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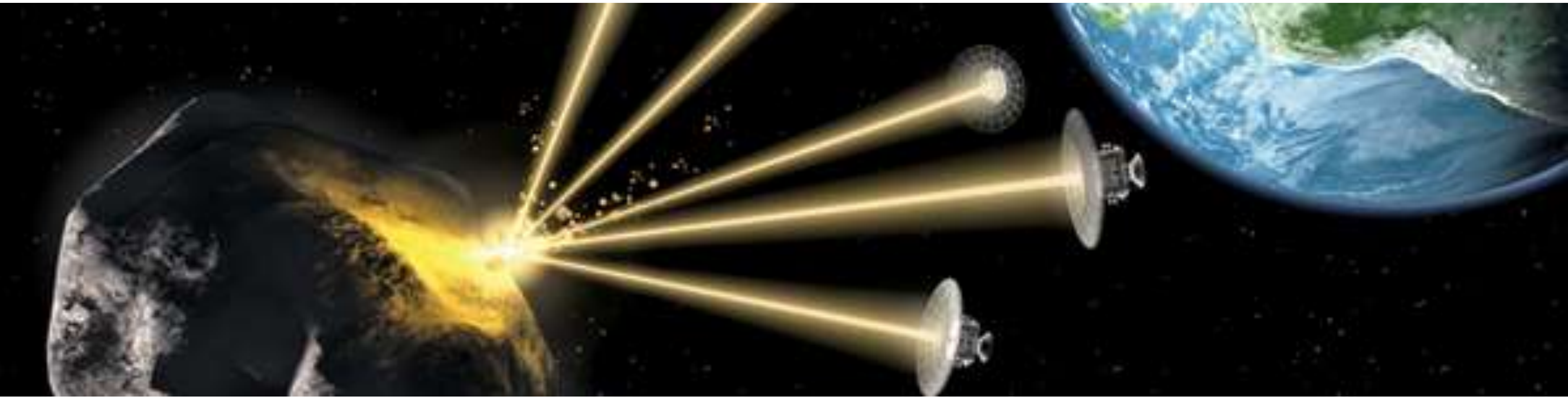


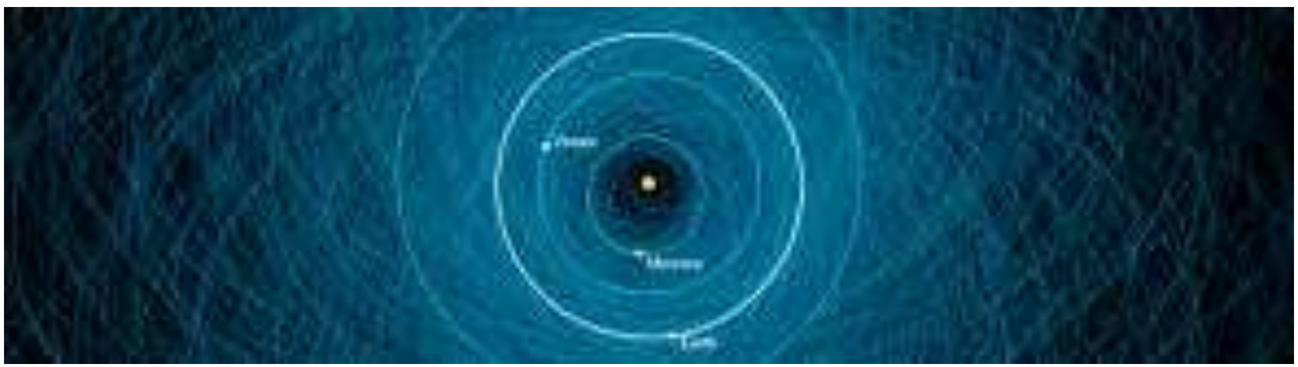
Methods and Techniques for Asteroid Deflection

Massimiliano Vasile, Mechanical & Aerospace Department

University of Strathclyde

Outline

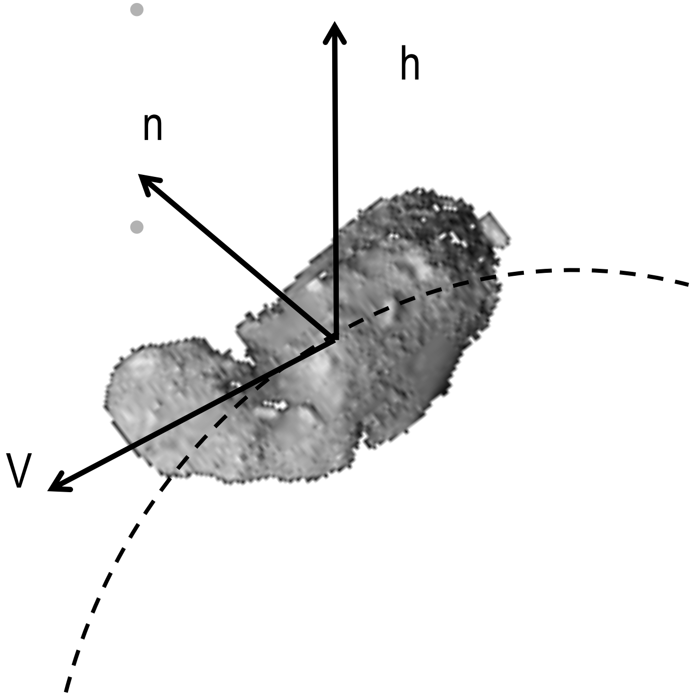
- **Part 1**
  - Basic Deflection Principles and Computational Tools
  - Analytical Propagation of Low-Thrust Motion
  - Trajectory Modelling
- **Part 2**
  - Deflection Technologies
  - Momentum Coupling and System Mass Consideration
  - Uncertainty Quantification



PART 1

In Which Way Can We Deflect an Asteroid?

We can change the orbit of the asteroid in many different ways depending on the direction in which we push.



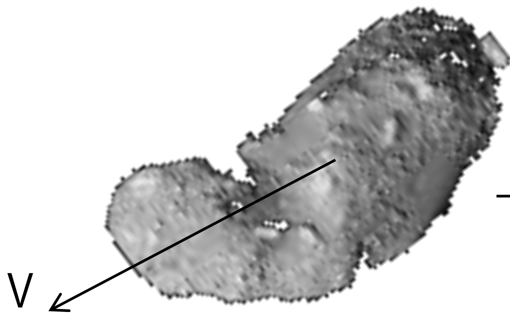
V – increase or decrease the velocity of the asteroid

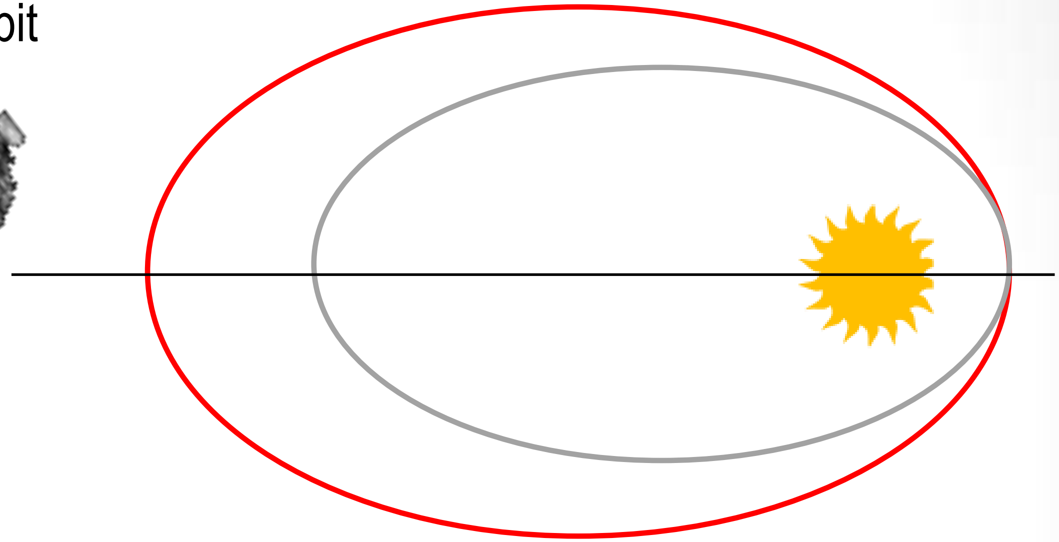
n – push in the direction perpendicular to the velocity in the orbit plane

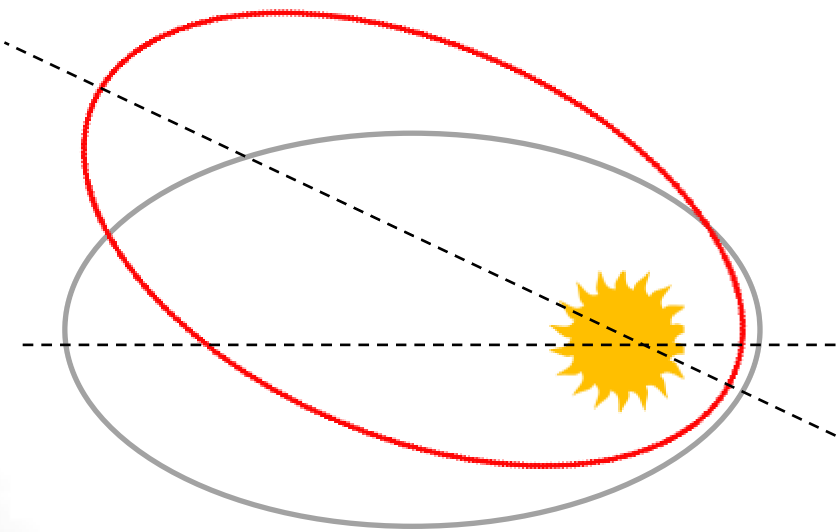
h – push in the direction perpendicular to the velocity and to the orbit plane

In Which Way Can We Deflect an Asteroid?

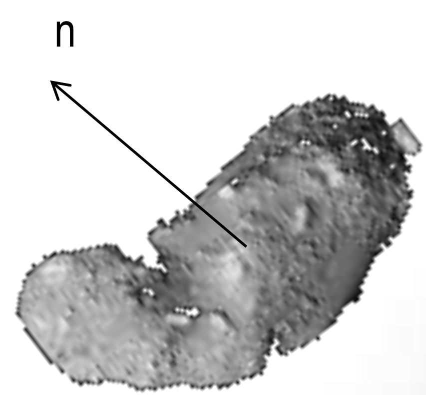
Change the energy of the orbit







Rotate the orbit in its plane



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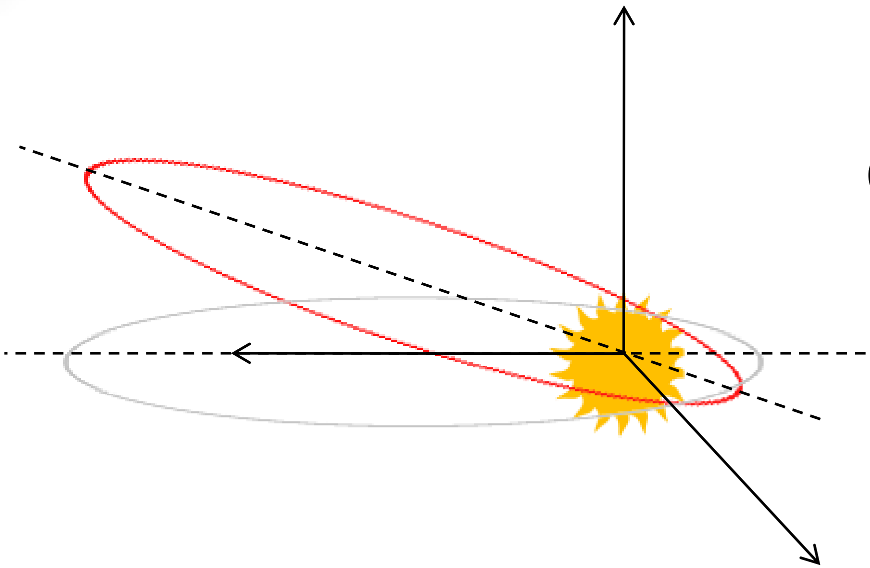
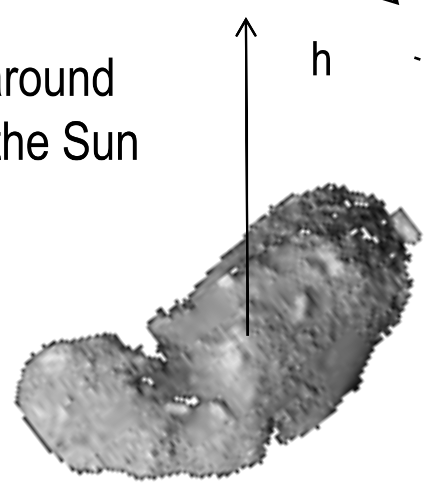
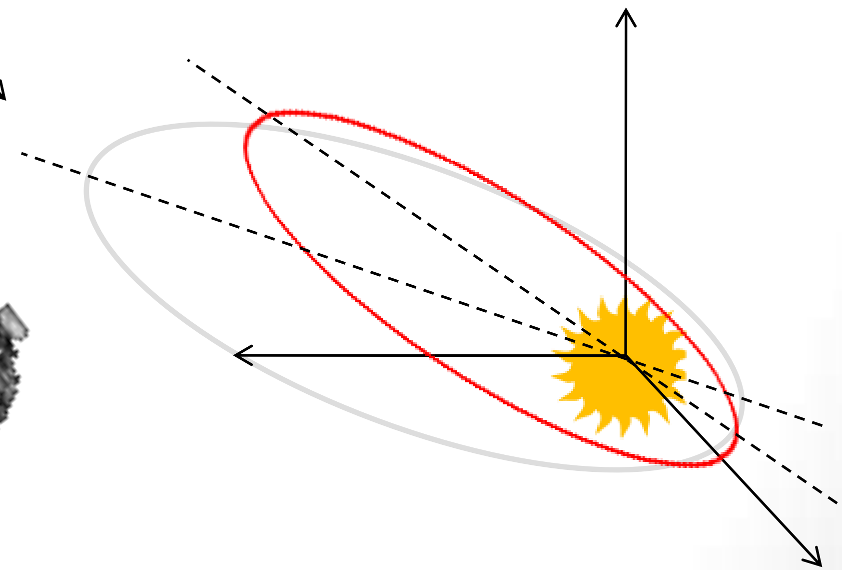
In Which Way Can We Deflect an Asteroid?



Change the inclination of the orbit

Rotate the orbit around
the polar axis of the Sun

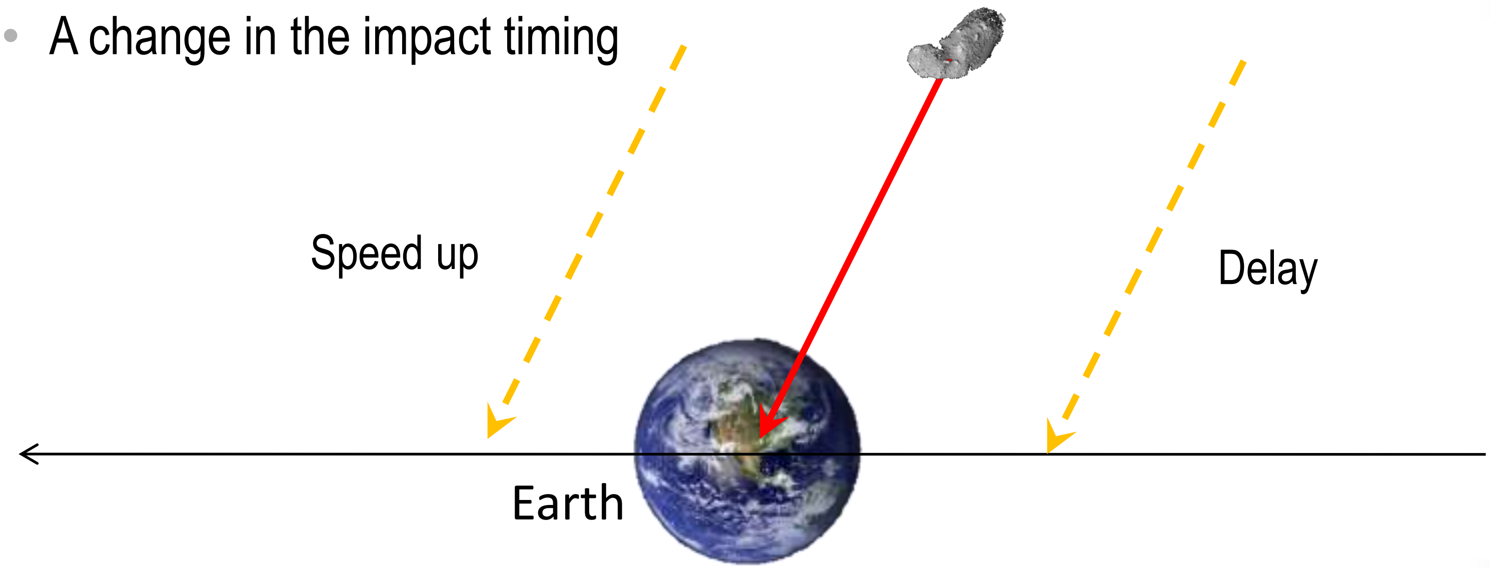




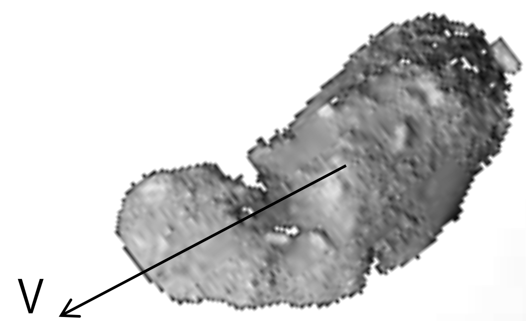
Vasile, Colombo 2008

What happens when we deflect?

- A deflection action generally has two consequences :
  - A change in the geometry /shape of the orbit
  - A change in the impact timing

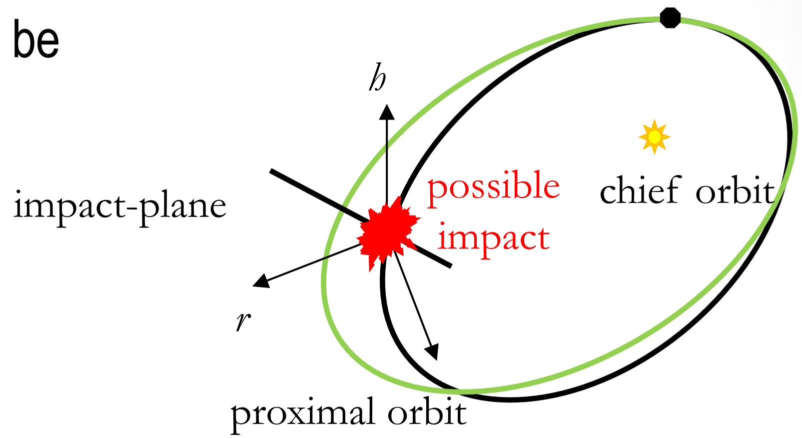


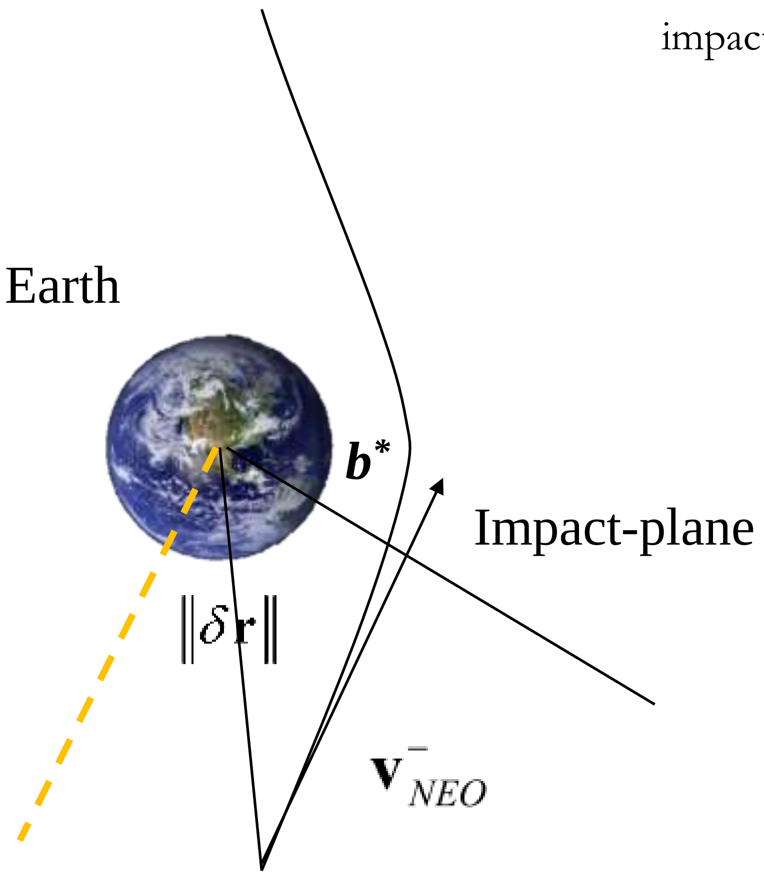
- For example if we push in the direction of the velocity of the asteroid we produce a delay in the arrival at the impact point.



What happens when we deflect?

The deflected orbit of the asteroid will be proximal to the undeflected one.





- We normally measure the deflection on a so–called impact plane on which we define the impact parameter b^*.
- When b^* is smaller than the radius of the Earth we have an impact.

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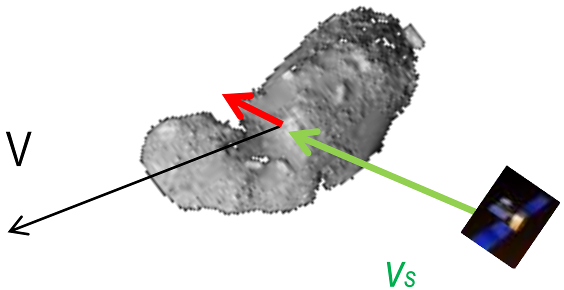
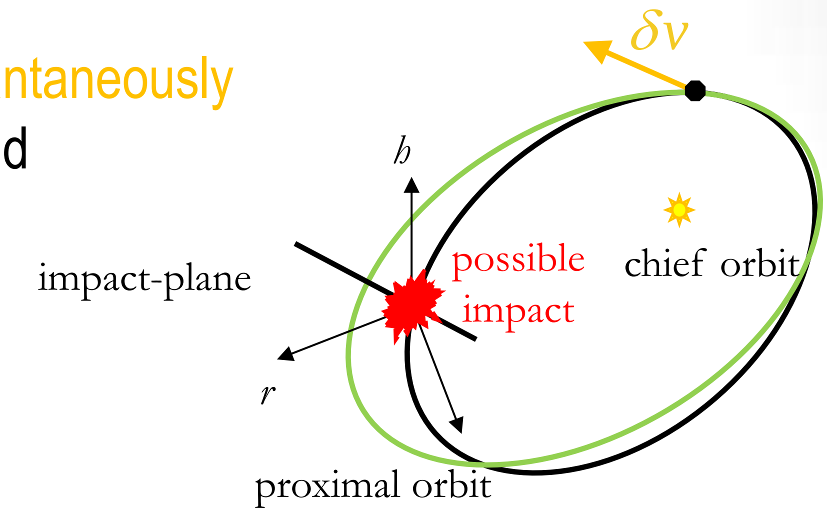
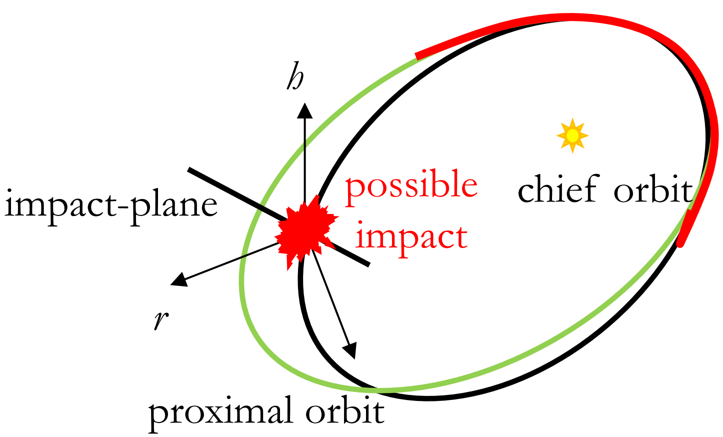
Types of Deflection Actions

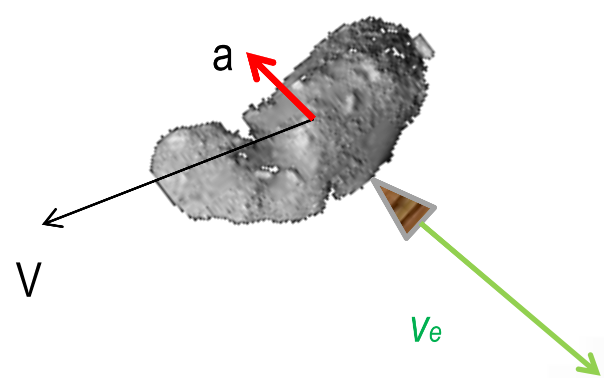
Impulsive deflection actions instantaneously change the velocity of the asteroid







Slow push deflections act along extended arcs, for months or years.



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Computing the Deflection- Linear Model

Proximal position equations

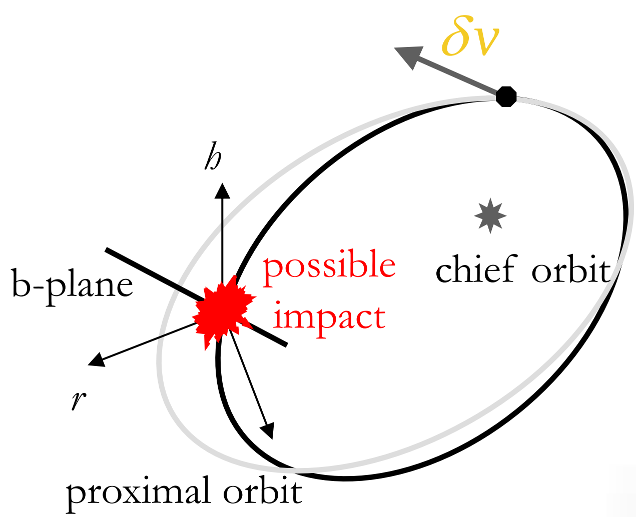
$$\eta = \sqrt{1-e^2}$$

$$\delta s_r \approx \frac{r}{a}\delta a + \frac{ae\sin f}{\eta}\delta M - a\cos f\,\delta e$$

$$\delta s_\vartheta \approx \frac{r}{\eta^3}(1+e\cos f)^2\,\delta M + r\delta\omega + \frac{r\sin f}{\eta^2}(2+e\cos f)\delta e + r\cos i\,\delta\Omega$$

$$\delta s_h \approx r(\sin\vartheta\,\delta i - \cos\vartheta\sin i\,\delta\Omega)$$

The deflection manoeuvre generates a change in the orbital parameters of the asteroid.



The achieved deviation $\delta\mathbf{r}=[\delta s_r\ \delta s_q\ \delta s_h]^T$ is computed at the expected Minimum Orbit Interception Distance (MOID).

Vasile and Colombo 2008

Computing the Deflection- Impulsive Deflection

- Gauss' equations

$$\delta a = \frac{2a^2}{h}\left(e \sin f \delta v_r + \frac{p}{r}\delta v_\vartheta\right)$$

$$\delta e = \frac{1}{h}\left\{p \sin f \delta v_r + \left[(p+r)\cos f + re\right]\delta v_\vartheta\right\}$$

$$\delta i = \frac{r \cos f}{h \sin i}\delta v_h$$

$$\delta\Omega = \frac{r \sin \vartheta}{h \sin i}\delta v_h$$

$$\delta\omega = \frac{1}{he}\left[-p \cos f \delta v_r + (p+r)\sin f \delta v_\vartheta\right] - \frac{r \sin\vartheta \cos i}{h \sin i}\delta v_h$$

- Variation in mean anomaly

$$\delta M = \overbrace{\frac{b}{ahe}\left[(p\cos f - 2re)\delta v_r - (p+r)\sin f \delta v_\vartheta\right]}^{\text{through Gauss' equations}} + \overbrace{\Delta n\left(t_{MOID} - t_d\right)}^{\text{due to a variation in } a}$$

Vasile and Colombo 2008

Computing the Deflection- Slow-Push Deflection

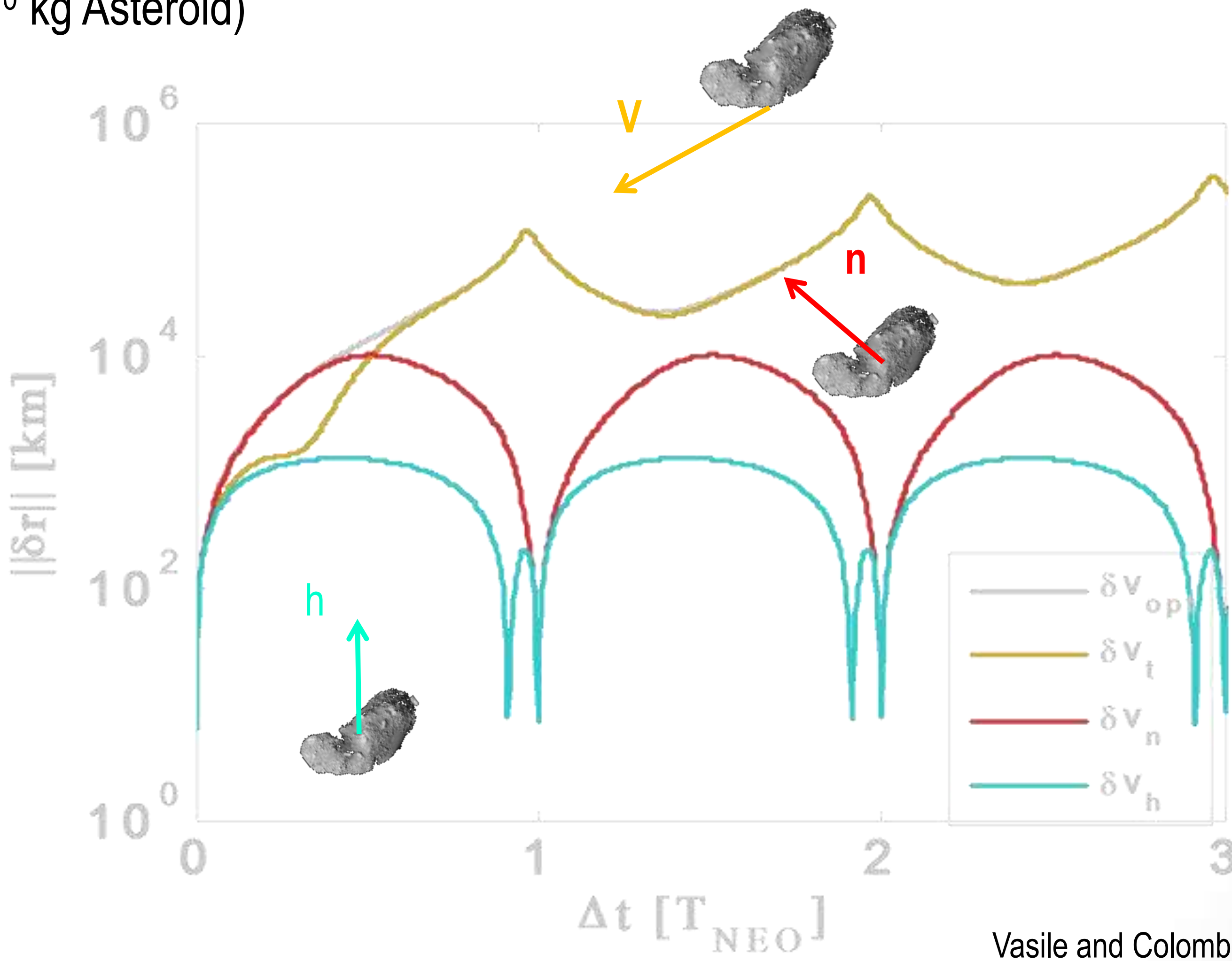
- The variation in mean anomaly is modified to take into account the thrust arc:

$$\delta M = \tilde{M}_{MOID} - M_{MOID} = \underbrace{(n_e - n_i)t_{MOID} + n_i t_i - n_e t_e}_{\text{due to a variation in } a} + \underbrace{\Delta M}_{\text{through Gauss' equations}}$$

Vasile and Colombo 2009

What is the most efficient way of deflecting?

Deviation achieved with δv=0.07 m/s for 1979XB (60mt impactor, 30km/s impact, $2.7 \cdot 10^{10}$ kg Asteroid)



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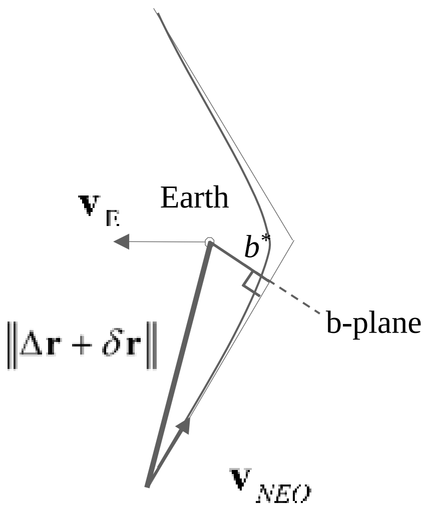
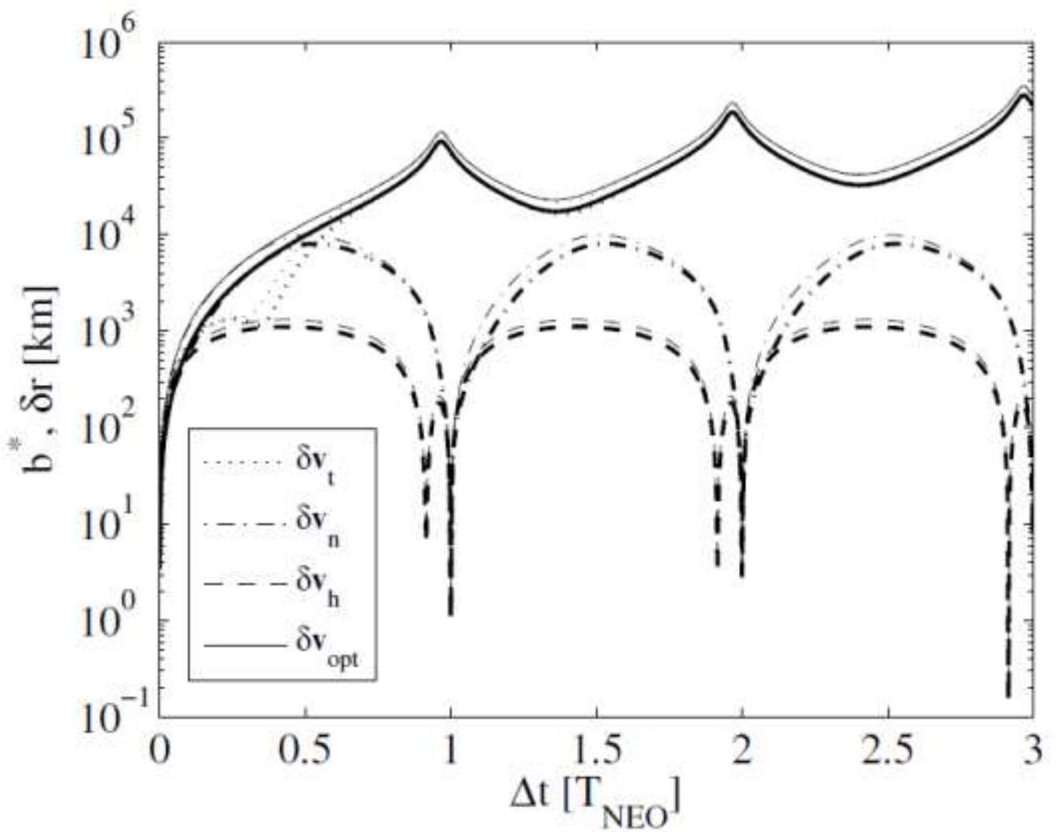
Deflection and minimum interception distance



B-Plane Analysis of the Deflection



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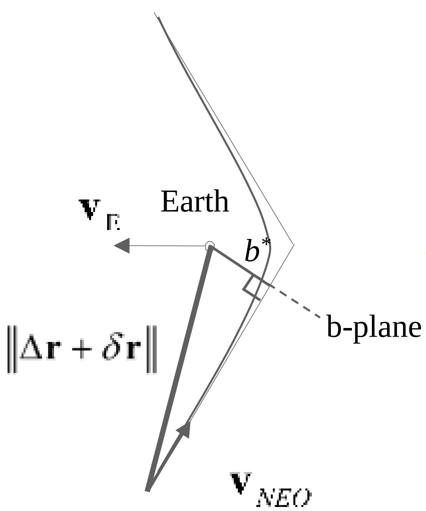
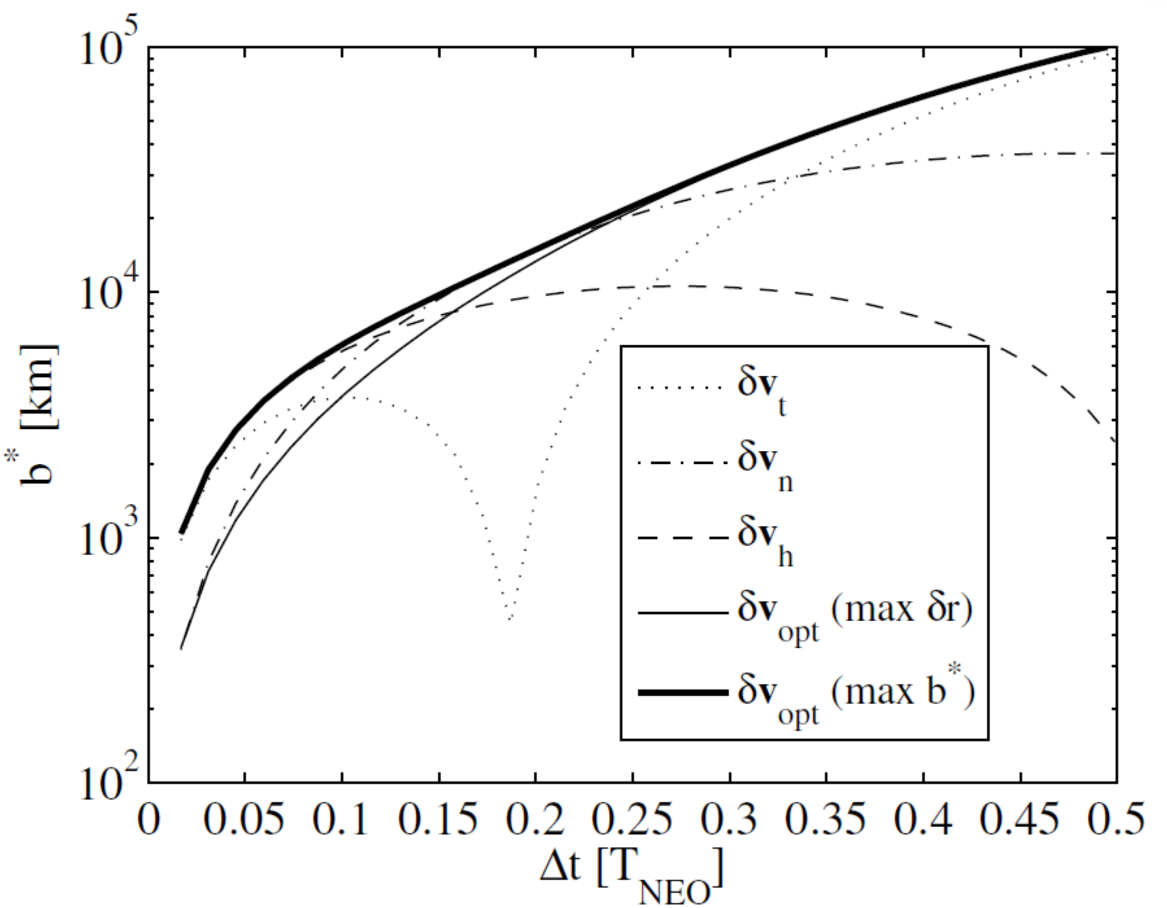
Does the manoeuvre change if we maximise b?





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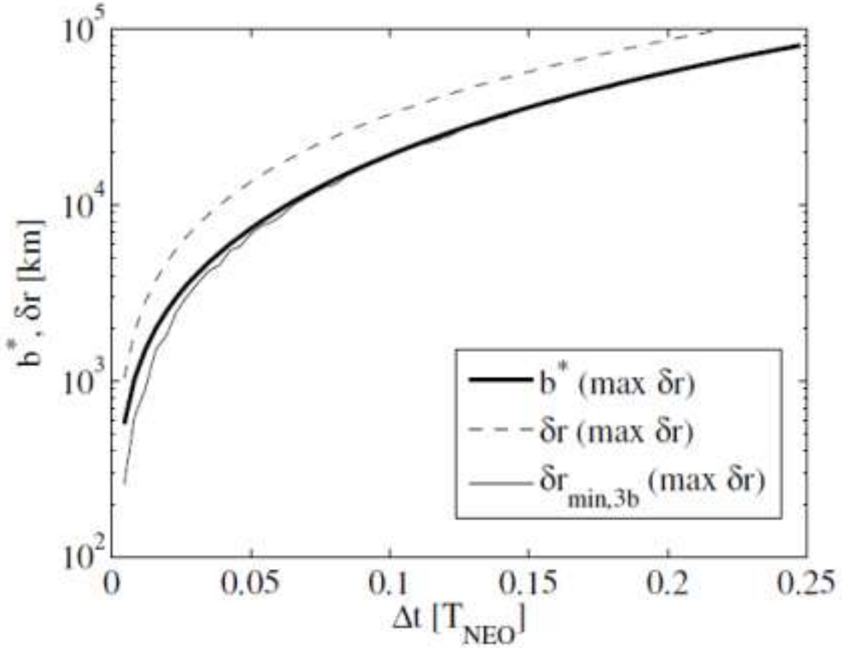
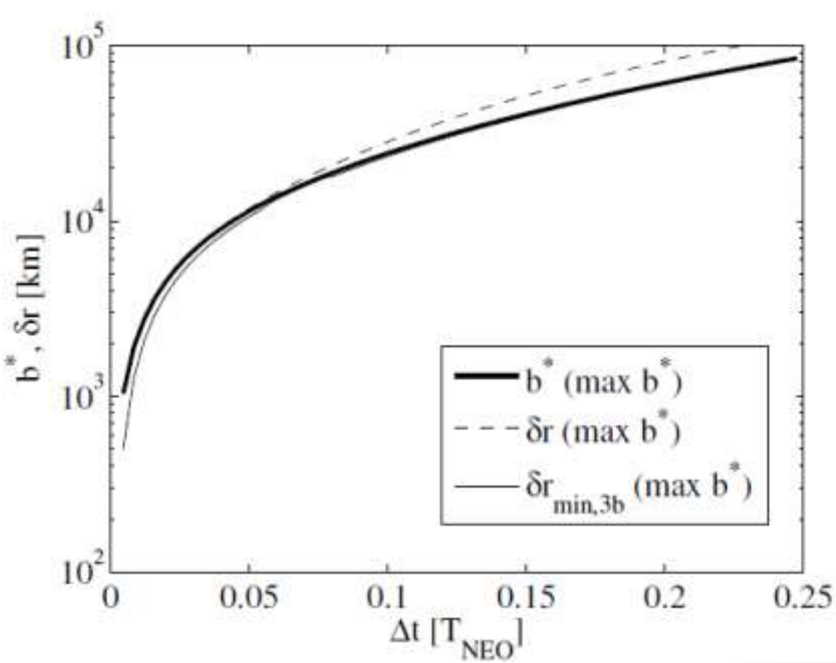
What happens if we consider 3rd body effects?

Comparison between linear and non-linear model





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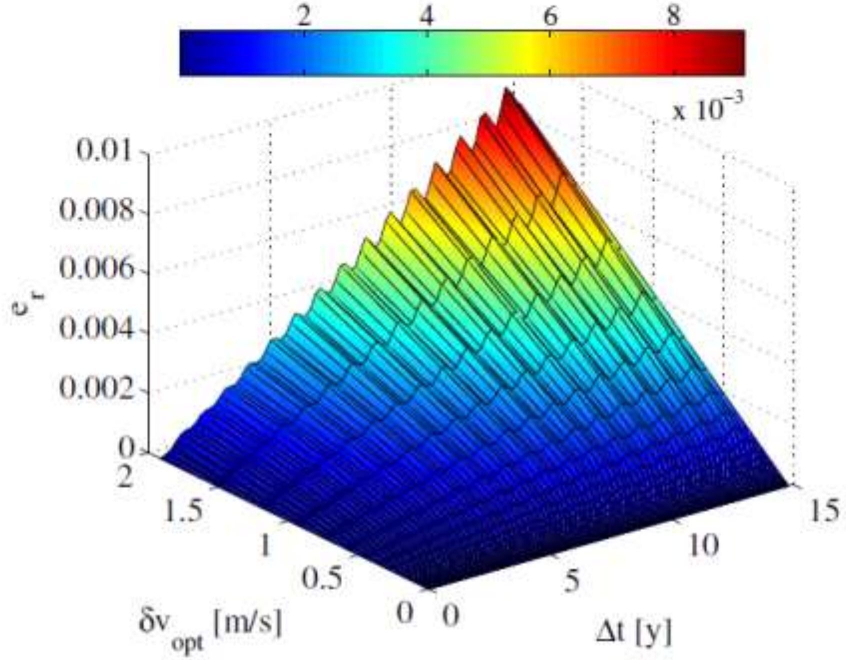
Computing the Deflection- Linear Model

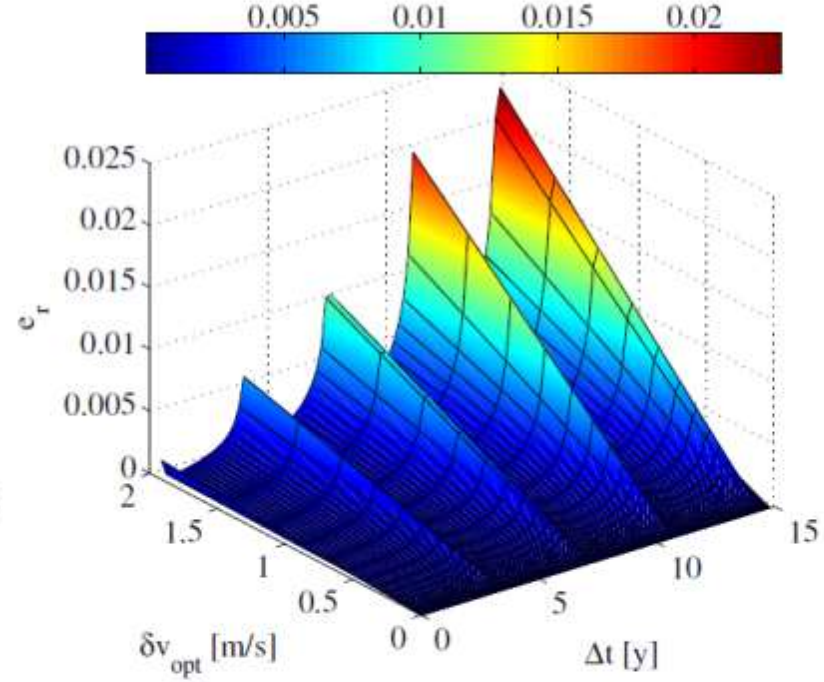
- Verification of the analytical expression

$e < 0.1$ $i < 10°$	$e_r = \dfrac{\left\| \delta r_{propagated} - \delta r_{estimated} \right\|}{\left\| \delta r_{propagated} \right\|}$	$e > 0.1$ $i > 10°$

2000SG344 deviation



1979XB deviation



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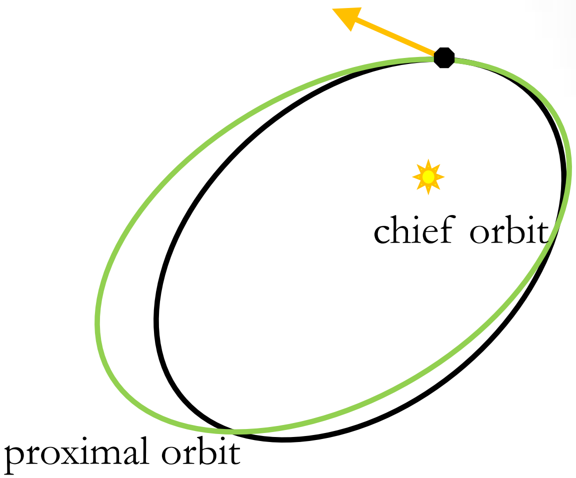
Nonlinear Proximity Position Equations

- Geometric difference between two orbits

$$\Delta \mathbf{r} = r_{A_{dev}} \Psi - \begin{bmatrix} r_{A_0} \\ 0 \\ 0 \end{bmatrix}$$



- Expressed in terms of difference of orbital elements

$$\Psi = \begin{bmatrix} \xi \cos(\theta_{A_0} + \Delta\theta) + \sin(\theta_{A_0} + \Delta\theta)\left(\rho \sin\theta_{A_0} - \cos(i_{A_0} + \Delta i)\sin\Delta\Omega\cos\theta_{A_0}\right) \\ -\zeta \cos(\theta_{A_0} + \Delta\theta) + \sin(\theta_{A_0} + \Delta\theta)\left(\rho \sin\theta_{A_0} - \cos(i_{A_0} + \Delta i)\sin\Delta\Omega\sin\theta_{A_0}\right) \\ -\cos(\theta_{A_0} + \Delta\theta)\sin\Delta\Omega\sin i_{A_0} + \varpi \sin(\theta_{A_0} + \Delta\theta) \end{bmatrix}$$

Vasile and Maddock 2010

Nonlinear Proximity Motion Equations

- With the additional terms

$$\varpi = \cos i_{A_0} \sin\left(i_{A_0} + \Delta i\right) - \cos\Delta\Omega \cos\left(i_{A_0} + \Delta i\right) \sin i_{A_0}$$

$$\rho = \sin i_{A_0} \sin\left(i_{A_0} + \Delta i\right) + \cos\Delta\Omega \cos\left(i_{A_0} + \Delta i\right) \cos i_{A_0}$$

$$\xi = \cos\Delta\Omega \cos\theta_{A_0} + \cos i_{A_0} \sin\Delta\Omega \sin\theta_{A_0}$$

- If the Delta elements are small enough then $\Delta \rightarrow \delta$ and if one retains only the first order terms then the nonlinear equations becomes the linear equations.

Vasile and Maddock 2010

A Perturbation Theory Approach

- Gauss planetary equations in Equinoctial non-singular elements:

$$\frac{da}{dt}=\frac{2}{B}\sqrt{\frac{a^3}{\mu}}\left[\left(P_2\sin L-P_1\cos L\right)a_r+\Phi(L)a_\theta\right]$$

$$\frac{dP_1}{dt}=B\sqrt{\frac{a}{\mu}}\left[-a_r\cos L+\left(\frac{P_1+\sin L}{\Phi(L)}+\sin L\right)a_\theta-P_2\frac{Q_1\cos L-Q_2\sin L}{\Phi(L)}a_h\right]$$

$$\frac{dP_2}{dt}=B\sqrt{\frac{a}{\mu}}\left[a_r\sin L+\left(\frac{P_2+\cos L}{\Phi(L)}+\cos L\right)a_\theta+P_1\frac{Q_1\cos L-Q_2\sin L}{\Phi(L)}a_h\right]$$

$$\frac{dQ_1}{dt}=\frac{B}{2}\sqrt{\frac{a}{\mu}}\left(1+Q_1^2+Q_2^2\right)\frac{\sin L}{\Phi(L)}a_h$$

$$\frac{dQ_2}{dt}=\frac{B}{2}\sqrt{\frac{a}{\mu}}\left(1+Q_1^2+Q_2^2\right)\frac{\cos L}{\Phi(L)}a_h$$

$$a$$

$$P_1=e\sin(\Omega+\omega)$$

$$P_2=e\cos(\Omega+\omega)$$

$$Q_1=\tan\frac{i}{2}\sin\Omega$$

$$Q_2=\tan\frac{i}{2}\cos\Omega$$

$$L=(\Omega+\omega)+\vartheta$$

A Perturbation Theory Approach

- Under the hypothesis of small, constant, perturbing acceleration:

$$\frac{dt}{dL} \approx \frac{r^2}{h} = \sqrt{\frac{a^3}{\mu}} \frac{B^3}{\Phi^2(L)}$$

with Gauss equations:

$$\frac{da}{dL} = \frac{2a^3B^2}{\mu}\left[\frac{(P_2 \sin L - P_1 \cos L)}{\Phi^2(L)} a_r + \frac{1}{\Phi(L)} a_\theta\right]$$

$$\frac{dP_1}{dL} = \frac{B^4a^2}{\mu}\left[-a_r \frac{\cos L}{\Phi^2(L)} + \left(\frac{P_1 + \sin L}{\Phi^3(L)} + \frac{\sin L}{\Phi^2(L)}\right) a_\theta - P_2 \frac{Q_1 \cos L - Q_2 \sin L}{\Phi^3(L)} a_h\right]$$

$$\frac{dP_2}{dL} = \frac{B^4a^2}{\mu}\left[\frac{\sin L}{\Phi^2(L)} a_r + \left(\frac{P_2 + \cos L}{\Phi^3(L)} + \frac{\cos L}{\Phi^2(L)}\right) a_\theta + P_1 \frac{Q_1 \cos L - Q_2 \sin L}{\Phi^3(L)} a_h\right]$$

$$\frac{dQ_1}{dL} = \frac{B^4a^2}{2\mu}\left(1 + Q_1^2 + Q_2^2\right) \frac{\sin L}{\Phi^3(L)} a_h$$

$$\frac{dQ_2}{dL} = \frac{B^4a^2}{2\mu}\left(1 + Q_1^2 + Q_2^2\right) \frac{\cos L}{\Phi^3(L)} a_h$$

A Perturbation Theory Approach

- Let's assume that the control acceleration can be expressed as:

$$
\begin{aligned}
a_r &= \varepsilon \cos\alpha \cos\beta \\
a_t &= \varepsilon \sin\alpha \cos\beta \\
a_h &= \varepsilon \sin\beta
\end{aligned}
$$

- Where ε is a small number compared to the local gravity field and the two angles α and β are with respect to a local *r-t-h* reference frame.
- Let's further assume that ε is constant along a thrust arc.

A Perturbation Theory Approach

- Following Perturbation Theory, one can express each non-singular element and the time as an expansion in the small parameter ε:

$$a = a_0 + \varepsilon a_1 + \varepsilon^2 a_2 + \varepsilon^3 a_3 + \ldots$$
$$P_1 = P_{10} + \varepsilon P_{11} + \varepsilon^2 P_{12} + \varepsilon^3 P_{13} + \ldots$$
$$P_2 = P_{20} + \varepsilon P_{21} + \varepsilon^2 P_{22} + \varepsilon^3 P_{23} + \ldots$$
$$Q_1 = Q_{10} + \varepsilon Q_{11} + \varepsilon^2 Q_{12} + \varepsilon^3 Q_{13} + \ldots$$
$$Q_2 = Q_{20} + \varepsilon Q_{21} + \varepsilon^2 Q_{22} + \varepsilon^3 Q_{23} + \ldots$$
$$t = t_{00} + \varepsilon t_1 + \varepsilon^2 t_2 + \varepsilon^3 t_3 + \ldots$$

- In the following only the linear terms in ε will be retained (linear theory).

Zuiani and Vasile 2010

A Perturbation Theory Approach

- The time equation needs a special consideration

$$\frac{dt}{dL} \approx \frac{r^2}{h} = \sqrt{\frac{a^3}{\mu}}\frac{B^3}{\Phi^2(L)} = H(a,P,P,L)$$

$$\frac{dt_0}{dL} + \varepsilon\frac{dt_1}{dL} = H(a_0,P_{10},P_{20},L) + \varepsilon\left(\left.\frac{dH}{da}\right|_{a_0,P_{10},P_{20}} a_1 + \left.\frac{dH}{dP_1}\right|_{a_0,P_{10},P_{20}} P_{11} + \left.\frac{dH}{dP_2}\right|_{a_0,P_{10},P_{20}} P_{21}\right)$$

$$t_0(L) = t_{00} + \int_{L_0}^{L}\sqrt{\frac{a_0^{\,3}}{\mu}}\frac{B_0^{\,3}}{\Phi_0^{\,2}(\mathcal{L})}d\mathcal{L}$$

$$t_1(L) = \int_{L_0}^{L}\sqrt{\frac{a_0}{\mu}}B_0\left\{\frac{3B_0^{\,2}}{2}\frac{a_1(\mathcal{L})}{\Phi_0^{\,2}(\mathcal{L})} - a_0\left[\begin{array}{l}\left(\frac{3P_{10}}{\Phi_0^{\,2}(\mathcal{L})} + 2B_0^{\,2}\frac{\sin\mathcal{L}}{\Phi_0^{\,3}(\mathcal{L})}\right)P_{11}(\mathcal{L}) + \\ \left(\frac{3P_{20}}{\Phi_0^{\,2}(\mathcal{L})} + 2B_0^{\,2}\frac{\cos\mathcal{L}}{\Phi_0^{\,3}(\mathcal{L})}\right)P_{21}(\mathcal{L})\end{array}\right]\right\}d\mathcal{L}$$

Zuiani and Vasile 2010

A Perturbation Theory Approach

- Thus the first order approximated solution of the perturbed Keplerian motion takes the form:

$$a = a_0 + \varepsilon \frac{2B_0^{\,2} a_0^{\,3}}{\mu} \cos\beta \left[\left(P_{20} I_{s2} - P_{10} I_{c2} \right) \cos\alpha + I_{11} \sin\alpha \right]$$

$$P_1 = P_{10} + \varepsilon \frac{B_0^{\,4} a_0^{\,2}}{\mu} \left[\cos\beta \left(-I_{c2} \cos\alpha + \left(I_{13} P_{10} + I_{s3} + I_{s2} \right) \sin\alpha \right) - P_{20} \left(Q_{10} I_{c3} - Q_{20} I_{s3} \right) \sin\beta \right]$$

$$P_2 = P_{20} + \varepsilon \frac{B_0^{\,4} a_0^{\,2}}{\mu} \left[\cos\beta \left(I_{s2} \cos\alpha + \left(I_{13} P_{20} + I_{c3} + I_{c2} \right) \sin\alpha \right) + P_{10} \left(Q_{10} I_{c3} - Q_{20} I_{s3} \right) \sin\beta \right]$$

$$Q_1 = Q_{10} + \varepsilon \frac{B_0^{\,4} a_0^{\,2}}{2\mu} \left(1 + Q_1^2 + Q_2^2 \right) I_{s3} \sin\beta$$

$$Q_2 = Q_{20} + \varepsilon \frac{B_0^{\,4} a_0^{\,2}}{2\mu} \left(1 + Q_1^2 + Q_2^2 \right) I_{c3} \sin\beta$$

with integrals:

$$I_{cn} = \int_{L_0}^{L} \frac{\cos\mathcal{L}}{\Phi_0^n(\mathcal{L})} d\mathcal{L}; \quad I_{sn} = \int_{L_0}^{L} \frac{\sin\mathcal{L}}{\Phi_0^n(\mathcal{L})} d\mathcal{L}; \quad I_{1n} = \int_{L_0}^{L} \frac{1}{\Phi_0^n(\mathcal{L})} d\mathcal{L}$$

Zuiani and Vasile 2010

A Perturbation Theory Approach

- The time equation can be further simplified :

$$t_1 = 3\sqrt{\frac{a_0^7}{\mu^3}} B_0^5 \cos\beta\left(\cos\alpha I_{t1} + \sin\alpha I_{t2}\right)$$

with integrals:

$$I_{t1} = \int_{L_0}^{L} \frac{1}{\Phi_0^2(\mathcal{L})}\left(\frac{1}{\Phi_0(\mathcal{L})} - \frac{1}{\Phi_0(L_0)}\right) d\mathcal{L} \quad I_{t2} = \int_{L_0}^{L} \frac{I_{11}(\mathcal{L})}{\Phi_0^2(\mathcal{L})} d\mathcal{L}$$

Zuiani and Vasile 2010

A Perturbation Theory Approach

- Equivalent formulas can be derived for an inertially fixed acceleration and can be used to model the effect of solar pressure:

$$a^{In} = a_0 + \varepsilon^{In} \frac{2B_0^2 a_0^3}{\mu} \cos\beta_0 \left[-\left(P_{10} I_{12} + I_{s2}\right)\cos\gamma_0 + \left(P_{20} I_{12} + I_{c2}\right)\sin\gamma_0 \right]$$

$$P_1^{In} = P_{10} + \varepsilon^{In} \frac{B_0^4 a_0^2}{\mu} \left\{ \cos\beta_0 \left[-\left(P_{10} I_{s3} + I_{12} + I_{2s3}\right)\cos\gamma_0 + \left(P_{10} I_{c3} + I_{1c1s3}\right)\sin\gamma_0 \right] - \sin\beta_0 P_{20} \left(Q_{10} I_{c3} - Q_{20} I_{s3}\right) \right\}$$

$$P_2^{In} = P_{20} + \varepsilon^{In} \frac{B_0^4 a_0^2}{\mu} \left\{ \cos\beta_0 \left[-\left(P_{20} I_{s3} + I_{1c1s3}\right)\cos\gamma_0 + \left(P_{20} I_{c3} + I_{12} + I_{2c3}\right)\sin\gamma_0 \right] + \sin\beta_0 P_{10} \left(Q_{10} I_{c3} - Q_{20} I_{s3}\right) \right\}$$

$$Q_1^{In} = Q_{10} + \varepsilon^{In} \frac{B_0^4 a_0^2}{2\mu} \left(1 + Q_1^2 + Q_2^2\right) \sin\beta_0 I_{s3}$$

$$Q_2^{In} = Q_{20} + \varepsilon^{In} \frac{B_0^4 a_0^2}{2\mu} \left(1 + Q_1^2 + Q_2^2\right) \sin\beta_0 I_{c3}$$

$$t_1^{In} = 3\sqrt{\frac{a_0^7}{\mu^3}} B_0^5 \cos\beta_0 \left\{ -\frac{\cos\gamma_0}{P_{20}} \left[\left(I_{13} + P_{10} I_{s3}\right) - \frac{\left(1 + P_{10}\sin L_0\right)}{\Phi_0\left(L_0\right)} I_{12} \right] + \sin\gamma_0 \left(I_{s3} - \frac{\sin L_0}{\Phi_0\left(L_0\right)} I_{12} \right) \right\}$$

Zuiani and Vasile 2011

A Perturbation Theory Approach

- The same formulas derived for a tangent-normal-binormal reference frame require the calculation of a some elliptic integrals:

$$a^{t} = a_0 + \varepsilon^{t} \frac{2B_0^{\,2} a_0^{\,3}}{\mu} \int_{L_0}^{L} \frac{1}{D_0(\mathcal{L})} \left(\frac{2}{\Phi_0(\mathcal{L})} - \frac{B_0^{\,2}}{\Phi_0(\mathcal{L})^2} \right) d\mathcal{L}$$

$$P_1^{t} = P_{10} + \varepsilon^{t} \frac{B_0^{\,4} a_0^{\,2}}{\mu} \int_{L_0}^{L} \frac{2\left(P_{10} + \sin\mathcal{L}\right)}{D_0(\mathcal{L})\Phi_0(\mathcal{L})^2} d\mathcal{L}$$

$$P_2^{t} = P_{20} + \varepsilon^{t} \frac{B_0^{\,4} a_0^{\,2}}{\mu} \int_{L_0}^{L} \frac{2\left(P_{20} + \cos\mathcal{L}\right)}{D_0(\mathcal{L})\Phi_0(\mathcal{L})^2} d\mathcal{L}$$

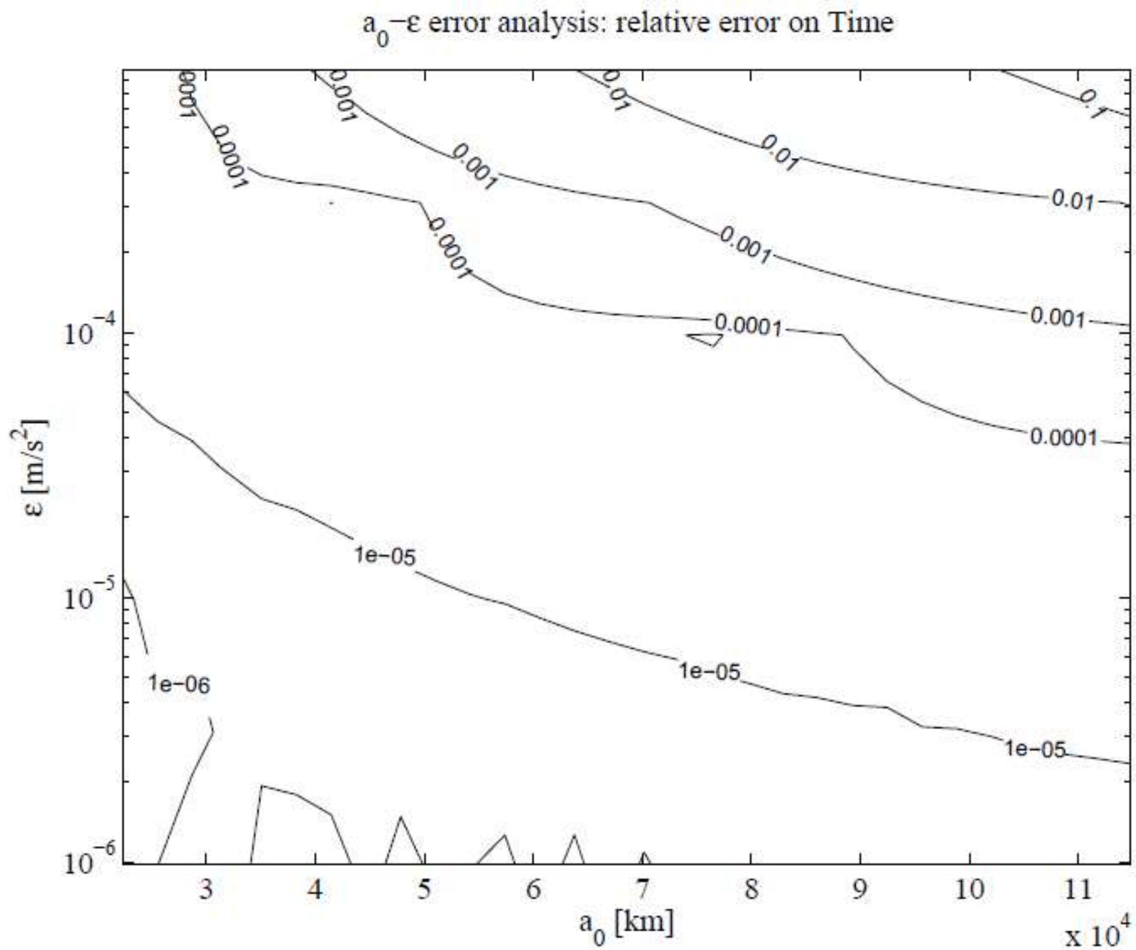
$$D = \sqrt{1 + P_1 + P_2 + 2\left(P_1 \sin L + P_2 \cos L\right)}$$

- The time equation, in this case, can be more conveniently integrated with a Gauss formula using 6 nodes per revolution.

Zuiani and Vasile 2011

A Perturbation Theory Approach

- Accuracy of the approximation over a single revolution.

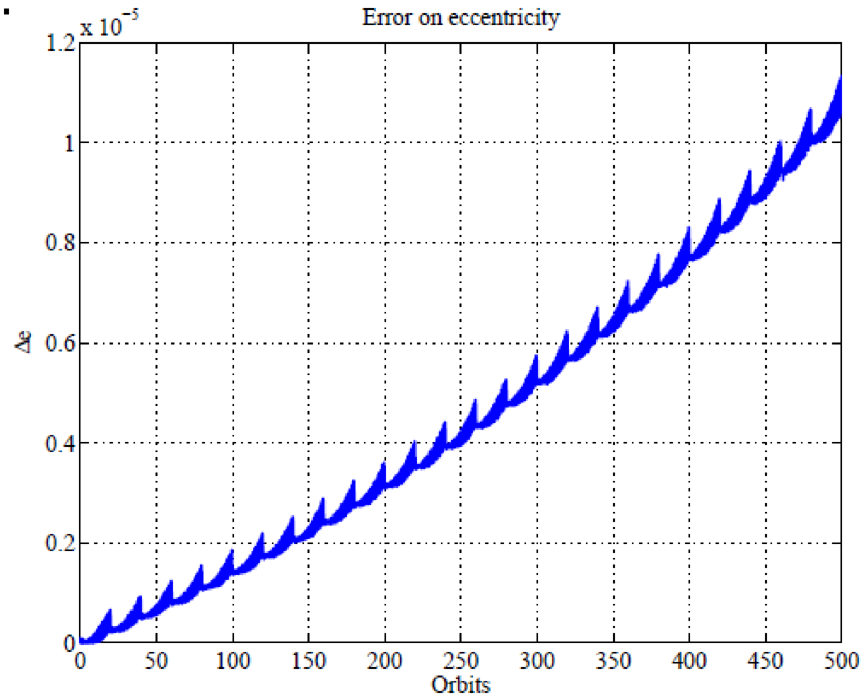


Zuiani and Vasile 2011

Error Control Over Long Spirals

- The propagation error can be controlled by updating the reference orbital elements E_0 every n orbits.
- The update simply consists in taking the value for $E(L)$ computed at the n-th orbit as the new reference condition E_0 for the following n orbits (see Colombo et al. 2009).

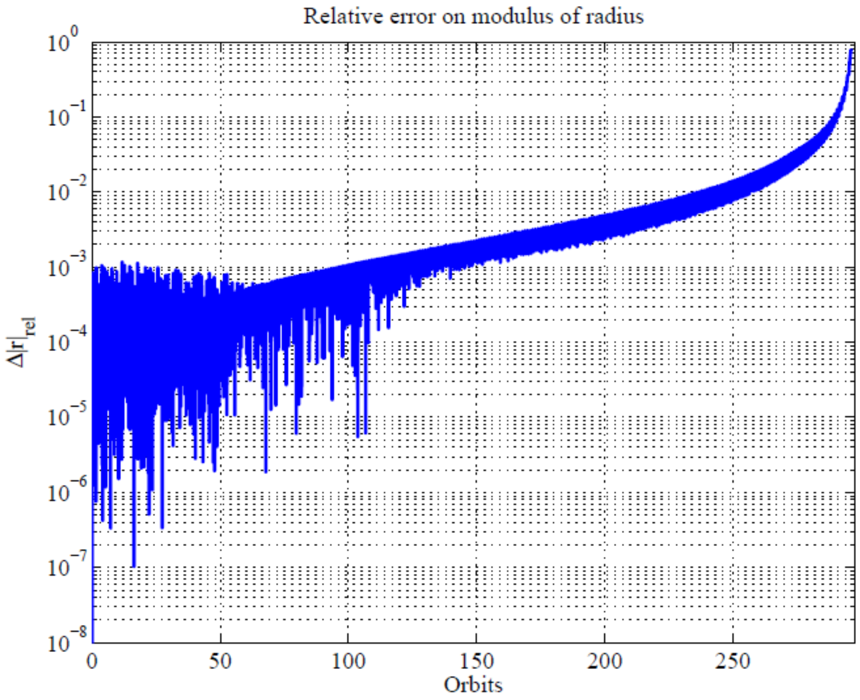
- Example: LEO spiral
  - Acceleration 1e-4 m/s^2
  - Rectification every n=20
  - Computation time **0.02s**



Zuiani and Vasile 2011

Fast Propagation Over Long Spirals

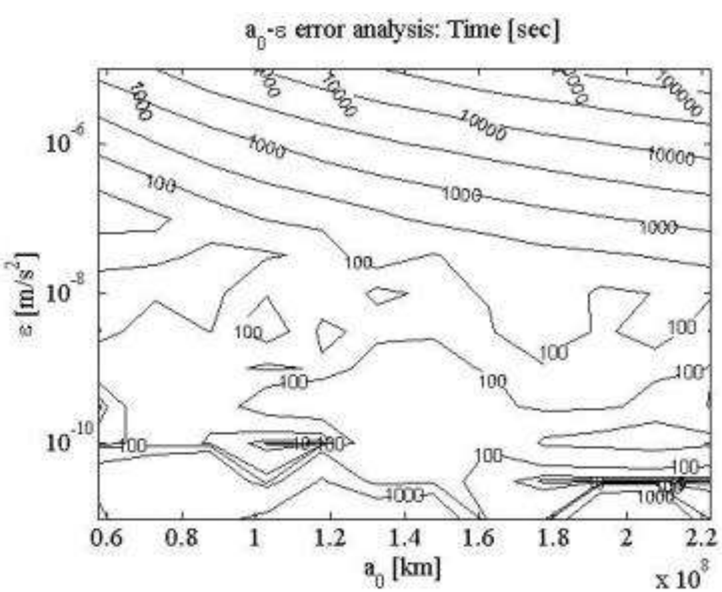
- Consider an upward spiral from GTO with constant thrust and J_2.
- The acceleration is 1e-4 m/s^2
- The frequency of rectification dynamically adjusted from *n=1 to n=8.*
- The analytical propagation required about **0.6 s** while a numerical one with *ode113* (Adams-Bashfort, with tolerance set at 10^{-13}) took about 15 s

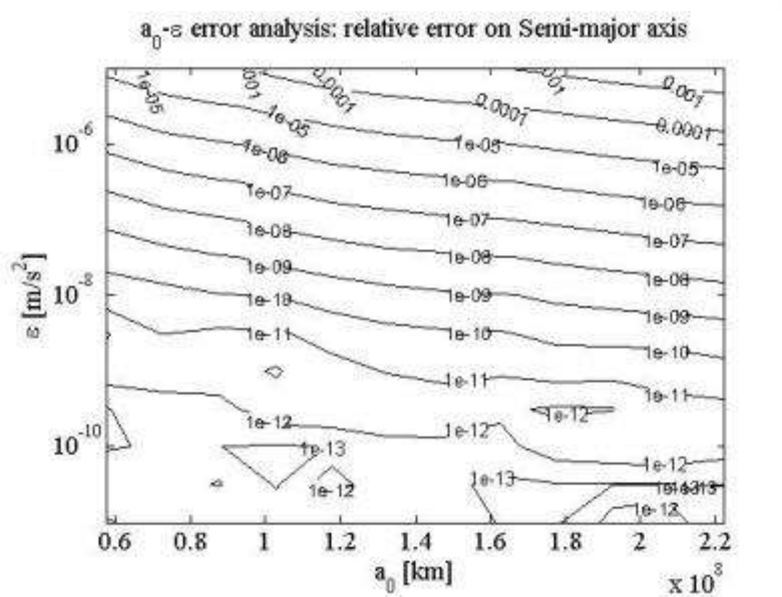


Zuiani and Vasile 2011

Relative Error in the Solar System

- For asteroid deflection the level of acceleration is very small (10^{-12}-10^{-8}) which makes the linear expansions accurate enough to correctly predict the deflection and the miss distance.





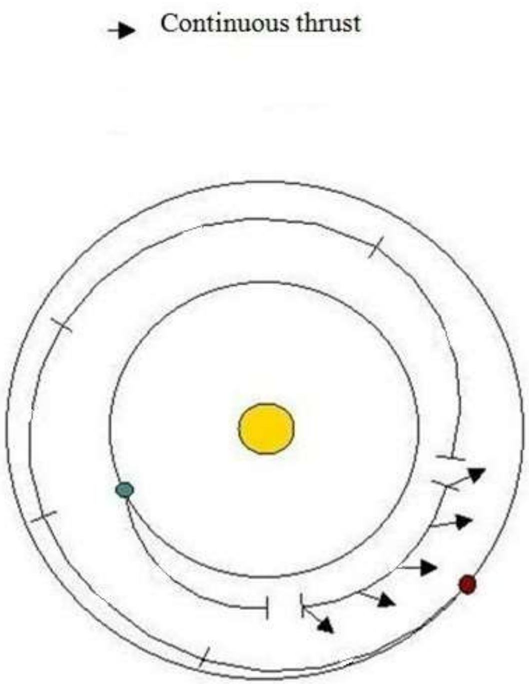
Zuiani and Vasile 2012

Transcription into FPET

- To propagate the motion, the trajectory is subdivided into Finite Perturbative Elements.

- On each element, thrust is continuous

- ~10 times faster compared to numerical integration and with comparable accuracy.

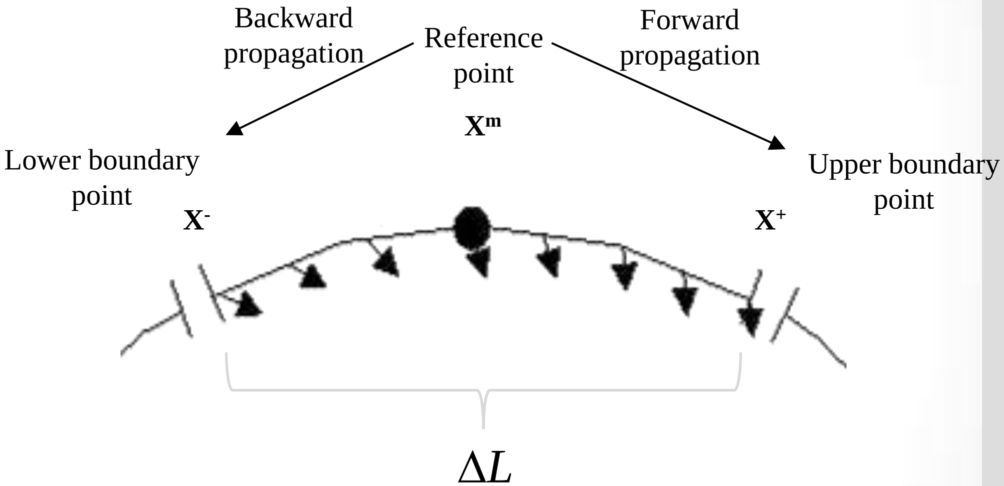


Zuiani and Vasile 2010

Transcription into FPET

- A Direct Transcription Method based on Finite Perturbed Elements in Time (FPET) has been designed using the perturbative approach.
- Each transfer trajectory is divided into *n* subarcs:

- Amplitude of arc is ΔL.
- Perturbed motion propagated using analytical solution.
- Constant thrust vector in the r-t-h reference frame.
- Reference node for propagation is the midpoint of the arc.
- Motion is propagated analytically backwards and forwards by $\pm\frac{\Delta L}{2}$ from the midpoint to obtain boundary nodes (better accuracy compared to a single sided propagation)



$$\begin{cases} \mathbf{X}^{+} = f\left(\mathbf{X^m}, \frac{\Delta L}{2}, \varepsilon, \alpha, \beta\right) \\ \mathbf{X}^{-} = f\left(\mathbf{X^m}, -\frac{\Delta L}{2}, \varepsilon, \alpha, \beta\right) \end{cases}$$

Zuiani and Vasile 2010

The Low Thrust Two-Points Boundary Value Problem with FPET

- The FPET transcription method is used to solve the LT boundary problem:

$$\min_{\mathbf{u}} J = \sum_{i=1}^{n_{FPET}} \varepsilon_i \Delta t_i$$

$$s.t.\ \mathbf{C_{eq}} = \begin{Bmatrix} \mathbf{X}_1^- - \mathbf{X}_0 \\ \mathbf{X}_i^+ - \mathbf{X}_{i+1}^-, i = 2, \dots, n_{FPET} - 1 \\ \mathbf{X}_{n_{FPET}}^+ - \mathbf{X}_f \\ \overline{ToF} - \sum_{i=1}^{n_{FPET}} \Delta t_i \end{Bmatrix} = 0$$

$$\varepsilon_i \le \varepsilon_{max},\ i = 1, \dots, n_{FPET}$$

Total ΔV

Departure conditions

Inter-element matching conditions

Arrival conditions

Fixed time of flight

Upper bounds on acceleration

- Decision variables for each of the n FPET:
  - Position of the reference point (5 scalars).
  - Acceleration magnitude, azimuth and elevation (3 scalars).
- 8n decision variables and 5(n+1)+1 scalar constraints.
- The problem is efficiently solved with a gradient-based local optimizer (*fmincon active-set*).

Zuiani and Vasile 2010

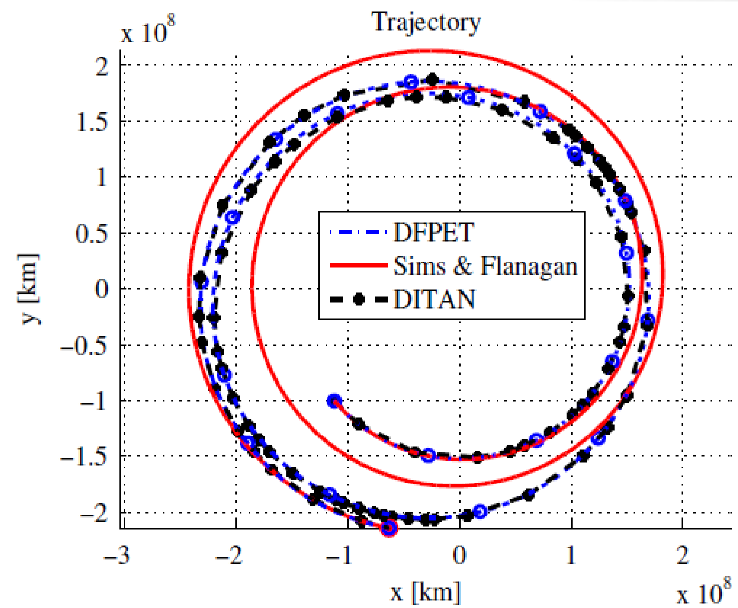
Earth-Mars Direct transfer (1)

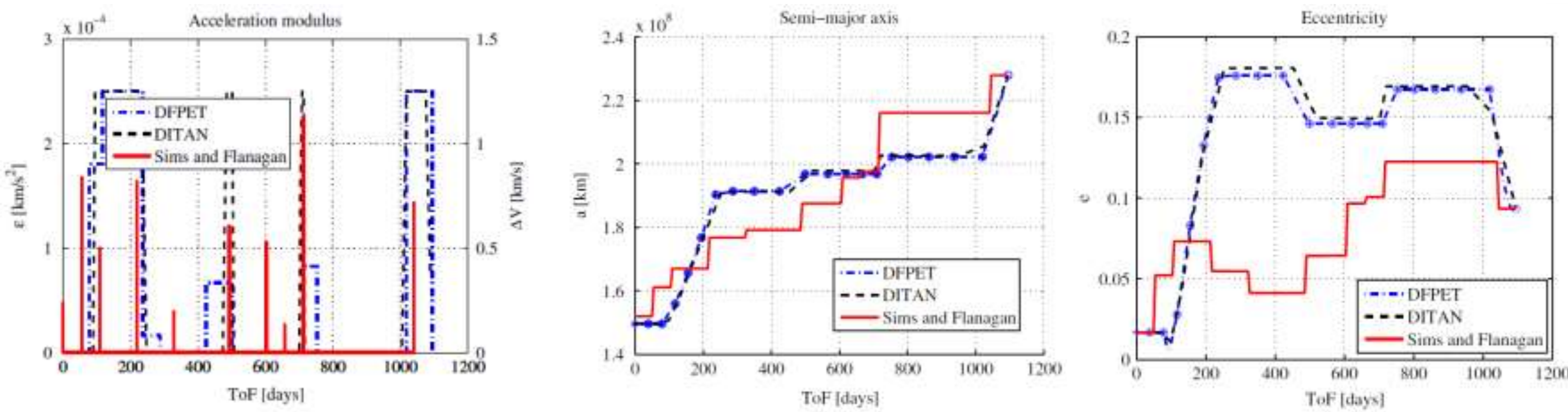
- Boundary problem:
  - Departure from Earth at 5600 MJD2000.
  - Rendezvous with Mars after a transfer time of 3 years.
  - 2 complete revolutions.
- Maximum acceleration: 2.5 $x10^{-5}$ m/s^2.
- 40 FPET.
- Initial guess for the local optimizer: constant thrust profile.

Zuiani and Vasile 2010

Earth-Mars Direct transfer (2)

- Results:
  - Total ΔV: 5.63 km/s.
  - Relative error 10^{-3}.
  - Solution found with DITAN: 5.71 km/s.
  - Hohmann Transfer: 5.49 km/s.



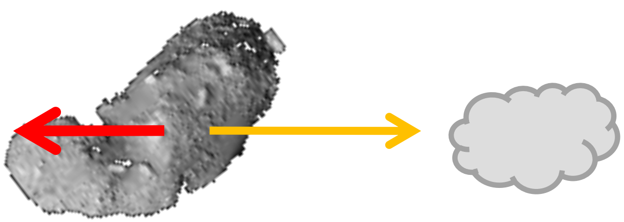


Zuiani and Vasile 2010

Asteroid's Prospective on the Deflection Action

- If one considers the asteroid-spacecraft system all the deflection methods can be grouped in two classes:

- Momentum change due to mass expulsion
  - **Low-thrust Tug**
    - Scheduled thrust, or dual engine
  - **Gravity Tug**
    - Mass expulsion function of the gravity attraction
  - **Laser/Solar Ablation**
    - Mass expulsion from ablated asteroid
  - **Mass Driver**
    - Asteroid mass expulsion
  - **Nuclear Blast**
    - Expulsion of the ablated surface of the asteroid



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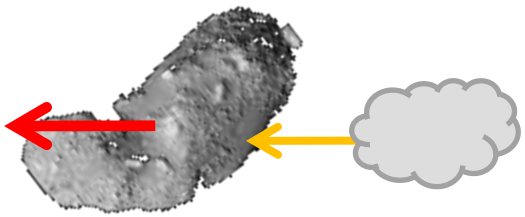
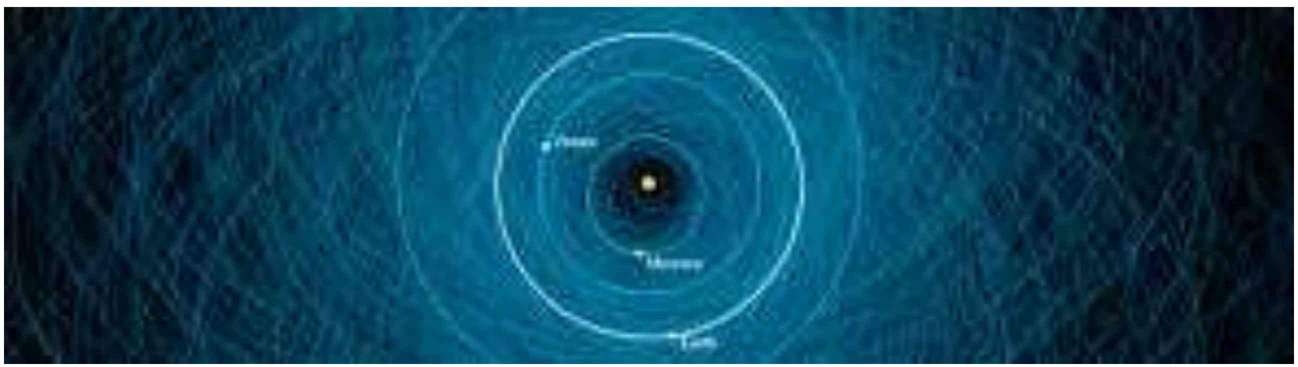
Asteroid's Prospective on the Deflection Action

- Momentum change due to mass acquisition or mix expulsion/acquisition:
  - **Ion Beam Shepherd**
    - Flow of accelerated gas impacting the asteroid
  - **Kinetic Impactor**
    - Inelastic impact with mass ejection
  - **Smart Clouds**
    - Inelastic impact with particles
  - **Light Tug**
    - Light absorption and enhanced Yarkovsky





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- Zuiani F., Vasile M. *Preliminary Design of Debris Removal Missions by Means of Simplified Models for Low-Thrust, Many-Revolution Transfers*. International Journal of Aerospace Engineering, Volume 2012 (2012), Article ID 836250, 22 pages, doi:10.1155/2012/836250

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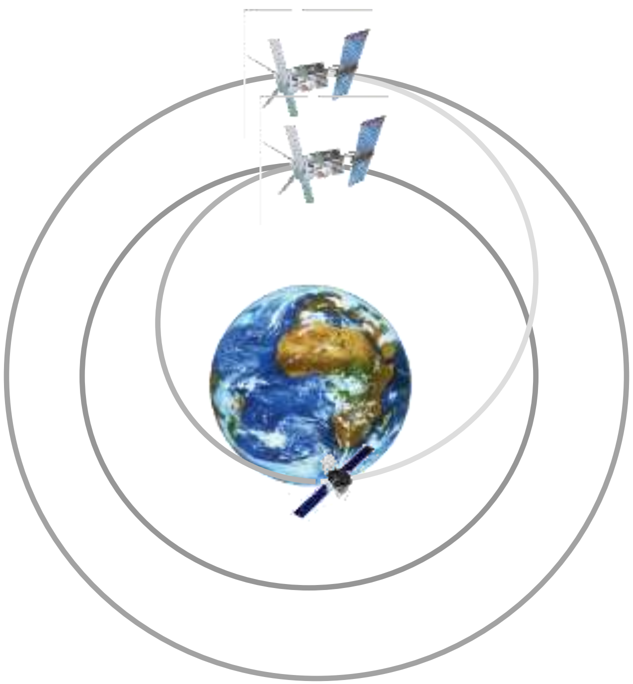
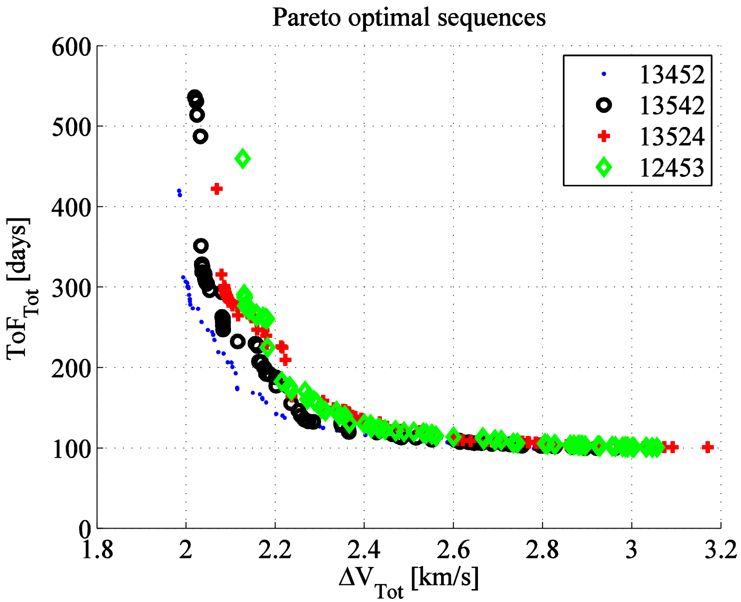
Excursus on Space Debris

- Zuiani F., Vasile M. *Preliminary Design of Debris Removal Missions by Means of Simplified Models for Low-Thrust, Many-Revolution Transfers*. International Journal of Aerospace Engineering, Volume 2012 (2012), Article ID 836250, 22 pages, doi:10.1155/2012/836250



$$\min_{\mathbf{u}} \left[ToF, \Delta V \right]^T$$



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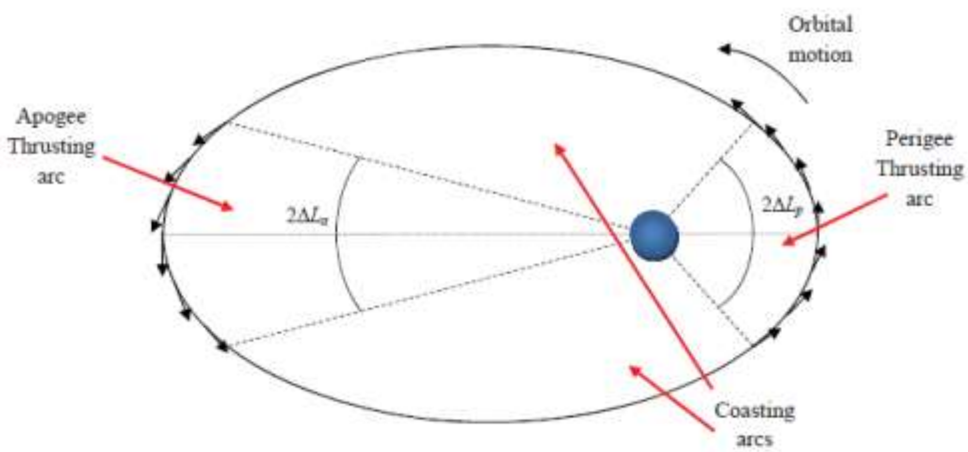
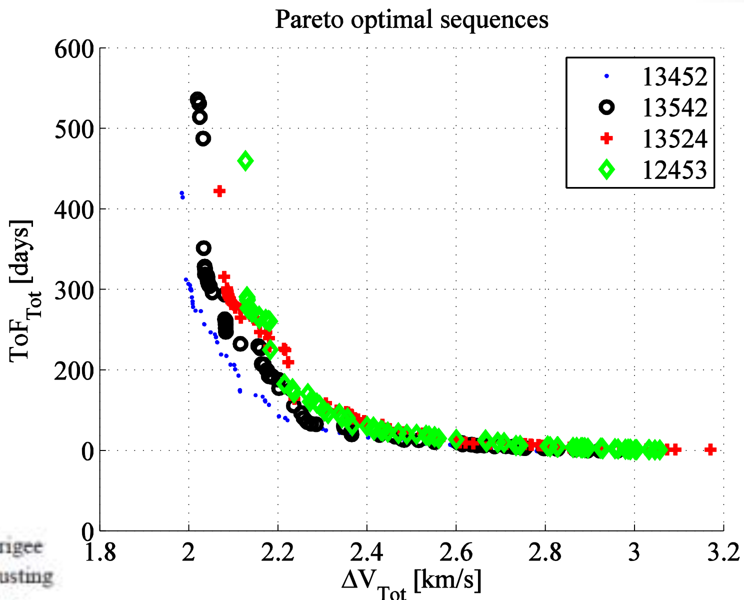
$$\min_{\mathbf{u}} \left[ToF, \Delta V \right]^T$$





The controls **u** are the amplitudes of the thrust arcs and the thrust direction per arc

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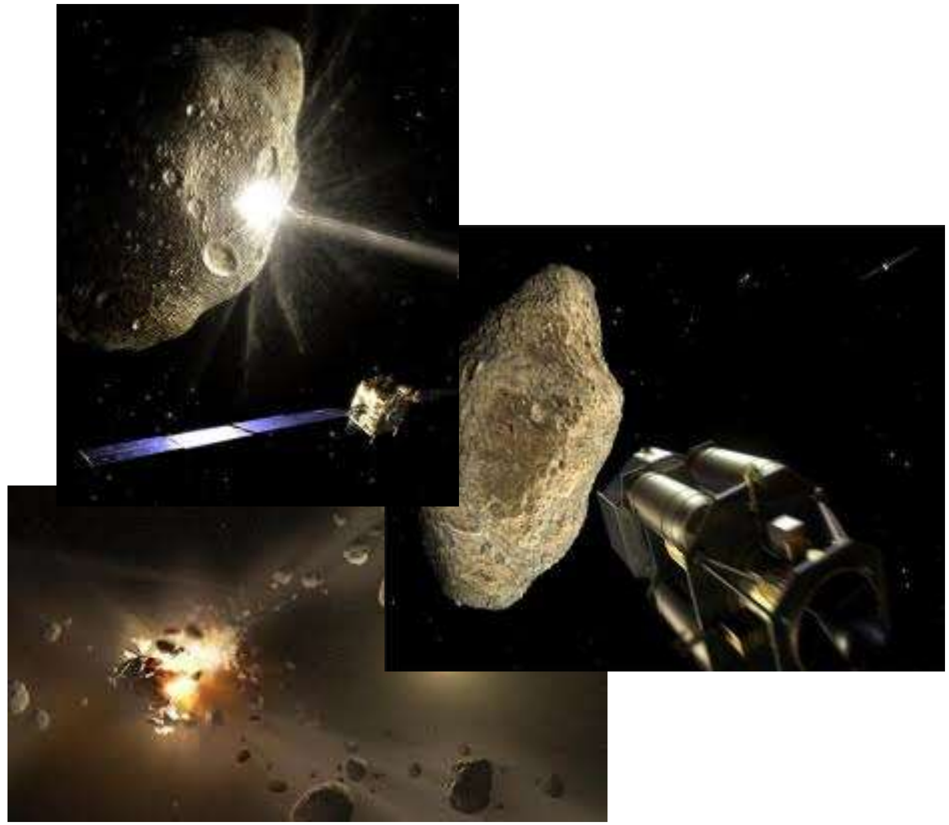
PART 2

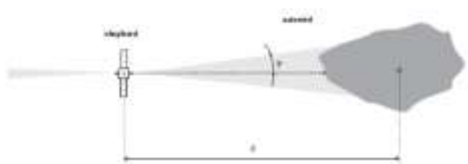
Impulsive Deflection Techniques

- Nuclear Blast
- Kinetic Impact
- Smart Clouds



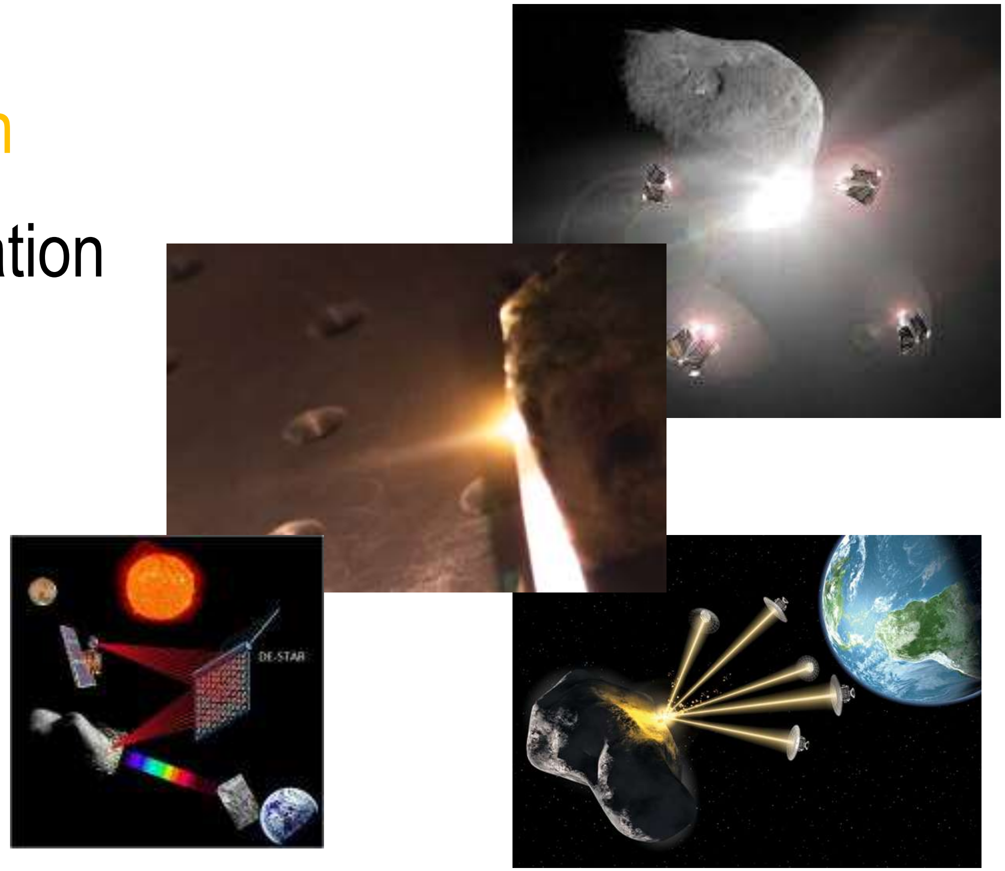
Slow-push Deflection Techniques

- Low-thrust tug
- Gravity tug
- Solar sail formation
- Enhancement by charging the asteroid
- Ion Beam Shepherd

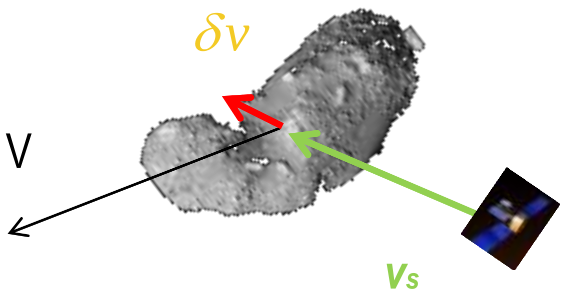


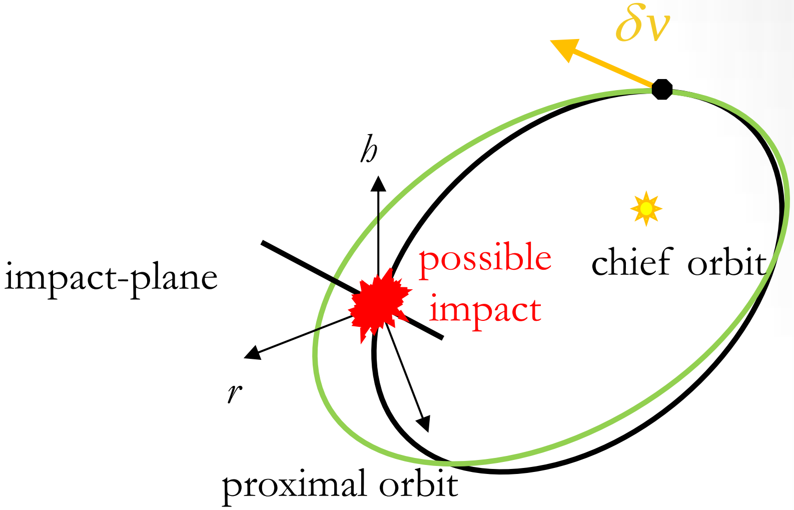
Slow-push Deflection Techniques

- Laser Ablation
- Solar Sublimation
- Light Tug



Kinetic Impactor





- If we assume that the impacting spacecraft is on an orbit similar to the one of the asteroid but rotates, the cost of the transfer is not different from the one of LT tugs or solar sublimation.
- The mass efficiency is therefore only a function of the impacting mass:

$$\eta_m = \frac{m_d}{m_{s/c}}; \quad \delta\mathbf{v} = v_f \frac{m_d}{m_A + m_d} \Delta\mathbf{v}_{s/c}$$

Fragmentation Hazard

- At the level of energy required to deflect an asteroid with a kinetic impact, an asteroid could fragment unpredictably

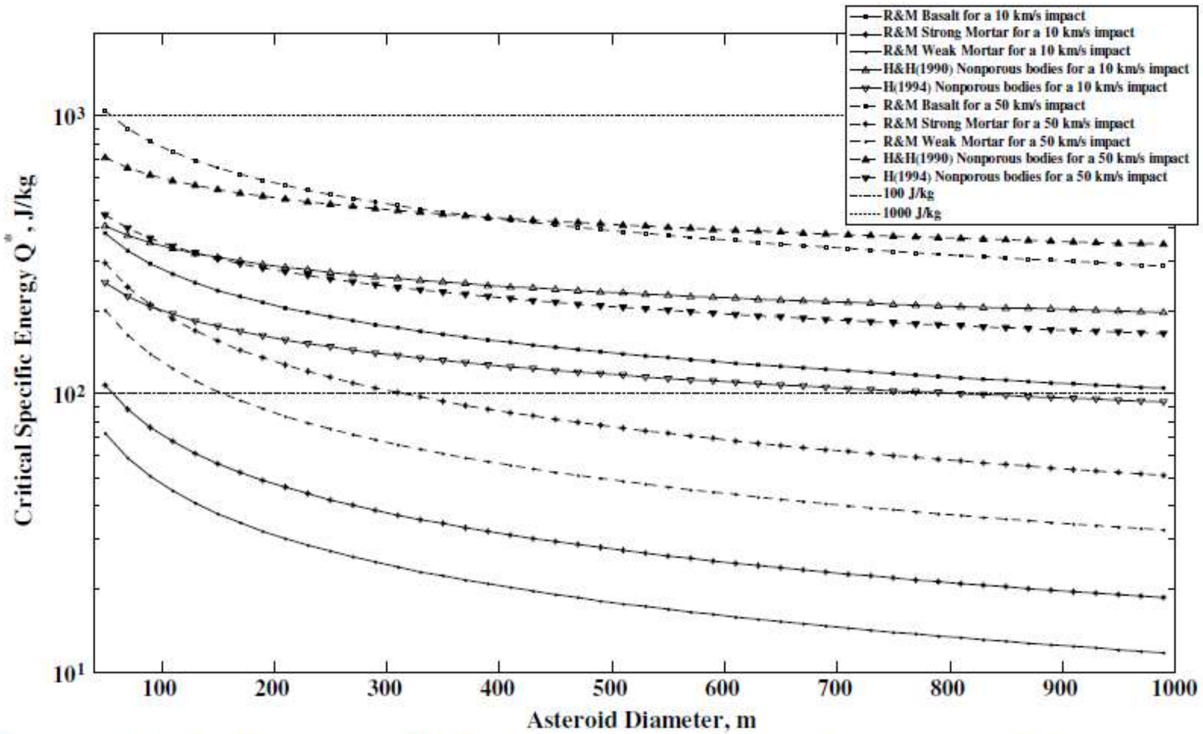
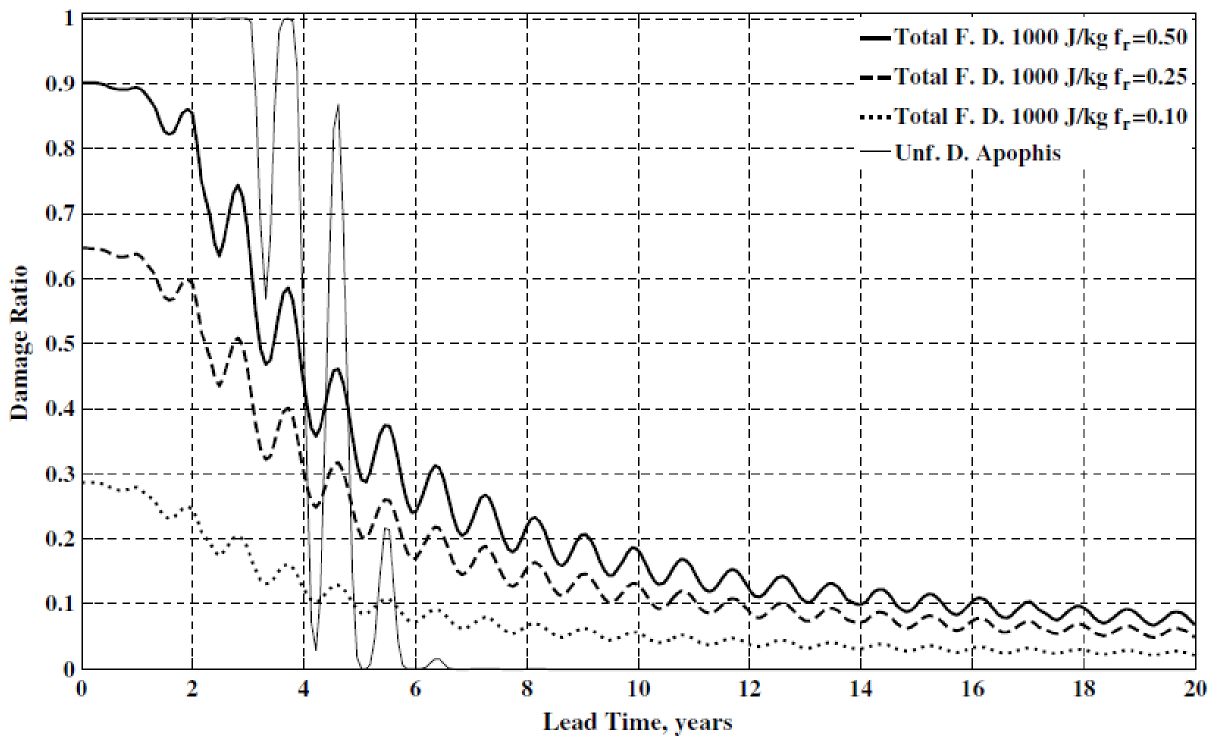


Fig. 1 Critical specific energy Q^* for barely catastrophically disrupting asteroids with a diameter ranging from 40 m to 1 km, calculated using the work of Ryan and Melosh (R&H) [3], Housen and Holsapple [H&H (1990)] [4], and Holsapple [H (1994)][20].

Sanchez et al. 2009

Fragmentation Hazard

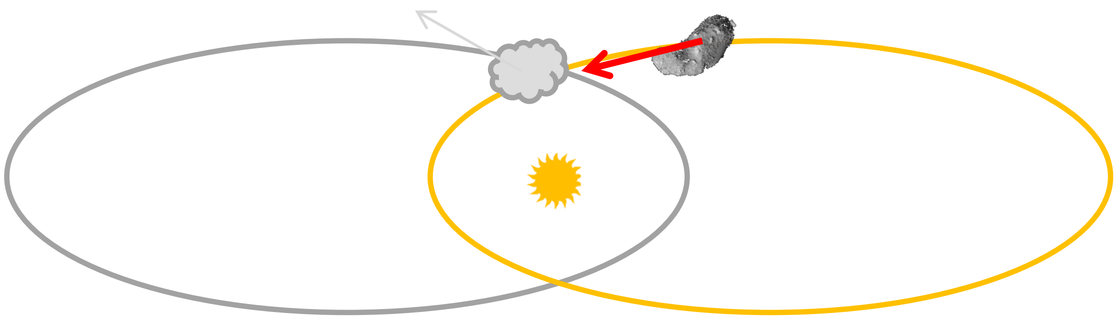
- To have an idea of the consequences of a fragmentation we can look at the expected damaged area due to an impact with the whole asteroid or with the resulting fragments after the deviation attempt.



Sanchez et al. 2009

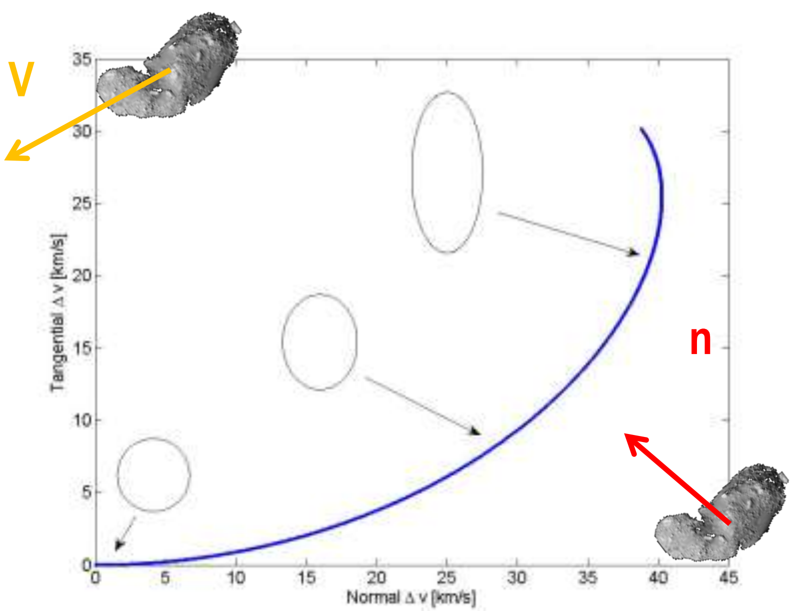
A Fractionated Kinetic Impactor Idea

- Kinetic impactors are the most mature method for asteroid deflection.
- The drawback of kinetic impactors is that the impact can fragment the asteroid.
- The idea is to impact with a cloud of particles with a total mass equivalent to a single spacecraft.



Smart Cloud vs Kinetic Impactor

- Unlike the Kinetic Impactor the cloud provides a distributed impact, more like a drag, thus no fragmentation.
- The relative velocity between the cloud and the asteroid can be very high, up to 50 km/s for highly eccentric asteroids.
- Unlike the Kinetic Impactor there is no need to pin-point the asteroid as an extended area is covered by the cloud.



The change of velocity of a $2.7 \cdot 10^{10}$ kg asteroid hit by a 500 kg cloud is 0.1 cm/s

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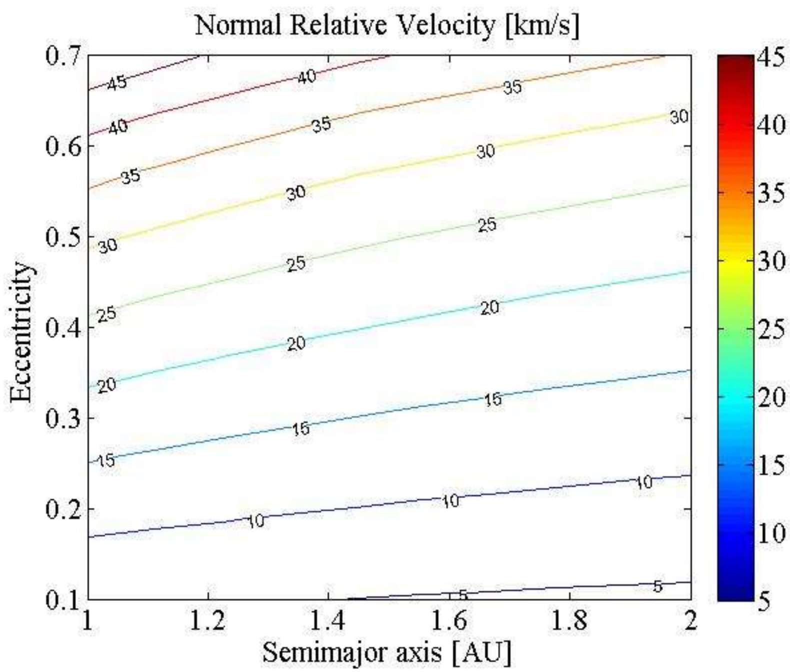
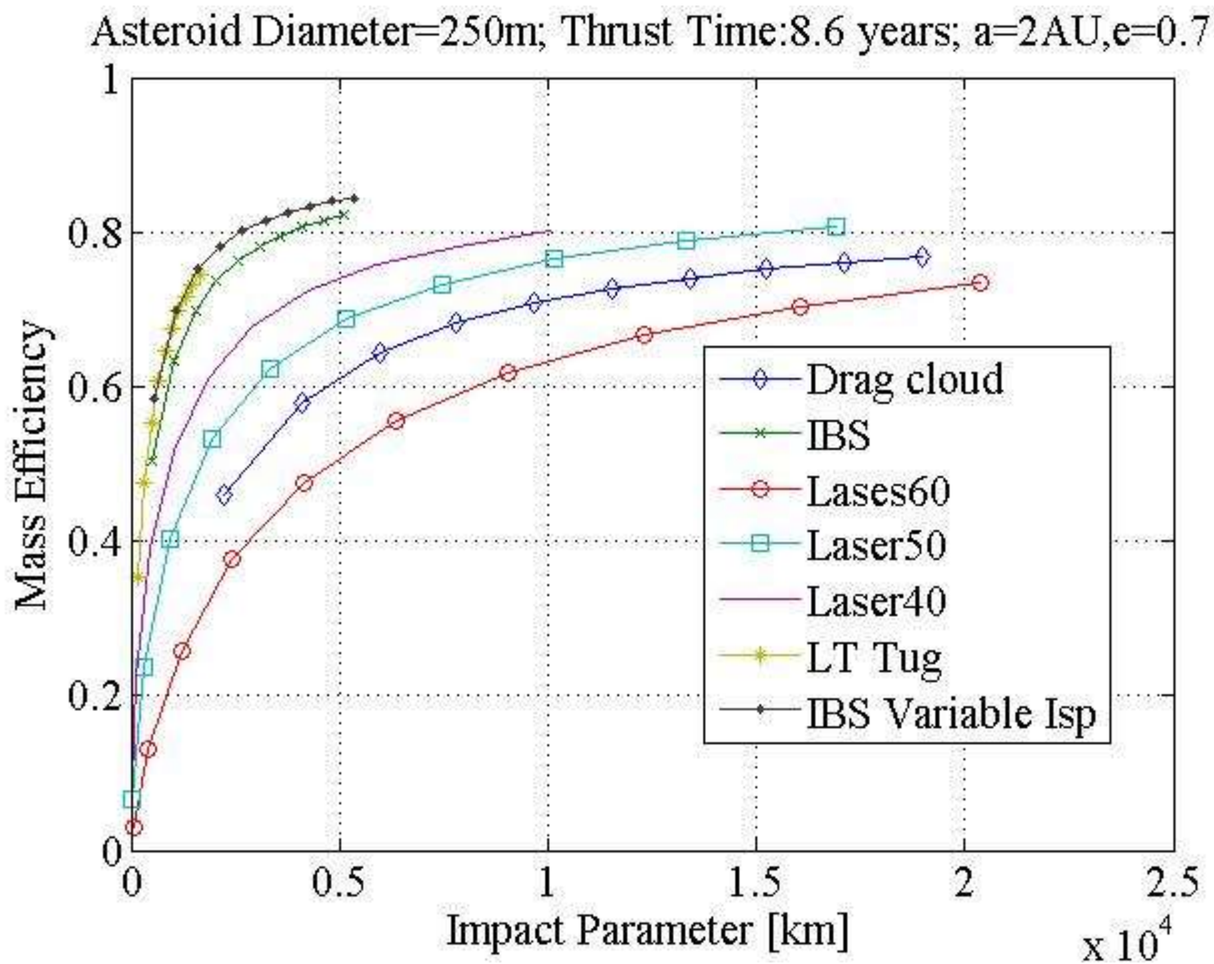
Smart Cloud vs Kinetic Impactor

- Deployment of a smart dust cloud with a total mass equivalent to the propellant of a low-thrust tug.
- High elliptical orbit with steep intersection with the orbit of the asteroid.
- Orders of magnitude lower impact energy of individual particles.
- High relative velocity for deep crosser
- No need of extra energy to accelerate the cloud



	NEAs(%)	PHAs (%)	Impactors
Q<1.05AU	1%	1%	11%
Q>0.95AU	8%	22%	38%
Deep Crossers	61%	77%	53%
Low Inc. (<5)	6%	25%	38%

Smart Clouds – Deep Crossers



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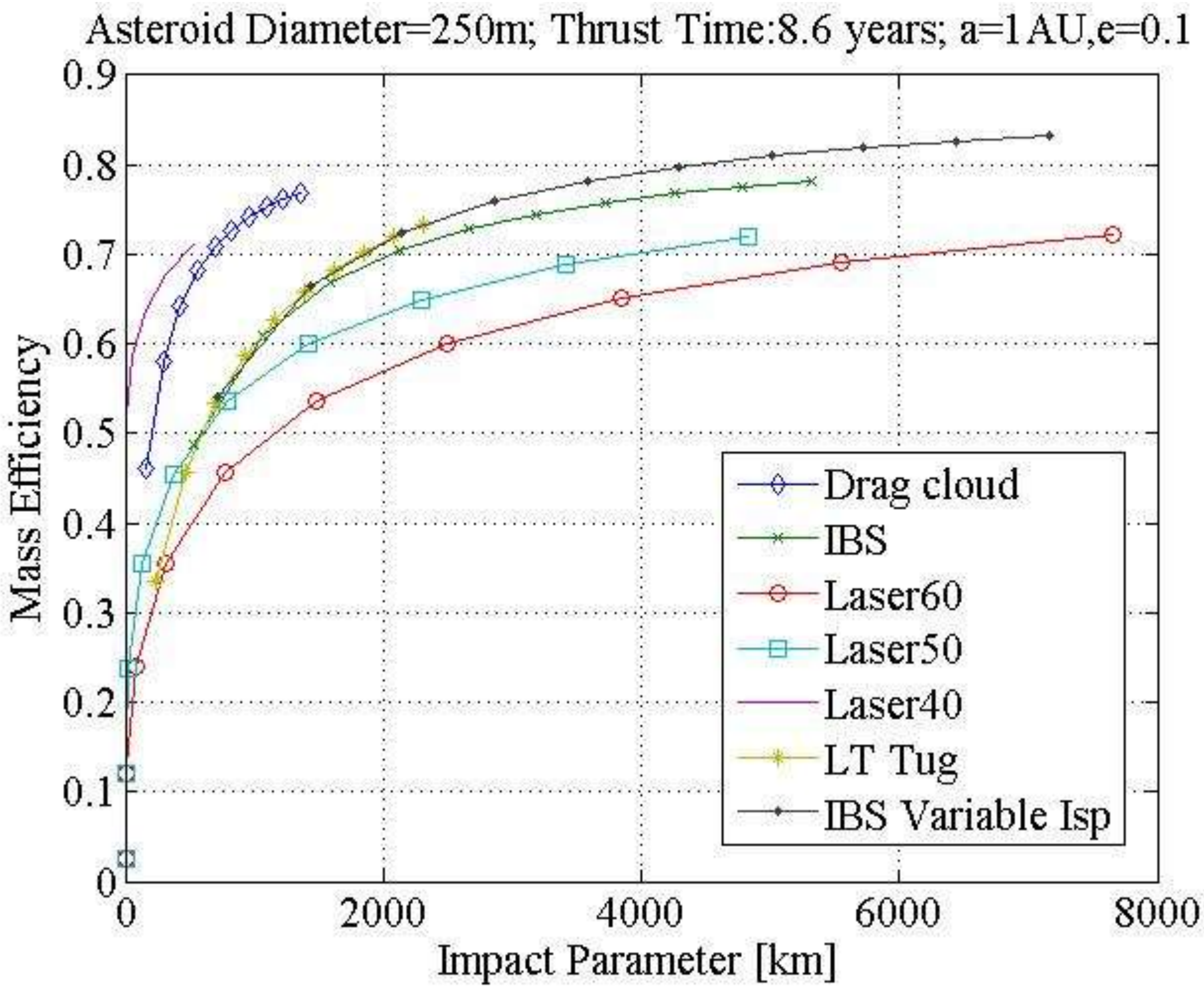
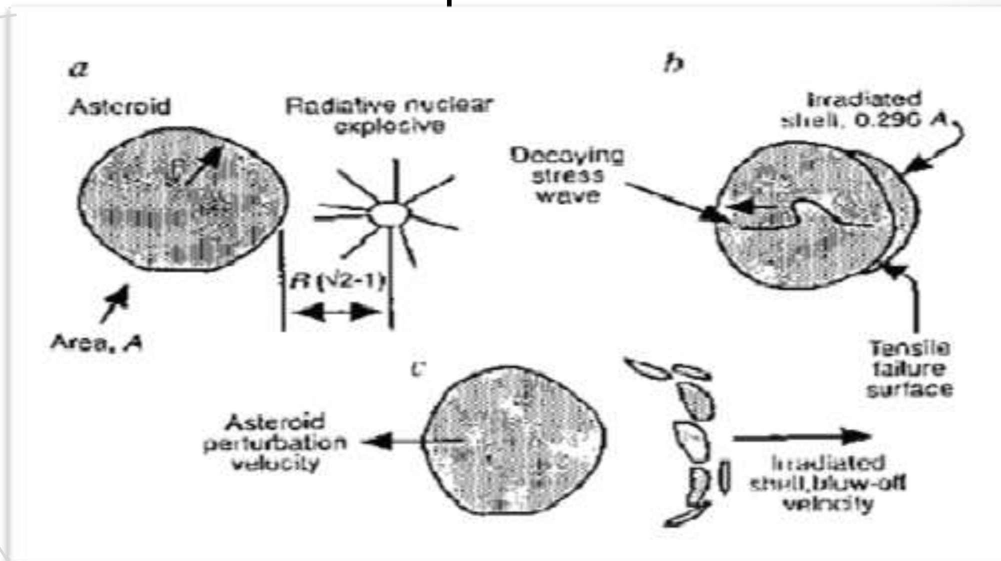
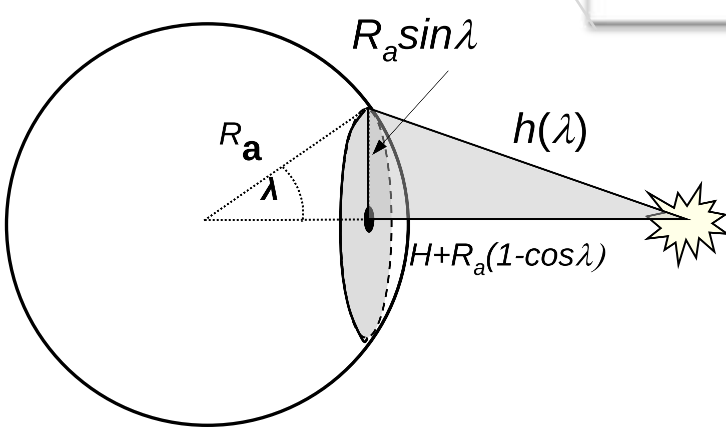
Smart Clouds – Shallow Crossers



Nuclear Blast

- Different variants of the nuclear blast concept:
  - Standoff
  - Buried
  - Surface





- Ejection velocity:

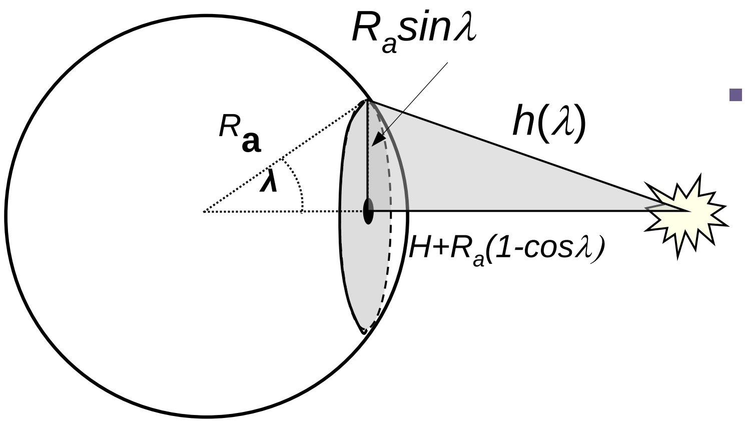
$$v_e = \sqrt{2\left(\mu_o E_A(z) - E_v\right)}$$

Sanchez et al. 2009

Nuclear Blast

- The amount of linear momentum dp_z per layer of material dz is:

$$dp_A = \rho_a v_e dz$$



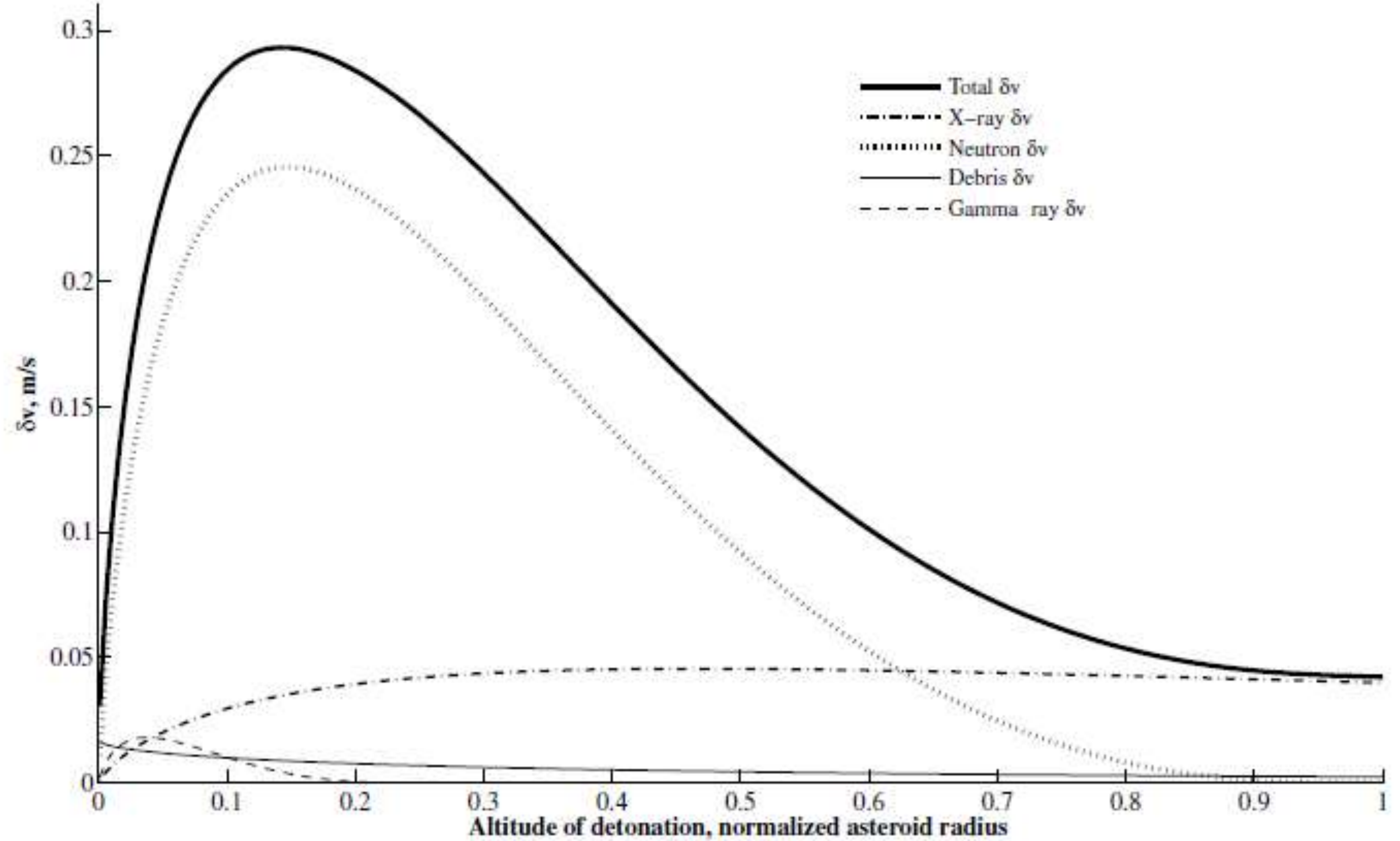
- One can now integrate over the surface area and over the thickness the total energy per type of radiation and get the total momentum per each type of radiation:

$$P = \sqrt{8}\pi R_a^2 \rho_a \int_0^{\lambda_{\max}} \left(\int_0^{Z_{\max}\sin\varepsilon(\lambda)} \left(\mu_o \frac{f_{radiation} E_t}{4\pi\left[h(\lambda)\right]^2} e^{-\frac{\rho_a \mu_o z}{\sin\varepsilon(\lambda)}} - E_v \right)^{1/2} dz \right) \sin\lambda \, d\lambda$$

Sanchez et al. 2009

Nuclear Blast

- Δ*v* delivered by each type of radiation/particles releases



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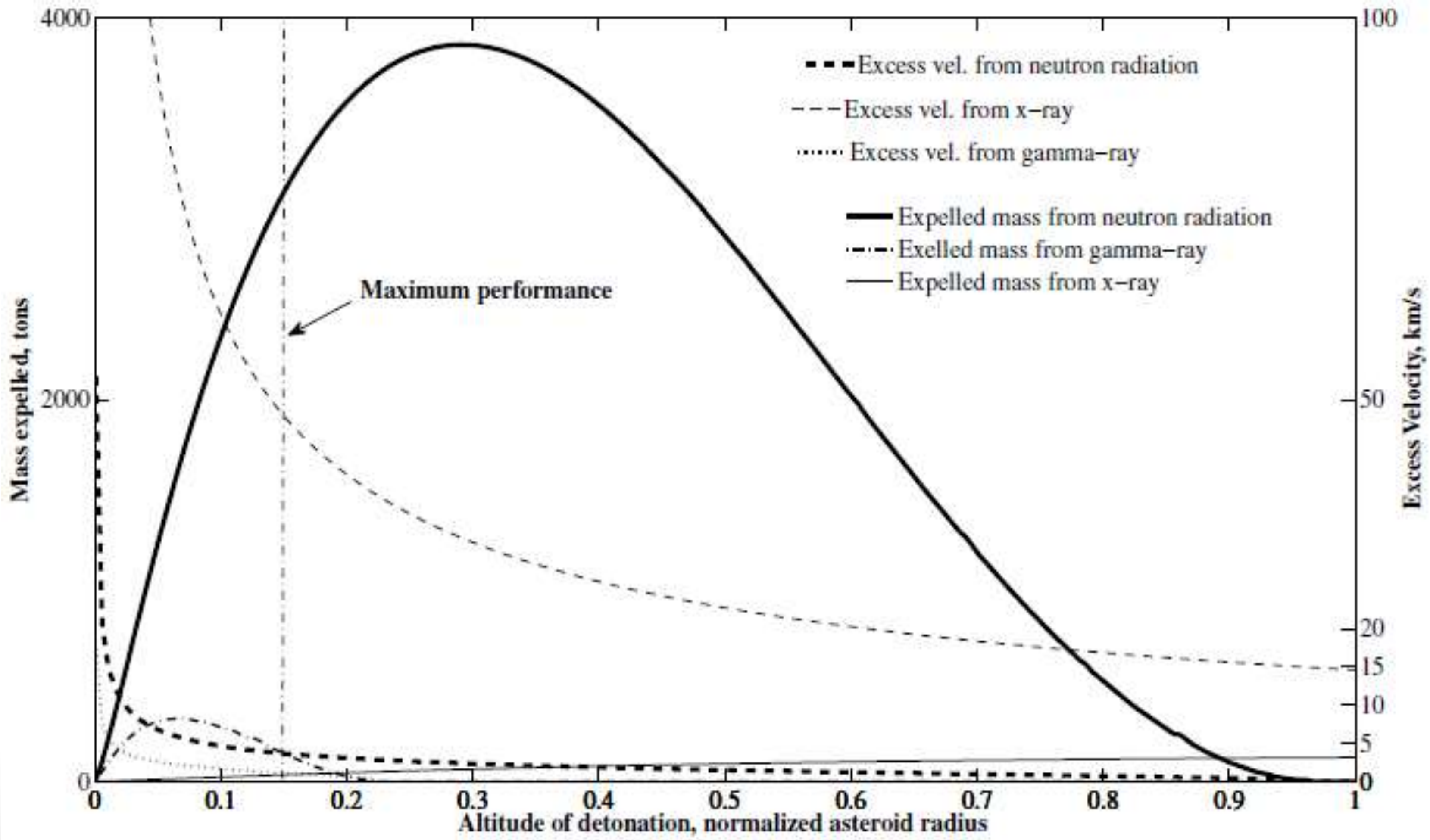
Nuclear Blast



Sanchez et al. 2009

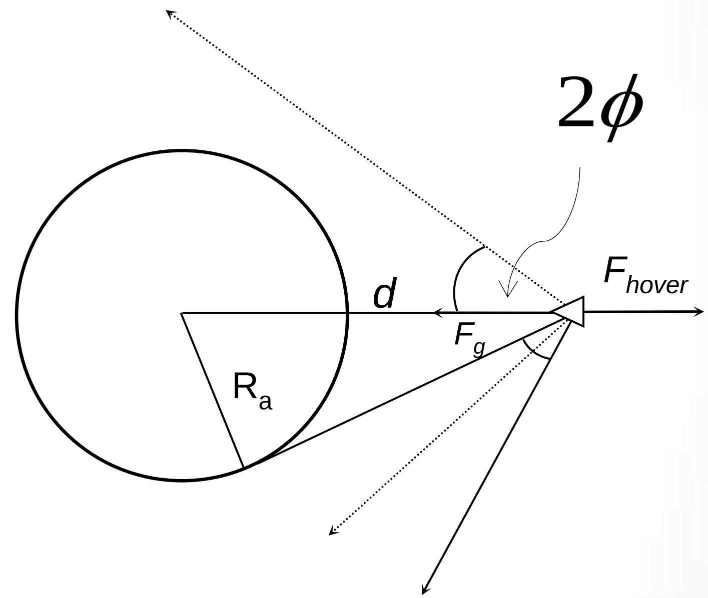
Gravity Tractor

- The effective thrust that the gravity tractor can deliver depends on the mass of the spacecraft, the exhaust angle of the plume 2ϕ, and the hovering radius *d:*

$$F_{hover} = T_n \cos\left(\arcsin\left(\frac{R_a}{d}\right) + \phi\right)$$

$$F_g = \frac{GM_a m(t)}{d^2}$$

$$F_{hover} = F_g$$



and the tugging acceleration is simply:

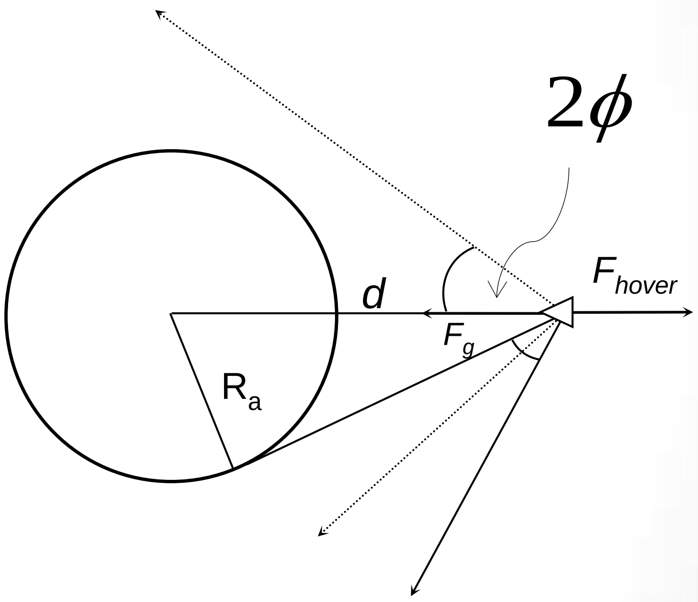
$$a_{gtug}(t) = \frac{Gm(t)}{d^2}$$

Sanchez et al. 2009

Gravity Tractor

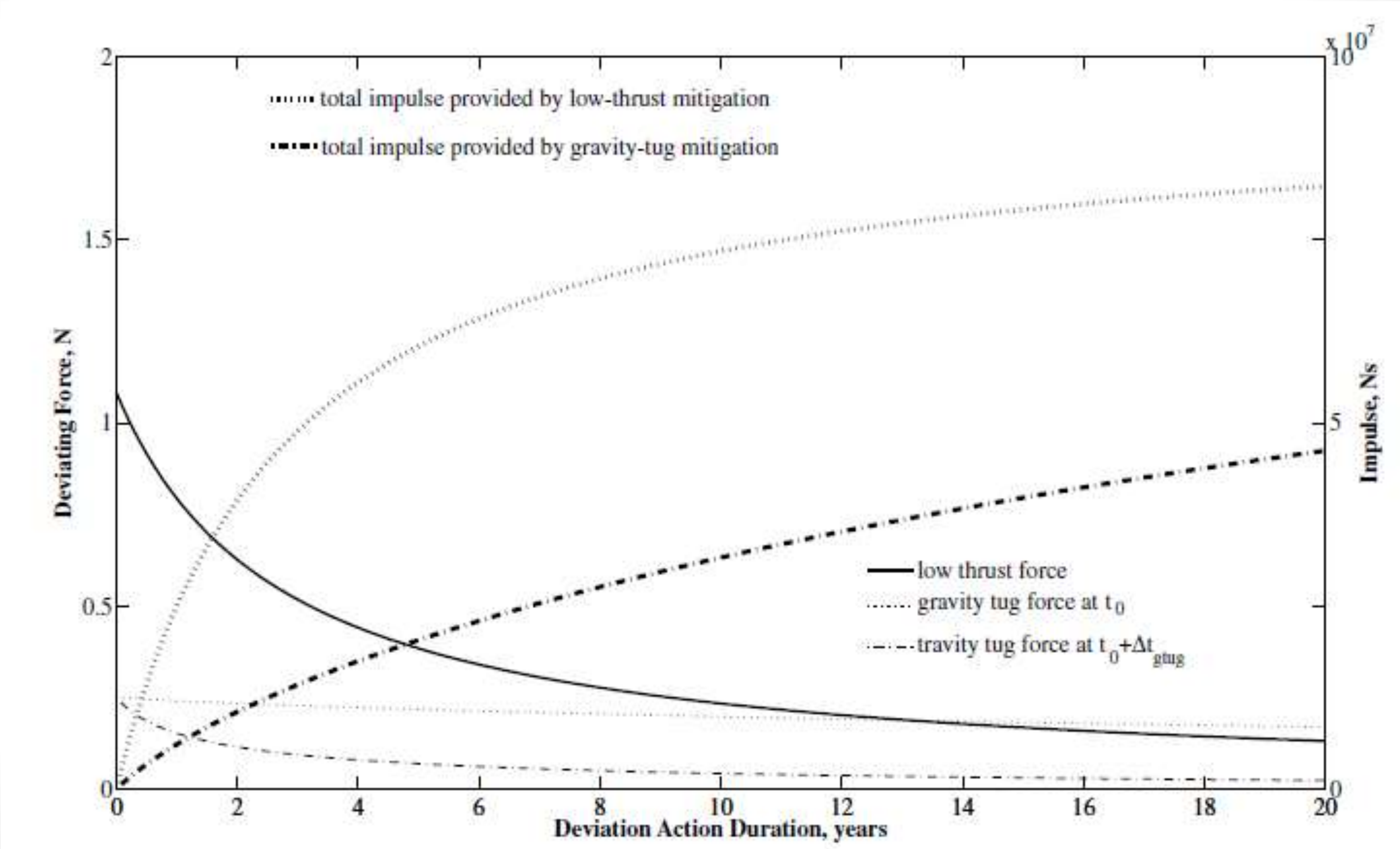
- If the engines are assumed to be always on and the initial mass of the spacecraft is m_i, the mass of the spacecraft at time t can be expressed as:

$$m(t) = m_i e^{-\left(\frac{GM_a(t-t_0)}{d^2 \cos\left(\arcsin\left(\frac{R_a}{d}\right)+\phi\right) I_{sp} g_0}\right)}$$



Sanchez et al. 2009

Gravity Tractor



Sanchez et al. 2009

Ablation Process

- Derive the mass flow rate, per unit area, of the sublimated material
  - One dimensional energy-balance at the illuminated spot
  - Absorbed laser beam per unit area P_l, the heat losses of conduction Q_{COND} and radiation Q_{RAD} respectively and the sublimation enthalpy of the target material E_v

$$\left(E_v + \frac{1}{2}\bar{v}^2 + C_p\left(T_s - T_0\right) + C_v\left(T_s - T_0\right)\right)\dot{\mu} = P_I - Q_{RAD} - Q_{COND}$$

Equivalent to increasing the enthalpy of sublimation by ~ 1-2 MJ/kg

$$Q_{RAD} = \sigma_{SB}\varepsilon\left(T_{SUB}^4 - T_{amb}^4\right)$$

Heating the gaseous ejecta from 3100-4747 K would consume ~ 2 MW/m^2 energy

$$k_A = k_{A0}\left(\frac{298}{T_{SUB}}\right)^{0.5}$$

[assuming a specific heat of 1361 J/kgK]

$$Q_{COND} = \left(T_{SUB} - T_0\right)\sqrt{\frac{C_V \rho_A k_A}{\pi t}}$$

Ablation Process

- Ablation temperature is also related to local pressure
  - Clausius-Clapeyron equation

$$\ln \frac{p_s}{p_{ref}} = \frac{E_V}{R}\left(\frac{1}{T_{ref}} - \frac{1}{T_{SUB}}\right)$$

T_{sub} corresponds to ρ_s

T_{ref} corresponds to ρ_{ref}

- Vapour pressure will increase with the temperature of the irradiated asteroid.

ABLATION MODEL

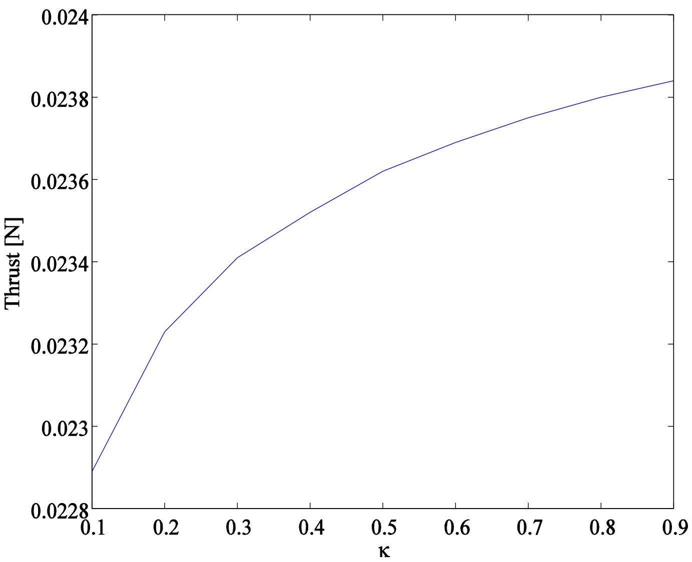
- Mass flow rate is also dependent on the local pressure at the interface between the Knudsen layer and ablated material
  - Hertz-Knudsen equation

$$\dot{\mu} = (1-k)\, p_s \left(\frac{1}{2\pi R_S T_{SUB}} \right)^{\frac{1}{2}}$$

where *k* is the fraction of molecules that re-condense at the interphase. p_s is the vapor pressure and R_s is the specific gas constant.

The fraction of molecules that re-condense is expected to increase with the local pressure.

However the change in thrust due to the re-condensation is limited. Over a wide range of k value the maximum variation in thrust is only 4 %



ABLATION MODEL

- Absorbed laser power per unit area P_I

$$P_I = \frac{\tau \tau_g \alpha_M \eta_L P_{IN}}{A_{spot}}$$

η_L efficiency of the laser system

P_{IN} input power to the laser,

α_M is the absorption at the spot

$$\alpha_M = \left(1 - \varepsilon_\alpha \alpha_s\right)$$

Albedo α_s of the asteroid and the increment in reflectively ε_α at the frequency of the laser beam.

τ_g accounts for the absorption of the laser in the rapidly expanding and absorbing plume of ejecta.

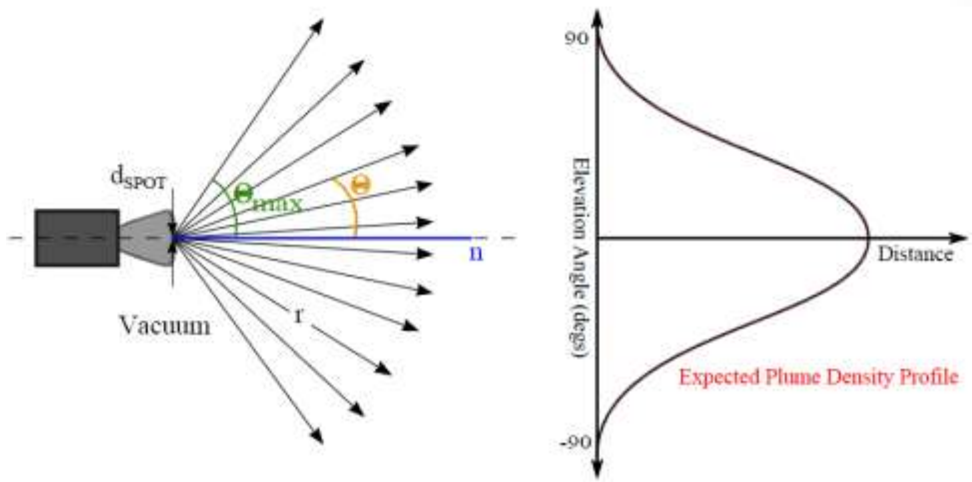
Expected, based on the experiment, to be 10-15 %

τ is the degradation factor caused by the re-condensed deposited ejecta

Contamination Process

- Degradation is computed by first calculating the plume density:

$$\rho(r,\theta) = \rho^* A_P \frac{d_{SPOT}^2}{\left(2r + d_{SPOT}^2\right)^2}\left[\cos\left(\frac{\pi\theta}{2\theta_{MAX}}\right)\right]^{\frac{2}{k-1}}$$



- Accumulative ejecta thickness:

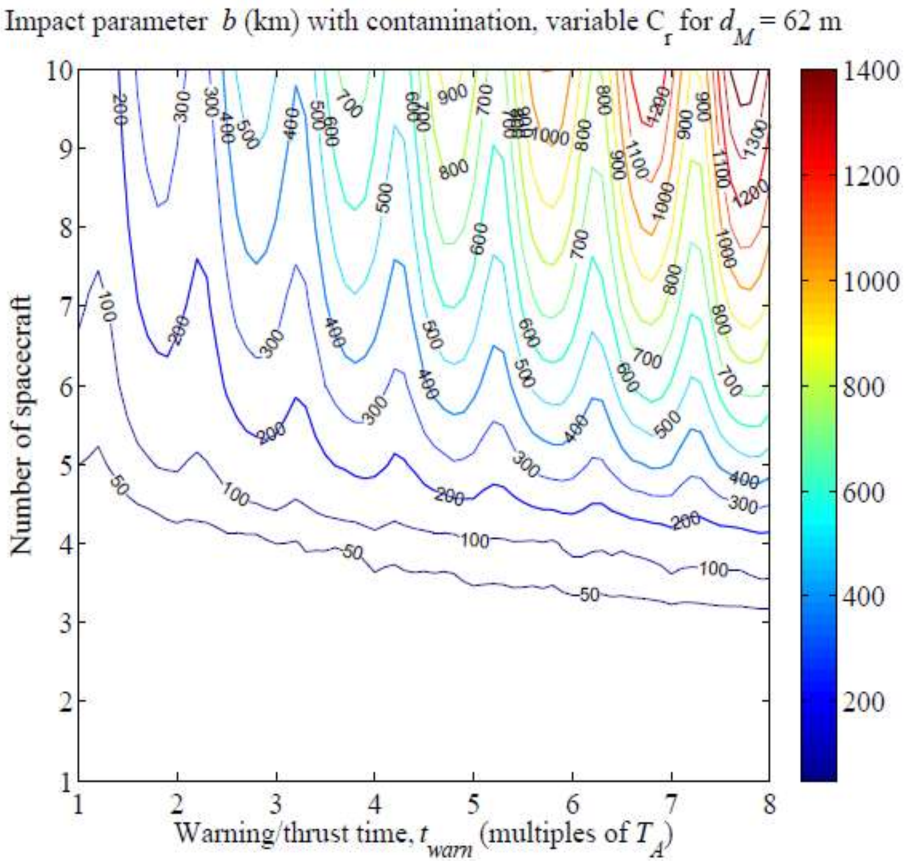
$$\frac{dh}{dt} = \frac{2\overline{v}\rho}{\rho_l}\cos\psi_{vf}$$

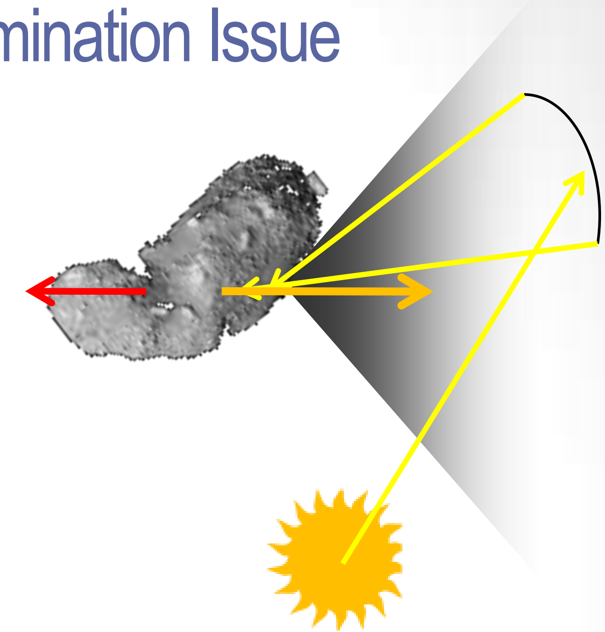
- Degradation factor:
  - Beer-Lambert-Bougier law

$$\tau = e^{-\eta h}$$

Direct Solar Concentration and the Contamination Issue

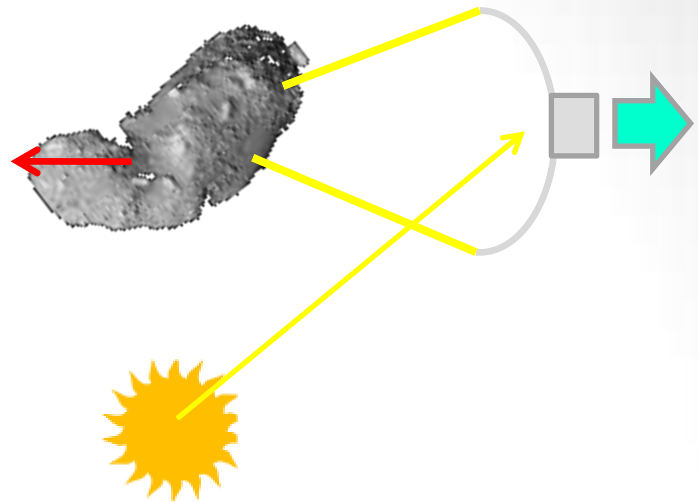
- Mirrors hovering at AEP, 800m from the target
- Distance limited by focusing capabilities
- High efficiency – up to 90%



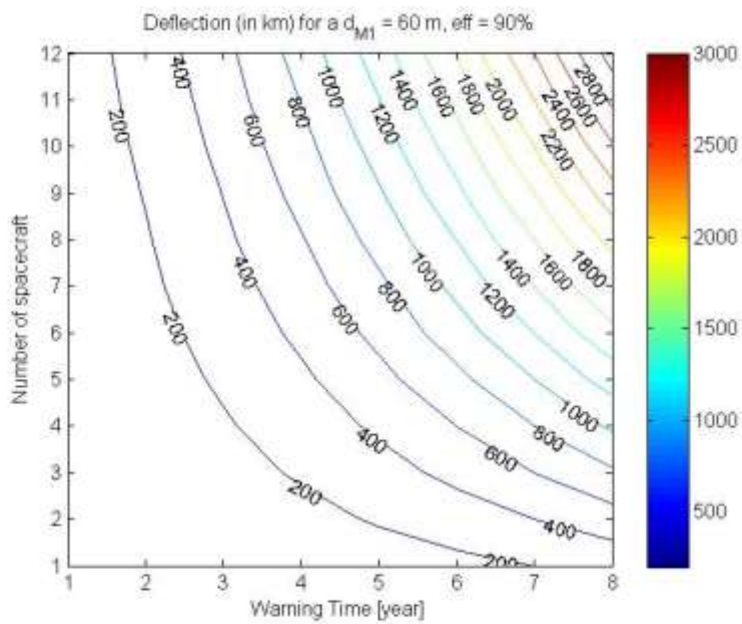


- Complete condensation of the whole contamination flow hitting the mirror

Light Tug Idea



- Combined light pressure and enhanced Yarkovsky effect.
- No contamination
- High energy efficiency
- Large deployable structures
- Propellant consumption to counterbalance light pressure



$$F_{light} = (2-\alpha_A)\sigma_M C_r \pi \frac{P_0 d_{spot}^2}{4}\left(\frac{r_{AU}}{r_S}\right)^2$$

$$F_{ir} = 2\frac{\varepsilon_A \sigma T^4}{c}\frac{d_{spot}^2}{4}$$

- Thermal balance:

$$\frac{K_A}{\rho_A c_A}\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

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Momentum Coupling and Deflection System Mass

Deflection System Mass

- Given a minimum deflection δ and time T available to achieve it, one can define the system mass ratio as:

$$\eta_m = \frac{m_d + m_E}{m_{s/c}}$$

where m_d is the mass required to deflect the asteroid, m_E is the energy required to deflect the asteroid, $m_{s/c}$ is the total mass of the spacecraft at launch.

- For example for a kinetic impactor we have:

$$\delta \mathbf{v}_A = \frac{m_d}{m_A + m_d} \Delta \mathbf{v}_{s/c}; \quad m_E = f(\Delta \mathbf{v}_{s/c})$$

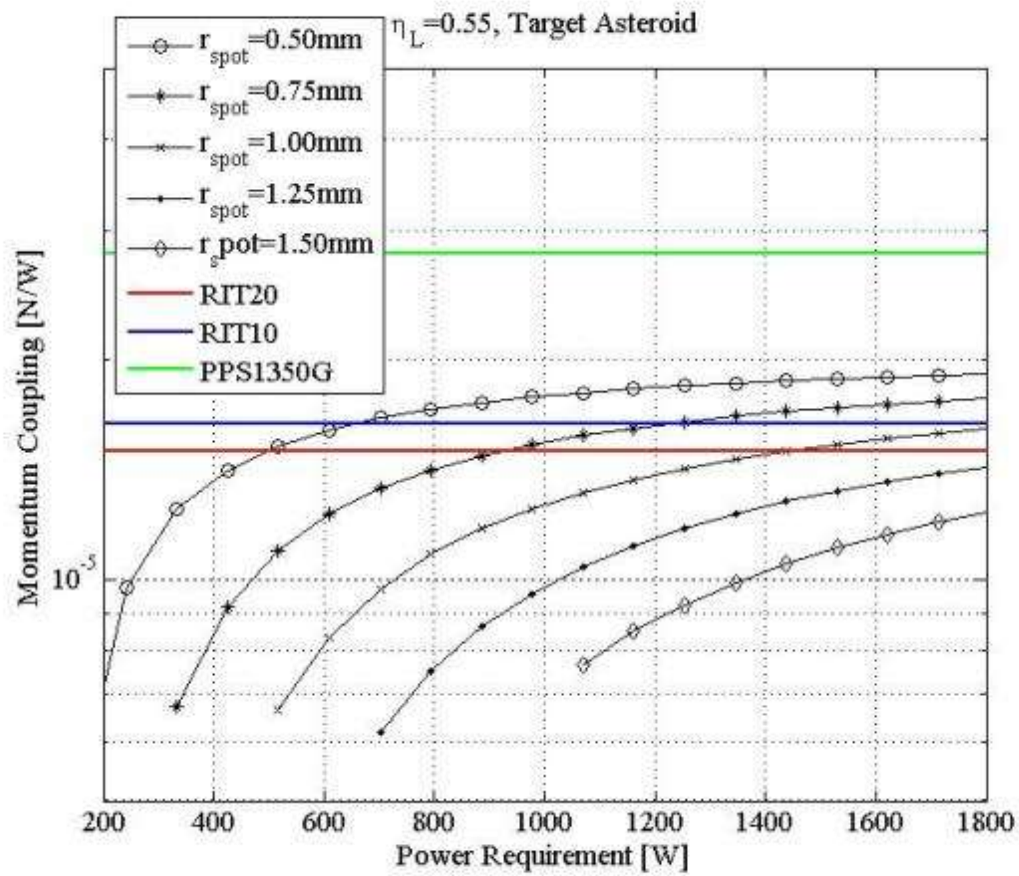
Momentum Coupling as a Performance Metric

- Let's take the ratio between the amount of power required to produce a deflection action P_{in} and the resulting variation of linear momentum F_d:

$$C_m = \frac{F_d}{P_{IN}}$$

- We call this coefficient the momentum coupling of the deflection action.
- This coefficient applies to all deflection methods for which the mass of the deflection system is a function of the installed power.

Momentum Coupling as a Performance Metric



$$C_m = \frac{F_{sub}}{P_{IN}}$$

$$P_{IN} = \tau \eta_P \eta_S \frac{P_{1AU} A_{SA}}{R_{AU}^2}$$

RIT20: Isp 4500 s, 150 mN, 5000 W
RIT10: Isp 3325, 15 mN,460 W; Green line - Hall Thruster

Deflection System Mass as a Performance Metric

- Evaluated the deflection-only mass of the laser ablation system to achieved a given Δv.

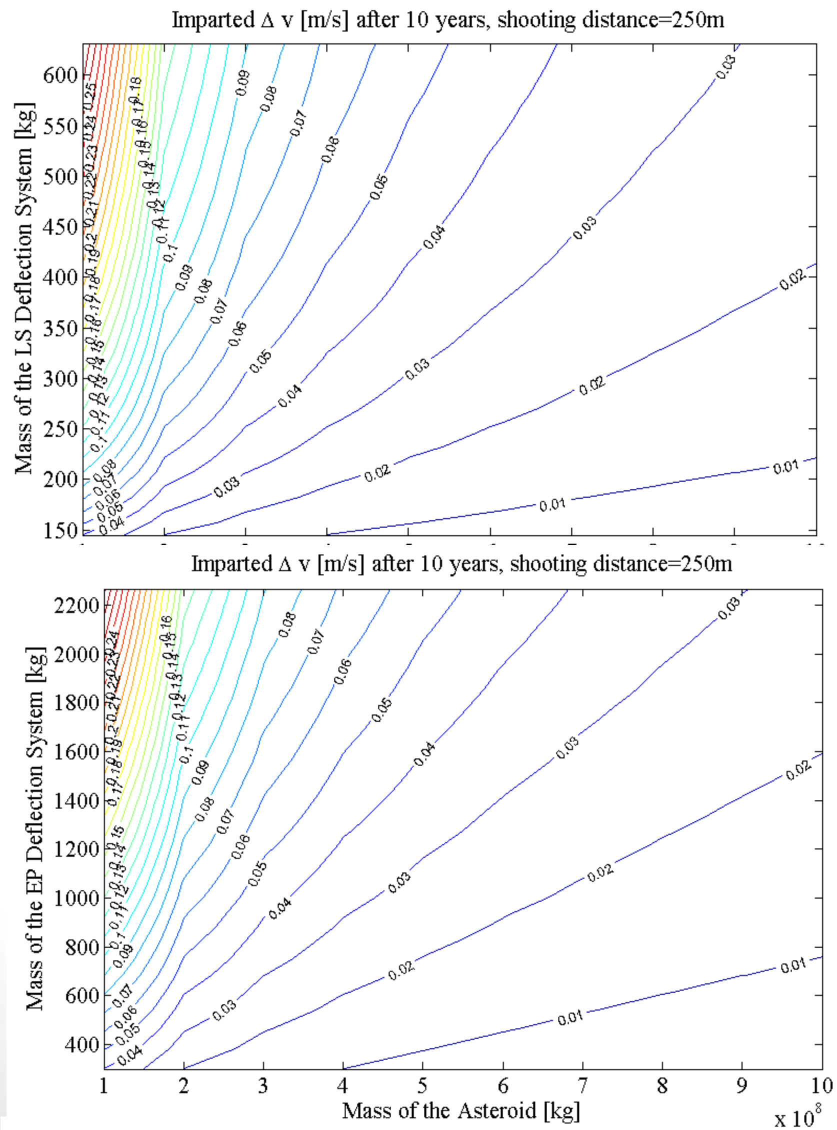
$$m_{LS} = \alpha_P P_{IN} + \rho_R A_R (1-\eta_L) \frac{P_{IN}}{\sigma \varepsilon_R T_R^4} + m_L$$

- Included:
  - Mass, emissivity and operating temperatures of the radiators;
  - The area and specific mass of the power system
  - Input power and efficiency of the laser system

- Compared to electrical propulsion, which has the same installed power of the laser and produces the same Δv.
- Included: the mass of two engines, power system, radiators, tanks and propellant

$$m_{EP} = 2.2 \frac{F_{EP}}{g_0 I_{sp}} \Delta t_{thrust} + \alpha_P P_{IN} + \rho_R A_R (1-\eta_{EP}) \frac{P_{IN}}{\sigma \varepsilon_R T_R^4} + 2 m_e$$

Low-Thrust vs. Sublimation



- Deflection mass for the same power input and delivering the same Δv.

$$C_m = \frac{F_{sub}}{P_{IN}}$$

- Low efficiency processes, such as laser ablation, can still generate a high thrust, for a low Isp:

$$\frac{2\eta P_{IN}}{g_0} = F I_{sp}$$

Optimal System Mass

- The mass of the last system can be modelled with a single analytic expression if one assumes that the contamination induces an exponential increase in the size of the arrays and the mass of the laser is proportional to the input power:

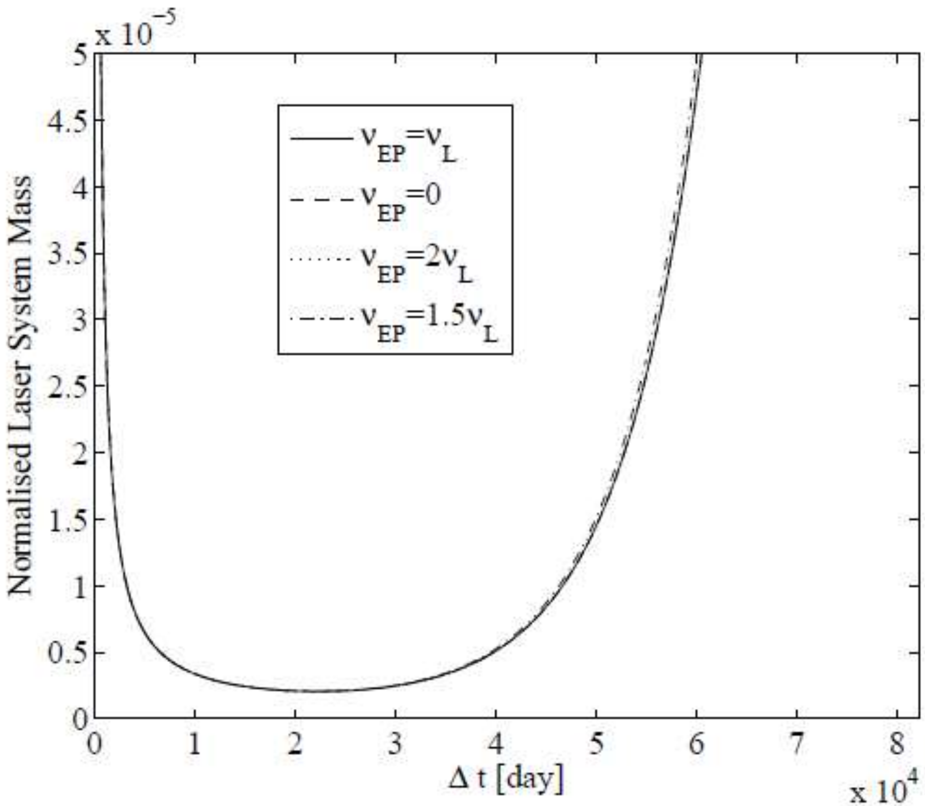
$$m_{LS} = \left[\alpha_P + \alpha_{SP} e^{-\nu_L \Delta t} + \frac{\rho_R A_R (1-\eta_L)}{\sigma \varepsilon_R T_R^4} + \mu_L \right] \frac{\Delta v m_A}{\Delta t C_m}$$

- Likewise the mass of an Electric Propulsion system can be expressed in a similar fashion including also the propellant consumption:

$$m_{EP} = \left[2.2 \frac{1}{g_0 I_{sp}} + \left(\alpha_P + \alpha_{SP} e^{-\nu_{EP} \Delta t} \right) \frac{I_{sp}}{2\eta \Delta t} + \rho_R A_R (1-\eta_{EP}) \frac{I_{sp}}{2\Delta t \eta \sigma \varepsilon_R T_R^4} + \frac{\mu_e I_{sp}}{\eta \Delta t} \right] \Delta v m_A$$

Optimal System Mass

$$\eta_{LS} = \left[\alpha_P + \alpha_{SP} e^{-\nu_L \Delta t} + \frac{\rho_R A_R (1-\eta_L)}{\sigma \varepsilon_R T_R^4} + \mu_L \right] \frac{m_A}{\Delta t C_m}$$



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Optimal System Mass

- One can now try to compute the derivative of the mass ratios for both the laser and the electric propulsion system to get:

$$\frac{d\eta_{LS}}{d\Delta t} = (\nu_L \Delta t - 1)\,\alpha_{SP} e^{\nu_L \Delta t} - \left[\alpha_P + \frac{\rho_R(1-\eta_L)}{\sigma \varepsilon_R T_R^4} + \mu_L\right] = 0$$

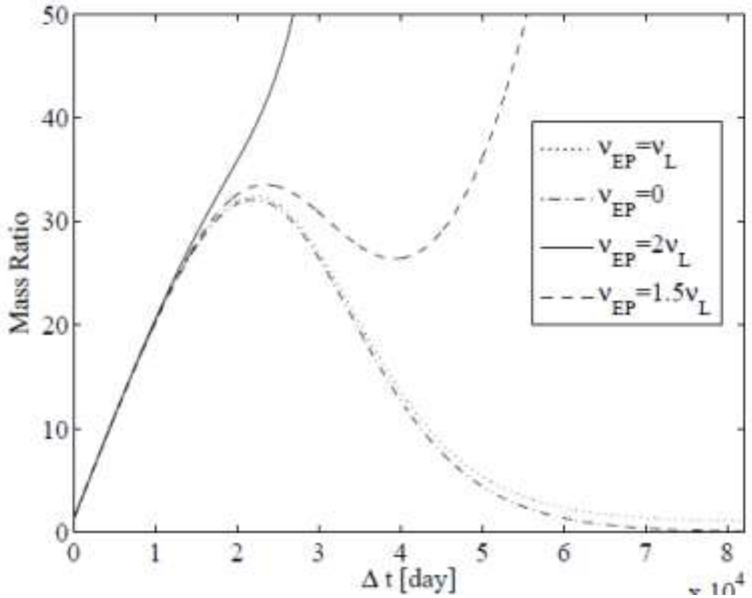
- The two derivatives can be put to zero to find the optimal time that minimises the mass ratio:

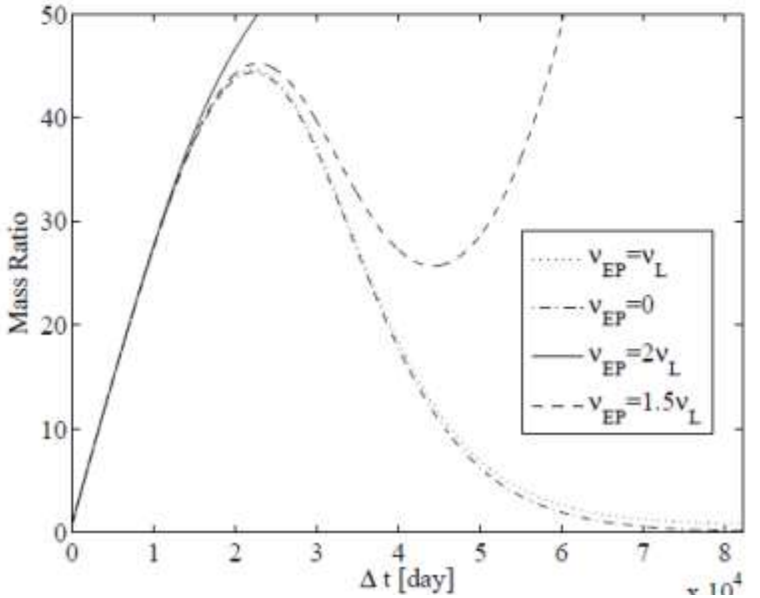
$$\frac{d\eta_{EP}}{d\Delta t} = (\nu_{EP} \Delta t - 1)\,\alpha_{SP} e^{\nu_{EP} \Delta t} - \left(\alpha_P + \frac{\rho_R(1-\eta_{EP})}{\sigma \varepsilon_R T_R^4} + 2\mu_e\right) = 0$$

- The two expressions are essentially identical and lead to the same conclusion on the optimal system mass.

Optimal System Mass

- One can also check the ratio between the mass of the EP system and the mass of the laser system for different contaminations.





$$\frac{m_{EP}}{m_{LS}} = \frac{2.2\frac{\Delta t}{g_0 I_{sp}} + \left[\alpha_P + \alpha_{SP} e^{\nu_{EP}\Delta t} + \frac{\rho_R(1-\eta_{EP})}{\sigma \varepsilon_R T_R^4} + 2\mu_e\right]\frac{I_{sp} g_0}{\eta_{EP}}}{\left[\alpha_P + \alpha_{SP} e^{\nu_L \Delta t} + \frac{\rho_R(1-\eta_L)}{\sigma \varepsilon_R T_R^4} + \mu_L\right]\frac{1}{C_m}}$$

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Uncertainty Quantification

Uncertainty in the Ablation Model

- The thrust is a function of the rate of mass expulsion:

$$\frac{dm_{\exp}}{dt} = 2n_{sc}v_{rot}\int_{y_0}^{y_{rot}}\int_{t_{in}}^{t_{out}}\frac{1}{H}\left(P_{in} - Q_{rad} - P_{cond}\right)dtdy$$

- The power input due to the solar concentrator is:

$$P_{in} = \eta_{sys}C_r\left(1-\varsigma_A\right)S_0\left(\frac{r_{AU}}{r_A}\right)^2$$

- The Black Body radiation loss and the conduction loss are:

$$Q_{rad} = \sigma\varepsilon_{bb}T^4 \qquad Q_{cond} = \left(T_{subl} - T_0\right)\sqrt{\frac{c_A k_A \rho_A}{\pi t}}$$

- The average velocity of the ejecta is given by:

$$\bar{v} = \sqrt{\frac{8k_B T_{subl}}{\pi M_{Mg\,2SiO_4}}}$$

- Thus the sublimation thrust is computed, under the assumption of tangential thrust, as:

$$\mathbf{u}_{sub} = \frac{\Lambda\bar{v}\dot{m}_{\exp}}{m_A}\hat{\mathbf{v}}_A$$

- Physical properties of the asteroid are known with a degree of uncertainty

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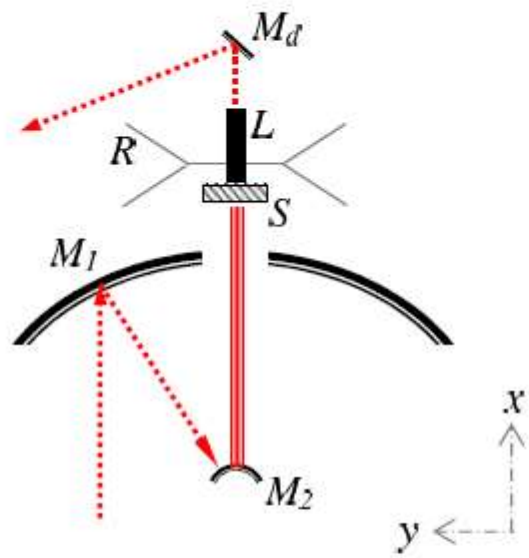
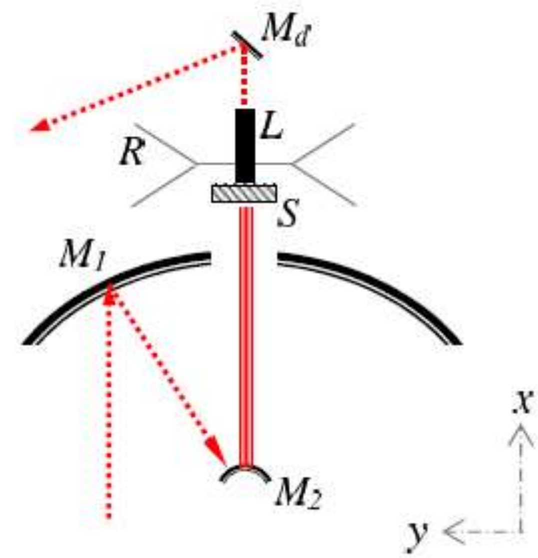
Spacecraft System Sizing



- Each spacecraft consists of:
  - A primary mirror M_1 which focuses the solar rays on the secondary mirror M_2.
  - A set of solar arrays S, which collect the radiation from the secondary mirror.
  - A semiconductor laser L.
  - A steering mirror M_d, which directs the Laser light on the target.
  - A set of radiators, which dissipate energy to maintain the Solar arrays and the Laser within acceptable limits.

Spacecraft System Sizing

- System sizing procedure:
  - The number of spacecraft n_{sc}, the primary mirror di… concentration ratio C_r are specified as design parar…
  - The radiator area is computed through steady state… the solar input power and the irradiated power.
  - The total mass of the spacecraft: $m_{sc} = m_{dry} + 1.1 m_p$
  - The dry mass: $m_{dry} = 1.2\left(m_C + m_S + m_M + m_L + m_R + m_{bus}\right)$



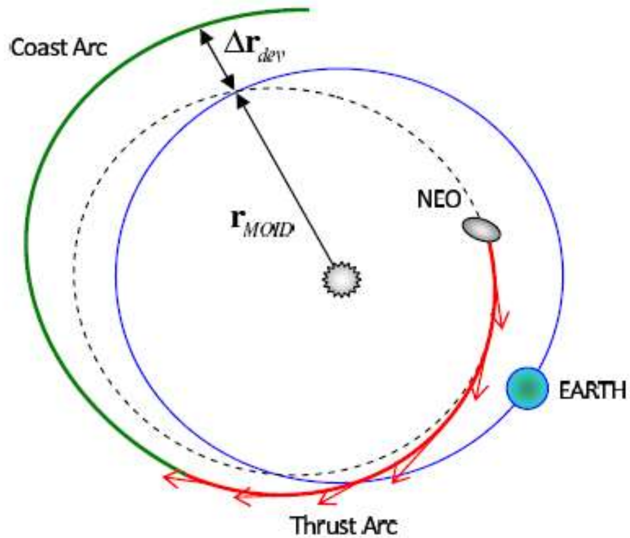
$$m_L = 1.5 \rho_L P_L \eta_L \qquad m_M = 1.25 \rho_M \left(A_d + A_{M_1} + 2A_{M_2}\right)$$

$$m_S = 1.15 \rho_S A_S \qquad m_R = \rho_R A_R \qquad \eta_{sys} = \eta_L \eta_{SA} \eta_P \varepsilon_M$$

- These quantities are the result of assumptions on technological readiness

Maximum Impact Parameter Problem

- Given a **spacecraft mass $m_{s/c}$** producing a deviation action $\mathbf{a}_d$ for a **time $\Delta t = t_e - t_i$** **maximise the impact parameter** on the b-plane at the expected time of the impact.



- In the Hill reference frame, this is computed as:

$$\Delta \mathbf{r}_{dev} = r_{A_{dev}} \boldsymbol{\Psi}\left(\mathbf{k}_{A_{dev}}, \mathbf{k}_{A_0}\right) - \begin{bmatrix} r_{A_0} \\ 0 \\ 0 \end{bmatrix} \rightarrow b^*$$

- With $\mathbf{k}_{A_0}$ and $\mathbf{k}_{A_{dev}}$ as the Keplerian elements of nominal and deflected asteroid orbits.

- **To compute $\mathbf{k}_{A_{dev}}$ one can use the analytical solution to the Gauss' Variational equations.**

Introduction to Evidence-based Reasoning

- Evidence Theory could be viewed as a generalisation of classical Probability Theory.
- Both aleatory (stochastic) and epistemic (incomplete knowledge) uncertainty can be modelled.
- Uncertain parameters $\mathbf{u}$ are given as intervals U_p and a probability m is associated to each interval.

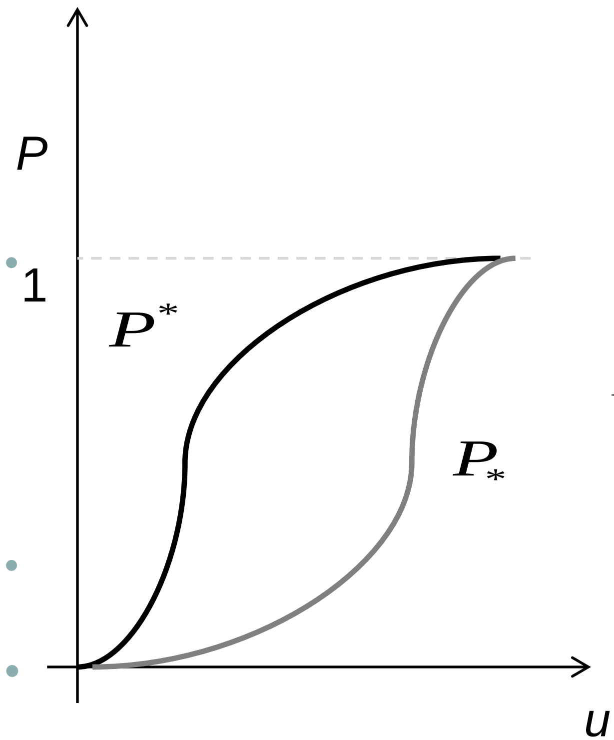
$$U_p = \left\{ \forall p : p \in [\underline{p}, \overline{p}] \right\}; \quad m(U_p) \in [0,1]$$

$$m(U_{p1}) + m(U_{p2}) + m(U_{p1} \cup U_{p2}) = 1$$

- Different uncertain intervals can be disconnected from each other or even overlapping.

Upper and Lower Expectations

Given the proposition (in set form):



$$A = \left\{ \mathbf{u} \mid f(\bar{\mathbf{d}}, \mathbf{u}) < v, \bar{\mathbf{d}} \in D, \mathbf{u} \in U \right\}$$

Lower Expectation (prevision):

$$E_* = \int_U I\left[f(u)\right] P^*(du)$$

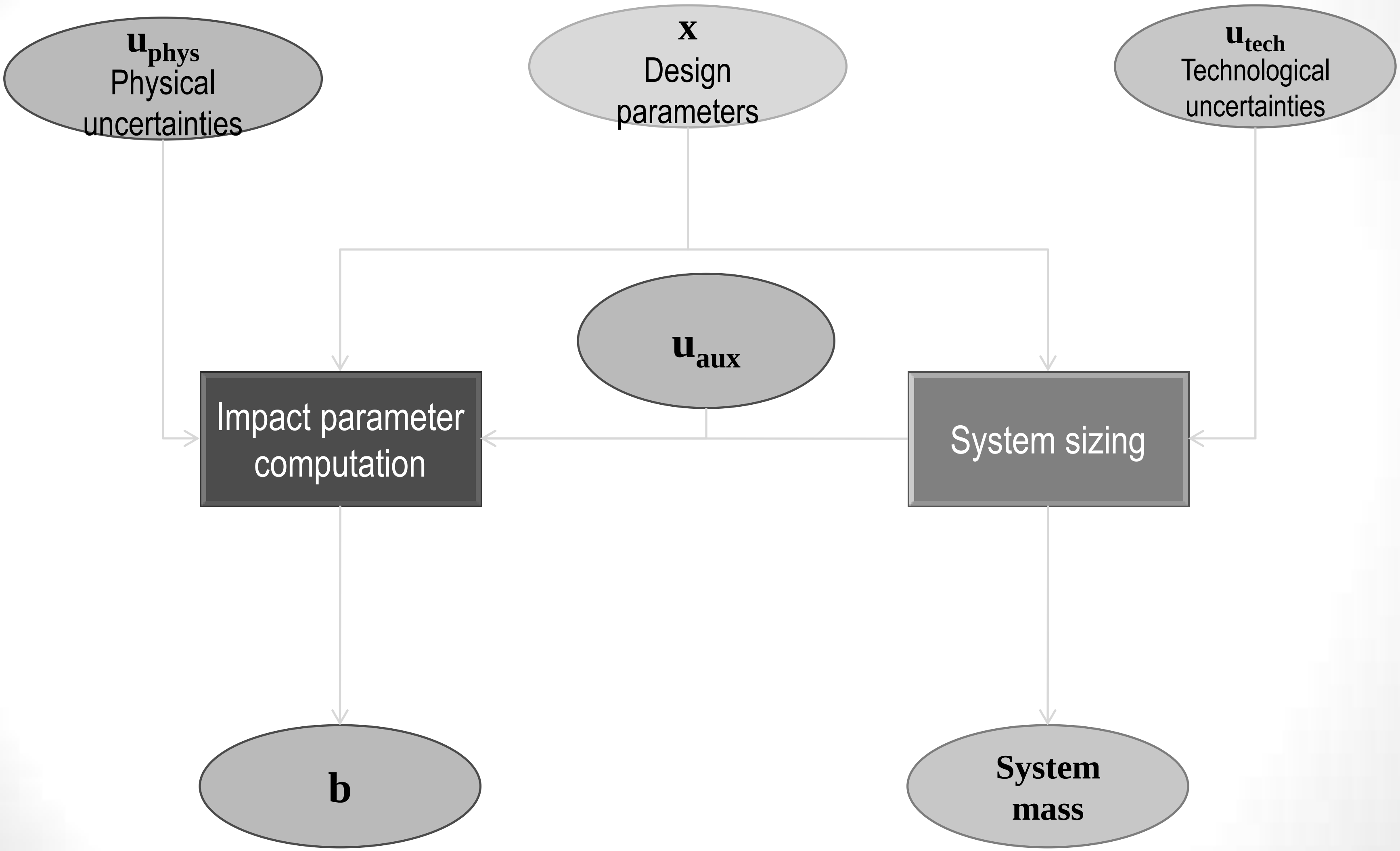
$$\rightarrow E_* = \int_U I_*\left[f\right] P(du)$$

Upper Expectation (prevision):

$$E^* = \int_U I\left[f(u)\right] P_*(du)$$

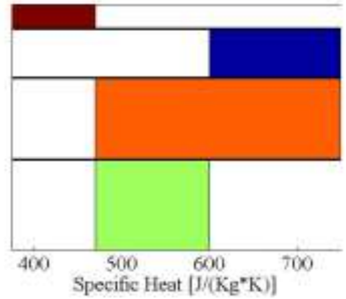
$$\rightarrow E^* = \int_U I^*\left[f\right] P(du)$$

Deflection and System Model Coupling

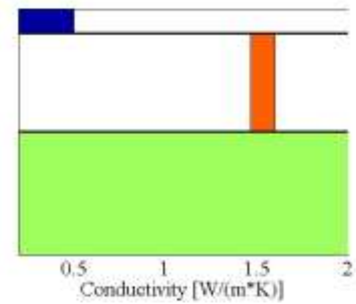


Interval summary (1): asteroid physical characteristics

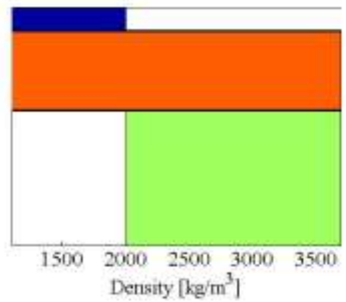
- Specific heat:



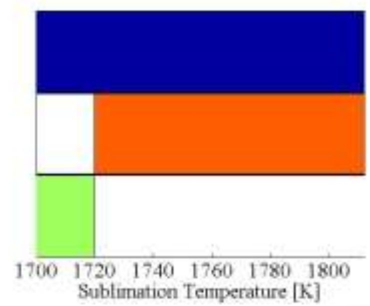
- Thermal conductivity:



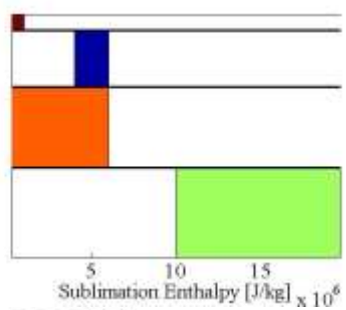
- Density:



- Sublimation Temperature:

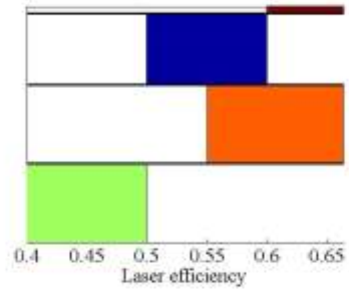


- Sublimation enthalpy:

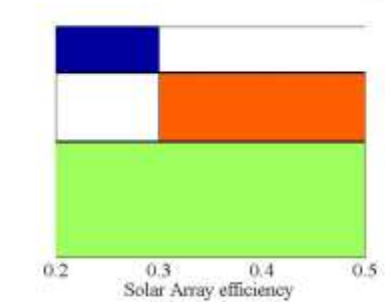


Interval summary (2): technological properties

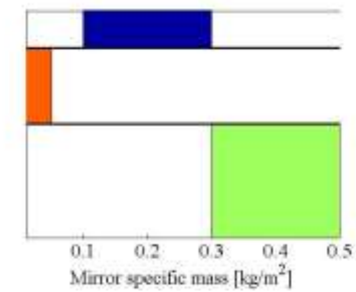
- Laser efficiency:



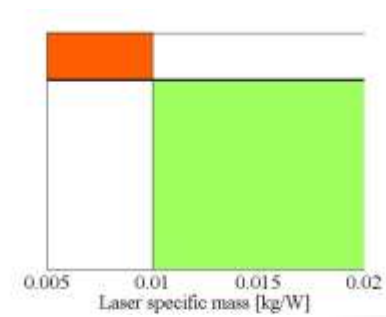
- Solar array efficiency:



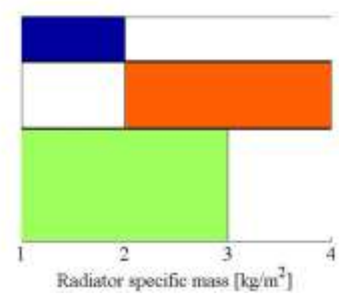
- Mirror specific mass:



- Laser specific mass:



- Radiator specific mass:



Integrated System and Trajectory Optimisation

- Minimum total spacecraft mass and maximum impact parameter variation:

$$\min_{\mathbf{x}\in D}\left[m_{system} \quad -b\right]$$

- Where **x** is given by the 3 design parameters:
  - Diameter of the primary mirror: $d_m \in [2, 20]m$
  - Number of spacecraft in the formation: $n_{SC} \in [1,10]$
  - Concentration ratio: $C_r \in [1000, 3000]$

- Mixed integer-nonlinear multiobjective optimisation problem
- Solution with Multi-Agent Collaborative Search (MACS2) a memetic stochastic optimiser.

Integrated System and Trajectory Optimisation Under Uncertainty

- Collection of focal elements are mapped into a unit hypercube $\bar{U}$

- The maximum over the hypercube defines the worst case values of the cost functions under uncertainty.
  - "minmax", i.e. optimised worst case scenario

$$\min_{\mathbf{x}\in D}\left[\max_{\mathbf{u}\in\bar{U}} m_{system} \quad \max_{\mathbf{u}\in\bar{U}}(-b)\right]$$

- The minimum over the hypercube defines the best case values of the cost functions under uncertainty.
  - "minmin", i.e. optimised best case scenario

$$\min_{\mathbf{x}\in D}\left[\min_{\mathbf{u}\in\bar{U}} m_{system} \quad \min_{\mathbf{u}\in\bar{U}}(-b)\right]$$

- Minimax mixed integer nonlinear programming problems. Solution with minmax version of MACS.

Deterministic vs Robust

- Deterministic multi-objective optimization problem:

$$\min_{\mathbf{x}\in D}\left[m_{system}\ \ -b\right]$$

- “minmax” case:

$$\min_{\mathbf{x}\in D}\left[\max_{\mathbf{u}\in U}\left(m_{system}\right)\ \ \max_{\mathbf{u}\in U}\left(-b\right)\right]$$

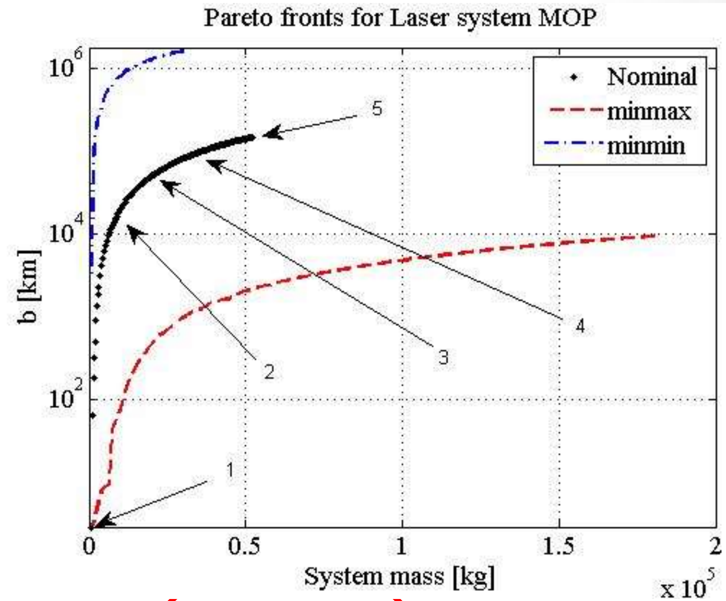
- “minmin” case:

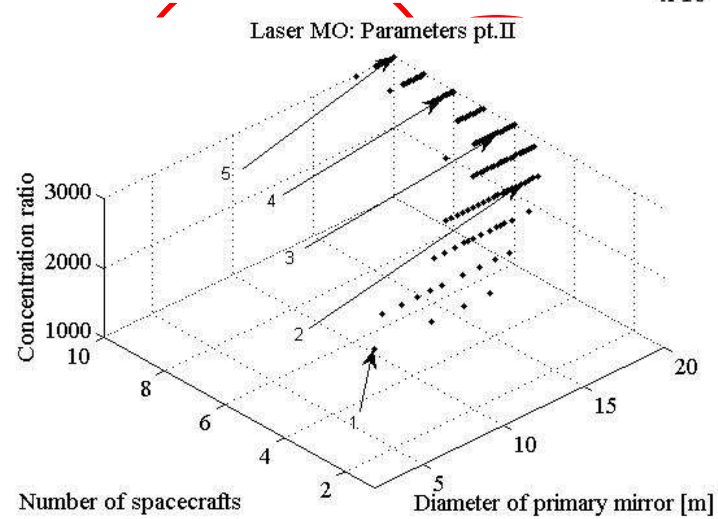
$$\min_{\mathbf{x}\in D}\left[\min_{\mathbf{u}\in U}\left(m_{system}\right)\ \ \min_{\mathbf{u}\in U}\left(-b\right)\right]$$

Performance parameters could be significantly sensitive to uncertainties on physical and technological parameters

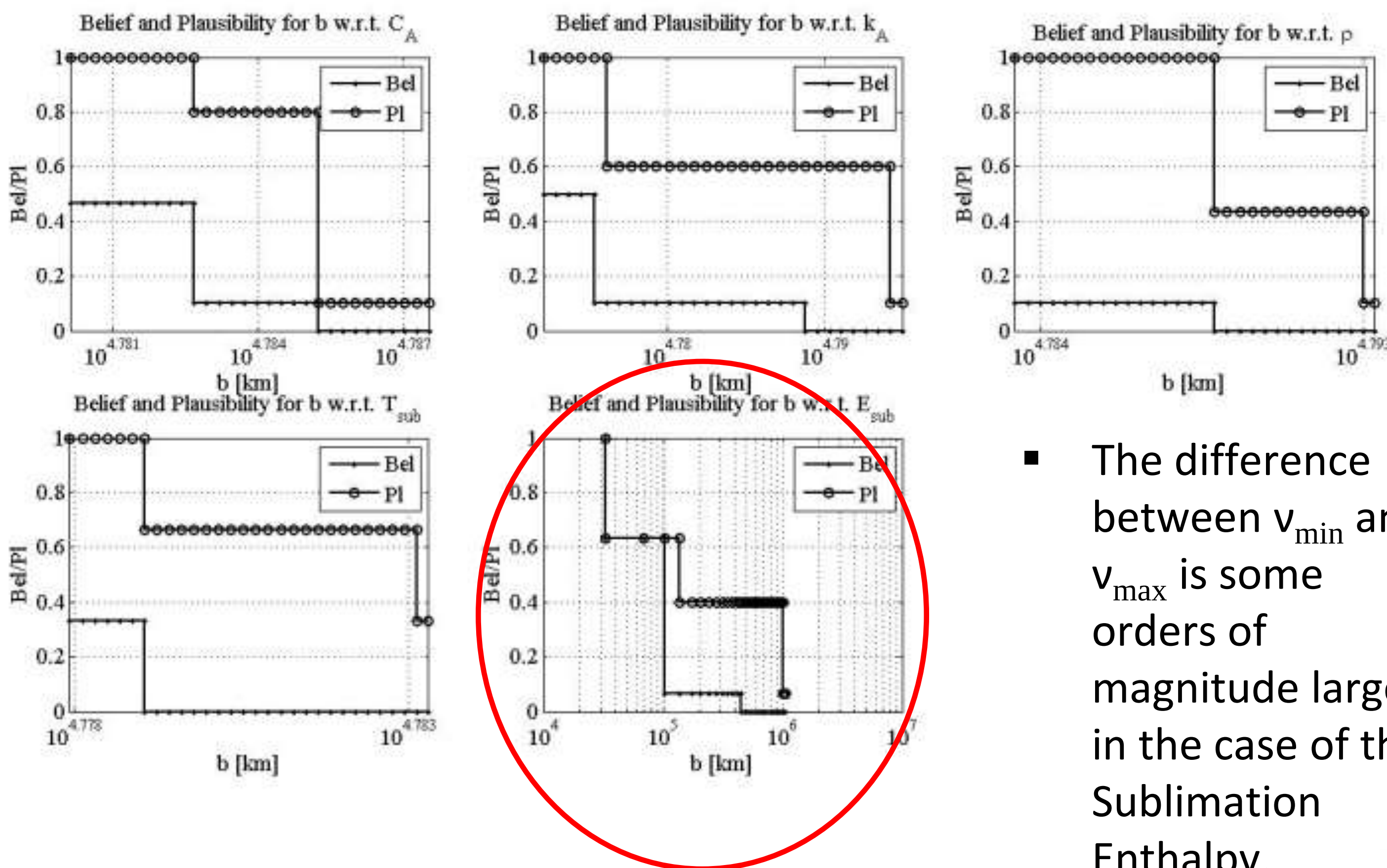
- The Pareto Sets show a switch between two families of designs:
  - In the “minmax” case, solutions with a high number of spacecraft and a small primary mirror are preferred (Many spacecraft to compensate for their low individual efficiency).
  - In the “minmin” case, solutions with a low number of spacecraft and a large primary mirror are preferred (Few spacecraft but very efficient).

Five design points are selected for further analysis.





Belief/Plausibility Curves for Single Uncertain Parameter



- The difference between ν_{min} and ν_{max} is some orders of magnitude larger in the case of the Sublimation Enthalpy.

Reference Material

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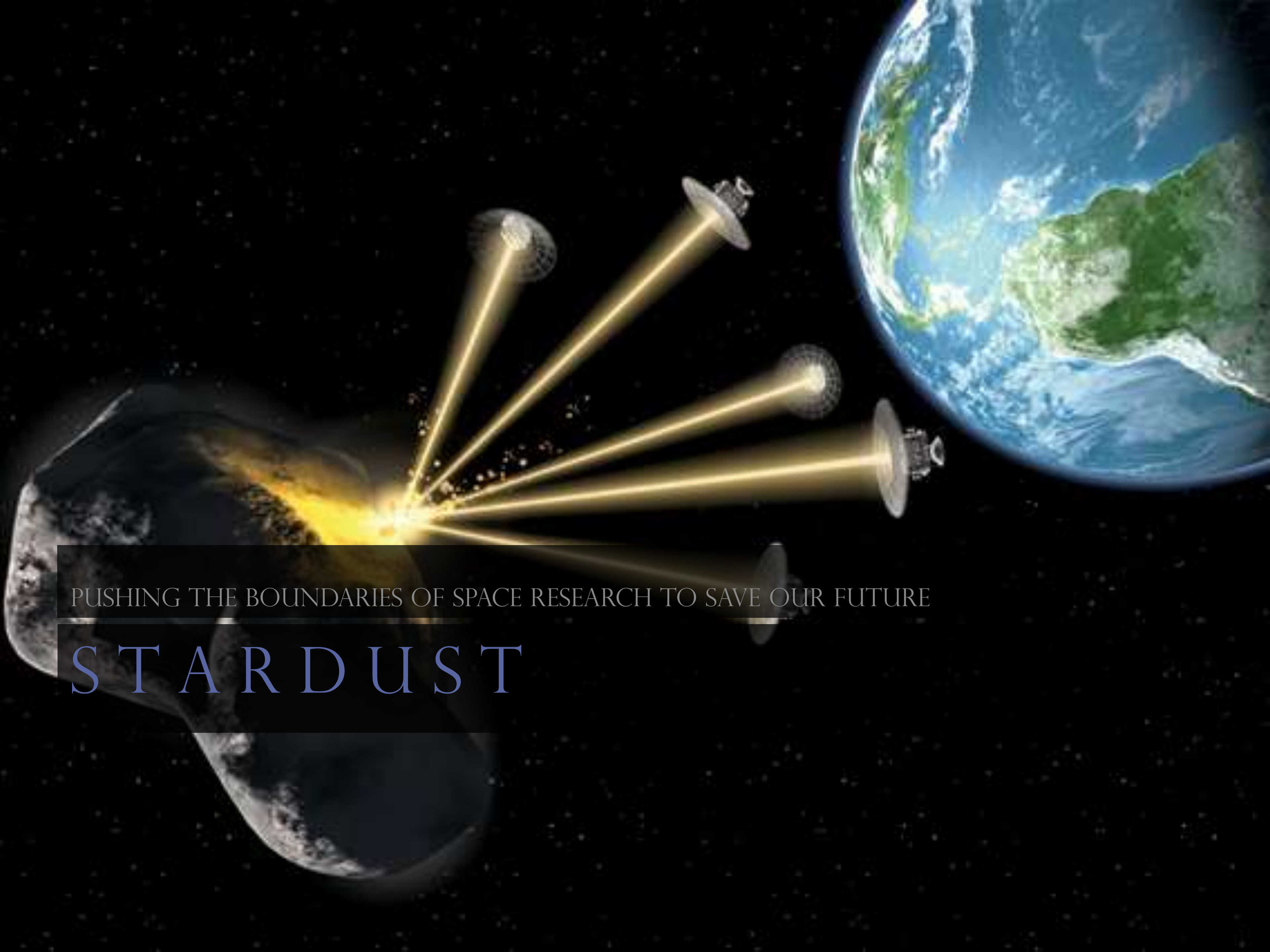
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