Metode Elemen Hingga TKM4204

3 SKS (WAJIB)

Dosen Pengampu

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Prasyarat

TKM4111 : Mekanika Kekuatan Bahan 2

TKM4202 : Matematika Teknik 1

Tujuan

Agar mahasiswa :

- Mampu menjelaskan konsep dasar metode elemen hingga dan memformulasikan problem teknik dalam model
- Dapat menyelesaikan pemodelan problem tersebut dalam struktur, frame, shell/plat pada matra garis, 2D, 3D.

Pokok Bahasan

Konsep dasar metode elemen hingga

Metode kekakuan

Prinsip energi potensial minimum

Problem MEH Solid Mechanic 1D (truss, beam, frame, heat transfer)

Problem MEH 2D (elemen segi tiga, elemen segi empat; Elemen Linier dan non linier)

Problem MEH 3D (*axisymetric,* elemen solid)

Referensi

Referensi

- Modul Ajar Metode Elemen Hingga. Teknik mesin Universitas Brawijaya.
- Zienkiewicz, O.C. "*The Finite Element Method*". London: Mc.Graw-Hill.
- Cook, Robert D. "*Concepts and Aplications of Finite Element Analysis*". New York: John Willey & Sons Inc.
- Atkinson, Kendall. "Elementary Numerical Analysis". New York: John Willey & Sons.
- Atkinson, Kendall. "*An Introduction to Numercial Analysis*". New York: John Willey & Sons.

Penilaian

Kehadiran → 10%

Quiz → 25%

Tugas → 25%

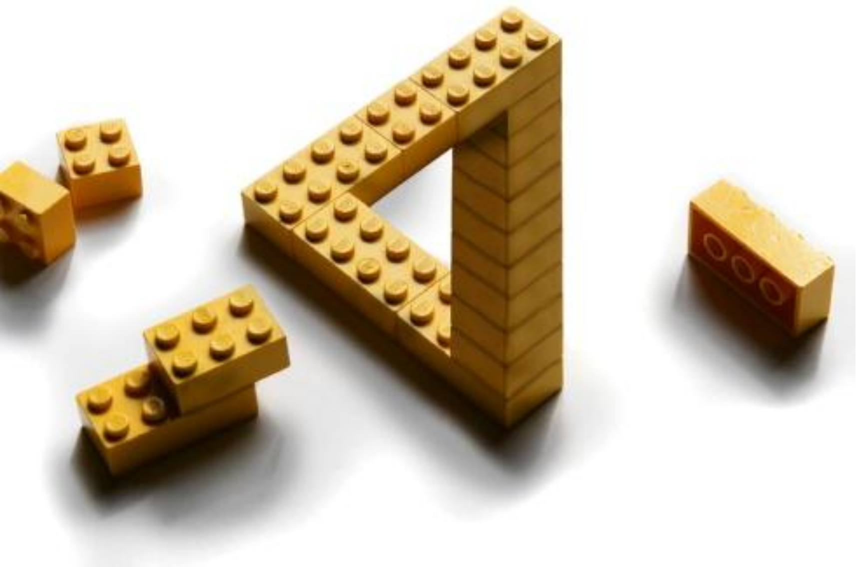
UAS → 40%

HP WAJIB DI *SILENT* ATAU DI NON-AKTIFKAN!!!

Introduction

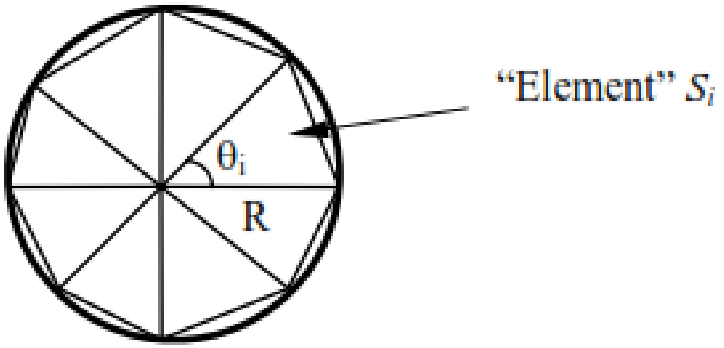
Konsep Dasar Metode Elemen Hingga

- Membangun benda yang komplek dari beberapa komponen kecil dan sederhana
- Membagi benda yang komplek menjadi beberapa bagian yang kecil dan bisa ditata



Introduction

Menghitung luas lingkaran



Area of one triangle: $S_i = \frac{1}{2} R^2 \sin\theta_i$

Area of the circle: $S_N = \sum_{i=1}^{N} S_i = \frac{1}{2} R^2 N \sin\left(\frac{2\pi}{N}\right) \rightarrow \pi R^2 \;\; as\, N \rightarrow \infty$

where N = total number of triangles (elements).

Introduction

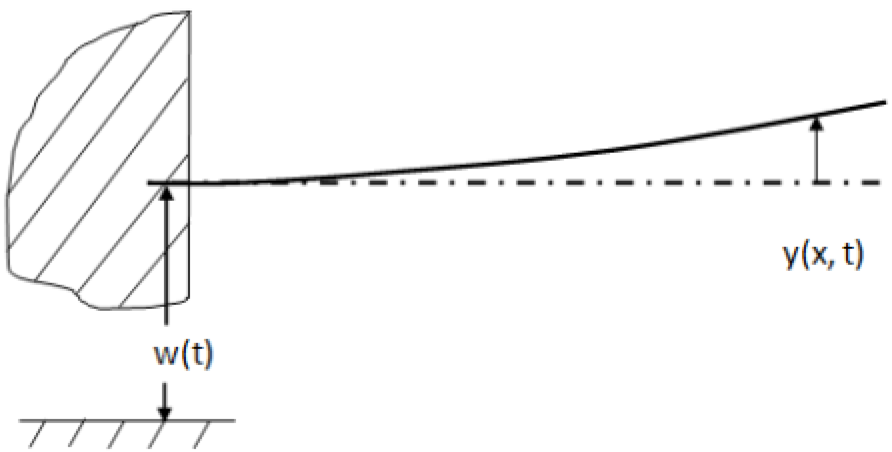
Mengapa harus menggunakan metode elemen hingga

- *Design analysis*: hand calculations, experiments, and computer simulations
- FEM/FEA is the most widely applied computer simulation method in engineering
- Closely integrated with CAD/CAM applications

Introduction

Sejarah Perkembangan Metode Elemen Hingga

- Hrennikoff dan McHenry (1941) menggunakan elemen satu dimensi berupa elemen garis, yang sekarang dikenal sebagai elemen batang



Introduction

Sejarah perkembangan metode elemen hingga

- 1943 ----- Courant (Variational methods)
- 1956 ----- Turner, Clough, Martin and Topp (Stiffness)
- 1960 ----- Clough ("Finite Element", plane problems)

- 1970s ----- Applications on mainframe computers
- 1980s ----- Microcomputers, pre- and postprocessors
- 1990s ----- Analysis of large structural systems

Introduction

Penerapan Metode Elemen Hingga Menggunakan Software

- Preprocessing (build FE model, loads and constraints)
- FEA solver (assemble and solve the system of equations)
- Postprocessing (sort and display the results)

Introduction

Hal – Hal Penting dalam Pengerjaan Metode Elemen Hingga

- Divide structure into pieces (elements with nodes)
- Describe the behavior of the physical quantities on each element
- Connect (assemble) the elements at the nodes to form an approximate system of equations for the whole structure
- Solve the system of equations involving unknown quantities at the nodes (e.g., displacements)
- Calculate desired quantities (e.g., strains and stresses) at selected elements

Introduction

Software Berbasis Elemen Hingga

- *ANSYS* (General purpose, PC and workstations)
- *SDRC/I-DEAS* (Complete CAD/CAM/CAE package)
- *NASTRAN* (General purpose FEA on mainframes)
- *ABAQUS* (Nonlinear and dynamic analyses)
- *COSMOS* (General purpose FEA)
- *ALGOR* (PC and workstations)
- *PATRAN* (Pre/Post Processor)
- *HyperMesh* (Pre/Post Processor)
- *Dyna-3D* (Crash/impact analysis)

Introduction

Persamaan Linier ➔ Review

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
\ldots\ldots\ldots & \\
a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n
\end{aligned}
\qquad (1)
$$

where x_1, x_2, ..., x_n are the unknowns.

Introduction

Bentuk Matrik

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{2}$$

where

$$\mathbf{A} = \left[a_{ij}\right] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$\mathbf{x} = \{x_i\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \qquad \mathbf{b} = \{b_i\} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix} \tag{3}$$

A is called a $n \times n$ (square) matrix, and **x** and **b** are (column) vectors of dimension n.

Introduction

Row and Column Vectors

$$\mathbf{v} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \qquad \mathbf{w} = \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}$$

Matrix Addition and Subtraction

For two matrices **A** and **B**, both of the *same size* ($m \times n$), the addition and subtraction are defined by

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \quad \text{with} \quad c_{ij} = a_{ij} + b_{ij}$$
$$\mathbf{D} = \mathbf{A} - \mathbf{B} \quad \text{with} \quad d_{ij} = a_{ij} - b_{ij}$$

Introduction

Scalar Multiplication

$$\lambda\mathbf{A} = \left[\lambda a_{ij}\right]$$

Matrix Multiplication

For two matrices $\mathbf{A}$ (of size $l \times m$) and $\mathbf{B}$ (of size $m \times n$), the product of $\mathbf{AB}$ is defined by

$$\mathbf{C} = \mathbf{AB} \quad \text{with } c_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj}$$

where $i = 1, 2, ..., l$; $j = 1, 2, ..., n$.

Note that, in general, $\mathbf{AB} \neq \mathbf{BA}$, but $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$ (associative).

Introduction

Transpose of a Matrix

If $\mathbf{A} = [a_{ij}]$, then the transpose of $\mathbf{A}$ is

$$\mathbf{A}^T = \left[a_{ji}\right]$$

Notice that $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$.

Symmetric Matrix

A *square* $(n \times n)$ matrix $\mathbf{A}$ is called symmetric, if

$$\mathbf{A} = \mathbf{A}^T \qquad \text{or} \qquad a_{ij} = a_{ji}$$

Unit (Identity) Matrix

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 \end{bmatrix}$$

Note that $\mathbf{AI} = \mathbf{A}$, $\mathbf{Ix} = \mathbf{x}$.

Introduction

Determinant of a Matrix

The determinant of *square* matrix $\mathbf{A}$ is a scalar number denoted by $\det \mathbf{A}$ or $|\mathbf{A}|$. For 2×2 and 3×3 matrices, their determinants are given by

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13}$$

$$- a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}$$

Singular Matrix

A *square* matrix $\mathbf{A}$ is *singular* if det $\mathbf{A} = 0$, which indicates problems in the systems (nonunique solutions, degeneracy, etc.)

Matrix Inversion

For a *square* and *nonsingular* matrix $\mathbf{A}$ (det $\mathbf{A} \neq 0$), its *inverse* $\mathbf{A}^{-1}$ is constructed in such a way that

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

The *cofactor matrix* $\mathbf{C}$ of matrix $\mathbf{A}$ is defined by

$$C_{ij} = (-1)^{i+j} M_{ij}$$

where M_{ij} is the determinant of the smaller matrix obtained by eliminating the *i*th row and *j*th column of $\mathbf{A}$.

Thus, the inverse of $\mathbf{A}$ can be determined by

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \mathbf{C}^{T}$$

We can show that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.

Introduction

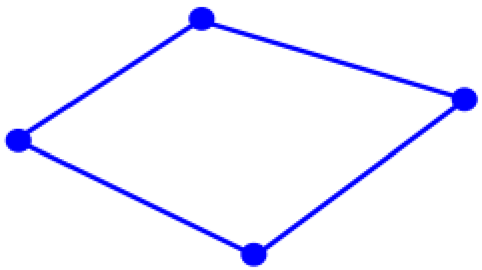
Prosedur Umum Metode Elemen Hingga

1. Memilih jenis elemen dan diskritisasi

1-D (Line) Element

(Spring, truss, beam, pipe, etc.)

2-D (Plane) Element

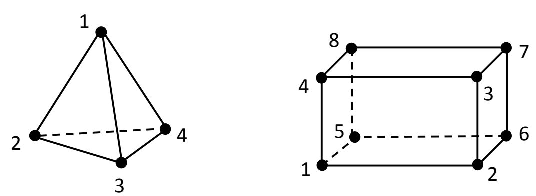


(Membrane, plate, shell, etc.)

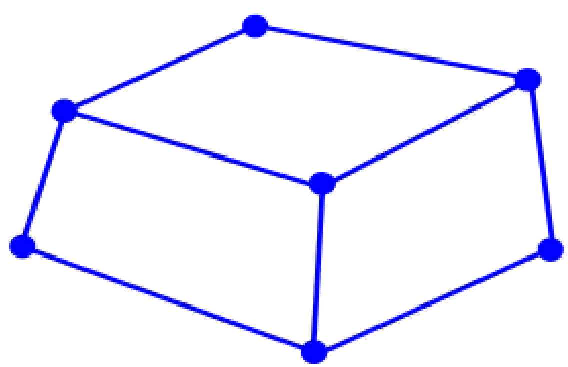
Introduction

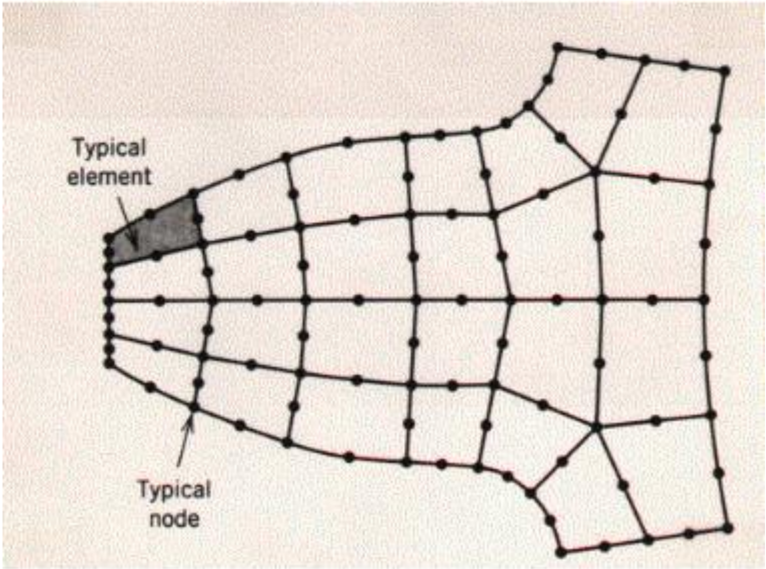
Prosedur Umum Metode Elemen Hingga

1. Memilih jenis elemen dan diskritisasi



3-D (Solid) Element



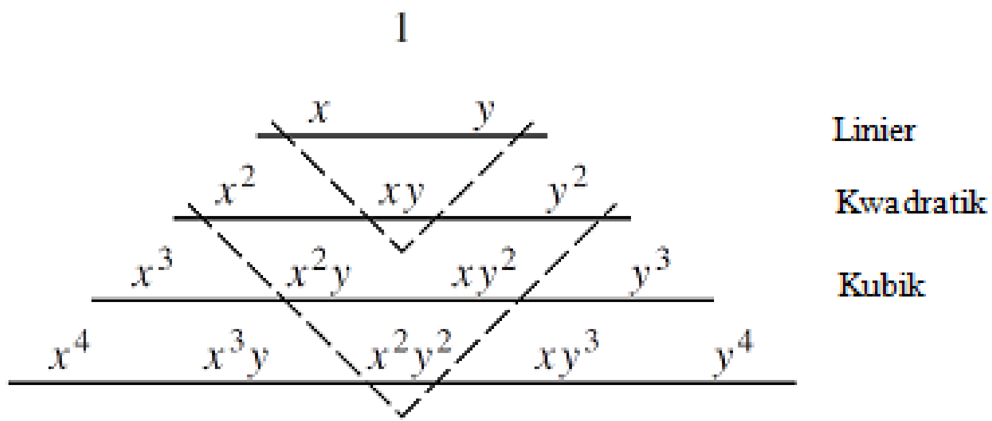


(3-D fields - temperature, displacement, stress, flow velocity)

Introduction

Prosedur Umum Metode Elemen Hingga

2. Memilih fungsi perpindahan (F=kΔx)

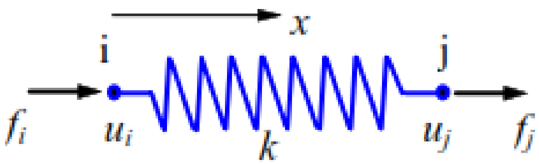


Introduction

Prosedur Umum Metode Elemen Hingga

3. Mendefinisikan hubungan antara regangan/perpindahan dan tegangan/regangan
4. Menurunkan rumus dan matrik kekakuan elemen
5. Menggabungkan rumus elemen untuk mendapat rumus global dan menentukan kondisi batas.
6. Menyelesaikan atau memecahkan derajat kebebasan yang tidak diketahui.
7. Menghitung harga tegangan dan regangan pada elemen
8. Menginterprestasikan hasil

Pegas



Two nodes: i, j

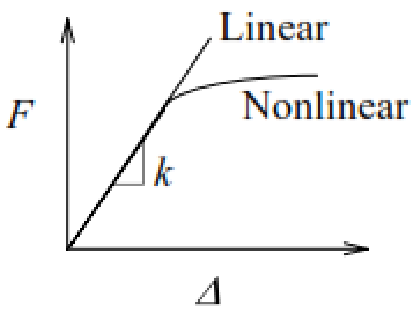
Nodal displacements: u_i, u_j (in, m, mm)

Nodal forces: f_i, f_j (lb, Newton)

Spring constant (stiffness): k (lb/in, N/m, N/mm)

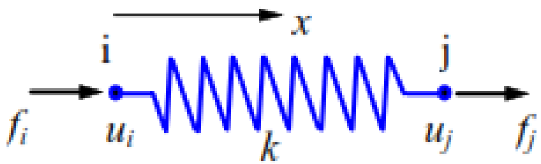
Spring force-displacement relationship:

$$F = k\Delta \qquad \text{with } \Delta = u_j - u_i$$



$k = F / \Delta$ (> 0) is the force needed to produce a unit stretch.

Pegas



Consider the equilibrium of forces for the spring. At node i, we have

$$f_i = -F = -k(u_j - u_i) = ku_i - ku_j$$

and at node j,

$$f_j = F = k(u_j - u_i) = -ku_i + ku_j$$

In matrix form,

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ f_j \end{Bmatrix}$$

or,

$$\mathbf{ku} = \mathbf{f}$$

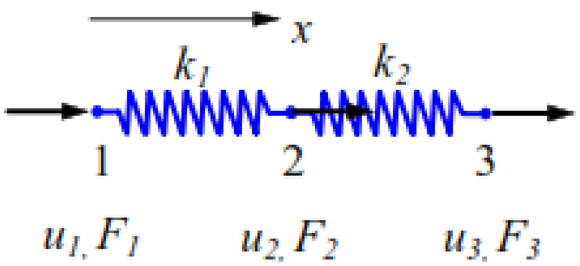
where

$\mathbf{k}$ = (element) stiffness matrix

$\mathbf{u}$ = (element nodal) displacement vector

$\mathbf{f}$ = (element nodal) force vector

Sistem Pegas



For element 1,

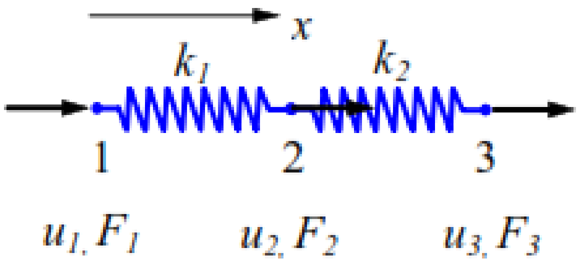
$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1^1 \\ f_2^1 \end{Bmatrix}$$

element 2,

$$\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} f_1^2 \\ f_2^2 \end{Bmatrix}$$

where f_i^m is the (internal) force acting on *local* node i of element m ($i = 1, 2$).

Sistem Pegas



Assemble the stiffness matrix for the whole system:

Consider the equilibrium of forces at node 1,

$$F_1 = f_1^1$$

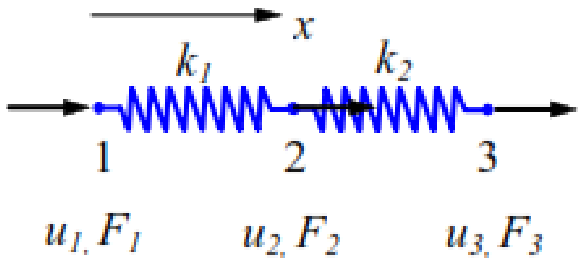
at node 2,

$$F_2 = f_2^1 + f_1^2$$

and node 3,

$$F_3 = f_2^2$$

Sistem Pegas



That is,

$$F_1 = k_1u_1 - k_1u_2$$
$$F_2 = -k_1u_1 + (k_1 + k_2)u_2 - k_2u_3$$
$$F_3 = -k_2u_2 + k_2u_3$$

In matrix form,

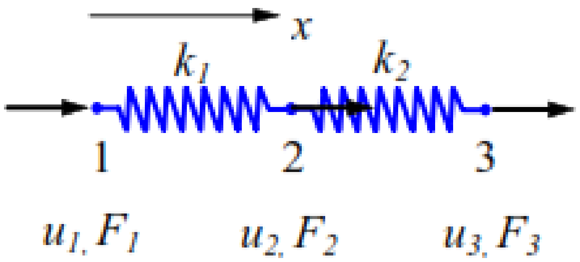
$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

or

$$\mathbf{KU} = \mathbf{F}$$

K is the stiffness matrix (structure matrix) for the spring system.

Sistem Pegas



Boundary and load conditions:

Assuming, $u_1 = 0$ and $F_2 = F_3 = P$

we have

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ P \\ P \end{Bmatrix}$$

which reduces to

$$\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} P \\ P \end{Bmatrix}$$

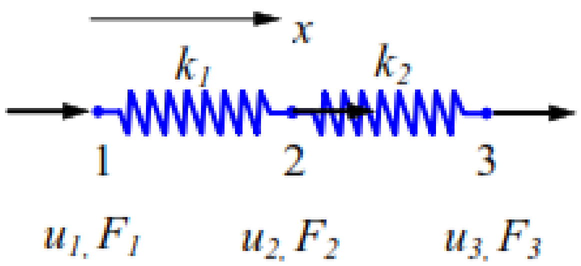
and

$$F_1 = -k_1 u_2$$

Unknowns are

$$\mathbf{U} = \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$ and the reaction force F_1 (if desired).

Sistem Pegas



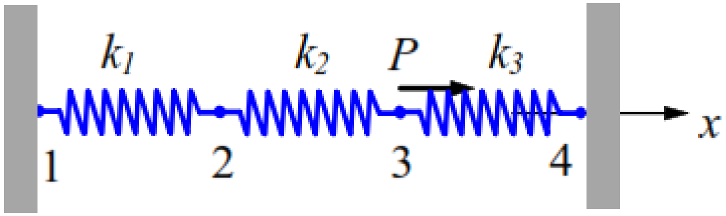
Solving the equations, we obtain the displacements

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 2P/k_1 \\ 2P/k_1 + P/k_2 \end{Bmatrix}$$

and the reaction force

$$F_1 = -2P$$

Contoh 1.1



Given: For the spring system shown above,

$k_1 = 100 \text{ N / mm}, \;\; k_2 = 200 \text{ N / mm}, \;\; k_3 = 100 \text{ N / mm}$

$P = 500 \text{ N}, \;\; \mathrm{u}_1 = u_4 = 0$

Find: (a) the global stiffness matrix

(b) displacements of nodes 2 and 3

(c) the reaction forces at nodes 1 and 4

(d) the force in the spring 2

Contoh 1.1

(a) The element stiffness matrices are

$$\mathbf{k}_1 = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \quad (\text{N/mm}) \qquad (1)$$

$$\mathbf{k}_2 = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \quad (\text{N/mm}) \qquad (2)$$

$$\mathbf{k}_3 = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \quad (\text{N/mm}) \qquad (3)$$

Contoh 1.1

Applying the superposition concept, we obtain the global stiffness matrix for the spring system as

$$
\begin{array}{cc}
 & \begin{array}{cccc} \quad u_1 \quad & \quad u_2 \quad & \quad u_3 \quad & \quad u_4 \quad \end{array} \\
\mathbf{K} = & \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 100+200 & -200 & 0 \\ 0 & -200 & 200+100 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix}
\end{array}
$$

or

$$
\mathbf{K} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 300 & -200 & 0 \\ 0 & -200 & 300 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix}
$$

Contoh 1.1

Equilibrium (FE) equation for the whole system is

$$\begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 300 & -200 & 0 \\ 0 & -200 & 300 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \\ P \\ F_4 \end{Bmatrix} \qquad (4)$$

(b) Applying the BC ($u_1 = u_4 = 0$) in Eq(4), or deleting the 1st and 4th rows and columns, we have

Contoh 1.1

$$\begin{bmatrix} 300 & -200 \\ -200 & 300 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ P \end{Bmatrix} \quad (5)$$

Solving Eq.(5), we obtain

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} P/250 \\ 3P/500 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 3 \end{Bmatrix} \text{ (mm)} \quad (6)$$

(c) From the 1st and 4th equations in (4), we get the reaction forces

$$F_1 = -100u_2 = -200 \text{ (N)}$$

$$F_4 = -100u_3 = -300 \text{ (N)}$$

Contoh 1.1

(d) The FE equation for spring (element) 2 is

$$\begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ f_j \end{Bmatrix}$$

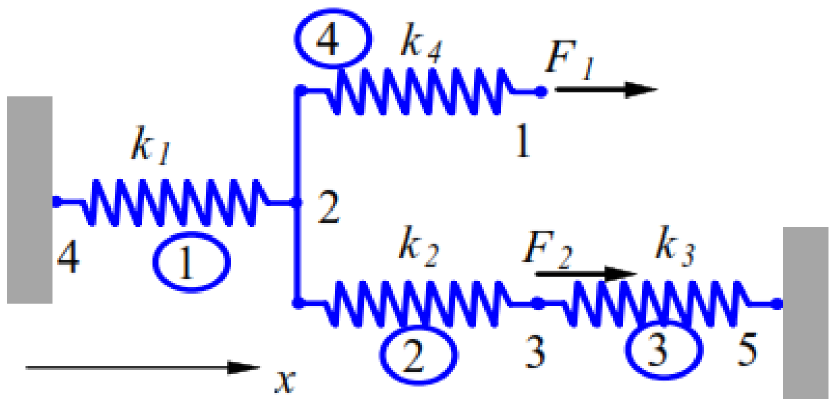
Here $i = 2, j = 3$ for element 2. Thus we can calculate the spring force as

$$F = f_j = -f_i = [-200 \quad 200] \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$= [-200 \quad 200] \begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$$

$$= 200 \ \ (\text{N})$$

Contoh 1.2



Problem: For the spring system with arbitrarily numbered nodes and elements, as shown above, find the global stiffness matrix.

Contoh 1.2

Solution:

First we construct the following

Element Connectivity Table

Element	*Node i (1)*	*Node j (2)*
1	4	2
2	2	3
3	3	5
4	2	1

which specifies the *global* node numbers corresponding to the *local* node numbers for each element.

Contoh 1.2

Then we can write the element stiffness matrices as follows

$$\begin{array}{cc} & \begin{array}{cc} u_4 & u_2 \end{array} \\ \mathbf{k}_1 = & \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} u_2 & u_3 \end{array} \\ \mathbf{k}_2 = & \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} u_3 & u_5 \end{array} \\ \mathbf{k}_3 = & \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} u_2 & u_1 \end{array} \\ \mathbf{k}_4 = & \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} \end{array}$$

Contoh 1.2

Finally, applying the superposition method, we obtain the global stiffness matrix as follows

$$\mathbf{K} = \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} k_4 & -k_4 & 0 & 0 & 0 \\ -k_4 & k_1+k_2+k_4 & -k_2 & -k_1 & 0 \\ 0 & -k_2 & k_2+k_3 & 0 & -k_3 \\ 0 & -k_1 & 0 & k_1 & 0 \\ 0 & 0 & -k_3 & 0 & k_3 \end{bmatrix} \end{array}$$

The matrix is *symmetric*, *banded*, but *singular*.