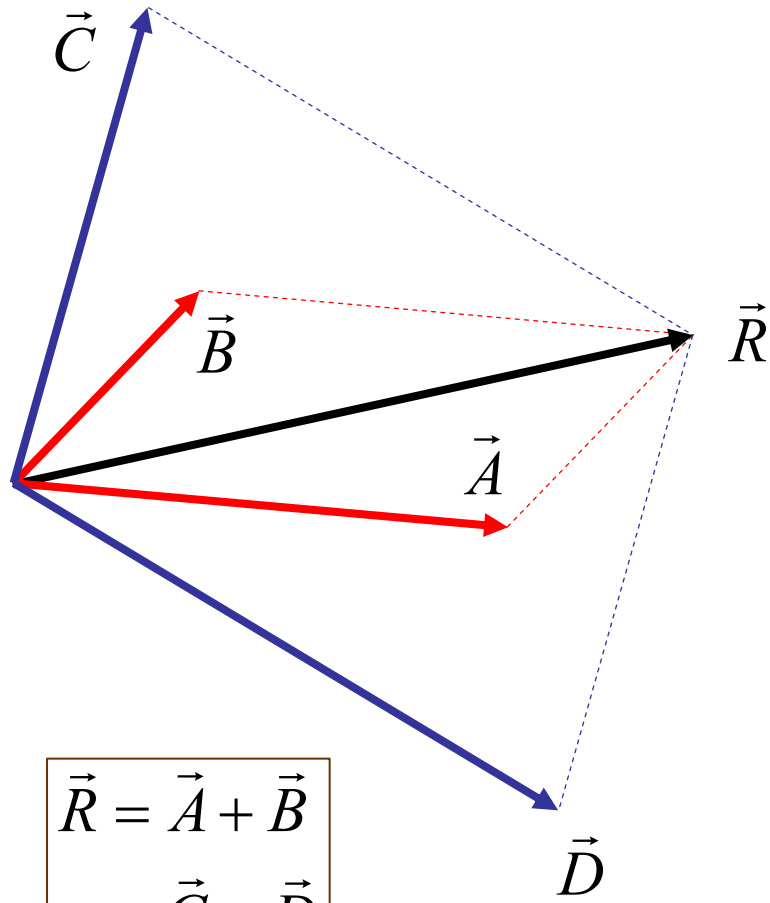
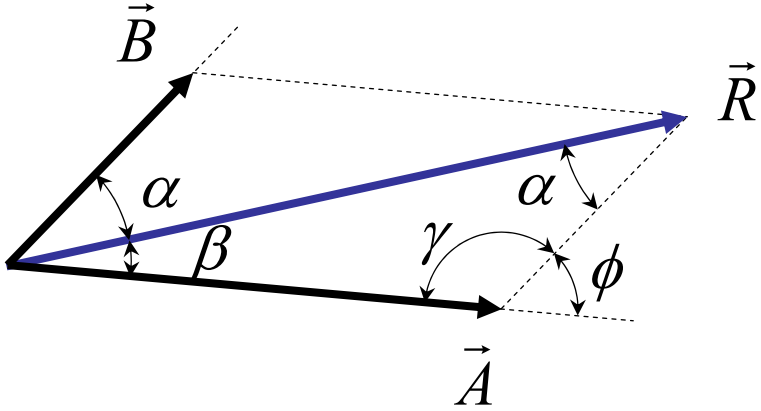
2.5 Resolution of A Force into Components

(Parallelograms and Laws of Sines and Cosines)



$$\boxed{\begin{aligned}\vec{R} &= \vec{A} + \vec{B} \\ &= \vec{C} + \vec{D}\end{aligned}}$$



$$\phi = \alpha + \beta \qquad \gamma = \pi - \alpha - \beta = \pi - \phi$$

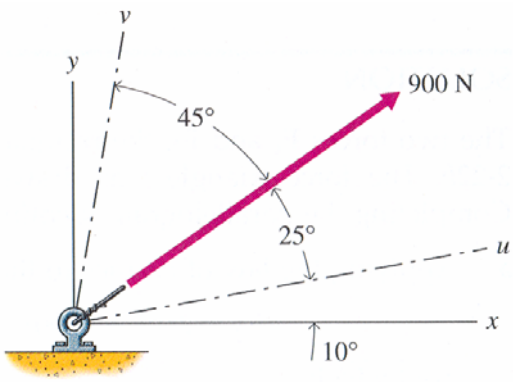
$$\text{Law of Sines}: \frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{R}{\sin\gamma}$$

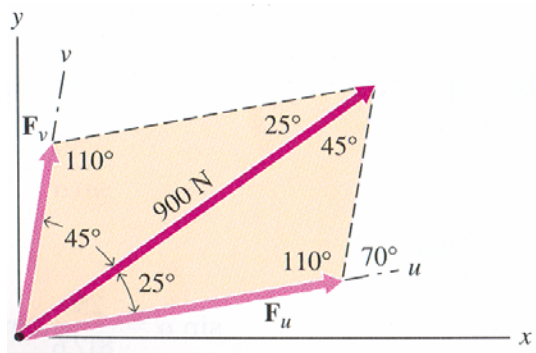
$$A = \frac{\sin\alpha}{\sin\gamma} R \qquad B = \frac{\sin\beta}{\sin\gamma} R$$

2.5 Resolution of A Force into Components

(Parallelograms and Laws of Sines and Cosines - **Examples**)

Resolve 900 N into *u*- and *v*-components as shown



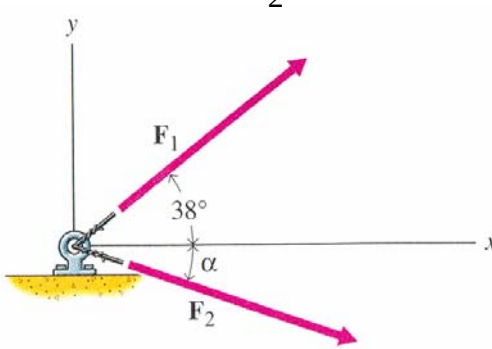


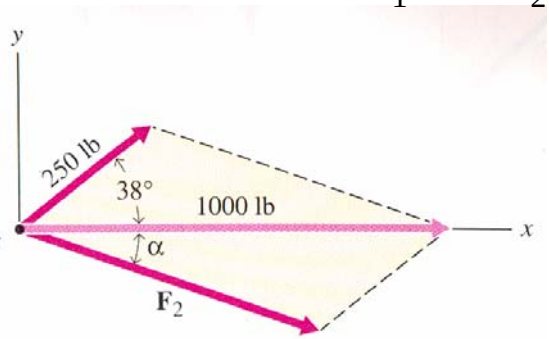
$$\frac{F_u}{\sin 45^\circ} = \frac{F_v}{\sin 25^\circ} = \frac{900}{\sin 110^\circ}$$

$$F_u = \frac{900 \sin 45^\circ}{\sin 110^\circ} = 677 \text{ N}$$

$$F_v = \frac{900 \sin 25^\circ}{\sin 110^\circ} = 405 \text{ N}$$

Determine F_2 and α so that the resultant of F_1 and F_2 is a 1,000 lb force in the x-direction





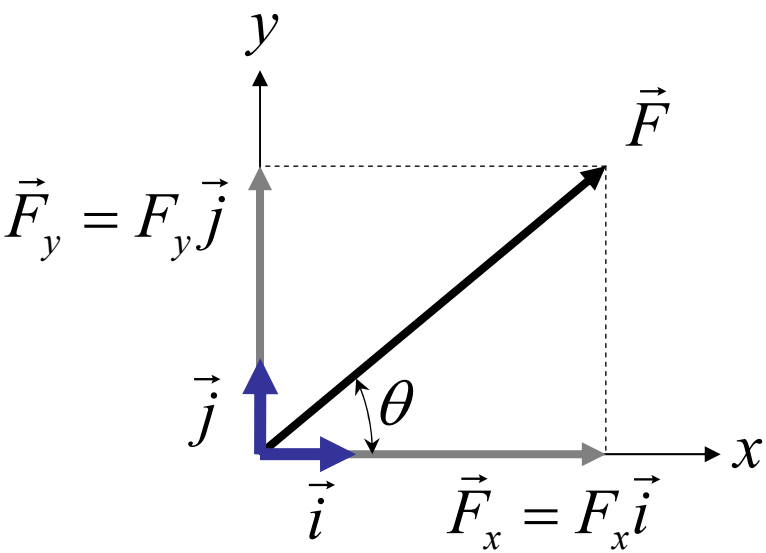
$$F_2 = \sqrt{F_1^2 + R^2 - 2F_1 R \cos 38^\circ} = 818 \text{ lb}$$

$$\frac{250}{\sin \alpha} = \frac{818}{\sin 38^\circ} \Rightarrow \alpha = \sin^{-1}\left(\frac{250}{818}\sin 38^\circ\right) = 10.85^\circ$$

Homework: Problems 2-31, 2-33, 2-37, 2-40

2.6 Rectangular Components of A Force

Two Dimensions



$\vec{i}$ = A unit vector in x - direction

$\vec{j}$ = A unit vector in y - direction

$F_x = F\cos\theta$: x scalar component of $\vec{F}$

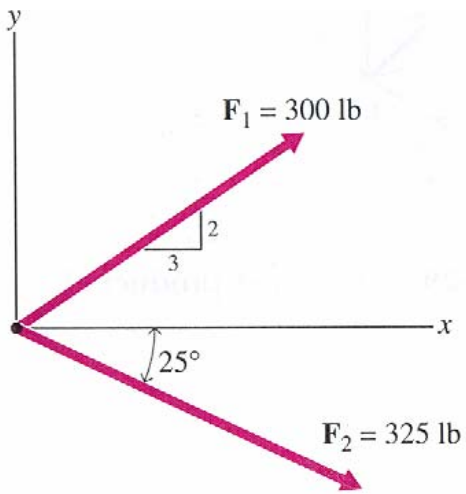
$F_y = F\sin\theta$: y scalar component of $\vec{F}$

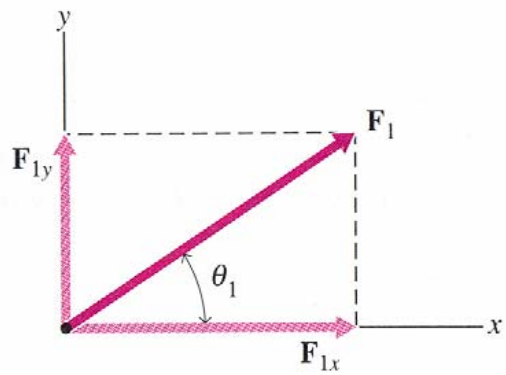
$$\boxed{\vec{F} = F_x\vec{i} + F_y\vec{j} = (F\cos\theta)\vec{i} + (F\sin\theta)\vec{j}}$$

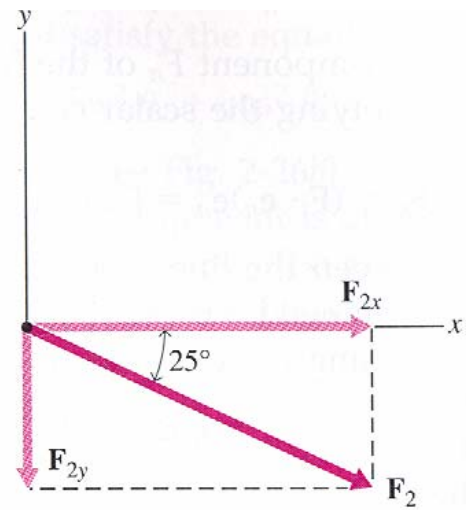
$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

2.6 Rectangular Components of A Force

Two Dimensions - Example







Determine the x and y scalar components for the two forces shown.

$$\theta_1 = \tan^{-1}\frac{2}{3} = 33.69^\circ$$

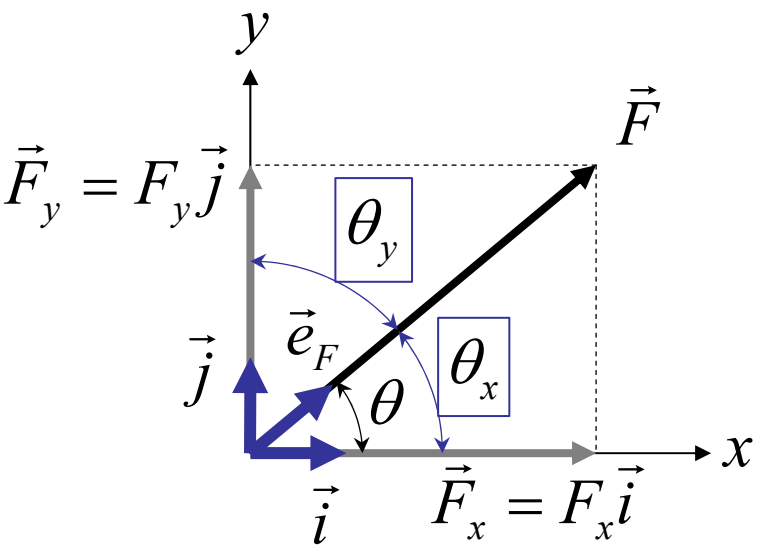
$$F_{1x} = F_1 \cos\theta_1$$
$$= 300\cos 33.69^\circ = 250 \text{ lb}$$
$$F_{1y} = F_1 \sin\theta_1$$
$$= 300\sin 33.69^\circ = 166.4 \text{ lb}$$
$$\vec{F}_1 = 250\vec{i} + 166.4\vec{j}$$

$$\theta_2 = -25^\circ$$

$$F_{2x} = F_2 \cos\theta_2$$
$$= 325\cos(-25^\circ) = 295 \text{ lb}$$
$$F_{2y} = F_2 \sin\theta_2$$
$$= 325\sin(-25^\circ) = -137.4 \text{ lb}$$
$$\vec{F}_2 = 295\vec{i} - 137.4\vec{j}$$

2.6 Rectangular Components of A Force

Two Dimensions



$$\vec{F} = F_x\vec{i} + F_y\vec{j} = F\vec{e}_F$$

$$\begin{aligned}\vec{F} &= F_x\vec{i} + F_y\vec{j} = F\vec{e}_F \\ &= F\cos\theta\vec{i} + F\sin\theta\vec{j} \\ &= F\cos\theta_x\vec{i} + F\cos\theta_y\vec{j}\end{aligned}$$

$F = \left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2}$: Magnitude of $\vec{F}$

$\vec{e}_F$: A unit vector in the direction of $\vec{F}$

$$\vec{e}_F = \frac{F_x}{F}\vec{i} + \frac{F_y}{F}\vec{j} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j}$$

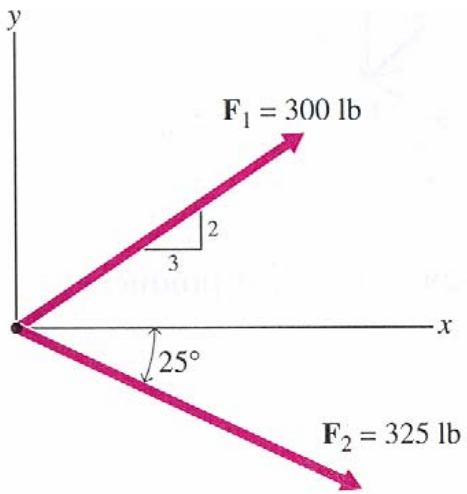
$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) \qquad \theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$$

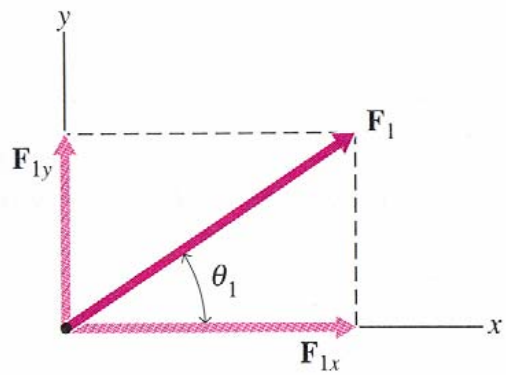
$$F_x = F\cos\theta_x = F\cos\theta$$

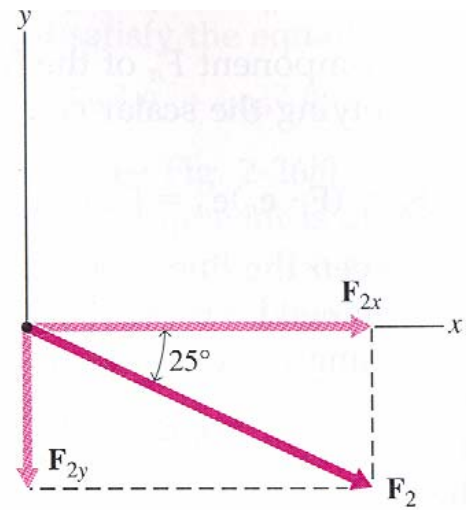
$$F_y = F\cos\theta_y = F\cos\left(\frac{\pi}{2} - \theta\right) = F\sin\theta$$

2.6 Rectangular Components of A Force

Two Dimensions - Example







Determine the x and y scalar components for the two forces shown.

$$\theta_{1x} = 33.69^\circ \quad \theta_{1y} = 56.31^\circ$$

$$F_{1x} = F_1 \cos\theta_{1x} = 250 \text{ lb}$$

$$F_{1y} = F_1 \cos\theta_{1y} = 166.4 \text{ lb}$$

$$\vec{e}_{F_1} = \cos\theta_{1x}\vec{i} + \cos\theta_{1y}\vec{j}$$

$$= 0.832\vec{i} + 0.555\vec{j}$$

$$\vec{F}_1 = F_1\vec{e}_{F_1} = 250\vec{i} + 166.4\vec{j}$$

$$\theta_{2x} = -25^\circ \quad \theta_{2y} = 90^\circ - \left(-25^\circ\right) = 115^\circ$$

$$F_{2x} = F_2 \cos\theta_{2x} = 295 \text{ lb}$$

$$F_{2y} = F_2 \cos\theta_{2y} = -137.4 \text{ lb}$$

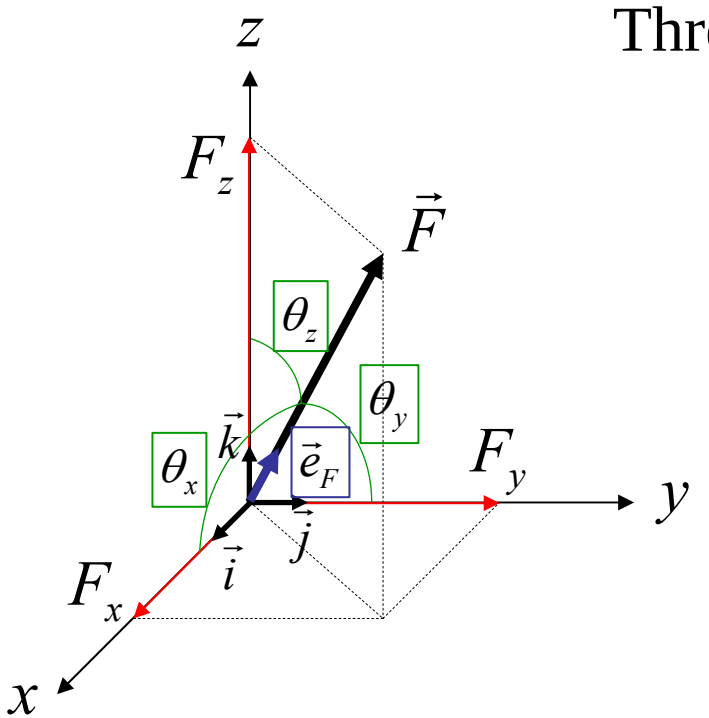
$$\vec{e}_{F_2} = \cos\theta_{2x}\vec{i} + \cos\theta_{2y}\vec{j}$$

$$= 0.906\vec{i} - 0.423\vec{j}$$

$$\vec{F}_2 = F_2\vec{e}_{F_2} = 295\vec{i} - 137.4\vec{j}$$

2.6 Rectangular Components of A Force

Three Dimensions



F : Magnitude of $\vec{F}$

F_x, F_y, F_z : Scalar components

$\vec{e}_F$: A unit vector along $\vec{F}$

$\theta_x, \theta_y, \theta_z$: Angles between $\vec{F}$ and positive coordinate axes

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = F\vec{e}_F$$

$$F_x = F\cos\theta_x \qquad \theta_x = \cos^{-1}\left(\frac{F_x}{F}\right)$$

$$F_y = F\cos\theta_y \qquad \theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$$

$$F_z = F\cos\theta_z \qquad \theta_z = \cos^{-1}\left(\frac{F_z}{F}\right)$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

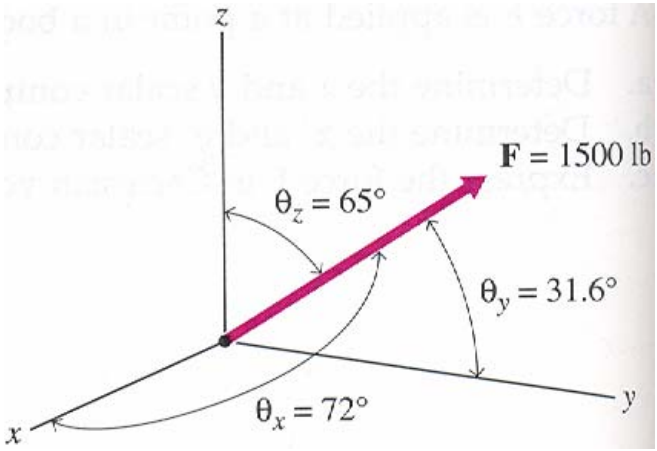
$$\vec{e}_F = \frac{F_x}{F}\vec{i} + \frac{F_y}{F}\vec{j} + \frac{F_z}{F}\vec{k}$$

$$= \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

NOTE : $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

2.6 Rectangular Components of A Force

Three Dimensions - Example



$$F_x = F\cos\theta_x = 1{,}500\cos 72.0^\circ = 464\text{ lb}$$

$$F_y = F\cos\theta_y = 1{,}500\cos 31.6^\circ = 1{,}278\text{ lb}$$

$$F_z = F\cos\theta_z = 1{,}500\cos 65.0^\circ = 634\text{ lb}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 1{,}500\text{ lb}$$

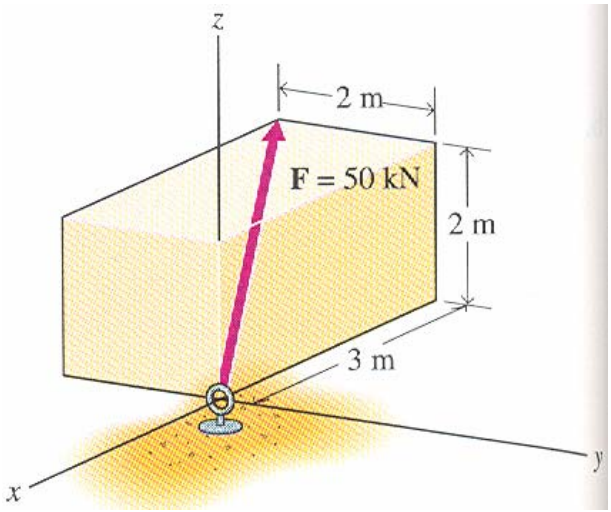
$$\vec{e}_F = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

$$= 0.3090\vec{i} + 0.8517\vec{j} + 4226\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = F\vec{e}_F = 464\vec{i} + 1{,}278\vec{j} + 634\vec{k}\text{ lb}$$

2.6 Rectangular Components of A Force

Three Dimensions - Example



$$d = \sqrt{(-3)^2 + (-2)^2 + (2)^2} = 4.123\,\text{m}$$

$$\theta_x = \cos^{-1}\frac{x}{d} = \cos^{-1}\frac{-3}{4.123} = 136.7^\circ$$

$$\theta_y = \cos^{-1}\frac{y}{d} = \cos^{-1}\frac{-2}{4.123} = 119.0^\circ$$

$$\theta_z = \cos^{-1}\frac{z}{d} = \cos^{-1}\frac{2}{4.123} = 61.0^\circ$$

$$F_x = F\cos\theta_x = 50\cos 136.7^\circ = -36.4\,\text{kN}$$

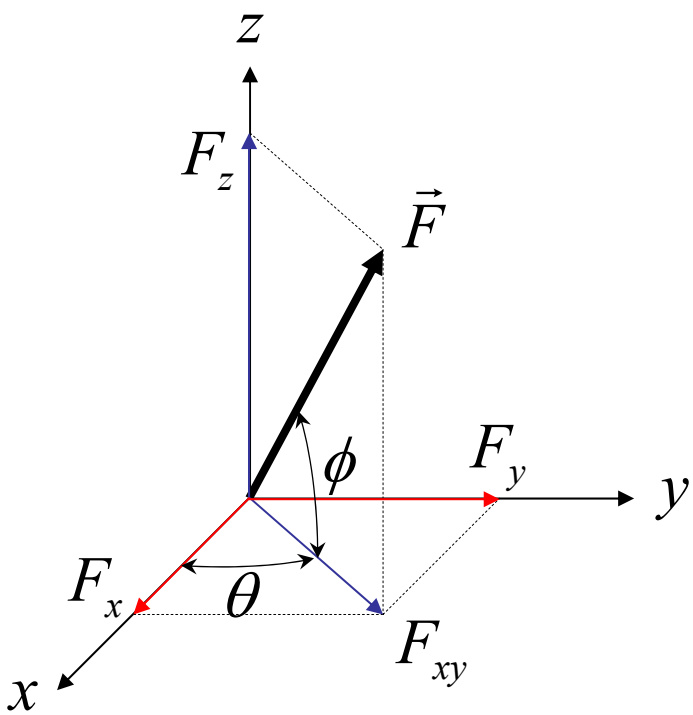
$$F_y = F\cos\theta_y = 50\cos 119.0^\circ = -24.3\,\text{kN}$$

$$F_z = F\cos\theta_z = 50\cos 61.0^\circ = 24.3\,\text{kN}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = -36.4\vec{i} - 24.3\vec{j} + 24.3\vec{k}\ \text{lb}$$

2.6 Rectangular Components of A Force

Alternative Method: Azimuth and Elevation Angles



θ : Azimuth Angle

ϕ : Elevation Angle

$$F_{xy} = F\cos\phi$$

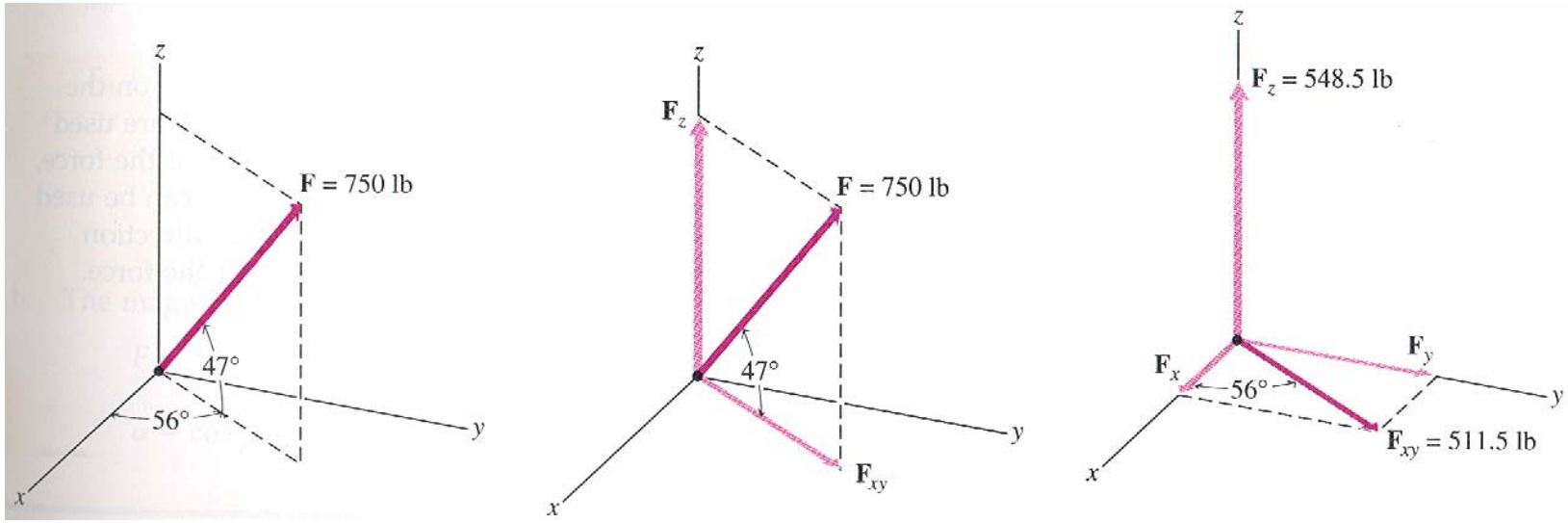
$$F_z = F\sin\phi$$

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$

$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$

2.6 Rectangular Components of A Force

Alternative Method: Azimuth and Elevation Angles – Example



$$\theta = 56^\circ \qquad \phi = 47^\circ$$

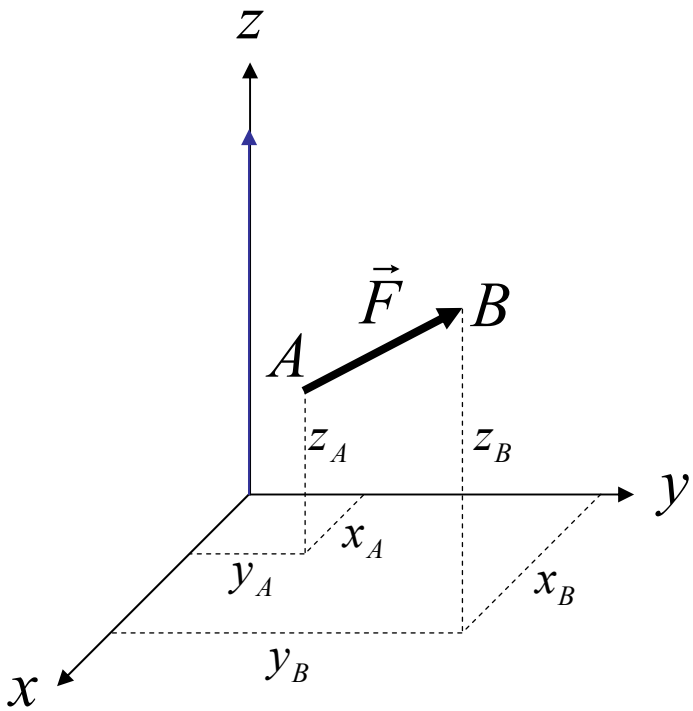
$$F_{xy} = F\cos\phi = 750\cos 47^\circ = 512 \text{ lb} \qquad F_z = F\sin\phi = 750\sin 47^\circ = 549 \text{ lb}$$

$$F_x = F_{xy}\cos\theta = 512\cos 56^\circ = 286 \text{ lb} \qquad F_y = F_{xy}\sin\theta = 512\sin 56^\circ = 424 \text{ lb}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = 286\vec{i} + 424\vec{j} + 549\vec{k} \text{ lb}$$

2.6 Rectangular Components of A Force

A Force Defined by Two Points, $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$



$$\vec{F} = F\vec{e}_F$$

$$F = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

$$\vec{e}_F = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$

$$\cos\theta_x = \frac{x_B - x_A}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

$$\cos\theta_y = \frac{y_B - y_A}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

$$\cos\theta_z = \frac{z_B - z_A}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

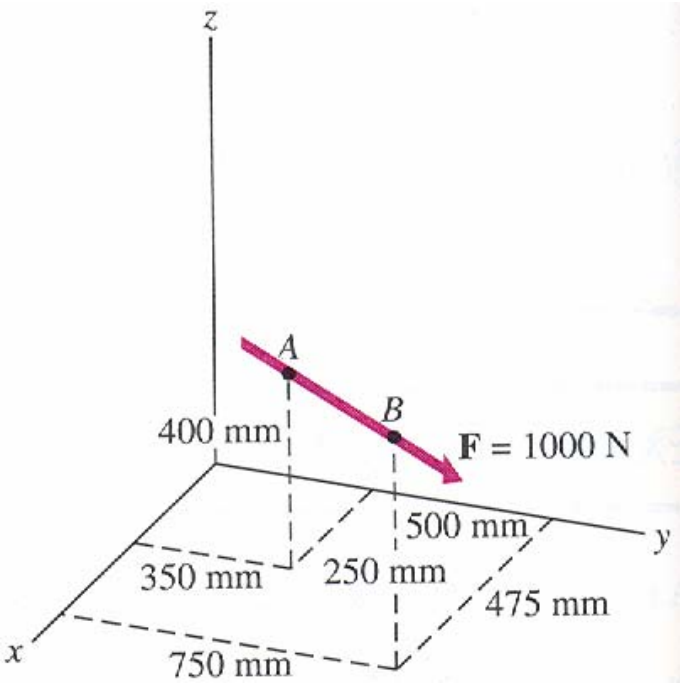
$$F_x = F\cos\theta_x = x_B - x_A$$

$$F_y = F\cos\theta_y = y_B - y_A$$

$$F_z = F\cos\theta_z = z_B - z_A$$

2.6 Rectangular Components of A Force

A Force Defined by Two Points, $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$ - Example



$$x_A = 250 \text{ mm} \quad y_A = 350 \text{ mm} \quad z_A = 400 \text{ mm}$$

$$x_B = 475 \text{ mm} \quad y_B = 750 \text{ mm} \quad z_B = 500 \text{ mm}$$

$$\cos\theta_x = \frac{x_B - x_A}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}} = 0.3907$$

$$\cos\theta_y = \frac{y_B - y_A}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}} = 0.8930$$

$$\cos\theta_z = \frac{z_B - z_A}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}} = 0.2233$$

$$F_x = F\cos\theta_x = 391 \text{ N}$$

$$F_y = F\cos\theta_y = 893 \text{ N}$$

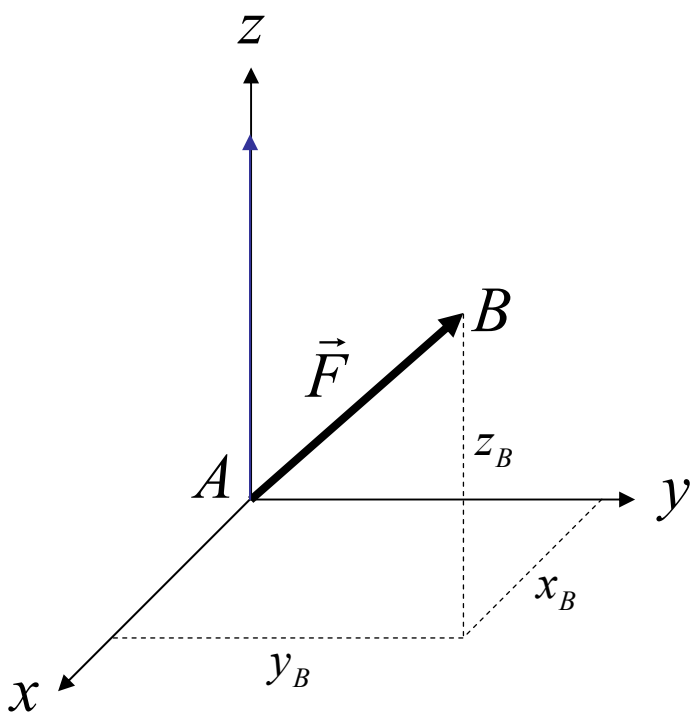
$$F_z = F\cos\theta_z = 223 \text{ N}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = 391\vec{i} + 893\vec{j} + 223\vec{k} \text{ lb}$$

2.6 Rectangular Components of A Force

A Force Defined by Two Points, $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$

If A is at origin, $x_A = y_A = z_A = 0$



$$\vec{F} = F\vec{e}_F$$

$$F_x = F\cos\theta_x = x_B$$

$$F_y = F\cos\theta_y = y_B$$

$$F_z = F\cos\theta_z = z_B$$

$$F = \sqrt{x_B^2 + y_B^2 + z_B^2}$$

$$\vec{e}_F = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

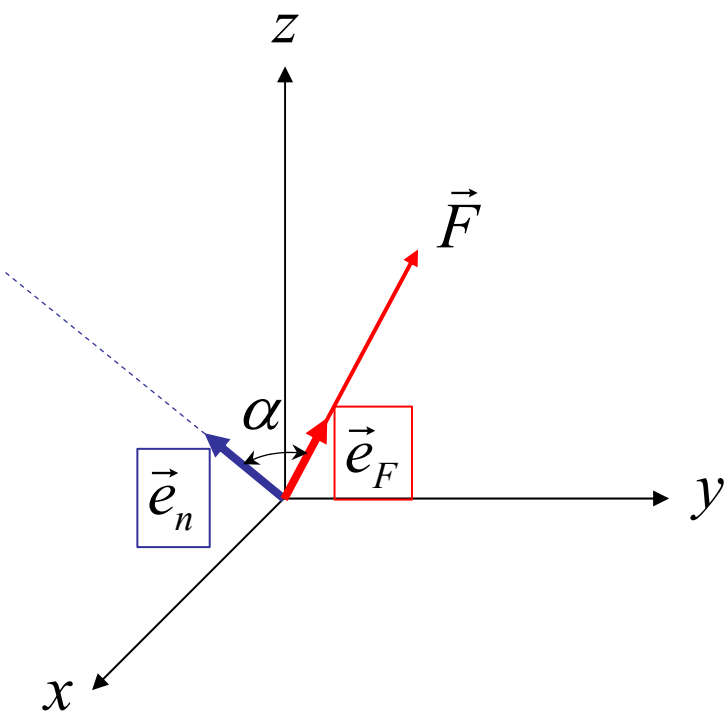
$$\cos\theta_x = \frac{x_B}{\sqrt{x_B^2 + y_B^2 + z_B^2}}$$

$$\cos\theta_y = \frac{y_B}{\sqrt{x_B^2 + y_B^2 + z_B^2}}$$

$$\cos\theta_z = \frac{z_B}{\sqrt{x_B^2 + y_B^2 + z_B^2}}$$

2.6 Rectangular Components of A Force

Along an arbitrary direction n



$$\vec{F} = F\vec{e}_F \qquad \vec{e}_n : \text{Direction of } n$$

$$F_n = \vec{F} \cdot \vec{e}_n = F\vec{e}_F \cdot \vec{e}_n = F\left(\vec{e}_F \cdot \vec{e}_n\right)$$

$$\vec{F}_n = F_n\vec{e}_n = \left(\vec{F} \cdot \vec{e}_n\right)\vec{e}_n = F\left(\vec{e}_F \cdot \vec{e}_n\right)\vec{e}_n$$

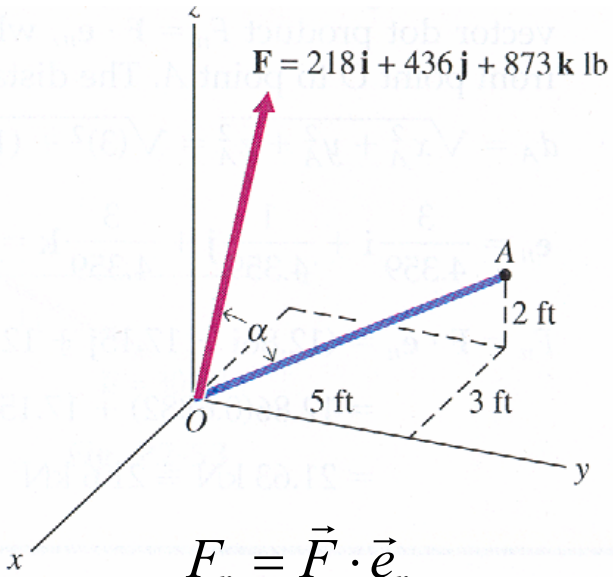
$$\vec{F} \cdot \vec{e}_n = (F)(1)\cos\alpha = F_n$$

$$\Rightarrow \alpha = \cos^{-1}\frac{\vec{F} \cdot \vec{e}_n}{F} = \cos^{-1}\frac{F_n}{F}$$

2.6 Rectangular Components of A Force

Along an arbitrary direction n - Example

Determine the rectangular scalar component F_n of $\boldsymbol{F}$ along line OA



$$\vec{F} = F\vec{e}_F = 218\vec{i} + 436\vec{j} + 873\vec{k} \text{ lb}$$

$$F = \sqrt{(218)^2 + (436)^2 + (873)^2} = 1{,}000 \text{ lb}$$

$$\vec{e}_n = \frac{-3\vec{i} + 5\vec{j} + 2\vec{k}}{\sqrt{(-3)^2 + (5)^2 + (2)^2}}$$

$$= -0.4867\vec{i} + 0.8112\vec{j} + 0.3245\vec{k}$$

$$F_n = \vec{F} \cdot \vec{e}_n$$

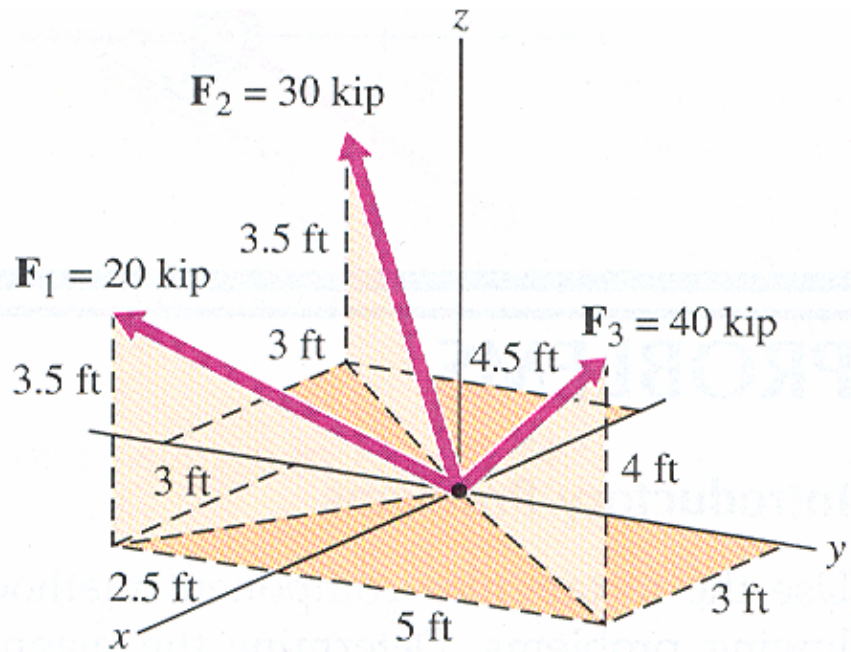
$$= \left(218\vec{i} + 436\vec{j} + 873\vec{k}\right) \cdot \left(-0.4867\vec{i} + 0.8112\vec{j} + 0.3245\vec{k}\right)$$

$$= (218)(-0.4867) + (436)(0.8112) + (873)(0.3245) = 531 \text{ lb}$$

$$\vec{F} \cdot \vec{e}_n = (F)(1)\cos\alpha = F_n \Rightarrow \alpha = \cos^{-1}\frac{F_n}{F} = \cos^{-1}\frac{531}{1{,}000} = 57.9^\circ$$

2.7 Resultants by Rectangular Components

Example



$$\vec{e}_1 = \frac{3\vec{i} - 2.5\vec{j} + 3.5\vec{k}}{\sqrt{(3)^2 + (-2.5)^2 + (3.5)^2}}$$

$$= 0.5721\vec{i} - 0.4767\vec{j} + 0.6674\vec{k}$$

$$\vec{e}_2 = \frac{-3\vec{i} - 4.5\vec{j} + 3.5\vec{k}}{\sqrt{(-3)^2 + (-4.5)^2 + (3.5)^2}}$$

$$= -0.4657\vec{i} - 0.6985\vec{j} + 0.5433\vec{k}$$

$$\vec{e}_3 = \frac{3\vec{i} + 5\vec{j} + 4\vec{k}}{\sqrt{(3)^2 + (5)^2 + (4)^2}}$$

$$= 0.4243\vec{i} + 0.7071\vec{j} + 0.5657\vec{k}$$

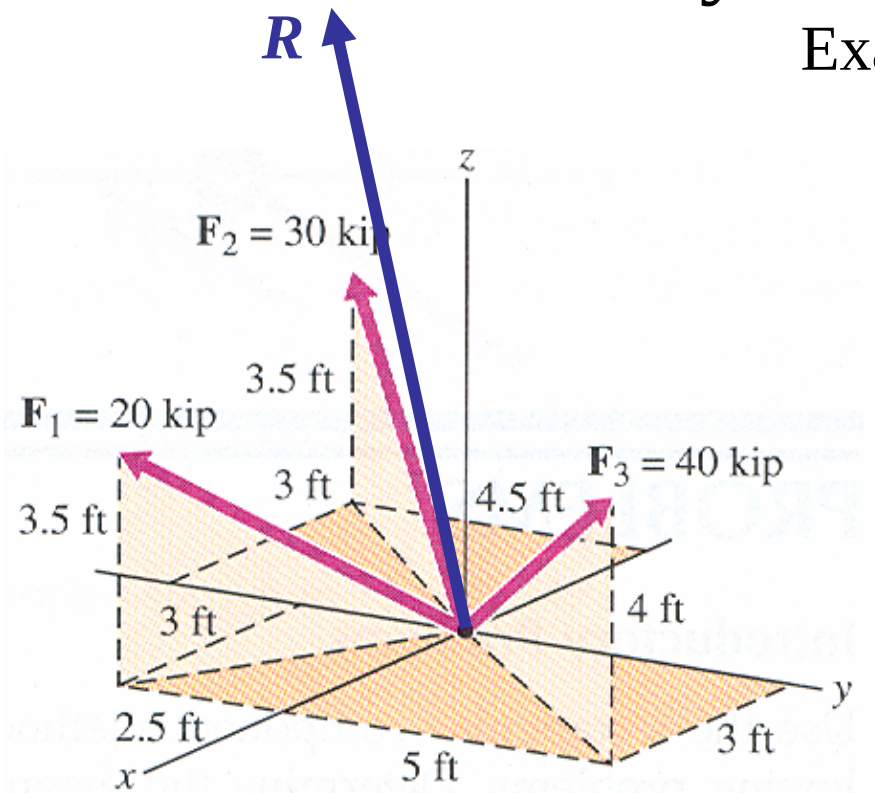
$$\vec{F}_1 = F_1\vec{e}_1 = 11.442\vec{i} - 9.534\vec{j} + 13.348\vec{k} = F_{1x}\vec{i} + F_{1y}\vec{j} + F_{1z}\vec{k} \text{ kip}$$

$$\vec{F}_2 = F_2\vec{e}_2 = -13.971\vec{i} - 20.955\vec{j} + 16.299\vec{k} = F_{2x}\vec{i} + F_{2y}\vec{j} + F_{2z}\vec{k} \text{ kip}$$

$$\vec{F}_3 = F_3\vec{e}_3 = 16.972\vec{i} + 28.284\vec{j} + 22.628\vec{k} = F_{3x}\vec{i} + F_{3y}\vec{j} + F_{3z}\vec{k} \text{ kip}$$

2.7 Resultants by Rectangular Components

Example



$$\vec{F}_1 = F_1\vec{e}_1 = 11.442\vec{i} - 9.534\vec{j} + 13.348\vec{k} \text{ kip}$$

$$\vec{F}_2 = F_2\vec{e}_2 = -13.971\vec{i} - 20.955\vec{j} + 16.299\vec{k} \text{ kip}$$

$$\vec{F}_3 = F_3\vec{e}_3 = 16.972\vec{i} + 28.284\vec{j} + 22.628\vec{k} \text{ kip}$$

$$\vec{R} = R_x\vec{i} + R_y\vec{j} + R_z\vec{k}$$

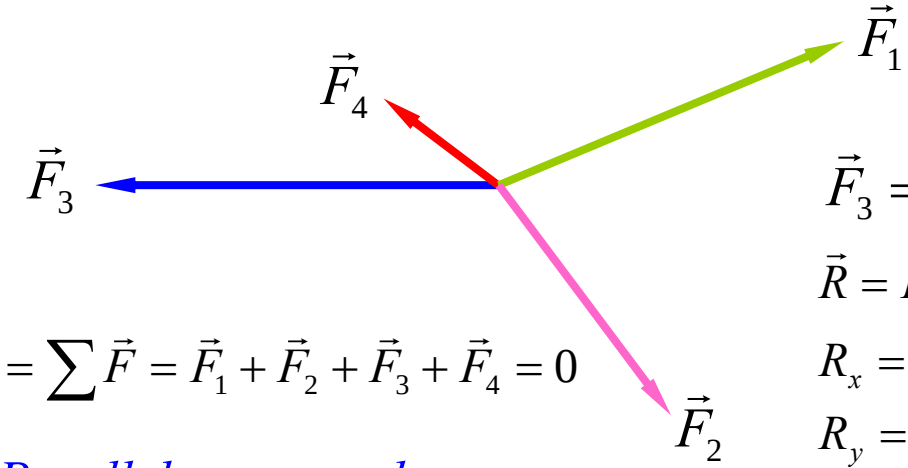
$$= \left(\sum F_{ix}\right)\vec{i} + \left(\sum F_{iy}\right)\vec{j} + \left(\sum F_{iz}\right)\vec{k}$$

$$= 14.443\vec{i} - 2.205\vec{j} + 52.275\vec{k} \text{ kip}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = 54.3 \text{ kip}$$

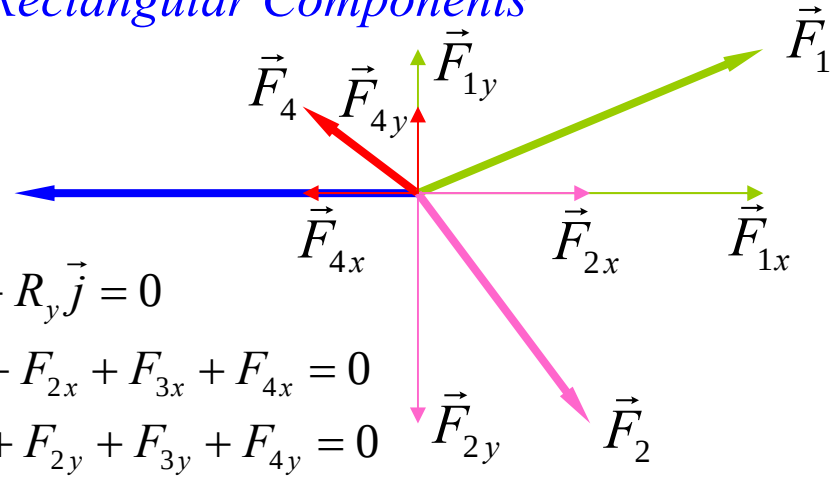
$$\theta_x = \cos^{-1}\frac{R_x}{R} = 74.6^\circ \qquad \theta_y = \cos^{-1}\frac{R_y}{R} = 92.3^\circ \qquad \theta_z = \cos^{-1}\frac{R_z}{R} = 15.6^\circ$$

Concurrent Force Systems – A Summary (2-D)



$$\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

Rectangular Components



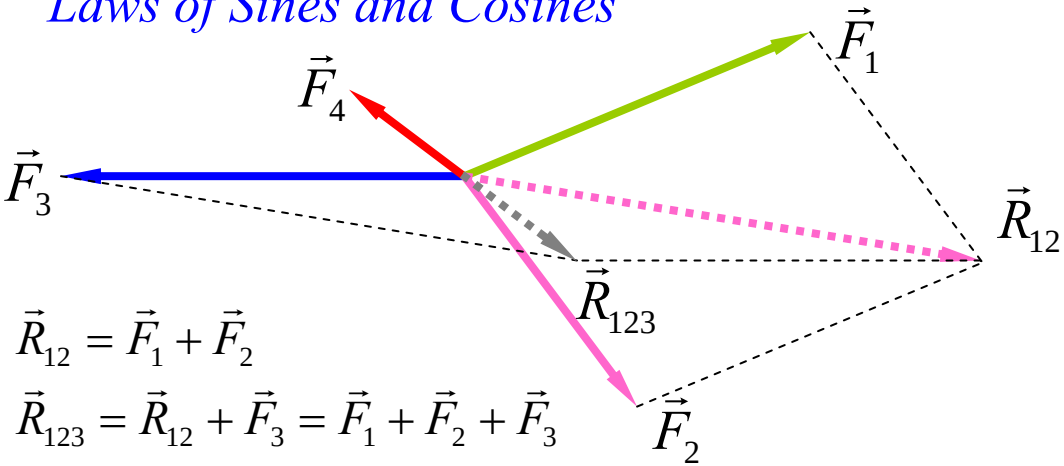
$$\vec{F}_3 = \vec{F}_{3x}$$

$$\vec{R} = R_x\vec{i} + R_y\vec{j} = 0$$

$$R_x = F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0$$

$$R_y = F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0$$

Parallelogram and Laws of Sines and Cosines

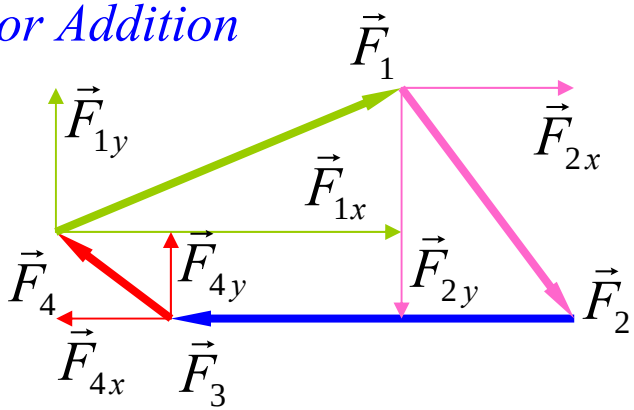


$$\vec{R}_{12} = \vec{F}_1 + \vec{F}_2$$

$$\vec{R}_{123} = \vec{R}_{12} + \vec{F}_3 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{R} = \vec{R}_{123} + \vec{F}_4 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

Vector Addition



$$\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

Concurrent Force Systems – A Summary (3-D)

Rectangular Components

Forces in components

$$\vec{F}_1 = F_{1x}\vec{i} + F_{1y}\vec{j} + F_{1z}\vec{k}$$

$$\vec{F}_2 = F_{2x}\vec{i} + F_{2y}\vec{j} + F_{2z}\vec{k}$$

$$\vdots$$

$$\vec{F}_n = F_{nx}\vec{i} + F_{ny}\vec{j} + F_{nz}\vec{k}$$

Resultant in components

$$\vec{R} = R_x\vec{i} + R_x\vec{i} + R_x\vec{i}$$

$$R_x = F_{1x} + F_{2x} + \cdots + F_{nx} = \sum F_x$$

$$R_y = F_{1y} + F_{2y} + \cdots + F_{ny} = \sum F_y$$

$$R_z = F_{1z} + F_{2z} + \cdots + F_{nz} = \sum F_z$$

Magnitude and direction of the resultant

$$\vec{R} = R\,\vec{e}_R$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$\vec{e}_R = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{\sum F_x}{R}$$

$$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\frac{\sum F_y}{R}$$

$$\theta_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\frac{\sum F_z}{R}$$