Lecture 20.

Methods for solving the radiative transfer equation with multiple scattering. Part 2: Inclusion of surface reflection and emissivity.

Objectives:

1. Surface reflection (reflectance).
2. Inclusion of the surface reflection into the radiative transfer equation.
3. Surface emissivity. Inclusion of the surface emissivity into the radiative transfer equation.

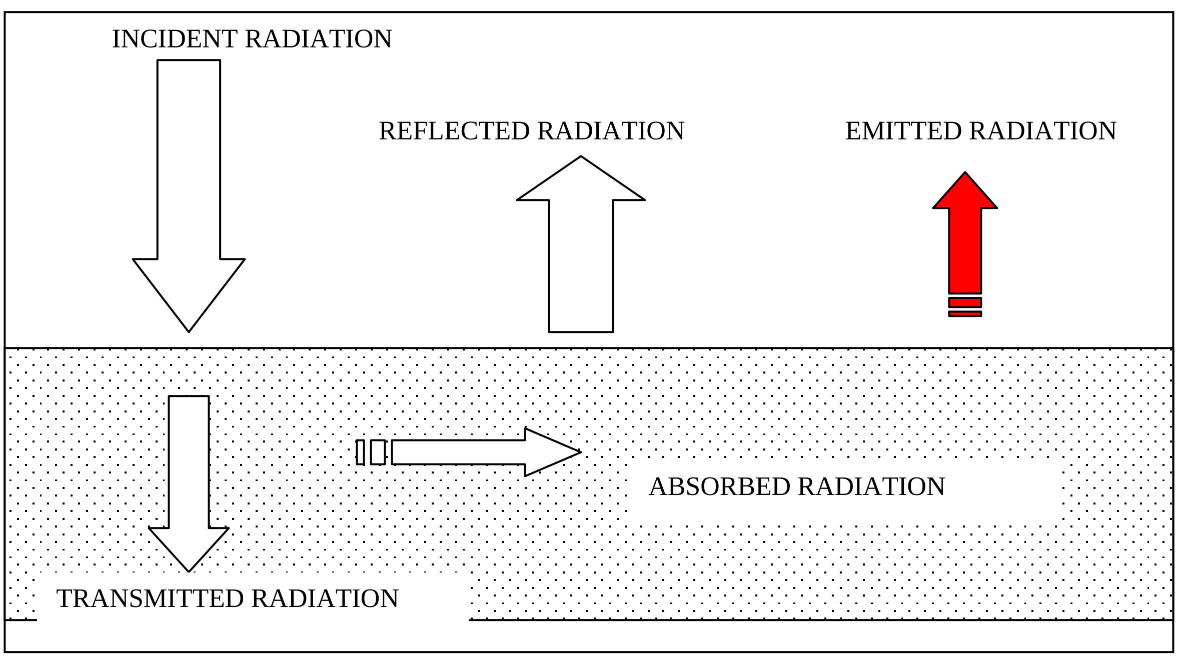
Required reading:

L02: 6.3.5

1. Surface reflection.

The ocean and land surfaces can modify the atmospheric radiation field by

a) reflecting a portion of the incident radiation back into the atmosphere;
b) transmitting some incident radiation;
c) absorbing a portion of incident radiation (Kirchhoff's law);
d) emitting the thermal radiation (Kirchhoff's law);



Types of reflection

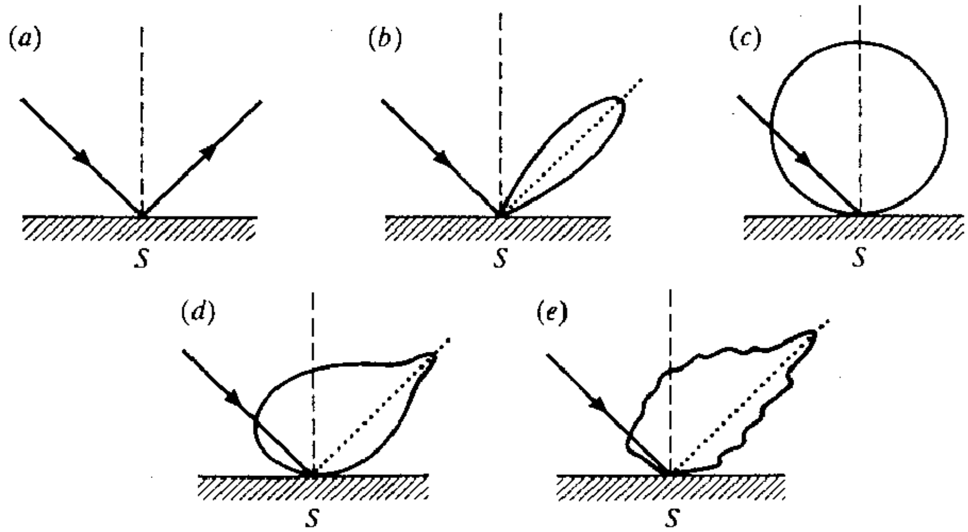


Figure 20.1 Schematic illustration of different types of surface scattering. The lobes are polar diagrams of the scattered radiation: (a) specular, (b) quasi-specular, (c) Lambertian, (d) quasi-Lambertian, (e) complex.

Two extreme types of the surface reflection:

specular reflectance and **diffuse reflectance**.

Specular reflectance is the reflectance from a perfectly smooth surface (e.g., a mirror):

Angle of incidence =Angle of reflectance

- Reflection is generally **specular** when the "roughness" of the surface is smaller than the wavelength used. In the solar spectrum (0.4 to 2 μm), reflection is therefore specular on smooth surfaces such as polished metal, still water or mirrors.

NOTE: While incoming solar light is unpolarized, reflected waves are generally polarized and Fresnel's laws can be used to determine polarization.

- Practically all real surfaces are not smooth and the surface reflection depends on the incident angle and the angle of reflection. Reflectance from such surfaces is called the **diffuse reflectance**.

Bi-directional reflectance distribution function (BRDF), $\rho(\mu,\varphi,-\mu',\varphi')$ is introduced to characterize the angular dependence in the surface reflection and defined as the ratio of the reflected intensity to the energy flux in the incident beam:

$$\rho(\mu,\varphi,-\mu',\varphi') = \frac{\pi dI^{\uparrow}(\tau^*,\mu,\varphi)}{I^{\downarrow}(\tau^*,-\mu',\varphi')\mu' d\Omega'} \qquad [20.1]$$

NOTE: Each type of surfaces has a specific spectral **BRDF**.

Reciprocity law: $\rho(\mu,\varphi,-\mu',\varphi') = \rho(-\mu',\varphi',\mu,\varphi)$

Upwelling radiance is an integral over BRDF and downwelling radiance

$$I_r^{\uparrow}(\tau^*,\mu,\varphi) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \rho(\mu,\varphi,-\mu',\varphi') I^{\downarrow}(\tau^*,-\mu',\varphi')\mu' d\mu' d\varphi' \qquad [20.2]$$

Surface albedo is defined as the ratio of the surface upwelling to downwelling flux:

$$r_{sur} = \frac{F^{\uparrow}}{F^{\downarrow}} = \frac{1}{\pi}\frac{\int_0^{2\pi}\int_0^1\int_0^{2\pi}\int_0^1 \rho(\mu,\varphi,-\mu',\varphi') I^{\downarrow}(-\mu',\varphi')\mu' d\mu' d\varphi' \mu d\mu d\varphi}{\int_0^{2\pi}\int_0^1 I^{\downarrow}(-\mu',\varphi')\mu' d\mu' d\varphi'} \qquad [20.3]$$

NOTE: Eq.[20.3] is similar to Eq.[19.4] , except that the latter is written for the collimated (direct) incident field.

- A surface is called the **Lambert surface** if it obeys **the Lambert's Law.**

Lambert's Law of diffuse reflection: the diffusely reflected light is isotropic and unpolarized independently of the state of polarization and the angle of the incidence light. **For the Lambert surface,** BRDF is independent on the directions of incident and observed light beam.

$$\rho(\mu,\varphi,-\mu',\varphi') = \rho_L \qquad [20.3]$$

For the Lambert surface, from Eq.[20.2], we have

$$I_r^{\uparrow}(\tau^*,\mu,\varphi) = \frac{\rho_L}{\pi} F^{\downarrow} \qquad [20.4]$$

and from Eq.[20.3], we have $\quad r_{sur} = \rho_L \qquad$ [20.5]

- ***Reflectance from ocean surfaces***

Ocean reflection depends on the ocean surface (and near surface) conditions: waves, whitecaps and suspended particulates. Many models have been developed to account for these factors by introducing corrections to the Fresnel reflection. One of the most widely used models is the Cox and Munk (1954) model.

- ***Reflectance from land surfaces (soils, vegetation):***

BRDFs are controlled by the physical structure of the surface (*e.g.*, the density and three-dimensional arrangement of plant leaves and stems, and the surface roughness of the soil substrate) and the optical properties of its component elements (*e.g.*, the spectral reflectance and transmittance of leaves, stems and soil facets). Numerous models have been developed to describe and account for these relationships. These models are generally formulated as follows

$$\rho_\lambda = f_{\lambda,iso} + f_{\lambda,geo} k_{geo} + f_{\lambda,vol} k_{vol}$$

where k_{geo} and k_{vol} are the model `kernels', and $f_{\lambda\ \text{iso}}$, $f_{\lambda\ \text{geo}}$ and $f_{\lambda\ \text{vol}}$ are spectrally-dependent weighting factors. The `kernels' are trigonometric functions that describe the shape of the BRDF in terms of the solar illumination and sensor view angles. They are derived from approximations to, and simplifications of, the principles of geometrical-optics (k_{geo}) and radiative-transfer theory (k_{vol}). Each kernel is multiplied by a factor, f_λ, that weights the relative contribution of surface-scattering ($f_{\lambda\ \text{geo}}$) and volume-scattering ($f_{\lambda\ \text{vol}}$) to the measured BRDF. The term $f_{\lambda\ \text{iso}}$ is included to account for isotropic scattering from the surface (in practice, this includes contributions from both single-scattering and multiple-scattering).

Thus, the model is formulated so that bidirectional reflectance is a linear combination of three terms weighted by three parameters: an **isotropic** function accounting for bidirectional reflectance with nadir viewing and the overhead sun; a **geometric** function accounting for the effects of shadows and the geometrical structure of protrusions from a background surface; and a **volume scattering** function based on radiative transfer accounting for reflectance by a collection of randomly-dispersed facets.

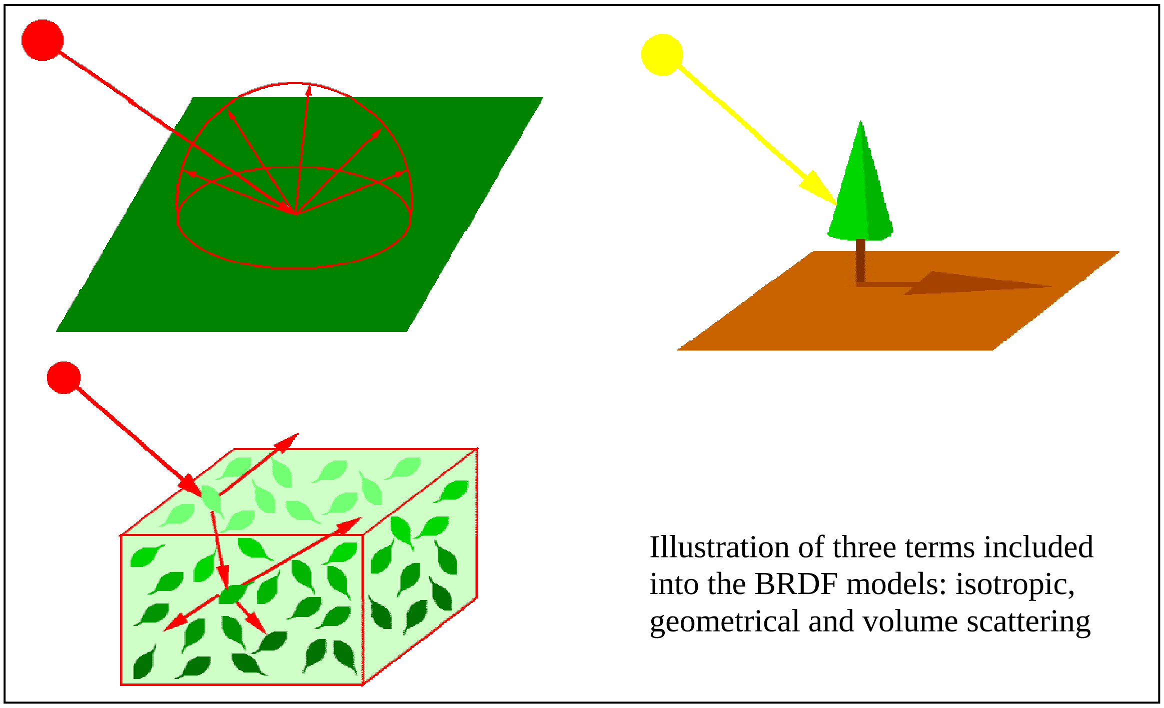
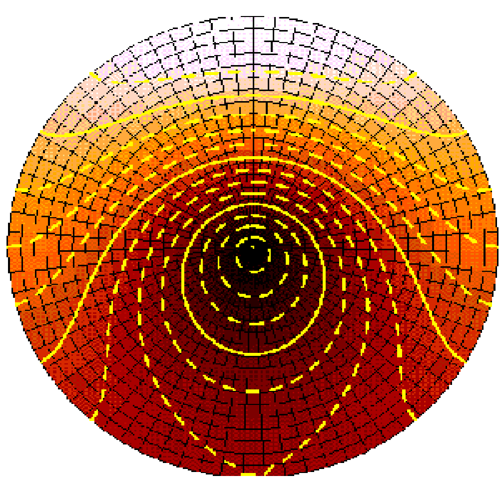


Illustration of three terms included into the BRDF models: isotropic, geometrical and volume scattering

✓ A notable feature of the BRDF for natural surfaces is the **hot spot** - a peak in reflectance for direct backscatter ($\Theta=180^0$).

Hot spot are caused by:

1) the lack of observed shadows and
2) specular reflection from oriented leaves



- **In general, the surface reflectance is a function of wavelength**.

Examples of the surface albedo at about 550 nm: fresh snow/ice =0.8-0.9; desert=0.3, soils=0.1-0.25; ocean=0.05.

Examples of the spectral surface reflectance.

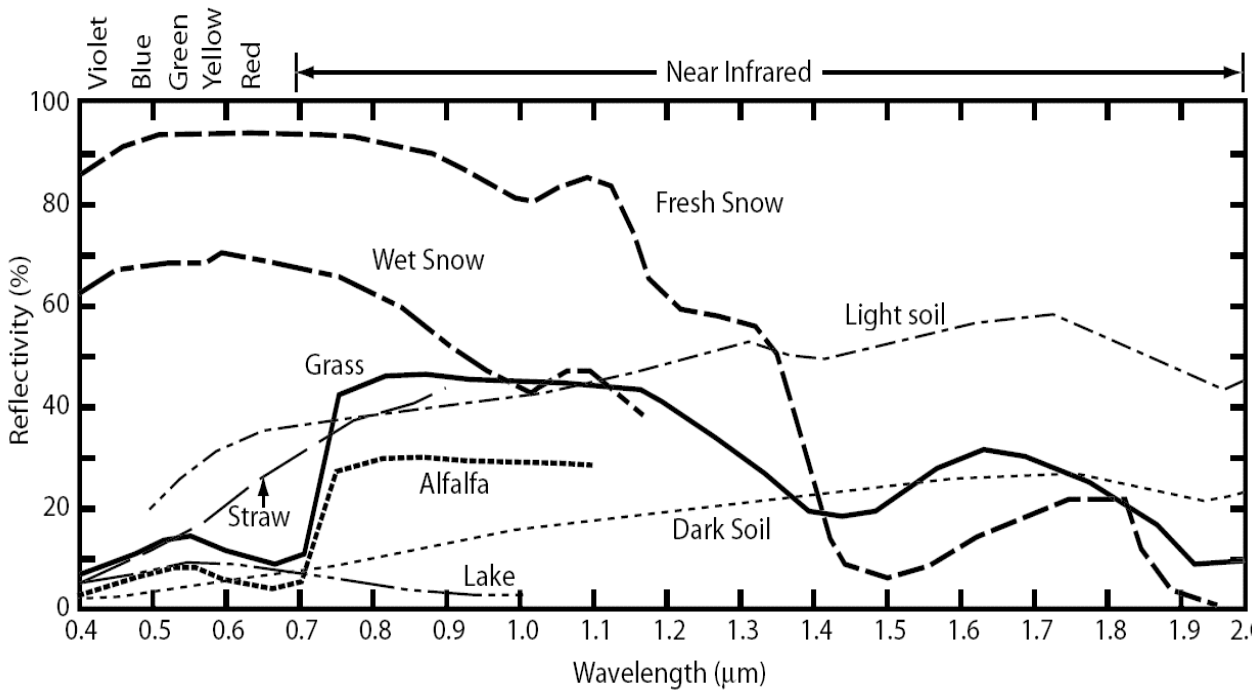


Figure 20.2 Typical shortwave spectral reflectances of various natural surfaces.

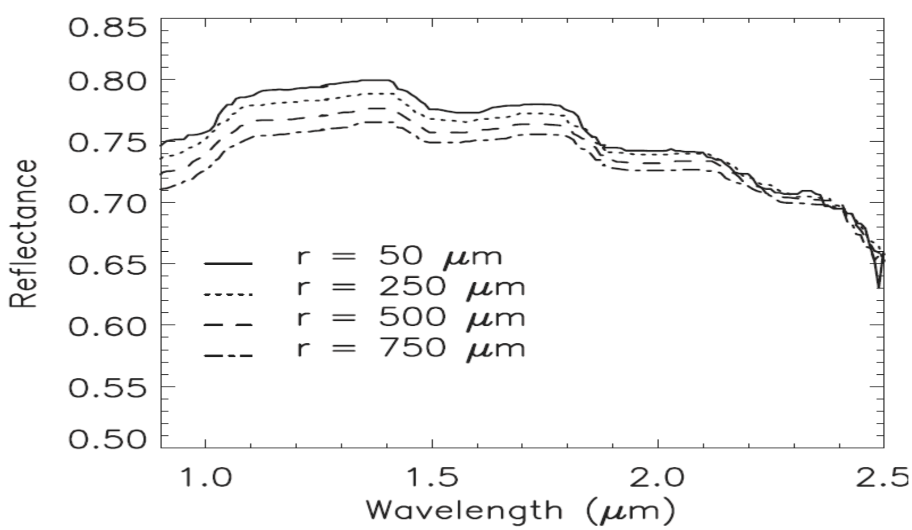


Figure 20.3 Reflectance of soil quartz grains of different sizes (model results, Okin and Painter, 2004)

2. Inclusion of the surface reflection into the radiative transfer equation.

Let's include the contribution from the Lambert surface.

Lambert surface: $I^{\uparrow}(\tau^*, \mu, \varphi) = I_{sur} = const$ [20.6]

Generalizing the definitions for the reflection and transmission functions (i.e., Eqs.[19.1—[19.2]), we may express the reflected diffuse intensity $I_r^{\uparrow}(0, \mu, \varphi)$ and transmitted diffuse intensity $I_t^{\downarrow}(\tau^*, -\mu, \varphi)$ as

$$I_r^{\uparrow}(0, \mu, \varphi) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 R(\mu, \varphi, \mu', \varphi') I_{inc}(-\mu', \varphi')\mu' d\mu' d\varphi' \qquad [20.7]$$

$$I_t^{\downarrow}(\tau^*, -\mu, \varphi) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 T(\mu, \varphi, \mu', \varphi') I_{inc}(-\mu', \varphi')\mu' d\mu' d\varphi' \qquad [20.8]$$

The reflected intensity at the top of the layer including the surface reflection may be written as

$$I^*(0,\mu,\varphi) = I^{\uparrow}(0,\mu,\varphi) + \frac{1}{\pi}\int_0^{2\pi}\int_0^1 T(\mu,\varphi,\mu',\varphi') I_{sur}\mu' d\mu' d\varphi' + I_{sur}\exp(-\tau^*/\mu) \qquad [20.9]$$

NOTE: The second term on the right-hand side gives the contribution from the surface reflected intensity which is diffusely transmitted to the top of the layer, whereas the third term gives the contribution from the surface reflected intensity which is the directly transmitted.

We can re-write Eq.[20.9] as

$$I^*(0,\mu,\varphi) = \mu_0 F_0 R(\mu,\varphi,\mu_0,\varphi_0) + I_{sur}\gamma(\mu) \qquad [20.10]$$

where $\gamma(\mu) = \exp(-\tau^*/\mu) + t(\mu)$

Now, let's consider the diffuse transmitted intensity. Isotropic intensity ***I_{sur}***, propagating upward in the layer after being scattered by the Lambertion surface, can be partially reflected back to the surface and, hence, contribute to the downward intensity in the additional amount

$$I_{add}^{\downarrow}(-\mu) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 R(\mu,\varphi,\mu',\varphi') I_{sur}\, \mu' d\mu' d\varphi' = I_{sur}\, r(\mu)$$

Thus, the transmitted intensity including the surface contribution is

$$I^*(\tau^*,-\mu,\varphi) = I^{\downarrow}(\tau^*,-\mu,\varphi) + I_{sur}\, r(\mu) = \mu_0 F_0 T(\mu,\varphi,\mu_0,\varphi_0) + I_{sur}\, r(\mu)$$

[20.11]

Both Eqs.[20.10] and [20.11] have $\boldsymbol{I_{sur}}$. Thus, we need to find $\boldsymbol{I_{sur}}$.

$$\pi I_{sur} = (Surface\ albedo)\ x(Downward\ \ flux)$$

The downward flux has three components:

(1) Transmitted direct flux = $\mu_0 F_0 \exp(-\tau^*/\mu_0)$

(2) Transmitted diffuse flux=

$$\int_0^{2\pi}\int_0^1 I^{\downarrow}(\tau^*,-\mu,\varphi)\mu d\mu d\varphi = \int_0^{2\pi}\int_0^1 \frac{\mu_0 F_0}{\pi} T(\mu,\varphi,\mu_0,\varphi_0)\mu d\mu d\varphi = \mu_0 F_0 t(\mu_0)$$

(3) Fraction of $\boldsymbol{I_{sur}}$ reflected by the atmosphere back to the surface =

$$\int_0^{2\pi}\int_0^1 I_{add}^{\downarrow}(-\mu)\mu d\mu d\varphi = \pi I_{sur}\, \bar{r}$$

Therefore

$$\pi I_{sur} = r_{sur}\left(\mu_o F_0 \exp(-\tau^*/\mu_0) + \mu_0 F_0 t(\mu_0) + \pi I_{sur}\bar{r}\right)$$

and rearranging term, we have

$$I_{sur} = \frac{r_{sur}}{1 - r_{sur}\bar{r}} \frac{\mu_0 F_0}{\pi}\gamma(\mu_0)$$

Therefore, the diffuse reflected and transmitted intensities, accounting for the surface contribution are

$$\boxed{I^*(0,\mu,\varphi) = I^{\uparrow}(0,\mu,\varphi) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \frac{\mu_0 F_0}{\pi}\gamma(\mu_0)\gamma(\mu)} \quad [20.12a]$$

$$\boxed{I^*(\tau^*,-\mu,\varphi) = I^{\downarrow}(\tau^*,-\mu,\varphi) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \frac{\mu_0 F_0}{\pi}\gamma(\mu_0) r(\mu)} \quad [20.12b]$$

Integrating Eq.[20.12a, b] over the solid angle, we find diffuse fluxes

$$F^{*}(0) = F^{\uparrow}(0) + \frac{r_{sur}}{1 - r_{sur}\,\bar{r}}\,\mu_0 F_0 \gamma(\mu_0)\bar{\gamma} \qquad [20.13a]$$

$$F^{*}(\tau^*) = F^{\downarrow}(\tau^*) + \frac{r_{sur}}{1 - r_{sur}\,\bar{r}}\,\mu_0 F_0 \gamma(\mu_0)\bar{r} \qquad [20.13a]$$

where $\bar{\gamma} = \bar{t} + 2\int_0^1 \exp(-\tau^*/\mu_0)\mu_0 d\mu_0$

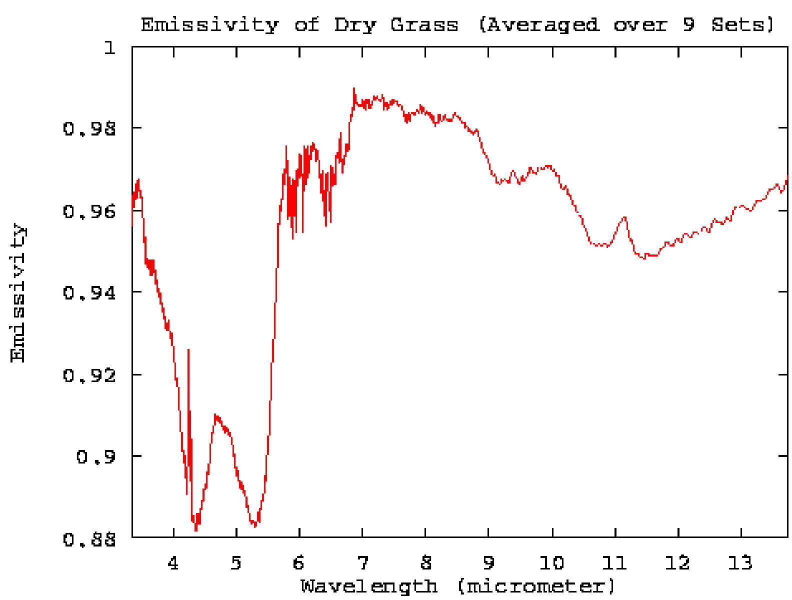
NOTE: $\bar{t}$ and $\bar{r}$ were defined in Lecture 19 (see Eq.[19.8] and [19.9]).

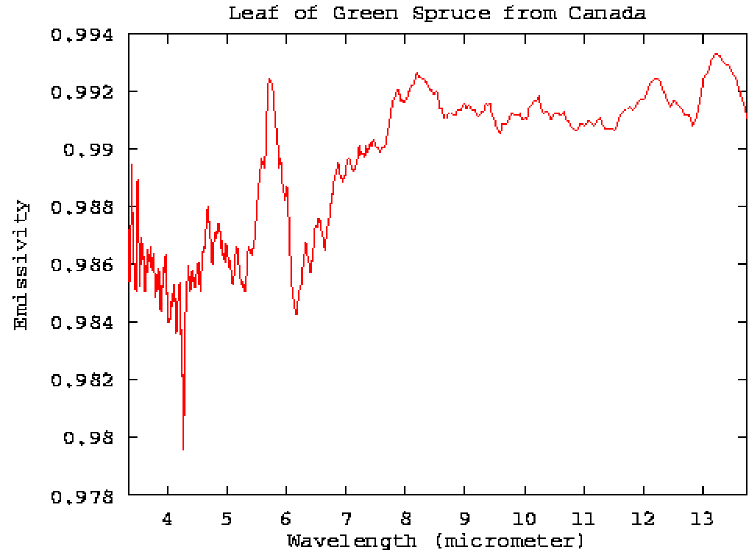
NOTE: For non-Lambert surface, the inclusion of the surface reflection is a complex boundary problem.

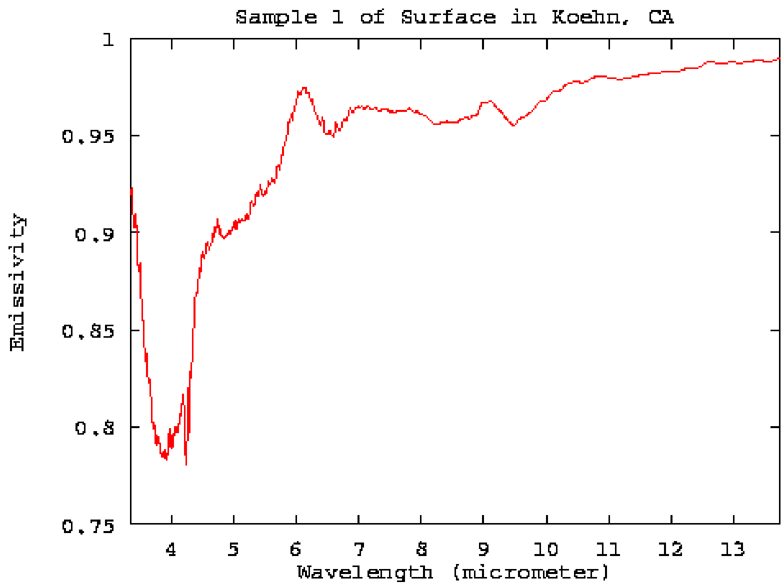
3. Surface emissivity.

- ✓ In general, emissivity depends on the direction of emission, surface temperature, wavelength and some physical properties of the surface (e.g., the refractive index).
- ✓ In the thermal IR (λ> 4 μm), nearly all surfaces are efficient emitters with the emissivity > 0.8 and their emissivity depends little on the direction (about 1-3% angular variation)

Examples of spectral emissivity (MODIS UCSB Emissivity Library)







<u>Inclusion of the surface emissivity into the radiative transfer equation:</u>

Recall the general solution of the upwelling radiance in the thermal IR (see Lecture 7, Eq.[7.2])

$$I_\nu^{\uparrow}(\tau;\mu) = I_\nu^{\uparrow}(\tau^*;\mu)\exp(-\frac{\tau^*-\tau}{\mu}) + \frac{1}{\mu}\int_{\tau}^{\tau_*}\exp(-\frac{\tau'-\tau}{\mu})\ B_\nu(T(\tau'))\,d\tau'$$

where $I_\nu^{\uparrow}(\tau^*;\mu)$ is the contribution from the surface.

In general, contribution from the surface = emission + reflection

For *a specular surface*:

$$I_\nu^{\uparrow}(\tau^*;\mu) = \varepsilon_\nu B(T_s) + (1-\varepsilon_\nu) I_\nu^{\downarrow}(\tau^*,\mu) \qquad [20.14]$$

where ε_ν is the surface emissivity $r_{suf} = 1- \varepsilon_\nu$, and $I_\nu^{\downarrow}(\tau^*,\mu)$ is the downwelling radiances reaching the surface.

In the IR window, r_{suf} is negligibly small for the land and ocean surfaces => it is common in the radiative transfer modeling to keep only the first term in Eq.[20.14].

Table 20.1 Broadband emissivities of some surfaces in the IR window (10-to12 μm).

Surface	***Emissivity***
Water	**0.993-0.998**
Ice	**0.98**
Green grass	**0.975-0.986**
Sand	**0.949-0.962**
Frozen soil	**0.93**
Concrete	**0.94**
Snow	**0.969-0.997**
Granite	**0.898**

NOTE: There are several databases that have been developed to provide the spectral emission data of natural surfaces. Both surface emissivity and reflectance (BRDF and albedo) can be retrieved from satellite observations (sensors differ in the spatial footprint/coverage and spectral resolution)

Example: *ASTER spectral library* (http://speclib.jpl.nasa.gov/): includes data from three other spectral libraries: the Johns Hopkins University (JHU) Spectral Library the Jet Propulsion Laboratory (JPL) Spectral Library, and the United States Geological Survey (USGS - Reston) Spectral Library. Current Version 2.0 of the ASTER spectral library includes over 2400 spectra of natural and man made materials.