1. **(Problem 8.23 in the Book)**

   An experimental nuclear core simulation apparatus consists of a long thin-walled metallic tube of diameter *D* and length *L*, which is electrically heated to produce the sinusoidal heat flux distribution

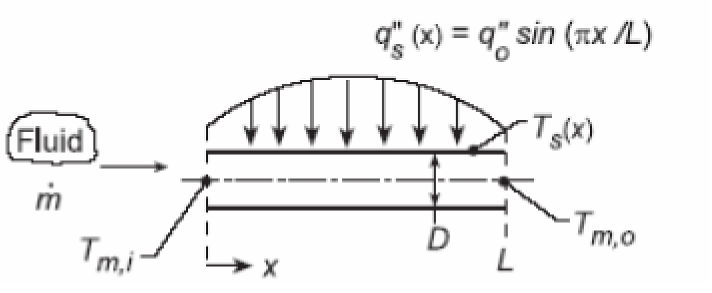
$$q_s''(x) = q_o'' \sin\left(\frac{\pi x}{L}\right)$$

   where *x* is distance measured from the tube inlet. Fluid at an inlet temperature, $T_{m,i}$ flows through the tube at a rate of *m*. Assuming the flow is turbulent and fully developed over the entire length of the tube, develop expressions for:

   a) the total rate of heat transfer, *q*, from the tube to the fluid;
   b) the fluid outlet temperature, $T_{m,o}$;
   c) the axial distribution of the wall temperature, $T_s(x)$ and
   d) the magnitude and position of the highest wall temperature.
   e) Consider a 40-mm-diameter tube of 4-m length with a sinusoidal heat flux distribution for which q"=10,000 W/m^2. Fluid passing through the tube has a flow rate of 0.025 kg/s, a specific heat of 4180 kJ/kgK, an entrance temperature of 25 C, and a convection coefficient of 1000 W/m^2 K. Plot the mean fluid and surface temperatures as a function of distance along the tube. Identify important features of the distributions. Explore the effect of ±25% changes in the convection coefficient and the heat flux on the distributions.

SOLUTION

Schematic



Assumptions

1) Steady state conditions, 2) Applicability of Eq. 8.34 and 3) Turbulent, fully developd flow.

Analysis

(a) The total rate of heat transfer from the tube to the fluid is

$$q = \int_0^L q_s'' P dx = q_o'' \pi D \int_0^D \sin(\pi x/L) dx = q_o'' \pi D (L/\pi) \left[-\cos(\pi x/L)\right]_0^L = 2DLq_o'' \quad (1)$$

(b) the fluid outlet temperature follows from the overall energy balance with knowledge of the total heat rate

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 2DLq_o'' \qquad T_{m,o} = T_{m,i} + \left(2DLq_o''/\dot{m}c_p\right) \tag{2}$$

(c) The axial distribution of the wall temperature can be determined from the rate equation

$$q_s'' = h\left[T_s(x) - T_m(x)\right] \qquad T_{s,x} = T_{m,x}(x) + q_s''/h \tag{3}$$

Where, by combining expressions of part (a) and (b), $T_{m,x}(x)$ is

$$\int_o^x q_s'' P dx = \dot{m}c_p\left(T_{m,x} - T_{m,i}\right)$$

$$T_{m,x} = T_{m,i} + \frac{q_o''\pi D}{\dot{m}c_p}\int_0^x \sin(\pi x/L)dx = T_{m,i} + \frac{DLq_o''}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] \tag{4}$$

Hence, substituting Eq (4) into (3), find

$$T_s(x) = T_{m,i} + \frac{DLq_o''}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] + \frac{q_o''}{h}\sin(\pi x/L) \tag{5}$$

(d) To determine the location of the maximum wall temperrature *x'*, where $T_x(x') = T_{s,max}$, set

$$\frac{dT_s(x)}{dx} = 0 = \frac{d}{dx}\left\{\frac{DLq_o''}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] + \frac{q_o''}{h}\sin(\pi x/L)\right\}$$

$$\frac{DLq_o''}{\dot{m}c_p}\cdot\frac{\pi}{L}\cdot\sin(\pi x'/L) + \frac{q_o''}{h}\cdot\frac{\pi}{L}\cdot\cos(\pi x'/L) = 0 \qquad \tan(\pi x'/L) = -\frac{q_o''/h}{DLq_o''/\dot{m}c_p} = -\frac{\dot{m}c_p}{DLh}$$

$$x' = \frac{L}{\pi}\tan^{-1}\left(-\dot{m}c_p/DLh\right) \tag{6}$$

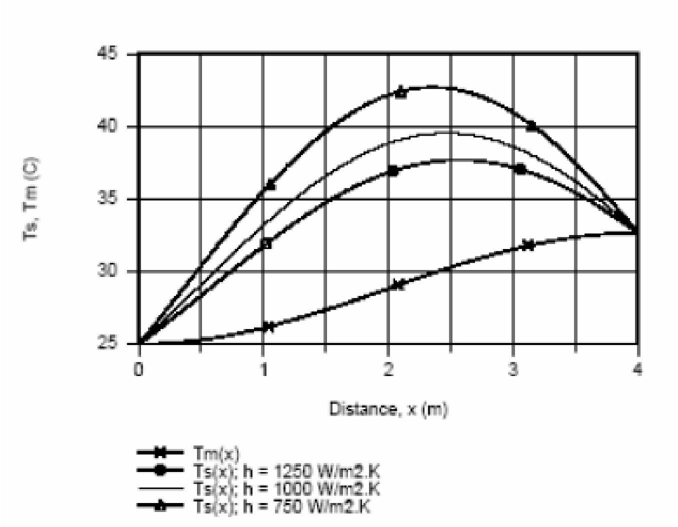
At this location, the wall temperature is

$$T_{s,max} = T_s(x') = T_{m,i} + \frac{DLq_o''}{\dot{m}c_p}\left[1 - \cos(\pi x'/L)\right] + \frac{q_o''}{h}\sin(\pi x'/L) \tag{7}$$

(e) Consider the prescribed conditions for which to compute and plot $T_m(x)$ and $T_s(x)$,

$D = 40$ mm	$\dot{m} = 0.025$ kg/s	$h = 1000$ W/m^2	$q_o'' = 10{,}000$ W/m^2
$L = 4$ m	$c_p = 4180$ J/kg·K	$T_{m,i} = 25°C$	

Using Eqs. (4) and (5), the results are plotted as follows



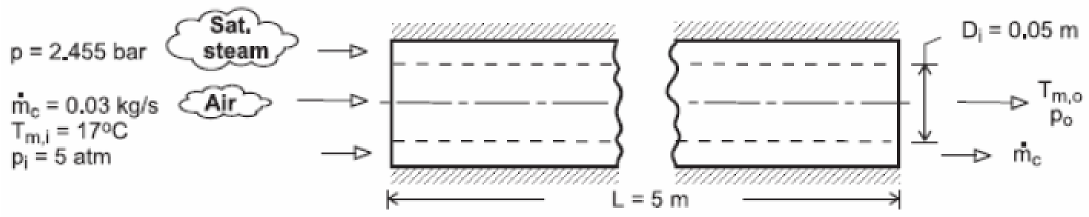
The effect of a lower convection coefficient is to increase the wall temperaure. The position of the maximum temperature, $T_{s,max}$, moves away from the tube exit with decreasing convection coefficient.

Comments

1. Because the fow is fully developed and turbulent, assuming h is constant along the entire length of the tube is reasonable.

2. To determine whether the $T_x(x)$ distribution has a maximum (rather than a minimum), one should evaluate $d^2T_s(x)/dx^2$ to show the value is indeed negative.

2. (**Problem 8.38 in the book**) An air heater for an industrial application consists of an insulated, concentric tube annulus, for which air flows through a thin-walled inner tube. Saturated steam flows through the outer annulus, and condensation of the steam maintains a uniform temperature Ts on the surface.



Consider conditions for which air enters a 50-mm-diameter tube at a pressure of 5 atm, a temperature of $T_{m,i}$ = 17 C , and a flow rate of $\dot{m}$ = 0.03kg/s , while saturated steam at 2.455 bars condenses on the outer surface of the tube. If the length of the annulus is L = 5 m, what are the outlet temperature $T_{m,o}$ and pressure p_o of the air? What is the mass rate at which condensate leaves the annulus.

SOLUTION

Assumptions

1) Steady state, 2) Outer surface of annulus is adiabatic, 3) Ideal gas with negligible viscous dissipation and pressure variation, 4) Fully-developed flow through the tube, 5) Smooth tube surface and 6) Constant properties.

Properties

Table A-4, air ($\overline{T}_m \approx 325\,\text{K},\ p = 5\,\text{atm}$): $\rho = 5\times\rho(1\,\text{atm}) = 5.391\,\text{kg/m}^3$, $c_p = 1008\,\text{J/kg}\cdot\text{K}$, $\mu = 196.4\times10^{-7}\,\text{N}\cdot\text{s/m}^2$, $k = 0.0281\,\text{W/m}\cdot\text{K}$, $Pr = 0.703$. *Table A-6*, sat. steam (p = 2.455 bars): $T_s = 400\,\text{K}$, $h_{fg} = 2183\,\text{kJ/kg}$.

Analysis

With a uniform surface temperature, the air outlet temperrature is

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi D_i L}{\dot{m} c_p}\overline{h}\right)$$

With $Re_D = 4\dot{m}/\pi D_i\,\mu = 0.12\,\text{kg/s}/\pi(0.05\text{m})196.4\times10^{-7}\,\text{kg/s}\cdot\text{m} = 38{,}980$, the flow is turbulent, and the Dittus-Boelter correlation yields

$$\overline{h} \approx h_{fd} = \left(\frac{k}{D_i}\right)0.023\,Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.0281\,\text{W/m}\cdot\text{K}}{0.05\text{m}}\right)0.023\left(38{,}980\right)^{4/5}\left(0.703\right)^{0.4} = 52.8\,\text{W/m}^2\cdot\text{K}$$

$$T_{m,o} = 127^\circ\text{C} - \left(110^\circ\text{C}\right)\exp\left(-\frac{\pi\times0.05\text{m}\times5\text{m}\times52.8\,\text{W/m}^2\cdot\text{K}}{0.03\,\text{kg/s}\times1008\,\text{J/kg}\cdot\text{K}}\right) = 99^\circ\text{C}$$

The pressure drop is $\Delta p = f\left(\rho u_m^2 / 2D_i\right)L$, where, with $A_c = \pi D_i^2/4 = 1.963\times10^{-3}\,m^2$, $u_m = \dot{m}/\rho A_c = 2.83\,m/s$, and with $Re_D = 38{,}980$, Fig. 8.3 yields $f \approx 0.022$. Hence,

$$\Delta p \approx 0.022\times5.391\,kg/m^3 \frac{(2.83\,m/s)^2\,5m}{2\times0.05m} = 47.5\,N/m^2 = 4.7\times10^{-4}\,atm$$

The rate of heat transfer to the air is

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 0.03\,kg/s\times1008\,J/kg\cdot K\,(82°C) = 2480\,W$$

And the rate of condenstaion is then

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{2480\,W}{2.183\times10^6\,J/kg} = 1.14\times10^{-3}\,kg/s$$

Comments

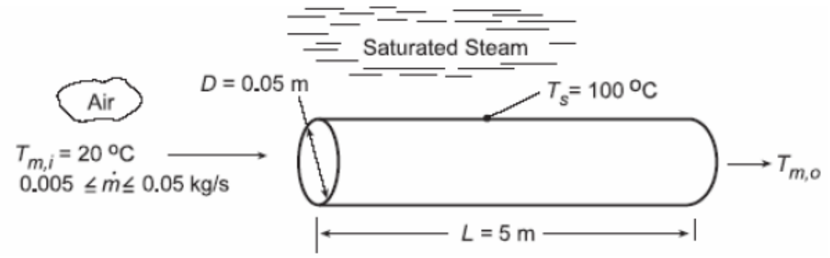
1. With $\overline{T}_m = (T_{m,i} + T_{m,o})/2 = 331K$, the initial estimate of 325 K is reasonable and iteration is not necessary.
2. For a steam flow rate of 0.01 Kg/Sec, approximately 10 % of the outflow would be in the form of saturated liquid.
3. With $L/D_i = 100$, it is reasonable to assume fully developed flow throughout the tube.

3. (**Problem 8.53 in the book**) Heated air required for a food-drying process is generated by passing ambient air at 20 C through long, circular tubes (D = 50 mm, L = 5 m) housed in a steam condenser. Saturated steam at atmospheric pressure condenses on the outer surface of the tubes, maintaining a uniform surface temperature of 100 C.

   a) If an air flow rate of 0.01 kg/s is maintained in each tube, determine the air outlet temperature $T_{m,o}$ and the total hear rate q for the tube.

   b) The air outlet temperature may be controlled by adjusting the tube mass flow rate. Compute and plot $T_{m,o}$ as a function of $\dot{m}$ for $0.005 \leq \dot{m} \leq 0.050$ kg/s. If a particular drying process requires approximately 1 kg/s of air at 75 C, what design and operating conditions should be prescribed for the air heater, subject to the constraint that the tube diameter and length be fixed at 50 mm and 5 m, respectively?

Schematic



Assumptions

1) Steady state, 2) Ideal gas with negligible viscous dissipation and pressure variation, and 3) Negligible tube wall thermal resistance

Properties

Table A.4, air (assume $\overline{T}_m$ = 330 K): $c_p = 1008$ J/kg·K, $\mu = 198.8 \times 10^{-7}$ N·s/m², k = 0.0285 W/m·K, Pr = 0.703.

Analysis

(a) For $\dot{m} = 0.01$Kg/s the Reynold number is 12,810. Hence the flow is turbulent. If fully developed flow is assumed throughout the tube, the Dittus-Boelter correlation may be used to obtain the average Nusselt number.

$$\overline{Nu}_D \approx Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(12,810)^{0.8}(0.703)^{0.4} = 38.6$$

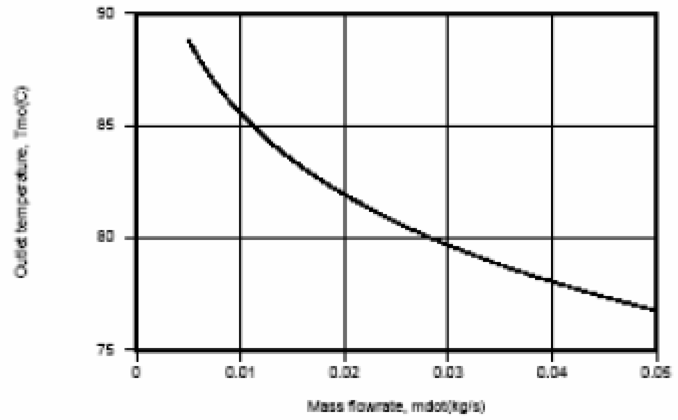
$$\overline{h} = \overline{Nu}_D (k/D) = 38.6(0.0285\,W/m\cdot K/0.05\,m) = 22.0\,W/m^2\cdot K$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\pi D L \overline{h}}{\dot{m} c_p}\right) = \exp\left(-\frac{\pi \times 0.05\,m \times 5\,m \times 22\,W/m^2\cdot K}{0.01\,kg/s \times 1008\,J/kg\cdot K}\right) = 0.180$$

$$T_{m,o} = T_s - 0.180(T_s - T_{m,i}) = 100^\circ C - 0.180(80^\circ C) = 85.6^\circ C$$

$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) = 0.01\,kg/s(1008\,J/kg\cdot K)65.6\,K = 661\,W$$

(b) The effect of flowrate on the outlet temperature is plotted below



Although $\bar{h}$ and hence the heat rate increase with increasing $\dot{m}$, the increase in q is not linearly proportional to the increase in $\dot{m}$ and $T_{m,o}$ decreases with increasing $\dot{m}$.

A flowrate of $\dot{m}$ = 0.05 Kg/s is not large enough to provide the desired outlet temperautre of 75 C and to achieve this value, a flowrate of 0.0678 Kg/s would be needed. At such a flowrate, N = 1 Kg/s/0.0678Kg/s = 14.75 ≈ 15 tubes would be needed to satisfy the process air requirement. Alternatively, a lower flowrate could be supplied to a larger number of tubes and the discharge mixed with ambient air to satisfy the desired conditions. Requirements of this option are that

$$N\dot{m} + \dot{m}_{amb} = 1\,kg/s$$

$$\left(N\dot{m} + \dot{m}_{amb}\right)c_p\left(T_{m,o} - T_{m,i}\right) = 1\,kg/s \times 1008\,J/kg \cdot K\left(75 - 20\right)K = 55,400\,W$$

Where $\dot{m}$ is the flowrate per tube. Using a larger number of tubes with a smaller flowrate per tube would reduce flow pressure losses and hence provide for reduced operaing costs.

Comments

With L/D = 5 m/0.05 m = 100, the assumption of fully developed conditions throughout the tube is reasonable.

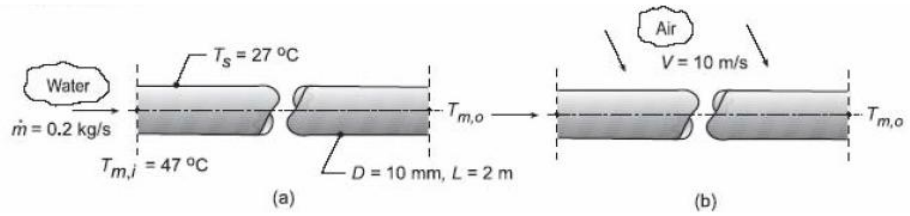
4. (**Problem 8.61 in the book**) Consider a thin-walled tube of 10-mm diameter and 2-m length. Water enters the tube from a large reservoir at $\dot{m}$ = 0.2 kg/s and $T_{m,i}$ = 47 C.

a) If the tube surface is maintained at a uniform temperature of 27 C, what is the outlet temperature of water, $T_{m,o}$? To obtain the properties of water, assume an average mean temperature of $\bar{T}_m$ = 300K.

b) What is the exit temperature of the water if it is heated by passing air at T_∞ =100 C and V = 10 m/s in cross flow over the tube? The properties of air may be evaluated at an assumed film temperature of T_f = 350 K.

c) In the foregoing calculations, were the assumed values of $\bar{T}_m$ and T_f appropriate? If not, use properly evaluated properties and recomputed $T_{m,o}$ for the conditions of part (b).

Schematic



Assumptions

1) Steady state, 2) Incompressible liquid with negligible viscous dissipation and negligible axial conduction, 3) Fully developed flow and thermal conditions for internal flow, and 4) Negligible tube wall thermal resistance.

Properties

Table A.6, Water ($\bar{T}_m$ = 300 K): ρ = 997 kg/m^3, c_p = 4179 J/kg·K, μ = 855 × 10^{-6} N·s/m^2, k = 0.613 W/m·K, Pr = 5.83; *Table A.4*, Air ($\bar{T}_f$ = 350 K, 1 atm): ν = 20.92 × 10^{-6} m^2/s, k = 0.030 W/m·K, Pr = 0.700.

Analysis

(a) For the constant wall temperature cooling process, Ts = 27 C, the water outlet temperature can be determined from Eq 8.41b, with P = πD

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}_i\right) \quad (1)$$

To estimate the convection coefficient, characterize the flow evaluating properties at $\bar{T}_m$ = 300 K

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.2\,\text{kg/s}}{\pi\times 0.010\,\text{m}\times 855\times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 29,783$$

Hence, the flow is turbulent and assuming fully developed (L/D = 200) and using Dittus-Boelter correlation, Eq. 8.60, find $\bar{h}_i$

$$Nu_D = \frac{\bar{h}_i D}{k} = 0.023 Re_D^{0.8} Pr^{0.3} \quad \bar{h}_i = \frac{0.613\,W/m\cdot K}{0.010\,m} 0.023(29,783)^{0.8}(5.83)^{0.3} = 9080\,W/m^2\cdot K \quad (2)$$

Substitute $\bar{h}_i$ into Eq. 1, find

$$\frac{(27 - T_{m,o})}{(27-47)^\circ C} = \exp\left(-\frac{\pi \times 0.010\,m \times 2\,m}{0.2\,kg/s \times 4179\,J/kg\cdot K} \times 9080\,W/m^2\cdot K\right) \qquad T_{m,o} = 37.1°C$$

(b) For the air heating process, T_∞ =100 C, the water outlet temperature follows from Eq. 8.45a,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot{m}c_p}\bar{U}\right) \quad (3)$$

where the overall coefficient is $\bar{U} = (1/\bar{h}_i + 1/\bar{h}_o)$ (4)

To estimate $\bar{h}_o$, use the Churchill-Bernstein correlation Eq. 7.54, for cross flow over a cylinder using properties at $\bar{T}_f$ = 350 K

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.010\,m}{20.92\times 10^{-6}\,m^2/s} = 4780 \quad (5)$$

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} \quad (6)$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(4780)^{1/2}(0.700)^{1/3}}{\left[1+(0.4/0.700)^{2/3}\right]^{1/4}}\left[1+\left(\frac{4780}{282,000}\right)^{5/8}\right]^{4/5} = 35.76$$

$$\bar{h}_o = \frac{\overline{Nu}_D k}{D} = \frac{0.030\,W/m\cdot K}{0.010\,m}\times 35.76 = 107\,W/m^2\cdot K$$

Assuming $\bar{h}_i$ = 9080 W/m^2-K as calculated from part (a), fing $\bar{U}$ and then $T_{m,o}$

$$\bar{U} = (1/9080 + 1/107)^{-1}\,W/m^2\cdot K = 106\,W/m^2\cdot K$$

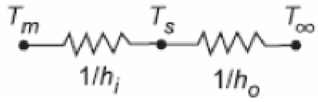
$$\frac{100 - T_{m,o}}{(100-47)^\circ C} = \exp\left(-\frac{\pi\times 0.010\,m\times 2\,m}{0.2\,kg/s\times 4179\,J/kg\cdot K}\times 106\,W/m^2\cdot K\right) \qquad T_{m,o} = 47.4^\circ C$$

(c) The analyses of part (b) are performed considering the appropriate temperatures to evaluate thermophysical properties. For internal and ecternal flow, respectively,

$$\bar{T}_m = (T_{m,i} + T_{m,o})/2 \qquad \bar{T}_f = (\bar{T}_s + T_\infty)/2 \quad (7,8)$$

Where the average tube wall temperature is evaluated from the thermal circuit,

$$\frac{\overline{T}_m - \overline{T}_s}{1/\overline{h}_i} = \frac{\overline{T}_s - T_\infty}{1/\overline{h}_o} \qquad (9)$$



The results of the analyses are summarized in the table along with the results from parts (a) and (b)

Condition	$\overline{T}_m$ (K)	$\overline{h}_i$ (W/m²·K)	$\overline{T}_f$ (K)	$\overline{h}_o$ (W/m²·K)	$\overline{U}$ (W/m²·K)	$T_{m,o}$ (°C)
T_s = 27°C	300	9080	---	---	---	37.1°C
T_∞ = 100 °C, T_f = 350°C	300	9080	350	107	106	47.4°C
Exact solution	320	11,420	347	107.3	106.3	47.4°C

Note that since $\overline{h}_o << \overline{h}_i$, $\overline{U}$ is controlled by the value of $\overline{h}_o$ which is evaluated near 350 K for both parts (b) and (c). Hence, it follows that $T_{m,o}$ is not very sensitive to $\overline{h}_i$ which, as seen above, is sensitive to the value of $\overline{T}_m$.