MENS, a mathematical model for cognitive systems

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Abstract

How do higher mental processes, learning, intentions, thoughts, feelings, arise from the functioning of the brain? That is the question we attempt to approach in the Memory Evolutive Neural Systems (or MENS). This theory proposes a formal unified model for the investigation of the mind, translating ideas of neuroscientists such as Changeux and Edelman in a mathematical language based on Eilenberg and Mac Lane's theory of categories (which unifies the main mathematical operations). MENS is an application for cognitive systems of our general model MES for autonomous complex hierarchical systems, such as biological or social systems. The 'complexification process', introduced in MES to model the formation of increasingly complex objects, is related to the "binding problem" of neuroscience and it characterizes how higher cognitive processes, the development of a semantic memory, and consciousness, may emerge from physical states of the brain, thus supporting an emergentist monism. In particular, the existence of consciousness is related to the development of a global invariant, the archetypal core that integrates and merges the lasting corporal and mental experiences, giving a basis at the notion of self.

Keywords

Neuron, mental object, memory, cognition, consciousness, emergence, category, colimit, complexification.

1 Introduction

The Memory Evolutive Neural Systems (or MENS) studied in this paper are a model for cognitive systems of animals, up to a theory of mind for man, which incorporates a basic level **Neur** formed by the neural system, and higher levels, deduced from it, representing an 'algebra of mental objects' (in the terms of Changeux, 1983). The main idea is that these higher levels emerge from the basis through iterative binding processes, so that a mental object appears as a family of synchronous assemblies of neurons, then of assemblies of assemblies of neurons, and so on. They develop over time through successive 'complexification processes', up to the formation of higher cognitive processes and consciousness. Their evolution is internally self-regulated and relies on the formation of a memory in which the different data, experiences, procedures can be stored in a flexible manner, to be later recalled or actualized for a better adaptation. The model takes account of the exchanges with the physical environment, through receptors and effectors which confer to it a kind of "embodiement" (Varela, 1989), and, for higher animals, through education and cultural activities, stressing the role of the society in the development of higher processes. The notion of self relies on the development of a permanent global invariant, the archetypal core, which integrates the main corporal, perceptual, behavioral, procedural and semantic experiences, with their emotional overtones; its self-maintained activation is at the root of consciousness, characterized in particular by temporal extension processes.

MENS is a particular case of the Memory Evolutive Systems which we have developed in a series of papers during the last 25 years (cf. our book, Ehresmann and Vanbremeersch, 2007). Initially, it has been influenced by works on several domains (Bunge, 1979; Laborit, 1983; Merleau-Ponty, 1945; Minsky, 1986; Morin, 1977; Piaget, 1940), and more particularly on neuroscience (Changeux, 1983; Crick, 1994; Edelman, 1989). It could also be applied to artificial systems such as robots equipped with means to sense their environment, but here we focus on the case of higher animals, up to man.

Though the main ideas can be explained in ordinary language (as we try to do as much as possible in this article, referring to the Appendix for rigorous definitions), MENS is a mathematical model based on the theory of categories. This theory is a relational domain of mathematics, introduced in the forties by Eilenberg & Mac Lane (1945) to unify some problems in algebra and topology, and which accounts for the various operations of the "working mathematician" (Mac Lane, 1971). In MENS, it gives tools for modeling the main human capacities: formation, comparison and analysis of the relations between interacting objects, synthesis of complex objects binding more elementary objects (colimit operation), formation of a hierarchy of increasingly complex objects (complexification process) and their later recognition, classification of objects into invariance classes (projective limit operation), allowing for the development of a semantic. In particular, we model a mental object by what we call a category-neuron (abbreviated in cat-neuron), iteratively constructed as the binding of synchronous assemblies of (cat-)neurons. We show that the "degeneracy of the neuronal coding" emphasized by Edelman (1989) implies that a cat-neuron has several such "physical" realizations; it follows that the links between cat-neurons are not only simple links binding clusters of links between their components of the lower level, but also complex links which emerge by composition of simple links binding non-adjacent clusters. The complex links reflect global properties of the lower level which are not observable locally at this lower level. It is the precise mechanism at the root of the emergence of mental objects and processes of increasing complexity.

MENS brings up philosophical problems related to emergence *vs.* reductionism, mind-brain correlation, self and consciousness.

2 Neurons, mental objects, category-neurons

MENS is a model for the cognitive system of an animal. It intends to describe the development of mental objects and cognitive processes of increasing complexity based on the functioning of his neural system. First we recall some physiological data on this system and its functioning, and we model it by the evolutive system of neurons **Neur**, which is at the basis of MENS.

2.1 The neural system

The neural system consists of neurons and synapses between them, its dynamics results from the propagation of an action potential from a neuron to other neurons through synapses. It slowly evolves during the life of the animal. There are several types of neurons. For some (e.g., intermediate neurons), their activity is entirely dependent from their connections with other neurons, for others, it is modulated by external or internal events. The receptor neurons, in the various perceptual areas are in contact with the environment of the system and are triggered by changes in this environment; this allows the animal to recognize (innately or after learning) some external features (appropriate foods, predators,...) and develop adaptive responses to them. These responses are realized by effector neurons, in the motor areas, which act on the environment through their action on muscles.

The state N(t) at an instant t of a neuron N is determined by its activity around t, which is a function of its instantaneous rate of firing and of its threshold (related to the difference of potential between inside and outside the cell necessary for starting an action potential). We say that an item (external object or neuron) *activates* N at t if it causes an increase in the activity of N at this date; and we think of the resulting activation as a kind of information transmitted by the item to N.

The state at t of a synapse s from N (the presynaptic neuron) to N' (the post-synaptic neuron) is determined by its *strength*, that is related to the capacity of s to transmit an action potential from N to N' around t. There are excitatory synapses and inhibitory ones. The strength of an excitatory synapse is inversely proportional to the number of spikes of N necessary to start a firing of N', supposing that N' does not receive inputs from other neurons. The synapse s has also a *propagation delay* which measures the delay between the firing of N and its possible transmission to N'. Since each impulse has a specific duration and there is a temporal summation of the impulses, the propagation delay is inversely proportional to the strength of s. An inhibitory synapse decreases the activity of the post-synaptic neuron; its strength is negative, with its absolute value inversely proportional to the number of spikes of N necessary to inhibit the activity of N'. While the activity of a neuron N varies quickly, the strength and propagation delay of a synapse vary much more slowly.

There is a spatial summation of impulses: if N receives simultaneously inputs from several neurons N_i through synapses s_i from N_i to N, the activity of N is an upper-bounded function of the sum of the activities of the N_i and of N, weighted by the strength of the synapses s_i.

2.2 The evolutive system of neurons Neur

Given two neurons N and N', there can be several "parallel" synapses from N to N'; they can also be linked by a sequence of synapses, say s_1 from N to N_1, then s_2 from N_1 to N_2, and so on up to N'. Such a sequence is called a *synaptic path* (or, more briefly, a *link*) from N to N'; its strength is a function of the product of the strengths of its components, and its propagation delay is the sum of their propagation delays. In particular a synaptic path (s_1, s_2) in which one synapse is inhibitory and the other excitatory has a negative strength, while its strength is positive if both are of the same kind. Synaptic paths are composed by concatenation (meaning one succeeding to the other). A synaptic path from N to N' is *activated* at *t* if N activates N' at *t* along it.

We model the configuration of the neural system at *t* by a (multi)graph: its vertices model the states N(*t*) of the neurons N existing at *t*, an arrow from N(*t*) to N(*t'*), also called *link* from N to N' at *t*, models the state at *t* of a synaptic path *s* from N to N'; it is determined by the strength $w_s(t)$ and the propagation delay $d_s(t)$ of *s* at *t*, which are real numbers. Equipped with the composition defined by concatenation, this graph becomes the *category* Neur_t *of neurons* at *t* (for the definition of a category, cf. the Appendix).

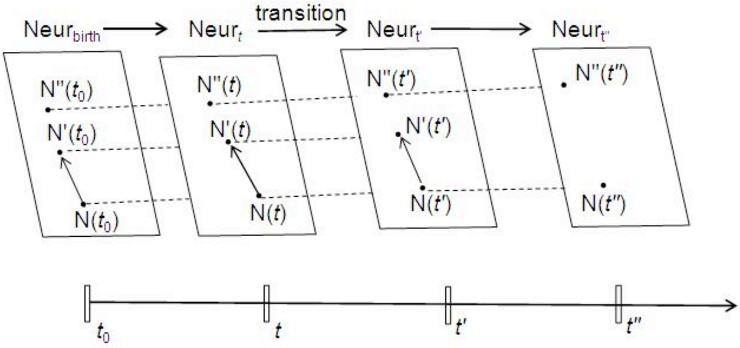

FIGURE 1. For each instant *t* of the life of the animal, the category of neurons Neur_t consists of the states N(*t*) *of* the neurons existing at *t*, and of the synaptic paths between them at this time. The transition from *t* to *t'* correlates N(*t*) and N(*t'*) if N still exists at *t'*. The categories Neur_t and the transitions between them form the evolutive system of neurons, **Neur**.
A neuron N appears as a component of **Neur**, consisting of its successive states.
Here the neuron N' does no more exist at *t''*.
To an interval (*t*, *t''*) we associate the category formed by the neurons and the synaptic paths between them which exist during this period.

Neurons and synapses have a long life. Over time, say from *t* to a later time *t'*, some neurons are lost, while a few new neurons can grow, and the number of synapses can vary. This change of configuration is modeled by a partial map from Neur_t to $\mathrm{Neur}_{t'}$, called the *transition* from *t* to *t'*, sending the state of a neuron N at *t* to its state at *t'* if N still exists, and similarly for a link. This transition defines a functor from a sub-category of Neur_t to $\mathrm{Neur}_{t'}$. The *Evolutive system of neurons* (Ehresmann and Vanbremeersch, 1987), denoted **Neur**, is formed by the categories Neur_t and the transitions between them, during the life of the system (cf. Figure 1). Its components model the neurons (via their successive states); they are still called neurons (or, later, cat-neurons of level 0). The operations are not instantaneous but require some period of time; thus what is particularly interesting is the category of neurons and their links existing during such a period, and we generally operate in this category.

2.3 Mental images

Neur models the physical structure of the brain of the animal and its elementary neural dynamics. How can it generate MENS, a model of his cognitive system accounting for the mental operations he can perform, and their evolution over time? The neurons will figure among the components of MENS, but MENS has also other more conceptual objects, which we call *category-neurons* (abbreviated in *cat-neuron*), and which model mental objects (in the terms of Changeux, 1983) of various kinds associated to features of the environment, sensory and motor inputs, internal states, motor skills and various procedures, sensations and emotions, particular events, and so on. A cat-neuron can be thought of as a 'higher order' neuron, The problem is to describe exactly what is a cat-neuron, and how the evolutive system MENS is generated by its sub-system **Neur**, in particular how living and learning leads to the emergence of a hierarchy of cat-neurons modeling more and more complex mental objects and processes.

First we consider a particular kind of mental object, namely a mental image. A mental image corresponds to a long term memory of an item perceived by the sensory organs, say an object in the environment, through which the item can later be recognized or recalled by the animal. It will be represented in MENS by a cat-neuron which gives a record of the item, while keeping some plasticity.

For a simple object, say a small segment of a specific orientation, there is a neuron (the "simple cells" discovered by Hubel and Wiesel, 1962) in a visual area whose firing is specifically triggered by the sight of the object. Such a dedicated neuron may also exist for some complex objects, if they are often met by the animal and/or particularly important for him; for instance an angle triggers the firing of a "complex cell", and there are "place cells" in the hippocampus which have a direct firing with location specific areas (O'Keefe and Dostrovski, 1971). However there is no "grand-mother neuron" (Barlow, 1972).

Brain imagery has shown that more complex items are recognized through the coordinated activation of a whole pattern of neurons more or less distributed in the brain and interconnected by distinguished links (which can be synapses or synaptic paths). This pattern corresponds to an internal memory of the item (Stryker, 1989); its characteristic is that it can act as a *synchronous assembly of neurons* (Hebb, 1949). Here 'synchronous' means that all the neurons of the pattern are activated during the same cycle of the natural oscillation of the neural activity of the brain area to which they belong (e.g. 40Hz in the hippocampus, Fisahn *et al,* 1998); anyway the synchronization lasts only a short time (cf. Miltner *et al.*, 1999; Rodriguez *et al.*, 1999; Usher and Donnelly, 1998).

To act synchronously, the distinguished links of the pattern must have short propagation delays, and therefore great strengths, since delays and strengths are inversely proportional. The formation of such a pattern relies on the following rule, proposed by Hebb in 1949, and experimentally confirmed for synapses in many areas of the brain (*e.g.*, Engert and Bonhoeffer, 1997; Frey and Morris, 1997; Zhang *et al.*, 1998):

> *Hebb rule*: If s is a synapse from N to N' and if the activities of N and N' are simultaneously increasing, the strength w_s of s increases at the same rate. Conversely if the activities of N and N' vary in opposite ways, w_s decreases.

The mental image of an unknown item O will be generated as follows: the perception of O at a given time t activates the neurons of a specific pattern P, thus forming a short-term memory of O. By Hebb rule, their coordinated activation at t increases the strength of the distinguished links between them; and the same repeats at each successive perception of O. Thus, there is a progressive decrease of the propagation delays, which facilitates a coordinated firing of the whole pattern; and over time, the pattern P will take its own identity, being able to act as a synchronous assembly of neurons. In this way the short-term image of O has been consolidated in a long-term memory.

However we cannot identify the pattern as such with the mental image of O. Indeed, depending on the context, the same object O can activate more or less different patterns acting as synchronous assemblies of neurons, and these patterns are not necessarily interconnected (Edelman, 1989, p. 50). The importance of this property has been emphasized by Edelman who speaks of the "*degeneracy* of the neuronal encoding"; we will see later how it is at the root of the emergence of higher mental processes. These other patterns also participate in the mental image of O. This image must be thought of as the invariant that all these patterns P have in common, namely they all have the same functional role, meaning that they can activate the same neurons, and with the same strength; we say that they are *homologous*. This invariant will be modeled by a category-neuron.

2.4 Category-neurons

If O is a simple object or a complex object of importance for the well-being of the animal), the invariant corresponds to a particular neuron. Indeed, in this case there is a neuron which activates the same neurons, and with the same strength, as anyone of the patterns P which are activated by the perception of O, hence which participate in its mental image. This neuron, called the *binding* of P, becomes the mental image (or *record*) ImO of O, and O is later recognized or recalled through the firing of ImO.

For more complex objects, there is no such neuron, and the mental image will be modeled by a cat-neuron, component of MENS. How does one explicitly define a cat-neuron?

In **Neur**, a *pattern* P of neurons is a family of neurons P_i interconnected by some distinguished links (*i.e.*, synaptic paths) through which they may transmit their activation to each other. A collective interaction of P is modeled by a *collective link* from P to a (cat-)neuron N'; it is a family of links s_i from P_i to N', correlated by the distinguished links of P, so that they may collectively activate N'. We model the fact that two patterns are homologous (i.e., have the same functional role) by the fact that there is a 1-1 correspondence between their collective links to any (cat-)neuron N'. If P has no binding *neuron,* it may have a *binding cat-neuron* N in the following sense: the collective links (s_i) from P to any cat-neuron N' are in 1-1 correspondence with the links s from N to N' (in categorical terms, N is a *colimit* of the pattern in MENS; cf. Appendix); in other terms, P as a whole and its binding N have the same functional role (cf. Figure 2). In this case, N is also the binding of any pattern Q homologous to P. The pattern P, as well as each other pattern Q that N binds is called a *decomposition* of N, and the passage from P to Q is called a *complex switch.* (cf. Figure 3).

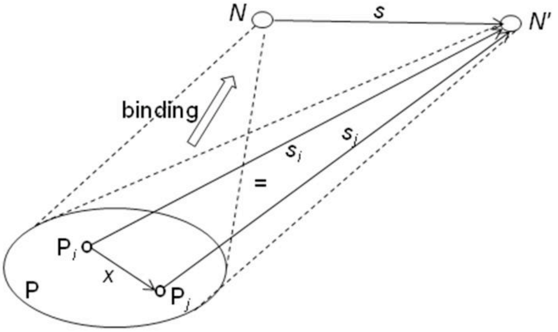

FIGURE 2. P is a pattern of neurons P_i with some distinguished links (synaptic paths) x between them. A collective link from P to a cat-neuron N' is a family of links s_i from each P_i to N' correlated by the distinguished links x. The pattern P admits a cat-neuron N as its binding if each collective link (s_i) from P to an N' binds into a link s from N to N', so that the whole pattern P and the cat-neuron N activate the same cat-neurons, with the same strength.

Let us come back to the item O, and let P be one of the patterns participating in its mental image. This image will be modeled by a new component ImO of MENS (hence a cat-neuron), called the *record* of O. This cat-neuron emerges as the binding of P; by definition of the binding, it will also bind the other homologous patterns which participate in the image. Since two decompositions P and Q of ImO are not necessarily interconnected (as said above), we say that ImO is a *multifold* component of MENS. Its activation at a time t (allowing to recognize or recall O) will consist in the synchronous activation of one of its decompositions, say P, and its activity is then a function of the global activity of the neurons of P.

Over time, ImO takes its own identity, independent of a particular decomposition. It may 'lose' one of its decompositions, for instance if lesions in the brain destroy a number of neurons of P, what remains of P may become too small for keeping the same functional role, and ImO will no longer be its binding. Conversely ImO may acquire a new decomposition Q; for instance if O progressively changes (e.g., a person who ages), the assembly of neurons synchronously activated by O changes slowly, though remaining a decomposition of ImO. However if the change in O becomes too large or sudden, ImO will not remain the image of (what has become) O.

To sum up, ImO is initially constructed to bind a particular pattern P of neurons and thus become the image of O. Later it takes its identity and can even disassociate from P. Thus it is not a rigid record (as in a computer), but offers a flexible memory which adapts to changing situations. The multiplicity of its decompositions ensures that the animal is able to recognize or recall the object under different forms, even new forms he has not yet met, as long as the change is progressive enough.

ImO is a cat-neuron 'of level 1'. More generally a *cat-neuron N of level* 1 will bind a class of homologous patterns of neurons. It is constructed at a time *t*, through a complexification process (cf. section 4) to bind a given pattern of neurons, so that it can act as a synchronous assembly of neurons. Later on, *N* takes its own identity, possibly acquiring other decompositions which are not necessarily interconnected with P.

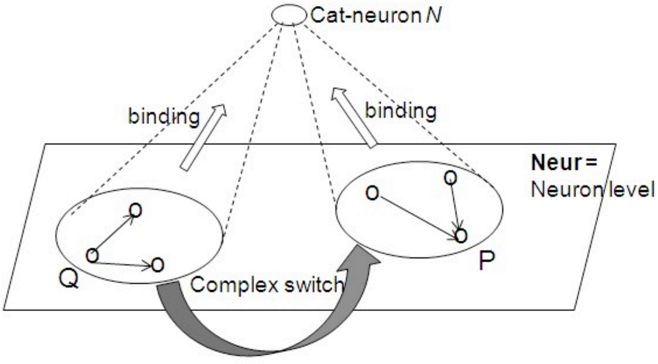

FIGURE 3. The cat-neuron *N* is the binding (or colimit) of the two non-interconnected patterns P and Q. However the fact that P and Q are homologous cannot be observed 'locally' at the level of the neurons in P and Q. The complex switch from P to Q emerges at the higher level of cat-neurons (as indicated by the twisted "Môbius band' style of arrow that crosses the Neur level); however it expresses a global property of Neur, namely that, for any cat-neuron *N'*, there is a 1-1 correspondence between the collective links from P to *N'* and the collective links from Q to *N'*.

It is important to realize that, as a multifold component of MENS, a cat-neuron *N* has 'emergent' properties, that is, properties not observable locally from inside the neuron level **Neur**, though a consequence of its global structure. Indeed, if P is a decomposition of *N*, to recognize that another pattern Q is also a decomposition of *N*, we must verify that P and Q are homologous, meaning that P and Q activate the same neurons and with the same strength. If P and Q are non-interconnected (there is no cluster of links between them), this verification has to take account of the whole structure of **Neur**, and not only of the links between neurons of P and Q. Thus the existence of a complex switch between P and Q expresses a 'global' property of **Neur**, which emerges as something new at the cat-neuron level 1; in the figures a complex switch will be represented by a twisted ('Möbius band' type) arrow between P and Q, crossing over the neuron level. In spite of its emergent properties, the cat-neuron relies on the physical basis of its different decompositions, and may produce physical effects through the synchronous activation of anyone of its decompositions.

3 The category-neurons and their links

The neurons will be identified to cat-neurons of level 0. The cat-neurons of level 1 that we have just defined bind patterns of neurons. To model more complex mental objects, we have to iterate the construction and form cat-neurons of increasing complexity levels, binding patterns of cat-neurons of lower complexity. This confronts us with two problems:

1. The "binding problem" which in MENS translates into: how do cat-neurons interact?
2. How do the "algebra of mental objects" emerge from the neural system?

3.1 Clusters and Simple links

If we think of a cat-neuron as a kind of 'virtual' higher order neuron modeling a mental object, what will correspond to 'virtual' synapses or synaptic paths between cat-neurons? A cat-neuron *N* emerges from the neuron level to bind a pattern P of neurons; the links between cat-neurons should also emerge from this level. The first idea is that a link from *N* to another *N′* will bind (in some sense) a *cluster* of links between the neurons of P and those of a decomposition P' of *N′*. So the first step is to find the properties that such a cluster should verify to be able to collectively activate P'. In particular, these clusters model the 'good' interactions between synchronous assemblies of neurons, thus giving a solution to the binding problem as it has been stressed by several neuroscientists, in particular von Malsburg (1995; von Malsburg and Bienenstock; 1986).

If P' is reduced to one neuron N', the pattern P activates N' if all its neurons simultaneous activate N in a coherent way; thus a cluster from P *to* N' is just a collective link from P to N' (as defined above), that is a family of links from the neurons P_i of P to N' correlated by the distinguished links of P which may operate synchronously to activate N'. If P has a binding cat-neuron *N*, this collective link binds into a link from *N* to N'.

If P is reduced to a neuron N, a cluster from N *to* P', also called a *perspective* of N for P' (this terminology will be explained later) is a maximal set of links from N to some neurons of P' which are correlated by a zig-zag of distinguished links of P', so that N can synchronously activate a well connected sub-pattern of P' (but not necessarily the whole of P').

In the general case, we define a *cluster* from P to P' as a collective link of perspectives from the different neurons of P to P' (cf. Appendix).

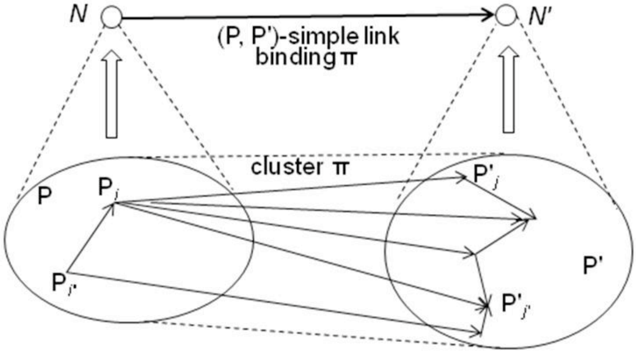

FIGURE 4. P and P' are two patterns; a cluster π from P to P' is a maximal set of links such that: (i) the links in the cluster from each P_i of P are correlated by a zig-zag of distinguished links in P' (they form a perspective of P_i for P'); and (ii) it is closed by composition with distinguished links of P.
If *N* and *N' are* cat-neurons binding P and P', the cluster binds into a (P, P')-simple link from *N* to *N'*; this link activates *N'* with the same strength as P acting collectively through the cluster.

If P and P' are respectively decompositions of the cat-neurons *N* and *N'*, a cluster from P to P' binds into a link from *N* to *N'* in MENS; we call this link a (P, P')-*simple link* (cf. Figure 4). Such a link just sums up the activation that the links in the cluster individually transmit from neurons of P to neurons of P', thus it is entirely reducible to the neuron level. It is completely dependent on the chosen decompositions P and P' and takes no account of the possible other decompositions of the cat-neurons: a (P, P')-simple link may not be (Q, Q')-simple if Q and Q' are other decompositions of *N* and *N'*. In particular the identity of *N* is (P, P)-simple for each decomposition P of *N*, but it is (P, Q)-simple only if P and Q are interconnected (meaning more precisely that there is a cluster between P and Q binding into the identity of *N*).

3.2 Emergence of complex links

The simple links have nothing to do with the emergent properties of *N* and *N'* due to their multiple decompositions. However, the existence of multiple decompositions accounts for the emergence of other links from *N* to *N'*, called *complex* links, which are not simple for any decomposition of *N* and *N'*. Their existence comes from the fact that a multifold cat-neuron *M* may have two non-interconnected decompositions Q and Q'. If we have a (P, Q)-simple link S from *N* to *M* and a (Q', P')-simple link S' from *M* to *N'*, they compose into a complex link SS' from *N* to *N'*; this link transmits the same activation to *N'* as that transmitted by S' when M is activated by S (cf. Figure 5).

More generally, a composite of simple links binding non-adjacent clusters, connected by complex switches is a complex link. A composite of complex links is generally a complex link (though it can sometimes be simple for particular decompositions of the extreme cat-neurons it connects).

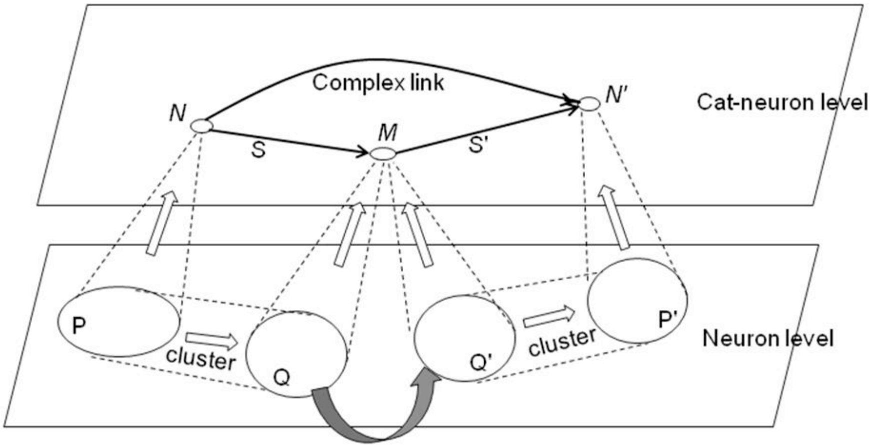

FIGURE 5. Q and Q' are two non-interconnected decompositions of the cat-neuron *M*, the complex link is the composite SS' of the (P, Q)-simple link S and the (Q', P')-simple link S'. It emerges at the cat-neuron level 1 and has properties which are not deducible from local properties of links between P and P', though coming from global properties of the neuron level.

We define the *propagation delay* of a (P, P')-simple link as the maximum of the propagation delays of the links in the cluster, and its strength is an increasing function of the strengths of these links. The propagation delay of a complex link is the sum of the propagation delays of its composing simple links, and its strength is an increasing function of their strengths. We have shown that *Hebb rule* generalizes to cat-neurons connected by such links (cf. Ehresmann and Vanbremeersch, 1999, 2007).

A (P, P')-simple link S is entirely determined by the cluster it binds, hence can be observed directly at the level of the neurons in P and P', and transmits only information already mediated through them. On the other hand, a complex link SS' conveys more 'global' information, since it makes use of a complex switch between the two non-interconnected decompositions Q and Q' of the intermediate cat-neuron *M*. As explained above, the existence of a complex switch is an emergent property of the global structure of **Neur**, even if it can sometimes be experimentally observed (e.g. if *M* models the mental image of an object O, its decompositions correspond to the patterns activated by O). Thus, a complex link from *N* to *N′* relies on properties of the whole level of neurons; it is not a reflection of local properties with respect to particular decompositions of *N* and *N′*. In this sense, it 'emerges' at the level of cat-neurons, but it does not appear 'ex machina', it just actualizes at the higher level a *global* property of the lower level.

The category of neurons is extended in a larger category by adding the cat-neurons of level 1 and the simple and complex links so constructed.

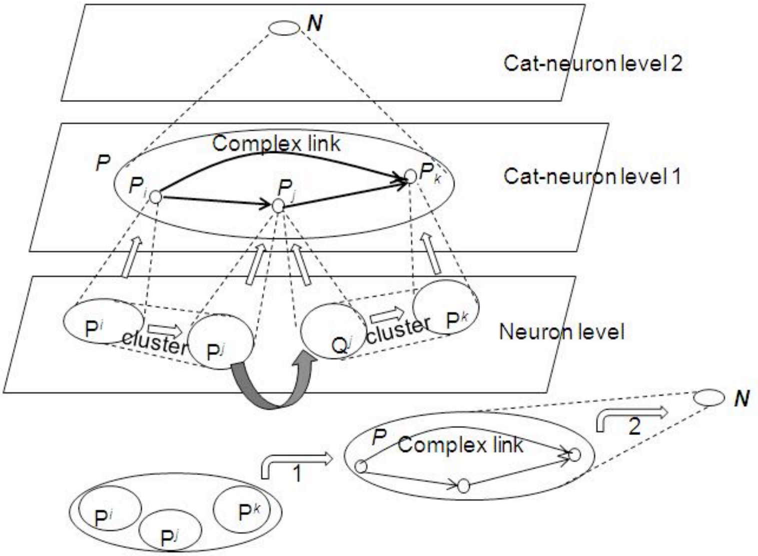

FIGURE 6. ***N*** is a cat-neuron of level 2 binding a pattern *P* of cat-neurons P_i of level 1, one of which P_j has 2 non-interconnected decompositions P^j and Q^j. Thus ***N*** admits 2 ramifications down to the neuron level, one with P^j at the end, the other with Q^j; and it can be activated by anyone of them, for example the first one. This activation necessitates 2 steps: (i) Simultaneous activation of one decomposition P^i of each cat-neuron P_i which forces the activation of this cat-neuron; (ii) The synchronous activation of all the cat-neurons of *P* leads to the activation of *P* (directly at the level 1 because of the complex link), and therefore of ***N***.

3.3 Higher level cat-neurons

The mental image of an item has been constructed as a cat-neuron of level 1, which binds the synchronous assemblies of neurons activated by the item. Higher animals are able to operate on mental objects to form more complex ones. For instance they can form a mental image of a complex object by decomposing it in smaller parts which are recognized, and combining the images of these parts. Or they can learn to perform a new motor skill, by combining more elementary skills already known. In MENS, this corresponds to the construction of a cat-neuron of level 2. Since we have defined what the simple and complex links between cat-neurons of level 1 are, we can speak of patterns of such cat-neurons, of their collective links, and of their binding (literally defined as for neurons), and we easily imitate the construction of cat-neurons of level 1, just replacing the patterns of neurons by patterns of cat-neurons of level < 2. Roughly we can 'compute' with cat-neurons of level 1 as if they were neurons.

Let us develop the construction of a cat-neuron of level 2 modeling a mental image of an object C formed by the juxtaposition of several objects O_i that the animal can already recognize (cf. Figure 6). When the animal perceives it for the first time, C simultaneously activates synchronous assemblies of neurons P^i corresponding to the various O_i. It follows an activation of their records ImO_i. A scan of the object C shows how the O_i are associated in C, and in MENS their inter-relations are modeled by links between the records ImO_i. The records and these links form a pattern *P* of cat-neurons synchronously activated by C. The mental image of C will be modeled by a new cat-neuron ImC added to MENS for binding this pattern (in categorical terms it becomes a colimit of *P*); we say that (P, (P^i)) is a *ramification* of ImC. As for cat-neurons of level 1, the record ImC takes its own identity and may acquire various homologous ramifications obtained by replacing each P^i by a homologous pattern of neurons, or *P* by a homologous pattern of cat-neurons. Thus it is not a rigid record, but can adapt to small modifications of C. The later recognition of C consists in the activation of ImC through the unfolding of one of its ramifications, which necessitates 2 steps:

1. first simultaneous activation of the various P^i which leads to the activation of their bindings P_i;
2. then activation of the distinguished links between the P_i to synchronously activate the pattern *P*.

More generally, a cat-neuron ***N*** of level 2 is the binding of a pattern of cat-neurons of level < 2, the distinguished links between them being either simple or complex. It emerges (in a complexification process, cf. Section 4) to bind such a pattern acting as a synchronous assembly of cat-neurons. It also binds all the patterns homologous to *P*; among them some can be non-interconnected. Later it takes its own identity, possibly independent from *P*, and may acquire other decompositions. As above, ***N*** has *ramifications* down to the neuron level; a ramification (P, (P^i)) consists in a decomposition *P* of ***N*** in cat-neurons P_i of level < 2, and for each P_i one of its decompositions P^i in neurons. In other terms, ***N*** binds the synchronous assembly of assemblies (or *super-assembly* of neurons) formed by the neurons of the different assemblies P^i with their distinguished links in *P*. Thus ***N***, as a component of MENS, is a conceptual unit, but its later activation (or recall) corresponds to the dynamic unfolding of one of its ramifications which activates the corresponding super-assembly of neurons; the unfolding is done in two steps: simultaneous activation of the various P^i, followed by their synchronization through the links of *P* to activate ***N***. An experimental example of this process has been observed in odor encoding (Wehr and Laurent, 1996). Let us note that at each step we have multiple choices: choice of a decomposition *P* of ***N***, then choice of a decomposition P^i for each cat-neuron P_i of *P*; that gives numerous degrees of freedom to the cat-neuron ***N***, allowing for an adaptation to various contexts.

Simple links between cat-neurons of level ≤ 2 are defined as for cat-neurons of level 1. Since a cat-neuron of level 2 may have non-interconnected decompositions, there will also exist complex links composing simple links binding non-adjacent clusters. The propagation delay and strength of these (simple or complex) links are computed as in the level 1. Thus the construction can be iterated to construct cat-neurons of level 3, and progressively cat-neurons of increasing levels, modeling more and more complex mental objects.

4 The memory evolutive neural system MENS

Above we have iteratively defined cat-neurons as binding synchronous assemblies of cat-neurons of lower levels. These cat-neurons model more or less complex mental objects, such as mental images of features in the environment, behaviors or internal states. Now we have to explain how the comportment of the animal in his environment, through his successive physical, affective or social experiences, promotes the construction of a particular cat-neuron rather than another, and how it leads to the progressive construction of a hierarchical evolutive system MENS over his life.

4.1 MENS as an evolutive system

In Section 2 we have seen that **Neur** is an evolutive system, its timescale being the life of the animal. It is the same for MENS. At a given time t of his life, the category MENS_t models the (neural and) cognitive system of the animal; its objects are the states at t of the cat-neurons existing at this date, its links are the simple and complex links connecting them around this date. The transition from t to t' keeps trace of the change of state of these cat-neurons and links.

NEUR	MENS
Neurons	Cat-neurons
Synapses	Simple links
Synaptic paths	Complex links

4.2 The complexification process

At birth, the neural system of the animal is practically formed, and he has some innate mental objects associated to some simple actions (breathing, eating, sleeping,...), to specific features of his environment important for his survival (e.g. cues to recognize predators or preys) and to adapted behaviors (sucking, running away from a predator, catching a prey). Thus, the category MENS_0 modeling his cognitive system at this date contains, in addition to Neur_0, the (states of the) corresponding cat-neurons (probably of level at most 2) and their links. The evolution of the neural and mental system of the animal during his life depends on his successive sensory, proprioceptive, motor, mental, affective and cognitive experiences; they give rise to the formation of new mental objects (e.g., formation of new mental images of objects he perceives) and processes (new motor skills, behaviors), and possibly to modifications or even destruction of others (if they are no more adapted).

In terms of cat-neurons, this evolution consists of realizing some objectives of the following kinds:

1. formation (or preservation, if it already exists) of a cat-neuron binding a given pattern P of cat-neurons; it forces the strengthening of the distinguished links of P, so that P can act as a synchronous assembly of cat-neurons; the formation of a mental image ImO is an example;
2. formation of a new neuron or of new links;
3. elimination of a cat-neuron (e.g., loss of a neuron, destruction or modification of a record if it is no more adapted).

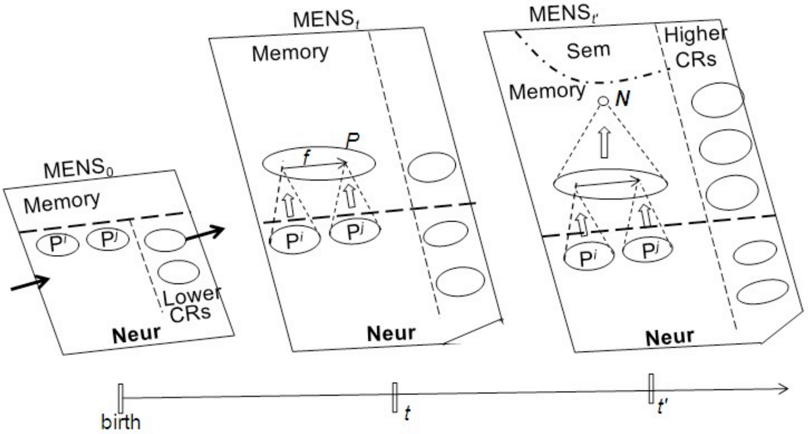

FIGURE 7. Development of MENS during the life. At birth MENS consists essentially of neurons in **Neur** and a few innate cat-neurons in the memory, there are only lower coregulators in Neur. By complexification, MENS extends; at t, various patterns P^i have acquired a binding, and simple or complex links such as f have emerged between their bindings; a few neurons have been lost. Another complexification process adds a cat-neuron N of level 2, binding the pattern P. More higher coregulators emerge, and the records begin to be classified in the semantic memory Sem.

To model the evolution of MENS, say from a date t to a later date t', we suppose that the category $\text{MENS}_{t'}$ is constructed as the *complexification* of MENS_t with respect to a procedure having specific objectives of the above kinds (cf. Figure 7); it means that these objectives are realized in the 'best way' in $\text{MENS}_{t'}$. We have given an explicit description of this category and of the corresponding transition functor from MENS_t to $\text{MENS}_{t'}$ (Ehresmann and Vanbremeersch, 1987, 2007) partially recalled in the appendix. Essentially the complexification fulfills these objectives in the 'most economical' way (in terms of energy and time); the links between the cat-neurons are both the simple links and the complex links as defined in Section 3.

Finally MENS is deduced from **Neur** by successive complexification processes. For higher animals, the procedures may have another kind of objective which will be explained later (Section 6).

4.3 The hierarchy of cat-neurons and their complexity order

The animal has a hierarchy of mental objects, from the mental image of a simple object modeled by a single neuron, to mental objects activated by a synchronous assembly of neurons, up to more complex mental objects combining more elementary ones. Translated in MENS, it means that MENS is a *hierarchical evolutive system* (cf. Appendix). Indeed, by construction:

- the cat-neurons are divided into several levels: the neurons figure at the level 0, the cat-neurons binding an assembly of neurons figure at level 1, the cat-neurons binding a super-assembly of neurons (constructed in 2 steps) at the level 2, and so on;
- a cat-neuron of level n binds at least one pattern of strictly lower levels cat-neurons.

A cat-neuron ***N*** of level n emerges in a complexification process to bind a pattern of cat-neurons of strictly lower levels (hence $< n$) which acts as a synchronous assembly, as well as all the homologous patterns. Later it takes its own identity and may acquire new decompositions in patterns of cat-neurons of lower levels. It is multifold, in the sense that two of its decompositions, say P and P', can be non-interconnected (categorically, the identity of ***N*** is not a (P, P')-simple link). As explained for cat-neurons of level 2, a cat-neuron of level n admits *ramifications* down to the neuron level. A ramification consists of a decomposition P of **N,** then a decomposition P^i of each cat-neuron P_i of P, then a decomposition of each cat-neuron of the various P^i, and so on down to decompositions in patterns of neurons. The activation of **N,** at a given time, consists in the unfolding of one of these ramifications, through a stepwise process, with multiple choices at each intermediate level, ultimately activating a synchronous assembly of assemblies... of assemblies of neurons, abbreviated in *synchronous hyper-assembly of neurons*. For example, to activate the record (and recognize) an ambiguous image, such as the duck-rabbit, we can use either a ramification that activates the duck record, or one that activates the rabbit record.

Thus, the plasticity of a cat-neuron, or of the mental object which it models, increases with the length of its ramifications, each step adding new degrees of freedom. For a cat-neuron ***N*** of level n, this length is generally n, since we have constructed the cat-neurons of level n in n steps: first cat-neurons of level 1 binding patterns of neurons, then cat-neurons of level 2, up to those of level n binding patterns of strictly lower levels. However ***N*** may have (or later acquire) ramifications of length strictly less than n. For example a cube C may have initially be decomposed in 6 squares, each modeled by a cat-neuron of level 1, and ImC which binds the pattern they form will be of level 2; but the cube can also be decomposed in its 12 edges, each having for record a unique neuron (a 'simple cell'), and ImC can be obtained in 1 step, as the binding of the pattern of these neurons. Thus, a cat-neuron of level n may sometimes be activated (through some of its ramifications) in less than n steps, and the 'real' complexity of a cat-neuron is related the minimum length of a ramification down to the neurons rather than the level.

To define this complexity, we define the *(complexity) order* of a cat-neuron ***N*** as the smallest *k* such that ***N*** binds (in one step) a pattern of cat-neurons of levels strictly less than *k*. If this order is strictly less than the level, the cat-neuron is *k-reducible*. The above example of a cube shows that its record is 1-reducible, and its order is 1. An example of a cat-neuron of order 2 is given by the record of a Möbius band, obtained by binding a pattern of triangles (this example is taken from Ryan, 2007). The total record of an ambiguous image, such as the duck-rabbit, is of strictly higher order than the records of the duck and of the rabbit taken separately.

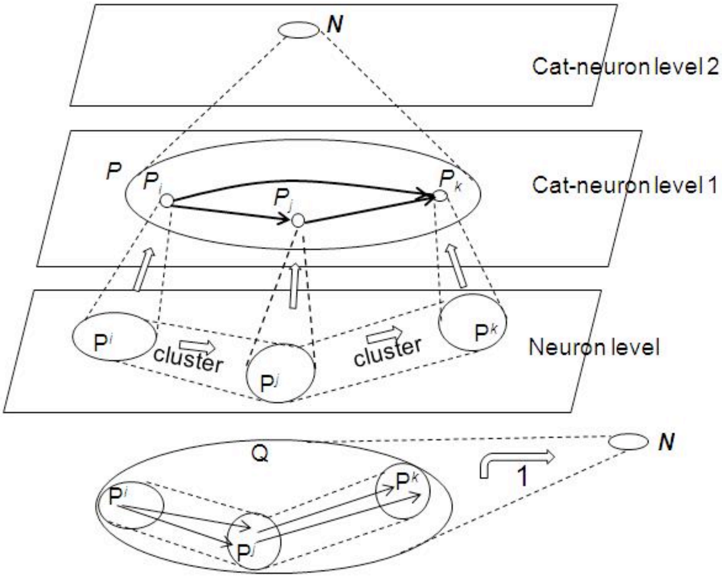

FIGURE 8. ***N*** is a cat-neuron of level 2 binding a pattern *P* of cat-neurons of level 1 in which all the distinguished links are simple. It is also the binding of the pattern Q of neurons containing all the decompositions P^i of the cat-neurons P_i of *P* and the links of all the clusters that the links of *P* bind. Thus ***N*** is of complexity order 1, and it can be activated in one step through the activation of the large pattern Q.

More generally we have proved the following result (Reduction Theorem, Ehresmann and Vanbremeersch, 2007, page 104): If ***N*** is a cat-neuron of level *n* binding a pattern *P* of cat-neurons in which all the distinguished links are simple links, its complexity order is strictly less than *n*; The following figure (Figure 8) illustrates this case. On the other hand, the cat-neuron ***N*** of level 2 binding *P* in the figure 6 of Section 3 is of order 2; it is not 1-reducible because one of the distinguished links of *P* is complex.

4.4 The multiplicity principle as the source of emergence

The construction of cat-neurons binding patterns of cat-neurons of lower levels, and of their simple and complex links, can be interpreted as a computation on mental objects, comparing and combining them to form more complex ones, thus it explains how to develop an algebra of mental objects (following the proposal of Changeux, 1983, p. 181).

It seems probable that most animals will only develop cat-neurons of complexity order less then, or equal to, 2. A characteristic of man is that he has the capacity of forming mental objects and cognitive processes of increasing complexity order. As we have seen, it relies on the possibility for a cat-neuron to admit several decompositions in non-interconnected patterns. This property generalizes to cat-neurons the degeneracy of the neuronal coding emphasized by Edelman (1989). Instead of 'degeneracy', we prefer to speak of 'multiplicity', saying that MENS satisfies the *Multiplicity Principle* (Ehresmann and Vanbremeersch, 1996). We have explained how the multiplicity (or degeneracy) at the neuron level extends to the cat-neurons of level 1. By iteration it extends to all the levels. (Categorically, the complexification process respects the multiplicity principle; cf. Appendix.)

To sum up, the root of the emergence of higher cognitive processes, up to consciousness (cf. Section 7) is the degeneracy of the neuronal coding. We had already shown this in 1996; later, Edelman and Gally (2001) have also insisted on the relation between degeneracy and emergence.

The neo-connectionist models of neural systems (following Hopfield, 1982), which operate at the sub-symbolic level, can only account for cat-neurons of level 1 (under the form of attractors of the dynamics). They cannot describe the interactions between attractors necessary to iterate the process and solve the binding problem at their level, leading to mental objects of increasing complexity. By contrast, the complexification process gives an explicit construction of the links, both simple and complex, between cat-neurons of any level, allowing for the binding of patterns of cat-neurons to construct more complex ones. Since cat-neurons model mental objects, it gives a solution, not only to the binding problem at the first level, but to a binding problem extended to each level. At the same time, the construction explains why usual methods in terms of assemblies of neurons fail for higher levels. Indeed, the correlation between a cat-neuron of level more than 1 and a synchronous hyper-assembly of neurons is intricate and non-univocal:

- It is intricate because the hyper-assembly of neurons is obtained via the dynamic stepwise unfolding of one of the ramifications of the cat-neuron **N** down to the neuron level; let us recall that a ramification consists of a pattern P of cat-neurons P_i of level $\leq$ n having **N** for its binding, each P_i binding a pattern P^i of cat-neurons of lower levels, and so on down to patterns of neurons. For instance for a ramification $(\mathrm{P}, (\mathrm{P}^i))$ of length 2, the neurons of the hyper-assembly are all the neurons of the various patterns P^i; but to recover the cat-neuron **N** we must also take into consideration the distinguished links of the pattern P; if some of these links are complex, they reflect global properties of the neuron level, not observable at this neuron level but actualized at the higher level (cf. Section 3).

- It is non-univocal (or 'degenerate') because a cat-neuron may have several ramifications, not necessarily interconnected.

In terms of mental states and brain states, this correlation gives a new approach to the *brain-mind problem* (cf. Section 8).

5 Self-regulation of the dynamics

MENS is more than an evolutive system, In Ehresmann and Vanbremeersch (2007) it is presented as an example of a memory evolutive system. We have introduced the memory evolutive systems in 1991 to model autonomous complex natural systems, such as biological or social systems. They are evolutionary systems with an auto-organization directed by a net of internal regulatory organs, called coregulators with only partial information on the system; they have the capacity to learn from their experiences by recording them in a memory from which they can be recalled later in analog situations. The coregulators direct the dynamics of the system in a more or less cooperative/competitive manner, with recourse to the central memory.

5.1 The memory and the coregulators

In MENS, the memory models the long-term memory of the animal. It is a hierarchical evolutive sub-system whose cat-neurons are called *records*, It is divided into:

- the empirical memory, itself divided into: (i) the perceptual memory whose records model the mental images of items perceived by the sense organs, or internal states (hunger, pain, joy, ...) and (ii) the episodic memory with records of particular events or personal experiences,
- the procedural memory with records of motor skills, behaviors or procedures. A record Pr in it operates through the activation (via 'commands') of a pattern E_{Pr} of cat-neurons modeling its effectors; these cat-neurons operate internally by activating other cat-neurons, or act externally (e.g., by activating muscles). The result of the activation by Pr of its effectors is recorded by a cat-neuron N which binds the pattern E_{Pr} (cf. Figure 9).
- Higher animals develop other kinds of memory: a semantic memory, and the archetypal core, a personal memory at the basis of the self (cf. sections 6 and 7).

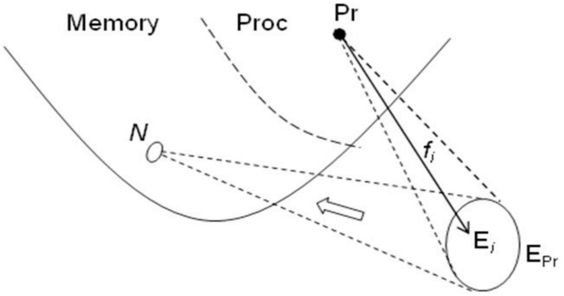

FIGURE 9. A record Pr in the procedural memory Proc can activate a pattern E_{Pr} of cat-neurons, its effectors, through links f_j (commands) from Pr to effectors E_j. The result of this activation of its effectors is memorized by the record *N* which binds the pattern. $E_{Pr.}$

As said above, at birth MENS is somewhat reduced, and the animal must progressively learn to recognize more objects, to perform more complex comportments, possibly to evaluate their results and remember them for best adapting to various situations. As a result, MENS is progressively extended by the formation of more cat-neurons, obtained through successive complexifications. This extension is not pre-programmed, but will be internally directed by the animal.

The whole comportment of the animal, his actions, his internal states, the formation of his mental objects are all dependent on his nervous system, hence reflected in the evolutive system MENS which it generates. The regulation system responsible for his comportment must be modeled in MENS. However there is no internal 'homunculus' able to have a global vision and to impose its choices. The control is distributed among a net of internal regulatory organs, the *coregulators*, able to collect partial information on the internal and external situation, select appropriate procedures, command their realization, evaluate their results and later participate in their recording in the memory.

A *coregulator* is an evolutive sub-system of MENS based on a particular more or less extended part of the neural system; here based means that its cat-neurons (called its *agents*) have ramifications whose lowest cat-neurons are in this part, so that they are activated by (hyper-)assemblies of neurons situated in this part. The existence of a kind of modular organization in the brain (as emphasized by Fodor, 1983) is now generally accepted; among the possible bases for a coregulator we distinguish various modules: systems of receptors (for vision, audition, smell,...); systems of internal or external effectors (connected to muscles); more or less specialized dedicated areas (in the visual areas, motor cortex, hippocampus, temporal cortex, brain stem, limbic system,...), but also smaller ones (e.g., the treatment units considered by Crick (1994) in vision, such as a color module processing colors). The coregulators are more or less complex depending on the complexity order of their agents; we speak of lower coregulators (their agents model neurons or assemblies of neurons), and higher coregulators.

Each coregulator operates by steps, delimited by its own discrete timescale; the duration of a step is related to the propagation delays of the links which activate its agents and to their refractory periods. It has a differential access to the (central) memory and has a characteristic 'function', determined by its *admissible procedures*, which model the actions it may command:

- For a lower coregulator, they may just consist in an automatic transmission of the information received (through the activation of some agents) from other more complex coregulators or from effectors. For example, a color module will transmit the various characteristics of the colors it perceives to higher visual areas.
- For other coregulators, there are particular admissible procedures modeled by records in the procedural memory with the following properties: they can activate some of the agents of the coregulator; and conversely (some of) the effectors of their objectives can be commanded through agents of the coregulator. The coregulator also participates in the later storage in the memory of the new information it has received, the responses it has triggered and their result.

The dynamics of MENS depends on the 'local' procedures of its coregulators, but the 'global' operative procedure which will really be implemented at a given time is the outcome of an equilibration process which makes the commands sent by the various coregulators as coherent as possible, with a possible fracture for those whose procedure will not be realized.

5.2 Local dynamics of a coregulator.

We have said that a coregulator has its own discrete timescale; a *step* of the coregulator extends between two consecutive instants, say t to t', of this timescale. Let us describe the unfolding of this step for a particular regulator. During this step, the coregulator can be looked at as the pattern CR of its agents and their distinguished links. The step is divided in 3 more or less overlapping phases, the two first ones add up to the *actual present* of CR during which its agents are activated, the last one corresponding to their refractory periods.

5.2.1 First phase: Collect of information (or decoding)

During this phase, CR collects information on the internal state of the animal and/or the external situation, thus forming what we call its *landscape* at t. This information consists in the activation of some agents by various cat-neurons. For instance, if CR is based on a color module and the animal perceives a blue object at t, the 'blue' agents are activated by the record of the object, but a 'red' agent is not. The aspects of the system which can be seen by the coregulator are modeled by the links b from a cat-neuron N to an agent of CR; this aspect is *t-activated* if N activates this agent along b during the actual present. The distinguished links between agents are supposed to be strong enough to transmit this information, so that the whole perspective b^* of N generated by a t-activated aspect b is formed of t-activated aspects; we say that the perspective is *t-activated.* The same N may have several t-activated perspectives for CR. Two t-activated perspectives of N and of N' are correlated if there is a link from N to N' correlating their aspects. If there is a cat-neuron binding the coregulator, a t-activated perspective b^* of N binds into a link cb^* from N to cr in MENS; this link activates cr during the actual present.

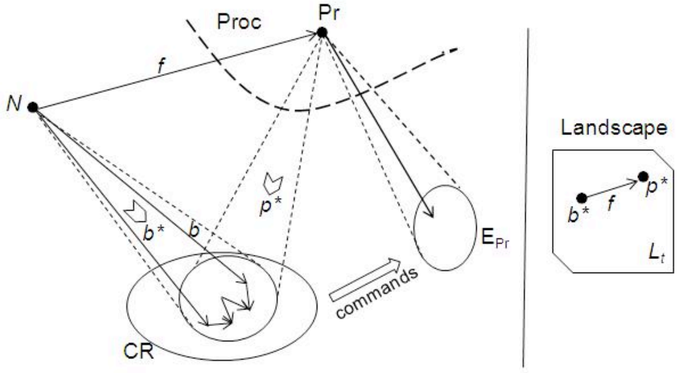

FIGURE 10. A perspective b^* of N for a coregulator CR is a maximal set of links (or aspects) b from N to agents of CR which are correlated by zig-zag of links in CR. It is t-activated if N activates the agents along all its aspects around t. A link f correlates two t-activated perspectives if it correlates their aspects. Here f is an activator link from N to the record Pr of an admissible procedure for CR. When b^* is t-activated, N activates Pr along f, and the perspective p^* of Pr is t-activated. CR can select Pr, and send the corresponding commands to effectors of Pr. The t-activated perspectives and their links form the landscape L_t of CR at t, modéled (on the right) by a category.

The t-activated perspectives and the links which correlate them form the *landscape* of the coregulator at t (cf. Figure 10); it can be compared to the 'perspective space' of Russell, 1971, from which we have taken the word perspective. The landscape accounts only for the part of MENS which is perceived by the coregulator during its actual present; CR can only collect information from the pattern of the cat-neurons N which have a t-activated perspective for CR. Categorically, we model the landscape by a category L_t and measure the loss of information for the CR by the *difference functor* from L_t to MENS which maps a t-activated perspective of N on N.

5.2.2 Second phase: selection of a procedure

Depending on the information received by the coregulator in its landscape, an admissible procedure is selected to respond to the situation. For a lower coregulator, the selection is automatic; for instance a color module will recognize a blue object via a perspective, and transmit this information to higher visual areas. Higher coregulators have admissible procedures recorded in the procedural memory. Some of them may have a t-activated perspective; in this case, one of them (generally the one whose perspective has the greatest strength) is selected.

In particular, the selection takes account of earlier experiences which have been memorized. If a similar situation has already occurred and a successful procedure has been used, this result is recorded via the formation of an *activa-*

tor link f from a record N of the situation to the record Pr of the procedure; this link correlates their perspectives for CR. In this case the activation of N leads to that of Pr, so that the perspective of Pr is t-activated, prompting CR to select the procedure. For instance, the recognition of a predator (activation of its record) activates an escape procedure.

If the situation is unknown and there is no admissible procedure with a t-activated perspective, a procedure already used in a not too different situation can be adapted, or a new procedure formed. There is a *fracture* if the step must be interrupted because no procedure is found.

5.2.3 Third phase: command and evaluation

The agents transmit the objectives of the selected procedure, in particular activating the effectors which they can command. This activation is carried through the unfolding of a particular ramification; for instance a specifically adapted motor gesture is chosen to uplift an object. The result is evaluated at the end of the step, by comparing the new landscape $L_{t'}$ with the *anticipated landscape,* in which the objectives of the procedures would be realized. If all the objectives are not achieved, they are more or less different, and this difference will have to be compensated later on to avoid a fracture. (In categorical terms, the anticipated landscape is the complexification AL_t of L_t with respect to the procedure, and the difference is measured by a *comparison functor* (if it can be formed) from AL_t to $L_{t'}$).

At the next step, the coregulator will participate to the formation of a record of the situation (if it had not yet been learned), of the procedure used and of its result. This result can be recorded by the formation (or, if it already exists, the strengthening) of an activator link from the record of the situation to that of the procedure.

One important cause of fractures is the non-respect of the *structural temporal constraints* which relate the length of the step, the mean propagations delays of the links in the t-activated perspectives, and the stability spans of the activated cat-neurons (cf. Ehresmann and Vanbremeersch, 1996, 2007). Indeed, there must be time enough for circulating the information among agents, selecting a procedure and sending commands to effectors, during which the necessary cat-neurons must be able to be activated. For instance, if the animal does not see a predator soon enough, a procedure of running will not be started in time for the animal to escape.

If these constraints cannot be met during successive steps, there is a *dyschrony*, which might necessitate a change in the period (mean duration of the successive steps) of the coregulator; we speak of a *resynchronization*. For instance, when he ages, the animal will move more slowly.

5.3 Global dynamics

At an instant t of his life the animal is confronted with various events in his environment and must produce adapted responses. The situation is analyzed through his different coregulators, which select a procedure and transmit its objectives to effectors. If all these objectives are compatible, they are all carried out. However, they are not always compatible, because some objectives of the various procedures can be conflicting or not realizable; *e.g.* if a coregulator sends a command to activate an effector which is simultaneously inhibited by another coregulator. For instance, a motor gesture to uplift an object can be interrupted by a higher coregulator which measures that it is not well directed, or because the object is too heavy. The problem is that the coregulators must operate coherently while there are many reasons for their selected procedures not to be compatible: the coregulators receive only partial information on the global situation through their landscapes; they function at their own rhythm and with specific structural temporal constraints; and they compete for the common resources of the system.

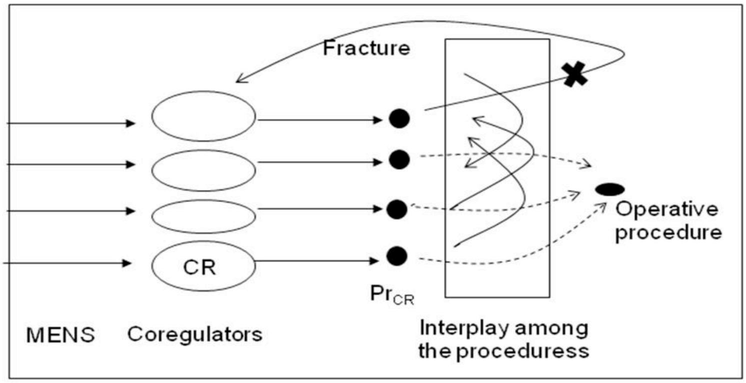

FIGURE 11. At a given time, there is an interplay among the procedures selected by the various coregulators to harmonize their objectives. The operative procedure which will be carried out is the result of this interplay. It will cause a fracture to a coregulator when (some of) the objectives of its procedure are not retained.

Thus there is a need for an equilibration process, or *interplay,* among the procedures, to determine a global *operative procedure* keeping as much as possible of the objectives of the various coregulators (cf. Figure 11). It is this operative procedure which will be finally carried out, possibly causing fractures to coregulators whose objectives are not realized. Categorically, the transition is modeled by the complexification of $MENS_t$ with respect to this operative procedure.

There is no general rule to determine the operative procedure. The interplay among the procedures takes into account:

- the strengths of the commands sent by the various coregulators; the objectives of coregulators with higher order agents generally prevail (the role of 'intentional' coregulators will be discussed later on, cf. Section 7);
- the plasticity of the cat-neurons allowing the unfolding of one ramification rather than another for activating a particular cat-neuron. For

instance, to catch a flying ball, there must be an adaptation of the various motor commands to the position of the ball, which amounts to complex switches between ramifications of the motor effectors;

- the temporal constraints which impose a kind of dialectic between two coregulators with very different rhythms. The coregulator CR, with a longer period, cannot be informed in real time of the small changes due to a lower coregulator with a much shorter period, because of the propagation delays or because they do not individually affect the stability of higher cat-neurons. However, the long term accumulation of small changes makes the unchanging landscape of CR more and more unreliable, ultimately causing a fracture to CR; for instance a fall can be due to a sequence of ill-adapted small equilibration gestures. To repair its fracture, CR will have to initiate a new procedure, which may retroact sooner or later on the lower coregulator (cf. Ehresmann and Vanbremeersch, 1996).

5.4 Development of the memory

During his life, the memory of the animal develops; he learns to recognize more items, to perform new skills; he memorizes successive experiences, his responses to them and their results. Let us see how the different coregulators participate in the formation of a new record. It will be a good illustration of the interplay among the procedures discussed above.

How can the animal memorize a new item O? In section 2 we have said that O synchronously activates a pattern of neurons (or of cat-neurons for more complex items); this pattern is consolidated at each later occurrence of O, its links being strengthened according to Hebb rule. The record of O will be a binding of this pattern. The formation of this record is done by the various coregulators which get different information on O; for instance for a physical object, different coregulators treat its color, shape, direction, possibly its odor, the noises it emits; for a motor skill, motor modules associated to different parts of the body must cooperate. Each coregulator CR will record the information received in its landscape and form a new cat-neuron called the CR-*record* of O; the operative procedure resulting from the interplay among the procedures will bind these partial records associated to the different coregulators into the global record of O.

Let CR be one of the coregulators. At the instant t, O synchronously activates a pattern P of cat-neurons; only a sub-pattern R of P has t-activated perspectives for CR; for instance if O is a blue object and CR a coregulator based on the color module, R activates the agents treating the characteristics of the color blue. The t-activated perspectives and the links of R correlating them form a pattern R* in the landscape L_t of CR at t (cf. Figure 12). The procedure selected by CR will be to bind this pattern (in categorical terms, form a colimit of R* in a complexification of L_t). This objective is transmitted (via the difference functor) and participates in the interplay among the procedures. Once retained in the operative procedure, it is realized by the formation of a cat-neuron binding R, called the CR-*record* of O. This CR-record may have no aspect for any agent of the coregulator. However, if CR (as a pattern) has a binding cr, each t-

activated perspective binds into a link to cr, and the pattern R* of these links defines a collective link from R to cr; the collective links bind into a link *c* from the CR-record to cr, which can be thought of as a 'generalized' aspect of the CR-record, along which O can 'globally' activate the coregulator.

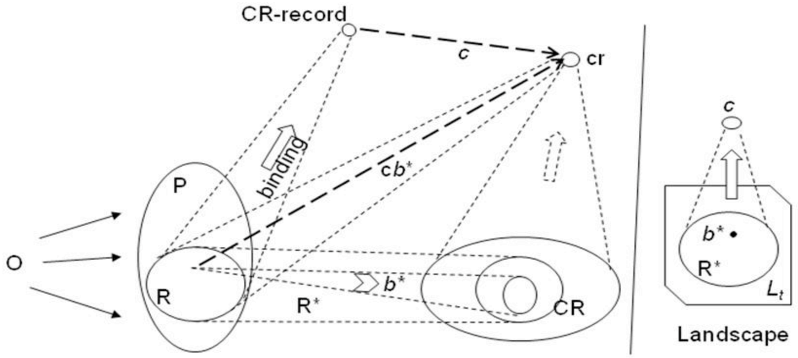

FIGURE 12. The item O activates a pattern P of cat-neurons; R is the sub-pattern whose cat-neurons have a *t*-activated perspective *b** for the coregulator CR. The selected procedure of CR will have for objective to bind the pattern R* of these perspectives in its landscape L_t. It is transmitted (via the difference functor) into the objective of binding R, retained in the operative procedure; the binding of R is the CR-record of O. If there is a cat-neuron cr binding CR, each perspective *b** binds into a link *cb** to cr; these links form a collective link from R to cr which binds into a link *c* from the CR-record to cr. This link is a 'generalized' aspect of the CR-record.

Each coregulator which receives information from the item O selects a procedure to form its own record of O at its own rhythm. Once all the coregulators have transmitted their objectives, the interplay among procedures collects them in the operative procedure. It leads not only to the formation of the various CR-records, but also to the formation of a cat-neuron which simultaneously:

- binds the pattern P of all the cat-neurons activated by O; and
- binds the pattern formed by the partial records.

This cat-neuron is the *record* of O. Its later recall consists in the simultaneous activation of its partial records, leading to the activation of the record through the unfolding of one of its ramifications.

6 Semantic memory

What we have said up to now can be applied to many animals, in particular mammals and birds; even if there are differences in the complexity of the mental objects they can form. Higher animals alone (particular mammals and birds, and specially man), are able to form cat-neurons of complexity order greater than 2; and these animals are also able to develop another capacity to detect invariants in their environment. It is easy to compare various objects when they are simultaneously observed and to classify them according to some common feature or attribute, for instance to classify some geometric figures according to their shape; experiments with the most varied animals have proved they are capable, given a little number of triangles and circles, to distinguish between them. What is more complicated is to classify these objects through their records, and still more to form a mental object representing the invariant they have in common; this mental object, or rather the cat-neuron which models it, will be called a *concept* (following Changeux, 1983; Edelman, 1989); for instance the concept 'blue' characterizes the class of all the objects which have this color. The concepts are at the basis of the development of a semantic memory. For us, the formation of concepts does not necessitate the use of language, hence is possible for higher animals; naturally for man, language will allow the formation of more elaborate concepts.

6.1 How to classify?

The notion of invariant depends on what features or attributes of the objects are considered; a blue triangle and a blue circle are similar with respect to the color, but not with respect to the shape. How to represent an attribute in MENS? Here, an attribute will correspond to the kind of information which can activate particular coregulators; such as the color (for a coregulator based on a color module), the odor, the shape, the orientation, and so on. In other terms, the attributes are associated to the function of a coregulator, and to classify some items according to an attribute will mean to classify these items (or rather their records) in function of the information they transmit to the corresponding coregulator.

It should be noted that the division of the brain in modules relies on the characteristic operations that these modules can perform, and these have been determined with the help of experiments, for instance, the neurons of a color module discriminate between the colors; but these experiments presuppose a classification into colors, and prove that such a neuron is activated by such a color. For other animals (e.g. dolphins) which have different environments and capacities, the 'attributes' can be different, and other kinds of coregulators should be taken into account for the formation of concepts. Here we see the difficulty to study our own mind (modeled by MENS); it is the 'self-reflection' problem which underlies all discussion or theory about the mind.

Finally, we interpret a classification according to an attribute as meaning a classification with respect to a particular coregulator CR. This classification is done in 2 steps: first a 'pragmatic' classification indicating if two items should be considered as 'CR-similar', then a more formal classification (operated through a higher coregulator) associating a CR-concept to a class of CR-similar records. After that, we describe how these CR-concepts with respect to various coregulators can be combined to get more elaborate concepts.

6.2 Classifier cat-neurons

The CR-concepts and the general concepts are mental objects which will be modeled by cat-neurons of a different type than those constructed in the preceding sections. While the 'binding' cat-neurons model classes of synchronous assemblies which *activate* the same cat-neurons, the *classifier cat-neurons* will model classes of synchronous assemblies which *are activated* by the same cat-neurons. Roughly, they are characterized not by the information that they can transmit, but by the information that they can receive.

Only higher animals have the capacity to form such classifier cat-neurons. Up to now, the only requirement for the neural system was the capacity to transmit information through the activation of neurons, to bind synchronous assemblies of neurons, and repeat these operations at least once. Now a supplementary capacity is required: to compare the information transmitted by two items (not necessarily at the same time), and to have coregulators of a sufficient complexity level to detect what is common to both and classify it; thus the animal must be endowed of a kind of partial 'self-reflection' over its internal operations.

Given a pattern Q of cat-neurons, the information globally received by Q from a cat-neuron *N* is modeled by a *distributed link* from *N* to Q, which is defined as a family of links from *N* to each Q_j well correlated by the distinguished links of Q; we say that it globally activates Q. It should not be confounded with a perspective of *N* for Q which may only activate some of the cat-neurons of Q, while a distributed link really 'distributes' the activation between all the Q_j.

A *classifier* of Q is a cat-neuron *C* with the following property: there is a 1-1 correspondence between the distributed links from a cat-neuron *N* to Q and the links from *N* to *C*. (In categorical terms, the classifier of Q is a *projective limit* of Q.) Roughly *C* receives the same information as the pattern Q as a whole; both have the same role as receptors. While a binding cat-neuron may bind several patterns, similarly a classifier cat-neuron may classify several patterns. As for binding cat-neurons, there are simple and complex links between classifier cat-neurons (cf. Appendix).

Classifiers will emerge from a *mixed complexification* process (cf. appendix) with respect to a *mixed procedure.* A mixed procedure is a procedure which has a supplementary objective: to form the classifier of some given patters. (Cf. Ehresmann and Vanbremeersch, 2007; and also the Appendix.)

For animals able to perform some kind of classification, the procedures selected by the coregulators, as well as the operative procedure carried out on MENS, can be mixed; their realization necessitates a mixed complexification process. In this case, MENS is deduced from **Neur** by successive mixed complexification processes, so that not only bindings but also classifiers can emerge.

6.3 CR-*concept*

Let CR be a coregulator, P the pattern of cat-neurons activated at a given time *t* by an item O, so that P is a decomposition of the record of O if it exists. What trace will it imprint on the coregulator? In some case, there is no trace, therefore no CR-concept: a musical sound does not activate a visual coregulator, and it will have no color-concept. A trace will be formed if some of the cat-neurons of P have *t*-activated perspectives for CR and we denote by R the sub-pattern of P they form. R activates a sub-pattern of CR which we call the CR-*trace* of P and denote by trP; it consists of the agents activated by a cat-neuron of R, indexed by the aspects along which they are activated, its distinguished links are the links of CR correlating these aspects.

Another pattern P' is said to be CR-*similar* to P if its trace trP' is globally activated by the same cat-neurons, and with the same strength, as trP (so that the same cat-neurons send distributed links to P and P'). If O has a record, all its decompositions are CR-similar. However this CR-similarity is not limited to the decompositions of the same record. Another item O' could have a record whose decompositions are CR-similar to those of O; in this case we say that O and O' (or their records) are CR-*similar*. For instance two blue objects will be similar for a color module.

The relation of CR-similarity leads to a classification of items (or records) into classes of CR-similar items. This classification is only 'acted' by the coregulator, meaning that all the records in one class leave CR-similar traces. It is not 'internalized' at its level: CR cannot itself distinguish the CR-similarity of two records, because it would imply a kind of self-reflection of CR on its operations. The CR-similarity can only be apprehended if there is a higher coregulator which receives perspectives from the CR-traces and can recognize their CR-similarity. For a lower enough coregulator CR, the existence of such a higher coregulator does not impose too stringent conditions on the neural system. In fact, CR-records are essentially constructed for lower coregulators of complexity order less than 2.

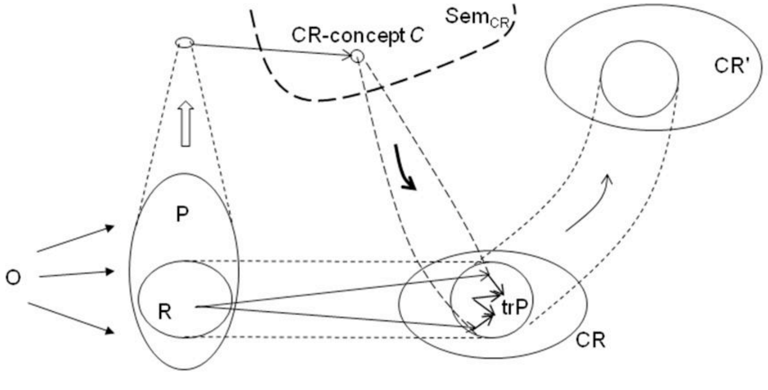

FIGURE 13. The item O activates a pattern P of cat-neurons; R is the sub-pattern whose cat-neurons have *t*-activated perspectives for the coregulator CR. The CR-trace trP of P is the pattern of agents activated by R and the links correlating them in CR. The CR-concept *C* of the record *M* of O is the classifier of trP added through a mixed complexification process directed by a higher coregulator CR'. The defining link *d* from *M* to *C* defines *C* as the 'best approximation' of *M* in the CR-semantic memory Sem$_{CR}$.

The classification into CR-similarity classes is consolidated by the formation of a cat-neuron, called a CR-*concept*, associated to a class of CR-similar items, for instance the color-concept 'blue' to the class of objects whose color is blue. Formally, the CR-*concept* of O, or of the record *M* of O, is defined as the classifier of the CR-trace of a decomposition P of *M* (cf. EV 1992, 2007); it does not depend on the choice of the decomposition since all its decompositions are CR-similar, hence have the same classifier if it exists. The formation of the CR-concept will be initiated by a higher coregulator, which will take the formation of the classifier as one of the objectives of the mixed procedure it selects (cf. Figure 13).

The constructions of the CR-record *M* of the item O, and of the CR-trace of its decomposition P should be well distinguished.:

- To construct the CR-record, we are interested in the *t*-activated perspectives of O, taken globally (corresponding to links toward the binding of CR if it exists); two aspects in the same perspective are not treated separately; what is important is the global information that CR itself can collect from O and can later use for selecting its procedures and sending commands.
- On the other hand, in the CR-trace we are interested not in perspectives taken as a whole, but in the agents activated along all the different aspects in these perspectives, and the links correlating them in CR. The CR-trace compares with the shape of P with respect to CR (in the sense of Borsuk shape theory; Borsuk, 1975); and the CR-concept converts this shape in an invariant.

Briefly, the CR-record retains how the object can activate other cat-neurons, while the CR-concept extracts what remains invariant in the traces it impresses on CR. These 'dual' viewpoints explain that O and O' may have the same CR-concept, while O and O' have not the same CR-record. Two blue objects have the same color-concept 'blue' whatever their other attributes (different shapes, sizes,...).

The different items (or their records) which have the same CR-concept are called *instances* of the concept. The 2-step construction sketched above shows that:

- We first recognize the CR-similarity of two records (through their traces) without having yet the corresponding concept; roughly, a class of CR-similar items is formed of items having "a family resemblance" (in the terminology of Wittgenstein, 1953).
- It is only in the second step that this class is consolidated through the formation of the CR-concept, which becomes an instance of itself; as such, it can play the role of a prototype for the class (in the terminology of Rosch, 1973).

As any cat-neuron, the concept will take its own identity, and it can acquire new instances later. As said above, at the time a concept emerges, only a few of its instances are already known, and it is initially formed to extract their invariant (as the classifier of their traces). Over time, the classification will be made more precise, e.g. by adding new instances or suppressing other ones. For instance, the child first will form a concept of moving objects encompassing cars and trains, then refine it by distinguishing between them.

A CR-concept emerges in the course of a mixed complexification process with respect to a procedure having its formation as an objective. This process also constructs the links between them and other cat-neurons (cf. Ehresmann and Vanbremeersch, 2007, Chapter 4). The CR-concepts and the links through which they communicate (in the role of classifiers) constitute the CR-*semantic memory*, which is modeled by an evolutive sub-system Sem_{CR} of the memory of MENS. For each record *M* of an item O which admits a CR-concept *C*, there is a link *d* from *M* to *C*, called the *defining link* which characterizes *C* as the 'best approximation' of *M* in Sem_{CR} in the sense that *C* activates each CR-concept which is activated by O (categorically this link defines *C* as a reflection of *M* in Sem_{CR}; cf. Appendix).

6.4 The semantic memory

The CR-concepts are 'concrete' in the sense that they reflect some specific property of the items they classify, for instance their color. Other concepts can be deduced from them, either more specific ones (a concept of 'blue circle'), or more general ones (a concept of 'dog') or more abstract (a concept of 'justice'). They will be successively formed by combining CR-concepts associated to various coregulators in different ways; leading to the development of the *semantic memory* which is an evolutive sub-system of MENS generated by the various Sem_{CR}.

The formation of a CR-concept starts from a small set of instances; then a mental object, the CR-concept, is formed to extract their similarities; later other instances are added to the concept by comparing their similarity with the concept. And the CR-concept of a record can be characterized in two ways:

- as the CR-concept which 'best approximates' the record (in the strict sense of a reflection in Sem_{CR});
- by the class of its instances, but this class varies over time; new instances can be found while some instances can be later eliminated; this elimination can be forced by a fracture in some higher coregulator if the instances have been wrongly classified because of a lack of knowledge or a poor observation.

The first characterization pre-supposes the existence of the concept, the second one is partly contextual.

The formation of more elaborate concepts will be different; it consists in combining already constructed concepts, and their instances will be recognized afterwards. For these concepts, the classes of instances are not separated: a record can be an instance of several concepts; for instance a spaniel is an instance of the concept 'dog', but also of the concept 'mammal'. More precisely, at a given time *t*, we consider the sub-evolutive system Sem_t of the memory of MENS containing the various Sem_{CR} associated to various coregulators CR with their links. Two methods are used to form new concepts (cf. Figure 14):

1. We define concepts with respect to several coregulators (or 'attributes' as explained at the beginning of the section). For instance, the concept 'blue triangle' will classify physical objects depending on both their

color and shape; it is obtained as the classifier of a pattern consisting of the color-concept and the shape-concept. More generally, given a pattern P of concepts P_i in Sem_t, a new concept C can be constructed as the classifier of this pattern, which will be added as a result of a following mixed complexification process. Its instances are the records M which are instances of each P_i and whose defining links d_i are correlated by the links in the pattern (thus forming a distributed link from M to the pattern); and the defining link d from M to C classifies them.

2. We define concepts binding several already constructed concepts; for instance the concept of mammals includes the concepts of dogs, cats, men,.... They are obtained as the binding C' of a pattern P' in Sem_t; an instance M' of C' is an instance of one of the concepts P'_j of the pattern; the defining link d' from M' to C' is the composite of the defining link of M' to P'_j with the attachment link of P'_j to C'.

In both cases, the new concepts emerge as the result of a mixed complexification process directed by the cooperation between higher coregulators with the objectives: in the first case, to classify the given pattern of concepts, in the second case to bind it. Thus, Sem_t is extended in a larger evolutive sub-system of the memory. And this process of constructing classifiers and/or binding of patterns of concepts already formed can be iterated. It leads to the formation of the *semantic memory* Sem.

Like any cat-neuron, a concept C takes its own identity over time, possibly acquiring more instances. Each instance M of C can activate the concept along its defining link. Conversely C can recall another instance M' by a 'priming effect' if M' is independently activated via a diffuse activation of the memory, the simultaneous activation of M' and of C strengthening the defining link (Hebb rule). Thus the activation of M can be transmitted to M' via C; we speak of a *shift* between the two instances.

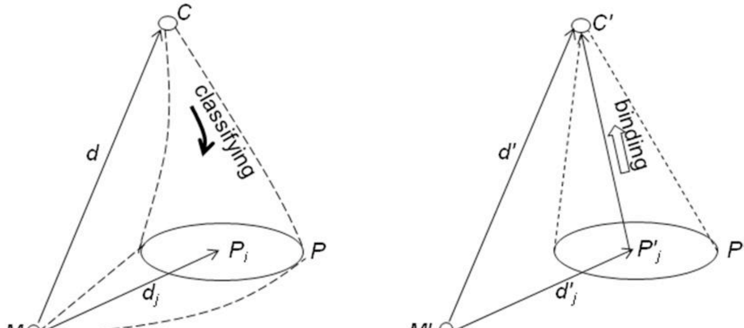

FIGURE 14. C is the concept which classifies a pattern P of concepts P_i. An instance M of C is a record which is an instance of each P_i and such that its defining links d_i to P_i form a distributed link to P'; this distributive link is classified into the defining link d to C. C' is the concept binding a pattern P' of concepts; its instances M' are all the instances of the various P'_j

The shifts between instances of a concept increase the plasticity, in particular in the selection of procedures and the interplay among them. The procedures of a higher coregulator can be recorded as concepts, and then commanded through the effectors of the most adapted instance: a movement such as walking can activate different patterns of muscles depending on the ground. And the interplay among the procedures acquires two kinds of degrees of freedom: possibility of shifts between instances of the different procedures, then complex switches between ramifications of their effectors.

We accept that higher animals other than man can develop a (more or less extended) semantic memory, since its construction is independent from language. For man, the language allows a still more abstract operation: to 'name' the concepts, that permits an economy of means (replace a concept by a name), and thus to handle still more abstract concepts.

7 Archetypal core. Conscious processes

The development of a semantic memory allows the development of a more complex and personal memory, the archetypal core, at the basis of the self. Though higher animals will develop such a memory, it will be particularly important for man, and we first consider this case.

7.1 Development of the archetypal core

At birth, the baby has an innate memory accessible by some lower coregulators which can command simple archaic motor reflexes and sucking reflexes. In the first days, the activation of one of these coregulators, each experience, each emotion, will be memorized, as well as the cause of this activation (internal or external sensorial stimulation) and its possible results if they are perceived. For instance *evaluating coregulators* (based on the emotive brain) are able to evaluate homeostatic drives and states related to pleasure or pain and to measure the consequences on the homeostasis and well-being of the baby; they form partial records which develop the value-dominated memory (Edelman, 1989, p. 99).

All these records extend the innate memory, so that more and more items can be recognized, in particular the emotive memory extends, and the power of action of the baby increases. Many simple and complex links are created in the memory, such as activator links towards records in the procedural memory; and the lower coregulators (in particular the evaluating coregulators), become connected to associative cortical areas, with creation of higher coregulators based on these areas.

During the first months, most of the experiences will be physical and/or affective, but they progressively are completed by a semantic approach, with a pragmatic classification in concepts with respect to coregulators based on the sensory and limbic systems. For the baby:

"I am hungry, I cry, I suck"; "I am hungry, I cry, I suck, it is good";
"I am hungry, I cry, I suck a breast which has a good odor"; and so on

Thus, a primal hard core is formed in the memory, with records of often repeated sensations or behaviors, and of notable experiences. We call it the *archetypal core*; it is an evolutive sub-system of the memory, with numerous powerful internal links whose strengths increase through their constant reactivation. The simple reflexes are replaced by more elaborate skills which are recorded in its part of the procedural memory. Each instant of the baby life activates part of this archetypal core which is the resultant of all his/her experiences and which reflects a memorization of the body through these experiences. The various sensations are remembered, and begin to be classified with formation of sensory concepts.

After a few months, the small child acquires other capacities, such as using some words, recognizing some music. However, throughout this period, the experiences are mainly corporal. The archetypal core is no more restricted to some adaptive reflexes, but has become a rich, more or less stable archive of what the body can do and feel. The later changes will enrich it by adjunction of details; for instance a biting cold may be pleasant if it is associated to games, the feeling of a caress will take a new color at the time of the first love around 10 years. These changes are very progressive, except when there are serious fractures which modify the corporal image such as pain, illness, or violent emotions.

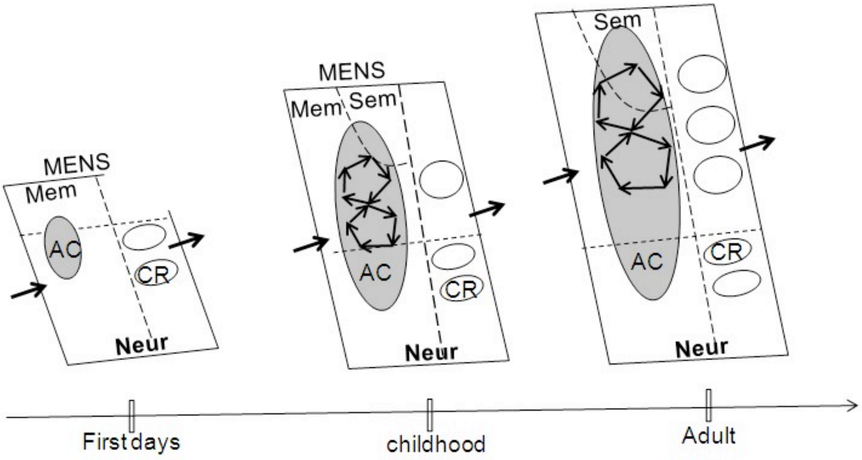

FIGURE 15. Development of the archetypal core AC, a sub-evolutive system of the memory which plays a central role. At birth AC contains a few innate records. During childhood, AC integrates the records of the main persistent physical, mental and emotional experiences, with strong and fast links between them, the links in fans. These links form 'archetypal loops' through which the activation of AC is self-maintained for a long time. AC extends;mo(re slowly during adulthood; its stability is at the root of the notion of Self.

Over time the archetypal core will extend by recruiting more cat-neurons (cf. Figure 15). A record which has a preferential link to an archetypal record will become archetypal if this link strengthens; for instance the memory of a very emotional event, or of an object evoking significant childhood experiences.

However, the extension is much slower than during the childhood, and the archetypal core remains stable enough.

Higher animals other than man also develop an archetypal core, but it has less extension and plasticity, since the animal has to obey to a larger number of innate procedures (means of defense, flight, moving, eating...), he becomes an adult more quickly and probably cannot form cat-neurons of complexity order more than 2, nor abstract concepts. On the other hand, the human baby has not many innate procedures and comportments (hunger, sleep, pain,..). The prime childhood, then the childhood being very long compared with those of the animal, (s)he has more time to construct the archetypal core on experiences related to the environment, education, games.

7.2 Role and structure of the archetypal core

The archetypal core is a permanent memory, developed from the first days on, with often reactivated records intermingling strong memories of the body, its sensations, feelings, emotions, and of the basic procedures associated to them. The links between them are strong and fast, and they are continuously strengthened up to a threshold. It keeps its identity over time, with only slight modifications, thus contributing to the notion of self. It remains in the background, where it acts as a filter, a referent. Each experience activates a semiotic search in it through several higher coregulators, which allows accounting for its sensory and emotional overtones (their importance has been stressed by Damasio, 1999). Being both a filter and a mirror, it modifies the experience in the light of past experiences, as Proust's "madeleine" well illustrates. It circulates the information in loops between various areas, acting as a kind of intranet in the middle of the diffuse neural noise which ensures its permanence.

We had introduced the archetypal core in 1999, as a hypothesis which seemed a natural consequence of our model, compatible with Edelman's view on the importance of the thalamo-cortical loop which supports reentrant activity among various areas. Recently this hypothesis has been confirmed by neuroscientists (P. Hagmann *et al.*, 2008) who have discovered an area in the median posterior cortex to which they attribute exactly the properties needed for its development. This area, which they call "neural connection core", seems the most densely connected zone of the brain, it has the largest energy consumption, even at rest and it is fed by a double artery; they suppose that it is related to consciousness since its activity decreases under anesthesia. And for these authors, it plays an essential role in the integration of information, exactly what we suppose.

In agreement with this, in MENS, the archetypal core is modeled by an evolutive sub-system AC of the memory, based on the neural connection core, which integrates and intertwines recurring sensorial, proprioceptive, motor, emotional, procedural memories and their concepts, as well as notable experiences. Initially it would consist of neurons in this neural core, then higher cat-neurons emerge (through successive mixed complexifications) as the bindings of patterns of these neurons, and later they are classified into concepts also in AC. Each archetypal record (i.e., cat-neuron in AC) has multiple, possibly non-interconnected, ramifications down to this neural core, each archetypal concept has instances which are archetypal records.

An archetypal record is linked to other archetypal records by very strong links whose activation is self-maintained through specific loops. More precisely, we suppose that, for each cat-neuron *A* in AC, there is a bundle F(*A*), called a *fan*, of strong complex links activating *A* in AC with the following property: there are loops formed of successive links belonging to fans which propagate very quickly the activation of *A* back to itself; such a loop will be called an *archetypal loop.* [Categorically, we suppose that the fans are covering families for a Grothendieck topology they generate (cf. Appendix), so that AC becomes a site; cf. Ehresmann and Vanbremeersch, 2007.] When an archetypal record is activated, links in fans propagate this activation to other archetypal records. This activation resonates to lower levels via the unfolding of a ramification, and, through complex switches, to other, possibly non-interconnected, ramifications. The activation of an archetypal concept resonates to an instance and, through shifts, to other instances and their ramifications. Thus, an extended part of the archetypal core resonates (this stochastic resonance has been experimentally observed in the brain; cf. Collins *et al.*, 1996; Levin and Miller, 1996; Wiesenfeld and Moss, 1995).

FIGURE 16. A metaphoric representation of the development of the archetypal core. First il is reduced to one Möbius band. An archetypal loop (sequence of successive links in fans) appears as a circuit drawn on it; such a loop propagates and maintains the activation. Later another Möbius band appears; it is glued to the first one so that an archetypal loop can propagate the activation through the two bands. A third band is later glued to the two first ones, and an archetypal loop propagates the activation to the three bands. And so on.

A geometric metaphoric image of the situation could be given by the surface gluing together several Möbius bands. Initially the archetypal core would be reduced to one Möbius band, on which an archetypal loop appears as a circuit. Later it extends by adjunction of records, concepts and their links; another Möbius band is added, and it is glued to the first one; now an archetypal loop can go from one band to the other by crossing their common part. And the process goes on, gluing together more and more Möbius bands (cf. Figure 16); the activation propagates along the circuits formed by the archetypal loops, bouncing back and forth between the various bands.

7.3 The halo and the intentional coregulators

The activation of part of the archetypal core can extend to some cat-neurons *N* outside, but 'near enough' of AC for being integrated in a loop crossing AC. More precisely there is a loop from *N* to *N* consisting of a link from *N* to an archetypal record *A*, then link(s) of fan(s), and finally a link from an archetypal record *A* to *N* by which the activation of *A* can be transmitted to *N* and then self-maintained through the loop (cf. Figure 17). These cat-neurons *N* form the *halo* of AC and may later be integrated in AC, the loop becoming an archetypal loop.

At a given time *t*, an activation of some archetypal records triggers, through archetypal loops, a self-maintained activation of a large domain of the archetypal core, which propagates to part of its halo; if the activation of a cat-neuron is strong enough, it bounces down through the unfolding of a ramification, with possible complex switch to another; the activation of a concept goes back to an instance, with possible shifts between instances. All these activated cat-neurons and the links transmitting the activation form a sub-system D_t of MENS, the *t-activated domain*; its activation is sustained by the long-term activation of the archetypal core, so that it persists for a long enough period which we call the *specious present* (in reference to James, 1890).

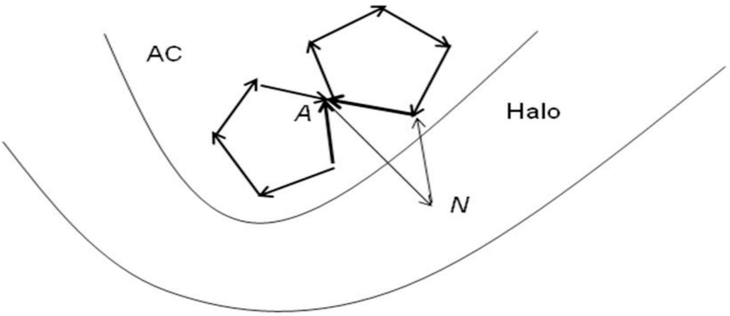

FIGURE 17. Two archetypal loops (formed of links in fans) maintain the self-activation of the archetypal record *A* for a long time. *N* is a cat-neuron outside AC activated by *A* and with the property: there is a loop of links from *N* to *N* containing at least one link in the fan of *A*; this loop self-maintains the activation of *N*. The cat-neurons *N* with this property form the halo of AC; if *N* is a record, it can later become included in AC.

We have explained how the functioning, dynamics and self-regulation of MENS depend on its net of coregulators and the interplay among their procedures. We have not explained how the procedures are selected; we have only said that lower coregulators have a few automatic procedures, and that other coregulators have a number of admissible procedures (recorded in the procedural memory). Higher coregulators can form and/or learn new procedures, and memorize them in the procedural memory and possibly the semantic memory.

The development of the archetypal core and its halo allows the formation of higher coregulators based on the associative cortical areas and with agents in the halo. Thanks to a large access to this core and their participation in the activated domain, they collect more information and retain it during a longer period, have more opportunity to select and possibly create complex procedures, and evaluate their results, in particular through the feedback received from lower evaluating coregulators. Thus they have some capacity to internally control their own functioning, and we call such a coregulator an *intentional coregulator,* in reference to the "intentional systems" of Dennett (1990). An example is given by the "conscious units" of Crick (1994, p. 336). The *intentional net I,* consisting of these intentional coregulators and links connecting their agents, is essential in the emergence and development of conscious processes.

7.4 The global landscape

The archetypal core, with its persistent activity, gives a dynamic archive of the whole life, reflecting in the present the recurrent salient corporal, sensorial, proprioceptive, procedural or emotional experiences, and strongly interconnecting them. Thus, it reflects (as a mirror) the various components of the *self,* which we propose to define as the (virtual) binding of AC. Its role will be essential in the development of conscious processes. We consider the case of man, though we suppose that higher animals will also develop some kind of consciousness.

The model for consciousness to which MENS leads (cf. Ehresmann and Vanbremeersch, 1992, 2002, 2007) enters the frame of the "global neural network space paradigm" (in the terminology of Wallace, 2004, p. 2), and it has some relation with the models of Edelman (1989) and of Dehaene *et al.* (1998, 2003). In particular it relies on a long-term elaborate memory (afforded by the archetypal core) and on a modular control system intermittently acting on it, modeled by the intentional net.

A conscious process is initiated at a time *t* by an arousing event, of internal or external origin, which has no automatic response. For instance, it could be a fracture in one of the intentional coregulators, the start of a voluntary action, a sudden sensation of pain. The first response is an increase of attention (Edelman, 1989, p. 205) which activates cat-neurons based on several zones (in particular the reticular formation) connected to the archetypal core. It triggers a self-maintained activation of a large domain of the archetypal core, which extends the *t*-activated domain. For instance, an unexpected noise arouses all our senses and recalls similar noises and the associated events, and we try to identify it (e.g., going to the window to look for its causes). The *t*-activated domain constitutes a large 'working memory' (to be compared to the "theater" of Baars, 1997). The intentional coregulators will use it to collectively extend their landscapes, both:

- 'spatially', collecting more diverse information, in particular in lower levels,
- 'temporally' going back to the recent past to find the causes of the arousing event ('retrospection'), and selecting more long-term procedures for the future ('prospection').

To model this, we define the *global landscape* GL_t *at t.* The intentional net I plays the role of a large higher coregulator in D_t; its agents are all the agents of the various intentional coregulators, and its actual present at t is the specious present. GL_t is its landscape. As for another landscape, it consists of the perspectives of the cat-neurons in D_t for the various intentional agents (in I) which remain activated during the specious present; the links are the links in D_t correlating them.

It is important to realize that a perspective of a cat-neuron N in the global landscape can be different from the perspective of N in the landscape L of one of the intentional coregulators, say CR. In L a t–activated perspective consists of aspects for the agents of CR; in the global landscape, the perspective may also include aspects for intentional agents which are not in CR, as soon as they communicate with agents of CR along a zig-zag of links in the activated domain. This extension of the perspectives allows for more cooperation between the intentional coregulators, which may exchange their information.

For instance, an intentional coregulator can observe a record M of the actual situation in its own landscape, but have no admissible procedure to respond. At the same time, M may have an activator link f toward (the record) Pr of an admissible procedure for another intentional coregulator CR, while M itself is not observable in the landscape of CR. Because of the surge of attention, the activator link enters in the t-activated domain D_t, hence figures in the global landscape. Thus Pr can be selected by the joint operation of the two coregulators.

7.5 Conscious processes: Retrospection and Prospection

For Merleau-Ponty, "consciousness unfolds or constitutes time", and time "is not an object of our knowledge, but a dimension of our being" (translated from Merleau-Ponty, 1945, p. 474-475). We agree with him and our hypothesis is that consciousness is characterized by two temporal processes: the retrospection (toward the past) and prospection (toward the future). Both will be operated in the global landscape.

The global landscape is maintained during a specious present through the self-activation originating from the archetypal core; thus it has a larger temporal span than the individual landscapes of the intentional coregulators. The retrospection process allows to reactivate ('intentionally' or not) events of the recent past, which are not observable in the individual landscapes, either because they have already faded from them, or even because they occurred at a lower level. The prospection process makes use of the greater stability of the global landscape to select more adapted and/or complex procedures, possibly extending on a longer period, even planning on the long-term..

To explain the retrospection process, let us suppose that an arousing event S, say a fracture, occurs at *t* in the landscape of at least one of the intentional coregulators ("flashlight" in Baars' theater), say CR. For instance, S can be an unusual sound caused by the fall of an object in another room. As said above, it causes a surge of attention, which extends the *t*-activated domain D_t. The cooperation of the intentional coregulators is strengthened, and together they form the global landscape GL_t. The agents of CR have thus access to more information for identifying the nature of the original event and its possible causes. They can select (as their procedure) to operate an 'abduction' process (in the sense of Pierce, 1903) to recall similar past events and what had been their causes; it is done through a series of loops in D_t, among them archetypal loops. (Cf. Figure 18)

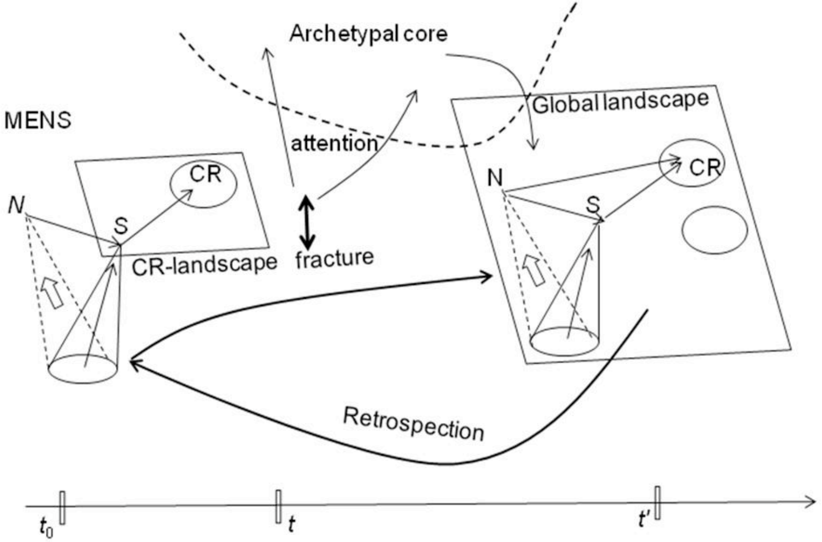

FIGURE 18. *N* activates S at t_0 which causes a fracture to an intentional coregulator CR by appearing in its landscape at *t*. In response there is a surge of attention which propagates through the archetypal core, increases the *t*-activated domain and leads to the formation of a global landscape. It allows a retrospection process recalling various components of S (possibly through the unfolding of ramifications of S), hence also their binding *N*. Thus the link from *N* to S becomes observable in the global landscape which unites and extends the landscapes of the various intentional coregulators.

First they try to recall as many as possible characteristics of the event; some of them were too weakly activated to be observable in the initial landscape of CR, but the increase of activation makes them observable in the global landscape. For instance, a retrospective analysis of the traces left by the sound S gives cues on the direction from where it came and its auditory characteristics. Have such characteristics already been associated to a sound in the past? This question re-activates a search for records of events having caused such a sound; it is done via loops in the activated domain, in particular archetypal loops, all reflected in the global landscape. If this search recalls a unique record with the same characteristics, it is probably the cause of the sound, and the search stops. If several possible records are activated, a new search begins to trace back other weaker characteristics of the sound, thus refining its probable nature; and so on up to the retrieval of a unique possible cause *N*; for instance the fall of a book in the living room. If no similar event had occurred in the past, more or less different events can be recalled for, by comparison with them, trying to discover (or rather 'reconstitute') the real cause of S.

Anyway, the retrospection allows for a search in the past and on various levels, including lower levels inside the activated domain, unfolding ramifications, activating one instance of a concept or another. In the above example, the retrospection is directed by an intentional coregulator. However, it is not always the case. An example is given by the priming effect, which may orient the prospection process as follows. An object, say an apple, is subliminally presented to a subject so that its record has only a briefly activated perspective for a lower coregulator. Soon after (during the specious present), it is followed by the full presentation of a set of objects, among them an orange, and the subject is asked to select one of these objects. Which one will he select? Experiments prove that it is the orange. This selection is explained by the fact that the record of the apple is still weakly activated when the cognitive effort of the subject causes a surge of attention; the record of the apple is then reactivated and enters in the *t*-animated domain, where it recalls the concept 'fruit'. On the other hand, the view of the orange also activates the concept 'fruit', causing a shift between the two instances, and a stronger activation of the orange with respect to the other objects in the global landscape, whence the selection of the orange by the subject.

While the retrospection process is oriented toward the past, the prospection process makes use of the global landscape to select procedures extending on the long-term future. This landscape gives a space where to 'virtually' select a procedure, evaluate its probable results as they have been recorded from former similar situations; and the process can be iterated during the specious present; it allows selecting a sequence of procedures to be successively realized. For instance, we can plan series of actions in advance by anticipating their results; however the anticipation relies on our former experiences and there is always the risk of fracture because of a non-anticipated change in the context.

A long term procedure can also be selected under the form of a sequence of procedures to be alternatively commanded by several coregulators, each one depending on the anticipated result of the preceding one (as it is retained in the activated domain). Or procedures can be imposed to lower coregulators who command them while intentional coregulators perform another one; for instance, speaking while driving a car. In this case, the higher procedure can be interrupted by a fracture caused by the lower coregulators (a road obstacle forces the attention of the driver). We speak of "consciousness spikes".

7.6 Consciousness and thought

Thought appears as series of mental images activated through a succession of intertwined retrospection and prospection processes; they have another dimension than a simple film, being colored by the temporal dimension of the self. Indeed they rely (through the global landscape) on the archetypal core which conjugates in the specious present a sketch of the past, keeping trace of the successive consciousness spikes, attributing a kind of instantaneous semantic which helps as referent if the intentional coregulators select to initiate a retrospection process. Let us give an illustrative example in which the several operations are described both in usual language, and in terms of the global landscape.

7.6.1 The situation

Collect of information in the landscapes of several lower coregulators while an intentional coregulator CR pursues a specific procedure:

The sight errs. The 'eye' records several images of objects, of color, of light, of shapes. Simultaneously the mind performs another task, for instance looks for the advent of something of interest (a prey or a predator for an animal, an object of curiosity or study for a man…).

7.6.2 Fracture

New stimuli at time *t* cause a fracture for lower coregulators and activate a pragmatic semantic classification, while CR pursues its current procedure.

A spot in the sky appears as a vague form, not very definite. It can be semantically associated with many things:

- undefined without interest (indistinct spot, fly or other insect,...),
- not very distinct nor interesting (very far small bird, flying leaf, airplane),
- more determined and interesting, but still imprecise (distant bird).

In each of these three cases, a pragmatic classification slightly activates the concepts of insect, small bird, leaf, more defined bird, plane, but differentially depending on the observer being an animal or a man (plane only for the later).

Some semantic circuits are more activated, depending on the interest of the observer, whether he looks for anything new, or for a special kind of things. In the first case, his attention will converge on the spot. In the second case, only spots similar to the object of his search will arouse his attention. The surge of attention at the time *t* produces a consciousness spike, which allows for a synchronous oscillation between the circuits pre-activated by the procedure of CR and the circuits which have stored in the memory the several attributes of the spot. The flight of a plane is far off and linear, with a deep sound, the flight of a bird of prey is curved, slow and easy, alternately coming and going away with small cries, the flight of a small bird will be sinuous, hopping, with specific sounds, the flight of an insect is uncertain, near the grass, and so on. The phenomena may be a fugitive, brief awareness, or prolonged, leading to a fracture.

7.6.3 The response

A fracture for CR extends the *t*-activated domain and leads to the formation of the global landscape; an iterative retrospection-prospection process is started for identifying the spot by recovering its different attributes.

In the case of a fracture, a virtual momentarily autonomous internal world is formed, (the global landscape) which takes hold of all the capacity of attention, memorization and observation of the observer; the retrospection permits multiple comparisons:

- between the spot at successive instants *t*, *t*+1, *t*+2, … *t*+*f*,
- between the spot at successive instants and the circuits pre-activated by the search of the observer,
- between the results of the above comparisons and the pre-activated circuits.

It constitutes an iterated reflex loop of the type: "classification, comparison, perceptive memories of the stimuli, their semantic extensions and corresponding procedures, classification, comparison….", each newly recalled attribute leading to new possible causes, and thence to more adequate procedures to respond. For instance the man thinks:

"It is a bird, similar to a bird of prey, even if it is small, it might be a small bird with the flight of a bird of prey, some swifts have a light resemblance with a bird of prey, but his flight is too slow for a swift; it is a bird of prey like the one I saw some days ago above the pasture; I will photograph it"

The procedure for identifying the spot has first activated the general concept of bird, then its attributes are refined: small, with the flight of a bird of prey, whence two possibilities: swift or bird of prey. A new retrospection permits to retrieve some data: the flight is slow, which causes a fracture in the virtual landscape corresponding to the choice 'swift'; thus this choice is excluded. It remains to confirm the choice 'bird of prey' by searching for instances of this concept, thus reactivating a recently activated instance (a kind of priming effect), whence the final identification of a bird of prey and the next procedure: to photograph it.

8 Discussion

We propose MENS as a theory of mind, in which an algebra of mental objects emerges from the functioning of the neural system. It accounts for the development of a hierarchy of mental objects of increasing complexity by an iterative process, based on the two main operations that man can perform: binding mental objects into a more complex one, and classifying mental objects with the formation of concepts. The mental objects and processes are modeled by category-neurons, constructed from the neuronal level up, through a sequence of complexification processes. The construction is done in the frame of category theory, which gives a rigorous description of the binding and classifying processes (formation of a colimit or of a projective limit). Philosophically, MENS amounts to an emergentist reductionism (Bunge, 1979).

The cat-neurons are iteratively constructed: a cat-neuron of a given level is constructed as the binding of a pattern P of cat-neurons of strictly lower levels, and it takes its own identity as an independent component of MENS. It may have or later acquire other decompositions than P, possibly non-interconnected with P. The cat-neuron has several ramifications obtained by descending the levels down to the neuron level, and its activation consists in the unfolding of one of them, corresponding to the activation of a synchronous hyper-assembly (or assembly of assemblies... of assemblies) of neurons.

8.1 The brain-mind problem

What is the correlation between a mental state (modeled by a cat-neuron) and a brain state?

For a cat-neuron of level 1, the correlation is given by the fact that the cat-neuron binds an assembly of neurons, and conversely is activated by the synchronous activation of this assembly; however even at this level the correlation is non-univocal, since the cat-neuron may bind several non-interconnected assemblies of neurons. This multiplicity (or "degeneracy") is a consequence of the degeneracy of the neuronal coding (Edelman, 1989); it is at the root of the emergence of complex links and, thanks to them, of the emergence of cat-neurons of increasing complexity order.

A mental object modeled by such a cat-neuron "supervenes" on physical brain processes via the stepwise construction of a ramification from the neuron level up; later it will cause physical brain states through the unfolding of this ramification down to the neuron level, leading to a synchronous hyper-assembly of neurons. However, as we have explained in Section 4.4, this unfolding is intricate, necessitating a stepwise construction accounting for emergent properties at each step; and it is multiple (or 'degenerate') since a cat-neuron may have several non-interconnected ramifications. As Kim (1998) has explained, this "multiple realizability" (in his terms) makes mental causation possible while preserving the physical closure of the world.

8.2 Development of higher mental processes

The dynamics of MENS is modulated by the cooperation/competition between a net of internal regulation organs, the coregulators. Each coregulator forms its own landscape where it selects a procedure; the objectives of the various procedures participate in the interplay among procedures, an equilibration process leading to the operative procedure whose objectives (formation of the binding of some patterns and of the classifier of others, possible elimination of some cat-neurons) will be carried out via a (mixed) complexification process. In the interplay the multiplicity of ramifications of a cat-neuron gives much latitude to try to make coherent the procedures of the various coregulators. It also allows the development of a memory whose records are not rigid, but flexible enough to adapt to progressive changes.

Higher animals have a supplementary capacity: classifying their records, and formalizing such a class by a concept. It leads to the development of a semantic memory. A concept has several instances and can be activated by anyone of them, with a possible shift between concepts. They develop the archetypal core, a personal memory integrating the persistent experiences of any nature; as it merges the past and the present in a dynamic way, it is at the basis of the self. This core has an internal organization which allows for its self-activation via loops of strong and fast links, namely the links in fans. Its activation spreads to the cat-neurons in its halo, and possibly to their ramifications, forming a large 'activated domain' which persists for a long time. It allows the formation of a global landscape by a net of higher coregulators, the intentional coregulators in the halo of the archetypal core; these coregulators receive feedbacks from lower coregulators evaluating the homeostatic drives and hedonic states; and they cooperate in the global landscape.

Conscious processes rely on the global landscape which is formed following an arousing event which activates the archetypal core and extends the activated domain. We have characterized consciousness by two more or less intermingled temporal processes: the retrospection to retrieve the possible cause of the arousing event, and the prospection for selecting long term procedures. The global landscape reflects our conscious experiences. In the global landscape an object is not apprehended as such, but by the intermediary of an activated perspective; it gives an internal perception of the object, different from that an external observer would have. Could this difference be at the origin of the qualia, thus giving an approach to the "hard problem" (Chalmers, 1996)?

The great stability of the global landscape and its very progressive change over time, with overlapping successive global landscapes, can explain the development of self-consciousness: the occurrence of fractures reveals the existence of constraints and, by opposition, leads to the differentiation of the self. For man, language allows developing a more elaborate thought, allowing for extended communication with others at the basis of education, higher learning and culture.

Animals with a nervous system are able to develop at least a primary consciousness (Edelman, 1989). Consciousness extends for higher animals; they can even acquire self-consciousness and, we suppose, have some kind of thought. The usefulness of the temporal dimension of consciousness for the well-being of the animal may explain the development of consciousness through natural selection.

8.3 Possible developments and generalizations.

MENS proposes essentially a *qualitative* model for a theory of mind, even if the energetic and temporal constraints (via the strengths and propagation delays of the links) play an important role in the development of the memory (via

Hebb rule), the temporal constraints of the coregulators, the selection of procedures and the conscious processes. It would be interesting to make it more 'computable', in particular:

- to find some general rules for the selection process (perhaps using cohomological operations, as suggested by R. Guitart, 2009);
- to develop simulations; this is presently tried by Monteiro *et al*. 2009, using the model of Izhikevich *et al.* (2004) for neural systems.

Several generalizations are possible. In MENS, the binding operation (and its opposite, classifying), is essential since it is iteratively applied from the neuron level up to construct cat-neurons modeling mental objects of increasing complexity order; we have attributed their emergence to the multiple realizability of the binding: the same cat-neuron is the binding of several non-interconnected patterns of strictly lower level cat-neurons. The binding operation has been modeled by the categorical colimit operation which has the advantage to allow for an explicit description, via the (mixed) complexification process, of the 'good' links between cat-neurons, making possible the iteration of the process. In some cases it could be interesting to somewhat 'refine' it along one of the following ways:

- The categories could be equipped with a supplementary structure, for instance a topology accounting for the topography of the brain, and making rigorous the geometric metaphor of the archetypal core as gluing together Möbius bands (Section 7.2). The complexification process extends to categories equipped with a compatible enough structure (Ehresmann, 1967), so that our model could easily be translated if we replace the categories by, for instance, topological categories or multiple categories. This last case has been suggested by Brown (2003) (cf. also Changeux and Connes, 1989).
- Colimits could be replaced by "local colimits" (Ehresmann, 2002), or more generally by Baas "hyperstructures" (Baas, 1997; Baas, Ehresmann and Vanbremeersch, 2004), or in the multiple categories case above, by lax colimits. However, to be able to iterate the process, it would be necessary to find precise constructions generalizing the complexification process, and this raises difficult problems.

Appendix: Mathematical definitions

A.1. Categories

For a general theory of categories we refer to Mac Lane's 1971 book. Here we just recall the definitions used in this article.

A (multi-)graph consists of a set of objects (its vertices), and a set of oriented edges between them, represented by arrows f: N → N'. There can exist several parallel edges from N to N'.

A category K is defined as a graph equipped with an internal (partial) composition law associating to the pair of 2 consecutive arrows f: N → N' and g: N'

$\to$ N'', a 'composite' arrow fg: N $\to$ N'', this composition being associative; moreover each object N has an 'identity' id_N: N $\to$ N. The arrows are called morphisms or, more simply, links.

A (partial) functor from K to K' is a homomorphism of graphs from (a sub-category of) K to K' which respects the composition and the identities.

If K is a category and K' a sub-category, a *reflection* of an object N of K in K' is an object N' of K' with a morphism d: N $\to$ N' such that any other morphism f from N to an object in K' factors in a unique way as $f = df'$ with f' in K'.

A.2. Evolutive Systems (Ehresmann and Vanbremeersch, 1987)

An Evolutive System K consists of the following items:

- a timescale T (finite or infinite subset of the real numbers) modeling its lifetime;
- for each t in T, a category K_t representing the configuration of the system at t;
- for each $t < t'$ in T, a partial functor $K_{t,t'}$ from K_t to $K_{t'}$, called *transition*, which represents the change of configuration from t to t'; we suppose that, for $t < t' < t''$ in T, the transition $K_{t,t''}$ is the composite of $K_{t,t'}$ and $K_{t,'t'}$.

A *component* N of **K** is defined as a maximal family (N_t), indexed by an interval T_N of T, where N_t is an object of K_t and $N_{t'}$ is the image of N_t by the transition from t to t'; the links between components are defined similarly.

For each interval U of T, the components N of **K** such that U is contained in T_N and their links form a category $\mathbf{K}_U$. These categories on the different U form a sheaf of categories on T. When we speak of the colimit of a pattern of components, it is computed in one of these categories.

An evolutive sub-system **K'** of **K** is an evolutive system whose timescale T' is a sub-set of T, its configuration categories $K'_{t'}$ being sub-categories of $K_{t'}$ and its transitions restrictions of those of **K.**

A.3. Colimits (or binding)

Let K be a category. A *pattern* (often called a *diagram*) of objects P in the category consists of a family $(N_i)_{i\varepsilon I}$ of objects N_i and some distinguished links x: $N_i \to N_j$ between them (thus defining a homomorphism of a graph G to K, the set of vertices of G being I). A *collective link* (or cone) from P to an object N' is a family $(f_i\colon N_i \to N')_{i\varepsilon I}$ of links correlated by the distinguished links of P, *i.e.*, for each x: $N_i \to N_j$ in P, we have $xf_j = f_i$.

A pattern P admits the object cP as a *colimit* (or inductive limit, Kan, 1958) if there is a collective link (c_i) from P to cP such that each collective link (f_i) from P to any N' *binds* into a unique link f from cP to N' satisfying the equations $f_i = c_i f$ for each i. In this case, we also say that P admits cP as its *binding*, and that P is a *decomposition* of cP. A pattern may have no colimit; if it exists, the colimit is unique (up to an isomorphism).

Two patterns are homologous if there is a 1-1 correspondence between their collective links to any object N'. In this case, either they both have the same colimit, or none of them has a colimit.

A.4. Simple and complex links

Let P and P' be two patterns in the category K. A cluster from P to P' is a maximal family π of morphisms from objects of P to objects of P' such that:

- For each object P_i of P there is at least one g in π from P_i to an object of P', and if there are several such morphisms, they are correlated by a zig-zag of distinguished links of P'.
- The composite of a morphism in π with a distinguished link of P (on the left) or of P' (on the right) is also in the cluster.

If π is a cluster from P to P' and if P and P' admit colimits cP and cP' respectively, there is a unique link cπ from cP to cP' binding the collective link (gc'_j) from P to cP', where g varies in π and (c'_j) is the collective link from P' to cP'; it is called the (P, P')-*simple link* binding π. A (P, P')-simple link might not be (Q, Q')-simple for other decompositions Q of cP and Q' of cP'.

Two decompositions P and Q of the object N are *interconnected* if the identity of N is a (P, Q)-simple link or a (Q, P)-simple link. Otherwise, we say that P and Q are *non-interconnected* and the passage from P to Q is called a *complex switch.*

Dually P admits a *projective limit* (or *classifier*) C in K if C is a colimit of the pattern opposite to P in the category K^{op} opposite to K (obtained by changing the direction of its morphisms). A *pro-cluster* from P to P' is a cluster from P' to P in K^{op}; if P and P' have classifiers, the pro-cluster is 'classified' by a (P, P')-simple link from C to C'; complex links are obtained by composing simple links classifying non-adjacent pro-clusters.

A.5. Hierarchical categories

A category K is hierarchical if its objects are divided up into in a finite number of complexity levels so that an object N of level $n+1$ is the colimit of at least one pattern of objects of strictly lower levels. If N admits several such non-interconnected decompositions, it is said to be n-multifold. If K has multifold objects, we say that K satisfies the multiplicity principle. In this case there are complex links obtained by composing simple links binding non-adjacent clusters, with complex switches between the different decompositions of the intermediate multifold objects. The complexity order of an object N of level $n+1$ is the smallest k such that N is the colimit of a pattern of objects of levels strictly lower than k; we have $k \leq n$ and we have given conditions for having $k < n$. (Cf. Ehresmann and Vanbremeersch, 1987.)

An evolutive system is hierarchical if the configuration categories are hierarchical and the transitions respect the levels.

A.6. The complexification process

A pattern P of objects in a category K may have no colimit. In this case the category K can be extended into a larger category in which P acquires a colimit. This is the basis of the complexification process.

Given a category K, a procedure (or 'option' in Ehresmann and Vanbremeersch, 2007) is a list of objectives for modifying K by means of some of the following actions:

- Binding a set B of patterns P of K: if P has a colimit in K, this colimit should be preserved, and if P has no colimit, a new object should be added to become its colimit.
- Eliminating a set S of objects, possibly thus dissociating some bindings.
- Adding a set E of external objects.

The complexification of K with respect to this procedure (Ehresmann and Vanbremeersch, 1987) is a category K' which is a universal solution of the problem of realizing the objectives of the procedure Pr. For an explicit construction of K', we refer to our 2007 book (Chapter 4). The objects of K' are the objects of K except those of S, the objects of E, and, for each P in B which has no colimit in K a new object cP which becomes its colimit in K'; if two patterns in B are homologous, the same cP is chosen for binding them. The links between two added objects cP and cP' are the (P, P')-simple links binding clusters from P to P', and complex links composing simple links. While a (P, P')-simple link only depends on the 'local' interactions between P and P', the complex links depend on the whole structure of the initial category, and they represent properties 'emerging' in its complexification.

A *mixed procedure* is a procedure whose objectives also require classifying a set of patterns Q by adding a new object which becomes the projective limit, or classifier, of Q. The corresponding *mixed complexification* is also described in Ehresmann and Vanbremeersch (2007, Chapter 4).

An important theorem on complexifications (Ehresmann and Vanbremeersch, 1996, 2002) asserts that: *If a category* K *satisfies the multiplicity principle, then so does any of its complexifications*. In this case, we have proved that an iteration of the complexification process leads to the emergence of a whole hierarchy of objects with strictly increasing complexity orders.

A.7. Grothendieck topologies

A *sieve* on an object N of the category K is a family of morphisms f_i: $N_i \rightarrow N$, closed by composition with morphisms to the N_i's. A *Grothendieck topology* (Grothendieck and Verdier, 1972) on K associates to each object N of K a class of sieves on N (called its *covering sieves*), so that N acts as an open set of a topological space and the covering sieves on N as the open coverings of N. With this topology, K becomes a *site.*

A Grothendieck topology on K may be generated by the data, for each object N of K, of a set F(N) of morphisms f_i: $N_i \rightarrow N$, called a *covering family* of N. It is obtained by taking first as covering sieves on N the sieve F*(N) generated by F(N) and the sieve I*(N) generated by the identity of N (its elements are all the morphisms with codomain N); and then constructing all the sieves *g**(R) where *g* is a morphism from N' to N, R a covering sieve on N', and *g**(R) has for elements the composites of the elements of R with *g*.

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