MOODY'S ANALYTICS

# MODELING METHODOLOGY

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# Credit Transition Model 2017 Update: Methodology and Performance Review


## Summary

The Credit Transition Model (CTM) is Moody's proprietary issuer-level model of rating transitions and default. It provides projections of probabilities of rating transitions and default for over 7000 bond and loan issuers. CTM belongs to the family of discrete time, multiple destination and proportional hazards models. It conditions on issuer-specific information coupled with forward-looking macroeconomic views to assign probabilities of default, withdrawal, upgrade and downgrade to individual issuers, portfolios of issuers, or rating categories. CTM facilitates scenario analysis for credit transitions and defaults at both the issuer and portfolio levels, and model-generated PDs can be conditioned on a wide selection of both standard and user-defined economic scenarios. The 2017 version of the Credit Transition Model described in this document maintains the framework from the previous version while adding a few enhancements. The parameters are re-estimated, for instance, using Moody's updated senior unsecured issuer ratings and more recent data. A version of the model is estimated separately for European issuers. Herein we present the full methodology, provide detailed information about the model input and output, and review the model's performance via a series of validation metrics.

## Table of Contents

## 1 Introduction

Credit ratings are generally derived from fundamental analysis of firms' credit quality, and serve as relative assessments of expected loss. With a history of over 100 years, Moody's ratings have been proved an effective ordinal measure for relative credit risk. Probabilities associated with ratings transitions, in particular to the default state, are key concepts in many credit risk management frameworks. Such metrics can be used by lenders for evaluation of borrowers, by corporations for assessment of their business partners' creditworthiness, by asset managers for investment screening and by other risk practitioners for credit risk surveillance. Importantly, the probability of default is an essential input to calculate expected loss and economic capital. This concept is also crucial in the calculation of capital requirements under the Basel framework.

A common practice is to collect historical frequencies for a given horizon in a transition matrix, such as the one presented in Table 1. This approach associates the transition probabilities, including default probabilities (PD), with internal ratings or credit ratings published by agencies, such as Moody's Investor Service. However, ratings are not intended to capture a particular default probability over a particular time horizon. How to translate the ordinal content of credit ratings into cardinal default probabilities has always been of interest to fixed income investors and risk managers.

**Table 1 Global 4-Quarter Transition Matrix from June 1, 2016 to May 31, 2017 (percent)** [1]

| From\To | Aaa | Aa1 | Aa2 | Aa3 | A1 | A2 | A3 | Baa1 | Baa2 | Baa3 | Ba1 | Ba2 | Ba3 | B1 | B2 | B3 | Caa1 | Caa2 | Caa3 | Ca-C | WR | DEF |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Aaa | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Aa1 | 0 | 79 | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Aa2 | 0 | 0 | 91 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 |
| Aa3 | 0 | 0 | 1 | 64 | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 |
| A1 | 0 | 0 | 0 | 2 | 84 | 8 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 |
| A2 | 0 | 0 | 0 | 0 | 17 | 74 | 3 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 |
| A3 | 0 | 0 | 0 | 0 | 0 | 4 | 80 | 6 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 0 |
| Baa1 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 85 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 |
| Baa2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 82 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 |
| Baa3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 80 | 4 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 6 | 0 |
| Ba1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 14 | 72 | 4 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 0 |
| Ba2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 11 | 72 | 3 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 7 | 0 |
| Ba3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 4 | 7 | 71 | 6 | 1 | 1 | 1 | 0 | 0 | 0 | 7 | 0 |
| B1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 9 | 67 | 4 | 3 | 0 | 0 | 1 | 0 | 12 | 0 |
| B2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 7 | 59 | 8 | 3 | 3 | 0 | 0 | 15 | 0 |
| B3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 9 | 62 | 9 | 3 | 0 | 0 | 14 | 1 |
| Caa1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 6 | 62 | 11 | 2 | 1 | 15 | 2 |
| Caa2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 7 | 57 | 9 | 3 | 19 | 3 |
| Caa3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 6 | 51 | 12 | 13 | 14 |
| Ca-C | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 2 | 3 | 41 | 16 | 35 |

Using historical transition matrix to generate default rate is certainly reasonable, as any more sophisticated statistical model is centered around the same averages, but the limitations of using ratings as a predictor of future credit migrations are also fairly obvious. First, actual transitions are pro-cyclical: there is abundant evidence showing that credit transitions correlate with credit and economic cycles. Figure 1 compares US speculative grade one-year forward default rates with the one-year forward change in the U.S. unemployment rate.[2] Second, the transitions are generally non-Markovian, meaning that the rating migration in the future depends not only on the current state, but also on the behavior in the past. Figure 2 compares the cumulative probability of downgrading for newly issued Ba issuers, those downgraded, and those upgraded. The probability of downgrading further is substantially higher for those credits which were just downgraded themselves, and substantially lower for upgraded issuers.

Besides resorting to transition matrices, default and transition models are also widely used to answer questions about rating migrations. Such models, as well as other general transition models are essentially of two basic types: aggregate time series or issuer based. Time series models exploit the correlation between default/transition rates and the macroeconomy by regressing the former on indicators of the latter. They usually perform well to fit the average default rate with economic variables. But this success comes with two major drawbacks. Time series models cannot be applied to a single credit or a portfolio and they are horizon dependent, meaning that they can only forecast a probability of default or transition for a given horizon. In order to achieve the goal of multi-horizon forecast, additional models have to be estimated, without guarantee of consistency among separate models. Issuer-based models focus more on explaining why some issuers are more likely to default than others over a given horizon. Logit or Probit models rely on issuer characteristics, and thus they avoid portfolio dependence embedded in the

[1] Monthly Default Report, Moody's Investor Services, May 2017

[2] Throughout this paper, the default rate for an issuer cohort over a given horizon is the share of those issuers which are observed to enter default at some point within that horizon. See Hamilton and Cantor (2006). No adjustment for withdrawal is made. Consequently, the default rate statistics presented in this paper will not generally correspond to those presented in other Moody's publications.

time series models. However, they are still horizon dependent. No matter time series or issuer based models, they usually focus only on one type of transition, default. In reality, default is not the only event of interest. Upgrades and downgrades, particularly from investment grade to speculative grade, attract investors' attention, too.

Figure 1 One-year Forward US Speculative Grade Default Rates and the Change in the US Unemployment Rate: Defaults are Procyclical

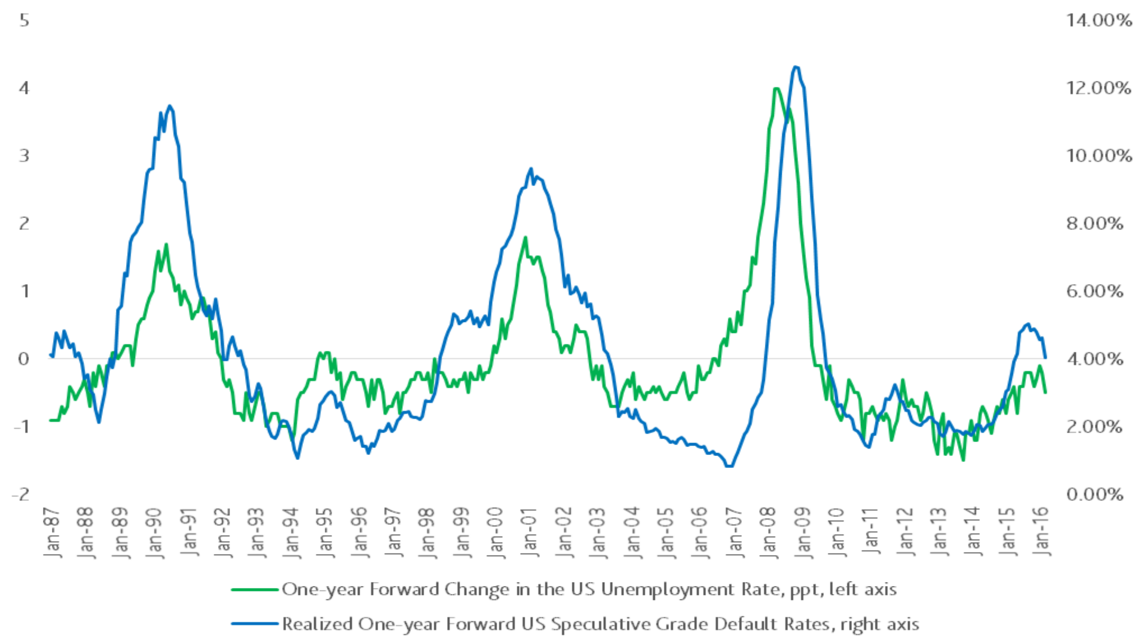
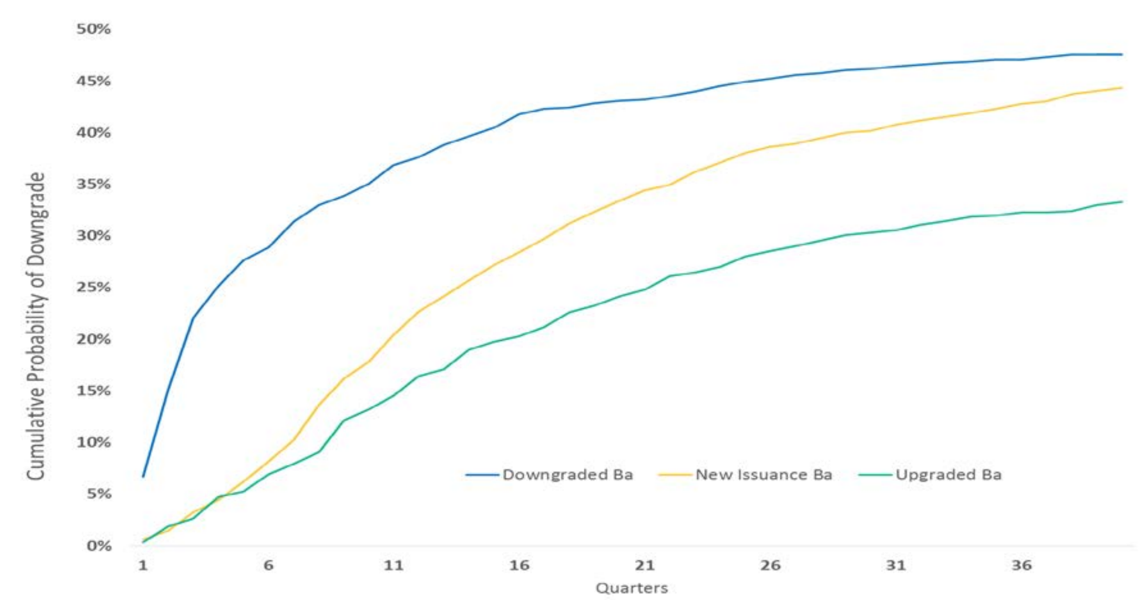



Figure 2 Cumulative Probability of Downgrade for US Ba Issuers[3]: Rating Transitions are Non-Markovian




[3] Based on US rating data from 1987 to 2017

In this paper we present a chain of similarly structured models of rating transitions which can be applied at multiple horizons. We study the complete set of transitions, including to the states default and withdrawal. The result is a transition matrix which is conditional on issuer-specific factors and macroeconomic drivers. One can thus quantify for a particular rated credit, and hence any portfolio of credits, the probability of upgrading to a particular rating, or downgrading, or defaulting, or withdrawing, all over given horizons of interest and under certain economic scenarios.

The model itself is of the familiar proportional hazards type. It conditions on a number of issuer-specific factors. These include: the current rating and rating history, whether the issuer was upgraded or downgraded into its current rating, how long the issuer has maintained its current rating, how long the issuer has consecutively maintained any Moody's credit rating, the rating outlook, whether the issuer is on review for upgrade or downgrade, how the outlook or review status has changed, the issuer's industry, whether it is a loan issuer or bond issuer, and the cumulative change in the unemployment rate since the current rating was assigned. Any two issuers with the same attributes across all these dimensions will generate the same modeled transition probabilities under the same economic scenarios. We restrict ourselves to a parsimonious representation of the macroeconomy and consider just two drivers: the unemployment rate and high yield spreads over Treasuries.[4]

A model of very similar structure is Koopman, Lucas and Monteiro (2006), in which rating transitions are driven by a dynamic latent factor as opposed to any observable factors. This may be seen as advantageous, since it avoid misspecification of the macro proxy for the credit cycle, or disadvantageous, as forecasts and scenario evaluation become purely statistical exercises. Figlewski, Frydman and Liang (2006) is another duration model of rating transitions which evaluates the explanatory power of various macroeconomic drivers. However, they do not specify the baseline transition intensities, whereas our primary interest is in constructing a forecasting tool, for which we need to parameterize baseline transition intensities and perform full maximum likelihood estimation.[5] Duffie and Wang (2007) condition on a Merton variable, a firm's distance to default, rather than credit ratings. This almost certainly offers improved discrimination in considering the transition to default, which is the sole focus of their study. But our focus is really all rating transitions.[6] We do not include a Merton variable for a couple of reasons. First, doing so would require us to model the dynamics of the Merton variable in addition to those of ratings. Second, we would need to ensure at each point in time that a reasonable correspondence between the Merton variable and ratings was maintained.[7] Nesting the two remains an avenue of interesting future research, but is beyond the scope of the current model.

On the front of proportional hazards models, some interesting research has been conducted on modeling the dynamics of an unobserved factor, such as the work by Duffie, Eckner, Buillaume and Saita (2006) and Koopman, Kraussl, Lucas and Monteiro (2006). This is motivated from the observation that most macroeconomic specifications are unable to account fully for correlation across defaults, and it is well known that failing to account for unobserved heterogeneity can significantly bias subsequent parameter estimates. In fact, earlier versions of this research allowed for a time-invariant unobserved mixing factor, but in recent iterations this factor has not been significant and has subsequently been omitted. Future efforts may include incorporating a richer, dynamic frailty factor, but the modeling cost of needing to forecast the value of such a factor under scenarios might prove greater than the benefits derived from including it. As a practical point, we estimate our model based on quarterly data, as the quarterly frequency permits reasonable model estimation times and guarantees a sufficiently rich mix of rating transition events between observations in the time series.

With the model described in this paper, users are able to forecast the probability of rating transitions and default at the issuer level and at the portfolio level. The portfolio forecast can be equally-weighted across issuers as well as volume weighted. It can be applied to study rating transitions of a portfolio of credits based on industry and geography, or any subgroups of interest. Forecast of defaults and other transitions are mutually consistent. Users can also generate consistent forecasts for any time horizons, with a chain of five similarly structured models. As the CTM conditions on the expected future path of the economy, performing scenario analysis and stress testing is straight forward.

The CTM is also a practical tool to calculate the first passage probability, which is the probability that an issuer first encounters a rating threshold. In reality, portfolio managers sometimes face certain investment requirements. For example, if a credit falls to speculative grade, it has to be removed from the portfolio. The fund manager would be interested in knowing the probability of a particular credit falling below Baa3 in a three year horizon. In this situation, not only the final rating of an issuer matters, but also

[4] We found that including other candidates did not sufficiently improve the model performance relative to the costs of accurately forecasting them, e.g.GDP growth rate.

[5] For forecasting transitions, the baseline temporal pattern is often more important than the economic factors especially over short horizons. To take just one (extreme) example, an issuer that was assigned a rating yesterday is unlikely to change its rating tomorrow, no matter what the state of the business cycle.

[6] There are many applications, such as CDO valuation, in which understanding the upgrade and downgrade dynamics of credit ratings is important in and of itself.

[7] There is also the added advantage that our model is applicable to entities that don't otherwise have a Merton variable.

the path of its credit rating is important. With the CTM, we can calculate the probability that an issuer first becomes a speculative grade in a particular quarter conditioning on not having done that before, and from there the probability that the credit will become speculative grade at some point over the next three years.

The Credit Transition Model[8] was first developed by Albert Metz at Moody's Investor Service in 2007 and the forecast results have been published on Moody's Annual Default Study and Monthly Default Report since then. In the subsequent five years, the model was modified and re-estimated. The model has also been validated annually in recent years. In this 2017 update, we re-estimate the parameters with more recent data, and more importantly, using the updated senior unsecured ratings. We also adopt an additional layer of empirical calibration for the European model, and incorporate the MCMESA tool for the model implementation.

This paper is organized as follows. Section 2 describes the estimation dataset. Section 3 explains the model structure, including both the estimation step and the projection step. In particular, Section 3 discusses the model framework, model parameterization and how forecasts are generated. Section 4 evaluates the performance of the model conditional on the perfect foresight of the economy. We examine the cardinal and ordinal accuracy of the model, the stability of the forecast and the sensitivity of the projections. Section 5 concludes.

## 2 Data

The CTM is built on two sources of information: Moody's ratings and macroeconomic time series. The rating data are issuer ratings, more specifically, Moody's senior unsecured ratings. The economic time series include the unemployment rate and high yield spreads.

### 2.1 Rating Data

The ratings data behind the model are Moody's senior unsecured ratings, which allow for meaningful comparison of credit quality across entities, regardless of their capital structure. In brief, a company's estimated senior rating is set equal to its actual senior unsecured debt rating when it exists, or estimated on the basis of rated subordinated or secured debt. This estimation process that derives issuer level ratings from particular debt obligation level ratings is called the senior rating algorithm (SRA). The process is designed to ensure that the derived ratings are consistent with Moody's notching practices, therefore theoretically equivalent to a senior unsecured bond rating. Moody's redesigned the SRA in 2015[9], so that the notching rules are determined dynamically and are consistent with its current rating practices. The issuer universe has also been expanded to include banks that only have deposit ratings under the 2015 SRA. The latest CTM estimation is based on the updated SRA.

In survival analysis, observations are called censored when the information about their survival time is incomplete. Left censoring occurs when we cannot observe the time when the event occurred, while right censoring occurs when the true unobserved event is to the right of the censoring time. We collect rating transitions beginning July 1, 1982 and use data beginning January 1, 1987 for estimation. As such, there is no left censoring. The data are right censored as of Dec. 31, 2015, and we treat this as random. We study transitions at the quarterly frequency. More specifically, we sample the ratings database on the first day of January, April, July and October of each year to determine if a rating transition has occurred.[10]

In the estimation of the US model, the issuer universe includes all rated North American, Latin America and Asia Pacific companies, including financial institutions, utilities and corporates.[11] There are 9,565 issuers with 29,434 observed transitions or initial assignments. US companies represent the majority of the sample. As European rated issuer universe is much smaller than that of the US, we include 12,633 global issuers with 38,599 observed transitions or initial assignments in the optimization step of the estimation for the European model. It is worth noting that besides issuers in Americas and Asia Pacific, issuers in Middle East and Africa have also been included in the sample of the European model. We cover this issue in more detail in the parameterization section.

The ratings data provide essentially all of our issuer-specific information: the current rating, whether the issuer was upgraded or downgraded into this rating, how long the issuer has maintained its current rating, how long the issuer has maintained any Moody's rating, the rating outlook, whether the issuer is under review for upgrade or downgrade and whether the outlook or

[8] United States Patent Publication No. US20090276234 A1

[9] See more detailed information regarding the new SRA in the appendix of the Annual Default Study: Corporate Default and Recovery Rates, 1920-2015

[10] Any rating spells which occur entirely within the quarter will not be observed, but these are very few in number. What are most likely to be missed are those cases where an issuer rating transitions to C just prior to default.

[11] Public sector and structured finance issuers are excluded, as are any government-related issuers.

review status has changed. The ratings data also include information about the issuer's industry and whether the rating is for bond or for loan. We use "age" to describe how long one issuer has maintained any Moody's rating.[12]

Defaults are identified through Moody's proprietary default database, based on Moody's Investors Service's definition. Defaults are defined to include missed or delayed interest or principal payments, bankruptcies, and distressed exchanges, etc. Appendix I provide more detail on Moody's definition of default. In some cases, issuer ratings are withdrawn. We identify this as a separate exiting state but do not distinguish the reasons for withdrawal. Cantor & Hamilton (2007) present a thorough discussion.

Tables 2, 3 and 4 present summary statistics of the ratings transition data. We divide issuers into three categories: newly assigned to a rating category, upgraded into a rating category, and downgraded into a category. Table 2 displays summary statistics of durations for firms with newly assigned ratings. Table 3 and 4 show summary statistics of durations for firms upgraded and downgraded to their current ratings, respectively. Percentages in each row do not add up to 1 because of right censoring.

Issuers downgraded to their current ratings have a higher chance of direct default. For example, Table 4 reports that issuers downgraded to Caa2 have a 25.4% chance of direct default, which is much higher than that of newly issued Caa2 issuers (4.2%) or issuers upgraded to Caa2 (6.6%). The summary statistics also indicate that newly assigned issuers have longest average durations in every rating category, followed by the upgraded issuers, and the downgraded issuers have the shortest average durations. For example, the average duration for a newly issued B2 issuer is 10 quarters, compared to 7.2 quarters for issuers upgraded to B2 and 6.8 quarters for issuers downgraded to B2.

There is not much difference in expected durations until an upgrade or downgrade occurs for newly assigned ratings, as displayed in Table 2. The average duration conditional on withdrawal is slightly longer, which could serve as evidence that the longer one remains in a category, the more likely it is to withdraw (Table 2). For the upgraded or downgraded issuers (Table 3 and 4), there is strong evidence of momentum in the form of shorter durations conditional on exiting in the same direction and much longer durations conditional on reversing direction. In other words, upgraded issuers have shorter durations conditional on being upgraded again than those conditional on exiting to downgrade.

Table 2 Global Rating Transition Data Summary Statistics (1987-2015): Initial Rating Assignments

| | | | Upgrade | | Downgrade | | Withdrawal | | Default | |
|---|---|---|---|---|---|---|---|---|---|---|
| Rating | # | Duration | % | Duration | % | Duration | % | Duration | % | Duration |
| Aaa | 443 | 31.8 | | | 58.5 | 22.0 | 26.4 | 31.6 | | |
| Aa1 | 387 | 18.4 | 8.0 | 16.7 | 68.5 | 17.2 | 16.8 | 22.2 | | |
| Aa2 | 495 | 15.3 | 21.1 | 17.1 | 60.0 | 14.3 | 14.6 | 17.3 | | |
| Aa3 | 815 | 17.0 | 22.7 | 20.0 | 49.3 | 14.8 | 23.7 | 18.7 | | |
| A1 | 805 | 15.9 | 23.6 | 15.0 | 48.7 | 16.1 | 17.1 | 16.9 | 0.1 | 10.0 |
| A2 | 917 | 15.9 | 27.2 | 14.8 | 46.1 | 14.7 | 17.1 | 18.2 | | |
| A3 | 979 | 14.3 | 33.7 | 14.0 | 38.3 | 13.5 | 14.8 | 16.9 | | |
| Baa1 | 846 | 14.4 | 27.7 | 14.5 | 42.3 | 12.1 | 13.5 | 16.5 | 0.7 | 11.8 |
| Baa2 | 1059 | 15.0 | 29.7 | 14.5 | 36.8 | 14.0 | 15.2 | 17.5 | 0.3 | 7.3 |
| Baa3 | 994 | 13.7 | 34.6 | 14.7 | 29.1 | 11.2 | 15.6 | 15.6 | 0.4 | 15.5 |
| Ba1 | 736 | 13.2 | 31.8 | 13.6 | 39.8 | 11.8 | 16.2 | 12.9 | 0.5 | 7.8 |
| Ba2 | 727 | 10.7 | 28.6 | 10.6 | 40.7 | 9.2 | 19.3 | 13.7 | 1.1 | 9.3 |
| Ba3 | 1104 | 11.0 | 24.9 | 10.7 | 39.1 | 10.3 | 24.6 | 12.5 | 2.1 | 12.6 |
| B1 | 1603 | 10.7 | 24.7 | 10.1 | 40.7 | 10.1 | 24.4 | 12.4 | 2.5 | 9.4 |
| B2 | 1662 | 10.0 | 25.1 | 9.4 | 40.0 | 10.3 | 23.2 | 10.5 | 3.6 | 9.5 |
| B3 | 1785 | 9.4 | 22.1 | 8.5 | 34.9 | 8.8 | 27.1 | 10.4 | 4.6 | 9.1 |
| Caa1 | 1558 | 8.1 | 16.6 | 8.0 | 30.6 | 7.8 | 25.7 | 8.7 | 1.9 | 8.7 |
| Caa2 | 773 | 6.9 | 24.8 | 6.5 | 20.3 | 7.2 | 18.7 | 7.3 | 4.2 | 8.6 |
| Caa3 | 165 | 6.0 | 23.0 | 6.2 | 18.2 | 5.5 | 20.6 | 6.0 | 3.0 | 6.4 |
| Ca | 46 | 5.4 | 47.8 | 4.6 | 21.7 | 5.7 | 19.6 | 7.4 | 4.4 | 4.5 |
| C | 8 | 9.6 | 12.5 | 7.5 | | | 12.5 | 3.0 | 12.5 | 16.0 |

[12] In a few cases, issuers have had their rating withdrawn (or defaulted) only to have another rating assigned later. In these cases, we reset the Age to 0.

Table 3 Global Rating Transition Data Summary Statistics (1987-2015): Upgraded Assignments

| | | | Upgrade | | Downgrade | | Withdrawal | | Default | |
|---|---|---|---|---|---|---|---|---|---|---|
| Rating | # | Duration | % | Duration | % | Duration | % | Duration | % | Duration |
| Aaa | 264 | 19.4 | | | 63.3 | 14.6 | 25.0 | 14.2 | | |
| Aa1 | 431 | 15.0 | 12.1 | 12.8 | 59.6 | 14.8 | 20.9 | 10.8 | | |
| Aa2 | 612 | 12.3 | 27.5 | 13.6 | 45.8 | 12.2 | 19.9 | 10.1 | | |
| Aa3 | 815 | 13.9 | 31.0 | 13.4 | 39.8 | 15.6 | 22.7 | 10.4 | 0.3 | 13.0 |
| A1 | 1037 | 14.3 | 31.9 | 12.4 | 31.8 | 18.8 | 24.2 | 9.9 | 0.2 | 19.0 |
| A2 | 1186 | 14.5 | 33.2 | 12.1 | 35.4 | 16.8 | 17.3 | 12.0 | 0.1 | 15.0 |
| A3 | 1171 | 12.6 | 35.6 | 11.7 | 25.5 | 16.9 | 19.3 | 9.6 | | |
| Baa1 | 1168 | 11.1 | 39.0 | 9.2 | 24.7 | 15.2 | 16.6 | 10.4 | 0.2 | 7.0 |
| Baa2 | 1215 | 11.4 | 42.2 | 10.3 | 21.5 | 14.4 | 17.8 | 9.7 | 0.2 | 7.5 |
| Baa3 | 991 | 11.3 | 43.6 | 10.6 | 18.9 | 12.1 | 20.2 | 9.7 | 0.2 | 28.5 |
| Ba1 | 835 | 9.0 | 49.9 | 7.9 | 19.6 | 10.7 | 22.2 | 8.9 | 0.1 | 6.0 |
| Ba2 | 949 | 8.2 | 43.0 | 6.8 | 21.9 | 10.7 | 24.7 | 8.3 | 0.7 | 9.1 |
| Ba3 | 909 | 7.9 | 39.4 | 7.4 | 25.7 | 9.7 | 23.5 | 7.2 | 1.1 | 3.1 |
| B1 | 964 | 7.6 | 37.8 | 6.7 | 24.5 | 8.8 | 26.2 | 7.7 | 1.0 | 9.6 |
| B2 | 746 | 7.2 | 38.7 | 6.1 | 27.2 | 8.5 | 20.5 | 7.2 | 1.7 | 8.7 |
| B3 | 706 | 6.8 | 29.7 | 6.2 | 25.1 | 8.1 | 26.6 | 6.1 | 2.7 | 6.7 |
| Caa1 | 429 | 6.4 | 31.5 | 5.6 | 16.1 | 6.6 | 30.8 | 6.7 | 2.8 | 6.0 |
| Caa2 | 166 | 5.9 | 41.0 | 5.2 | 13.9 | 5.9 | 23.5 | 6.4 | 6.6 | 6.4 |
| Caa3 | 61 | 4.9 | 37.7 | 4.1 | 4.9 | 5.3 | 26.2 | 3.6 | 21.3 | 6.5 |
| Ca | 10 | 3.5 | 60.0 | 3.3 | 30.0 | 2.6 | 10.0 | 7.0 | | |
| C | | | | | | | | | | |

Table 4 Global Rating Transition Data Summary Statistics (1987-2015): Downgraded Assignments

| | | | Upgrade | | Downgrade | | Withdrawal | | Default | |
|---|---|---|---|---|---|---|---|---|---|---|
| Rating | # | Duration | % | Duration | % | Duration | % | Duration | % | Duration |
| Aaa | | | | | | | | | | |
| Aa1 | 318 | 9.7 | 7.9 | 19.5 | 76.4 | 8.3 | 10.4 | 12.6 | | |
| Aa2 | 583 | 11.0 | 11.7 | 18.1 | 66.7 | 9.4 | 11.3 | 10.8 | | |
| Aa3 | 1010 | 11.1 | 12.1 | 20.7 | 68.5 | 9.1 | 11.5 | 8.7 | | |
| A1 | 1252 | 10.9 | 14.6 | 16.6 | 60.9 | 9.1 | 13.3 | 11.1 | 0.2 | 4.0 |
| A2 | 1567 | 10.6 | 16.5 | 14.5 | 56.9 | 8.0 | 12.4 | 10.5 | 0.1 | 3.0 |
| A3 | 1635 | 10.9 | 19.6 | 15.0 | 56.8 | 8.1 | 13.2 | 11.4 | 0.1 | 1.0 |
| Baa1 | 1542 | 10.5 | 21.6 | 13.3 | 51.4 | 7.9 | 14.3 | 10.0 | 0.1 | 9.0 |
| Baa2 | 1587 | 10.5 | 23.2 | 14.2 | 49.3 | 7.3 | 13.9 | 11.4 | 0.4 | 4.3 |
| Baa3 | 1551 | 8.9 | 25.7 | 13.4 | 47.3 | 5.9 | 12.3 | 10.0 | 0.8 | 2.3 |
| Ba1 | 1078 | 6.9 | 24.4 | 11.3 | 49.8 | 4.4 | 12.3 | 7.2 | 1.0 | 3.4 |
| Ba2 | 879 | 6.9 | 20.6 | 10.7 | 57.6 | 5.0 | 11.4 | 8.3 | 2.0 | 2.3 |
| Ba3 | 1091 | 7.2 | 24.7 | 9.9 | 51.1 | 5.4 | 15.1 | 7.9 | 2.1 | 5.7 |
| B1 | 1273 | 7.1 | 23.6 | 9.6 | 50.4 | 5.5 | 16.1 | 9.1 | 2.8 | 3.7 |
| B2 | 1271 | 6.8 | 21.5 | 8.4 | 51.4 | 5.4 | 15.7 | 9.1 | 5.6 | 4.9 |
| B3 | 1667 | 6.4 | 19.7 | 8.7 | 42.2 | 4.9 | 18.7 | 8.6 | 13.6 | 4.0 |
| Caa1 | 1525 | 5.6 | 20.4 | 7.2 | 38.1 | 4.0 | 19.8 | 7.3 | 11.0 | 3.5 |
| Caa2 | 1404 | 4.9 | 18.9 | 7.0 | 28.0 | 3.8 | 17.7 | 6.7 | 25.4 | 3.2 |
| Caa3 | 783 | 4.1 | 15.6 | 5.9 | 25.0 | 3.5 | 14.4 | 5.9 | 35.1 | 2.6 |
| Ca | 580 | 3.4 | 15.3 | 5.3 | 6.7 | 2.9 | 19.5 | 4.9 | 54.0 | 2.2 |
| C | 196 | 3.5 | 10.7 | 5.1 | | | 30.6 | 4.2 | 54.6 | 2.7 |

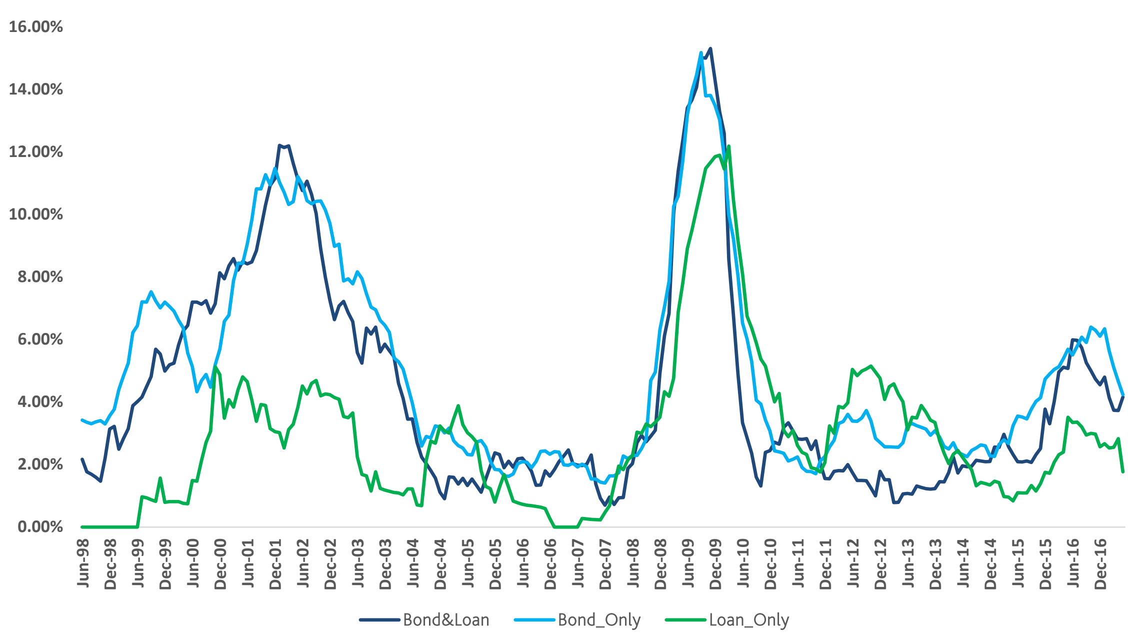
Moody's previous research shows that rating changes and default rates are strongly correlated with outlook/watchlist status. A Moody's rating outlook is an opinion regarding the likely rating direction over the medium term. Rating outlooks mainly fall into three categories: Positive (POS), Negative (NEG) and Stable (STA). If a Moody's rating is placed on review for upgrade (UPG), or downgrade (DNG), it is said to be on Moody's "watchlist", which means that it is under consideration for a change in the near term. Hamilton and Cantor (2005) find that the average duration of outlooks is about one year to 18 months and rating changes including defaults are strongly correlated with outlook status. Table 5 shows that B1 issuers placed on review for upgrade (UPG) have a much higher probability to be upgraded within a year than others.

Table 5 Average One-Year Rating Transition Probabilities Conditional on Outlook Status for US B1 Issuers, %

| Outlook | Cohort Rating | Issuer - Cohorts | Aaa | Aa1 | Aa2 | Aa3 | A1 | A2 | A3 | Baa1 | Baa2 | Baa3 | Ba1 | Ba2 | Ba3 | B1 | B2 | B3 | Caa1 | Caa2 | Caa3 | Ca | C | Def |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DNG | B1 | 1789 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.28 | 0.50 | 2.35 | 1.29 | 43.21 | 19.45 | 16.94 | 4.75 | 2.57 | 1.17 | 0.95 | 0.17 | 6.04 |
| NEG | B1 | 6144 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.05 | 0.13 | 0.08 | 0.75 | 1.07 | 5.27 | 56.87 | 16.60 | 8.97 | 3.86 | 1.22 | 0.62 | 0.99 | 0.05 | 3.43 |
| STA | B1 | 61079 | 0.01 | 0.02 | 0.03 | 0.01 | 0.05 | 0.04 | 0.04 | 0.08 | 0.18 | 0.30 | 0.71 | 2.54 | 6.57 | 74.34 | 6.04 | 4.45 | 1.25 | 0.86 | 0.23 | 0.18 | 0.00 | 2.07 |
| POS | B1 | 4777 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.19 | 0.00 | 0.23 | 0.13 | 1.65 | 7.95 | 19.68 | 58.17 | 5.13 | 3.96 | 1.51 | 0.48 | 0.10 | 0.06 | 0.00 | 0.73 |
| UPG | B1 | 972 | 0.41 | 0.00 | 0.00 | 0.41 | 1.95 | 0.41 | 0.62 | 2.06 | 1.03 | 3.19 | 9.57 | 25.31 | 20.58 | 26.44 | 5.14 | 0.82 | 0.72 | 0.51 | 0.21 | 0.00 | 0.00 | 0.62 |

In the model, we distinguish four types of issuers, issuers who currently only have rated bonds (bond-only); issuers who have both rated bonds and loans (bond and loan); issuers who currently have only rated loans, but had rated bonds in the past (loan-ex-bonds), and issuers who have never had rated bonds (loan-no-bond). Moody's research by Zhang and Metz (2010) find that these four types of issuers differ primarily in their withdrawal patterns. Loan-only issuers are the most likely to have their ratings withdrawn, and issuers who have both rated bonds and loans are far less likely to have their ratings withdrawn than issuers with only rated bonds. Even though loan-only issuers have defaulted at lower rates, after conditioning on rating, outlook and momentum, the differences are not significant. Figure 3 displays a comparison for global speculative grade 12-month trailing default rates for bond and loan issuers, bond-only issuers and loan-only issuers. It shows that loan-only issuers have defaulted at lower rates most of the time. This could be due to the fact that rating distribution of loan-only issuers is different from that of bond issuers.

Figure 3 Global Speculative-Grade Trailing 12-Month Default Rates for Bond and Loan Issuers, Bond-Only Issuers and Loan-Only Issuers[13]




[13] Monthly Default Report, May 2017, Moody's Investors Service

### 2.2 Economic Data

The economic data series in the model are the unemployment rate and high yield spreads. The US unemployment rate in the estimation step comes from the Current Population Survey conducted by the Bureau of Labor Statistics, while the historical Eurozone unemployment rate is calibrated by the Economic and Consumer Credit Analytics (ECCA) division within Moody's Analytics. Barclays' US and European high yield spreads data are used in the model estimation step. They are option-adjusted spreads over US Treasury and Germany Treasury curves, respectively. We average the monthly economic data to obtain quarterly values.

## 3 The Model

### 3.1 Model Framework

The Credit Transition Model (CTM) belongs to a class of survival models. It is a discrete time, multiple-destination proportional hazard type. Survival models usually analyze the expected duration of time until one or more events happen. In CTM, the events are rating transitions. As we are not only interested in when rating transitions happen, but also which exact exiting state they go to, we resort to a multiple destination proportional hazard type of model.

The model does not directly estimate the probability associated with the rating class which the issuer will exit to. Instead, it first estimates the transition dynamics to five exiting states: upgrade, downgrade above C, downgrade to C, withdrawal and default. We then rely on historical transition matrices to connect the exiting states and the rating classes. We construct a US model and a European model with very similar structures, which are estimated separately. Section 2.1 provides more detail on the estimation data of these two models.

We begin by establishing basic notational conventions. Consider the instantaneous transition rate from a rating category *r* to an exiting state *s,* after elapsed time *u* conditional on no prior exit from *r*. We define the hazard rate as a function of elapsed time *u* in the rating category *r*[14] and observed (time-varying) covariates $x_t$.

$$h_s^r(u|x_t) \equiv \psi_s^r(u)\theta_s^r(x_t)$$

To limit the number of parameters, we do not estimate a free transition intensity from every rating category to every viable exit state, but instead impose certain restrictions across categories. In particular, we define six classes of ratings, the investment grade (IG) *Aaa and Aa* process(IG-I), the IG A process (IG-II), the IG Baa process (IG-III), the speculative grade (SG) Ba process (SG-I), the SG B process (SG-II), and the SG *C* process (SG-III). Individual rating categories within these larger processes are distinguished only as scalar transformations of their underlying process. For example, the transition intensity for an Aaa rating is given by:

$$h_s^{Aaa}(u|x_t) \equiv \propto_s^{Aaa} \cdot h_s^{IG-I}(u|x_t) = \propto_s^{Aaa} \cdot \psi_s^{IG-I}(u)\theta_s^{IG-I}(x_t)$$

Where $\psi_s^{IG}(u)$ denote the baseline transition intensity for rating category *r* to exit state *s* and $\theta_s^r(x_t)$ is a strictly positive function of the observed covariates for rating category *r* and exit state *s*. It is worth noting that there are virtually no restrictions imposed across processes.

These transition intensities are used, to obtain the densities $f_s^r$ governing the probability of exiting from rating *r* to state *s* at time *u* and the survival probability $\bar{F}$, which is the probability of exiting after elapsed time *u* or no prior exit from *r*:

$$f_s^r(u|x_t) = h_s^r(u|x_t)\bar{F}(u|x_t)$$

$$\bar{F}(u|x_t) = \exp(-\sum_{s=1}^{S}\int_0^u h_s^r(\tau|x_t)d\tau)$$

[14] We denote elapsed time by u to distinguish it from calendar time t.

This will be familiar as a standard application of the multiple-destination mixed proportional hazards model. Also familiar is the assumption of conditional independence across issuers.[15] In our application, time is measured discretely, thus we use the above equation to obtain the probability of starting with a rating r and exiting to a particular state s, which occurs within a window of time T ∈ $(a, a{+}\Delta t]$ as:

$$Pr(S = s, a < T \leq a + \Delta t | r, x_t) = \int_a^{a+\Delta t} f_s^r(u|x_t)du$$

This can be further simplified to:

$$= \frac{h_s^r(a|x_t)}{\sum_{i=1}^S h_i^r(a|x_t)} \cdot (1 - exp\left(-\sum_{i=1}^S h_i^r(a|x_t)\right)) \cdot \bar{F}(a|x_t, r)$$

A mathematical derivation of the above equation can be found in Appendix II.

If we summarize all the parameters in the above equation in a vector $\beta$, the above transition probability can also be written as $l_j(\beta|X)$, where *j* denotes the *j*th transition in history, X includes information of issuer characteristics, rating history and macroeconomic trend. If we define $L(\beta|X) = \sum_{j=1}^n \ln l_j(\beta|X)$ , and our estimation task becomes:

$$\max_\beta L(\beta|X)$$

### 3.2 Exiting States

One could define exiting states to be all other rating categories as well as the absorbing states *default* and *withdrawal*. This is proved impractical due to data limitations, and we instead distinguish the states *upgrade*, *downgrade above C*, *downgrade to C*, *default* and *withdrawal*.

Not every rating process transitions to every exiting state. In particular from *IG*, one can only *upgrade*, *downgrade above C* or *withdraw*.[16] All exiting states are viable from SG. Obviously from C, downgrade above C is undefined, but all other exit states are viable.

These exiting states apply to individual rating categories, not just whole aggregate processes. In other words, a particular IG rating can "exit to downgrade" to a different IG rating - not just downgrade from one aggregate process to another. Our use of aggregate processes simply imposes some structural discipline on our estimates: up to scale, all IG-I (Aaa and Aa) ratings are identical, having the same baseline transition shapes and the same betas to the macroeconomic drivers. But we track movements within the broader processes as upgrades and downgrades.

Of course, this implies certain rating category specific restrictions beyond those described above. The Aaa category cannot exit to upgrade, even though the broader IG process has a defined upgrade transition. Similarly, the B3 category cannot exit to *downgrade above C*.

### 3.3 Historical Conditional Transition Matrix

We have defined exiting states, but by themselves these do not specify to which rating category the issuer upgrades or downgrades to. To make that determination, we use conditional historical transition frequencies. For the first quarter, the transition matrices are conditional only on:

1. the exit state: upgrade, downgrade above C, or downgrade to C
2. watchlist/outlook status

After the first quarter, the transition matrices are conditional on several dimensions:

1. the exit state: upgrade, downgrade above C, or downgrade to C

[15] For an interesting discussion of the conditional independence assumption, see Das et al (2005).

[16] In our data set, which covers 29 years from 1987-2015, there have been only 50 instances of transitioning from an IG rating to default globally when transitions are measured at the quarterly frequency. Similarly, there are only 47 cases of transitioning from IG to C globally. In each case there are too few observations to separately estimate a transition from IG to default or downgrade to C. Instead, we score all of these events as downgrades in the estimation. For the purposes of forecast simulations, we include a non-zero probability that when downgrading from an IG rating, an issuer could transition directly to C or to default.

2. whether the transition happens before 1997
3. the quarterly change in the unemployment rate: we categorize this into four buckets, reflecting improving, stable, slightly deteriorating and significantly deteriorating economic environment
4. whether the issuer was upgraded or downgraded to the current rating, or it was an initial rating assignment

Table 6 is an example of such conditional transition matrices. If an issuer, which has been downgraded to its current rating, is projected to be downgraded again when the unemployment rate gradually increases, we consult Table 6 to determine the probability of transitioning from its current rating to each specific rating after the first forecast quarter. Trivially, an issuer currently rated Aa1 can only transition to Aaa conditional on exiting to *upgrade*.

Clearly, the macroeconomy and the issuer rating history impact not only the probability of upgrade and downgrade, but also the probability of d-notch change.

**Table 6 Historical Transition Matrix Conditional on Downgrade Above C for Issuers Downgraded to its Current Rating under Slightly Deteriorating Economic Environment Post 1997**

| | | | | | | | | | | To Rating | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| From Rating | Aaa | Aa1 | Aa2 | Aa3 | A1 | A2 | A3 | Baa1 | Baa2 | Baa3 | Ba1 | Ba2 | Ba3 | B1 | B2 | B3 | Caa1 | Caa2 | Caa3 | Ca | C |
| Aaa | | 100 | | | | | | | | | | | | | | | | | | | |
| Aa1 | | | 33 | 67 | | | | | | | | | | | | | | | | | |
| Aa2 | | | | 81 | 13 | 6 | | | | | | | | | | | | | | | |
| Aa3 | | | | | 32 | 61 | 6 | | | | | | | | | | | | | | |
| A1 | | | | | | 70 | 16 | 14 | | | | | | | | | | | | | |
| A2 | | | | | | | 78 | 13 | 8 | 3 | | | | | | | | | | | |
| A3 | | | | | | | | 71 | 22 | 2 | 5 | | | | | | | | | | |
| Baa1 | | | | | | | | | 63 | 24 | 6 | 3 | | 3 | 1 | | | | | | |
| Baa2 | | | | | | | | | | 63 | 17 | 10 | 2 | 4 | 2 | 2 | | | | | |
| Baa3 | | | | | | | | | | | 41 | 26 | 11 | 8 | 7 | 3 | | | | | |
| Ba1 | | | | | | | | | | | | 43 | 29 | 18 | 4 | 4 | | | | | |
| Ba2 | | | | | | | | | | | | | 34 | 38 | 13 | 16 | | | | | |
| Ba3 | | | | | | | | | | | | | | 52 | 24 | 24 | | | | | |
| B1 | | | | | | | | | | | | | | | 57 | 43 | | | | | |
| B2 | | | | | | | | | | | | | | | | 100 | | | | | |
| B3 | | | | | | | | | | | | | | | | | | | | | |
| Caa1 | | | | | | | | | | | | | | | | | | | | | |
| Caa2 | | | | | | | | | | | | | | | | | | | | | |
| Caa3 | | | | | | | | | | | | | | | | | | | | | |
| Ca | | | | | | | | | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | | | | | | | | | |

## 3.4 Conditioning Information

### 3.4.1 ISSUER SPECIFIC INFORMATION

The issuer-specific conditioning information includes, in addition to elapsed time $u$:

- *Rating Category*

  As discussed above, the specific rating category determines the scale of the underlying process.

- *Watchlist/Outlook*

  Incorporating issuers' watchlist/outlook assignment has significantly enhanced the model's ability to identify which issuers are likely to upgrade, downgrade or eventually default. There are five watchlist/outlook status: Watchlist for a Downgrade, Negative Outlook, Stable, Positive Outlook and Watchlist for an Upgrade. The credit transition dynamics depend on issuers' outlook status, as well as the changes of the watchlist/outlook status.

- *Rating History*

  We condition on whether (and when) an issuer was upgraded or downgraded into its current rating. We allow a flexible dynamic response for up to 12 quarters following the upgrade or downgrade.

- *Age*

  Age is here defined as how long the issuer has continuously maintained *any* Moody's rating. In those few cases where an issuer defaults or withdraws and later re-enters the data set, its *Age* is reset to 0. *Age* enters as a quadratic function, and is capped at 40 quarters.

- *Industry*

  We also condition on the issuer's industry, as financial firms and utility firms react differently to economic and financial shocks, compared to others.

- *Loan and/or bond issuer*

  The current version of the model is built on Moody's proprietary senior unsecured ratings, which is derived from a bond rating or a loan rating, or both. It is worth noting that the rated loan market has a much shorter history than the bond market, and there are fairly stable differences between loan and bond issuers during credit cycles. The model distinguish four types of issuers: bond-only (issuers who currently ONLY have rated bonds)[17], bond-and-loan (issuers who currently have both rated bonds and loans), loan-ex-bond (issuers who currently have only rated loans but have, in the past, had rated bonds), and loan-no-bond (issuers who have never had rated bonds). We regard this as one of the issuer characteristics impacting the credit transition.

- *Cumulative Change in Unemployment*

  The transition also conditions on the cumulative change in unemployment since the issuer entered its current rating. The intuition behind this is that having endured a period of increasing unemployment should increase the probability of default and downgrade even if the current economic state is relatively strong.

#### 3.4.2 ECONOMIC INFORMATION

The model conditions on two pieces of economic information, the unemployment rate and high yield spreads. There are several reasons why we pick only the unemployment rate among many macroeconomic variables in the model. First, most macroeconomic indicators are highly correlated. Given the number of parameters we need to estimate, we would prefer to achieve a certain level of parsimony with the model. Second, some macro variables are revised constantly, such as GDP. Quarterly GDP numbers are revised twice in the subsequent two quarters after the initial release, and they are under separate annual revisions and comprehensive revisions every five years, with new sources of data. Historical values are also included in such revisions. These will impact the accuracy of near-term GDP forecast, as well as model estimation and back testing. In contrast, the unemployment rate revisions are minor.

For the unemployment rate, we utilized the information from two dimensions, the cyclical part and the incremental part. We first take averages of the monthly unemployment rate to convert it to quarterly series, and then take the logarithm of the quarterly values before applying the Hodrick-Prescott filter ($\lambda = 1600$) to extract its cyclical component and long-run trend. We believe the cyclical component correlates more with credit transition cycles. We also calculate the quarterly change of the unemployment in logarithm terms, and this will capture the short-term evolution of labor market.

We smooth the logarithm of high yield spreads to filter out the highest frequency noise in the series.[18] With the smoothed series, we organize this information into four sub categories: the first two sub categories describe whether this is a high credit cost environment (high yield spreads are greater than 9 percent) or a low credit cost environment (high yield spreads are less than 3 percent), and two other subcategories quantify the current deviation of high yield spreads from its long-term mean, which we assume is 5 percent in the model.

### 3.5 Parameterization

In this section we discuss details of our parameterization of the baseline intensities and covariate functions. We also discuss structural shifts in rating definitions that occurred in 1997 and in watchlist/outlook database expansion in 2004.

- Baseline transition intensities, $\psi_s^r(u)$:

[17] Such issuers may have loans, but they are not rated by Moody's.

[18] The US high yield spread series dates from January 1987 and the European high yield spread series starts from August 2000 .

We specify step-function baseline intensities where the values are given by a piecewise linear process. We estimate break points of the linear process at quarters {1, 2, 3, 4, 5, 6, 8, 12, 16} and impose the scale normalization that $\psi_s^r(u) = 1$ for u ≥20 quarters. The baseline intensities are set as 0s when the transition is not viable, such as the transition from investment grade to default.

- Covariate functions, $\theta_s^r(x_t) = exp(x_t\beta_s^r)$, where $x_t$ is a vector summarizing the issuer specific information listed in section 3.4.1 and economic information in section 3.4.2. It also includes the dummy variables explaining the structural changes specified below.
- Structural changes:

  In 1997, Moody's introduced Caa rating modifiers and the subsequent expansion in the use of all C rating categories. Some issuers which previously might have been rated B3 were moved into one of the new C categories. This, of course, changed the upgrade, downgrade and default dynamics of several rating categories. We capture this by allowing all means - the scale effects of rating categories in all transitions - to change post-1997. Moody's "live" outlook database was not available until late 2003. Before that, the outlook information was only stored in Moody's press releases, which was later manually collected and entered into the database. In the model, we use two dummy variables in $x_t$ to reflect such structurally changes.

Even though we know that watchlist/outlook information is important in projecting transition probabilities, including such information in the model is nontrivial, as we have to predict an issuer's rating outlook in the future first. Building a model to forecast issuers' rating outlook requires further reconciliation with the rating assignments. For example, an issuer which is downgraded is more likely to be downgraded again and is also more likely to carry a Negative Outlook or to be put on Review for Downgrade.

Instead, we have adopted a novel approach to avoid this additional task. The current version of the Credit Transition Model chains together five separate models with similar structure. The first model, conditioning on the issuer's current watchlist/outlook assignment (t), is used to forecast rating transitions over the first quarter (t+1). The second model conditions on the issuer's watchlist/outlook assignment one quarter ago to generate the projections for the second quarter (t+2). Similarly, the third and fourth models, with two-quarter lag and three-quarter lag watchlist/outlook information, predict transitions for the third and fourth quarters (t+3 and t+4). In contrast, the fifth model does not condition on watchlist/outlook information. It is used to generate rating transitions for the fifth to the twentieth quarters (t+5 to t+20). Table 7 summarizes the above design.

**Table 7 Forecast Models with Watchlist/Outlook information**

| Forecast Model | Watchlist/Outlook Information | Forecast Quarter |
|---|---|---|
| M1 | At time t | t+1 |
| M2 | At time t | t+2 |
| M3 | At time t | t+3 |
| M4 | At time t | t+4 |
| M5 | N/A | from t+5 to t+20 |

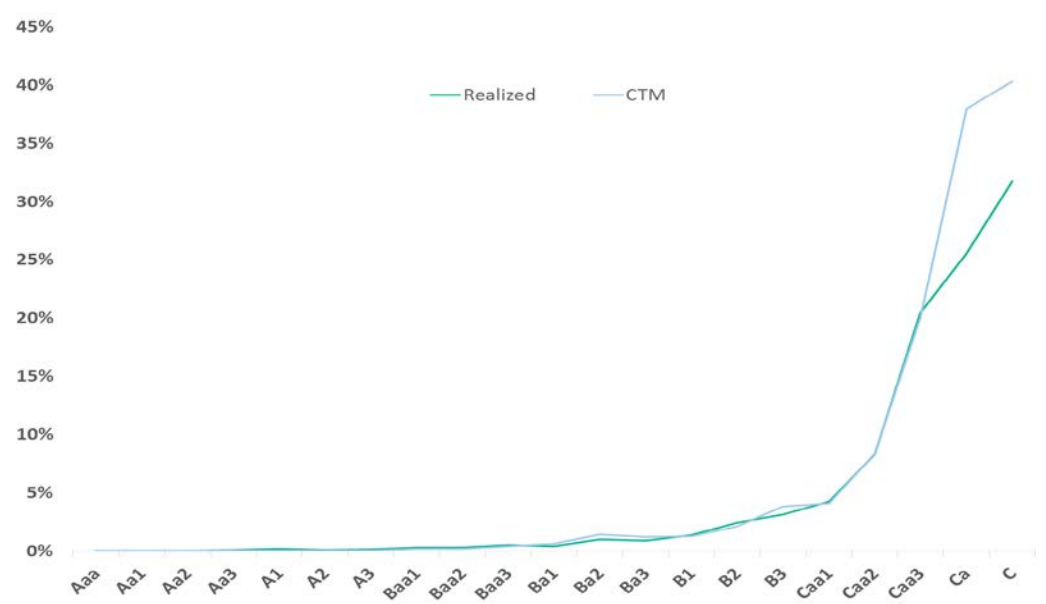
Given different credit and economic cycles, we estimate the parameters governing rating transitions in the US and in Europe separately. For each region, we estimate over 4000 parameters. In order to achieve convergence with the Newton method in optimization, we need a relatively large sample size. Thus, it would not be feasible to estimate separate models for Asia and Latin America, even though that might otherwise be desirable. Given this constraint, we include APAC and LATAM rating transitions in estimating the parameters in the US model, assuming that the US economy and financial conditions drive global demand and the rating transitions in Asia Pacific and Latin America.

The sample size in estimating the European parameters is much smaller, which creates a huge hurdle to achieve the convergence in optimization. This is due to two facts. First, the number of Moody's rated European issuers is much smaller than that of the US issuers, especially for high yield issuers in 2000s. Figure 4 shows the issuer count comparison as of January 2005 as an example. Second, the European high yield spreads series only starts from Aug 2000.

Figure 4 Number of Issuers Comparison by Rating Group as of January 2005


In order to maintain a consistent estimation framework, we employ the global rating dataset for the optimization process when estimating the European parameters. Importantly, we differentiate the EMEA issuers from issuers in the other regions by using a dummy variable in the covariate function. Issuers in Europe, the Middle East and Africa are given a value of 1, while others are associated with 0. As a result, the parameter estimation for this model is impacted not only by the European data, but also by the US data. Such modeling treatment could potentially reduce the accuracy of the forecast. For example, the model overestimates the default rate for the two lowest rating categories. Figure 5 displays the comparison of 12-month default rates between the CTM model projections and realized default rates.

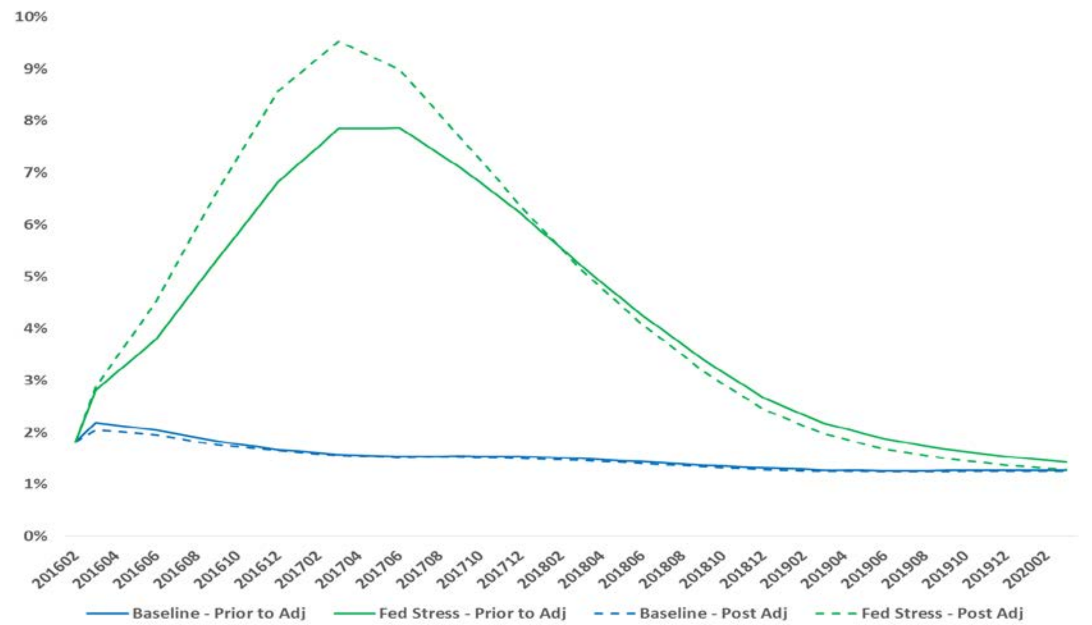
Figure 5 12-Month EMEA Default Rates by Rating Category before Empirical Calibration of the EMEA Parameters




We therefore adjust the coefficients associated with the current rating category Ca and C, in order to lower their default transition intensities. Similarly, we adjust the coefficients on the change of the unemployment rate and the dummy variable, which is equal to one pre-2004, corresponding to the expansion of outlook database in 2004, to enhance the cardinal accuracy of the forecast. The empirical calibration is processed in a parsimonious way in the sense that we only adjust a very small set of parameters after the optimization step. The values of these parameters are determined through simulations. The improvement in cardinal accuracy of the European model through targeted calibration of selected coefficients can be clearly observed when comparing Figure 5 and Figure 12.

Figure 6 illustrates the differences in the CTM projection of 12-month default rate between the model prior to the empirical adjustment and the model after the adjustment. We pick two scenarios, Moody's Analytics' March 2017 baseline scenario and the 2017 Fed Severely Adverse scenario. As we see in Figure 6, with the adjustments, the default rate projections become more sensitive to the change in the unemployment rate in the severely adverse scenario. In the baseline scenario, the difference in default rate projections are much smaller. More model performance test results are provided in Section 4.

Figure 6 CTM European Default Rate Projections Based on Pre-Adjustment Parameters and Post-Adjustment Parameters




## 3.6 Forecasts

For any issuer, based on the model framework discussed in Section 3.1, we can forecast its transition probability to any exit state at any particular quarter with issuer characteristic information, rating history including watchlist/outlook status, and economic projections. In order to determine which exact rating category the issuer will migrate to, we rely on historical conditional transition matrices. With a specific set of parameters and conditional transition matrices, we are able to project transition probabilities to each rating category for each quarter, as well as cumulative transition probabilities over a multi-quarter horizon.

The model also equips us with the capability to forecast credit transitions under different economic scenarios. Sometimes, CTM users would like to explore the credit transition outlook based on a certain macroeconomic view on GDP growth, or they do not have a strong opinion on one of the CTM macro inputs. Using an internally developed tool, a simplified and modified version of MCMESA, we can translate clients' view on one of the three economic variables, GDP, the unemployment rate, or high yield spread, into CTM required macro input paths in a consistent way. The tool leverages on Moody's Analytics' carefully designed S0-S4 scenarios and two "bookends" scenarios to calibrate the unspecified paths of the economic inputs.

### 3.6.1 ECONOMIC FORECAST

As discussed in the previous sections, the CTM explores the relationship between default rates and issuers' rating history, as well as economic/financial factors. In order to generate default forecast of the highest quality, we need robust and consistent economic projections as input. Most of these economic forecasts are directly obtained through Moody's Analytics' Economic & Consumer Credit Analytics (ECCA) database, including US and Eurozone unemployment rates and US high yield spreads.

ECCA maintains a database of over 280 million economic, financial and demographic historical time series from over 600 sources, covering 180 countries and their subregions. Forecasts are available for more than 50 countries, including the US and all of its states and metropolitan areas, plus countries, states, and cities around the globe. To ensure high levels of data quality, ECCA has in place an internally built system designed to achieve accuracy, completeness, auditability, timeliness, and consistency. Automated accuracy checks include, for example, checks that the unemployment rate is never negative, that total employment always exceeds manufacturing employment, or that the GDP accounting identity is never violated. They are augmented manually by the specialist in charge of updating each series. Specialists also examine completeness, or the extent to which the expected attributes of a data series are provided, daily, for each series in the database. The system monitors all activity to ensure that it can be tracked to its originating transaction and checks that new data releases are published in a timely manner, with the highest priority releases updated within 15 minutes of release by the source. Two parallel databases are maintained, using FAME and SQL repositories, and replicated externally, ensuring data consistency across the enterprise. ECCA's team of more than 80 economists also produce, on a monthly basis, quarterly forecasts for core economic measures for over 50 countries. The global economic forecasting models seek to provide forecasts for a set of economic measures that is as consistent as possible across countries.[19]

We separately project the European high yield spread with the US high yield spread and VIX forecast, as ECCA does not forecast this series. The forecasts of US high yield spread and VIX are also acquired from ECCA's forecast database.

#### 3.6.2 SCENARIOS

Based on different paths of macroeconomic variables, CTM can generate transition/default forecast under different economic scenarios on the issuer and portfolio level. Moody's Analytics publishes on a monthly basis baseline and alternative economic scenario forecasts, as well as regulatory driven CCAR scenarios. Table 8 summarizes the scenarios and its associated probabilities. CTM ingests economic scenario forecasts from Moody's Analytics' own macro forecast database, and generates the corresponding transition matrices on the web interface.

Table 8 **Economic Scenario Descriptions**[20]

| Scenario | Description | Probability of Worse Outcome |
|---|---|---|
| Moodys-S0 | Baseline | 50% |
| Moodys-S1 | Stronger Near-Term Growth | 90% |
| Moodys-S2 | Slower Near-Term Growth | 25% |
| Moodys-S3 | Moderate Recession | 10% |
| Moodys-S4 | Protracted Slump | 4% |
| Fed-CCAR Baseline | Baseline | N.A. |
| Fed-CCAR Adverse | Adverse | N.A. |
| Fed-CCAR Severely Adverse | Severely Adverse | N.A. |

#### 3.6.3 MCMESA

In order to forecast future credit migrations, the CTM needs complete paths of the unemployment rate and high yield spreads as macro inputs. Sometimes, users do not have much of an opinion on one the above series, or they have strong preference to express their view on GDP growth. Therefore, we develop a calculation tool to meet users' need. This tool is indeed a simplified and modified version of the original MCMESA. MCMESA is the acronym for Moody's Multi-Country Macroeconomic Scenario Accelerator, which was created by Kyle Hillman and Tony Hughes from Moody's Analytics[21]. The algorithm makes use of ECCA's baseline and S1-S4 scenarios and two "bookend" scenarios, all of which are updated monthly. The bookend scenarios represent the outer ranges of highly unlikely but still plausible economic outcomes, serving as boundaries. Both of the upside and downside bookend scenarios are viewed as 1-in-10,000 probability events. The core algorithm is applied to the US and Europe separately. Instead of supplying two paths for the unemployment rate and high yield spreads, with the help of MCMESA, users now can run CTM when specifying only one of the two series for each region, GDP or the unemployment rate. The tool then will generate the remaining necessary inputs for CTM. Table 9 summarizes the process.

We use the following example to describe our computing process. Suppose, a user provides the path of GDP during the forecast horizon, which lies between Moody's S2 and S3 scenarios, but close to S3 scenario. Then the calculated weights are 0.6 on S3 and

[19] A detailed description of the macroeconomic modeling approach used can be found in Zandi (2011).

[20] The scenario descriptions and probabilities may change over time. These described here reflect the 2017 forecast cycle.

[21] See Hillman and Hughes (2013) for a detailed explanation of the original MESA algorithm and a discussion for its benefits and limitations.

0.4 on S2[22], which are then used to generate the paths for other variables. If users provide paths for both GDP and the unemployment rate, the weights will be calibrated on both the GDP path and the unemployment rate path to generate the projection for high yield spreads. User supplied assumptions will not be overridden. It is worth noting that users have to provide at least one of GDP or the unemployment rate path for each region to run CTM properly.

Table 9 **MCMESA Process Summary**

| GDP | UE | HYS |
|---|---|---|
| ✓ | ✓ | Calibrated based on both GDP and UE path |
| ✓ | Calibrated based on GDP and HYS path | ✓ |
| No need to calibrate | ✓ | ✓ |
| ✓ | Calibrated based on GDP path | Calibrated based on GDP path |
| Calibrated based on UE path | ✓ | Calibrated based on UE path |

# 4 Validation

During the model development stage, the methodology was subjected to rigorous testing and validation procedures, including backtesting with perfect economic foresight, tests of ordinal accuracy, as well as stability and sensitivity analysis. In this section, we discuss these procedures.

## 4.1 Backtesting – Perfect Economic Foresight Exercise

CTM forecasts the issuer-level probabilities of default, upgrade, downgrade and defaults over a user-chosen horizon, up to 20 quarters. Such forecasts are conditional forecasts, meaning they represent expected probabilities given specific economic scenarios. Therefore, the performance of CTM depends not only on CTM itself, but also on the accuracy of the economic assumptions. As our purpose here is to validate the CTM methodology, it is necessary to eliminate the forecast errors in the economic assumptions. We achieve this by applying the realized values of the economic and financial variables when calculating the CTM projections. In this way, any deviation in credit transition forecasts with "perfect economic foresight" is purely due to CTM itself.

We first compare the CTM predicted default rates and realized default rates. In the calculation, we form the cohort of issuers at the beginning of every calendar month. Then the 12-month transition/default rate at every month is calculated based on the formed cohort. For all the validation exercise in this paper, the issuer universe of the "US" model is composed of all Moody's rated companies in North America, Latin America and Asia Pacific, while the issuer universe of the "European" model includes all Moody's rated companies in Europe, the Middle East and Africa. Figure 7 and Figure 8 compare show high correlations between the realized and model projected 12-month cumulative default rates[23]. The horizontal axis displays the cohort formation date and the default rates in the chart represent realized or projected values in the 12-month horizon starting from the cohort forming date.

More specifically, the CTM forecast of the 12-month default rate is calculated as:

$$\bar{D} = 1 - \prod_{q=1}^{4}(1 - \overline{d^q})$$

where $\bar{D}$ is the 12-month default rate forecast and $\overline{d^q}$ is the projected default rate in the quarter q within the 12 months.

The comparisons in Figure 7 and Figure 8 have demonstrated a high level of cardinal accuracy of the forecasts. Comparatively, the US model performs better when forecasting default than the European model.

It is worth mentioning that these forecast are not out-of-sample, neither are they a fully in-sample fit. First, actual rating transition data in 2016, which are outside of the estimation sample, are used in part to generate the realized default rate for cohort formed at Feb 2015 and later. Second and more importantly, we calculate the direct default rate for the first quarter within the 12-month

[22] We assume the weights here are constant across the forecast horizon for simplicity.

[23] Issuers in Americas but outside of the US are included when forming the US cohorts; while issuers in Middle East and Africa are included when forming the European cohorts.

horizon, as well as the probability of upgrade, downgrade, withdrawal and no rating change. From each of the new ratings, we need to make a similar set of calculations for the second quarter. We then repeat this process for the third and fourth quarter to get the 12-month default rate. This is quite different if compared to a fit line in a typical regression.

Figure 7 12-Month U.S. Speculative-Grade Default Rate: Realized vs. CTM[24]

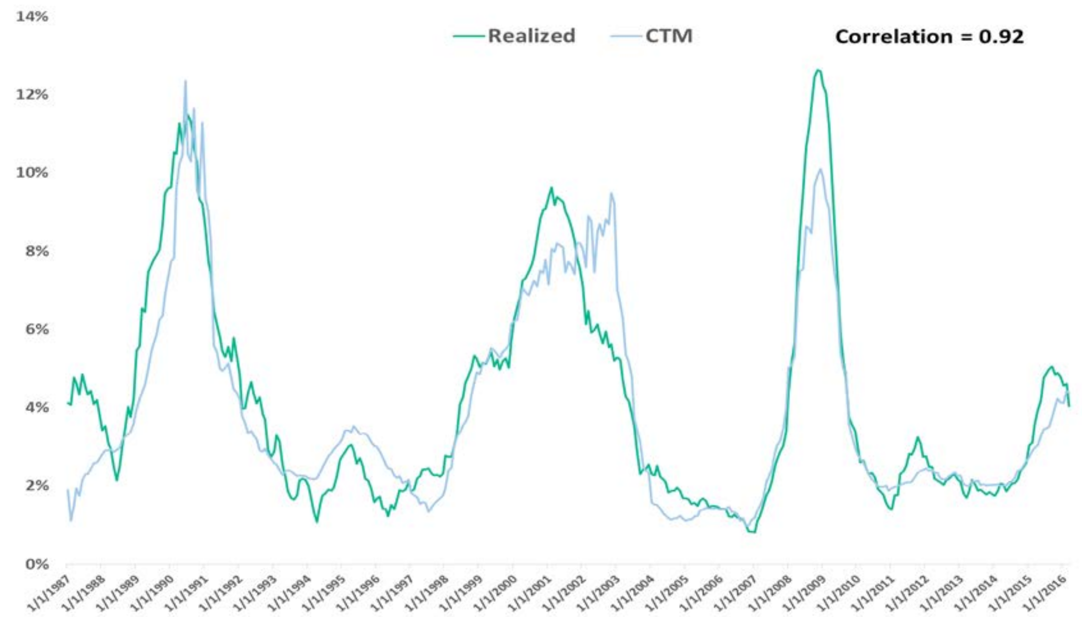
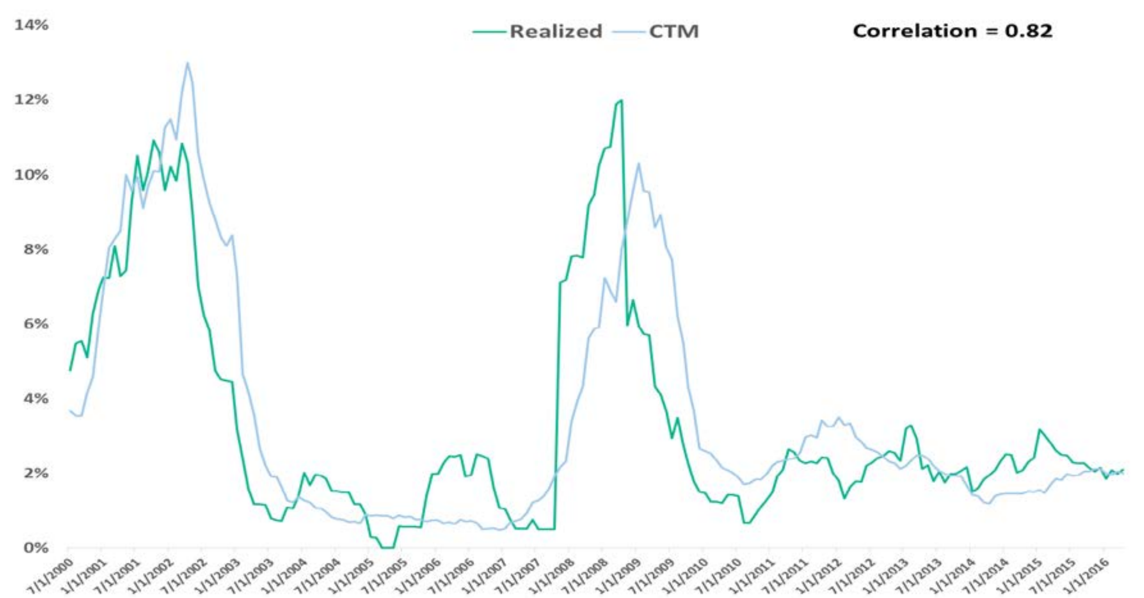



Figure 8 12-Month European Speculative-Grade Default Rate: Realized vs. CTM




Similarly, Figure 9 and Figure 10 below show high correlations between the realized and projected downgrade rates.

[24] The default rates and other transition rates calculated in this paper are not adjusted for withdrawals.

Figure 9 12-Month US Downgrade Rate: Realized vs. CTM

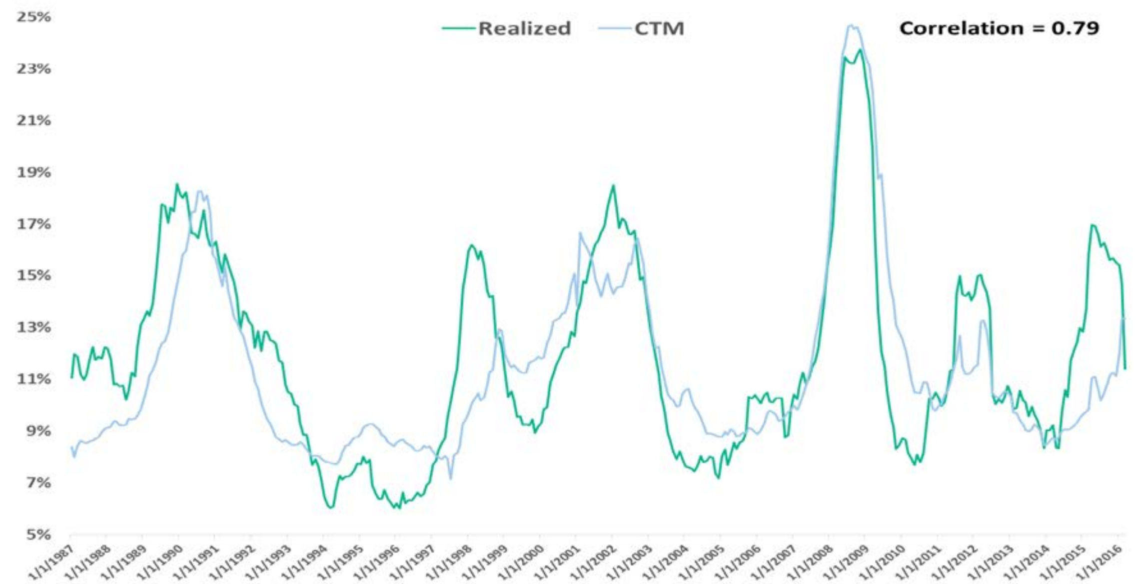
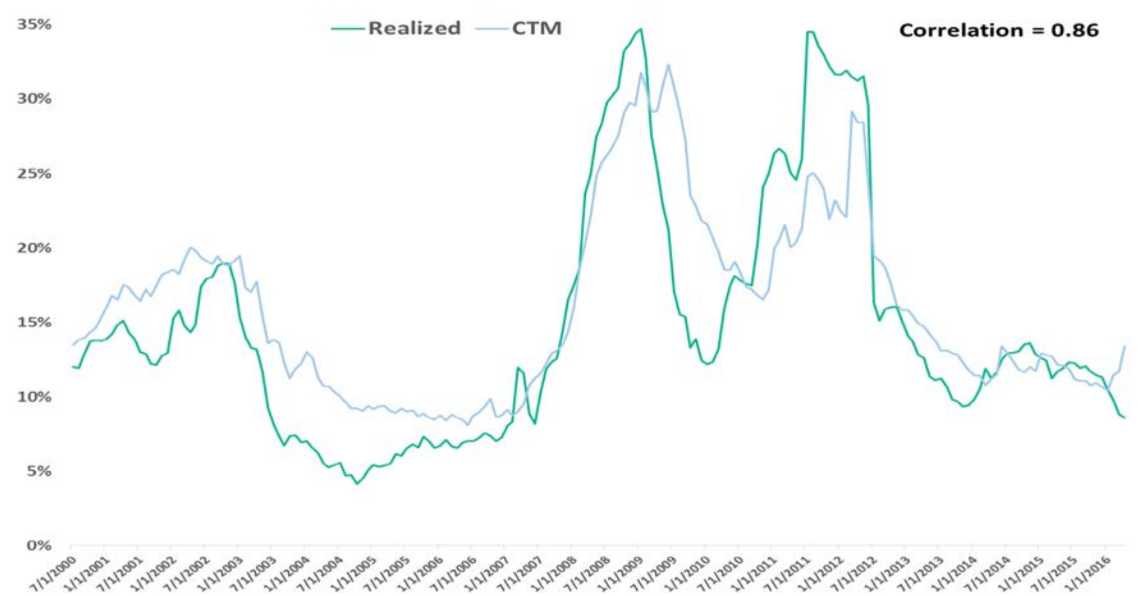



Figure 10 12-Month European Downgrade Rate: Realized vs. CTM




We also examined the forecast accuracy for default by rating categories, as shown in Figure 11 and Figure 12.

Figure 11 12-Month U.S. Default Rate by Rating Categories: Realized vs. CTM

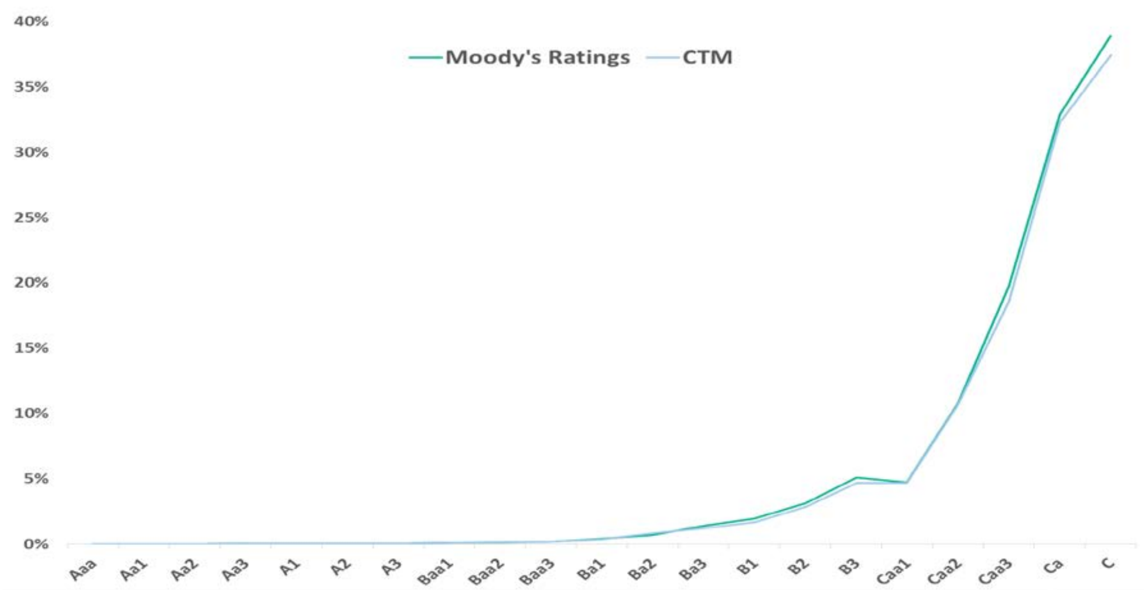



Figure 12 12-Month European Default Rate by Rating Categories: Realized vs. CTM


## 4.2 Accuracy Ratio

In section 4.1, we examine the cardinal accuracy of CTM's forecast. Because CTM is a model based on rating history, we are also keen to explore the ordinal accuracy of the model, and evaluate its efficacy to distinguish between defaulters and non-defaulters. Additionally, we are interested to see the comparison of the discriminatory power between CTM and Moody's ratings.

Moody's Investor Service has been using average default position (ADP) to measure the accuracy of its ordinal rating system. In order to use the ADP to measure the rating performance, we have to create a cohort on a specific date and then measure the performance of that cohort during a period of time after the cohort formation date. The ADP weighs both Type I and Type II errors. Type I errors occur when ratings assigned relatively risky issuers are too high; while Type II errors occur when relatively safe issuers are placed too low in the rank order. Both sources of error cause the ADP to fall. In other words, a high ADP indicates that defaulters are rated low relative to non-defaulters and the rating system is effective in discriminating between defaulters and non-defaulters.

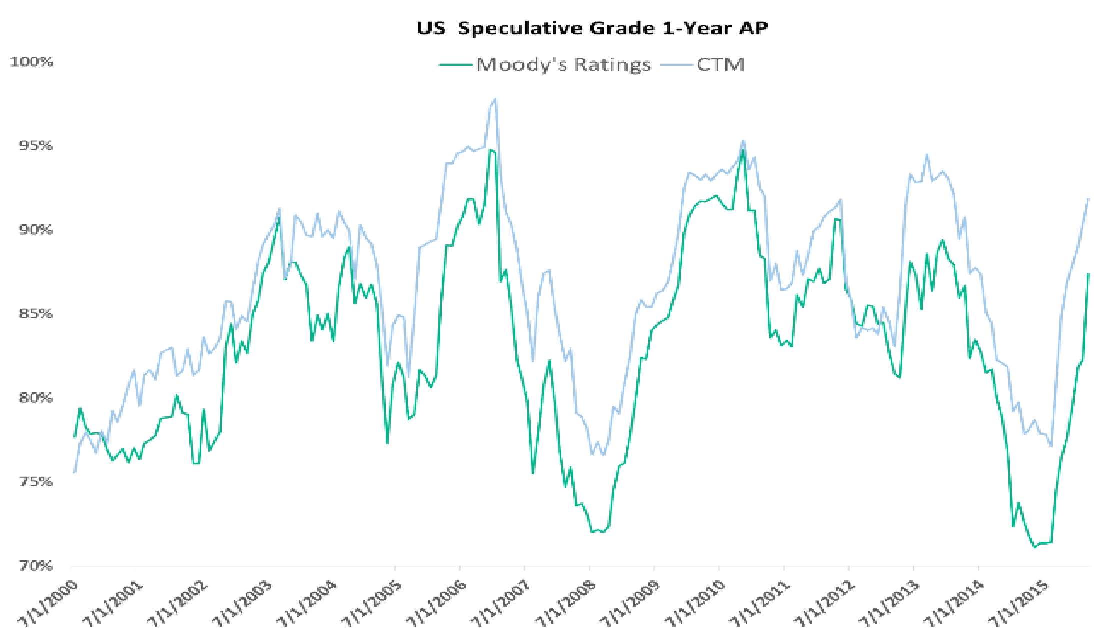
A commonly used technique to measure the accuracy of a default prediction model is the Cumulative Accuracy Profile (CAP). One popular statistic based on the CAP is the Accuracy Ratio (AR). Mathmatically, the accuracy ratio is formally defined as the area under the Cumulative Accuracy Profile (CAP). Typically, the higher the AR, the higher the prediction power of the underlying model. As there is a linear relationship between ADP and AR[25], AR = 2 * ADP – 1, we can compare CTM and Moody's ratings with the same measure, the average default position (ADP).

Table 10 shows the comparison of the 1-Year ADP in CTM and in Moody's ratings for speculative grade credits. The sample period describes the time horizon, during which the issuer cohorts are formed at the beginning of each month. We then take the simple average of the individual cohort's ADPs. Although the ADPs implied by Moody's ratings are already at high levels, CTM is able to add an extra 3-4% to ADP during the years between 2000 and 2016. The improvement comes from incorporating rating history and outlook information, as well as economic assumptions. Figure 13 displays how the ADP moves over time from 2000 to 2016. During the three months between March 2005 and May 2005, there was no defaults for the European issuers, which is why data on the ADP is missing for those months.

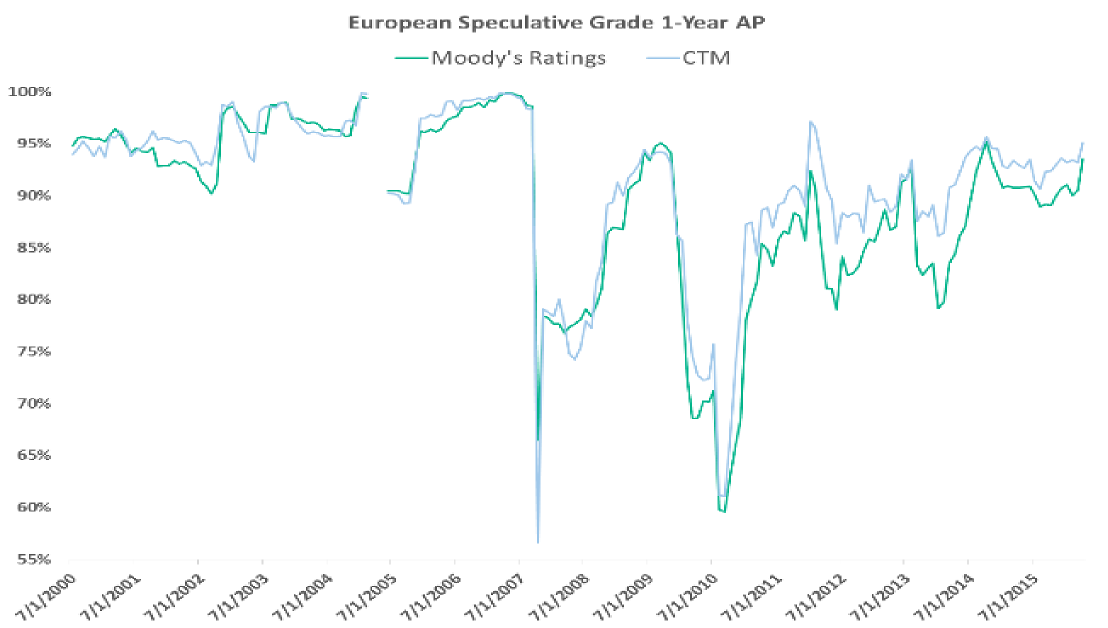
**Table 10 Comparison of 1-Year Average Default Position between CTM and Moody's Ratings for Speculative Grade Credits (%)**

| Sample Period | Region | Moody's Ratings | CTM |
|---|---|---|---|
| 2000.07 - 2016.04 | US | 82.93 | 86.60 |
| 2000.07 - 2007.12 | US | 83.84 | 86.40 |
| 2008.01 - 2016.04 | US | 83.01 | 86.78 |
| 2000.07 - 2016.04 | Europe | 79.47 | 83.06 |
| 2000.07 - 2007.12 | Europe | 82.94 | 83.92 |
| 2008.01 - 2016.04 | Europe | 76.45 | 82.31 |

Figure 13 1-Year Average Default Position , Moody's Ratings vs. CTM




[25] For more details, see "Measuring Rating Accuracy Using the Average Default Position" (2011) by Christopher Mann and Richard Cantor and "Measuring the Discriminative Power of Rating Systems" by Bernd Engelmann , Evelyn Hayden and Dirk Tasche.




## 4.3 Stability

One difference among various credit risk measures is the degree to which each measure has a point-in-time (PIT) or through-the-cycle (TTC) orientation. A point-in-time measure takes all available and pertinent information as of a given date to estimate a firm's expected PD over a certain time horizon. The PIT PDs react to all the news affecting the firm's credit risk, making them not only a timely signal, but also highly volatile and procyclical. High frequency reversals and fluctuations in PD measures are unhelpful for capital management. In comparison, a through-the-cycle measure reflects a firm's long-run, enduring trend of credit risk. TTC risk measures are associated with a high degree of stability over the credit cycle. A typical example of TTC PDs are the ones generated by mapping ratings with historical default data. Such PDs show much less volatility over the cycle with the cost of reduced timeliness and lower forecast accuracy for defaults.

On one hand, the PD projections generated by the CTM incorporate cyclical information from the macroeconomy and the financial market to capture the credit risk trend in a timely fashion. Backtesting results in Section 4.1 show that the CTM projections are able to catch the sharply rising trend of PD during recessions. One the other hand, as the CTM relies on ratings and rating histories, which are based on fundamental credit analysis, the PD projections have displayed a high degree of smoothness, compared to a typical point-in-time measure. Table 11 compares the 1-Year probability of maintaining current ratings between the CTM output based on perfect economic foresight and Moody's ratings for the US issuers during 1987-2016, and for the European issuers during 2000-2016.

Table 11 **Probability of Maintaining Current Ratings over a 1-Year Horizon**[26]

| | US | | Europe | |
|---|---|---|---|---|
| Rating Catogory | Moody's Rating | CTM | Moody's Rating | CTM |
| Aaa | 88.4% | 84.6% | 85.4% | 77.6% |
| Aa1 | 77.9% | 78.6% | 74.3% | 73.3% |
| Aa2 | 75.1% | 76.8% | 67.4% | 68.5% |
| Aa3 | 76.1% | 77.0% | 71.6% | 71.7% |
| A1 | 76.6% | 77.4% | 71.8% | 73.3% |
| A2 | 77.5% | 77.7% | 71.0% | 72.3% |
| A3 | 75.7% | 75.0% | 72.7% | 72.7% |
| Baa1 | 74.8% | 75.0% | 70.9% | 72.1% |
| Baa2 | 75.8% | 75.8% | 70.2% | 71.6% |
| Baa3 | 72.7% | 73.4% | 65.8% | 67.5% |
| Ba1 | 65.2% | 65.8% | 61.4% | 64.8% |
| Ba2 | 63.6% | 62.0% | 60.5% | 59.7% |
| Ba3 | 63.4% | 64.0% | 62.0% | 61.7% |
| B1 | 63.6% | 63.4% | 61.0% | 59.6% |
| B2 | 63.1% | 62.9% | 57.8% | 59.1% |
| B3 | 59.7% | 60.6% | 61.9% | 61.3% |
| Caa1 | 57.3% | 58.8% | 54.2% | 55.2% |
| Caa2 | 51.2% | 53.8% | 49.5% | 48.7% |
| Caa3 | 44.7% | 44.2% | 31.4% | 41.5% |
| Ca | 39.1% | 34.8% | 33.7% | 34.2% |
| C | 50.6% | 32.9% | 48.1% | 34.1% |

## 4.4 Scenario Forecasts

As the CTM conditions on the future path of the economy, performing sensitivity analysis and scenario testing is straightforward. Model users are usually interested in understanding the macroeconomic sensitivity of the default forecast. In the figures below, we display the 12-month default rate forecast and examine the paths during a five-year horizon under eight different economic scenarios, which are listed in Table 8. Appendix III are provided for users who need to further examine the forecast paths of the macroeconomic inputs under each of the scenarios. As shown in Figure 14, The US speculative grade 12-Month Default Rate rises to its peak at 13.9% from 4.4% under the 2017 Fed Severely Adverse Scenario. As a comparison, the European 12-Month Default Rate rises to its peak at 9.5% (March 2017 cohort) from 1.8% (Feb 2016 cohort) under the 2017 Fed Severely Adverse Scenario. By specifying different economic assumptions, financial institutions are able to understand better the risk profile of their loan and bond portfolio in different economic situations. The PDs in the stressed cases are also important inputs for regulatory stress testing.

[26] The stability calculation for the US model is based on CTM outputs between 1987 and 2016; the stability calculation for the European model is based on CTM projections between 2000 and 2016. On the mathematical front, it should be not be read as the probabilities of no rating change over a 1-Year horizon, because ratings may move up and down during the specified horizon, and return to the original one assigned at the beginning of the horizon.

Figure 14 US Speculative Grade 12-Month Default Rate Under Different Economic Scenarios

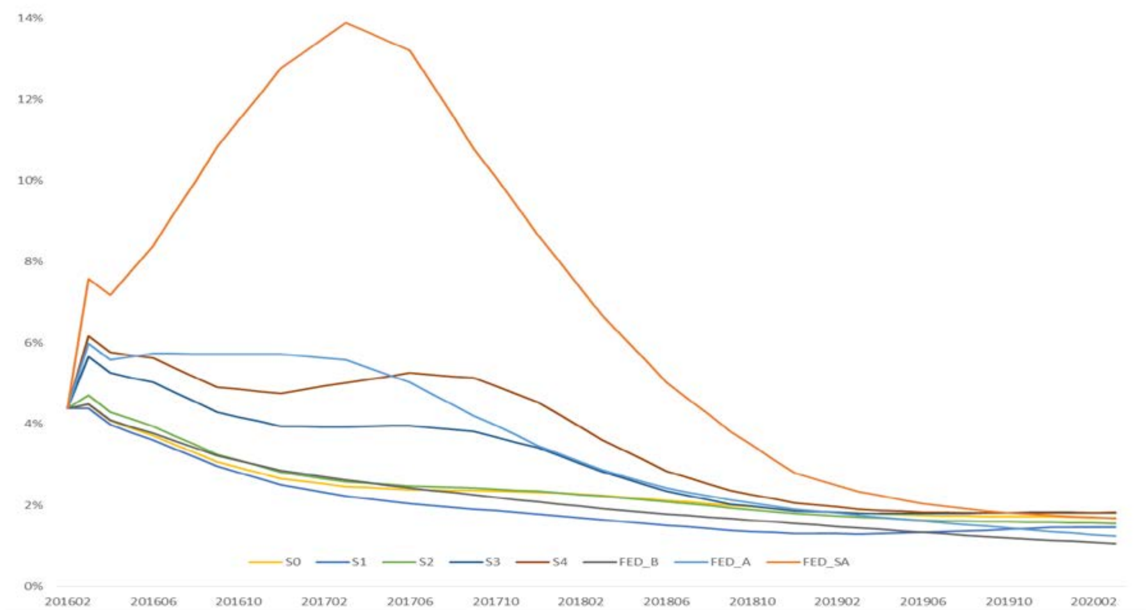
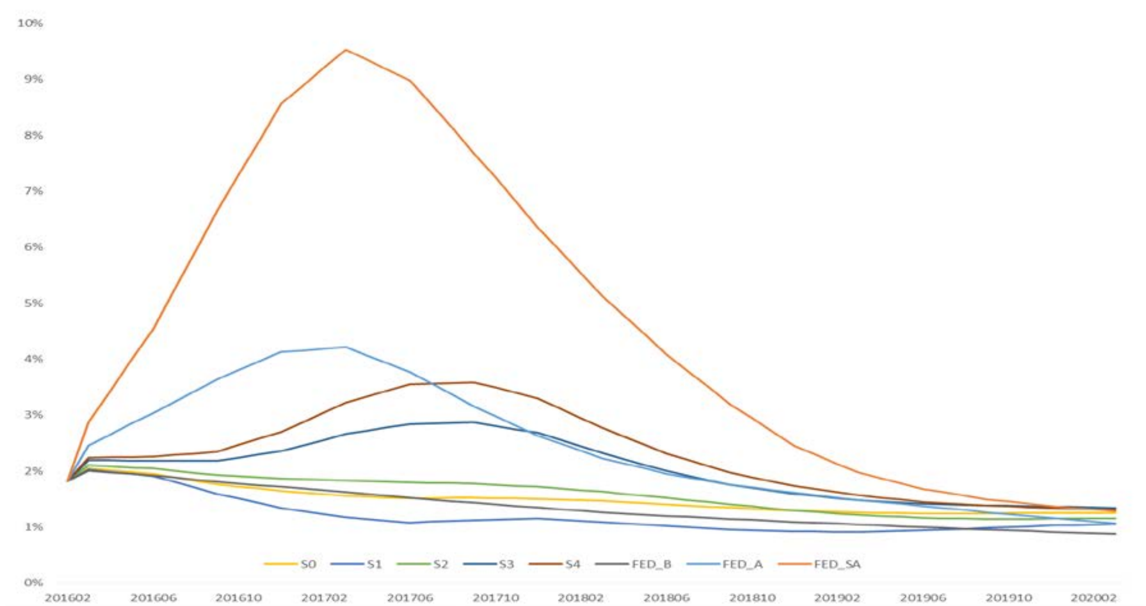



Figure 15 European Speculative Grade 12-Month Default Rate Under Different Economic Scenarios




## 5 Conclusion

In this model methodology paper, we have presented the theory and mechanics behind the Credit Transition Model (CTM). In short, it is an issuer-level model of rating transition probabilities conditioning on issuer characteristics, rating history and the future paths of the unemployment rate and high yield spread. The model belongs to the multi-destination proportional hazards type. With a chain of five similarly structured models, the forecasts can be extended to any time horizon. Since the model conditions on the expected path of the economy, performing scenario analysis and stress testing is straightforward. In practice, the performance of CTM depends in part on the quality of the economic forecasts on which it conditions.

One of CTM's most important outputs is the probability of default, which is a key concept in the credit risk management framework. Such metrics can be used by lenders for evaluation of borrowers; by corporations for assessment of their business partners' creditworthiness; by asset managers for investment screening and by risk practitioners for credit risk surveillance. Importantly, the probability of default is an essential input to calculate expected loss and economic capital. This concept is also crucial in the calculation of capital requirements under the Basel framework. Therefore, the Credit Transition Model can be widely utilized in the area of credit risk analytics and investment management.

There are remaining important avenues of research to be explored. Further work might include introducing oil price to the list of macro drivers, or financial data as part of the issuer-specific information.

## Appendix I Definition of Default by Moody's Investors Service

For events constitute a debt default under Moody's Investor Service definition:

1. A missed or delayed disbursement of a contractually-obligated interest or principal payment (excluding missed payments cured within a contractually allowed grace period), as defined in credit agreements and indentures;

2. A bankruptcy filing or legal receivership by the debt issuer or obligor that will likely cause a miss or delay in future contractually-obligated debt service payments;

3. A distressed exchange whereby

a) an issuer offers creditors a new or restructured debt, or a new package of securities, cash or assets, that amount to a diminished value relative to the debt obligation's original promise and

b) the exchange has the effect of allowing the issuer to avoid a likely eventual default;

4. A change in the payment terms of a credit agreement or indenture imposed by the sovereign that results in a diminished financial obligation, such as a forced currency re-denomination (imposed by the debtor, or the debtor's sovereign) or a forced change in some other aspect of the original promise, such as indexation or maturity.

Distressed exchanges is included in the definition of default in order to capture credit events whereby issuers effectively fail to meet their debt service obligations but do not actually file for bankruptcy or miss an interest or principal payment. Moody's employs fundamental analysis in assessing the likelihood of future default and considers various indicators in assessing loss relative to the original promise, which may include the yield to maturity of the debt being exchanged.

Moody's definition of default does not include so-called "technical defaults," such as maximum leverage or minimum debt coverage violations, unless the obligor fails to cure the violation and fails to honor the resulting debt acceleration which may be required. Also excluded are payments owed on long-term debt obligations which are missed due to purely technical or administrative errors which are 1) not related to the ability or willingness to make the payments and 2) are cured in very short order (typically, 1-2 business days). Finally, in select instances based on the facts and circumstances, missed payments on financial contracts or claims may be excluded if they are the result of legal disputes regarding the validity of those claims.

## Appendix II Mathematical Derivation in Section 3.1

$$Pr(S = s, a < T \leq a + \Delta t | r, x_t) = \int_a^{a+\Delta t} f_s^r(u|x_t) du$$

$$= Pr(S = s | a < T \leq a + \Delta t, r, x_t) \cdot Pr(a < T \leq a + \Delta t | r, x_t)$$

$$= \frac{h_s^r(T|x_t)}{\sum_{i=1}^S h_i^r(T|x_t)} \cdot \frac{F(a + \Delta t | x_t, r) - F(a|x_t, r)}{\bar{F}(a|x_t, r)} \cdot \bar{F}(a|x_t, r)$$

$$= \frac{h_s^r(T|x_t)}{\sum_{i=1}^S h_i^r(T|x_t)} \cdot \frac{(1 - \bar{F}(a + \Delta t | x_t, r) - (1 - \bar{F}(a|x_t, r))}{\bar{F}(a|x_t, r)} \cdot \bar{F}(a|x_t, r)$$

$$= \frac{h_s^r(T|x_t)}{\sum_{i=1}^S h_i^r(T|x_t)} \cdot (1 - \frac{\bar{F}(a + \Delta t | x_t, r)}{\bar{F}(a|x_t, r)}) \cdot \bar{F}(a|x_t, r)$$

$$= \frac{h_s^r(T|x_t)}{\sum_{i=1}^S h_i^r(T|x_t)} \cdot (1 - \frac{exp(-\sum_{i=1}^S \int_0^{a+\Delta t} h_i^r(u|x_t) du)}{exp(-\sum_{i=1}^S \int_0^a h_i^r(u|x_t) du)}) \cdot \bar{F}(a|x_t, r)$$

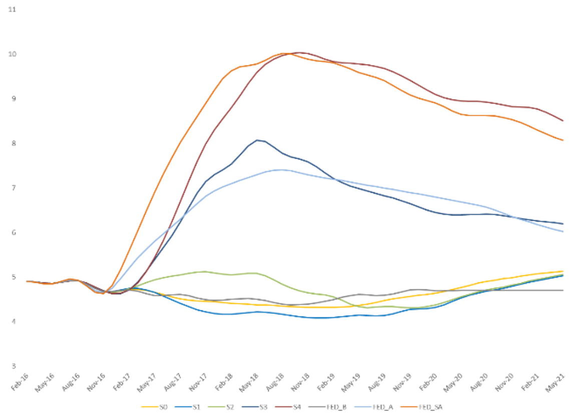
$$= \frac{h_s^r(T|x_t)}{\sum_{i=1}^S h_i^r(T|x_t)} \cdot (1 - ex\,p\left(-\sum_{i=1}^S \int_a^{a+\Delta t} h_i^r(u|x_t) du\right)) \cdot \bar{F}(a|x_t, r)$$

This can be further simplified to:

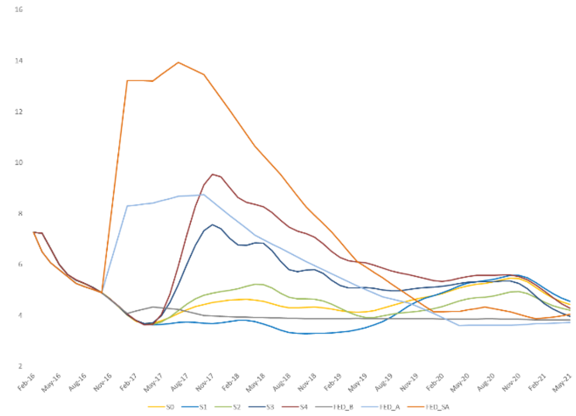
$$= \frac{h_s^r(a|x_t)}{\sum_{i=1}^S h_i^r(a|x_t)} \cdot (1 - ex\,p\left(-\sum_{i=1}^S h_i^r(a|x_t)\right)) \cdot \bar{F}(a|x_t, r)$$

## Appendix III Assumptions for Economic Scenarios

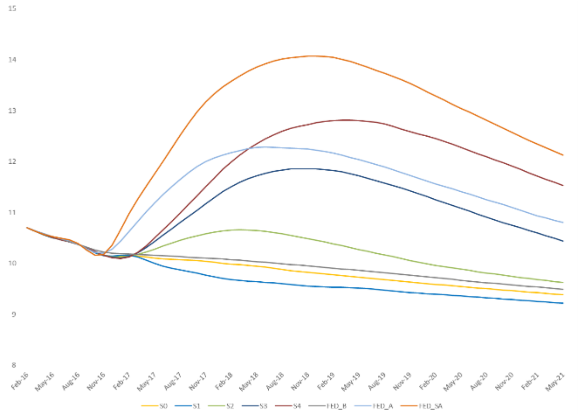
US Unemployment Rate under Different Scenarios (%)



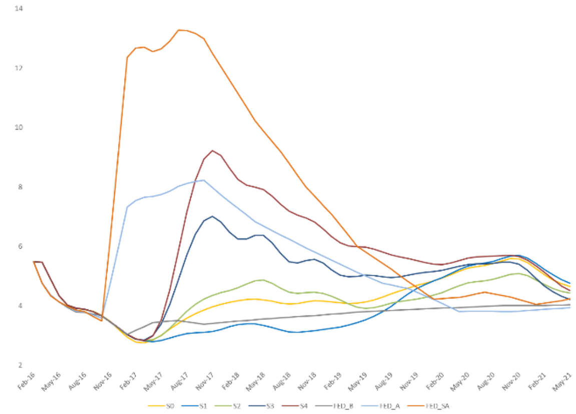

US High Yield Spreads under Different Scenarios (%)




Europe Unemployment Rate under Different Scenarios (%)




Europe High Yield Spreads under Different Scenarios (%)



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