	0	0	0
B2	0	0	0	0
B3	467	792	1,261	3,510
Y	1,517	794	3,438	125,371

percentiles, baseline VE dominates the reform VE. Therefore, no unambiguous conclusion can be made about whether the reform has increased or decreased VE.

[*Figure 3*]

Thus, despite significant changes in two main instruments—Personal income tax and Child benefit—the VE of the analysed fiscal system remains roughly unchanged. Figure 4 compares the vertical effects of key instruments in two scenarios. T1 and T2 have very similar vertical effects; for $\alpha = 1$ and $\alpha = 1.5$, the baseline slightly dominates the reform scenario at the top quantiles.

[*Figure 4*]

For the analytical purposes of Figure 4, the new variable of 'B3-B1' is created. This variable shows the difference between the *Reformed child benefit* and the *TASC benefit*. For $\alpha = 0$ and $\alpha = 0.5$, B3 has a significantly smaller effect than B2. However, the vertical effect of 'B3-B1' dominates B2 at the median and above. Thus, B3 is less redistributive than B2; this is unsurprising given that the latter benefit is income-tested and provides much larger support to families with many children (through the 'pronatality supplement'). However, the good of B2 is largely nullified by the inequity of B1 (at least for $\alpha \leq 1$); and overall VE is not jeopardised through its replacement with B3.

4.4.3 S-Gini based indices

Table A3 in Appendix presents S-Gini indices for the baseline and reform scenarios. The indices are calculated for twelve combinations of the parameters α and κ and are used to obtain the information presented in Figure 5. All of the various effects are divided by the corresponding indexes of pre-fiscal income inequality. VE(T1) is the most interesting curve; it says that the share of T1's vertical effect in GX falls from 16% for $\alpha = 0; \kappa = 1.5$ to only 2% for $\alpha = 1.5; \kappa = 3$. Thus, the share of T1 in GX decreases both in α and κ. The trends are the opposite for benefits. The share of B1 ranges from -2% for $\{\alpha = 0; \kappa = 1.5\}$ to 0.5% for $\{\alpha = 1.5; \kappa = 1.5\}$. VE of the overall fiscal system is 20% of GX, and this percentage is quite stable across the pairs of $\alpha; \kappa$ values. However, IE decreases in κ because HE has higher shares in GX for larger values of κ.

[*Figure 5*]

5 Conclusion

Distributive aspects of the design and redesign of the tax-benefit system are widely discussed not just only among policy makers and academic researchers but also by the general public, media and interest groups. People have different views on how various taxes and benefits should be distributed to satisfy equity norms. Although a given policy instrument may be equitable for some people, it might be unacceptable from the equity perspective of other people. The differences in views affect the debate on the shape of the tax-benefit system.

To epitomise various ethical stances, income-redistribution analysts may choose from a plethora of inequality measures, inequality concepts, equivalence scales, etc. However, the practice of the last several decades demonstrates that the typical 'ethical mix' is based on the *relative* inequality concept and *relative* equivalence scales. This paper encourages the use of alternative inequality concepts by extending the relative inequality apparatus of Kakwani (1977) and Lambert (1985, 2001) to the Bosmans et al. (2014)'s α- and the Ebert (2004)'s d-inequality concepts. Subsequent research will devote attention to the issue of equivalence scales.

Empirical analysis captured the Croatian fiscal subsystem, which consists of personal income tax and non-pension cash social benefits. The results indicate that the roles of various taxes and benefits in income redistribution significantly change based on different inequality views: the contribution of taxes to the overall vertical effect increases from 36% for the relative inequality view to 73% for the absolute inequality view.

Particular attention was given to the tax allowance for supported children; this allowance is usually deemed inequitable. The analysis proves the 'regressivity' of this instrument for a wide range of inequality views, but not for all of them. Namely, for Ebert (2004)'s 'reference or minimum income' view ($d > 0$), tax allowance has a positive vertical effect. This view is compatible with a highly pro-rich concentrated income tax such as that prevailing in Croatia, which demonstrates that the advocates of the current system are consistent in their views.

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Appendix

Proofs

To simplify the presentation, without loss of generality, in all subsequent derivations we assume that $w_{i:X} = 1, \forall i = 1, ..., n$; therefore, $N = n$.

Equation 2

RCCs of pre-fiscal income, post-tax income, post-benefit income and post-fiscal income are respectively,

$$\begin{aligned} C(X_{i:X}) &= \frac{1}{n\mu_X}\sum_{j=1}^{i} X_{j:X} \\ C(Y^T_{i:X}) &= \frac{1}{n(1-\phi_T)\mu_X}\sum_{j=1}^{i} Y^T_{j:X} \\ C(Y^B_{i:X}) &= \frac{1}{n(1+\phi_B)\mu_X}\sum_{j=1}^{i} Y^B_{j:X} \\ C(Y^F_{i:X}) &= \frac{1}{n(1-\phi_T+\phi_B)\mu_X}\sum_{j=1}^{i} Y^F_{j:X} \end{aligned}$$

First recall that $Y_{i:X} = X_{i:X} - T_{i:X} + B_{i:X}$, from which it follows that:

$$Y_{i:X} - X_{i:X} = [(X_{i:X} - T_{i:X}) - X_{i:X}] + [(X_{i:X} + B_{i:X}) - X_{i:X}] \tag{24}$$

From equation 24, it follows that:

$$\begin{gathered} (1-\phi_T+\phi_B)C(Y^F_{i:X}) - C(X_{i:X}) = \\ [(1-\phi_T)C(Y^T_{i:X}) - C(X_{i:X})] + [(1+\phi_B)C(Y^B_{i:X}) - C(X_{i:X})] \end{gathered} \tag{25}$$

Add $(\phi_T - \phi_B)C(X_{i:X})$ to both sides of equation 25, to obtain:

$$\begin{gathered} (1-\phi_T+\phi_B)C(Y^F_{i:X}) - C(X_{i:X}) + \phi_T C(X_{i:X}) - \phi_B C(X_{i:X}) = \\ (1-\phi_T)C(Y^T_{i:X}) - C(X_{i:X}) + \phi_T C(X_{i:X}) + (1+\phi_B)C(Y^B_{i:X}) - C(X_{i:X}) - \phi_B C(X_{i:X}), \end{gathered}$$

$$\begin{gathered} (1-\phi_T+\phi_B)[C(Y^F_{i:X}) - C(X_{i:X})] = \\ (1-\phi_T)[C(Y^T_{i:X}) - C(X_{i:X})] + (1+\phi_B)[C(Y^B_{i:X}) - C(X_{i:X})], \end{gathered}$$

and, finally:

$$C(Y^F_{i:X}) - C(X_{i:X}) = \frac{(1-\phi_T)[C(Y^T_{i:X}) - C(X_{i:X})] + (1+\phi_B)[C(Y^B_{i:X}) - C(X_{i:X})]}{1-\phi_T+\phi_B},$$

which corresponds to equation 2.

Equation 4

The ACCs of pre-fiscal income, post-tax income, post-benefit income and post-fiscal income are respectively:

$$\begin{aligned}
C^a(X_{i:X}) &= -\frac{1}{n\mu_X}\sum\nolimits_{j=1}^{i}(X_{j:X}-\mu_X)\\
C^a(Y^T_{i:X}) &= -\frac{1}{n\mu_X}\sum\nolimits_{j=1}^{i}(Y^T_{j:X}-\mu_{Y^T})\\
C^a(Y^B_{i:X}) &= -\frac{1}{n\mu_X}\sum\nolimits_{j=1}^{i}(Y^B_{j:X}-\mu_{Y^B})\\
C^a(Y^F_{i:X}) &= -\frac{1}{n\mu_X}\sum\nolimits_{j=1}^{i}(Y^F_{j:X}-\mu_{Y^F})
\end{aligned}$$

Equation 4 says that:

$$C^a(Y^T_{i:X})-C^a(X_{i:X}) = [C^a(Y^T_{i:X})-C^a(X_{i:X})]+[C^a(Y^B_{i:X})-C^a(X_{i:X})] \tag{26}$$

Ignoring $-1/(n\mu_X)$ that appears in all ACCs, we can rewrite the three terms from equation 26 in an extended way, as follows:

$$\begin{aligned}
C^a(Y^T_{i:X})-C^a(X_{i:X}) \rightarrow\\
A=\sum\nolimits_{j=1}^{i}(Y^F_{j:X}-\mu_{Y^F})-\sum\nolimits_{j=1}^{i}(X_{j:X}-\mu_X)=\\
\sum\nolimits_{j=1}^{i}\{[(X_{j:X}-T_{j:X}+B_{j:X})-(1-\phi_T+\phi_B)\mu_X]-[X_{j:X}-\mu_X]\}=\\
\sum\nolimits_{j=1}^{i}[-T_{j:X}+B_{j:X}+(\phi_T-\phi_B)\mu_X]\\
\dots\\
C^a(Y^T_{i:X})-C^a(X_{i:X}) \rightarrow\\
B=\sum\nolimits_{j=1}^{i}(Y^T_{i:X}-\mu_{Y^T})-\sum\nolimits_{j=1}^{i}(X_{j:X}-\mu_X)=\\
\sum\nolimits_{j=1}^{i}\{[(X_{j:X}-T_{j:X})-(1-\phi_T)\mu_X]-[X_{j:X}-\mu_X]\}=\\
\sum\nolimits_{j=1}^{i}[-T_{j:X}+\phi_T\mu_X]\\
\dots\\
C^a(Y^B_{i:X})-C^a(X_{i:X}) \rightarrow\\
C=\sum\nolimits_{j=1}^{i}(Y^B_{j:X}-\mu_{Y^B})-\sum\nolimits_{j=1}^{i}(X_{j:X}-\mu_X)=\\
\sum\nolimits_{j=1}^{i}\{[(X_{j:X}+B_{j:X})-(1+\phi_B)\mu_X]-[X_{j:X}-\mu_X]\}=\\
\sum\nolimits_{j=1}^{i}[B_{j:X}-\phi_B\mu_X]
\end{aligned}$$

It is obvious that $A=B+C$ that which proves the identities in equations 26 and 4.

Equation 13

Observe that the following identity holds:

$$Y_{i:X}-X^{F;\alpha}_{i:X} \;=\; [(X_{i:X}-T_{i:X})-X^{T;\alpha}_{i:X}]+[(X_{i:X}+B_{i:X})-X^{B;\alpha}_{i:X}] \tag{27}$$

From equation 27, it follows that:

$$\begin{aligned}
(1-\phi_T+\phi_B)C(Y^F_{i:X})-(1-\phi_T+\phi_B)C(X^{F;\alpha}_{i:X})=\\
(1-\phi_T)[C(Y^T_{i:X})-C(X^{T;\alpha}_{i:X})]+(1+\phi_B)[C(Y^B_{i:X})-C(X^{B;\alpha}_{i:X})]
\end{aligned} \tag{28}$$

After the rearrangement of equation 28 we obtain the following identity:

$$C(Y^F_{i:X}) - C(X^{F;\alpha}_{i:X}) = \frac{(1-\phi_T)[C(Y^T_{i:X}) - C(X^{T;\alpha}_{i:X})] + (1+\phi_B)[C(Y^B_{i:X}) - C(X^{B;\alpha}_{i:X})]}{1-\phi_T+\phi_B},$$

which is exactly what equation 13 shows.

Equation 15

Observe that for some $d \in \mathbb{R}$ the following identity holds:

$$\begin{aligned} (Y_{i:X} - d) - (X_{i:X} - d) = \\ \langle (X_{i:X} - T_{i:X} - d) - (X_{i:X} - d)\rangle + \langle (X_{i:X} + B_{i:X} - d) - (X_{i:X} - d)\rangle \end{aligned} \tag{29}$$

From equation 29, it follows that:

$$\begin{aligned} (1-\phi_T+\phi_B-\delta)C^d(Y^F_{i:X}) - (1-\delta)C^d(X_{i:X}) = \\ [(1-\phi_T-\delta)C^d(Y^T_{i:X}) - (1-\delta)C^d(X_{i:X})] + [(1+\phi_B-\delta)C^d(Y^B_{i:X}) - (1-\delta)C^d(X_{i:X})] \end{aligned} \tag{30}$$

Add $(\phi_T - \phi_B)C^d(X_{i:X})$ to both sides of equation 30 and proceed similarly to the proof of equation 2, to obtain the following:

$$C^d(Y^F_{i:X}) - C^d(X_{i:X}) = \frac{(1-\phi_T-\delta)[C^d(Y^T_{i:X}) - C^d(X_{i:X})] + (1+\phi_B-\delta)[C^d(Y^B_{i:X}) - C^d(X_{i:X})]}{1-\phi_T+\phi_B-\delta},$$

which corresponds to equation 15.

Proposition 2

Concentration curves of T^α and B^α are given by equation 18, as $C(T^\alpha_{i:X}) = C(B^\alpha_{i:X}) = \alpha C(X_{i:X}) + (1-\alpha)q_{i:X}$. Concentration curves of $X^{T;\alpha}$ and $X^{B;\alpha}$ are as follows:

$$\begin{aligned} C(X^{T;\alpha}_{i:X}) &= \frac{1}{1-\phi_T}[C(X_{i:X}) - \phi_T\alpha C(X_{i:X}) - (1-\alpha)q_{i:X}] \\ C(X^{B;\alpha}_{i:X}) &= \frac{1}{1+\phi_B}[C(X_{i:X}) + \phi_B\alpha C(X_{i:X}) + (1-\alpha)q_{i:X}] \end{aligned}$$

Let f, g and h be defined as follows:

$$\begin{aligned} f &= C(T^{\alpha_2}_{i:X}) - C(T^{\alpha_1}_{i:X}) = \\ &= [\alpha_2 C(X_{i:X}) + (1-\alpha_2)q_{i:X}] - [\alpha_1 C(X_{i:X}) + (1-\alpha_1)q_{i:X}] = \\ &= \alpha_2 C(X_{i:X}) - \alpha_1 C(X_{i:X}) + (1-\alpha_2)q_{i:X} - (1-\alpha_1)q_{i:X} = \\ &= (\alpha_2-\alpha_1)C(X_{i:X}) - (\alpha_2-\alpha_1)q_{i:X} = \\ &= -(\alpha_1-\alpha_2)C(X_{i:X}) + (\alpha_1-\alpha_2)q_{i:X} = \\ &= (\alpha_1-\alpha_2)[q_{i:X} - C(X_{i:X})] \end{aligned}$$

$$\begin{aligned} g &= \frac{1}{1-\phi_T}[C(X^{T;\alpha_2}_{i:X}) - C(X^{T;\alpha_1}_{i:X})] = \\ &= [C(X_{i:X}) - \phi_T\alpha_2 C(X_{i:X}) - (1-\alpha_2)q_{i:X}] - [C(X_{i:X}) - \phi_T\alpha_1 C(X_{i:X}) - (1-\alpha_1)q_{i:X}] = \\ &= \phi_T\alpha_1 C(X_{i:X}) - \phi_T\alpha_2 C(X_{i:X}) + (1-\alpha_1)q_{i:X} - (1-\alpha_2)q_{i:X} = \\ &= \phi_T C(X_{i:X})(\alpha_1-\alpha_2) + q_{i:X}(\alpha_2-\alpha_1) = \\ &= -\phi_T C(X_{i:X})(\alpha_2-\alpha_1) + q_{i:X}(\alpha_2-\alpha_1) = \\ &= (\alpha_2-\alpha_1)[q_{i:X} - \phi_T C(X_{i:X})] \end{aligned}$$

$$
\begin{aligned}
h &= \frac{1}{1+\phi_B}[C(X_{i:X}^{B;\alpha_2}) - C(X_{i:X}^{B;\alpha_1})] = \\
&= [C(X_{i:X}) + \phi_B\alpha_2 C(X_{i:X}) + (1-\alpha_2)q_{i:X}] - [C(X_{i:X}) + \phi_B\alpha_1 C(X_{i:X}) + (1-\alpha_1)q_{i:X}] = \\
&= \phi_B\alpha_2 C(X_{i:X}) - \phi_B\alpha_1 C(X_{i:X}) + (1-\alpha_2)q_{i:X} - (1-\alpha_1)q_{i:X} = \\
&= \phi_B C(X_{i:X})(\alpha_2-\alpha_1) - q_{i:X}(\alpha_2-\alpha_1) = \\
&= -\phi_B C(X_{i:X})(\alpha_1-\alpha_2) + q_{i:X}(\alpha_1-\alpha_2) = \\
&= (\alpha_1-\alpha_2)[q_{i:X} - \phi_B C(X_{i:X})]
\end{aligned}
$$

We know that $q_{i:X} \geqslant C(X_{i:X})$, $q_{i:X} \geqslant \phi_T C(X_{i:X})$ and $q_{i:X} \geqslant \phi_B C(X_{i:X})$, $\forall i$. Also, recall that $\alpha_1 < \alpha_2$. Therefore, it follows that (1) $f < 0 \Rightarrow C(T_{i:X}^{\alpha_2}) < C(T_{i:X}^{\alpha_1})$, which is a proof of Proposition 2(a) and (c); (2) $g > 0 \Rightarrow C(X_{i:X}^{T;\alpha_2}) > C(X_{i:X}^{T;\alpha_1})$, which proves Proposition 2(b), and (3) $h < 0 \Rightarrow C(X_{i:X}^{B;\alpha_2}) < C(X_{i:X}^{B;\alpha_1})$, which is a proof of Proposition 2(d).

Tables and figures

Table A1: Making equity conclusions

	Fiscal system	Tax	Benefit	(source)
Inequality reducing				
RIC	$r_i^{F;r} > 0$	$r_i^{T;r} > 0$	$r_i^{B;r} > 0$	Table 2
AIC	$r_i^{F;a} > 0$	$r_i^{T;a} > 0$	$r_i^{B;a} > 0$	Table 3
α-*IC*	$r_i^{F;\alpha} > 0$	$r_i^{T;\alpha} > 0$	$r_i^{B;\alpha} > 0$	Table 4
d-IC	$r_i^{F;d} > 0$	$r_i^{T;d} > 0$	$r_i^{B;d} > 0$	Table 5
Vertically equitable				
RIC	$v_i^{F;r} > 0$	$v_i^{T;r} > 0$	$v_i^{B;r} > 0$	Table 2
AIC	$v_i^{F;a} > 0$	$v_i^{T;a} > 0$	$v_i^{B;a} > 0$	Table 3
α-*IC*	$v_i^{F;\alpha} > 0$	$v_i^{T;\alpha} > 0$	$v_i^{B;\alpha} > 0$	Table 4
d-IC	$v_i^{F;d} > 0$	$v_i^{T;d} > 0$	$v_i^{B;d} > 0$	Table 5
Horizontally inequitable				
RIC	$h_i^{F;r} > 0$	$h_i^{T;r} > 0$	$h_i^{B;r} > 0$	Table 2
AIC	$h_i^{F;a} > 0$	$h_i^{T;a} > 0$	$h_i^{B;a} > 0$	Table 3
α-*IC*	$h_i^{F;\alpha} > 0$	$h_i^{T;\alpha} > 0$	$h_i^{B;\alpha} > 0$	Table 4
d-IC	$h_i^{F;d} > 0$	$h_i^{T;d} > 0$	$h_i^{B;d} > 0$	Table 5
Progressive				
RIC		$p_i^{T;r} > 0$	$p_i^{B;r} > 0$	Sect. 3.3
α-*IC*		$p_i^{T;\alpha} > 0$	$p_i^{B;\alpha} > 0$	Sect. 3.3

Table A2: Inequality, vertical and horizontal effects

	Fiscal system	Tax	Benefit
		RIC	
IE	$R_\rho^{F;r} = D_\rho(X_X) - D_\rho(Y_{Y^F}^F)$	$R_\rho^{T;r} = D_\rho(X_X) - D_\rho(Y_{Y^T}^T)$	$R_\rho^{B;r} = D_\rho(X_X) - D_\rho(Y_{Y^B}^B)$
VE	$V_\rho^{F;r} = D_\rho(X_X) - D_\rho(Y_X^F)$	$V_\rho^{T;r} = D_\rho(X_X) - D_\rho(Y_X^T)$	$V_\rho^{B;r} = D_\rho(X_X) - D_\rho(Y_X^B)$
HE	$H_\rho^{F;r} = D_\rho(Y_{Y^F}^F) - D_\rho(Y_X^F)$	$H_\rho^{T;r} = D_\rho(Y_{Y^T}^T) - D_\rho(Y_X^T)$	$H_\rho^{B;r} = D_\rho(Y_{Y^B}^B) - D_\rho(Y_X^B)$
		AIC	
IE	$R_\rho^{F;a} = D_\rho^a(X_X) - D_\rho^a(Y_{Y^F}^F)$	$R_\rho^{T;a} = D_\rho^a(X_X) - D_\rho^a(Y_{Y^T}^T)$	$R_\rho^{B;a} = D_\rho^a(X_X) - D_\rho^a(Y_{Y^B}^B)$
VE	$V_\rho^{F;a} = D_\rho^a(X_X) - D_\rho^a(Y_X^F)$	$V_\rho^{T;a} = D_\rho^a(X_X) - D_\rho^a(Y_X^T)$	$V_\rho^{B;a} = D_\rho^a(X_X) - D_\rho^a(Y_X^B)$
HE	$H_\rho^{F;a} = D_\rho^a(Y_{Y^F}^F) - D_\rho^a(Y_X^F)$	$H_\rho^{T;a} = D_\rho^a(Y_{Y^T}^T) - D_\rho^a(Y_X^T)$	$H_\rho^{B;a} = D_\rho^a(Y_{Y^B}^B) - D_\rho^a(Y_X^B)$
		α-IC	
IE	$R_\rho^{F;\alpha} = D_\rho(X_X^{F;\alpha}) - D_\rho(Y_{Y^F}^F)$	$R_\rho^{T;\alpha} = D_\rho(X_X^{T;\alpha}) - D_\rho(Y_{Y^T}^T)$	$R_\rho^{B;\alpha} = D_\rho(X_X^{B;\alpha}) - D_\rho(Y_{Y^B}^B)$
VE	$V_\rho^{F;\alpha} = D_\rho(X_X^{F;\alpha}) - D_\rho(Y_X^F)$	$V_\rho^{T;\alpha} = D_\rho(X_X^{T;\alpha}) - D_\rho(Y_X^T)$	$V_\rho^{B;\alpha} = D_\rho(X_X^{B;\alpha}) - D_\rho(Y_X^B)$
HE	$H_\rho^{F;\alpha} = D_\rho(Y_{Y^F}^F) - D_\rho(Y_X^F)$	$H_\rho^{T;\alpha} = D_\rho(Y_{Y^T}^T) - D_\rho(Y_X^T)$	$H_\rho^{B;\alpha} = D_\rho(Y_{Y^B}^B) - D_\rho(Y_X^B)$
		d-IC	
IE	$R_\rho^{F;d} = D_\rho^d(X_X) - D_\rho^d(Y_{Y^F}^F)$	$R_\rho^{T;d} = D_\rho^d(X_X) - D_\rho^d(Y_{Y^T}^T)$	$R_\rho^{B;d} = D_\rho^d(X_X) - D_\rho^d(Y_{Y^B}^B)$
VE	$V_\rho^{F;d} = D_\rho^d(X_X) - D_\rho^d(Y_X^F)$	$V_\rho^{T;d} = D_\rho^d(X_X) - D_\rho^d(Y_X^T)$	$V_\rho^{B;d} = D_\rho^d(X_X) - D_\rho^d(Y_X^B)$
HE	$H_\rho^{F;d} = D_\rho^d(Y_{Y^F}^F) - D_\rho^d(Y_X^F)$	$H_\rho^{T;d} = D_\rho^d(Y_{Y^T}^T) - D_\rho^d(Y_X^T)$	$H_\rho^{B;d} = D_\rho^d(Y_{Y^B}^B) - D_\rho^d(Y_X^B)$

Table A3: Fiscal subsystem for Croatia

α	0			0.5			1			1.5		
κ	1.5	2	3	1.5	2	3	1.5	2	3	1.5	2	3
						Baseline and reform scenarios						
GX	0.2328	0.3560	0.4931	0.1164	0.1780	0.2465	0.2328	0.3560	0.4931	0.3491	0.5340	0.7396
VE(B4)	0.2	0.1	0.1	0.4	0.4	0.3	0.7	0.7	0.6	1.0	0.9	0.9
VE(B5)	0.2	0.2	0.1	0.4	0.4	0.3	0.7	0.6	0.6	0.9	0.9	0.8
VE(B6)	0.5	0.6	0.8	0.7	0.8	1.0	0.9	1.0	1.2	1.1	1.2	1.4
VE(B7)	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2
VE(B8)	1.5	1.9	2.6	2.0	2.4	3.1	2.4	2.8	3.5	2.8	3.2	3.9
VE(B9)	0.9	1.0	1.2	1.4	1.5	1.7	1.8	2.0	2.2	2.3	2.5	2.7
VE(B10)	0.3	0.3	0.5	0.6	0.7	0.9	1.0	1.1	1.2	1.4	1.5	1.6
						Baseline scenario						
VE(F)	20.4	19.8	19.6	20.4	19.8	19.6	20.4	19.8	19.6	20.5	19.9	19.6
IE(F)	19.1	18.2	17.4	19.1	18.2	17.4	19.1	18.2	17.4	19.1	18.2	17.5
HE(F)	1.3	1.6	2.2	1.3	1.6	2.2	1.3	1.6	2.2	1.3	1.6	2.2
VE(T1)	16.0	14.4	12.6	12.3	10.7	9.0	8.6	7.1	5.3	5.0	3.4	1.7
VE(B1)	-1.8	-1.9	-2.0	-1.0	-1.2	-1.3	-0.3	-0.4	-0.5	0.5	0.3	0.2
VE(B2)	2.6	3.0	3.6	3.4	3.9	4.5	4.3	4.8	5.4	5.2	5.7	6.3
						Reform scenario						
VE(F)	20.6	20.1	19.7	20.6	20.1	19.7	20.7	20.1	19.7	20.7	20.2	19.8
IE(F)	19.2	18.4	17.5	19.2	18.4	17.5	19.3	18.5	17.6	19.4	18.5	17.6
HE(F)	1.4	1.7	2.2	1.4	1.7	2.2	1.4	1.7	2.2	1.4	1.7	2.2
VE(T2)	15.8	14.4	12.7	12.1	10.7	9.0	8.4	7.0	5.3	4.7	3.3	1.6
VE(B3)	1.2	1.4	1.6	2.9	3.1	3.3	4.6	4.7	4.9	6.3	6.4	6.6

Notes: GX= $D^a_\rho(X_X)$ for $\alpha = 0$; GX= $D^d_\rho(X_X)$ for $\alpha > 0$

VE(B*)= 100%$V^{B*;a}_\rho / D^a_\rho(X_X)$ for $\alpha = 0$; VE(B*)= 100%$V^{B*;d}_\rho / D^d_\rho(X_X)$ for $\alpha > 0$; etc.

Figure 1: TASC benefit for the four hypothetical family types

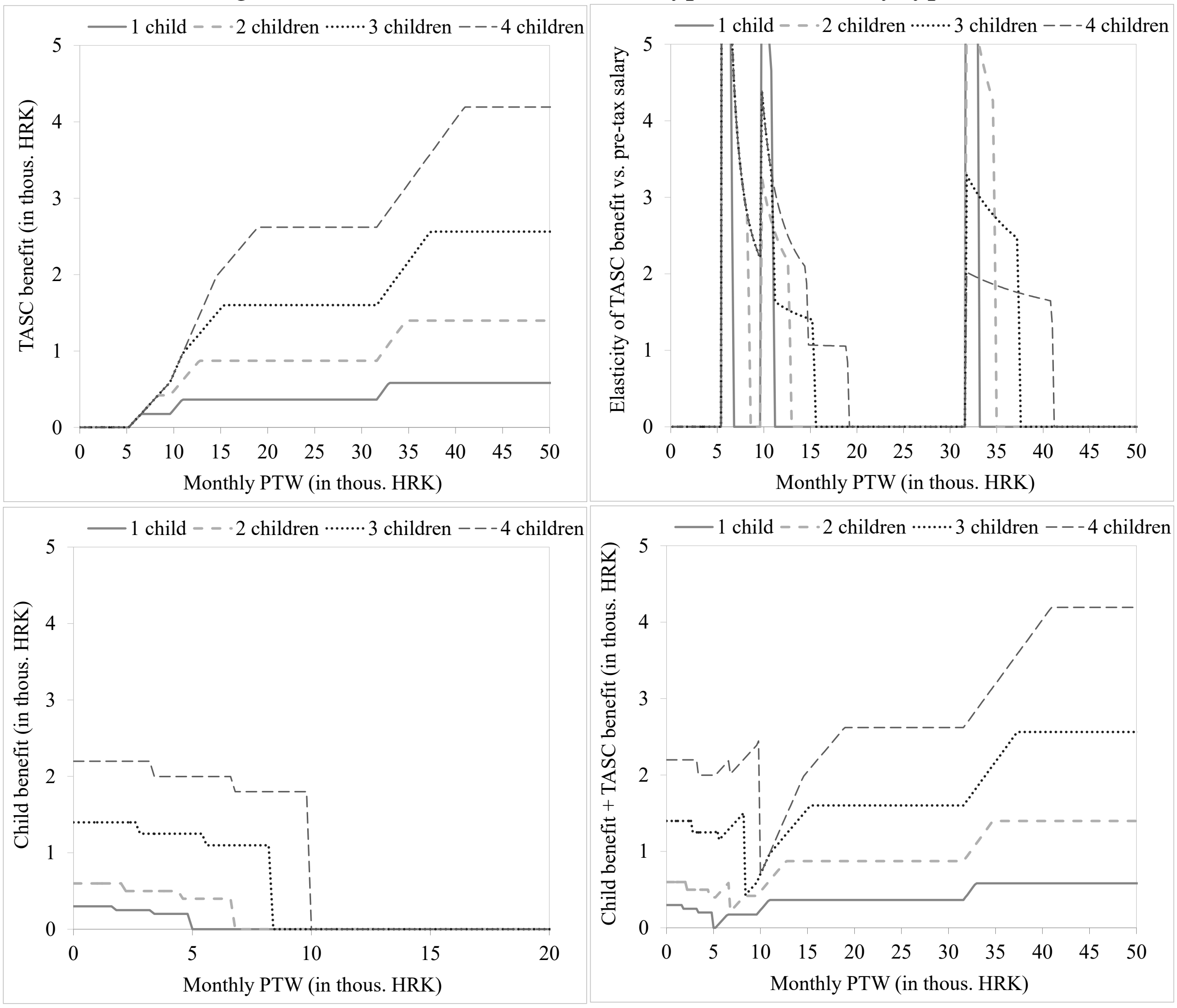

Figure 2: Vertical effects of taxes and benefits, baseline scenario

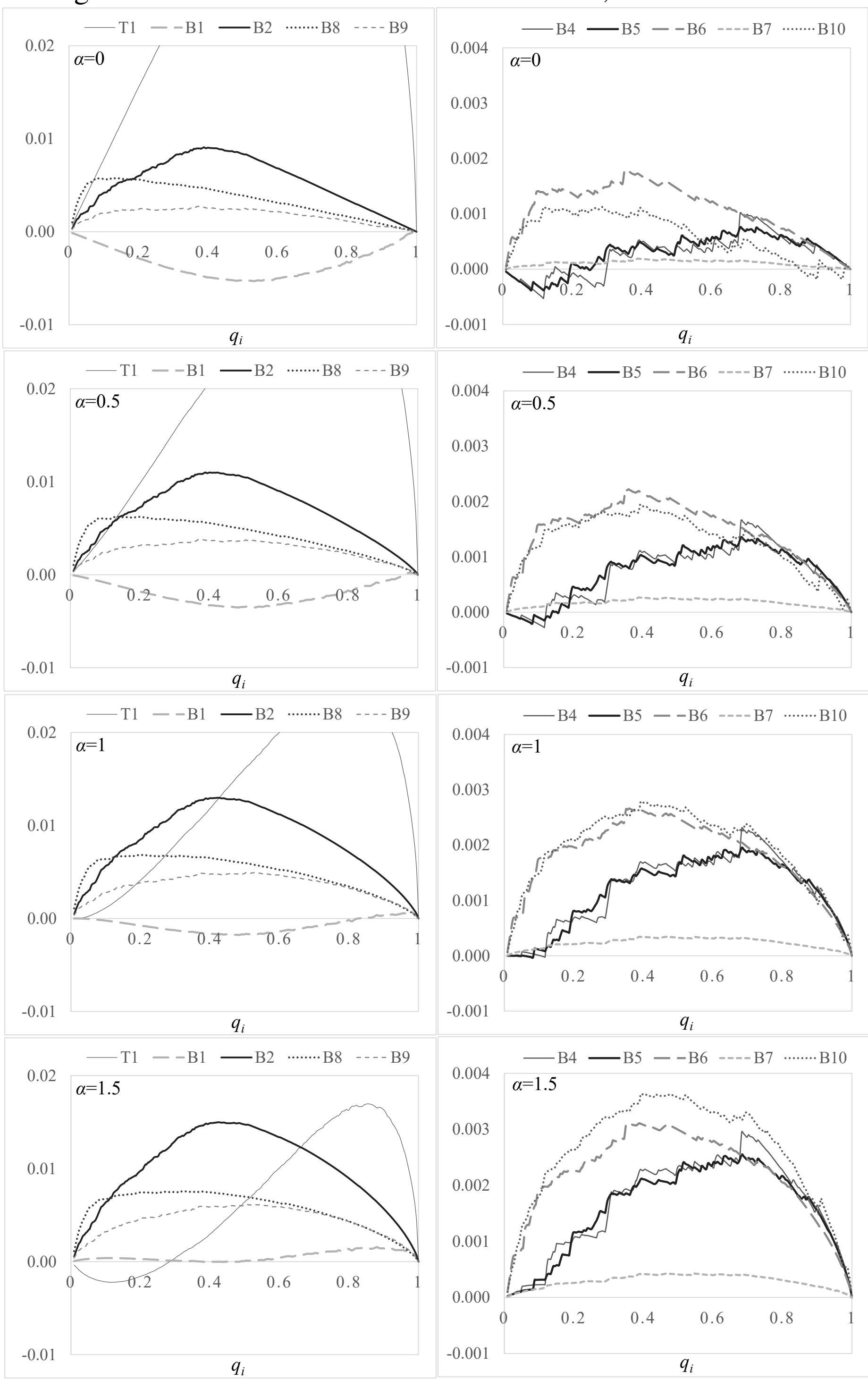

Figure 3: Vertical effect of the fiscal system, baseline and reform scenarios (left panel: $v_i^{F;a}$; right panel: $v_i^{F;d}$ for $\alpha = 1.5$)

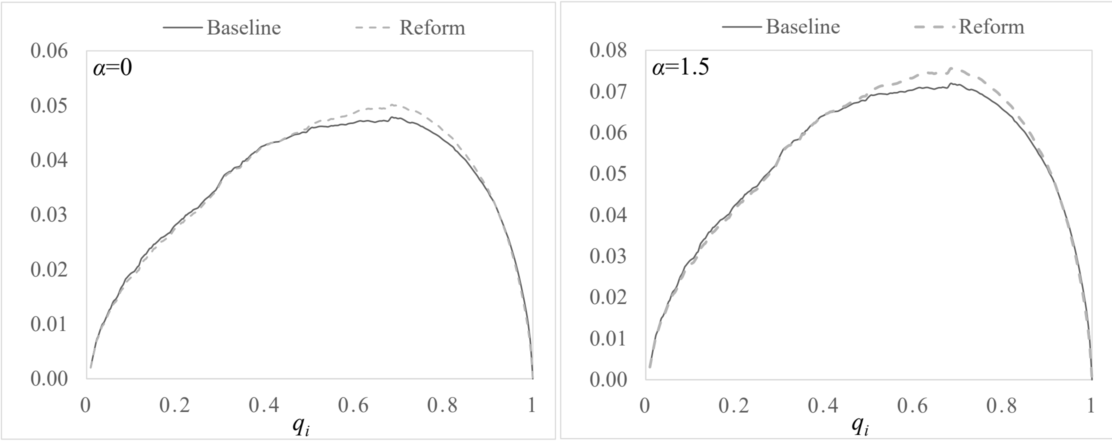

Figure 4: Vertical effect of taxes and benefits, baseline and reform scenarios

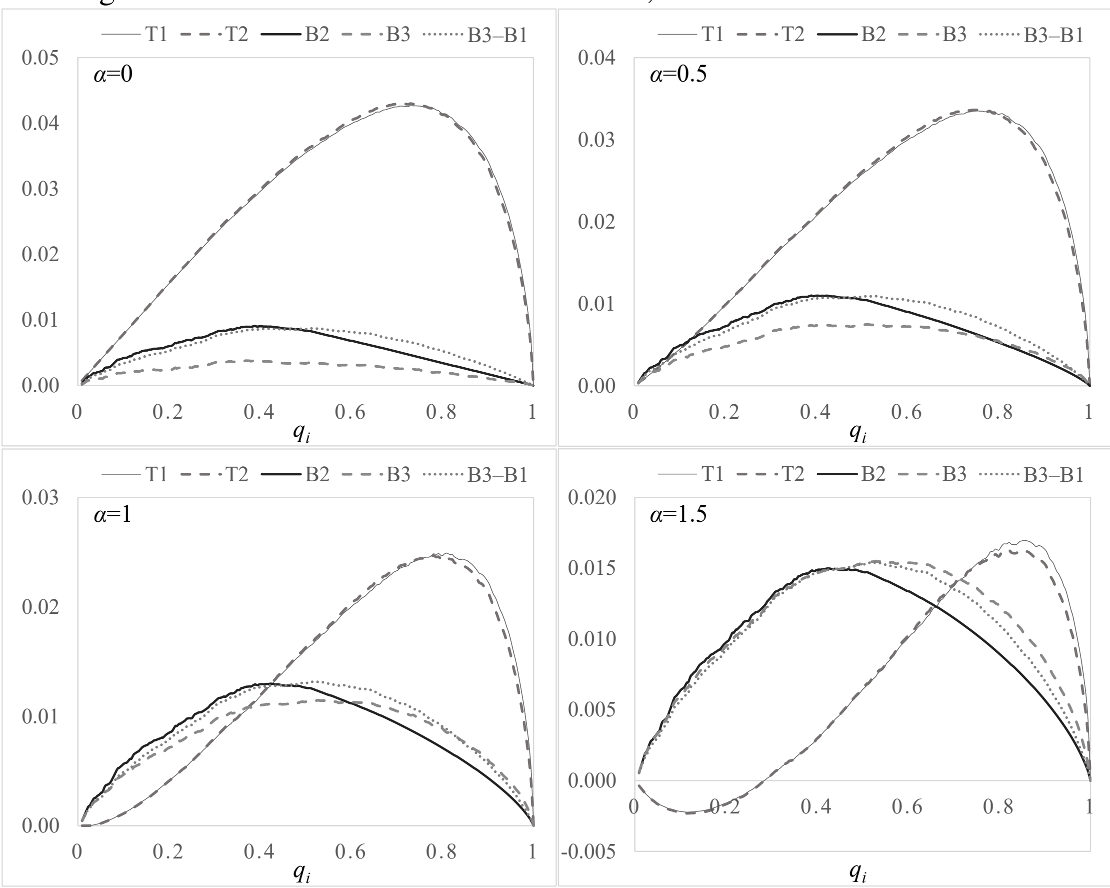

Figure 5: Vertical effects of taxes and benefits, baseline scenario