Measuring the Rotational Speed of Spiral Galaxies and Discovering Dark Matter

Activity UCIObs – 9
Grade Level: College

What's This Activity About?

In this activity, students will measure the rotation of a spiral galaxy using the Doppler shift and learn that galaxies are mostly made up of non-luminous material called dark matter.

What Will Students Do?

Students will measure the mass in dark matter in a nearby spiral galaxy by calculating the speeds at which gas clouds rotate around the center of the galaxy. They will (a) use the Doppler shift equation to convert the observed wavelength of a hydrogen emission line to the line-of-sight velocity, (b) convert line-of-sight velocity to rotation velocity, and (c) use the equation for circular velocity to solve for the 'dynamical' mass of the galaxy. They will then compare the dynamical mass of the galaxy to the mass of the galaxy in stars and gas and learn that 85% of the galaxy's total mass is made of dark matter.

Required Skills: Algebra, Graphing

Reading Assignment:
The relevant sections of your textbook, "*21st Century Astronomy*", by J. Hester, et al. (2007), that you should review before completing this exercise are:

- The Doppler Effect – Is it Moving Toward Us or Away From Us?, pgs. 116-118

- Section 18.4 – Galaxies are Mostly Dark Matter, pgs. 535-538

- Section 19.2 – Measuring the Milky Way, pgs. 558-561

What Will Students Learn?
Concepts: Doppler Shift, Galaxy Rotation, Dark Matter
Inquiry Skills: Applying, Calculating, Graphing, Comparing, Evaluating, Inferring
Big Ideas: Matter, Scale, Systems

What you will need for each student or team of students:
Scientific calculator, Ruler, Scratch Paper for Calculations

Instructions:
1. Do the reading assignment listed above.
2. Read the text and follow the instructions on the next pages.

Student Name: _____   **Date: _____**

Discussion Section: _____

Measuring the Rotational Speed of Spiral Galaxies and Discovering Dark Matter

In this exercise, you are going to measure the mass in dark matter in a nearby spiral galaxy (UGC 477).   You will use the Doppler shift of a hydrogen emission line to measure the speeds at which gas clouds rotate around the center of the galaxy. Using your measured velocities and the equation for circular speed, you will calculate the total dynamical mass of the galaxy. You will then compare the mass of the galaxy in stars and gas (the mass in normal matter) to the dynamical mass and calculate what percentage of a galaxy's total mass is made of dark matter. Dark matter has mass, but does not interact electromagnetically to produce light. It is currently thought to be an unusual elementary particle that we have yet to detect in laboratory experiments.

A. Mass Components of a Galaxy

Galaxies are large collections of stars, gas and dark matter that are held together by their own gravity.   There are three general types of galaxies: spirals, ellipticals and irregulars. Spiral galaxies are flattened disks in which the stars form a spiral pattern. The stars and gas in a spiral galaxy trace circular orbits around the center of the galaxy. Elliptical galaxies are smooth, featureless galaxies that range in shape from spherical to football-shaped; irregular galaxies have a chaotic, random appearance.

Galaxies are made up of three major mass (M) components: stars, gas, and dark matter such that

$$M_{total} \;=\; M_{stars} + M_{gas} + M_{darkmatter} \;\;.$$ (1)

We can detect the stars and gas directly with telescopes (infrared telescopes, optical telescopes, radio telescopes, x-ray telescopes, etc), but we can only indirectly detect the dark matter by the gravitational effect it has on the stars and gas. Because we cannot directly see dark matter, it has been very difficult to figure out what it is.

In units of solar masses (M_{sun} = 2×10^{30} kg), the mass of your spiral galaxy in stars is M_{stars} = 3.8×10^9 M_{sun} and the mass in gas is M_{gas} = 6.5×10^9 M_{sun}. You will calculate the total mass, M_{total}, and then infer the mass in dark matter, $M_{darkmatter}$. Note that this is the first step in trying to identify what dark matter might be made up of.

B. Doppler Shift

When a source of light is moving toward or away from an observer, the measured wavelength of the received light ($\lambda_{observed}$) will be different than the wavelength of the light if the source were at rest (λ_{rest}). The measured wavelength of the observed light and the line-of-sight velocity (V_{LOS}) of the source are related through the Doppler equation:

$$\frac{(\lambda_{observed} - \lambda_{rest})}{\lambda_{rest}} = \frac{V_{LOS}}{c} \qquad (2)$$

where the speed of light c is 3×10^5 km/s.

In the disk of the spiral galaxy that we will examine, massive O and B stars emit intense ultraviolet radiation, which ionizes the surrounding gas, most of which is hydrogen. The excited hydrogen electrons later drop back down to lower energy levels. When an electron jumps from the n=3 level to the n=2 level, a photon at a wavelength of 6563 Angstroms (Å; 1 Å = 10^{-9} m) is emitted. We call this light "Halpha" emission. This emission line is in the red region of the visible spectrum. Because most of the gas in galaxies is made of hydrogen, this can be a very strong emission line if young, massive, hot stars are present, as they are in the disks of spiral galaxies.

Halpha photons have a rest wavelength (λ_{rest}) 6563 Å (Angstroms; 1Å = 10^{-9} m). This gas rotates around the center of the galaxy. Because we are viewing the galaxy close to edge-on (like looking at the side of a rotating CD), some of the gas is moving away from us and some of it is moving toward us. You can use the observed wavelength of the light ($\lambda_{observed}$) and the Doppler equation to determine how fast the gas is moving toward or away from us (V_{LOS}) at different distances from the center of the galaxy.

C. Rotation Velocity

The line-of-sight velocity that is calculated from the Doppler equation tells you how fast the gas is moving directly toward or away from you; what you really need to know, however, is how fast the gas is rotating around the galaxy center. The equation you need for that is:

$$V_{rotation} = \frac{(V_{LOS} - V_{recession})}{\sin(i)} \qquad (3)$$

where $V_{rotation}$ is the rotation velocity, V_{LOS} is the velocity you determined from the Doppler shift, $V_{recession}$ is how fast the galaxy is moving away from us, and sin(i) is the sine of the inclination angle i of the galaxy. The inclination angle is a measure of how close to edge-on you are viewing a galaxy. A galaxy viewed face-on (like looking straight down at a plate) has i = 0º; a galaxy viewed edge-on (like looking at the edge of a plate) has i = 90º. Let's take a closer look at this equation.

It is important to recognize that as a whole, the spiral galaxy is moving away from us. This velocity is called recession velocity. In addition to that movement, the gas clouds in the galaxy are rotating around the center of the galaxy with a rotational speed called the rotation velocity. The Doppler shift velocity is a combination of both the recession velocity and the rotation velocity. We need to separate these two velocities.

In Figure 1 is the Hubble Space Telescope image of the spiral galaxy M101 (http://hubblesite.org/gallery/album/galaxy/spiral/pr2006010a/). We can assume that the very center of the galaxy (star 2) is not rotating and is only moving in a straight line away from us. The velocity we measure here is the recession velocity. The recession velocity of your spiral galaxy is $V_{recession}$ = 2675 km/s. The rest of the galaxy is moving away from us *and* rotating. If we view the galaxy close to edge-on, this means the gas near

star 1 will be moving toward us, the gas near star 3 will be moving away from us, and the gas near stars 4 and 5 will be moving across our line-of-sight. The measured velocity of a gas cloud near star 3 will therefore be the recession velocity plus a positive rotation velocity. Near star 1 the measured velocities will be the recession velocity plus a negative rotation velocity.   Because the gas near stars 4 and 5 do not have any movement *along* our line of sight, the velocities measured there will be only the recession velocity. Going back to the top right side of Equation 3, we see that to isolate

Figure1: Motion of gas in a galaxy viewed edge-on. The entire galaxy is moving away from us. Gas near star 1(3) is rotating toward(away) from us. Gas near stars 4 & 5 is moving across our line-of-sight.

the rotational contribution to the measured velocity, we subtract the recession velocity from the measured velocity.

Now let's discuss the bottom part of the right side of Equation 3: the division by sin(*i*). In this equation, *i* is the inclination of the galaxy. The inclination of your spiral galaxy is *i* = 82°. Remember that Doppler shift only measures an object's motion *directly* toward or away from you. If an object is moving *across* your line of sight (like the gas near stars 4 and 5), you will not detect any additional Doppler shift over $V_{recession}$. So unless your galaxy is inclined by 90° (and assuming the inclination is not 0°), the Doppler shift velocity you measure will only be a fraction of the total motion. To correct for this unmeasurable velocity, we divide by sin(*i*).

D. Dynamical Mass from the Circular Velocity

Now let's review how the rotational velocity tells us the mass enclosed within the orbit. Consider a gas cloud of mass m orbiting in a perfect circle with radius r and constant speed V around the center of a galaxy. Combining Newton's 2nd Law (F = ma) with Newton's Law of Gravity ($F = GmM/r^2$), where M is the total mass of the galaxy interior to radius r, and the fact that the centripetal acceleration $a = v^2/r$ for the gas cloud, we find that the equation for the rotational velocity is related to the enclosed mass of the galaxy by:

$$F = ma = \frac{GMm}{r^2}$$

$$m\left(\frac{v^2}{r}\right) = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$V_{rotation} = \sqrt{(GM/r)}$$

Thus the rotation speed of an object ($V_{rotation}$) depends on the mass of the object being orbited (M), the distance between the object and the center of the circular orbit (r), and the gravitational constant (G):

$$V_{rotation} = \sqrt{(GM)/r} \tag{4}$$

Note that in the units that are convenient for this problem, Newton's gravitational constant $G = 1.33 \times 10^{11}$ km^3 M$_{sun}^{-1}$ s^{-2}. You can use a little bit of algebra to rearrange the equation and solve for the mass M. Then by measuring $V_{rotation}$ and r for any type of object that orbits the galaxy (stars, gas clouds, star clusters or even satellite galaxies), you can calculate the mass of the galaxy enclosed within r. Masses that are determined using this technique are typically called 'dynamical' masses:

$$M_{dynamical} = \frac{(V_{rotation})^2 \; r}{G} \tag{5}$$

The dynamical mass is the *total* mass inside of r; it is the mass in stars, gas, and dark matter. If we measure $V_{rotation}$ at the edge of the galaxy, r_{edge}, then we can calculate the total mass of the galaxy ($M_{dynamical} = M_{total}$). If we know the mass of the galaxy in stars and gas (M_{stars} and M_{gas}), we can use the total dynamical mass to calculate the mass in dark matter ($M_{darkmatter}$).

E. An Example

We will define the center of the galaxy to be located at r = 0. Radii measured on the approaching side of the galaxy, the side rotating towards us, will be defined to be negative (r < 0), and radii on the receding side, the side rotating away from us, will be defined to be positive (r > 0).

At a radius of r = -0.25 kiloparsecs (kpc; 1 kpc = 3.08 × 10^{16} km), the observed wavelength of the hydrogen gas is $\lambda_{observed}$ = 6621.2 Å. A negative radius indicates that the gas is on the side of the galaxy that is moving toward us.

To calculate the line-of-sight velocity at this radial point, we solve for V_{LOS}:

$$V_{LOS} = \frac{c \; (\lambda_{observed} - \lambda_{rest})}{\lambda_{rest}}$$

$$V_{LOS} = \frac{(3 \times 10^5 \; km/s)(6621.2 \text{Å} - 6563 \text{Å})}{6563.0 \text{ Å}}$$

$$V_{LOS} = 2660.4 \; km/s$$

Using V_{LOS}, $V_{recession}$ = 2675 km/s, and i = 82°, calculate $V_{rotation}$:

$$V_{rotation} = \frac{V_{LOS} - V_{recession}}{\sin(i)}$$

$$V_{rotation} = \frac{2660.4 \; km/s - 2675.0 \; km/s}{\sin(82°)}$$

$$V_{rotation} = -14.7 \; km/s$$

A negative rotation velocity means the gas is rotating toward us.

Calculate the dynamical mass inside of r = 0.25 kpc.
Remember that 1 kpc = 3.08 × 10^{16} km and G = 1.33 × 10^{11} km^3 M$_{sun}$$^{-1}$ s^{-2}.

$$M_{dynamical} = \frac{(V_{rotation})^2 \, r}{G}$$

$$M_{dynamical} = \frac{(-14.7 \; km/s)^2 (0.25 \; kpc)}{1.33 \times 10^{11} \; km^3 \; M_{sun}^{-1} \; s^{-2}} \frac{(3.08 \times 10^{16} \; km)}{(1 \; kpc)}$$

$$M_{dynamical} = 1.25 \times 10^7 \; M_{sun}$$

Thus total mass of this galaxy inside r = 0.25 kpc is 12.5 million solar masses.

F. Calculate the Dark Matter Mass

Following this example, calculate V_{LOS}, $V_{rotation}$, and $M_{dynamical}$ for each radial point in the Data Table on the following page. Next, plot your points on the rotation curve graph shown at the bottom of the next page. You may assume that your galaxy rotates symmetrically about the center and therefore plot the absolute value of r and $V_{rotation}$ for each radial bin.

Then use the dynamical mass calculated at the largest radius ($M_{dynamical}$ at r_{edge} = M_{total}) to make the four calculations shown and thus determine the mass in dark matter, $M_{darkmatter}$. Finally, answer the accompanying questions.

What should we expect for the plot of $V_{rotation}$ versus r? Let's find out.

-Imagine a (spherical) galaxy that has a constant mass density ρ.

$$\rho = (Mass \, / \, Volume)$$

$$Volume = (4/3)\,\pi\,r^3$$

-The velocity of a star on a circular orbit *inside* the galaxy changes as a function of radius in the following way:

$$\rho = (Mass\,/\,Volume) = (M\,/\,(4/3)\pi r^3) \rightarrow M = (4/3)\pi r^3 \rho \quad \text{where } \rho \text{ is constant.}$$

$$V_{rotation} = (GM\,/\,r)^{1/2} = [(G(4/3)\pi r^3 \rho)\,/\,r]^{1/2} = [(4/3)G\pi\rho r^2]^{1/2} \rightarrow V_{rotation} \propto r$$

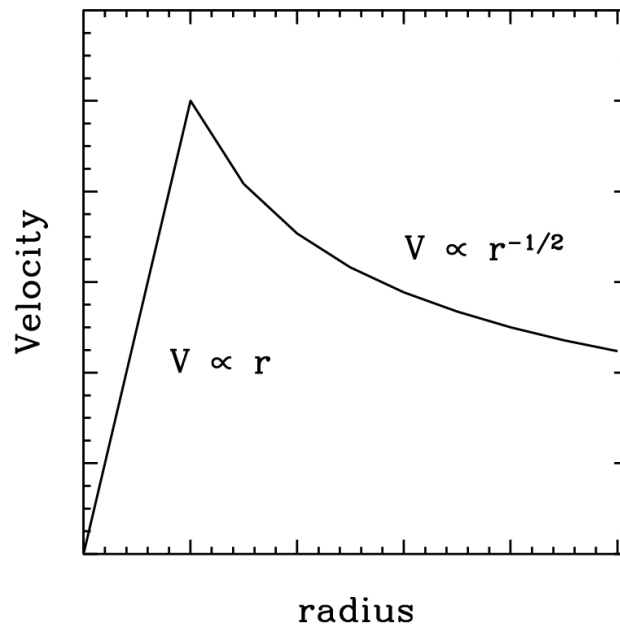(i) *For radii inside the galaxy:* $\boxed{V_{rotation} \propto r}$

-The velocity of a star on a circular orbit *outside* most of the mass of the galaxy changes as a function of radius in the following way:

$$V_{rotation} = (GM_{TOTAL}\,/\,r)^{1/2} \rightarrow V_{rotation} \propto (1/r)^{1/2}$$

(ii) *For radii outside of the galaxy:* $\boxed{V_{rotation} \propto r^{-1/2}}$
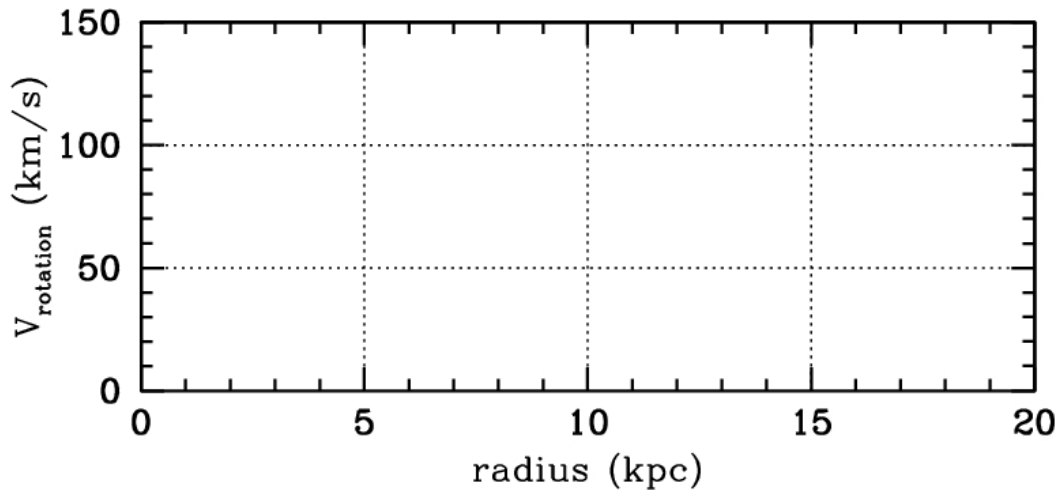
The expected rotation curve of a spiral galaxy therefore looks like:

where the transition between the *r* and *r* $^{-1/2}$ behavior indicates the edge of the majority of the total mass.

Data Table

R (kpc)	$\lambda_{observed}$ (Å)	V_{LOS} (km/s)	$V_{rotation}$ (km/s)	Enclosed $M_{Dynamical}$ (M_{sun})
-0.25	6621.2	2660.4	-14.7	1.25×10^7
-0.75	6621.1			
-1.0	6621.0			
-2.0	6620.5			
-3.0	6620.4			
-3.25	6620.3			
-3.5	6620.4			
-3.75	6620.2			
-4.75	6619.9			
-9.0	6619.4			
-10.75	6619.2			
-18.0	6618.7			
1.75	6622.2			
4.0	6622.8			
5.25	6622.9			
5.5	6623.2			
6.0	6623.2			
6.5	6623.3			
7.75	6623.5			
12.0	6623.8			
15.0	6624.0			

CALCULATIONS:

Be sure to show your work and to include units!

1. The total dynamical mass of the galaxy is:

$M_{total} =$ _____

2. Calculate the mass of the galaxy that is in dark matter. Remember that:
   $M_{stars} = 3.8 \times 10^9 \; M_{sun}$
   $M_{gas} = 6.5 \times 10^9 \; M_{sun}$

   $M_{total} = M_{stars} + M_{gas} + M_{darkmatter}$

   $M_{darkmatter} =$ _____

3. What percentage of the total mass of the galaxy is in dark matter?

   $\dfrac{M_{darkmatter}}{M_{total}} =$ _____ $\times \; 100 =$ _____%

4. What is the ratio of dark matter mass to visible mass (stars and gas)?

   $\dfrac{M_{darkmatter}}{(M_{stars} + M_{gas})} =$ _____ $=$ _____

5. Given the graph that you have obtained of $V_{rotation}$ versus r, do you think your data extend past the majority of the total mass? (Hint: Consider the discussion in Section F.) Thus, do you think you have determined the total mass of the system? Explain why or why not.

Questions:

Answer the following questions. Use complete sentences, and justify your answers using data from your calculations.

     Question #1:
     How does the visible mass of the galaxy ($M_{visible} = M_{stars} + M_{gas}$) compare to the
     dynamical mass of the galaxy? Compute the % of the mass that is visible matter.

     Question #2:
     How does the visible mass compare to the dark matter mass ($M_{darkmatter}$)?

     Question #3:
     What does this tell you about how much of the galaxy you can directly study with
     telescopes?

     Question #4:
     How does the shape of the *observed* galaxy rotation curve compare to the
     expected shape of a galaxy rotation curve? Do they look the same or different?

Question #5:
What does the shape tell you about how the mass is distributed? Is most of the mass in the central bulge of the galaxy, or is there significant non-luminous mass in the galaxy halo?

Question #6:
While astronomers know that dark matter exists, they do not yet know exactly what it is. They do, however, have a pretty good idea of what it is not. Astronomers have suggested many things that we are familiar with that could be the dark matter (objects like planets or very dim stars), but have so far been able to rule them out. To explore why planets cannot be the dark matter, calculate the mass of the Sun in units of Earth masses to determine how many Earths it takes to equal the mass of the Sun. Then figure out how many Earths it would take to equal the mass of dark matter in this spiral galaxy.

$M_{Sun} = 2 \times 10^{30}$ kg
$M_{Earth} = 6 \times 10^{24}$ kg

M_{Sun} = _____ M_{Earth}

Now take the mass you computed for the dark matter halo of the galaxy and compute how many Earths that would be:

$M_{darkmatter}$ = _____ M_{Sun} x _____ $\underline{M_{Earth}}$
                                                                              1 M_{Sun}

$M_{darkmatter}$ = _____$\underline{M_{Earth}}$

This number is *huge* and is one reason why astronomers do not think that dark matter is made of planets. To put this number into perspective, the Milky Way currently has about 2×10^{10} stars. This means that each star in the galaxy would have how many planets?

Planets = _____

These planets are also ruled out as the dark matter because of the amount of infrared radiation that this many planets would generate. Instead, it is currently thought that dark matter is an undiscovered, new elementary particle such as the WIMP envisioned to exist by theorists, but not yet detected in laboratory experiments.