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Mechanics 2
Collisions

Collisions M2

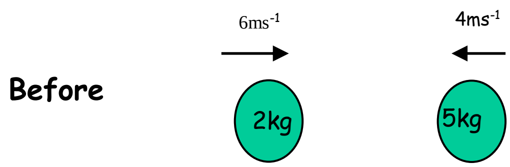
Points to remember from earlier work on collisions

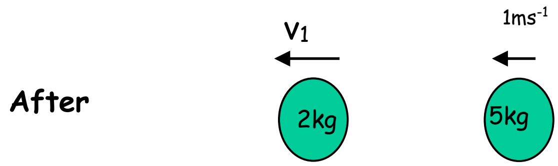
- In the absence of any other forces acting on the two bodies, the changes in momentum of A and B will be equal in magnitude, but opposite in direction.
- Momentum is a vector.
- The gain in momentum of a body will equal the loss in momentum of the other body. Hence the sum of momentum before the impact will equal the sum of momentum after the impact (**Conservation of Linear Momentum**)
- The **impulse** of a force = change in momentum produced

$$\mathbf{I} = m\mathrm{v} - m\mathrm{u}$$

Reminder of M1 Type question.

The two bodies shown collide on a horizontal surface. Find the speed v_1 of the lighter body after the impact.
Always draw two diagrams, one highlighting the situation before the collision and the other showing the situation after the collision.





Assuming that velocities to the right are positive, then;

Momentum before collision = $(2 \times 6) + (5 \times (-4))$
Momentum after collision = $(2 \times (-v_1)) + (5 \times (-1))$

$$12 - 20 = -2v_1 - 5$$
$$v_1 = 1.5\text{ms}^{-1}$$

Therefore by the Conservation of Momentum
The speed of the lighter body after the impact is 1.5ms^{-1}, and its direction is reversed.

More realistic collisions.

In M2 the time of collision is split into two parts
1. The period of compression
2. The period of restitution (shape recovery)

The property that allows for compression and restitution is called the ***elasticity.*** For perfectly elastic collisions there is no loss of Kinetic Energy and for inelastic collisions the particles coalesce. Obviously in the real world there is some loss in Kinetic Energy by way of noise or heat. Collisions between two particles have a coefficient of restitution e and this determines the speed of separation of the particles. This leads to a further Newton's Law.

Newton's Law of Restitution for Direct Impact.

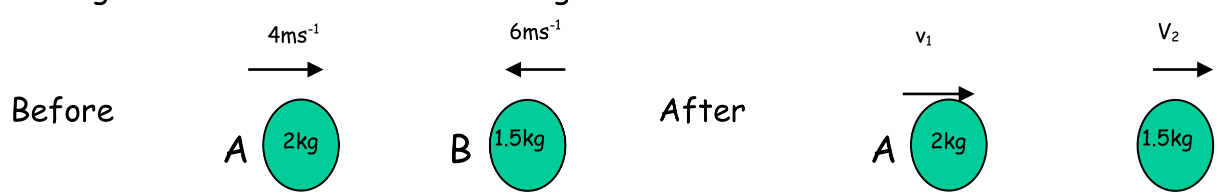
Newton suggested that the speed after a collision depends on the nature of the particles and the speed at which they collide.

$$\frac{\textbf{Speed of separation of particles}}{\textbf{Speed of approach of particles}} = e$$

Obviously;
$0 \le e \le 1$, for perfectly elastic collisions e = 1 and for inelastic collisions e = 0
Another assumption is that the surface is smooth. The presence of friction would be accompanied by spinning of the particles.

Examples

1 Two spheres A and B, are of equal radii and masses 2kg and 1.5kg respectively. The spheres A and B move towards each other along the same straight line on a smooth horizontal surface with speeds $4ms^{-1}$ and $6ms^{-1}$ respectively.
If the coefficient of restitution between the spheres is 0.4, find their speeds and directions after the impact.
Once again remember to draw two diagrams.



It is worth remembering that at this point one could not be certain of the direction of v_1.
Newton's Law gives:

$$\frac{2}{5} = \frac{v_2 - v_1}{4+6}$$

$$\therefore 4 = v_2 - v_1 (1)$$

By Conservation of Momentum (assuming that speeds to the right are positive)

$$(2 \times 4) + ((1.5 \times (-6)) = (2 \times v_1) + (1.5 \times v_2)$$

$$-1 = 2\,v_1 + 1.5\,v_2 \quad (2)$$

By adding equation (2) to twice equation (1) this gives:

$$7 = 3.5\,v_2$$

$$\therefore v_2 = 2$$

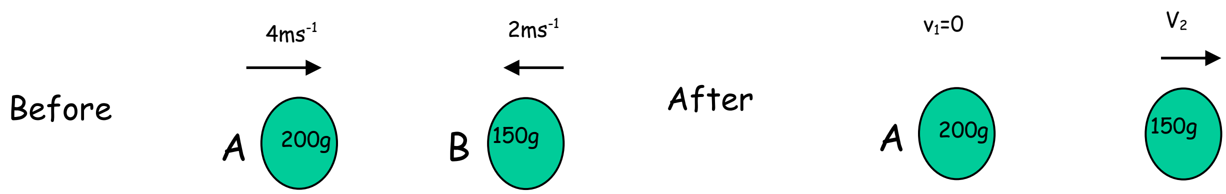
By substituting v_2 into equation (1) gives:

$$v_1 = -2\text{ms}^{-1}$$

(note that particle A has reversed its direction).

Questions involving the collision of two particles should always be approached in the same manner. Students appear to struggle with collisions questions on exam papers but there should be no excuse for them not starting the question. Setting up the equations for Conservation of Momentum and Newton's Law of Restitution will score marks in an exam and to score full marks only requires solving simultaneous equations. Questions dealing with the range of value of e will be discussed later.

2 Two spheres A and B of equal radii and masses 200g and 150g are travelling towards each other along a straight line on a smooth horizontal surface. Initially, A has a speed of 4 ms^{-1} and B has a speed of 2ms^{-1}. The spheres collide and the collision reduces particle A to rest. Find the coefficient of restitution between the spheres.



Once again the standard approach is to set up the two equations.

Conservation of momentum gives:

$$(200 \times 4) + (150 \times (-2)) = (150 \times v_2)$$

$$500 = 150v_2$$

$$v_2 = \frac{10}{3} \quad (1)$$

Newton's Law gives:

$$e = \frac{v_2}{6}$$

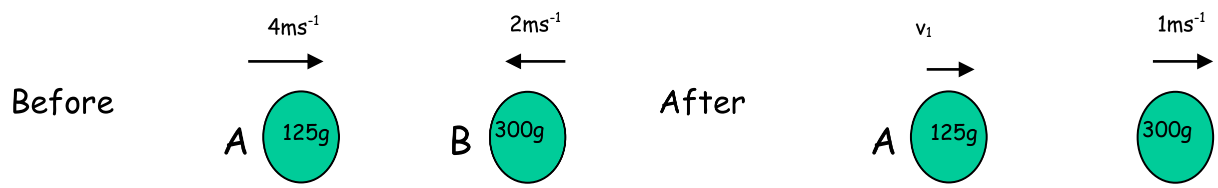
$$\therefore e = \frac{5}{9}$$

Loss of Mechanical Energy

When particles collide there is no loss of momentum but there is a loss of Kinetic Energy. Some of the KE is transformed into other forms of energy at the impact, e.g. heat and sound energy.

Examples

1 Two spheres A and B of equal radii and masses 125g and 300g are travelling towards each other along a straight line on a smooth horizontal surface. Initially, A has speed of $4ms^{-1}$ and B has speed of $2ms^{-1}$. After the collision the direction of B is reversed and it is travelling at a speed of $1ms^{-1}$. Find the speed of A after the collision and the loss of Kinetic energy due to the collision.



Conservation of momentum gives:

$$(0.125 \times 4) + (0.3 \times (-2)) = (0.125 \times v_1) + (0.3 \times 1)$$

$$-0.4 = 0.125v_1$$

$$v_1 = -3.2 \quad (1)$$

Note again that the direction for v_1 in the second diagram is incorrect as v_1 is negative.

Loss of Kinetic Energy

$$= \frac{1}{2} \times 0.125(4)^2 + \frac{1}{2} \times 0.3(2)^2 - \left[\frac{1}{2} \times 0.125(3.2)^2 + \frac{1}{2} \times 0.3(1)^2\right]$$
$$= 0.81J$$

2 A smooth sphere *A* of mass *m* is moving with speed *u* on a smooth horizontal table when it collides directly with another smooth sphere *B* of mass 3*m*, which is at rest on the table. The coefficient of restitution between *A* and *B* is *e*. The spheres have the same radius and are modelled as particles.

(*a*) Show that the speed of *B* immediately after the collision is $\frac{1}{4}(1 + e)u$.

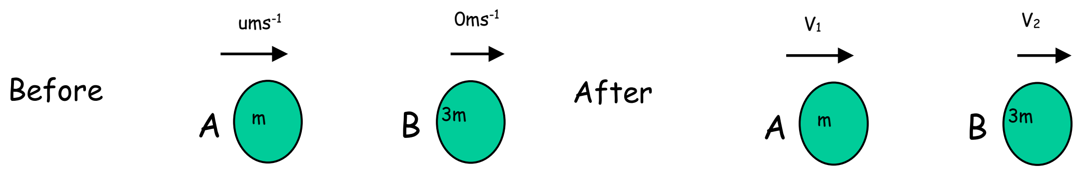
(*b*) Find the speed of *A* immediately after the collision.

Immediately after the collision the total kinetic energy of the spheres is $\frac{1}{6}mu^2$.

(*c*) Find the value of *e*.

(*d*) Hence show that *A* is at rest after the collision.

a)



Conservation of momentum gives:

$$mu = mv_1 + 3mv_2$$

$$v_1 = u - 3v_2 \quad (1)$$

Newton's Law gives:

$$e = \frac{v_2 - v_1}{u}$$

$$\therefore eu = v_2 - v_1 (2)$$

Substituting (1) into (2) gives:

$$eu = v_2 - u + 3v_2$$

$$(1 + e)u = 4\,v_2$$

$$v_2 = \frac{1}{4}(1+e)u$$

b) Using equation (1)

$$v_1 = u - \frac{3}{4}(1+e)u$$

$$v_1 = \frac{u}{4} - \frac{3}{4}eu = \frac{u}{4}(1-3e)$$

c) After collision

$$KE = \frac{1}{6}mu^2$$

$$\frac{1}{6}mu^2 = \frac{1}{2}m(v_1)^2 + \frac{3}{2}m(v_2)^2$$

$$\frac{1}{6}u^2 = \frac{1}{2}(\frac{u}{4}(1-3e))^2 + \frac{3}{2}(\frac{1}{4}(1+e)u)^2$$

$$\frac{1}{6} = \frac{1}{32}(1-3e)^2 + \frac{3}{32}(1+e)^2$$

$$\frac{16}{3} = (1-3e)^2 + 3(1+e)^2$$

$$36e^2 = 4$$

$$\therefore e = \frac{1}{3}$$

At first glance the above proof is a little complicated. In line 3, the values of v_1 and v_2 have been substituted and the m's have been cancelled. In line 4, the u's have been cancelled and the fractions have been simplified. Lines 5 to the end involve multiplying out two brackets and simplifying. Note that e can't be negative.

d) From part b we know:

$$v_1 = u - \frac{3}{4}(1+e)u$$

Substituting for e gives:

$$v_1 = u - \frac{3}{4} \times \frac{4}{3}u$$
$$\therefore v_1 = 0$$

Hence A is at rest after the collision.

Collisions and Vector Notation

The only thing to remember is to only work with like terms and that the magnitude of the velocity vector is the speed.

Example

A body A of mass 250g, is moving with velocity $(-2\mathbf{i} + 3\mathbf{j})\ ms^{-1}$ when it collides with a body B, of mass 750g, moving with velocity $(5\mathbf{i} + 8\mathbf{j})\ ms^{-1}$. Immediately after the collision the velocity of A is $(\mathbf{i} + 9\mathbf{j})ms^{-1}$. Find:

a) The velocity of B after the collision
b) The loss in kinetic energy of the system due to the collision
c) The impulse of A on B due to the collision.

a) Conservation of momentum gives:

$$0.25 \times (-2\mathbf{i} + 3\mathbf{j}) + 0.75 \times (5\mathbf{i} + 8\mathbf{j}) = 0.25 \times (\mathbf{i} + 9\mathbf{j}) + 0.75 \times (x\mathbf{i} + y\mathbf{j})$$

$$(3\mathbf{i} + 4.5\mathbf{j}) = 0.75(x\mathbf{i} + y\mathbf{j})$$

Velocity of B = $(4\mathbf{i} + 6\mathbf{j})ms^{-1}$

b) Loss of kinetic energy

$$= \frac{1}{2} \times 0.25 \times ((2^2 + 3^2) - (1^2 + 9^2)) + \frac{1}{2} \times 0.75 \times ((5^2 + 8^2) - (4^2 + 6^2))$$
$$= 5.25\text{J}$$

c) Impulse of A on B

Impulse = change in momentum

$= 0.75 \times ((4\mathbf{i} + 6\mathbf{j}) - (5\mathbf{i} + 8\mathbf{j}))$

$= (-0.75\mathbf{i} - 1.5\mathbf{j})$Ns

Questions A

1 Two spheres A and B are of equal radii and masses 2kg and 6kg respectively. The spheres move towards each other along the same straight line on a smooth horizontal surface with speeds 8ms^{-1} and 4ms^{-1} respectively. After the collision the particle A rebounds with speed 7ms^{-1}. Find the velocity of B after the collision.

2 Two spheres A and B of equal radii and masses 3kg and 1kg are travelling towards each other along a straight line on a smooth horizontal surface. Initially, A has a speed of 6ms^{-1} and B has a speed of 2ms^{-1}. After the collision the direction of B is reversed and it is travelling at a speed of 4ms^{-1}. Find the speed of A after the collision and the loss of Kinetic energy due to the collision.

3 A particle A, of mass 1.5kg, is moving with speed 2ms^{-1} on a smooth horizontal surface when it collides directly with another particle B, of mass 1kg, which is moving with speed 1ms^{-1} in the same direction. After the collision A continues to move in the same direction with speed 1.4ms^{-1}. Find:

a) The speed of B after the impact,
b) The coefficient of restitution between A and B.

4 Two spheres A and B of equal radii and masses 175g and 100g are travelling towards each other along a straight line on a smooth horizontal surface. Initially, A has a speed of 2ms^{-1} and B has a speed of 6ms^{-1}. After the collision both particles have reversed their original directions of motion and B now has a speed of 3ms^{-1}. Find the speed of A after the collision and the loss of Kinetic energy due to the collision.

5 Two spheres A and B of equal radii and masses m and 2m are travelling on a smooth horizontal surface. Initially, A and B are moving in the same direction with speeds 3u and 2u respectively. There is a direct collision between A and B, the coefficient of restitution between them being e.

a) Find, in terms of u and e, the speeds of A and B after the collision.

b) Show that, whatever the value of e, the speed of B cannot exceed $\frac{8}{3}u$

Given that e is 0.8,

c) Find the magnitude of the impulse exerted by A on B in the collision.

6 A body A of mass 5kg, is moving with velocity $(-12\mathbf{i} + 6\mathbf{j})\text{ms}^{-1}$ when it collides with a body B, of mass 4kg, moving with velocity $(10\mathbf{i} - 5\mathbf{j})\text{ms}^{-1}$. Immediately after the collision the velocity of A is $(-8\mathbf{i} + 4\mathbf{j})\text{ms}^{-1}$. Find the velocity of B immediately after the collision.

Impact of a particle with a fixed surface.

It is assumed that the collisions occur normally (i.e. along a perpendicular line)

Newton's Law becomes:

$$\frac{\textbf{Speed of rebound}}{\textbf{Speed of approach}} = e$$

As well as horizontal collisions with fixed surfaces one can also be asked to consider particles falling from a height and rebounding to a new height.

Example

1 A small smooth ball falls from a height of 3m above a fixed smooth horizontal surface. It rebounds to a height 1.2m. Find the coefficient of restitution between the ball and the plane.

The first task is to calculate the time it takes to fall to the surface and the speed at the time of contact.

Using $s = ut + \frac{1}{2}at^2$ with s = 3m, u = 0 and a = 9.8

$$3 = \frac{1}{2} \times 9.8t^2$$

$$t = \sqrt{\frac{6}{9.8}}$$

Then using v = u +at

$$V = 9.8 \times \sqrt{\frac{6}{9.8}} = 7.668\,ms^{-1}$$

The particle strikes the plane with a velocity of 7.668 ms^{-1}. After the rebound the particle only manages to reach a height of 1.2m. The next task is to calculate the speed with which the particle leaves the surface.

Using $v^2 = u^2 + 2as$ with s =-1.2m, v = 0, a = -9.8

$$u^2 = 2 \times 9.8 \times 1.2$$

$$u = 4.850 ms^{-1}$$

Therefore using

$$\frac{\textbf{Speed of rebound}}{\textbf{Speed of approach}} = \mathbf{e}$$

$$e = \frac{4.850}{7.668}$$

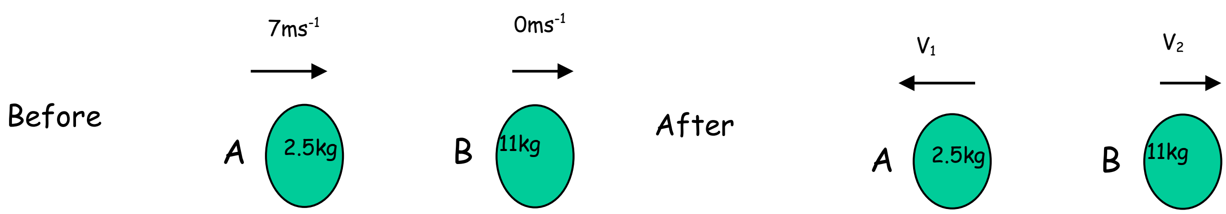
$$e = 0.632$$

Problems involving a range in the value of e

When attacking problems of this nature students need to think about the fundamentals of the situation. Simply thinking of what is necessary for further collisions to occur will lead to the right answer.

Example

1 A small smooth sphere of mass 2.5kg moving on a smooth horizontal plane with speed $7\,ms^{-1}$ collides directly with a sphere of mass 11kg which is at rest. Given that the spheres move in opposite directions after the collision, obtain the inequality satisfied by e.



Conservation of momentum gives:

$$\frac{35}{2}=-\frac{5}{2}v_1+11v_2$$

$$v_2=\frac{35}{22}+\frac{5}{22}v_1(1)$$

Newton's Law gives:

$$e=\frac{v_2+v_1}{7}$$

$$7e=v_2+v_1(2)$$

Substituting equation (1) into (2) gives:

$$7e = v_1 + \frac{35}{22} + \frac{5}{22}v_1$$

$$7e - \frac{35}{22} = \frac{27}{22}v_1$$

$$154e - 35 = 27v_1$$

$$\frac{7}{27}(22e - 5) = v_1$$

The question states that the sphere A moves to the left, and the diagram has taken this into consideration, therefore v_1 must be positive.

$$\frac{7}{27}(22e - 5) > 0$$

$$\therefore 22e > 5$$

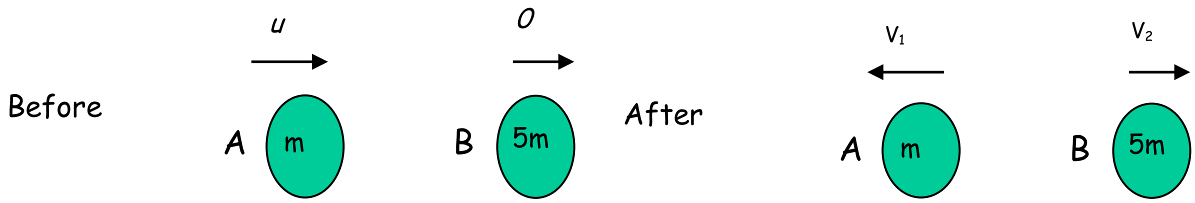
$$e > \frac{5}{22}$$

Note also that e < 1.
This question is not mathematically difficult, there may be some tricky algebra and fraction manipulation but the solution depends upon getting the expression for v_1. All previous questions have involved finding v_1 and v_2, so students shouldn't worry too much when an exam question asks for the range in values in e.

2 Two small smooth spheres, *A* and *B*, of equal radius, have masses *m* and 5*m* respectively. The sphere *A* is moving with speed *u* on a smooth horizontal table when it collides directly with *B*, which is at rest on the table. As a result of the collision the direction of motion of A is reversed The coefficient of restitution between *A* and *B* is *e.*

a) Find the speed of *A* and *B* immediately after the collision
b) Find the range of possible values of e.

a)



Conservation of momentum gives:

$$mu = -mv_1 + 5mv_2$$

$$v_1 = 5v_2 - u \quad (1)$$

Newton's Law gives:

$$e = \frac{v_2 + v_1}{u}$$

$$eu = v_2 + v_1$$

$$v_2 = eu - v_1 \quad (2)$$

Substituting equation (1) into (2) gives:

$$v_2 = eu - 5v_2 + u$$

$$6v_2 = u(1 + e)$$

$$v_2 = \frac{u}{6}(1 + e)$$

Substituting back into equation (1) gives:

$$v_1 = \frac{5}{6}u(1+e) - u$$

$$v_1 = \frac{u}{6}(5e - 1)$$

b) Once again v_1 must be positive therefore;

$$5e - 1 > 0$$

$$e > 0.2$$

$$\therefore 0.2 < e \le 1$$

Multiple collisions and problems involving three particles.

Once again these questions only require the same approach as before and students should be able to set up the first set of simultaneous equations and find the values for v_1 and v_2.

Example

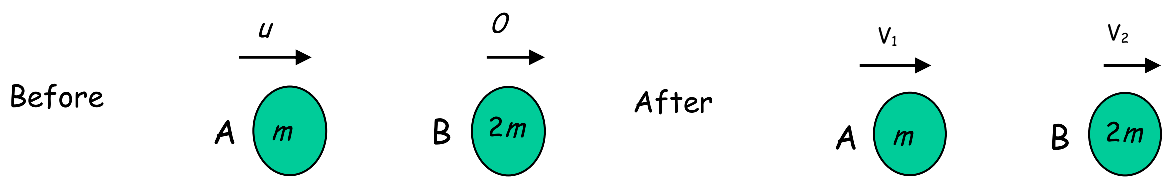
1 A uniform sphere *A* of mass *m* is moving with speed *u* on a smooth horizontal table when it collides directly with another uniform sphere *B* of mass 2*m* which is at rest on the table. The spheres are of equal radius and the coefficient of restitution between them is *e*. The direction of motion of *A* is unchanged by the collision.

(*a*) Find the speeds of *A* and *B* immediately after the collision.
(*b*) Find the range of possible values of *e*.

After being struck by *A*, the sphere *B* collides directly with another sphere *C* of mass 4*m* and of the same size as *B*. The sphere *C* is at rest on the table immediately before being struck by *B*. The coefficient of restitution between *B* and *C* is also *e*.

(*c*) Show that, after *B* has struck *C*, there will be a further collision between *A* and *B*.

a) Remember to always draw a diagram



Conservation of momentum gives:

$$u = v_1 + 2v_2 \ (1)$$

Newton's Law gives:

$$e = \frac{v_2 - v_1}{u}$$

$$eu = v_2 - v_1 \quad (2)$$

Adding the two equations gives:

$$u(e+1) = 3v_2$$

$$v_2 = \frac{u}{3}(e+1)$$

And by substituting back into equation (1) gives:

$$v_1 = \frac{u}{3}(1-2e)$$

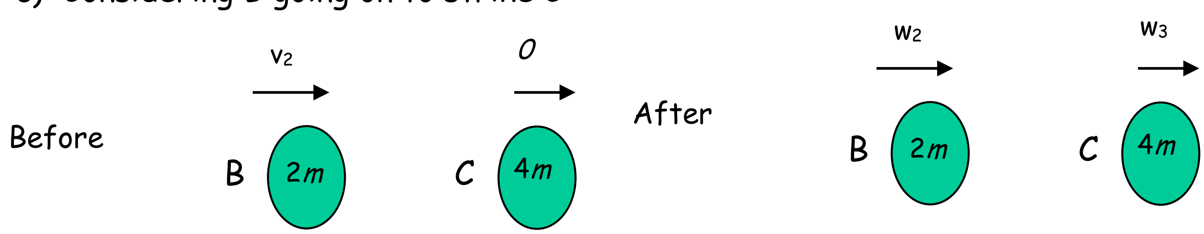
b) Range of values of e

The question states that v_1 is positive, therefore:

$$1 - 2e > 0$$

$$e < 0.5$$

c) Considering B going on to strike C



Before attempting to find a value for w_2 one needs to consider where the question is going. For A to collide with B, v_1 must be greater than w_2.

Conservation of momentum gives:

$$2v_2 = 2w_2 + 4w_3$$

$$v_2 = w_2 + 2w_3 \ (1)$$

Newton's Law gives:

$$e = \frac{w_3 - w_2}{v_2}$$

$$ev_2 = w_3 - w_2 \ (2)$$

Equation (1) - 2 x Equation (1) gives:

$$v_2 - ev_2 = 3w_2$$

$$v_2(1-2e) = 3w_2$$

Substituting v_2 into the above equation gives:

$$\frac{u}{3}(e+1)(1-2e) = 3w_2$$

$$\frac{u}{3}(e+1) \times \frac{1}{3} \times (1-2e) = w_2 \ (3)$$

Remembering to note that the aim is to show that for A to collide with B, v_1 must be greater than w_2 and that $v_1 = \frac{u}{3}(1-2e)$

Equation (3) becomes:

$$w_2 = v_1 \times \frac{1}{3}(e+1)$$

$$\therefore \frac{1}{3}(e+1) < \frac{1}{2}$$

$e < 0.5$

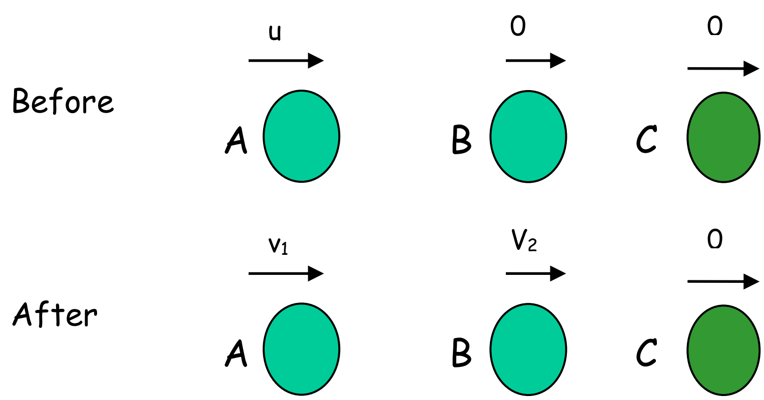
Hence $v_1 > 2w_2$ and therefore there will be a further collision between A and B.

2 Three identical spheres A, B and C lie on a smooth horizontal surface with their centres in a straight line and with B between A and C. Given that A is projected towards B with speed u, show that, after impact, B moves with speed 0.5u(1+e), where e is the coefficient of restitution between each of the spheres. When C first moves, it is found to have speed $\frac{9u}{16}$.

a) Find e.

b) Show that when C begins to move, $\frac{75}{128}$ of the initial kinetic energy has been lost.

a) A strikes B



Conservation of momentum gives:

$$u = v_1 + v_2$$

Newton's Law gives:

$$e = \frac{v_2 - v_1}{u}$$

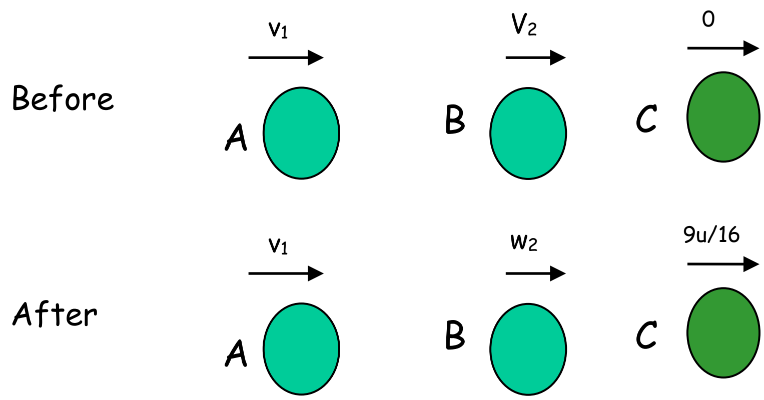
$$eu = v_2 - v_1$$

Adding the two equations gives:

$$eu + u = 2v_2$$

$$\frac{1}{2}u(e+1) = v_2$$

b) Considering B colliding with C



Conservation of momentum gives:

$$v_2 = w_2 + \frac{9u}{16} \quad (1)$$

Newton's Law gives:

$$e = \frac{\frac{9u}{16} - w_2}{v_2}$$

$$ev_2 = \frac{9u}{16} - w_2 \quad (2)$$

Adding together equations (1) and (2) gives:

$$ev_2 + v_2 = \frac{9u}{8}$$

$$v_2(e+1) = \frac{9u}{8} \quad (3)$$

We know v_2 from part a)

$$v_2 = \frac{1}{2}u(e+1)$$

Therefore equation (3) becomes:

$$\frac{1}{2}u(e+1)(e+1) = \frac{9}{8}u$$

$$e^2 + 2e + 1 = \frac{9}{4}$$

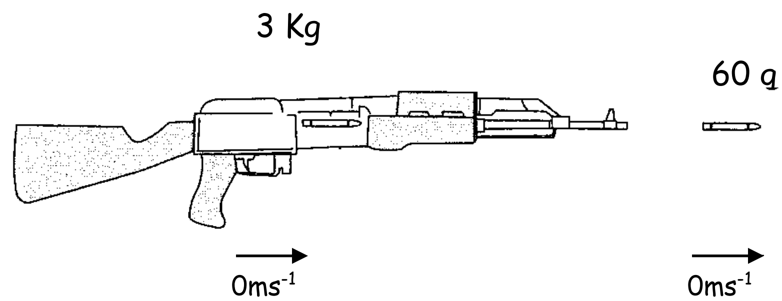
$$e^2 + 2e - \frac{5}{4} = 0$$

The only valid solution is $e = 0.5$

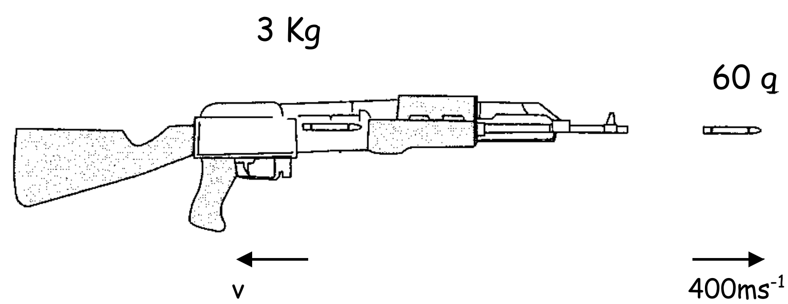
Recoil of a Gun

A bullet is fired from a gun with a horizontal velocity of 400 ms^{-1}. The mass of the gun is 3Kg and the mass of the bullet is 60g. Find the initial speed of recoil of the gun and the gain in kinetic energy of the system.
The rifle is brought to rest by a horizontal force exerted by the soldier's shoulder, against which the rifle is pressed. This force is assumed to be constant. Given that the rifle recoils a distance of 3cm before coming to rest, find the magnitude of the horizontal force exerted on the rifle by the soldier in bringing it to rest.

Before



After



Conservation of momentum gives:

$$3v = 0.06 \times 400$$

$$v = 8ms^{-1}$$

Initial Ke = 0

$$\text{Final KE } = \frac{1}{2} \times 3 \times 8^2 + \frac{1}{2} \times 0.06 \times 400^2$$
$$= \frac{1}{2} \times 192 + \frac{1}{2} \times 9600$$
$$= 4896\text{J}$$

The initial speed of recoil of the gun is $8ms^{-1}$ and the gain in kinetic energy due to the explosion is 4896J.

The work done by the horizontal force must equal the kinetic energy of the rifle (96J).

Work done = force x distance moved.

96J = F x 0.03

F = 3200N

What does this force mean in real terms?

Questions B

1 A pile driver of mass 1000kg falls from a height of 1.8m on to a pile of mass 300kg. After the impact the pile and the driver move on together. Given that the pile is driven a distance of 0.35m into the ground, find:

a) The speed at which the pile starts to move into the ground,

b) The magnitude of the resistance of the ground, in Kn (assumed constant).

2 Three identical spheres A, B and C each of mass 2.5Kg lie at rest on a smooth horizontal table. Sphere A is projected with speed 20ms^{-1} to strike sphere B directly. Sphere B then goes on to strike sphere C directly. Given that the coefficient of restitution between any two spheres is $\frac{1}{3}$

a) Find the speeds of the spheres after these two collisions.

b) Find the total loss of kinetic energy due to these two collisions

3 A particle P of mass 0.6kg is moving with a speed 2.5ms^{-1} on a smooth horizontal table of height 1m. Another particle Q of mass Mkg is at rest on the edge of the top of the table. Particle P strikes particle Q and they coalesce into a single particle R. The particle R then falls to the floor. The horizontal displacement of R is 0.4m.

a) Find the time it takes for R to reach the floor.

b) Find the value of M

4 A uniform sphere *A* of mass *m* is moving with speed *v* on a smooth horizontal table when it collides directly with another uniform sphere *B* of mass 2*m* which is at rest on the table. The spheres are of equal radius and the coefficient of restitution between them is *e.*

a) Find the speeds of the two spheres after the impact.

Given that one half of the kinetic energy is lost in the impact,

b) Find the value of e.

5 A particle A, of mass m, is moving with speed u on a smooth horizontal surface when it collides directly with a stationary particle B, of mass 3m. The coefficient of restitution between A and B is *e.* The direction of motion of is reversed by the collision.

a) Show that the speed of B after the collision is $\frac{1}{4}u(e+1)$.

b) Find the speed of A after the collision.

Subsequently B hits a wall fixed at right angles to the direction of motion of A and B. The coefficient of restitution between B and the wall is 0.5. After B rebounds from the wall there is another collision between A and B.

c) Show that $\frac{1}{3} < e < \frac{3}{5}$

d) In the case where e = 0.5, find the magnitude of the impulse exerted on B by the wall.

6 Two particles P and Q each of mass 3.5kg are at rest 1.5m apart on a rough horizontal table. The coefficient of friction between the particles and the surface is 0.45. Particle P is projected towards Q with a speed of 8.5 ms^{-1}.

a) Calculate the speed of P just before it strikes Q.

Particle P collides with Q and the two particles coalesce into a single particle S.

b) Calculate the distance that S travels before coming to rest.

1. The unit vectors **i** and **j** lie in a vertical plane, **i** being horizontal and **j** vertical. A ball of mass 0.1 kg is hit by a bat which gives it an impulse of (3.5**i** + 3**j**) Ns. The velocity of the ball immediately after being hit is (10**i** + 25**j**) m s^{-1}.

(*a*) Find the velocity of the ball immediately before it is hit.

(3)

In the subsequent motion the ball is modelled as a particle moving freely under gravity. When it is hit the ball is 1 m above horizontal ground.

(*b*) Find the greatest height of the ball above the ground in the subsequent motion.

(3)

The ball is caught when it is again 1 m above the ground.

(*c*) Find the distance from the point where the ball is hit to the point where it is caught.

(4)

June 2001, Q4

2. A particle A of mass $2m$ is moving with speed $2u$ on a smooth horizontal table. The particle collides directly with a particle B of mass $4m$ moving with speed u in the same direction as A. The coefficient of restitution between A and B is $\frac{1}{2}$.

(*a*) Show that the speed of B after the collision is $\frac{3}{2}u$.

(6)

(*b*) Find the speed of A after the collision.

(2)

Subsequently B collides directly with a particle C of mass m which is at rest on the table. The coefficient of restitution between B and C is e. Given that there are no further collisions,

(*c*) find the range of possible values for e.

(8)

June 2001, Q6

3. A smooth sphere P of mass m is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass $2m$ is at rest on the table. The sphere P collides directly with Q. After the collision the direction of motion of P is unchanged. The spheres have the same radii and the coefficient of restitution between P and Q is e. By modelling the spheres as particles,

(*a*) show that the speed of Q immediately after the collision is $\frac{1}{3}(1 + e)u$,

(5)

(*b*) find the range of possible values of e.

(4)

Given that $e = \frac{1}{4}$,

(*c*) find the loss of kinetic energy in the collision.

(4)

(*d*) Give one possible form of energy into which the lost kinetic energy has been transformed.

(1)

Jan 2002,Q6

4. A small smooth ball A of mass m is moving on a horizontal table with speed u when it collides directly with another small smooth ball B of mass $3m$ which is at rest on the table. The balls have the same radius and the coefficient of restitution between the balls is e. The direction of motion of A is reversed as a result of the collision.

(*a*) Find, in terms of e and u. the speeds of A and B immediately after the collision.

(7)

In the subsequent motion B strikes a vertical wall, which is perpendicular to the direction of motion of B, and rebounds. The coefficient of restitution between B and the wall is $\frac{3}{4}$.

Given that there is a second collision between A and B,

(*b*) find the range of values of e for which the motion described is possible.

(6)

June 2002,Q6

5. A smooth sphere P of mass $2m$ is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass m is at rest on the table. The sphere P collides directly with Q. The coefficient of restitution between P and Q is $\frac{1}{3}$. The spheres are modelled as particles.

(*a*) Show that, immediately after the collision, the speeds of P and Q are $\frac{5}{9}u$ and $\frac{8}{9}u$ respectively.

(7)

After the collision, Q strikes a fixed vertical wall which is perpendicular to the direction of motion of P and Q. The coefficient of restitution between Q and the wall is e. When P and Q collide again, P is brought to rest.

(*b*) Find the value of e.

(7)

(*c*) Explain why there must be a third collision between P and Q.

(1)

Jan 2003,Q6

6. A tennis ball of mass 0.2 kg is moving with velocity $(-10\mathbf{i})$ m s^{-1} when it is struck by a tennis racket. Immediately after being struck, the ball has velocity $(15\mathbf{i} + 15\mathbf{j})$ m s^{-1}. Find

(*a*) the magnitude of the impulse exerted by the racket on the ball, **(4)**

(*b*) the angle, to the nearest degree, between the vector **i** and the impulse exerted by the racket,

(2)

(*c*) the kinetic energy gained by the ball as a result of being struck.

(2)

June 2003,Q2

7. A uniform sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another uniform sphere B of mass $2m$ which is at rest on the table. The spheres are of equal radius and the coefficient of restitution between them is e. The direction of motion of A is unchanged by the collision.

(*a*) Find the speeds of A and B immediately after the collision.

(7)

(*b*) Find the range of possible values of e.

(2)

After being struck by A, the sphere B collides directly with another sphere C, of mass $4m$ and of the same size as B. The sphere C is at rest on the table immediately before being struck by B. The coefficient of restitution between B and C is also e.

(*c*) Show that, after B has struck C, there will be a further collision between A and B.

(6)

June 2003,Q7

8. A smooth sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another smooth sphere B of mass $3m$, which is at rest on the table. The coefficient of restitution between A and B is e. The spheres have the same radius and are modelled as particles.

(*a*) Show that the speed of B immediately after the collision is $\frac{1}{4}(1 + e)u$.

(5)

(*b*) Find the speed of A immediately after the collision.

(2)

Immediately after the collision the total kinetic energy of the spheres is $\frac{1}{6}mu^2$.

(*c*) Find the value of e.

(6)

(*d*) Hence show that A is at rest after the collision.

(1)

Jan 2004,Q6

9. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A ball has mass 0.2 kg. It is moving with velocity $(30\mathbf{i})$ m s^{-1} when it is struck by a bat. The bat exerts an impulse of $(-4\mathbf{i} + 4\mathbf{j})$ Ns on the ball.

Find

(*a*) the velocity of the ball immediately after the impact, **(3)**

(*b*) the angle through which the ball is deflected as a result of the impact,

(2)

(*c*) the kinetic energy lost by the ball in the impact.

(4)

June 2004,Q2

10. Two small smooth spheres, P and Q, of equal radius, have masses $2m$ and $3m$ respectively. The sphere P is moving with speed $5u$ on a smooth horizontal table when it collides directly with Q, which is at rest on the table. The coefficient of restitution between P and Q is e.

(*a*) Show that the speed of Q immediately after the collision is $2(1 + e)u$.

(5)

After the collision, Q hits a smooth vertical wall which is at the edge of the table and perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f, $0 < f \leq 1$.

(*b*) Show that, when $e = 0.4$, there is a second collision between P and Q.

(3)

Given that $e = 0.8$ and there is a second collision between P and Q,

(*c*) find the range of possible values of f. **(3)**

June 2004,Q5

11. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal table. The particle P collides with a particle Q of mass $2m$ moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(*a*) Show that the speed of Q after the collision is $\frac{1}{5}u(9e + 4)$.

(5)

As a result of the collision, the direction of motion of P is reversed.

(*b*) Find the range of possible values of e.

(5)

Given that the magnitude of the impulse of P on Q is $\frac{32}{5}mu$,

(*c*) find the value of e.

(4)

Jan 2005,Q6

12. Two small spheres A and B have mass $3m$ and $2m$ respectively. They are moving towards each other in opposite directions on a smooth horizontal plane, both with speed $2u$, when they collide directly. As a result of the collision, the direction of motion of B is reversed and its speed is unchanged.

(*a*) Find the coefficient of restitution between the spheres.

(7)

Subsequently, B collides directly with another small sphere C of mass $5m$ which is at rest. The coefficient of restitution between B and C is $\frac{3}{5}$.

(*b*) Show that, after B collides with C, there will be no further collisions between the spheres.

(7)

June 2005,Q5

13. A particle A of mass $2m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle collides directly with a particle B of mass m moving with speed $2u$ in the opposite direction to A. Immediately after the collision the speed of B is $\frac{8}{3}u$ and the direction of motion of B is reversed.

(*a*) Calculate the coefficient of restitution between A and B.

(6)

(*b*) Show that the kinetic energy lost in the collision is $7mu^2$.

(3)

After the collision B strikes a fixed vertical wall that is perpendicular to the direction of motion of B. The magnitude of the impulse of the wall on B is $\frac{14}{3}mu$.

(*c*) Calculate the coefficient of restitution between B and the wall.

(4)

Jan 2006,Q4

14. Two particles A and B move on a smooth horizontal table. The mass of A is m, and the mass of B is $4m$. Initially A is moving with speed u when it collides directly with B, which is at rest on the table. As a result of the collision, the direction of motion of A is reversed. The coefficient of restitution between the particles is e.

(*a*) Find expressions for the speed of A and the speed of B immediately after the collision.

(7)

In the subsequent motion, B strikes a smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{4}{5}$. Given that there is a second collision between A and B,

(*b*) show that $\frac{1}{4} < e < \frac{9}{16}$.

(5)

Given that $e = \frac{1}{2}$,

(*c*) find the total kinetic energy lost in the first collision between A and B.

(3)

June 2006,Q8

15. A particle P of mass m is moving in a straight line on a smooth horizontal table. Another particle Q of mass km is at rest on the table. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. After the collision, the speed of P is v and the speed of Q is $3v$. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(*a*) Find, in terms of v only, the speed of P before the collision.

(3)

(*b*) Find the value of k.

(3)

After being struck by P, the particle Q collides directly with a particle R of mass $11m$ which is at rest on the table. After this second collision, Q and R have the same speed and are moving in opposite directions. Show that

(*c*) the coefficient of restitution between Q and R is $\frac{3}{4}$,

(4)

(*d*) there will be a further collision between P and Q.

(2)

Jan 2007,Q4

16. Two small spheres P and Q of equal radius have masses m and $5m$ respectively. They lie on a smooth horizontal table. Sphere P is moving with speed u when it collides directly with sphere Q which is at rest. The coefficient of restitution between the spheres is e, where $e > \frac{1}{5}$.

(*a*) (i) Show that the speed of P immediately after the collision is $\dfrac{u}{6}(5e - 1)$.

(ii) Find an expression for the speed of Q immediately after the collision, giving your answer in the form λu, where λ is in terms of e.

(6)

Three small spheres A, B and C of equal radius lie at rest in a straight line on a smooth horizontal table, with B between A and C. The spheres A and C each have mass $5m$, and the mass of B is m. Sphere B is projected towards C with speed u. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.

(*b*) Show that, after B and C have collided, there is a collision between B and A.

(3)

(*c*) Determine whether, after B and A have collided, there is a further collision between B and C.

(4)

June 2007,Q7

17. A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P. The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$.

(5)

(b) Find the total kinetic energy lost in the collision.

(5)

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

(c) Calculate the range of values of e for which there will be a second collision between P and Q.

(7)

Jan 2008,Q7

18. A particle A of mass $4m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle A collides directly with a particle B of mass $3m$ moving with speed $2u$ in the same direction as A. The coefficient of restitution between A and B is e. Immediately after the collision the speed of B is $4eu$.

(a) Show that $e = \frac{3}{4}$.

(5)

(b) Find the total kinetic energy lost in the collision.

(4)

May 2008,Q2

19. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$. **(5)**

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$. **(4)**

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall. **(4)**

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(d) find, in terms of d, the distance of the point B from the wall.

(4)

Jan 2009,Q7

20. Particles A, B and C of masses $4m$, $3m$ and m respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particles A and B are projected towards each other with speeds u m s^{-1} and v m s^{-1} respectively, and collide directly. As a result of the collision, A is brought to rest and B rebounds with speed kv m s^{-1}. The coefficient of restitution between A and B is $\frac{3}{4}$.

(a) Show that $u = 3v$. **(6)**

(b) Find the value of k. **(2)**

Immediately after the collision between A and B, particle C is projected with speed $2v$ m s^{-1} towards B so that B and C collide directly.

(c) Show that there is no further collision between A and B. **(4)**

May 2009,Q8

21. Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $2u$ and the speed of Q is u. The coefficient of restitution between the particles is e, where $e < 1$. Find, in terms of u and e,

(i) the speed of P immediately after the collision,

(ii) the speed of Q immediately after the collision.

(7)

22.

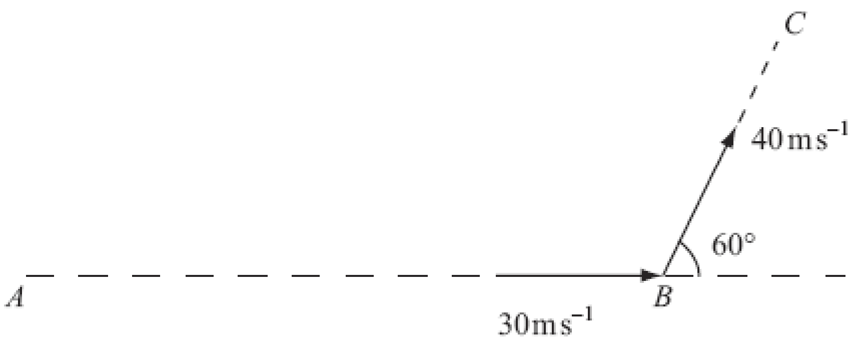


Figure 1

The points A, B and C lie in a horizontal plane. A batsman strikes a ball of mass 0.25 kg. Immediately before being struck, the ball is moving along the horizontal line AB with speed 30 m s^{-1}. Immediately after being struck, the ball moves along the horizontal line BC with speed 40 m s^{-1}. The line BC makes an angle of 60° with the original direction of motion AB, as shown in Figure 1.

Find, to 3 significant figures,

(i) the magnitude of the impulse given to the ball,

(ii) the size of the angle that the direction of this impulse makes with the original direction of motion AB.

(8)

Jan 2010,Q4

23. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A ball of mass 0.5 kg is moving with velocity (10**i** + 24**j**) m s^{-1} when it is struck by a bat. Immediately after the impact the ball is moving with velocity 20**i** m s^{-1}.
Find

(*a*) the magnitude of the impulse of the bat on the ball, **(4)**

(*b*) the size of the angle between the vector **i** and the impulse exerted by the bat on the ball, **(2)**

(*c*) the kinetic energy lost by the ball in the impact. **(3)**

June 2010,Q5

24. A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(*a*) Find

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision. **(7)**

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

(*b*) Find the speed of B immediately after hitting the wall. **(2)**

The first collision between A and B occurred at a distance $4a$ from the wall. The balls collide again T seconds after the first collision.

(*c*) Show that $T = \dfrac{112a}{15u}$.

(6)

Jan 2011,Q8

25. A particle P of mass m kg is moving with speed 6 m s^{-1} in a straight line on a smooth horizontal floor. The particle strikes a fixed smooth vertical wall at right angles and rebounds. The kinetic energy lost in the impact is 64 J. The coefficient of restitution between P and the wall is $\frac{1}{3}$.

(*a*) Show that $m = 4$.

(6)

After rebounding from the wall, P collides directly with a particle Q which is moving towards P with speed 3 m s^{-1}. The mass of Q is 2 kg and the coefficient of restitution between P and Q is $\frac{1}{3}$.

(*b*) Show that there will be a second collision between P and the wall.

(7)

June 2011,Q8

26. Three identical particles, A, B and C, lie at rest in a straight line on a smooth horizontal table with B between A and C. The mass of each particle is m. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$.

(a) Find, in terms of u,

(i) the speed of A after this collision,

(ii) the speed of B after this collision. **(7)**

(b) Show that the kinetic energy lost in this collision is $\frac{5}{36}mu^2$. **(4)**

After the collision between A and B, particle B collides directly with C.

(c) Find, in terms of u, the speed of C immediately after this collision between B and C. **(4)**

Jan 2012,Q6

27. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. The particle P collides directly with a particle Q of mass $4m$ moving on the plane with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Find the speed of Q immediately after the collision. **(6)**

Given that the direction of motion of P is reversed by the collision,

(b) find the range of possible values of e. **(5)**

May 2012,Q2

28.

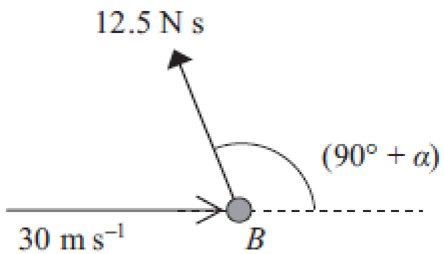


Figure 3

A small ball B of mass 0.25 kg is moving in a straight line with speed 30 m s^{-1} on a smooth horizontal plane when it is given an impulse. The impulse has magnitude 12.5 N s and is applied in a horizontal direction making an angle of $(90° + \alpha)$, where $\tan \alpha = \frac{3}{4}$, with the initial direction of motion of the ball, as shown in Figure 3.

(i) Find the speed of B immediately after the impulse is applied.

(ii) Find the direction of motion of B immediately after the impulse is applied. **(6)**

May 2012,Q5

29. At time t seconds the velocity of a particle P is $[(4t - 5)\mathbf{i} + 3\mathbf{j}]$ m s^{-1}. When $t = 0$, the position vector of P is $(2\mathbf{i} + 5\mathbf{j})$ m, relative to a fixed origin O.

(*a*) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$. **(1)**

(*b*) Find an expression for the position vector of P at time t seconds. **(4)**

A second particle Q moves with constant velocity $(-2\mathbf{i} + c\mathbf{j})$ m s^{-1}. When $t = 0$, the position vector of Q is $(11\mathbf{i} + 2\mathbf{j})$ m. The particles P and Q collide at the point with position vector $(d\mathbf{i} + 14\mathbf{j})$ m.

(*c*) Find

(i) the value of c,

(ii) the value of d. **(5)**

Jan 2013, Q4

30. A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with another particle B, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is e. The direction of motion of A is reversed by the collision.

(*a*) Find, in terms of e and u,

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After being struck by A the particle B collides directly with another particle C, of mass $4m$, which is at rest on the floor. The coefficient of restitution between B and C is $2e$. Given that the direction of motion of B is reversed by this collision,

(*b*) find the range of possible values of e,

(6)

(*c*) determine whether there will be a second collision between A and B.

(3)

Jan 2013, Q7

31. A particle P of mass 2 kg is moving with velocity $(\mathbf{i} - 4\mathbf{j})$ m s^{-1} when it receives an impulse of $(3\mathbf{i} + 6\mathbf{j})$ N s.

Find the speed of P immediately after the impulse is applied.

(5)

June 2013, Q1

32. A particle P of mass 3 kg moves from point A to point B up a line of greatest slope of a fixed rough plane. The plane is inclined at 20° to the horizontal. The coefficient of friction between P and the plane is 0.4.

Given that $AB = 15$ m and that the speed of P at A is 20 m s^{-1}, find

(*a*) the work done against friction as P moves from A to B,

(3)

(*b*) the speed of P at B.

(4)

June 2013, Q2

33. Three particles P, Q and R lie at rest in a straight line on a smooth horizontal table with Q between P and R. The particles P, Q and R have masses $2m$, $3m$ and $4m$ respectively. Particle P is projected towards Q with speed u and collides directly with it. The coefficient of restitution between each pair of particles is e.

(*a*) Show that the speed of Q immediately after the collision with P is $\frac{2}{5}(1 + e)u$.

(6)

After the collision between P and Q there is a direct collision between Q and R.

Given that $e = \frac{3}{4}$, find

(*b*) (i) the speed of Q after this collision,

(ii) the speed of R after this collision. **(6)**

Immediately after the collision between Q and R, the rate of increase of the distance between P and R is V.

(*c*) Find V in terms of u.

(3)

June 2013, Q7

34. Two particles P and Q, of masses $2m$ and m respectively, are on a smooth horizontal table. Particle Q is at rest and particle P collides directly with it when moving with speed u. After the collision the total kinetic energy of the two particles is $\frac{3}{4}mu^2$. Find

(*a*) the speed of Q immediately after the collision,

(10)

(*b*) the coefficient of restitution between the particles.

(3)

June 2013_R, Q5

35. A ball of mass 0.4 kg is moving in a horizontal plane when it is struck by a bat. The bat exerts an impulse $(-5\mathbf{i} + 3\mathbf{j})$ N s on the ball. Immediately after receiving the impulse the ball has velocity $(12\mathbf{i} + 15\mathbf{j})$ m s^{-1}.

Find

(*a*) the speed of the ball immediately before the impact,

(4)

(*b*) the size of the angle through which the direction of motion of the ball is deflected by the impact.

(3)

June 2014_R, Q2

36. A particle P of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal table. A second particle Q of mass $3m$ is moving in the opposite direction to P along the same straight line with speed u. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. The coefficient of restitution between P and Q is e.

(*a*) Show that the speed of Q immediately after the collision is $\frac{u}{5}(8e+3)$.

(6)

(*b*) Find the range of possible values of e.

(4)

The total kinetic energy of the particles before the collision is T. The total kinetic energy of the particles after the collision is kT. Given that $e = \frac{1}{2}$,

(*c*) find the value of k.

(4)

June 2014_R, Q7

37. A particle of mass m kg lies on a smooth horizontal surface. Initially the particle is at rest at a point O midway between a pair of fixed parallel vertical walls. The walls are 2 m apart. At time $t = 0$ the particle is projected from O with speed u m s^{-1} in a direction perpendicular to the walls. The coefficient of restitution between the particle and each wall is $\frac{2}{3}$. The magnitude of the impulse on the particle due to the first impact with a wall is λmu N s.

(*a*) Find the value of λ.

(3)

The particle returns to O, having bounced off each wall once, at time $t = 3$ seconds.

(*b*) Find the value of u.

(6)

June 2014, Q5

38. A particle P of mass 0.75 kg is moving with velocity $4\mathbf{i}$ m s^{-1} when it receives an impulse $(6\mathbf{i} + 6\mathbf{j})$ N s. The angle between the velocity of P before the impulse and the velocity of P after the impulse is $\theta°$.

Find

(*a*) the value of θ,

(5)

(*b*) the kinetic energy gained by P as a result of the impulse.

(3)

June 2015, Q3

39. Three identical particles P, Q and R, each of mass m, lie in a straight line on a smooth horizontal plane with Q between P and R. Particles P and Q are projected directly towards each other with speeds $4u$ and $2u$ respectively, and at the same time particle R is projected along the line away from Q with speed $3u$. The coefficient of restitution between each pair of particles is e. After the collision between P and Q there is a collision between Q and R.

(a) Show that $e > \dfrac{2}{3}$.

(7)

It is given that $e = \dfrac{3}{4}$.

(b) Show that there will not be a further collision between P and Q.

(6)

June 2015, Q8

40. A particle of mass 0.6 kg is moving with constant velocity $(c\mathbf{i} + 2c\mathbf{j})$ m s^{-1}, where c is a positive constant. The particle receives an impulse of magnitude $2\sqrt{10}$ N s.

Immediately after receiving the impulse the particle has velocity $(2c\mathbf{i} - c\mathbf{j})$ m s^{-1}.

Find the value of c.

(6)

June 2016, Q3

41. Two particles A and B, of mass $2m$ and $3m$ respectively, are initially at rest on a smooth horizontal surface. Particle A is projected with speed $3u$ towards B. Particle A collides directly with particle B. The coefficient of restitution between A and B is $\dfrac{3}{4}$.

(a) Find

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After the collision B hits a fixed smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is e. The magnitude of the impulse received by B when it hits the wall is $\dfrac{27}{4}mu$.

(b) Find the value of e.

(3)

(c) Determine whether there is a further collision between A and B after B rebounds from the wall.

(2)

June 2016, Q7

42. A particle P of mass 2 kg is moving with velocity $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1} when it receives an impulse. Immediately after the impulse is applied, P has velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

(*a*) Find the magnitude of the impulse.

(5)

(*b*) Find the angle between the direction of the impulse and the direction of motion of P immediately before the impulse is applied.

(3)

Jan2014, IAL, Q1

43. Three particles A, B and C, each of mass m, lie at rest in a straight line L on a smooth horizontal surface, with B between A and C. Particles A and B are projected directly towards each other with speeds $5u$ and $4u$ respectively. Particle C is projected directly away from B with speed $3u$. In the subsequent motion, A, B and C move along L. Particles A and B collide directly. The coefficient of restitution between A and B is e.

(*a*) (i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

Given that the direction of motion of A is reversed in the collision between A and B, and that there is no collision between B and C,

(*b*) find the set of possible values of e.

(4)

Jan2014, IAL, Q7

44. A particle of mass 0.5 kg is moving on a smooth horizontal surface with velocity $12\mathbf{i}$ m s^{-1} when it receives an impulse $K(\mathbf{i} + \mathbf{j})$ N s, where K is a positive constant. Immediately after receiving the impulse the particle is moving with speed 15 m s^{-1} in a direction which makes an acute angle θ with the vector $\mathbf{i}$.

Find

(i) the value of K,

(ii) the size of angle θ.

(7)

June2014, IAL, Q5

45. Three particles P, Q and R have masses $3m$, km and $7.5m$ respectively. The three particles lie at rest in a straight line on a smooth horizontal table with Q between P and R. Particle P is projected towards Q with speed u and collides directly with Q. The coefficient of restitution between P and Q is $\frac{1}{9}$.

(*a*) Show that the speed of Q immediately after the collision is $\dfrac{10u}{3(3+k)}$.

(6)

(*b*) Find the range of values of k for which the direction of motion of P is reversed as a result of the collision.

(3)

Following the collision between P and Q there is a collision between Q and R. Given that $k = 7$ and that Q is brought to rest by the collision with R,

(*c*) find the total kinetic energy lost in the collision between Q and R.

(5)

June2014, IAL, Q5

46. A particle P of mass 0.6 kg is moving with velocity $(4\mathbf{i} - 2\mathbf{j})$ m s^{-1} when it receives an impulse $\mathbf{I}$ N s. Immediately after receiving the impulse, P has velocity $(2\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

Find

(*a*) the magnitude of $\mathbf{I}$,

(4)

(*b*) the kinetic energy lost by P as a result of receiving the impulse.

(3)

Jan2015, IAL, Q1

47. Three particles P, Q and R lie at rest in a straight line on a smooth horizontal surface with Q between P and R. Particle P has mass m, particle Q has mass $2m$ and particle R has mass $3m$. The coefficient of restitution between each pair of particles is e. Particle P is projected towards Q with speed $3u$ and collides directly with Q.

(*a*) Find, in terms of u and e,

(i) the speed of Q immediately after the collision,

(ii) the speed of P immediately after the collision.

(6)

(*b*) Find the range of values of e for which the direction of motion of P is reversed as a result of the collision with Q.

(2)

Immediately after the collision between P and Q, particle R is projected towards Q with speed u so that R and Q collide directly. Given that $e = \frac{2}{3}$

(*c*) show that there will be a second collision between P and Q.

(6)

Jan2015, IAL, Q7

48. A particle of mass 0.3 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j})$ m s^{-1} when it receives an impulse $(-3\mathbf{i} + 3\mathbf{j})$ N s. Find the change in the kinetic energy of the particle due to the impulse.

(6)

June2015, IAL, Q1

49. Three particles A, B and C lie at rest in a straight line on a smooth horizontal table with B between A and C. The masses of A, B and C are $3m$, $4m$, and $5m$ respectively. Particle A is projected with speed u towards particle B and collides directly with B. The coefficient of restitution between A and B is $\frac{1}{3}$.

(*a*) Show that the impulse exerted by A on B in this collision has magnitude $\frac{16}{7}mu$.

(7)

After the collision between A and B there is a direct collision between B and C.

After this collision between B and C, the kinetic energy of C is $\frac{72}{245}mu^2$.

(*b*) Find the coefficient of restitution between B and C.

(6)

June2015, IAL, Q5

50. A particle P of mass 0.7 kg is moving in a straight line on a smooth horizontal surface. The particle P collides with a particle Q of mass 1.2 kg which is at rest on the surface. Immediately before the collision the speed of P is 6 m s^{-1}. Immediately after the collision both particles are moving in the same direction. The coefficient of restitution between the particles is e.

(*a*) Show that $e < \frac{7}{12}$

(7)

Given that $e = \frac{1}{4}$

(*b*) find the magnitude of the impulse exerted on Q in the collision.

(3)

Jan2016, IAL, Q2

51. A particle of mass 3 kg is moving with velocity (3**i** + 5**j**) m s^{-1} when it receives an impulse (–4**i** + 3**j**) N s.

Find

(*a*) the speed of the particle immediately after receiving the impulse,

(5)

(*b*) the kinetic energy gained by the particle as a result of the impulse.

(3)

June2016, IAL, Q1

52. Two particles *A* and *B*, of mass *m* and 2*m* respectively, are moving in the same direction along the same straight line on a smooth horizontal surface, with *B* in front of *A*. Particle *A* has speed 3 m s^{-1} and particle *B* has speed 2 m s^{-1}. Particle *A* collides directly with particle *B*. The coefficient of restitution between *A* and *B* is $\frac{2}{3}$. The direction of motion of both particles is not changed by the collision. Immediately after the collision, *A* has speed *v* m s^{-1} and *B* has speed *w* m s^{-1}.

(*a*) (i) Show that $w = \frac{23}{9}$

(ii) Find the value of *v*.

(7)

When *A* and *B* collide they are 3 m from a smooth vertical wall which is perpendicular to their direction of motion. After the collision with *A*, particle *B* hits the wall and rebounds. The coefficient of restitution between *B* and the wall is $\frac{1}{2}$.

There is a second collision between *A* and *B* at a point *d* m from the wall.

(*b*) Find the value of *d*.

(7)

June2016, IAL, Q7

53. A particle of mass 2 kg is moving with velocity 3**i** m s^{-1} when it receives an impulse (λ**i** – 2λ**j**) Ns, where λ is a constant. Immediately after the impulse is received, the speed of the particle is 6 m s^{-1}.

Find the possible values of λ.

(8)

Oct 2016, IAL, Q2

54. Particles A, B and C, of masses $4m$, km and $2m$ respectively, lie at rest in a straight line on a smooth horizontal surface with B between A and C. Particle A is projected towards particle B with speed $3u$ and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$

Find

(a) the speed of A immediately after the collision with B, giving your answer in terms of u and k,

(6)

(b) the range of values of k for which A and B will both be moving in the same direction immediately after they collide.

(2)

After the collision between A and B, particle B collides directly with C.
Given that $k = 4$,

(c) show that there will not be a second collision between A and B.

(6)

Oct 2016, IAL, Q8

55. A particle P of mass 0.2 kg is moving with velocity $(20\mathbf{i} - 16\mathbf{j})$ m s^{-1} when it receives an impulse $(-6\mathbf{i} + 8\mathbf{j})$ N s.

(a) Find the speed of P immediately after it receives the impulse.

(5)

(b) Find the size of the angle between the direction of motion of P before the impulse is received and the direction of motion of P after the impulse is received.

(4)

Jan2017, IAL, Q4

56. Two particles P and Q, of masses $2m$ and $3m$ respectively, are moving in opposite directions along the same straight line on a smooth horizontal plane. The particles collide directly and, as a result of the collision, the direction of motion of P is reversed and the direction of motion of Q is reversed. Immediately after the collision, the speed of P is v and the speed of Q is $\frac{3v}{2}$. The coefficient of restitution between P and Q is $\frac{1}{5}$.

(a) Find

(i) the speed of P immediately before the collision,

(ii) the speed of Q immediately before the collision.

(7)

After the collision with P, the particle Q moves on the plane and strikes at right angles a fixed smooth vertical wall and rebounds. The coefficient of restitution between Q and the wall is e. Given that there is a further collision between the particles,

(*b*) find the range of possible values of e.

(3)

Jan2017, IAL, Q5