Mechanics of Materials

Chapter 6

Bending

Tishk International University
Civil Engineering Department
Second Year (2020-2021)
Mechanics of Materials
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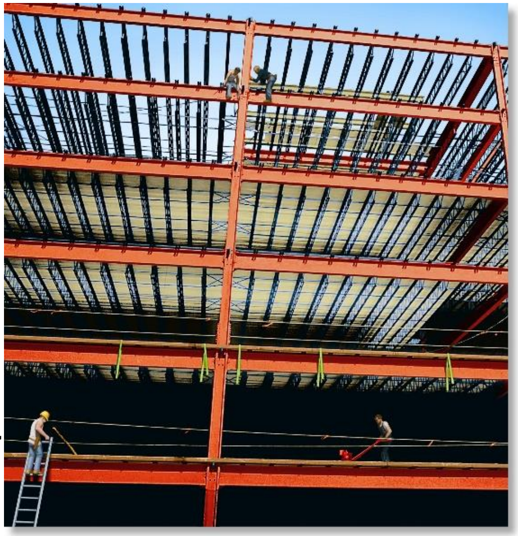
CHAPTER OBJECTIVES



- Determine stress in members caused by bending
- Discuss how to establish shear and moment diagrams for a beam or shaft
- Determine largest shear and moment in a member, and specify where they occur
- Consider members that are straight, symmetric x-section and homogeneous linear-elastic material
- Consider special cases of unsymmetrical bending and members made of composite materials

CHAPTER OUTLINE

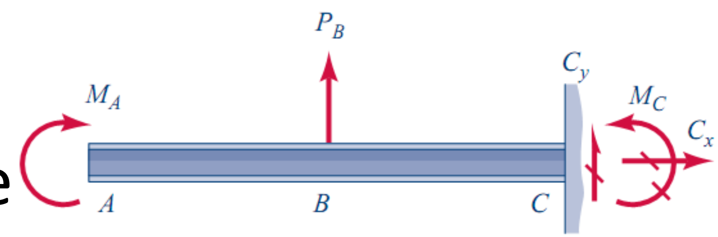
1. Shear and Moment Diagrams
2. Graphical Method for Constructing Shear and Moment Diagrams
3. Bending Deformation of a Straight Member
4. The Flexure Formula
5. Unsymmetrical Bending
6. Composite Beams
7. Reinforced Concrete Beams
8. Stress Concentrations

Shear and Moment Diagrams

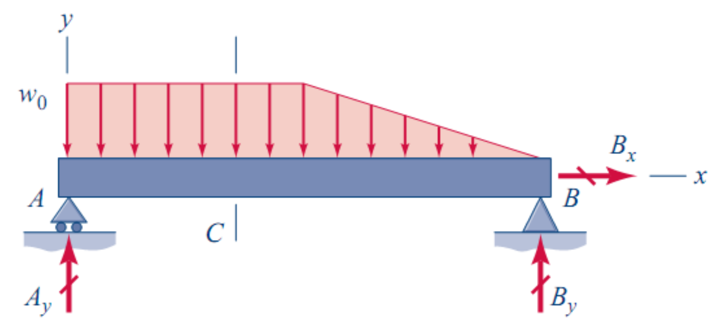
BEAMS AND LOADINGS

The member that resists transverse load (perpendicular to longitudinal axis) and slender are called beams. In general, beams are long, straight having a constant cross sectional area. The applied loads on the induced an internal shear force and bending moment. In order to design a beam, the maximum shear and moment at different location must be determined. Shear, V, and moment, M, can be expressed as a function at arbitrary position x along beam`s axis. The shear and moment function must be determined for each region of the beam located between any discontinuities of loading.

A cantilever beam: with a concentrated at B and a couple at A.



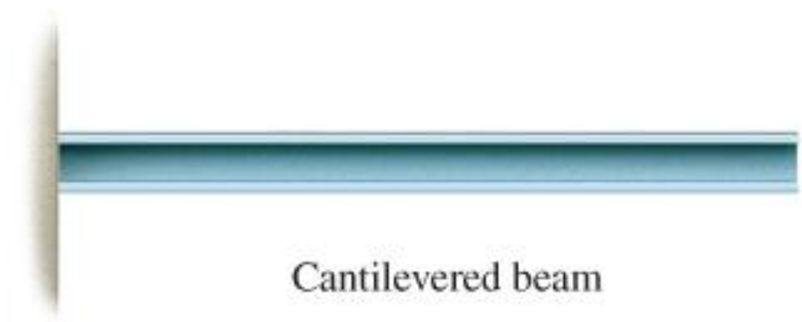
A simply supported: beam with distributed load.



6.1 SHEAR AND MOMENT DIAGRAMS

- Members that are slender and support loadings applied perpendicular to their longitudinal axis are called *beams*

Simply supported beam

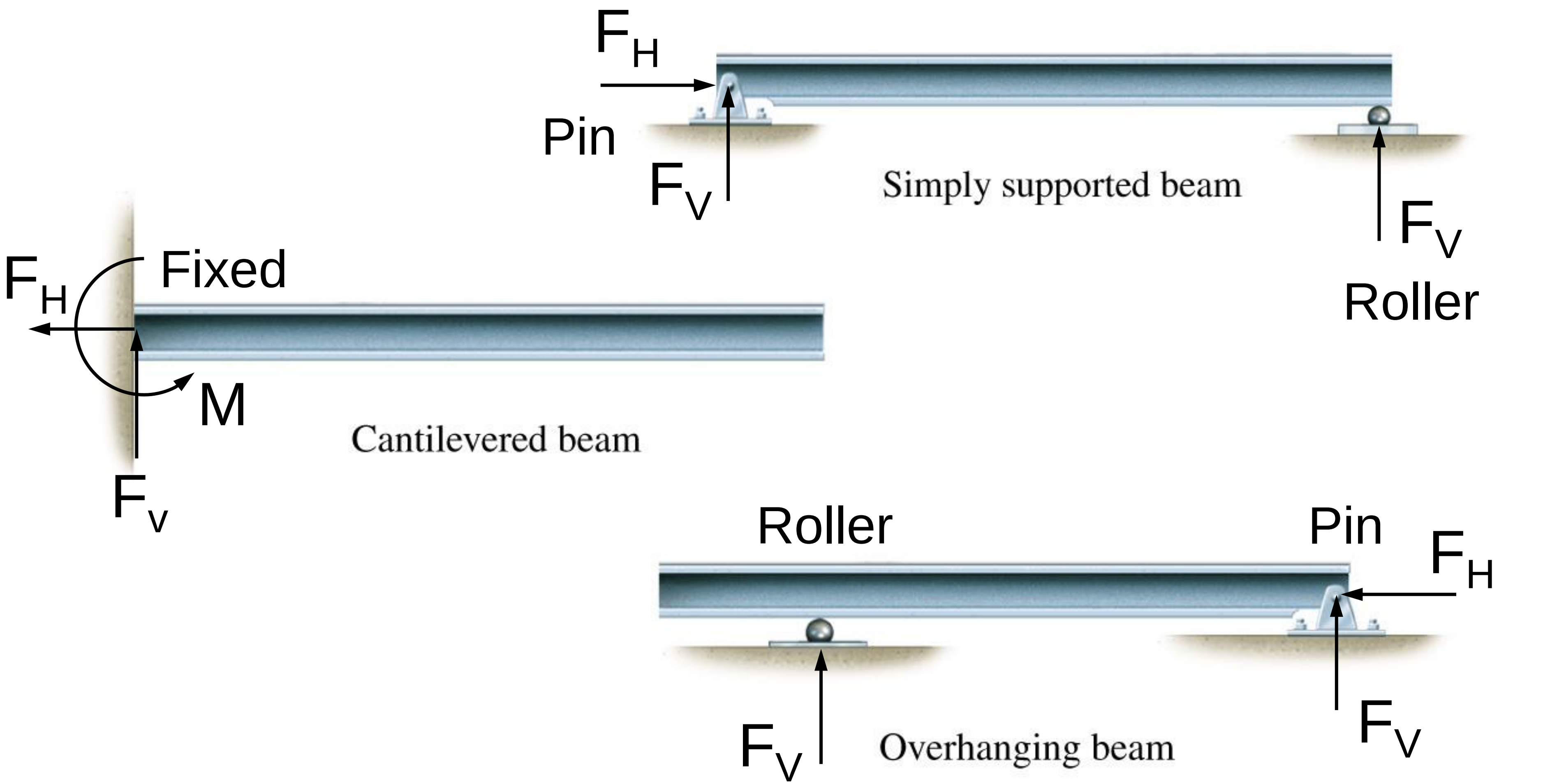


Cantilevered beam

Overhanging beam

6.1 SHEAR AND MOMENT DIAGRAMS

- Depends on the support configuration



6.1 SHEAR AND MOMENT DIAGRAMS

- In order to design a beam, it is necessary to determine the maximum shear and moment in the beam
- Express **V** and **M** as functions of arbitrary position x along axis.
- These functions can be represented by graphs called *shear and moment diagrams*
- Engineers need to know the *variation* of shear and moment along the beam to know where to reinforce it

Internal loading at a specified Point

In General

- **The loading for coplanar structure will consist of a normal force N, shear force V, and bending moment M.**
- **These loading actually represent the** *resultants of the stress distribution* **acting over the member's cross-sectional.**

Shear & Moment Diagrams (By Section Method)

Shear Force Diagram (SFD):

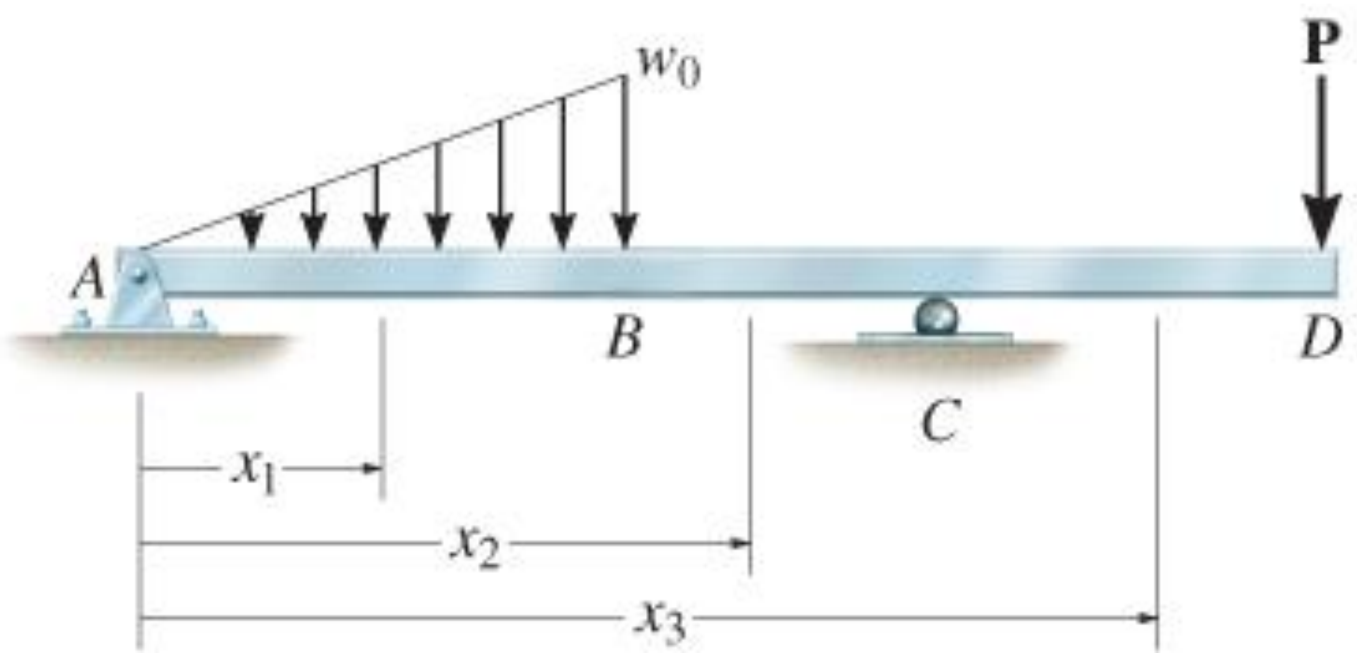
The diagram which shows the variation of shear force along the length of the beam is called ***Shear Force Diagram (SFD).***

Bending Moment Diagram (BMD):

The diagram which shows the variation of bending moment along the length of the beam is called ***Bending Moment Diagram (BMD).***

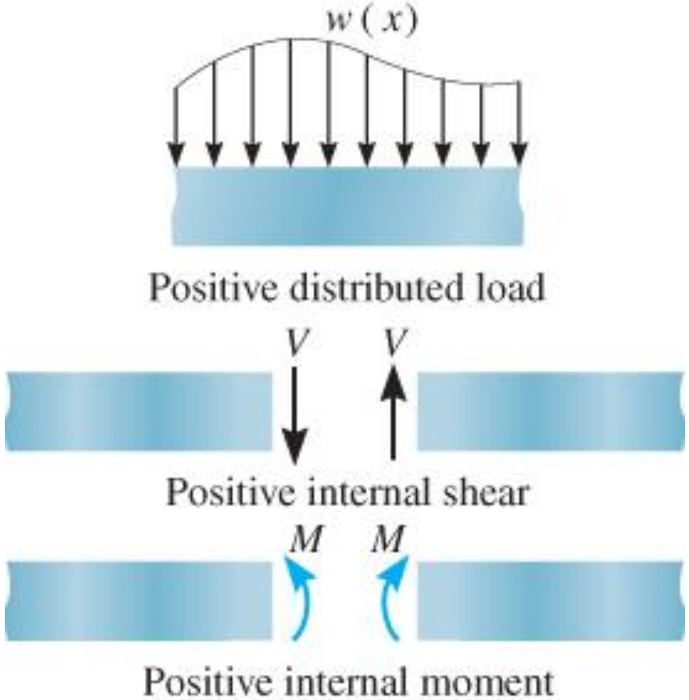
6.1 SHEAR AND MOMENT DIAGRAMS

- Shear and bending-moment functions must be determined for each *region* of the beam *between* any two discontinuities of loading



Beam sign convention

- Although choice of sign convention is arbitrary, in this course, we adopt the one often used by engineers:

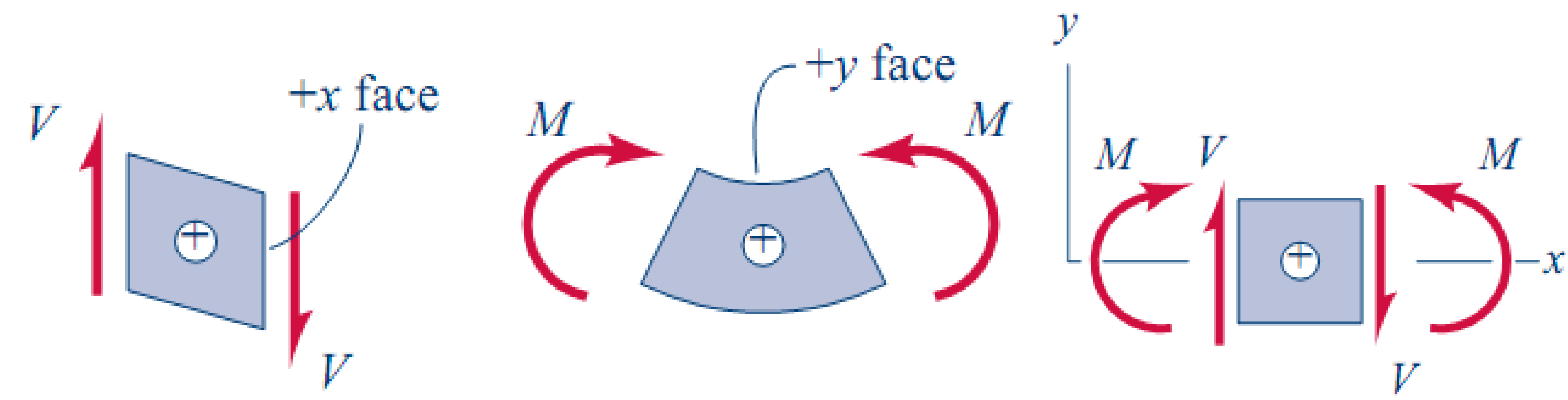


Beam sign convention

6.1 SHEAR AND MOMENT DIAGRAMS

SIGN CONVENTION

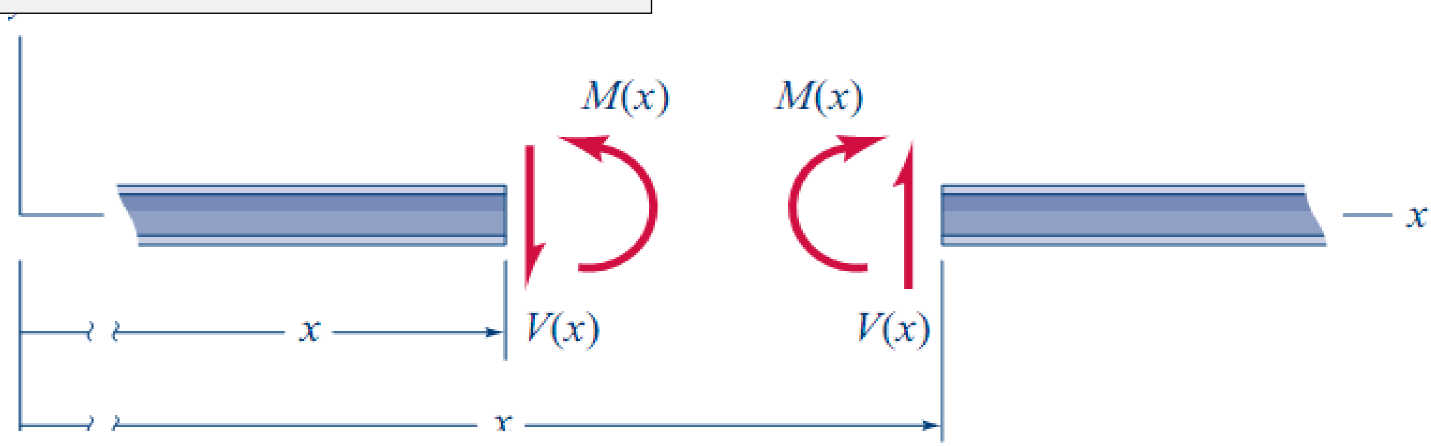
The positive directions are as follows: the internal shear force causes a **clockwise rotation** of the beam segment on which it acts; and the internal moment causes **compression in the top fibers** of the segment such that it bends the segment such that it holds water

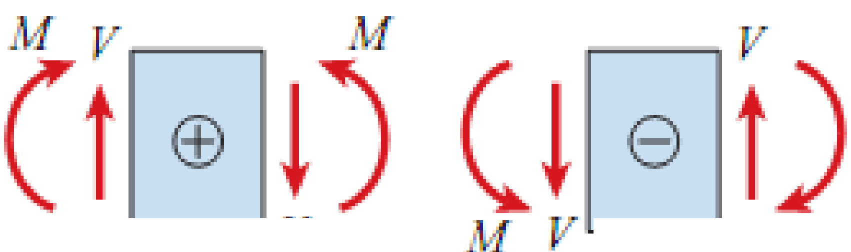


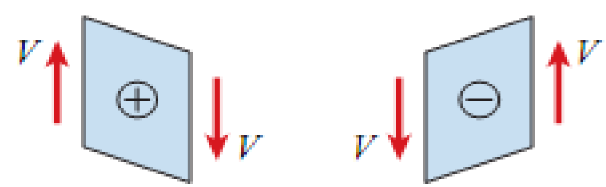
(b) Positive shear. (c) Positive moment. (d) Positive V and M.

6.1 SHEAR AND MOMENT DIAGRAMS

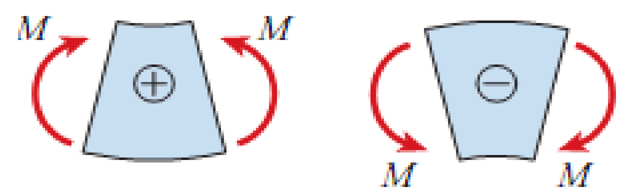
SIGN CONVENTION







(a)

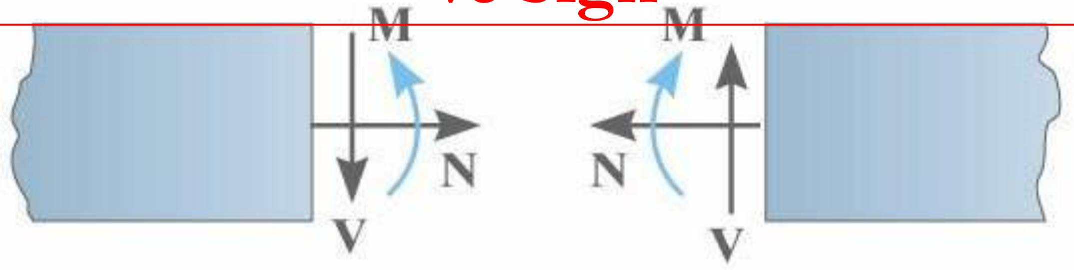


(b)

"Happy" Beam is +VE **"Sad" Beam is -VE**

Sign Convention

+ve Sign



6.1 SHEAR AND MOMENT DIAGRAMS

IMPORTANT

- Beams are long straight members that carry loads perpendicular to their longitudinal axis. They are classified according to how they are supported
- To design a beam, we need to know the variation of the shear and moment along its axis in order to find the points where they are maximum
- Establishing a sign convention for positive shear and moment will allow us to draw the shear and moment diagrams

6.1 SHEAR AND MOMENT DIAGRAMS

Procedure for analysis

Support reactions

- Determine all reactive forces and couple moments acting on beam
- Resolve all forces into components acting perpendicular and parallel to beam's axis
- Free-Body Diagram
- Equation of Equilibrium

Shear and moment functions

- Specify separate coordinates x having an origin at beam's left end, and extending to regions of beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading

Procedure for analysis

Shear and moment functions

- Section beam perpendicular to its axis at each distance x
- Draw free-body diagram of one segment
- Make sure **V** and **M** are shown acting in positive sense, according to sign convention
- Sum forces perpendicular to beam's axis to get shear
- Sum moments about the sectioned end of segment to get moment

6.1 SHEAR AND MOMENT DIAGRAMS

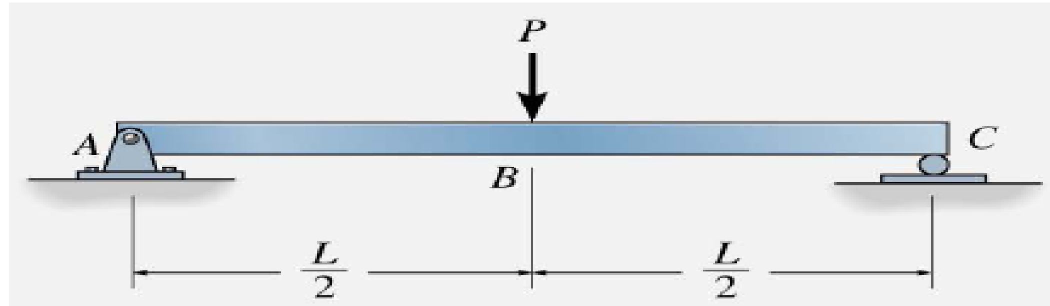
Procedure for analysis

Shear and moment diagrams

- Plot shear diagram (**V** vs. x) and moment diagram (**M** vs. x)
- If numerical values are positive, values are plotted above axis, otherwise, negative values are plotted below axis
- It is convenient to show the shear and moment diagrams directly below the free-body diagram

6.1 SHEAR AND MOMENT DIAGRAMS

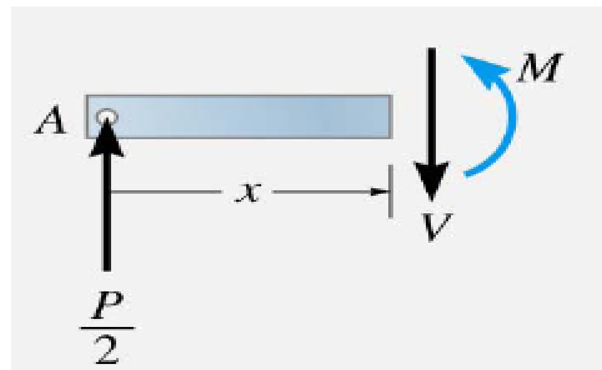
$$Ay = Cy = \frac{p}{2}$$



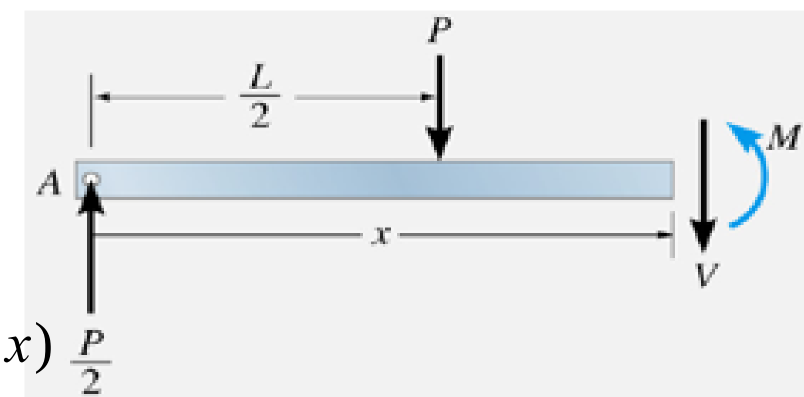
Segment AB 0 ≤ x ≤ L/2

$$\sum fy = 0 \Rightarrow \frac{p}{2} - v = 0 \Rightarrow v = \frac{p}{2}$$

$$\sum M = 0 \Rightarrow \frac{p}{2}x - M = 0 \Rightarrow M = \frac{p}{2}x$$



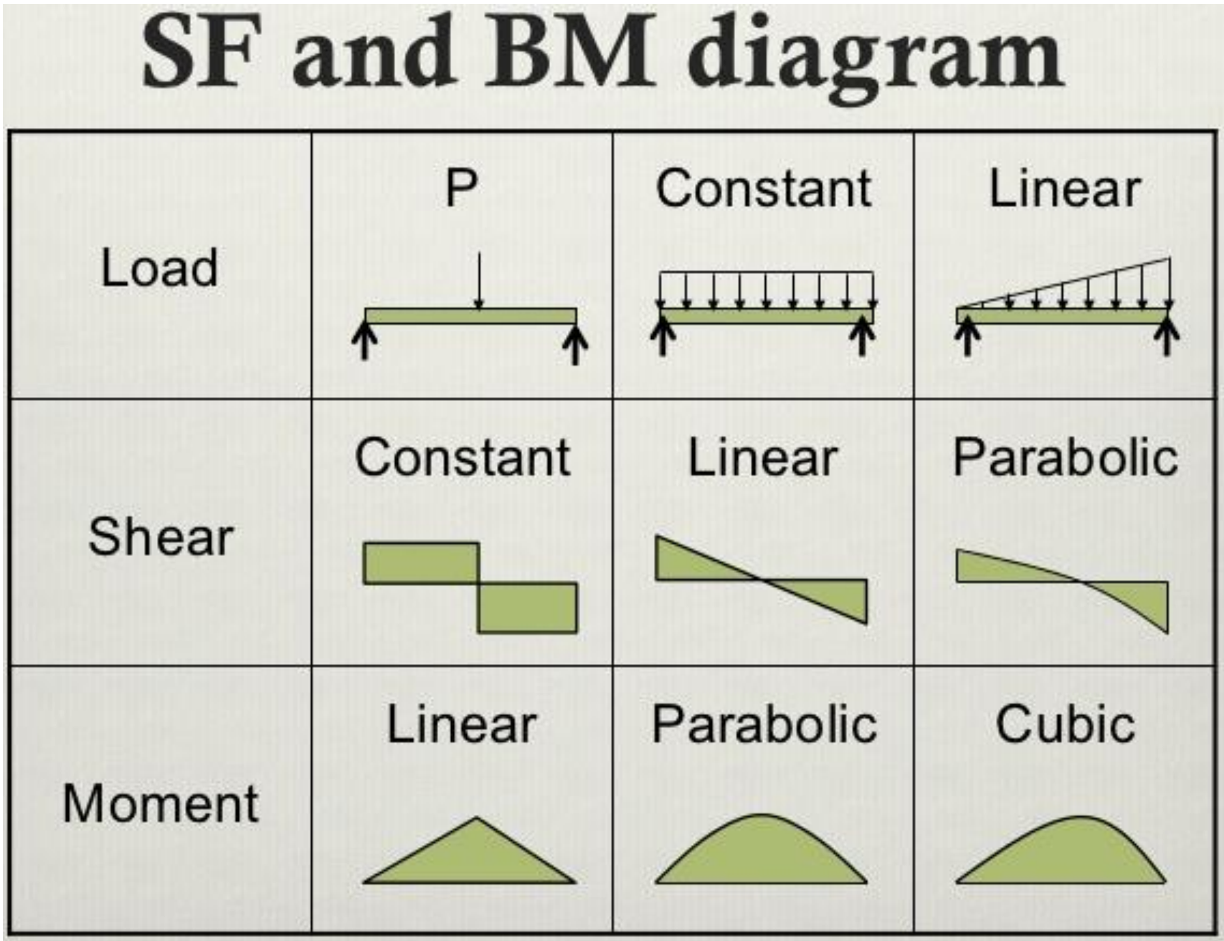
Segment BC L/2 ≤ x ≤ L



$$\sum fy = 0 \Rightarrow \frac{p}{2} - p - v = 0 \Rightarrow v = -\frac{p}{2}$$

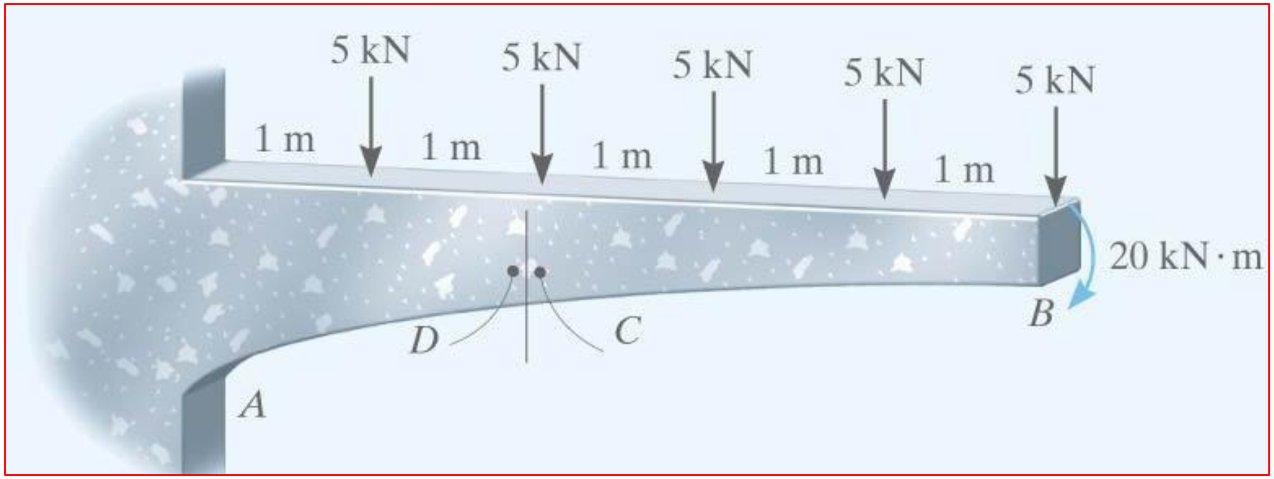
$$\sum M = 0 \Rightarrow \frac{p}{2}x - M - p(x - \frac{L}{2}) = 0 \Rightarrow M = \frac{p}{2}(L - x)$$

COMMON RELATIONSHIPS

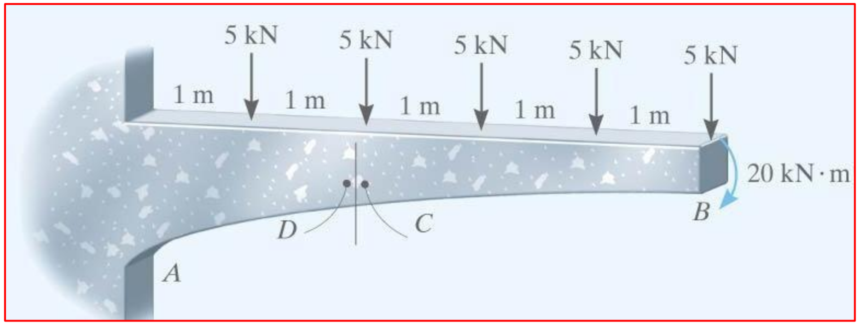


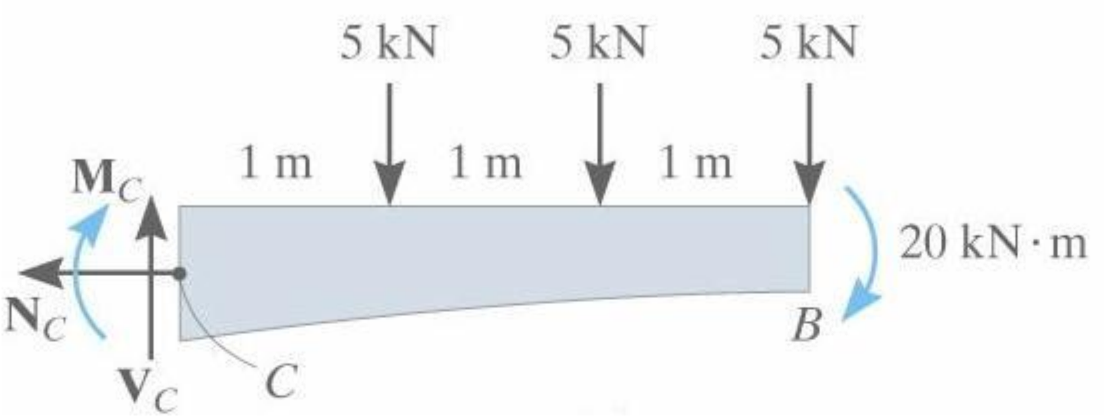
EXAMPLE 6.1

Determine the internal shear and moment acting in the cantilever beam shown in figure at sections passing through points C & D



EXAMPLE 6.1 (Cont.)





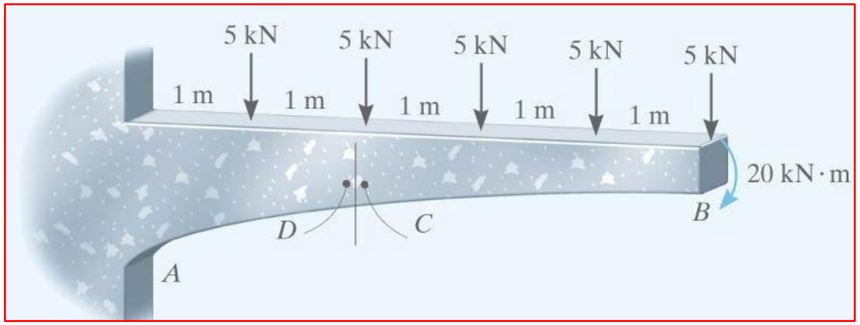
$$\sum F_y = 0 \quad \Rightarrow -V_C - 5 - 5 - 5 = 0$$

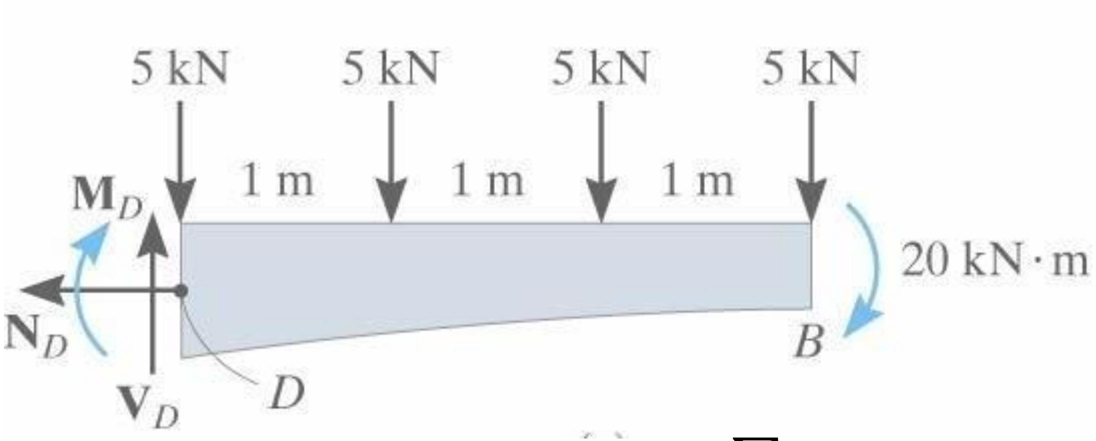
$$V_C = 15kN$$

$$\sum M_C = 0 \quad \Rightarrow -M_c - 5(1) - 5(2) - 5(3) - 20 = 0$$

$$M_c = -50kN.m$$

EXAMPLE 6.1 (Cont.)





$$\sum F_y = 0 \quad \Rightarrow V_D - 5 - 5 - 5 - 5 = 0$$

$$V_C = 20kN$$

$$\sum M_C = 0 \quad \Rightarrow -M_D - 5(1) - 5(2) - 5(3) - 20 = 0$$

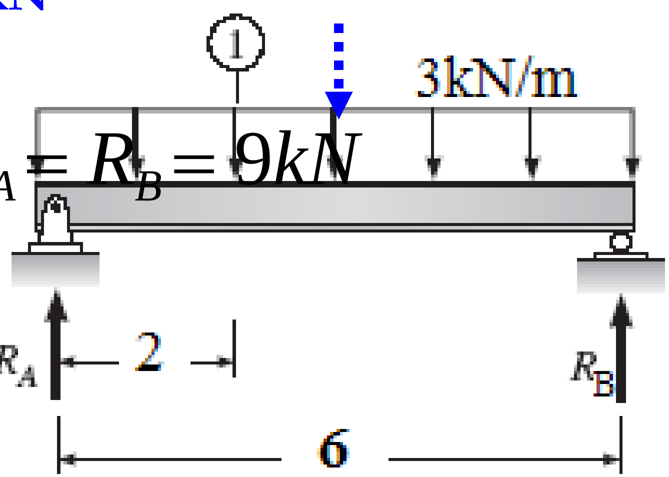
$$M_D = -50kN.m$$

EXAMPLE 6.2

Determine the internal shear and moment acting in section 1 in the beam as shown in figure

18kN

$$R_A = R_B = 9kN$$

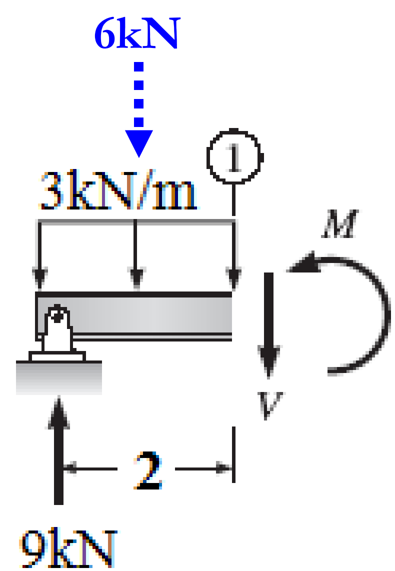


$$\sum F_y = 0 \quad \Rightarrow -V + 9 - 6 = 0$$

$$V = 3kN$$

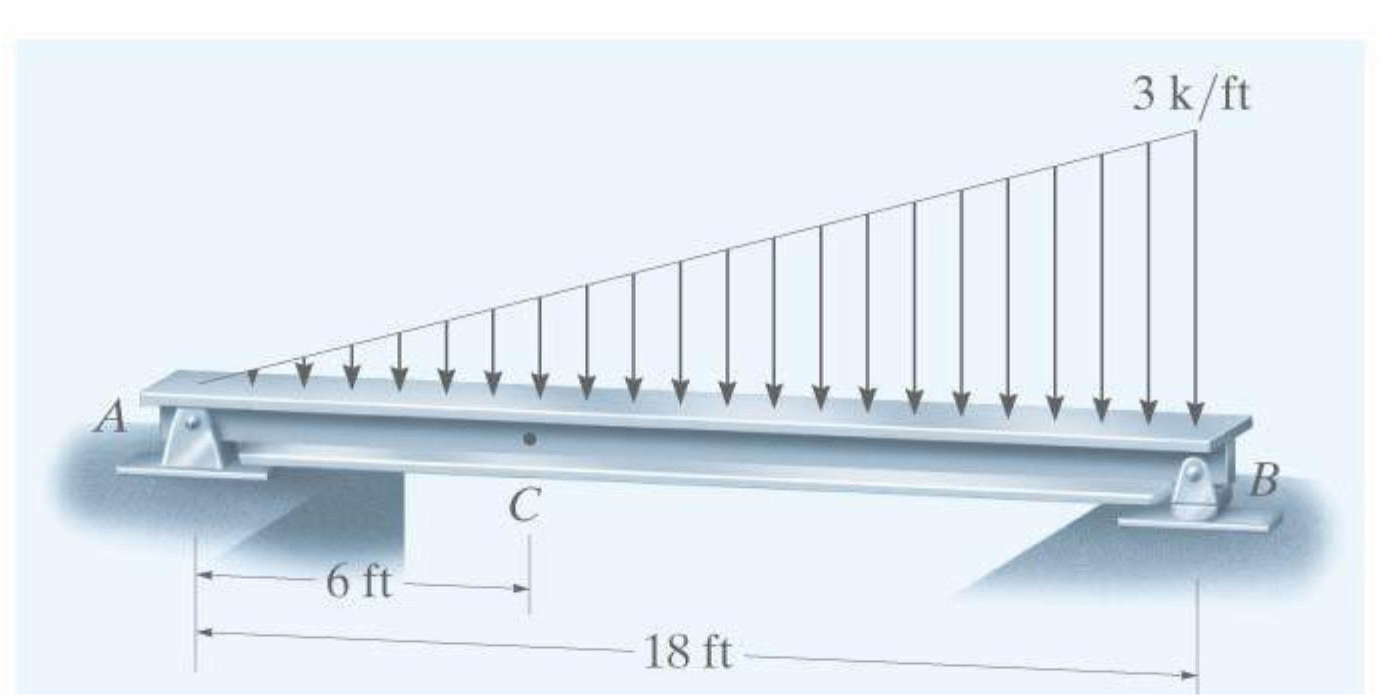
$$\sum M_{\text{at section}} = 0 \quad \Rightarrow M + 6(1) - 9(2) = 0$$

$$M_D = 12kN.m$$

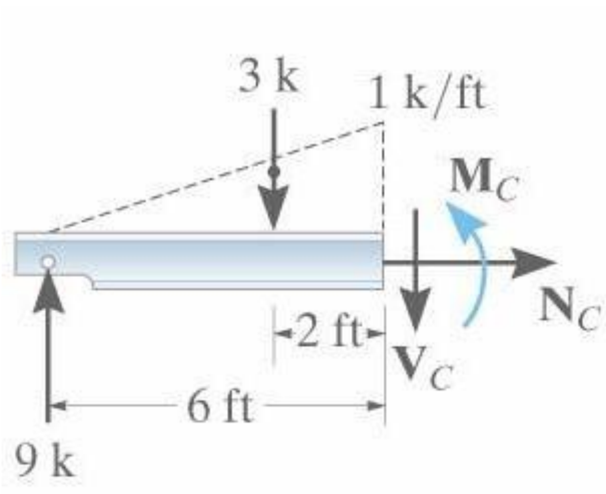


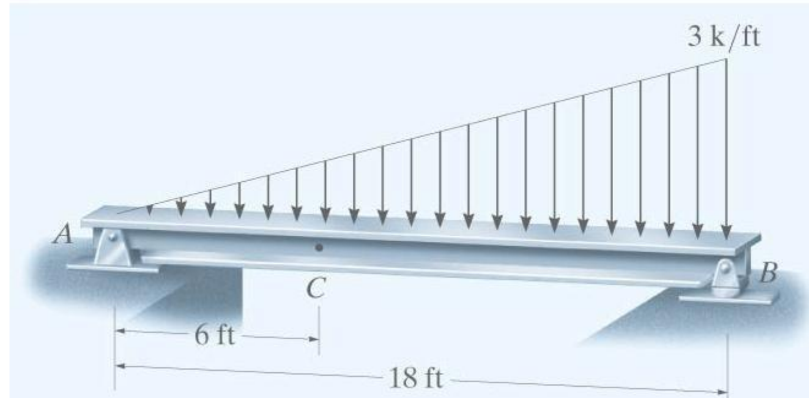
EXAMPLE 6.3

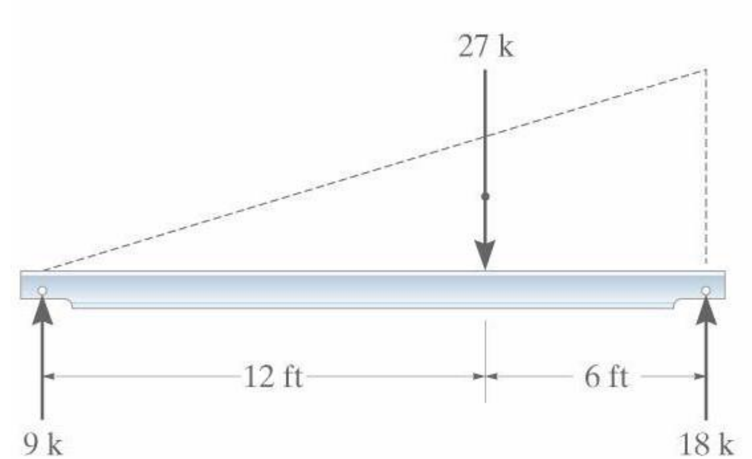
Determine the internal shear and moment acting in the cantilever beam shown in figure at sections passing through points C



EXAMPLE 6.3 (Cont.)







$$\sum F_y = 0 \quad \Rightarrow -V_C + 9 - 3 = 0$$

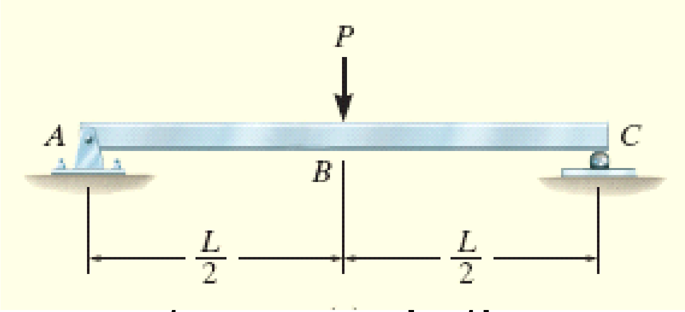
$$V = 6k$$

$$\sum M_c = 0 \quad \Rightarrow M_c + 3(2) - 9(6) = 0$$

$$M_D = 48k.\ ft$$

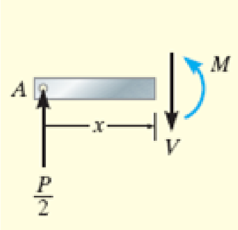
EXAMPLE 6.4

Draw the shear and moment diagrams for the beam shown.



Solution:

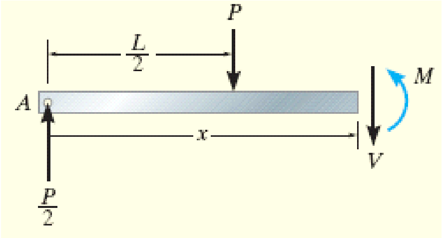
From the free-body diagram of the left segment, we apply the equilibrium equations,



$$+\uparrow \sum F_y = 0; \quad V = \frac{P}{2} \qquad (1)$$

$$+\uparrow \sum M = 0; \quad M = \frac{P}{2}x \qquad (2)$$

Left segment of the beam extending a distance *x* within region *BC* is as follow.



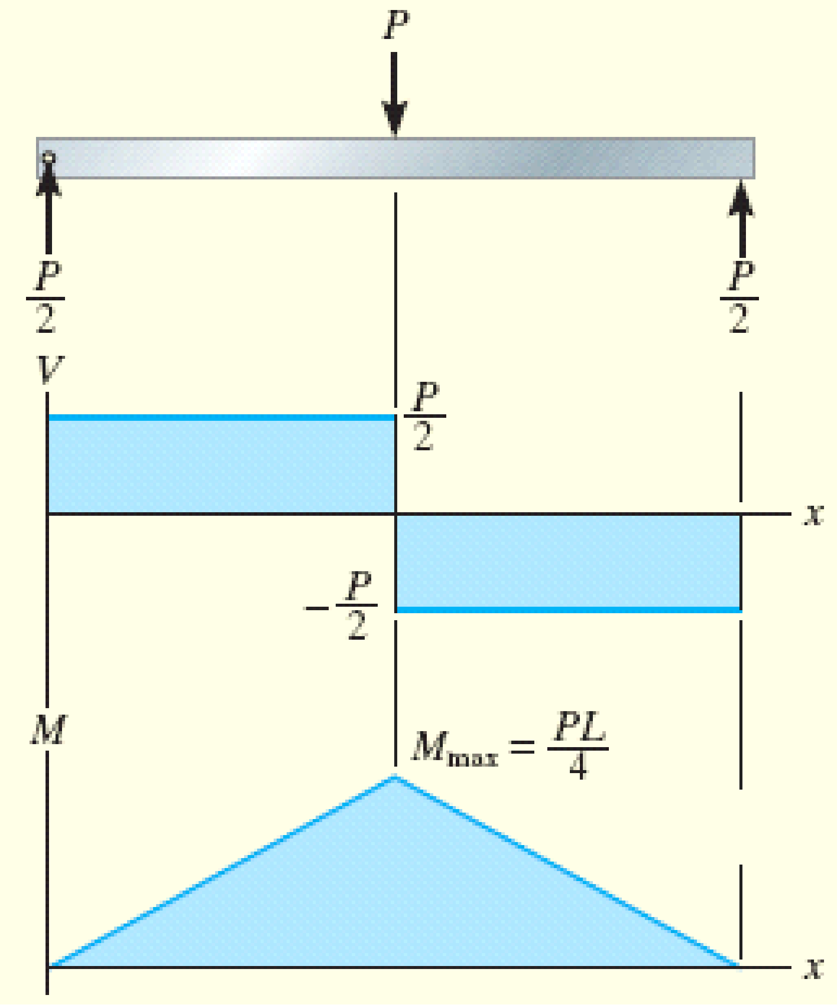
$$\uparrow \sum F_y = 0; \quad \frac{P}{2} - P - V = 0 \Rightarrow V = -\frac{P}{2} \qquad (3)$$

$$\curvearrowleft \sum F_y = 0; \quad M + P\left(x - \frac{L}{2}\right) - \frac{P}{2}x \Rightarrow M = \frac{P}{2}(L\text{-}x) \qquad (4)$$

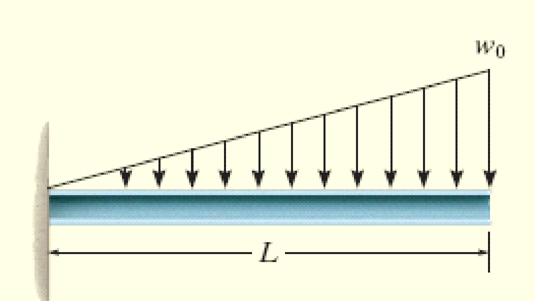
EXAMPLE 6.4 (Cont.)

The shear diagram represents a plot of Eqs. 1 and 3 ➔

The moment diagram represents a plot of Eqs. 2 and 4 ➔



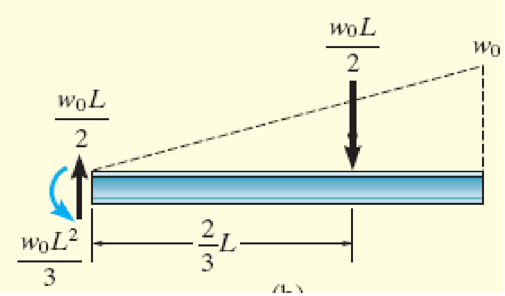
EXAMPLE 6.5



Draw the shear and moment diagrams for the beam shown

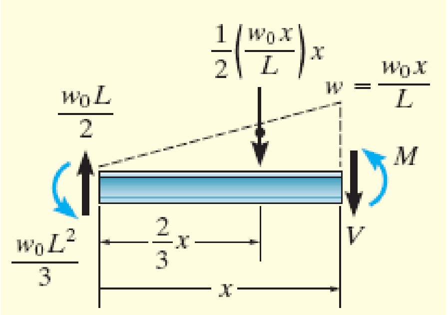
Solution:

The distributed load is replaced by its resultant force and the reactions.



Intensity of the triangular load at the section is found by proportion, $w/x = w_0/L$ or $w = w_0 x/L$

Resultant of the distributed loading is determined from the area under the diagram,



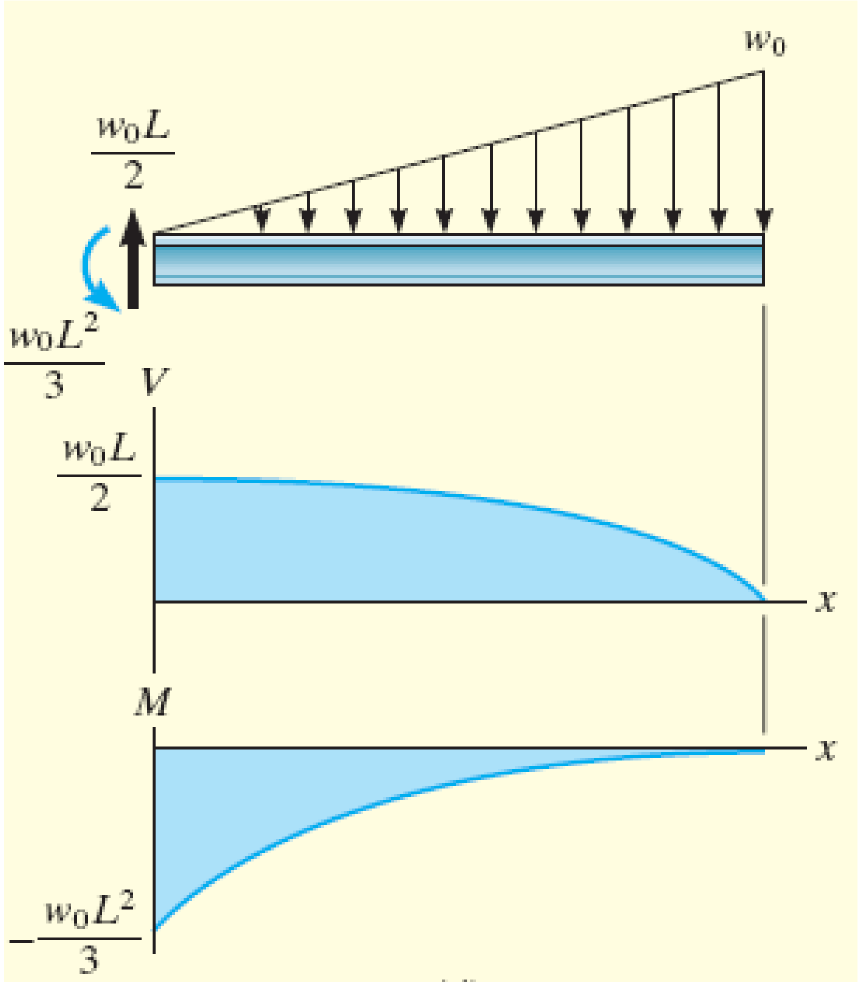
$$+\uparrow \sum F_y = 0; \quad \frac{w_0 L}{2} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)x - V = 0 \Rightarrow V = \frac{w_0}{2L}\left(L^2 - x^2\right) \quad (1)$$

$$+\sum M = 0; \quad \frac{w_0 L^2}{3} - \frac{w_0 L}{2}(x) + \frac{1}{2}\left(\frac{w_0 x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0 \quad (2)$$

EXAMPLE 6.5 (Cont.)

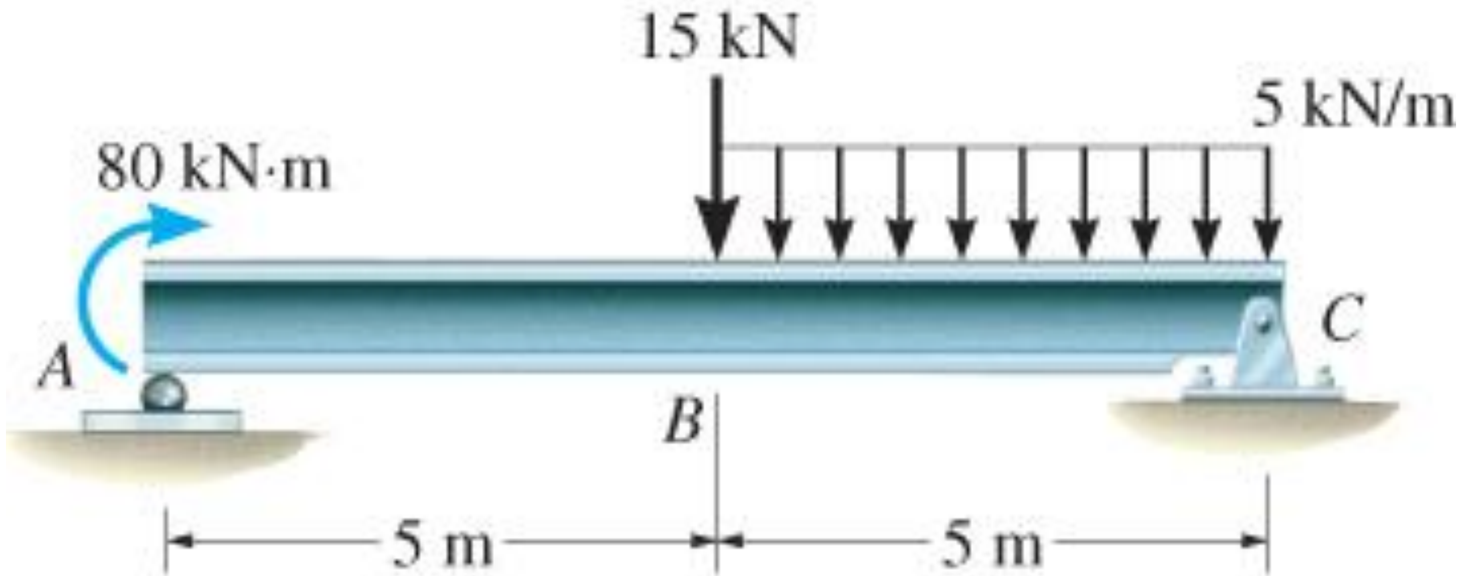
The shear diagram represents a plot of Eqs. 1 ➔

The shear diagram represents a plot of Eqs. 1 ➔



EXAMPLE 6.6

Draw the shear and moment diagrams for beam shown below.



(a)

EXAMPLE 6.6 (Cont.)

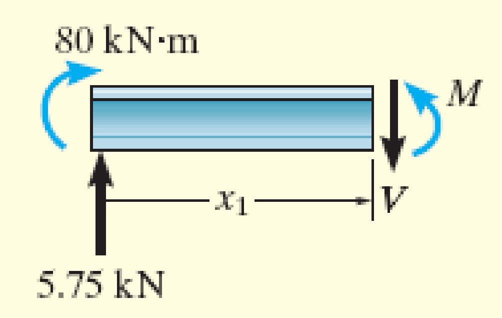
Solution:

2 regions of *x* must be considered in order to describe the shear and moment functions for the entire beam.

$0 \le x_1 < 5\,\text{m},$

$+\uparrow \sum F_y = 0; \quad 5.75 - V = 0 \Rightarrow V = 5.75\,\text{kN} \quad (1)$

$+\sum M = 0; \quad -80 - 5.75x_1 + M = 0 \Rightarrow M = (5.75x_1 + 80)\,\text{kNm} \quad (2)$

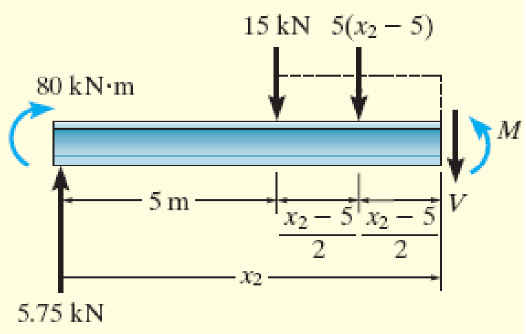


$5\,\text{m} \le x_1 < 10\,\text{m},$

$+\uparrow \sum F_y = 0; \quad 5.75 - 15 - 5(x_2 - 5) - V = 0 \Rightarrow V = (15.75 - 5x_2)\,\text{kN} \; (3)$

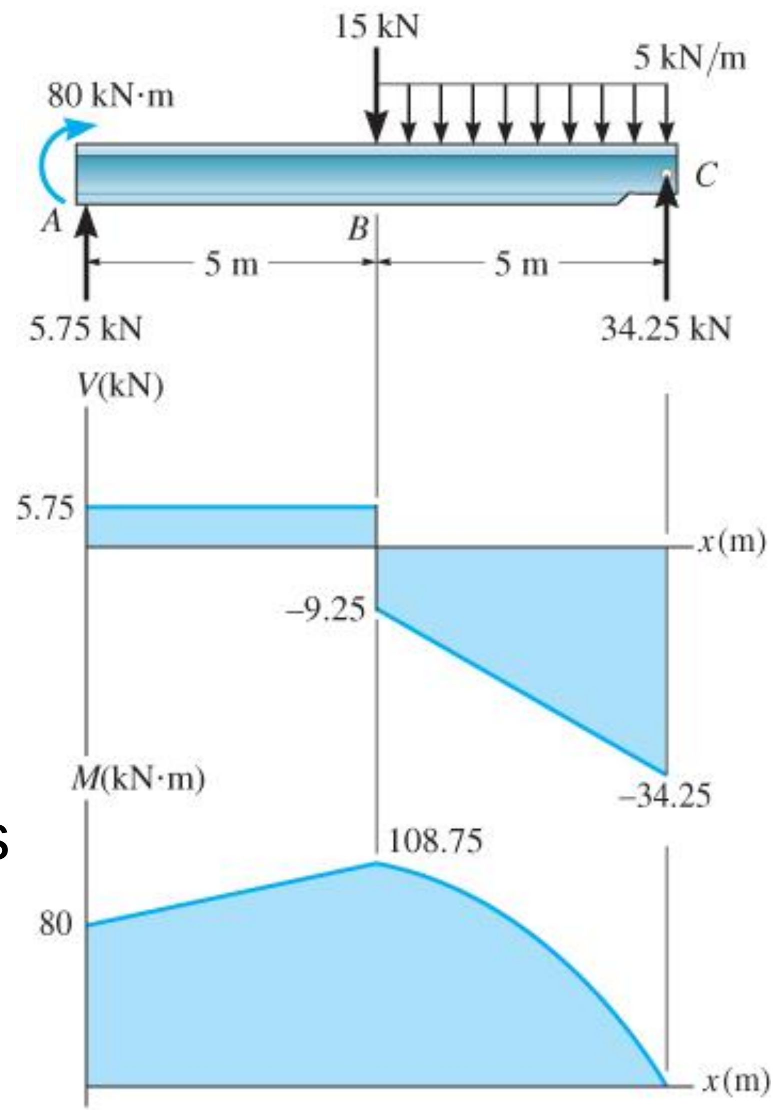
$+\sum M = 0; \quad -80 - 5.75x_1 + +15 + 5(x_2 - 5)\left(\frac{x_2 - 5}{2}\right) + M = 0$

$M = (-2.5x_2^2 + 15.75x_2 + 92.5)\,\text{kNm} \quad (4)$



EXAMPLE 6.6 (Cont.)

Shear and moment diagrams



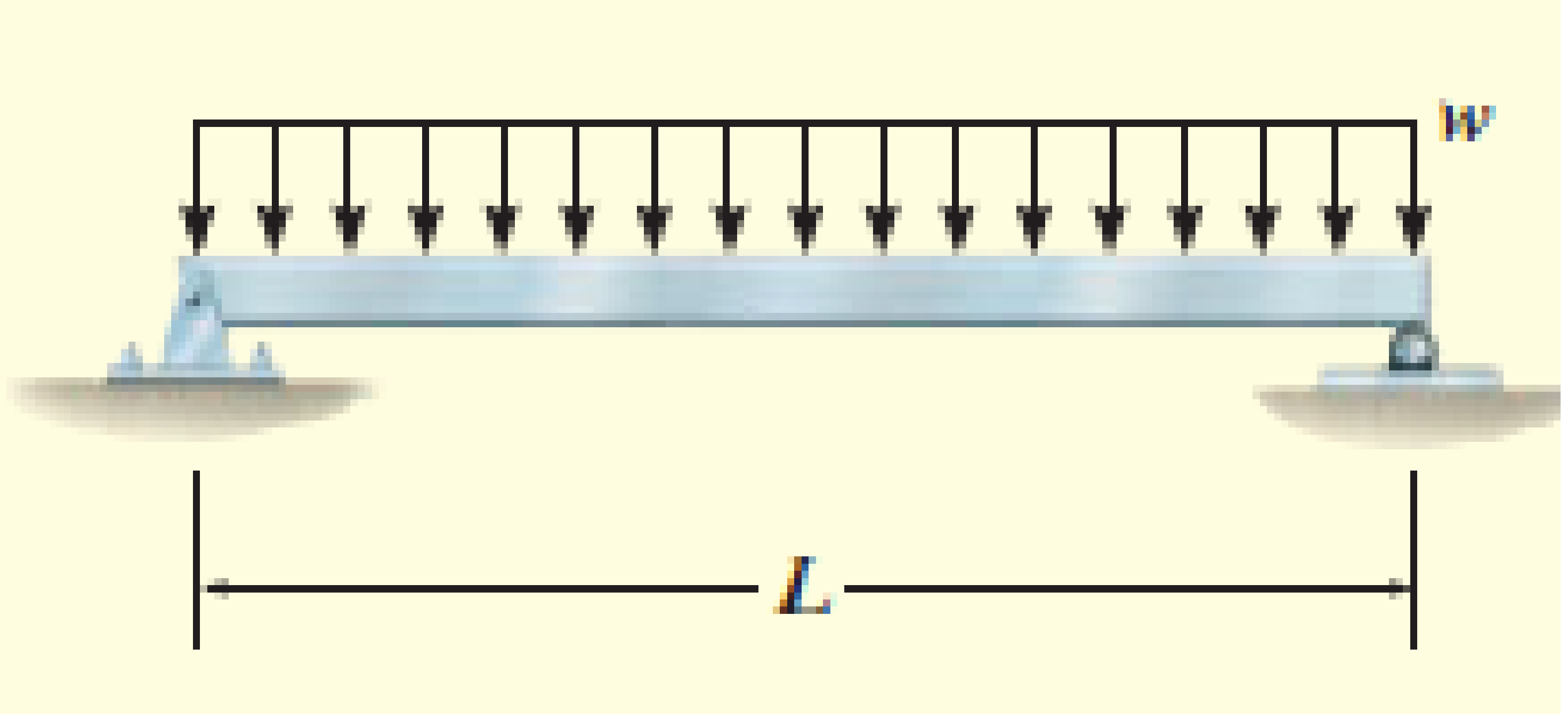
(d)

The shear diagram represents a plot of Eqs. 1 and 3 ➔

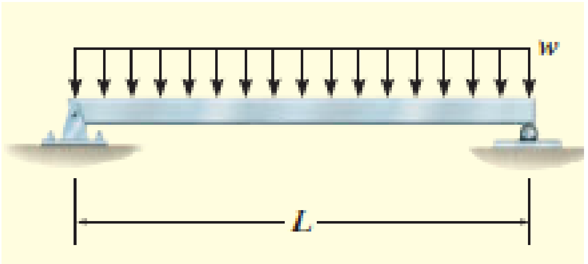
The moment diagram represents a plot of Eqs. 2 and 4 ➔

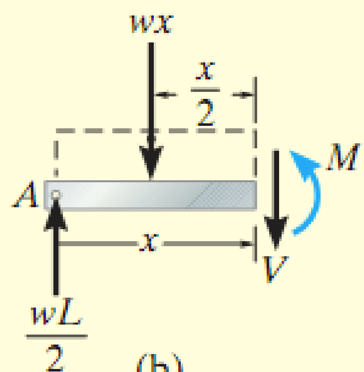
EXAMPLE 6.7

Draw the shear and moment diagrams for beam shown below.



EXAMPLE 6.7 (Cont.)





(b)

$$\sum fy = 0 \Rightarrow \frac{wL}{2} - wx - v = 0$$

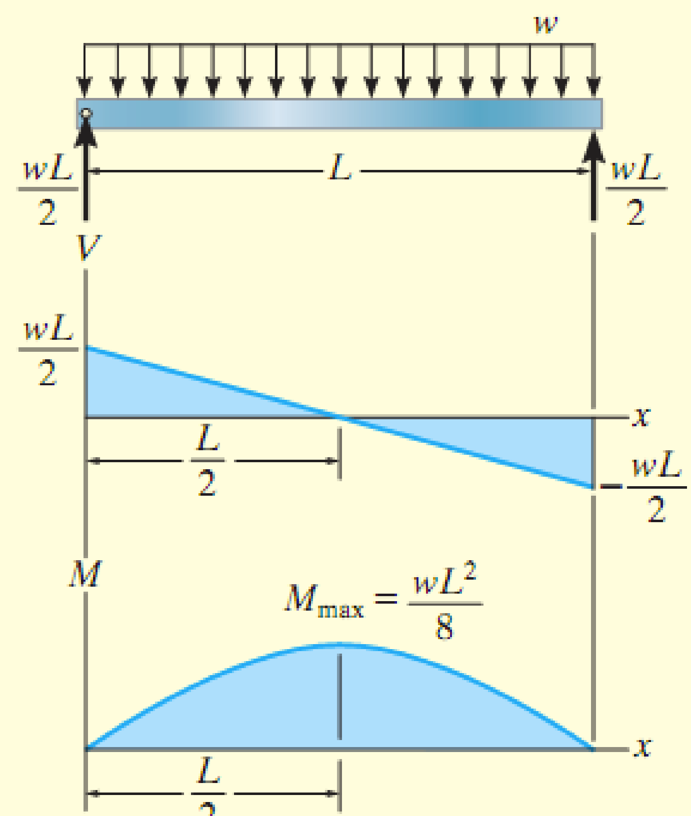
$$v = w(\frac{L}{2} - x)$$

$$\sum M = 0 \Rightarrow (\frac{wL}{2})x - (wx)(\frac{x}{2}) - M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

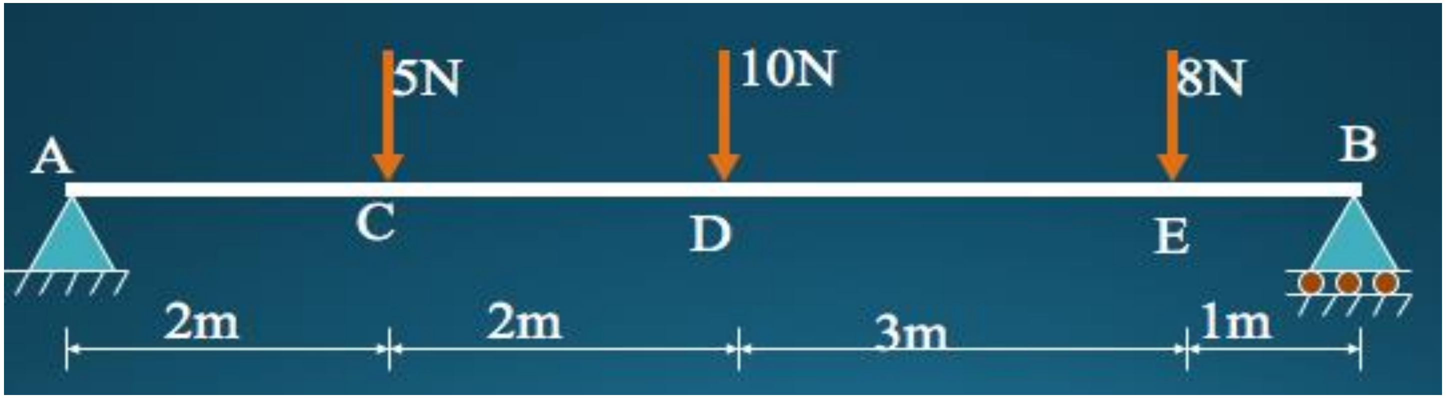
$$v = w(\frac{L}{2} - x) = 0 \Rightarrow x = \frac{L}{2}$$

$$M_{\max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right] = \frac{wL^2}{8}$$



EXAMPLE 6.8

Draw shear force and bending moment diagrams [SFD and BMD] for a simply supported beam subjected to three point loads as shown in the Fig. given below.



Solution:

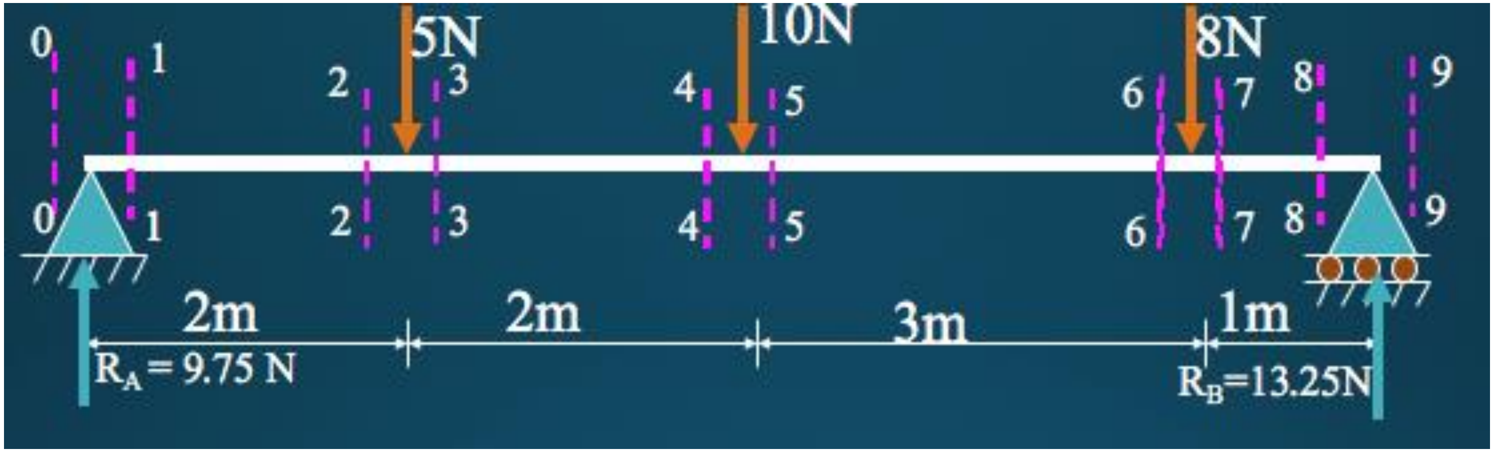
Using the condition: $\Sigma M_A = 0$

$- R_B \times 8 + 8 \times 7 + 10 \times 4 + 5 \times 2 = 0 \rightarrow R_B = 13.25\ N$

Using the condition: $\Sigma F_y = 0$

$R_A + 13.25 = 5 + 10 + 8 \rightarrow R_A = 9.75\ N$

EXAMPLE 6.8 (Cont.)



Shear Force at the section 1-1 is denoted as V_{1-1}

Shear Force at the section 2-2 is denoted as V_{2-2} and so on...

$V_{0-0} = 0$; $V_{1-1} = +9.75$ N

$V_{2-2} = +9.75$ N

$V_{3-3} = +9.75 - 5 = 4.75$ N

$V_{4-4} = +4.75$ N

$V_{5-5} = +4.75 - 10 = -5.25$ N

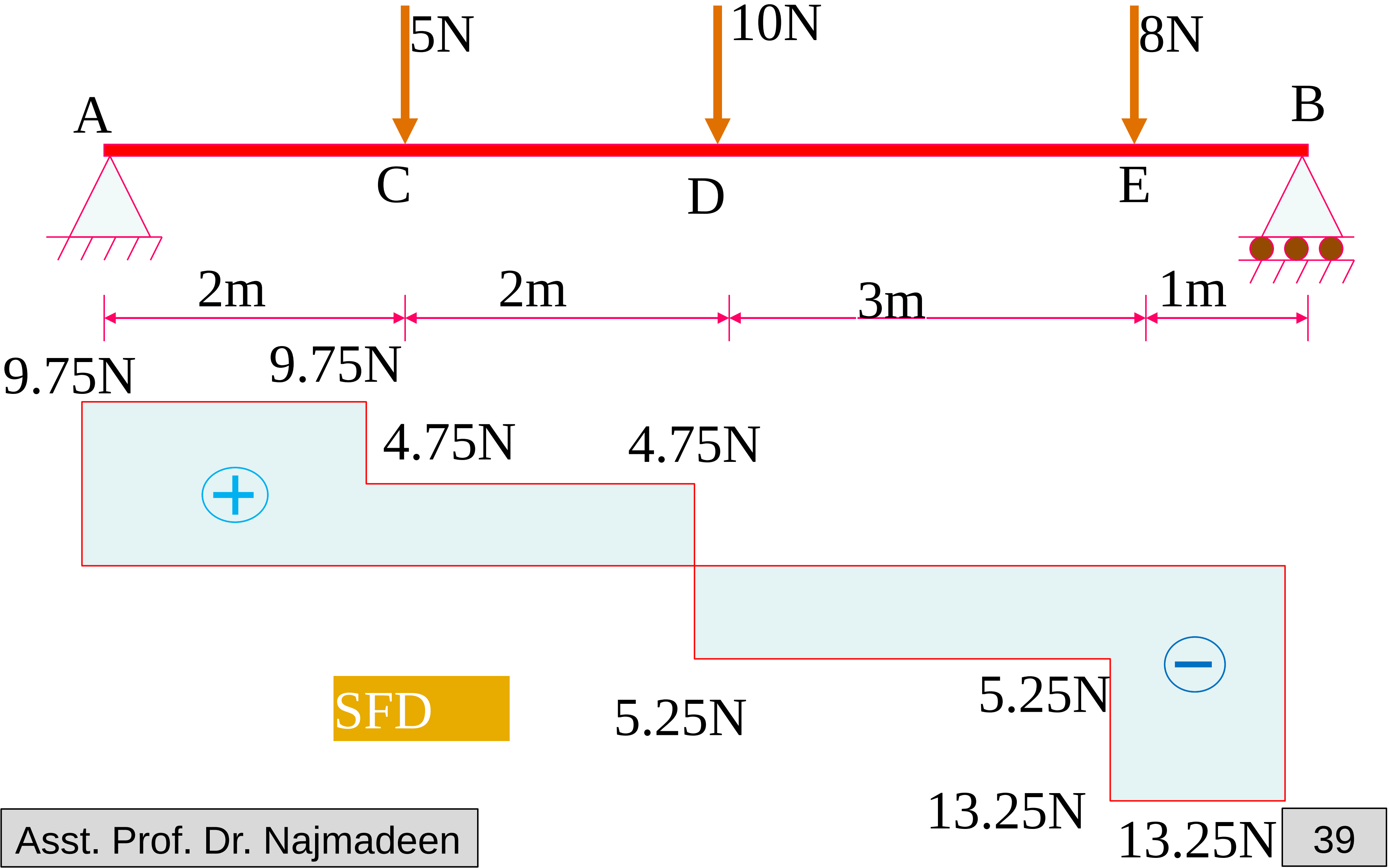
$V_{6-6} = -5.25$ N

$V_{7-7} = 5.25 - 8 = -13.25$ N

$V_{8-8} = -13.25$

$V_{9-9} = -13.25 + 13.25 = 0$

EXAMPLE 6.8 (Cont.)



EXAMPLE 6.8 (Cont.)

Bending moment at A is denoted as M_A

Bending moment at B is denoted as M_B

and so on…

$M_A = 0$ [since it is simply supported]

$M_C = 9.75 \times 2 = 19.5$ Nm

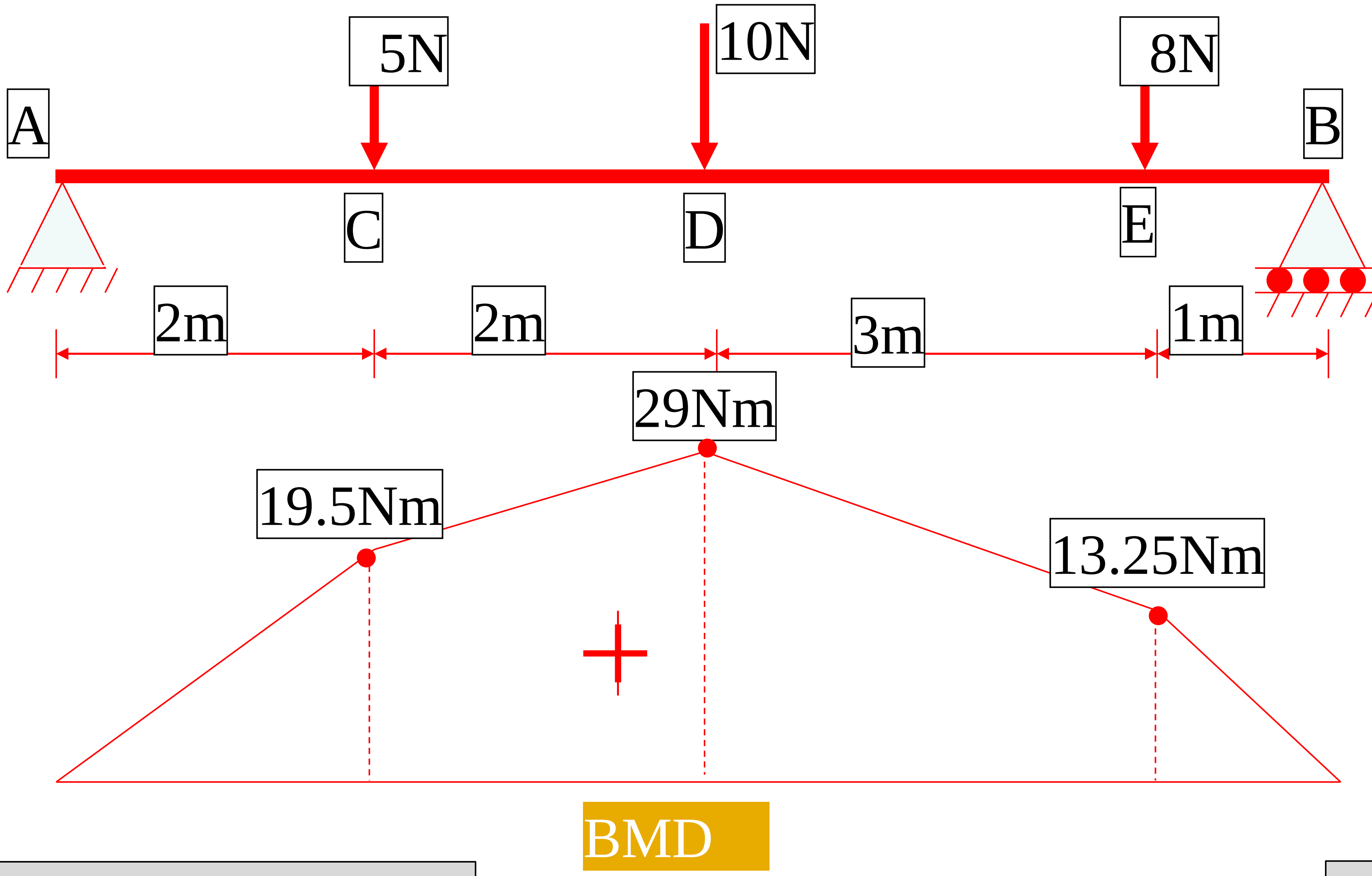
$M_D = 9.75 \times 4 - 5 \times 2 = 29$ Nm

$M_E = 9.75 \times 7 - 5 \times 5 - 10 \times 3 = 13.25$ Nm

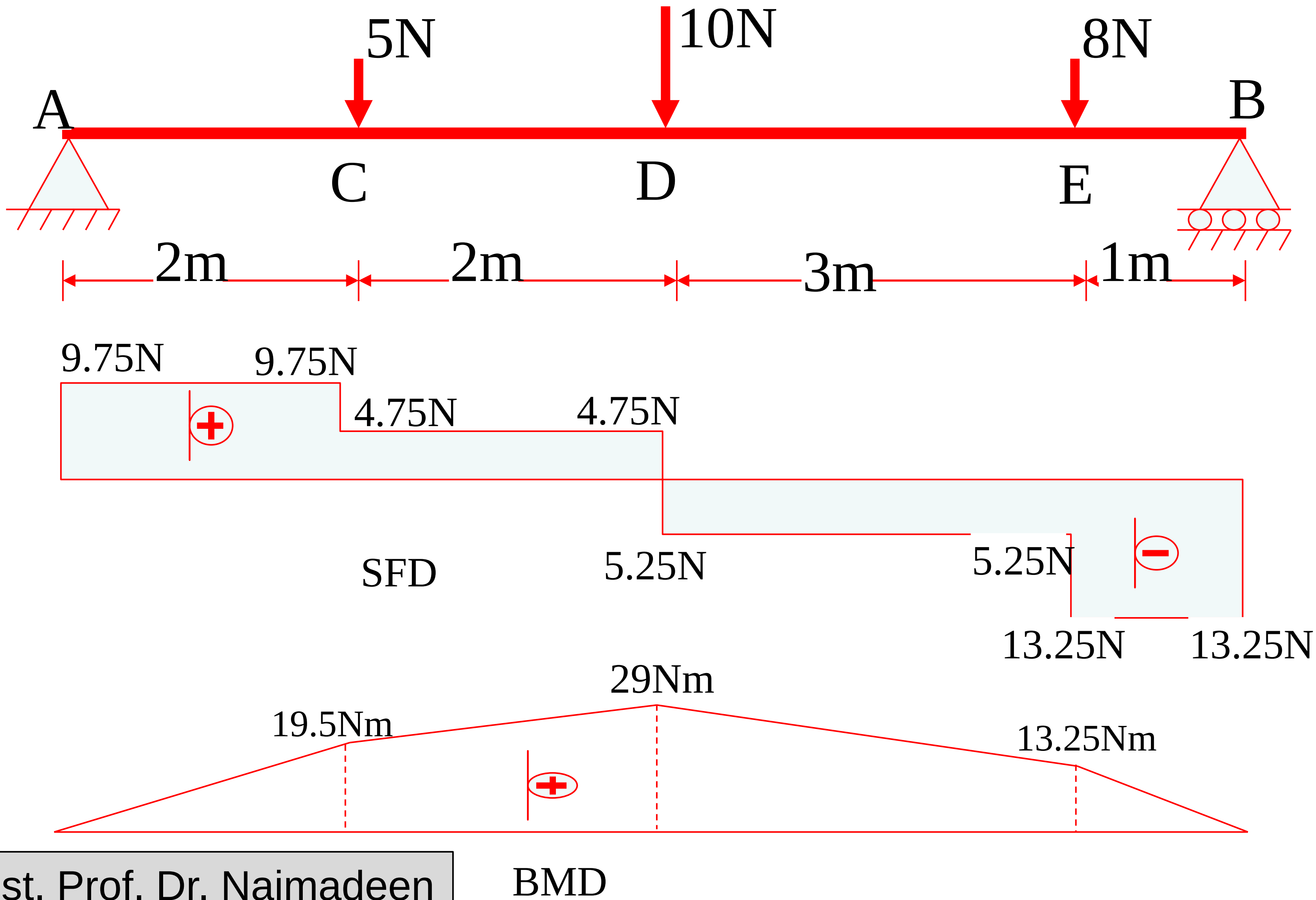
$M_B = 9.75 \times 8 - 5 \times 6 - 10 \times 4 - 8 \times 1 = 0$

or $M_B = 0$ [since it is simply supported]

EXAMPLE 6.8 (Cont.)

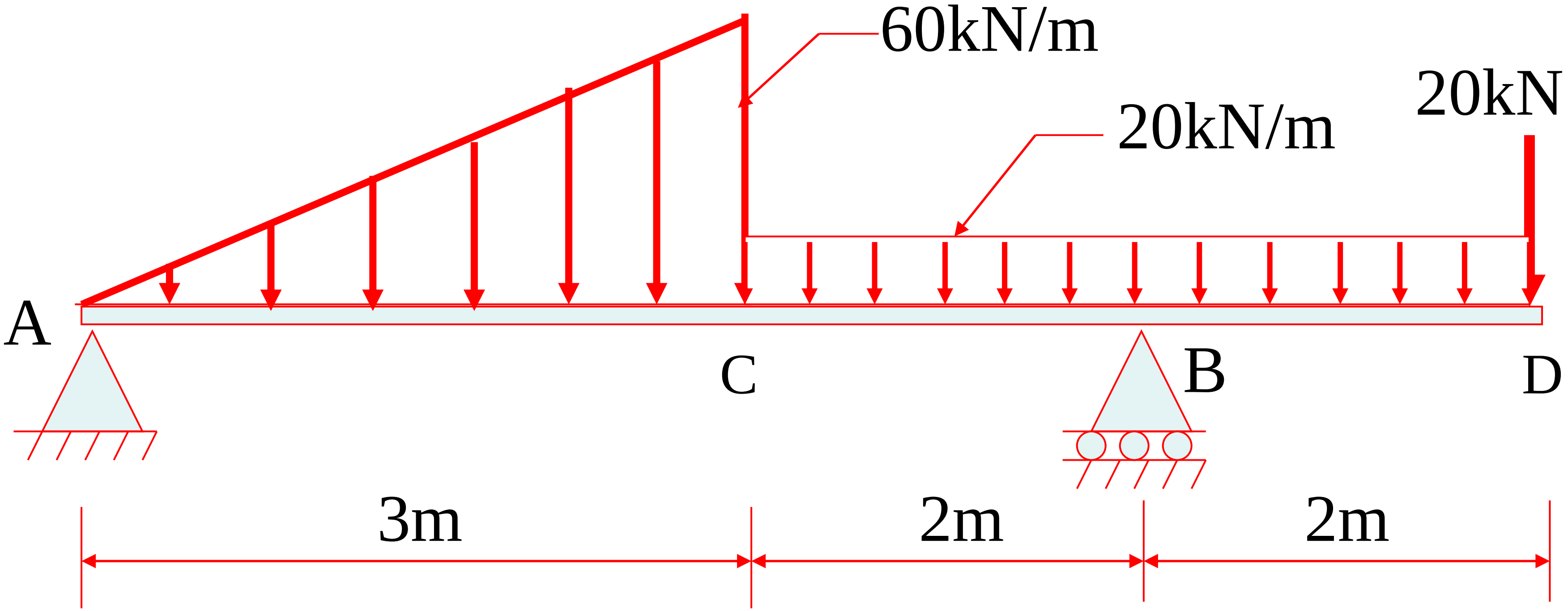


EXAMPLE 6.8 (Cont.)



EXAMPLE 6.9

Draw SFD and BMD for the single side overhanging beam subjected to loading as shown below. Mark salient points on SFD and BMD.



EXAMPLE 6.9 (Cont.)

Solution: Calculation of reactions:

$\Sigma MA = 0$

$-R_B \times 5 + \frac{1}{2} \times 3 \times 60 \times (2/3) \times 3 + 20 \times 4 \times 5 + 20 \times 7 = 0 \rightarrow R_B = 144 kN$

$\Sigma Fy = 0$

$R_A + 144 - \frac{1}{2} \times 3 \times 60 - 20 \times 4 - 20 = 0 \rightarrow R_A = 46 kN$

Shear Force Calculations:

$V_{0-0} = 0$; $V_{1-1} = + 46$ kN

$V_{2-2} = +46 - \frac{1}{2} \times 3 \times 60 = - 44$ kN

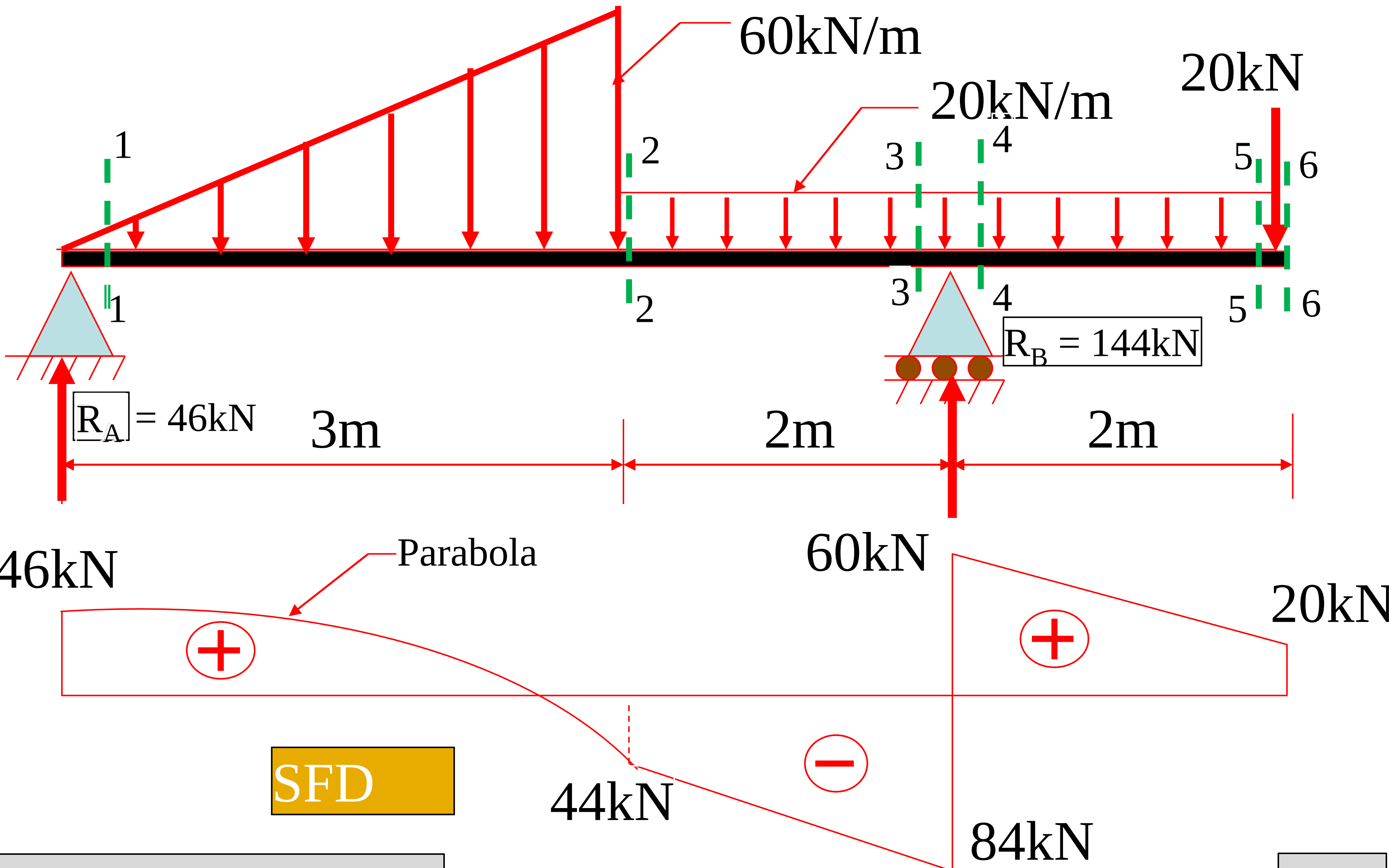
$V_{3-3} = - 44 - 20 \times 2 = - 84$ kN

$V_{4-4} = - 84 + 144 = + 60$ kN

$V_{5-5} = +60 - 20 \times 2 = + 20$ kN

$V_{6-6} = 20 - 20 = 0$ (Check)

EXAMPLE 6.9 (Cont.)



EXAMPLE 6.9 (Cont.)

Max. bending moment will occur at the section where the shear force is zero. The SFD shows that the section having zero shear force is available in the portion AC. Let that section be X-X, considered at a distance 'x' from support A as shown above. The shear force expression at that section should be equated to zero. i.e.,

$$V_{x-x} = 46 - \tfrac{1}{2} .x. (60/3)x = 0 \quad \rightarrow \quad x = 2.145 \text{ m}$$

<u>Calculation of bending moments</u>:

$M_A = M_D = 0$

$M_C = 46 \times 3 - \tfrac{1}{2} \times 3 \times 60 \times (1/3 \times 3) = 48$ kNm[Considering LHS of section]

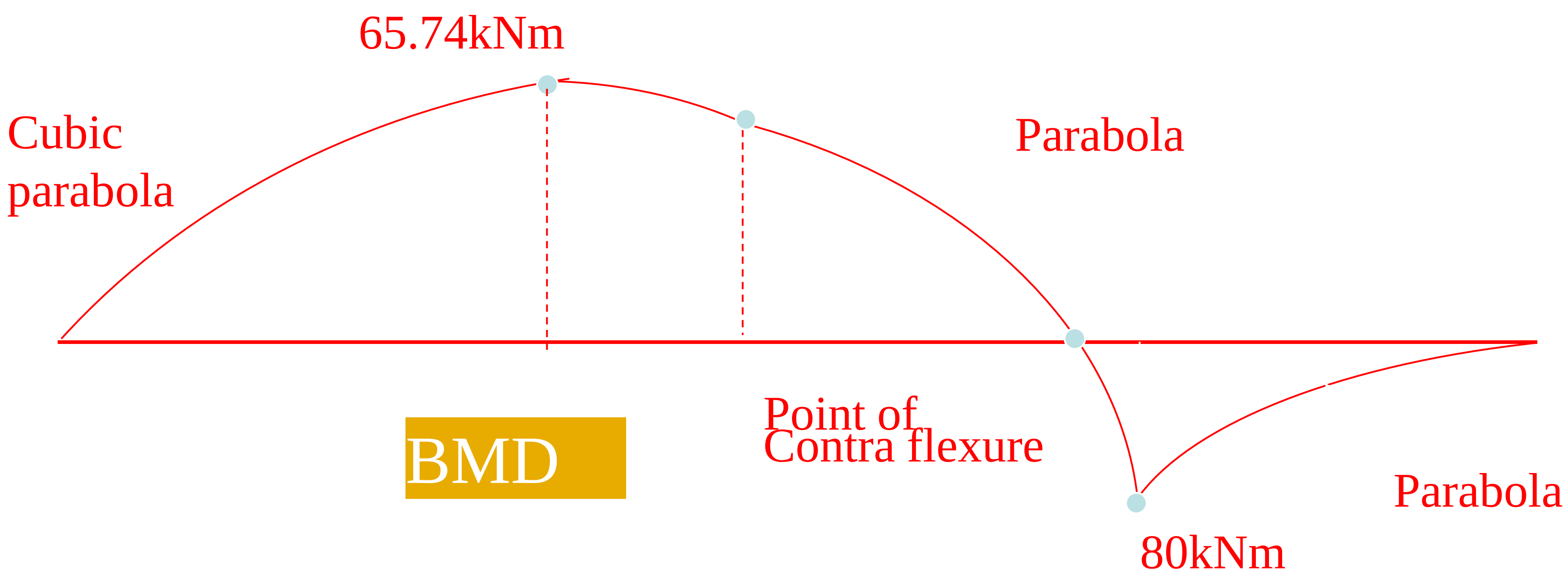
$M_B = -20 \times 2 - 20 \times 2 \times 1 = -80$ kNm [Considering RHS of section]

<u>Absolute Maximum Bending Moment</u>,

$M_{max} = 46 \times 2.145 - \tfrac{1}{2} \times 2.145 \times (2.145 \times 60/3) \times (1/3 \times 2.145)$

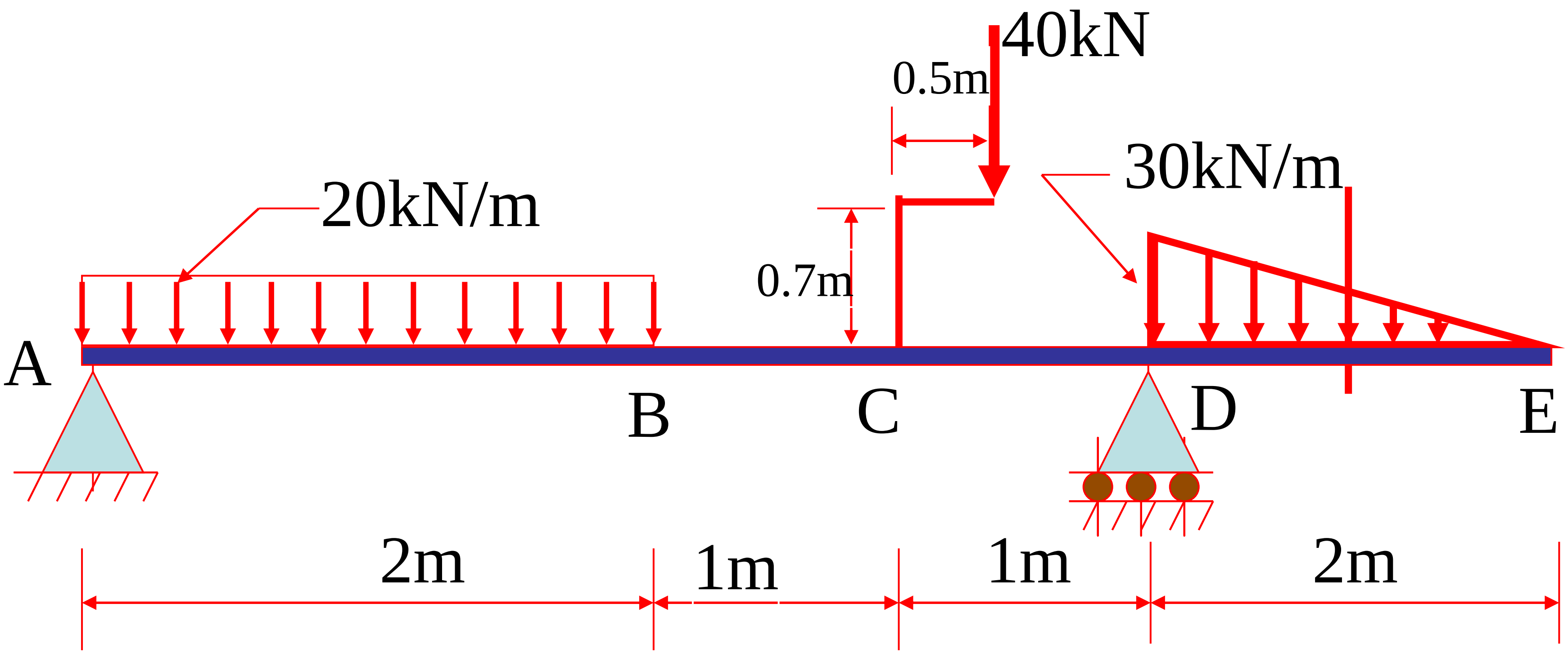
$= 65.74$ kNm

EXAMPLE 6.9 (Cont.)

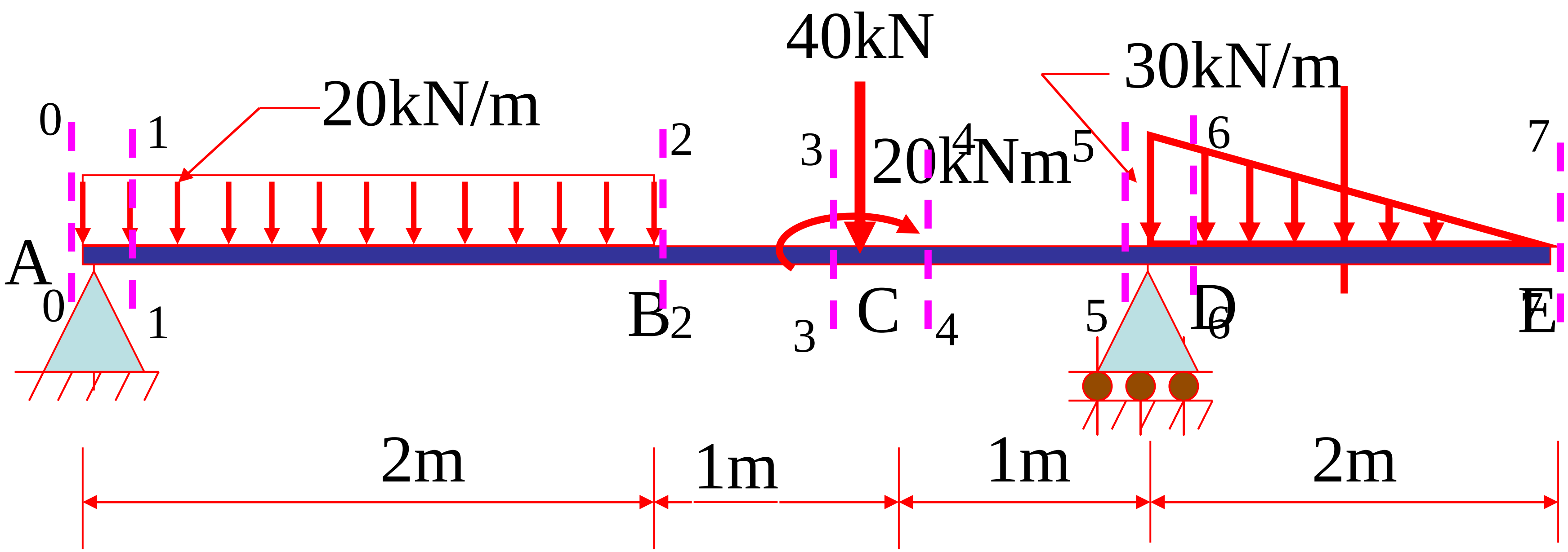


EXAMPLE 6.10

Draw SFD and BMD for the single side overhanging beam subjected to loading as shown below. Mark salient points on SFD and BMD.



EXAMPLE 6.10 (Cont.)



Solution: Calculation of reactions:

$\Sigma M_A = 0$

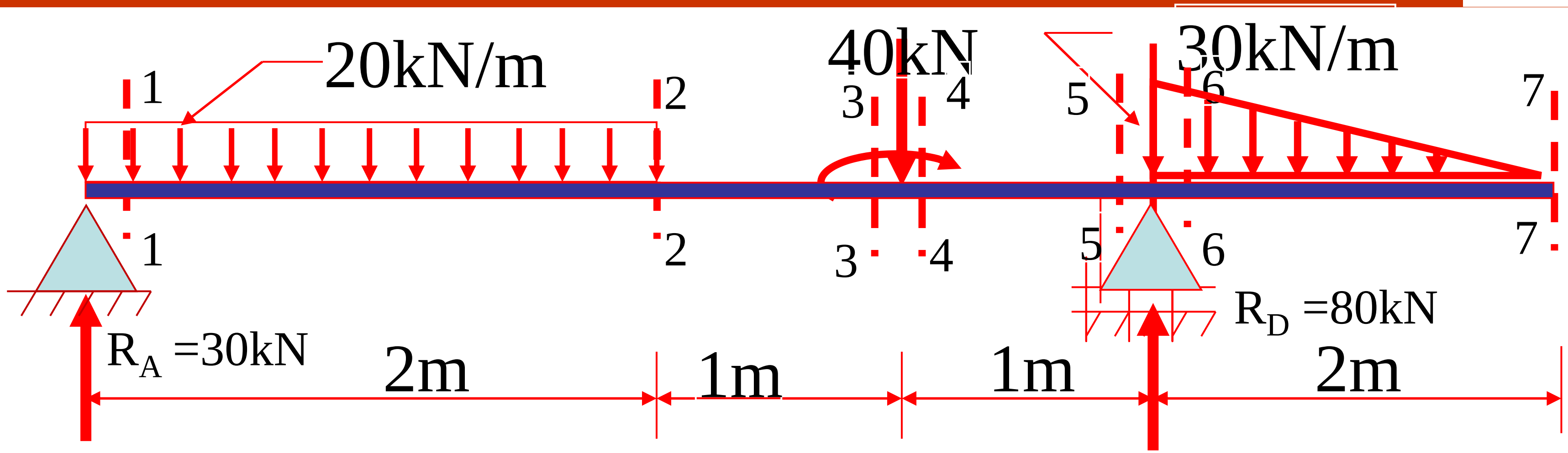
$-R_D \times 4 + 20 \times 2 \times 1 + 40 \times 3 + 20 + \frac{1}{2} \times 2 \times 30 \times (4+2/3) = 0$

$\rightarrow R_D = 80kN$

$\Sigma Fy = 0$

$R_A + 80 - 20 \times 2 - 40 - \frac{1}{2} \times 2 \times 30 = 0 \quad \rightarrow \quad R_A = 30\ kN$

EXAMPLE 6.10 (Cont.)



Calculation of Shear Forces: $V_{0\text{-}0} = 0$

$V_{1\text{-}1} = 30$ kN

$V_{2\text{-}2} = 30 - 20 \times 2 = -10$kN

$V_{3\text{-}3} = -10$kN

$V_{4\text{-}4} = -10 - 40 = -50$ kN

$V_{5\text{-}5} = -50$ kN

$V_{6\text{-}6} = -50 + 80 = +30$kN

$V_{7\text{-}7} = +30 - ½ \times 2 \times 30 = 0$(check)

EXAMPLE 6.10 (Cont.)

Calculation of Shear Forces: $V_{0-0} = 0$

$V_{1-1} = 30$ kN

$V_{2-2} = 30 - 20 \times 2 = -10$kN

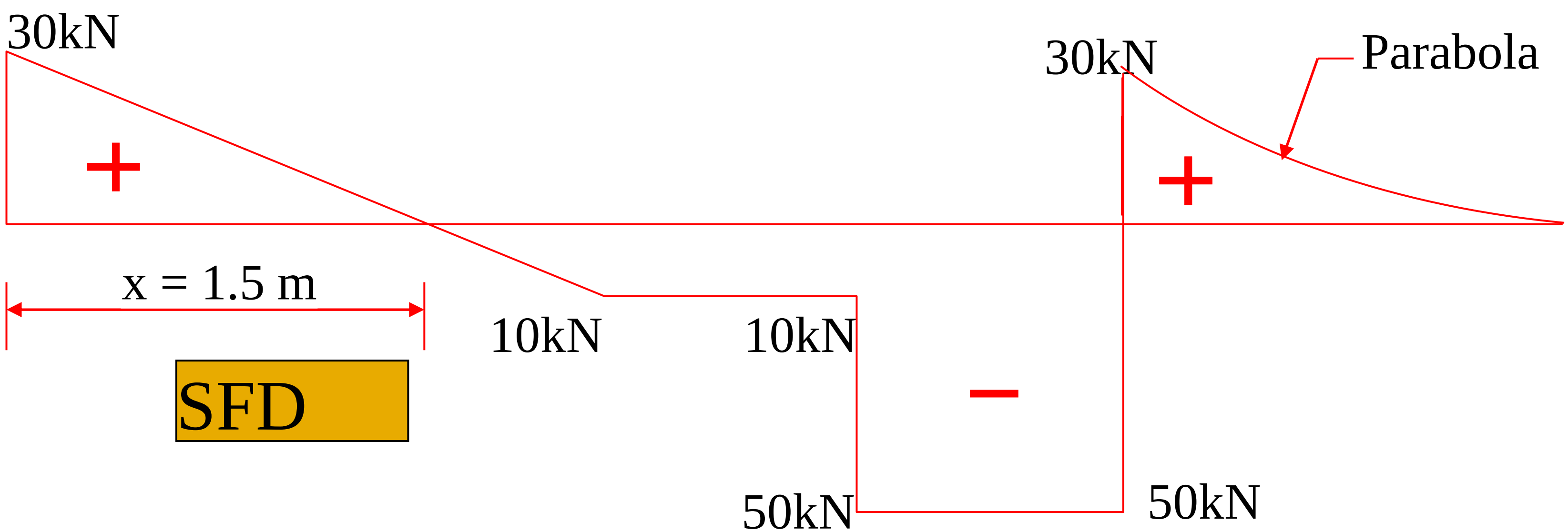
$V_{3-3} = -10$kN

$V_{4-4} = -10 - 40 = -50$ kN

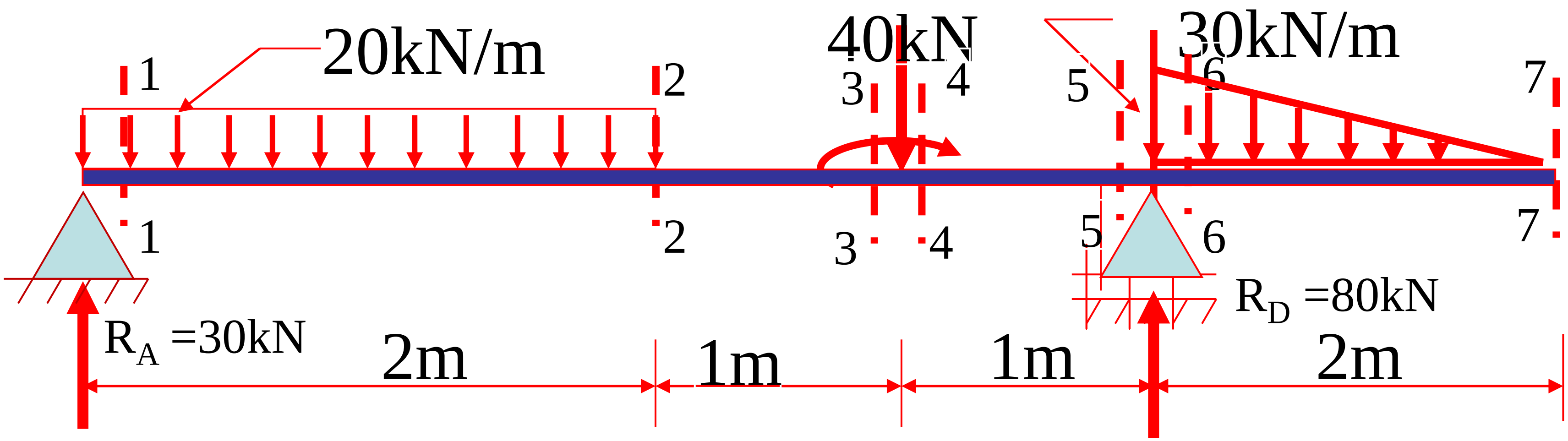
$V_{5-5} = -50$ kN

$V_{6-6} = -50 + 80 = +30$kN

$V_{7-7} = +30 - ½ \times 2 \times 30 = 0$(check)



EXAMPLE 6.10 (Cont.)



<u>Calculation of bending moments</u>:

$M_A = M_E = 0$

$M_X = 30 \times 1.5 - 20 \times 1.5 \times 1.5/2 = 22.5$ kNm

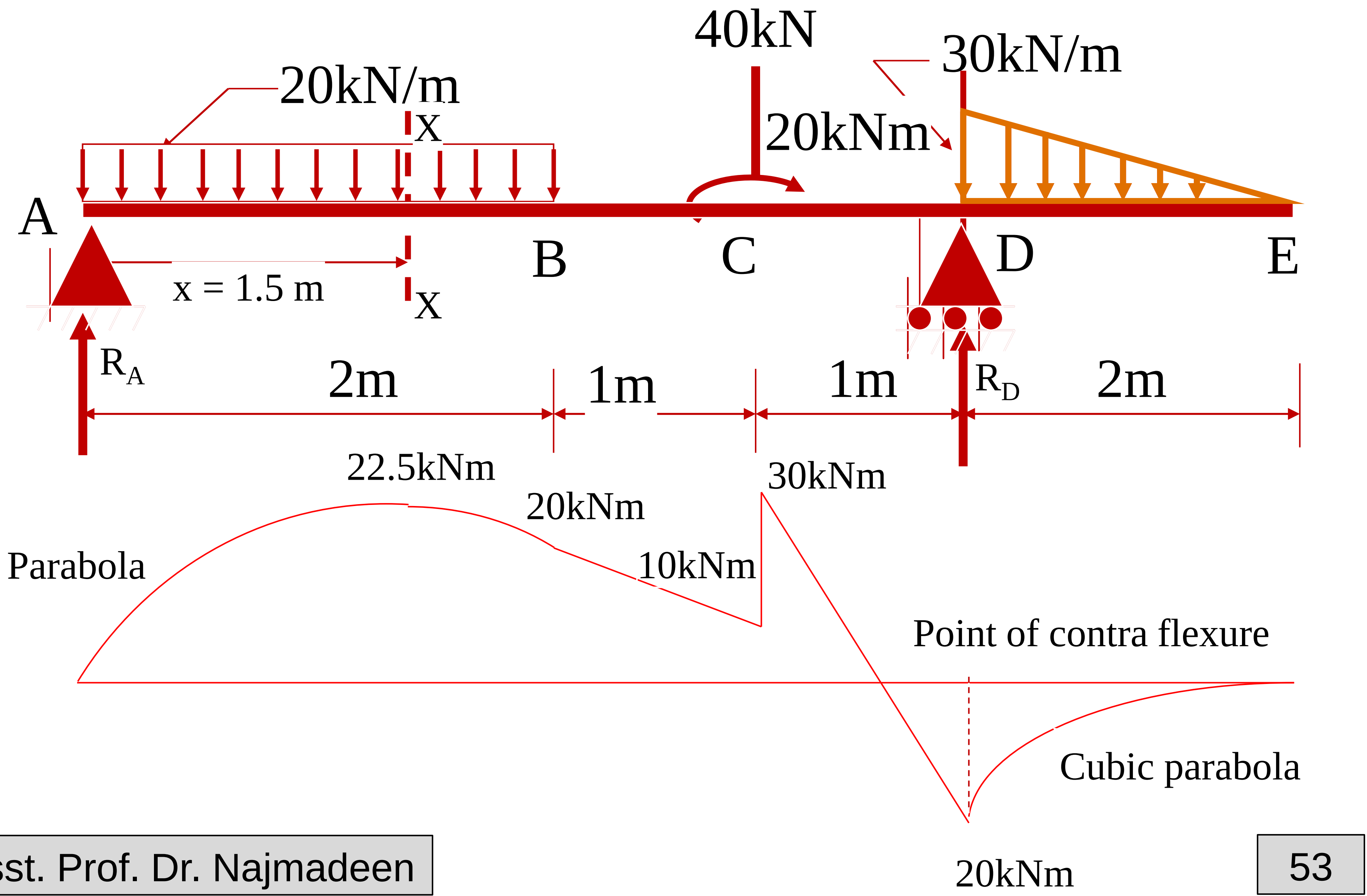
$M_B = 30 \times 2 - 20 \times 2 \times 1 = 20$ kN.m

$M_C = 30 \times 3 - 20 \times 2 \times 2 = 10$ kN.m (section before the couple)

$M_C = 10 + 20 = 30$ kN.m (section after the couple)

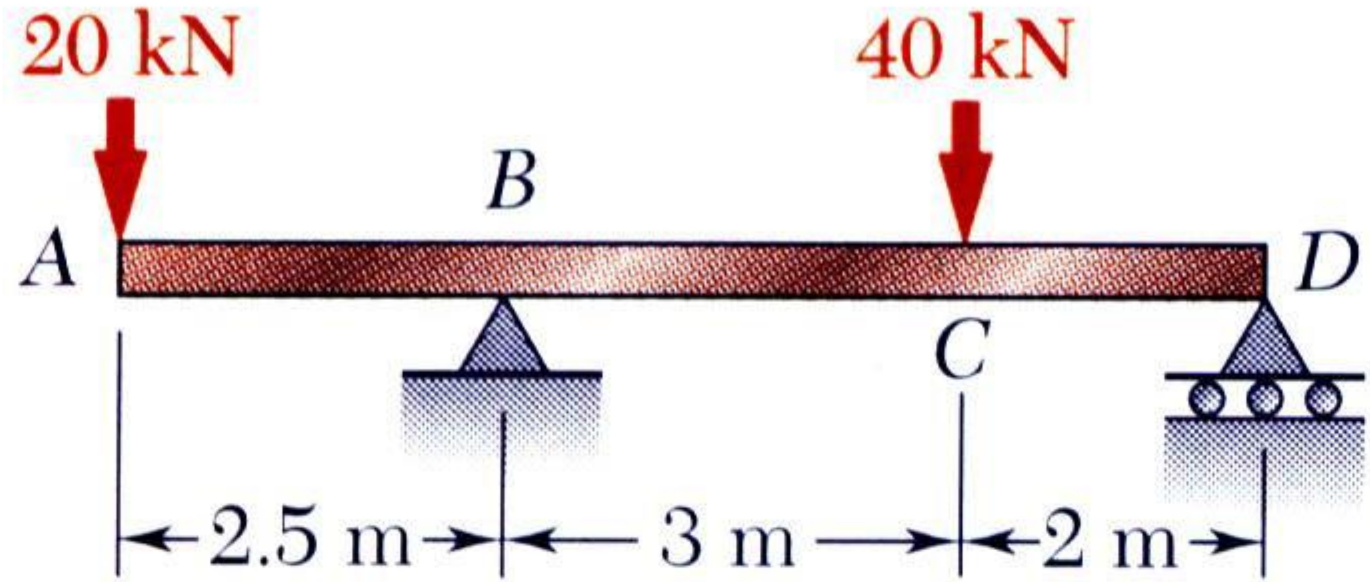
$M_D = - ½ \times 30 \times 2 \times (1/3 \times 2) = - 20$ kN.m (Considering RHS of the section)

EXAMPLE 6.10 (Cont.)

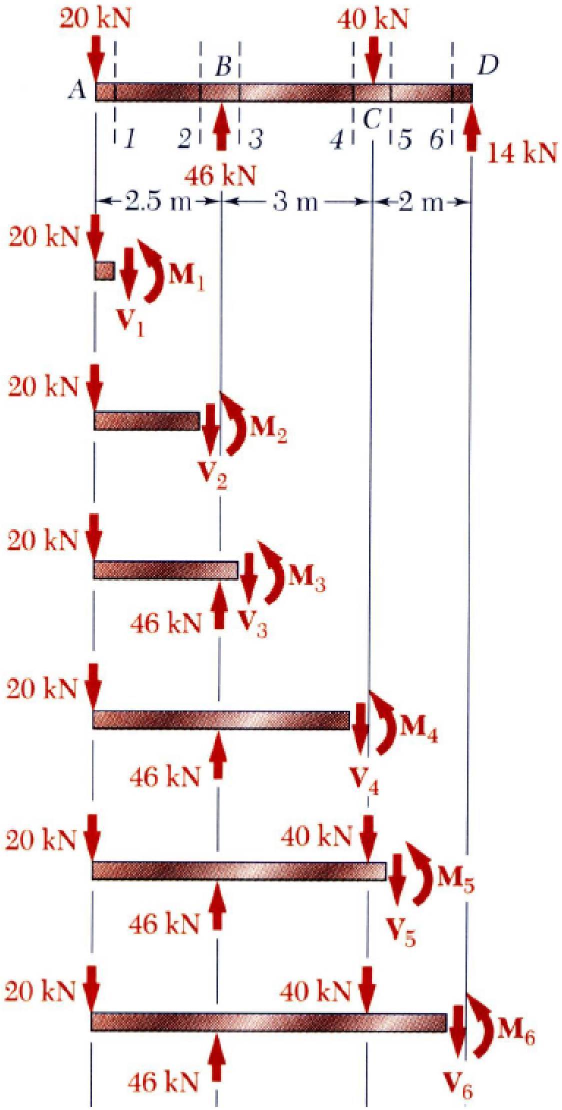


EXAMPLE 6.11

For the timber beam and loading shown, draw the shear and bending moment diagrams and determine the maximum moment.



EXAMPLE 6.11 (Cont.)



SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
  from $\sum F_y = 0 = \sum M_B: \quad R_B = 46\,\text{kN} \quad R_D = 14\,\text{kN}$

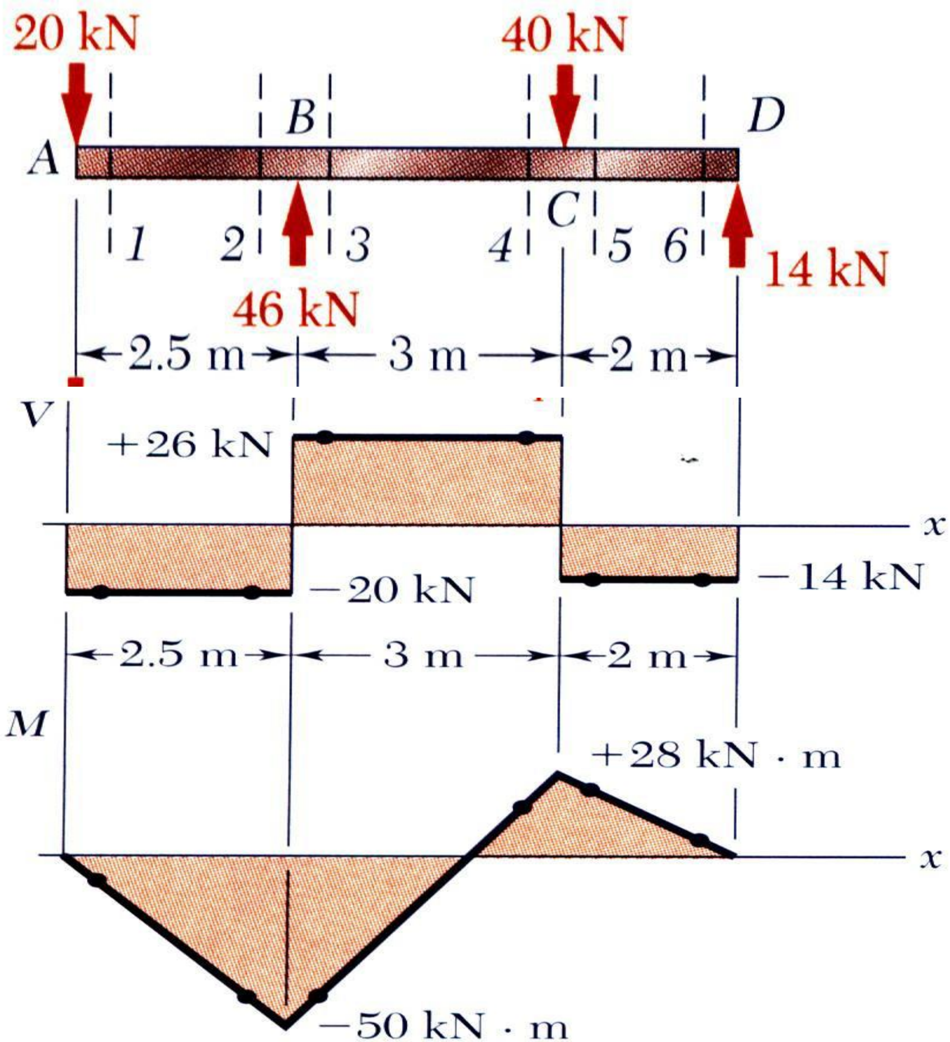
- Section the beam and apply equilibrium analyses on resulting free-bodies

$$\sum F_y = 0 \quad -20\text{ kN} - V_1 = 0 \qquad V_1 = -20\,\text{kN}$$
$$\sum M_1 = 0 \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\text{ kN} - V_2 = 0 \qquad V_2 = -20\,\text{kN}$$
$$\sum M_2 = 0 \quad (20\,\text{kN})(2.5\,\text{m}) + M_2 = 0 \quad M_2 = -50\,\text{kN}\cdot\text{m}$$

$$V_3 = +26\,\text{kN} \quad M_3 = -50\,\text{kN}\cdot\text{m}$$
$$V_4 = +26\,\text{kN} \quad M_4 = +28\,\text{kN}\cdot\text{m}$$
$$V_5 = -14\,\text{kN} \quad M_5 = +28\,\text{kN}\cdot\text{m}$$
$$V_6 = -14\,\text{kN} \quad M_6 = 0$$

EXAMPLE 6.11 (Cont.)



Identify the maximum shear and bending moment from plots of their distributions.

$Vm = 26\text{kN}$

$Mm = MB = 50\text{kN·m}$

Graphical Method For Constructing Shear And Moment Diagrams

Thank you!