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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

Wednesday 14 May 2014 (morning)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	4

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

12 pages

Please **do not** write on this page.

Answers written on this page will not be marked.

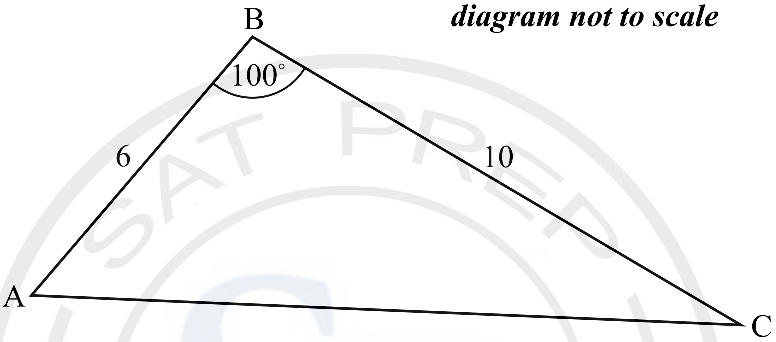
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows triangle ABC.



$AB = 6\text{ cm}$, $BC = 10\text{ cm}$, and $A\hat{B}C = 100^\circ$.

(a) Find AC. *[3]*

(b) Find $B\hat{C}A$. *[3]*

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2. *[Maximum mark: 5]*

Consider the expansion of $(x+3)^{10}$.

(a) Write down the number of terms in this expansion. *[1]*

(b) Find the term containing x^3. *[4]*

. . . .

3. *[Maximum mark: 7]*

The following table shows the average weights (y kg) for given heights (x cm) in a population of men.

Heights (x cm)	165	170	175	180	185
Weights (y kg)	67.8	70.0	72.7	75.5	77.2

(a) The relationship between the variables is modelled by the regression equation $y = ax + b$.

(i) Write down the value of a and of b.

(ii) Hence, estimate the weight of a man whose height is 172 cm. *[4]*

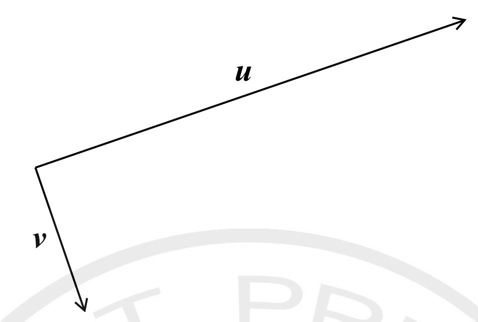
(b) (i) Write down the correlation coefficient.

(ii) State which **two** of the following describe the correlation between the variables. *[3]*

strong	zero	positive
negative	no correlation	weak

4. *[Maximum mark: 6]*

The following diagram shows two perpendicular vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.



(a) Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Represent $\boldsymbol{w}$ on the diagram above. *[2]*

(b) Given that $\boldsymbol{u} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find n. *[4]*

5. *[Maximum mark: 6]*

The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

(a) Find the number of deer in the reserve on 1 May 2014. *[3]*

(b) (i) Find the rate of change of the deer population on 1 May 2014.

(ii) Interpret the answer to part (i) with reference to the deer population size on 1 May 2014. *[3]*

6. *[Maximum mark: 8]*

Ramiro and Lautaro are travelling from Buenos Aires to El Moro.

Ramiro travels in a vehicle whose velocity in m s^{-1} is given by $V_R = 40 - t^2$, where t is in seconds.

Lautaro travels in a vehicle whose displacement from Buenos Aires in metres is given by $S_L = 2t^2 + 60$.

When $t = 0$, both vehicles are at the same point.

Find Ramiro's displacement from Buenos Aires when $t = 10$.

7. *[Maximum mark: 7]*

Let $f(x) = \dfrac{g(x)}{h(x)}$, where $g(2) = 18$, $h(2) = 6$, $g'(2) = 5$, and $h'(2) = 2$. Find the equation of the normal to the graph of f at $x = 2$.

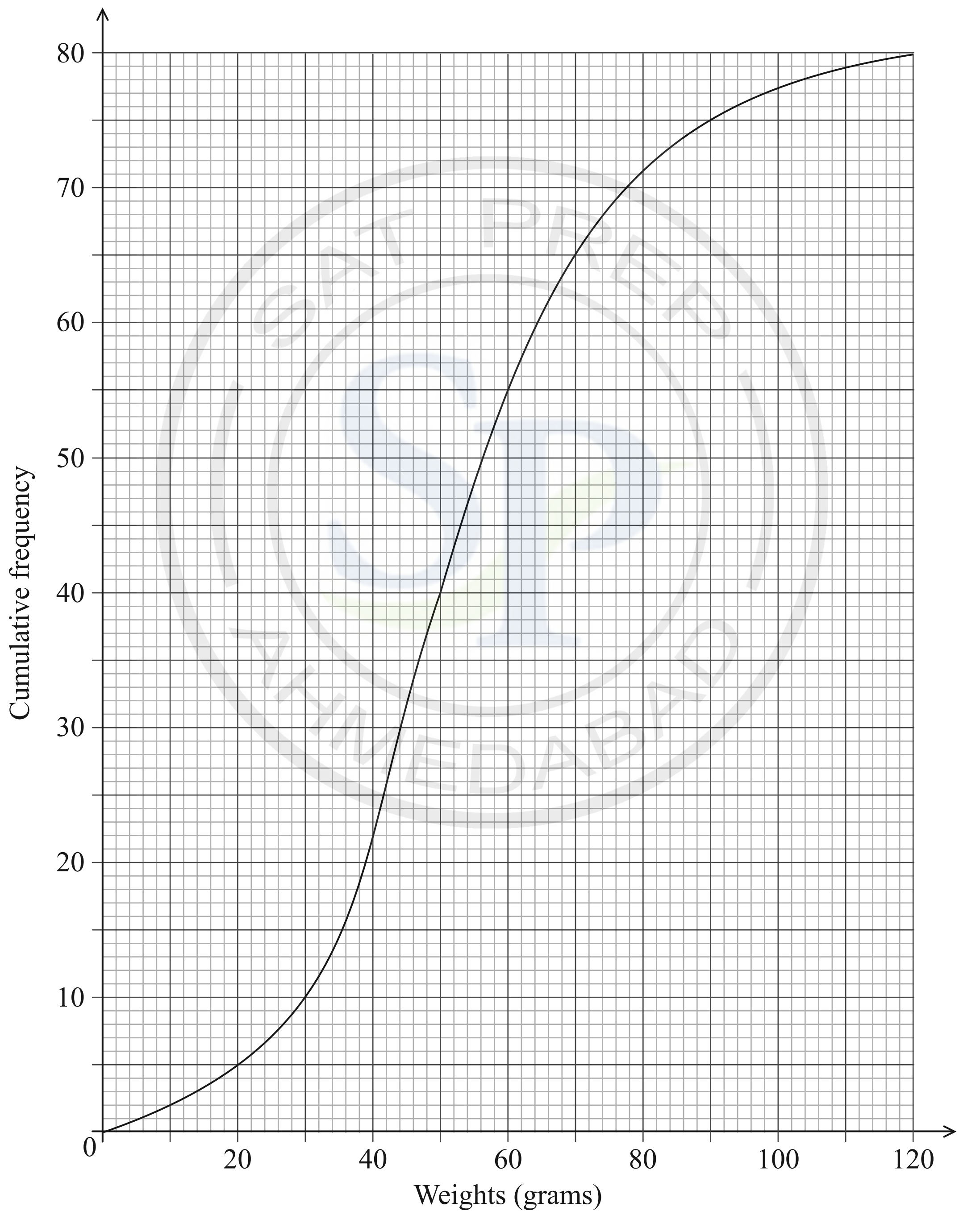
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 16]*

The weights in grams of 80 rats are shown in the following cumulative frequency diagram.



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*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) (i) Write down the median weight of the rats.

(ii) Find the percentage of rats that weigh 70 grams or less. *[4]*

The same data is presented in the following table.

Weights w grams	$0 \le w \le 30$	$30 < w \le 60$	$60 < w \le 90$	$90 < w \le 120$
Frequency	p	45	q	5

(b) (i) Write down the value of p.

(ii) Find the value of q. *[4]*

(c) Use the values from the table to estimate the mean and standard deviation of the weights. *[3]*

Assume that the weights of these rats are normally distributed with the mean and standard deviation estimated in part (c).

(d) Find the percentage of rats that weigh 70 grams or less. *[2]*

(e) A sample of five rats is chosen at random. Find the probability that at most three rats weigh 70 grams or less. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right)$, for $-4 \le x \le 4$.

(a) Sketch the graph of f. *[3]*

(b) Find the values of x where the function is decreasing. *[5]*

(c) The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \le c \le 2$. Find the value of

(i) a;

(ii) c. *[7]*

10. *[Maximum mark: 14]*

Let $f(x) = \frac{3x}{x-q}$, where $x \ne q$.

(a) Write down the equations of the vertical and horizontal asymptotes of the graph of f. *[2]*

The vertical and horizontal asymptotes to the graph of f intersect at the point $\text{Q}(1, 3)$.

(b) Find the value of q. *[2]*

(c) The point $\text{P}(x, y)$ lies on the graph of f. Show that $\text{PQ} = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$. *[4]*

(d) Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$. *[6]*

M14/5/MATME/SP2/ENG/TZ2/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

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1 hour 30 minutes

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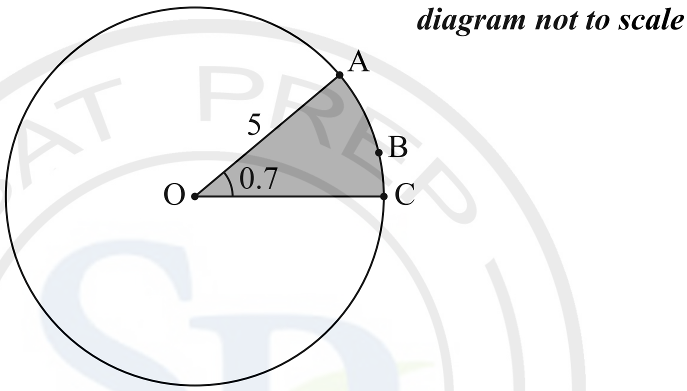
SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows a circle with centre O and radius 5 cm.



The points A, B and C lie on the circumference of the circle, and $A\hat{O}C = 0.7$ radians .

(a) (i) Find the length of the arc ABC.

(ii) Find the perimeter of the shaded sector. *[4]*

(b) Find the area of the shaded sector. *[2]*

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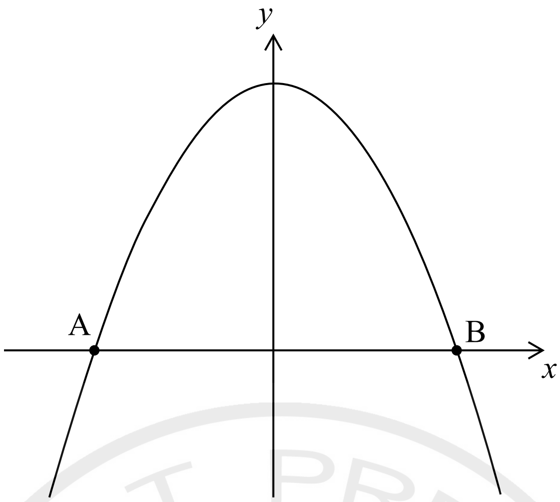
(Question 1 continued)

2. *[Maximum mark: 6]*

Let $f(x) = 5 - x^2$. Part of the graph of f is shown in the following diagram.



The graph crosses the x-axis at the points A and B.

(a) Find the x-coordinate of A and of B. *[3]*

(b) The region enclosed by the graph of f and the x-axis is revolved 360° about the x-axis. Find the volume of the solid formed. *[3]*

3. *[Maximum mark: 5]*

The following table shows the amount of fuel (y litres) used by a car to travel certain distances (x km).

Distance (x km)	40	75	120	150	195
Amount of fuel (y litres)	3.6	6.5	9.9	13.1	16.2

This data can be modelled by the regression line with equation $y = ax + b$.

(a) (i) Write down the value of a and of b.

(ii) Explain what the gradient a represents. *[3]*

(b) Use the model to estimate the amount of fuel the car would use if it is driven 110 km. *[2]*

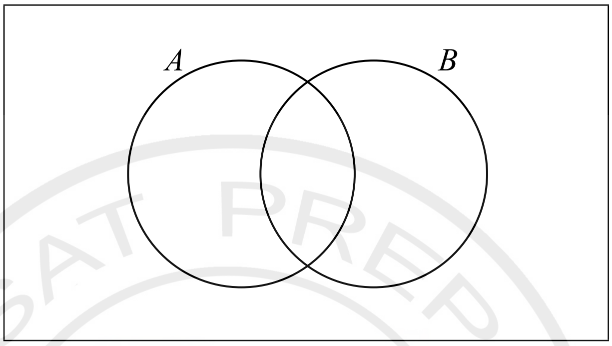
4. *[Maximum mark: 7]*

Let A and B be independent events, where $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.6$.

(a) Find $\mathrm{P}(A \cap B)$. *[2]*

(b) Find $\mathrm{P}(A \cup B)$. *[2]*

(c) (i) On the following Venn diagram, shade the region that represents $A \cap B'$.



(ii) Find $\mathrm{P}(A \cap B')$. *[3]*

5. *[Maximum mark: 7]*

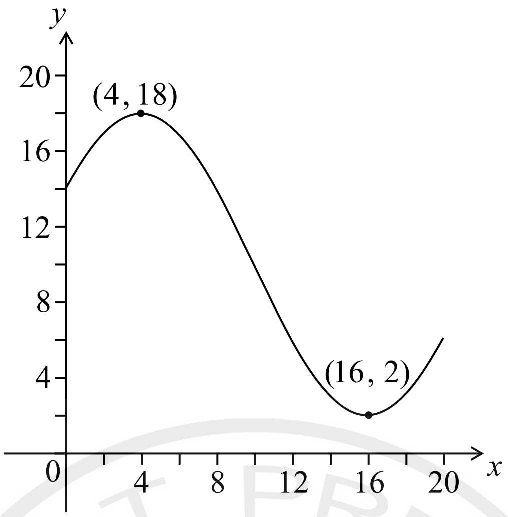
In triangle ABC, $AB = 6\,cm$ and $AC = 8\,cm$. The area of the triangle is $16\,cm^2$.

(a) Find the two possible values for $\hat{A}$. *[4]*

(b) Given that $\hat{A}$ is obtuse, find BC. *[3]*

6. *[Maximum mark: 8]*

Let $f(x) = p\cos\left(q(x+r)\right)+10$, for $0 \le x \le 20$. The following diagram shows the graph of f.



The graph has a maximum at $(4, 18)$ and a minimum at $(16, 2)$.

(a) Write down the value of r. *[2]*

(b) (i) Find p.

(ii) Find q. *[4]*

(c) Solve $f(x) = 7$. *[2]*

7. *[Maximum mark: 7]*

Consider the expansion of $x^2\left(3x^2+\frac{k}{x}\right)^8$. The constant term is 16 128.

Find k.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The number of bacteria in two colonies, A and B, starts increasing at the same time.

The number of bacteria in colony A after t hours is modelled by the function $A(t) = 12e^{0.4t}$.

(a) Find the initial number of bacteria in colony A. *[2]*

(b) Find the number of bacteria in colony A after four hours. *[3]*

(c) How long does it take for the number of bacteria in colony A to reach 400? *[3]*

The number of bacteria in colony B after t hours is modelled by the function $B(t) = 24e^{kt}$.

(d) After four hours, there are 60 bacteria in colony B. Find the value of k. *[3]*

(e) The number of bacteria in colony A first exceeds the number of bacteria in colony B after n hours, where $n \in \mathbb{Z}$. Find the value of n. *[4]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

A particle moves in a straight line. Its velocity, $v\,\text{ms}^{-1}$, at time t seconds, is given by

$$v = (t^2 - 4)^3, \text{ for } 0 \le t \le 3.$$

(a) Find the velocity of the particle when $t = 1$. *[2]*

(b) Find the value of t for which the particle is at rest. *[3]*

(c) Find the total distance the particle travels during the first three seconds. *[3]*

(d) Show that the acceleration of the particle is given by $a = 6t(t^2 - 4)^2$. *[3]*

(e) Find all possible values of t for which the velocity and acceleration are both positive or both negative. *[4]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 14]*

A forest has a large number of tall trees. The heights of the trees are normally distributed with a mean of 53 metres and a standard deviation of 8 metres. Trees are classified as giant trees if they are more than 60 metres tall.

(a) A tree is selected at random from the forest.

(i) Find the probability that this tree is a giant.

(ii) Given that this tree is a giant, find the probability that it is taller than 70 metres. *[6]*

(b) Two trees are selected at random. Find the probability that they are both giants. *[2]*

(c) 100 trees are selected at random.

(i) Find the expected number of these trees that are giants.

(ii) Find the probability that at least 25 of these trees are giants. *[6]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

Thursday 13 November 2014 (morning)

1 hour 30 minutes

Examination code

8	8	1	4	–	7	3	0	2

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13 pages

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $f(x) = 2x + 3$ and $g(x) = x^3$.

(a) Find $(f \circ g)(x)$. *[2]*

(b) Solve the equation $(f \circ g)(x) = 0$. *[3]*

2. *[Maximum mark: 6]*

The following table shows the Diploma score x and university entrance mark y for seven IB Diploma students.

Diploma score (x)	28	30	27	31	32	25	27
University entrance mark (y)	73.9	78.1	70.2	82.2	85.5	62.7	69.4

(a) Find the correlation coefficient. *[2]*

The relationship can be modelled by the regression line with equation $y = ax + b$.

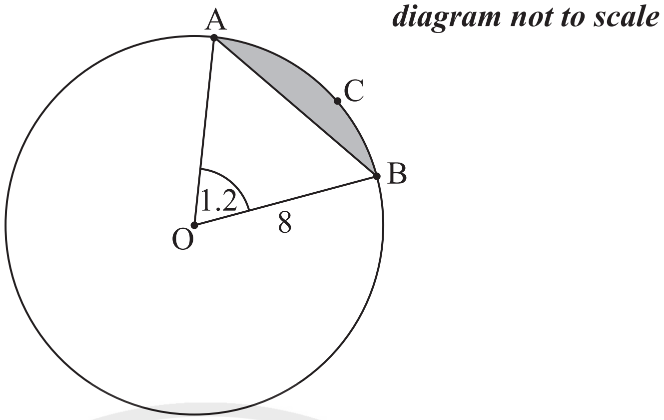
(b) Write down the value of a and of b. *[2]*

Rita scored a total of 26 in her IB Diploma.

(c) Use your regression line to estimate Rita's university entrance mark. *[2]*

3. *[Maximum mark: 7]*

The following diagram shows a circle with centre O and radius 8 cm.



The points A, B and C are on the circumference of the circle, and $A\hat{O}B = 1.2$ radians.

(a) Find the length of arc ACB. *[2]*

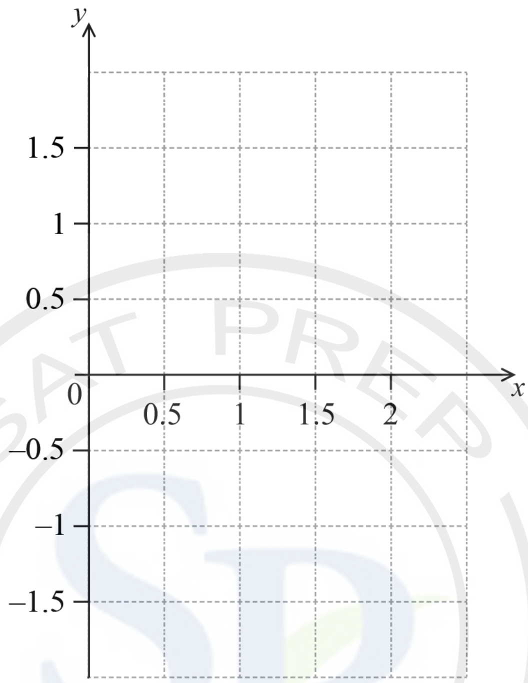
(b) Find AB. *[3]*

(c) Hence, find the perimeter of the shaded segment ABC. *[2]*

4. *[Maximum mark: 8]*

Let $f(x) = -x^4 + 2x^3 - 1$, for $0 \le x \le 2$.

(a) Sketch the graph of f on the following grid. *[3]*



(b) Solve $f(x) = 0$. *[2]*

(c) The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed. *[3]*

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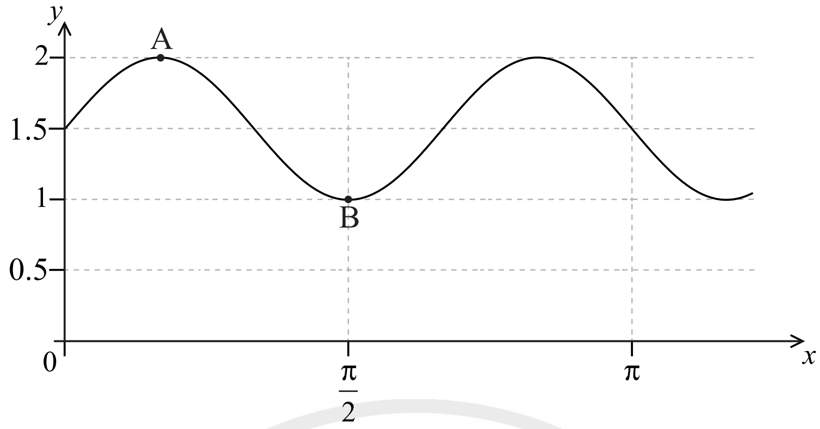
5. *[Maximum mark: 7]*

The following diagram shows part of the graph of $y = p\sin(qx) + r$.



The point $A\left(\frac{\pi}{6}, 2\right)$ is a maximum point and the point $B\left(\frac{\pi}{2}, 1\right)$ is a minimum point. Find the value of

(a) p; *[2]*

(b) r; *[2]*

(c) q. *[3]*

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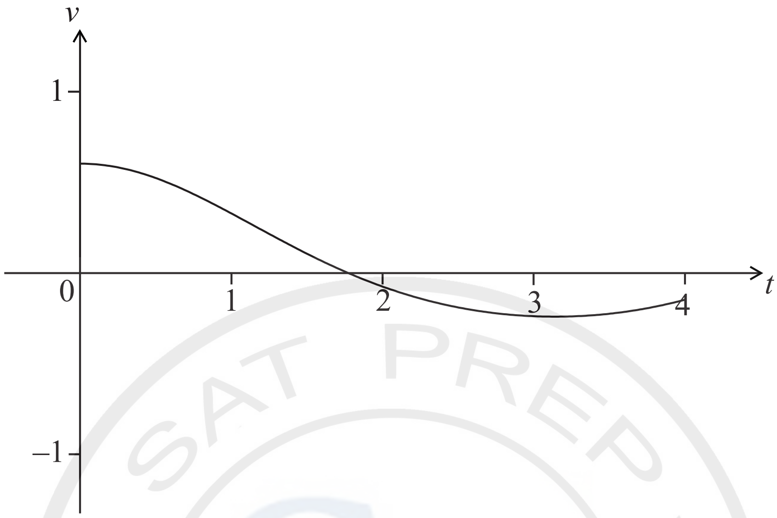
6. *[Maximum mark: 6]*

Consider the expansion of $\left(\frac{x^3}{2}+\frac{p}{x}\right)^8$. The constant term is 5103. Find the possible values of p.

7. *[Maximum mark: 6]*

A particle starts from point A and moves along a straight line. Its velocity, $v\,\text{m s}^{-1}$, after t seconds is given by $v(t) = e^{\frac{1}{2}\cos t} - 1$, for $0 \le t \le 4$. The particle is at rest when $t = \frac{\pi}{2}$. The following diagram shows the graph of v.



(a) Find the distance travelled by the particle for $0 \le t \le \frac{\pi}{2}$. *[2]*

(b) Explain why the particle passes through A again. *[4]*

Turn over

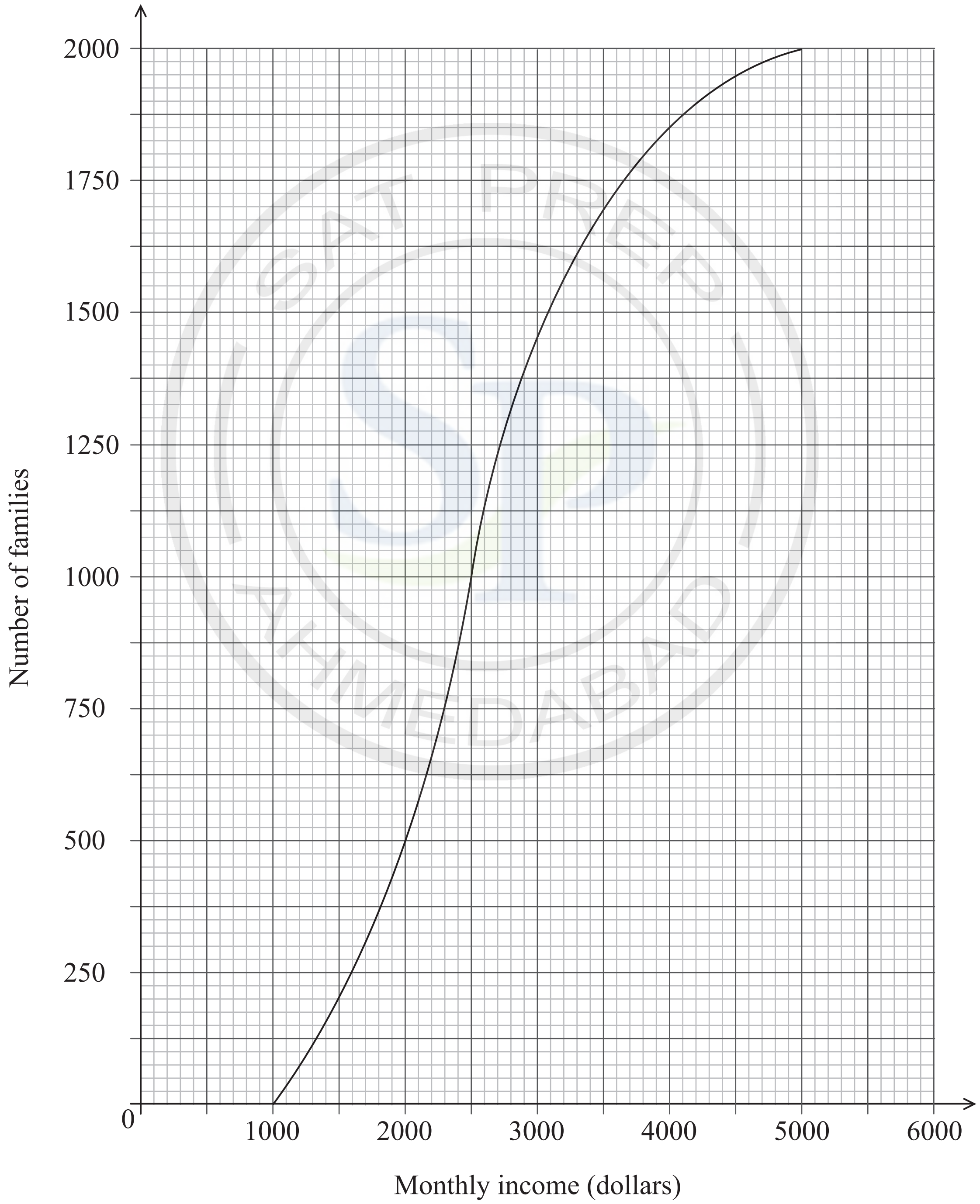
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The following cumulative frequency graph shows the monthly income, I dollars, of 2000 families.



(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) Find the median monthly income. *[2]*

(b) (i) Write down the number of families who have a monthly income of 2000 dollars or less.

(ii) Find the number of families who have a monthly income of more than 4000 dollars. *[4]*

The 2000 families live in two different types of housing. The following table gives information about the number of families living in each type of housing and their monthly income I.

	$1000 < I \le 2000$	$2000 < I \le 4000$	$4000 < I \le 5000$
Apartment	436	765	28
Villa	64	p	122

(c) Find the value of p. *[2]*

(d) A family is chosen at random.

(i) Find the probability that this family lives in an apartment.

(ii) Find the probability that this family lives in an apartment, given that its monthly income is greater than 4000 dollars. *[4]*

(e) Estimate the mean monthly income for families living in a villa. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

The first two terms of a geometric sequence u_n are $u_1 = 4$ and $u_2 = 4.2$.

(a) (i) Find the common ratio.

(ii) Hence or otherwise, find u_5. *[5]*

Another sequence v_n is defined by $v_n = an^k$, where $a, k \in \mathbb{R}$, and $n \in \mathbb{Z}^+$, such that $v_1 = 0.05$ and $v_2 = 0.25$.

(b) (i) Find the value of a.

(ii) Find the value of k. *[5]*

(c) Find the smallest value of n for which $v_n > u_n$. *[4]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

The weights of fish in a lake are normally distributed with a mean of 760 g and standard deviation σ. It is known that 78.87% of the fish have weights between 705 g and 815 g.

(a) (i) Write down the probability that a fish weighs more than 760 g.

(ii) Find the probability that a fish weighs less than 815 g. *[4]*

(b) (i) Write down the standardized value for 815 g.

(ii) Hence or otherwise, find σ. *[4]*

A fishing contest takes place in the lake. Small fish, called tiddlers, are thrown back into the lake. The maximum weight of a tiddler is 1.5 standard deviations below the mean.

(c) Find the maximum weight of a tiddler. *[2]*

(d) A fish is caught at random. Find the probability that it is a tiddler. *[2]*

(e) 25% of the fish in the lake are salmon. 10% of the salmon are tiddlers. Given that a fish caught at random is a tiddler, find the probability that it is a salmon. *[4]*

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will not be marked.

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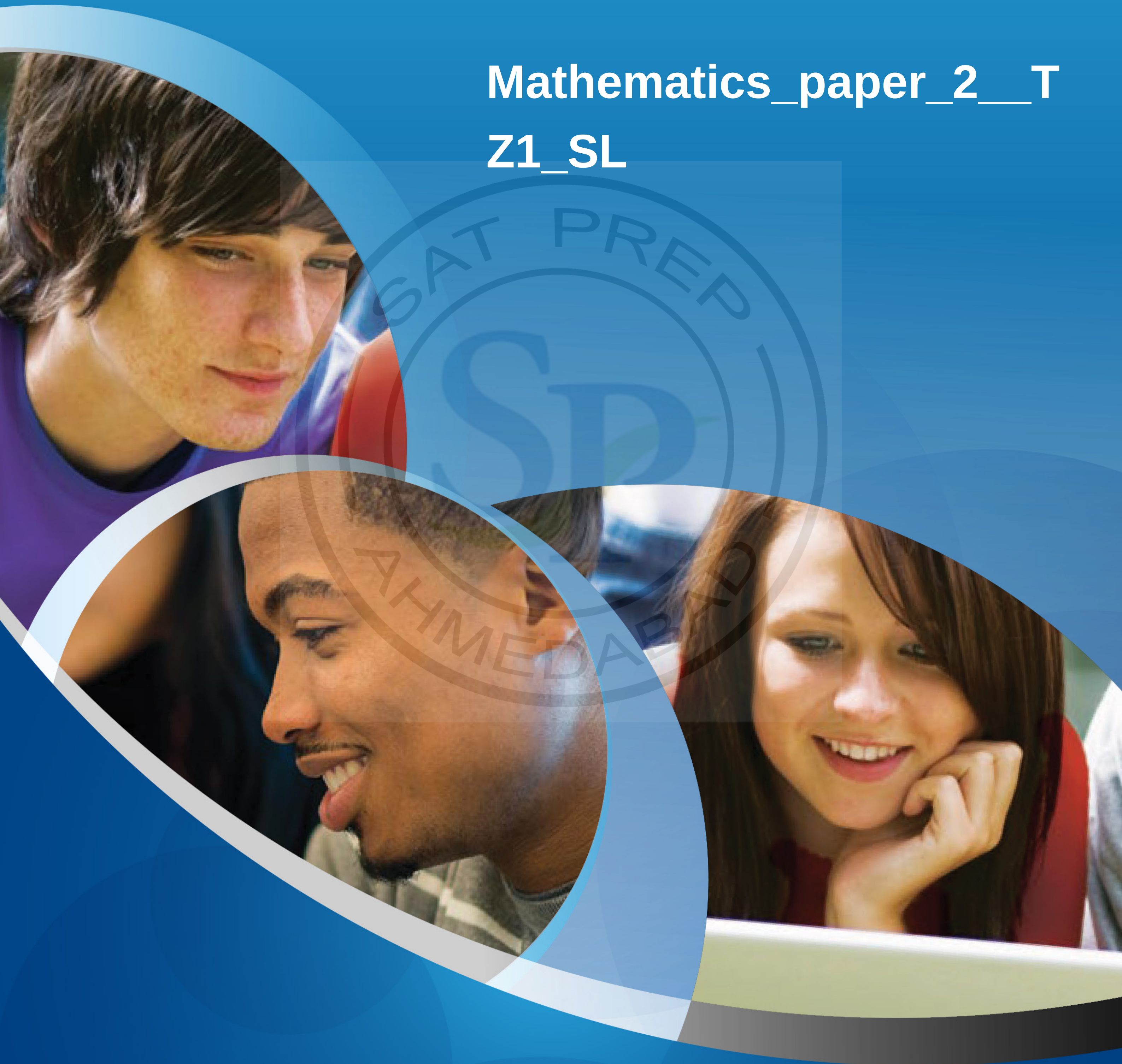

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 13 May 2015 (afternoon)

1 hour 30 minutes

Candidate session number

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Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

The following table shows the average number of hours per day spent watching television by seven mothers and each mother's youngest child.

Hours per day that a mother watches television (x)	2.5	3.0	3.2	3.3	4.0	4.5	5.8
Hours per day that her child watches television (y)	1.8	2.2	2.6	2.5	3.0	3.2	3.5

The relationship can be modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the correlation coefficient.

(ii) Write down the value of a and of b. [4]

Elizabeth watches television for an average of 3.7 hours per day.

(b) Use your regression line to predict the average number of hours of television watched per day by Elizabeth's youngest child. Give your answer correct to one decimal place. [3]

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2. [Maximum mark: 5]

Consider the expansion of $(2x+3)^8$.

(a) Write down the number of terms in this expansion. [1]

(b) Find the term in x^3. [4]

3. [Maximum mark: 6]

In an arithmetic sequence $u_{10} = 8$, $u_{11} = 6.5$.

(a) Write down the value of the common difference. [1]

(b) Find the first term. [3]

(c) Find the sum of the first 50 terms of the sequence. [2]

4. [Maximum mark: 7]

Let $f(x) = \frac{2x-6}{1-x}$, for $x \neq 1$.

(a) For the graph of f

(i) find the x-intercept;

(ii) write down the equation of the vertical asymptote;

(iii) find the equation of the horizontal asymptote. [5]

(b) Find $\lim_{x \to \infty} f(x)$. [2]

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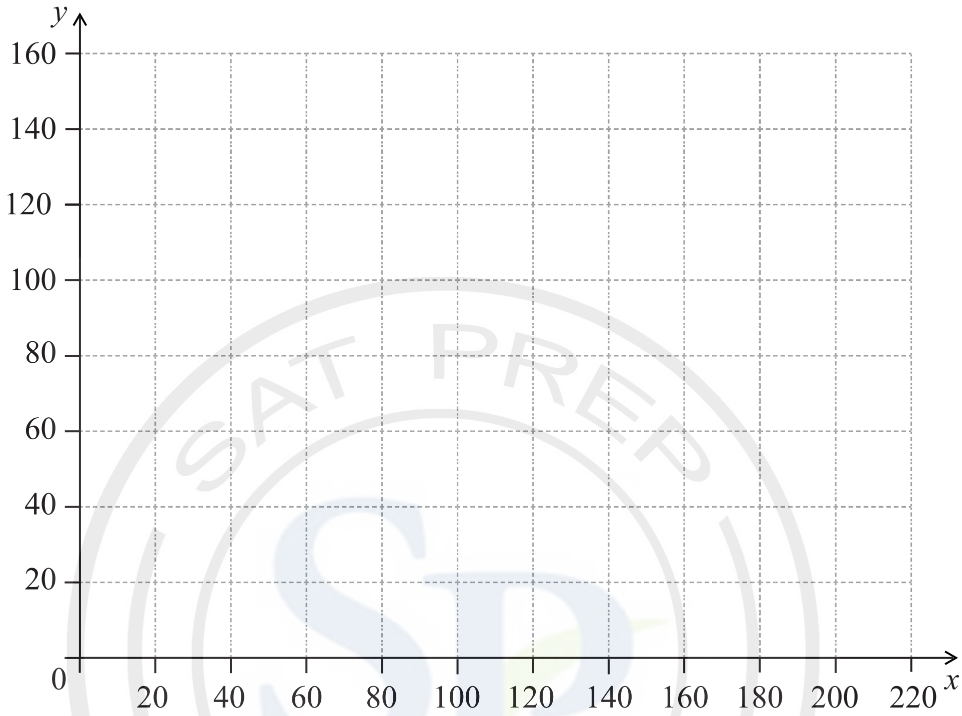
5. [Maximum mark: 6]

Let $G(x) = 95\mathrm{e}^{(-0.02x)} + 40$, for $20 \le x \le 200$.

(a) On the following grid, sketch the graph of G. [3]



(b) Robin and Pat are planning a wedding banquet. The cost per guest, G dollars, is modelled by the function $G(n) = 95\mathrm{e}^{(-0.02n)} + 40$, for $20 \le n \le 200$, where n is the number of guests.

Calculate the **total** cost for 45 guests. [3]

(This question continues on the following page)

(Question 5 continued)

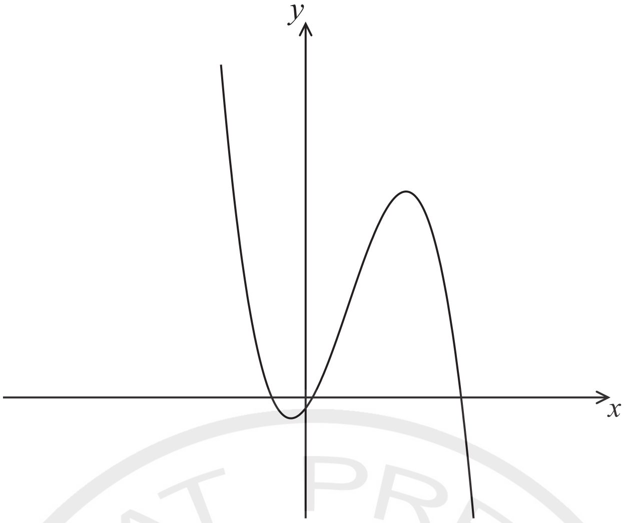
6. [Maximum mark: 7]

Let $f(x) = \frac{\ln(4x)}{x}$, for $0 < x \leq 5$.

Points $P(0.25, 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q.

7. [Maximum mark: 7]

The following diagram shows part of the graph of $f(x) = -2x^3 + 5.1x^2 + 3.6x - 0.4$.



(a) Find the coordinates of the local minimum point. [2]

(b) The graph of f is translated to the graph of g by the vector $\begin{pmatrix} 0 \\ k \end{pmatrix}$. Find all values of k so that $g(x) = 0$ has exactly one solution. [5]

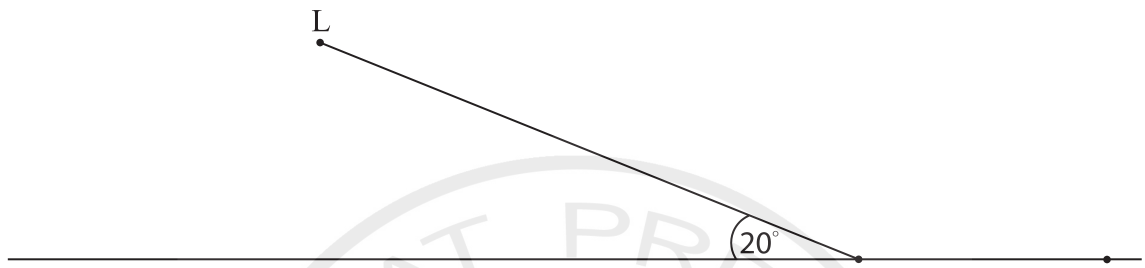
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

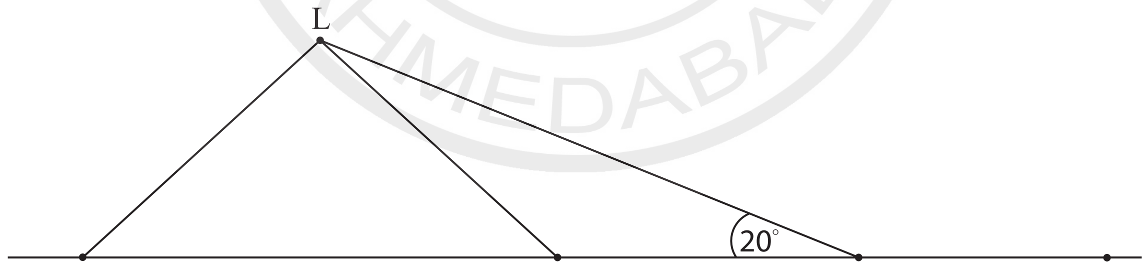
The following diagram shows a straight shoreline, with a supply store at S, a town at T, and an island L.



A boat delivers supplies to the island. The boat leaves S, and sails to the island. Its path makes an angle of 20° with the shoreline.

(a) The boat sails at 6 km per hour, and arrives at L after 1.5 hours. Find the distance from S to L. [2]

It is decided to change the position of the supply store, so that its distance from L is 5 km. The following diagram shows the two possible locations P and Q for the supply store.



(b) Find the size of $\text{S}\hat{\text{P}}\text{L}$ and of $\text{S}\hat{\text{Q}}\text{L}$. [5]

(c) The town wants the new supply store to be as near as possible to the town.

(i) State which of the points P or Q is chosen for the new supply store.

(ii) Hence find the distance between the old supply store and the new one. [6]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A company makes containers of yogurt. The volume of yogurt in the containers is normally distributed with a mean of 260 ml and standard deviation of 6 ml.

A container which contains less than 250 ml of yogurt is **underfilled**.

(a) A container is chosen at random. Find the probability that it is underfilled. [2]

The company decides that the probability of a container being underfilled should be reduced to 0.02. It decreases the standard deviation to σ and leaves the mean unchanged.

(b) Find σ. [4]

The company changes to the new standard deviation, σ, and leaves the mean unchanged. A container is chosen at random for inspection. It passes inspection if its volume of yogurt is between 250 and 271 ml.

(c) (i) Find the probability that it passes inspection.

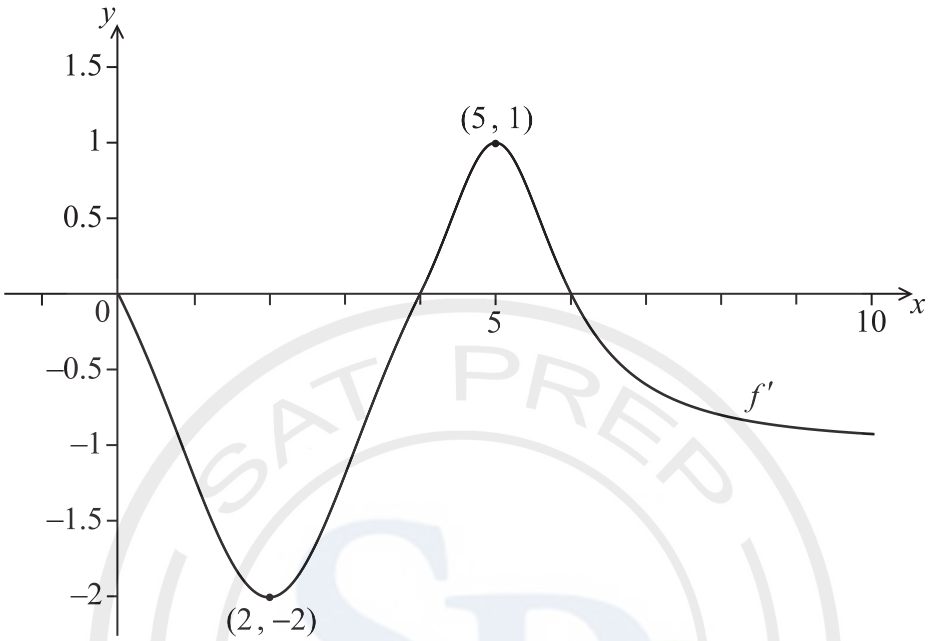
(ii) Given that the container is **not** underfilled, find the probability that it passes inspection. [6]

(d) A sample of 50 containers is chosen at random. Find the probability that 48 or more of the containers pass inspection. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

Consider a function f, for $0 \le x \le 10$. The following diagram shows the graph of f', the derivative of f.



The graph of f' passes through $(2, -2)$ and $(5, 1)$, and has x-intercepts at 0, 4 and 6.

(a) The graph of f has a local maximum point when $x = p$. State the value of p, and justify your answer . [3]

(b) Write down $f'(2)$. [1]

Let $g(x) = \ln(f(x))$ and $f(2) = 3$.

(c) Find $g'(2)$. [4]

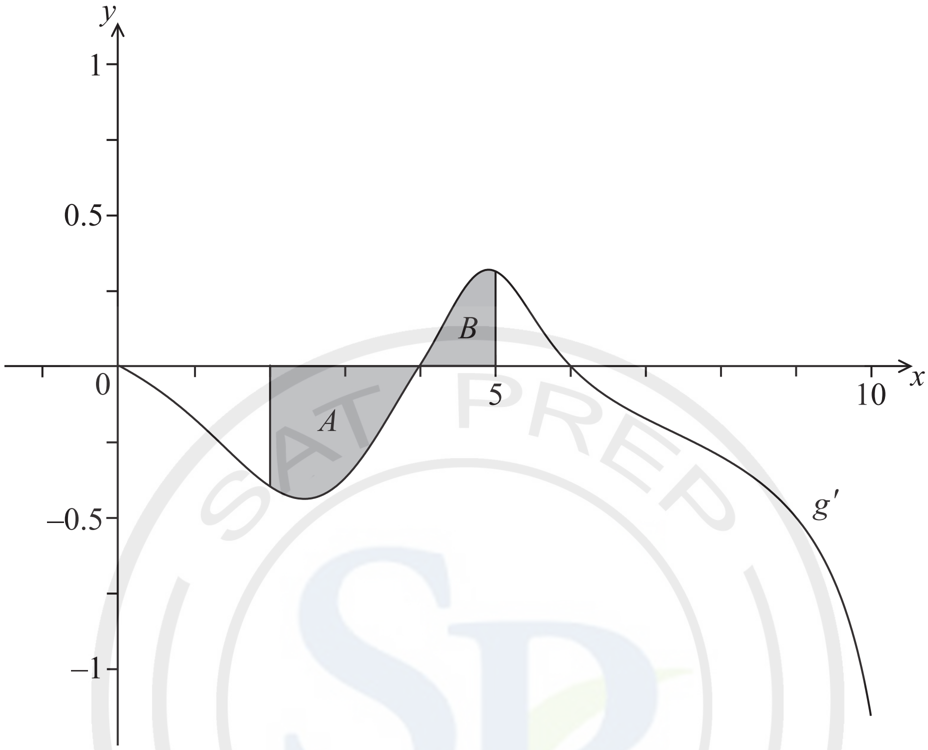
(d) Verify that $\ln 3 + \int_2^a g'(x)\,\mathrm{d}x = g(a)$, where $0 \le a \le 10$. [4]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

(e) The following diagram shows the graph of g', the derivative of g.



The shaded region A is enclosed by the curve, the x-axis and the line $x = 2$, and has area 0.66 units^2.

The shaded region B is enclosed by the curve, the x-axis and the line $x = 5$, and has area 0.21 units^2.

Find $g(5)$. [4]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Markscheme

May 2015

Mathematics

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions and the document "**Mathematics SL: Guidance for e-marking May 2015**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the RM assessor tool. Please check that you are entering marks for the right question. All the marks will be added and recorded by RM assessor.

If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.

- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal (see examples on next page).

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 ***N*** **marks**

If ***no*** *working shown, award* ***N*** *marks for* ***correct*** *answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (****M****,* ***A****,* ***R****). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award* ***N0****.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 **Implied and must be seen marks**

Implied marks appear in ***brackets*** *eg* ***(M1)****.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

Must be seen marks appear without ***brackets*** *eg* ***M1***.

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 **Follow through marks (only applied after an error is made)**

*Follow through (****FT****) marks are awarded where an incorrect answer (final or intermediate) from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the* ***final*** *answer, then* ***FT*** *marks should be awarded if appropriate. Examiners are expected to check student work in order to award* ***FT*** *marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation". Accept sloppy notation in the working, where this is followed by correct working eg $-2^2 = 4$ where they should have written $(-2)^2 = 4$.

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value, the exact value if applicable, and the correct 3 sf answer.

Units (which are generally not required) will appear in brackets at the end.

Section A

1. (a) (i) evidence of valid approach *(M1)*

eg 1 correct value for r, (or for a or b, seen in (ii))

0.946591

$r = 0.947$ ***A1*** ***N2***

(ii) $a = 0.500957$, $b = 0.803544$

$a = 0.501, b = 0.804$ ***A1A1*** ***N2***

[4 marks]

(b) substituting $x = 3.7$ into **their** equation *(M1)*

eg $0.501(3.7) + 0.804$

2.65708 (2 hours 39.4252 minutes) *(A1)*

$y = 2.7$ (hours)(**must** be correct 1 dp, accept 2 hours 39.4 minutes) ***A1*** ***N3***

[3 marks]

Total [7marks]

2. (a) 9 terms ***A1*** ***N1***

[1 mark]

(b) valid approach to find the required term *(M1)*

eg $\binom{8}{r}(2x)^{8-r}(3)^r$, $(2x)^8(3)^0 + (2x)^7(3)^1 + \ldots$, Pascal's triangle to 8th row

identifying correct term (may be indicated in expansion) *(A1)*

eg 6th term, $r = 5$, $\binom{8}{5}$, $(2x)^3(3)^5$

correct working (may be seen in expansion) *(A1)*

eg $\binom{8}{5}(2x)^3(3)^5$, $56 \times 2^3 \times 3^5$

$108864x^3$ (accept $109000x^3$) ***A1*** ***N3***

[4 marks]

Notes: Do not award any marks if there is clear evidence of adding instead of multiplying.
Do not award final ***A1*** for a final answer of 108864, even if $108864x^3$ is seen previously.
If no working shown award ***N2*** for 108864.

Total [5 marks]

3. (a) $d = -1.5$ *A1* *N1*

[1 mark]

(b) **METHOD 1**

valid approach *(M1)*

eg $u_{10} = u_1 + 9d$, $8 = u_1 - 9(-1.5)$

correct working *(A1)*

eg $8 = u_1 + 9d$, $6.5 = u_1 + 10d$, $u_1 = 8 - 9(-1.5)$

$u_1 = 21.5$ *A1* *N2*

METHOD 2

attempt to list 3 or more terms in either direction *(M1)*

eg $9.5, 11, 12.5, \ldots$; $5, 3.5, 2, \ldots$ …

correct list of 4 or more terms in **correct** direction *(A1)*

eg 9.5, 11, 12.5, 14

$u_1 = 21.5$ *A1* *N2*

[3 marks]

(c) correct expression *(A1)*

eg $\frac{50}{2}\left(2(21.5) + 49(-1.5)\right)$, $\frac{50}{2}(21.5 - 52)$, $\sum_{k=1}^{50} 21.5 + (k-1)(-1.5)$

$\text{sum} = -762.5$ **(exact)** *A1* *N2*

[2 marks]

Total [6 marks]

4. (a) (i) valid approach *(M1)*

eg sketch, $f(x) = 0$, $0 = 2x - 6$

$x = 3$ or $(3, 0)$ *A1* *N2*

(ii) $x = 1$ (must be equation) *A1* *N1*

(iii) valid approach *(M1)*

eg sketch, $\frac{2x}{-1x}$, inputting large values of x, L'Hopital's rule

$y = -2$ (must be equation) *A1* *N2*

[5 marks]

(b) valid approach *(M1)*

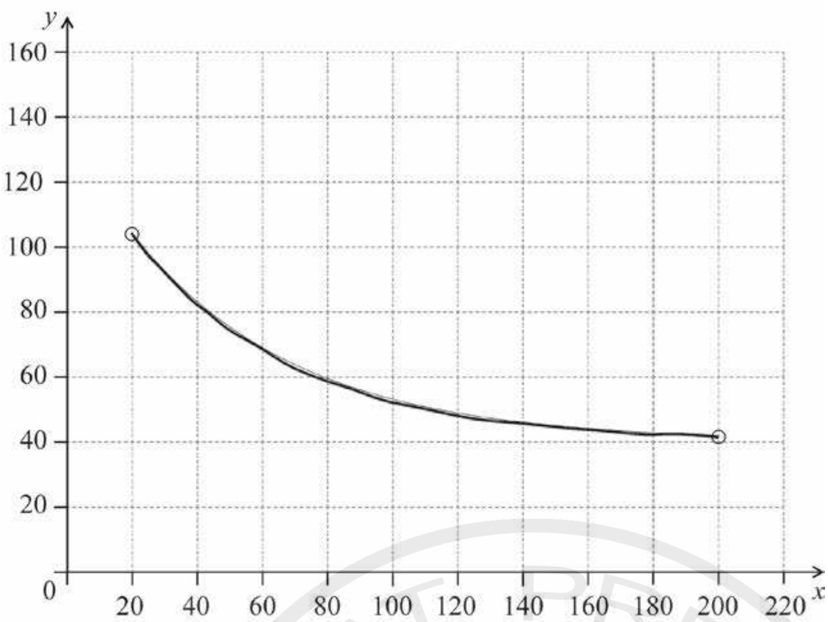
eg recognizing that $\lim\limits_{x \to \infty}$ is related to the horizontal asymptote, table with large values of x, their y value from (a)(iii), L'Hopital's rule

$\lim\limits_{x \to \infty} f(x) = -2$ *A1* *N2*

[2 marks]

Total [7 marks]

5. (a)



A1A1A1 ***N3***

Note:	Curve must be approximately correct exponential shape (concave up and decreasing). Only if the shape is approximately correct, award the following: ***A1*** for left endpoint in circle, ***A1*** for right endpoint in circle, ***A1*** for asymptotic to $y = 40$ (must not go below $y = 40$).

[3 marks]

(b) attempt to find $G(45)$ ***(M1)***

eg 78.6241, value read from **their** graph

multiplying cost times number of people ***(M1)***

eg 45×78.6241, $G(45) \times 45$

3538.08

3540 (dollars) ***A1*** ***N2***

[3 marks]

Total [6 marks]

6. recognizing that the gradient of tangent is the derivative *(M1)*
eg f'

finding the gradient of f at P *(A1)*
eg $f'(0.25) = 16$

evidence of taking negative reciprocal of **their** gradient at P *(M1)*
eg $\frac{-1}{m}, -\frac{1}{f'(0.25)}$

equating derivatives ***M1***
eg $f'(x) = \frac{-1}{16}, \; f' = -\frac{1}{m}, \; \frac{x\left(\frac{1}{x}\right) - \ln(4x)}{x^2} = 16$

finding the x-coordinate of Q, $x = 0.700750$
$\boldsymbol{x = 0.701}$ ***A1*** ***N3***

attempt to substitute **their** x into f to find the y-coordinate of Q *(M1)*
eg $f(0.7)$

$y = 1.47083$
$\boldsymbol{y = 1.47}$ ***A1*** ***N2***

[7 marks]

7. (a) $(-0.3, -0.967)$
$x = -0.3$ (exact), $y = -0.967$ (exact) ***A1A1*** ***N2***

[2 marks]

(b) y-coordinate of local maximum is $y = 11.2$ *(A1)*

negating the y-coordinate of one of the max/min *(M1)*
eg $y = 0.967, \; y = -11.2$

recognizing that the solution set has two intervals ***R1***
eg two answers,

$k < -11.2, \; k > 0.967$ ***A1A1*** ***N3N2***

[5 marks]

Notes: If working shown, do not award the final mark if strict inequalities are not used.
If no working shown, award ***N2*** for $k \le -11.2$ or ***N1*** for $k \ge 0.967$

Total [7 marks]

Section B

8. (a) valid approach *(M1)*

eg $\text{speed} = \frac{\text{distance}}{\text{time}}$, 6×1.5

$\text{SL} = 9 \text{ (km)}$ ***A1*** ***N2***

[2 marks]

(b) evidence of choosing sine rule *(M1)*

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\sin\theta = \frac{(\text{SL})\sin 20^\circ}{5}$

correct substitution *(A1)*

eg $\frac{\sin\theta}{9} = \frac{\sin 20^\circ}{5}$

37.9981

$\text{S}\hat{\text{P}}\text{L} = 38.0^\circ$ ***A1*** ***N2***

recognition that second angle is the supplement of first *(M1)*

eg $180 - x$

142.001

$\text{S}\hat{\text{Q}}\text{L} = 142^\circ$ ***A1*** ***N2***

[5 marks]

continued…

Question 8 continued

(c) (i) new store is at Q ***A1*** ***N1***

(ii) **METHOD 1**

attempt to find third angle ***(M1)***

eg $\mathrm{S\hat{L}P} = 180 - 20 - 38$, $\mathrm{S\hat{L}Q} = 180 - 20 - 142$

$\mathrm{S\hat{L}Q} = 17.998^\circ$ (seen anywhere) ***A1***

evidence of choosing sine rule or cosine rule ***(M1)***
correct substitution into sine rule or cosine rule ***(A1)***

eg $\dfrac{x}{\sin 17.998} = \dfrac{5}{\sin 20}\left(= \dfrac{9}{\sin 142}\right)$, $9^2 + 5^2 - 2(9)(5)\cos 17.998^\circ$

$4.51708\,\mathrm{km}$
4.52 (km) ***A1*** ***N3***

METHOD 2

evidence of choosing cosine rule ***(M1)***
correct substitution into cosine rule ***A1***

eg $9^2 = x^2 + 5^2 - 2(x)(5)\cos 142^\circ$

attempt to solve ***(M1)***
eg sketch; setting quadratic equation equal to zero;
$0 = x^2 + 7.88x - 56$

one correct value for x ***(A1)***
eg $x = -12.3973$, $x = 4.51708$

4.51708
4.52 (km) ***A1*** ***N3***
[6 marks]

Total [13 marks]

9. (a) 0.0477903
probability $= 0.0478$ ***A2*** ***N2***
[2 marks]

(b) $\mathrm{P}(\text{volume} < 250) = 0.02$ ***(M1)***

$z = -2.05374$ (may be seen in equation) ***A1***

attempt to set up equation with z ***(M1)***

eg $\frac{\mu - 260}{\sigma} = z$, $260 - 2.05(\sigma) = 250$

4.86914
$\sigma = 4.87$ (ml) ***A1*** ***N3***
[4 marks]

(c) (i) 0.968062
$\mathrm{P}(250 < \text{Vol} < 271) = 0.968$ ***A2*** ***N2***

(ii) recognizing conditional probability (seen anywhere, including in correct working) ***R1***

eg $\mathrm{P}\left(A \middle| B\right)$, $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$, $\mathrm{P}(A \cap B) = \mathrm{P}\left(A \middle| B\right)\mathrm{P}(B)$

correct value or expression for P(not underfilled) ***(A1)***
eg 0.98, $1 - 0.02$, $1 - P(X < 250)$

probability $= \frac{0.968}{0.98}$ ***A1***
0.987818

probability $= 0.988$ ***A1*** ***N2***
[6 marks]

continued…

Question 9 continued

(d) **METHOD 1**

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \quad \text{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

$\text{P}(X \le 47) = 0.214106$ ***(A1)***

evidence of using complement ***(M1)***
eg $1 - \text{P}(X \le 47)$

0.785894
probability $= 0.786$ ***A1 N3***

METHOD 2

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \quad \text{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

$\text{P}(\text{not pass}) = 1 - \text{P}(\text{pass}) = 0.0319378$ ***(A1)***
evidence of attempt to find P (2 or fewer fail) ***(M1)***
eg 0, 1, or 2 not pass, $\text{B}(50, 2)$

0.785894
probability $= 0.786$ ***A1 N3***

METHOD 3

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \quad \text{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

evidence of summing probabilities ***(M1)***
eg $\text{P}(X = 48) + \text{P}(X = 49) + \text{P}(X = 50)$

correct working
eg $0.263088 + 0.325488 + 0.197317$ ***(A1)***

0.785894
probability $= 0.786$ ***A1 N3***

[4 marks]

Total [16 marks]

10. (a) $p = 6$ *A1* *N1*

recognising that turning points occur when $f'(x) = 0$ *R1* *N1*

eg correct sign diagram

f' changes from positive to negative at $x = 6$ *R1* *N1*

[3 marks]

(b) $f'(2) = -2$ *A1* *N1*

[1 mark]

(c) attempt to apply chain rule *(M1)*

eg $\ln(x)' \times f'(x)$

correct expression for $g'(x)$ *(A1)*

eg $g'(x) = \frac{1}{f(x)} \times f'(x)$

substituting $x = 2$ into **their** g' *(M1)*

eg $\frac{f'(2)}{f(2)}$

-0.666667

$\mathbf{g'(2) = -\frac{2}{3}\ (exact), -0.667}$ *A1* *N3*

[4 marks]

(d) evidence of integrating $g'(x)$ *(M1)*

eg $g(x)\big|_2^a$, $g(x)\big|_a^2$

applying the fundamental theorem of calculus (seen anywhere) *R1*

eg $\int_2^a g'(x) = g(a) - g(2)$

correct substitution into integral *(A1)*

eg $\ln 3 + g(a) - g(2)$, $\ln 3 + g(a) - \ln(f(2))$

$\ln 3 + g(a) - \ln 3$ *A1*

$\ln 3 + \int_2^a g'(x) = g(a)$ *AG* *N0*

[4 marks]

continued…

Question 10 continued

(e) **METHOD 1**

substituting $a=5$ into the formula for $g(a)$ ***(M1)***

eg $\int_2^5 g'(x)\,dx$, $g(5)=\ln 3+\int_2^5 g'(x)\,dx$ (do not accept only $g(5)$)

attempt to substitute areas ***(M1)***

eg $\ln 3+0.66-0.21$, $\ln 3+0.66+0.21$

correct working ***(A1)***

eg $g(5)=\ln 3+(-0.66+0.21)$

0.648612

$g(5)=\ln 3-0.45$ (exact), $\mathbf{0.649}$ ***A1 N3***

METHOD 2

attempt to set up an equation for one shaded region ***(M1)***

eg $\int_4^5 g'(x)\,dx=0.21$, $\int_2^4 g'(x)\,dx=-0.66$, $\int_2^5 g'(x)\,dx=-0.45$

two correct equations ***(A1)***

eg $g(5)-g(4)=0.21$, $g(2)-g(4)=0.66$

combining equations to eliminate $g(4)$ ***(M1)***

eg $g(5)-[\ln 3-0.66]=0.21$

0.648612

$g(5)=\ln 3-0.45$ (exact), $\mathbf{0.649}$ ***A1 N3***

METHOD 3

attempt to set up a definite integral ***(M1)***

eg $\int_2^5 g'(x)\,dx=-0.66+0.21$, $\int_2^5 g'(x)\,dx=-0.45$

correct working ***(A1)***

eg $g(5)-g(2)=-0.45$

correct substitution ***(A1)***

eg $g(5)-\ln 3=-0.45$

0.648612

$g(5)=\ln 3-0.45$ (exact), $\mathbf{0.649}$ ***A1 N3***

[4 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 13 May 2015 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12 pages

2215 – 7306

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

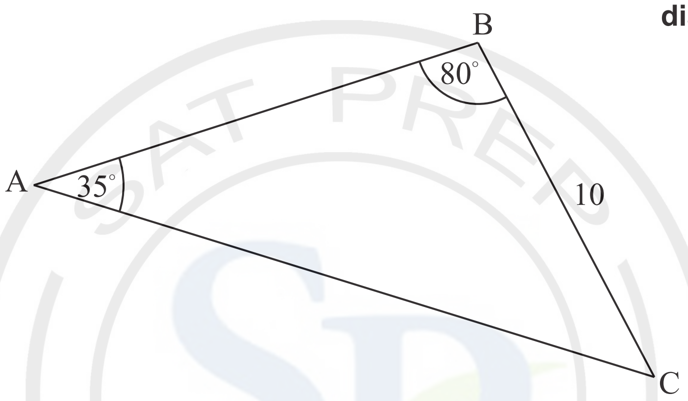
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows triangle ABC.

diagram not to scale



$BC = 10\,cm$, $A\hat{B}C = 80^\circ$ and $B\hat{A}C = 35^\circ$.

(a) Find AC. [3]

(b) Find the area of triangle ABC. [3]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 7]

Let $\boldsymbol{u} = 6\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{v} = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$.

(a) Find

(i) $\boldsymbol{u} \bullet \boldsymbol{v}$;

(ii) $|\boldsymbol{u}|$;

(iii) $|\boldsymbol{v}|$. [5]

(b) Find the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$. [2]

3. [Maximum mark: 6]

The following table shows the sales, y millions of dollars, of a company, x years after it opened.

Time after opening (x years)	2	4	6	8	10
Sales (y millions of dollars)	12	20	30	36	52

The relationship between the variables is modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the value of a and of b.

(ii) Write down the value of r. [4]

(b) Hence estimate the sales in millions of dollars after seven years. [2]

4. [Maximum mark: 5]

The third term in the expansion of $(x+k)^8$ is $63x^6$. Find the possible values of k.

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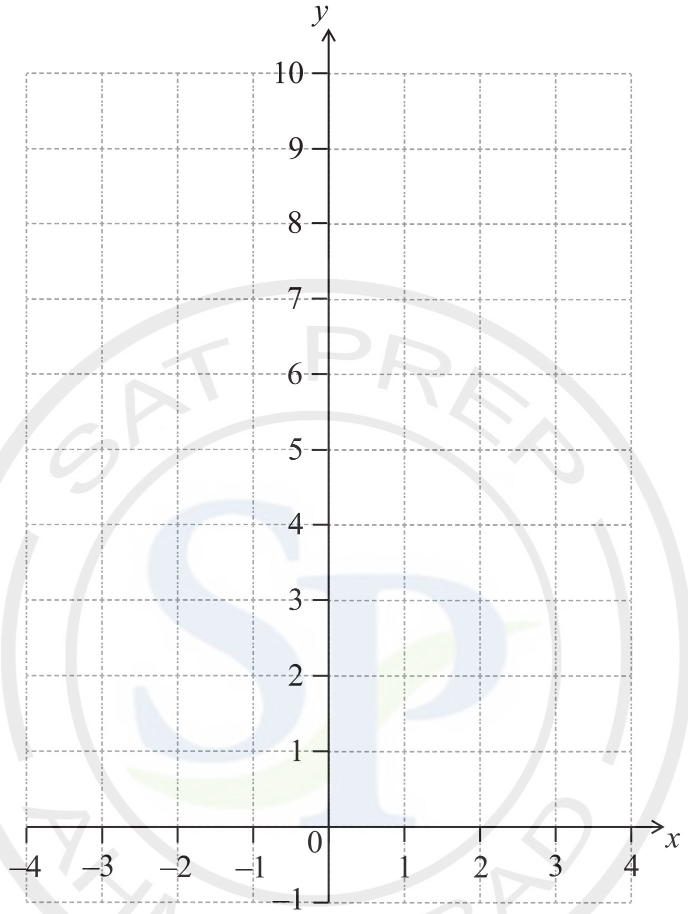
5. [Maximum mark: 6]

Let $f(x) = e^{x+1} + 2$, for $-4 \le x \le 1$.

(a) On the following grid, sketch the graph of f. [3]



(b) The graph of f is translated by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to obtain the graph of a function g.

Find an expression for $g(x)$. [3]

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6. [Maximum mark: 7]

Ramiro walks to work each morning. During the first minute he walks 80 metres. In each subsequent minute he walks $90\,\%$ of the distance walked during the previous minute. The distance between his house and work is 660 metres. Ramiro leaves his house at $08{:}00$ and has to be at work by $08{:}15$.

Explain why he will not be at work on time.

7. [Maximum mark: 8]

Let $f(x)=kx^2+kx$ and $g(x)=x-0.8$. The graphs of f and g intersect at two distinct points. Find the possible values of k.

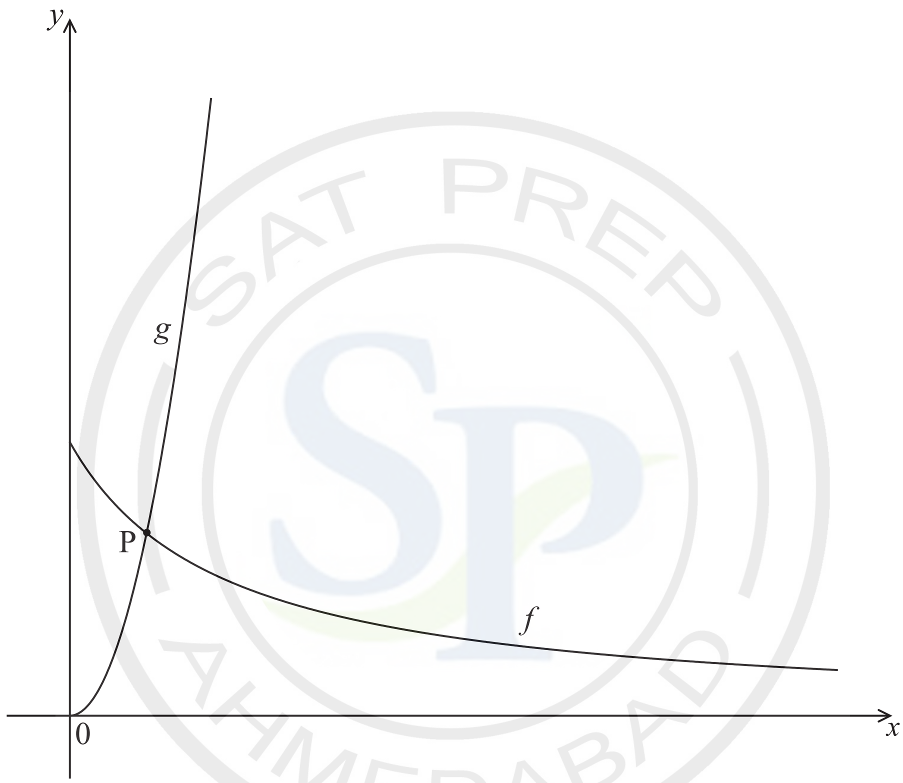
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

Let $f(x) = \frac{9}{x+2}$ and $g(x) = 3x^2$, for $x \geq 0$. Parts of the graphs of f and g are shown in the following diagram.



The graphs of f and g intersect at the point $\mathrm{P}(p, q)$.

(a) Find the value of p and of q. [3]

(b) Write down $f'(p)$. [2]

Let L be the normal to the graph of f at P.

(c) (i) Find the equation of L, giving your answer in the form $y = ax + b$.

(ii) Write down the y-intercept of L. [5]

(d) Let R be the region enclosed by the y-axis, the graph of g and the line L. Find the area of R. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A machine manufactures a large number of nails. The length, L mm, of a nail is normally distributed, where $L \sim \mathrm{N}\left(50, \sigma^2\right)$.

(a) Find $\mathrm{P}(50 - \sigma < L < 50 + 2\sigma)$. [3]

(b) The probability that the length of a nail is less than 53.92 mm is 0.975.
Show that $\sigma = 2.00$ (correct to three significant figures). [2]

All nails with length at least t mm are classified as large nails.

(c) A nail is chosen at random. The probability that it is a large nail is 0.75.
Find the value of t. [3]

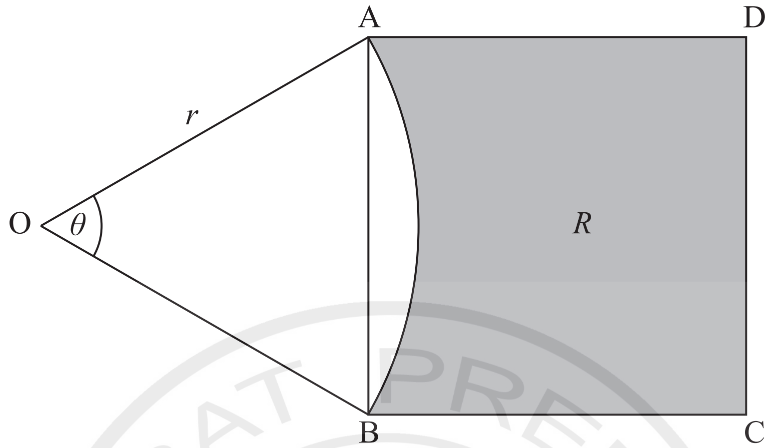
(d) (i) A nail is chosen at random from the large nails. Find the probability that the length of this nail is less than 50.1 mm.

(ii) Ten nails are chosen at random from the large nails. Find the probability that at least two nails have a length that is less than 50.1 mm. [8]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

The following diagram shows a square ABCD, and a sector OAB of a circle centre O, radius r. Part of the square is shaded and labelled R.



$A\hat{O}B = \theta$, where $0.5 \le \theta < \pi$.

(a) Show that the area of the square ABCD is $2r^2(1-\cos\theta)$. [4]

(b) When $\theta = \alpha$, the area of the square ABCD is equal to the area of the sector OAB.

(i) Write down the area of the sector when $\theta = \alpha$.

(ii) Hence find α. [4]

(c) When $\theta = \beta$, the area of R is more than twice the area of the sector. Find all possible values of β. [8]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Thursday 12 November 2015 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8815 – 7302

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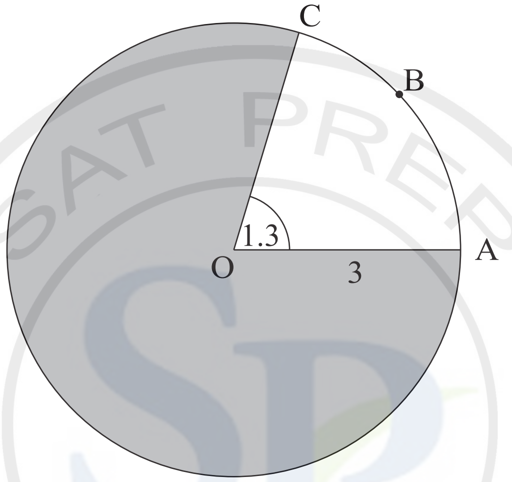
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 3 cm.

diagram not to scale



Points A, B, and C lie on the circle, and $\text{A}\hat{\text{O}}\text{C} = 1.3$ radians.

(a) Find the length of arc ABC. [2]

(b) Find the area of the shaded region. [4]

2. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X.

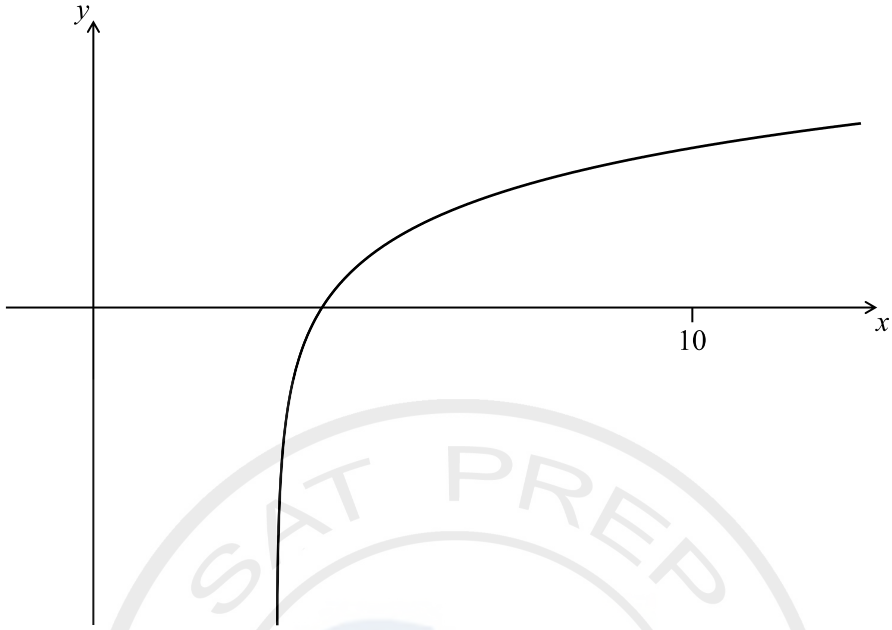
x	0	1	2	3
$P(X=x)$	0.15	k	0.1	$2k$

(a) Find the value of k. [3]

(b) Find $E(X)$. [2]

3. [Maximum mark: 7]

Let $f(x) = 2\ln(x-3)$, for $x > 3$. The following diagram shows part of the graph of f.



(a) Find the equation of the vertical asymptote to the graph of f. [2]

(b) Find the x-intercept of the graph of f. [2]

(c) The region enclosed by the graph of f, the x-axis and the line $x = 10$ is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

(This question continues on the following page)

(Question 3 continued)

4. [Maximum mark: 7]

The first three terms of a geometric sequence are $u_1 = 0.64$, $u_2 = 1.6$, and $u_3 = 4$.

(a) Find the value of r. [2]

(b) Find the value of S_6. [2]

(c) Find the least value of n such that $S_n > 75\,000$. [3]

5. [Maximum mark: 7]

Let C and D be independent events, with $\mathrm{P}(C) = 2k$ and $\mathrm{P}(D) = 3k^2$, where $0 < k < 0.5$.

(a) Write down an expression for $\mathrm{P}(C \cap D)$ in terms of k. [2]

(b) Given that $\mathrm{P}(C \cap D) = 0.162$, find k. [2]

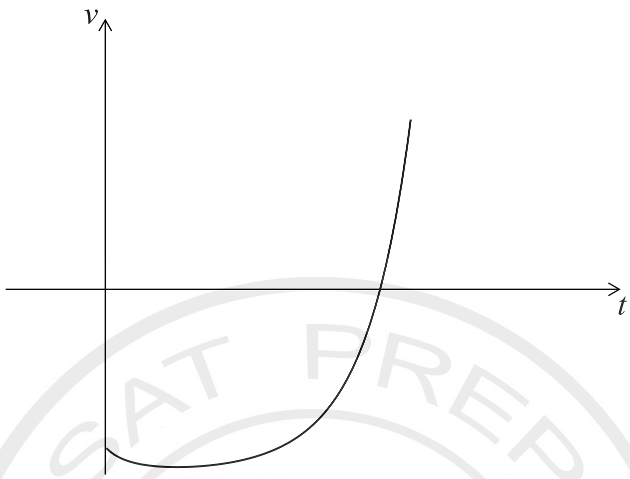
(c) Find $\mathrm{P}\left(C' \middle| D\right)$. [3]

6. [Maximum mark: 6]

The velocity $v\,\mathrm{m\,s^{-1}}$ of a particle after t seconds is given by

$$v(t) = (0.3t + 0.1)^t - 4 \text{, for } 0 \le t \le 5 .$$

The following diagram shows the graph of v.



(a) Find the value of t when the particle is at rest. [3]

(b) Find the value of t when the acceleration of the particle is 0. [3]

7. [Maximum mark: 8]

Let $f(x) = \ln(x^2)$, for $x \neq 0$.

(a) Show that $f'(x) = \frac{2}{x}$. [2]

(b) The tangent to the graph of f at a point $\mathrm{P}(d, f(d))$ passes through another point $\mathrm{Q}(1, -3)$. Find the value of d. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

The following diagram shows the quadrilateral ABCD.

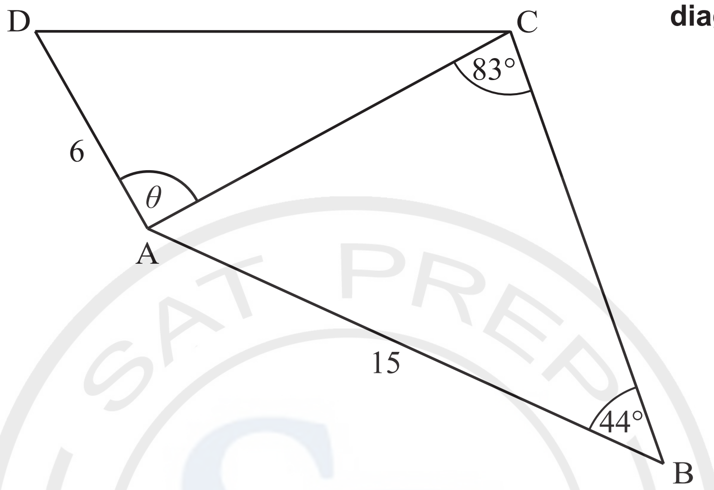


diagram not to scale

$\text{AD} = 6\,\text{cm}$, $\text{AB} = 15\,\text{cm}$, $\text{A}\hat{\text{B}}\text{C} = 44^\circ$, $\text{A}\hat{\text{C}}\text{B} = 83^\circ$ and $\text{D}\hat{\text{A}}\text{C} = \theta$

(a) Find AC. [3]

(b) Find the area of triangle ABC. [3]

The area of triangle ACD is half the area of triangle ABC.

(c) Find the possible values of θ. [5]

(d) Given that θ is obtuse, find CD. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

An environmental group records the numbers of coyotes and foxes in a wildlife reserve after t years, starting on 1 January 1995.

Let c be the number of coyotes in the reserve after t years. The following table shows the number of coyotes after t years.

number of years (t)	0	2	10	15	19
number of coyotes (c)	115	197	265	320	406

The relationship between the variables can be modelled by the regression equation $c = at + b$.

(a) Find the value of a and of b. [3]

(b) Use the regression equation to estimate the number of coyotes in the reserve when $t = 7$. [3]

Let f be the number of foxes in the reserve after t years. The number of foxes can be modelled by the equation $f = \dfrac{2000}{1 + 99e^{-kt}}$, where k is a constant.

(c) Find the number of foxes in the reserve on 1 January 1995. [3]

(d) After five years, there were 64 foxes in the reserve. Find k. [3]

(e) During which year were the number of coyotes the same as the number of foxes? [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

The masses of watermelons grown on a farm are normally distributed with a mean of $10\,\text{kg}$. The watermelons are classified as small, medium or large.

A watermelon is small if its mass is less than $4\,\text{kg}$. Five percent of the watermelons are classified as small.

(a) Find the standard deviation of the masses of the watermelons. [4]

The following table shows the percentages of small, medium and large watermelons grown on the farm.

small	medium	large
5%	57%	38%

A watermelon is large if its mass is greater than $w\,\text{kg}$.

(b) Find the value of w. [2]

All the medium and large watermelons are delivered to a grocer.

(c) The grocer selects a watermelon at random from **this** delivery. Find the probability that it is medium. [3]

(d) The grocer sells all the medium watermelons for $\$1.75$ each, and all the large watermelons for $\$3.00$ each. His costs on this delivery are $\$300$, and his total profit is $\$150$. Find the number of watermelons in the delivery. [5]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- The maximum mark for this examination paper is **[90 marks]**.

2216 – 7304

Please **do not** write on this page.

Answers written on this page will not be marked.

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Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A random variable X is distributed normally with a mean of 20 and standard deviation of 4.

(a) On the following diagram, shade the region representing $\mathrm{P}(X \le 25)$. [2]

(b) Write down $\mathrm{P}(X \le 25)$, correct to two decimal places. [2]

(c) Let $\mathrm{P}(X \le c) = 0.7$. Write down the value of c. [2]

2. [Maximum mark: 6]

Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

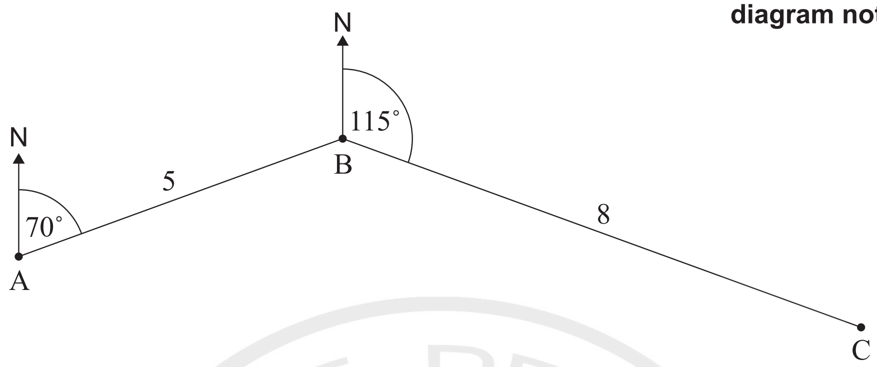
(a) Solve $f(x) = g(x)$. [3]

(b) Find the area of the region enclosed by the graphs of f and g. [3]

3. [Maximum mark: 7]

The following diagram shows three towns A, B and C. Town B is $5\,\text{km}$ from Town A, on a bearing of 070°. Town C is $8\,\text{km}$ from Town B, on a bearing of 115°.

diagram not to scale



(a) Find $\hat{ABC}$. [2]

(b) Find the distance from Town A to Town C. [3]

(c) Use the sine rule to find $\hat{ACB}$. [2]

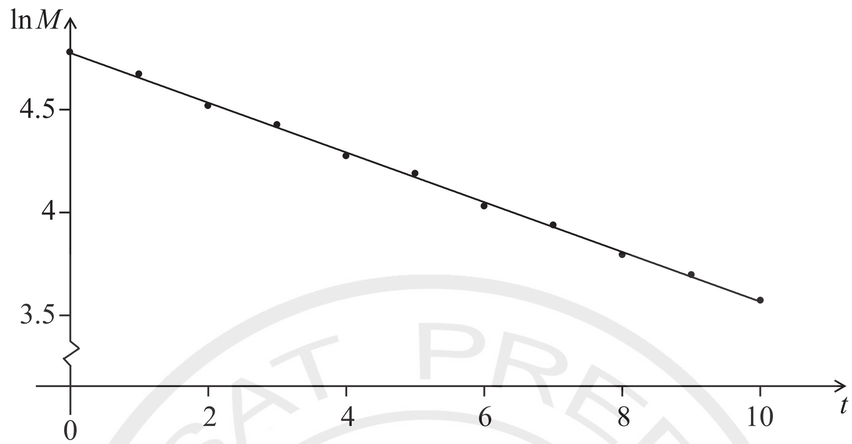
4. [Maximum mark: 6]

(a) Find the term in x^6 in the expansion of $(x+2)^9$. [4]

(b) Hence, find the term in x^7 in the expansion of $5x(x+2)^9$. [2]

5. [Maximum mark: 6]

The mass M of a decaying substance is measured at one minute intervals. The points $(t, \ln M)$ are plotted for $0 \le t \le 10$, where t is in minutes. The line of best fit is drawn. This is shown in the following diagram.



The correlation coefficient for this linear model is $r = -0.998$.

(a) State **two** words that describe the linear correlation between $\ln M$ and t. [2]

(b) The equation of the line of best fit is $\ln M = -0.12t + 4.67$. Given that $M = a \times b^t$, find the value of b. [4]

6. [Maximum mark: 6]

In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

7. [Maximum mark: 8]

Note: One decade is 10 years

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

(a) (i) Find the value of k.

(ii) Interpret the meaning of the value of k. [3]

(b) Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

A factory has two machines, A and B. The number of breakdowns of each machine is independent from day to day.

Let A be the number of breakdowns of Machine A on any given day. The probability distribution for A can be modelled by the following table.

a	0	1	2	3
$\mathrm{P}(A=a)$	0.55	0.3	0.1	k

(a) Find k. [2]

(b) (i) A day is chosen at random. Write down the probability that Machine A has no breakdowns.

(ii) Five days are chosen at random. Find the probability that Machine A has no breakdowns on exactly four of these days. [3]

Let B be the number of breakdowns of Machine B on any given day. The probability distribution for B can be modelled by the following table.

b	0	1	2	3
$\mathrm{P}(B=b)$	0.7	0.2	0.08	0.02

(c) Find $\mathrm{E}(B)$. [2]

On Tuesday, the factory uses both Machine A and Machine B. The variables A and B are independent.

(d) (i) Find the probability that there are exactly two breakdowns on Tuesday.

(ii) Given that there are exactly two breakdowns on Tuesday, find the probability that both breakdowns are of Machine A. [8]

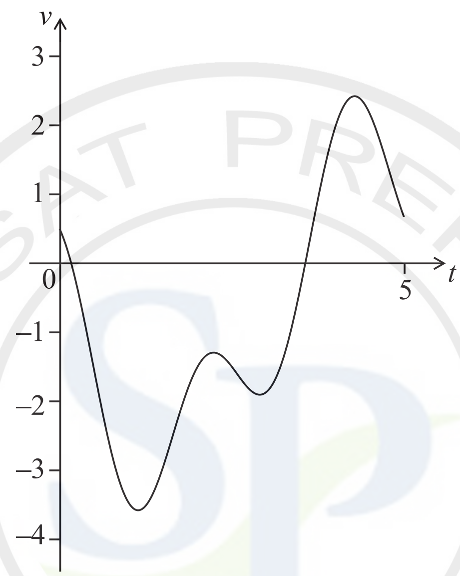
Do **not** write solutions on this page.

9. [Maximum mark: 14]

A particle P moves along a straight line so that its velocity, v ms^{-1}, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \le t \le 5$. The initial displacement of P from a fixed point O is 4 metres.

(a) Find the displacement of P from O after 5 seconds. [5]

The following sketch shows the graph of v.



(b) Find when P is first at rest. [2]

(c) Write down the number of times P changes direction. [2]

(d) Find the acceleration of P after 3 seconds. [2]

(e) Find the maximum speed of P. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

The points A and B lie on a line L, and have position vectors $\begin{pmatrix}-3\\-2\\2\end{pmatrix}$ and $\begin{pmatrix}6\\4\\-1\end{pmatrix}$ respectively.

Let O be the origin. This is shown on the following diagram.

diagram not to scale

(a) Find $\overrightarrow{AB}$. [2]

The point C also lies on L, such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

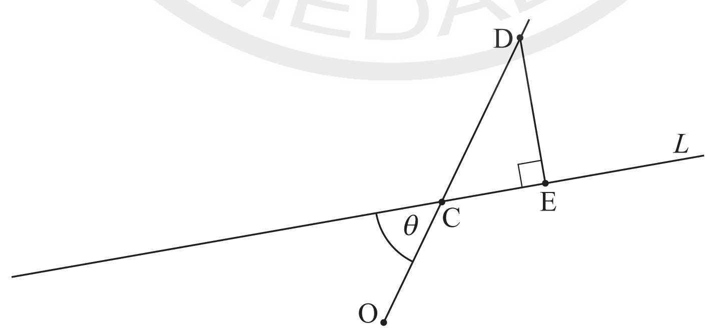
(b) Show that $\overrightarrow{OC} = \begin{pmatrix}3\\2\\0\end{pmatrix}$. [3]

Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{OC}$.

(c) Find θ. [5]

Let D be a point such that $\overrightarrow{OD} = k\overrightarrow{OC}$, where $k > 1$. Let E be a point on L such that $\text{C}\hat{\text{E}}\text{D}$ is a right angle. This is shown on the following diagram.

diagram not to scale



(d) (i) Show that $\left|\overrightarrow{DE}\right| = (k-1)\left|\overrightarrow{OC}\right|\sin\theta$.

(ii) The distance from D to line L is less than 3 units. Find the possible values of k. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12 pages

2216 – 7306

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The first three terms of an arithmetic sequence are $u_1 = 0.3$, $u_2 = 1.5$, $u_3 = 2.7$.

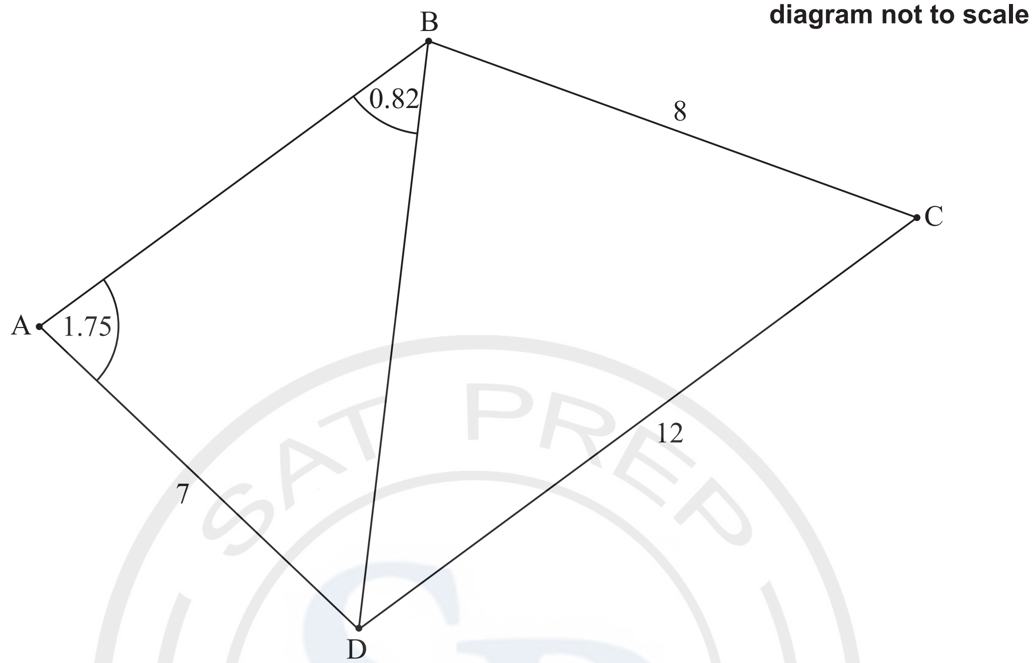
(a) Find the common difference. [2]

(b) Find the 30th term of the sequence. [2]

(c) Find the sum of the first 30 terms. [2]

2. [Maximum mark: 6]

The following diagram shows a quadrilateral ABCD.



$AD = 7\text{ cm}$, $BC = 8\text{ cm}$, $CD = 12\text{ cm}$, $D\hat{A}B = 1.75$ radians, $A\hat{B}D = 0.82$ radians.

(a) Find BD. [3]

(b) Find $D\hat{B}C$. [3]

3. [Maximum mark: 7]

Let $f(x) = e^{0.5x} - 2$.

(a) For the graph of f

(i) write down the y-intercept;

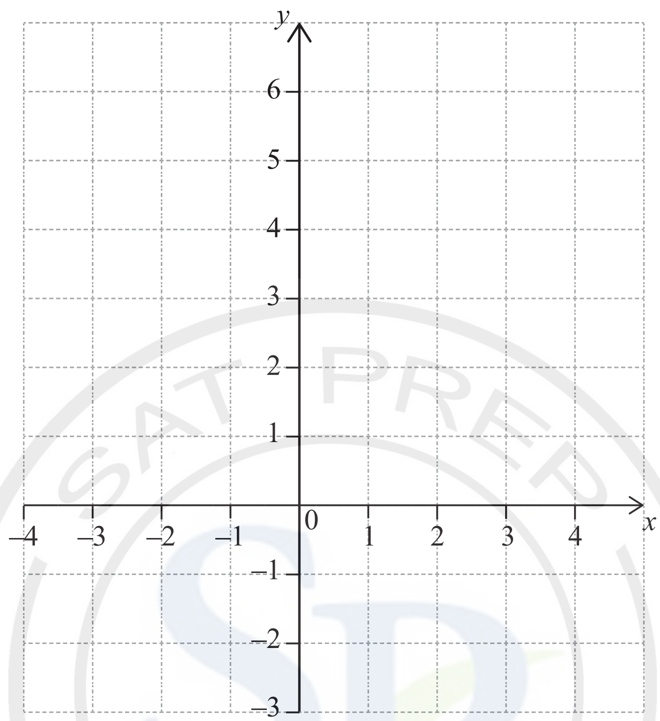
(ii) find the x-intercept;

(iii) write down the equation of the horizontal asymptote. [4]

(This question continues on the following page)

(Question 3 continued)

(b) On the following grid, sketch the graph of f, for $-4 \le x \le 4$. [3]



4. [Maximum mark: 8]

The height, h metres, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15\cos 1.2t + 17, \text{ for } t \geq 0.$$

(a) Find the height of the seat when $t = 0$. [2]

(b) The seat first reaches a height of $20\,\text{m}$ after k minutes. Find k. [3]

(c) Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place. [3]

5. [Maximum mark: 6]

Consider the expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$.

(a) Write down the number of terms of this expansion. [1]

(b) Find the coefficient of x^8. [5]

6. [Maximum mark: 6]

A competition consists of two independent events, shooting at 100 targets and running for one hour.

The number of targets a contestant hits is the S score. The S scores are normally distributed with mean 65 and standard deviation 10.

(a) A contestant is chosen at random. Find the probability that their S score is less than 50. [2]

The distance in km that a contestant runs in one hour is the R score. The R scores are normally distributed with mean 12 and standard deviation 2.5. The R score is independent of the S score.

Contestants are disqualified if their S score is less than 50 **and** their R score is less than $x\ \text{km}$.

(b) Given that $1\,\%$ of the contestants are disqualified, find the value of x. [4]

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7. [Maximum mark: 7]

A particle moves in a straight line. Its velocity $v \text{ m s}^{-1}$ after t seconds is given by

$$v = 6t - 6, \text{ for } 0 \le t \le 2.$$

After p seconds, the particle is 2 m from its initial position. Find the possible values of p.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y dollars	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(a) (i) Find the correlation coefficient.

(ii) Write down the value of a and of b. [4]

On 1 January 2010, Lina buys a car which has travelled 11 000 km.

(b) Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. [3]

The price of a car decreases by 5 % each year.

(c) Calculate the price of Lina's car after 6 years. [4]

Lina will sell her car when its price reaches 10 000 dollars.

(d) Find the year when Lina sells her car. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Let $f(x) = \frac{1}{x-1} + 2$, for $x > 1$.

(a) Write down the equation of the horizontal asymptote of the graph of f. [2]

(b) Find $f'(x)$. [2]

Let $g(x) = ae^{-x} + b$, for $x \geq 1$. The graphs of f and g have the same horizontal asymptote.

(c) Write down the value of b. [2]

(d) Given that $g'(1) = -e$, find the value of a. [4]

(e) There is a value of x, for $1 < x < 4$, for which the graphs of f and g have the same gradient. Find this gradient. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

Consider the points $\mathrm{A}(1, 5, -7)$ and $\mathrm{B}(-9, 9, -6)$.

(a) Find $\overrightarrow{\mathrm{AB}}$. [2]

Let C be a point such that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$.

(b) Find the coordinates of C. [2]

The line L passes through B and is parallel to (AC).

(c) Write down a vector equation for L. [2]

(d) Given that $\left|\overrightarrow{\mathrm{AB}}\right| = k\left|\overrightarrow{\mathrm{AC}}\right|$, find k. [3]

(e) The point D lies on L such that $\left|\overrightarrow{\mathrm{AB}}\right| = \left|\overrightarrow{\mathrm{BD}}\right|$. Find the possible coordinates of D. [6]

N16/5/MATME/SP2/ENG/TZ0/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Friday 11 November 2016 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8816 – 7302

16EP01

Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x - 5$, for $x \in \mathbb{R}$.

(a) Find $f(8)$. [2]

(b) Find $(g \circ f)(x)$. [2]

(c) Solve $(g \circ f)(x) = 0$. [3]

2. [Maximum mark: 7]

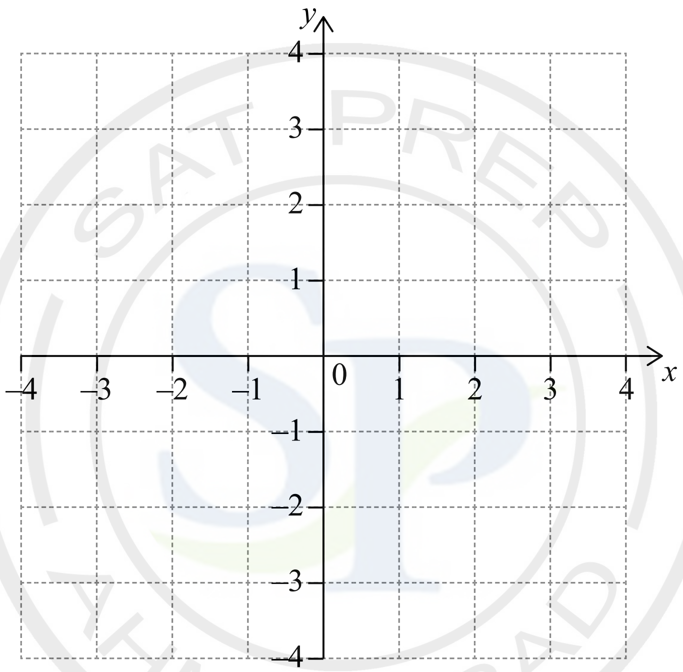
Let $f(x) = 0.225x^3 - 2.7x$, for $-3 \le x \le 3$. There is a local minimum point at A.

(a) Find the coordinates of A. [2]

(b) On the following grid,

(i) sketch the graph of f, clearly indicating the point A;

(ii) sketch the tangent to the graph of f at A. [5]



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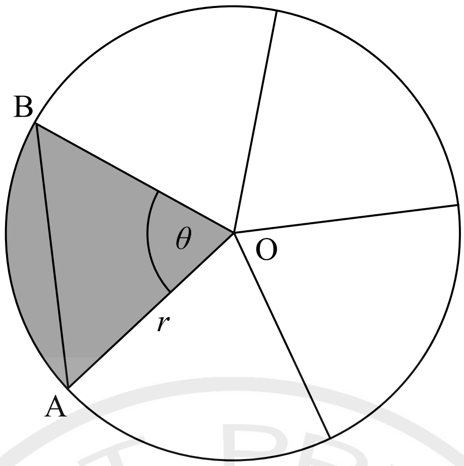
3. [Maximum mark: 7]

The following diagram shows a circle, centre O and radius r mm. The circle is divided into five equal sectors.

diagram not to scale



One sector is OAB, and $\hat{AOB} = \theta$.

(a) Write down the **exact** value of θ in radians. [1]

The area of sector AOB is $20\pi\,\text{mm}^2$.

(b) Find the value of r. [3]

(c) Find AB. [3]

4. [Maximum mark: 6]

Let $f(x) = x\mathrm{e}^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

(a) Find the value of p and of q. [3]

(b) Hence, find the area of the region enclosed by the graphs of f and g. [3]

5. [Maximum mark: 6]

The weights, W, of newborn babies in Australia are normally distributed with a mean 3.41 kg and standard deviation 0.57 kg. A newborn baby has a low birth weight if it weighs less than $w\text{ kg}$.

(a) Given that 5.3% of newborn babies have a low birth weight, find w. [3]

(b) A newborn baby has a low birth weight.
Find the probability that the baby weighs at least 2.15 kg. [3]

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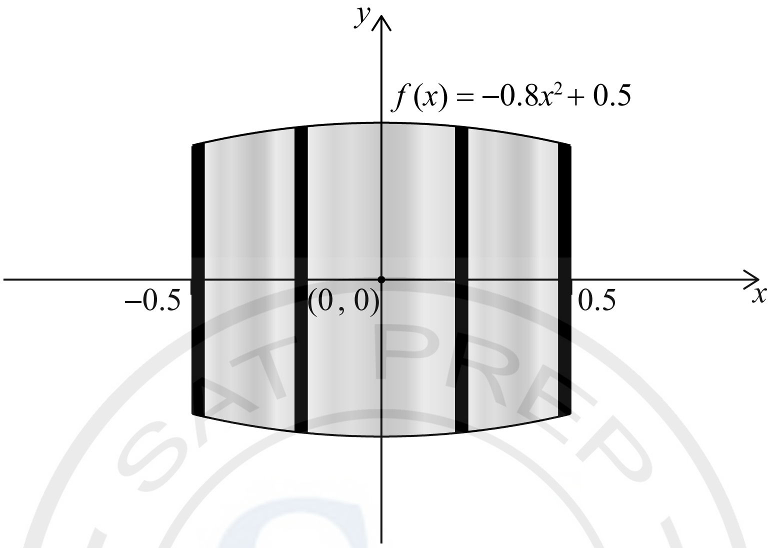
6. [Maximum mark: 6]

All lengths in this question are in metres.

Let $f(x) = -0.8x^2 + 0.5$, for $-0.5 \le x \le 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of f, the x-axis, the line $x = -0.5$ and the line $x = 0.5$ is rotated 360° about the x-axis. This is shown in the following diagram.



(a) Use the model to find the volume of the barrel. [3]

(b) The empty barrel is being filled with water. The volume $V\ \mathrm{m}^3$ of water in the barrel after t minutes is given by $V = 0.8\left(1 - \mathrm{e}^{-0.1t}\right)$. How long will it take for the barrel to be half-full? [3]

7. [Maximum mark: 6]

A jar contains 5 red discs, 10 blue discs and m green discs. A disc is selected at random and replaced. This process is performed four times.

(a) Write down the probability that the first disc selected is red. [1]

(b) Let X be the number of red discs selected. Find the smallest value of m for which $\text{Var}(X) < 0.6$. [5]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Ten students were surveyed about the number of hours, x, they spent browsing the Internet during week 1 of the school year. The results of the survey are given below.

$$\sum_{i=1}^{10} x_i = 252,\ \sigma = 5 \text{ and median} = 27.$$

(a) Find the mean number of hours spent browsing the Internet. [2]

(b) During week 2, the students worked on a major project and they each spent an additional five hours browsing the Internet. For week 2, write down

(i) the mean;

(ii) the standard deviation. [2]

(c) During week 3 each student spent 5% less time browsing the Internet than during week 1. For week 3, find

(i) the median;

(ii) the variance. [6]

(d) During week 4, the survey was extended to all 200 students in the school. The results are shown in the cumulative frequency graph on the following page.

(i) Find the number of students who spent between 25 and 30 hours browsing the Internet.

(ii) Given that 10% of the students spent more than k hours browsing the Internet, find the maximum value of k. [6]

(This question continues on the following page)

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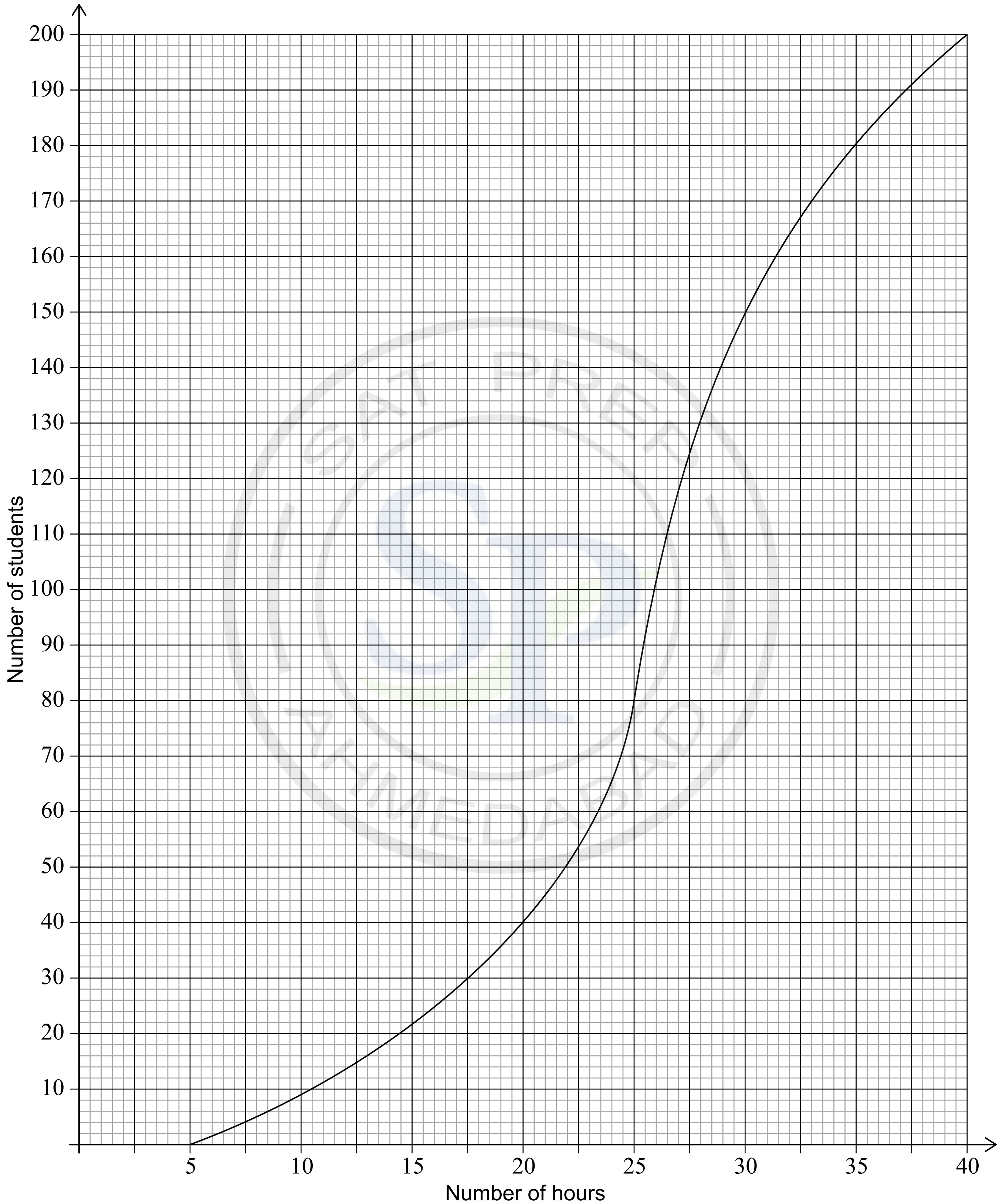
(Question 8 continued)



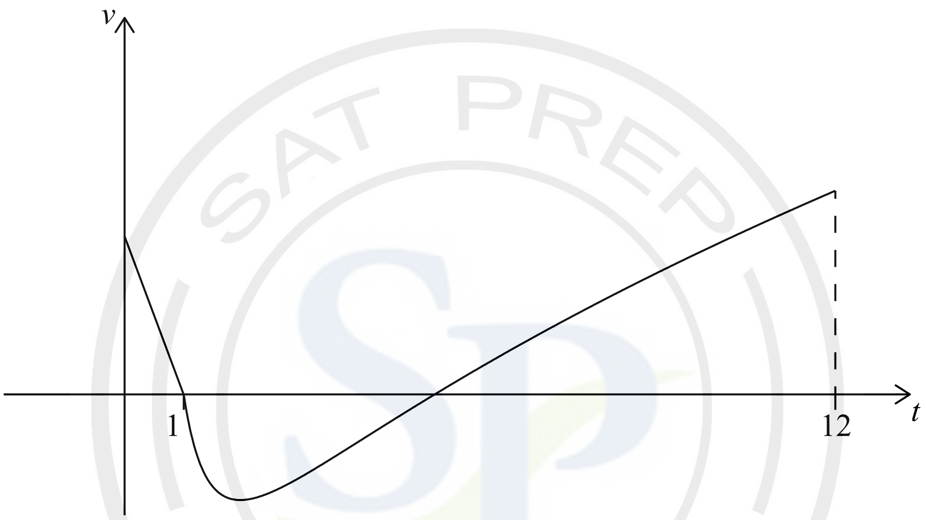
Do **not** write solutions on this page.

9. [Maximum mark: 14]

A particle P starts from a point A and moves along a horizontal straight line.
Its velocity $v\ \mathrm{cm\,s^{-1}}$ after t seconds is given by

$$v(t)=\begin{cases}-2t+2, \text{ for } 0\le t\le 1\\ 3\sqrt{t}+\dfrac{4}{t^2}-7, \text{ for } 1\le t\le 12\end{cases}$$

The following diagram shows the graph of v.



(a) Find the initial velocity of P. [2]

P is at rest when $t=1$ and $t=p$.

(b) Find the value of p. [2]

When $t=q$, the acceleration of P is zero.

(c) (i) Find the value of q.

(ii) Hence, find the **speed** of P when $t=q$. [4]

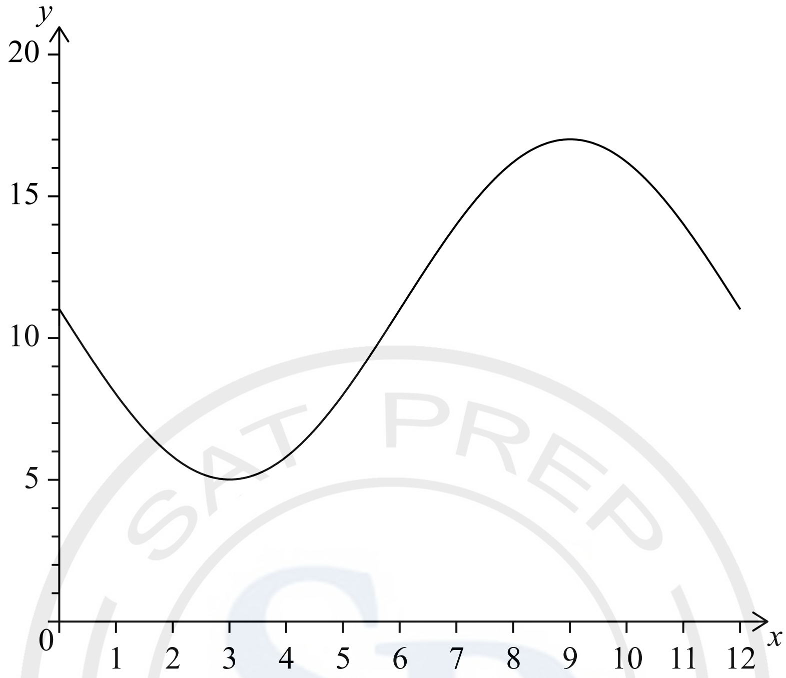
(d) (i) Find the total distance travelled by P between $t=1$ and $t=p$.

(ii) Hence or otherwise, find the displacement of P from A when $t=p$. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

The following diagram shows the graph of $f(x) = a \sin bx + c$, for $0 \le x \le 12$.



The graph of f has a minimum point at $(3, 5)$ and a maximum point at $(9, 17)$.

(a) (i) Find the value of c.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a. [6]

The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5, 17)$.

(b) (i) Write down the value of k.

(ii) Find $g(x)$. [3]

The graph of g changes from concave-up to concave-down when $x = w$.

(c) (i) Find w.

(ii) Hence or otherwise, find the maximum positive rate of change of g. [6]

Please **do not** write on this page.

Answers written on this page will not be marked.

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