MHT-CET TRIUMPH

PHYSICS

Based on Std. XI & XII Syllabus of MHT-CET

HINTS TO MULTIPLE CHOICE QUESTIONS, EVALUATION TESTS & MHT-CET 2019 (6^{th} May, Afternoon) PAPER

CONTENT

Textbook Chapter No.

01 Measurements

Hints

Classical Thinking

13. Temperature is a fundamental quantity.

26. 1 dyne = 10^{-5} N, 1 cm^2 = 10^{-4} m^2

$\therefore$ 10^3 dyne/cm^2 = $10^3 \times 10^{-5}/10^{-4}$ N/m^2

= 10^2 N/m^2

OR

Using quick conversion for pressure,

1 dyne/cm^2 = 0.1 N/m^2

$\therefore$ 10^3 dyne/cm^2 = $10^3 \times 0.1 = 10^2$ N/m^2

57. Percentage error = $\left(\frac{\Delta d}{d} \times 100\right)\%$

$= \left(\frac{0.01}{1.03} \times 100\right)\%$

= 0.97%

Critical Thinking

1. Physical quantity (M)
= Numerical value (n) × Unit (u)
If physical quantity remains constant then $n \propto 1/u \therefore n_1u_1 = n_2u_2$.

2. Because in S.I. system, there are seven fundamental quantities.

3. $\frac{\text{mass} \times \text{pressure}}{\text{density}} = \frac{m \times (F/A)}{(m/V)} = \frac{F \times V}{A}$

$= \frac{F \times (A \times s)}{A} = F \times s = \text{work}$

6. $mv = kg\left(\frac{m}{sec}\right)$

7. Curie = disintegration/second

8. Bxt is unitless.
$\therefore$ Unit of B is m^{-1}s^{-1}.

9. $Y = \frac{F}{A} \cdot \frac{L}{\Delta L} = \frac{\text{dyne}}{\text{cm}^2} = \frac{10^{-5}\,\text{N}}{10^{-4}\,\text{m}^2} = 0.1\ \text{N/m}^2$

10. Parallactic angle, $\theta = 57'$

$= \left(\frac{57}{60}\right)^{o} = \left(\frac{57}{60}\right) \times \frac{\pi}{180}$ rad

b = Radius of earth = 6.4×10^6 m

Distance of the moon from the earth,

$s = \frac{b}{\theta} = \frac{6.4 \times 10^6 \times 60 \times 180}{57 \times \pi}$ $s = 3.86 \times 10^8$ m

11. Distance of sun from earth, s = 1.5×10^{11} m

Angular diameter of sun,

$\theta = 1920'' = \left(\frac{1920}{60 \times 60}\right)^{o} = \frac{1920}{3600} \times \frac{\pi}{180}$ rad

Diameter of sun, D = s × θ

$= 1.5 \times 10^{11} \times \frac{1920}{3600} \times \frac{\pi}{180}$

$D \approx 1.4 \times 10^9$ m

12. Torque = $[M^1L^2T^{-2}]$,
Angular momentum = $[M^1L^2T^{-1}]$
So mass and length have the same dimensions.

13. According to Poiseuille's formula,

$\eta = \frac{\pi P r^4}{8l(dV/dt)}$

$\therefore$ $[\eta] = \frac{[M^1L^{-1}T^{-2}][L^4]}{[L^1][L^3/T^1]} = [M^1L^{-1}T^{-1}]$

15. [Dipole moment] = $[M^0L^1T^1A^1]$
[Electric flux] = $[M^1L^3T^{-3}A^{-1}]$
[Electric field] = $[M^1L^1T^{-3}A^{-1}]$

16. $\frac{1}{2}Li^2$ = energy stored in an inductor

= $[M^1L^2T^{-2}]$

17. The dimension of a quantity is independent of changes in its magnitude.

21. $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ = c = velocity of light

23. $\frac{mg}{\eta r} = \frac{[M^1][L^1T^{-2}]}{[L^{-1}M^1T^{-1}][L^1]} = [L^1T^{-1}]$

24. From $F = at + bt^2$

$$a = \frac{F}{t} = \frac{[M^1L^1T^{-2}]}{[T^1]} = [M^1L^1T^{-3}]$$

$$b = \frac{F}{t^2} = \frac{[M^1L^1T^{-2}]}{[T^2]} = [M^1L^1T^{-4}]$$

25. $F = a\sqrt{x}$

$$\therefore \quad a = \frac{F}{\sqrt{x}} = \frac{[M^1L^1T^{-2}]}{[L^{1/2}]} = [M^1L^{1/2}T^{-2}]$$

$bt^2 = F$

$$\therefore \quad b = \frac{F}{t^2} = \frac{[M^1L^1T^{-2}]}{[T^2]} = [M^1L^1T^{-4}]$$

$$\frac{a}{b} = \frac{[M^1L^{1/2}T^{-2}]}{[M^1L^1T^{-4}]} = [L^{-1/2}\ T^2]$$

26. $[M^1L^1T^{-2}] = [L^2]^a\ [L^1T^{-1}]^b\ [M^1L^{-3}]^c$
$= [L^{2a}]\ [L^bT^{-b}]\ [M^cL^{-3c}]$
$= [M^c\ L^{2a+b-3c}\ T^{-b}]$
Comparing powers of M, L and T,
$c = 1,\ 2a + b - 3c = 1,\ -b = -2$
$\therefore$ $b = 2$
$2a + 2 - 3(1) = 1$
$\therefore$ $2a = 2$
$\therefore$ $a = 1$

27. Let $T^2 = \dfrac{4\pi^2a^x}{G^yM^z}$

$4\pi^2$ being pure number is dimensionless.

$$\therefore \quad [M^0L^0T^2] = \frac{[M^0L^1T^0]^x}{[M^{-1}L^3T^{-2}]^y\ [M^1L^0T^0]^z}$$

$\Rightarrow [M^0L^0T^2] = [L^x]\ [M^{-1}L^3T^{-2}]^{-y}\ [M^1]^{-z}$
Comparing powers of M, L and T
$y - z = 0,$
$x - 3y = 0$ and $2y = 2$
$\therefore$ $y = 1$
Substituting value of y,
$z = 1,\ x = 3$

Thus, $T^2 = \dfrac{4\pi^2a^3}{GM}$

28. $T = P^aD^bS^c$
$[M^0L^0T^1] = [M^1L^{-1}T^{-2}]^a\ [M^1L^{-3}T^0]^b\ [M^1L^0T^{-2}]^c$
Comparing powers of M, L, T
$a + b + c = 0,$
$-a - 3b = 0$ and $-2a - 2c = 1$

Solving, $a = -\dfrac{3}{2}$, $b = \dfrac{1}{2}$ and $c = 1$.

29. In the given wave equation x denotes displacement. Thus $\left(\dfrac{x}{v}\right)$ has dimensions of T. Hence from the principle of homogenity k has dimensions of T.

30. $P = \dfrac{a - t^2}{bx}$

$a = [t^2] = [T^2]$

$\therefore$ $P = \dfrac{T^2}{bx}$

$$b = \frac{T^2}{Px} = \frac{[T^2]}{[M^1L^{-1}T^{-2}][L^1]} = \frac{[T^4]}{[M^1]}$$

$$\frac{a}{b} = [T^2]\frac{[M^1]}{[T^4]} = [M^1T^{-2}]$$

31. By principle of dimensional homogeneity

$$\left[\frac{a}{V^2}\right] = [P]$$

$\therefore$ $[a] = [P]\ [V^2] = [M^1L^{-1}T^{-2}] \times [L^6]$
$= [M^1L^5T^{-2}]$
Dimensions of b are same as that of V,
$[b] = [L^3]$

$$\therefore \quad \left[\frac{a}{b}\right] = \frac{[M^1L^5T^{-2}]}{[L^3]} = [M^1L^2T^{-2}]$$

32. Let $G \propto c^xg^yp^z$
Substituting dimensions,
$[M^{-1}L^3T^{-2}] = [M^0L^1T^{-1}]^x\ [M^0L^1T^{-2}]^y\ [M^1L^{-1}T^{-2}]^z$
Comparing powers of M, L, T
$-1 = z,$
$x + y - z = 3$ and
$-x - 2y - 2z = -2$
Solving, $x = 0,\ y = 2$

33. Acceleration due to gravity = $g = \dfrac{s}{t^2}$

$g = [L^1T^{-2}]$
$a = 1,\ b = -2$

1st system	2nd system
$L_1 = 1$ cm $= 10^{-5}$ km	$L_2 = 1$ km
$T_1 = 1\text{ s} = \dfrac{1}{60}$ min	$T_2 = 1$ min

$$n = \left[\frac{L_1}{L_2}\right]^a \left[\frac{T_1}{T_2}\right]^b = 980 \times \left[\frac{10^{-5}\text{ km}}{1\text{ km}}\right]^1 \left[\frac{1/60\text{ min}}{1\text{ min}}\right]^{-2}$$

$= 980 \times 10^{-5} \times 3600$
$= 35.28\text{ km min}^{-2}$

39. The number of significant figures in all of the given number is 4.

41. A vernier calliper has a least count 0.01 cm. Hence measurement is accurate only upto three significant figures.

42. In multiplication or division, final result should retain the same number of significant figures as there are in the original number with the least significant figures.

∴ Area of rectangle = 6 × 12 = 72 m^2

43. $a_m = \frac{20.17+21.23+20.79+22.07+21.78}{5}$

$a_m = 21.21$

$|\Delta a_1| = |21.21 - 20.17| = 1.04$

$|\Delta a_2| = |21.21 - 21.23| = 0.02$

$|\Delta a_3| = 0.42$

$|\Delta a_4| = 0.86$

$|\Delta a_5| = 0.57$

$|\Delta a_m| = \frac{|\Delta a_1|+|\Delta a_2|+|\Delta a_3|+|\Delta a_4|+|\Delta a_5|}{5}$

$= \frac{1.04+0.02+0.42+0.86+0.57}{5} = 0.58$

45. Percentage error = $\left(\frac{\Delta d}{d}\times 100\right)\%$

$= \left(\frac{0.005}{0.020}\times 100\right)\% = 25\%$

46. $\frac{\Delta r}{r}\times 100 = 0.1\%$ and $V = \frac{4}{3}\pi r^3$

Percentage error in volume $= \frac{\Delta V}{V}\%$

$= \frac{3\Delta r}{r} = 0.3\%$

47. $P = \frac{F}{A} = \frac{F}{l^2}$

so maximum error in pressure (P)

$\left(\frac{\Delta P}{P}\times 100\right)_{max} = \frac{\Delta F}{F}\times 100 + 2\frac{\Delta l}{l}\times 100$

$= 4\% + 2 \times 2\% = 8\%$

48. Percentage error in K.E $= \left(\frac{\Delta m}{m}+\frac{2\Delta v}{v}\right)\%$

$= (0.75 + 2 \times 1.85)\%$

$= 4.45\%$

49. Maximum possible error in measurement of

$\frac{L}{T^2} = \left(\frac{\Delta L}{L}+2\frac{\Delta T}{T}\right)\%$

$= (0.1 + 2 \times 3)\% = 6.1\%$

50. $T = 2\pi\sqrt{l/g} \Rightarrow T^2 = 4\pi^2 l/g \Rightarrow g = \frac{4\pi^2 l}{T^2}$

% error in $l = \frac{1\,mm}{100\,cm}\times 100 = \frac{0.1}{100}\times 100 = 0.1\%$

and error in T $= 2\left[\frac{0.1}{100}\times 100\right] = 0.2\%$

∴ % error in g = % error in l + % error in T
= 0.1 + 0.2 = 0.3 %

51. $\frac{\Delta V}{V}\times 100 = \left(\frac{\Delta l}{l}+\frac{\Delta b}{b}+\frac{\Delta h}{h}\right)\times 100\%$

$= \left(\frac{0.02}{13.12}+\frac{0.01}{7.18}+\frac{0.02}{4.16}\right)\times 100\%$

$= 0.77\%$

52. $H = \frac{I^2Rt}{4.2}$

% Error, $\frac{\Delta H}{H}\times 100 = \left(2\frac{\Delta I}{I}+\frac{\Delta R}{R}+\frac{\Delta t}{t}\right)\%$

$= 2 \times 2 + 1 + 1 = 6\%$

53. [Energy] $= [M^1L^2T^{-2}]$

$= [M^1L^1T^{-1}]\,[L^1]\,[T^{-1}]$

$= [P^1A^{1/2}T^{-1}]$

54. Avogadro number (N) represents the number of atoms in 1 gram mole of an element. i.e., it has the dimensions of $mole^{-1}$.

55. As the graph is a straight line , P ∝ Q, or

P = Constant × Q i.e., $\frac{P}{Q}$ = constant.

Competitive Thinking

3. The van der Waals equation for 'n' moles of the gas is,

$\left(P+\frac{n^2a}{V^2}\right) \times [V - nb] = nRT$

Pressure correction   Volume correction

∴ $a = \frac{PV^2}{n^2} = \frac{\frac{F}{A}\times V^2}{n^2} = \frac{FlV}{n^2} = \frac{Fl^4}{n^2}$

Thus, S.I.units of a is N m^4/mol^2.

4. From Van der Waal's equation,
nb has dimensions of volume.

$\therefore$ $b = \frac{V}{n}$

Thus, S.I. units of b is m^3/mol.

12. $[x] = [bt^2]$

unit of b $= \frac{x}{t^2} = \frac{\text{metre}}{(\text{hour})^2} = \frac{m}{hr^2}$

13. Energy = force × distance, so if both are increased by 4 times then energy will increase by 16 times.

14. 1 dyne = 10^{-5} N and 1 cm = 10^{-2} m
$\Rightarrow$ 1 dyne/cm = 10^{-3} N/m
$\therefore$ 10^8 dyne/cm = 10^5 N/m

15. RC is the time constant of RC circuit and $\left(\frac{L}{R}\right)$ is the time constant of LR circuit. Hence, both RC and $\left(\frac{L}{R}\right)$ have the dimensions of time

Alternate method:

$$RC = \text{ohm} \times \text{farad} = \text{ohm} \times \frac{\text{coulomb}}{\text{volt}}$$

$$= \frac{\text{volt}}{\text{ampere}} \times \frac{\text{coulomb}}{\text{volt}} = \frac{\text{coulomb}}{\text{ampere}}$$

$$= \text{second} = [T]$$

Now, $\frac{L}{R} = \frac{\text{henry}}{\text{ohm}} = \frac{\text{ohm} \times \text{second}}{\text{ohm}}$

$= \text{second} = [T]$

Both RC and $\frac{L}{R}$ have the dimensions of time.

16. $[\varepsilon_0 L]=[C]$

$\therefore$ $X = \frac{\varepsilon_0 LV}{t} = \frac{C \times V}{t} = \frac{Q}{t} =$ Current

20. $F = \frac{Gm_1m_2}{r^2}$

$\Rightarrow G = \frac{Fr^2}{m_1m_2}$

$\therefore$ $[G] = \frac{[M^1L^1T^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

22. $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow \mu_0\epsilon_0 = \frac{1}{C^2}$

$[C] = [M^0L^1T^{-1}]$

$\left[\frac{1}{C^2}\right] = [M^0L^{-2}T^2]$

25. $W = \frac{1}{2}Kx^2$

$\therefore$ $[K] = \frac{[W]}{[x^2]} = \left[\frac{M^1L^2T^{-2}}{L^2}\right] = [M^1T^{-2}]$

26. $F \propto v$
$F = kv$

$k = \frac{F}{v} = \left[\frac{M^1L^1T^{-2}}{L^1T^{-1}}\right] = [M^1L^0T^{-1}]$

28. $R = \frac{PV}{T} = \left[\frac{M^1L^{-1}T^{-2} \times L^3}{\theta}\right] = [M^1L^2T^{-2}\theta^{-1}]$

29. $F = \frac{x}{\sqrt{d}}$

$\therefore$ $[x] = [F][d]^{1/2}$

$= [M^1L^1T^{-2}][M^1L^{-3}T^0]^{1/2} = [M^{3/2}L^{-1/2}T^{-2}]$

30. $F = \frac{X}{\text{Linear density}}$

Linear density is mass per unit length

$\therefore$ $[M^1L^1T^{-2}] \times \left[\frac{M^1}{L^1}\right] = [X]$

$\Rightarrow [X] = [M^2L^0T^{-2}]$

31. The van der Waals equation for 'n' moles of the gas is,

$$\underbrace{\left(P + \frac{n^2a}{V^2}\right)}_{\text{Pressure correction}} \times \underbrace{[V - nb]}_{\text{Volume correction}} = nRT$$

$\therefore$ $a = \frac{PV^2}{n^2} = \frac{\frac{F}{A} \times V^2}{n^2} = \frac{FlV}{n^2} = \frac{Fl^4}{n^2}$

$\Rightarrow [a] = \left[\frac{Fl^4}{n^2}\right] = [M^1L^5T^{-2}mol^{-2}]$

32. $\varepsilon_0 = \frac{q_1q_2}{4\pi Fr^2}$

$[\varepsilon_0] = \frac{A^2T^2}{(M^1L^1T^{-2})L^2} = [M^{-1}L^{-3}T^4A^2]$

33. Electric Field $= \frac{\text{Force}}{\text{Charge}} = \frac{[M^1L^1T^{-2}]}{[A^1T^1]}$

$[E] = [M^1L^1T^{-3}A^{-1}]$

34. Capacitance (C) = $\frac{\text{Charge(Q)}}{\text{Voltage(V)}}$

But, Voltage (V) = $\frac{\text{Work (W)}}{\text{Charge(Q)}}$

∴ $C = \frac{Q}{\frac{W}{Q}} = \frac{Q^2}{W}$

∴ $C = \frac{[Q^2]}{[ML^2T^{-2}]} = [M^{-1}L^{-2}T^2Q^2]$

35. $[\varepsilon_0 E^2] = [\varepsilon_0]\,[E]^2$
$= [M^{-1}L^{-3}T^4A^2]\,[M^1L^1T^{-3}A^{-1}]^2$
$= [M^1L^1T^{-2}A^0]$
OR
$\frac{1}{2}\varepsilon_0 E^2 = u$
where u is energy density and has dimensions $[M^1L^1T^{-2}]$

36. Magnetic flux = ϕ = BA,
where, B = magnetic flield, A = area
Permeability = $\mu = \frac{B}{H}$,
where, H = magnetic intensity

∴ $\frac{\phi}{\mu} = \frac{BA}{\left(\frac{B}{H}\right)}$ = Area × magnetic intensity

Now,
[Area] = [A] = $[L^2]$
Magnetic intensity = H = nI
$= \frac{\text{number of turns}}{\text{metre}} \times$ current

$[H] = \left[\frac{A}{L}\right]$ ....(∵ [Current] = Ampere [A])

∴ $\left[\frac{\phi}{\mu}\right] = \left[L^2 \times \frac{A}{L}\right]$ = [LA]

∴ $\left[\frac{\phi}{\mu}\right] = [M^0L^1T^0A^1]$

37. Energy density is given by $U = \frac{1}{2}\frac{B^2}{\mu_0}$

Also,

Energy density = $\frac{\text{Energy}}{\text{Volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$

∴ $\frac{B^2}{2\mu_0} = [ML^{-1}T^{-2}]$

39. Mobility = $\frac{\text{Drift velocity}}{\text{Electric field}} = \frac{v_d}{E}$
$= \frac{[M^0L^1T^{-1}]}{[M^1L^1T^{-3}A^{-1}]}$
$= [M^{-1}L^0T^2A^1]$

40. Units of solar constant : $\frac{W}{m^2}$

$= \text{kg}\,\frac{m^2}{s^3} \times \frac{1}{m^2} = \frac{\text{kg}}{s^3}$

∴ Dimension ⇒ $[M^1\,L^0T^{-3}]$

41. c = [T]
$a = \frac{v}{t} = \frac{[L^1\,T^{-1}]}{[T^1]} = [L^1T^{-2}]$
$b = v(t + c) = [L^1T^{-1}] \times T^1 = [L^1]$

42. $\frac{EJ^2}{M^5G^2} = \frac{[M^1L^2T^{-2}][M^1L^2T^{-1}]^2}{[M^1]^5[M^{-1}L^3T^{-2}]^2} = [M^0L^0T^0]$
The dimensions of angle are $[M^0L^0T^0]$.

43. F = A cos(Bx) + C cos(Dt)
But,
$F = A\cos\left(\frac{2\pi x}{\lambda}\right) + C\cos\left(\frac{2\pi t}{T}\right)$
on comparing we get,
$B = \frac{2\pi}{\lambda}$ = metre^{-1}

and, $D = \frac{2\pi}{T}$ = second^{-1}

i.e. $\left[\frac{D}{B}\right] = \frac{\text{second}^{-1}}{\text{metre}^{-1}} = \frac{\text{metre}}{\text{second}}$ = velocity

44. $Y = \frac{X}{3Z^2} = \frac{[M^{-1}L^{-2}T^4A^2]}{[M^1L^0T^{-2}A^{-1}]^2} = [M^{-3}L^{-2}T^8A^4]$

45. $[G] = [M^{-1}\,L^3\,T^{-2}]$
$[c] = [M^0\,L^1\,T^{-1}]$
$[h] = [M^1\,L^2\,T^{-1}]$
Now, let the relation between given quantities and length be,
$L = G^x c^y h^z$
∴ $[L^1] = [M^{-1}\,L^3\,T^{-2}]^x\,[M^0\,L^1\,T^{-1}]^y\,[M^1\,L^2\,T^{-1}]^z$
∴ We get,
$-x + z = 0$
i.e., z = x …(i)
$3x + y + 2z = 1$ ...(ii)
$-2x - y - z = 0$ ...(iii)
∴ y = – 3x ...[from (i) and (iii)]

Substituting the value in equation (ii),

$\therefore$ $3x - 3x + 2z = 1$

i.e., $z = \frac{1}{2}$

Substituting this value we get,

$x = \frac{1}{2}$ and $y = \frac{-3}{2}$

$\therefore$ $L = \frac{\sqrt{Gh}}{c^{3/2}}$

46. $T = kr^x\rho^yS^z$

Time (T) = $[L^0M^0T^1]$

Radius (r) = $[L^1M^0T^0]$

Density (ρ) = $[L^{-3}M^1T^0]$

Surface tension (S) = $[L^0M^1T^{-2}]$

$\therefore$ $[T^1] = k[L]^x[ML^{-3}]^y[MT^{-2}]^z$

$\Rightarrow T^1 = kL^{x-3y}M^{y+z}T^{-2z}$

$-2z = 1 \Rightarrow z = -\frac{1}{2}$;

$y + z = 0 \Rightarrow y = -z = +\frac{1}{2}$;

$x - 3y = 0 \Rightarrow x = 3y = \frac{3}{2}$;

$\therefore$ $T = kr^{3/2}\rho^{1/2}S^{-1/2} = k\sqrt{\frac{\rho r^3}{S}}$

47. Let the physical quantity formed of the dimensions of length be given as,

$[L] = [c]^x[G]^y\left[\frac{e^2}{4\pi\in_0}\right]^z$ ….(i)

Now,

Dimensions of velocity of light $[c]^x = [LT^{-1}]^x$

Dimensions of universal gravitational constant $[G]^y = [L^3T^{-2}M^{-1}]^y$

Dimensions of $\left[\frac{e^2}{4\pi\in_0}\right]^z = [ML^3T^{-2}]^z$

Substituting these in equation (i)

$[L] = [LT^{-1}]^x[M^{-1}L^3T^{-2}]^y[ML^3T^{-2}]^z$

$= L^{x+3y+3z}M^{-y+z}T^{-x-2y-2z}$

Solving for x, y, z

$x + 3y + 3z = 1$

$-y + z = 0$

$x + 2y + 2z = 0$

Solving the above equation,

$x = -2, y = \frac{1}{2}, z = \frac{1}{2}$

$\therefore$ $L = \frac{1}{c^2}\left[G\frac{e^2}{4\pi\in_0}\right]^{1/2}$

48. In the given equation, $\frac{\alpha Z}{k\theta}$ should be dimensionless,

$\therefore$ $\alpha = \frac{k\theta}{Z}$

$\Rightarrow [\alpha] = \frac{[M^1L^2T^{-2}K^{-1}\times K^1]}{[L^1]} = [M^1L^1T^{-2}]$

And $P = \frac{\alpha}{\beta}$

$\Rightarrow [\beta] = \left[\frac{\alpha}{P}\right] = \frac{[M^1L^1T^{-2}]}{[M^1L^{-1}T^{-2}]}$

$\Rightarrow [\beta] = [M^0L^2T^0]$

49. $[R] \equiv [M^1L^2T^{-3}A^{-2}]$ using $R = \frac{V}{I}$

$[V] \equiv [M^1L^2T^{-3}A^{-1}]$ using $V = \frac{U}{q}$

$[\rho] \equiv [M^1L^3T^{-3}A^{-2}]$ using $\rho = \frac{RA}{l}$

$[\sigma] \equiv [M^{-1}L^{-3}T^3A^2]$ using $\sigma = \frac{1}{\rho}$

50. Boltzmann constant (k_B) = $\frac{PV}{NT}$

S.I. unit: $J\,K^{-1} \equiv [M^1L^2T^{-2}K^{-1}]$

Coefficient of viscosity (η) = $\frac{F}{A\left(\frac{dv}{dx}\right)}$

S.I. unit : $\frac{Ns}{m^2} \equiv [M^1L^{-1}T^{-1}]$

Water equivalent is the mass of water that will absorb or lose same quantity of heat as that of the substance for the same change in temperature.

S.I. unit : kg $\equiv [M^1L^0T^0]$

Coefficient of thermal conductivity (K)

$= \frac{Q}{At\left(\frac{\Delta\theta}{\Delta x}\right)}$

S.I. unit : J/ m s K $\equiv [M^1L^1T^{-3}K^{-1}]$

53. 30 VSD = 29 MSD

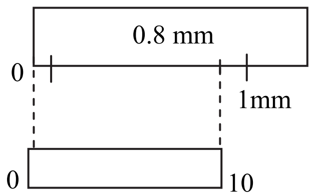
1 VSD $= \frac{29}{30}$ MSD

L.C. = 1 MSD – 1 VSD

$= \left(1 - \frac{29}{30}\right)$ MSD $= \frac{1}{30} \times 0.5°$

= 1 minute

54. 20 VSD = 16 MSD
1 VSD = 0.8 MSD
Least count = MSD – VSD
= 1 mm – 0.8 mm
= 0.2 mm



55. For a given vernier callipers,
1 MSD = 5.15 – 5.10 = 0.05 cm
1 VSD $= \frac{2.45}{50} = 0.049$ cm
∴ L.C = 1 MSD – 1VSD = 0.001 cm
Thus, the reading = 5.10 + (0.001 × 24)
= 5.124 cm
⇒ diameter of cylinder = 5.124 cm

56. As per the question, the measured value is 3.50 cm. Hence the least count must be 0.01 cm = 0.1 mm
For vernier scale, where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm
1 MSD = 1 mm and 9 MSD = 10 VSD,
Least count = 1 MSD – 1 VSD = 0.1 mm
Hence, correct option is (B).

57. One main scale division, 1 M.S.D. = x cm
One vernier scale division,
1 V.S.D. $= \frac{(n-1)x}{n}$
Least count = 1 M.S.D. – 1 V.S.D.
$= \frac{nx - nx + x}{n} = \frac{x}{n}$ cm.

58. Least count of screw gauge $= \frac{1}{100}$ mm
= 0.01 mm
Diameter = Main scale reading + (Divisions on circular scale × least count)
$= 0 + \left(52 \times \frac{1}{100}\right) = 0.52$ mm
Diameter = 0.052 cm.

59. 30 VSD = 29 MSD
1 VSD $= \frac{29}{30}$ MSD
Least count of vernier = 1 M.S.D. – 1 V.S.D.
$= 0.5° - \left(\frac{29}{30} \times 0.5°\right)$
$= \frac{0.5°}{30}$
Reading of vernier = M.S. reading + V.S. reading × L.C.
$= 58.5° + 9 \times \frac{0.5°}{30}$
= 58.65°

61. $A = 4\pi r^2$
∴ Fractional error $\frac{\Delta A}{A} = \frac{2\Delta r}{r}$
$\frac{\Delta A}{A} \times 100 = 2 \times 0.3\% = 0.6\%$

63. Volume of sphere (V) $= \frac{4}{3}\pi r^3$
% error in volume $= 3 \times \frac{\Delta r}{r} \times 100$
$= \left(3 \times \frac{0.1}{5.3}\right) \times 100$

64. $R = \frac{V}{I} \Rightarrow \pm\frac{\Delta R}{R} = \pm\frac{\Delta V}{V} \pm \frac{\Delta I}{I}$
= 3 + 3 = 6%

65. Given that: $P = \frac{a^3b^2}{cd}$
error contributed by a $= 3 \times \left(\frac{\Delta a}{a} \times 100\right)$
= 3 × 1% = 3%
error contributed by b $= 2 \times \left(\frac{\Delta b}{b} \times 100\right)$
= 2 × 2% = 4%
error contributed by c $= \left(\frac{\Delta c}{c} \times 100\right) = 3\%$
error contributed by d $= \left(\frac{\Delta d}{d} \times 100\right) = 4\%$
∴ Percentage error in P is given as,
$\frac{\Delta p}{p} \times 100$ = (error contributed by a)+(error contributed by b) + (error contributed by c) + (error contributed by d)
= 3% + 4% + 3% + 4%
= 14%

66. Given: $x = \frac{a^2b^2}{c}$

Percentage error is given by,

$$\frac{\Delta x}{x} = \frac{2\Delta a}{a} + \frac{2\Delta b}{b} + \frac{\Delta c}{c}$$

$= (2 \times 2) + (2 \times 3) + 4$

$= 4 + 6 + 4 = 14$

$\therefore \quad \frac{\Delta x}{x}\% = 14\%$

67. Least count $= \frac{\text{Pitch}}{\text{No. of div. in circular scale}}$

$= \frac{0.5}{50}$

$= 0.01$ mm

Actual reading $= 0.01 \times 35 + 3 = 3.35$ mm

Taking error into consideration

$= 3.35 + 0.03$

$= 3.38$ mm.

68. Zero error $= 5 \times \frac{0.5}{50} = 0.05$ mm

Actual measurement

$= 2 \times 0.5 \text{ mm} + 25 \times \frac{0.5}{50} - 0.05$ mm

$= 1 \text{ mm} + 0.25 \text{ mm} - 0.05$ mm

$= 1.20$ mm

69. Main Scale Reading (MSR) = 0.5 mm

Circular Scale Division (CSD) = 25^{th}

Number of divisions on circular scale = 50

Pitch of screw = 0.5 mm

$\therefore$ LC of screw gauge $= \frac{0.5}{50} = 0.01$ mm

$\therefore$ zero error $= -5 \times \text{LC} = -0.05$ mm

$\therefore$ zero correction = +0.05 mm

Observed reading $= 0.5 \text{ mm} + (25 \times 0.01)$ mm

$= 0.75$ mm

Corrected reading $= 0.75 \text{ mm} + 0.05$ mm

$= 0.80$ mm

70. Least count of screw gauge = 0.001 cm

$= 0.01$ mm

Main scale reading = 5 mm,

Zero error = – 0.004 cm

= – 0.04 mm

Zero correction = +0.04 mm

$$\begin{matrix}\text{Observed} \\ \text{reading}\end{matrix} = \begin{matrix}\text{Mainscale} \\ \text{reading}\end{matrix} + (\text{Division} \times \text{least count})$$

Observed reading $= 5 + (25 \times 0.01) = 5.25$ mm

$$\begin{matrix}\text{Corrected} \\ \text{reading}\end{matrix} = \begin{matrix}\text{Observed} \\ \text{reading}\end{matrix} + \begin{matrix}\text{zero} \\ \text{correction}\end{matrix}$$

Corrected reading $= 5.25 + 0.04$

$= 5.29 \text{ mm} = 0.529$ cm

71. Least count $= \frac{0.5}{50} = 0.01$ mm

Diameter of ball $D = 2.5 \text{ mm} + (20)(0.01)$

$D = 2.7$ mm

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$$

$$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta M}{M} + 3\frac{\Delta D}{D}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = 2\% + \left[3\left(\frac{0.01}{2.7}\right)\times 100\%\right]$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = 3.1\%$$

72. Pressure $(P) = \frac{\text{Force (F)}}{\text{Area (A)}}$

$= \frac{F}{L^2}$ .... ($\because$ Area = length^2)

Percentage error in pressure is given by,

$$\therefore \quad \frac{\Delta P}{P} \times 100 = \frac{\Delta F}{F} \times 100 + 2\frac{\Delta L}{L} \times 100$$

$= 4\% + 2(3\%) = 4\% + 6\% = 10\%$

73. Density $(\rho) = \frac{\text{mass}}{\text{Volume}} = \frac{m}{l^3}$

…(for cube $V = l^3$)

Percentage relative error in density will be,

$$\frac{\Delta\rho}{\rho}\times 100 = \frac{\Delta m}{m}\times 100 + 3\frac{\Delta l}{l}\times 100$$

$= 1.5 + (3 \times 1) = 1.5 + 3 = 4.5\%$

74. Least count of both instrument

$$\Delta d = \Delta l = \frac{0.5}{100} \text{ mm} = 5 \times 10^{-3} \text{mm}$$

$$Y = \frac{4MLg}{\pi l d^2}$$

$$\left(\frac{\Delta Y}{Y}\right)_{max} = \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$$

Error due to l measurement $\frac{\Delta l}{l}$

$$= \frac{0.5/100 \text{mm}}{0.25 \text{mm}} = 2\%$$

Error due to d measurement,

$$2\frac{\Delta d}{d} = \frac{2\times\frac{0.5}{100}}{0.5\,\text{mm}} = \frac{0.5/100}{0.25} = 2\%$$

75. We have;

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring

$$T^2 = 4\pi^2\left(\frac{l}{g}\right)$$

$\therefore$ $g = 4\pi^2\frac{l}{T^2}$

Fractional error in g is

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

76. $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\left(\frac{\Delta T}{T}\right)$

$\therefore$ $\Delta g = g\left[\frac{\Delta L}{L} + 2\left(\frac{\Delta T}{T}\right)\right]$

Time for 20 oscillations = 40 s

$\therefore$ Time for 1 oscillation = $\frac{40}{20}$

$\therefore$ T = 2 s

$$g = \frac{4\pi^2 L}{T^2} = \frac{4(3.14)^2\times 0.98}{(2)^2} = 9.68\ \text{m/s}^2$$

$\therefore$ $\Delta g = 9.68\left[\frac{0.1}{98} + 2\left(\frac{0.1}{2}\right)\right]$

$\therefore$ $\Delta g = 9.68\left[\frac{0.1}{98} + 0.1\right]$

77. Given : $T = 2\pi\sqrt{\frac{L}{g}}$

$\Rightarrow g = 4\pi^2.\frac{L}{T^2}$

% Accuracy in determination of g,

$$\frac{\Delta g}{g}\times 100 = \frac{\Delta L}{L}\times 100 + 2\frac{\Delta T}{T}\times 100$$

$$= \frac{\Delta L}{L}\times 100 + 2\frac{\Delta t}{t}\times 100$$

$$= \frac{0.1}{20}\times 100 + 2\times\frac{1}{90}\times 100$$

$$= \frac{100}{200} + \frac{200}{90} = 0.5 + 2.22$$

$= 2.72 \approx 3\%$

78. $D = 1.25\times 10^{-2}$ m; $h = 1.45\times 10^{-2}$ m

The maximum permissible error in D

$= \Delta D = 0.01\times 10^{-2}$ m

The maximum permissible error in h

$= \Delta h = 0.01\times 10^{-2}$ m

g is given as a constant and is errorless.

$$T = \frac{rhg}{2}\times 10^3\ \text{N/m} = \frac{dhg}{4}\times 10^3\ \text{N/m}$$

$\therefore$ % error $\frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h}$

$\therefore$ $\frac{\Delta T}{T}\times 100 = \frac{\Delta d}{d}\times 100 + \frac{\Delta h}{h}\times 100$

$$= \left(\frac{0.01\times 10^{-2}}{1.25\times 10^{-2}} + \frac{0.01\times 10^{-2}}{1.45\times 10^{-2}}\right)\times 100$$

$$= \frac{100}{125} + \frac{100}{145}$$

$\therefore$ $\frac{\Delta T}{T} = 0.8\ \% + 0.7\ \% = 1.5\ \%$

79. $g = \frac{4\pi^2 l}{T^2}$

$\therefore$ % error in g $= \frac{\Delta g}{g}\times 100$

$$= \left(\frac{\Delta l}{l}\right)\times 100 + 2\left(\frac{\Delta T}{T}\right)\times 100$$

$$E_I = \frac{0.1}{64}\times 100 + 2\left(\frac{0.1}{16}\right)\times 100 = 1.406\%$$

$$E_{II} = \frac{0.1}{64}\times 100 + 2\left(\frac{0.1}{16}\right)\times 100 = 1.406\%$$

$$E_{III} = \frac{0.1}{20}\times 100 + 2\left(\frac{0.1}{9}\right)\times 100 = 2.72\%$$

81. $\frac{ML^2}{Q^2} = \frac{\left[M^1L^2\right]}{\left[A^1T^1\right]^2} = [M^1L^2T^{-2}A^{-2}]$

These are the dimensions of unit Henry.

82. Given: $P = \frac{x^2 - b}{at}$

From principle of homogeneity, ‘b’ will have the dimensions of x^2

$\therefore$ $[b] = [L^2]$ ….(i)

Also,

$[P] = [M^1L^2T^{-3}]$

$[t] = [T^1]$

$\therefore$ $[a] = \frac{[b]}{[P][t]} = \frac{[L^2]}{[M^1L^2T^{-3}][T^1]}$

$[a] = [M^{-1}T^2]$ ….(ii)

$\therefore \quad \frac{[b]}{[a]} = \frac{[L^2]}{[M^{-1}T^2]} = [M^1L^2T^{-2}]$ ….(iii)

Torsional constant $K = \frac{\tau}{\theta}$

$\therefore \quad [K] = [\tau]$

$[K] = [M^1L^2T^{-2}]$ ….(iv)

From (iii) and (iv),

$\frac{[b]}{[a]} = [K]$

83. $F = ma = \frac{mv}{t}$

$\therefore \quad m = \frac{Ft}{v}$

$\therefore \quad [m] = \left[\frac{Ft}{v}\right] = [F^1\ V^{-1}\ T^1]$

84. $[\text{Surface tension}] = \left[\frac{F}{L}\right] = \left[\frac{E}{L^2}\right] = \left[\frac{E}{(VT)^2}\right]$

$= [E^1V^{-2}T^{-2}]$

85. $[T] = [M^1T^{-2}]$

$[\eta] = [M^1L^{-1}T^{-1}]$

$[\rho] = [M^1L^{-3}]$

From the options given,

$\frac{[T]}{[\eta]} = \frac{[M^1T^{-2}]}{[M^1L^{-1}T^{-1}]} = [L^1T^{-1}] = [v]$

$\therefore \quad v = \frac{T}{\eta}$

86. Planck constant (h) is related to angular momentum (L) as, $\frac{nh}{2\pi} = L$

$\Rightarrow [h] = [L] = [mvr]$

Moment of inertia $I = mr^2$

$\therefore \quad \frac{h}{I} = \frac{mvr}{mr^2} = \frac{v}{r} = \omega$

But $\omega = 2\pi f$

$\Rightarrow \left[\frac{h}{I}\right] = [2\pi f] = [f]$

Evaluation Test

1. A given dimensional formula may represent two or more physical quantities. But given a physical quantity, it has unique dimensions…

2. $[A] = [M^1L^1T^{-2}]$

$[m] = \frac{[\text{mass}]}{[\text{length}]}$

$= [M^1L^{-1}T^0]$

$[B] = \frac{[A]}{[m]} = \frac{[M^1L^1T^{-2}]}{[M^1L^{-1}T^0]}$

$= [M^0L^2T^{-2}]$

This is a dimensional formula for latent heat.

3. An instrument is said to have a high degree of precision if measured value remains unchanged over number of readings repeated. Here readings are constant upto three significant figures. Hence average measurement is precise. But, as zero error is not considered readings are inaccurate.

5. Capacitance, $C = \frac{q}{V} = \frac{q}{\text{work / charge}} = \frac{q^2}{W}$

$\therefore \quad [C] = \frac{[C^1]^2}{[M^1L^2T^{-2}]} = [M^{-1}L^{-2}T^2C^2]$

6. Two full turns of circular scale covers distance of 1 mm. Hence one full turn will cover distance of 0.5 mm.

$\therefore$ L.C. of given instrument $= \frac{0.5}{50}$

Diameter = Zero error + MSR + CSR × LC

$= 0.02 + 4 + 37 \times \frac{0.5}{50}$

$= 4.39$ mm

7. In the expression, $U = \frac{A\sqrt{x}}{x^2 + B}$

B must have the dimensions of x^2 i.e., $[L^2]$

Dimensions of $A = \frac{Ux^2}{\sqrt{x}} = \frac{[M^1L^2T^{-2}]L^2}{L^{1/2}}$

$= [M^1L^{7/2}T^{-2}]$

$\therefore \quad AB = [M^1L^{7/2}T^{-2}]\ [L^2] = [M^1L^{11/2}T^{-2}]$

8. The number 37800 has three significant digits because the terminal zeros in a number without a decimal point are not significant. All zeros occuring between two non-zero digits are significant.

9. $\because A = B + \dfrac{C}{D+E}$

$\therefore$ [D] = [E]

$\therefore$ $[A] = [B] = \left[\dfrac{C}{D+E}\right] = \left[\dfrac{C}{D}\right] = \left[\dfrac{C}{E}\right]$

$\therefore$ $[A] = [B] = [M^0LT^{-1}]$

$\left[\dfrac{C}{D}\right] = [A] = [M^0LT^{-1}]$

$[D] = [E] = \left[\dfrac{C}{LT^{-1}}\right] = \left[\dfrac{M^0LT^0}{M^0LT^{-1}}\right] = [T]$

10. Given $X = \dfrac{A^2B}{C^{1/3}D^3}$

Taking logarithm of both sides,

$\log X = 2\log A + \log B - \dfrac{1}{3}\log C - 3\log D$

Partially differentiating,

$\dfrac{\delta X}{X} = 2\dfrac{\delta A}{A} + \dfrac{\delta B}{B} - \dfrac{1}{3}\dfrac{\delta C}{C} - 3\dfrac{\delta D}{D}$

Percentage error in A $= 2\dfrac{\delta A}{A} = 2 \times 1\% = 2\%$

Percentage error in B $= \dfrac{\delta B}{B} = 3\%$

Percentage error in C $= \dfrac{1}{3}\dfrac{\delta C}{C} = \dfrac{1}{3} \times 4\% = \dfrac{4}{3}\%$

Percentage error in D $= 3\dfrac{\delta D}{D} = 3 \times 5\% = 15\%$

The minimum percentage error is contributed by C. Hence the correct choice is (C).

11. $E = [M^1L^2T^{-2}]$, $G = [M^{-1}L^3T^{-2}]$, $I = [M^1L^1T^{-1}]$

$\therefore$ $\left[\dfrac{GI^2M}{E^2}\right] = \dfrac{[M^{-1}L^3T^{-2}][M^1L^1T^{-1}]^2[M^1]}{[M^1L^2T^{-2}]^2}$

$= [M^0L^1T^0]$

This is the dimension of wavelength.

12. Mere dimensional correctness of an equation does not ensure its physical correctness. A dimensionally correct equation may or may not be physically correct but a dimensionally incorrect equation is definitely incorrect.

13. Percentage error

$= 3\dfrac{\Delta r}{r} \times 100$

$= 3 \times \dfrac{0.4}{6.2} \times 100 = 19.35\%$

Nearest answer is option (C).

14. Here $[N] = [M^0L^{-2}T^{-1}]$

$[n_1] = [n_2] = [M^0L^{-3}T^0]$

and $[z_1] = [z_2] = [M^0L^1T^0]$

Hence $[D] = \dfrac{[N]}{[n_1]} \times [z_1]$

$= \dfrac{[M^0L^{-2}T^{-1}]}{[M^0L^{-3}T^0]} \times [M^0L^1T^0]$

$= [M^0L^2T^{-1}]$

15. As zero of circular scale is above the reference line of graduation, zero correction is positive and zero error is negative

$\therefore$ Zero error $= -4 \times 10^{-3}$ cm

16. Relative velocity is defined as the time rate of change of relative position of one object with respect to another. It is not the ratio of similar quantities.

17. M.S.D. = 3.48 cm, V.S.D. = 6

L.C. = 0.01 cm

$\therefore$ observed internal diameter of calorimeter

D_0 = M.S.D. + (V.S.D. × L.C.)

$= 3.48 + (6 \times 0.01) = 3.48 + 0.06$

$D_0 = 3.54$ cm

zero error = –0.03 cm

Since, zero error is negative, it is added into observed reading.

Corrected internal diameter,

$D = D_0$ + zero error

$D = 3.54 + 0.03 = 3.57$ cm

18. M.S.D. = 6.4 cm, V.S.D. = 4

L.C = 0.01 cm

$\therefore$ observed depth of beaker

= M.S.D. + (V.S.D. × L.C.) = 6.4 + (4 × 0.01)

= 6.4 + 0.04 = 6.44 cm

Here zero error = 0

$\therefore$ Actual depth of beaker = observed depth of beaker = 6.44 cm.

Textbook Chapter No.

02 Scalars and Vectors

Hints

Classical Thinking

19. $\vec{A} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$|\vec{A}| = \sqrt{(3)^2 + (2)^2 + (-4)^2} = \sqrt{29}$

20. $\vec{P} = 3\hat{i} + \hat{j} + 2\hat{k}$

Length in XY plane $= \sqrt{(3)^2 + (1)^2} = \sqrt{10}$ unit

21. Magnitude of $\vec{A} = |\vec{A}| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{14}$

$\therefore$ Direction cosine $= -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}$ and $\frac{3}{\sqrt{14}}$

22. $\overline{PQ} = \vec{Q} - \vec{P}$

$= \left(-2\hat{i} - 5\hat{j} + 7\hat{k}\right) - \left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$

$= -4\hat{i} - 8\hat{j} + 13\hat{k}$

23. Resultant vector $= \vec{A} + \vec{B} + \vec{C}$

$= \left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) + \left(\hat{i} + \hat{j} + 3\hat{k}\right) + \left(4\hat{i} + 5\hat{j} + 3\hat{k}\right)$

$= 9\hat{i} + 8\hat{j} + 3\hat{k}$

24. Resultant of vectors $\vec{A}$ and $\vec{B}$

$\vec{R} = \vec{A} + \vec{B}$

$= 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$

$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

25. $\vec{A} + \vec{B} = \left(3\hat{i} + 2\hat{j} - 4\hat{k}\right) + \left(5\hat{i} - 7\hat{j} + 2\hat{k}\right)$

$\vec{A} + \vec{B} = 8\hat{i} - 5\hat{j} - 2\hat{k}$

Let $\vec{P}$ be the vector when added to $\vec{A} + \vec{B}$ gives a unit vector along X-axis.

$\therefore$ $\vec{P} + 8\hat{i} - 5\hat{j} - 2\hat{k} = \hat{i}$

$\Rightarrow \vec{P} = -7\hat{i} + 5\hat{j} + 2\hat{k}$

27. $14.14 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 90^\circ}$

But $F_1 = F_2 = F$

$\therefore$ $14.14 = \sqrt{2F^2}$

$\therefore$ $199.94 = 2F^2$

$\therefore$ $F = 9.99 \approx 10$ N

$\therefore$ $F_1 = F_2 = 10$ N

28. $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$

$= \sqrt{\left(\sqrt{2}\right)^2 + (3)^2 + 2\left(\sqrt{2}\right)(3)\cos 45^\circ}$

$F = \sqrt{2 + 9 + 6} = \sqrt{17}$ N

29. Vertical component of velocity,

$v_y = v \sin\theta = 20 \times \sin 30^\circ$

$v_y = 10$ m/s

30. Component of force of gravity $= F_y = F \sin\theta$

$F_y = mg \sin 30^\circ = 10 \times 9.8 \times \frac{1}{2} = 49$ N

31. $\vec{A} = 3(2\hat{i} + 3\hat{j} - \hat{k}) = 2\vec{B}$

As $\vec{A}$ is scalar multiple of $\vec{B}$, $\vec{A}$ and $\vec{B}$ are parallel.

34. Electric flux $d\phi = \vec{E} \cdot \overline{ds}$

35. $\phi = \vec{B} \cdot \vec{A}$

where, $\vec{B}$ is magnetic induction and $\vec{A}$ is area vector.

37. $\vec{P} \cdot \vec{Q} = 0$ $\left(\because \vec{P} \perp \vec{Q}\right)$

$\left(5\hat{i} + 7\hat{j} - 3\hat{k}\right) \cdot \left(2\hat{i} + 2\hat{j} - a\hat{k}\right) = 0$

$(5)(2) + (7)(2) + (-3)(-a) = 0$

$10 + 14 + 3a = 0$

$\therefore$ $a = -8$

38. Power $= \vec{F} \cdot \vec{v}$

$= \left(5\hat{i} + 6\hat{j}\right) \cdot \left(4\hat{j} - 2\hat{k}\right) = 24$ unit

39. $W = \vec{F}\cdot\vec{s}$

$= (2\hat{i}+3\hat{j}+5\hat{k}).(3\hat{i}+2\hat{j}+2\hat{k}) = 6 + 6 + 10$

$\therefore$ W = 22 J

40. $\cos\theta = \dfrac{\vec{P}\cdot\vec{Q}}{|\vec{P}||\vec{Q}|}$

$= \dfrac{(3\hat{i}+\hat{j}+2\hat{k})(\hat{i}-2\hat{j}+3\hat{k})}{\left(\sqrt{(3)^2+(1)^2+(2)^2}\right)\left(\sqrt{(1)^2+(-2)^2+(3)^2}\right)}$

$= \dfrac{3-2+6}{\sqrt{14}\sqrt{14}} = \dfrac{7}{14} = \dfrac{1}{2}$

$\therefore$ $\cos\theta = \dfrac{1}{2}$

$\therefore$ $\theta = 60°$

41. $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

$= \dfrac{(\hat{i}+\hat{j}+\hat{k}).(-\hat{i}-\hat{j}+2\hat{k})}{\sqrt{(1)^2+(1)^2+(1)^2}.\sqrt{(-1)^2+(-1)^2+(2)^2}}$

$= \dfrac{-1-1+2}{\sqrt{18}} = 0$

$\therefore$ $\theta = 90°$

42. $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\vec{A}\cdot\vec{B} = 3\times5\times\cos 60° = 15\times\dfrac{1}{2}$

$\therefore$ $\vec{A}\cdot\vec{B} = 7.5$

46. $\vec{\tau} = \vec{r}\times\vec{F}$

47. $\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 3 & -4\\ 3 & -4 & 5\end{vmatrix}$

$= \hat{i}(15-16) - \hat{j}(-10+12) + \hat{k}(8-9)$

$= -\hat{i} - 2\hat{j} - \hat{k}$

48. $\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ -3 & 1 & -2\end{vmatrix}$

$= \hat{i}(-2-1) - \hat{j}(-2+3) + \hat{k}(1+3)$

$= -3\hat{i} - \hat{j} + 4\hat{k}$

49. $\vec{P}\times\vec{Q} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ 3 & 1 & -1\end{vmatrix}$

$= \hat{i}(-2-1) - \hat{j}(-1-3) + \hat{k}(1-6)$

$= -3\hat{i} + 4\hat{j} - 5\hat{k}$

50. Angular momentum

$= \vec{r}\times\vec{p} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 2\\ 2 & 4 & 5\end{vmatrix}$

$= \hat{i}(-5-8) - \hat{j}(15-4) + \hat{k}(12+2)$

$= -13\hat{i} - 11\hat{j} + 14\hat{k}$

52. Area of triangle $= \dfrac{1}{2}|\vec{A}\times\vec{B}|$

$\therefore$ $\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & -2\\ 2 & 2 & 3\end{vmatrix}$

$= \hat{i}(-6+4) - \hat{j}(3+4) + \hat{k}(2+4)$

$= -2\hat{i} - 7\hat{j} + 6\hat{k}$

$\therefore$ $|\vec{A}\times\vec{B}| = \sqrt{(-2)^2+(-7)^2+(6)^2} = \sqrt{89}$

Area of triangle $= \dfrac{\sqrt{89}}{2} = 4.717$ sq. unit

53. Let $\vec{P} = \hat{i}+2\hat{j}+3\hat{k}$, $\vec{Q} = (\hat{i}-3\hat{j}+\hat{k})$

Area of parallelogram $= |\vec{P}\times\vec{Q}|$

$\vec{P}\times\vec{Q} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 1 & -3 & 1\end{vmatrix}$

$= \hat{i}(2+9) - \hat{j}(1-3) + \hat{k}(-3-2)$

$= 11\hat{i} + 2\hat{j} - 5\hat{k}$

$\therefore$ $|\vec{P}\times\vec{Q}| = \sqrt{(11)^2+(2)^2+(-5)^2}$

$= \sqrt{121+4+25} = \sqrt{150}$ m^2

54. $\vec{P}+\vec{Q} = (\hat{i}+2\hat{j}-4\hat{k}) + (\hat{i}+2\hat{j}-\hat{k})$

$= 2\hat{i}+4\hat{j}-5\hat{k}$

$\vec{P}-\vec{Q} = (\hat{i}+2\hat{j}-4\hat{k}) - (\hat{i}+2\hat{j}-\hat{k}) = -3\hat{k}$

$(\vec{P}+\vec{Q}).(\vec{P}-\vec{Q}) = (2\hat{i}+4\hat{j}-5\hat{k})(-3\hat{k}) = 15$

55. $\left(2\vec{A}-\vec{B}\right) = 2\left(2\hat{i}+3\hat{j}+4\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$= 3\hat{i}+4\hat{j}+5\hat{k}$

$\left(\vec{A}+2\vec{B}\right) = \left(2\hat{i}+3\hat{j}+4\hat{k}\right)+2\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$= 4\hat{i}+7\hat{j}+10\hat{k}$

$\left(2\vec{A}-\vec{B}\right).\left(\vec{A}+2\vec{B}\right)$

$=\left(3\hat{i}+4\hat{j}+5\hat{k}\right)\left(4\hat{i}+7\hat{j}+10\hat{k}\right)$

$= 12 + 28 + 50 = 90$

56. $\vec{s} = \vec{s}_2-\vec{s}_1$

$= \left(14\hat{i}+13\hat{j}+9\hat{k}\right)-\left(3\hat{i}+2\hat{j}-6\hat{k}\right)$

$= 11\hat{i}+11\hat{j}+15\hat{k}$

$W = \vec{F}\cdot\vec{s}$

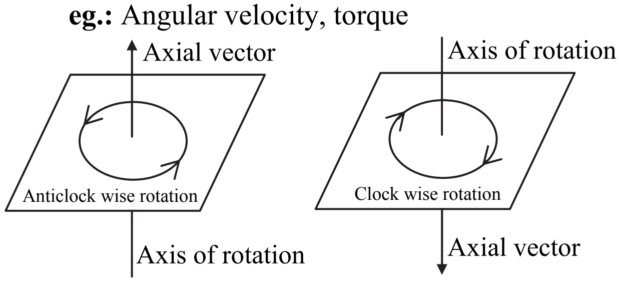
$= \left(4\hat{i}+\hat{j}+3\hat{k}\right)\left(11\hat{i}+11\hat{j}+15\hat{k}\right)$

$= 44 + 11 + 45 = 100$ J

Critical Thinking

4. The vectors acting along parallel straight lines are called collinear vectors. When they are in same direction, angle between them is 0^c and they are said to be parallel vectors. When they are in opposite direction, angle between them is π^c and they are said to be antiparallel vectors.

6. A vector representing rotational effects and is always along the axis of rotation in accordance with right hand screw rule is called an axial vector.
eg.: Angular velocity, torque



9. Resultant of forces will be zero when they can be represented by the sides of a triangle taken in same order. In such a case, the sum of the two smaller sides of the triangle is more than the third side.
Only in option (D), sum of the first two forces is smaller than third force, thus not forming a possible triangle.

11. As the multiple of $\hat{j}$ in the given vector is zero therefore this vector lies in XZ plane and projection of this vector on Y-axis is zero.

12. $\vec{R} = 3\hat{i}+\hat{j}+2\hat{k}$

∴ Length in XY plane $= \sqrt{R_x^2+R_y^2} = \sqrt{3^2+1^2}$

$= \sqrt{10}$

13. If two vectors $\vec{A}$ and $\vec{B}$ are given then the resultant R_{max} = A + B = 7 N and R_{min} = 4 − 3 = 1 N i.e., net force on the particle is in between 1 N and 7 N.

14. $5 = \sqrt{F_1^2+F_2^2+2F_1F_2.\cos 90°}$

$25 = F_1^2+F_2^2$ …..(i)

When θ = 120°

$\sqrt{13} = \sqrt{F_1^2+F_2^2+2F_1F_2\cos 120°}$

$13 = 25 + 2F_1F_2\left(-\frac{1}{2}\right)$

$13 = 25 - F_1F_2$

$F_1F_2 = 12$

$F_2 = \frac{12}{F_1}$ …..(ii)

Substituting equation (ii) in (i)

$F_1^2+\frac{144}{F_1^2} = 25$

$F_1^4 + 144 = 25F_1^2$

$F_1^4 - 25F_1^2 + 144 = 0$

$(F_1^2 - 9)(F_1^2 - 16) = 0$

$F_1, F_2 = 3, 4$

15. $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$

16. Magnitude of vector = 1

$\sqrt{a_x^2+a_y^2+a_z^2} = 1$

∴ $\sqrt{0.5^2+0.8^2+c^2} = 1$

$\sqrt{c^2+0.89} = 1$

∴ $c^2 = 0.11$

∴ $c = \sqrt{0.11}$

17. Negative of the given vector be $\vec{A}$.

∴ $\vec{A} = -\left(-\hat{i}+\hat{j}-\hat{k}\right)$

Unit vector in direction of $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

$$= \frac{-\left(-\hat{i}+\hat{j}-\hat{k}\right)}{\sqrt{(1)^2+(-1)^2+(1)^2}}$$

$$= \frac{-1}{\sqrt{3}}\left(-\hat{i}+\hat{j}-\hat{k}\right)$$

18. Magnitude of vector $\vec{A} = |\vec{A}|$

$$= \sqrt{(2)^2+(6)^2}$$

$$= \sqrt{4+36} = \sqrt{40}$$

Unit vector parallel to $\vec{A}$ is $\dfrac{\vec{A}}{|\vec{A}|} = \dfrac{2\hat{i}+6\hat{j}}{\sqrt{40}}$

Magnitude of vector $\vec{B} = |\vec{B}|$

$$= \sqrt{(4)^2+(3)^2} = 5$$

Let $\vec{p}$ be the required vector then $\dfrac{\vec{p}}{p} = \hat{p}$

$$\vec{p} = \hat{p}\,p = \left(\frac{2\hat{i}+6\hat{j}}{\sqrt{40}}\right)5$$

$$= \frac{\sqrt{10}}{4}\left[2\left(\hat{i}+3\hat{j}\right)\right] = \frac{\sqrt{10}}{2}\left(\hat{i}+3\hat{j}\right)$$

19. Let $\hat{n}_1$ and $\hat{n}_2$ be the two unit vectors, then the sum is

$\hat{n}_S = \hat{n}_1 + \hat{n}_2$

$n_S^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta = 1 + 1 + 2\cos\theta$

As n_S is also a unit vector,

$\Rightarrow 1 = 1 + 1 + 2\cos\theta$

$\therefore\ \cos\theta = -\dfrac{1}{2} \Rightarrow \theta = 120°$

Let the difference vector be $\hat{n}_d = \hat{n}_1 - \hat{n}_2$

$n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta$

$= 1 + 1 - 2\cos(120°)$

$\therefore\ n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3$

$\therefore\ n_d = \sqrt{3}$

20. From the figure, $|\overline{OA}| = a$ and $|\overline{OB}| = a$

Also from triangle rule, $\overline{OB} - \overline{OA} = \overline{AB} = \Delta\vec{a}$

$\therefore\ |\Delta\vec{a}| = AB$

since $d\theta = \dfrac{\text{arc}}{\text{radius}} \Rightarrow AB = a\,d\theta$

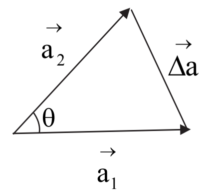
$\therefore\ |\Delta\vec{a}| = a d\theta$

Δa means change in magnitude of vector i.e., $|\overline{OB}| - |\overline{OA}|$

$\therefore\ a - a = 0$

Hence, $\Delta a = 0$

21. From the figure,



$\vec{a}_2 = \vec{a}_1 + \Delta\vec{a}$

$\Rightarrow \Delta\vec{a} = \vec{a}_2 - \vec{a}_1$

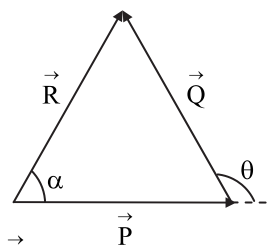
Also $|\vec{a}_2| = |\vec{a}_1| = a$

$\therefore\ \Delta a = \left|\vec{a}_2 - \vec{a}_1\right| = \left[a_2^{\,2} + a_1^{\,2} - 2a_2a_1\cos\theta\right]^{1/2}$

$$= \left[2a^2\left(1-\cos\theta\right)\right]^{1/2}$$

$$= \left[2a^2(2\sin^2\theta/2)\right]^{\frac{1}{2}} = 2\,a\sin\theta/2.$$

22.



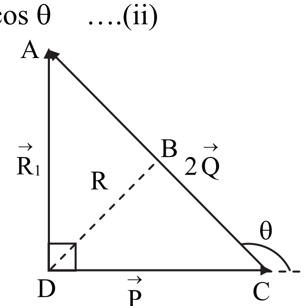
$\vec{P} + \vec{Q} = \vec{R}$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ….(i)

and $\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

When Q is doubled, resultant is perpendicular to $\vec{P}$

$\therefore\ R_1^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ ….(ii)



From right angled triangle ADC

$4Q^2 = R_1^2 + P^2$

$R_1^2 = 4Q^2 - P^2$

Substituting in (ii) and solving,

$P^2 + 2PQ\cos\theta = 0$ ….(iii)

Substituting (iii) in (i),

$R = Q$

23. Mass $= \dfrac{\text{Force}}{\text{Acceleration}} = \dfrac{|\vec{F}|}{a}$

$$= \frac{\sqrt{36+64+100}}{1} = 10\sqrt{2}\ \text{kg}$$

24. $\vec{A}$ and $\vec{B}$ are parallel to each other. This implies $\vec{A} = m\vec{B}$. comparing X-component, $m = \frac{1}{6}$. Comparing Y-component, b = 18 and comparing Z-component a = 1.

25. Let $\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k}$ and $\vec{B} = -4\hat{i} + 4\hat{j} + m\hat{k}$.

For $\vec{A}$ perpendicular to $\vec{B}$,

$\vec{A}.\vec{B} = 0$

$\therefore$ $(2\hat{i}+3\hat{j}+8\hat{k})(-4\hat{i}+4\hat{j}+m\hat{k}) = 0$

$\therefore$ $-8 + 12 + 8m = 0$

$\therefore$ $m = -\frac{1}{2}$

26. $W = \vec{F}\cdot\vec{s}$

$\therefore$ $6 = (3\hat{i}+c\hat{j}+2\hat{k})(4\hat{i}+2\hat{j}+3\hat{k})$

$\therefore$ $6 = 12 + 2c + 6$

$6 = 18 + 2c$

$2c = -12$

$c = -6$

27. $W = \vec{F}\cdot\vec{s} = Fs\cos\theta$

For force causing displacement in its own direction $\theta = 0°$

$\therefore$ $W = Fs = \left(\sqrt{(7)^2+(-4)^2+(-4)^2}\right)\times 10$

$= \left(\sqrt{49+16+16}\right)\times 10 = 9\times 10 = 90$ J

28. $\vec{P} + \vec{Q} = 5\hat{i}-4\hat{j}+3\hat{k}$

Let α be the angle made by $\vec{P}+\vec{Q}$ with X-axis

$\therefore$ $\cos\alpha = \frac{(\vec{P}+\vec{Q})\cdot\hat{i}}{|\vec{P}+\vec{Q}||\hat{i}|}$

$= \frac{5}{\sqrt{5^2+(-4^2)+3^2}} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$

$\therefore$ $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

30. $\vec{A} = 3\hat{i}+\hat{j}+2\hat{k}$, $\vec{B} = 2\hat{i}-2\hat{j}+4\hat{k}$

$\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$

$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ 2 & -2 & 4\end{vmatrix}$

$= \hat{i}(4+4) - \hat{j}(12-4) + \hat{k}(-6-2)$

$= 8\hat{i} - 8\hat{j} - 8\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{8^2+(-8)^2+(-8)^2} = \sqrt{192}$

$|\vec{A}| = \sqrt{(3)^2+(1)^2+(2)^2} = \sqrt{14}$

$|\vec{B}| = \sqrt{(2)^2+(-2)^2+(4)^2} = \sqrt{24}$

$\sin\theta = \frac{\sqrt{192}}{\sqrt{14}\sqrt{24}} \approx 0.76$

32. Let the two vectors be $\vec{A}$ and $\vec{B}$.

$\sin\theta = \frac{\vec{A}\times\vec{B}}{|\vec{A}||\vec{B}|}$

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

$\cot\theta = \frac{\vec{A}\cdot\vec{B}}{\vec{A}\times\vec{B}} = \sqrt{3}$ $\therefore$ $\theta = 30°$

34. $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = (\vec{A}\times\vec{A}) - (\vec{A}\times\vec{B}) + (\vec{B}\times\vec{A}) - (\vec{B}\times\vec{B})$

$= -(\vec{A}\times\vec{B}) + (\vec{B}\times\vec{A})$

$+ (\vec{B}\times\vec{A}) + (\vec{B}\times\vec{A})$

$= 2(\vec{B}\times\vec{A})$

35. $\vec{P}.(\vec{P}+\vec{Q}) = P^2$

$\therefore$ $\vec{P}\cdot\vec{P} + \vec{P}\cdot\vec{Q} = P^2$ $\Rightarrow$ $P^2 + \vec{P}\cdot\vec{Q} = P^2$

$\therefore$ $\vec{P}\cdot\vec{Q} = 0$ $\Rightarrow$ $PQ\cos\theta = 0$

$\therefore$ $\cos\theta = 0$ $\Rightarrow$ $\theta = 90°$

36. $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta = 0$ and

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta = 0$

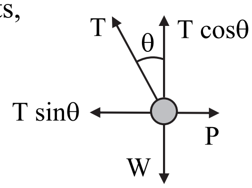
If $\vec{A}$ and $\vec{B}$ are not null vectors then sin θ and cos θ both should be zero simultaneously. This is not possible so it is essential that one of the vector must be null vector.

37. Cross product of two vectors is perpendicular to the plane containing both the vectors.

38. As the ball is in equilibrium under the effect of three forces,

$\vec{T} + \vec{P} + \vec{W} = 0$. Hence, option (B) is true.

Resolving tension into two rectangular components,



From the figure,

T cosθ = W and T sinθ = P

$\frac{T\cos\theta}{T\sin\theta} = \frac{W}{P} \Rightarrow$ P = W tanθ. Hence, option (A) is true.

Also, $(T \sin\theta)^2 + (T \cos\theta)^2 = T^2$

$\Rightarrow \quad P^2 + W^2 = T^2$

Hence option, (C) is true.

But T = $\sqrt{(T\sin\theta)^2 + (T\cos\theta)^2} = \sqrt{P^2 + W^2}$

Hence, option (D) is wrong.

Competitive Thinking

4. Resultant of two vectors $\vec{A}$ and $\vec{B}$ can be given by, $\vec{R} = \vec{A} + \vec{B}$

$|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

If θ = 0 then $|\vec{R}| = A + B = |\vec{A}| + |\vec{B}|$

5. Initial position vector

$\vec{r_1} = (-3\hat{i} + 4\hat{j} - 3\hat{k})$ m

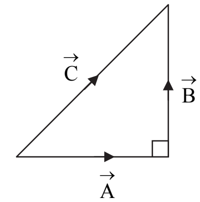
Final position vector

$\vec{r_2} = (7\hat{i} - 2\hat{j} - 3\hat{k})$ m

Displacement $\vec{r} = \vec{r_1} - \vec{r_2}$

$= (7\hat{i} - 2\hat{j} - 3\hat{k}) - (-3\hat{i} + 4\hat{j} - 3\hat{k}) = 10\hat{i} - 6\hat{j}$

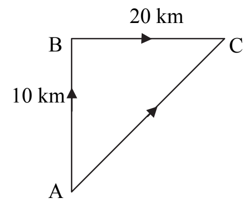
6.



$C = \sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2} = 5$

∴ Angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$

7.



$\overline{AC} = \overline{AB} + \overline{BC}$

$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(10)^2 + (20)^2}$

$= \sqrt{100 + 400} = \sqrt{500} = 22.36$ km

8. From figure we have,

$\vec{A} = 4\hat{i} + 3\hat{j}$ ….(i)

$\vec{B} = 3\hat{i}$ ….(ii)

$\vec{C} = 2\hat{j}$ ….(iii)

Resultant is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$\vec{R} = (4\hat{i} + 3\hat{j}) + 3\hat{i} + 2\hat{j}$

$\vec{R} = 7\hat{i} + 5\hat{j}$

Magnitude of resultant vector is

$|\vec{R}| = \sqrt{49 + 25}$

$|\vec{R}| = \sqrt{74} = 8.6$ m

Angle made by $\vec{R}$ with X-axis is,

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{5}{7}\right) = 35.5°$

9. Velocity of A

$\vec{v_A} = 10$ km/h

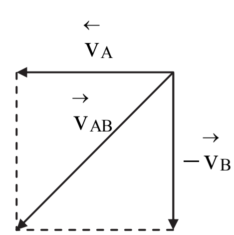
Velocity of B

$\vec{v_B} = 10$ km/h

velocity of A w.r.t. B

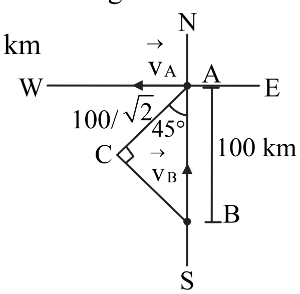
$\vec{v_{AB}} = \vec{v_A} - \vec{v_B}$

$|\vec{v_{AB}}| = \sqrt{(10)^2 + (10)^2}$



$|\vec{v}_{AB}| = 10\sqrt{2}$ km/h directed along AC

displacement AC = $\frac{100}{\sqrt{2}}$ km



∴ time $t = \frac{AC}{|\vec{v}_{AB}|} = \frac{\left(\frac{100}{\sqrt{2}}\right)}{10\sqrt{2}}$

t = 5 h

10. Particle B moves making an angle of 60° with X-axis. Hence resolving it into components,

$\vec{v}_B = 20\cos60°\,\hat{i} + 20\sin60°\,\hat{j}$

Relative velocity, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

$= \left(20\cos60°\,\hat{i} + 20\sin60°\,\hat{j}\right) - 10\hat{i}$

$= \left(10\hat{i} + 10\sqrt{3}\,\hat{j}\right) - 10\hat{i} = 10\sqrt{3}\,\hat{j}$

11. $R = \sqrt{12^2 + 5^2 + 6^2} = \sqrt{144 + 25 + 36}$

$= \sqrt{205} \approx 14.31$ m

12. $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

∴ $40\sqrt{3} = \sqrt{F^2 + F^2 + 2F^2\cos60°} = \sqrt{3F^2}$

$\Rightarrow F = 40$ N

13. $F_{max} = 5 + 10 = 15$ N and $F_{min} = 10 - 5 = 5$ N
Range of resultant force is $5 \le F \le 15$

14. $R_{max} = A + B = 17$ when $\theta = 0°$
$R_{max} = A - B = 7$ when $\theta = 180°$
by solving, A = 12 and B = 5
When $\theta = 90°$
then $R = \sqrt{A^2 + B^2}$
$\Rightarrow R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$

15. $\vec{r} = \vec{a} + \vec{b} + \vec{c} = 4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - \hat{k}$

$= \hat{i} + \hat{j} - \hat{k}$

$\hat{r} = \frac{\vec{r}}{|r|} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

16. Here, $\vec{B} + \vec{C} = \left(\hat{i} - 3\hat{j} + 5\hat{k}\right) + \left(2\hat{i} + \hat{j} - 4\hat{k}\right)$

$= 3\hat{i} - 2\hat{j} + \hat{k} = \vec{A}$

As, $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$,

$\left|\vec{A}\right| = \sqrt{9 + 4 + 1} = \sqrt{14}$ ....(i)

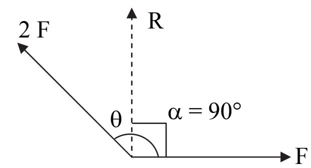
Similarly,

$\left|\vec{B}\right| = \sqrt{1 + 9 + 25} = \sqrt{35}$ ....(ii)

$\left|\vec{C}\right| = \sqrt{4 + 1 + 16} = \sqrt{21}$ ....(iii)

From equations (i), (ii) and (iii), we get,
$B^2 = A^2 + C^2$

17. $\tan\alpha = \frac{2F\sin\theta}{F + 2F\cos\theta} = \infty$ (as $\alpha = 90°$)



$F + 2F\cos\theta = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = 120°$

18. $|\vec{A}| + |\vec{B}| = 18$ ....(i)

$12 = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ....(ii)

$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \tan 90°$

$\Rightarrow \cos\theta = -\frac{A}{B}$ ....(iii)

By solving (i), (ii) and (iii),
A = 13 N and B = 5 N

19. $F_{net}^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$\left(\frac{F}{3}\right)^2 = F^2 + F^2 + 2F^2\cos\theta$

$\frac{F^2}{9} = 2F^2(1 + \cos\theta)$

∴ $1 + \cos\theta = \frac{1}{18}$

$\cos\theta = \left(-\frac{17}{18}\right)$

20. $|\vec{F}_R| = |\vec{F} + \vec{F}| = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$

$= \left[2F^2(1 + \cos\theta)\right]^{\frac{1}{2}}$

$= \left[2F^2(2\cos^2\theta/2)\right]^{\frac{1}{2}}$

$= 2F\cos\theta/2$

21. Since, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

A = B = R

∴ $A^2 = 2A^2 + 2A^2\cos\theta$

$\cos\theta = -\frac{1}{2} = \cos 120°$

∴ $\theta = 120°$

22. $R^2 = (3P)^2 + (2P)^2 + 2 \times 3P \times 2P \times \cos\theta$
$R^2 = 9P^2 + 4P^2 + 12P^2 \cos\theta$
$R^2 = 13P^2 + 12P^2 \cos\theta$ ….(i)
$(2R)^2 = (6P)^2 + (2P)^2 \times 2 \times 6P \times 2P \times \cos\theta$
$4R^2 = 40P^2 + 24P^2 \cos\theta$
$R^2 = 10P^2 + 6P^2 \cos\theta$ ….(ii)
From (i) and (ii)
$13P^2 + 12P^2 \cos\theta = 10P^2 + 6P^2 \cos\theta$
$3P^2 = -6P^2 \cos\theta$
$\therefore$ $\cos\theta = -\dfrac{1}{2}$
$\theta = 120°$

23. As $\left|\vec{A}+\vec{B}\right| = \left|\vec{A}-\vec{B}\right|$,
$\therefore$ $A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$
$\therefore$ $4AB\cos\theta = 0$, i.e. $\cos\theta = 0 = \cos 90°$,
$\therefore$ $\theta = 90°$

25. Let θ be the angle between $\vec{A}$ and $\vec{B}$.
Given: $|\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$
$\therefore$ $|\vec{A}+\vec{B}|^2 = n^2|\vec{A}-\vec{B}|^2$
$\therefore$ $A^2 + B^2 + 2AB\cos\theta = n^2[A^2 + B^2 - 2AB\cos\theta]$
$\therefore$ $A^2 + A^2 + 2A^2\cos\theta = n^2[A^2 + A^2 - 2A^2\cos\theta]$
($\because$ A = B)
$\therefore$ $2A^2(1 + \cos\theta) = n^2 2A^2(1 - \cos\theta)$
$\therefore$ $1 + \cos\theta = n^2(1 - \cos\theta)$
$\therefore$ $(n^2 + 1)\cos\theta = (n^2 - 1)$
$\therefore$ $\cos\theta = \left(\dfrac{n^2-1}{n^2+1}\right)$ $\therefore$ $\theta = \cos^{-1}\left(\dfrac{n^2-1}{n^2+1}\right)$

26. Unit vector $= 0.8\hat{i} + b\hat{j} + 0.4\hat{k}$
$\therefore$ $\sqrt{(0.8)^2 + b^2 + (0.4)^2} = 1$
$\therefore$ $0.64 + b^2 + 0.16 = 1$
$\therefore$ $0.80 + b^2 = 1$ $\therefore$ $b^2 = 1 - 0.8 = 0.2$
$\therefore$ $b = \sqrt{0.2}$

27. The angle α which the resultant R makes with A is given by
$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$
$\tan\left(\dfrac{\theta}{2}\right) = \dfrac{B\sin\theta}{A + B\cos\theta}$ $\left(\because \alpha = \dfrac{\theta}{2}\right)$
$\Rightarrow \dfrac{\sin\left(\dfrac{\theta}{2}\right)}{\cos\left(\dfrac{\theta}{2}\right)} = \dfrac{2B\sin\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{\theta}{2}\right)}{A + B\cos\theta}$
Which gives $A + B\cos\theta = 2B\cos^2\left(\dfrac{\theta}{2}\right)$
$\Rightarrow A + B\left[2\cos^2\left(\dfrac{\theta}{2}\right) - 1\right] = 2B\cos^2\left(\dfrac{\theta}{2}\right)$
Which gives A = B.

28. Let $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{k}$
$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \Rightarrow \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$
$\therefore$ $|\vec{A}|\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|} = \dfrac{(2\hat{i} + 3\hat{j} + \hat{k})\cdot(3\hat{i} + 4\hat{k})}{\sqrt{3^2 + 4^2}}$
$= \dfrac{10}{5} = 2$

29. $\vec{A}\cdot\vec{B} = AB\cos\theta$
$\therefore$ $|\vec{A}|\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{B} = \dfrac{2+3}{\sqrt{2}} = \dfrac{5}{\sqrt{2}}$

30. $\vec{A}\cdot\vec{B} = 0$
$\therefore$ $[2 \times (-4)] + (3 \times 4) + (8 \times \alpha) = 0$
$\therefore$ $-8 + 12 + 8\alpha = 0$ $\therefore$ $8\alpha + 4 = 0$
$\therefore$ $\alpha = -\dfrac{4}{8} = -\dfrac{1}{2}$

31. $\vec{P}\cdot\vec{Q} = 0$
$\therefore$ $a^2 - 2a - 3 = 0 \Rightarrow a = 3$

32. As the vectors are mutually perpendicular,
$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{C} = 0$
$\therefore$ $(a\hat{i} + \hat{j} + \hat{k})\cdot(\hat{i} + b\hat{j} + \hat{k}) = 0$
$\therefore$ $a + b + 1 = 0$ ….(i)
Similarly,
$1 + b + c = 0$ ....(ii)
$a + 1 + c = 0$ ....(iii)
Adding equations (i), (ii) and (iii), we get,
$2(a + b + c) + 3 = 0$
$\therefore$ $a + b + c = -\dfrac{3}{2}$
$\therefore$ $-1 + c = \dfrac{-3}{2}$ ....[from (i)]
$\therefore$ $c = -\dfrac{1}{2}$
Substituting in equation (ii) and (iii), we get,
$a = b = -\dfrac{1}{2}$

33. Vectors are orthogonal, i.e., $\vec{A}\cdot\vec{B} = 0$

∴ $\cos\omega t \cos\left(\frac{\omega t}{2}\right) + \sin\omega t \sin\left(\frac{\omega t}{2}\right) = 0$

∴ $\cos\left[\omega t - \frac{\omega t}{2}\right] = 0$ ∴ $\cos\left(\frac{\omega t}{2}\right) = 0$

$\Rightarrow \frac{\omega t}{2} = \frac{\pi}{2}$

∴ $t = \frac{\pi}{\omega}$

34. $\vec{F_1}\cdot\vec{F_2} = (2\hat{j} + 5\hat{k})(3\hat{j} + 4\hat{k}) = 6 + 20 = 26$

35. $\vec{F} = \hat{i} + 5\hat{k}$

$\vec{s} = -4\hat{i} + 4\hat{j} + 2\hat{k}$

$W = \vec{F}\cdot\vec{s} = 1 \times (-4) + 5 \times 2 = -4 + 10 = 6$ J

36. $P = \vec{F}\cdot\vec{v} = (4\hat{i} + \hat{j} - 2\hat{k}).(2\hat{i} + 2\hat{j} + 3\hat{k})$

$= (8 + 2 - 6)$ W $= 4$ W

37. Given: $\vec{F} = \left(2t\hat{i} + 3t^2\hat{j}\right)$

But, F = ma

As mass m = 1 kg ,

∴ $a = \frac{F}{1}$

$\vec{a} = 2t\hat{i} + 3t^2\hat{j}$

$v = \int_0^t a\,dt = t^2\hat{i} + t^3\hat{j}$

$P = \vec{F}\cdot\vec{v} = \left(2t\hat{i} + 3t^2\hat{j}\right)\cdot(t^2\hat{i} + t^3\hat{j})$

$= 2t\cdot t^2 + 3t^2\cdot t^3 = (2t^3 + 3t^5)$ W

38. $\vec{A}\cdot\vec{B} = AB\cos\theta$

Given, $\vec{A}\cdot\vec{B} = -|\vec{A}||\vec{B}|$

i.e., $\cos\theta = -1$

∴ $\theta = 180°$

i.e., $\vec{A}$ and $\vec{B}$ act in the opposite direction.

39. $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)\cdot\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)}{\sqrt{3^2 + 4^2 + 5^2}\sqrt{3^2 + 4^2 + (-5^2)}}$

$= \frac{9 + 16 - 25}{\sqrt{25}\sqrt{25}} = 0$

$\Rightarrow \theta = 90°$

40. Unit vector along X-axis is $\hat{i}$.

$\cos\theta = \frac{\vec{p}\cdot\hat{i}}{|\vec{p}||\hat{i}|} = \frac{\left(\hat{i} + \hat{j} + \hat{k}\right).\left(\hat{i}\right)}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{1^2}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

41. $(\hat{i} + \hat{j}).(\hat{j} + \hat{k}) = 0 + 0 + 1 + 0 = 1$

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{1}{\sqrt{2}\times\sqrt{2}} = \frac{1}{2}$

∴ $\theta = 60°$

42. $p_x = 2\cos t$, $p_y = 2\sin t$

∴ $\vec{p} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

$\vec{F} = \frac{d\vec{p}}{dt} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$

$\vec{F}\cdot\vec{p} = (-2\sin t\,\hat{i} + 2\cos t\,\hat{j})\cdot(2\cos t\,\hat{i} + 2\sin t\,\hat{j})$

$\vec{F}\cdot\vec{p} = 0$

∴ $\theta = 90°$

43. $\vec{AB} = 3\hat{i} + \hat{j} + \hat{k}$

$\vec{AC} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{CB} = \vec{AB} - \vec{AC} = \left(3\hat{i} + \hat{j} + \hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right) = 2\hat{i} - \hat{j}$

∠ABC is angle between $\vec{AB}$ and $\vec{CB}$,

∴ Consider,

$\vec{AB}\cdot\vec{CB} = \left|\vec{AB}\right|\left|\vec{CB}\right|\cos\theta$ ....(i)

∴ $\vec{AB}\cdot\vec{CB} = \left(3\hat{i} + \hat{j} + \hat{k}\right)\cdot\left(2\hat{i} - \hat{j}\right) = 6 - 1 = 5$

$\left|\vec{AB}\right| = \sqrt{(3)^2 + (1)^2 + (1)^2} = \sqrt{11}$

and $\left|\vec{CB}\right| = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

∴ $5 = \sqrt{11}\times\sqrt{5}\times\cos\theta$ ....[from (i)]

∴ $\cos\theta = \frac{\sqrt{5}}{\sqrt{11}}$

∴ $\theta = \cos^{-1}\left(\sqrt{\frac{5}{11}}\right)$

45. $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

Where, $\hat{n}$ is a unit vector indicating the direction of $\vec{A}\times\vec{B}$.

Vector product is non commutative,

$\therefore$ $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

46. Direction of vector A is along Z-axis

$\therefore$ $\vec{A} = a\hat{k}$

Direction of vector B is towards north

$\therefore$ $\vec{B} = b\hat{j}$

Now $\vec{A}\times\vec{B} = a\hat{k}\times b\hat{j} = ab(-\hat{i})$

$\therefore$ The direction of $\vec{A}\times\vec{B}$ is along west.

47. Vector product is non commutative,

$\therefore$ $\vec{v} = \vec{r}\times\vec{\omega}$ and $\vec{v} = -\left(\vec{\omega}\times\vec{r}\right)$

48. $$\vec{v} = \vec{r}\times\vec{\omega} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & -6 & 6\\ 3 & -4 & 1\end{vmatrix}$$

$$= \hat{i}(-6+24) - \hat{j}(5-18) + \hat{k}(-20+18)$$

$$\vec{v} = 18\hat{i} + 13\hat{j} - 2\hat{k}$$

49. AB sin θ = –AB sin θ

2AB sin θ = 0

sin θ = 0 or θ = 180°

50. $$\vec{r} = \left(2\hat{i} - 3\hat{k}\right) - \left(2\hat{i} - 2\hat{j} - 2\hat{k}\right) = 2\hat{j} - \hat{k}$$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$= \left(2\hat{j} - \hat{k}\right)\times\left(4\hat{i} + 5\hat{j} - 6\hat{k}\right)$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 2 & -1\\ 4 & 5 & -6\end{vmatrix}$$

$$= \hat{i}\left[-12-(-5)\right] - \hat{j}\left[0-(-4)\right] + \hat{k}\left[0-8\right]$$

$$= -7\hat{i} - 4\hat{j} - 8\hat{k}$$

51. For angular momentum to be conserved,

$\vec{\tau}_{ext} = 0$

$\therefore$ $\vec{r}\times\vec{F} = 0$

$\therefore$ $$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -6 & -12\\ \alpha & 3 & 6\end{vmatrix} = 0$$

$$\hat{i}(-36+36) - \hat{j}(12+12\alpha) + \hat{k}(6+6\alpha) = 0$$

$\therefore$ $12 + 12\alpha = 0$ or $6 + 6\alpha = 0$

$\therefore$ $\alpha = -1$

52. Angular momentum

$\vec{L} = \vec{r}\times\vec{p}$ in terms of component becomes

$$\vec{L} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ p_x & p_y & p_z\end{vmatrix}$$

As motion is in x-y plane ($z = 0$ and $p_z = 0$), hence

$\vec{L} = \vec{k}(xp_y - yp_x)$

Here $x = vt$, $y = b$, $p_x = mv$ and $p_y = 0$

$\therefore$ $\vec{L} = \vec{k}[vt\times 0 - bmv] = -mvb\hat{k}$

53. Here, $\vec{r}_1 = 1.5\,\hat{j}$, $\vec{r}_2 = 2.8\,\hat{i}$

$\vec{p}_1 = 6.5\times 2.2\hat{i} = 14.3\hat{i}$

$\vec{P}_2 = 3.1\times 3.6\hat{j} = 11.16\hat{j}$

$\vec{L} = \vec{r}\times\vec{p}$

$\therefore$ $\vec{L} = \left(\vec{r}_1\times\vec{p}_1\right) + \left(\vec{r}_2\times\vec{p}_2\right)$

$$= \left[1.5\hat{j}\times 14.3\hat{i}\right] + \left[2.8\hat{i}\times 11.16\hat{j}\right]$$

$$= 21.45\,(-\hat{k}) + 31.248\hat{k} = 9.798\,\hat{k}\text{ kg m}^2/\text{s}$$

54. $\vec{L} = \vec{r}\times\vec{p} = \vec{r}\times(m\vec{v}) = m\left[\vec{r}\times\vec{v}\right]$

$$\vec{r}\times\vec{v} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 4 & 6\\ 5 & 4 & 6\end{vmatrix}$$

$$= \hat{i}[24-24] - \hat{j}[-12-30] + \hat{k}[-8-20]$$

$$= 42\hat{j} - 28\hat{k}$$

$\therefore$ $\vec{L} = m(42\hat{j} - 28\hat{k})$

55. Area of parallelogram = $|\vec{A}\times\vec{B}|$

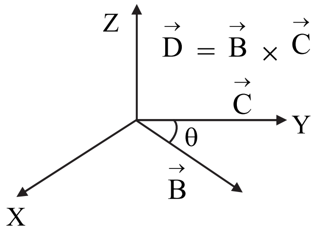
$$\vec{A}\times\vec{B} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\times\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 3 & -2 & 1\end{vmatrix}$$

$$= (8)\hat{i} + (8)\hat{j} - (8)\hat{k}$$

$\therefore$ $|\vec{A}\times\vec{B}| = \sqrt{64+64+64} = 8\sqrt{3}$

56. $\vec{A}\cdot\vec{B} = 0;\ \vec{A}\cdot\vec{C} = 0$



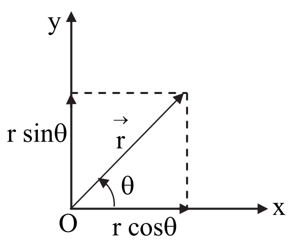
$\vec{A}$ is perpendicular to $\vec{B}$ as well as $\vec{C}$.

Let $\vec{D} = \vec{B}\times\vec{C}$

The direction of $\vec{D}$ is perpendicular to the plane containing $\vec{B}$ and $\vec{C}$.

Hence, $\vec{A}$ is parallel to $\vec{D}$ i.e., $\vec{A}$ is parallel to $\vec{B}\times\vec{C}$.

57.



Component of vector $\vec{r}$ along x-axis is r cosθ.

∴ $\vec{r}_x = r\cos\theta$

Now r_x will have maximum value if $\cos\theta = 1$

∴ $\theta = \cos^{-1}(1)$

∴ $\theta = 0$

Hence, component of $\vec{r}$ along x-axis will have maximum value if $\vec{r}$ is along +ve x-axis.

58. $|\vec{A}\times\vec{B}| = \sqrt{3}\ \vec{A}\cdot\vec{B}$

$AB\sin\theta = \sqrt{3}\,AB\cos\theta$

∴ $\tan\theta = \sqrt{3}$

∴ $\theta = 60^\circ$

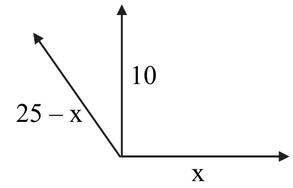
Now $|\vec{R}| = |\vec{A}+\vec{B}|$

$= \sqrt{A^2+B^2+2AB\cos\theta}$

$= \sqrt{A^2+B^2+2AB\left(\frac{1}{2}\right)}$

$= (A^2+B^2+AB)^{1/2}$

59.



$(25-x)^2 = 10^2 + x^2$

$625 + x^2 - 50x = 100 + x^2$

∴ $x = 10.5$ N

∴ $25 - x = 14.5$ N

60. The net force acting on particle,

$\vec{F} = \vec{F}_1 + \vec{F}_2 = 5\hat{i} - 3\hat{j} + \hat{k}$

Displacement,

$\vec{s} = \vec{r}_2 - \vec{r}_1 = -20\hat{i} - 15\hat{j} + 7\hat{k}$ cm

∴ $W = \vec{F}.\vec{s}$

$= (-100 + 45 + 7)\times 10^{-2}$

$= -0.48$ J

61. Given,

$\vec{s} = \vec{r}_2 - \vec{r}_1 = \left(4\hat{j} + 3\hat{k}\right) - \left(-2\hat{i} + 5\hat{j}\right)$

$= 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{F} = 4\hat{i} + 3\hat{j}$

∴ $W = \vec{F}\cdot\vec{s} = \left(4\hat{i} + 3\hat{j}\right)\cdot\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = 8 - 3 + 0$

$= 5$ J

62. For motion of the particle from (0, 0) to (a, 0)

$\vec{F} = -K(y\hat{i} + x\hat{j}) \Rightarrow \vec{F} = -K a\hat{j}$

Displacement $\vec{r} = (a\hat{i} + 0\hat{j}) - (0\hat{i} + 0\hat{j}) = a\hat{i}$

So work done from (0, 0) to (a, 0) is given by

$W = \vec{F}\cdot\vec{r} = -Ka\hat{j}\cdot a\hat{i} = 0$

For motion (a, 0) to (a, a)

$\vec{F} = K(a\hat{i} + a\hat{j})$ and displacement

$\vec{r} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j}) = a\hat{j}$

So work done from (a, 0) to (a, a),

$W = \vec{F}\cdot\vec{r}$

$= -K(a\hat{i} + a\hat{j})\cdot a\hat{j} = -Ka^2$

So total work done $= -Ka^2$

63. The sum of the two forces be,

$\vec{F}_1 = \vec{A} + \vec{B}$ ….(i)

The difference of the two forces be,

$\vec{F}_2 = \vec{A} - \vec{B}$ ….(ii)

Since sum of the two forces is perpendicular to their difference,

$\vec{F}_1 \cdot \vec{F}_2 = 0$

$\Rightarrow \left(\vec{A} + \vec{B}\right) \cdot \left(\vec{A} - \vec{B}\right) = 0$

$\Rightarrow A^2 - A \cdot B + B \cdot A - B^2 = 0$

$\therefore$ $A^2 = B^2$

$\Rightarrow |A| = |B|$

Thus, the forces are equal to each other in magnitude.

65. $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$= \vec{a} \times \vec{a} - \vec{a} \times \vec{b} + \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$ ….(i)

As, cross product of parallel vectors is zero

$\therefore$ $\vec{a} \times \vec{a} = \vec{b} \times \vec{b} = \vec{0}$

As, $\vec{a} \times \vec{b} = (ab \sin \theta)\,\hat{n} = -[(ba \sin \theta)\,\hat{n}]$

$= -\vec{b} \times \vec{a}$

Substituting the values in relation (i),

$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = 2(\vec{b} \times \vec{a})$

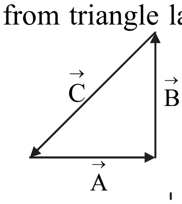
66. Let $\vec{A} \cdot \left(\vec{B} \times \vec{A}\right) = \vec{A} \cdot \vec{C}$

$\vec{C} = \vec{B} \times \vec{A}$ which is perpendicular to both vectors $\vec{A}$ and $\vec{B}$

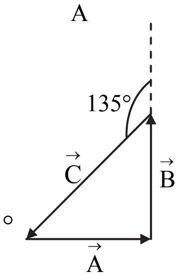
$\therefore$ $\vec{A} \cdot \vec{C} = 0$

67. As $\vec{A} + \vec{B} + \vec{C} = 0$, it means $\vec{A}$, $\vec{B}$ and $\vec{C}$ form a closed triangle and hence from triangle law, resultant is zero.

Also, As $\left|\vec{A}\right| = \left|\vec{B}\right|$,



and $\left|\vec{C}\right| = \sqrt{2}\left|\vec{A}\right|$



$\therefore$ $\vec{A} \perp \vec{B}$ and angle between $\vec{B}$ and $\vec{C}$ is 180° – 45° = 135°

68. $\vec{r} = 3t\hat{i} - 4t^2\hat{j} + 5\hat{k}$

$\therefore$ $\vec{v} = \dfrac{d\vec{r}}{dt} = 3\hat{i} - 4(2t)\hat{j}$

For t = 2 s,

$|\vec{v}| = \sqrt{3^2 + [4(2 \times 2)]^2} = \sqrt{9 + 256} = \sqrt{265}$ m/s

Evaluation Test

1. $\vec{F} = 4\hat{i} + 3\hat{j} - 2\hat{k}$,

$\vec{r} = 1\hat{i} + 1\hat{j} + 0\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 4 & 3 & -2 \end{vmatrix}$$

$= [\hat{i}(-2) - \hat{j}(-2) + \hat{k}(3 - 4)]$

$= -2\hat{i} + 2\hat{j} - \hat{k}$

2. $|\vec{a}| = \sqrt{1^2 + 2^2 + (-2)^2} = 3$ and

$|\vec{b}| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$

$\therefore$ $|\vec{a}| \neq |\vec{b}|$

But two unequal vectors may have same magnitude.

eg.: if $\vec{P} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{Q} = \hat{i} - \hat{j} + \hat{k}$, then two vectors are unequal but $|\vec{P}| = |\vec{Q}|$

3. For the given two forces, magnitude of resultant is maximum if 2 forces act along same direction, i.e., $|\vec{R}_{max}| = |\vec{A} + \vec{B}|$ and magnitude of resultant is minimum if 2 forces act in opposite direction, i.e., $|\vec{R}_{min}| = |\vec{A} - \vec{B}|$

For all other directions,

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ where, θ is the angle between $\vec{A}$ and $\vec{B}$.

Therefore the magnitude of the resultant between 3 N and 5 N will be between 8 N and 2 N.

4. $\vec{A} = 3$ units due east.

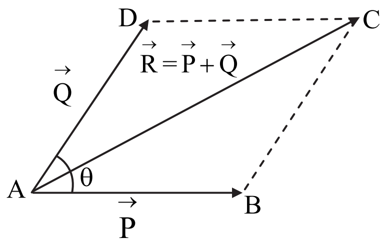
∴ $-4\vec{A} = -4$ (3 units due east)
$= -12$ units due east $= 12$ units due west.

5. The displacement is along the Z direction, i.e., $\vec{s} = 10\hat{k}$

Work done $W = \vec{F}\cdot\vec{s}$

$W = (-2\hat{i} + 15\hat{j} + 6\hat{k})\cdot 10\hat{k} = 60$ J

6.



$R^2 = P^2 + Q^2 + 2PQ\cos\theta$
From this relation, it is clear that
$R^2 = P^2 + Q^2$, when $\theta = 90°$
$R^2 > P^2 + Q^2$, when $\theta < 90°$
$R^2 < P^2 + Q^2$, when $\theta > 90°$

7. $1 = \sqrt{a^2 + b^2} \Rightarrow a^2 + b^2 = 1$ ….(i)

and $\left(a\hat{i} + b\hat{j}\right)\cdot\left(2\hat{i} + \hat{j}\right) = 0 \Rightarrow 2a + b = 0$

$\Rightarrow b = -2a$
Substituting for b in (i)

$a^2 + (-2a)^2 = 1 \Rightarrow a = \frac{1}{\sqrt{5}}$ and $b = -\frac{2}{\sqrt{5}}$

8. $\vec{r} = \hat{i} - \hat{j}$

Torque at that point, $\vec{\tau} = \vec{r}\times\vec{F} = \left(\hat{i} - \hat{j}\right)\times(-4F)\hat{k}$

$\hat{i}\times\hat{k} = -\hat{j}$ and $\hat{j}\times\hat{k} = \hat{i}$

∴ $\vec{\tau} = -4F\left(\hat{i}\times\hat{k}\right) + 4F\left(\hat{j}\times\hat{k}\right)$

$= -4F\left(-\hat{j}\right) + 4F\left(\hat{i}\right)$

$= 4F\hat{i} + 4F\hat{j}$

$= 4F\left(\hat{i} + \hat{j}\right)$

9. $\cos\theta = \frac{\left(\hat{i} + \hat{j} + \hat{k}\right)\cdot\hat{j}}{\left(\hat{i}^2 + \hat{j}^2 + \hat{k}^2\right)^{1/2}\times\left(\hat{j}^2\right)^{1/2}}$

$= \frac{\hat{i}\cdot\hat{j} + \hat{j}\cdot\hat{j} + \hat{k}\cdot\hat{j}}{(1+1+1)^{1/2}\times 1} = \frac{0+1+0}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$(\hat{i}\cdot\hat{j} = \hat{k}\cdot\hat{j} = 0$ and $\hat{j}\cdot\hat{j} = 1)$

10. To find the net force, we vectorially add the three vectors. The x-component is

$F_{net\,x} = -F_1 - F_2\sin 60° + F_3\cos 30°$

$= -3 - 4\sin 60° + 10\cos 30°$

$= -3 - 4\times\frac{\sqrt{3}}{2} + 10\times\frac{\sqrt{3}}{2} = 2.196$ N

and the y-component is

$F_{net\,y} = -F_2\cos 60° + F_3\sin 30°$

$= -4\cos 60° + 10\sin 30° = 3$ N

The magnitude of net force is

$F_{net} = \sqrt{F_{net\,x}^2 + F_{net\,y}^2} = \sqrt{(2.196)^2 + 3^2}$

$= 3.72$ N

The work done by the net force is

$W = F_{net\,x}\,\Delta x = (3.72)(5) \approx 18.6$ J

11. $\frac{\vec{A}.\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{AB\cos\theta}{AB\sin\theta} = \cot\theta$

Given, $\frac{\vec{A}.\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{1}{\sqrt{3}}$

∴ $\cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = 60° = \frac{\pi^c}{3}$

12. The three vectors not lying in one plane cannot form a triangle, hence their resultant cannot be zero. Also, their resultant will neither be in the plane of $\vec{P}$ or $\vec{Q}$ nor in the plane of $\vec{R}$. Hence option (D) is correct.

13. Net force on the body $\vec{F} = \vec{F}_1 + \vec{F}_2$

$= \left(5\hat{i} + \hat{j} - 2\hat{k}\right) + \left(2\hat{i} + \hat{j} - 2\hat{k}\right)$

$= 7\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{s} = 6\hat{i} + 4\hat{j} - 2\hat{k} - \left(2\hat{i} + 2\hat{j} - 4\hat{k}\right)$

$= 4\hat{i} + 2\hat{j} + 2\hat{k}$

$W = \vec{F}.\vec{s} = \left(7\hat{i} + 2\hat{j} - 4\hat{k}\right)\cdot\left(4\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$= 28 + 4 - 8 = 24$ units.

14. Let, $\vec{A} = \hat{i}A_x + \hat{j}A_y$, $\vec{B} = \hat{i}B_x + \hat{j}B_y$

∴ $\vec{A} + \vec{B} = \left(\hat{i}A_x + \hat{j}A_y\right) + \left(\hat{i}B_x + \hat{j}B_y\right)$

$= \hat{i}\left(A_x + A_x\right) + \hat{j}\left(A_y + B_y\right)$

Given $A_x = 4$ m, $A_y = 6$ m

$A_x + B_x = 12$ m , $A_y + B_y = 10$ m

∴ $B_x = 12\text{ m} - A_x = 12\text{ m} - 4\text{ m} = 8$ m

$B_y = 10\text{ m} - A_y = 10\text{ m} - 6\text{ m} = 4$ m.

15. The angle subtended is

$$\sin\theta = \frac{3}{\sqrt{6^2+3^2+4^2}} = \frac{3}{\sqrt{61}}$$

$$\theta = \sin^{-1}\left(\frac{3}{\sqrt{61}}\right)$$

16. $\vec{p} = \hat{i}p_x + \hat{j}p_y$

$= \hat{i}[3\cos t] + \hat{j}[3\sin t]$

According to Newton's second law of motion,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

∴ $\vec{F} = \frac{d}{dt}\left[\hat{i}\,3\cos t + \hat{j}\,3\sin t\right]$

$= \hat{i}(-3\sin t) + \hat{j}(3\cos t)$

∴ $|\vec{F}| = \sqrt{(-3\sin t)^2 + (3\cos t)^2} = 3$

17. $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \Rightarrow \cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|}$

The component of $\vec{A}$ in the direction of $\vec{B}$

$= |\vec{A}|\cos\theta = |\vec{A}|\cdot\dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

$= \dfrac{3+2}{\sqrt{2}} = \dfrac{5}{\sqrt{2}}$ along $\vec{B}$

18. The vector product of two non-zero vectors is zero if they are in the same direction or in the opposite direction. Hence vector $\vec{B}$ must be parallel to vector $\vec{A}$, i.e. along ± x-axis.

19. Area of the triangle $= \dfrac{1}{2}|\vec{A}\times\vec{B}|$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 1 & 0 & -2 \end{vmatrix}$$

$$= \frac{1}{2}\left|\left[\hat{i}(6-0) - \hat{j}(-4-4) + \hat{k}(+3)\right]\right|$$

$$= \frac{1}{2}\left|6\hat{i} + 8\hat{j} + 3\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{36+64+9}$$

$$= \frac{1}{2}\times\sqrt{109} = 5.22 \text{ units}$$

20. $\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Magnitude of vector $\vec{A} = |\vec{A}| = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2}$

where, a_x, a_y and a_z are the magnitudes of projections of $\vec{A}$ along three coordinate axes x, y and z respectively.

$|\hat{j} - \hat{k}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

∴ Component of vector $\vec{A}$ along the direction of $(\hat{j} - \hat{k}) = \dfrac{a_y - a_z}{\sqrt{2}}$

Textbook Chapter No.

04 Force

Hints

Classical Thinking

5. $F = m\left(\frac{v-u}{t}\right)$

$= 0.25\left(\frac{-15-20}{0.1}\right)$ (ball rebounds $\therefore$ v = 0)

$F = -87.5$ N

6. $|\vec{F}| = \sqrt{(6)^2+(-8)^2+(10)^2} = \sqrt{200} = 10\sqrt{2}$

Also F = ma

$\therefore$ $m = \frac{F}{a} = \frac{10\sqrt{2}}{1} = 10\sqrt{2}$ kg

15. $F = \frac{G m_1 m_2}{r^2}$

$= \frac{6.67\times10^{-11}\times6\times10^{24}\times4.8\times10^{24}}{\left(2.5\times10^{10}\right)^2}$

$= 3.1\times10^{18}$ N

22. $v = \frac{MV}{m} = \frac{1000\times30}{3} = 10^4$ cm/s

23. $V = -\frac{mv}{M} = -\frac{0.01\times100}{2.5} = -0.4$ m/s

24. $MV = m_1v_1 + m_2v_2$

$v_2 = \frac{MV}{m_2}$ $[\because v_1 = 0 \text{ m/s}]$

$= \frac{30\times48}{12} = 120$ m s^{-1}

25. For ball 'A',

Initial momentum = 0.05 × 6 = 0.3 kg m s^{-1}

Final momentum = (0.05)(–6) = –0.3 kg m s^{-1}

$\therefore$ Change in momentum = –0.3 – 0.3

= –0.6 kg m s^{-1}

For ball 'B',

initial momentum = 0.05 × (–6)

= –0.3 kg m s^{-1}

Final momentum = (0.05) × (6)

= +0.3 kg m s^{-1}

$\therefore$ Change in momentum = 0.3 – (–0.3)

= 0.3 + (0.3)

= 0.6 kg m s^{-1}

31. $W = F\cdot s = Fs\cos180^\circ$

$= -Fs = -200\times10 = -2000$ J

37. As $m_2 < m_1$, $v_2 > v_1$

39. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$mu + 2m\times0 = m\times0 + 2m\times v_2$

$\therefore$ $v_2 = \frac{u}{2}$

$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\frac{u}{2}-0}{u-0}$

$e = \frac{u/2}{u} = \frac{1}{2} = 0.5$

40. $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$\therefore$ $0.1\times5 + 0.2\times1.2 = (0.1+0.2)\,v$

$\therefore$ $v = \frac{0.5+0.24}{0.3}$

$= \frac{0.74}{0.3}$

= 2.467 m/s

41. $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$\therefore$ $v = \frac{m_1u_1+m_2u_2}{m_1+m_2}$

$= \frac{3\times m + 2m\times0}{m+2m}$

= 1 km h^{-1}

55. $\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 3\\ 2 & -3 & -4\end{vmatrix}$

$= \hat{i}(-8+9) - \hat{j}(-12-6) + \hat{k}(-9-4)$

$= \left(\hat{i}+18\hat{j}-13\hat{k}\right)$ N m

56. $r = \frac{d}{2} = 20$ cm = 0.2 m,

$\vec{\tau} = \vec{r}\times\vec{F} = rF\sin\theta$

In this case, motion of wheel is perpendicular to the axis of rotation. Hence, θ = 90°

$\therefore$ $\tau = rF = 0.2\times10\times9.8 = 19.6$ N m

64. $X_{C.M} = \dfrac{\sum_{i=1}^{3} m_i x_i}{\sum_{i=1}^{3} m_i} = \dfrac{0\times50+50\times5+0\times50}{50+50+50}$

$= \dfrac{250}{150} = \dfrac{5}{3}$ cm

$Y_{CM} = \dfrac{\sum_{i=1}^{3} m_i y_i}{\sum_{i=1}^{3} m_i} = \dfrac{0\times50+0\times50+5\times50}{50+50+50}$

$= \dfrac{250}{150} = \dfrac{5}{3}$ cm

65. $x = \dfrac{(2\times0)+(3\times0)+(5\times1)+(7\times1)}{2+3+5+7} = \dfrac{12}{17}$ m

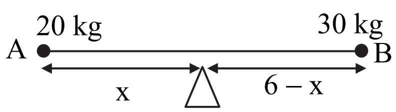
$y = \dfrac{(2\times0)+(3\times1)+(5\times1)+(7\times0)}{2+3+5+7} = \dfrac{8}{17}$ m

66. $m_1r_1 = m_2r_2$

$5\ r_1 = 35\ (0.7 - r_1)$

∴ $r_1 = 0.6125$ m

72.



$20 \times x = 30(6 - x)$

$20x = 180 - 30x$

∴ $50x = 180$

∴ $x = 3.6$ m from 20 kg

74. From the law of conservation of momentum

$3 \times 16 = 6 \times v$

∴ $v = 8$ m/s

∴ K.E. $= \dfrac{1}{2} \times 6 \times (8)^2 = 192$ J

75. Loss of K.E. $= \dfrac{1}{2} \times 0.02 \times (250)^2 = 625$ J

Loss of K.E. = W = F × 0.12

∴ $625 = 0.12$ F

∴ $F = \dfrac{625}{0.12}$

∴ $F = 5.2 \times 10^3$ N

Critical Thinking

3. As the mass of 10 kg has acceleration 12 m/s^2, therefore it applies 120 N force on mass 20 kg in a backward direction.

∴ Net forward force on 20 kg mass = 200 – 120 = 80 N

∴ Acceleration $= \dfrac{80}{20} = 4$ m/s^2

4. Internal force of the system cannot change the momentum.

5. Force, F = (M kg s^{-1}) (v m s^{-1})

= Mv kg m s^{-2} = Mv N

6. Impulse = change in momentum = 2 mv

$= 2 \times 0.06 \times 4 = 0.48$ kg m/s

7. Impulse = Ft = change in momentum

$= mv - (-mv) = 2\ mv = 2 \times 0.01 \times 5 = 0.1$

∴ $F = \dfrac{0.1}{0.01} = 10$ N

8. $F = \dfrac{dp}{dt} = \dfrac{d}{dt}(a + bt^2) = 2bt \therefore F \propto t$

9. When tension in the rope is zero it breaks and acceleration of lift equals g.

10. Given that $\vec{p} = p_x\hat{i} + p_y\hat{j} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

∴ $\vec{F} = \dfrac{d\vec{p}}{dt} = 2\sin t\ \hat{i} - 2\cos t\,\hat{j}$

Here, $\vec{F}\cdot\vec{p} = 0$ hence angle between $\vec{F}$ and $\vec{p}$ is 90°.

11. The pressure on the rear side would be more due to fictitious force (acting in the opposite direction of acceleration) on the rear face. Consequently, the pressure in the front side would be lowered.

15. Law of conservation of linear momentum is correct when no external force acts. When bullet is fired from a rifle then both should possess equal momentum but different kinetic energy. $E = \dfrac{p^2}{2m}$ ∴ Kinetic energy of the rifle is less than that of bullet because $E \propto 1/m$

16. Momentum of one piece $= \dfrac{M}{4} \times 3$

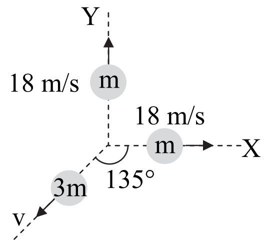
Momentum of the other piece $= \dfrac{M}{4} \times 4$

∴ Resultant momentum $= \sqrt{\dfrac{9M^2}{16} + M^2} = \dfrac{5M}{4}$

The third piece should also have the same momentum. Let its velocity be v, then

$\dfrac{5M}{4} = \dfrac{M}{2} \times v$ or $v = \dfrac{5}{2} = 2.5$ m/s

17. Let two pieces have equal mass m and third piece has a mass of 3 m.



According to law of conservation of linear momentum, since the initial momentum of the system was zero, therefore final momentum of the system must be zero. i.e., the resultant of momentum of two pieces must be equal to the momentum of third piece.

If two particle possess same momentum and angle between them is 90°, then resultant will be given by

$p\sqrt{2} = mv\sqrt{2} = 18\sqrt{2}$ m.

Let the velocity of mass 3m is v. So $3mv = 18m\sqrt{2}$

$\therefore$ $v = 6\sqrt{2}$ m/s and angle 135° from either.

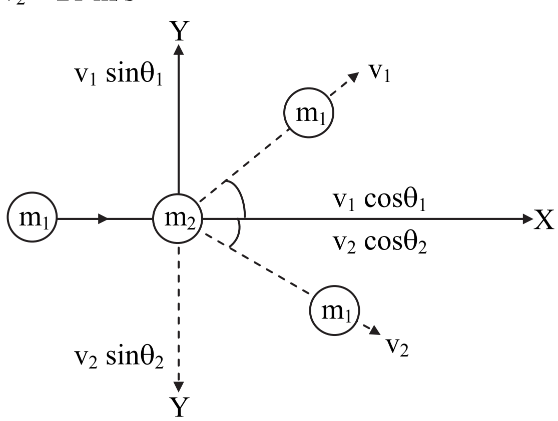
18. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.25 \times 400 + 4.75 \times 0 = 400 \times 0 + 4.75 \times v_2$

$100 = 4.75 \times v_2$

$\therefore$ $v_2 \approx 21$ m/s

19.



According to law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 \cos\theta_1 + m_2v_2 \cos\theta_2$

In this case, $m_1 = m_2 = m$, $u_2 = 0$ and $\theta_1 = \theta_2 = 45°$

$\therefore$ $mu_1 = mv_1 \cos\theta + mv_2 \cos\theta$

$m \times 10 = mv_1 \cos 45° + mv_2 \cos 45°$

$$10\text{ m} = \frac{m}{\sqrt{2}}(v_1 + v_2)$$

$v_1 + v_2 = 10\sqrt{2}$

By conservation of momentum along the direction perpendicular to the original line.

$m \times 0 + m \times 0 = mv_1 \sin 45° - mv_2 \sin 45°$

$$0 = \frac{m(v_1 - v_2)}{\sqrt{2}}$$

$\therefore$ $v_1 = v_2$ $\quad\therefore$ $2v_1 = 10\sqrt{2}$

$\therefore$ $v_1 = 5\sqrt{2}$ m/s $\quad\therefore$ $v_2 = 5\sqrt{2}$ m/s

20. $W = \int dw = \int_A^B \vec{F}\cdot\overline{ds} = \int_{S_1}^{S_2} \vec{F}\cdot\overline{ds}$

21. $W = \int_{x=0}^{x=4} F\,dx = \int_{x=0}^{x=4} (0.5x + 12)\,dx$

$$= \int_{x=0}^{x=4} 0.5x\,dx + \int_{x=0}^{x=4} 12\,dx$$

$$= 0.5\left[\frac{x^2}{2}\right]_{x=0}^{x=4} + 12\,[x]_{x=0}^{x=4}$$

$$= 0.5\left[\frac{4^2 - 0}{2}\right] + 12[4 - 0]$$

$W = 4 + 48 = 52$ J

22. $v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2$

$\therefore$ $0 = \frac{(m_1 - m_2)u_1}{m_1 + m_2}$ $\quad(\because u_2 = 0)$

$\Rightarrow m_1 - m_2 = 0$

$\therefore$ $m_1 = m_2$

$\therefore$ $\frac{m_1}{m_2} = 1$

23. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$2 \times 4 - 1 \times 2 = 2v_1 + v_2$

$\therefore$ $2v_1 + v_2 = 6$

$\therefore$ $v_2 = 6 - 2v_1$

Also $\frac{1}{2}[m_1u_1^2 + m_2u_2^2] = \frac{1}{2}[m_1v_1^2 + m_2v_2^2]$

$\therefore$ $2 \times (4)^2 + 1 \times (-2)^2 = 2(v_1^2) + (v_2^2)$

$\therefore$ $32 + 4 = 2v_1^2 + v_2^2$

$\therefore$ $36 = 2v_1^2 + v_2^2$

$\therefore$ $2v_1^2 + (6 - 2v_1)^2 = 36$

$\therefore$ $v_1 = 0$ or $v_1 = 4$

When $v_1 = 0$, $v_2 = 6$ and $v_1 = 4$, $v_2 = -2$

$\therefore$ $v_1 = 0$, $v_2 = 6$ m/s

24. $v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2$

$$-\frac{u_1}{3} = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}$$

$$= \frac{(0.1 - m_2)u_1 + 2 \times m_2 \times 0}{0.1 + m_2}$$

$\therefore \quad -\frac{u_1}{3} = \frac{(0.1 - m_2)u_1}{0.1 + m_2}$

$\therefore \quad -\frac{1}{3} = \frac{(0.1 - m_2)}{0.1 + m_2}$

$\therefore \quad 0.1 + m_2 = -0.3 + 3m_2$

$\therefore \quad 2m_2 = 0.4$

$\therefore \quad m_2 = 0.2$ kg

25. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\therefore \quad 1 \times 5 - 2 \times 1.5 = 1 \times v_1 + 2v_2$

$\therefore \quad v_1 + 2v_2 = 2 \qquad$ ....(i)

$e = \frac{v_2 - v_1}{u_1 - u_2}$

$\therefore \quad 0.8\ (5 + 1.5) = v_2 - v_1$

$\therefore \quad v_2 - v_1 = 5.2 \qquad$ ....(ii)

Solving equation (i) and (ii) simultaneously

$\therefore \quad v_1 = -2.8$ m/s , $v_2 = +2.4$ m/s

26. $\tau = \frac{dL}{dt}$, if $\tau = 0$ then L = constant

27. No external force is acting on the system so C.M. will not shift.

28. Assuming point A as origin, let AE be along Y-axis and AF be along X-axis. Due to uniform density, let mass of AF be m and mass of AE be 2 m.
The centre of mass of AE is at a distance of l from A and the centre of mass of AF is at a distance of $l/2$ from A.
Hence distance of centre of mass of the metal strip from A is

$X_{c.m.} = \frac{m \times (l/2) + 2m(0)}{m + 2m} = l/6$

$Y_{c.m.} = \frac{m \times (0) + 2m(l)}{m + 2m} = \frac{2l}{3}$

Thus, the coordinates of centre of mass of strip = $(l/6, 2l/3)$
In the given figure, point 'c' is the only point having approximately same coordinates.

29. $x_{CM} = \frac{m_1x_1 + m_2x_2 + ...}{m_1 + m_2 + ...}$

$= \frac{ml + 2m.2l + 3m.3l + ...}{m + 2m + 3m + ...}$

$= \frac{ml(1 + 4 + 9 + ...)}{m(1 + 2 + 3 + ...)} = \frac{\frac{l\,n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}$

$= \frac{l(2n+1)}{3}$

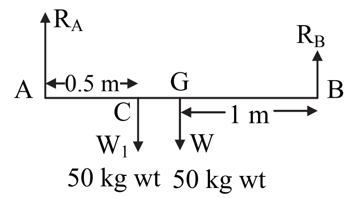
31. For the given body $\sum \vec{F} = 0$. Hence body is in translational equilibrium.
Also, $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$

$= \left(3F \times \frac{d}{2}\right) + \left(-F \times \frac{3d}{2}\right)$

(considering sense of rotation)

$= 0$

Hence body is in rotational equilibrium.

32.


For equilibrium,
Considering moments about point A,

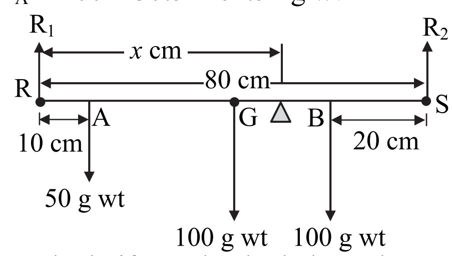
$R_A \times 0 - W_1 \times AC - W \times AG + R_B \times AB$

$R_A \times 0 - 50 \times 0.5 - 50 \times 1 + R_B \times 2 = 0$

$\therefore \quad 2R_B = 75$

$\therefore \quad R_B = 37.5$ kg wt

$R_A = 100 - 37.5 = 62.5$ kg wt

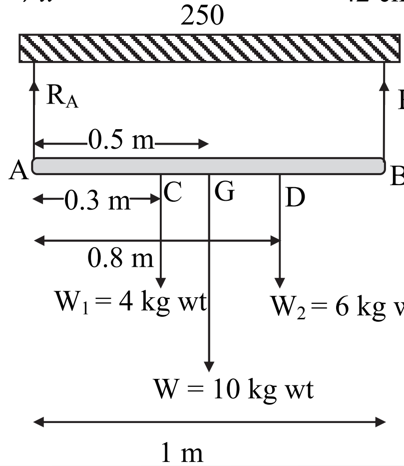
33.


Let the knife – edge be balanced at x cm from point R. For equilibrium, considering moments about point R,

$W_1 \times RA + W \times RG + W_2 \times RB = (W_1 + W + W_2) \times x$

$50 \times 10 + 100 \times 40 + 100 \times 60 = (50 + 100 + 100)\,x$

$\Rightarrow x = \frac{500 + 4000 + 6000}{250} = 42$ cm

34.


For translational equilibrium

∴ $R_A + R_B - W - W_1 - W_2 = 0$

∴ $R_A + R_B = W + W_1 + W_2$

$= (10 + 4 + 6)$ kg wt

∴ $R_A + R_B = 20$ kg wt

For rotational equilibrium, considering moments about A,

$R_A \times 0 - W_1 \times AC - W \times AG - W_2 \times AD + R_B \times AB = 0$

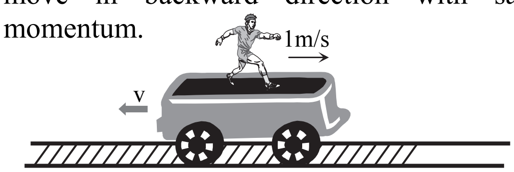
$-4 \times 0.3 - 10 \times 0.5 - 6 \times 0.8 + R_B \times 1 = 0$

$\Rightarrow R_B = 11$ kg wt

∴ $R_A = 9$ kg wt

$= 9 \times 9.8$ N $= 88.2$ N

35. If the man starts walking on the trolley in the forward direction then whole system will move in backward direction with same momentum.



Momentum of man in forward direction

= Momentum of system (man + trolley) in backward direction

∴ $80 \times 1 = (80 + 320) \times v$

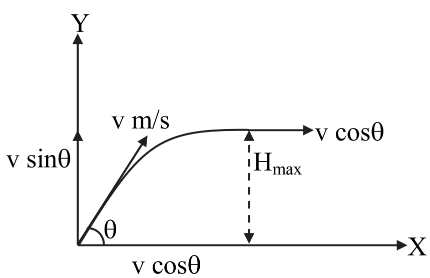
∴ $v = 0.2$ m/s

So the velocity of man w.r.t. ground $1.0 - 0.2 = 0.8$ m/s

∴ Displacement of man w.r.t. ground, $= 0.8 \times 4 = 3.2$ m

36. Gravitational field is a conservative field. Therefore work done in moving a particle from A to B is independent of path chosen.

37.



Let the mass of shell be m. At the highest point it has only horizontal component of velocity.

Hence its momentum at that point = mv cosθ

It breaks into two equal mass. One piece traces its path with speed v cosθ.

Let speed of other piece just after explosion be v′ then,

Final momentum $= \frac{m}{2} v\cos\theta + \frac{m}{2} v'$

By the principle of conservation of momentum,

$mv\cos\theta = \frac{m}{2} v\cos\theta + \frac{m}{2} v'$

$\left(1 + \frac{1}{2}\right) mv\cos\theta = \frac{m}{2} v'$

$v' = 3v\cos\theta$

38. From the principle of momentum conservation,

$m_g v_g = m_b v_b$ (considering magnitudes)

∴ $v_g = \frac{0.05 \times 400}{5} = 4$ m/s

($\because m_b = 50$ g $= 0.05$ kg)

The gun fires 30 bullets in 1 minute i.e., in 60 s. This means 1 bullet is fired every 2 s.

From Newton's second law,

$F = \frac{p_2 - p_1}{t}$

Where p_2, p_1 is final and intial momentum of gun respectively.

∴ $F = \frac{m_g v_g - 0}{2} = \frac{5 \times 4}{2} = 10$ N

39. Let 'm_0' be the initial mass of rocket. Its ejection speed of gases $\left(\frac{dm}{dt}\right) = 16$ kg/s

Hence after t = 1 min = 60 s, its mass will be

$m = m_0 - \left(\frac{dm}{dt}\right)t = 6000 - (16) \times 60$

∴ $m = 5040$ kg

At this time instant thrust on the rocket is,

$F = u\frac{dm}{dt}$

where u is constant relative speed.

$ma = u\frac{dm}{dt}$

$a = \frac{(u\,dm/dt)}{m}$

$= \frac{11 \times 10^3 \times 16}{5040} = 34.92$ m/s^2

≈ 35 m/s^2

40. To hold the gun stable, rate of change of momentum of the gun should not exceed maximum exerted force.

Hence $F = \frac{\Delta p}{t}$

For t = 1 s, $F = \Delta p \Rightarrow 144$ N s

From the principle of conservation of momentum,
Momentum of gun = momentum of bullet

$$\Delta p' = 40 \times 10^{-3} \times 12 \times 10^2 = 48 \text{ kg m/s.}$$

So, number of bullets that can be fired per second,

$$\frac{\Delta p}{\Delta p'} = \frac{144}{48} = 3$$

41. As $\vec{v} = 5t\hat{i} + 2t\hat{j}$

∴ $\vec{a} = a_x\hat{i} + a_y\hat{j} = 5\hat{i} + 2\hat{j}$

$\vec{F} = ma_x\hat{i} + m(g + a_y)\hat{j}$

∴ $|\vec{F}| = m\sqrt{a_x^2 + (g + a_y)^2} = 26$ N

42. $\vec{F}\Delta t = m\Delta\vec{v} \Rightarrow F = \dfrac{m\Delta\vec{v}}{\Delta t}$

By doing so time of change in momentum increases and impulsive force on knees decreases.

43. Momentum of vehicle = 100 × 0.02 = 2 kg m/s ….(i)

Momentum of weight = $4 \times 10^{-3} \times 10^3 \times 10^{-2}$
= 4×10^{-2} kg m/s ….(ii)

For 200 g weight, K.E. = $\frac{1}{2}mv^2 = 10^{-6}$ J

∴ $v = \left(\dfrac{2\times10^{-6}}{0.2}\right)^{1/2} = 10^{-5/2}$

Hence, its momentum = $0.2 \times 10^{-5/2}$ kg m/s ….(iii)

For a weight falling from h = 1 km = 10^3 m

$(P.E)_{max} = (K.E)_{max}$

$mgh = \frac{1}{2}mv^2$

∴ $v = \sqrt{2gh} = \sqrt{2\times9.8\times10^3} = 140$ m/s

Hence, its momentum = 0.2 × 140
= 28 kg m/s ….(iv)

Comparing the values, momentum of a 200 g weight after falling through 1 km has maximum value.

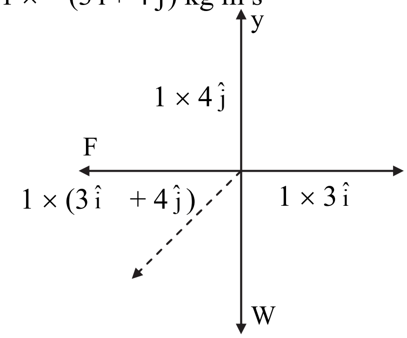
44. If man slides down with some acceleration, then its apparent weight decreases. For critical condition, rope can bear only 2/3 of his weight. If a is the minimum acceleration then,
Tension in the rope = m (g – a) = Breaking strength

∴ $m(g - a) = \frac{2}{3}mg$

∴ $a = g - \frac{2g}{3} = \frac{g}{3}$

45. Gas will come out with sufficient speed in forward direction, so reaction of this forward force will change the reading of the spring balance.

46. According to law of conservation of momentum the third piece has momentum
$= 1 \times -(3\hat{i} + 4\hat{j})$ kg m s^{-1}



Impulse = Average force × time

$\Rightarrow$ Average force $= \dfrac{\text{Impulse}}{\text{time}}$

$= \dfrac{\text{Change in momentum}}{\text{time}}$

$= \dfrac{-(3\hat{i} + 4\hat{j})}{10^{-4}}$

$= -(3\hat{i} + 4\hat{j}) \times 10^4$ N.

Competitive Thinking

2. If a large force F acts for a short time dt the impulse imparted J is

$J = F\,dt = \frac{dp}{dt}dt$

J = dp = change in momentum

3. Impulse = change in linear momentum
= 0.5 × 20 – 0.5 × (–10)
= 10 + 5 = 15 N s

4. Change in momentum = Area below the F versus t graph in that interval

$= \left(\frac{1}{2}\times2\times6\right) - (2\times3) + (4\times3)$

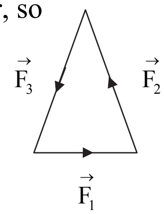
= 6 – 6 + 12 = 12 N s

5. Since all three blocks are moving up with a constant speed v, acceleration a is zero.

$\Rightarrow F = 0$

∴ Net force is zero.

7. Given three forces are acting along the three sides of triangle in same order, so



$\therefore \quad \vec{F}_{net} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$\Rightarrow \vec{F}_{net} = 0$

$\Rightarrow \vec{a} = 0$,

Velocity will remain constant

8. Initial thrust must be m[g + a]

$= 3.5 \times 10^4 (10 + 10) = 7 \times 10^5$ N

9. $\Delta p = p_i - p_f = mv - (-mv) = 2$ mv

10. Given: m = 5 g, v = 4 cm/s, t = 2.5 s

we know,

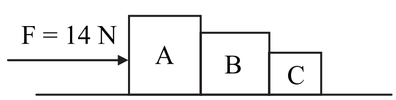
$a = \frac{v}{t}$

and F = ma

$\therefore \quad a = \frac{4}{2.5}$ cm/s^2

$F = 5 \times \frac{4}{2.5} = 8$ dyne

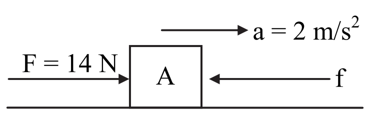
12.



Acceleration of system of blocks is

$a = \frac{F}{m_A + m_B + m_C} = \frac{14}{4+2+1} = 2$ m/s^2

Let contact force between A and B be f then,



$14 - f = m_A \times a$

$\therefore \quad 14 - f = 4 \times 8$

$\therefore \quad f = 14 - 8 = 6$ N

13. $a_{net} = \frac{F}{m_1 + m_2 + m_3}$

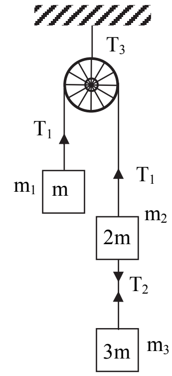
$a_{net} = \frac{24}{6} = 4$ m/s^2

$F_{net} = m\, a_{net}$

$F_{net} = 2 \times 4$

$F_{net} = 8$ N

14. From the figure, tension between masses 2m and 3m is T_2.

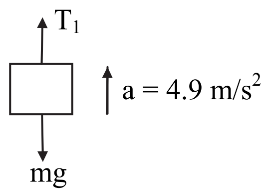


We know that,

$T_2 = \left(\frac{2\, m_1 m_3}{m_1 + m_2 + m_3}\right) g$

$\therefore \quad T_2 = \frac{(2m)(3m)}{6\,m} g = mg$

15. **First case:**

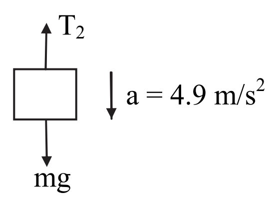


$T_1 - mg = ma$

$T_1 - mg = \frac{mg}{2} \quad \ldots.\left(\because a = 4.9 \text{ ms}^{-2} = \frac{g}{2}\right)$

$T_1 = \frac{3mg}{2} \quad \ldots.(i)$

Second case:



$mg - T_2 = ma$

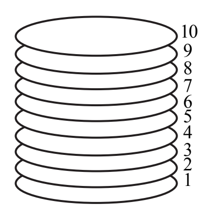
$mg - T_2 = \frac{mg}{2} \quad \ldots.\left(\because a = 4.9 \text{ ms}^{-2} = \frac{g}{2}\right)$

$T_2 = \frac{mg}{2} \quad \ldots.(ii)$

Dividing equation (i) by (ii),

$\frac{T_1}{T_2} = \frac{3mg}{2} \times \frac{2}{mg} = \frac{3}{1}$

16. (A) is correct as 6^{th} coin has four coins on its top which exert a force 4 mg on it.



(B) is correct as 7^{th} coin has three coins, placed over it. Thus 7^{th} coin exerts a force 4 mg on 6^{th} coin (downwards)

(C) is correct, as the reaction of 6^{th} coin on the 7^{th} coin is 4 mg (upwards)

(D) is wrong as 10^{th} coin, which is the topmost coin, experiences a reaction force of mg (upwards) from all the coins below it.

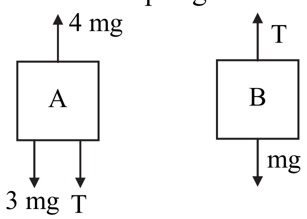
17. For a freely falling lift, (a = g)

Apparent weight = m(g – a)
= m (g – g)
= 0

18. F = m × g = 0.05 × 9.8 = 0.49 N. As the weight of ball acts downwards, the net force will act vertically downward.

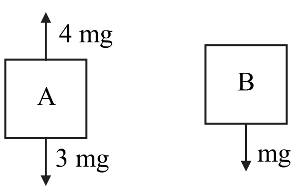
19. F = m(g – a)
= 60(9.8 – 1.8)
= 480 N

20. Tension in spring before cutting the strip



∴ T = mg

After cutting the strip



Acceleration in brick A

$a_A = \frac{4mg - 3mg}{3m} = \frac{g}{3}$

Acceleration in block B

$a_B = \frac{mg}{m} = g$

21. For mass m_1, $a_1 = 6 = \frac{F}{m_1}$

∴ $m_1 = \frac{F}{6}$ ....(i)

For mass m_2, $a_2 = 3 = \frac{F}{m_2}$

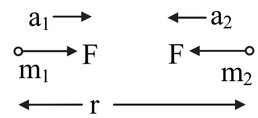
∴ $m_2 = \frac{F}{3}$ ....(ii)

∴ $a = \frac{F}{m_1 + m_2}$

From equations (i) and (ii),

$a = \frac{F}{F/6 + F/3} = 2\ m/s^2$

23.



$F = \frac{Gm_1m_2}{r_2} \Rightarrow a_1 = \frac{F}{m_1} = \frac{Gm_2}{r^2}$

∴ $a_1 \propto m_2$

24. The weight of body changes but its mass remains the same.

25. From law of conservation of momentum,

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

Let $\vec{p}_1$ and $\vec{p}_2$ go off at right angles to each other.

∴ $|\vec{p}_3| = \sqrt{p_1^2 + p_2^2}$

∴ $m_3 \times 4 = \sqrt{(1\times12)^2 + (2\times8)^2} = \sqrt{12^2 + 16^2} = 20$

∴ $m_3 = \frac{20}{4} = 5$ kg

26. Let, $\vec{P}_A = -3P\hat{i}$ and $\vec{P}_B = 2P\hat{j}$

According to law of conservation of momentum,

$\vec{P}_A + \vec{P}_B + \vec{P}_C = 0$

∴ $-3P\hat{i} + 2P\hat{j} + \vec{P}_C = 0$

∴ $\vec{P}_C = 3P\hat{i} - 2P\hat{j}$

∴ $|P_C| = \sqrt{9P^2 + 4P^2} = \sqrt{13}\,P$

27. As the bullet explodes at highest point of trajectory, it only has horizontal velocity.

$v_H = v\cos 60° = 30 \times \frac{1}{2} = 15$ m/s

According to law of conservation of momentum, momentum before and after explosion must be same.

$(m_1 + m_2)\ v_H = m_1v_1 + m_2v_2$

But, $m_1 = m$ and $m_2 = 3m$ (given)

∴ $4m \times 15 = m \times 0 + 3m\ v_2$

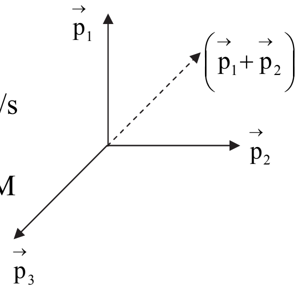
$\Rightarrow v_2 = \frac{15\times4}{3} = 20$ m/s

28. By law of conservation of momentum,

$|\vec{p}_1 + \vec{p}_2| = |\vec{p}_3|$

∴ $\sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta} = |\vec{p}_3|$

as $p_1 = p_2 = p$ and $p_1 \perp p_2$, $\theta = 90°$.



$\therefore$ $\sqrt{2p^2} = |\vec{p}_3|$

$\therefore$ $\sqrt{2}\,p = |\vec{p}_3|$

as $v_1 = v_2 = v = 30$ m/s

$\therefore$ $30\sqrt{2}\,m = m_3v_3$

also, m + m + 3m = M

$\therefore$ $m = \dfrac{M}{5}$

$m_3 = 3m = \dfrac{3M}{5}$

$\therefore$ $30\sqrt{2}\,\dfrac{M}{5} = \dfrac{3M}{5}\,v_3$

$\therefore$ $v_3 = 10\sqrt{2}$ m/s

29. By conservation of linear momentum,

initial momentum of bullet = final momentum of system

$\therefore$ $mv_b = (M + m)\,v_{sys}$

here, m = mass of bullet = 0.016 kg

M = mass of block = 4 kg

v_{sys} = velocity of system = $\sqrt{2gh}$

h = 0.1 m

$\therefore$ $0.016\,v_b = 4.016 \times \left(\sqrt{2 \times 9.8 \times 0.1}\right)$

$\therefore$ $v_b = 351.4$ m/s

30. By law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

here, m_1 = mass of bullet = 10 g = 0.01 kg

m_2 = mass of block = 2 kg

u_1 = initial velocity of bullet = 400 ms^{-1}

u_2 = initial velocity of block = 0

v_1 = final velocity of bullet

v_2 = final velocity of block = $\sqrt{2gh}$

$\therefore$ $(0.01) \times 400 + 0 = 0.01\,v_1 + 2 \times \sqrt{2 \times 9.8 \times 0.1}$

$4 = 0.01v_1 + 2 \times \sqrt{1.96}$

$\therefore$ $v_1 = \dfrac{4 - 2 \times 1.4}{0.01} = 120$ m/s

31. Mass of each piece (m) = 1 kg.

Initial momentum = 0.

Final momentum = $p_1 + p_2 + p_3$.

From the principle of conservation of momentum, we have

$p_1 + p_2 + p_3 = 0$

$p_3 = -(p_1 + p_2)$

$= -\,(mv_1 + mv_2) = -m(v_1 + v_2)$

$= -1\text{ kg} \times (2\hat{i} + 3\hat{j})\text{ m s}^{-1} = -\,(2\hat{i} + 3\hat{j})\text{ kg m s}^{-1}$

Force $F = \dfrac{p_3}{t} = \dfrac{-(2i + 3j)}{10^{-5}}$

$= -\,(2\hat{i} + 3\hat{j}) \times 10^5$ newton

32. $K.E. = \dfrac{p^2}{2m} = \dfrac{36}{2 \times 4} = 4.5$ J

33. $(K.E.)_1 = \dfrac{1}{2}m_1v_1^2$

$= \dfrac{1}{2}\dfrac{m_1^2v_1^2}{m_1}$

$= \dfrac{1}{2}\dfrac{p_1^2}{m_1}$

$(K.E.)_2 = \dfrac{1}{2}\dfrac{p_2^2}{m_2}$

$\therefore$ $(K.E.)_1 = (K.E.)_2$ ….(given)

$\therefore$ $\dfrac{p_1^2}{2m_1} = \dfrac{p_2^2}{2m_2}$

$\therefore$ $\dfrac{p_1}{p_2} = \sqrt{\dfrac{m_1}{m_2}}$

34. From principle of conservation of momentum,

Final momentum = Initial momentum

$\therefore$ $m_1v_1 - m_2v_2 = 0 \Rightarrow v_1 = \dfrac{m_2v_2}{m_1}$

$$\frac{(K.E.)_1}{(K.E.)_2} = \frac{E_1}{E_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{m_1v_1^2}{m_2v_2^2}$$

Substituting for v_1

$$\frac{E_1}{E_2} = \frac{m_1\dfrac{m_2^2v_2^2}{m_1^2}}{m_2v_2^2} = \frac{m_2}{m_1}$$

35. From conservation of linear momentum,

$MV = m_1v_1 + m_2v_2$

As bomb is at rest initially, its initial momentum will be zero.

$\therefore$ $m_1v_1 + m_2v_2 = 0$

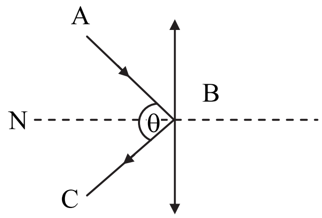
$\therefore$ $20 + 8v_2 = 0$

$\Rightarrow v_2 = -\dfrac{20}{8} = -\dfrac{5}{2}$ m/s

$\therefore$ Kinetic energy of the 8 kg piece is,

$K.E. = \dfrac{1}{2}m_2v_2^2 = \dfrac{1}{2} \times 8 \times \left(\dfrac{25}{4}\right) = 25$ J

36. Let the point B represents the position of bat. The ball strikes the bat with velocity v along the path AB and gets deflected with same velocity along BC, such that $\angle ABC = \theta$



Initial momentum of the ball = $mv \cos\left(\frac{\theta}{2}\right)$ (along NB)

Final momentum of the ball = $mv \cos\left(\frac{\theta}{2}\right)$ (along BN)

Hence, Impulse = change in momentum

$$= mv\cos\left(\frac{\theta}{2}\right) - \left[-mv\cos\left(\frac{\theta}{2}\right)\right]$$

$$= 2mv\cos\left(\frac{\theta}{2}\right)$$

37. Impulse = change in momentum

∴ $I = p_f - p_i$

Resultant of two vectors having same magnitude and separated by angle θ,

$R = 2A\cos\frac{\theta}{2}$

here, θ = 60° + 60° = 120°

∴ $I = 2p\cos\left(\frac{120}{2}\right)^{o} = 2mV\cos(60°) = mV$

38. $\vec{F} = \left(3\hat{i} + \hat{j}\right)$

$\vec{s} = \left(\vec{r}_2 - \vec{r}_1\right)\left[2\hat{i} + 3\hat{j} - 2\hat{k}\right]$

$W = \vec{F}\cdot\vec{s} = \left(3\hat{i} + \hat{j}\right)\left[2\hat{i} + 3\hat{j} - 2\hat{k}\right] = 6 + 3 + 0$

∴ W = 9 J

39. $F = \frac{K}{v}$

∴ $W = Fs\cos\theta$

∴ $W = \frac{K}{v}s$ $(\because \theta = 0°)$

∵ $v = \frac{s}{t}$

∴ $W = K \times \frac{t}{s} \times s$

∴ W = Kt

40. Displacement is in x direction and force is in y –direction,

∴ Force is perpendicular to displacement, hence work done will be zero.

41. $s = \frac{t^2}{4}$

∴ $ds = \frac{2t}{4}dt = \frac{t}{2}dt$

$a = \frac{d^2s}{dt^2} = \frac{d}{dt}\left[\frac{ds}{dt}\right] = \frac{d}{dt}\left[\frac{t}{2}\right] = \frac{1}{2}$

∴ $F = ma = 6 \times \frac{1}{2} = 3\ N$

Now, $W = \int_0^2 F\,ds = \int_0^2 3\frac{t}{2}dt$

$$= \frac{3}{2}\left[\frac{t^2}{2}\right]_0^2 = \frac{3}{4}\left[(2)^2 - (0)^2\right] = 3\ J$$

42. $dW = \vec{F}\cdot\overline{dx}$

$$= K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i} + \frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]\cdot\left[dx\,\hat{i} + dy\,\hat{j}\right]$$

$$= K\left[\frac{xdx + ydy}{\left(x^2+y^2\right)^{3/2}}\right] \quad \text{….(i)}$$

Let $x^2 + y^2 = r^2$

∴ 2xdx + 2ydy = 2rdr

∴ xdx + ydy = rdr

Substituting in equation (i),

$dw = K\left[\frac{rdr}{r^3}\right] = \frac{K}{r^2}dr$

Integrating, $W = \int_{r_1}^{r_2}\frac{K}{r^2}dr = \left[\frac{-K}{r}\right]_{r_1}^{r_2}$

$r_1 = \sqrt{x_1^2 + y_1^2} = a$, $r_2 = \sqrt{x_2^2 + y_2^2} = a$

∴ W = 0

43. Work done by the net force

= change in kinetic energy of the particle

44. Using Work-Energy Theorem,

$W = \frac{1}{2}m(v^2 - u^2)$

$\therefore$ Final K.E.,

$\frac{1}{2}mv^2 = W + \frac{1}{2}mu^2$

$= -0.1\int_{20}^{30} x\,dx + \frac{1}{2}\times 10\times 10^2$ $\quad (\because W = \vec{F}\cdot\vec{s})$

(Negative sign indicates retardation)

$= -0.1\left[\frac{x^2}{2}\right]_{20}^{30} + 500 = -0.1\left[\frac{30^2}{2} - \frac{20^2}{2}\right] + 500$

$= -25 + 500 = 475$ J

45. $F = 6t = ma$

$m = 1$ kg

$\therefore$ $a = 6t$

$\therefore$ $\frac{dv}{dt} = 6t$

Integrating we get,

$\int_0^v dv = \int_0^1 6t\ dt$

$v = \left(3t^2\right)_0^1 = 3$ m/s

From work energy theorem,

$W = \frac{1}{2}(v^2 - u^2)$

$= \frac{1}{2}(1)(9 - 0)$

$= 4.5$ J

46. Work done by gravitation force is given by (W_g)

$W_g = mgh = 10^{-3} \times 10 \times 10^3 = 10$ J

According to work energy theorem

$W_g + W_{res} = \Delta KE$

$10 + W_{res} = \frac{1}{2} \times 10^{-3} \times 50 \times 50$

$10 + W_{res} = \frac{5}{4}$

$W_{res} = -8.75$ J

47. From graph, work done is area under the curve

$\therefore$ $W = \left(\frac{1}{2}\times 3\times 20\right) + (3\times 20) + \left(\frac{1}{2}\times 3\times 20\right)$

$= 30 + 60 + 30 = 120$ J

From work energy theorem,

$\frac{1}{2}mv^2 = W = 120$

$\therefore$ $\frac{1}{2} \times 2.4 \times v^2 = 120$

$\therefore$ $v^2 = 100$

$\therefore$ $v = 10$ m/s

48. 20% of fat burned is converted into mechanical energy

Here, mechanical energy is potential energy

$\therefore$ P.E. = mgh

When person lifts the mass 1000 times,

Total P.E. = $U = 10 \times 9.8 \times 1 \times 1000 = 9.8 \times 10^4$ J

Let total fat burned be x kg,

Hence the energy supplied by x kg fat is

$E = x \times 3.8 \times 10^7$

20 % of which is converted to U

$\therefore$ $x \times 3.8 \times 10^7 \times \frac{20}{100} = 9.8 \times 10^4$

$76\ x = 9.8 \times 10^{-1}$

$\therefore$ $x = 12.89 \times 10^{-3}$ kg

49. This is a case of a perfectly inelastic collision in which linear momentum is conserved but kinetic energy is not conserved.

52. When two bodies with same mass collide elastically, their velocities get interchanged.

53. Let mass of bullet be m and mass of ice be M. According to the conservation of linear momentum,

$m \times 300 + M \times 0 = m \times 0 + Mv$

$0.01 \times 300 + 0 = 5v$

$\therefore$ $v = \frac{3}{5} = 0.6$ m/s = 60 cm/s

54. For collision, the relative velocity of one particle should be directed towards the relative position of other particle.

Let $\hat{v}_R$ be direction of relative velocity of B w.r.t. A and $\hat{r}_R$ be direction of relative position of A w.r.t. B.

$\therefore$ $\hat{v}_R = \frac{\vec{v}_2 - \vec{v}_1}{\left|\vec{v}_2 - \vec{v}_1\right|}$ and $\hat{r}_R = \frac{\vec{r}_1 - \vec{r}_2}{\left|\vec{r}_1 - \vec{r}_2\right|}$

$\because$ $\hat{v}_R = \hat{r}_R \Rightarrow \frac{\vec{r}_1 - \vec{r}_2}{\left|\vec{r}_1 - \vec{r}_2\right|} = \frac{\vec{v}_2 - \vec{v}_1}{\left|\vec{v}_2 - \vec{v}_1\right|}$

55. Before collision:

$(K.E.)_1 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

After collision:

$(K.E.)_2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

Total energy being conserved in collision,

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

56. Coefficient of restitution is a ratio of same physical quantity viz., velocity. Hence, it has no dimensions.

57. Coefficient of restitution:

$e = \frac{v_2 - v_1}{u_1 - u_2}$

Given: $u_1 = v$, $u_2 = 0$

$\therefore \quad e = \frac{v_2 - v_1}{v} = \frac{v_2}{v} - \frac{v_1}{v}$ ….(i)

By law of conservation of momentum,

$mu_1 + mu_2 = mv_1 + mv_2$

$mv = mv_1 + mv_2$

$v = v_1 + v_2$

$1 = \frac{v_1}{v} + \frac{v_2}{v}$

$\therefore \quad \frac{v_1}{v} = 1 - \frac{v_2}{v}$ ….(ii)

From equation (i) and (ii),

$e = \frac{v_2}{v} - \left(1 - \frac{v_2}{v}\right)$

$\therefore \quad e = \frac{v_2}{v} - 1 + \frac{v_2}{v}$

$e = \frac{2v_2}{v} - 1$

$\therefore \quad \frac{2v_2}{v} = e + 1$

$\therefore \quad \frac{v_2}{v} = \frac{e+1}{2}$

58. Given: $m_1 = m$, $m_2 = 4m$, $u_1 = v$, $u_2 = 0$, $v_1 = 0$

According to law of conservation of momentum,

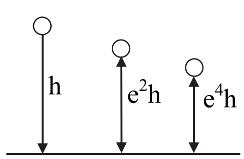
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$mv + 4m \times 0 = m \times 0 + 4mv_2$

$\therefore \quad v_2 = \frac{v}{4}$

$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\frac{v}{4} - 0}{v - 0} = \frac{1}{4} = 0.25$

59.



Total distance = $h + 2e^2h + 2e^4h$ ….

$= h + 2e^2h\,(1 + e^2 + ….)$

Using binomial expansion,

$(1 + e^2 + e^4 + …) = \frac{1}{(1 - e^2)}$

$\therefore \quad$ Total distance $= h + 2e^2h\left(\frac{1}{1 - e^2}\right)$

$= h + \frac{2e^2h}{1 - e^2}$

$= \frac{h - e^2h + 2e^2h}{(1 - e^2)}$

$= \frac{h(1 + e^2)}{(1 - e^2)}$

60. As $e = \sqrt{\frac{h_1}{h_0}}$

$\therefore \quad h_1 = e^2h_0$

For n number of bouncing, $h_n = e^{2n}h$

$\therefore \quad t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2he^2}{g}} + 2\sqrt{\frac{2he^4}{g}} + ...$

$= \sqrt{\frac{2h}{g}}\left(1 + 2e + 2e^2 + ...\right)$

$= \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]$

$\therefore \quad 10 = \sqrt{\frac{2 \times 0.4}{10}}\left(\frac{1+e}{1-e}\right)$

$\therefore \quad e = \frac{25\sqrt{2} - 1}{25\sqrt{2} + 1} \approx \frac{17}{18}$

61. Let v_1 and v_2 be their respective velocities after collision.

Applying the law of conservation of linear momentum,

$\therefore \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\therefore \quad m \times 2 + 2m \times 0 = m \times v_1 + 2m \times v_2$

$2m = mv_1 + 2mv_2$

$2 = (v_1 + 2v_2)$ …(i)

By definition of coefficient of restitution,

$e = \frac{v_2 - v_1}{u_1 - u_2}$

$e(u_1 - u_2) = v_2 - v_1$

$0.5(2 - 0) = v_2 - v_1$ …(ii)

$1 = v_2 - v_1$

Solving equations (i) and (ii),

$v_1 = 0$ m/s, $v_2 = 1$ m/s

62. Total mechanical energy of ball,

$T = \frac{1}{2}mv^2 + mgh$

Total energy after the collision,

$\frac{2}{3}\left(\frac{1}{2}mv^2 + mgh\right)$

The ball rebounds back to the same height after collision,

$\therefore \quad \frac{2}{3}\left(\frac{1}{2}mv^2 + mgh\right) = mgh$

$\therefore \quad \frac{2}{3}\left(\frac{1}{2}v^2 + gh\right) = gh,$

$\frac{1}{3}v^2 + \frac{2}{3}gh = gh,$

$\therefore \quad \frac{v^2}{3} = gh - \frac{2gh}{3}$

$\frac{v^2}{3} = \frac{gh}{3}$

$v = \sqrt{gh} = \sqrt{40 \times 10} = 20$ m/s

63. Velocity v after rebound can be given as,

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20$ m/s

$\therefore$ kinetic energy just after collision is,

$K = \frac{1}{2}mv^2 = \frac{1}{2}m \times (20)^2 = \frac{400m}{2} = 200m$

As the ball loses 50% of energy in collision, its initial energy would be 400m

By conservation of energy,

$\frac{1}{2}mv_0^2 + mgh = 400m$

$\therefore \quad \frac{1}{2}mv_0^2 + m \times 10 \times 20 = 400m$

$\therefore \quad v_0^2 + 400 = 800$

$\therefore \quad v_0 = 20$ m/s

64. In elastic collision

$(K.E)_{\text{before collision}} = (K.E)_{\text{After collision}}$

speed of second body after collision v_2 can be found as

$\frac{1}{2}mv^2 + 0 = \frac{1}{2}m\left(\frac{v}{3}\right)^2 + \frac{1}{2}m(v_2)^2$

$\therefore \quad v^2 = \frac{v^2}{9} + v_2^2 \quad \Rightarrow \quad \frac{8v^2}{9} = v_2^2$

$\therefore \quad v_2 = \frac{2\sqrt{2}}{3}v$

65. According to law of conservation of momentum,

$mv_0 = mv_1 + mv_2$

$\therefore \quad v_0 = v_1 + v_2 \quad$ ....(i)

Initial KE, $(K.E.)_i = \frac{1}{2}mv_0^2$

Final KE, $(K.E.)_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

Given that,

$(K.E.)_f = 0.5\,(K.E.)_i + (K.E.)_i = \frac{3}{2}(K.E.)_i$

$\therefore \quad \frac{1}{2}m\left(v_1^2 + v_2^2\right) = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$

$\therefore \quad v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \quad$ ....(ii)

On squaring equation (i) and subtracting equation (ii) from it, we get,

$\left(v_1^2 + v_2^2 + 2v_1v_2\right) - \left(v_1^2 + v_2^2\right) = v_0^2 - \frac{3}{2}v_0^2$

$\therefore \quad -2v_1v_2 = \frac{1}{2}v_0^2$

$\therefore \quad -4v_1v_2 = v_0^2 \quad$ ....(iii)

Now, $(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$

$\therefore \quad (v_1 - v_2)^2 = v_0^2 + v_0^2 = 2v_0^2$

....[using equations (i) and (iii)]

$\therefore \quad v_{rel} = |v_1 - v_2| = v_0\sqrt{2}$

66. During collision of ball with the wall, horizontal momentum changes (vertical momentum remains constant)

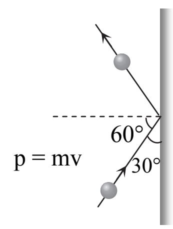
$\therefore \quad F = \frac{\text{Change in horizontal momentum}}{\text{Time of contact}}$

$= \frac{2p\cos\theta}{0.1}$

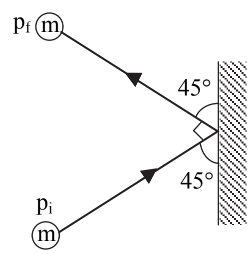
$= \frac{2mv\cos\theta}{0.1}$

$= \frac{2 \times 0.1 \times 10 \times \cos 60^\circ}{0.1}$

$= 10$ N



67.



$\vec{\Delta p} = \vec{p}_f - \vec{p}_i$

$\left|\vec{\Delta p}\right| = \sqrt{p_f^2 + p_i^2 + 2p_fp_i\cos\theta}$

$$\left|\overrightarrow{\Delta p}\right| = \sqrt{p^2 + p^2}$$

$$= \sqrt{p_f^2 + p_i^2} \qquad (\because \theta = 90°)$$

$$= p\sqrt{2}$$

$$= 5 \times \sqrt{2}$$

$$\left|\overrightarrow{\Delta p}\right| = 7.07 \text{ kg ms}^{-1}$$

68. In case of inelastic collision

$$\Delta K.E. = \frac{m_1 m_2}{2(m_1 + m_2)}(1 - e^2)(u_1 - u_2)^2$$

$$= \frac{1(0.5)}{2(1 + 0.5)}\left(1 - \frac{1}{9}\right)[6 - (-9)]^2$$

$$= \frac{1}{6}\left(\frac{8}{9}\right)225$$

$$= \frac{8 \times 225}{54}$$

$$= 33.33 \text{ J}$$

69. $$KE_{loss} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(1 - e^2)(u_1 - u_2)^2$$

$$= \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(1 - 0^2)(v - 0)^2$$

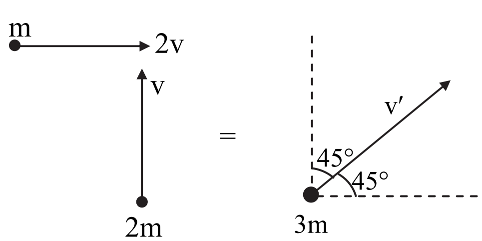
$$= \frac{1}{2}\left(\frac{4.2 \times 10^{-2} \times 9 \times 4.2 \times 10^{-2}}{42 \times 10^{-2}}\right)(300)^2$$

$$= 1701 \text{ J}$$

$$= \frac{1701}{4.2}$$

$$= 405 \text{ cal}$$

70.



Before collision   After collision

Collision being perfectly inelastic,

$m(2v)\cos 45° + 2m(v)\cos 45° = (m + 2m)v'$

$\therefore \quad 2mv\frac{1}{\sqrt{2}} + 2mv\frac{1}{\sqrt{2}} = 3mv'$

$\therefore \quad \frac{2\sqrt{2}\,mv}{3m} = v' \qquad \Rightarrow \frac{2\sqrt{2}}{3}v = v'$

Loss in K.E. = total initial K.E. – total final K.E.

$$= \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 - \frac{1}{2}\times(3m)\left(\frac{2v\sqrt{2}}{3}\right)^2$$

$$= 2mv^2 + mv^2 - \left[\frac{3}{2}m\left(\frac{8}{9}v^2\right)\right] = \frac{5}{3}mv^2$$

$$\text{Percentage loss in K.E.} = \frac{\frac{5}{3}mv^2}{2mv^2 + mv^2}\times 100$$

$$= \frac{5}{9}\times 100 = 55.56\%$$

$$\approx 56\%$$

71. Say mass of 2 kg is at rest initially, then

$3 \times 15 + 2 \times 0 = 3v_1 + 2v_2$

$\therefore \quad 45 = 3v_1 + 2v_2 \qquad$ ….(i)

$\therefore \quad e = \frac{5}{15} = \frac{1}{3}$

$\therefore \quad \frac{v_2 - v_1}{15 - 0} = \frac{1}{3}$

$\therefore \quad 15 = 3v_2 - 3v_1 \qquad$ ….(ii)

Solving equations (i) and (ii),

$v_1 = 7$, $v_2 = 12$

$\therefore$ Loss of kinetic energy

$$= \frac{1}{2}\times 3 \times 15^2 - \frac{1}{2}\times 3 \times 7^2 - \frac{1}{2}\times 2 \times 12^2$$

$$= 337.5 - 73.5 - 144 = 120 \text{ J}$$

72. The frame of reference which are at rest or in uniform motion are called inertial frames while frames which are accelerated with respect to each other are non-inertial frames. Spinning or rotating frames are accelerated frames.

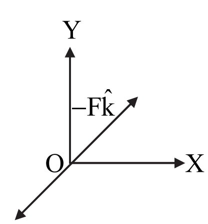
74. $\vec{\tau} = \vec{r} \times \vec{F}$

Vector $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$.

$\therefore \quad \vec{r}\cdot\vec{\tau} = 0$ and $\vec{F}\cdot\vec{\tau} = 0$

75. $\vec{F} = F\hat{k}$

$\vec{r} = (\hat{i} - \hat{j})$



$\vec{\tau} = \vec{r}\times\vec{F} = (\hat{i} - \hat{j})(-F\hat{k})$

$= -F(\hat{i}\times\hat{k}) + F(\hat{j}\times\hat{k})$

$= -F(-\hat{j}) + F(\hat{i}) \Rightarrow F\hat{j} + F\hat{i} = F(\hat{i} + \hat{j})$

76. $\vec{r} = \vec{r}_1 - \vec{r}_2 = (\hat{i} + 2\hat{j} + 3\hat{k}) - (3\hat{i} - 2\hat{j} - 3\hat{k})$

$= -2\hat{i} + 4\hat{j} + 6\hat{k}$

Now $\vec{\tau} = \vec{r} \times \vec{F}$

$= (-2\hat{i} + 4\hat{j} + 6\hat{k}) \times (4\hat{i} - 5\hat{j} + 3\hat{k})$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 4 & 6 \\ 4 & -5 & 3 \end{vmatrix} = \hat{i}(12+30) - \hat{j}(-6-24) + \hat{k}(10-16)$$

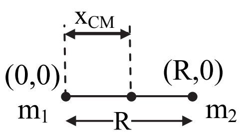
$= (42\hat{i} + 30\hat{j} - 6\hat{k})$ N m

77. Couple consists of two equal and opposite forces which causes pure rotational motion.

78. Depends on the distribution of mass in the body.

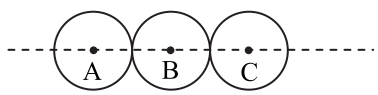
79. Centre of mass always lies towards heavier mass.

81.



$$x_{CM} = \frac{m_1 \times 0 + m_2 R}{m_1 + m_2} = \frac{m_2 R}{m_1 + m_2}$$

82. Considering A as origin



∴ For 1[st] sphere = $x_1 = 0$

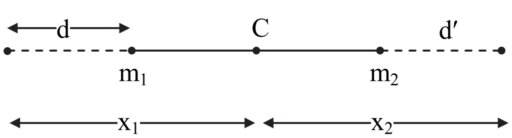
2[nd] sphere = x_2 = AB

3[rd] sphere = x_3 = AC

∴ $$x = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{0 + m(AB) + m(AC)}{3m} = \frac{AB + AC}{3}$$

83.



$m_1x_1 = m_2x_2$ ....(i)

$m_1(x_1 - d) = m_2(x_2 - d')$ ....(ii)

∴ $m_1x_1 - m_1d = m_2x_2 - m_2d'$

$m_1d = m_2d'$ ....[From (i)]

∴ $d' = \frac{m_1}{m_2} d$

84. The (x, y, z) co-ordinates of masses 1 g, 2 g, 3 g and 4 g are

$(x_1 = 0, y_1 = 0, z_1 = 0)$, $(x_2 = 0, y_2 = 0, z_2 = 0)$

$(x_3 = 0, y_3 = 0, z_3 = 0)$,

$(x_4 = \alpha, y_4 = 2\alpha, z_4 = 3\alpha)$

∴ $$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$X_{CM} = \frac{1\times0 + 2\times0 + 3\times0 + 4\times\alpha}{1+2+3+4}$$

$1 = \frac{4\alpha}{10} \Rightarrow \alpha = \frac{5}{2}$

Similarly,

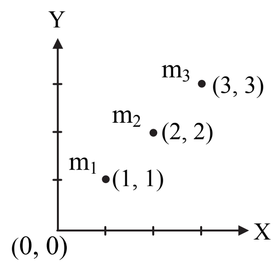
$$Y_{CM} = \frac{1\times0 + 2\times0 + 3\times0 + 4\times2\alpha}{1+2+3+4}$$

$2 = \frac{8\alpha}{10} \Rightarrow \alpha = \frac{20}{8} = \frac{5}{2}$

$$Z_{CM} = \frac{1\times0 + 2\times0 + 3\times0 + 4\times3\alpha}{1+2+3+4}$$

$3 = \frac{12\alpha}{10} \Rightarrow \alpha = \frac{30}{12} = \frac{5}{2}$

85.



The co-ordinates of the centre of mass are

$$X_{C.M} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m\times1 + m\times2 + m\times3}{m + m + m} = 2$$

$$Y_{C.M} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m\times1 + m\times2 + m\times3}{m + m + m} = 2$$

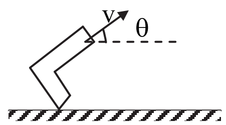
Hence, the co-ordinates of centre of mass are (2, 2).

86. Since the object has only translational motion without rotation therefore the centre of mass of the object is the point where the force has been applied. To find the centre of mass of the object, let C be taken as the origin and CD to be along Y-axis. If m be the mass of AB, then the mass of CD is 2m. The centre of mass of AB is at a distance $2l$ from C. The centre of mass of CD is at a distance l from C.

Distance of centre of mass of the object from C

$$= \frac{2m \times l + m \times 2l}{2m + m} = \frac{4ml}{3m} = \frac{4l}{3}$$

87. Velocity of centre of mass in X-direction is zero since there is no external force in X-direction. This means centre of mass can't change its position in X-direction. In other words, gun and bullet move in opposite direction along X-axis to maintain same position of C.M. in horizontal direction.



In Y-direction, external force is exerted by horizontal surface on gun and hence gun is at rest and only bullet moves with velocity mv sinθ in Y-direction.
velocity of C.M. is

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$\therefore \quad v_y = \frac{mv\sin\theta + m\times 0}{m + M}$$

$$\Rightarrow v_y = \frac{mv\sin\theta}{M + m}$$

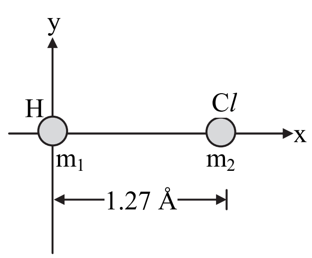
88. $v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{10\times 14 + 4\times 0}{4 + 10} = 10 \text{ m s}^{-1}$

89. $\vec{r}_1 = \frac{m_1\vec{r}_1 + m_1\vec{r}_2}{m_1 + m_2}$

$$\vec{r} = \frac{35.5\times 1.27}{1 + 35.5}\hat{i}$$

$$\vec{r} = \frac{35.5}{36.5}\times 1.27\hat{i}$$

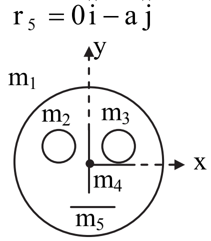
$$= 1.24\hat{i}$$



90. $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{2\times 3 + 3\times 2}{2 + 3}$

$$= \frac{12}{5} = 2.4 \text{ m/s}$$

91. According to problem
$m_1 = 6$ m, $m_2 = m_3 = m_4 = m_5 = m$
$\vec{r}_1 = 0\hat{i} + 0\hat{j}$, $\vec{r}_2 = -a\hat{i} + a\hat{j}$, $\vec{r}_3 = a\hat{i} + a\hat{j}$;
$\vec{r}_4 = 0\hat{i} + 0\hat{j}$, $\vec{r}_5 = 0\hat{i} - a\hat{j}$



Position vector of centre of mass

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4 + m_5\vec{r}_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

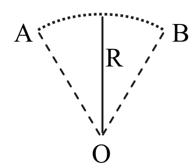
$$\vec{r}_{cm} = \frac{0 + m(-a\hat{i} + a\hat{j}) + m(a\hat{i} + a\hat{j}) + 0 + m(-a\hat{j})}{10m}$$

$$= 0\hat{i} + \frac{a}{10}\hat{j}$$

So, the coordinate of centre of mass = $\left(0, \frac{a}{10}\right)$.

92. Centre of mass is closer to massive part of the body therefore the bottom piece of bat has larger mass.

93. As particles are placed around origin they form arc.
If arc length → 0, centre of mass is at a distance R from the origin.
But as the arc length AB increases, centre of mass starts moving down.



94. $m_1r_1 = m_2r_2$

$$\Rightarrow \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

$$\therefore \quad r \propto \frac{1}{m}$$

95. The position of centre of mass remains unaffected because breaking of mass into two parts is due to internal forces.

96. Centre of mass lies always on the line that joins the two particles.
For the combination cd and ab this line does not pass through the origin.
For combination bd, initially it passes through the origin but later on its moves towards negative X-axis.
But for combination ac it will always pass through origin. So we can say that centre of mass of this combination will remain at origin.

97. $\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

$$= \frac{1(i + 2\hat{j} + \hat{k}) + 3(-3\hat{i} - 2\hat{j} + \hat{k})}{1 + 3}$$

$$\Rightarrow \vec{r}_{cm} = -2\hat{i} - \hat{j} + \hat{k}$$

98. $X = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$

$X = \dfrac{0 + 40x_4}{100} \Rightarrow 3 = \dfrac{40x_4}{100}$

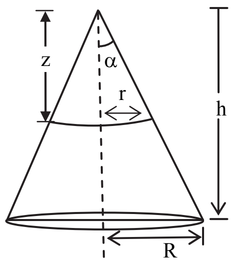
$x_4 = \dfrac{300}{40} = 7.5$

Similarly $y_4 = 7.5$ and $z_4 = 7.5$.

100. Mass = density × volume

$dm = \rho\pi r^2 dz$

From the figure,



$\tan\alpha = \dfrac{r}{z} = \dfrac{R}{h}$

∴ $r = \dfrac{R}{h}z$

Now,

$z_{CM} = \dfrac{\int z\,dm}{\int dM} = \dfrac{\int_0^h \rho\pi r^2 z\,dz}{\frac{1}{3}\pi R^2 h\rho}$

where, dM = mass element of entire cone.

∴ $z_{CM} = \dfrac{3}{R^2h}\int_0^h \left(\dfrac{R}{h}z\right)^2 z\,dz$

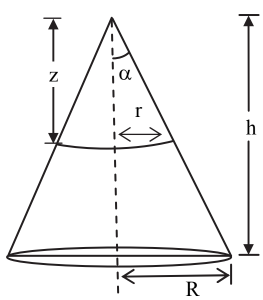
$= \dfrac{3}{hR^2}\left(\dfrac{R^2}{h^2}\right)\int_0^h z^3\,dz$

$= \dfrac{3}{h^3}\left[\dfrac{z^4}{4}\right]_0^h = \dfrac{3h}{4}$

101. Mass = density × volume

$dm = \rho\pi r^2 dz$

From the figure,



$\tan\alpha = \dfrac{r}{z} = \dfrac{R}{h}$

∴ $r = \dfrac{R}{h}z$

Now,

$z_{CM} = \dfrac{\int z\,dm}{\int dM} = \dfrac{\int_0^h \rho\pi r^2 z\,dz}{\frac{1}{3}\pi R^2 h\rho}$

where dM = mass element of entire cone

∴ $z_{CM} = \dfrac{3}{R^2h}\int_0^h \left(\dfrac{R}{h}z\right)^2 z\,dz = \dfrac{3}{hR^2}\left(\dfrac{R^2}{h^2}\right)\int_0^h z^3\,dz$

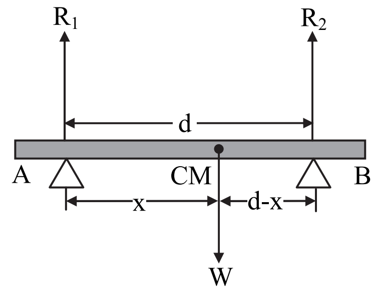
$= \dfrac{3}{h^3}\left[\dfrac{z^4}{4}\right]_0^h = \dfrac{3h}{4}$

∴ distance of centre of mass from base is

$h - \dfrac{3h}{4} = \dfrac{h}{4}$

∴ centre of mass has co-ordinates $\left(0, 0, \dfrac{h}{4}\right)$

102.

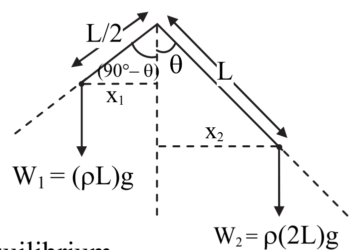


For equilibrium,

$N_1d = W(d - x)$

∴ $N_1 = \dfrac{W(d - x)}{d}$

103. The rule hanging from a peg is at equilibrium, hence, the principle of moments applies here.



For equilibrium,

$W_1x_1 = W_2x_2$

Where, $x_1 = \dfrac{L}{2}\sin(90° - \theta)$ and $x_2 = L\sin\theta$

∴ $(\rho L)g \times \dfrac{L}{2}\sin(90 - \theta) = 2(\rho L)g \times L\sin\theta$

∴ $\cos(\theta) = 4\sin\theta$ ....[∵ $\sin(90 - \theta) = \cos\theta$]

∴ $\tan\theta = \dfrac{1}{4}$

∴ $\theta = \tan^{-1}\left(\dfrac{1}{4}\right)$

104. (a) Centre of mass of a body not always coincides with the centre of gravity of the body.

(c) A couple on a body produces purely rotational motion.

Hence, (b) and (d) are correct.

105. Total initial momentum of balls = mnu

Total final momentum of balls = –mnu

Force experienced by the surface = Rate of change of momentum

= mnu – (– mnu) ....(Assuming unit time)

= 2mnu

106. $F = ma = kt$

Since $m = 1$ kg,

$a = kt$

$\therefore \quad \frac{dv}{dt} = kt$

$dv = kt\,dt$

Integrating both sides,

$v = \frac{kt^2}{2}$

$\therefore \quad dx = \frac{kt^2}{2}dt \quad .... \left(\because v = \frac{dx}{dt}\right)$

Integrating both sides,

$x = \frac{kt^3}{6} = 1 \times \frac{6 \times 6 \times 6}{6} = 36\,m$

107. $\vec{v} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

$= \frac{3\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right) + 4\left(3\hat{i} + 2\hat{j} - 3\hat{k}\right)}{3 + 4}$

$= \frac{18\hat{i} + 17\hat{j} - 3\hat{k}}{7}$

108. $\vec{p} = A\cos kt\ \hat{i} - A\sin kt\ \hat{j}$

$\therefore \quad \vec{F} = \frac{d\vec{p}}{dt} = -Ak\sin kt\ \hat{i} - Ak\cos kt\ \hat{j}$

Now, to find angle between $\vec{F}$ and $\vec{p}$

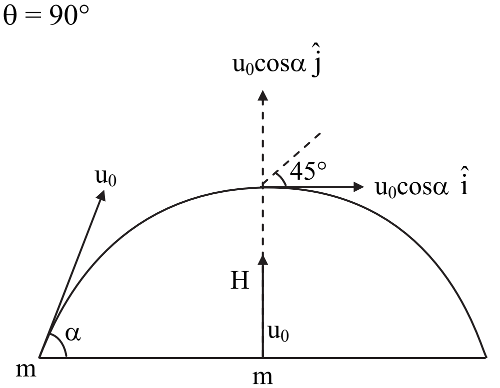
$\vec{F}\cdot\vec{p} = (-Ak\sin kt)(A\cos kt) + (-Ak\cos kt)$
$(-A\sin kt)$

$\therefore \quad Fp\cos\theta = A^2k\sin kt(-\cos kt + \cos kt)$
$= A^2k\sin kt(0)$

$\therefore \quad \cos\theta = 0$

$\therefore \quad \theta = 90°$

109.



Speed of 1st particle at highest point = $u_0\cos\alpha$

Speed of 2nd particle at highest point =

$\sqrt{u_0^2 - 2gH} \quad ....(i)$

By formula, maximum height

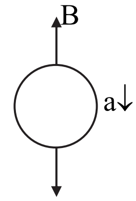
$H = \frac{u_0^2\sin^2\alpha}{2g}$ substituting in (i) and solving,

Speed of 2nd particle = $u_0\cos\alpha$

Collision being inelastic, final momentum of composite system = $mu_0\cos\alpha\,\hat{i} + mu_0\cos\alpha\,\hat{j}$

Hence angle made w.r.t. horizontal = $\frac{\pi}{4}$

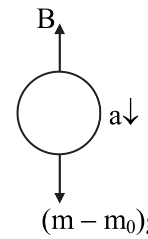
110.



$mg - B = ma \quad ....(i)$

(B is buoyant force)

Let m_0 be the mass that should be removed then



$B - (m - m_0)g = (m - m_0)a \quad ....(ii)$

Adding equations (i) and (ii),

$\Rightarrow mg - mg + m_0g = ma + ma - m_0a$

$\Rightarrow m_0 = \frac{2ma}{g + a}$

111. If monkey moves downward with acceleration a then its apparent weight decreases. In that condition

Tension in string = $m(g - a)$

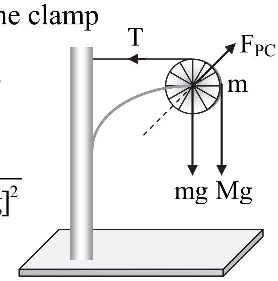
This should not exceed the breaking strength of the rope i.e.,

$360 \geq m(g - a)$

$\Rightarrow 360 \geq 60(10 - a)$

$\Rightarrow a \geq 4\ m/s^2$

112. Force on the pulley by the clamp

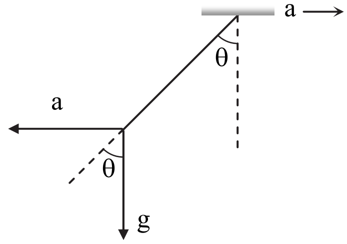


$F_{PC} = \sqrt{T^2 + [(M + m)g]^2}$

$F_{PC} = \sqrt{(Mg)^2 + [(M + m)g]^2}$

$F_{PC} = \sqrt{(M + m)^2 + M^2}\ g$

113. When car moves towards right with acceleration 'a' then due to pseudo force the plumb line will tilt in backward direction making an angle θ with the vertical.
From the figure,
$\tan\theta = a/g$
∴ $\theta = \tan^{-1}(a/g)$

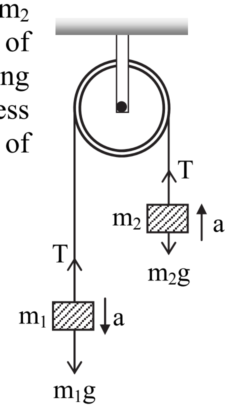


114. $F = ma = m\dfrac{dv}{dt}$
$m\,dv = F\,dt$
integrating on both side,
$$m\int_{v_1}^{v_2} dv = \int_0^t (3t^2 - 30)dt$$
$$m(v_2 - v_1) = \left[\frac{3t^3}{3} - 30t\right]_0^5$$
∴ $10(v_2 - 10) = 125 - 30 \times 5$
∴ $v_2 = 7.5$ m/s

115. Given:
$$v \propto \frac{1}{\sqrt{x}}$$
∴ $\dfrac{dv}{dx} = \dfrac{-1}{2x^{3/2}}$
Dividing throughout by dt, we get,
$$\frac{dv/dt}{dx/dt} = \frac{-1}{2x^{3/2}}$$
∴ $\dfrac{dv}{dt} = \dfrac{-1}{2x^{3/2}}\dfrac{dx}{dt}$
But $v = \dfrac{dx}{dt} = k\dfrac{1}{x^{1/2}}$
∴ $\dfrac{dv}{dt} \propto \dfrac{1}{2x^{3/2}} \times \dfrac{1}{x^{1/2}}$ ....[Considering constant of proportionality as (–1)]
∴ $\dfrac{dv}{dt} \propto \dfrac{1}{x^2}$
∴ $F \propto \dfrac{1}{x^2}$ ....$\left(F = ma = m\dfrac{dv}{dt}\right)$

116. Let v_1 and v_2 be initial and final velocity of body
Final K.E. $= \dfrac{1}{8}m v_1^2$ ....(given)
$$\frac{1}{2}m v_2^2 = \frac{1}{8}m v_1^2$$
$$v_2 = \frac{v_1}{2} = \frac{10}{2} = 5 \text{ m/s}$$
frictional force is given as, $F = -kv^2$
$ma = -kv^2$
$$m\frac{dv}{dt} = -kv^2$$
$$10^{-2}\frac{dv}{dt} = -kv^2$$
$$\int_{10}^{5}\frac{dv}{v^2} = -100k\int_0^{10} dt$$
$$\frac{1}{5} - \frac{1}{10} = 100k\,(10)$$
$k = 10^{-4}$ kgm^{-1}

117. When two masses m_1 and m_2 are connected to the two ends of an inextensible string passing over a smooth frictionless pulley, then the acceleration of the masses is given by:

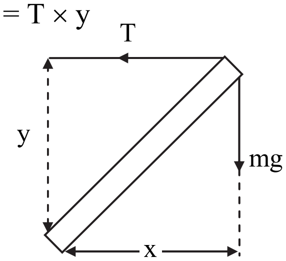


$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$
Here $a = g/8$
∴ $\dfrac{g}{8} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)g$
∴ $\dfrac{m_1}{m_2} = \dfrac{9}{7}$.

118. In terms of three significant figure
Momentum $p = mv = 3.513 \times 5.00 = 17.6$ kg m/s

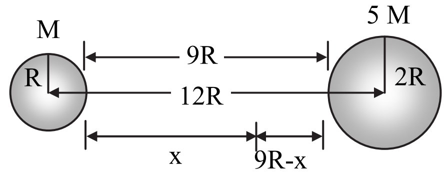
119. According to conservation of linear momentum,
$p_f = p_i$
here, uranium at rest decays,
∴ $p_f = p_i = 0$
i.e., $p_{He} - p_{Th} = 0$
∴ $p_{He} = p_{Th}$
As, $K = \dfrac{p^2}{2m}$
$K \propto \dfrac{1}{m}$
∴ $K_{He} > K_{Th}$ ($\because m_{He} < m_{Th}$)

120. For equilibrium, $mgx = T \times y$
$\Rightarrow T = \dfrac{mgx}{y}$



For T to be minimum y should be maximum.

121. As the spherical bodies have their own size so the distance covered by both the body 12R – 3R = 9R, but individual distance covered by each body depends upon their masses.



These bodies are moving under the effect of mutual attraction only, so their position of centre of mass remains unaffected.

Let smaller body cover distance x just before collision

From $m_1r_1 = m_2r_2$,

$\Rightarrow$ M x = 5 M (9R – x) $\Rightarrow$ x = 7.5R

122. For free fall, $s_n = u + \frac{a}{2}(2n-1)$

Where, s_n = distance covered during nth second.

$\therefore$ $h_n \propto (2n - 1)$

When the ball is released from the top of tower, then ratio of distances covered by the ball in first, second and third second is

$\therefore$ $h_I : h_{II} : h_{III} = 1:3:5$

$\therefore$ Ratio of work done,

$mgh_I : mgh_{II} : mgh_{III} = 1:3:5$

123. According to law of conservation of momentum,

M × 20 = (M + m)V

$V = \frac{M \times 20}{M+m}$ ….(i)

Work done in penetration,

$W = \frac{1}{2} \times (M+m)V^2$

But W = f × s where f is resistive force and s = 1 cm = 10^{-2} m.

$\therefore$ $\frac{1}{2}(M+m)V^2 = f \times 10^{-2}$

Substituting for V using equation (i),

$\frac{10^2}{2}(M+m) \times \left(\frac{M\times 20}{M+m}\right)^2 = f$

$\frac{400M^2 \times 10^2}{2(M+m)} = f$

$\therefore$ $f = \frac{2M^2}{M+m} \times 10^4$

124. Initially both the particles are at rest, so velocity of centre of mass is equal to zero and no external force acts on the system, therefore its velocity of centre of mass remains constant i.e., zero.

125. Initial velocity of C.M in X-direction

$u_x = \frac{m_1 u_{X_1} + m_2 u_{X_2}}{m_1 + m_2} = \frac{m(2+0)}{2m} = 1$

acceleration of C.M in X-direction

$a_x = \frac{m_1 a_{X_1} + m_2 a_{X_2}}{m_1 + m_2} = \frac{m(3+0)}{2m} = \frac{3}{2}$

From v = u + at, final velocity of C.M in X-direction is

$v_x = u_x + a_x t$ $\therefore$ $v_x = 1 + \frac{3}{2}t$

Initial velocity of C.M in Y-direction

$u_y = \frac{m_1 u_{Y_1} + m_2 u_{Y_2}}{m_1 + m_2} = \frac{m(0+2)}{2m} = 1$

acceleration of C.M in Y-direction

$a_y = \frac{m_1 a_{Y_1} + m_2 a_{Y_2}}{m_1 + m_2} = \frac{m(3+0)}{2m} = \frac{3}{2}$

Now, $v_y = u_y + a_y t$

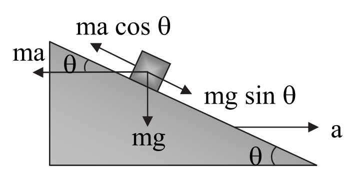
$\therefore$ $v_y = 1 + \frac{3}{2}t$

As C.M travels with same velocity in X and Y direction, it must be travelling in straight line.

126. Velocity of centre of mass of a body is constant when no external force acts on the body. If there is no external torque, it does not mean that no external force acts on it.

127. According to law of inertia (Newton's first law), when cloth is pulled from a table, the cloth comes in state of motion but dishes remain stationary due to inertia. Thus we can pull the cloth from table without dislodging the dishes.

128.

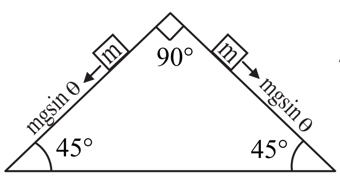


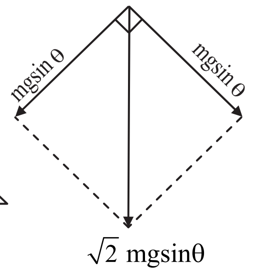
Let the mass of block be m. It will remain stationary if forces acting on it are in equilibrium i.e., ma cos θ = mg sin θ

Here, ma = Pseudo force on block.

$\therefore$ a = g tan θ

129.





Acceleration of the centre of mass of the system is given by,

$a = \frac{F_{ext}}{M}$ …(M ≡ mass of the system)

$= \frac{\sqrt{2}mg\sin\theta}{m+m} = \frac{\sqrt{2}\,g\sin\theta}{2}$

$= \frac{10\times\sin45°}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}}$

$a = 5$ m/s^2 vertically downward

130. Initial momentum = $p_i = 0$

Final momentum $p_f = 0 = mv\hat{i} + mv\hat{j} + \vec{p}_3$

$\Rightarrow p_3 = mv\sqrt{2}$ and $m_3 = 4m - 2(m) = 2m$

K.E. of 3rd piece = $\frac{p_3^2}{2m_3} = \frac{p_3^2}{2\times2m}$

Total KE = $\frac{p_3^2}{2\times2m} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$= \frac{2m^2v^2}{4m} + mv^2 = \frac{3mv^2}{2}$

131. $P = \frac{W}{t} = \frac{F\times s}{t} = \frac{ma\times s}{t} = m\frac{dv}{dt}\times\frac{s}{t}$

Here P = k

∴ $k = mv\frac{dv}{dt}$ ....(i)

$\frac{v^2}{2} = \frac{k}{m}t$

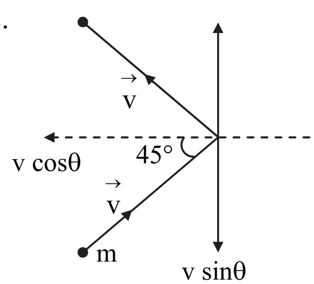
$v = \sqrt{\frac{2kt}{m}}$

$F = m\frac{dv}{dt}$

Using (i)

$F = \frac{k}{v} = \frac{k}{\sqrt{\frac{2kt}{m}}} = \sqrt{\frac{mk}{2}}\,t^{\frac{-1}{2}}$

132.



Change in momentum of one molecule,

$\Delta P' = 2mv\cos45° = \sqrt{2}$ mv

Force $F = \frac{\Delta P}{\Delta t} = n\times\Delta P'$

Where, n = no. of molecules incident per unit time

Pressure $P = \frac{\text{Force}}{\text{Area}}$

∴ $P = \frac{n\times\sqrt{2}mv}{A}$

$P = \frac{10^{23}\times\sqrt{2}\times3.32\times10^{-27}\times10^3}{2\times10^{-4}}$

$P = \frac{3.32}{1.41}\times10^3 = 2.35\times10^3$ N/m^2

133. $P = \vec{F}\cdot\vec{v}$ ….(i)

F = mg = 250 × 9.8 = 2450 N

v = 0.2 m/s

From equation (i),

P = 2450 × 0.2 = 490 W

As, 1 hp = 746 W

∴ $P = \frac{490}{746}$ hp = 0.65 hp

134. Mass of deuterium is twice that of a neutron.

Now, according to law of conservation of momentum,

$mu = mv_1 + 2m\,v_2$

∴ $u = v_1 + 2v_2$ ….(i)

Coefficient of restitution for perfectly elastic collision,

$e = \frac{v_2 - v_1}{u} = 1$

∴ $v_2 - v_1 = u$ ….(ii)

On solving equations (i) and (ii),

$v_2 = \frac{2u}{3}$ and $v_1 = -\frac{u}{3}$

Initial K.E. of neutron is, $(K.E.)_i = \frac{1}{2}mu^2$

Final K.E. of neutron,

$(K.E.)_f = \frac{1}{2}mv_1^2 = \frac{1}{2}m\left(\frac{-u}{3}\right)^2 = \frac{1}{9}\left(\frac{1}{2}mu^2\right)$

∴ Loss in K.E. = $(K.E.)_f - (K.E.)_i$

$= \Delta E = \frac{8}{9}\left(\frac{1}{2}mu^2\right)$

Fractional loss $\frac{\Delta E}{(K.E.)_i} = p_d = \frac{8}{9} = 0.89$

Mass of carbon nucleus

= 12 × (mass of a neutron)

∴ In case of collision of neutron with carbon nucleus,

$mv_1 + 12\,mv_2 = mu$

$v_1 + 12v_2 = u$ ….(iii)

On Solving equations (ii) and (iii)

$v_2 = \frac{2u}{13}$ and $v_1 = -\frac{11u}{13}$

For neutron,

Final K.E. = $\frac{1}{2}\, m\left(\frac{-11u}{13}\right)^2 = \frac{121}{169}\left(\frac{1}{2}mu^2\right)$

∴ Loss in K.E. = $\frac{48}{169}\left(\frac{1}{2}mu^2\right)$

Fractional loss = $p_c = \frac{48}{169} = 0.28$

Evaluation Test

1. The momentum of hammer = $m_1^2\sqrt{2gh}$

Also, momentum of (hammer + pile)

= $(m_1 + m_2)\, v$

According to law of conservation of momentum,

$(m_1 + m_2)\, v = m_1\sqrt{2gh}$

$\Rightarrow v = \frac{m_1\sqrt{2gh}}{m_2 + m_1}$ ….(i)

Let opposition to penetration be F, then from work energy theorem,

Work done = Change in K.E.

$(m_2 + m_1)gd - Fd = 0 - \frac{1}{2}(m_1 + m_2)v^2$

$\Rightarrow F = \frac{1}{2d}(m_1 + m_2)v^2 + (m_2 + m_1)g$

∴ $F = \frac{1}{2d}(m_1 + m_2)\frac{m_1^2 \cdot 2gh}{(m_2 + m_1)^2} + (m_2 + m_1)g$

….[Using (i)]

$\Rightarrow F = \frac{m_1^2 gh}{(m_2 + m_1)d} + (m_2 + m_1)g$

2. Initial velocity $u = \frac{\text{momentum}}{\text{mass}}$

$= \frac{20}{8} = 2.5$ m/s

Acceleration $= \frac{\text{Force}}{\text{mass}} = \frac{12}{8} = 1.5\ \text{m/s}^2$

From equation of motion

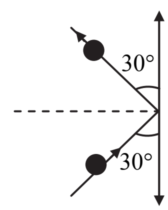
$s = ut + \frac{1}{2}at^2 = 2.5 \times 4 + \frac{1}{2} \times 1.5 \times 4 \times 4$

= 10 + 12 = 22 m

According to work energy theorem,

Increase in kinetic energy = work done.

∴ Work done = 12 × 22 = 264 J.

3. 

Components of momentum parallel to the wall add each other and components of momentum perpendicular to the wall are opposite to each other.

∴ Change in momentum

= Final momentum – initial momentum

= $mv\sin\theta - (-mv\sin\theta) = 2\,mv\sin\theta$ ….(i)

(After collision) (Before collision)

Also, change in momentum = F × t ….(ii)

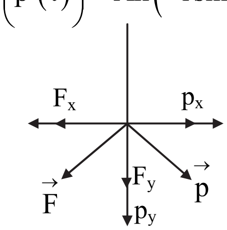
From (i) and (ii)

∴ $F = \frac{2mv\sin\theta}{t} = \frac{2 \times 1 \times 20 \times \sin 30°}{0.5} = 40$ N

4. Originally, centre of mass is at the centre O. After square 1 is removed, C.M. lies in quadrant 3. After squares 1 and 2 are removed, C.M. lies on Y-axis below below O. When squares 1 and 3 are removed, C.M. will remain at O. When squares 1, 2, 3 are removed, C.M. will shift to fourth quadrant. When all the four squares are removed, C.M. will shift back to O.

5. $\vec{p}(t) = A\left[\hat{i}\cos kt - \hat{j}\sin kt\right]$

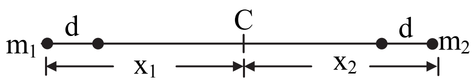
$\vec{F} = \frac{d}{dt}\left(\vec{p}(t)\right) = Ak\left(-\hat{i}\sin kt - \hat{j}\cos kt\right)$



$\vec{F}\cdot\vec{p} = A^2k(-\cos kt \sin kt + \sin kt \cos t) = 0$

∴ The momentum and force are perpendicular to each other at 90°.

6. Suppose x_1 is distance of m_1 and x_2 is distance of m_2 from centre of mass C, as shown in figure below. Let m_1 be pushed towards C through a distance d. If m_2 is pushed through a distance d′ to keep the centre of mass at C, then taking C as the origin, we have



$m_1 x_1 = m_2 x_2$ ....(i)

and $m_1(x_1 - d) = m_2(x_2 - d')$ ....(ii)

$m_1 d = m_2 d'$

∴ $d' = \dfrac{m_1}{m_2} d$

7. Let $m_1 = 2$ kg, $m_2 = 12$ kg and $m_3 = 4$ kg. If 'a' is acceleration of the system to the right, then the equations of motion of the three bodies are

$m_1 a = T_1 - m_1 g$,

$m_2 a = T_2 - T_1$ and

$m_3 a = m_3 g - T_2$

Adding the three equations,

$(m_1 + m_2 + m_3)a = (m_3 - m_1)g$

$a = \dfrac{(m_3 - m_1)g}{m_1 + m_2 + m_3}$

$= \dfrac{(4-2)10}{2+12+4} = 1.11 \text{ m/s}^2$

8. $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\times\dfrac{m(mv^2)}{m} = \dfrac{(mv)^2}{2m}$

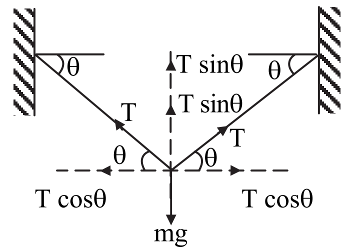
$\Rightarrow K = \dfrac{p^2}{2m}$

∴ $\dfrac{K_1}{K_2} = \dfrac{p_1^2}{2m_1}\times\dfrac{2m_2}{p_2^2} \Rightarrow \dfrac{5}{2} = \left(\dfrac{p_1}{p_2}\right)^2 \times \dfrac{10}{4}$

∴ $p_1 : p_2 = 1 : 1$

9. Torque is given by, $\vec{\tau} = \vec{r}\times\vec{F}$. Hence option (A) is incorrect.
Though torque and work have same dimensions and unit, they are different physical quantities. Hence option (B) is incorrect.
The direction of moment of force is perpendicular to the plane of figure. Hence option (C) is incorrect.

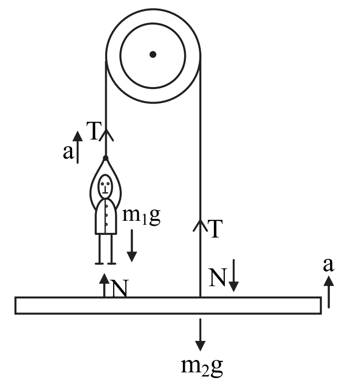
10. Mass of rope, m = 0.2 kg, θ = 30°



From figure, 2Tsinθ = mg

∴ $T = \dfrac{mg}{2\sin\theta} = \dfrac{0.2\times9.8}{2\sin30^\circ} = 1.96\text{ N}$

11. From the F.B.D.,



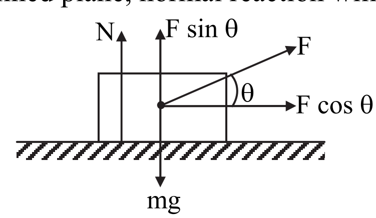
For m_1 : $N + T - m_1 g = m_1 a$ ....(i)

For m_2 : $T - N - m_2 g = m_2 a$ ....(ii)

From (i) and (ii),

$N = \dfrac{(m_1 - m_2)}{2}(g + a)$.

12. From F.B.D., at the moment of breaking off the inclined plane, normal reaction will be zero.



F sin θ = mg ⇒ kt sinθ = mg

∴ $t = \dfrac{mg}{k\sin\theta}$

Since F cos θ = ma ⇒ kt cos θ = $m\dfrac{dv}{dt}$

∴ $\int_0^v dv = \dfrac{k\cos\theta}{m}\int_0^t t\,dt \Rightarrow v = \left(\dfrac{k\cos\theta}{2m}\right)t^2$

At the time of breaking off $t = \dfrac{mg}{k\sin\theta}$

∴ $v = \dfrac{k\cos\theta}{2m}\times\left(\dfrac{mg}{k\sin\theta}\right)^2 = \dfrac{mg^2\cos\theta}{2k\sin^2\theta}$

13. Initial momentum = 0

Final momentum = $2m\vec{v} - 2m\vec{v} = 0$

Relative velocity of one with respect to the other = 2v

Final K.E. = $2 \times \frac{1}{2} \times 2mv^2 = E \Rightarrow v = \sqrt{\frac{E}{2m}}$

∴ Relative velocity = $2v = 2\sqrt{\frac{E}{2m}}$

$= \sqrt{\frac{4E}{2m}} = \sqrt{\frac{2E}{m}}$

14. Assertion is true, but the Reason is not true. Infact, the centre of mass is related to the distribution of mass of the body.

15. According to law of conservation of momentum,

$m_P u_1 + m_Q \times 0 = m_P v_1 + m_Q(-v_1)$

$m_P u_1 = (m_P - m_Q)v_1$ …(i)

∴ $\frac{u_1}{v_1} = \frac{m_P - m_Q}{m_P}$ …(ii)

According to law of conservation of kinetic energy,

$\frac{1}{2}m_P u_1^2 = \frac{1}{2}\left(m_P + m_Q\right)v_1^2$ …(iii)

Dividing (iii) by (i),

$u_1 = \frac{\left(m_P + m_Q\right)v_1}{m_P - m_Q}$

$\Rightarrow \frac{u_1}{v_1} = \frac{m_P + m_Q}{m_P - m_Q}$ …(iv)

From (ii) and (iv),

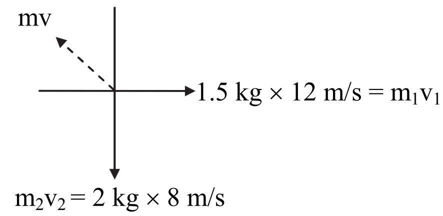
$\Rightarrow \frac{m_P - m_Q}{m_P} = \frac{m_P + m_Q}{m_P - m_Q}$

On solving, $\frac{m_P}{m_Q} = \frac{1}{3}$.

16. Earth revolves around the sun in almost circular orbit and has spinning motion about its axis. Due to this, the velocity of earth is changing with time. Hence Newton's first law of motion does not hold good for the earth. Thus, Reason is correct.

But for the object moving on the earth, the earth can be taken at rest and the frame of reference attached to motion on the earth is taken as inertial.

17. When an explosion breaks a rock, its initial momentum is zero. Hence, according to the law of conservation of momentum, final momentum will be zero.



Total momentum of the two pieces of 1.5 kg and 2 kg

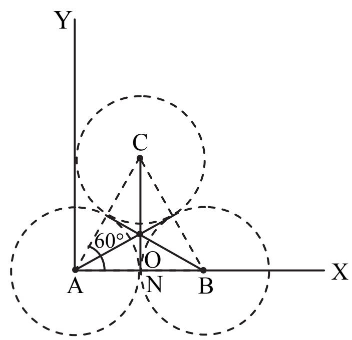
$= \sqrt{18^2 + 16^2} \approx 24$ kg m s^{-1}.

The third piece will have the same momentum but in direction opposite to the resultant of these two momenta.

∴ Momentum of the third piece = 24 kg m s^{-1} velocity = 6 m s^{-1}.

∴ Mass of the 3rd piece = $\frac{mv}{v} = \frac{24}{6} = 4$ kg

18.



Taking A as the origin, the co-ordinates of the three vertices of the triangle are:

$A(x_1, y_1) = (0, 0)$; $B(x_2, y_2) = (2\,r, 0)$ and

$C(x_3, y_3) = \left(r, r\sqrt{3}\right)$

∴ Co-ordinates of centre of mass O are

$x = \frac{m\left(x_1 + x_2 + x_3\right)}{3m} = \frac{0 + 2r + r}{3} = r$

$y = \frac{m\left(y_1 + y_2 + y_3\right)}{3m} = \frac{0 + 0 + r\sqrt{3}}{3} = \frac{r}{\sqrt{3}}$

19. The velocity of ball hitting the bat = v m/s

The velocity of recoil in the opposite direction = (v/2) m/s

∴ Change of momentum = $mv - \left(-\frac{mv}{2}\right)$

$\Rightarrow \Delta p = \frac{3mv}{2}$.

∴ Force on the ball = $\frac{3mv}{2t}$.

20. Common acceleration, $a = \frac{F}{m_1 + m_2 + m_3}$

$a = \frac{5}{10 + 8 + 2} = 2.5 m/s^2$

Equation of motion of m_3 is $T_3 - T_2 = m_3 a$

$\Rightarrow 50 - T_2 = 2 \times 2.5 \Rightarrow T_2 = 45$ N

21. Amongst the given balls, glass balls have maximum coefficient of restitution i.e., e = 0.94.

22. For the completely filled bob, C.G. coincides with its centre. As the liquid flows out, C.G. shifts downward. When more than half of liquid flows out, it starts shifting upwards and when the bob gets emptied completely, C.G. is at centre again.

Textbook Chapter No.

05 Friction in Solids and Liquids

Hints

Classical Thinking

15. $\mu_S = \frac{F_S}{mg} = \frac{294}{50\times9.8} = 0.6$

18. $\mu_k = \frac{16}{4\times9.8} = 0.41$

25. $h = \frac{P}{\rho g} = \frac{10^5}{10^3\times10} = 10$ m

26. $P = P_0 + h\rho g = 1.01\times10^5 + (3\times10^3\times1030\times9.8)$
$\approx 3\times10^7$ Pa

34. $P_{avg} = \frac{F}{A} = \frac{40\times9.8}{2\times10\times10^{-4}} = 1.96\times10^5$ Pa

35. $70\times13.6\times g = h\times3.4\times g$
$\therefore$ $h = \frac{70\times13.6}{3.4} = 280$ cm

41. $\pi(2R)^2\times v_1 = \pi(R)^2 v_2$
$\therefore$ $v_2 = \frac{4Rv_{1^2}}{R^2} = 4v_1$

42. $A_1v_1 = A_2v_2$
$\pi(1)^2\times5 = \pi(0.5)^2\times v_2$ $(\because A = \pi r^2)$
$\therefore$ $v_2 = \frac{1\times5}{0.5\times0.5} = 20$ cm/s

46. Force of adhesion is more between the liquid layer and bottom of vessle. Hence velocity of liquid layer of bottom is least and velocity increases towards the surface.

50. $\frac{dv}{dx} = \frac{12}{0.8} = 15$/s

51. velocity gradient $= \frac{dv}{dx}$
$5 = \frac{dv}{2.5}$
dv = 12.5 cm/s

54. $F = \eta A\frac{dv}{dx} = 2\times0.04\times\frac{0.05}{0.0005} = 8$ N

55. $F = \eta A\left(\frac{dv}{dx}\right)$
$\therefore$ $\eta = \frac{F}{A\left(\frac{dv}{dx}\right)} = \frac{2000}{10\times\frac{1}{0.1}} = \frac{2000\times0.1}{10} = 20$ poise

57. $\frac{F_1}{F_2} = \frac{6\pi\eta r_1 v}{6\pi\eta r_2 v} = \frac{r_1}{r_2} = \frac{r}{2r} = \frac{1}{2}$

58. $F = 6\pi\,\eta rv$
$= 6\times3.142\times1.8\times10^{-4}\times0.05\times200$
$= 0.034$ dyne

61. $v = \frac{\frac{2}{9}r^2g(\rho-\sigma)}{\eta}$
$= \frac{2}{9}\frac{(0.1\times10^{-2})^2\times9.8\times(8000-1330)}{8.33\times10^{-1}}$
≈ 0.01743 m/s
$= 17.43\times10^{-3}$ m/s

62. Neglecting buoyancy due to air,
$v = \frac{2r^2\rho g}{9\eta} = \frac{2\times(2\times10^{-5})^2\times1.2\times10^3\times9.8}{9\times1.8\times10^{-5}}$
$= 5.81\times10^{-2}$ m/s
$v \approx 5.8$ cm/s

63. $\eta = \frac{2r^2\rho g}{9v}$ (Neglecting density of air)
$= \frac{2\times(10^{-5})^2\times1000\times9.8}{9\times1.21\times10^{-2}}$
$\eta = 1.8\times10^{-5}$ N s/m^2

66. $N = \frac{v_C\rho D}{\eta} = \frac{8\times1\times1}{10^{-2}} = 800$
Since 800 < 2000
$\therefore$ The flow is streamline.

73. $v = \sqrt{2gh} = \sqrt{2\times10\times20} = 20$ m s^{-1}

74. $v = \sqrt{2gh} = \sqrt{2\times9.8\times0.1} = 1.4$ m/s

77. $P_1 - P_2 = \rho g(h_2 - h_1)$
$= 1040\times9.8\,(0.5)$
$P_1 - P_2 = 5096$ N m^{-2}

78. From the Bernoulli's Principle

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$= \frac{1}{2} \times 1.3 \times [(120)^2 - (90)^2]$$

$$= 4095 \text{ N/m}^2 \text{ or pascal}$$

82. $a_{max} = \mu_S g$

$\therefore$ $a_{max} = 0.15 \times 10 = 1.5 \text{ m/s}^2$

83. $W = \vec{F}\cdot\vec{ds} = (\mu N)S$

84. $P = P_0 + h\rho g$

$= 1.01 \times 10^5 + 0.20 \times 1000 \times 10$

$= 1.01 \times 10^5 + 0.02 \times 10^5 = 1.03 \times 10^5$ Pa

$\because$ $F = PA$

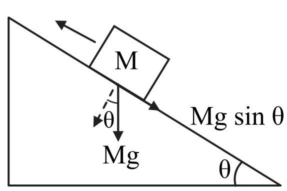
$\therefore$ $F = 1.03 \times 10^5 \times 1 = 1.03 \times 10^5$ N

Critical Thinking

2. Friction is non-conservative force. Also frictional force = μmg i.e., it depends upon the mass of the body.

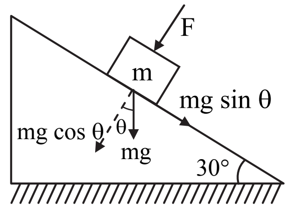
6. When a bicycle is in motion, two cases may arise:
   i. When the bicycle is being pedalled, the applied force has been communicated to rear wheel. Due to which the rear wheel pushes the earth backwards. Now the force of friction acts in the forward direction on the rear wheel but front wheel moves forward due to inertia, so force of friction works on it in backward direction.
   ii. When the bicycle is not being pedalled, both the wheels move in forward direction, due to inertia. Hence force of friction on both the wheels acts in backward direction.

7.



The component of weight Mg of the block along the inclined plane = Mg sinθ.
The minimum frictional force to overcome is also Mg sinθ.
To make the block just move up the plane the minimum force applied must overcome the component Mg sinθ of gravitational force as well as the frictional force Mg sinθ is 2 Mg sinθ.

8.



Here, $\theta = 30°$

$N = mg\cos\theta$

$F_S = \mu N = \mu\,(mg\cos\theta) = 0.5$

$(mg)\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}mg$

While the downward force component,

$mg\sin\theta = mg\sin 30° = \frac{1}{2}mg$

$\Rightarrow mg\sin\theta > \mu\,(mg\cos\theta)$

This means force of static friction is not sufficient to stop the block from slipping downwards.

Let F be the minimum force applied on it, so that it does not slip. Then $N = F + mg\cos 30°$

$\therefore$ $mg\sin 30° = \mu N = \mu(F + mg\cos 30°)$

$$\Rightarrow F = \frac{mg\sin 30°}{\mu} - mg\cos 30°$$

$$= \left[\frac{(2)(10)(1/2)}{0.5}\right] - (2)\,(10)\left(\frac{\sqrt{3}}{2}\right)$$

$\therefore$ $F = 2.68$ N

10. For block A to just move,

$(F_S)_A \propto m_A$

For block A and B to just move,

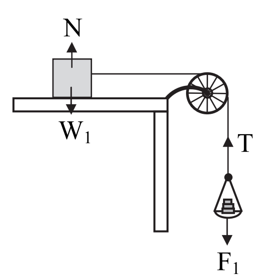
$(F_S)_{AB} \propto (m_A + m_B)$

Taking ratio,

$$\frac{(F_S)_{AB}}{(F_S)_A} = \frac{(m_A + m_B)}{m_A}$$

$\therefore$ $(F_S)_{AB} = 12 \times \frac{(4+8)}{4} = 36$ N

11.

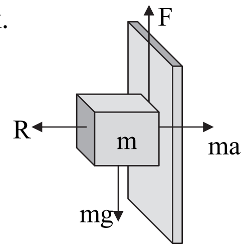


As shown in free body diagram,

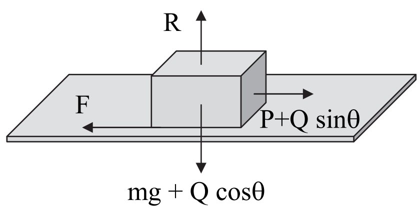
Weight of block = N

$F_1 = T$ = Force due to static friction between block and surface = F_{S_1}

12. For the limiting condition, upward frictional force between board and block will balance the weight of the block.
i.e., F > mg
$\Rightarrow \mu(R) > mg$
$\Rightarrow \mu(ma) > mg$
$\Rightarrow \mu > \frac{g}{a}$



13. Resolving force Q into its components, the free body diagram of the block is given by



$F = \mu R$
$\Rightarrow P + Q \sin\theta = \mu(mg + Q\cos\theta)$
$\therefore \quad \mu = \frac{P + Q\sin\theta}{mg + Q\cos\theta}$

14. Acceleration of block on horizontal surface
$a = (100 - \mu R)/m$
$= (100 - 0.5 \times 100)/10 = 5 \text{ m/s}^2$
Note: g is gravitational acceleration and motion is along horizontal. Hence g will not play any role in this case.

15. At a point, pressure acts in all directions and a definite direction is not associated with it, so pressure is a scalar quantity.

16. When two holes are made in the tin, air keeps entering through the other hole. Due to this the pressure inside the tin does not become less than atmospheric pressure which happens when only one hole is made.

18. Pressure depends on depth alone.

19. pressure $(P_g) = 200$ kPa,
P_0 = atmospheric pressure $= 1.01 \times 10^5$ Pa
$= 101$ kPa
Absolute pressure $(P) = P_0 + P_g$
$= 101 + 200 = 301$ kPa

20. Total pressure $= P_a + \rho gh$
$[\because \rho_{water} = 10^3 \text{ kg/ms}^2]$
$= 1.01 \times 10^5 + 10^3 \times 10 \times 10$
$= 2.01 \times 10^5$ Pa
≈ 2 atm

21. $r_1 = \frac{5}{100}\text{ m}, r_2 = \frac{10}{100}\text{ m}$,
$F_2 = 1350 \text{ kg f} = 1350 \times 9.8$ N;
As, $\frac{F_1}{a_1} = \frac{F_2}{a_2}$
$\therefore \quad F_1 = \frac{a_1}{a_2}F_2 = \frac{\pi r_1^2}{\pi r_1^2}F_2$
$\Rightarrow \quad F_1 = \frac{r_1^2}{r_1^2}F_2 = \frac{(5/100)^2}{(10/100)^2} \times 1350 \times 9.8$
$= 1470$N
Pressure, $P = \frac{F_1}{a_1} = \frac{F_1}{\pi r_1^2}$
$= \frac{1470}{(22/7)(5/100)^2}$
$= 1.87 \times 10^5$ Pa

24. Hydraulic brakes work as per Pascal's law. Hence change in liquid pressure is transmitted equally to wheels.

29. According to the equation of continuity,
Av = constant
The speed of still water is very small and hence area will be large. This makes the still water run deep.

30. The equation of continuity is derived on the basis of the principle of conservation of mass and it is true in every case, whether tube is kept horizontal or vertical.

31. If the liquid is incompressible then mass of liquid entering through left end, should be equal to mass of liquid coming out from the right end.
$\therefore \quad M = m_1 + m_2$
$\Rightarrow Av = A_1v_1 + A_2v_2$
$\Rightarrow 24 \times 10 = 12 \times 6 + 8 \times v_2$
$\Rightarrow v_2 = \frac{240 - 72}{8} = 21$ m/s

32. Volume of big drop = 2 (Volume of small drop)
$\frac{4}{3}\pi r_2^3 = 2 \times \frac{4}{3}\pi r_1^3$
$\therefore \quad r_2 = 2^{1/3} r_1$
Also $v_1 \propto r_1^2$, $v_2 \propto r_2^2$
$\therefore \quad \frac{v_2}{v_1} = \frac{r_2^2}{r_1^2}$
$\therefore \quad v_2 = \frac{r_2^2}{r_1^2} \times v_1 = \frac{2^{\frac{2}{3}} r_1^2}{r_1^2} \times 0.15$
$v_2 = 0.15 \times 2^{2/3}$ cm/s

35. Velocity gradient = $\frac{dv}{dx} = \frac{8}{0.1} = 80$ / s

37. A lubricating oil is generally used between the various parts of the machine to reduce the friction. In winter, since the temperature is low, the viscosity of oil between the machine part increases considerably resulting in jamming of the machine parts.

40. Terminal velocity is caused due to viscosity, which is absent in vacuum.

41. $v_C = N\frac{\eta}{\rho D}$

For laminar flow, Reynold's number N = 2000

$\therefore$ $v_C = \frac{2000 \times 10^{-3}}{10^3 \times 2 \times 10^{-2}} = 0.1\ m\ s^{-1}$

43. As air under the pan is blown, pressure below the pan decreases. This as per Bernoulli's theorem causes downward motion.

44. According to Bernoulli's theorem, when wind velocity over the wings is larger than the wind velocity under the wings, pressure of wind over the wings becomes less than the pressure of wind under the wings. This provides the necessary lift to the aeroplane.

45. According to Bernoulli's theorem, when velocity of liquid flow increases, pressure decreases and vice-versa. When two boats move parallel to each other, close to one another, the stream of water between the boats is set into vigorous motion. As a result, the pressure exerted by the water in between the boats becomes less than the pressure of water outside the boats. Due to this pressure difference, the boats are pulled towards each other.

47. $P + \rho_1 gh_1 + \rho_2 gh_2$

$h = h_1 + h_2$ = height of free surface above hole

While at hole, horizontal velocity will be zero

$P + \rho_1\ gh_1 + \rho_2\ gh_2 = P + \frac{1}{2}\ \rho_1 v^2$

$\therefore$ $v = \sqrt{2g\left(\frac{\rho_1 h_1 + \rho_2 h_2}{\rho_1}\right)} = \sqrt{2g\left(h_1 + h_2\frac{\rho_2}{\rho_1}\right)}$

48. $P_1 + \frac{1}{2}\rho\, v_1^2 = P_2 + \frac{1}{2}\rho\, v_2^2$

$\therefore$ $\frac{2(P_1 - P_2)}{\rho} = v^2$

$\therefore$ $v = \sqrt{\frac{2(P_1 - P_2)}{\rho}} = \sqrt{\frac{2(3.5-3)\times 10^5}{10^3}} = 10$ m/s

52. According to Bernoulli's theorem,

$h = \frac{v^2}{2g} \Rightarrow h = \frac{(2.45)^2}{2 \times 10} = 0.300 = 30.0$ cm

$\therefore$ Height of jet coming from orifice

$= 30.0 - 10.6 = 19.4$ cm

53. The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second.

Volume of water flowing out per second

$= Av = A\sqrt{2gh}$ ....(i)

Volume of water flowing in per second

$= 70\ cm^3/s$ ....(ii)

From (i) and (ii)

$A\sqrt{2gh} = 70$

$\Rightarrow 1 \times \sqrt{2gh} = 70 \Rightarrow 1 \times \sqrt{2 \times 980 \times h} = 70$

$\therefore$ $h = \frac{4900}{1960} = 2.5$ cm.

56. Kinetic energy = work done

$\frac{1}{2}mv^2 = \mu mgs$

$s = \frac{v^2}{2\mu g} = \frac{(21)^2}{2 \times 0.3 \times 9.8} = 75$ m

57. $a = \frac{v-u}{t} = \frac{0-2}{10} = -0.2\ m\ s^{-2}$

$a = \mu g$

$\therefore$ $\mu = \frac{a}{g} = \frac{-0.2}{9.8}$

$F_S = \mu \times m \times g = -\frac{0.2}{9.8} \times 1 \times 9.8$

$\therefore$ $F_S = -0.2$ N

58. With rise in temperature, viscosity of liquid decreases while viscosity of gases increases.

59. $v^2 = u^2 + 2as$; Here v = 0 and $a_{max} = \mu g$

$\therefore$ $0^2 = v^2 - 2\mu gs$

$\therefore$ $s = \frac{v^2}{2\mu g}$

60. From free body diagram,

For a given figure,

$m_1 a = T - m_1 g \sin\theta$

$\therefore$ $T = m_1 a + m_1 g \sin\theta$ ....(i)

Also, $m_2 a = m_2 g - T$

$\therefore$ $T = m_2 g - m_2 a$ ....(ii)

Equating (i) and (ii),

$m_1a + m_1g \sin\theta = m_2g - m_2a$

$m_2 (g - a) = m_1 (a + g \sin\theta)$

Given $\frac{m_2}{m_1} = \sin\theta$

$\therefore$ $\sin\theta = \frac{a + g\sin\theta}{g - a}$

$\Rightarrow 2a = 0 \Rightarrow a = 0 \Rightarrow$ No motion.

61. $s = \frac{v^2}{2\mu g} = \frac{m^2v^2}{2\mu g m^2} = \frac{P^2}{2\mu m^2 g}$

62. Limiting friction = $\mu_S R = \mu_S mg$

$= 0.5 \times 60 \times 10 = 300$ N

Kinetic friction = $\mu_k R = \mu_k mg$

$= 0.4 \times 60 \times 10 = 240$ N

Force applied on the body = 300 N and if the body is moving then,

Net accelerating force

= Applied force – kinetic friction

$\Rightarrow ma = 300 - 240 = 60$

$\therefore$ $a = \frac{60}{60} = 1$ m/s^2

63. Velocity of ball when it strikes the water surface $v = \sqrt{2gh}$ ....(i)

Terminal velocity of ball inside the water

$v = \frac{2}{9} r^2 g \frac{(\rho - 1)}{\eta}$ ....(ii)

Equating (i) and (ii)

$\sqrt{2gh} = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - 1)$

$\Rightarrow h = \frac{2}{81} r^4 g \left(\frac{\rho - 1}{\eta}\right)^2$

64. A part of pressure energy is dissipated in doing work against friction.

65. Area of each wing = 20 m^2

Speed, v_1 = 216 km h^{-1} = $216 \times \frac{5}{18}$ = 60 m s^{-1}

Speed, v_2 = 180 km h^{-1} = $180 \times \frac{5}{18}$ = 50 m s^{-1}

Let P_1 and P_2 be the pressures of air at the upper and lower wings of plane respectively, then

$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2$

$\therefore$ $P_1 - P_2 = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = \frac{1}{2} \times 1 \times (60^2 - 50^2)$

= 550 Pa

(Air density, $\rho = 1$ kg m^{-3})

$\text{pressure} = \frac{\text{Force}}{\text{area}}$

$\Rightarrow$ Force = pressure $\times$ area

$ma = 550 \times 20$ (m = mass of a wing)

$m = \frac{11000}{10} = 1100$ kg ($\because$ a = g)

Assuming mass of the plane is mostly due to its wings,

Mass of plane = 2m = 1100 $\times$ 2 = 2200 kg.

67. Let 'A' be the area of cross-section of the tank, 'a' be tha area of hole, 'v_e' be the velocity of efflux. 'V' be the speed with which level decreases.

So according to equation of continuity

$av_e = AV$ [i.e., area (a) $\times$ velocity (v) = constant]

$V = \frac{av_e}{A}$

Now applying Bernoulli's theorem,

$\rho_0 + H\rho g + \frac{1}{2}\rho\left[\frac{av_e}{A}\right]^2 = \rho_0 + \frac{1}{2}\rho v_e^2$

$\Rightarrow \rho\left[Hg + \frac{1}{2}\left(\frac{av_e}{A}\right)^2\right] = \frac{1}{2}\rho v_e^2$

$\Rightarrow Hg + \frac{1}{2}\left[\frac{av_e}{A}\right]^2 = \frac{1}{2}v_e^2$

$2Hg = v_e^2\left[1 - \left(\frac{a}{A}\right)^2\right]$

$v_e^2 = \frac{2Hg}{1 - \left(\frac{a}{A}\right)^2} = \frac{2 \times (4 - 0.6) \times 10}{1 - (0.2)^2} = 71$ m^2/s^2.

$\therefore$ $v_e = \sqrt{71} = 8.4$ m/s

68. Acceleration down a rough inclined plane

$a = g (\sin\theta - \mu\cos\theta)$ and this is less than g.

Competitive Thinking

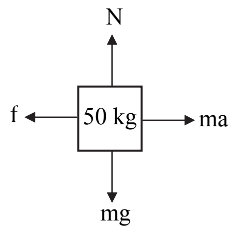
1. Sand is used to increase the friction.
2. There is increase in normal reaction when the object is pushed and there is decrease in normal reaction when object is pulled.
3. $F = \mu R = 0.3 \times 250 = 75$ N
4. $\mu_S = \frac{m_B}{m_A}$

$\Rightarrow 0.2 = \frac{m_B}{2}$

$\Rightarrow m_B = 0.4$ kg

5. Since the lift is moving downwards with acceleration equal to g, the effective weight and hence the normal reaction of the body is zero. Therefore, the force of friction is also zero.

6. Box is stationary on floor of train i.e., it is moving with acceleration same as that of train.



$\therefore$ f = ma

$\therefore$ μN = ma

$\therefore$ μmg = ma

a = μg = 0.3 × 10

a = 3 ms^{-2}

7. As the block is momentarily pushed,

$$\mu = \frac{a}{g} \Rightarrow a = \mu g$$

If the block comes to rest in time t, then

$$a = \frac{v_{Final} - v_{initial}}{t} = \frac{0 - v}{t} = \frac{-v}{t}$$

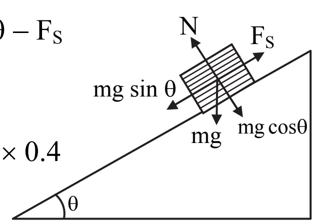
$$t = \frac{v}{a} \text{ (neglecting negative sign)}$$

$$= \frac{v}{\mu g} \quad (\because a = \mu g)$$

8. F = mg sin θ

$$m = \frac{F}{g\sin\theta} = \frac{10}{10 \times \frac{1}{2}} = 2 \text{ kg}$$

9. Net force = mg sinθ – F_S



$\therefore$ ma = mg sinθ – F_S

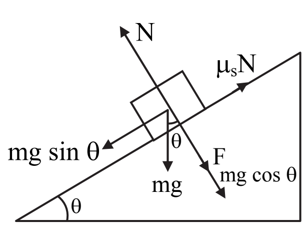
$\therefore$ F_S = mg sin θ – ma

= 80 sin 30° – 8 × 0.4

= 40 – 3.2

= 36.8 N

10.



mg sin θ = μ_sN

But N = F + mg cos θ

mg sin θ = μ_s(F + mg cos θ)

$$F = \frac{mg\sin\theta}{\mu_s} - mg\cos\theta$$

$$= mg\left[\frac{\sin\theta}{\mu_s} - \cos\theta\right]$$

$$= 10\left[\frac{1}{2\times 0.2} - \frac{\sqrt{3}}{2}\right] \quad ....(\because \theta = 30°)$$

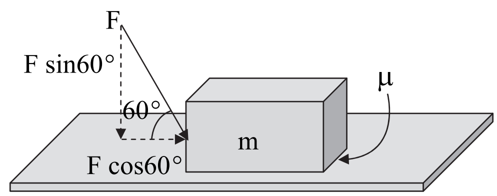
$$= 5\left[5 - \sqrt{3}\right]$$

= 5 [5 – 1.732]

= 16.34

11. As the block does not move, maximum force equals force of friction.

$\therefore$ F = μR



Resolving applied force into its components,

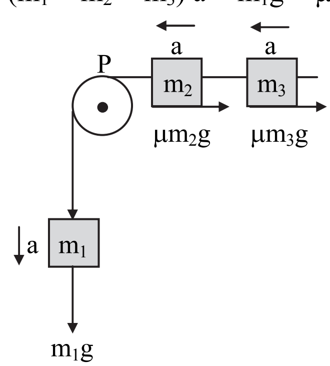
F cos 60° = μ(mg + F sin 60°)

$$\frac{F}{2} = \frac{1}{2\sqrt{3}}\left(\sqrt{3}\times 10 + F \times \frac{\sqrt{3}}{2}\right)$$

$$\frac{F}{2} = 5 + \frac{F}{4}$$

$\therefore$ F = 20 N.

12. Let downward acceleration of mass m_1 be a, then $(m_1 + m_2 + m_3)$ a = m_1g – $\mu(m_2 + m_3)$ g



$$a = \frac{m_1 g - \mu(m_2 + m_3)g}{m_1 + m_2 + m_3} = \frac{m[g - 2\mu g]}{3m}$$

$$= \frac{g}{3}[1 - 2\mu]$$

14. Coefficient of sliding friction has no dimensions.

15. When there is no friction, minimum force on body = R

In presence of frictional force,

Maximum force on body = $\sqrt{f^2 + R^2}$

$= \sqrt{(\mu R)^2 + R^2}$

$= R\sqrt{\mu^2 + 1}$

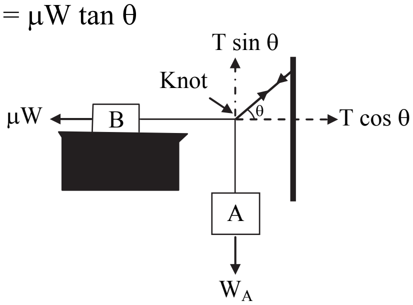
Thus force ranges such that,

$R \le F \le R\sqrt{\mu^2 + 1}$

i.e., $Mg \le F \le Mg\sqrt{\mu^2 + 1}$

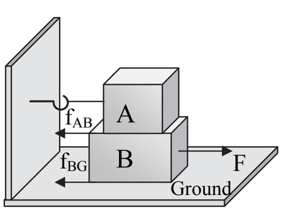
16. From law of conservation of energy,
$mgh = \mu \times mgd$
$d = \dfrac{h}{\mu}$

17. $T \sin \theta = W_A$
$T \cos \theta = \mu W$
Dividing two equations,
∴ $W_A = \mu W \tan \theta$



18. Let $\dfrac{M}{L}$ be the mass per unit length of chain.
Let the length of chain that hangs is L′, so the length of chain that rests on table is L – L′.
Thus, mass of the chain that hangs and that rests are $\dfrac{M}{L}$ L′ and $\dfrac{M}{L}$ (L – L′) respectively.
Let frictional force due to chain on table balance the gravitational force on hanging chain
∴ $f = m'g$
∴ $\mu mg = m'g$
∴ $\mu \dfrac{M}{L}(L - L') = \dfrac{M}{L}(L')$
∴ $0.25 (L - L') = L'$
∴ $\dfrac{L'}{L} = \dfrac{0.25}{1.25} = 0.2 \equiv 20\ \%$

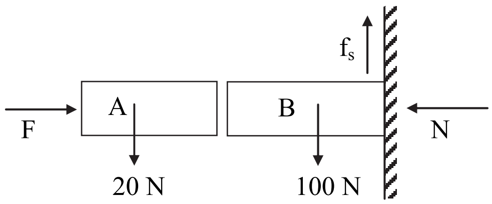
19.



$F = f_{AB} + f_{BG}$
$= \mu_{AB} m_a g + \mu_{BG}(m_A + m_B)g$
$= 0.2 \times 100 \times 10 + 0.3(300) \times 10$
$= 200 + 900 = 1100$ N

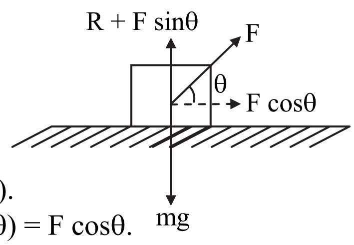
20. Applied force = 2.5 N
Limiting friction = $\mu mg = 0.4 \times 2 \times 9.8$
$= 7.84$ N
For the given condition applied force is very smaller than limiting friction.
∴ Static friction on a body = Applied force
= 2.5 N

21.



Here force of friction exerted by the wall is along vertical direction. Hence if the system is in vertical equilibrium then,
$f_s = W_A + W_B = 20 + 100 = 120$ N

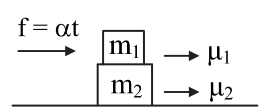
22. From the figure, vertical component of F is F sinθ and the horizontal component is F cosθ.
Thus,
$R + F \sin\theta = mg$
∴ $R = mg - F \sin\theta$



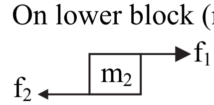
Frictional force,
$\mu R = \mu(mg - F \sin\theta)$.
Also, $\mu(mg - F \sin \theta) = F \cos\theta$.
∴ $F = \dfrac{\mu\, mg}{(\mu \sin\theta + \cos\theta)}$
F will be minimum if the denominator is maximum, i.e., if
$\dfrac{d}{d\theta}(\mu \sin\theta + \cos\theta) = 0$
$\Rightarrow\ \mu \cos\theta - \sin\theta = 0 \Rightarrow \mu = \tan\theta$.

23. $v = u - at \Rightarrow u - \mu gt = 0$
∴ $\mu = \dfrac{u}{gt} = \dfrac{6}{10 \times 10} = 0.06$

24.



On lower block (m_2)



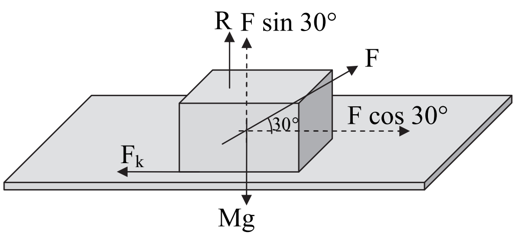
f_2 = force of friction between lower block & the table.
f_1 = force of friction between lower block & upper block.
So $f_1 \le f_2$ ....($\because$ m_2 never moves)

$\therefore$ $\mu_1 m_1 g \le \mu_2(m_1 + m_2)g$

$\frac{\mu_1}{\mu_2} \le \frac{m_1 + m_2}{m_1}$

$\frac{\mu_1}{\mu_2} \le 1 + \frac{m_2}{m_1}$

So maximum value of $\frac{\mu_1}{\mu_2} = 1 + \frac{m_2}{m_1}$

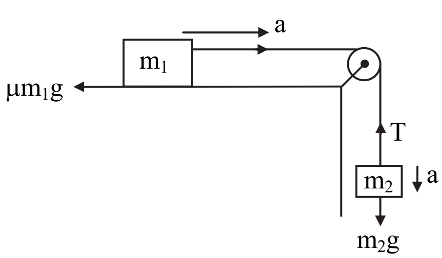
25. 

Kinetic friction $= \mu_k R = 0.2(Mg - F \sin 30°)$
$= 0.2(5 \times 10 - 40 \times \frac{1}{2})$
$= 0.2(50 - 20) = 6$ N

acceleration of the block
$= \frac{F\cos 30° - \text{Kinetic friction}}{\text{Mass}}$

$a = \frac{40 \times \frac{\sqrt{3}}{2} - 6}{5} = 5.73$ m/s^2

26. For a block of 3 kg,
$F = mg - T$
$= 3 \times 10 - 27$
$F = 3$ N
$\therefore$ $ma = 3$
$a = 1$ m/s^2
For a block of 20 kg,
$ma = 27 - \mu_k mg$
$20 \times 1 = 27 - \mu_k \times 20 \times 10$
$\therefore$ $\mu_k = \frac{7}{200} = 0.035$

27. 

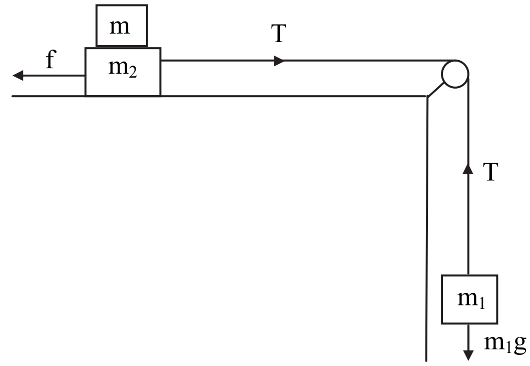
Net force acting, on the first body
$T - \mu m_1 g = m_1 a$ ....(i)
Net force acting on the second body
$m_2 g - T = m_2 a$ ....(ii)

$\Rightarrow a = \frac{m_2 g - T}{m_2}$

Substituting in equation (i),

$T - \mu m_1 g = \frac{m_1(m_2 g - T)}{m_2}$

$\therefore$ $T = \frac{m_1 m_2 g(\mu_k + 1)}{(m_1 + m_2)}$

28. 

For mass m_1, $T = m_1 g$
For mass m_2, at equilibrium,
$f = T = m_1 g$
$f_{max} = \mu\ (m_2 + m)g$
$\therefore$ $\mu(10 + m)\ g = 5g$
$\therefore$ $10 + m = \frac{5}{0.15}$
$\therefore$ $10 + m = \frac{100}{3}$
$\therefore$ $m = \frac{70}{3}$ kg = 23.3 kg.
The minimum weight in the given options is 27.3 kg.

29. When air is blown through a hole on a closed pipe containing liquid, then the pressure will increase in all directions.

30. $P = \rho gh$
Hence, pressure is independent of area of liquid surface.

32. Pressure difference between lungs and atmosphere = 760 mm – 750 mm
= 10 mm = 1 cm of Hg
Also, Pressure difference $= 1 \times 13.6 \times g$
i.e., one can draw from a depth of 13.6 cm of water.

33. Pressure at bottom of the lake = $P_0 + h\rho g$

Pressure at half the depth of a lake = $P_0 + \frac{h}{2}\rho g$

According to given condition,

$$P_0 + \frac{1}{2}h\rho g = \frac{2}{3}(P_0 + h\rho g)$$

$$\frac{1}{3}P_0 = \frac{1}{6}h\rho g$$

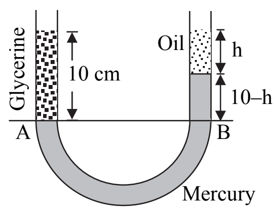
$$h = \frac{2P_0}{\rho g} = \frac{2\times10^5}{10^3\times10} = 20 \text{ m}$$

34. $$\frac{P}{P_a} = \frac{h\rho g + P_a}{P_a} = \frac{(10\times10^3\times10)+1\times10^5}{1\times10^5} = 2$$

35. The system is in equilibrium and pressure on both sides is equal.
This means,
$h_w\rho_w g = h_o\rho_o g$

$\therefore$ $$\rho_o = \frac{h_w\rho_w}{h_o} = \frac{130\times10^{-3}\times10^3}{140\times10^{-3}} = 928.6 \text{ kg/m}^3$$

36.



At the condition of equilibrium
Pressure at point A = Pressure at point B
$P_A = P_B$

$\therefore$ $10 \times 1.3 \times g = h \times 0.8 \times g + (10 - h) \times 13.6 \times g$

$\Rightarrow h = 9.6$ cm

37. $$\frac{P_1 - P_2}{\rho g} = \frac{v^2}{2g} \Rightarrow \frac{4.5\times10^5 - 4\times10^5}{10^3\times g} = \frac{v^2}{2g}$$

$\therefore$ v = 10 m/s

39. From kinetic theory point of view viscosity represents transport of momentum.

43. In steady flow of incompressible liquid rate of flow remains constant i.e., V = av = constant. This is equation of continuity.
When pipe is placed vertically upward velocity of flow decreases with height so area of cross section increases and when pipe is placed vertically downward, velocity of flow increases in downward direction so area of cross section decreases i.e., it becomes narrower.

44. A streamlined body offers less resistance to air.

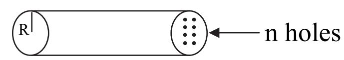
45. If velocities of water at entry and exit points are v_1 and v_2, then according to equation of continuity,

$$A_1v_1 \Rightarrow A_2v_2 \Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

46. $A_1v_1 = A_2v_2$

$\therefore$ $$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{10}{5}\right)^2 = 4 : 1$$

47.



Using equation of continuity,
av = constant

$$\pi R^2V = n\pi r^2v \Rightarrow v = \frac{VR^2}{nr^2}$$

48. $$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2\times10\times0.2} = 2 \text{ m/s}$$

$A_1v_1 = A_2v_2$

$$\pi\left(\frac{8\times10^{-3}}{2}\right)^2 \times 0.4 = \pi \times \frac{d^2}{4} \times 2$$

$\Rightarrow d \approx 3.6 \times 10^{-3}$ m

50. $$F = \eta A\frac{dv}{dx}$$

$\therefore$ $$\text{shearing stress} = \frac{F}{A} = \eta\frac{dv}{dx}$$

$\therefore$ $$\text{shearing stress} = 10^{-2} \times \frac{9\times\frac{5}{18}}{10}$$

$$= 0.25 \times 10^{-3} \text{ N/m}^2$$

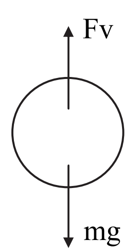
51. $$F = \eta A\frac{dv}{dx} = 0.9 \times 500 \times 10^{-4} \times \frac{2\times10^{-2}}{0.5\times10^{-3}}$$

$$= 1.8 \text{ N}$$

52. Since $F = 6\pi\eta r v$

$\therefore$ $F \propto r v$

53.

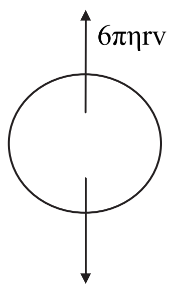


$F_v = mg$

$\therefore$ $$6\pi r\eta v = \frac{4}{3}\pi r^3\rho g$$

$$\eta = \frac{4r^2\rho g}{3\times6\times v} = \frac{4\times1^2\times1.75\times980}{3\times6\times0.35} = 1089 \text{ poise}$$

54.



$6\pi\eta rv = \frac{4}{3}\pi r^3\rho g$

$\therefore \quad v = \frac{2}{9\eta} r^2\rho g$

$\therefore \quad v = \frac{2\times 0.9^2\times 10^{-6}\times 10^3\times 9.8}{9\times 1.8\times 10^{-5}} = 98\ ms^{-1}$

55. $F \propto r^3 \propto V$
As volume becomes doubled, F changes to 2F.

57. For a given material, terminal velocity is independent of mass of the body but depends on density of the material.

58. In the first 100 m, body starts from rest and its velocity goes on increasing and after 100 m it acquires maximum velocity (terminal velocity). Further, air friction i.e., viscous force which is proportional to velocity is low in the beginning and maximum at $v = v_T$.
Hence, work done against air friction in the first 100 m is less than the work done in next 100 m.

59. Using $v = \frac{2r^2}{9\eta}(\rho - \sigma)$, $v \propto (\rho - \sigma)$

$\frac{v_{gold}}{v_{silver}} = \frac{19.5 - 1.5}{10.5 - 1.5} = \frac{18}{9} = 2$

$\therefore \quad v_{silver} = \frac{v_{gold}}{2} = \frac{0.2}{2} = 0.1\ m\ s^{-1}$

60. Mass = Volume × Density $\Rightarrow M = \frac{4}{3}\pi r^3 \times \rho$

As the density remains constant

$\therefore \quad M \propto r^3$

$\therefore \quad \frac{r_1}{r_2} = \left(\frac{M_1}{M_2}\right)^{1/3} = \left(\frac{M}{8M}\right)^{1/3} = \frac{1}{2}$ ….(i)

Terminal velocity, $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

$\therefore \quad v \propto r^2$

$\therefore \quad \frac{vT_1}{vT_2} = \left(\frac{r_1}{r_2}\right)^2$

$\frac{v}{nv} = \left(\frac{r_1}{r_2}\right)^2$ or $\frac{1}{n} = \left(\frac{1}{2}\right)^2$ [Using (i)]

$\Rightarrow n = 4$

61. $v \propto r^2\rho$ (neglecting density of liquid)
where ρ = density of material of sphere.

Now, $\frac{4}{3}\pi r_1^3\rho_1 = \frac{4}{3}\pi r_2^3\rho_2 \Rightarrow \frac{\rho_1}{\rho_2} = \frac{r_2^3}{r_1^3}$

$\therefore \quad \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2}\times\frac{\rho_1}{\rho_2} = \frac{r_1^2}{r_2^2}\times\frac{r_2^3}{r_1^3} = \frac{r_2}{r_1}$

62. Terminal speed $v \propto r^2$

$\therefore \quad \frac{v_1}{v_2} = \frac{r^2}{R^2} = \frac{r^2}{(2^{1/3}r)^2}$

$\therefore \quad v_2 = v_1\frac{(2^{1/3}r)^2}{r^2} = 5\times 2^{2/3} = 5\times 4^{1/3}\ cm\ s^{-1}$

63. The onset of turbulence in a liquid is determined by a dimensionless parameter called as Reynold's number.

65. $v_c = \frac{N\eta}{\rho D} = \frac{3000\times 10^{-3}}{10^3\times 0.02} = 0.15\ m/s$

66. $v_c = \frac{N\eta}{\rho D} = \frac{2\times 10^3\times 6\times 10^{-3}\times 10^{-1}}{720\times 5\times 10^{-3}}$

$= 0.33\ m/s$

Flow becomes turbulent, if the velocity is above 0.33 m/s.

67. Reynold's number $N_R = \frac{v\rho D}{\eta}$ ….(i)

where v is the speed of flow.
Rate of flow of water Q = Area of cross section × speed of flow

$Q = \frac{\pi D^2}{4}\times v$ or $v = \frac{4Q}{\pi D^2}$

Substituting the value of v in equation (i),

$N_R = \frac{4Q\rho D}{\pi D^2\eta} = \frac{4Q\rho}{\pi D\eta}$

Substituting the values,

$N_R = \frac{4\times 5\times 10^{-5}\times 10^3}{\left(\frac{22}{7}\right)\times 1.25\times 10^2\times 10^3} = 5100$

For $N_R > 3000$, the flow is turbulent.
Hence, the flow of water is turbulent with Reynold's number 5100.

69. $P + \frac{1}{2}\rho v^2 = P' + \frac{1}{2}\rho \times 4v^2$

$P' = P + \frac{\rho}{2} v^2 (1-4)$

$P' = P - \frac{3}{2}\rho v^2$

70. Using Bernoulli's theorem,

$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$

$\frac{4.5\times10^5}{\rho g} + 0 = \frac{4\times10^5}{\rho g} + \frac{1v^2}{2g}$

$v_2^2 = \frac{10^5}{\rho} = \frac{10^5}{10^3}$

$v_2 = 10$ m/s

71. According to Bernoulli's principle,

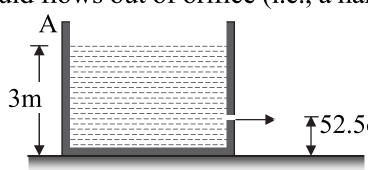
$P = \frac{F}{A} = \frac{1}{2}\rho v^2$

$\Rightarrow F = \frac{1}{2}\rho v^2 A = \frac{1}{2} \times 1.2 \times (40)^2 \times 250$

$= 2.4 \times 10^5$ N

Also, net force acting on the roof is upward.

72. Let A = cross-section of tank
a = cross-section hole
V = velocity with which level decreases
v = velocity of efflux i.e., velocity with which the liquid flows out of orifice (i.e., a narrow hole)



From equation of continuity av = AV

$\Rightarrow V = \frac{av}{A}$

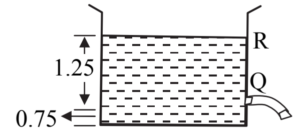
By using Bernoulli's theorem for energy per unit volume

Energy per unit volume at point A = Energy per unit volume at point B

$P + \rho gh + \frac{1}{2}\rho V^2 = P + 0 + \frac{1}{2}\rho v^2$

$\Rightarrow v^2 = \frac{2gh}{1-\left(\frac{a}{A}\right)^2} = \frac{2\times10\times(3-0.525)}{1-(0.1)^2} = 50 \text{ m}^2/\text{s}^2$

73. Using equation of continuity,
av = AV ….(i)
where, V is velocity with which liquid level decreases and v is velocity of efflux.



According to Bernoulli's theorem,

Energy per unit volume at point R = Energy per unit volume at point Q

$P + \rho gh + \frac{1}{2}\rho V^2 = P + 0 + \frac{1}{2}\rho v^2$

But, $V = \frac{av}{A}$ ….[from equaion (i)]

$\therefore \quad \rho gh + \frac{1}{2}\rho\left(\frac{av}{A}\right)^2 = \frac{1}{2}\rho v^2$

$\therefore \quad v^2 = \frac{2gh}{1-(a/A)^2} = \frac{2\times9.8\times1.25}{1-\left(\sqrt{0.2}\right)^2} = 30.625$

$\therefore \quad v = 5.53$ m/s ≈ 5.5 m/s

74. Horizontal range will be maximum when

$h = \frac{H}{2} = \frac{90}{2}$ …(***Using Shortcut 4***)

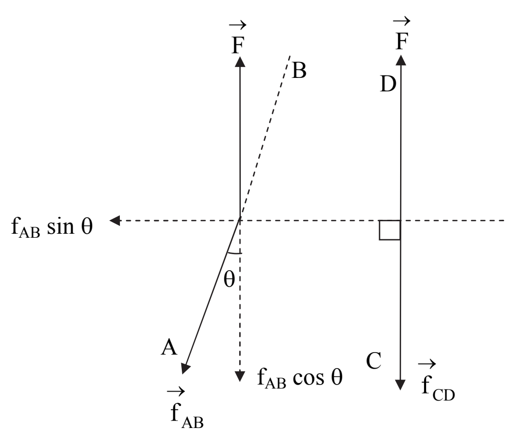
= 45 cm i.e., hole 3.

75. For maximum range, height of the hole

$= \frac{\text{Total height}}{2} = \frac{h+\frac{h}{2}}{2} = \frac{3h}{4}$

From PQ level, hole number 2 is at height of $\frac{3h}{4}$.

76.



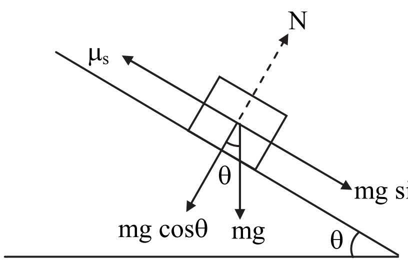
As the roller is given force $\vec{F}$, it acts perpendicular to axis of roller. In case of rail CD, the frictional force developed, $\vec{f}_{CD}$ exactly balances $\vec{F}$. But, in case of rail AB, $\vec{f}_{AB}$ is not balanced by $\vec{F}$.

Cosine component of $\vec{f}_{AB}$ i.e.,
$f_{AB}\cos\theta$ balances $\vec{F}$, whereas sine component of $\vec{f}_{AB}$ remains unbalanced. As $f_{AB}\sin\theta$ is directed towards left, roller will turn left.

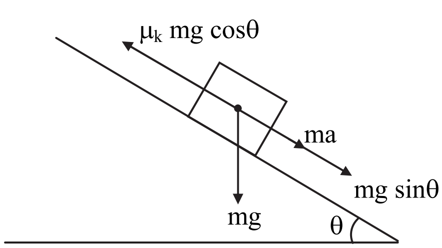
77. For the body just sliding with constant acceleration a,
$F_S = F_k + ma$
$\therefore$ $ma = \mu_S R - \mu_k R$
$= mg\ (0.75 - 0.5)$ $(\because R = mg)$
$\therefore$ $a = g\ (0.25) = \dfrac{g}{4}$

78. For force of friction,
$F = \mu N = \mu Mg$
Given: $F = F_0 t$
At time t = T,
$T = \dfrac{F}{F_0} = \dfrac{\mu Mg}{F_0}$

79.



Referring to diagram,
$mg\sin\theta = \mu_s N$ and
$N = mg\cos\theta$
$\therefore$ $\mu_s.mg\cos\theta = mg\sin\theta$
i.e. $\mu_s = \tan\theta = \tan 30° = \dfrac{1}{\sqrt{3}}$
$\therefore$ $\mu_s = 0.6$
Now, while sliding, let acceleration of box be a,



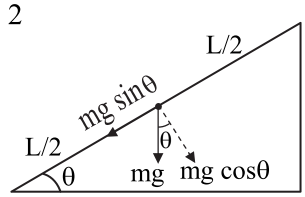
$mg\sin\theta - \mu_k\ mg\cos\theta = ma$
$\therefore$ $a = g\sin 30° - \mu_k g\cos 30°$
$a = \dfrac{g}{2} - \dfrac{\mu_k g\sqrt{3}}{2}$

Using kinematical equation of motion,
$s = ut + \dfrac{1}{2}at^2$
$4 = (0)(4) + \dfrac{1}{2}\left[\dfrac{g}{2} - \dfrac{\mu_k g\sqrt{3}}{2}\right](4)^2$
$16 = \left(g - \mu_k g\sqrt{3}\right)16$
$1 = g\left(1 - \mu_k\sqrt{3}\right)$
$\therefore$ $\mu_k = \dfrac{1 - 0.1}{\sqrt{3}}$
$\therefore$ $\mu_k = 0.52$

80. Pressure at the bottom of tank
$P = h\rho g = 3 \times 10^5$
Pressure due to liquid column
$P_1 = 3 \times 10^5 - 1 \times 10^5 = 2 \times 10^5$
and velocity of water $v = \sqrt{2gh}$
$\therefore$ $v = \sqrt{\dfrac{2P_1}{\rho}} = \sqrt{\dfrac{2 \times 2 \times 10^5}{10^3}} = \sqrt{400}$ m/s

82. $[v_c] = [\eta^x \rho^y r^z]$
$[M^0 L^1 T^{-1}] = [M^1 L^{-1} T^{-1}]^x [M^1 L^{-3}]^y [L^1]^z$
$[M^0 L^1 T^{-1}] = [M^{x+y}] [L^{-x-3y+z}][T^{-x}]$
Comparing both sides,
$x + y = 0, -x - 3y + z = 1, -x = -1$
$\Rightarrow x = 1, y = -1, z = -1$

83. $v^2 = u^2 + 2as$ ….(i)
Now, initial velocity at midpoint
$u = \sqrt{2g\dfrac{L}{2}\sin\theta}$
and final velocity for the lower half = v = 0
At lower half acceleration = $g\sin\theta - \mu g\cos\theta$
and $s = \dfrac{L}{2}$



$\therefore$ From equation (i),
$0^2 - 2g\dfrac{L}{2}\sin\theta = 2\ [g\sin\theta - \mu g\cos\theta] \times \dfrac{L}{2}$
$\therefore$ $-2g\dfrac{L}{2}\sin\theta = gL\sin\theta - \mu gL\cos\theta$
$\therefore$ $2gL\sin\theta = \mu gL\cos\theta$
$\therefore$ $\mu = 2\tan\theta$

84. mass of bullet = m = 0.05 kg
also, velocity = v = 210 m/s
Mass of block = M = 1 kg
After collision, the block-bullet system will move with velocity v_{sys}
According to conservation of linear momentum,
$mv = (m + M)\, v_{sys}$
∴ $0.05 \times 210 = 1.05 \times v_{sys}$
∴ $v_{sys} = 10$ m/s
Also, The K.E. acquired by system is converted to work done in moving the system i.e. according to work-energy theorem,
$\frac{1}{2}(m + M)(v_{sys})^2 = F \times s$
∴ $\frac{1}{2}(m + M)(10)^2 = \mu\,(m + M)g \times s$
∴ $s = \frac{100}{2 \times 0.5 \times 10} = 10$ m

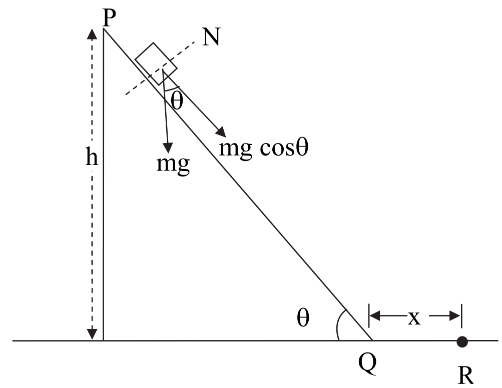
85. Weight of the body = 64 N
so mass of the body, m = 6.4 kg,
Net acceleration
$= \frac{\text{Applied force} - \text{Kinetic friction}}{\text{Mass of the body}}$
$= \frac{\mu_S mg - \mu_k mg}{m} = (\mu_S - \mu_k)g = (0.6 - 0.4)g$
= 0.2 g

86. Three vessels have same base area and equal volumes of liquid are added in them. Considering the geometry of vessles, liquid in vessle 'C' will rise to maximum height amongst the three.
Force on base, F ∝ Pressure exerted on base, P ∝ height of liquid (h)
Hence, the force on the base will be maximum at vessel C.

87. Net downward acceleration
$= \frac{\text{Weight} - \text{Friction force}}{\text{Mass}} = \frac{(mg - \mu R)}{m}$
$= \frac{60 \times 10 - 0.5 \times 600}{60} = \frac{300}{60} = 5\ \text{m/s}^2$

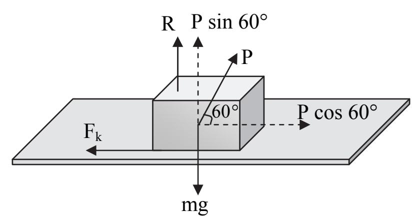
88. Time taken by water to reach the bottom,
$t = \sqrt{\frac{2(H - D)}{g}}$ and
velocity of water coming out of hole,
$v = \sqrt{2gD}$
∴ Horizontal distance covered,
$x = v \times t = \sqrt{2gD} \times \sqrt{\frac{2(H - D)}{g}} = 2\sqrt{D(H - D)}$

89. Work done by friction along PQ = work done by friction along QR



$W_{PQ} = F \times PQ$
$= \mu mg\cos\theta \times \frac{h}{\sin\theta} = \mu mg\left(\frac{\sqrt{3}}{2} \times 4\right)$
∴ $W_{PQ} = 2\sqrt{3}\ \mu mg$
Hence, $W_{QR} = F \times QR = \mu mg\, x$
∴ $\mu mg\ 2\sqrt{3} = \mu mg\, x$
∴ x = 3.5 m
According to work energy theorem,
$mgh = W_{PQ} + W_{QR}$
But, $W_{PQ} = W_{QR}$
∴ $mg(2) = 2 \times 2\sqrt{3}\ \mu mg$
∴ $\mu = \frac{1}{2\sqrt{3}}$ ∴ μ = 0.29

90.



Let body be dragged with force P, making an angle 60° with the horizontal.
F_k = Kinetic friction in the motion = $\mu_k R$
From the figure $F_k = P\cos 60°$ and
$R = mg - P\sin 60°$
∴ $P\cos 60° = \mu_k(mg - P\sin 60°)$
$\Rightarrow \frac{P}{2} = 0.5\left(60 \times 10 - \frac{P\sqrt{3}}{2}\right)$
$\Rightarrow$ P = 315.1 N
∴ $F_k = P\cos 60° = \frac{315.1}{2}$ N
Work done $= F_k \times s = \frac{315.1}{2} \times 2 = 315$ joule

92. From Bernoulli's theorem,

$$P_1 + \frac{\rho v_1^2}{2} + \rho g h_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g h_2$$

Here $h_1 \approx h_2$

$\therefore$ $P_1 - P_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) = 0.6\ (70^2 - 60^2) = 780$ Pa

This pressure difference causes uplift of plane

$\therefore$ Net force = upward force – downward force

$= (P_1 - P_2) \times$ area – weight

$= 780 \times 14 - 1000 \times 10$ ($\because$ weight = mg)

= 920 N (in upward direction)

93. When the liquid escapes through the orifice it has zero initial velocity in vertical direction.

Using $s = ut + \frac{1}{2}at^2$ in vertical direction,

$$h = 0 + \frac{1}{2}gt^2$$

time taken to be emptied for h height,

$$t = \sqrt{\frac{2h}{g}}$$

and for $\frac{h}{2}$ height, $t' = \sqrt{\frac{2h/2}{g}} = \sqrt{\frac{h}{g}}$

$\therefore$ $\frac{t'}{t} = \frac{1}{\sqrt{2}}$

$\Rightarrow t' = \frac{t}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7$ minute

94. Velocity of efflux for A: $v_1 = \sqrt{2gh}$

Velocity of efflux for B: $v_2 = \sqrt{2g \times 3h} = \sqrt{6gh}$

Water flowing out from A = Water flowing out from B. ....(Given)

$\therefore$ $v_1A_1 = v_2A_2$

Since, Area of square $(A_1) = L^2$

Area of circle $(A_2) = \pi r^2$

$\therefore$ $\sqrt{2gh} \times L^2 = \sqrt{6gh} \times \pi r^2$

$\therefore$ $L^2 = \frac{\sqrt{6gh}}{\sqrt{2gh}} \times \pi r^2 = \sqrt{3}\pi r^2$

$L = 3^{\frac{1}{4}}\pi^{\frac{1}{2}}r = r(\pi)^{\frac{1}{2}}(3)^{\frac{1}{4}}$

95. Velocity of efflux when the hole is at depth h,

$v = \sqrt{2gh}$

Rate of flow of water from square hole

$Q_1 = a_1v_1 = L^2\sqrt{2gy}$

Rate of flow of water from circular hole

$Q_2 = a_2v_2 = \pi R^2\sqrt{2g(4y)}$

According to problem $Q_1 = Q_2$

$\Rightarrow L^2\sqrt{2gy} = \pi R^2\sqrt{2g(4y)} \Rightarrow R = \frac{L}{\sqrt{2\pi}}$

96. Work done = Force × Displacement

$= \mu\, mg \times (v \times t)$

$W = (0.2) \times 2 \times 9.8 \times 2 \times 5$ joule

Heat generated $Q = \frac{W}{J} = \frac{0.2 \times 2 \times 9.8 \times 2 \times 5}{4.2}$

= 9.33 cal

97. Weight of the ball

= Buoyant force + Viscous force

$V\rho_1 g = V\rho_2 g + kv^2$

$\Rightarrow kv^2 = V(\rho_1 - \rho_2)g$

$\Rightarrow v = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

98. Power of heart = F × velocity

$= \frac{F}{\text{Area}} \times \text{Area} \times \text{velocity}$

$= \text{Pressure} \times \left(\frac{\text{area} \times \text{displacement}}{\text{time}}\right)$

$= \text{Pressure} \times \frac{\text{volume}}{\text{time}}$

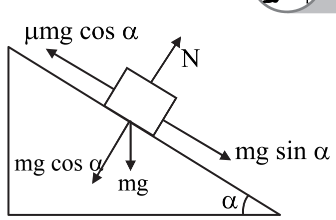
$= P.\frac{dV}{dt} = h\rho g \times \frac{dV}{dt}$

$= (0.15) \times (13.6 \times 10^3)\ (10) \times \frac{5 \times 10^{-3}}{60}$

$= \frac{13.6 \times 5 \times 0.15}{6}$

= 1.70 watt

Evaluation Test

1. 

Resultant acceleration downwards

$a = g\sin\alpha - \mu g\cos\alpha$

$= 10 \times \sin 45° - 0.2 \times 10 \times \cos 45°$

$= 5.66\ m/s^2$

$v = u + at$

Here, u = 0 and v = 50 km/hr = 13.88 m/s

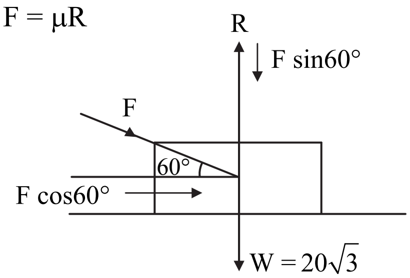
$\therefore$ $t = \dfrac{13.88}{5.66} = 2.45$ s

2. Fluids move from higher pressure to lower pressure. In a fluid, pressure increases with depth, so pressure at the top P_a (the atmospheric pressure) is lesser than at the bottom [$P_a + d\rho g$]. Hence the air bubble will move from bottom to top. (It cannot move side ways as the pressure at the same level in a fluid is same). In coming from bottom to top, pressure decreases, so in accordance with Boyle's law i.e., PV = constant, volume V will increase. Thus, the air bubble will grow in size and its radius will increase.

3. $P = h\rho g$

h and ρ being constant pressure in all four containers is same.

4. $F = \mu R$



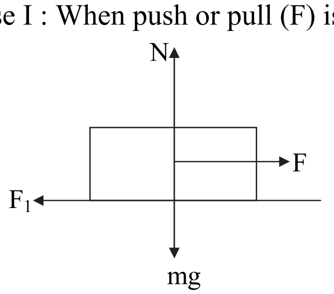
Thus, $F\cos 60° = \mu(W + F\sin 60°)$

$\therefore$ $\dfrac{F}{2} = \dfrac{1}{2\sqrt{3}}\left(20\sqrt{3} + \dfrac{\sqrt{3}F}{2}\right)$

$\therefore$ $\dfrac{F}{2} = 10 + \dfrac{F}{4}$

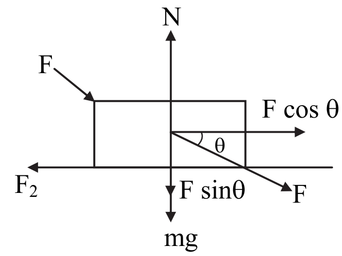
$\Rightarrow F = 40$ N

5. Case I : When push or pull (F) is horizontal.



$F_1 = \mu_k mg$ ….(i)

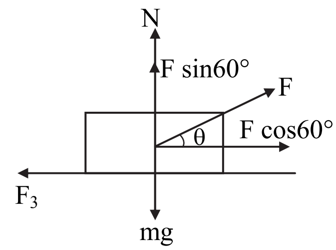
Case II : When push on the block is downward at angle 0 < θ < 90° with horizontal.



$N = mg + F\sin\theta$

$\therefore$ $F_2 = \mu k(mg + F\sin\theta)$ ….(ii)

Case III : When pull F on the block is upward at angle θ (0 < θ < 90°) with horizontal



$N = (mg - F\sin\theta)$

$\therefore$ $F_3 = \mu k(mg - F\sin\theta)$ ….(iii)

Then, $F_3 < F_1 < F_2$. Hence, option (B) is correct.

6. As the block moves with uniform velocity, the resultant force is zero. Resolving F into horizontal component Fcosθ and vertical component F sinθ,

$R + F\sin\theta = mg \Rightarrow R = mg - F\sin\theta$

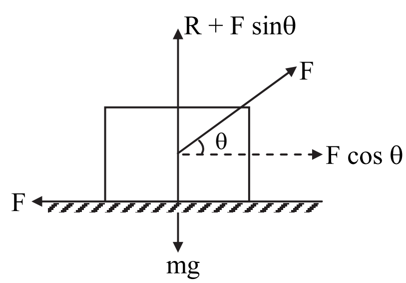
Also, $F = \mu R = \mu(mg - F\sin\theta)$

But $F\cos\theta = F$

$\therefore$ $F\cos\theta = \mu(mg - F\sin\theta)$

$F(\cos\theta + \mu\sin\theta) = \mu mg$

$\therefore$ $F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$



Work $W = Fs\cos\theta$

$\therefore$ $W = \dfrac{\mu mgs\cos\theta}{\cos\theta + \mu\sin\theta}$

7. Initial kinetic energy of the car = $\dfrac{1}{2}mv^2$

Work done against friction = μmgs

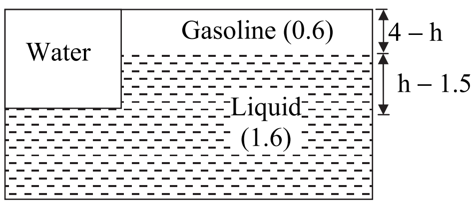
From conservation of energy

$\mu mgs = \frac{1}{2}mv^2$

Stopping distance, $s = (v^2/2\mu g)$

$v = 32 \text{ km/h} = 72 \times \frac{5}{18} = 10 \text{ m/s}$

$\therefore \quad s = \frac{10 \times 10}{2 \times 0.4 \times 10} = 12.5 \text{ m}$

8. 

$P_{\text{left side}} = P_{\text{right side}}$

$\rho_w \times g \times 2.5 = \rho_{gas} \times g \times (4 - h) + \rho_{liq}g(h - 1.5)$

$1000 \times g \times 2.5 = 600g\,(4 - h) + 1600g(h - 1.5)$

$2500 = 2400 - 600\,h + 1600\,h - 2400$

$\therefore \quad h = \frac{2500}{1000} = 2.5 \text{ m}$

9. Mass of liquid in AB = $yA\sigma$

Net force = mass × acceleration

$= (yA\sigma) \times x$ ….(i)

Also, pressure at A = $h_2\sigma g$,

pressure at B = $h_1\sigma g$

Net force = Net pressure × area

$= (h_2\sigma g - h_1\sigma g) \times A$ ….(ii)

Equating (ii) and (i)

$\therefore \quad (h_2 - h_1)\,\sigma gA = (yA\sigma)\,x$

$\therefore \quad h_2 - h_1 = \frac{xy}{g}$

10. $v_1 = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$ ….(i)

From Bernoulli's theorem,

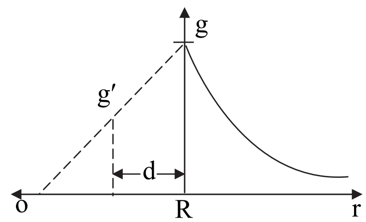
$2\rho gh + 4\rho g\left(\frac{h}{2}\right) = \frac{1}{2}(4\rho)v_2^2$

$\therefore \quad v_2 = \sqrt{2gh}$ ….(ii)

$\therefore \quad \frac{v_2}{v_1} = \sqrt{2}$

11. Assertion is true but reason is false.
In the first few steps, work has to be done against limiting friction and afterwards, work is to be done against dynamic friction, which is smaller than the limiting friction.

12. Below the surface of the earth, pressure increases with increase in depth. Hence pressure in the mine is higher than atmospheric pressure.
The acceleration due to gravity below the surface of the earth decreases uniformly with the distance from the centre, as shown in the figure below.



13. Gauge pressure at point A = $h\rho g$
Total pressure at point A
= atmospheric pressure + gauge pressure
$= P_a + h\rho g$

14. Using Bernoulli's equation,

$P_1 + \frac{1}{2}\rho_1 v_1^2 = P_2 + \frac{1}{2}\rho_2 v_2^2$ ….(i)

Also, $P_1 - P_2 = \rho g \times 6$ ….(ii)

From (i) and (ii),

$v_2^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho} = \frac{2\rho g \times 6}{\rho} = (2g) \times 6$

$= 2 \times 980 \times 6$

$v_2^2 - v_1^2 = 12 \times 980 \text{ cm}^2\text{/s}$ ….(iv)

From equation of continuity,

$A_1v_1 = A_2v_2$

$\therefore \quad \frac{v_1}{v_2} = \frac{A_2}{A_1}$

$= \frac{\pi \times 0.5^2}{\pi \times 1^2} = 0.25$

$v_1^2 = 0.25^2 \times v_2^2$

Substituting in (iv),

$v_2^2\left[1 - (0.25)^2\right] = 12 \times 980$

$v_2 = \sqrt{\frac{12 \times 980}{0.9375}}$

Quantity of water flowing

$= A_1v_1 = A_2v_2$

$= \pi \times 0.5^2 \times \sqrt{\frac{12 \times 980}{0.9375}}$

≈ 88 c.c per s

15. The pressure of water at the base of aquarium
$P = h\rho g$
Pressure being linear function of height, average pressure is half of the maximum pressure.

Hence force on the lateral wall,

$F = P_{av} \times A$

$= P_{av} \times (h \times l)$

$= \frac{h\rho g}{2} \times h \times l$

$= \frac{0.4 \times 10^3 \times 10}{2} \times 0.4 \times 0.5$

$= 400$ N

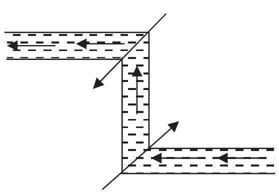
16. According to equation of continuity, Av = constant.
By attaching a jet, area of cross-section is reduced. This results into increasing the velocity of water flowing out of the pipe.

17. From equation of terminal velocity $v \propto r^2$
This represents equation of straight line.

18. For a freely falling body, g = 0 Hence v = 0.

19. When the snow accumulates on the wings of an aeroplane, the upper surface of the wing becomes flat. It means the curvature of the surface decreases. Pressure difference which causes the lift off of the aeroplane depends on the curvature of the wing. Thus, due to the decrease in curvature, the lift-off of the aeroplane also decreases.

20. The water undergoes change in momentum, only at the bends of tube. Hence the water and tube exert forces on each other at these locations. The forces exerted by the water at the bends are shown in figure. The two forces form a couple causing an anticlockwise torque.



21. Velocity of efflux, $v = \sqrt{2gd}$

Time taken for the range $r = \sqrt{\frac{2H}{g}}$

$r = \sqrt{2gd} \times \sqrt{\frac{2H}{g}}$

$\therefore \quad r^2 = 2dg \times \frac{2H}{g} = 4dH$

$\Rightarrow d = \frac{r^2}{4H}$

22. According to equation of continuity,
Av = constant
At A, area is larger than B hence v is smaller at A than at B.
Also, from Bernoulli's principle,

$P + \frac{1}{2}\rho v^2 = \text{constant}$

This means where v is small, P is more.
At A, pressure is higher. Hence liquid at point A will raise to greater height than at point B. Hence option (B) is incorrect.
Now, pressure at A, $P_1 = P_a + h_A rg$
Pressure at B, $P_2 = P_a + h_B rg$
$P_1 - P_2 = (h_A - h_B)rg = hrg$
Hence option (A) is correct.
Bernoulli's principle is applicable for non-viscous, streamlined flow of liquid. Hence option (C) is also correct.

Textbook Chapter No.

08 Refraction of Light

Hints

Classical Thinking

22. $_a\mu_g = {}_a\mu_w \times {}_w\mu_g$

$\therefore \quad {}_w\mu_g = \dfrac{_a\mu_g}{_a\mu_w} = \dfrac{3/2}{4/3} = \dfrac{9}{8}$

23. $_d\mu_g = \dfrac{1}{_g\mu_a \times {}_a\mu_d}$

$= \dfrac{1}{\frac{2}{3}\times\frac{12}{5}}$

$= \dfrac{5}{8}$

24. $i = 90° - 30° = 60°$

$_w\mu_g = \dfrac{\sin i}{\sin r}$

$\therefore \quad \sin r = \dfrac{\sin i}{_w\mu_g} = \dfrac{\sin 60°}{\sqrt{3}} = \dfrac{\sqrt{3}}{2}\times\dfrac{1}{\sqrt{3}} = \dfrac{1}{2}$

$\therefore \quad r = 30°$

25. $\mu = \dfrac{\lambda_a}{\lambda_g}$

$\therefore \quad 1.5 = \dfrac{4800}{\lambda_g}$

$\therefore \quad \lambda_g = \dfrac{4800}{1.5} = 3200$ Å

26. $_w\mu_a = \dfrac{1}{_a\mu_w} = \dfrac{1}{5/3} = \dfrac{3}{5}$

$\sin r = \dfrac{\sin i}{_w\mu_a} = \dfrac{\sin 32°}{3/5} = \dfrac{0.5299\times5}{3}$

$\therefore \quad r = \sin^{-1}(0.8832)$

$r = 62°2'$

37. $\sin i_C = \dfrac{1}{\mu}$ therefore i_C will be maximum when μ is minimum which is for red light.

38. $\mu = \dfrac{1}{\sin i_C} = \dfrac{1}{\sin 60°} = \dfrac{2}{\sqrt{3}} = \dfrac{2}{1.732} = 1.15$

47. $\mu \propto \dfrac{1}{\lambda}$

$\lambda_R > \lambda_Y > \lambda_G > \lambda_V$

$\mu_R < \mu_Y < \mu_G < \mu_V$

60. $\delta = A(\mu - 1)$

$\therefore \quad 2.4 = 4(\mu - 1)$

$\therefore \quad \mu - 1 = 0.6$

$\therefore \quad \mu = 1.6$

63. $e = 0$

$\therefore \quad r_2 = 0,\ A = r_1$ since 'i' is small

$\mu = \dfrac{i}{r_1}$

$\therefore \quad i = \mu r_1 = \mu A$

68. $\delta_v - \delta_r = A(\mu_v - \mu_r) = 5°(1.665 - 1.645)$

$\delta_v - \delta_r = 0.1°$

70. $\omega = \dfrac{\mu_v - \mu_r}{\left(\dfrac{\mu_v + \mu_r}{2}\right) - 1} = \dfrac{1.7 - 1.65}{1.675 - 1}$

$= \dfrac{0.05}{0.675} = 0.074$

74. $A(\mu - 1) = A'(\mu' - 1)$

$\therefore \quad 4(1.54 - 1) = A'(1.72 - 1)$

$\therefore \quad A' = 3°$

96. $\mu = \dfrac{c}{v} = \dfrac{100}{100 - 30} = \dfrac{100}{70}$

$= 1.43$

97. $_a\mu_w = \dfrac{\lambda_a}{\lambda_w}$

$\therefore \quad \lambda_w = \dfrac{\lambda_a}{_a\mu_w}$

$= \dfrac{6500}{1.3} = 5000$ Å

change in wavelength = 6500 – 5000
= 1500 Å

$\therefore$ percentage change in wavelength

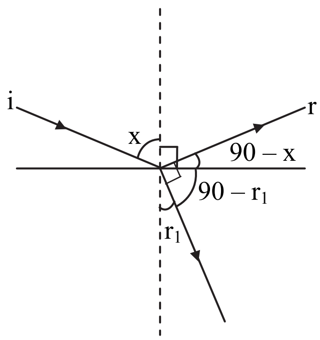
$= \dfrac{1500}{6500}\times 100 = 23\%$

100. A completely transparent material will be invisible in vacuum when its refractive index will equal refractive index of vacuum.
(Refer Mindbender 4.)

Critical Thinking

2. ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3$

$$= \frac{\mu_1}{\mu_2} \times \frac{\mu_2}{\mu_3} \times \frac{\mu_3}{\mu_4} = \frac{\mu_1}{\mu_4} = {}_4\mu_1 = \frac{1}{{}_1\mu_4}$$

3. 

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin x}{\sin(90° - x)}$$

$[\because (90° - x) + (90° - r_1) = 90°]$

$$= \frac{\sin x}{\cos x} = \tan x$$

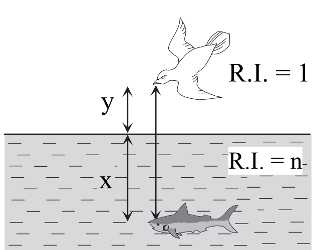
$\Rightarrow x = \angle i = \tan^{-1}(\mu)$

6. $\mu = \frac{h}{h'} \Rightarrow h' = \frac{h}{n}$

7. $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

$\therefore$ Apparent depth $= \frac{\text{Real depth}}{\mu} = \frac{46}{4/3}$

$= 34.5$ cm

8. 

The distance of the surface of water for the fish = x
For reference frame of fish, as light rays will travel from denser to lighter (air) medium, they will bend away from normal and bird will appear farther.
Thus, apparent height = n × real height = ny.

9. For prism, $\mu = 1.5$

$\therefore$ $i_C \approx 42°$
For ray B, angle of incidence in the prism is 45°.
Hence, for ray B angle of incidence is greater than critical angle.

12. When incident angle is greater than critical angle, then total internal reflection takes place and will come back in same medium. To signal light out he has to direct the beam at an angle lesser than the critical angle.

13. For glass, $\mu = \sqrt{2}$

$$\Rightarrow i_C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

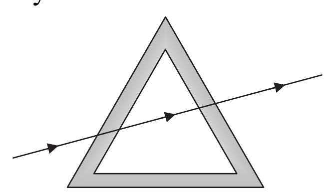
This means the ray is incident at critical angle hence will come out just grazing the surface, i.e., angle of refraction equal to 90°.

14. Critical angle $= \sin^{-1}\left(\frac{1}{\mu}\right)$

$$\theta = \sin^{-1}\left(\frac{1}{\mu_{\lambda_1}}\right) \text{ and } \theta' = \sin^{-1}\left(\frac{1}{\mu_{\lambda_2}}\right)$$

Since $\mu_{\lambda_2} > \mu_{\lambda_1}$, hence $\theta' < \theta$

16. Effectively there is no deviation or dispersion.



17. Net deviation caused by prisms Q and R is zero hence the ray suffers same deviation.

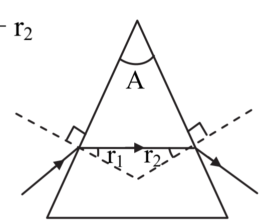
18. For a prism in water, its refractive index

$${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.5}{1.33} = 1.13$$

As relative refractive index of prism reduces, the angle of minimum deviation decreases.

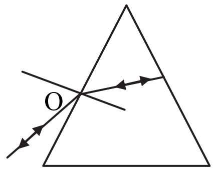
19. Angle of prism, $A = r_1 + r_2$
For minimum deviation
$r_1 = r_2 = r$
$A = 60°$

$\therefore$ $r = \frac{A}{2} = \frac{60°}{2} = 30°$



20. $r_2 = 0 \qquad A = r_1 + r_2$

$\therefore \quad A = r_1 = 30°$



$\mu = \dfrac{\sin i}{\sin r_1} = \dfrac{\sin i}{\sin A}$

$\therefore \quad \sqrt{2} = \dfrac{\sin i}{\sin 30°}$

$\therefore \quad \sin i = \sqrt{2} \times \sin 30°$

$= \sqrt{2} \times \dfrac{1}{2} = \dfrac{1}{\sqrt{2}}$

$i = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45°$

21. $i - e = 10°$ ....(i)

$i + e = A + \delta$

$\therefore \quad i + e = 60° + 30° = 90°$

$i + e = 90°$ ....(ii)

Solving equation (i) and (ii)

$i = 50°, r = 50° - 10° = 40°$

22. $e = 0$

$\therefore \quad r_2 = 0$

Also $r_1 = 30°$ and $\mu = \dfrac{\sin i}{\sin r}$

$\therefore \quad 1.5 = \dfrac{\sin i}{\sin 30°}$

$\therefore \quad \sin i = 1.5 \times \sin 30° = 1.5 \times 0.5$

$\therefore \quad i = \sin^{-1}(0.75)$

23. Since, $i + e = A + \delta$

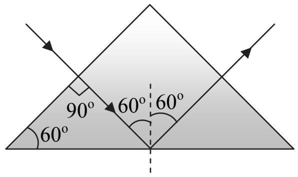
$e = (A + \delta) - i$

$= (30° + 30°) - 60°$

$= 0$

This means if angle of emergence (measured with respect to normal to the second face) is zero, therefore angle made by emergent ray with the second face of prism is 90°.

24.



25. When angle of refraction exceeds value of critical angle, no emergent ray is observed.

Thus, $\angle r > \angle C$

but, $r = \dfrac{A}{2}$ where, A is angle of prism.

$\Rightarrow \dfrac{A}{2} > C$

$\therefore \quad A > 2C$

26. $i = 0$

$\therefore \quad r_1 = 0$

$\therefore \quad e = A + \delta$ and $A = r_2$

$\mu = \dfrac{\sin e}{\sin r_2} = \dfrac{\sin(A+\delta)}{\sin(A)}$

27. By formula $\delta = (\mu - 1)A \Rightarrow 34 = (\mu - 1)A$

and in the second position $\delta' = (\mu - 1)\dfrac{A}{2}$

$\therefore \quad \dfrac{34}{\delta'} = \dfrac{(\mu-1)A}{(\mu-1)\dfrac{A}{2}}$ or $\delta' = \dfrac{34}{2} = 17°$

28. $\delta = A(\mu - 1)$, $\delta_b = A(\mu_b - 1)$, $\delta_r = A(\mu_r - 1)$

$\therefore \quad D_2 = A(1.525 - 1)$

$\therefore \quad D_1 = A(1.520 - 1)$

$\Rightarrow D_2 > D_1$

29. $\dfrac{\delta_w}{\delta_a} = \dfrac{({}_w\mu_g - 1)}{({}_a\mu_g - 1)} = \dfrac{\left(\dfrac{9}{8} - 1\right)}{\left(\dfrac{3}{2} - 1\right)} = \dfrac{1}{4}$

30. At P, $\delta = 0$

For thin prism $\delta = \delta_m$

$\therefore \quad \delta = A(\mu - 1)$

$\Rightarrow 0 = A(\mu - 1)$

$\Rightarrow \mu = 1$

Thus, option (A) is correct.

Also, $\delta = (\mu - 1)A = A\mu - A$

Comparing with $y = mx + c$

Slop of line PQ = m = A

Thus, option (C) is correct.

31. $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \dfrac{\sin\left\{\dfrac{[A + (180 - 2A)]}{2}\right\}}{\sin\left(\dfrac{A}{2}\right)}$

$\therefore \quad \mu = \dfrac{\sin\left[90 - \left(\dfrac{A}{2}\right)\right]}{\sin\left(\dfrac{A}{2}\right)} = \dfrac{\cos\left(\dfrac{A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$\therefore \quad \mu = \cot\left(\dfrac{A}{2}\right)$

35. $\mu = 1.5 = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

Since $A = \delta_m$

$$1.5 = \frac{\sin\left(\frac{2A}{2}\right)}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\cos\frac{A}{2} = 0.75$$

$$\frac{A}{2} = \cos^{-1}(0.75) = 41°$$

∴ $A = 2 \times 41° = 82°$

36. $$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Here $A = \delta_m$

∴ $$\mu = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\frac{A}{2}}$$

$$\sqrt{3} = \frac{\sin A}{\sin\frac{A}{2}}$$

$$\sqrt{3} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$$

$$\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

∴ $$\frac{A}{2} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

∴ $$\frac{A}{2} = 30°$$

∴ $A = 60°$

37. Net angular dispersion = $\delta(\omega - \omega')$. As $\omega' > \omega$, net angular dispersion is negative.

38. Since $\delta > \delta'$ ∴ $A(\mu - 1) > A'(\mu' - 1)$ and $(\mu' - 1) > (\mu - 1)$ ∴ $A > A'$

39. $$A' = -\frac{A(\mu_b - \mu_r)}{(\mu'_b - \mu'_r)} = -\frac{6°(1.531 - 1.520)}{(1.684 - 1.662)}$$

$A' = -3°$.

Negative sign for opposite manner of flint glass prism.

Hence refracting angle = 3°

Net deviation = $A(\mu - 1) + A'(\mu' - 1)$

$$= 6°\left[\frac{1.531+1.520}{2} - 1\right] + (-3°)\left[\frac{1.684+1.662}{2} - 1\right]$$

$= 1.134°$

40. The dispersive power for crown glass

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

$$= \frac{1.5318 - 1.5140}{(1.5170 - 1)} = \frac{0.0178}{0.5170} = 0.034$$

Dispersive power for flint glass,

$$\omega' = \frac{1.6852 - 1.6434}{(1.6499 - 1)} = 0.064$$

41. $$\frac{(\mu_v - \mu_r)}{(\mu - 1)} = \omega \quad \therefore (\mu_v - \mu_r) = \omega(\mu - 1)$$

∴ $\Delta = A(\mu_v - \mu_r) = A\,\omega(\mu - 1)$

∴ Achromatic combination

$(\mu - 1)\,\omega A = (\mu' - 1)\,\omega' A'$

∴ $$\frac{A'}{A} = \frac{(\mu - 1)\omega}{(\mu' - 1)\omega'} = \frac{0.517 \times 0.03}{0.621 \times 0.05} = 0.50 \quad \ldots(i)$$

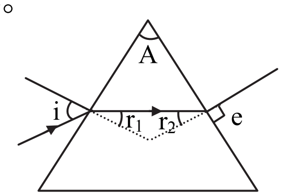
Net deviation = $\delta - \delta' = (\mu - 1)A - (\mu' - 1)A'$

∴ $1° = 0.517\,A - 0.621\,A'$ …(ii)

On solving equations (i) and (ii),

$A = 4.8°$ and $A' = 2.4°$.

43. $e = 90°$, $r_2 = i_C = 45°$



$A = r_1 + r_2$

$r_1 = A - r_2$

$r_1 = A - i_C$

$= 75° - 45° = 30°$

$$\mu = \frac{\sin e}{\sin r_2} = \frac{\sin e}{\sin i_C}$$

∴ $$\sqrt{2} = \frac{1}{\sin i_C}$$

∴ $$i_C = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

∴ $i_C = 45°$

$$\mu = \frac{\sin i}{\sin r_1}$$

∴ $$\sqrt{2} = \frac{\sin i}{\sin 30°}$$

∴ $$\sin i = \sqrt{2} \times \sin 30° = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

∴ $$i = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

44. $i = 2r$

$\mu = \dfrac{\sin i}{\sin r}$

$\sqrt{2} = \dfrac{\sin 2r}{\sin r} = \dfrac{2\sin r \cos r}{\sin r} = 2\cos r$

$\therefore$ $\cos r = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

$\therefore$ $r = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

$\therefore$ $r = 45°$

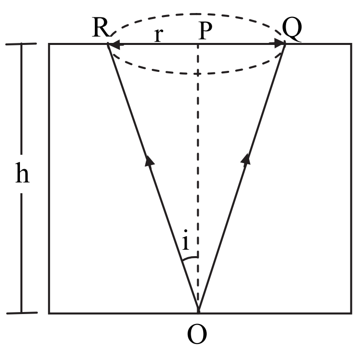
But $A = 2r$ for minimum deviation

$\therefore$ $A = 2 \times 45° = 90°$

45. ${}_1\mu_2 = \dfrac{v_1}{v_2} = \dfrac{s_1/t}{s_2/t} = \dfrac{s_1}{s_2}$

Also ${}_1\mu_2 = \dfrac{c/v_2}{c/v_1} = \dfrac{\mu_2}{\mu_1} = \dfrac{s_1}{s_2} = \dfrac{3}{2}$

46.



Let the bulb be placed at point O. The light rays originating from it will spread at the surface of water as shown in the figure, forming a circle. Angle of semi vertex (∠ i) here equals critical angle of water i.e., $\angle i = \angle i_C$

From the figure, PQ = PR = r, say,

then, $r = h \tan i_C$

$$r = \frac{h \sin i_C}{\cos i_C} = \frac{h\left(\frac{1}{\mu_w}\right)}{\sqrt{1-\sin^2 i_C}} = \frac{h\left(\frac{1}{\mu_w}\right)}{\sqrt{1-\left(\frac{1}{\mu_w}\right)^2}}$$

$\left(\because \sin i_C = \dfrac{1}{\mu_w}\right)$

For h = 80 cm = 0.8 m and $\mu_w = 1.33$,

$$r = \frac{0.8\left(\frac{1}{1.33}\right)}{\sqrt{1-\left(\frac{1}{1.33}\right)^2}} = 0.912 \text{ m}$$

Area of circle $= \pi r^2 = 3.142 \times (0.912)^2 = 2.61 \text{ m}^2$.

47. $\mu = \dfrac{d_{real}}{d_{apparent}}$

$\mu_w = \dfrac{12.5}{9.4} = 1.33$

When water is replaced by liquid,

$d'_{apparent} = \dfrac{d_{real}}{\mu_l} = \dfrac{12.5}{1.63} \approx 7.7$ cm

The distance by which microscope should be moved,

$d = d_{real} - d_{apparent}$

$= 9.4 - 7.7 = 1.7$ cm

48. $h' = \dfrac{d}{\mu_1} + \dfrac{d}{\mu_2} = d\left(\dfrac{1}{\mu_1} + \dfrac{1}{\mu_2}\right)$

49. Apparent depth $= \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2}$

$\therefore$ $\dfrac{36}{7} = \dfrac{5}{\frac{5}{3}} + \dfrac{3}{\mu_2}$

$\therefore$ $\mu_2 = \dfrac{7}{5} = 1.4$

50. From the figure,

$\alpha + 2\beta = 180°$

and $\beta = 2\alpha$

$\therefore$ $\alpha = 36°$

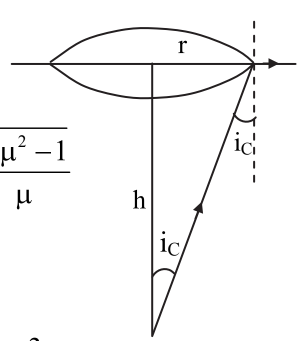
51. ${}_w\mu_g = \dfrac{{}_a\mu_g}{{}_a\mu_w} = \dfrac{3/2}{4/3} = \dfrac{9}{8}$

$\therefore$ $\delta_a = A\left(\dfrac{3}{2} - 1\right), \delta_w = A\left(\dfrac{9}{8} - 1\right)$

$\delta_a = \dfrac{A}{2}$ and $\delta_w = \dfrac{A}{8}$

$\therefore$ $\dfrac{\delta_w}{\delta_a} = \dfrac{A/8}{A/2} = \dfrac{1}{4}$

52. $\tan i_C = \dfrac{r}{h}$



$\therefore$ $r = h \tan i_C$

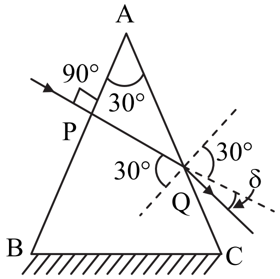
$\sin i_C = \dfrac{1}{\mu} \Rightarrow \cos i_C = \dfrac{\sqrt{\mu^2 - 1}}{\mu}$

$\therefore$ $\tan i_C = \dfrac{1}{\sqrt{\mu^2 - 1}}$

$\therefore$ $r = \dfrac{h}{\sqrt{\mu^2 - 1}} = \dfrac{\sqrt{7}}{\sqrt{\frac{16}{9} - 1}} = 3$ cm

53. At point Q of ray PQ

$$\frac{\sin i}{\sin r} = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$$



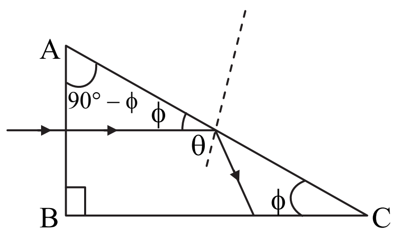
$$\frac{\sin 30°}{\sin r} = \frac{1}{\sqrt{2}}$$

$\therefore$ $\sin r = \sqrt{2}\ \sin 30°$

$= \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$

$\therefore$ $r = 45°$, $\delta = r - i = 45° - 30° = 15°$

54.



$\sin i_C = \frac{1}{\mu} = \frac{1}{1.5} = \frac{2}{3}$

$\theta > i_C \therefore \sin\theta > \sin i_C \therefore \sin\theta > \frac{2}{3}$

But $\theta + \phi = 90°$

$\therefore$ $\theta = 90° - \phi$

$\therefore$ $\sin(90° - \phi) > \frac{2}{3}$

$\cos\phi > \frac{2}{3}$

$\therefore$ $\phi < \cos^{-1}\left(\frac{2}{3}\right)$

$\therefore$ Largest value of ϕ is $\cos^{-1}\left(\frac{2}{3}\right)$

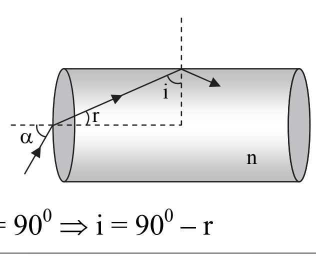
55. Refractive index $\propto \frac{1}{(\text{Temperature})}$

56. Snell's law in vector form is $\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$

57. All colours are reflected.

58. $\underset{\text{(Primary)}}{\text{Yellow}} + \underset{\text{(Primary)}}{\text{Blue}} = \underset{\text{(Secondary)}}{\text{Green}}$

59. From the following figure



$r + i = 90^0 \Rightarrow i = 90^0 - r$

For ray not to emerge from curved surface $i > i_C$

$\Rightarrow \sin i > \sin i_C \Rightarrow \sin(90^o - r) > \sin i_C$

$\Rightarrow \cos r > \sin i_C$

$\Rightarrow \sqrt{1 - \sin^2 r} > \frac{1}{n}$ ....(i) $\left[\because \sin i_C = \frac{1}{n}\right]$

From Snell's law,

$n = \frac{\sin\alpha}{\sin r}$

$\therefore$ $\sin^2 r = \frac{\sin^2\alpha}{n^2}$

Substituting in equation (i),

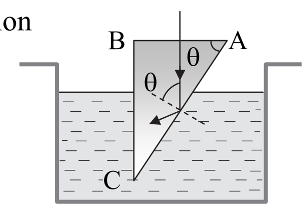
$\Rightarrow 1 - \frac{\sin^2\alpha}{n^2} > \frac{1}{n^2} \Rightarrow 1 > \frac{1}{n^2}(1 + \sin^2\alpha)$

$\Rightarrow n^2 > 1 + \sin^2\alpha$

$\Rightarrow n > \sqrt{2}$ (as $\sin\alpha \to 1$)

$\Rightarrow$ Least value = $\sqrt{2}$

60. For total internal reflection at AC



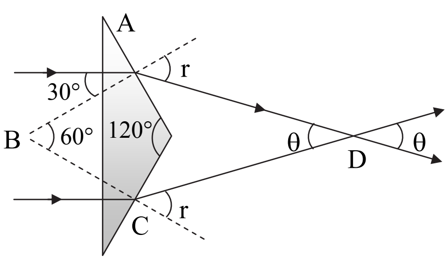
$\theta > i_C$

$\Rightarrow \sin\theta \geq \sin i_C$

$\Rightarrow \sin\theta \geq \frac{1}{{}_w\mu_g}$

$\Rightarrow \sin\theta \geq \frac{\mu_w}{\mu_g} \Rightarrow \sin\theta \geq \frac{8}{9}$

61.



At point A, $\frac{\sin 30°}{\sin r} = \frac{1}{1.44}$

$\Rightarrow r = \sin^{-1}(0.72)$ also $\angle BAD = 180° - \angle r$

In quadrilateral ABCD,

$\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow (180° - r) + 60° + (180° - r) + \theta = 360°$

$\Rightarrow \theta = 2[\sin^{-1}(0.72) - 30°]$

62. From graph, $\tan 30° = \frac{\sin r}{\sin i} = \frac{1}{{}_1\mu_2}$

$\Rightarrow {}_1\mu_2 = \sqrt{3} \Rightarrow \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = 1.73 \Rightarrow v_1 = 1.73\,v_2$

Thus, option (B) is correct.

Also from $\mu = \frac{1}{\sin i_C}$

$\Rightarrow \sin i_C = \frac{1}{_{Rarer}\mu_{Denser}}$

$\Rightarrow \sin i_C = \frac{1}{_1\mu_2} = \frac{1}{\sqrt{3}}$.

63. Refractive index of liquid C is same as that of glass piece. So, it will not be visible in liquid C.

Competitive Thinking

2. All the rays will be incident normally on the surface of the sphere. Hence, the rays will not be refracted but will pass through the sphere undeviated.

3. $_\omega\mu_g = \frac{_a\mu_g}{_a\mu_\omega} = \frac{1.5}{1.3}$

4. Frequency is independent of medium.

5. $_v\mu_w > 1$

$_v\mu_w = \frac{\lambda_{vacuum}}{\lambda_{water}}$

$\Rightarrow \lambda_{vacuum} > \lambda_{water}$

7. $\lambda \propto \frac{1}{\mu}$

$\frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1} = \frac{\mu}{1}$

8. Refractive index of medium 2 w.r.t. medium 1 is,

$\frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2}$ ….(i)

Also, according to Snell's law,

$\frac{\mu_2}{\mu_1} = \frac{\sin\alpha_1}{\sin\alpha_2}$ ….(ii)

$\therefore \frac{\lambda_1}{\lambda_2} = \frac{\sin\alpha_1}{\sin\alpha_2}$ ….[from (i) and (ii)]

$\lambda_2 = \lambda_1 \frac{\sin\alpha_2}{\sin\alpha_1}$

9. Incident and reflected waves propagate in same medium hence have same wavelength.

$\therefore \mu = \frac{\lambda_{incident}}{\lambda_{refracted}} = \frac{\lambda_{reflected}}{\lambda_{refracted}} = 1.5$

10. $_a\mu_w = \frac{\lambda_a}{\lambda_w}$

$\therefore \lambda_w = \frac{\lambda_a}{_a\mu_w} = \frac{4200}{(4/3)} = \left(\frac{3}{4}\right) \times 4200$

$\therefore \lambda_w = 3150$ Å

11. $\lambda_1 = \frac{c}{v_1} = \frac{3\times10^8}{4\times10^{14}} = 0.75 \times 10^{-6}$ m

$\mu = \frac{\lambda_1}{\lambda_2} \Rightarrow \lambda_2 = \frac{0.75\times10^{-6}}{1.5} = 0.5 \times 10^{-6}$ m

$\Delta\lambda = \lambda_1 - \lambda_2 = 0.25 \times 10^{-6} = 2.5 \times 10^{-7}$ m

12. $\mu = \frac{\lambda_1}{\lambda_2}$

$\therefore \lambda_2 = \frac{\lambda_1}{1.5}$

$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{0.5\lambda_1}{1.5}$

$\% \Delta\lambda = \frac{1}{3}\times 100\lambda_1$

= 33.33 % of original wavelength.

13. $_w\mu_g = \frac{v_w}{v_g}$

$\Rightarrow v_w = {_w\mu_g} = v_g = \frac{9}{8}\times2\times10^8 = 2.25 \times 10^8$ m/s

14. $\frac{v_g}{v_w} = \frac{c/v_w}{c/v_g} = \frac{\mu_w}{\mu_g} = \frac{1.33}{1.5} = 0.8867 : 1$

15. $\mu \propto \frac{1}{v}$

$\frac{\mu_g}{\mu_w} = \frac{v_w}{v_g}$

$\frac{\left(\frac{3}{2}\right)}{\left(\frac{4}{3}\right)} = \frac{v_w}{2\times10^8}$

$\therefore v_w = 2.25 \times 10^8$ m/s

16. $_m\mu_g = \frac{4}{3}$

$\frac{v_m}{v_g} = \frac{4}{3} \Rightarrow v_m = \frac{4}{3}v_g$

$\therefore v_m - v_g = \left(\frac{4}{3} - 1\right)v_g = 6.25 \times 10^7$ m/s

$v_g = 18.75 \times 10^7$ m/s

$\Rightarrow v_m = 6.25 \times 10^7 + 18.75 \times 10^7$

$= 25 \times 10^7 = 2.5 \times 10^8$ m/s

17. $\mu = \frac{\sin i}{\sin r} = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}$

$v = \frac{c}{\mu} = \frac{3\times 10^8}{\sqrt{2}} = 2.12 \times 10^8$ m/s

18. Refraction at air-oil interface, $\mu_{oil} = \frac{\sin i}{\sin r_1}$

$\therefore$ $\sin r_1 = \frac{\sin 40^\circ}{1.45} = 0.443$

Refraction at oil-water interface,

${}_{oil}\mu_{water} = \frac{\sin r_1}{\sin r}$

$\therefore$ $\frac{1.33}{1.45} = \frac{0.443}{\sin r}$

$\Rightarrow \sin r = \frac{0.443 \times 1.45}{1.33}$

$\Rightarrow r = 28.9^\circ$

19. $i = 2r$

$\frac{\sin i}{\sin r} = \mu$

$\frac{\sin 2r}{\sin r} = \mu$

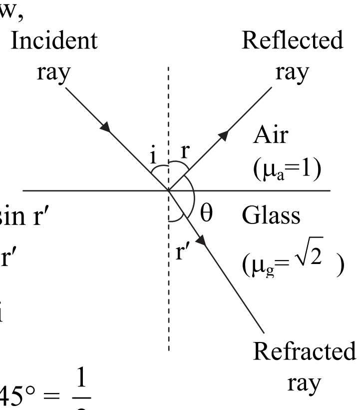
$\frac{2\sin r\cos r}{\sin r} = \mu$

$\therefore$ $\cos r = \frac{\mu}{2} \Rightarrow r = \cos^{-1}\left(\frac{\mu}{2}\right)$

20. Using Snell's law,

${}_a\mu_g = \frac{\sin i}{\sin r'}$

but ${}_a\mu_g = \frac{\mu_g}{\mu_a}$



$\Rightarrow \mu_a \sin i = \mu_g \sin r'$

$1 \sin i = \sqrt{2} \sin r'$

$\sin r' = \frac{1}{\sqrt{2}} \sin i$

$= \frac{1}{\sqrt{2}} \sin 45^\circ = \frac{1}{2}$

$r' = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

From figure, $r + \theta + r' = 180^\circ$

$i + \theta + 30^\circ = 180^\circ$ $[\because i = r]$

$45^\circ + \theta + 30^\circ = 180^\circ$

$\Rightarrow \theta = 180^\circ - 75^\circ = 105^\circ$

Hence, the angle between reflected and refracted rays is 105°.

21. Of all the colours in spectrum, red shows least deviation.

23. $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

$\therefore$ In case of water filled beaker,

$\mu_w = \frac{h}{h'_w}$ ….(i)

Similarly for oil filled beaker,

$\mu_o = \frac{h}{h'_o}$ ….(ii)

Dividing equation (i) by (ii)

$\frac{\mu_w}{\mu_o} = \frac{h}{h'_w} \times \frac{h'_o}{h}$

$\therefore$ $\frac{4}{3\times 1.6} = \frac{h'_o}{h'_w}$

$\therefore$ $h'_w = 1.2\, h'_o$

i.e., apparent depth of water is 1.2 times greater than that of oil

24. $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

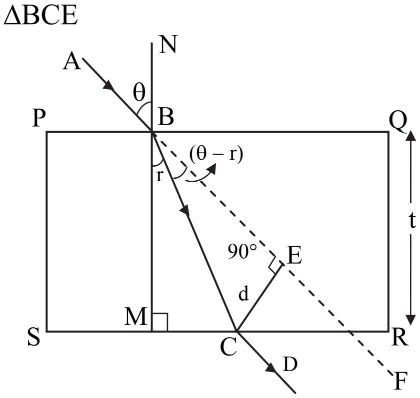
Let t be the real thickness of the slab,

Given apparent thickness = 3 + 5 = 8 cm

$\therefore$ $\mu = \frac{t}{8}$

i.e. $t = 8 \times 1.5 = 12$ cm

25. In ΔBCE



$\sin(\theta - r) = \frac{CE}{BC} \Rightarrow CE = BC\sin(\theta - r)$

$\Rightarrow d = BC\sin(\theta - r)$ ….(i)

In ΔBMC

$\cos r = \frac{BM}{BC} \Rightarrow BC = \frac{BM}{\cos r} = \frac{t}{\cos r}$ ….(ii)

From equations (i) and (ii),

$d = \frac{t}{\cos r}\sin(\theta - r)$

$d = \frac{t}{\cos r}(\sin\theta\cos r - \cos\theta\sin r)$

$= t(\sin\theta - \cos\theta\tan r)$

If n is the refractive index of material of slab (glass) w.r.t.air, then

$n = \frac{\sin\theta}{\sin r}$

For small angle,

$n \approx \frac{\theta}{r} \Rightarrow r = \frac{\theta}{n}$ and $d = t(\theta - 1.r)$

[$\because \sin\theta \approx \theta$ and $\cos\theta \approx 1$ if θ is small]

$d = t\left(\theta - \frac{\theta}{n}\right) = t\theta\left(1 - \frac{1}{n}\right)$

$\Rightarrow d = \frac{t\theta(n-1)}{n}$

26. The emergent ray will be parallel to incident ray only if the mediums have same refractive indices.

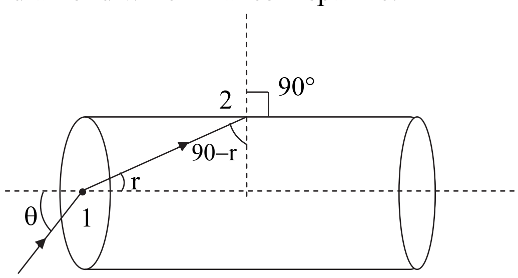
27. For total internal reflection $i > i_C$

$\sin i > \sin i_C$

$\sin i > \frac{1}{\mu}$ $\quad\therefore\quad \frac{1}{\sin i} < \mu$

28. Due to large refractive index of diamond (μ = 2.42), critical angle of diamond is very small. This causes total internal reflection in diamond which makes it sparkle.

29.



At interface 1: $\mu = \frac{\sin\theta}{\sin r}$

$\Rightarrow \sin\theta = \mu\sin r$ ….(i)

At interface 2: $(90 - r) = i_C$

$\Rightarrow \sin(90 - r) = \sin i_C$

$\Rightarrow \cos r = \frac{1}{\mu}$ $\quad\left[\because \sin i_C = \frac{1}{\mu}\right]$

$\Rightarrow \cos r = \frac{1}{2/\sqrt{3}} = \sqrt{3}/2 \Rightarrow r = 30°$

From equation (i), $\sin\theta = \frac{2}{\sqrt{3}}\sin 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

30. $i_C = \sin^{-1}\left(\frac{1}{\mu}\right)$ and $\mu \propto \frac{1}{\lambda}$

Yellow, orange and red have higher wavelength than green, so μ will be less for these rays, consequently critical angle for these rays will be high, hence if green is just totally internally reflected then yellow, orange and red rays will emerge out.

31. $\sin i_C = \frac{1}{\mu}$ and $\mu \propto \frac{1}{\lambda}$

For greater wavelength (i.e., lesser frequency) μ is less. Hence, i_C would be more. Thus, these wavelengths will not suffer internal reflection and come out at angles less than 90°.

32. ${}_a\mu_g = \frac{1}{\sin i_C}$

$\sin i_C = \frac{1}{{}_a\mu_g}$

As μ for violet colour is maximum, so $\sin i_C$ is minimum and hence critical angle i_C is minimum for violet colour.

33. $\sin i_C = \frac{1}{\mu}$

$\therefore \quad \mu = \frac{1}{\sin(24.5°)} = \frac{1}{0.414} = 2.41$

34. As the beam just suffers total internal reflection at interface of region III and IV, it almost grazes region IV

$\Rightarrow i \approx 90°$

Hence,

$n_0\sin\theta = \frac{n_0}{2}\sin\theta_1 = \frac{n_0}{6}\sin\theta_2 = \frac{n_0}{8}\sin 90°$

$\therefore \quad \sin\theta = \frac{1}{8} \Rightarrow \theta = \sin^{-1}\frac{1}{8}$

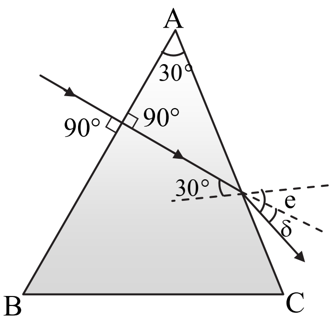
35. $i_C = \sin^{-1}\left(\frac{1}{{}_{rarer}\mu_{denser}}\right) = \sin^{-1}\left(\frac{1}{{}_r\mu_d}\right)$

$\therefore \quad i_C = \sin^{-1}\left(\frac{1}{\frac{\mu_d}{\mu_r}}\right) = \sin^{-1}\left(\frac{\mu_r}{\mu_d}\right)$

$i_C = \sin^{-1}\left(\frac{1.5}{1.6}\right) = \sin^{-1}\left(\frac{15}{16}\right)$

38. After refraction at two parallel faces of a glass slab, a ray of light emerges in a direction parallel to the direction of incidence of white light on the slab. As rays of all colours emerge in the same direction (of incidence of white light), hence there is no dispersion, but only lateral displacement.

39.



For surface AC, $\frac{1}{\mu} = \frac{\sin 30°}{\sin e}$

$\Rightarrow \sin e = \mu \sin 30° = 1.5 \times \frac{1}{2} = 0.75$

$\Rightarrow e = \sin^{-1}(0.75) = 48° \, 36'$

From figure, $\delta = e - 30°$

$= 48°36' - 30° = 18°36'$

40. In minimum deviation condition

$r = \frac{A}{2} = \frac{60°}{2} = 30°$

41. In minimum deviation position,

$\angle i = \angle e$

42. Angle of deviation decreases initially with increase in angle of incidence, attains minimal value. On further increase in angle of incidence, angle of deviation increases.

43. In minimum deviation position refracted ray inside the prism is parallel to the base of the prism.

44. $i = \frac{A + \delta_m}{2} = 50°$

45. Angle of prism, A = 60°

For minimum deviation, Angle of refraction,

$r = \frac{A}{2} = \frac{60°}{2}$

= 30° for both the colours

46. Given $i = e = \frac{3}{4}A = \frac{3}{4} \times 60 = 45°$

In the position of minimum deviation

$2i = A + \delta_m$ or $\delta_m = 2i - A = 90° - 60° = 30°$

47. By the hypothesis, we know that

$i + e = A + \delta \Rightarrow 55° + 46° = 60° + \delta$

$\Rightarrow \delta = 41°$

But $\delta_m < \delta$, so $\delta_m < 41°$

48. $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$= \frac{\sin\left(\frac{60+30}{2}\right)}{\sin\left(\frac{60}{2}\right)}$

$= \frac{\sin 45°}{\sin 30°}$

$= \sqrt{2}$

$= 1.414$

49. Given: $i = 60°$, $A = 60°$

At minimum deviation position,

$i = \frac{A+\delta_m}{2}$

$\therefore \quad \delta_m = 2\,i - A = 60°$

Using prism formula,

$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin(60°)}{\sin(30°)} = \sqrt{3} = 1.732$

50. $\delta_m = A = 60°$

$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$= \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$= \frac{\sin(A)}{\sin\left(\frac{A}{2}\right)}$

$= \frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$= 2\cos\left(\frac{A}{2}\right)$

$$\mu = 2\cos\left(\frac{60^\circ}{2}\right) \qquad \text{(As A = 60°)}$$

$$\mu = 2\cos(30^\circ) = \sqrt{3}$$

51. $\delta_m = A$, $\mu = 1.5$

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin\left(\frac{2A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin A}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$\therefore$ $\mu = 2\cos\left(\frac{A}{2}\right)$

$\therefore$ $1.5 = 2\cos\left(\frac{A}{2}\right)$

$\therefore$ $\frac{3}{4} = \cos\frac{A}{2}$

$\therefore$ $\frac{A}{2} = \cos^{-1}(0.75)$

$= 90^\circ - \sin^{-1}(0.75)$

$= 90^\circ - 48^\circ 36'$

$= 41^\circ 24'$

$\therefore$ $A = 82^\circ 48'$

52. $$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Substituting the values,

$$\sqrt{2}\sin\left(\frac{60^\circ}{2}\right) = \sin\left(\frac{60^\circ+\delta_m}{2}\right)$$

….($\because$ Prism is equilateral)

$$\sqrt{2}\times\frac{1}{2} = \sin\left(\frac{60^\circ+\delta_m}{2}\right)$$

$$\frac{1}{\sqrt{2}} = \sin\left(\frac{60^\circ+\delta_m}{2}\right)$$

$$\sin(45^\circ) = \sin\left(\frac{60^\circ+\delta_m}{2}\right)$$

$90^\circ = 60^\circ + \delta_m$ or $\delta_m = 30^\circ$

$$i = \frac{A+\delta_m}{2} = \frac{60^\circ+30^\circ}{2}$$

$\therefore$ $i = 45^\circ$

53. At the minimum deviation δ_m the refracted ray inside the prism becomes parallel to its base.

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\sqrt{3} = \frac{\sin\left(\frac{60^\circ+\delta_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$\sqrt{3}\sin 30^\circ = \sin\left(\frac{60^\circ+\delta_m}{2}\right)$$

$$\frac{\sqrt{3}}{2} = \sin\left(\frac{60^\circ+\delta_m}{2}\right)$$

$$60^\circ = \frac{60^\circ+\delta_m}{2}$$

$\delta_m = 60^\circ$

As $\delta_m = 2i - A$,

where i is the angle of incidence

$\therefore$ $i = \theta$

$\therefore$ $$\theta = \frac{\delta_m + A}{2} = \frac{60^\circ+60^\circ}{2} = 60^\circ$$

54. $$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\sin\left(\frac{A+\delta_m}{2}\right) = \mu\sin\left(\frac{A}{2}\right)$$

$$\sin\left(\frac{60^\circ+\delta_m}{2}\right) = 1.6\sin\left(\frac{60^\circ}{2}\right)$$

$$\sin\left(\frac{60^\circ+\delta_m}{2}\right) = 0.8$$

$$45^\circ < \frac{60^\circ+\delta_m}{2} < 60^\circ$$

$90^\circ < 60^\circ + \delta_m < 120^\circ$

$30^\circ < \delta_m < 60^\circ$

55. Using prism formula,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad \left(\because \mu = \cot\frac{A}{2}\right)$$

$\therefore$ $\sin\left(\frac{A+\delta_m}{2}\right) = \cot\frac{A}{2}\ \sin\frac{A}{2}$

$\therefore$ $\sin\left(\frac{A+\delta_m}{2}\right) = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}\sin\frac{A}{2}$

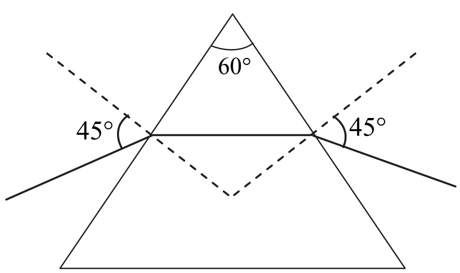
$\therefore$ $\sin\left(\frac{A+\delta_m}{2}\right) = \sin\left(\frac{\pi}{2}-\frac{A}{2}\right)$

$\Rightarrow \frac{A+\delta_m}{2} = \frac{\pi}{2}-\frac{A}{2}$

$\therefore$ $A + \delta_m = \pi^c - A$

$\Rightarrow \delta_m = 180° - 2A$

56.



As ray suffers minimum deviation,

i = e

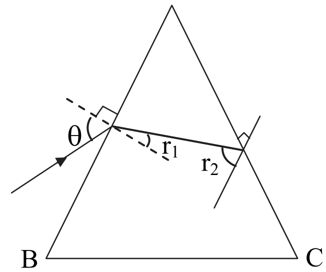
$\therefore$ $\delta_m = (i + e) - A = (45° + 45°) - 60° = 30°$

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin\left(\frac{60+30}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$= \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

57.



Using Snell's law,

$\sin\theta = \mu \sin r_1$

$\Rightarrow \sin r_1 = \frac{\sin\theta}{\mu}$

$\therefore$ $r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$

$\Rightarrow r_2 = A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$ ....(i)

$\therefore$ $r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$

Substituting for r_2 in equation (i),

$\Rightarrow A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\frac{1}{\mu}\right)$

$\therefore$ $A - \sin^{-1}\left(\frac{1}{\mu}\right) < \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$

$\therefore$ $\sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right] < \frac{\sin\theta}{\mu}$

$\therefore$ $\mu\left\{\sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]\right\} < \sin\theta$

$\therefore$ $\sin^{-1}\left\{\mu\sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]\right\} < \theta$

58. From the given data,

$i + e = A + \delta$

$A = i + e - \delta = 35° + 79° - 40° = 74°.$

Now, $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} < \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\therefore$ $\mu < \frac{\sin\left(\frac{74°+40°}{2}\right)}{\sin\left(\frac{74°}{2}\right)}$ $\therefore$ $\mu < \frac{\sin 57°}{\sin 37°}$

$\mu < 1.39$

The nearest value amongst given options is 1.5

59. For thin prism,
$\delta = (\mu - 1)A$
$\therefore$ $3.6 = (1.6 - 1)A$
$\therefore$ $A = 6°$

60. $\theta = (\mu_v - \mu_r)A = (1.66 - 1.64) \times 10° = 0.2°$

61. $\dfrac{\delta_v - \delta_r}{\delta_{mean}} = \omega$
Angular dispersion $= \delta_v - \delta_r = \omega\delta_{mean}$

62. To have dispersion without deviation,
$(\mu - 1)A + (\mu' - 1')A' = 0$
$(1.5 - 1)3 + (1.6 - 1)A' = 0$
$A' = \dfrac{(1.5-1)\times 3}{(1.6-1)} = \dfrac{0.5\times 3}{0.6}$
(Neglecting negative sign)
$A' = 2.5°$

63. The condition for no deviation is given by,
$\dfrac{A'}{A} = \dfrac{(\mu-1)}{(\mu'-1)}$
$\therefore$ $A(\mu - 1) = A'(\mu' - 1)$
$\therefore$ $10(1.42 - 1) = A'(1.7 - 1)$
$\therefore$ $A' = 6°$

64. $(\mu_v - \mu_r) + A'(\mu'_v - \mu'_r) = 0°$
$A' = 5°$

65. ω depends only on nature of material.

66. $\theta_{net} = \theta + \theta' = 0 \Rightarrow \omega d + \omega' d' = 0$
(θ = Angular dispersion $= \omega\delta_y$)

67. In Rainbow formation, dispersion and total internal reflection both take place.

70. According to Rayleigh's law of scattering, for scattering of light, size of particle must be comparable to wavelength of light. Hence, small size dust particle scatter smaller wavelength.

72. According to the theory of scattering by Rayleigh, the scattered intensity $\propto \dfrac{1}{\lambda^4}$.

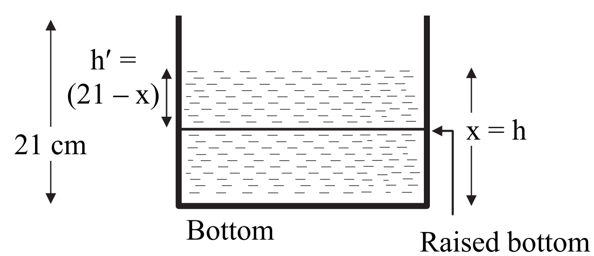
73. A red object looks red because it reflects only red colour and absorbs all other colours present in the white light. So, when red object is seen in the yellow light, it absorbs yellow colour falling on it and appears dark.
According to Rayleigh scattering, intensity of scattered light is inversely proportional to fourth power of wavelength. Since, red colour has largest wavelength, therefore this colour will be scattered least as compared to other colours.

74. According to Rayleigh scattering,
$I \propto \dfrac{1}{\lambda^4}$
$\therefore$ $\dfrac{I_2}{I_1} = \left(\dfrac{\lambda_1}{\lambda_2}\right)^4 = \left(\dfrac{8000\times 10^{-10}}{4000\times 10^{-10}}\right)^4 = 2^4 = 16$

79. $\mu = \dfrac{1}{\sin i_C} = \dfrac{1}{\sin 30°} = 2$
$\therefore$ $v = \dfrac{3\times 10^8}{2} = 1.5 \times 10^8$ m/s

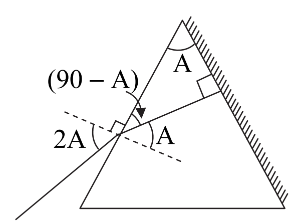
81. Apparent depth of bottom,
$= \dfrac{H/4}{n_1} + \dfrac{H/4}{n_2} + \dfrac{H/4}{n_3} + \dfrac{H/4}{n_4}$
$= \dfrac{H}{4}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2} + \dfrac{1}{n_3} + \dfrac{1}{n_4}\right)$

82. To see the container half-filled from the top, water should be filled up to height x so that bottom of the container should appear to be raised upto height (21 – x).
As shown in figure apparent depth
$h' = (21 - x)$
Real depth $h = x$



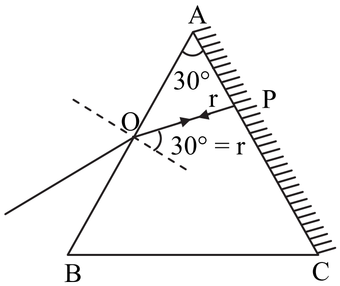
$\therefore$ $\mu = \dfrac{h}{h'} \Rightarrow \dfrac{4}{3} = \dfrac{x}{21 - x} \Rightarrow x = 12$ cm

83. Normal incidence at silvered surface



$\therefore$ $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin 2A}{\sin A} = \dfrac{2\sin A\cos A}{\sin A} = 2\cos A$

84. The ray will retrace its path from mirror if it falls normal to the surface of the mirror.



∴ $\angle AOP = 60°$ and angle of refraction = 30°

Using Snell's law of refraction,

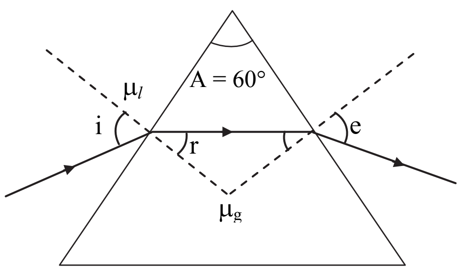
$$\mu = \frac{\sin i}{\sin r}$$

∴ $\sin i = \mu \times \sin r = \sqrt{2} \times \sin 30°$

$$= \frac{1}{\sqrt{2}}$$

∴ $i = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

85.



At minimum deviation,

$$_l\mu_g = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\frac{\mu_g}{\mu_l} = \frac{\sin\left(\frac{60° + 30°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$= \frac{\sin 45°}{\sin 30°} = \frac{1}{\sqrt{2}} \times 2$$

i.e., $\frac{\mu_l}{\mu_g} = \frac{1}{\sqrt{2}}$ ….(i)

Now critical angle for prism – medium interface,

$\sin(i_C) = \frac{\mu_l}{\mu_g} = \frac{1}{\sqrt{2}}$ ….[from (i)]

∴ $i_C = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

∴ $i_C = 45°$

86. In case of critical angle,

$$\mu = \frac{1}{\sin i_C}$$

For symmetry $i_C = 45°$

∴ $\mu = \frac{1}{\sin 45°} = \sqrt{2} = 1.414$

$\mu_r = 1.39$

$\mu_g = 1.44$

$\mu_v = 1.47$

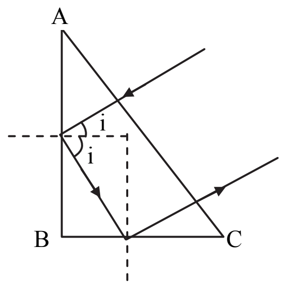
$\mu_r < \mu = 1.414$

while, $\mu_v > \mu$

$\mu_g > \mu$

Hence, only red colour part will not undergo total internal reflection and emerge out separately, while blue and green parts will suffer total internal reflection.

87.



For total internal reflection.

$i > i_C$ ….(i)

Also, from the symmetry of diagram,

⇒ $i = 45°$

∴ $\sin i > \sin i_C$ ….from(i)

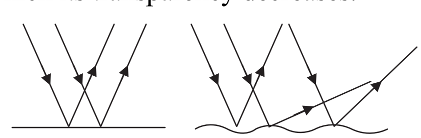
∴ $\frac{1}{\sin i} < \frac{1}{\sin i_C}$ but , $_a\mu_g = \frac{1}{\sin i_C}$

∴ $\frac{1}{\sin(45°)} < {}_a\mu_g$

∴ $\sqrt{2} < {}_a\mu_g$

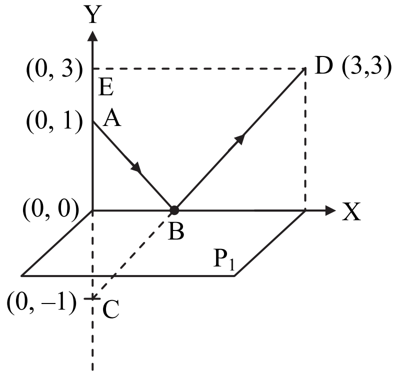
∴ Minimum value of $_a\mu_g = \sqrt{2}$

88. When glass surface is made rough then the light falling on it is scattered in different direction due to which its transparency decreases.



Smooth surface Rough surface

89.



Consider the ray AB is incident on plane P_1. After reflection the ray takes the path BD and passes through point D (3, 3). If the reflected ray is extended below X-axis, it intersects the Y-axis at point C (0, –1).
Hence, the path length of the ray can be calculated from C to D using Pythagoras theorem for ΔCED,
$CD^2 = CE^2 + DE^2$

∴ $CD = \sqrt{(4)^2 + (3)^2} = 5$ units

Evaluation Test

1. In total internal reflection, 100% of incident light is reflected back into the same medium and there is no loss of intensity. While in reflection from mirrors and refraction from lenses, there is some loss of intensity. Therefore images formed by total internal reflection are much brighter than those formed by mirrors or lenses.

2. $A = 60°$, $\delta_m = 40°$

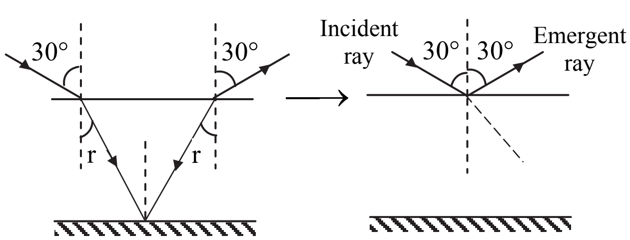
Hence,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60°+40°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$= 2 \sin 50° = 1.53$$

i.e., at $\mu = 1.53$, minimum deviation is 40° i.e., deviation ≥ 40° if $\mu \geq 1.53$.

3.



From the figure, it is clear that the angle between the incident ray and the emergent ray is 60°

4. $I = I_0^{e^{-\alpha x}}$ is an equation of decreasing exponential curve with I_0 as intercept on I-axis.

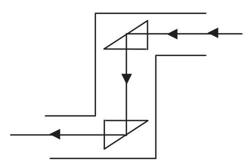
5. When light is incident from core (higher refractive index medium) to cladding (lower refractive index medium), the condition for total internal reflection of light is,

$$\frac{\mu_{core}}{\mu_{cladding}} = \frac{1}{\sin i_C}$$

If the angle of incidence of ray(y) in the core to cladding interface is greater than the critical angle i_C, the ray is totally internally reflected i.e., $y > i_C$.

Note: For this condition, x < the critical angle.

6.



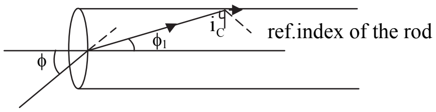
The principle of the periscope is that the image of an object (a ship for example) is formed at a lower level (in a submarine). Light is incident normal on a right angled prism which makes total internal reflection of the ray coming from the right at the hypotenuse of the prism. This is again reflected by another prism to give an image to a person in the lower level (say, in a submarine). This can be combined with telescopes.

7. If the distance travelled by a ray of light in two media are s_1 and s_2 in the same time 't_0' then the ratio of refractive index of the 2nd medium to 1st medium is given by

$${}_1\mu_2 = \frac{v_1}{v_2} = \frac{s_1}{s_2}$$

∴ $\frac{\mu_2}{\mu_1} = \frac{s_1}{s_2}$

∴ $\mu_2 = 1.5 \times \frac{4}{4.8} = 1.25$

8. 

Here incident angle is ϕ.
The light ray will graze along the rod, if it gets incident on rod at ciritical angle and will get reflected internally as shown in the figure above.

If i_C is the critical angle, $i_C = \sin^{-1}\frac{1}{\mu}$

But $i_C = 90° - \phi_1$.
From Snell's law,

$$\frac{\sin\phi}{\sin\phi_1} = \mu = \sqrt{3} \Rightarrow \frac{\sin\phi}{\cos i_C} = \mu.$$

But

$$\cos i_C = \frac{\sqrt{\mu^2 - 1}}{\mu} \qquad \left(\because \sin i_C = \frac{1}{\mu}\right)$$

$\therefore$ $$\sin\phi = \mu\frac{\sqrt{\mu^2-1}}{\mu} = \sqrt{\mu^2-1}$$

$$\Rightarrow \phi = \sin^{-1}\sqrt{3-1} = \sin^{-1}\left(\sqrt{2}\right)$$

Thus, for $\phi = \sin^{-1}\left(\sqrt{2}\right)$, light ray grazes along the wall of the rod.

9. The angle of deviation depends on the refractive index of prism. As μ decreases, δ decreases. Refractive index of prism relative to water is less than that relative to air. Hence, when a glass prism is immersed in water, the deviation caused by prism decreases.

10. As ABC is an isosceles right angled prism, angle of incidence of each ray is 45°. If critical angle for a colour, i_C is less than 45°, the ray of that colour will be totally internally reflected at AC. When $\angle i_C > 45°$, the ray will be transmitted through the face AC.
For red ray, $\mu = 1.39$

$$\sin i_{C_R} = \frac{1}{\mu} = \frac{1}{1.39} = 0.719 \Rightarrow i_{C_R} = 46.0°$$

Hence, red ray will be transmitted.
For blue ray, $\mu = 1.47$

$$\sin i_{C_B} = \frac{1}{\mu} = \frac{1}{1.47} = 0.68 \Rightarrow i_{C_B} = 42.8°$$

Hence, blue ray will be reflected at face AC.

11. For light waves, medium in which waves travel with lesser velocity is said to be denser medium. The velocity of light is more in water than in diamond. Hence water is rarer than diamond.

12. As both the diver as well as the fish are in water, refraction effects such as bending of light are not present.

13. Here, $\sin\theta_1 = \frac{1}{\mu_g} = \frac{1}{3/2} = 0.6666$

And $\sin\theta_2 = \frac{1}{\mu_w} = \frac{1}{4/3} = \frac{3}{4} = 0.75$

As $\mu_g > \mu_w$

$\therefore$ $\theta_1 < \theta_2$

If θ is the critical angle between glass and water then,

$$\sin\theta = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9} = 0.8888$$

$\therefore$ $\theta > \theta_2$.

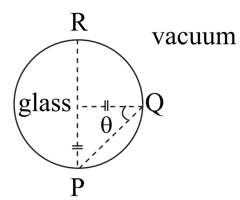
14. $\mu_v > \mu_b > \mu_g > \mu_y$

But $i_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow (i_C)_y > (i_C)_g > (i_C)_b > (i_C)_V$

15. This is case of total internal reflection.

$\therefore$ $\theta > i_C \left(= \sin^{-1}\frac{1}{\mu}\right)$



$\frac{1}{\mu} < \sin\theta$

$\frac{1}{\mu} < \sin 45°$

$\mu > \sqrt{2}$

$\because$ $v = \frac{c}{\mu}$

$\therefore$ $v < \frac{c}{\sqrt{2}} = \frac{3\times10^8}{\sqrt{2}}$

$v < 2.1 \times 10^8$

$\therefore$ only (B) is not possible.

17. Alexandar's dark band between the primary and secondary rainbows is because light scattered into this region interferes destructively. Further, primary rainbow subtends an angle of 41° – 43° at the eye of the observer w.r.t. the incident light, and secondary rainbow subtends an angle of about 51° to 54° at the eye of the observer w.r.t. the incident light. Therefore, the region between the angles of 41° to 51° is dark.

Textbook Chapter No.

09 Ray Optics

Hints

Classical Thinking

12. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{1}{-30}+\frac{1}{-10}=-\frac{1}{30}-\frac{1}{10}$

$\therefore$ $f=-\frac{15}{2}$ cm

and $R=2f=-\frac{15}{2}\times 2=-15$ cm

17. $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}=\frac{1}{-24}+\frac{1}{-40}=\frac{-1}{15}$

$\therefore$ f = –15 cm

25. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\therefore$ $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{+15}-\frac{1}{-12}=\frac{1}{15}+\frac{1}{12}$

$\therefore$ v = +6.7 cm

26. $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}=\frac{1}{5}+\frac{1}{-25}=\frac{1}{5}-\frac{1}{25}=\frac{4}{25}$

$\therefore$ f = +6.25 cm

f is positive therefore the mirror is convex.

31. $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$\therefore$ $\frac{1.5}{v}-\frac{1}{-20}=\frac{1.5-1}{5}$

$\therefore$ v = 30 cm

37. $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\therefore$ $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{0.15}+\frac{1}{-0.2}=\frac{5}{3}$

$\therefore$ $v=\frac{3}{5}=0.6$ m

38. $m=\frac{v}{-u}=3$

$\therefore$ v = –3u

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{-3u}-\frac{1}{u}=\frac{4}{-3u}$

$\therefore$ $u=-\frac{4f}{3}$

39. (–u) + v = 54 and $m=\frac{v}{-u}=2$

$\therefore$ v = – 2u

$\therefore$ (–u) + (–2u) = 54

$\therefore$ u = –18 cm

$\therefore$ v = –2 (–18) = 36 cm

Also $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\therefore$ $\frac{1}{36}-\frac{1}{-18}=\frac{1}{f}$

$\therefore$ $\frac{1}{36}+\frac{1}{18}=\frac{1}{f}$

$\therefore$ f = 12 cm

40. $\frac{1}{f}=\left({}_a\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$,

R_1 = R, R_2 = –R

$\therefore$ $\frac{1}{f}=(1.5-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=(0.5)\left(\frac{2}{R}\right)=\frac{1}{R}$

$\therefore$ f = R = 30 cm

41. $\frac{1}{f}=\left({}_a\mu_g-1\right)\left(\frac{1}{R}\right)$

$\therefore$ $R_2=\infty$ and R_1 = R

$\therefore$ $\frac{1}{20}=(1.5-1)\left(\frac{1}{R}\right)$

$\therefore$ $\frac{1}{20}=\frac{0.5}{R}$

$\therefore$ R = 10 cm

44. $m=\frac{v}{u}=\frac{1}{4}$

$\therefore$ $v=\frac{u}{4}$

Also $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\therefore$ $\frac{4}{u}-\frac{1}{u}=\frac{1}{+f}$

$\therefore$ $\frac{3}{u}=\frac{1}{f}$

$\therefore$ u = 3f

45. $\frac{1}{f} = \left({}_a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.6 - 1)\left(\frac{1}{20} - \frac{1}{30}\right)$

$\therefore$ $\frac{1}{f} = -(0.6)\left(\frac{1}{12}\right) = -\frac{1}{20}$

$\therefore$ f = –20 cm

46. $\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\therefore$ $|R_1| = |R_2| = R$

$\therefore$ $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-R} - \frac{1}{+R}\right)$

$\frac{1}{f} = -(0.5)\left(\frac{2}{R}\right)$

$\therefore$ f = –R = –30 cm

51. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{15} + \frac{1}{30} = \frac{1}{10}$

$\therefore$ f = 10 cm

52. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{f} = \frac{1}{+30} + \frac{1}{-20} = \frac{1}{30} - \frac{1}{20} = -\frac{1}{60}$

$\therefore$ f = –60 cm

53. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\therefore$ $\frac{1}{13} = \frac{1}{10} + \frac{1}{f_2}$

$\therefore$ $\frac{1}{f_2} = \frac{1}{13} - \frac{1}{10} = -\frac{3}{130}$

$\therefore$ $f_2 = -\frac{130}{3} = -43.33$ cm

63. $M.P = \left(\frac{D}{f} + 1\right) = \left(\frac{25}{2.5} + 1\right) = (10 + 1) = 11$

70. $M_o = \frac{M.P}{M_e} = \frac{35}{1 + \frac{D}{f_e}} = \frac{35}{1 + \frac{25}{8}}$

$\therefore$ $M_o \approx 8.48$

79. $M.P = -\frac{f_o}{f_e} = -\frac{2}{0.05} = -40$

80. $M.P = \left|\frac{f_o}{f_e}\right|$

If $f_e' = 2 f_e$,

then $M.P' = \left|\frac{f_o}{2f_e}\right| = \frac{1}{2}\left|\frac{f_o}{f_e}\right| = \frac{M.P}{2}$

81. $M.P = \frac{f_o}{f_e} = 10$

$\therefore$ $f_o = 10 f_e$

Also $L = f_o + f_e = 44$ cm

$\therefore$ $10f_e + f_e = 44$

$\therefore$ $11 f_e = 44$

$\therefore$ $f_e = 4$ cm and $f_o = 44 - 4 = 40$ cm

85. $P = P_1 + P_2 = (+15) + (-5) = +10$ D

$\therefore$ $f = \frac{1}{P} = \frac{1}{+10} = 0.1$ m = 10 cm

89. $M.P = \left(\frac{D}{f} + 1\right)$

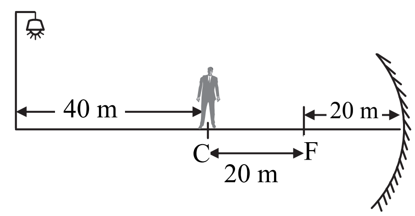
$\therefore$ $6 = \frac{25}{f} + 1$

$\therefore$ $f = 5$ cm $= 5 \times 10^{-2}$ m

$\therefore$ $P = \frac{1}{5 \times 10^{-2}} = \frac{100}{5} = 20$ D

Critical Thinking

3.



When the boy moves by 40 m towards the mirror, he reaches at centre of curvature (2F) of mirror. Hence his image formed is inverted and of same size. The lamp lies between infinity and centre of curvature hence image formed is inverted and diminished.

4. Concave mirror forms inverted and enlarged image when object is placed between focus and centre of curvature, while convex mirror always forms erect and diminished image. As the distance of person is not changed from the mirror, mirror B cannot be concave.

5. If plane mirror is rotated through ‘θ’, reflected ray would rotate through double the angle i.e., 2θ.

6. Given $u = (f + x_1)$ and $v = (f + x_2)$

The focal length $f = \frac{uv}{u+v} = \frac{(f + x_1)(f + x_2)}{(f + x_1) + (f + x_2)}$

On solving, $f^2 = x_1x_2$

$\Rightarrow$ $f = \sqrt{x_1x_2}$

7. At u = f, v = ∞
At u = 0, v =0 (i.e., object and image both lies at pole) Satisfying these two condition, only option (A) is correct.

8. $\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_a} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_a} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ….(i)

$\frac{1}{f_w} = \left(\frac{1.5}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ….(ii)

solving equations (i) and (ii),
$f_a (0.5) = f_w (0.125)$

∴ $f_w = \frac{10 \times 0.5}{0.125}$

∴ $f_w = 40$ cm

9. Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
u is always negative,
v is positive.

10. If $n_1 > n_g$ then the lens will be in more dense medium. Hence its nature will change and the convex lens will behave like a concave lens. ***(Refer Shortcut 2.)***

11. m = f/(u – f) = f/*x*.

12. $\frac{1}{v} - \frac{1}{-f/2} = \frac{1}{f}$

∴ v = – f

$m = \frac{v}{u} = \frac{-f}{-f/2} = 2$

The image is virtual, double the size.

13. For convex lens, $P = \frac{1}{f}$

Using lens maker's equation,

$\frac{1}{f} = (\mu_2 - 1)\left(\frac{2}{R}\right)$

$5 = (1.5 - 1)\frac{2}{R}$ ….(i)

$\Rightarrow \frac{1}{R} = 5$ /m

When the lens is placed in liquid, it acts like plano concave lens.

For concave lens, f = –100 cm = –1 m.

Using lens maker's equation,

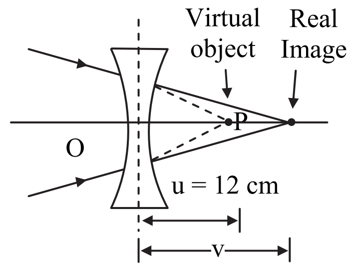
$\frac{1}{f} = (\mu_2 - 1)\left(\frac{1}{R}\right)$

Here $\mu_2 = {}_l\mu_g = \frac{{}_a\mu_g}{{}_a\mu_l} = \frac{1.5}{{}_a\mu_l}$

∴ $-1 = \left(\frac{1.5}{{}_a\mu_l} - 1\right)5$ [From (i)]

∴ ${}_a\mu_l = \frac{1.5}{\left(1 - \frac{1}{5}\right)} = \frac{1.5 \times 5}{4} = 1.875$

14.



By using lens formula

$\frac{1}{-16} = \frac{1}{v} - \frac{1}{(+12)} \Rightarrow \frac{1}{v} = \frac{1}{12} - \frac{1}{16} = \frac{4-3}{48}$

$\Rightarrow$ v = 48 cm

15. *l* = 90 cm, d = 20 cm

$f = \frac{l^2 - d^2}{4l}$ ***…(Using Shortcut 3)***

$= \frac{90^2 - 20^2}{4 \times 90} = \frac{8100 - 400}{360} \approx 21.4$ cm

17. As seen from a rarer medium (L_2 or L_3), the interface L_1L_2 is concave and L_2L_3 is convex. The divergence produced by concave surface is much smaller than the convergence produced by convex surface. Hence the arrangement corresponds to concavo-convex.

19. Let the resultant focal length of combination be f then,

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{f} = \frac{1}{20} + \frac{1}{(-20)} \Rightarrow f = \infty$

Hence, it behaves as a plane slab of glass.

20. For small value of f_o and f_e
$v_o \approx L$ and $u_o = f_o$
$$M.P. = -\frac{L}{f_o}\left(1+\frac{D}{f_e}\right)$$
$\therefore$ $$-375 = -\frac{15}{0.5}\left(1+\frac{25}{f_e}\right)$$
$\therefore$ $f_e = 2.17$ cm ≈ 2.2 cm.

21. The image distance from the eye lens remains constant because for healthy eye, image is always formed on retina.

22. $L = f_o + f_e = 1.53$ m ….(i)
$$|M.P| = \frac{f_o}{f_e} = 50$$
$50 f_e + f_e = 1.53$
$51 f_e = 1.53$ $\therefore$ $f_e = 0.03$ m
From equation (i) $f_o = 1.5$ m
$\therefore$ $f_o = 1.5$ m and $f_e = 0.03$ m

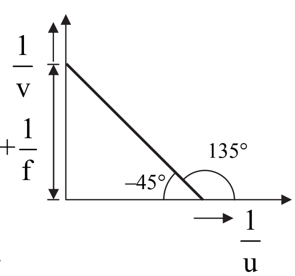
23. Telescope is used to observe distant object nearer.

24. $\frac{\beta}{\alpha} = \frac{f_o}{f_e} \Rightarrow \frac{\beta}{0.5^o} = \frac{100}{2} \Rightarrow \beta = 25°$

25. $$\frac{\text{Light gathered by A}}{\text{Light gathered by B}} = \frac{(D_A)^2}{(D_B)^2} = (3)^2 = 9$$

27. Since $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$
$$\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$$
Using the sign conventions,
$$\frac{1}{(-v)} = -\frac{1}{(-u)} + \frac{1}{(-f)}$$
$$\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$$
Comparing this equation with
y = mx + c
Slope = m = tanθ = –1
θ = 135° or –45° and intercept
$$c = +\frac{1}{f}$$

28. $u = -(75 - v)$
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$
$\therefore$ $$\frac{1}{v} - \frac{1}{-(75-v)} = \frac{1}{12}$$



$\therefore$ v = 60 cm or 15 cm
$\therefore$ | u | = 75 – 60 = 15 cm or
| u | = 75 – 15 = 60 cm

Magnification, $m = \frac{v}{-u} = \left|\frac{v}{u}\right| = \left|\frac{60}{15}\right| = 4$

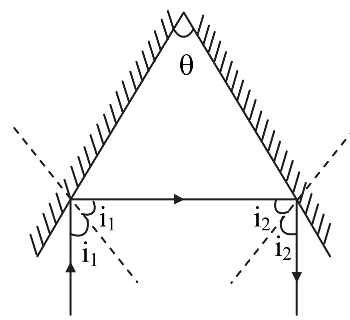
29. In each case two plane-convex lens are placed close to each other and $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$. Hence focal length is same for all given combinations.

30. Eye lens being convergent forms a real image of a virtual object (i.e., the virtual image being seen on the retina of the eye).

31. Reflection takes place in the same medium.

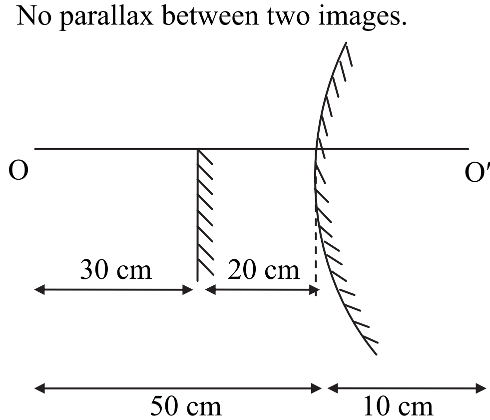
32. $\because$ $$P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
Thick lens has R less than thin lens, hence more power.

33.



Let the angle between the two mirrors be ‘θ’.
Total deviation $d = d_1 + d_2$
$= (180° - 2i_1) + (180° - 2i_2)$
$= 360° - 2(i_1 + i_2)$
Since the resultant ray is parallel
$\therefore$ d = 180°
$\Rightarrow 180° = 360° - 2(i_1 + i_2)$,
$\therefore$ $i_1 + i_2 = 90°$
But $i_1 + i_2 = \theta$
$\therefore$ θ = 90°

34. No parallax between two images.



$\frac{1}{f} = \frac{1}{-50} + \frac{1}{+10} = \frac{2}{25}$

$\therefore$ $f = \frac{25}{2}$, $R = 2f = 2 \times \frac{25}{2} = 25$ cm

35. $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-25} = -\frac{3}{50}$

$\therefore$ $v = -\frac{50}{3} = -16.67$ cm

$m = \frac{h_2}{h_1} = -\frac{v}{u}$

$\therefore$ $\frac{h_2}{+3} = \frac{\frac{-50}{3}}{-25} = -\frac{2}{3}$

$\therefore$ $h_2 = -2$ cm

Negative sign indicates real inverted image.

$\therefore$ Area $= 2 \times 2 = 4$ cm^2

36. From lens-maker's formula

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\therefore$ $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ and

$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

f_1 and f_2 are focal lengths corresponding to μ_1 and μ_2 respectively.

Hence, there are two focal lengths giving two images.

37. Since light transmitting area is same, there is no effect on intensity.

38. If a mirror is placed in a medium other than air its focal length does not change as $f = \frac{R}{2}$. But for the lens

$\frac{1}{f_a} = \left({}_a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_w} = \left({}_w\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

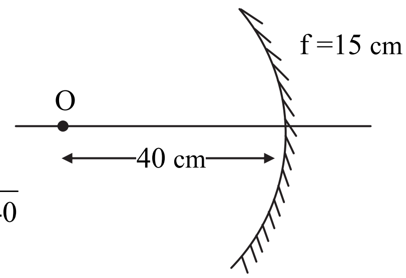
As ${}_w\mu_g < {}_a\mu_g$, hence focal length of lens in water increases.

The refractive index of water is $\frac{4}{3}$ and that of air is 1.

Hence, $\mu_w > \mu_a$.

Competitive Thinking

1. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$



$\therefore$ $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$= \frac{1}{-15} - \frac{1}{-40}$

$= \frac{-1}{24}$

$\therefore$ $v = -24$ cm

Negative sign indicates image is formed in front of the mirror.

Given: $u' = -20$ cm

Now, according to mirror formula,

$\frac{1}{v'} = \frac{1}{f} - \frac{1}{u'}$

$= \frac{1}{-15} - \frac{1}{-20}$

$= -\frac{1}{60}$

$\therefore$ $v' = -60$ cm

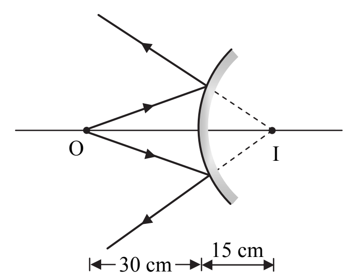
Negative sign indicates that image is formed in front of the mirror.

Displacement of image

$= v - v'$

= 36 cm away from mirror

2.



$u = -30$ cm, $f = +30$ cm

Using mirror formula

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$\frac{1}{30} = \frac{1}{v} + \frac{1}{(-30)}$

$v = 15$ cm, behind the mirror

4. As the medium has no role in focal length of mirror, it doesn't change.

5. Given: $R = -15$ cm,

$h_1 = 10$ cm,

$u = -10$ cm

From mirror formula,

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

∴ $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$= \frac{2}{R} - \frac{1}{u}$

$= \frac{2}{-15} - \frac{1}{-10}$

$= -\frac{1}{30}$

∴ v = –30

Magnification, $m = \frac{-v}{u} = \frac{-(-30)}{-10} = -3$

∴ The image formed is magnified and inverted.

6. For concave mirror,

$m = -3,\ f = \frac{R}{2} = -\frac{30}{2} = -15$ cm

Also for spherical mirrors,

$u = \left(\frac{m-1}{m}\right) f = \left(\frac{-3-1}{-3}\right)(-15) = (+4)(-5)$

∴ u = – 20 cm

7. From mirror formula,

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

We know,

$f = \frac{R}{2} = \frac{1}{2}$

∴ $\frac{1}{v} + \frac{1}{-1.5} = 2$

∴ v = s′ = 0.375 m

As the image distance is positive, image is virtual.

Magnification of a mirror,

$m = \frac{-v}{u} = \frac{-0.375}{-1.5} = \frac{3/8}{3/2} = \frac{1}{4} = 0.25$

As magnification is positive the image is erect (upright).

8. $f = \frac{R}{2}$

But for a plane mirror R = ∞

If f is expressed in metres, then power of mirror is given as

$P = \frac{1}{f} = \frac{1}{\infty} = 0$

9. From mirror formula,

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

∴ $v = \frac{fu}{u-f}$ ….(i)

Now, magnification of mirror is,

$m = \frac{v}{u}$

∴ $m = \frac{fu}{(u-f)u}$ ….[from (i)]

∴ $u - f = \frac{f}{m}$

∴ $u = \frac{(m+1)f}{m}$

u is kept same for both lenses,

∴ $u = \frac{(m_1+1)f_1}{m_1} = \frac{(m_2+1)f_2}{m_2}$

$\frac{f_1}{f_2} = \frac{m_1(1+m_2)}{m_2(1+m_1)}$

11. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

∴ $\frac{1.5}{v} - \frac{1}{(-15)} = \frac{(1.5-1)}{+30} \Rightarrow v = -30$ cm.

Negative sign shows that, image is obtained on the same side of object i.e., towards left.

12. Lens formula gives,

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

∴ $\frac{1}{f} = \frac{1}{75} - \frac{1}{-25}$

∴ $\frac{1}{f} = \frac{100}{75 \times 25}$

∴ $f = \frac{75}{4} = 18.75$ cm

As the focal length is positive, the lens is convex.

13. The convex lens can form enlarged and erect image only when the object is kept between pole and focus.

As f = 20 cm, u < 20 cm

14. According to lens maker's formula,

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

∴ $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{1}{R} \Rightarrow f = R$

15. $\mu_w = \frac{4}{3}$, $\mu_g = 1.5 = \frac{3}{2}$

$R_1 = +R$, $R_2 = -R$

According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ $$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R} - \frac{1}{(-R)}\right)$$

$$\frac{1}{f_a} = \left(\frac{1.5}{1} - 1\right)\left(\frac{2}{R}\right)$$

$$\frac{1}{f_a} = \frac{1}{R}$$

$$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R} - \frac{1}{(-R)}\right)$$

$$= \left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)\left(\frac{2}{R}\right)$$

$$= \frac{2}{8R}$$

$\therefore$ $f_w = 4R$

16. According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f_a} = \left(\frac{1.5}{1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f_a} = (0.5)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{….(i)}$$

$$\frac{1}{f_w} = \left(\frac{1.5}{\left(\frac{4}{3}\right)} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f_w} = (0.125)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{….(ii)}$$

Dividing equation (i) by equation (ii)

$\therefore$ $$\frac{f_w}{f_a} = \frac{0.5}{0.125}$$

$f_w = 4\ f_a = 4 \times 8 = 32$ cm

17. $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$R_1 = +20$ cm, $R_2 = -20$ cm, $\mu = 1.5$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-20}\right)$$

$$= 0.5\left(\frac{1}{20} + \frac{1}{20}\right)$$

$$= 0.5 \times \frac{2}{20} = \frac{0.5}{10}$$

$\therefore$ $f = 20$ cm.

Parallel rays converge at focus. Hence, L = f.

18. $$\frac{1}{f} = ({}_g\mu_a - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{2}{3} - 1\right)\left(\frac{2}{10}\right)$$

$f = -15$ cm, so behaves as concave lens.

19. The focal length of a plano-convex lens is,

$$f = \frac{R}{\mu - 1}$$

$\therefore$ $$f = \frac{60}{1.5 - 1} = \frac{60}{0.5} = 120 \text{ cm}$$

20. Focal length of combination,

$$\frac{1}{f} = \frac{1}{f_{concave}} + 2\left(\frac{1}{f_{convex}}\right)$$

$$= \frac{2(\mu_{oil} - 1)}{-R} + 2\left(\frac{\mu_{lens} - 1}{R}\right)$$

$$= \frac{2(1.7 - 1)}{-R} + 2\left(\frac{1.5 - 1}{R}\right)$$

$$= \frac{-1.4}{R} + \frac{1}{R} = \frac{-0.4}{R}$$

$\therefore$ $$f = -\frac{R}{0.4} = \frac{-20}{0.4} = -50 \text{ cm}$$

21. By Lens maker's formula,

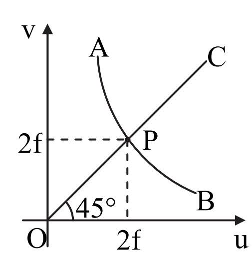
$$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{-1}{10}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\Rightarrow f_1 = 4f$ and $f_2 = -5f$

22. The experimental plot of v vs u is represented by curve AB. Let line OC meet the curve at point P.



23. For bifocal convex lens:

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$= \frac{(\mu-1)\times 2}{R} \qquad \ldots.(R_1 = R_2 = R)$$

For plane surface: $R_2 = \infty$

For half plane-convex lens:

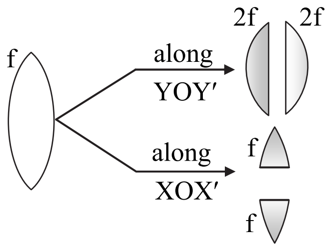
$$\frac{1}{f'} = (\mu-1)\frac{1}{R}$$

$$\frac{1/f}{1/f'} = \frac{(\mu-1)}{R} \times 2 \times \frac{R}{\mu-1} = 2$$

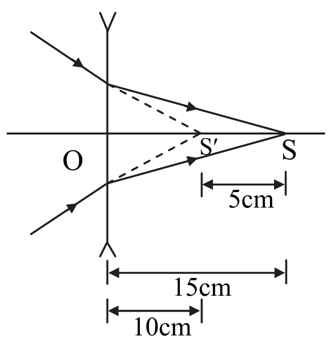
$$\frac{f'}{f} = 2$$

$\Rightarrow f' = 2f$

24.



25.



Using lens equation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Substituting u = 10 cm, v = 15 cm,

$$\frac{1}{15} - \frac{1}{10} = \frac{1}{f} \Rightarrow f = -30 \text{ cm}$$

26. $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{10} - \left(\frac{1}{-10}\right) = \frac{2}{10}$$

$\Rightarrow f = 5$ cm

$$f = \frac{uv}{u+v}$$

$$\frac{\Delta f}{f} = \left|\frac{\Delta u}{u}\right| + \left|\frac{\Delta v}{v}\right| + \frac{|\Delta u| + |\Delta v|}{|u| + |v|}$$

$\Delta f = 0.15$ [for f = 5 cm]

The most appropriate answer is 5.00 ± 0.10 cm

27. Using lens equation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Here, u = – 25 cm and v = – 75 cm

$$\therefore \quad -\frac{1}{75} - \left(\frac{-1}{25}\right) = \frac{1}{f}$$

$\Rightarrow f = 37.5$ cm

28. Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For first lens: $u_1 = -4$ m, $f_1 = 2$ m

$$\therefore \quad \frac{1}{v_1} = \frac{1}{2} + \frac{1}{(-4)} = \frac{1}{4}$$

$\Rightarrow v_1 = 4$ m

For 2nd lens:

$\therefore$ image formed by first lens will act like source.

$u_2 = 1$ m and $f_2 = 1$ m

$$\therefore \quad \frac{1}{v_2} = \frac{1}{1} + \frac{1}{1} = 2$$

$\Rightarrow v_2 = 0.5$ m

$\therefore$ Distance from object = 4 + 3 + 0.5 = 7.5 m

29. $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} + \frac{1}{-f}$$

$\Rightarrow f = \infty$

30. Focal length of first lens,

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R}$$

Focal length of second lens,

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{(\mu_1 - 1)}{R}$$

So focal length of the combination,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{\mu_1 - 1}{R} - \frac{(\mu_2 - 1)}{R}$$

$$\frac{1}{f} = \frac{\mu_1 - \mu_2}{R}$$

$$\therefore \quad f = \frac{R}{\mu_1 - \mu_2}$$

31. According to lens maker's formula, the focal length of plano-convex lens is

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\therefore \quad \frac{1}{f_1} = (1.6 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{0.6}{R}$$

Similarly focal length of concavo plane lens is

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$

$\therefore \quad \frac{1}{f_2} = (1.5 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{0.5}{R}$

For the combination of lenses,

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{0.6}{R} + \left(-\frac{0.5}{R}\right) = \frac{0.1}{R} \Rightarrow f = \frac{R}{0.1}$

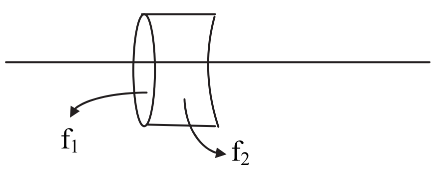
32. Power of the combination $P = P_1 + P_2$

$= 12 - 2 = 10$ D

$\therefore$ Focal length of the combination

$f = \frac{100}{P} = \frac{100}{10} = 10$ cm

33.



Given: $\frac{P_1}{P_2} = \frac{4}{3}$

As $P = \frac{1}{f}$

$\therefore \quad \frac{1}{f_1} \times f_2 = \frac{4}{3}$

$\therefore \quad \frac{f_2}{f_1} = \frac{4}{3}$

$\therefore \quad f_2 = \frac{4}{3} \times 12$ (Given: $f_1 = 12$ cm)

$f_2 = 16$ cm

As the lens is concave,

$f_2 = -16$ cm

then, focal length of combination is given by

$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2}$

$\therefore \quad \frac{1}{f_{eff}} = \frac{1}{12} - \frac{1}{16} = \frac{4-3}{48}$

$f_{eff} = 48$ cm

34. Focal length of combination,

$\frac{1}{f_{com}} = \frac{1}{f_{convex}} + \frac{1}{f_{concave}} + \frac{1}{f_{convex}}$

By lens maker's formula,

$\frac{1}{f_{convex}} = (ug - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$= \left(\frac{3}{2} - 1\right)\left[\frac{2}{R}\right]$

$\therefore \quad \frac{1}{f} = \frac{1}{R}$ ...(i)

$\frac{1}{f_{concave}} = (u_w - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$\therefore \quad \frac{1}{f_{concave}} = \left(\frac{1}{3}\right)\left[\frac{-2}{R}\right]$

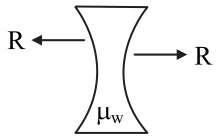
$= \frac{-2}{3R}$ ...(ii)

$\therefore \quad \frac{1}{f_{com}} = \frac{2}{R} - \frac{2}{3R}$ ...[from (i) and (ii)]

But $R = f$ ...[from (i)]

$\therefore \quad f_{com} = \frac{3R}{4} = \frac{3f}{4}$

35.



$\frac{1}{f_w} = (\mu_w - 1)\left(\frac{-1}{R} - \frac{1}{R}\right)$

$\therefore \quad \frac{1}{f_w} = (\mu_w - 1)\left(-\frac{2}{R}\right)$

But, $\frac{1}{f_l} = (\mu_l - 1)\left(\frac{1}{R} + \frac{1}{R}\right) = (\mu_l - 1)\left(\frac{2}{R}\right)$

$\therefore \quad \frac{2}{R} = \frac{1}{(\mu_l - 1) f_l}$

$\therefore \quad \frac{1}{f_w} = -\frac{\mu_w - 1}{\mu_l - 1}\left(\frac{1}{f_l}\right)$

$\frac{1}{f_{eq}} = \frac{1}{F} = \frac{2}{f_l} - \frac{1}{f_l}\left(\frac{\mu_w - 1}{\mu_l - 1}\right) = \frac{1}{f_l}\left(2 - \frac{\mu_w - 1}{\mu_l - 1}\right)$

Given: $\mu_l > \mu_w$

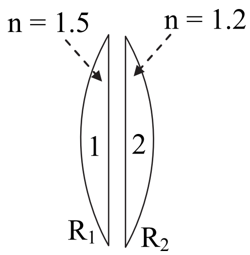
$\mu_l - 1 > \mu_w - 1$

$\Rightarrow \frac{\mu_w - 1}{\mu_l - 1} < 1$

$\therefore \quad \frac{1}{f_l} < \frac{1}{F} < \frac{2}{f_l}$

$\therefore \quad \frac{f}{2} < F < f$

36.



$$\frac{1}{f_1}(\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For $\mu = n = 1.5$ and $R_1 = 14$ cm

$$\frac{1}{f_1} = (1.5 - 1)\left[\frac{1}{14} - \frac{1}{\infty}\right] = \frac{0.5}{14}$$

For $\mu = n = 1.2$ and $R_2 = -14$ cm

$$\frac{1}{f_2} = (1.2 - 1)\left[\frac{1}{\infty} - \frac{1}{-14}\right] = \frac{0.2}{14}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{0.5}{14} + \frac{0.2}{14} = \frac{0.7}{14}$$

Using lens equation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{v} = \frac{7}{140} - \frac{1}{40} = \frac{1}{20} - \frac{1}{40}$$

$\therefore \quad \frac{1}{v} = \frac{2-1}{40}$

$\therefore \quad v = 40$ cm

37. For lens separated by distance d,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$\therefore \quad \frac{1}{f} = \frac{f_1 + f_2}{f_1 f_2} - \frac{d}{f_1 f_2}$

$\therefore \quad \frac{1}{f} = \frac{f_1 + f_2 - d}{f_1 f_2}$

But, $P = \frac{1}{f}$ (if focal length is measured in metres)

$\therefore \quad P = \frac{f_1 + f_2 - d}{f_1 f_2}$

Thus, for $P = 0$, $d = f_1 + f_2$

38. The image formed by diverging lens will be virtual and at a distance $v_1 = -25$ cm.

This image acts as an object for the converging lens.

$\therefore \quad u_2 = -25 + (-15) = -40$ cm

$\therefore$ By lens formula,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$\therefore \quad \frac{1}{v_2} - \frac{1}{(-40)} = \frac{1}{20}$

$\therefore \quad v_2 = +40$ cm from the converging lens.

39. Magnifying power for simple microscope when image is formed at infinity,

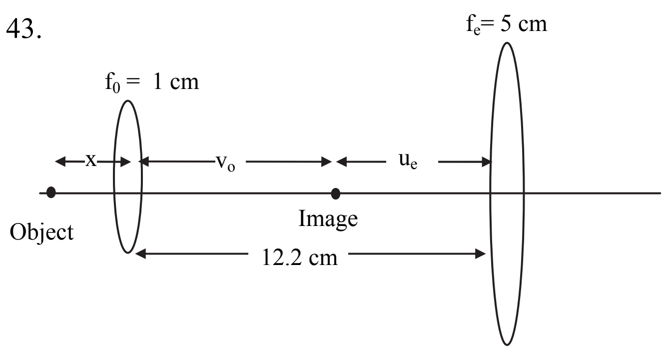
$$M = \frac{D}{f} = \frac{25}{12.5} = 2$$

40. $M.P = 1 + \frac{D}{f}$

$$M.P = 1 + \frac{25}{5} = 6$$

42. Intermediate image means the image formed by objective, which is real, inverted and magnified.

43.



Given: $f_0 = 1$ cm, $f_e = 5$ cm,

$L = v_0 + u_e = 12.2$ cm,

$v_e = -25$ cm

For eyepiece,

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \quad \text{....(i)}$$

$\therefore \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5} = \frac{-6}{25}$

$\therefore \quad u_e = \frac{-25}{6}$ cm

As u_e is on left side of eyepiece, from sign conventions, u_e is negative. Hence, neglecting negative sign,

$$u_e = \frac{25}{6} \text{ cm}$$

As, $L = v_0 + u_e = 12.2$ cm

$\therefore \quad v_0 = 12.2 - \frac{25}{6} = 8.03$ cm

For objective,

$$\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0}$$

$$\therefore \quad \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{8.03} - \frac{1}{1} = \frac{-7.03}{8.03}$$

$$\therefore \quad u_0 = -\frac{8.03}{7.03} = -1.14 \text{ cm}$$

45. $|m| = \frac{f_o}{f_e} = 5$

$\therefore$ $f_o = 5f_e$

$L = f_o + f_e = 36$

$\therefore$ $6f_e = 36$

$\therefore$ $f_e = 6$ cm, $f_o = 30$ cm

46. $m = \frac{f_o}{f_e} = \left(1 + \frac{f_e}{D}\right)$

47. Three lenses are: objective, eye piece and erecting lens.

49. Given: $f_o = 40$ cm, $f_e = 4$ cm

For objective,

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\therefore \quad \frac{1}{v_o} - \frac{1}{-200} = \frac{1}{40}$$

$$\frac{1}{v_0} = \frac{1}{40} - \frac{1}{200} = \frac{5-1}{200} = \frac{1}{50}$$

$v_o = 50$ cm

For normal adjustment, $L = v_o + f_e = 54$ cm

50. As, $m = f_o/f_e$, $f_o = m \times f_e = 10 \times 3 = 30$ cm

For an object at 180 cm from objective, the image formed by objective is at a distance v_o.

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\therefore \quad \frac{1}{v_o} = \frac{1}{30} + \frac{1}{-180}$$

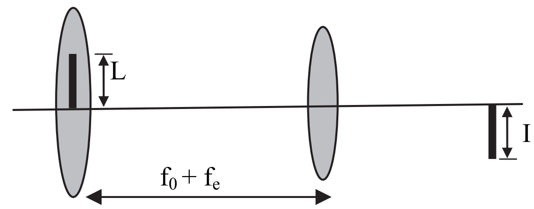
$$\therefore \quad \frac{1}{v_o} = \frac{1}{30} - \frac{1}{180}$$

$v_o = 36$ cm

Now if this image is made to fall at focus of eyepiece so that final image is at infinity, the total length of telescope would now be

$L = v_o + f_e = 36 + 3 = 39$ cm

51.



Magnification of telescope:

$$M = -\frac{f_o}{f_e}$$

For a convex lens:

$$M = \frac{f_e}{f_e + u} = -\frac{I}{O}$$

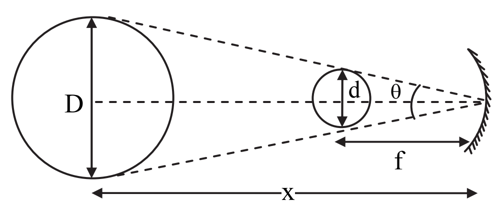
The object being line on objective, $u = f_o + f_e$ and $O = L$

$$\therefore \quad \frac{f_e}{f_e - (f_0 + f_e)} = -\frac{I}{L}$$

$$\therefore \quad -\frac{f_e}{f_o} = -\frac{I}{L}$$

$$\therefore \quad M = \frac{L}{I}$$

52.



From geometry of given figure,

$$\theta = \frac{D}{x} = \frac{d}{f}$$

$$\therefore \quad d = \frac{D}{x} \times f$$

$$= \frac{D}{x} \times \frac{R}{2}$$

….(R = radius of curvature of mirror)

$\therefore$ $d = 0.009 \times 0.2$ ….$\left(\because \text{Given: } \frac{D}{x} = 0.009\right)$

$= 1.8 \times 10^{-3}$ m

53. The rays incident from object on the lens travel parallel after refraction. These parallel rays are incident on plane mirror and trace back their path after reflection.

Hence, the final image will be formed on object itself.

54. $\frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

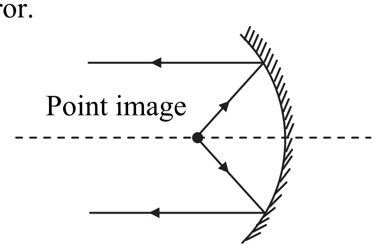
$\frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(\frac{1}{-R} - \frac{1}{R}\right)$

$= \frac{1}{3.5R}$

f = 3.5 R

In the medium it behaves as a convergent lens.

55. Object should be placed at focus of a concave mirror.



56. In case of mirrors, convex mirror always produces diminished and virtual images.

Hence, convex mirror cannot have magnification m > 1.

Also, in mirrors, virtual image is always formed on right hand side. Hence, magnification produced is always positive. (i.e., m for virtual image, $(m = +\frac{1}{2})$ or (m = +2). These conditions are satisfied by option (C).

57. By mirror formula:

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

As u > f, image formed is real,

$\therefore \quad \frac{1}{-u} + \frac{1}{-v} = -\frac{1}{f}$

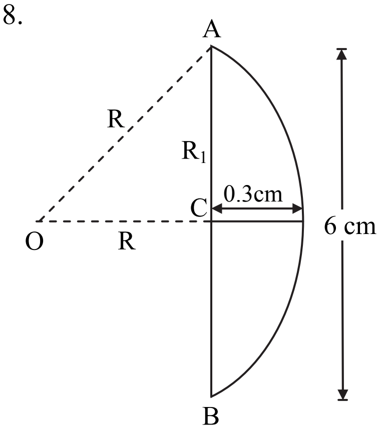
$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$\therefore \quad v = \frac{uf}{u-f}$

The image of the nearer end will be formed at distance v, while the other end of rod is at infinite distance, hence its image will be formed at focus.

$\therefore \quad L = |v| - |f| = \frac{uf}{u-f} - f = \frac{f^2}{u-f}$

58.



R. I. of lens, $\mu = \frac{c}{v} = \frac{3\times10^8}{2\times10^8} = 1.5$

As D_1 = 6 cm, R_1 = 3 cm.

$\therefore$ From ΔACO, radius of curvature of lens is,

$R^2 = R_1^2 + (R - 0.3)^2$

$R^2 = 3^2 + (R - 0.3)^2$

$R^2 = 9 + R^2 + 0.09 - 0.6\ R$

0.6 R = 9.09

R = 15.15 cm.

$F = \frac{R}{\mu - 1} = \frac{15.15}{1.5 - 1} = 30.3$ cm

59. $\mu = \frac{c}{v} = \frac{f\lambda_{air}}{f\lambda_{med}} = \frac{3}{2}$

Given, v = +8 m,

$m = \frac{-1}{3} = \frac{v}{u}$

$\therefore$ u = –24 m.

Using formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{8} - \left(\frac{1}{24}\right)$

$\frac{1}{f} = \frac{4}{24}$

f = 6 m.

Using lens maker's equation,

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For plano-convex lens, $R_1 = \infty$, $R_2 = -R$.

$\therefore \quad \frac{1}{f} = (\mu - 1)\frac{1}{R}$

$\Rightarrow R = f(\mu - 1) = 6(1.5 - 1) = 3$ m

60. When a convex lens is introduced, object forms two images.

One is diminished, $I_1 = \frac{2}{3}$ cm

and another is magnified, I_2 = 6 cm

Magnification for magnified image (m_2) and that for diminished image (m_1) are related as

$m_2 = \frac{1}{m_1}$

∴ $m_1 m_2 = 1$

∴ $\frac{I_1}{O} \times \frac{I_2}{O} = 1$

∴ $O^2 = I_1 I_2$

i.e., $O = \sqrt{I_1 I_2}$

hence, size of object $O = \sqrt{\frac{2}{3} \times 6} = 2$ cm

61. (lens + cornea) forms an image of distance object at retina.

∴ converging power (40+20) D = 60 D

From Lens equation,

$\frac{1}{v} - \frac{1}{\infty} = \frac{60}{100}$

∴ $v = \frac{5}{3}$ cm

∴ v = 1.67 cm.

62. The person to be able to see object at infinity, the image should be formed at 400 cm.

∴ $u = \infty$

$v = -400$ cm $= -4$ m

By lens formula,

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-4} - \frac{1}{\infty}$

$f = -4$ m

As focal length is negative, the lens used is concave.

$P = \frac{1}{f} = -0.25$ D

63. According to lens maker's formula,

the focal length of plano-convex lens is

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$

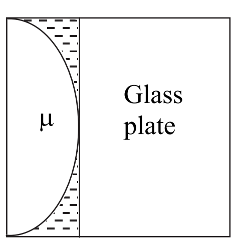
$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R}\right) = \frac{1}{2R} \Rightarrow R = \frac{f}{2}$ ....(i)

The focal length of liquid lens is

$\frac{1}{f_l} = (\mu_l - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$

$\frac{1}{f_l} = \left(\frac{\mu_l - 1}{R}\right)$

$\frac{1}{f_l} = \frac{2(\mu_l - 1)}{f}$ [using (i)]



Effective focal length of the combination is

$\frac{1}{2f} = \frac{1}{f} + \frac{1}{f_l}$

$\frac{1}{2f} = \frac{1}{f} - \frac{2(\mu_l - 1)}{f}$

$\Rightarrow 2(\mu_l - 1) = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow \mu_l - 1 = \frac{1}{4} \Rightarrow \mu_l = \frac{5}{4} = 1.25$

64. By lens maker's formula,

$\frac{1}{f_g} = \left({}_a\mu_g - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Also, $\frac{1}{f_{liquid}} = \left({}_l\mu_g - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Dividing above equations,

$\frac{f_{liquid}}{f_g} = \frac{\left({}_a\mu_g - 1\right)}{\left({}_l\mu_g - 1\right)} = \frac{\left(\mu_g - 1\right)}{\left(\frac{\mu_g}{\mu_l} - 1\right)} = \frac{(1.45 - 1)}{\left(\frac{1.45}{1.3} - 1\right)} = 3.9$

65. By lens maker's formula,

$\frac{1}{f_a} = ({}_a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{(-R_2)}\right]$

$= (1.5 - 1)\left(\frac{2}{20}\right)$

∴ $P_{air} = \frac{0.5}{10}$

Similarly, when the lens is immersed in a liquid,

$\frac{1}{f_{liquid}} = \left(\frac{\mu_g}{\mu_l} - 1\right)\left[\frac{1}{R_1} - \frac{1}{(-R_2)}\right]$

∴ $\frac{1}{f_{liquid}} = \left(\frac{1.5}{1.25} - 1\right)\left[\frac{1}{10}\right]$

∴ $P_{liquid} = \frac{1}{50}$

∴ $\frac{P_{air}}{P_{liquid}} = \frac{50}{10} \times 0.5 = \frac{5}{2}$

66. In telescope $f_0 >> f_e$ as compared to microscope.

67. For combination of lenses,

$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

$\frac{1}{F} = \frac{3}{3} = 1$

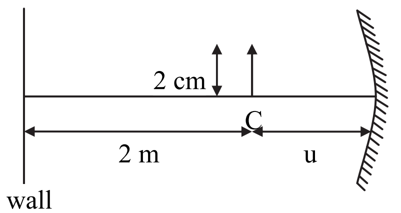
Total magnification,

$M = 1 + \frac{D}{F} = 1 + \frac{25}{1} = 26$

Evaluation Test

1. The field of view is maximum for convex mirror because the image of an object formed by a convex mirror is always diminished. Each image is thus confined to small area and many objects can be viewed in the mirror.

2.



Let the candle C be placed u metre away from pole of the mirror.
According to question, image distance
$v = u + 2$
Also, magnification of a concave mirror

$$m = \frac{-v}{u} = \frac{-(u+2)}{u} = \frac{\text{image height}}{\text{object height}}$$

Here, negative sign indicates, image is inverted.

$\therefore \quad |m| = \frac{u+2}{u} = \frac{6}{2} \Rightarrow u = 1\text{ m}$

Distance of the wall from the mirror is
$u + 2 = (1 + 2)\text{ m} = 3\text{ m} = 300\text{ cm}$.

3. For near end the bar, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Here, u and f are negative

$\therefore \quad |v| = \frac{uf}{u-f}$

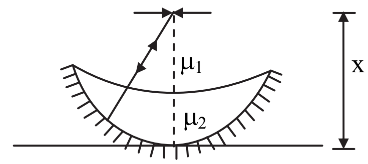
Far end of the bar is at infinity. Therefore, image will be formed at focus.

$\therefore$ Length of the image $= |v| - f$

$$= \frac{uf}{u-f} - f = \frac{f^2}{u-f}$$

4. We cannot interchange the objective and eye lens of a microscope to make a telescope. The focal lengths of lenses in microscope are very small, of the order of mm or a few cm and the difference ($f_o - f_e$) is also very small. While in the telescope, objective has a very large focal length.

5. Whenever any surface of convex or plano-convex or concavo-convex lens is silvered, it behaves like a concave mirror. Similarly whenever any surface of a concave or plano-concave or convexo-concave lens is silvered, it behaves like a convex mirror.



When ray travels from μ_1 to μ_2,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

The ray refracts from R_1 and falls normally of R_2. Let the pin be placed at distance x from lens. i.e., $u = x$.

$\therefore \quad \frac{1.5}{-10} - \frac{1}{-x} = \frac{1.5-1}{-30}$

$\therefore \quad \frac{1}{x} = \frac{-0.5}{30} + \frac{1.5}{10}$

$\therefore \quad x = 7.5\text{ cm}$

Image of object coincides with the object itself as the ray after refraction from first surface falls normally on second surface.

6. Focal length of convex/concave mirror depends only on radius of curvature (R) of the mirror. It does not depend upon u and v.

7. When the lens is in air,

$$\frac{1}{f_a} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{30} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{….(i)}$$

When lens is in water

$$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \quad \frac{1}{f_w} = \left(\frac{1.5 - 1.33}{1.33}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{….(ii)}$$

Dividing equation (i) by (ii),

$$\therefore \quad \frac{f_w}{30} = (1.5 - 1)\left(\frac{1.33}{1.5 - 1.33}\right)$$

$$f_w = 30 \times 0.5 \times \frac{1.33}{0.17} = 117.35\text{ cm}$$

The change in focal length
$= 117.35 - 30 = 87.35 \approx 87.4\text{ cm}$

8. Magnifying power of a telescope in normal adjustment $= \frac{f_a}{f_e}$

Tube length = Distance between objective and eyepiece

$= f_o + f_e$

$\frac{f_o}{f_e} = 9 \Rightarrow f_o = 9f_e$

Tube length = $f_o + f_e$

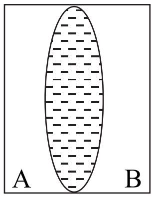
$60 = 9f_e + f_e = 10f_e$

$\therefore$ $f_e = 6$ cm and

$f_o = 9f_e = 9 \times 6 = 54$ cm

9. As shown in the figure, the system is equivalent to combination of three thin lenses in contact

$\therefore$ $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$



By lens maker's formula

$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{20}\right) = \frac{-1}{40}$

$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{20} + \frac{1}{10}\right) = \frac{1}{20}$

$\frac{1}{f_3} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{\infty}\right) = -\frac{1}{20}$

$\frac{1}{f} = -\frac{1}{40} + \frac{1}{20} - \frac{1}{20} = -\frac{1}{40}$

$\Rightarrow$ $f = -40$ cm

Hence system behaves as concave lens of focal length 40 cm.

11. By focussing a lens, energy can be concentrated into a small beam. This does not violate principle of conservation of energy, as lens does not generate energy but merely concentrates the available energy.

12. A dentist uses concave mirror to converge light and obtain enlarged image.

13. Let the closest distance be u and farthest distance be u′.

$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{5} = \frac{-6}{25}$ ($\because$ v = 25 cm)

$\therefore$ $u = \frac{-25}{6}$ cm

Also $\frac{1}{u'} = \frac{1}{v'} - \frac{1}{f} = \frac{1}{\infty} - \frac{1}{5}$ ($\because$ v = ∞)

$\therefore$ $u' = -5$ cm

Ratio, $\frac{u}{u'} = \frac{-25/6}{-5} = \frac{5}{6}$

14. For a given compound microscope,

M.P. = $\frac{v_o}{u_o} \times \frac{D}{u_e}$

and $L = v_o + u_e$

When L is increased, u_e increases as v_o is fixed. Hence, its magnifying power decreases.

15. The objective of a telescope must have large aperture to gather more light. It should also have large focal length $\left(m = \frac{f_0}{f_e}\right)$. Therefore, lens A is selected as objective lens.

The eyepiece should have small aperture and small focal length. Therefore, lens D is selected as eye lens.

16. Focus alone depends on whether the rays are paraxial or not. The rest of the three factors do not depend on whether the rays are paraxial or not.

17. As refractive index of lens is different for different colours/wavelengths, therefore, different colours are focussed at different points. Hence the image is coloured.

Textbook Chapter No.

12 Magnetic Effect of Electric Current

Hints

Classical Thinking

3. 1 G (gauss) = 10^{-4} T (tesla)

6. $dB = \frac{\mu_0}{4\pi}.\frac{Idl\sin\theta}{r^2}$

7. $B = \frac{\mu_0 I}{2\pi a} = \frac{4\pi\times10^{-7}\times10}{2\pi\times4\times10^{-2}} = 5\times10^{-5}$ N/A m

17. Element 'dl' and radius are perpendicular

18. $B = \frac{\mu_0 I}{2r} = \frac{4\pi\times10^{-7}}{2r}\times0.5 = \frac{\pi\times10^{-7}}{r}$

But $22\times10^{-2} = 2\pi r$

$\therefore\ r = \frac{11\times10^{-2}}{\pi}$

$\therefore\ B = \frac{\pi\times10^{-7}\times\pi}{11\times10^{-2}} \approx 9\times10^{-6}$ Wb/m^2

19. $B = \frac{\mu_0 n I a^2}{2(a^2+x^2)^{3/2}}$

$= \frac{4\pi\times10^{-7}\times50\times1\times(5\times10^{-2})^2}{2[(0.05)^2+(0.2)^2]^{3/2}}$

$= \frac{\pi\times10^{-5}\times25\times10^{-4}}{[(25+400)\times10^{-4}]^{3/2}}$

$\therefore\ B = 9\times10^{-6}$ Wb/m^2

24. $F = qvB\sin\theta$

$= 200\times10^{-6}\times2\times10^5\times5\times10^{-5}\times\sin30°$

$F = 10^{-3}$ N

25. $F = qvB\sin\theta$

For $\theta = 90°$ and $v = 10^{-3}c$,

$q = \frac{F}{vB} = \frac{1.732\times10^{-2}\times\sqrt{3}}{10^{-3}\times3\times10^8\times2\times10^{-5}}$

$= \frac{\sqrt{3}\times\sqrt{3}}{3\times2}\times10^{-2}$

$= 5\times10^{-3}$ C

27. $F = IlB\sin\theta$

$F = 1\times1\times10^{-4}\times\frac{1}{\sqrt{2}}$ ($\because$ 1 oersted = 10^{-4} T)

$= \frac{10^{-4}}{\sqrt{2}}$ N

28. $F_{max} = IlB$

$\therefore\ I = \frac{F_{max}}{Bl}$

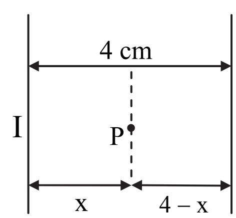
$\therefore\ I = \frac{3\times10^{-4}}{5\times10^{-5}\times15\times10^{-2}} = 40$ A

32. Since currents are flowing in opposite direction. Hence force of attraction does not exist.

33. $F = \frac{\mu_0 I_1 I_2 l}{2\pi a}$

$\frac{F}{l} = \frac{4\pi\times10^{-7}\times2\times4}{2\pi\times10^{-1}} = 1.6\times10^{-5}$ N/m

34.



At neutral point P, $B_1 = B_2$

$\therefore\ \frac{\mu_0}{2\pi}\frac{I}{x} = \frac{\mu_0}{2\pi}\frac{3I}{(4-x)}$

$\therefore\ x = 1$ cm

36. $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a} = \frac{4\pi\times10^{-7}\times1\times1}{2\pi\times1}$

$\therefore\ \frac{F}{l} = 2\times10^{-7}$ N/m

39. $\tau = nIAB\sin\theta = 20\times12\times(10^{-1})^2\times0.8\times\sin30°$

$= 0.96$ N m

40. $M = nIA = 5\times1\times(4\times10^{-2})^2$

$M = 8\times10^{-3}$ A m^2

42. $B = \frac{\mu_0 I}{2r}$

$1.76\times10^{-6} = \frac{4\pi\times10^{-7}\times1.4}{2r}$

$\therefore\ r = 0.5$ m

circumference = $2\pi r$ = 3.14 m

43. $F = qvB \sin\theta = qvB \sin 90°$

$F = evB$

$B = \dfrac{\mu_0 I}{2\pi a}$

$\therefore \quad F = \dfrac{ev\mu_0 I}{2\pi a} = \dfrac{1.6\times10^{-19}\times10^{7}\times4\pi\times10^{-7}\times50}{2\pi\times5\times10^{-2}}$

$F = 3.2 \times 10^{-16}$ N

44. $B = \dfrac{\mu_0 I}{2\pi a}$

$1.33\times 10^{-4} = \dfrac{4\pi\times10^{-7}\times I}{2\pi\times0.1}$

$\therefore \quad I = 66.5$

$n = \dfrac{It}{e}$

$n = \dfrac{66.5\times1}{1.6\times10^{-19}} \approx 4.16 \times 10^{20}$

Hence, the nearest option is (B).

Critical Thinking

2. $B = \dfrac{\mu_0 I}{2\pi a}$

$\therefore \quad B \propto \dfrac{1}{a}$

$\dfrac{B_1}{B_2} = \dfrac{a_2}{a_1}$

$\therefore \quad \dfrac{10^{-8}}{B_2} = \dfrac{12}{4}$

$\therefore \quad B_2 = 3.33 \times 10^{-9}$ T

3. $B = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2I}{r}$

Hence, if distance is same, field will be same

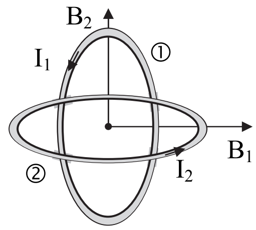
4. Magnetic field lies inside as well as outside the solid current carrying conductor.

5. Inside the pipe, I = 0

$\Rightarrow B_{inside} = \dfrac{\mu_0 I}{2\pi r} = 0$

6. $I = \dfrac{q}{t} = \dfrac{2\times1.6\times10^{-19}}{2} = 1.6 \times 10^{-19}$ A

$\therefore \quad B = \dfrac{\mu_0 I}{2r} = \dfrac{\mu_0\times1.6\times10^{-19}}{2\times0.8} = 10^{-19}\,\mu_0$

7. 

$B = \sqrt{B_1^2 + B_1^2} = \sqrt{2}\,B_1$

$\therefore \quad \dfrac{B}{B_1} = \sqrt{2}$

8. $L = n_1.2\pi r_1 = n_2.2\pi r_2$

$\therefore \quad \dfrac{r_1}{r_2} = \dfrac{n_2}{n_1}$

$\dfrac{B_1}{B_2} = \dfrac{\mu_0 n_1 I/2r_1}{\mu_0 n_2 I/2r_2} = \dfrac{n_1}{n_2}\dfrac{r_2}{r_1} = \left(\dfrac{n_1}{n_2}\right)^2 = \left(\dfrac{4}{2}\right)^2 = \dfrac{4}{1}$

9. $B = \dfrac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}} = \dfrac{\mu_0 I a^2}{2(a^2+a^2)^{3/2}}$

$= \dfrac{\mu_0 I a^2}{2(2)^{3/2}a^3} = \dfrac{4\pi\times10^{-7}\times1}{4\times\sqrt{2}\,a} = \dfrac{\pi}{\sqrt{2}\,a}\times10^{-7}$ T

10. $B_1 = \dfrac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}$,

But x >>> a

$\therefore \quad B_1 = \dfrac{\mu_0 I a^2}{2x^3}$

$B_2 = \dfrac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}$

But x = 0

$\therefore \quad B_2 = \dfrac{\mu_0 I a^2}{2a^3}$

$\therefore \quad \dfrac{B_1}{B_2} = \dfrac{a^3}{x^3}$

11. The radius of the circular loop $r = \dfrac{L}{2\pi}$.

Therefore, $B = \dfrac{\mu_0 I}{2r} = \dfrac{\pi\mu_0 I}{L}$

12. The magnetic field at the centre is,

$|\vec{B}_c| = \dfrac{\mu_0}{4\pi}\dfrac{2n\pi I}{r}$

For 2 turns: $|\vec{B}_c| = \dfrac{\mu_0}{4\pi}.\dfrac{4\pi I}{r/2}$

For 4 turns: $|\vec{B}_c| = \dfrac{\mu_0}{4\pi}.\dfrac{8\pi I}{r/4}$

$\therefore \quad |\vec{B'}_c| = 4|\vec{B}_c| = 4 \times 0.2 = 0.8$ T

13. Magnetic field at centre $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$

Magnetic field at a point on the axis,

$$B' = \frac{\mu_0}{4\pi} \times \frac{2\pi I r^2}{(r^2+x^2)^{3/2}}$$

Given $B' = \frac{B}{8} \Rightarrow \frac{B}{B'} = 8$

$$\therefore \quad \frac{\frac{\mu_0}{4\pi}\frac{2\pi I}{r}}{\frac{\mu_0}{4\pi}\times\frac{2\pi I r^2}{(r^2+x^2)^{3/2}}} = 8$$

$$\therefore \quad \frac{(r^2+x^2)^{3/2}}{r^3} = 8 \qquad \therefore \quad \frac{(r^2+x^2)^3}{r^6} = 64$$

$$\therefore \quad \frac{r^2+x^2}{r^2} = 4 \qquad \therefore \quad r^2 + x^2 = 4r^2$$

$$\therefore \quad 3r^2 = x^2 \qquad \therefore \quad x = \sqrt{3}\,r$$

14. The force experienced by a charge particle in a magnetic field is given by, $\vec{F} = q(\vec{v}\times\vec{B})$ which is independent of mass. As q, v and B are same for both the electron and proton, both will experience same force.

15. $F_m = qvB\sin\theta$,
Since $v = 0$
$\therefore$ $F_m = 0$

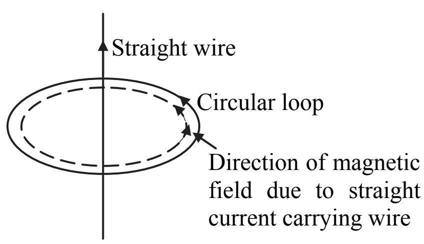
16. $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

Electron is a negatively charged particle, therefore force $\vec{F}$ will be acting in negative Z-direction.

17. $\vec{F} = q(\vec{v}\times\vec{B}) = 10^{-11}(10^8\,\hat{j}\times 0.5\,\hat{i})$

$= 5\times10^{-4}(\hat{j}\times\hat{i}) = 5 \times 10^{-4}$ N along $-\hat{k}$

18.



Magnetic field due to straight wire is either parallel or antiparallel to the current flow in loop depending on direction of current in wire. Thus force F exerted by this magnetic field B is,

$\vec{F} = I\,d\vec{l}\times\vec{B}$

$= IdlB\sin\theta = 0 \qquad (\because \theta = 0 \text{ or } 180°)$

19. As per the figure,
$F = q(E + v \times B) = qE + q(v \times B)$
Now, $F_e = qE$

$= -16 \times 10^{-18} \times 10^4 (-\hat{k})$

$= 16 \times 10^{-14}\,\hat{k}$

And $F_m = -16 \times 10^{-18} (10\,\hat{i} \times B\,\hat{j})$

$= -16 \times 10^{-17}\, B(\hat{k})$

$\therefore$ $F = F_e + F_m = 16 \times 10^{-14}\,\hat{k} - 16 \times 10^{-17}\, B\,\hat{k}$

Since, the particle will continue to move along + X-axis, so resultant force is zero. Therefore, $F_e + F_m = 0$

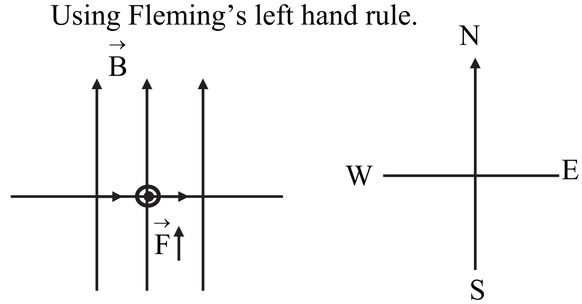
$\therefore$ $16 \times 10^{-14}\,\hat{k} = 16 \times 10^{-17}\, B\hat{k}$

$\therefore$ $B = 10^3$ Wb/m^2

20. Vertically up

$\vec{F} = I\,\vec{l}\times\vec{B}$

Using Fleming's left hand rule.



21. The force per unit length is,

$$F = \frac{\mu_0}{4\pi}\times\frac{2I^2}{R}$$

If R is increased to 2R and I is reduced to I/2, the force per unit length becomes,

$$F' = \frac{\mu_0}{4\pi}\times\frac{2(I/2)^2}{2R}$$

$$= \frac{\mu_0}{4\pi}\times\frac{2I^2}{2R}\times\frac{1}{8} = \frac{F}{8}$$

23. $F_1 = \frac{\mu_0 I_1 I_2 l}{2\pi a}$ and $B_2 = \frac{\mu_o I_2}{2\pi a}$

$F_1 = B_2 I_1 l$

$$B_2 = \frac{F_1}{I_1 l} = \frac{mg}{I_1 l} = \frac{7.5\times10^{-5}\times10}{4\times10^{-1}}$$

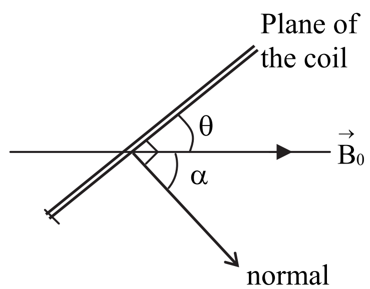
$B_2 = 1.875 \times 10^{-3}$ T

25. Current carrying coil is a closed loop. Net force acting on the coil due to uniform magnetic field is always zero. But there will be a non-zero torque acting on the coil, except when plane of the coil is perpendicular to the field.

26. $\tau = MB \sin\theta = N\,I\,B\,A \sin\theta$
τ does not depend upon shape of the loop.

27. τ = NBIA sin θ, so the graph between τ and θ is a sinusoidal graph.

28. $\vec{M} = NI\vec{A}$
$\vec{\tau} = \vec{M} \times \vec{B} = NI\vec{A} \times \vec{B}$ = NIAB sin α
but $\alpha = 90 - \theta$
$\therefore$ $\vec{\tau}$ = NIAB sin(90 – θ) = NIAB cos θ



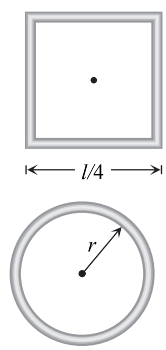
29. Suppose length of each wire is l. $r \propto \dfrac{1}{B}$

$A_{circle} = \pi r^2 = \pi\left(\dfrac{l}{2\pi}\right)^2 = \dfrac{l^2}{4\pi}$

$\because$ Magnetic moment $M = IA$

$\Rightarrow \dfrac{M_{square}}{M_{circle}} = \dfrac{A_{square}}{A_{circle}}$

$= \dfrac{l^2/16}{l^2/4\pi} = \dfrac{\pi}{4}$



30. $E_{kinetic} = \dfrac{1}{2}mv^2$

$2 \times 1.6 \times 10^{-13} = \dfrac{1}{2} \times 1.67 \times 10^{-27} \times v^2$

$v^2 = 3.83 \times 10^{14}$
$v = 0.196 \times 10^8$ m/s
$F = qvB \sin\theta$
$= 1.6 \times 10^{-19} \times 0.196 \times 10^8 \times 2.5 \times \sin 90°$
$F = 7.84 \times 10^{-12}$ N

31. $B = \dfrac{\mu_0 nI}{2r}$
For n = 1
$B = \dfrac{\mu_0 I}{2r}$
When same length is bent into 2 turns, radius is halved.

$B' = \dfrac{\mu_0 n' I}{2r'} = \dfrac{\mu_0 \times 2 \times I}{2 \times \frac{r}{2}} = 4\left(\dfrac{\mu_0 I}{2r}\right)$

$B' = 4B$

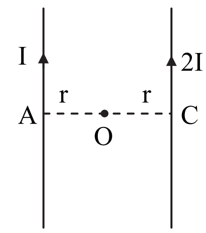
32. $B = \dfrac{\mu_0}{4\pi} \times \dfrac{2I}{r}$ (For infinitely long conductor)

$B \propto \dfrac{1}{r}$

Hence, graph (C) is correct.

33. Here magnetic force is zero, but the velocity increases due to electric force.

34.



Let two wires be A and C carrying current I and 2I respectively. The magnetic field produced by two wires at mid-point ‘O’ will be in opposite direction. Hence net magnetic field at O is, $B = B_1 - B_2 = \dfrac{\mu_0 I_1}{2r} - \dfrac{\mu_0 I_2}{2r}$

$= \dfrac{\mu_0}{2r}(I_1 - I_2) = \dfrac{\mu_0}{2r}(I - 2I) = \dfrac{\mu_0}{2r}(-I)$

Here negative sign indicates the direction of B.
Hence neglecting it, $|B| = |B_1|$ ….(i)
When 2I wire is switched off, field produced at point O will be only B_1, which referring to equation (i), equals B.
Thus, when 2I wire is switched off, field will be B.

35. Electrostatic force $F_e = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{e^2}{r^2}$

Magnetic force $F_m = \dfrac{\mu_0}{4\pi}\left(\dfrac{e^2 v^2}{r^2}\right)$

$\therefore$ $\dfrac{F_m}{F_e} = \mu_0\varepsilon_0 v^2 = \dfrac{v^2}{c^2}$ $\quad (\because \mu_0\varepsilon_0 = 1/c^2)$

36. $F = BIl = 2 \times 2 \times 3 \times 10^{-2} = 0.12$ N
Now, F = ma
$\therefore$ $a = \dfrac{0.12}{10 \times 10^{-3}} = 12$ m/s² (along Y-axis)

37. Force on the charge in motion in magnetic field,
$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$, implying $\vec{F}$ is perpendicular to $\vec{v}$.

Work done $W = \vec{F} \cdot \vec{v}$

$= F\,v\cos\theta$

$= F\,v\cos 90^\circ \quad \left(\because \vec{F} \perp \vec{v}\right)$

$= 0$

38. The perimeter in plane is two-dimensional. Amongst the given shapes, circle has maximum area. Hence, maximum torque will act on it.

39. The wires are in parallel and ratio of their resistances are 3 : 4 : 5. Hence currents in wires are $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$.

Force between top and middle wire,

$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r_1}$ (r_1 = distance between these wires)

$\therefore \quad F_1 = \frac{\mu_0 (1/3)(1/4)}{2\pi r_1}$

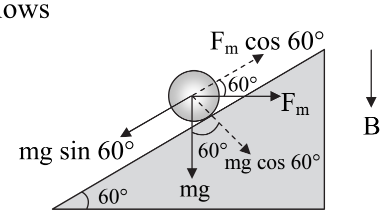
Force between bottom and middle wire,

$F_2 = \frac{\mu_0 (1/4)(1/5)}{2\pi r_2}$

As the forces are equal and opposite,

$F_1 = F_2 \Rightarrow \frac{1}{3r_1} = \frac{1}{5r_2} \Rightarrow \frac{r_1}{r_2} = \frac{5}{3}$

40. Let F_m be the force arising due to magnetic field, then the given situation can be drawn as follows



$F_m = BIl \Rightarrow mg\sin 60^\circ = BIl\cos 60^\circ$

$\Rightarrow B = \frac{0.01 \times 10 \times \sqrt{3}}{0.1 \times 1.73} = 1\ \text{T}$

41. The charge will not experience any force if $|\vec{F}_e| = |\vec{F}_m|$. This condition is satisfied in option (B) only.

42. $B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\frac{q}{t} = \frac{\mu_0 q f}{2r}$

$= \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 6.8 \times 10^{15}}{2 \times 0.5 \times 10^{-10}}$

$B = 13.7$ T

43. $F = \frac{\mu_0 I_1 I_2 l}{2\pi a}$

$\frac{mg}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$

$I_2 = \frac{20 \times 10^{-3} \times 9.8 \times 2\pi \times 2 \times 10^{-2}}{4\pi \times 10^{-7} \times 200}$

$\left(\because \frac{m}{l} = \text{Linear density}\right)$

$I_2 = 98$ A

Competitive Thinking

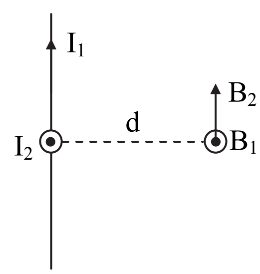
1. Magnetic field is produced by moving charge.

3. (By Biot-Savart's law $dB = \frac{\mu_0}{4\pi} = \frac{Idl\sin\theta}{r^2}$

i.e. $dB \propto \frac{1}{r^2}$)

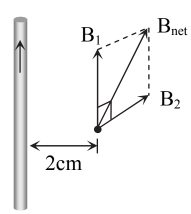
4. Every point on line AB will be equidistant from X and Y-axis. So magnetic field at every point on line AB due to wire 1 along X-axis is equal in magnitude but opposite in direction to the magnetic field due to wire along Y-axis. Hence B_{net} on AB = 0

5.



$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)^{1/2}$

6.

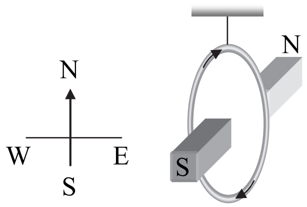


At the point, magnetic induction due to external magnetic field be $B_1 = 4 \times 10^{-4}$ T.

Now, due to wire carrying current magnetic induction produced at that point be $B_2 = \frac{\mu_0 I}{2\pi a}$

$= \frac{4\pi \times 10^{-7} \times 30}{2\pi \times 2 \times 10^{-2}} = 3 \times 10^{-4}$ T

7.



Current carrying loop, behaves as a bar magnet. A freely suspended bar magnet stays in the N – S direction.

8. $B = \frac{\mu_0 nI}{2r} = \frac{4\pi\times10^{-7}\times100\times0.1}{2\times5\times10^{-2}}$

$= 4\pi \times 10^{-5}$ tesla

9. When radius is doubled the resistance in the circuit is also doubled. Therefore the current in the circuit becomes halved.

Magnetic induction is given by,

$B = \frac{\mu_0 I}{2r}$

Now,

$B' = \frac{\mu_0 I'}{2r'}$ where $I' = \frac{I}{2}$ and $r' = 2r$

$\therefore$ $B' = \frac{\mu_0 I}{8r} = \frac{B}{4}$

10. Let the wire of length l be bent into circle of radius R.

$\therefore$ $B = \frac{\mu_0 nI}{2R}$

here, n = 1

$R = \frac{l}{2\pi}$

$\therefore$ $B = \frac{\mu_0 I}{2\left(\frac{l}{2\pi}\right)}$

$\therefore$ $B = \frac{\mu_0 \pi I}{2l}$ ....(i)

When the same wire is bent into coil of n turns, let R′ be the radius of the coil,

$\therefore$ $2\pi nR' = l$

$\therefore$ $R' = \frac{l}{2\pi n}$

$\therefore$ $B' = \frac{\mu_0 nI}{2R'} = \frac{\mu_0 nI}{2\left(\frac{l}{2\pi n}\right)} = \frac{\mu_0 \pi I}{2l} n^2$

$\therefore$ $B' = n^2 B$ ....[From (i)]

11. Dipole moment $M = nIA = I \times \pi R^2$

If dipole moment is doubled keeping current constant,

$M' = I \times \pi(R')^2$

$\therefore$ $2M = I \times \pi(R')^2$

$\therefore$ $2\,(I \times \pi R^2) = I \times \pi(R')^2$

$\therefore$ $R' = \sqrt{2}\,R$

Magnetic field at center of loop is,

$B = \frac{\mu_0 I}{2R}$

$\therefore$ $B \propto \frac{1}{R}$

$\therefore$ $\frac{B_1}{B_2} = \frac{R'}{R} = \frac{\sqrt{2}}{1}$

12. B at the centre of a coil carrying a current, I is

$B_{coil} = \frac{\mu_0 I}{2r}$ (upward)

B due to wire, $B_{wire} = \frac{\mu_0 I}{2\pi r}$ (downward)

Magnetic field at centre C

$B_c = B_{coil} + B_{wire}$

$= \frac{\mu_0 I}{2r}$ (upward) $+ \frac{\mu_0 I}{2\pi r}$ (downward)

$= \frac{\mu_0 I}{2r} - \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2r}\left(1-\frac{1}{\pi}\right)$ upward

$= \frac{4\pi\times10^{-7}\times8}{2\times10\times10^{-2}}\left(1-\frac{1}{\pi}\right)$ upward

$= \frac{4\pi\times10^{-7}\times8\times2.14}{2\times10\times10^{-2}\times\pi}$ upward

$= 3.424 \times 10^{-5}$ N/A m upward

13. $B = \sqrt{B_1^2+B_2^2} = \frac{\mu_0}{2r}\sqrt{I_1^2+I_2^2}$

$= \frac{4\pi\times10^{-7}}{2\times2\pi\times10^{-2}}\sqrt{3^2+4^2}$

$= 5 \times 10^{-5}$ Wb m^{-2}

14. Magnetic field on the axis of circular current

$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi nIa^2}{(a^2+x^2)^{3/2}}$

$B \propto \frac{na^2}{(a^2+x^2)^{3/2}}$

15. $B_{axis} = \dfrac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}$, $B_{centre} = \dfrac{\mu_0 I}{2a}$

$B_{axis} = B_{centre} \times \dfrac{a^3}{(a^2+x^2)^{3/2}}$

$B_{centre} = \dfrac{(B_{axis})(a^2+x^2)^{3/2}}{a^3}$

$B_{centre} = \dfrac{(54)(3^2+4^2)^{3/2}}{3^3} = \dfrac{54\times125}{27} = 250\ \mu T$

16. $B_{centre} = 5\sqrt{5}\ B_{axis}$

$\dfrac{\mu_0 n I}{2r} = 5\sqrt{5}\ \dfrac{\mu_0 n I r^2}{2(r^2+x^2)^{3/2}}$

$(r^2+x^2)^{3/2} = 5\sqrt{5}\ r^3$

$(r^2+x^2)^3 = 125\ r^6$

$r^2+x^2 = 5r^2$

$x^2 = 4r^2$

$x = 2r$

$x = 2 \times 0.1$ ....($\because r = 0.1$)

$x = 0.2$ m

17. Magnetic field at the centre: $B_c = \dfrac{\mu_0 n I}{2R}$

Magnetic field at the axial point:

$B_{axis} = \dfrac{\mu_0 n I R^2}{2(R^2+x^2)^{3/2}}$

Given: $B_{axis} = \dfrac{B_c}{8}$

$\dfrac{\mu_0 n I R^2}{2(R^2+x^2)^{3/2}} = \dfrac{\mu_0 n I}{8\times 2R}$

$\therefore \dfrac{R^2}{2(R^2+x^2)^{3/2}} = \dfrac{1}{16R}$

$\therefore (R^2+x^2)^{3/2} = 8R^3$

$\therefore (R^2+x^2)^{1/2} = 2R$

$\therefore R^2+x^2 = 4R^2$

$\therefore x^2 = 3R^2$

$\therefore x = \sqrt{3}R$

18. $B = \dfrac{\mu_0}{4\pi}\times\dfrac{2\pi n I a^2}{(a^2+x^2)^{3/2}} \Rightarrow B \propto \dfrac{1}{(a^2+x^2)^{3/2}}$

$\Rightarrow \dfrac{8}{1} = \dfrac{(a^2+x_2^2)^{3/2}}{(a^2+x_1^2)^{3/2}} \Rightarrow \left(\dfrac{8}{1}\right)^{2/3} = \dfrac{a^2+0.04}{a^2+0.0025}$

$\Rightarrow \dfrac{4}{1} = \dfrac{a^2+0.04}{a^2+0.0025}$,

On solving, a = 0.1 m

19. Maximum force will act on proton so it will move on a circular path. Force on electron will be zero because it is moving parallel to the field.

21. $\vec{v}$ is parallel to $\vec{B}$

$\therefore \theta = 0° \Rightarrow \vec{F} = 0$

22. Component of velocity parallel to the field, makes the particle move in direction of field and due to perpendicular component of velocity, particle follows circular path making combined path as helical.

23. The particle is released from rest

and $\vec{E} \parallel \vec{B}$

$\vec{F}_{net} = \vec{F}_E + \vec{F}_B$ ....(i)

$\vec{F}_E = \pm q\vec{E}$ and $\vec{F}_B = \pm q\left(\vec{v}\times\vec{B}\right)$

As $\vec{v} = 0$, $\vec{F}_B = 0$

hence, $\vec{F}_{net} = \vec{F}_E$

As $\vec{F}_E$ is acting along the direction of electric field, particle will always move in the direction of electric field. Also, $\vec{v}$ being parallel to $\vec{B}$, particle will not deviate.

24. $\vec{F} = \pm q\left(\vec{v}\times\vec{B}\right)$

As particle is projected towards east

$\hat{v} = \hat{i}$

Force is acting in north direction

$\therefore \hat{F} = +\hat{j}$

$\therefore \hat{j} = (\hat{i}\times\hat{B})$

But we know,

$\hat{i}\times(-\hat{k}) = \hat{j}$

$\therefore \hat{B} = -\hat{k}$

25. As the coil is perpendicular to magnetic field

$\vec{B} = 2$ T,

$\theta = 90^{o}$

The loop formed is circular,

$\therefore l = 2\pi r$

As the force acting on the loop will be along radius,

$r = \frac{l}{2\pi}$

∴ Tension developed is

$T = F = B\,I\,r \sin\theta = B\,I\,r$

$= \frac{2 \times 1.1 \times l}{2\pi} = \frac{1.1}{\pi} = 0.35$ N

26. $F = qvB \sin\theta$

$B = \frac{F}{qv\sin\theta}$

$B_{min} = \frac{F}{qv}$ (when $\theta = 90°$)

∴ $B_{min} = \frac{10^{-10}}{10^{-12} \times 10^{5}} = 10^{-3}$ tesla in $\hat{z}$ - direction

27. Force on moving charge in magnetic field is given by,

$F = qvB\sin\theta$

but, $\theta = 90^{\circ}$

∴ $\sin 90^{\circ} = 1$

∴ $F = qvB$

Kinetic Energy of proton is given by

$E = \frac{1}{2}mv^2$

∴ velocity (v) $= \sqrt{\frac{2E}{m}}$

∴ $F = q \times \sqrt{\frac{2E}{m}} \times B$

$= 1.6 \times 10^{-19} \times \sqrt{\frac{2 \times 2 \times 10^{6} \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27}}} \times 2.5$

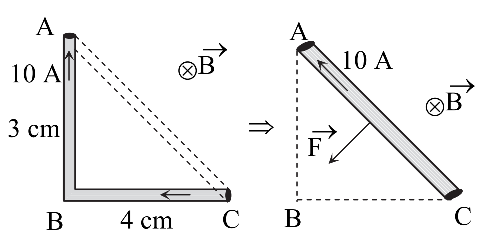
$F = 8 \times 10^{-12}$N

28. $F = BIl \sin\theta$

$7.5 = 2 \times 5 \times 1.5 \sin\theta$

$\theta = 30°$

29.



Force on the conductor ABC = Force on the conductor AC

$F = Il\,B \sin\theta$

$= I\,l\,B$ ($\because \theta = 90°$)

∴ $F = 5 \times 10 \times (5 \times 10^{-2}) = 2.5$ N

31. $\vec{F}_B = \pm q\left(\vec{v} \times \vec{B}\right)$

$\vec{F}_B = \pm q\left[a\hat{i} \times \left(b\hat{j} + c\hat{k}\right)\right]$

$= \pm q\left[ab\hat{k} + ac(-\hat{j})\right]$

$\vec{F}_B = qa\left(b\hat{k} - c\hat{j}\right)$

Taking magnitude on both sides,

$\left|\vec{F}_B\right| = qa\sqrt{b^2 + c^2}$

$F_B = qa(b^2 + c^2)^{1/2}$

32. From Fleming's left hand rule the force on electron is towards the east means it is deflected towards east.

35. Two wires, if carry current in opposite direction, they repel each other.

38. $F = \frac{\mu_0 I_1 I_2}{2\pi a} l$

$F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)I_2}{3a} l = -\frac{2}{3}\frac{\mu_0 I_1 I_2}{2\pi a} l = -\frac{2}{3}F$

39. Force per unit length $= \frac{\mu_0}{4\pi}.\frac{2I_1I_2}{a} = \frac{\mu_0}{2\pi}.\frac{I^2}{b}$

40. $\left(\frac{F}{l}\right) = \frac{\mu_0}{4\pi}.\frac{2I_1I_2}{a}$

$\left(\frac{F}{l}\right) = \frac{\mu_0}{4\pi}.\frac{2I^2}{d} = \frac{\mu_0 I^2}{2\pi d}$ (Attractive)

41. $\frac{F}{l} = \frac{\mu_0}{4\pi} \times \frac{2I_1I_2}{a} = \frac{\mu_0}{4\pi}\frac{2I^2}{a}$ ($\because I_1 = I_2 = I$)

∴ $2 \times 10^{-7} = 10^{-7} \times \frac{2I^2}{1}$

I = 1 A

42. $F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{a}$

$F_1 = \frac{\mu_0}{4\pi}\frac{2I^2}{x}$ (Attraction)

$F_2 = \frac{\mu_0}{4\pi}\frac{2I \times 2I}{2x} = \frac{\mu_0}{4\pi}\frac{2I^2}{x}$ (Repulsion)

Thus $F_1 = -F_2$

43. Force on wire Q due to wire P is

$F_P = 10^{-7} \times \dfrac{2\times30\times10}{0.1} \times 0.1$

$= 6 \times 10^{-5}$ N (Towards left)

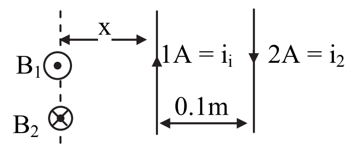
Force on wire Q due to wire R is

$F_R = 10^{-7} \times \dfrac{2\times20\times10}{0.02} \times 0.1$

$= 20 \times 10^{-5}$ N (Towards right)

$F_{net} = F_R - F_P = 14 \times 10^{-5}$ N $= 1.4 \times 10^{-4}$ N

44.



Here,

$B_1 = B_2$

$\dfrac{\mu_0(1)}{2\pi x} = \dfrac{\mu_0(2)}{2\pi(0.1+x)}$

$2x = 0.1 + x$

$x = 0.1$m

45. Magnetic field due to first wire is given by

$B_1 = \dfrac{\mu_0 I}{2\pi r}$

Magnetic field due to second wire is given by

$B_2 = \dfrac{\mu_0 I}{2\pi(3r)} = \dfrac{\mu_0 I}{6\pi r}$

Net Magnetic field at P is,

$B = B_1 + B_2$

$B = \dfrac{\mu_0 I}{2\pi r} + \dfrac{\mu_0 I}{6\pi r}$

$= \dfrac{3\mu_0 I + \mu_0 I}{6\pi r}$

$= \dfrac{4\mu_0 I}{6\pi r}$

$= \dfrac{2}{3}\dfrac{\mu_0 I}{\pi r}$

46. $\dfrac{F}{l} = \dfrac{\mu_0 I_1 I_2}{2\pi d} = 2 \times 10^{-7} \times \dfrac{5\times5}{1}$

$= 5 \times 10^{-6}$, attractive

47. $F = \dfrac{\mu_0 I_1 I_2 l}{2\pi a}$

$= \dfrac{4\pi\times10^{-7}\times5\times5\times5\times10^{-2}}{2\pi\times2.5\times10^{-2}}$

$= 10^{-5}$ N

48. Net force on wire B, $F_{net} = \sqrt{F_A^2 + F_C^2}$

$F_A = F_C = \dfrac{\mu_0 I_1 I_2}{2\pi d}$

$= \dfrac{\mu_0 i^2}{2\pi d}$ ....($\because I_1 = I_2 = i$)

$\therefore$ $F_{net} = \sqrt{2\left(\dfrac{\mu_0 i^2}{2\pi d}\right)^2}$

$= \dfrac{\sqrt{2}\mu_0 i^2}{2\pi d}$

$= \dfrac{\mu_0 i^2}{\sqrt{2}\pi d}$

49. According to Ampere's circuital law, the magnetic induction on axial line of a straight current carrying conductor is zero.

$\therefore$ The segments DE and AB do not produce a magnetic field at O.

For segments BC and EF,

$B_{BC} = \dfrac{\mu_0}{4\pi}\dfrac{I_C}{r_C}$, $B_{EF} = \dfrac{\mu_0}{4\pi}\dfrac{I_F}{r_F}$

$B_{net} = B_{BC} + B_{EF}$

$\therefore$ $B_{net} = 10^{-7} \times \left[\dfrac{4}{0.02} + \dfrac{9}{0.03}\right] = 5 \times 10^{-5}$ T

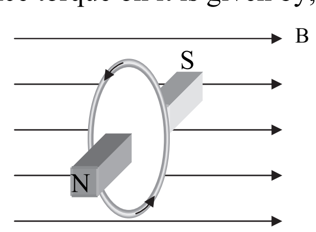
51. For $\theta = 90°$,

Area of equilateral triangle $= \dfrac{\sqrt{3}}{4} l^2$

$\Rightarrow \tau = NIAB = 1 \times I \times \left(\dfrac{\sqrt{3}}{4} l^2\right) B$

$= \dfrac{\sqrt{3}}{4} l^2 BI$

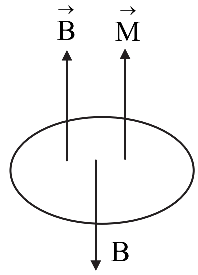
52. As shown in the following figure, the given situation is similar to a bar magnet placed in a uniform magnetic field perpendicularly. Hence torque on it is given by,



$\tau = MB \sin 90° = (I\pi r^2)B$

53. $M = niA = ni(\pi r^2) \Rightarrow M \propto r^2$

54.



When θ = 0° (parallel) it is in stable equilibrium.

When θ = 180° (anti-parallel), it is in unstable equilibrium.

55. M = nIA, thus independent of magnetic field of induction.

56. $B = \frac{\mu_0 nI}{2r}$

$M = nIA = 100 \times 5 \times 2 \times 10^{-2} = 10$

$\tau_1 = MB \sin\theta\ ;\ \tau_2 = MB\cos\theta$

$\tau_1^2 = M^2B^2\sin^2\theta,\ \tau_2^2 = M^2B^2\cos^2\theta$

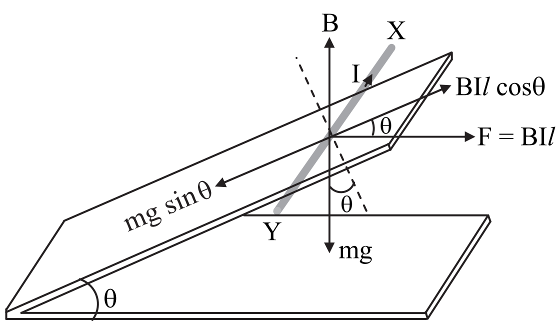
$\therefore \quad \tau_1^2 + \tau_2^2 = M^2B^2$

$\therefore \quad (0.09 + 0.16) = 10^2B^2$

$B^2 = \frac{0.25}{100} = 2.5 \times 10^{-3}$

B = 0.05 T

57. Rod will be stationary if component of magnetic field balances component of weight of rod as shown in the figure below.



To keep the rod stationary,

$BIl\cos\theta = mg\sin\theta$

$\therefore \quad I = \frac{mg\tan\theta}{Bl}$

$= \frac{\lambda g\tan\theta}{B}$ ....($\because m/l = \lambda$)

$I = \frac{0.5 \times 9.8 \times \tan 30°}{0.25} = \frac{19.6}{\sqrt{3}} = 11.32$ A

58. $B_1 = \frac{\mu_0}{4\pi}\frac{I\theta}{R_1}$

here, $\theta = 60° = \frac{\pi}{3}^c$

$\therefore \quad B_1 = \frac{\mu_0}{4\pi}\frac{\frac{\pi}{3}I}{R_1} = \frac{\mu_0 I}{12R_1}$

Similarly,

$B_2 = \frac{\mu_0}{4\pi}\frac{\frac{\pi}{3}I}{R_2} = \frac{\mu_0 I}{12R_2}$

$B_{net} = B_1 - B_2$

$= \frac{\mu_0 I}{12R_1} - \frac{\mu_0 I}{12R_2}$

$= \frac{\mu_0 I}{12}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

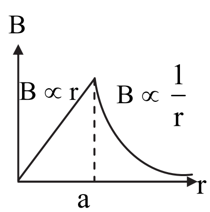
59. Magnetic fields due to a long straight wire of radius 'a' carrying current I at a point distant 'r' from the centre of the wire is given as follows,

$B = \frac{\mu_0 Ir}{2\pi a^2}$ for r < a

$B = \frac{\mu_0 I}{2\pi a}$ for r = a

$B = \frac{\mu_0 I}{2\pi r}$ for r > a

The variation of magnetic field B with distance r from the centre of wire is shown in the figure.



60. Electric field = $\frac{\text{Force}}{\text{Charge}}$

$= \frac{ma_0}{e}$ (in west direction)

Magnetic force = F_m

$= 3ma_0 - ma_0$

$= 2ma_0$ (in west direction)

($\because \vec{v} \times \vec{B}$ is directed towards west)

Since, $\vec{v}$ is directed towards north for positive charge, $\vec{B}$ is directed vertically down.

Now, $\vec{F} = q\vec{v} \times \vec{B}$

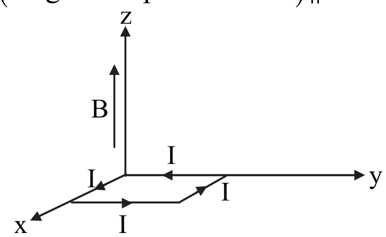
$\therefore \quad 2ma_0 = ev_0 \times B$

$\therefore \quad B = \frac{2ma_0}{ev_0}$ (vertically down)

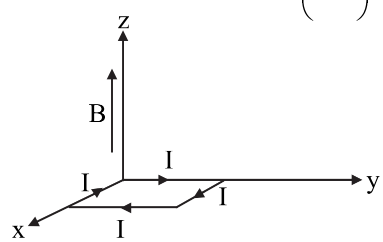
61. Since electron is moving parallel to the magnetic field, hence magnetic force on it $F_m = 0$.
The only force acting on the electron is electric force which reduces its speed.

62. For stable equilibrium

$\vec{M}$(magnetic dipole moment) $\| \vec{B}$

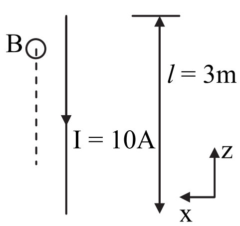


For unstable equilibrium $\vec{M} \| \left(-\vec{B}\right)$



63. Average Power = $\dfrac{\text{work}}{\text{time}}$

$W = \int_0^2 F dx$ ….(i)



Magnetic force on conductor

$F = B\,I\,l \sin\theta$

Here, $B = 3.0 \times 10^{-4}\, e^{-0.2x}$ T, I = 10 A and

$l = 1.5 - (1.5) = 3$ m

$\therefore$ $F = 3.0 \times 10^{-4}\, e^{-0.2x} \times 10 \times 3$

Substituting in equation (i),

$W = \int_0^2 3.0 \times 10^{-4} e^{-0.2x} \times 10 \times 3\, dx$

$= 9 \times 10^{-3} \int_0^2 e^{-0.2x}\, dx$

$= \dfrac{9 \times 10^{-3}}{0.2}\left[-e^{-0.2\times 2} + 1\right]$

$= \dfrac{9 \times 10^{-3}}{0.2} \times \left[1 - e^{-0.4}\right]$

$= 45 \times 10^{-3}\,[1 - 0.67]$

$\approx 14.84 \times 10^{-3}$ J

$P = \dfrac{14.84 \times 10^{-3}}{5 \times 10^{-3}}$

≈ 2.97 W

64. For given two coils, magnetic induction at their centres is same.

Let $B_1 = B_2$

$\dfrac{\mu_0 I_1}{2r} = \dfrac{\mu_0 I_2}{2(2r)}$

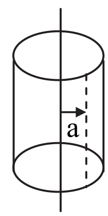
$\Rightarrow \dfrac{I_1}{I_2} = \dfrac{1}{2}$

Using Ohm's Law,

$V \propto \dfrac{1}{I}$

$\therefore$ $\dfrac{V_1}{V_2} = \dfrac{2}{1}$

65.



$B = \dfrac{\mu_0 I}{2\pi a} \times \dfrac{\pi a^2}{\pi r^2}$

$B = \dfrac{\mu_0 I}{2\pi r^2}\, a$

$B \propto a$

66. M = nIA

For coil, magnetic induction at the centre,

$B = \dfrac{\mu_0 n I}{2R}$

$\therefore$ $I = \dfrac{B \times 2R}{\mu_0 n}$

For n = 1, Area $A = \pi R^2$

$M = \dfrac{B \times 2R}{\mu_0} \times \pi R^2$

$= \dfrac{2\pi B R^3}{\mu_0}$

67. Number of revolutions completed by the electron in one second,

$n = \dfrac{v}{2\pi r}$

Also current,

$I = nq = \dfrac{v}{2\pi r} q$

Now, magnetic field,

$$B = \frac{\mu_0 I}{2r}$$

$$= \frac{\mu_0}{2r} \times \frac{v}{2\pi r} q$$

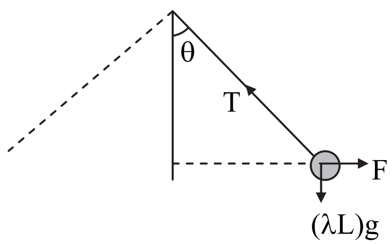
$$= \frac{\mu_0 vq}{4\pi r^2}$$

$$= \frac{4\pi \times 10^{-7} \times 2.2 \times 10^6 \times 1.6 \times 10^{-19}}{4\pi\left(5 \times 10^{-11}\right)^2}$$

∴ $B = 14.08$ T

68. For charged particles, if they are moving freely in space, electrostatic force is dominant over magnetic force between them. Hence due to electric force they repel each other.

69.



As the system is in equilibrium vertically,

$T \cos\theta = \lambda gL$ ....(i)

Along horizontal,

$$T \sin\theta = \frac{\mu_0}{2\pi} \frac{I \times I \times L}{(2L\sin\theta)} \quad ....(ii)$$

$$\left(\because F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \text{ and here } a = 2L\sin\theta\right)$$

$$\therefore \quad I^2 = \frac{4\pi L\sin\theta \times T\sin\theta}{\mu_0 L}$$

$$I = 2\sin\theta\sqrt{\frac{\pi T}{\mu_0}} \quad ....(iii)$$

Using equation (i),

$$T = \frac{\lambda gL}{\cos\theta}$$

Substituting for T in equation (iii),

$$I = 2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$$

Evaluation Test

1. $$B = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

$$108 \times 10^{-6} = \frac{\mu_0 I(6)^2 \times \left(10^{-2}\right)^2}{2\left(6^2 + 8^2\right)^{3/2} \times \left(10^{-4}\right)^{3/2}}$$

$$\therefore \quad \frac{\mu_0 I}{2} = \frac{108 \times 10^{-6} \times \left(10^2\right)^{3/2} \times 10^{-6}}{6^2 \times 10^{-4}} = \frac{108 \times 10^{-5}}{36} \quad ....(i)$$

At the centre of the coil, x = 0

$$\therefore \quad B = \frac{\mu_0 I}{2a} = \frac{\mu_0 I}{2 \times 6 \times 10^{-2}}$$

Using (i)

$$B = \frac{108 \times 10^{-5}}{36 \times 6 \times 10^{-2}} = 5 \times 10^{-4} \text{ T} = 500\ \mu\text{T}$$

2. Force between two long conductors carrying current,

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l \quad ....(i)$$

After carrying out changes,

$$F' = \frac{\mu_0}{2\pi} \frac{(-2\, I_1)(I_2)}{d'} l$$

From (i) and (ii),

$$\frac{F'}{F} = \frac{-2/d'}{1/d} = -2\left(\frac{d}{d'}\right) = -2\left(\frac{0.5}{1.5}\right) = \frac{-2}{3}$$

$$\Rightarrow F' = \frac{-2}{3} F$$

3. The net force on the particle is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad ....(i)$$

The solution of this problem can be obtained by resolving the motion along the three coordinate axes namely,

$$\left.\begin{aligned} a_x &= \frac{F_x}{m} = \frac{q}{m}(E_x + v_y B_z - v_z B_y) \\ a_y &= \frac{F_y}{m} = \frac{q}{m}(E_y + v_z B_x - v_x B_z) \\ a_z &= \frac{F_z}{m} = \frac{q}{m}(E_z + v_x B_y - v_y B_x) \end{aligned}\right\} \quad ....(ii)$$

For the given problem,

$E_x = E_y = 0$, $v_y = v_z = 0$ and $B_x = B_z = 0$

Substituting in equation (ii),

$a_x = a_y = 0$ and $a_z = \frac{q}{m}[-E_z + v_x B_y]$

Again $a_z = 0$, as the particle transverses through the region undeflected.

$\Rightarrow E_z = v_x B_y$

$\therefore \quad B_y = \dfrac{E_z}{v_x} = \dfrac{5\times10^4}{20} = 2.5 \times 10^3 \text{ Wb/m}^2$

4. $W = -MB\cos\theta = -MB(\cos\theta_2 - \cos\theta_1)$

$= -MB(\cos 60° - \cos 0°)$

$W = -MB\left(\dfrac{1}{2} - 1\right) = \dfrac{MB}{2} \qquad \text{....(i)}$

Now, when $\theta = 60°$, torque acting on dipole should be

$\tau = MB\sin\theta = MB\sin 60° = \dfrac{\sqrt{3}}{2}MB$

Using (i)

$\tau = \sqrt{3}\,W$

5. When a charged particle is moving in a region with uniform electric and magnetic field parallel to each other, it experiences force only due to electric field, along the direction of field, due to which the path of a charged particle will be a straight line.

6. The normal to the plane of the coil (X-Y plane) makes angle of 90° with the direction of the field.

$\therefore$ torque on the loop $\tau = BIA = BI(\pi r^2)$ ....(i)

Also the torque required to just raise an edge of the loop is

$\tau = Fr = \left(\dfrac{mg}{2}\right)r \qquad \text{....(ii)}$

Equating (i) and (ii),

$BI\pi r^2 = \dfrac{mgr}{2} \Rightarrow I = \dfrac{mg}{2\pi Br}$

7. $\tau \propto$ Area. The area of circle is largest.

8. Deflecting couple on magnet

$= MB\sin\theta = (2lm)B\sin\theta$

$= (10 \times 8) \times 0.32 \times \sin 45°$

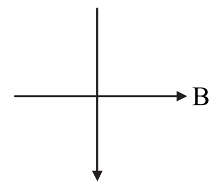
$= 18.1 \approx 18$ dyne cm

9. The kinetic energy of the proton, $\dfrac{1}{2}mv^2 = qV$

$\Rightarrow v^2 = \dfrac{2Vq}{m} \qquad \text{....(i)}$

If the proton is moving undeflected, then the deflection produced by the electric field must nullify the deflection produced by magnetic field.

As, the deflection of the proton caused by the magnetic field is upwards, deflection produced by the electric field should be into the paper. Hence the direction of the field is also into the paper.



$qE = qvB \Rightarrow v = E/B \qquad \text{....(ii)}$

$\therefore$ Equation (i) gives $v^2 = \dfrac{2Vq}{m} \Rightarrow \dfrac{E^2}{B^2} = \dfrac{2Vq}{m}$

$\Rightarrow V = \dfrac{mE^2}{2qB^2}$

10. The magnetic field on the axis of a coil carrying current I, having N turns, radius r and at a distance d from the centre of the coil, is given by:

$B = \dfrac{\mu_0}{4\pi} \times \dfrac{2\pi NIr^2}{(r^2+d^2)^{3/2}}$

The field at the centre is given by,

$B_c = \dfrac{\mu_0}{4\pi} \times \dfrac{2\pi NI}{r}$

$\therefore \quad \dfrac{B}{B_c} = \dfrac{r^3}{(r^2+d^2)^{3/2}}$

$= \dfrac{1}{\left[1 + \dfrac{3}{2}\dfrac{d^2}{r^2}\right]}$ ....(Using Binomial equation)

$\Rightarrow B\left[1 + \dfrac{3}{2}\dfrac{d^2}{r^2}\right] = B_c$

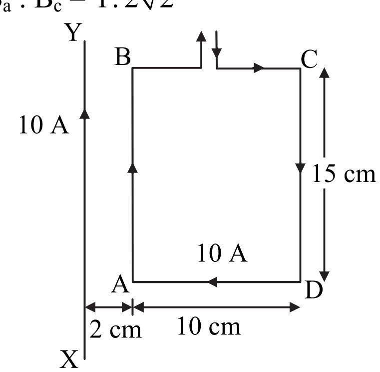
$\therefore \quad \dfrac{(B_c - B)}{B} = \dfrac{3}{2}\dfrac{d^2}{r^2}$

11. $B_c = \dfrac{\mu_0 I}{2r}$

$B_a = \dfrac{\mu_0 I r^2}{2\left(r^2 + r^2\right)^{3/2}} = \dfrac{\mu_0 I r^2}{2r^3(2^{3/2})} = \dfrac{\mu_0 I}{2r(2\sqrt{2})}$

$\therefore \quad B_a : B_c = 1 : 2\sqrt{2}$

12.



The effective force is only on AB and CD.
The force on AB is attractive and that on CD is repulsive.
Force between two current carrying conductors is F_1 between XY and AB $= \frac{\mu_0 I_1 I_2}{2\pi a} l$ attractive force and F_2 between XY and CD $= \frac{\mu_0 I_1 I_2 l}{2\pi d'}$ repulsive force.

$\therefore$ $F_1 - F_2 = \left(\frac{2\mu_0}{4\pi}\right)\frac{10\times10\times0.15}{0.02} - \frac{2\mu_0}{4\pi}\frac{10\times10\times0.15}{0.12}$

$\therefore$ $F_{resultant} = 2\times\frac{\mu_0}{4\pi}\times10\times10\times0.15\times\left[\frac{100}{2}-\frac{100}{12}\right]$

$= 2\times10^{-7}\times100\times\frac{0.15\times500}{12}$

$= 1.25\times10^{-4}$ N

13. For α-particle, q = 2 e

$\vec{F} = q(\vec{v}\times\vec{B}) = q[(6\times10^5\,\hat{i})\times(4\hat{i}-\hat{j})]$

$= q\times(-6\times10^5)\hat{k}$

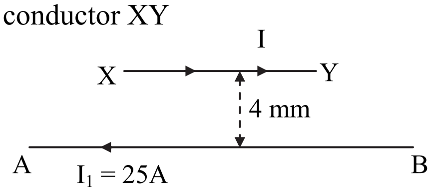
$= 2e\times(-6\times10^5)\hat{k}$

Negative sign indicates particle is moving along negative Z-axis.

$\therefore$ $|\vec{F}| = 2\times1.6\times10^{-19}\times-6\times10^5$

$= 1.92\times10^{-13}$ N

14. Mass per unit length of conductor XY, $m = 5\times10^{-2}$ kg/m
As magnetic repulsion is balancing the weight of conductor XY



$mg = \frac{F}{l}$

$\therefore$ $mg = \frac{\mu_0 I_1 I_2}{2\pi a} = \left(\frac{\mu_0}{4\pi}\right)\frac{2 I_1 I_2}{a}$

$mg = 10^{-7}\times\frac{2\times25\times I_2}{4\times10^{-3}}$

$5\times10^{-2}\times9.8 = \frac{25}{2}\times10^{-4}\,I_2$

$\therefore$ $I_2 = \frac{2\times5\times10^{-2}\times9.8}{25\times10^{-4}} = 392$ A

15. Magnetic field lines about a current carrying wire get crowded when the wire is bent into a circular loop.

Textbook Chapter No.

13 Magnetism

Hints

Classical Thinking

9. $B = \frac{\phi}{A}$

$= \frac{5\times10^{-4}}{25\times10^{-4}}$

$= 0.2\ Wb/m^2$

16. $M = m \times 2l$

$= 20 \times \frac{5}{6} \times 4.8 \times 10^{-2}$

$= 0.8\ A\ m^2$

17. $2\pi r = 4l$

$\Rightarrow r = \frac{2l}{\pi}$

$M = IA = I\pi r^2$

$= I\pi \times \frac{4l^2}{\pi^2}$

$= \frac{4Il^2}{\pi}$

22. $M = nIA$

$A = \frac{M}{nI}$

$= \frac{10}{75\times120\times10^{-3}}$

$= 1.1\ m^2$

23. $\tau = IAB \sin\theta$

$25 = I \times 5 \times 2 \times \frac{1}{2}$

$\Rightarrow I = 5\ A$

24. $B = \frac{\mu_0}{4\pi}\frac{M}{r^3}$

$= 10^{-7} \times \frac{2\times0.5}{(0.15)^3}$

$= 2.96 \times 10^{-5}$

$\therefore$ $B \approx 3 \times 10^{-5}\ Wb/m^2$

27. $\frac{\tau_1}{\tau_2} = \frac{MB\sin\theta_1}{MB\sin\theta_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 90°}{\sin 0°} = \frac{1}{0} = \infty$

29. $\tau = MB \sin\theta$

$\therefore$ $M = MB \sin\theta$

$\therefore$ $1 = B \sin 90°$

$\therefore$ $B = 1\ Wb/m^2$ $[\because \theta = 90°]$

36. $B_v = B \sin\delta = B \sin 30° = \frac{B}{2}$

37. $\tan\delta = \frac{B_V}{B_H} = 1$

$\therefore$ $\delta = \tan^{-1}(1) = 45°$

47. $\frac{B_{axis}}{B_{equator}} = \frac{2}{1}$

48. M remains constant.

$\therefore$ $\frac{B_1}{B_2} = \frac{r_2^3}{r_1^3} = \frac{(3x)^3}{(x)^3} = \frac{27}{1}$

49. $B_{eq} = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = \frac{10^{-7}\times10^{-1}}{(10^{-2})^3}$

$= \frac{10^{-8}}{10^{-6}}$

$= 10^{-2}\ Wb/m^2$

50. $B_{eq} = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}$

$= \frac{10^{-7}\times5\times10^{-3}\times6\times10^{-2}}{(0.1)^3}$

$B_{eq} = 3 \times 10^{-8}$ N/A m, directed from N-pole to S-pole.

56. $F = mB$

$m = \frac{20}{0.2} = 100\ A\ m$

$\therefore$ $M = m \times 2l$

$= 100 \times 20 \times 10^{-2}$

$= 20\ A\ m^2$

57. $F = mB$

$m = \frac{5.12\times10^{-5}}{3.2\times10^{-5}} = 1.6\ A\ m$

$M = m.2l$

$\therefore$ $2l = \frac{M}{m} = \frac{0.4}{1.6} = 0.25\ m = 25\ cm$

58. $B = \sqrt{B_H^2 + B_V^2}$

$B^2 - B_H^2 = B_V^2$

$B_V^2 = (5 \times 10^{-4})^2 - (3 \times 10^{-4})^2$

$B_V^2 = 16 \times 10^{-8}$

$B_V = 4 \times 10^{-4}$ T

59. $\tau = MB \sin\theta = 5 \times 1.5 \times 10^{-4} \times \sin 90°$

$\tau = 7.5 \times 10^{-4}$ N m

60. M = IA

$I = \frac{8}{2} = 4$

$n = \frac{It}{e} = \frac{4 \times 1}{1.6 \times 10^{-19}} = 25 \times 10^{18}$

Critical Thinking

2. In an atom, electrons revolve around the nucleus and as such the circular orbits of electrons may be considered as the small current loops. In addition to orbital motion, an electron has got spin motion also. So, the total magnetic moment of electron is the vector sum of its magnetic moments due to orbital and spin motion. Charge particles at rest do not produce magnetic field.

3. If a hole is made at the centre of a bar magnet, then its magnetic moment will not change as its pole strength and length remains same.

4. Magnetism of a magnet falls with rise of temperature and becomes practically zero above curie temperature.

5. One face of loop will behave as south pole and other as north pole. The face where current is anticlockwise will have north polarity and at other face where current is clockwise will have south polarity.

7. Magnetic moment of circular loop carrying current,

$$M = IA = I(\pi R^2) = I\pi\left(\frac{L}{2\pi}\right)^2 = \frac{IL^2}{4\pi}$$

∴ $L = \sqrt{\frac{4\pi M}{I}}$

8. Torque on a bar magnet in earth's magnetic field (B_H) is $\tau = MB_H \sin\theta$, τ will be maximum if sin θ = maximum i.e., θ = 90°. Hence, axis of the magnet is perpendicular to the field of earth.

9. $\tau = MB_H \sin\theta \Rightarrow \frac{d\tau}{d\theta} = MB_H \cos\theta$

This will be maximum when θ = 0°.

10. $\tau = MB \sin\theta = m \times 2l \times B \sin\theta$

$= 48 \times 25 \times 10^{-2} \times 0.15 \times \frac{1}{2}$

$\tau = 0.9$ N m

11. $\tau = MB \sin\theta \Rightarrow \tau \propto \sin\theta$

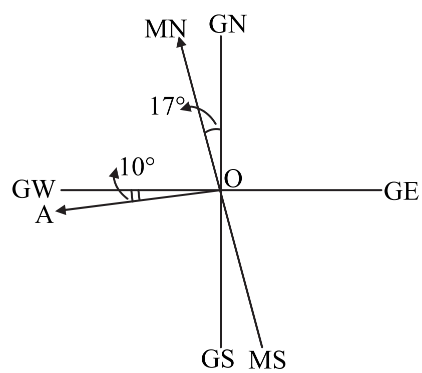
$\Rightarrow \frac{\tau_1}{\tau_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \frac{\tau}{\tau/2} = \frac{\sin 90}{\sin\theta_2}$

∴ $\sin\theta_2 = \frac{1}{2} \Rightarrow \theta_2 = 30°$

∴ angle of rotation = 90° – 30 = 60°

14. At poles, magnetic field is perpendicular to the surface of earth.

15.



As the ship is to reach a place 10° south of west i.e., along OA, it should be steered west of (magnetic) north at an angle of (90 – 17 + 10) = 83°.

16. $B_H = B\cos\delta$

∴ $B = \frac{B_H}{\cos\delta} = \frac{0.5}{\cos 30°} = \frac{0.5}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

17. $B_{eq} = \frac{\mu_0}{4\pi}\frac{M}{r^3}$

∴ $M = \frac{10 \times 10^{-6} \times (0.1)^3}{10^{-7}} = 0.1$ A m^2

18. $B_A = B_X + B_Y$, $B_X = 2B_Y$

(∵ X is along axis and Y along equator)

$= 2B_Y + B_Y$

$= 3B_Y$

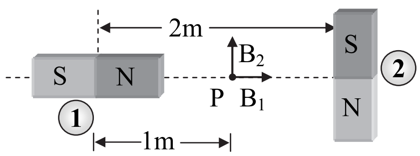
$B_Y = \frac{B_A}{3} = \frac{0.3 \times 10^{-4}}{3}$

$= 0.1 \times 10^{-4}$ T

$B_X = 2B_Y = 2 \times 0.1 \times 10^{-4}$
$= 0.2 \times 10^{-4}$ T
B is away from X
On reversing $B_X \rightarrow -B_X$
$\therefore$ $B = -B_X + B_Y$
$= -0.2 \times 10^{-4} + 0.1 \times 10^{-4}$
$= -0.1 \times 10^{-4}$ T
$\therefore$ $B = -1 \times 10^{-5}$ T
Negative sign shows change in direction of B
$\therefore$ $B = 1 \times 10^{-5}$ T (towards X)

19. With respect to 1st magnet, P lies in end side-on position.

$\therefore$ $B_1 = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$ (RHS)



With respect to 2nd magnet, P lies in broad side on position.

$\therefore$ $B_2 = \frac{\mu_0}{4\pi}\left(\frac{M}{r^3}\right)$ (Upward)

$B_1 = 10^{-7} \times \frac{2\times1}{1} = 2 \times 10^{-7}$ T,

$B_2 = \frac{B_1}{2} = 10^{-7}$ T

As B_1 and B_2 are mutually perpendicular, hence the resultant magnetic field

$B_R = \sqrt{B_1^2 + B_2^2}$

$= \sqrt{(2\times10^{-7})^2 + (10^{-7})^2}$

$\therefore$ $B_R = \sqrt{5} \times 10^{-7}$ T

20. When a bar magnet of pole strength 'm' and magnetic moment 'M' is cut into n equal parts longitudinally and transversely then pole strength of each piece = $\frac{m}{n}$ and magnetic moment of each piece = $\frac{M}{n^2}$

(Refer shortcut 3.)

21. $\cos\delta = \sin\delta \Rightarrow \tan\delta = 1$

$B_H = B_v = 5 \times 10^{-4}$ T

$5 \times 10^{-4} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$

$5 \times 10^{-4} = \frac{10^{-7} \times M}{1}$

$M = 5 \times 10^3$ A m^2

22. Magnetic intensity (H) = 1600 A/m

$\phi = BA$

$B = \frac{\phi}{A}$

$= \frac{2.4\times10^{-5}}{0.2\times10^{-4}}$

$= 1.2$ Wb/m^2

$\mu = \frac{B}{H}$

$= \frac{1.2}{1600}$

$= \frac{12}{16} \times 10^{-3}$

$\mu = 0.75 \times 10^{-3}$ T A^{-1} m

24. No, a stationary charge does not produce magnetic field.

25. Magnetic dipole moment,

$M = IA = \frac{e}{T} \times \pi r^2$

$= \frac{e}{\left(\frac{2\pi r}{v}\right)} \times \pi r^2$ $\left[\because T = \frac{2\pi r}{v}\right]$

$\therefore$ $M = \frac{evr}{2}$

26. Magnetic lines of force is a vector quantity.

28. Magnetic poles always exist in pairs. But, one can imagine magnetic field configuration with three poles. When north poles or south poles of two magnets are glued together. They provide a three pole field configuration. Hence, assertion is false. A bar magnet does not exert a torque on itself due to own its field. Hence, reason is also false.

29. In case of the electric field of an electric dipole, the electric lines of force originate from positive charge and end at negative charge, whereas isolated magnetic lines are closed continuous loops extending through out the body of the magnet.

30. Period of revolution of electron,

$T = \frac{2\pi r}{v}$

$= \frac{2\pi\times0.53\times10^{-10}}{2.3\times10^6}$

$= 1.448 \times 10^{-16}$ s

The orbital motion of electron is equivalent to current,

$I = \frac{e}{T} = \frac{1.6\times10^{-19}}{1.448\times10^{-16}} = 1.105 \times 10^{-3}$ A

Therefore, magnetic moment of the revolving electron,

M = I A

$= I \times \pi r^2$

$= 1.105 \times 10^{-3} \times \pi \times (0.53 \times 10^{-10})^2$

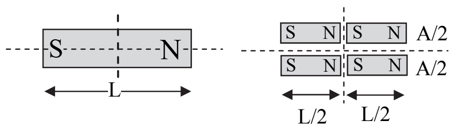
∴ $M = 9.75 \times 10^{-24}$ A m^2

Competitive Thinking

3. $2l = \frac{5}{6} \times$ geometric length

∴ Geometric length $= 2l \times \frac{6}{5} = 10 \times \frac{6}{5} = 12$ cm.

5. For each part $m' = \frac{m}{2}$



7. New magnetic moment

$M' = \frac{2M}{\pi} = \frac{2mL}{\pi}$

$= \frac{2\times0.8\times31.4\times10^{-2}}{3.14}$

$= 0.16$ A m^2

8. $L = \pi r$

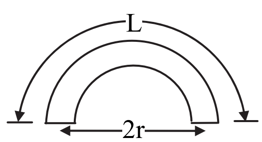
$r = L/\pi$

∴ New magnetic moment

$M' = m \times 2r$

$= m \times 2 \times L/\pi$

$M' = \frac{M}{L} \times 2 \times \frac{L}{\pi} = \frac{2M}{\pi}$



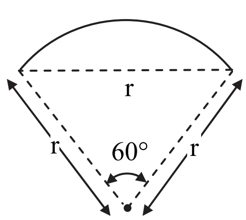
9. $L = \frac{\pi}{3} \times r$

$\Rightarrow r = \frac{3L}{\pi}$

$M' = m \times r$

$= m\left(\frac{3L}{\pi}\right)$

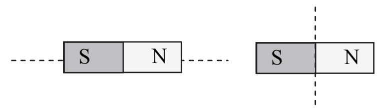
$= \frac{3M}{\pi}$ [∵ M = mL]



11. If a magnet is cut along the axis of magnet of length L, then new pole strength $m' = \frac{m}{2}$ and new length L′ = L.

∴ New magnetic moment,

$M' = \frac{m}{2} \times L = \frac{mL}{2} = \frac{M}{2}$



If a magnet is cut perpendicular to the axis of magnet, then new pole strength m′ = m and new length,

L′ = L/2

∴ New magnetic moment,

$M' = m \times \frac{L}{2} = \frac{mL}{2} = \frac{M}{2}$

12. For a coil M = iA

∴ $M \propto A \propto r^2$ ....(∵ $A = \pi r^2$)

But coil has length L,

$r = \frac{L}{2\pi}$ ....(∵ $L = 2\pi r$)

∴ $M \propto L^2$

13. $M = iA = i(\pi r^2)$

But $l = 2\pi r$

$\Rightarrow r = l / 2\pi$

∴ $M = i\left(\pi \times \frac{l^2}{4\pi^2}\right) = \frac{l^2 i}{4\pi}$

16. $\tau = MB \sin\theta$

$= 200 \times 0.25 \times \sin 30°$

∴ $\tau = 25$ N-m

17. $\tau = MB \sin\theta$

$= m \times (2l) \times B \sin\theta$

$= 10^{-4} \times 0.1 \times 30 \sin 30°$

$= 1.5 \times 10^{-4}$ N-m

18. $\tau = MB \sin\theta = [m (2l)] B \sin\theta$

$= 40 \times 10 \times 10^{-2} \times 2 \times 10^{-4} \times \sin 45°$

$\approx 5.656 \times 10^{-4}$

$= 0.5656 \times 10^{-3}$ N-m

19. M = nIA

$= 2000 \times 2 \times 1.5 \times 10^{-4}$

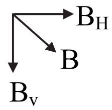
= 0.6 J/T

$\tau = MB \sin 30°$

$= 0.6 \times 5 \times 10^{-2} \times \frac{1}{2}$

$= 1.5 \times 10^{-2}$ N-m

21.



$B = \sqrt{B_v^2 + B_H^2}$

Where, 'B_H' and 'B_v' are the horizontal and vertical components of earth's magnetic induction 'B'.

25. $B_H = B \cos \delta$

$B = \dfrac{B_H}{\cos\delta} = \dfrac{B_0}{\cos 45^\circ} = \sqrt{2}B_0$

26. $B_H = 3.0$ G, $\delta = 30^\circ$

$B_H = B\cos\delta$

$B = \dfrac{B_H}{\cos\delta} = \dfrac{3}{\cos 30^\circ} = 3.46 \text{ G} \approx 3.5 \text{ G}$

27. $B_H = B \cos \delta$

$\cos \delta = \dfrac{B_H}{B} = \dfrac{0.22}{0.4}$

$\tan \delta = \dfrac{\sqrt{(0.4)^2 - (0.22)^2}}{0.22}$

$\delta = \tan^{-1} (1.518)$

28. Since $B_V = B_H \tan\theta$ and

$B_H = \sqrt{3}\, B_V$

$\therefore$ $B_V = \sqrt{3}\, B_V \tan\theta$

$\therefore$ $\tan \theta = \dfrac{1}{\sqrt{3}} = \tan 30^\circ$

$\therefore$ $\theta = 30^\circ$

29. Since $B_V = B_H \tan\theta$ and $B_V = \sqrt{3}\, B_H$

$\therefore$ $\sqrt{3}\, B_H = B_H \tan\theta$

$\Rightarrow \tan \theta = \sqrt{3} = \tan 60^\circ$

$\therefore$ $\theta = 60^\circ$

30. $\tan \delta = \dfrac{B_V}{B_H} = \dfrac{3}{4}$

$\therefore$ $B_V = \dfrac{3}{4} B_H$, $B_V = 6 \times 10^{-5}$ T

$B_H = \dfrac{4}{3} \times 6 \times 10^{-5} \text{ T} = 8 \times 10^{-5}$ T

$\therefore$ $B_{total} = \sqrt{B_V^2 + B_H^2}$

$= \sqrt{(36+64)} \times 10^{-5}$

$= 10 \times 10^{-5}$

$= 10^{-4}$ T

31. $B_E = \dfrac{\mu_0 M}{4\pi r^3}$

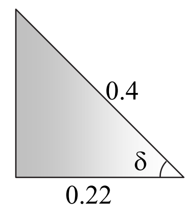
$\Rightarrow M = \dfrac{4\pi r^3 B_E}{\mu_0}$

$M = \dfrac{4\pi \times 0.5 \times (6.4\times10^6)^3 \times 10^{-4}}{4\pi\times10^{-7}}$

$= 1.31 \times 10^{23}$ Am^2

32. If δ_1, δ_2 are the observed angles of dip in two mutually perpendicular planes and δ is true value of dip, then

$\tan \theta_1 = \dfrac{B_V}{B_{H_1}}$, $\tan \theta_2 = \dfrac{B_V}{B_{H_2}}$ and $\tan \theta = \dfrac{B_V}{B_H}$



As B_{H_1} and B_{H_2} are horizontal components in two vertical planes perpendicular to each other,

$B_H^2 = B_{H_1}^2 + B_{H_2}^2$

$\left(\dfrac{B_V}{\tan\theta}\right)^2 = \left(\dfrac{B_V}{\tan\theta_1}\right)^2 + \left(\dfrac{B_V}{\tan\theta_2}\right)^2$

$\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2$

36. $B = \dfrac{\mu_0}{4\pi}\dfrac{2M}{r^3}$

$B = 10^{-7} \times \dfrac{2\times1.2}{(0.1)^3} = 2.4 \times 10^{-4}$ T

37. $B_{axis} = \dfrac{\mu_0}{4\pi}\dfrac{2M}{r^3}$

$\therefore$ $M = \dfrac{4\times10^{-5}\times(0.1)^3}{2\times10^{-7}} = 0.2$ A m^2

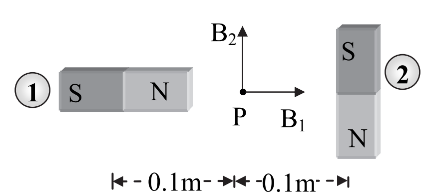
38. $(B_{axis})_P = (B_{eq})_Q$

$\dfrac{\mu_0}{4\pi}\dfrac{2M}{r_1^3} = \dfrac{\mu_0}{4\pi}\dfrac{M}{r_2^3}$

$\dfrac{r_1^3}{r_2^3} = \dfrac{2}{1}$

$\therefore$ $\dfrac{r_1}{r_2} = (2)^{1/3}$

39.



From figure, $B_{net} = \sqrt{B_a^2 + B_e^2}$

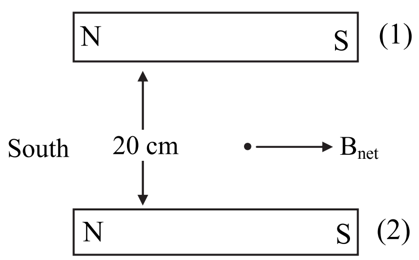
$$= \sqrt{\left(\frac{\mu_0}{4\pi}.\frac{2M}{r^3}\right)^2 + \left(\frac{\mu_0}{4\pi}.\frac{M}{r^3}\right)^2}$$

$$= \sqrt{5}.\frac{\mu_0}{4\pi}.\frac{M}{r^3}$$

$$= \sqrt{5} \times 10^{-7} \times \frac{10}{(0.1)^3}$$

∴ $B_{net} = \sqrt{5} \times 10^{-3}$ tesla.

40.



For a given arrangement,
$B_{net} = B_1 + B_2 + B_H$
For short bar magnets,

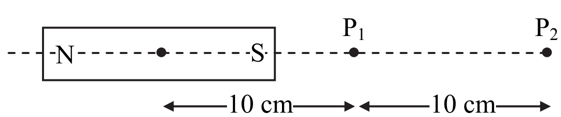
$$B = \frac{\mu_0}{4\pi} \times \frac{M}{r^3}$$

∴ $B_1 = 10^{-7} \times \frac{1.20}{10^{-3}} = 1.2 \times 10^{-1}$ Wb/m^2

and $B_2 = 10^{-7} \times \frac{1}{10^{-3}} = 1 \times 10^{-4}$ Wb/m^2

$B_{net} = (1.2 + 1 + 0.36)\ 10^{-4}$
$= 2.56 \times 10^{-4}$ Wb/m^2

41.



Magnetic field along the axis is given by,

$$B_{axis} = \frac{\mu_0}{4\pi} \times \frac{2Mr}{\left(r^2 - l^2\right)^2}$$

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{2M \times 0.1}{\left[(0.1)^2 - l^2\right]^2}$$

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{2M \times 0.2}{\left[(0.2)^2 - l^2\right]^2}$$

$$\frac{B_1}{B_2} = \frac{0.1}{\left[(0.1)^2 - l^2\right]^2} \times \frac{\left[(0.2)^2 - l^2\right]^2}{0.2}$$

∴ $\frac{25}{2} = \frac{\left[(0.2)^2 - l^2\right]^2}{2\left[(0.1)^2 - l^2\right]^2}$ ....$\left(\because \frac{B_1}{B_2} = \frac{25}{2}\right)$

∴ $5 = \frac{0.04 - l^2}{0.01 - l^2}$

$0.05 - 5l^2 = 0.04 - l^2$
$0.01 = 4l^2$
$0.1 = 2l$
$l = 0.05$ m = 5 cm
Magnetic length = $2l$ = 10 cm

43. M = n i A,

For circular loop $A_1 = \pi\left[\frac{L^2}{4\pi^2}\right] = \frac{L^2}{4\pi}$

For square loop $A_2 = \left(\frac{L}{4}\right)^2 = \frac{L^2}{16}$

∴ $\frac{M_1}{M_2} = \frac{niA_1}{niA_2} = \frac{L^2}{4\pi} \times \frac{16}{L^2} = \frac{4}{\pi}$

44. M = n i A

But, $i = \frac{q}{t} = q \times n = q\frac{\omega}{2\pi}$
Where n = frequency
ω = angular velocity

∴ $M = \left[q\frac{\omega}{2\pi}\right]\pi R^2 = \frac{q\omega R^2}{2}$

46. At magnetic poles the horizontal component of earth's field is zero, only vertical component exists.
So, a compass needle is free to rotate in horizontal plane and may stay in any direction.
The dip needle rotates in vertical plane and the angle of dip at poles is 90°. Hence, the dip needle will stand vertical at the north pole of earth.

47. W = –MB (cos 60° – cos 0°)

$$= -MB\left(\frac{1}{2} - 1\right)$$

$$= \frac{1}{2}MB$$

∴ MB = 2 W
Torque required τ = MB sin θ
= 2 W sin 60°

∴ $\tau = 2\text{ W}\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$ W

Evaluation Test

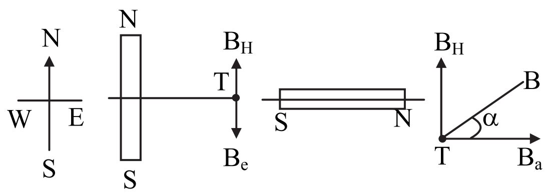
1. Flux = $\vec{B} \times \vec{A}$ = BA sin 45°

$= \sqrt{2} \times 10^{-4} \times \pi \times 5^2 \times 10^{-4} \times \frac{1}{\sqrt{2}}$ $(\because A = \pi r^2)$

$= 25\pi \times 10^{-8}$ Wb

3. At poles, angle of dip (δ) = 90°, B_H = zero, B_V = B. Magnetic field is almost vertical.

4. A neutral point is obtained on equatorial line when north pole of magnet points towards north of earth.
At neutral point,
field due to magnet = field due to Earth
i.e., numerically, $B_e = B_H$



As the magnet is rotated, the point T lies now on the axial line of magnet.
B_a = field due to magnet when ⊥ to earth's N-S direction.
For a short magnet, $B_a = 2\ B_e$
Field at T = B

$B^2 = B_H^2 + B_a^2$

$\Rightarrow B^2 = B_H^2 + (2B_e)^2 = B_H^2 + (2B_H)^2$

$\Rightarrow B^2 = B_H^2 + 4B_H^2 = 5B_H^2$

$\Rightarrow B = \sqrt{5}B_H$

5. $\tan\delta = \frac{\text{Vertical component}}{\text{Horizontal component}} = \frac{B_V}{B_H}$

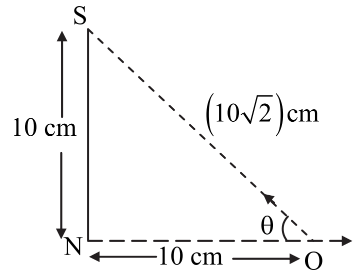
$\tan\delta_1 = \frac{B_V}{B_H \cos\theta} = \frac{\tan\delta}{\cos\theta}$

$= \tan\delta \sec\theta$

$\delta_1 = \tan^{-1}(\tan\delta \sec\theta)$

6. N-S is a magnet placed vertically on paper. O is a point 10 cm south of the lower N-pole. Let m be the pole strength

$\cos\theta = \frac{NO}{SO} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$



Magnetic induction at O due to N-pole

$= \frac{m}{(10)^2}\left(\text{along } \overline{NO}\right)$

Magnetic induction at O due to S-pole

$= \frac{m}{\left(10\sqrt{2}\right)^2}\left(\text{along } \overline{OS}\right)$

Resultant magnetic induction at O in the horizontal plane

$= \frac{m}{(10)^2} - \left[\frac{m}{\left(10\sqrt{2}\right)^2}\cos\theta\right]$

$= \frac{m}{(10)^2} - \left[\frac{m}{\left(10\sqrt{2}\right)^2}\frac{1}{\sqrt{2}}\right] = 6.46 \times 10^{-3}$ m

At neutral point, the magnetic induction B due to magnet is equal and opposite to the horizontal component of earth's magnetic induction

∴ 6.46×10^{-3}m = 0.5

∴ m = 77.4 ab-ampere × cm.

7. Period of revolution of the electron,

$T = \frac{2\pi r}{v}$

Current $I = \frac{e}{T} = \frac{ev}{2\pi r}$

Magnetic moment, M = IA = I × πr^2

$= \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$

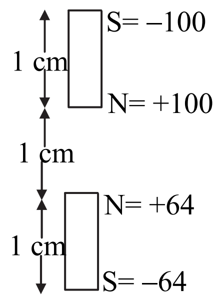
$= \frac{1.6 \times 10^{-19} \times 1.8 \times 10^{6} \times 1.52 \times 10^{-10}}{2}$

$= 2.19 \times 10^{-23}$ A m^2

8. Adding magnetic moments vectorially,

$M = \sqrt{M^2 + M^2 + 2MM\cos 60°} = \sqrt{3}M$

9.



In CGS system, $\frac{\mu_0}{4\pi} = 1$

In equilibrium,

net repulsion due to magnetic interaction = weight of upper magnet.

From the figure shown

$$\frac{100(64)}{1^2} + \frac{100(-64)}{2^2} - \frac{100(64)}{2^2} - \frac{100(-64)}{3^2} = m \times g$$

$\therefore$ $100 \times 64\left[\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{2^2} + \frac{1}{3^2}\right] = m \times 1000$

$\therefore$ $6.4\left(\frac{11}{18}\right) = m$

$\therefore$ m = 3.91 g

10. On magnetisation, the molecular magnets are aligned parallel to the field. Therefore, the length of the bar in the direction of magnetisation increases. This effect is called magnetostriction effect.

11. When a bar magnet is placed with its north pole towards geographic north, the neutral point lies on equatorial line of the magnet. When a bar magnet is placed with its north pole towards geographic south, the neutral point lies on axial line of the magnet.

12. Inner radius = 20 cm, outer radius = 22 cm.

$\therefore$ Mean radius r = $\frac{20+22}{2}$ = 21 cm.

Magnetic induction along axis

$$B_{axis} = \frac{\mu_0}{4\pi}\frac{2nIA}{x^3}$$

A = πr^2 and x = 2 m

$\therefore$ $B_{axis} = \frac{10^{-7} \times 2 \times 3000 \times 10 \times \pi \times (0.21)^2}{2^3}$

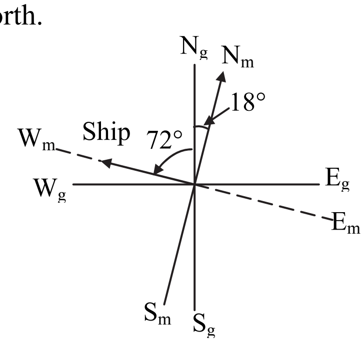
$= 1.04 \times 10^{-4}$ T

13. When a piece of a magnetic material like soft iron, cobalt, nickel etc. is placed near a bar magnet, it acquires magnetism. The magnetism so acquired is called induced magnetism and this property of magnetism is called inductive property. Hence, option (A) is correct.

The force of attraction or repulsion F between two magnetic poles of strengths m_1 and m_2 separated by a distance r in space is directly proportional to the product of pole strengths and inversely proportional to the square of the distance between their centres. This is called Coulomb's law of magnetic force. Hence, option (C) is correct.

In a bar magnet, attraction is minimum at centre and maximum at poles. Hence, option (D) is incorrect.

15. As shown in the figure, magnetic N-S is 18° east of geographic NS. As ship is sailing due west according to Mariner's compass, it is going (90°–18°) = 72° west of (geographic) north.



16. If we assume that earth's magnetic field is due to a bar magnet at the centre of earth held along the polar axis of earth, then the equatorial magnetic field is,

$$B_{equator} = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

where, r = R = radius of earth = 6.4×10^6 m

$\therefore$ $0.4 \times 10^{-4} = 10^{-7} \times \frac{M}{(6.4 \times 10^6)^3}$

$\therefore$ $M = \frac{0.4 \times 10^{-4}(6.4 \times 10^6)^3}{10^{-7}}$

$\approx 1.05 \times 10^{23}$ A m^2

Textbook Chapter No.

01 Circular Motion

Hints

Classical Thinking

3. $f = 300$ r.p.m. $= \frac{3000}{60}$ r.p.s;

$\theta = \omega.t = 2\pi \times \frac{3000}{60} \times 1 = 100\ \pi$ rad

5. For a seconds hand of a watch, T = 60 s

$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30}$ rad/s

6. $n = 100$ r.p.m. $= \frac{100}{60}$ r.p.s.

$\omega = 2\pi n = \frac{2\pi \times 100}{60} = 10.47$ rad/s

7. $n = 3.5$ r.p.s.

$\omega = 2\pi n = 2 \times \pi \times 3.5 = 7\pi$

$= 7 \times 3.14 \approx 22$ rad/s

8. For earth, $T = 24$ hr $= 24 \times 3600 = 86400$ s

$\omega = \frac{2\pi}{T} = \frac{2\pi}{24}$ rad/hr $= \frac{2\pi}{86400}$ rad/s

9. Using, $\omega = 2\pi n$

$\therefore$ $125 = 2\pi n$

$\therefore$ $n = \frac{125}{2\pi}$ $\qquad \therefore$ $n \approx 20$ Hz

10. For minute hand, $T_M = 60 \times 60$ s; for hour hand,

$T_H = 12 \times 3600$ s

$\therefore$ $\frac{\omega_M}{\omega_H} = \frac{T_H}{T_M} = \frac{12 \times 3600}{60 \times 60} = 12 : 1$ ....[$\because \omega \propto \frac{1}{T}$]

11. $\alpha = \frac{d\omega}{dt} = 0$ ....($\because \omega$ = constant)

12. $n_1 = 0$, $n_2 = 210$ r.p.m. $= \frac{210}{60}$ r.p.s.

$d\omega = 2\pi(n_2 - n_1) = 2\pi\left(\frac{210}{60} - 0\right) = 7\ \pi$ rad/s

$\alpha = \frac{d\omega}{dt} = \frac{2\pi \times 210}{60 \times 5} = 4.4$ rad/s^2

14. $C = 2\pi r$

$\therefore$ $r = \frac{C}{2\pi}$

$\therefore$ $v = r(2\pi n) = \frac{C}{2\pi} \times 2\pi \times f = fC$ ....[$\because \omega = 2\pi n$]

15. Using, $v = r\omega = 0.2 \times 10$ m/s $= 2$ m/s

16. Using, $v = r\omega$

$= r \times (2\pi n) = 0.4 \times 2\pi \times 5$

$= 0.4 \times 2 \times 3.14 \times 5 = 12.56 \approx 12.6$ m/s

17. Angular velocity of particle P about point A,

$\omega_A = \frac{v}{r_{AB}} = \frac{v}{2r}$

Angular velocity of particle P about point C,

$\omega_C = \frac{v}{r_{BC}} = \frac{v}{r}$

$\frac{\omega_A}{\omega_C} = \frac{v}{2r} \times \frac{r}{v}$

$\frac{\omega_A}{\omega_C} = \frac{1}{2}$

18. In U.C.M., direction of velocity and acceleration change from point to point.

22. At each point on circular path, the magnitude of velocity remains the same for any value of θ.

23. The particle performing circular motion flies-off tangentially.

27. $n = 1200$ r.p.m. $= \frac{1200}{60}$ r.p.s. $= 20$ r.p.s.

$a = \omega^2 r = (4\ \pi^2\ n^2)\ r = 4 \times (3.142)^2 \times (20)^2 \times 0.3$

≈ 4740 cm/s^2

28. $n = 900$ r.p.m. $= \frac{900}{60}$ r.p.s $= 15$ r.p.s,

$d = 1.2$ m $\Rightarrow r = \frac{1.2}{2} = 0.6$ m

$a = \omega^2 r = (2\ \pi n)^2 \times \frac{1.2}{2} = 540\ \pi^2$ m/s^2

29. $r = 10$ cm $= 0.1$ m, $a = 1000 \times 10$ m/s^2
$a = \omega^2 r$
$\therefore \quad \omega^2 = \frac{a}{r}$
$\therefore \quad \omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{1000\times10}{10\times10^{-2}}} \approx 316$ rad/s
$n = 316/2\pi = 50.3$ r.p.s. ≈ 50 r.p.s.
$\therefore \quad n = 3000$ r.p.m.

31. Using,
$a_r = \frac{v^2}{r} = \frac{20\times20}{10} = 40$ m/s^2, $a_t = 30$ m/s^2
$a = \sqrt{a_r^2 + a_t^2} = \sqrt{40^2 + 30^2} = 50$ m/s^2

39. $p = mv$; $F = \frac{mv^2}{r}$
$\therefore \quad \frac{F}{p} = \frac{mv^2}{r} \times \frac{1}{mv} = \frac{v}{r}$

40. Using, $F_s = \frac{mv^2}{r}$
$\therefore \quad v^2 = \frac{F_s r}{m} = \frac{10^5 \times 10}{10^2} = 10^4$
$\therefore \quad v = 100$ m/s

41. $F = \frac{mv^2}{r}$
If m and v are constants, then $F \propto \frac{1}{r}$
$\therefore \quad \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)$

42. Using, $F = \frac{mv^2}{r}$
$\therefore \quad r = \frac{mv^2}{F} = \frac{10\times(5)^2}{125} = \frac{250}{125} = 2$ m

43. Using, $v^2 = \frac{Tr}{m}$
Breaking tension $T = \frac{mv^2}{r}$
(r = length of the string)
$\therefore \quad v^2 = \frac{50\times1}{1}$
$\therefore \quad v = 5\sqrt{2}$ m/s

44. Using, $F = mr\omega^2 = m \times 4\pi^2 n^2 r$
$\therefore \quad m \times 4\pi^2 n^2 r = 6 \times 10^{-14}$
$\therefore \quad n^2 = \frac{6\times10^{-14}}{4\times1.6\times10^{-27}\times3.14^2\times0.12}$
$\therefore \quad n \approx 5 \times 10^6$ cycles/s

53. Centripetal acceleration,
$a_{cp} = \omega^2 r = \frac{g l \sin\theta}{l\cos\theta} = g\tan\theta$
$= 10 \times \tan 60° = 17.3$ m/s^2

54. Using,
$mr\omega^2 = T$ and $\omega = 2\pi n$
$n = \frac{1}{2\pi}\sqrt{\frac{T}{mr}} = 2\,\text{Hz}$

59. For looping the loop, minimum velocity at the highest point should be $\sqrt{gl}$.

60. Thrust at the lowest point of concave bridge
$= mg + \frac{mv^2}{r}$

61. $N = mg\cos\theta - \frac{mv^2}{R}$, θ = angle with vertical.
As vehicle descends, angle increases, its cosine decreases, hence N decreases.

64. $\mu\, mr\omega^2 \geq mg$; $\omega \geq \sqrt{\frac{g}{\mu r}}$

65. $v_1 = \sqrt{rg}$
$v_2 = \sqrt{5rg} = \sqrt{5} \times \sqrt{rg} = \sqrt{5} \times v_1$

66. Using,
$\alpha = \frac{\omega - \omega_0}{t} = \frac{2\pi(n - n_0)}{t}$
$= \frac{2\times3.14\times(350-0)}{220} \approx 10$ rad/s^2

67. Using,
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$= 4 \times 10 + \frac{1}{2} \times 2 \times (10)^2 = 140$ rad
$n = \frac{\theta}{2\pi} = \frac{140}{2\times3.142} \approx 22$

68. $v = 72$ km/hr $= 72\times\frac{5}{18} = 20$ m/s,
$d = 0.5$ m $\Rightarrow r = \frac{0.5}{2}$ m
$\therefore \quad \omega_0 = \frac{v}{r} = \frac{20}{0.5/2} = 80$ rad/s
$\omega^2 = \omega_0^2 + 2\alpha\theta$
$0 = (80)^2 + 2\alpha(2\pi \times 20)$
$-6400 = 80\pi\alpha$
$\alpha = \frac{-80}{\pi} = -25.48$ rad/s^2

69. Difference in tensions = 6 mg = 6 × 2 × 9.8
= 12 kg wt

70. $F = m\omega^2 R$

∴ $R \propto \frac{1}{\omega^2}$ (m and F are constant)

If ω is doubled, then radius will become 1/4 times i.e., R/4

Critical Thinking

1. Frequency of wheel, $n = \frac{300}{60} = 5$ r.p.s.

Angle described by wheel in one rotation
= 2π rad.
Therefore, angle described by wheel in 1 sec
θ = 2π × 5 radians = 10 π rad

2. In non-uniform circular motion, particle possesses both centripetal as well as tangential accelerations.

3. n = 2000, distance = 9500 m
Distance covered in 'n' revolutions = n(2πr)
= nπD

∴ 2000πD = 9500

∴ $D = \frac{9500}{2000 \times \pi} = 1.5$ m

4. Period of second hand = T_s = 60 s and
Period of minute hand = T_m = 60 × 60 = 3600 s

Angular speed of second hand $\omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{60}$

Angular speed of minute hand $\omega_m = \frac{2\pi}{T_m} = \frac{2\pi}{3600}$

∴ $\frac{\omega_s}{\omega_m} = \frac{2\pi}{60} \times \frac{3600}{2\pi} = 60 : 1$

5. For minute hand, T = 60 min = 60 × 60 s

Angular speed, $\omega = \frac{2\pi}{T} = \frac{2\pi}{60 \times 60}$ rad/s

$= \frac{\pi}{1800} \times \frac{180}{\pi} = 0.1$

....[∵ 1 rad = $\frac{180^\circ}{\pi}$]

6. $\omega = \frac{\text{angle described}}{\text{time taken}} = \frac{2\pi}{2} = \pi$ rad/s

7. $n = \frac{540}{60} = 9$ r.p.s., ω = 2πn = 18π rad/s

Angular acceleration

$= \frac{\text{Gain in angular velocity}}{\text{time}} = \frac{18\pi}{6} = 3\pi \text{ rad s}^{-2}$

8. Using, $\alpha = \frac{d\omega}{dt}$

∴ $\alpha = \frac{15\pi - 10\pi}{4-2} = \frac{5\pi}{2} = 2.5\ \pi\ \text{rad/s}^2$

9. Using,
$\theta = 2t + 3t^2$

∴ $\omega = \frac{d\theta}{dt} = 2 + 6t$

$\alpha = \frac{d\omega}{dt} = 6\ \text{rad/s}^2$

10. v = r.ω
where r is distance from axis of rotation.
At the north-pole, r = 0 ⇒ v = 0

11. A particle will describe a circular path if the angle between velocity, $\vec{v}$ and acceleration $\vec{a}$ is 90°.

12. Frequency = $\frac{n}{60}$ r.p.s., t = 1 min = 60 s

Angular velocity, $\omega = 2\pi \frac{n}{60}$

∴ Linear velocity, $v = \omega r = \frac{2\pi n \times \pi}{60} = \frac{2\pi^2 n}{60}$ cm/s

13. Using,
$v = r\omega = r \times \frac{2\pi}{T} = 60 \times \frac{2 \times 3.14}{60} = 6.28$ mm/s

$\Delta v = 6.28\sqrt{2}$ mm/s ≈ 8.88 mm/s

14. Speed of $C_1 = \omega R_1 = \frac{2\pi}{T} R_1$

Speed of $C_2 = \omega R_2 = \frac{2\pi}{T} R_2$

∴ $\frac{\text{Speed of } C_1}{\text{Speed of } C_2} = \frac{2\pi R_1 / T}{2\pi R_2 / T} = \frac{R_1}{R_2}$

15. r = 0.25 m, n = 15 r.p.m. = $\frac{15}{60}$ r.p.s.

$\omega = 2\pi n = \frac{2 \times \pi \times 15}{60} = \frac{\pi}{2}$ rad/s

$v = r\omega = 0.25 \times \frac{\pi}{2} = \frac{\pi}{8}$ m/s

16. $T = \frac{20}{40} = \frac{1}{2} = 0.5$ s

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi$ rad/s

Let r = 50 cm = 0.5 m
v = rω = 0.5 × 4π = 2π m/s

17. T = 24 hr, r = 6400 km

$$v = \omega r = \frac{2\pi}{T} \times r = \frac{2\pi}{24} \times 6400 = \frac{2 \times 3.14 \times 6400}{24}$$

$v \approx 1675$ km/hr

18. $$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$$

19. $\theta = 2t^3 + 0.5$

$\therefore$ $\omega = \frac{d}{dt}(2t^3 + 0.5) = 6t^2$

At t = 2 s, $\omega = 6 \times 2^2 = 24$ rad/s

22. While moving along a circle, the body has a constant tendency to regain its natural straight line path.
This tendency gives rise to a force called centrifugal force. The centrifugal force does not act on the body in motion, the only force acting on the body in motion is centripetal force. The centrifugal force acts on the source of centripetal force to displace it radially outward from centre of the path.

23. Tangential force acting on the car increases with the magnitude of its speed.

$\therefore$ a_t = time rate of change of its speed
= change in the speed of the car per unit time which is 3 m/s

$\therefore$ Tangential acceleration = 3 m /s^2

24. There is no relation between centripetal and tangential acceleration. Centripetal acceleration is a must for circular motion but tangential acceleration may be zero.

25. When a body is moving with constant speed, the tangential acceleration developed in a body is zero.

26. Radius of horizontal loop, r = 1 km = 1000 m

$$v = 900 \text{ km/h} = \frac{900 \times 10^3}{3600} = 250 \text{ m/s}$$

$$\therefore \quad a = \frac{v^2}{r} = \frac{250 \times 250}{1000} = 62.5 \text{ m/s}^2$$

$$\therefore \quad \frac{a}{g} = \frac{62.5}{10} = 6.25$$

27. Velocity, $v = \omega r$

$$\therefore \quad v' = \omega r' = \frac{\omega r}{2} = \frac{v}{2} = 10 \text{ cm/s}$$

$\therefore$ $a = \omega^2 r$

$$\therefore \quad a' = \omega^2 r' = \omega^2 \times \frac{r}{2} = \frac{a}{2} = 10 \text{ cm/s}^2$$

28. In uniform circular motion, acceleration is caused due to change in direction and is directed radially towards centre.

29. As ω is constant, acceleration is due to the change in direction of velocity = $\omega^2 r$

As $r_A > r_B \Rightarrow a_A > a_B$

30. In half a circle, the direction of acceleration is reversed.

It goes from $\frac{v^2}{r}$ to $\frac{-v^2}{r}$

Hence, change in centripetal acceleration

$$= \frac{v^2}{r} - \left(\frac{-v^2}{r}\right) = \frac{2v^2}{r}$$

31. If $a_r = 0$, there is no radial acceleration and circular motion is not possible

So $a_r \neq 0$

If $a_t \neq 0$ the motion is not uniform as angular velocity will change

So $a_r \neq 0$ and $a_t = 0$ for uniform circular motion

32. Centripetal force = $\frac{mv^2}{r}$ and is directed always towards the centre of circle. Sense of rotation does not affect magnitude and direction of this centripetal force.

33. The surface will rise from the sides, due to centrifugal force.

34. Distance covered, $s = \frac{\theta}{360^\circ} \times 2\pi r$

$$660 = \frac{90}{360} \times 2\pi r$$

r = 420 m

$$F = \frac{mv^2}{r} = \frac{840 \times 10 \times 10}{420} = 200 \text{ N}$$

35. Using, $F_{cp} = m\omega^2 r = m\left(\frac{2\pi}{T}\right)^2 r$

$$= 500 \times 10^{-3} \times \left(2 \times \frac{22}{7} \times \frac{1}{11}\right)^2 \times 0.49$$

$$= \frac{500 \times 10^{-3} \times 16 \times 0.49}{49} = 0.08 \text{ N}$$

36. Centripetal force on electrons is provided by electrostatic force of attraction.

$\therefore$ $F \propto \frac{1}{r^2}$ and $r \propto n^2$ where n is principal quantum no.

$\therefore$ $\frac{F_1}{F_2} = \frac{n_2^4}{n_1^4} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$

37. m = 2 kg, r = 1 m, F = 32 N

Force, $F = m\omega^2 r$

$\therefore$ $\omega^2 = \frac{32}{2\times1} = 16$ $\therefore$ $\omega = 4$ rad/s

$\therefore$ Frequency of revolution per minute

$n = \frac{\omega}{2\pi} \times 60 = \frac{4\times7}{2\times22} \times 60 \approx 38$ rev / min

38. r = 20 cm = 20×10^{-2} m = 0.2 m

Using, $F = \frac{mv^2}{r} = 10$

$\therefore$ $\frac{1}{2}mv^2 = 10 \times \frac{r}{2} = 10 \times \frac{0.20}{2} = 1$ J

39. $r_1 = 9$ cm

In the given condition, friction provides the required centripetal force and that is constant. i.e. $m\omega^2 r$ = constant.

$\therefore$ $r \propto \frac{1}{\omega^2}$ $\therefore$ $r_2 = r_1\left(\frac{\omega_1}{\omega_2}\right)^2 = 9\left(\frac{1}{3}\right)^2 = 1$ cm

40. Using,

$\mu_s mg \le mr\omega^2$

$\mu_s g = r\omega^2$ (For minimum angular speed)

$\omega^2 = \frac{\mu_s g}{r} = \frac{0.25\times9.8}{5\times10^{-2}} = \frac{25}{5} \times 9.8$

$= 9.8 \times 5 = 49.0$

$\therefore$ $\omega = 7$ rad/s

41. Breaking tension = 4 × 10 = 40 N

$\therefore$ $T = mr\omega^2$

$\therefore$ $\omega^2 = \frac{T}{mr} = \frac{40}{200\times10^{-3}\times1} = 200$

$\therefore$ $\omega \approx 14$ rad/s

42. Using,

$v = \sqrt{\mu rg} = \sqrt{0.4\times50\times9.8} = \sqrt{196}$

v = 14 m/s

$\omega = \frac{v}{r} = \frac{14}{50} = 0.28$ rad/s

43. Since car turns through 90° after travelling 471 m on the circular road, the distance 471 m is quarter of the circumference of the circular path. If R is the radius of the circular path, then

$\frac{1}{4}(2\pi R) = 471$

$\therefore$ $R = \frac{471\times2}{\pi} = \frac{471\times2}{3.14} = 300$ m

v = 12 m/s, m = 1000 kg

$\therefore$ Centripetal force,

$F_{cp} = \frac{mv^2}{R} = \frac{1000\times(12)^2}{300} = 480$ N

44. This horizontal inward component provides required centripetal force to negotiate the curve safely.

45. $\tan\theta = \frac{v^2}{rg} \Rightarrow \tan\theta \propto v^2$

$\therefore$ $\frac{\tan\theta_1}{\tan\theta_2} = \frac{v_1^2}{v_2^2} = \frac{v^2}{4v^2} = \frac{1}{4}$

$\therefore$ $\tan\theta_2 = 4\tan\theta_1$

46. $\sin\theta = \frac{h}{l}$ and $\tan\theta = \frac{v^2}{rg}$

$\therefore$ $\tan\left\{\sin^{-1}\left(\frac{h}{l}\right)\right\} = \frac{v^2}{rg}$

47. Reaction on inner wheel, $R_1 = \frac{1}{2}M\left[g - \frac{v^2h}{ra}\right]$

Reaction on outer wheel, $R_2 = \frac{1}{2}M\left[g + \frac{v^2h}{ra}\right]$

where, r = radius of circular path, 2a = distance between two wheels and h = height of centre of gravity of car.

48. Using,

$\mu mg = m\omega^2 r$

$\therefore$ $\omega = \sqrt{\frac{\mu g}{r}} = \sqrt{\frac{0.4\times10}{1}} = \sqrt{4} = 2$ rad/s

49. Using,

$v^2 = \mu rg = 0.8 \times 100 \times 9.8 = 784$

$\therefore$ v = 28 m/s

50. $v = \sqrt{\mu gr}$

When μ becomes $\frac{\mu}{2}$, v becomes $\frac{v}{\sqrt{2}}$ i.e. $\frac{10}{\sqrt{2}}$

$= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ ms^{-1}

51. $v = 36 \text{ km/hr} = \frac{36\times10^3}{3600} = 10 \text{ m/s}$

The speed with which the car turns is

$v^2 \geq \mu Rg$

$\therefore \quad R \leq (10)^2 \times \frac{1}{0.8\times10} = 12.5 \text{ m}$

$R \leq 12.5 \text{ m}$

$\therefore \quad R = 12 \text{ m}$

52. $v = 12 \text{ m/s}, v' = 4\sqrt{2} \text{ m/s}$

$v = \sqrt{\mu rg}$

$\therefore \quad 12 = \sqrt{\mu rg}, \; 4\sqrt{2} = \sqrt{\mu' rg}$

$\frac{12}{4\sqrt{2}} = \sqrt{\frac{\mu}{\mu'}} \Rightarrow \frac{3}{\sqrt{2}} = \sqrt{\frac{\mu}{\mu'}}$

$\therefore \quad \mu' = \frac{2}{9}\mu$

53. For the crate not to slide, the centripetal force should be $\frac{mv^2}{r} = \mu mg$

$\therefore \quad v^2 = \mu rg = 0.6 \times 35 \times 9.8 = 205.8$

$\therefore \quad v = 14.3 \text{ m/s}$

54. Using,

$\mu mg = \frac{mv^2}{r} \qquad \therefore \qquad 0.5 \text{ mg} = \frac{mv^2}{r}$

$v^2 = 0.5 \times r \times g = 0.5 \times 10 \times 9.8 = 49$

$\therefore \quad v = 7 \text{ m/s}$

55. Using,

$\tan\theta \approx \theta = \frac{h}{l}$

$h = l\,\theta = 1.5 \times 0.01 = 0.015 \text{ m}$

56. $l = 1 \text{ m}, g = 110 \text{ m/s}^2$

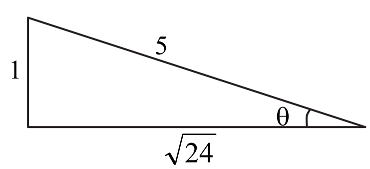
$r = 400 \text{ m}, v = 72 \text{ km/hr} = 72 \times \frac{5}{18} = 20 \text{ m/s},$

$\frac{v^2}{rg} = \frac{h}{l}$

$\therefore \quad h = \frac{v^2 l}{rg} = \frac{20\times20\times1}{400\times10} = 0.1 \text{ m} = 10 \text{ cm}$

57. $\theta = \sin^{-1}(0.2), N = 2000 \text{ N},$

$\sin\theta = 0.2 = \frac{1}{5}$



$mg = N\cos\theta$

$\therefore \quad \text{Weight} = N\cos\theta = \frac{\sqrt{24}}{5} \times 2000 = 1959.6 \text{ N}$

$\ldots\left[\because \cos\theta = \sqrt{1-\left(\frac{1}{5}\right)^2} = \frac{\sqrt{24}}{5}\right]$

58. Using,

$v = \sqrt{rg\tan\theta} = \sqrt{10\times10\times\tan\theta}$

$10 = 10\sqrt{\tan\theta}$

$\tan\theta = 1 \qquad \therefore \qquad \theta = 45°$

59. Using, $h = l\sin\theta$

$\therefore \quad \sin\theta \approx \tan\theta = \frac{h}{l} = \frac{1.2}{8} = 0.15$

$\therefore \quad \tan\theta = 0.15$

Now, $v = \sqrt{rg\tan\theta} = \sqrt{40\times9.8\times0.15} \approx 8 \text{ m/s}$

60. $r = 50 \text{ m}, l = 10 \text{ m}, h = 1.5 \text{ m}$

$\frac{v^2}{rg} = \frac{h}{l}$

$\therefore \quad v = \sqrt{\frac{rgh}{l}} = \sqrt{\frac{50\times9.8\times1.5}{10}} = 8.6 \text{ m/s}$

61. The maximum velocity for a banked road with friction,

$v^2 = gr\left(\frac{\mu+\tan\theta}{1-\mu\tan\theta}\right)$

$\therefore \quad v^2 = 9.8 \times 1000 \times \left(\frac{0.5+1}{1-0.5\times1}\right) \ldots [\because \tan 45 = 1]$

$\therefore \quad v \approx 172 \text{ m/s}$

62. Using,

$\tan\theta = \frac{v^2}{rg}$

$\therefore \quad v = \sqrt{\tan\theta \, rg}$

$= \sqrt{\tan 30^\circ \times 17.32 \times 10}$

$= \sqrt{\frac{1}{\sqrt{3}} \times 17.32 \times 10} = 10 \text{ m/s}$

63. Using,

$\tan\theta = \frac{v^2}{rg} = \frac{20\times20}{20\times9.8} = \frac{20}{9.8} = 2.04$

$\theta = \tan^{-1}(2.04) = 63.90°$

64. $v = 60\text{ km/h} = 60 \times \frac{5}{18} = \frac{50}{3}\text{ m/s}$,

$r = 0.1\text{ km} = 0.1 \times 1000 = 100\text{ m}$

$\tan\theta = \frac{v^2}{rg} = \left(\frac{50}{3}\right)^2 \times \frac{1}{0.1\times10^3\times9.8}$

$\therefore \quad \theta = \tan^{-1}\left[\frac{(50/3)^2}{100\times9.8}\right]$

65. $v = 180\text{ km/hr} = \frac{5}{18}\times 180 = 50\text{ m/s}$

Using,

$\tan\theta = \frac{v^2}{rg} = \frac{50\times50}{500\times10} = \frac{5}{10} = \frac{1}{2}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}(0.5)$

66. $m = 80\text{ kg}, v = 20\text{ m/s}, \theta = \tan^{-1}(0.5)$

In order for the cyclist to turn,

frictional force = centripetal force

$\therefore \quad \mu mg = m\left(\frac{v^2}{r}\right) = mg\frac{v^2}{rg}$

But $\frac{v^2}{rg} = \tan\theta$

$\therefore \quad \mu mg = mg\tan\theta = 80 \times 10 \times 0.5 = 400\text{ N}$

67. Let initial velocity $= v_1$

New velocity $v_2 = v\left(1+\frac{20}{100}\right) = \frac{6v}{5}$

$r_1 = 30\text{ m}, \tan\theta_1 = \frac{v_1^2}{r_1 g}, \tan\theta_2 = \frac{v_2^2}{r_2 g}$

As there is no change in angle of banking,

$\theta_1 = \theta_2$

$\therefore \quad \tan\theta_1 = \tan\theta_2$

$\therefore \quad \frac{v_1^2}{r_1 g} = \frac{v_2^2}{r_2 g}$

$\therefore \quad \frac{r_1}{r_2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{v_1}{\frac{6}{5}v_1}\right)^2 = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$

$\therefore \quad r_2 = \frac{36}{25}r_1 = \frac{36}{25}\times 30 = \frac{216}{5} = 43.2\text{ m}$

68. Using,

$F_s = \frac{mv^2}{r}$ But, $\tan\theta = \frac{v^2}{rg}$

$\frac{v^2}{r} = g\tan\theta$

$F_s = mg\tan\theta = 90 \times 10 \times \tan 30° \approx 520\text{ N}$

69. For banking of road, $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$\theta = \tan^{-1}(0.24)$

$\therefore \quad \tan\theta = 0.24$

Also, $\tan\theta = \frac{v^2}{rg} = \mu \Rightarrow \mu = 0.24$

70. $T = ma = mr\omega^2$

$T \propto \omega^2$

$\frac{\omega'^2}{\omega^2} = \frac{T'}{T} = \frac{4T}{T} = 4$

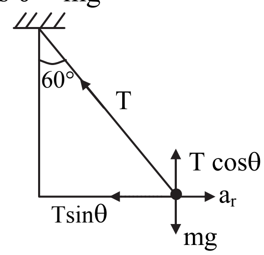
$\therefore \quad \omega'^2 = 4\omega^2 \qquad \therefore \quad \omega' = 2\omega$

$n' = 2n = 2 \times 5 = 10$ r.p.m.

71. Using,

$T\sin\theta = m\omega^2 r = m\omega^2 l\sin\theta$ ....(i)

$T\cos\theta = mg$ ....(ii)



From (i) and (ii), $\omega^2 = \frac{g}{l\cos\theta}$

$\therefore \quad \omega = \sqrt{\frac{g}{l\cos\theta}}$

$\therefore \quad$ Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

$= 2 \times 3.14 \times \sqrt{\frac{1\times\cos 60°}{10}} = 1.4\text{ s}$

72. Using,

$r = l\sin\theta$

$r = 10\sin 30° \Rightarrow r = 5\text{ m}, T = 3\text{ s}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$

Centripetal force $= m\omega^2 r$

$= 5 \times 10^{-2} \times \frac{4\pi^2}{9} \times 5$

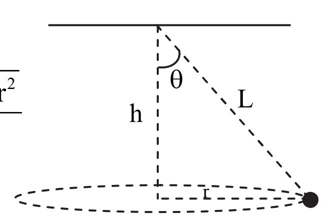
$= 25 \times 10^{-2} \times 4$

$= 100 \times 10^{-2} \approx 1\text{ N}$

73. $T = \frac{mg}{\cos\theta}$

$\cos\theta = \frac{h}{L} = \frac{\sqrt{L^2 - r^2}}{L}$

$\therefore \quad T = \frac{mg\,L}{\sqrt{L^2 - r^2}}$



74. At the highest point,

$$mg = \frac{mv^2}{r}$$

$\therefore$ $v = \sqrt{rg} = \sqrt{4000 \times 10} = 200$ m/s

75. r = 6.4 m

Minimum velocity at the bottom,

$v = \sqrt{5gr} = \sqrt{5 \times 9.8 \times 6.4} = \sqrt{313.6} = 17.7$ m/s

76. Using,

$$F = \frac{mv^2}{r} = m\omega^2 r = mg$$

$\therefore$ $\omega = \sqrt{\frac{g}{r}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{9.8}{4}}$

$\therefore$ $T = \frac{2\pi \times 2}{\sqrt{9.8}} \approx 4$ s

77. $T_L - T_H = \frac{m}{r}(u^2 + gr) - \frac{m}{r}(u^2 - 5gr)$

$= \frac{m}{r}(u^2 + gr - u^2 + 5gr)$

$= \frac{m}{r}(6gr) = 6$ mg

78. Using,

$$\frac{mv^2}{r} = \frac{2 \times (4)^2}{1} = 32 \text{ N}$$

It is clear that tension will be 52 N at the bottom of the circle because we know that,

$$T_{Bottom} = mg + \frac{mv^2}{r}$$

79. $T_L = 350$ N

Using,

$$\frac{mv^2}{r} = T_L - mg = (2 \times 350 - 40 \times 10) = 300$$

$\therefore$ $v^2 = \frac{300 \times 3}{40} = 22.5$ m/s

$v \approx 4.7$ m/s

80. At the highest point of the circle,

$$F = \frac{mv^2}{r} - mg = 70 \times \left[\frac{4 \times 10^4}{400} - 10\right] = 6300 \text{ N}$$

81. At the lowest point of the circle,

$$F = \frac{mv^2}{r} + mg = 70 \times \left[\frac{4 \times 10^4}{400} + 10\right] = 7700 \text{ N}$$

82. Using,

$$\frac{mv^2}{r} = mg$$

$\therefore$ $v^2 = gr$

$v = \sqrt{gr} = \sqrt{10 \times 12.1} = \sqrt{121} = 11$ m/s

83. Using,

$(K.E.)_L - (K.E.)_H = \frac{1}{2}m\left[v_L^2 - v_H^2\right] = \frac{1}{2}m[5rg - rg]$

$= 2$ mrg $= 2 \times 1 \times 1 \times 10 = 20$ J

84. Even though particle is moving in a vertical loop, its speed remain constant.

Tension at lowest point, $T_{max} = \frac{mv^2}{r} + mg$

Tension at highest point, $T_{min} = \frac{mv^2}{r} - mg$

$$\frac{T_{max}}{T_{min}} = \frac{\frac{mv^2}{r} + mg}{\frac{mv^2}{r} - mg} = \frac{5}{3}$$

By solving we get, $v = \sqrt{4gr} = \sqrt{4 \times 9.8 \times 2.5}$

$= \sqrt{98}$ m/s

85. Using,

$$mg - N_1 = \frac{mv_1^2}{r}$$

$\therefore$ $\frac{mv_1^2}{r} = 667 - 556 = 111$

Let $v_2 = 2v_1$

$\therefore$ $\frac{mv_2^2}{r} = \frac{4mv_1^2}{r} = 4 \times 111 = 444$

$$mg - N_2 = \frac{mv_2^2}{r}$$

$\therefore$ $N_2 = 667 - 444 = 223$ N

86. By conservation of energy,

$$\frac{1}{2}mv^2 = mgh$$

$v = \sqrt{2gh}$ ....(i)

For looping the loop, the lower velocity must be greater than $\sqrt{5gr}$

$v_{min} = \sqrt{5gr} = \sqrt{\frac{5gD}{2}}$ ....(ii)

From (i) and (ii),

$$2gh = \frac{5gD}{2}$$

$$h = \frac{5D}{4}$$

87. According to law of conservation of energy,

$mgh = \frac{1}{2}mv^2 = \frac{1}{2}m \times 5 \times Rg$

∴ $R = \frac{2}{5}h = \frac{2}{5} \times 5 = 2$ cm

89. Using,

$\alpha = \frac{\omega - \omega_0}{t} = \frac{36-0}{6} = 6$ rad/s^2

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 6 \times 6 \times 6 = 108$ rad

90. $n_2 = 1200$ r.p.m. $= \frac{1200}{60} = 20$ r.p.s.

$n_1 = 600$ r.p.m. $= \frac{600}{60} = 10$ r.p.s., t = 5 s

$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi(20-10)}{5}$

$= \frac{20\pi}{5} = 4\pi$ rad/s^2

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 20\pi \times 5 + \frac{1}{2} \times 4\pi \times 25$

$= 100\pi + 50\pi = 150\pi$

Number of revolutions $= \frac{\theta}{2\pi} = \frac{150\pi}{2\pi} = 75$

91. $\alpha = \frac{\omega}{t}$ and $\omega = \frac{\theta}{t}$

∴ $\alpha = \frac{\theta}{t^2}$

But α = constant $\Rightarrow \theta \propto t^2$

So, $\frac{\theta_1}{\theta_1+\theta_2} = \frac{(2)^2}{(2+3)^2}$

or $\frac{\theta_1}{\theta_1+\theta_2} = \frac{4}{25}$

or $\frac{\theta_1+\theta_2}{\theta_1} = \frac{25}{4}$

or $1 + \frac{\theta_2}{\theta_1} = \frac{25}{4}$

∴ $\frac{\theta_2}{\theta_1} = \frac{21}{4}$

92. By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha(2\pi n)$

∴ $\alpha = \frac{3}{4}\frac{\omega_0^2}{4\pi \times 36}$ ....(i)

Now let fan complete total n′ revolutions from the starting to come to rest

$0 = \omega_0^2 - 2\alpha(2\pi n')$

∴ $n' = \frac{\omega_0^2}{4\alpha\pi}$

Substituting the value of α from equation (i),

$n' = \frac{\omega_0^2}{4\pi}\frac{4 \times 4\pi \times 36}{3\omega_0^2} = 48$ revolutions

Number of rotations = 48 – 36 = 12

93. Let velocity at A = v_1

Velocity at B = v_2

∵ Velocity is constant,

∴ $v_1 = v_2 = v$ (say)

$\angle AOB = 60°$

∴ Change in velocity,

$|v_1 - v_2| = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$

$= \sqrt{v^2 + v^2 - 2v^2 \times \cos\theta}$

$= \sqrt{2v^2(1-\cos\theta)} = v\sqrt{2 \times 2\sin^2\frac{\theta}{2}}$

$= 2v\sin\frac{\theta}{2} = 2v\sin 30°$

(Note: Refer Shortcut 2.)

94. Using,

$v = \frac{2\pi r}{T}$

∴ $T = \frac{2\pi r}{v} = \frac{2\pi}{80} \times \frac{20}{\pi} = \frac{1}{2}$ s

∵ T = Time taken for one revolution

There are 2 revolutions ⇒ total time taken = 1 s

$\omega = \frac{2\pi}{T} = 4\pi$ ....(∵ T = 1)

$\alpha = \frac{d\omega}{dt} = \frac{4}{2}\pi = 2\pi$

$a_t = \alpha \cdot r$ i.e. $= 2\pi \times \frac{20}{\pi} = 40$ m/s^2

95. Using,

Maximum tension, $T_{max} = \frac{mv_1^2}{r} + mg$

Minimum tension, $T_{min} = \frac{mv_2^2}{r} - mg$

Using the law of conservation of energy,

$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + 2mgr$

∴ $v_1^2 = v_2^2 + 4rg$

Hence $\frac{T_{max}}{T_{min}} = \frac{\frac{v_1^2}{r}+g}{\frac{v_2^2}{r}-g} = \frac{v_1^2+rg}{v_2^2-rg}$

$= \frac{v_2^2+5rg}{v_2^2-rg} = \frac{4}{1}$ ....[$\because$ $v_1{}^2 = v_2{}^2 + 4rg$]

This gives, $4v_2^2 - 4rg = v_2^2 + 5rg$

$\therefore$ $3v_2^2 = 9\ rg = 9 \times \frac{10}{3} \times 10$

$\therefore$ $v_2^2 = \frac{9}{3} \times \frac{10}{3} \times 10$

$\therefore$ $v_2^2 = 100$

$\therefore$ $v_2 = 10$ m/s

Competitive Thinking

2. $T_E = 24$ hr, $T_H = 12$ hr

$\therefore$ $\frac{\omega_E}{\omega_H} = \frac{2\pi/T_E}{2\pi/T_H} = \frac{T_H}{T_E} = \frac{12}{24} = \frac{1}{2}$

3. $n_1 = 600$ r.p.m., $n_2 = 1200$ r.p.m.,
Using,
Increment in angular velocity, $\omega = 2\pi(n_2 - n_1)$
$\omega = 2\pi(1200 - 600)$ rad/min
$= (2\pi \times 600)/60$ rad/s
$\omega = 20\pi$ rad/s

4. For an hour hand, T = 12 hr = 12 × 3600 s

$\omega = \frac{2\pi}{T} = \frac{2\pi}{12 \times 3600} = \frac{\pi}{21600}$ rad/s

5. $\omega_{hour} = \frac{2\pi}{T_{hour}}$

$= \frac{2\pi}{12 \times 60 \times 60} \times \frac{180}{\pi}$

...{$\because$ $1^c = \frac{180°}{\pi}$}

$\omega_{hour} = \frac{1}{120}$ degree / s

6. Angular speed of second hand,

$\omega_1 = \frac{2\pi}{60}$ (T = 60 seconds)

Angular speed of hour hand,

$\omega_2 = \frac{2\pi}{12 \times 60 \times 60}$ (T = 12 hr)

$\frac{\omega_1}{\omega_2} = 12 \times 60 = \frac{720}{1}$

7. Angular speed of minute hand $\omega_m = \frac{2\pi}{60 \times 60}$

Angular speed of second hand $\omega_s = \frac{2\pi}{60}$

$\therefore$ $\omega_s - \omega_m = \frac{2\pi}{60} - \frac{2\pi}{3600} = \frac{59\pi}{1800}$ rad / s

9. Angular acceleration = $\frac{d^2\theta}{dt^2} = 2\theta_2$

10. $v = r\omega$

$\therefore$ $\omega = \frac{v}{r}$ = constant [As v and r are constant]

11. $T_1 = T_2 \Rightarrow \omega_1 = \omega_2$

$\omega = \frac{v}{r} \Rightarrow \frac{v}{r}$ = constant

$\therefore$ $\frac{v_1}{r_1} = \frac{v_2}{r_2} \Rightarrow \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{R}{r}$

12. For seconds hand, T = 60 s,
$r = 3\ cm = 3 \times 10^{-2}$ m

$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.1047$ rad/s

and $v = \omega r = 0.1047 \times 3 \times 10^{-2} = 0.00314$ m/s

13. n = 600 r.p.m. = $\frac{600}{60}$ r.p.s. = 10 r.p.s.

$v = r\omega = r \times 2\pi n = 10 \times 2 \times 3.142 \times 10$
$= 628.4$ cm/s.

14. Using,
$v = r\omega = 0.5 \times 70 = 35$ m/s

15. No. of revolutions = $\frac{\text{Total time}}{\text{Time period}} = \frac{140s}{40s}$

= 3.5 Rev.

So, distance = $3.5 \times 2\pi R = 3.5 \times 2\pi \times 10$
≈ 220 m

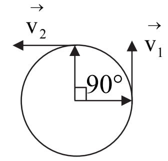
16. In 15 seconds hand rotates through 90°

Change in velocity $\left|\Delta \vec{v}\right| = 2v \sin\left(\frac{\theta}{2}\right)$

$= 2(r\omega) \sin\left(\frac{90°}{2}\right)$

$= 2 \times 1 \times \frac{2\pi}{T} \times \frac{1}{\sqrt{2}}$

$= \frac{4\pi}{60\sqrt{2}} = \frac{\pi\sqrt{2}}{30} \frac{cm}{s}$



(Note: Refer Shortcut 2.)

17. In circular motion,
Centripetal force ⊥ Displacement
∴ work done is zero.

18. $\vec{a}_r = \vec{\omega} \times \vec{v}$

19. $L = I\omega$. In U.C.M., ω = constant
∴ L = constant

20. Work done by centripetal force in uniform circular motion is always equal to zero.

22. Angular momentum is an axial vector. It is directed always in a fixed direction (perpendicular to the plane of rotation either outward or inward), if the sense of rotation remains same.

23. The instantaneous velocity of a body in U.C.M. is always perpendicular to the radius or along the tangent to the circle at the point.

24. In one complete revolution, total displacement is zero. So average velocity is zero.

25. $r = \pi$, $n = \left(\frac{p}{t}\right)$ r.p.s.

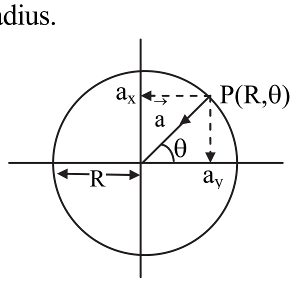
$v = r\omega = r \times 2\pi n = \pi \times 2\pi \times \frac{p}{t} = \frac{2\pi^2 p}{t}$

26. $E = \frac{1}{2} mv^2 \Rightarrow v^2 = \frac{2E}{m}$

$a = \frac{v^2}{r} = \frac{2E}{mr}$

29. The radius vector points outwards while the centripetal acceleration points inwards along the radius.

30.



$\vec{a} = -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

31. They have same angular speed ω.
Centripetal acceleration = $\omega^2 r$

$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$

32. $a = \omega^2 R = \left(\frac{2\pi}{0.2\pi}\right)^2 (5 \times 10^{-2}) = 5\ \text{m/s}^2$

33. Using,
$\omega = 2\pi n = 2\pi \times 1 = 2\pi$ rad/s
$a = r\omega^2 = 0.4 \times (2\pi)^2 = 0.4 \times 4\pi^2$
$a = 1.6\ \pi^2\ \text{m/s}^2$

34. Since, $n = 2$, $\omega = 2\pi \times 2 = 4\pi\ \text{rad/s}^2$

So acceleration $= \omega^2 r = (4\pi)^2 \times \frac{25}{100}\ \text{m/s}^2 = 4\pi^2$

35. Using,
$a = \omega^2 r = 4\pi^2 n^2 r = 4(3.14)^2 \times 1^2 \times 20 \times 10^3$
∴ $a \approx 8 \times 10^5\ \text{m/s}^2$

36. Tangential acceleration: $a_t = r\alpha$ …(i)

Radial acceleration: $a_r = \frac{v^2}{r}$ …(ii)

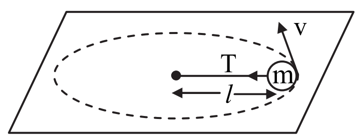
Dividing equation (i) by equation (ii),

∴ $\frac{a_t}{a_r} = \frac{r\alpha}{\left(\frac{v^2}{r}\right)} = \frac{\alpha r^2}{v^2}$

37. Net acceleration in non-uniform circular motion

$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left(\frac{900}{500}\right)^2} \approx 2.7\ \text{m/s}^2$

39.



Here, tension provides required centripetal force.

i.e., $\frac{mv^2}{l} = T$

40. Radial force $= \frac{mv^2}{r} = \frac{m}{r}\left(\frac{p}{m}\right)^2 = \frac{p^2}{mr}$

….[∵ p = mv]

42. Using, $T = mr\omega^2 \Rightarrow \omega^2 = \frac{T}{mr}$

∴ $\omega = \sqrt{\frac{6.4}{0.1 \times 6}} \approx 3$ rad/s

43. $F = \frac{mv^2}{r}$

∴ $F \propto v^2$. If v becomes double, then F (tendency to overturn) will become four times.

44. $L = r\,p\sin\theta = r\,p$ for U.C.M. [∵ θ = 90°]

∴ $\frac{L^2}{mr^3} = \frac{r^2 m^2 v^2}{mr^3} = \frac{mv^2}{r}$

45. Using, $T = m\omega^2 r$
∴ $10 = 0.25 \times \omega^2 \times 0.1$ ∴ $\omega = 20$ rad/s

46. $F = m\omega^2 r$

Substituting for $r = 2l$, $\omega = \frac{2\pi}{T}$

$kl = m(2l)\left(\frac{2\pi}{T}\right)^2$ ….(i)

….($\because$ F = kx and x = l here)

Upon speeding, $F_1 = m\omega_1^2 r_1$

Substituting for $r_1 = 3l$, $\omega_1 = \frac{2\pi}{T_1}$

$k(2l) = m(3l)\left(\frac{2\pi}{T_1}\right)^2$ ….(ii)

….($\because$ x = 2l here)

Dividing equation (i) by equation (ii),

$$\frac{kl}{k(2l)} = \frac{m(2l)(2\pi/T)^2}{m(3l)(2\pi/T_1)^2}$$

$\therefore$ $\left(\frac{T_1}{T}\right)^2 = \frac{3}{4}$

$\Rightarrow T_1 = \frac{\sqrt{3}}{2} T$

47. v = 36 km/h = 10 m/s

Using,

$\therefore$ $F = \frac{mv^2}{r} = \frac{500\times100}{50} = 1000$ N

48. m = 100 kg, v = 9 m/s, r = 30 m

Maximum force of friction = centripetal force

$\frac{mv^2}{r} = \frac{100\times(9)^2}{30} = 270$ N

49. Using, $F = mr\omega^2 = m\,4\pi^2 n^2 r$

$\therefore$ $m\,4\pi^2 n^2 r = 4\times10^{-13}$

$\therefore$ $n = \sqrt{\frac{4\times10^{-13}}{1.6\times10^{-27}\times4\times3.14^2\times0.1}}$

$\therefore$ $n = 0.08\times10^8$ cycles/second

50. The centripetal force, $F = \frac{mv^2}{r}$

$\therefore$ $r = \frac{mv^2}{F}$

$\therefore$ $r \propto v^2$ or $v \propto \sqrt{r}$

(If m and F are constant), $\frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{1}{2}}$

51. $r_1 = 4$ cm, $\omega_2 = 2\omega_1$

$r\omega^2$ = constant

$\therefore$ $r_1\omega_1^2 = r_2\omega_2^2$ $\therefore$ $r_1\omega_1^2 = r_1(2\omega_1)^2 = r_1 = 4r_2$

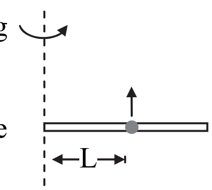
$\therefore$ $r_2 = \frac{r_1}{4} = \frac{4}{4} = 1$ cm

52. $F = \frac{mv^2}{r}$

$F \propto v^2$ i.e. force will become 4 times.

53. Let the bead starts slipping after time t

For critical condition, frictional force provides the centripetal force

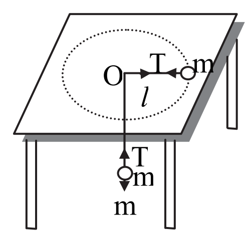


$m\omega^2 L = \mu R = \mu m \times a_1 = \mu L m\alpha$

$\Rightarrow m(\alpha t)^2 L = \mu m L\alpha$

$\Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$ ... [$\because$ $\omega = \alpha t$]

54.



Tension T in the string will provide centripetal force $\Rightarrow \frac{mv^2}{l} = T$ ….(i)

Also, tension T is provided by the hanging ball of mass m,

$\Rightarrow T = mg$ ….(ii)

$mg = \frac{mv^2}{l} \Rightarrow g = \frac{v^2}{l}$

56. Because the reaction on inner wheel decreases and becomes zero. So it leaves the ground first.

61. Using,

$v_{max} = \sqrt{\mu rg} = \sqrt{0.2\times100\times9.8} = 14$ m/s

62. Using,

$v = \sqrt{\mu rg} = \sqrt{0.4\times30\times9.8} = 10.84$ m/s

63. As the car moves on a plain horizontal circular track, the only force that can provide centripetal acceleration so that the car does not skid is frictional force.

$\therefore$ $\frac{mv^2}{r} = \mu mg$ $\Rightarrow \mu = \frac{v^2}{rg}$

$v = 60 \text{ km/hr} = 60 \times \frac{5}{18}$ m/s, r = 60 m, g = 10 m/s^2

$\mu = \left(60 \times \frac{5}{18}\right)^2 \Big/ 60 \times 10$

$\therefore \quad \mu = 25/54$

64. $C = 34.3 \text{ m} \Rightarrow r = \frac{34.3}{2 \times \pi}$,

$T = \sqrt{22} \text{ s} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{\sqrt{22}}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{r\omega^2}{g}\right) = \tan^{-1}\left(\frac{34.3}{2\pi} \times \frac{2\pi \times 2\pi}{22} \times \frac{1}{9.8}\right)$

$= \tan^{-1}\left(34.3 \times 2 \times \frac{22}{7 \times 22} \times \frac{1}{9.8}\right) = \tan^{-1}\left(\frac{4.9 \times 2}{9.8}\right)$

$= \tan^{-1}(1) = 45°$

65. Using, $\tan\theta = \frac{v^2}{rg}$

$\therefore \quad \tan 12° = \frac{(150)^2}{r \times 10}$

$\therefore \quad r = 10.6 \times 10^3 \text{ m} = 10.6 \text{ km}$

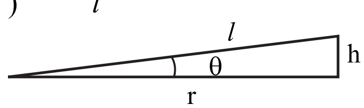
66. For banking, $\tan\theta = \frac{v^2}{Rg}$

$\tan 45 = \frac{v^2}{90 \times 10} = 1$

v = 30 m/s

67. $\tan\theta = \frac{h}{(l^2 - h^2)^{1/2}} \approx \frac{h}{l}$

$(l^2 >> h^2)$



$\tan\theta = \frac{v^2}{rg}$

$\frac{h}{l} = \frac{v^2}{rg}$

$\therefore \quad h = \frac{v^2 l}{rg}$

68. The inclination of person from vertical is given by,

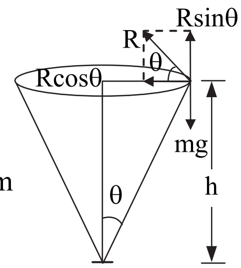
$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{50 \times 10} = \frac{1}{5}$

$\therefore \quad \theta = \tan^{-1}(1/5)$

69. The particle is moving in circular path.

From the figure, mg = R sin θ …(i)

$\frac{mv^2}{r} = R\cos\theta$ …(ii)



From equation (i) and (ii) we get

$\tan\theta = \frac{rg}{v^2}$ but $\tan\theta = \frac{r}{h}$

$\therefore \quad h = \frac{v^2}{g} = \frac{(0.5)^2}{10} = 0.025 \text{ m}$

$= 2.5 \text{ cm}$

71. Because tension is maximum at the lowest point.

72. When body is released from the position (inclined at angle θ from vertical), then velocity at mean position,

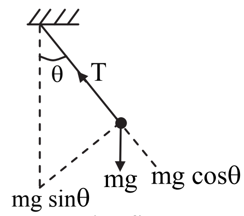
$v = \sqrt{2gl(1-\cos\theta)}$

$\therefore \quad$ Tension at the lowest point $= mg + \frac{mv^2}{l}$

$= mg + \frac{m}{l}[2gl(1 - \cos 60°)]$

$= mg + mg = 2mg$

73.



From the figure,

T = mg cosθ + mg sinθ

$\therefore \quad T = mg\cos\theta + mv^2/L$

74. Tension at mean position, $mg + \frac{mv^2}{r} = 3mg$

$v = \sqrt{2gl}$ ….(i)

and if the body displaces by angle θ with the vertical then $v = \sqrt{2gl(1-\cos\theta)}$ ….(ii)

Comparing (i) and (ii), cos θ = 0

$\therefore \quad \theta = 90°$

78. Tension, $T = \frac{mv^2}{r} + mg\cos\theta$

For, θ = 30°, $T_1 = \frac{mv^2}{r} + mg\cos 30°$

θ = 60°, $T_2 = \frac{mv^2}{r} + mg\cos 60°$

$\therefore \quad T_1 > T_2$

79. $T = mg + m\omega^2 r = m\{g + 4\pi^2 n^2 r\}$

….[$\omega = 2\pi n$]

$= m\left[g + \left(4\pi^2\left(\frac{n}{60}\right)^2 r\right)\right] = m\left[g + \left(\frac{\pi^2 n^2 r}{900}\right)\right]$

80. Minimum angular velocity,

$$\omega_{min} = \sqrt{\frac{g}{R}}$$

$\therefore$ $T_{max} = \frac{2\pi}{\omega_{min}} = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{2}{10}} = 2\sqrt{2} \approx 3$ s

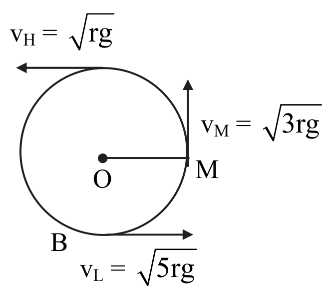
81. Using, $mr\omega^2 = mg$

$\therefore$ $r\left(\frac{2\pi}{T}\right)^2 = g \Rightarrow T^2 = \frac{4\pi^2 r}{g}$

$\therefore$ $T = 2\pi\sqrt{\frac{r}{g}} = 2 \times 3.14 \times \sqrt{\frac{4}{9.8}} \approx 4$ s

82. Critical velocity at highest point $= \sqrt{gR}$
$= \sqrt{10 \times 1.6}$
$= 4$ m/s

83. $v = \sqrt{3gr}$ and $a = \frac{v^2}{r} = \frac{3gr}{r} = 3g$

84.


Centripetal acceleration at midway point (M)

$$= \frac{v_M^2}{r} = \frac{3rg}{r} = 3g$$

85. $T_{max} = 30$ N
Using,

$$T_{max} = m\omega_{max}^2 r + mg$$

$\therefore$ $\frac{T_{max}}{m} = \omega^2 r + g$

$$\frac{30}{0.5} - 10 = \omega_{max}^2 r$$

$$\omega_{max} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5 \text{ rad/s}$$

86. Max. tension that string can bear = 3.7 kg-wt
= 37 N
Tension at lowest point of vertical loop
$= mg + m\omega^2 r = 0.5 \times 10 + 0.5 \times \omega^2 \times 4$
$= 5 + 2\omega^2$

$\therefore$ $37 = 5 + 2\omega^2$

$\therefore$ $\omega = 4$ rad/s

87. Using,

$$T_L = \frac{mv_L^2}{r} + mg = 6\,mg = 6 \times 5 \times 10 = 130 \text{ N}$$

$\therefore$ The mass is at the bottom position.

88. $(K.E)_L = \frac{5}{2}mgr$ …(i)

$(K.E)_H = \frac{1}{2}mgr$ …(ii)

$\therefore$ Divide equation (ii) by equation (i)

$\therefore$ $\frac{(K.E)_H}{(K.E)_L} = \frac{\left(\frac{1}{2}mgr\right)}{\left(\frac{5}{2}mgr\right)} = \frac{1}{5} = 0.2$

89. Change in momentum
$= Mv - (-Mv) = 2\,Mv$

90. Centripetal acceleration

$$\frac{v^2}{r} = K^2 t^2 r$$

$\therefore$ $v = K\,t\,r$

acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(K\,t\,r) = Kr$

$F = m \times a$

and $P = F \times v = mKr \times Ktr = mK^2 t r^2$

91. $n = \frac{2}{\pi}$ r.p.s.

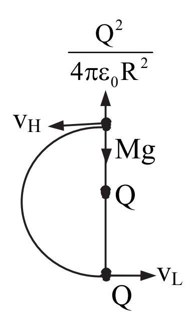
$T\sin\theta = M\omega^2 R$ ….(i)

$T\sin\theta = M\omega^2 L\sin\theta$ ….(ii)

From (i) and (ii),

$T = M\omega^2 L = M\,4\pi^2 n^2 L$

$= M\,4\pi^2\left(\frac{2}{\pi}\right)^2 L = 16\,ML$

92.


At highest point, T = 0

$\therefore$ $Mg - \frac{Q^2}{4\pi\varepsilon_0 R^2} = \frac{mv_H^2}{R}$

But $Mg = \frac{Q^2}{4\pi\varepsilon_0 R^2}$ …. (Given)

$\therefore$ $v_H = 0$

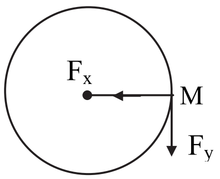
According to work- energy theorem

$\therefore$ $W = \Delta KE$

$mg(2R) = \frac{1}{2}mv_L^2 - \frac{1}{2}mv_H^2$

$= \frac{1}{2}mv_L^2$ ....($\because v_H = 0$)

$\therefore$ $v_L = 2\sqrt{gR}$

93. 

At midway point (M),

$F_y = mg$

$F_x = \frac{mv_M^2}{r} = 3mg$ ....$\left(v_M = \sqrt{3rg}\right)$

$F_{net} = \sqrt{F_y^2 + F_x^2}$

$= \sqrt{(mg)^2 + (3mg)^2}$

$= \sqrt{10}\ mg$

94. $\frac{mv^2}{r} = \frac{k}{r^2}$

$\therefore$ $mv^2 = \frac{k}{r}$

$\therefore$ K.E.$= \frac{1}{2}\ mv^2 = \frac{k}{2r}$

P.E. $= \int F\,dr = \int \frac{k}{r^2}\,dr = -\frac{k}{r}$

$\therefore$ Total energy = K.E + P.E $= \frac{k}{2r} - \frac{k}{r} = -\frac{k}{2r}$

95. $v^2 = u^2 + 2a_tS$

$\therefore$ $v^2 = 2a_tS$ ....{$\because$ u = 0}

$\therefore$ $a_t = \frac{v^2}{2S}$

At the end of second revolution, the particle travels a distance equal to twice the circumference of circle.

$\therefore$ $S = 2(2\pi r) = 4\pi r$

$\therefore$ $a_t = \frac{v^2}{2(4\pi r)}$

$\therefore$ $a_t = \frac{v^2}{8\pi r}$

96. m = 10 g = 0.01 kg

$r = 6.4$ cm $= 6.4 \times 10^{-2}$ m,

K.E. of particle $= \frac{1}{2}mv^2$

$\therefore$ $\frac{1}{2}mv^2 = 8 \times 10^{-4}$ J

$\therefore$ $v^2 = \frac{16\times10^{-4}}{0.01} = 16 \times 10^{-2}$

$v^2 = u^2 + 2a_ts$

$\therefore$ $v^2 = 2a_ts$ ....{$\because$ u = 0}

$s = 2(2\pi r)$

$\therefore$ $v^2 = 2a_t4\pi r$

$\therefore$ $a_t = \frac{v^2}{8\pi r} = \frac{16\times10^{-2}}{8\times3.14\times6.4\times10^{-2}} = 0.1\ m/s^2$

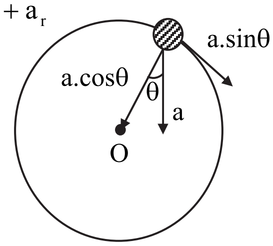
97. $\frac{mv_1^2}{r} = \frac{(2m)v_2^2}{\frac{r}{2}}$

$\Rightarrow v_1^2 = 4v_2^2$

$\Rightarrow v_1 = 2v_2$

98. In given figure,

Total acceleration $\vec{a} = \vec{a}_t + \vec{a}_r$



$\therefore$ $a_r = a.\cos\theta$

also, $a_r = \frac{v^2}{r}$

$\therefore$ $a.\cos\theta = \frac{v^2}{r}$

$\therefore$ $15\cdot\cos(30°) = \frac{v^2}{2.5}$

$\therefore$ $v^2 = 32.5$

v = 5.7 m/s

99. Linear velocity, $v = \omega r = 2\pi nr$

$= 2 \times 3.14 \times 3 \times 0.1$

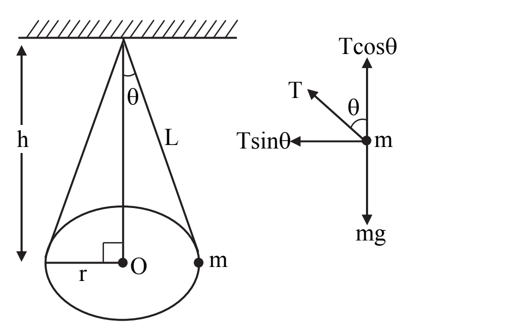
$= 1.88$ m/s

Acceleration, $a = \omega^2 r = (6\pi)^2 \times 0.1 = 35.5\ m/s^2$

Tension in string, $T = m\,\omega^2 r = 1 \times (6\pi)^2$

$= 1 \times (6\pi)^2 \times 0.1$

$= 35.5$ N

100. 

The centripetal force required for circular motion is given by

$\frac{mv^2}{r} = T\sin\theta$ ....(i)

Also we have,

$mg = T\cos\theta$ ....(ii)

Dividing eq(i) by eq(ii) we get,

$\frac{mv^2}{r}\cdot\frac{1}{mg} = \frac{T\sin\theta}{T\cos\theta}$

∴ $v^2 = rg\tan\theta$

∴ $v = \sqrt{rg\tan\theta}$ ....(iii)

From figure,

$\tan\theta = \frac{r}{h}$

∴ $\tan\theta = \frac{r}{\sqrt{L^2 - r^2}}$ ....(iv) $\{\because L^2 = r^2 + h^2\}$

Substituting equation (iv) in equation (iii) we get,

$v = \sqrt{rg\frac{r}{\sqrt{L^2 - r^2}}}$

∴ $v = r\sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$

102. Speed of the body after just reaching at the bottom is $v = \sqrt{2gh}$ ....(i)

It just completes a vertical circle using this velocity.

To complete vertical circle, speed required is v

$v = \sqrt{5g\frac{D}{2}}$ ....(ii)

From equation (i) and (ii),

∴ $\sqrt{2gh} = \sqrt{5g\frac{D}{2}}$

∴ $h = \frac{5}{4}D$

103. Centripetal acceleration,

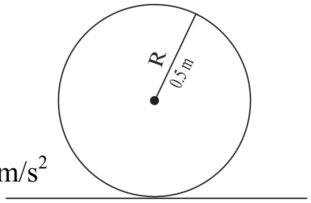
$a_c = \omega^2 r = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2}{(0.2\pi)^2} \times 5 \times 10^{-2} = 5\ ms^{-2}$

As particle is moving with constant speed, its tangential acceleration, $a_T = 0$.

The acceleration of the particle,

$a = \sqrt{a_c^2 + a_T^2} = \sqrt{5^2 + 0^2} = 5\ m/s^2$

104. Given



Angular acceleration $\alpha = 2\ rad\ s^{-2}$

∴ Angular speed $\omega = \alpha t$
$= (2)(2) = 4$ rad/s

$a_c = r\omega^2 = 0.5 \times 16 = 8\ m/s^2$

$a_t = \alpha r = 1\ m/s^2$

Resultant acceleration is given by,

$a = \sqrt{a_c^2 + a_t^2} = \sqrt{8^2 + 1^2} \approx 8\ m/s^2$

105. The centripetal force acting on the particle is provided by the central force,

∴ $\frac{mv^2}{R} = K \times \frac{1}{R^n}$

∴ $v^2 = K \times \frac{R}{mR^n} = K \times \frac{1}{mR^{n-1}}$

∴ $v = K' \times \frac{1}{R^{\frac{(n-1)}{2}}}$ .... $\left(K' = \sqrt{\frac{K}{m}}\right)$

The time period of rotation is,

$T = \frac{2\pi R}{v} = \frac{2\pi R \times R^{\frac{n-1}{2}}}{K'} = \frac{2\pi}{K'} \times R^{\frac{n+1}{2}}$

∴ $T \propto R^{\frac{n+1}{2}}$

106. Potential energy is given to be,

$U = -\frac{k}{2r^2}$ ....(i)

The force acting on the particle will be,

$F = \frac{dU}{dr} = \frac{-d}{dr}\left(\frac{-k}{2r^2}\right) = +\frac{k}{2}\left(\frac{-2}{r^3}\right)$

∴ $F = -\frac{k}{r^3}$

As the particle is moving in circular path, the force acting on it will be centripetal force.

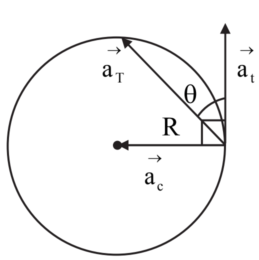
∴ $F = -\frac{mv^2}{r} = -\frac{k}{r^3}$ ∴ $mv^2 = \frac{k}{r^2}$

Now, K.E. $= \frac{1}{2}mv^2 = \frac{k}{2r^2}$ ....(ii)

∴ Total Energy E = K + U = 0

....[from (i) and (ii)]

107.



Velocity of object is given as

$V = K\sqrt{S}$ ….(i)

Centripetal acceleration of the object is,

$a_c = \frac{V^2}{R}$ ….(ii)

Tangential acceleration is given by,

$a_t = \frac{dV}{dt} = \frac{dV}{dS}\frac{dS}{dt}$

$= V\frac{dV}{dS}$

$= K\sqrt{S}\,\frac{d}{dS}\left(K\sqrt{S}\right)$ ….from (i)

$= K^2\sqrt{S}\,\frac{1}{2\sqrt{S}}$

$a_t = \frac{K^2}{2}$ ….(iii)

from figure,

$\tan\theta = \frac{a_c}{a_t} = \left(\frac{V^2}{R}\right)\frac{2}{K^2}$ ….From (ii) and (iii)

∴ $\tan\theta = \frac{2}{R}\frac{K^2S}{K^2}$ ….from (i)

∴ $\tan\theta = \frac{2S}{R}$

108. At an instant, speed of P = v, going in clockwise direction

Speed of Q = v, going in anticlockwise direction

Relative angular velocity of P w.r.t.

Q = ω– (–ω) = 2 ω

Relative angular separation of P and Q in time t,

θ = 2 ωt.

Relative speed between the points P and Q at time t

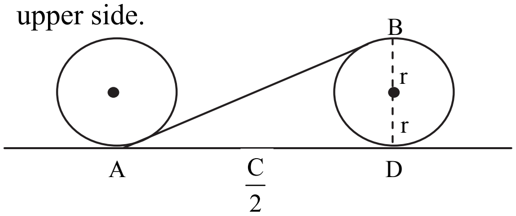
$|\vec{v}_r| = \sqrt{v^2 + v^2 - 2vv\cos(2\omega r)}$

$= \sqrt{2v^2(1-\cos 2\omega r)}$

$= \sqrt{2v^2 \times 2\sin^2 \omega r}$

= 2v sin ωr

Since, $|\vec{v}_r|$ will not have any negative value so the lower part of the sine wave will come upper side.

109. 

Let A be initial position of point of contact and B be its position after the wheel completes half revolution.

Distance travelled by the wheel in half revolution = $\frac{C}{2}$ = AD

∴ from figure ;

Displacement of initial point of contact after half revolution = AB

∴ $AB^2 = AD^2 + DB^2$

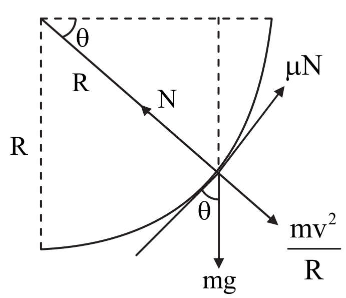
$AB^2 = \left(\frac{C}{2}\right)^2 + (2r)^2$

But $r = \frac{C}{2\pi}$

∴ $AB^2 = \left(\frac{C}{2}\right)^2 + \left(\frac{C}{\pi}\right)^2$

∴ $AB = \sqrt{\frac{C^2}{4} + \frac{C^2}{\pi^2}} = C\sqrt{\frac{1}{\pi^2} + \frac{1}{4}}$

Evaluation Test

1. 

$N = \frac{mv^2}{R} + mg\sin\theta$

For equilibrium,

$mg\cos\theta = \mu N = \mu\left(\frac{mv^2}{R} + mg\sin\theta\right)$ ….(i)

From energy conservation,

$\frac{1}{2}mv^2 = mgR(\sin\theta)$

∴ $\frac{mv^2}{R} = 2mg\sin\theta$ ….(ii)

∴ mg cos θ = μ (2 mg sin θ + mg sin θ) ….[From (i) and (ii)]

∴ $\mu = \frac{\cos\theta}{3\sin\theta}$

$\therefore \quad \tan\theta = \dfrac{1}{3\mu}$

$\therefore \quad \theta = 45°$ …. $\left[\because \mu = \dfrac{1}{3}\right]$

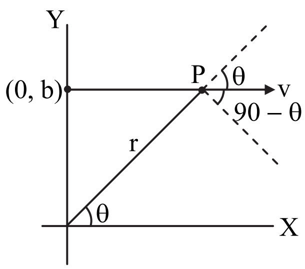
2. $r = \dfrac{b}{\sin\theta}$

$v' = v\sin\theta$

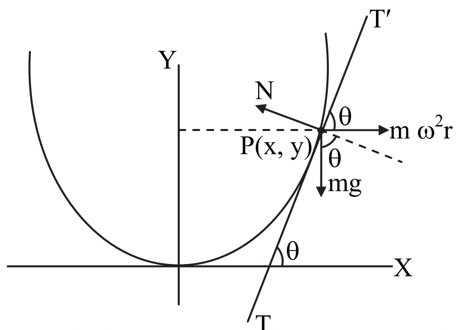
Now, $\omega = v'/r$

$= \dfrac{v\sin\theta}{\left(\dfrac{b}{\sin\theta}\right)}$

$= \dfrac{v}{b}\sin^2\theta$



3.



TT′ is the tangent to the curve at point P.

$mg\sin\theta = (m\omega^2 x)\cos\theta$ ….[along TT′]

$\therefore \quad \tan\theta = \dfrac{\omega^2 x}{g}$

$\dfrac{dy}{dx} = \dfrac{\omega^2 x}{g}$

But,

$\therefore \quad \dfrac{dy}{dx} = \dfrac{d}{dx}\left(a^3x^4\right) = 4a^3x^3$

$\therefore \quad 4a^3x^3 = \dfrac{\omega^2 x}{g} \Rightarrow \omega = 2x\sqrt{a^3 g}$

4. $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Now,

$\left|\vec{v}_{AB}\right| = \sqrt{v^2 + v^2 + 2v^2\cos(180-\theta)}$

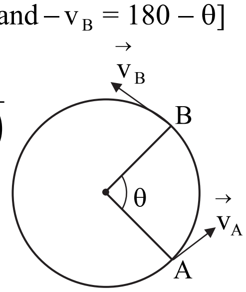
$\therefore$ [smaller angle between $\vec{v}_A$ and $-\vec{v}_B = 180 - \theta$]

$= \sqrt{2v^2(1-\cos\theta)}$

$= \sqrt{2v^2\left(2\sin^2(\theta/2)\right)}$

$= 2v\sin(\theta/2)$

$= 2R\omega\sin(\theta/2)$



5. Since this is not a case of a normal string, the velocity at the topmost point can be zero.

$\therefore \quad (T.E.)_{initial} = (T.E.)_{final}$

$\therefore \quad mgh + \dfrac{1}{2}mv^2 = mg(2R)$

$\therefore \quad v = \sqrt{2g(2R-h)}$

Note: *In case of a string,* v *at the topmost point should be equal to* $\sqrt{Rg}$ *to complete the vertical circle as* T = 0 *and ball will fall vertically down if* v = 0.

6. $\Delta P.E. = mgR(1-\cos\theta)$ and

$\Delta K.E. = \dfrac{1}{2}mv^2$

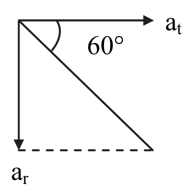
$(\text{Work done})_{pseudo\ force} = -mgR\sin\theta$

$\therefore \quad mgR(1-\cos\theta) + mgR\sin\theta = \dfrac{1}{2}mv^2$

$\therefore \quad mgR(1-\cos\theta+\sin\theta) = \dfrac{1}{2}mv^2$

$\therefore \quad v = \sqrt{2gR(1-\cos\theta+\sin\theta)}$

7.



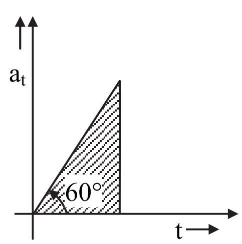
$\tan 60° = \dfrac{a_r}{a_t}$

$\therefore \quad a_r = a_t\sqrt{3}$

$\therefore \quad \dfrac{v^2}{r} = a_t\sqrt{3}$ ….(i)

v = area under graph.

$\therefore \quad v = \dfrac{a_t t}{2}$ ….(ii)



$\therefore \quad \dfrac{a_t^2 t^2}{4(1)} = a_t\sqrt{3}$ ….[From (i) and (ii)]

$\therefore \quad \dfrac{a_t . t^2}{4} = \sqrt{3}$ ….(iii)

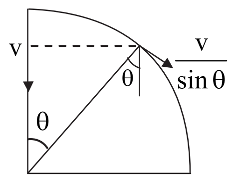
Also, $\tan(60°) = \dfrac{a_t}{t}$

$\therefore \quad \sqrt{3} = \dfrac{a_t}{t}$ or $a_t = t\sqrt{3}$ ….(iv)

$\therefore \quad \dfrac{t^3\sqrt{3}}{4} = \sqrt{3}$ ….[From (iii) and (iv)]

$\therefore \quad t^3 = 4 \Rightarrow t = 2^{2/3}$ s

8. $a_r = \dfrac{\left(\dfrac{v}{\sin\theta}\right)^2}{R} = \dfrac{v^2}{R\sin^2\theta}$ …[∵ $v_t = v/\sin\theta$]



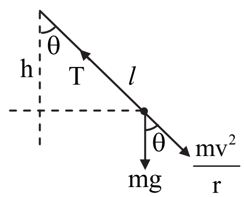
Also, $\dfrac{R(1-\cos\theta)}{v} = t$

∴ $\cos\theta = \left(1 - \dfrac{vt}{R}\right)$

∴ $a_r = \dfrac{v_t^2}{R}\ \dfrac{v^2}{R\left(1-\left(1-\dfrac{vt}{R}\right)^2\right)} = \dfrac{v^2}{R\left(\dfrac{2vt}{R} - \dfrac{v^2t^2}{R^2}\right)}$

$= \dfrac{Rv}{\left(2Rt - vt^2\right)}$

9. $mgh = \dfrac{1}{2}mv^2$



∴ $v = \sqrt{2gh}$

$\cos\theta = \dfrac{h}{l}$

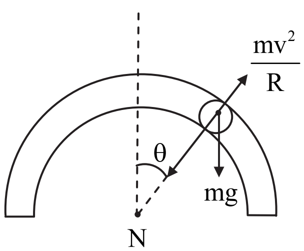
∴ $T = \dfrac{mv^2}{r} + mg\cos\theta$

∴ $T = \dfrac{2mgh}{l} + mg\dfrac{h}{l} = \left(\dfrac{3mg}{l}\right)h$

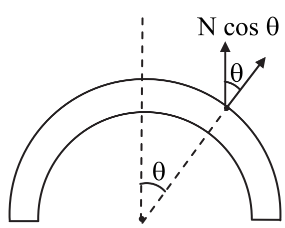
⇒ which implies a straight line graph.

10.

FBD of the ball



FBD of tube



$f_{avg} = \displaystyle\int_{\theta=0}^{\theta=\pi} (N\cos\theta)$

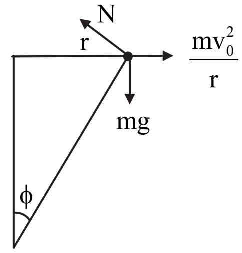
Here, integration is not possible.

So, we use the fact that we need to calculate f_{avg}

∴ $f_{avg} = \dfrac{\Delta p}{\Delta t}$

∴ $F_{avg} = \dfrac{(2mv)}{\left(\dfrac{\pi r}{v}\right)} = \dfrac{2mv^2}{\pi r}$

11. $N\cos\phi = \dfrac{mv_0^2}{r}$ and $N\sin\phi = mg$



∴ $\tan\phi = \dfrac{g}{\left(\dfrac{v_0^2}{r}\right)}$

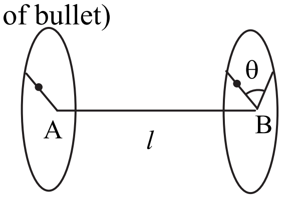
∴ $r = \dfrac{{v_0}^2}{g}\tan\phi$

12. Angle moved = θ in time t

$t = \dfrac{l}{v}$ ….(v = velocity of bullet)

Also, θ = ωt

∴ $\theta = \omega\left(\dfrac{l}{v}\right) \Rightarrow v = \dfrac{\omega l}{\theta}$



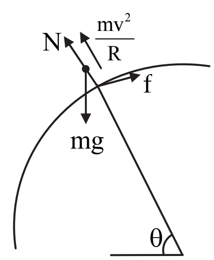
13. $\dfrac{d\omega}{dt} = \alpha = k \Rightarrow \omega = kt + c_1 = \dfrac{d\theta}{dt}$

∴ $\theta = \int (kt + c_1)\,dt$

$= \dfrac{kt^2}{2} + c_1t + c_2$

= quadratic equation which has a graph of parabola

14.



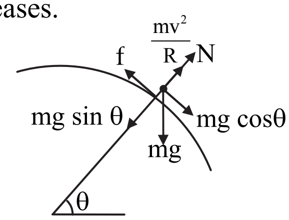
Friction will act in upward direction.

Since velocity is a constant,

$N = \left(mg\sin\theta - \dfrac{mv^2}{R}\right)$

$f = \mu\left(mg\sin\theta - \dfrac{mv^2}{R}\right) = mg\cos\theta$ $[a_t = 0]$

As θ increases, cos θ decreases ⇒ friction decreases.



Again, $a_t = 0$

∴ Friction $= \mu\left(mg\sin\theta - \frac{mv^2}{R}\right) = mg\cos\theta$

∴ As θ decreases, cos θ increases ⇒ friction increases.

15. The area under the α–t graph gives change in angular velocity.

Area $= \frac{\pi(2)^2}{2} = \frac{4\pi}{2} = 2\pi$

∴ $\omega_2 - \omega_1 = 2\pi$

∴ $\omega_2 = 2\pi + 2\pi = 4\pi$ rad/s

16. Velocity is a vector which changes but speed remains same for uniform circular motion.

In case A, radius of curvature remains same throughout hence $a = \frac{v^2}{r}$ remains constant.

However, in case of B, the radius of curvature keeps increasing hence $a = \frac{v^2}{r}$ keeps decreasing. Hence option (C) is the only correct option.

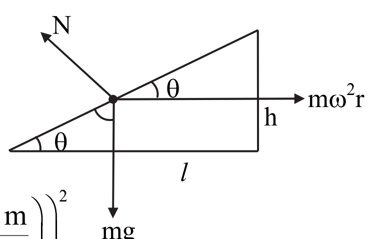
17. The direction of rotation is determined by the sign of angular velocity. In turn, the sign of angular velocity is determined by the sign of slope on angular displacement vs time plot. The sign of slope is negative for line OA, positive for line AC and zero for line CD.

The positive angular velocity indicates anti-clockwise rotation and negative angular velocity indicates clockwise rotation. The disk is stationary when angular velocity is zero.

18. $m\omega^2 r\cos\theta = mg\sin\theta$

∴ $\omega^2 = \frac{g\tan\theta}{r}$

∴ $\tan\theta = \frac{v^2}{rg}$



∴ $\frac{h}{l} = \frac{\left(72\times\left(\frac{1000\text{ m}}{3600\text{ s}}\right)\right)^2}{(400\text{m})(10\text{m/s})}$

$\frac{h}{1\text{m}} = \frac{1}{10}$

∴ h = 10 cm

19. At the highest point,

$\omega = \sqrt{\frac{g}{R}} = 2\pi n$

∴ $n = \frac{1}{2\pi}\sqrt{\frac{g}{R}} = \sqrt{\frac{g}{4\pi^2 R}}$

∴ r.p.m. $= 60n = 60\sqrt{\frac{g}{4\pi^2 n}} = \sqrt{\frac{900g}{\pi^2 R}}$

20. $\alpha = \omega\left(\frac{d\omega}{d\theta}\right)$ → So α is negative, if

$\omega > 0, \frac{d\omega}{d\theta} < 0$ or $\omega < 0, \frac{d\omega}{d\theta} > 0$

21. For option (A),
Net force $= Mv^2/r$ = Mass × acceleration
For option (B),
$\vec{a}_t$ and $\vec{\omega}$ are perpendicular hence cross product is not 0.
For option (C),
Angular velocity and angular accleration have the same direction or opposite direction according to the type of motion.
For option (D),
The correct statement is:
The resultant force acts always towards the centre.

22. Weight = Number of balls × centripetal force

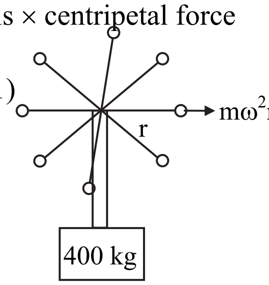
$(400)(10) = 8 \times m\,\omega^2 r$

$= 8 \times (5)\,\omega^2\,(1)$

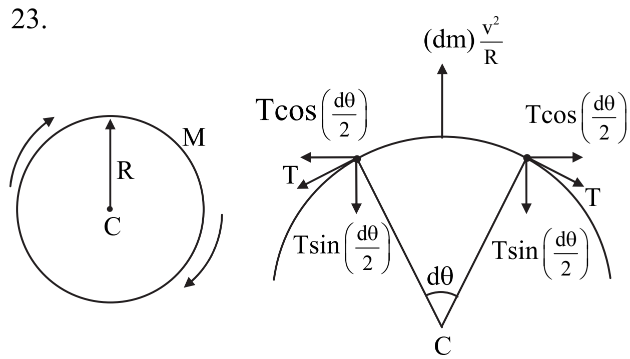
∴ $\omega^2 = \frac{4000}{40}$

$= 100$

∴ ω = 10 rad/s



23.



Take a small mass element dm
This element experiences a centripetal force along radial direction,

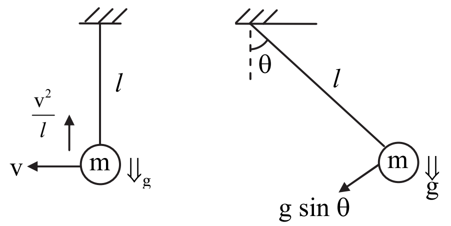
$F_d = (dm)\frac{v^2}{R}$

The components $T\cos\left(\frac{d\theta}{2}\right)$ cancel each other

$\therefore \quad 2T\sin\left(\frac{d\theta}{2}\right) = (dm)\ \frac{v^2}{R}$

$\therefore \quad T\,d\theta = \left(\frac{M}{2\pi R}\right)\times Rd\theta\times\frac{v^2}{R}\quad \left[\because \begin{matrix}\sin\theta\approx\theta\\ \text{as }\theta\to 0\end{matrix}\right]$

$\therefore \quad T = \frac{Mv^2}{2\pi R}$

24. 

Energy conservation, $mgl(1-\cos\theta) = \frac{1}{2}mv^2$

$\therefore \quad v = \sqrt{2gl(1-\cos\theta)}$

$\therefore \quad \frac{v^2}{l} = g\sin\theta$

$\therefore \quad 2g\,(1-\cos\theta) = g\sin\theta$

$\therefore \quad 2(1-\cos\theta) = \sin\theta$

$\therefore \quad 2 = \frac{\sin\theta}{1-\cos\theta} = \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\sin^2\left(\frac{\theta}{2}\right)}$

$\therefore \quad \cot\left(\frac{\theta}{2}\right) = 2$

i.e. $\theta = 53°$

25. $\omega = a(t^2)\,\hat{i} + b(e^{-t})\,\hat{j}$

$\alpha = \frac{d\omega}{dt} = 2a(t)\,\hat{i} + (-b)(e^{-t})\,\hat{j}$

at t = 1 s and $\omega = a\hat{i} + \frac{b\hat{j}}{e}$

$\alpha = 2a\hat{i} - \frac{b\hat{j}}{e}$

$\therefore \quad \bar{\omega}\cdot\bar{\alpha} = 2a^2 - \frac{b^2}{e^2} = \sqrt{a^2+\frac{b^2}{e^2}}\ \sqrt{4a^2+\frac{b^2}{e^2}}\ \cos\theta$

$\therefore \quad \theta = \cos^{-1}\left(\frac{2a^2-\frac{b^2}{e^2}}{\sqrt{a^2+\frac{b^2}{e^2}}\sqrt{4a^2+\frac{b^2}{e^2}}}\right)$

$\therefore \quad \alpha \approx 30° \qquad \ldots[\because a = b = 1]$

Textbook Chapter No.

02 Gravitation

Hints

Classical Thinking

3. $\vec{F} = \frac{Gm_1m_2\hat{r}}{r^2} = \frac{Gm_1m_2r}{r^3}\hat{r} = \frac{Gm_1m_2}{r^3}\vec{r}$

7. From Newton's law of gravitation,
$F = \frac{Gm_1m_2}{r^2}$
If $m_1 = m_2 = 1$ unit of mass
$r = 1$ unit of distance
$F = G$ = universal gravitational constant

9. $F = G\frac{m_1m_2}{r^2}$ $\therefore$ $G = \frac{Fr^2}{m_1m_2}$

$\therefore$ Units of G is $\frac{Nm^2}{kg^2}$

11. The value of universal gravitational constant is always same. As r varies, the force between the two bodies changes, but G remains constant.

12. Gravitational constant 'G' is independent of the medium intervening the two masses interacting gravitationally.

14. $F = G\frac{m_1 \times m_2}{r^2}$
$= 6.67 \times 10^{-11} \times \frac{m^2}{r^2}$
$= 6.67 \times 10^{-11} \times \left(\frac{1}{1}\right)^2$
$= 6.67 \times 10^{-11}$ N

15. $g' = G \times \frac{M}{10} \times \left(\frac{2}{R}\right)^2 = 0.4\frac{GM}{R^2}$
$= 0.4 \times 9.8 \text{ m s}^{-2} = 3.92 \text{ ms}^{-2}$

20. If it is not so, then the centrifugal force would exceed the force of attraction or we can say that gravitational force won't be able to keep the satellite in circular motion.

21. $F \propto \frac{1}{r} \Rightarrow F = \frac{K}{r} = \frac{mv^2}{r} \Rightarrow v = \text{constant}$

22. $v_c = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}}$ and $v_c = r\omega$

This gives $r^3 = \frac{R^2g}{\omega^2}$

23. $v = \sqrt{\frac{GM}{r}}$
v is independent of mass of the satellite.
$\therefore$ $\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} \Rightarrow r_1 > r_2 \Rightarrow v_2 > v_1$
Orbital speed of satellite does not depend upon the mass of the satellite

25. Longer period and slower velocity as
$T \propto \sqrt{r^3}$ and $v \propto \frac{1}{\sqrt{r}}$

26. $T = \frac{2\pi r}{v_c} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \sqrt{\frac{4\pi^2 r^3}{GM}}$
Since $r \approx R_p$
where R_p = Radius of the planet, put
$M = \frac{4}{3}\pi R_p^3 \rho$
$\therefore$ $T = \sqrt{\frac{4\pi^2 R_p^3}{G \times \frac{4}{3}\pi R_p^3 \rho}} = \sqrt{\frac{3\pi}{G\rho}}$
$\therefore$ $T \propto \frac{1}{\sqrt{\rho}}$

27. Kinetic and potential energies vary with position of earth w.r.t sun. Angular momentum remains constant everywhere.

28. From Kepler's second law of planetary motion, the velocity of a planet is maximum when its distance from sun is the least.

29. Kepler's third law is a consequence of law of conservation of angular momentum.

30. $T^2 \propto r^3$
$\therefore$ $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

36. $\omega = \frac{v}{r}$

For a star, angular velocity at which matter will start escaping from its equator is,

$$\omega = \frac{v_e}{r} = \frac{2}{R}\sqrt{\frac{4GM}{R}} \quad \ldots\left\{\because r = \frac{R}{2}\right\}$$

$$= \sqrt{\frac{16GM}{R^3}} = 4\sqrt{\frac{g}{R}} \quad \ldots\left\{\because \frac{GM}{R^2} = g\right\}$$

37. $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\left(\frac{4}{3}\pi R^3\rho\right)}$

$$v_e = \sqrt{\frac{8G\pi R^2\rho}{3}} = 2R\sqrt{\frac{2G\pi\rho}{3}}$$

38. $v_e = \sqrt{\frac{2GM}{R}}$, $v_e' = \sqrt{\frac{2GM}{R+h}}$

As $R + h > R \Rightarrow v_e > v_e'$

39. $\frac{v_1}{v_2} = \sqrt{\frac{2g_1R_1}{2g_2R_2}} = \sqrt{k_1k_2}$

41. Escape velocity,

$$v_e = \sqrt{\frac{2GM}{R}} = \left[\frac{2\times6.67\times10^{-11}\times6\times10^{24}}{6.4\times10^{6}}\right]^{1/2}$$

$$= 1.12 \times 10^4 \text{ m/s} = 11.2 \text{ km/s}$$

42. In a free fall, even near the earth, a body is in a state of weightlessness.

46. $g = \frac{GM}{R^2}$. If the earth shrinks, its mass remains unchanged and its radius decreases. So, the value of acceleration due to gravity increases.

47. At the centre of earth $g' = 0$;

Weight $= mg' = 100 \times 0 = 0$

48. When the earth stops rotating, the centripetal force of $mR\omega^2$ vanishes. As a result of this, the acceleration due to gravity increases.

49. $g_d = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right) \Rightarrow g_d = \frac{gr}{R}$

51. Geostationary satellite remains stationary with respect to the earth.

Since the time period of earth is 24 hours, therefore time period of a geostationary satellite is also 24 hours.

54. Let ρ be the density of the material of each sphere.

Then, $M_1 = \frac{4}{3}\pi r^3\rho$ and $M_2 = \frac{4}{3}\pi(2r^3)\rho$

Distance between their centres = r + 2 r = 3 r

Now, $F = \frac{GM_1M_2}{(3r)^2} = \frac{G\left(\frac{4\pi}{3}r^3\rho\right)\left(\frac{4}{3}\pi8r^3\rho\right)}{9r^2}$

This gives $F \propto r^4$

$\therefore$ Assertion is false.

55. Here Assertion is False, as

$$W = mg = \frac{GMm}{R^2}$$

and $W' = mg' = \frac{GMm}{(R+h)^2} = \frac{GMm}{(R+R/2)^2} = \frac{4}{9}X$

Critical Thinking

2. $R_p = \frac{R_e}{2}$, $M_p = \frac{M_e}{5}$

$\therefore$ $g_p = \frac{GM_p}{R_p^{\ 2}} = G \times \frac{1}{5}M_e \times \frac{4}{R_e^{\ 2}} = \frac{4}{5}g = 8 \text{ m s}^{-2}$

3. $r' = 2r$ ….[Given]

Now, $F \propto \frac{1}{r^2}$

$\therefore$ $F' \propto \frac{1}{(2r)^2} = \frac{1}{4r^2} \Rightarrow F' = \frac{F}{4}$

$\therefore$ Force is reduced to one-fourth.

4. $r = 50 \text{ cm} = 50 \times 10^{-2}$ m

$$F = G\frac{m_1m_2}{r^2} = \frac{6.67\times10^{-11}\times625\times625}{50\times50\times10^{-4}}$$

$= 1.042 \times 10^{-4}$ N = 10.42 dyne

5. $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

$$= 6.67 \times 10^{-11} \times \frac{10^5 \text{ dyne} \times 10^4 \text{ cm}^2}{10^6 \text{ g}}$$

$= 6.67 \times 10^{-11+3}$

$= 6.67 \times 10^{-8} \text{ dyne cm}^2/\text{g}^2$

6. $F_e = \frac{GMm}{R^2} = 50$ N ….(i)

$F_s = \frac{GMm'}{4R^2} = F$ ….(ii)

$\therefore$ Dividing equation (ii) by (i) we get

$$\frac{F}{50} = \frac{m'}{4m} = \frac{200}{4\times5}$$

$\therefore$ $F = 10 \times 50 = 500$ N

7. $F = \frac{Gm_1m_2}{r^2}$

$$= \frac{6.67\times10^{-11}\times1.9\times10^{27}\times1.99\times10^{30}}{(7.8\times10^{11})^2}$$

$= 4.14 \times 10^{23}$ N

8. $F = \frac{Gm_1m_2}{r^2}$ $\therefore$ $F' = \frac{Gm_1m_2}{(3r)^2} = \frac{F}{9}$

$\therefore$ % decrease in $F' = \left(\frac{F-F'}{F}\right)100$

$= \frac{8}{9} \times 100 \approx 89\%$

9. $F = mg = 81 = \frac{GMm}{R^2}$

$\therefore$ $F = mg = \frac{GMm}{\left(R+\frac{R}{2}\right)^2}$

$\therefore$ $F = \frac{4}{9}\frac{GMm}{R^2} = \frac{4}{9} \times 81 = 36$ N

10. $F = \frac{Gm_1m_2}{r^2}$

$\therefore$ $r^2 = \frac{Gm_1m_2}{F} = \frac{6.6\times10^{-11}\times1\times1}{10^{-9}\times9.8}$

$= \frac{2}{3} \times 10^{-2} = 0.673 \times 10^{-2}$

$\therefore$ $r \approx 0.08$ m ≈ 8 cm

11. $r = 20 \times 10^{-2}$ m, total mass = 5 kg

Let m and (5 – m) be the two masses

$F = \frac{G m_1 m_2}{r^2}$

$\therefore$ $1 \times 10^{-8} = \frac{6.67 \times 10^{-11} \times m \times (5-m)}{(2\times10^{-1})^2}$

$\therefore$ $1 \times 10^{-8} = 6.67 \times \frac{m(5-m)}{4} \times 10^{-9}$

$\therefore$ $10 = \frac{40}{6}\times\frac{m(5-m)}{4}$

$\therefore$ $m^2 - 5m + 6 = 0$ $\therefore$ $(m-2)(m-3) = 0$

$\therefore$ m = 3 or m = 2

12. $\frac{M_1}{M_2} = 2 : 3$, $\frac{R_1}{R_2} = 3 : 2$

$\therefore$ $\frac{g_1}{g_2} = \frac{GM_1/R_1^2}{GM_2/R_2^2} = \frac{M_1}{M_2}\times\left(\frac{R_2}{R_1}\right)^2$

$= \frac{2}{3}\times\left(\frac{2}{3}\right)^2 = \frac{8}{27}$

13. $\frac{M_m}{M_e} = \frac{1}{9}$, $\frac{R_m}{R_e} = \frac{1}{2}$

$W_e = mg_e = \frac{GM_em}{R_e^2}$

$W_m = mg_m = \frac{GM_mm}{R_m^2}$

$\therefore$ $\frac{W_m}{W_e} = \frac{M_m}{R_m^2}\times\frac{R_e^2}{M_e} = \left(\frac{M_m}{M_e}\right)\times\left(\frac{R_e}{R_m}\right)^2$

$= \left(\frac{1}{9}\right)\times(2)^2 = \frac{4}{9}$

$\therefore$ $W_m = \frac{4}{9}\times W_e = \frac{4}{9}\times 63 = 28$ kg-wt

14. $M_p = 2M_e$

$\therefore$ $\frac{4}{3}\pi R_P^3\rho = 2\times\frac{4}{3}\pi R_e^3\rho$

$\therefore$ $R_P^3 = 2R_e^3 \Rightarrow R_p = 2^{1/3}R_e$

$\therefore$ $g_p = \frac{GM_p}{R_p^2} = \frac{G[2M_e]}{[2^{1/3}R_e]^2} = 2^{1-\frac{2}{3}}\frac{GM_e}{R_e^2}$

$\therefore$ $g_p = 2^{1/3}g_e$

$\therefore$ $mg_P = 2^{1/3}\ mg_e = 2^{1/3}W$

15. $M_e = 20\ M_m$

$g_e = \frac{GM_e}{R_e^2}$ and $g_m = \frac{GM_m}{R_m^2}$

$\therefore$ $\frac{g_m}{g_e} = \frac{M_m}{M_e}\times\left(\frac{R_e}{R_m}\right)^2 = \frac{M_m}{20M_m}\times\left(\frac{6400}{3200}\right)^2$

$\therefore$ $\frac{mg_m}{mg_e} = \frac{4}{20}$

$\therefore$ Weight on Mars $= 500 \times \frac{4}{20} = 100$ N

16. $g = \frac{GM}{R^2}$

$\therefore$ $M = \frac{gR^2}{G} = \frac{9.8\times(6\times10^6)^2}{6.67\times10^{-11}}$

$\therefore$ $M = \frac{9.8\times36}{6.67}\times10^{23} = 52.89\times10^{23}$ kg

$\therefore$ $M \approx 5.3\times10^{24}$ kg

17. $v_c = \sqrt{\frac{GM_S}{r}}$

Orbital speed of all planets depends upon the mass of Sun and the separation. So,

$v_c \propto \frac{1}{\sqrt{r}}$

Since Jupiter is having more orbital radius in comparison to earth, so orbital speed of Jupiter is less than that of earth.

18. Critical velocity of a satellite is independent of mass of a satellite.

19. $r_1 = 4r$, $r_2 = r$

Orbital speed $v_c \propto \frac{1}{\sqrt{r}}$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{r}{4r}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

(Note: Refer to Shortcut 13.)

20. $R_A = 9R$, $R_B = R$

$$v = \sqrt{\frac{GM}{R}}$$

$$\therefore \frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{9R}} = \frac{1}{3}$$

$$\therefore \frac{v_A}{v_B} = \frac{4v}{v_B} = \frac{1}{3} \Rightarrow v_B = 12v$$

(Note: Refer to Shortcut 13.)

21. $v_1 = \sqrt{\frac{GM}{R+h}}$, $v_2 = \sqrt{\frac{GM}{R}}$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{R}{R+h}} = \sqrt{\frac{R}{R+7R}} = \frac{1}{2\sqrt{2}}$$

$$\therefore v_1 = \frac{v}{2\sqrt{2}}$$

(Note: Refer to Shortcut 13.)

22. $T = \sqrt{\frac{3\pi}{G\rho}} = \sqrt{\frac{3\times3.14}{6.67\times10^{-11}\times8\times10^3}}\ s \approx 4200\ s$

(Note: Refer to Shortcut 11.v)

23. $T_1 = T$, $T_2 = 8T$

$$\therefore R_2 = R_1\left(\frac{T_2}{T_1}\right)^{2/3} = R\left(\frac{8\,T}{T}\right)^{2/3} = 4R$$

24. Time period of satellite which is very near to planet

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{G\frac{4}{3}\pi R^3\rho}}$$

$$\therefore T \propto \sqrt{\frac{1}{\rho}}$$

i.e. Time period of nearest satellite does not depend upon the radius of planet, it only depends upon the density of the planet.
In the problem, density is same so time period will remain the same.

25. $T = 83$ min, $R' = 4R$

$$\therefore \frac{T'}{T} = \left[\frac{R'}{R}\right]^{3/2} = \left[\frac{4R}{R}\right]^{3/2}$$

T is increased by a factor of $[4]^{3/2}$ i.e. 8 times.
$T' = 8 \times 83$ minutes $= 664$ minutes

26. For a satellite circling around the Earth, the time period is given by $T = 2\pi\sqrt{\frac{(R+h)^2}{GM}}$.
As it is clear from the above equation, the time period is independent of the mass of the satellite. Hence ratio of time periods is 1 : 1

27. $$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{2}{16}\right)^2 = \frac{(10^4)^3}{r_2^3}$$

$r_2^3 = (10^{12}) \times (8)^2 = 64 \times 10^{12} = (4 \times 10^4)^3$

$r_2 = 4 \times 10^4$ km

28. $r_2 = \frac{1}{4}r_1$, $T_1 = 1$ year

Now, $T^2 \propto r^3$

$$\therefore T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = 1\left(\frac{1}{4}\right)^{3/2} = \left(\frac{1}{8}\right)\text{year}$$

29. According to Kepler's law $T^2 \propto R^3$
If n is the frequency of revolution then
$n^2 \propto (R)^{-3}$

$$\therefore \frac{n_2}{n_1} = \left(\frac{R_2}{R_1}\right)^{-3/2} \Rightarrow \frac{R_1}{R_2} = \left(\frac{n_2}{n_1}\right)^{2/3}$$

30. Angular momentum,

$$L = 2m\frac{\Delta A}{\Delta t} \Rightarrow \frac{\Delta A}{\Delta t} = \frac{L}{2\,m}$$

31. $T_A = 8\,T_B$

Using Kepler's third law, $\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$

$$\therefore \frac{(8T_B)^2}{T_B^2} = \left(\frac{r_A}{r_B}\right)^3 \qquad \text{....}[\because T_A = 8T_B]$$

$$\left(\frac{r_A}{r_B}\right)^3 = (4)^3 \Rightarrow \frac{r_A}{r_B} = 4 \text{ or } r_A = 4r_B$$

32. $r_M = 1.525\ r_E$

$$\therefore \frac{r_M}{r_E} = 1.525$$

$$\therefore \left(\frac{T_M}{T_E}\right)^2 = \left(\frac{r_M}{r_E}\right)^3 = (1.525)^3$$

$$\therefore T_M^2 = T_E^2 \times (1.525)^3 = (1)^2\,(1.525)^3$$

$$\therefore T_M = (1.525)^{3/2} = 1.883 \text{ years}$$

33. U = Loss in gravitational energy
= gain in K.E.

So, $U = \frac{1}{2}mv^2 \Rightarrow m = \frac{2U}{v^2}$

34. Orbital radius of satellites $r_1 = R + R = 2R$
$r_2 = R + 7R = 8R$

$P.E_1 = \frac{-GMm}{r_1}$ and $P.E_2 = \frac{-GMm}{r_2}$

$K.E._1 = \frac{GMm}{2r_1}$ and $K.E_2 = \frac{GMm}{2r_2}$

$T.E_1 = \frac{-GMm}{2r_1}$ and $T.E_2 = \frac{-GMm}{2r_2}$

$\therefore \frac{P.E_1}{P.E_2} = \frac{K.E_1}{K.E_2} = \frac{T.E_1}{T.E_2} = 4$

35. $U = -\frac{GMm}{r}$ and

Kinetic energy $= \frac{GMm}{2r}$

$\therefore U = (-2)\frac{GMm}{2r} = -2 \times$ Kinetic energy

$= -2 \times \frac{1}{2}mv^2 = -mv^2$

36. $P.E. = \frac{-GMm}{r}$

$\therefore P.E. \propto \frac{-1}{r}$

Similarly,

$T.E. \propto \frac{-1}{2r}$

And $K.E. \propto \frac{1}{2r}$

37. $B.E_1 = \frac{GMm}{2R} = \frac{1}{2}mgR$ and

$B.E_2 = \frac{GMm}{R} = mgR$

$\therefore B.E_2 - B.E_1 = mgR - \frac{1}{2}mgR = \frac{1}{2}mgR$

38. $v_e = \sqrt{\frac{2GM}{R}}$

$\therefore K.E_1 = \frac{1}{2}mv_e^2$

$= \frac{1}{2}m \times \frac{2GM}{R}$

$= \frac{1}{R^2}m(2gR) = mgR$

$\therefore v_c = \sqrt{\frac{GM}{R}}$ at $h \cong R$

$\therefore K.E_2 = \frac{1}{2}mv_c^2 = \frac{1}{2}mgR$

$\therefore \frac{K.E_1}{K.E_2} = \frac{2mgR}{mgR} = \frac{2}{1}$

Alternate method:

$K.E_1 = \frac{1}{2}mv_e^2$

$= \frac{1}{2}m \times 2gR = mgR$ ....[$v_e = \sqrt{2gR}$]

When orbit is close to Earth, $v_0 = \sqrt{gR}$

$K.E_2 = \frac{1}{2}mv^2 = \frac{1}{2}mgR$

$\therefore \frac{K.E_1}{K.E_2} = \frac{(mgR)}{\frac{1}{2}mgR} = 2$

39. $R_m = \frac{R_e}{4}$, $\rho_m = \frac{2}{3}\rho_e$

Energy spent $= mg_eh_e = mg_m h_m$

$\therefore h_m = g_eh_e / g_m$

$\therefore h_m = \frac{\left(\frac{4}{3}\pi R_e\rho_eG\right)\times h_e}{\frac{4}{3}\pi R_m\rho_mG}$

$\therefore h_m = \frac{R_e}{R_m}\times\frac{\rho_e}{\rho_m}\times h_e = \frac{3}{2}\times\frac{4}{1}\times 0.5 = 3$ m

40. $M_A = 2M_B$, $R_A = 2R_B$

$v_e = \sqrt{\frac{2GM}{R}}$

$\therefore \frac{(v_e)_A}{(v_e)_B} = \sqrt{\frac{2M_B / 2R_B}{M_B / R_B}} = 1$

$\therefore (v_e)_A = (v_e)_B$

41. $v = \sqrt{\frac{2GM}{R}}$

$\therefore v_e = R\sqrt{\frac{8}{3}\pi G\rho}$ ....($\because M = \frac{4}{3}\pi R^3\rho$)

Now, $v_e \propto R$ and $v_p \propto 2R$

$\therefore \frac{v_p}{v_e} = 2$ or $v_e = \frac{v_p}{2}$

42. $v_e = \sqrt{\frac{2GM}{R+h}}$

∴ $(v_e)_1 = \sqrt{\frac{2GM}{2R}} = v$ and

$(v_e)_2 = \sqrt{\frac{2GM}{8R}}$

∴ $\frac{(v_e)_2}{(v_e)_1} = \sqrt{\frac{2GM}{8R} \times \frac{2R}{2GM}} = \sqrt{\frac{1}{4}} = \frac{1}{2}(v_e)_1 = v/2$

43. $v_e = \sqrt{2}\, v_c = 1.414\, v_c$

$= v_c + 0.414\, v_c$

∴ $\frac{v_e - v_c}{v_c} = 0.414$

∴ % increase in speed = 0.414 × 100 = 41.4%

(Note: Refer to Note 16.)

44. $v_e = \sqrt{2}\, v_c$. Clearly, if v_c becomes 36%, v_e will also become 36%

∴ $v_e' = \frac{36}{100} \times 11.2\ \text{km s}^{-1} = \frac{9}{25} \times 11.2\ \text{km s}^{-1}$

45. Since $v_e = \sqrt{2}\, v_c = 1.414\, v_c$

Additional velocity $= v_e - v_c = v_c\left(\sqrt{2} - 1\right)$

$= v_c\,(1.414 - 1)$

$= 1 \times 0.414 = 0.414$ km/s

46. $v_e = \sqrt{\frac{2GM}{R}}$

$v_e \propto \frac{1}{\sqrt{R}}$

∴ $v_e \propto R^{-1/2}$

∴ $dv_e \propto -\frac{1}{2}\, dR\, R^{-3/2}$

∴ $\frac{dv_e}{v_e} = -\frac{1}{2}\frac{dR}{R} = -\frac{1}{2} \times -4\,\% = 2\%$

∴ As radius decreases, escape velocity increases

47. Weight is least at the equator.

48. $g' = g\left(1 - \frac{d}{R}\right) = 10\left(1 - \frac{80}{6400}\right)$

$= 10\left(1 - \frac{1}{80}\right) = \frac{10 \times 79}{80}$

$= 9.87\ \text{m/s}^2 \approx 990\ \text{cm/s}^2$

49. $\rho_p = 2\rho_e$, $g_p = g_e$

$g = \frac{4}{3}\pi\rho G R$

∴ $\frac{R_p}{R_e} = \left(\frac{g_p}{g_e}\right)\left(\frac{\rho_e}{\rho_p}\right) = (1) \times \left(\frac{1}{2}\right)$

∴ $R_p = \frac{R_e}{2} = \frac{R}{2}$

50. For scientist A who goes down in mine,

$g' = g\left(1 - \frac{d}{R}\right)$

For scientist B, who goes up in air,

$g' = g\left(1 - \frac{2h}{R}\right)$

So, it is clear that value of g measured by each will decrease at different rates.

51. $g' = 16\%\ g = \frac{16g}{100} \Rightarrow \frac{g'}{g} = \frac{16}{100}$

∴ $\frac{R^2}{(R+h)^2} = \frac{16}{100} \Rightarrow \frac{R+h}{R} = \frac{5}{2}$

∴ $\frac{h}{R} = \frac{3}{2} \Rightarrow h = \frac{3}{2} \times 6300 = 9450$ km

52. $g_d = g\left(1 - \frac{d}{R}\right)$,

For $d = \frac{R}{2}$,

$g_d = g\left(1 - \frac{R/2}{R}\right) = \frac{g}{2} = 0.5\, g$

53. Given,

$g_d = g'$

∴ $g\left[1 - \frac{d}{R}\right] = g - R\omega^2\cos^2\phi$

∴ $g - \frac{gd}{R} = g - R\omega^2\cos^2\phi$

∴ $\frac{gd}{R} = R\omega^2\cos^2\phi$

∴ $\cos^2\phi = \frac{gd}{R^2\omega^2}$

∴ $\cos\phi = \frac{\sqrt{gd}}{R\omega}$

∴ $\phi = \cos^{-1}\left[\frac{\sqrt{gd}}{R\omega}\right]$

54. $g' = g - R\omega^2\cos^2\phi$; When $\phi = 45°$,

$g' = g - R\omega^2\left(\frac{1}{2}\right)$

When earth stops rotating, g = 0,

so $g' = \frac{R\omega^2}{2}$

Hence the weight of the body increases by $\frac{R\omega^2}{2}$.

55. Gravitational pull depends upon the acceleration due to gravity on that planet.

$M_m = \frac{1}{81} M_e$, $g_m = \frac{1}{6} g_e$

$g = \frac{GM}{R^2} \Rightarrow R = \left(\frac{GM}{g}\right)^{1/2}$

$\therefore \quad \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e}\right)^{1/2} = \left(81 \times \frac{1}{6}\right)^{1/2}$

$\therefore \quad R_e = \frac{9}{\sqrt{6}} R_m$

56. $mg_d = mg\left[1 - \frac{d}{R}\right]$

$\therefore \quad 31.5 = 63\left[1 - \frac{d}{R}\right]$

$\therefore \quad 1 - \frac{d}{R} = \frac{31.5}{63} = \frac{1}{2}$

$\therefore \quad \frac{d}{R} = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore \quad 2d = R$ or $d = \frac{R}{2} = 0.5R$

57. $g_d = g\left[1 - \frac{R/2}{R}\right]$ or $g_d = \frac{g}{2} = \frac{10\ ms^{-2}}{2} = 5\ ms^{-2}$

58. $g \propto \frac{1}{R^2}$

$\therefore$ Percentage change in g = 2 × (Percentage change in R)
= 2 × 1% = 2%

59. $\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2$

$\therefore \quad \frac{g_h}{g} = \frac{1}{100}$

$\therefore \quad \frac{R}{R+h} = \frac{1}{10}$

$\therefore$ h = 9R = 9 × 6400 = 57600 km

60. $x = \frac{GM}{R^2}$

$\therefore \quad \frac{x}{16} = \frac{GM}{(R+h)^2}$

$\therefore \quad x = GM\left(\frac{4}{R+h}\right)^2$

$\therefore \quad \frac{1}{R^2} = \left(\frac{4}{R+h}\right)^2 \Rightarrow \frac{1}{R} = \frac{4}{R+h}$

$\therefore$ R + h = 4R or h = 3R

61. $g' = G\frac{0.99M}{(0.99R)^2} = 1.01\frac{GM}{R^2} = 1.01\ g$

$\therefore \quad \frac{g'}{g} - 1 = 0.01$

$\therefore \quad \frac{g' - g}{g} \times 100 = 1\%$

62. $g = g_p - R\omega^2 \cos^2\phi = g_p - \omega^2 R \cos^2 60°$

$= g_p - \frac{1}{4} R\omega^2$

63. $\phi = 0°$, $g' = g - R\,\omega^2 \cos^2 \phi = 0$

$\therefore \quad \omega = \sqrt{g/R} = \sqrt{10/(6400 \times 10^3)} = 1/800$

64. In pendulum clock, the time period depends on the value of g while in spring watch, the time period is independent of the value of g.

65. Because value of g decreases with increasing height.

71. Apparent weight = actual weight – upthrust force
$Vdg' = Vdg - V\rho g$

$\therefore \quad g' = \left(\frac{d - \rho}{d}\right) g$

72. Weight of the body at equator
$= \frac{3}{5}$ of initial weight

$\therefore \quad g' = \frac{3}{5} g$ (because mass remains constant)

$g' = g - \omega^2 R \cos^2\phi$

$\frac{3}{5} g = g - \omega^2 R \cos^2 (0°)$

$\therefore \quad \omega^2 = \frac{2g}{5R}$

$\therefore \quad \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2 \times 10}{5 \times 6400 \times 10^3}}$

$= \sqrt{62.5 \times 10^{-8}} = 7.9 \times 10^{-4}$ rad/s

73. Since, $F = Mr\omega^2$,

$\therefore \quad T \propto \sqrt{\frac{R}{F}} \Rightarrow T^2 \propto \frac{R}{F}$

$\therefore \quad T^2 \propto \frac{R}{\left(R^{-\frac{3}{2}}\right)} \Rightarrow T^2 \propto R^{\frac{5}{2}}$

76. $T^2 \propto r^3$

$\therefore \quad \frac{T_s}{T_m} = \left(\frac{r_s}{r_m}\right)^{3/2} = \left(\frac{r/2}{r}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2}$

Let $T_s = n\,T_m \Rightarrow \dfrac{T_s}{T_m} = n$

$\therefore\quad n = \dfrac{1}{2^{3/2}} = 2^{-3/2}$

77. Gravitational potential at a point on the surface of Earth $= -\dfrac{GM}{R}$

If Earth is assumed to be a solid sphere, then the gravitational potential at the centre of Earth $= \left(\dfrac{3}{2}\dfrac{GM}{R}\right)$

$\therefore$ Decrease in gravitation potential

$= \dfrac{1}{2} \times \dfrac{GM}{R} = \dfrac{Rg}{2}$

$\therefore$ Loss in potential energy $= \dfrac{Rg}{2} \times m$

Now, gain in kinetic energy = loss in potential energy

$\therefore\quad \dfrac{1}{2}mv^2 = \dfrac{1}{2}mgR$ or $v = \sqrt{gR}$

78. $g_h = g\left(1 - \dfrac{2h}{R}\right)$

$\therefore\quad 9 = g\left[1 - \dfrac{2\left(\dfrac{R}{20}\right)}{R}\right] = g\left(1 - \dfrac{1}{10}\right)$

$\therefore\quad 9 = \dfrac{9g}{10} \Rightarrow g = 10\ ms^{-2}$

$\therefore\quad g_d = g\left(1 - \dfrac{d}{R}\right) = 10\left[1 - \dfrac{\left(\dfrac{R}{20}\right)}{R}\right] = 10\left(\dfrac{19}{20}\right)$

$\therefore\quad g_d = 9.5\ ms^{-2}$

Competitive Thinking

2. Under mutual gravitational force, astronauts move towards each other with very small acceleration.

5. $F = \dfrac{G \times m \times m}{(2R)^2} = \dfrac{G \times \left(\dfrac{4}{3}\pi R^3 \rho\right)^2}{4R^2}$

$= \dfrac{4}{9}\pi^2\rho^2R^4$

$\therefore\quad F \propto R^4$

6. $\dfrac{GM^2}{(2R)^2} = \dfrac{Mv^2}{R}$ or $\dfrac{GM}{4R} = v^2$

$\therefore\quad v = \dfrac{1}{2}\sqrt{\dfrac{GM}{R}}$

7. As $g = \dfrac{GM}{R^2}$,

On the planet,

$g_p = \dfrac{GM/7}{R^2/4} = \dfrac{4g}{7}$

$\therefore$ Hence weight on the planet $= 700 \times \dfrac{4}{7}$

$= 400$ gm-wt

8. $g = \dfrac{GM}{R^2}$ and $M = \dfrac{4}{3}\pi R^3 \times \rho$

$\therefore\quad g = \dfrac{4}{3}\dfrac{\pi R^3 \times G\rho}{R^2}$ $\qquad\therefore\quad \rho = \dfrac{3g}{4\pi RG}$

9. $\rho_2 = 2\rho_1$, $R_1 = R_2$

$g \propto \rho R \Rightarrow g_1 \propto \rho_1 R_1$ and $g_2 \propto \rho_2 R_2$

$\therefore\quad \dfrac{g_1}{g_2} = \dfrac{\rho_1}{\rho_2} \times \dfrac{R_1}{R_2} = \dfrac{1}{2} \times 1 = \dfrac{1}{2}$

$\therefore\quad g_2 = 2 \times 9.8 = 19.6\ m/s^2$

10. $M' = 2M$, $R' = 2R$ and $g = \dfrac{GM}{R^2}$

$\therefore\quad \dfrac{g'}{g} = \dfrac{M'}{M}\left(\dfrac{R}{R'}\right)^2 = \left(\dfrac{2M}{M}\right)\left(\dfrac{R}{2R}\right)^2 = \dfrac{1}{2}$

$\therefore\quad g' = \dfrac{g}{2} = \dfrac{9.8}{2} = 4.9\ m/s^2$

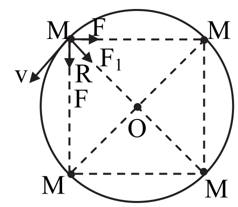
11. $R_m = \dfrac{R_e}{4}$, $M_m = \dfrac{M_e}{80}$

Using $g = \dfrac{GM}{R^2}$ we get $g_m = \dfrac{g}{5}$

$\therefore\quad \dfrac{g_m}{g_e} = \dfrac{M_m}{M_e} \times \left(\dfrac{R_e}{R_m}\right)^2 = \dfrac{1}{80} \times (4)^2$

$\therefore\quad g_m = \dfrac{g}{5}$

12. $\dfrac{F}{\sqrt{2}} + \dfrac{F}{\sqrt{2}} + F_1 = \dfrac{Mv^2}{R}$



$\therefore\quad \dfrac{2 \times GM^2}{\sqrt{2}\left(R\sqrt{2}\right)^2} + \dfrac{GM^2}{4R^2} = \dfrac{Mv^2}{R}$

$\therefore\quad \dfrac{GM^2}{R}\left[\dfrac{1}{4} + \dfrac{1}{\sqrt{2}}\right] = Mv^2$

$\therefore\quad v = \sqrt{\dfrac{GM}{R}\left(\dfrac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \dfrac{1}{2}\sqrt{\dfrac{GM}{R}\left(1 + 2\sqrt{2}\right)}$

13. $g = \frac{GM}{R^2} = \frac{GM_0}{(D_0/2)^2} = \frac{4GM_0}{D_0^2}$

14. $\frac{\rho_1}{\rho_2} = \frac{2}{3}, \frac{R_1}{R_2} = \frac{1}{2}$

$g \propto \rho R \Rightarrow g_1 \propto \rho_1 R_1$ and $g_2 \propto \rho_2 R_2$

$\therefore \frac{g_1}{g_2} = \frac{\rho_1}{\rho_2} \times \frac{R_1}{R_2} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

15. Force on satellite is only gravitational force, which will always be towards the centre of earth.

16. $v \propto \frac{1}{\sqrt{r}}$

$\therefore$ % increase in speed $= \frac{1}{2}$ (% decrease in radius)

$= \frac{1}{2}(1\%)$

$= 0.5\%$

i.e. speed will increase by 0.5%

17. $v_c = \sqrt{\frac{GM}{r}}$

Thus, critical velocity is independent of mass of satellite.

18. $\frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{4R}{R}} = 2$

$\Rightarrow v_B = 2 \times v_A = 2 \times 3v = 6v$

(Note: Refer to Shortcut 13.)

20. $T \propto r^{\frac{3}{2}}$ i.e. $r \propto T^{\frac{2}{3}}$; K.E. $\propto \frac{1}{r} \propto \frac{1}{T^{\frac{2}{3}}}$

$\therefore$ K.E. $\propto T^{\frac{-2}{3}}$

21. $r = 1.5 \times 10^8 \times 10^3$ m

When orbiting, gravitational force

$F = m\omega^2 r$

$= 6 \times 10^{24} \times (2 \times 10^{-7})^2 \times 1.5 \times 10^8 \times 10^3$

$= 36 \times 10^{21}$ N

22. $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\therefore T^2 = \frac{4\pi^2}{GM}(R+h)^3$

$\therefore R + h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3}$

$\therefore h = \left[\frac{GMT^2}{4\pi^2}\right]^{\frac{1}{3}} - R$

23. $\frac{T^2}{r^3} =$ constant

$\therefore T^2 r^{-3} =$ constant

24. $r_2 = 2r_1$

$\therefore \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = (2)^{3/2} = 2\sqrt{2}$

$\therefore T_2 = 2\sqrt{2}$ years

25. $r_2 = \frac{1}{4} r_1$

$T \propto r^{\frac{3}{2}} \Rightarrow T_1 \propto r^{\frac{3}{2}}$ and $T_2 \propto r_2^{\frac{3}{2}}$

$\therefore \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{\frac{3}{2}} \Rightarrow T_2 = T_1\left(\frac{1}{4}\right)^{\frac{3}{2}}$

$\therefore T_2 = 24 \times \frac{1}{8} = 3$hr

26. In the problem, orbital radius is increased by 1%.

Time period of satellite $T \propto r^{3/2}$

Percentage change in time period

$= \frac{3}{2}$ (% change in orbital radius)

$= \frac{3}{2}(1\%) = 1.5\%$.

27. $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{\frac{3}{2}} \Rightarrow T_2 = 24\left(\frac{6400}{36000}\right)^{\frac{3}{2}} \cong 2$ hour.

28. Point A indicates perihelion position while point C represents aphelion position.

This means point A is closest to the sun followed by point B and C.

Hence, $v_A > v_B > v_C$

$\therefore K_A > K_B > K_C$

29. we know

$T^2 \propto r^3$

$T^2 = kr^3$

Take *l*n on both side

$lnT^2 = lnkr^3$

$2lnT^2 = lnk + 3lnr$

Differentiate both side w.r.t. x

$2\frac{1}{T}\frac{dT}{dx} = \frac{1}{k}\frac{dk}{dx} + 3\frac{1}{r}\frac{dr}{dx}$

$\frac{2\Delta T}{T} = \frac{\Delta k}{k} + 3\frac{\Delta r}{r}$

$\frac{2\Delta T}{T} = 3\frac{\Delta r}{r}$

$\Delta T = \frac{3}{2}T\frac{\Delta r}{r}$

30. K.E. (K) = $\frac{GMm}{2r}$ and P.E. (V) = $\frac{-GMm}{r}$

∴ $E = K + V = -\frac{GMm}{2r}$

$\Rightarrow K = -\frac{V}{2}$

31. Binding energy of a satellite on the surface of the earth is,

B.E. = $\frac{GMm}{R}$

Binding energy of satellite revolving around the earth at height h is,

$(B.E.)_h = \frac{GMm}{R}$

∴ $\frac{B.E.}{(B.E.)_h} = \frac{2(R+h)}{R}$

33. Because it does not depend on the mass of particle.

34. $\omega = \frac{v_e}{R} = \frac{1}{R}\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^3}}$

35. $v_e = \sqrt{\frac{2GM}{R}}$

∴ $v_e \propto \sqrt{M}$ if R = constant

∴ If the mass of the planet becomes four times then escape velocity will become 2 times.

36. $v_e = \sqrt{\frac{2GM}{R}}$ ∴ $v_e \propto \sqrt{\frac{M}{R}}$

If mass and radius of the planet are three times than that of earth then escape velocity will remain same.

37. $v_e \propto \sqrt{\rho} \Rightarrow v_1 \propto \sqrt{\rho_1}$ and $v_2 \propto \sqrt{\rho_2}$

∴ $\frac{v_1}{v_2} = \sqrt{\frac{\rho_1}{\rho_2}}$

38. Escape velocity, $v_e = R\sqrt{\frac{8}{3}\pi G\rho}$

$\Rightarrow \frac{v_e}{v_p} = \frac{R\sqrt{\rho}}{R_p\sqrt{\rho_p}}$

Given: $R_p = 2R$ and $\rho_p = 2\rho$

∴ $\frac{v_e}{v_p} = \frac{1}{2\sqrt{2}}$

39. Orbital velocity of satellite $v_0 = \sqrt{gR}$

Escape velocity of satellite $v_e = \sqrt{2gR}$

Minimum increase required,

$\Delta v = v_e - v_0 = \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\left(\sqrt{2}-1\right)$

40. $v_e = \sqrt{2}v$

$\Rightarrow K.E. = \frac{1}{2}mv_e^2 = \frac{1}{2}m\left(\sqrt{2}v\right)^2 = mv^2$

41. On earth, $v_e = \sqrt{\frac{2GM}{R}} = 11.2$ km/s

On moon, $v_m = \sqrt{\frac{2GM \times 4}{81 \times R}} = \frac{2}{9}\sqrt{\frac{2GM}{R}}$

$= \frac{2}{9} \times 11.2 = 2.5$ km/s

42. $v_e = \sqrt{\frac{2Gm}{r}}$

Thus, escape velocity is independent of mass of satellite and depends on the radius of orbit. Hence they have equal escape velocities.

43. $M_p = 2M_e$, $R_p = 3R_e$

$\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}} = \sqrt{2 \times \frac{1}{3}} = \sqrt{\frac{2}{3}}$

∴ $v_p = \sqrt{\frac{2}{3}}v_e$

44. If body is projected with velocity v $(v < v_e)$ then

height up to which it will rise, $h = \frac{R}{\left(\frac{v_e^2}{v^2} - 1\right)}$

$v = \frac{v_e}{2}$ (Given)

∴ $h = \frac{R}{\left(\frac{v_e}{v_e/2}\right)^2 - 1} = \frac{R}{4-1} = \frac{R}{3}$

45. $v_e = \sqrt{\frac{2GM}{R}} = c$

$\Rightarrow R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(3 \times 10^8)^2}$

$= \frac{2 \times 6.67 \times 5.98}{9} \times 10^{-3}$ m

$= 8.86 \times 10^{-3}$ m $\approx 10^{-2}$ m

46. $R_p = \frac{R_E}{4}$, $g_p = 2g_E$

$v_e = \sqrt{2gR}$

$\therefore \quad \frac{v_{eP}}{v_{eE}} = \sqrt{\frac{g_p}{g_e} \times \frac{R_p}{R_E}} = \sqrt{2 \times \frac{1}{4}} = \frac{1}{\sqrt{2}}$

47. K.E. = P.E.

$\frac{1}{2}mv_s^2 = \frac{GMm}{2R}$

$v_s^2 = \frac{GM}{R}$

$v_s = \sqrt{gR}$ $\quad (\because GM = gR^2)$

But $v_e = \sqrt{2gR}$

$v_e = \sqrt{2}v_s$

$v_s = \frac{v_e}{\sqrt{2}}$

48. $v_c = \sqrt{\frac{GM}{R+h}}$

$v_e = \sqrt{\frac{2GM}{R}}$

But, $4v_c = v_e$ ….(given)

$4\sqrt{\frac{GM}{R+h}} = \sqrt{\frac{2GM}{R}}$

$\frac{16GM}{R+h} = \frac{2GM}{R}$

$\therefore \quad 8R = R + h$

$\therefore \quad h = 7R$

49. P.E. $= -\frac{GMm}{(R+nR)}$

Change in potential energy

$P.E._2 - P.E._1 = \frac{-GMm}{R+nR} - \left(-\frac{GMm}{R}\right)$

$= \frac{GMm}{R} - \frac{GMm}{R(n+1)}$

$= \frac{GMm}{R}\left(1 - \frac{1}{n+1}\right)$

$= \frac{GMm}{R}\left(\frac{n+1-1}{n+1}\right)$

$= \frac{GMm}{R} \times \frac{n}{(n+1)}$

$= \frac{GMm \times R}{R^2}\left(\frac{n}{n+1}\right)$

$= mgR\left(\frac{n}{n+1}\right)$

(Note: One may refer shortcut 25.ii for solving certain problem/s from this section.)

50. Change in potential energy in displacing a body from r_1 to r_2 is given by

$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = GMm\left(\frac{1}{2R} - \frac{1}{3R}\right)$

$= \frac{GMm}{6R}$

51. $P.E._1 = 0$

$P.E._2 = -\frac{GmM}{2R}$

$\therefore$ Change in P.E. $= GmM\left[\frac{1}{R} - \frac{1}{2R}\right] = \frac{GmM}{2R}$

$= \frac{GM}{R^2} \times \frac{mR}{2} = \frac{1}{2}mgR$

52. $\Delta U = \frac{mgh}{\left(1+\frac{h}{R}\right)} = \frac{mg \times 3R}{\left(1+\frac{3R}{R}\right)} = \frac{3}{4}mgR$

53. P.E.(U) $= -\frac{GMm}{R+h}$

$\therefore$ Increase in P.E. $= U_2 - U_1$

$= \frac{-GMm}{R+10R} + \frac{GMm}{R}$

$= \frac{GMm}{R}\left[1 - \frac{1}{11}\right] = \frac{10\,GMm}{11R}$

54. The change in potential energy is given as,

$\Delta U = U_f - U_i$

$= \frac{-GMm}{R+2R} - \frac{-GMm}{R}$

$= \frac{GMm}{R}\left[1 - \frac{1}{3}\right] = \frac{2}{3}\frac{GMm}{R}$

$= \frac{2}{3}\frac{GMm \times R}{R^2} = \frac{2}{3}\left(\frac{GM}{R^2}\right)mR$

$\therefore \quad \Delta U = \frac{2}{3}mgR$

55. $U_S = \frac{-GMm}{R}$ …(at surface)

$U_T = \frac{-GMm}{2R}$ …(at target)

$W = U_T - U_S = \frac{-GMm}{2R} + \frac{GMm}{R}$

$= \frac{GMm}{2R} = \frac{gR^2m}{2R}$ …$(GM = gR^2)$

$= \frac{mgR}{2}$

56. Increase in the P.E. is given by,

$\Delta U = U_B - U_A$

$U_B = -\frac{GMm}{R+h} = -\left(\frac{GMm}{R+R/5}\right) = -\frac{5GMm}{6R}$

$U_A = -\frac{GMm}{R}$

$\therefore$ $\Delta U = -\frac{5GMm}{6R} + \frac{GMm}{R} = \frac{GMm}{R}\left(1-\frac{5}{6}\right)$

$\Delta U = \frac{GMm}{6R}$

$\therefore$ $\Delta U = \frac{mgR^2}{6R}$ $(\because GM = gR^2)$

$\therefore$ $\Delta U = \frac{mgR}{6}$

$\therefore$ $\Delta U = \frac{5}{6}\,mgh$ $(\because R = 5h)$

Alternate method (I):

$\Delta U = \frac{mgh}{1+h/R}$

Substituting R = 5h

we get $\Delta U = \frac{mgh}{1+1/5} = \frac{5}{6}mgh$

57. Orbital Energy $E_0 = \frac{-GMm}{2(R+h)}$

$\therefore$ $E_0 = \frac{-GMm}{2(R+2R)} = \frac{-GMm}{6R}$ ….$[\because h = 2R]$

Energy at surface $E = \frac{-GMm}{R}$

$\therefore$ Min. energy required $= E_0 - E$

$= \frac{-GMm}{6R} - \left(\frac{-GMm}{R}\right)$

$= \frac{5\,GMm}{6R}$

58. Total energy of a satellite is,

$T.E. = -\frac{GMm}{2(R+h)}$ ...(i)

$\therefore$ Multiplying and dividing the eq (i) by R^2.

$T.E. = -\frac{GMmR^2}{2(R+h)R^2}$

$\therefore$ $T.E. = -\frac{g_0mR^2}{2(R+h)}$ ... $(\because g_0 = \frac{GM}{R^2})$

59. $B.E. = \frac{GmM}{2r} = \frac{GM}{R^2} \times \frac{mR^2}{2r} = \frac{mgR^2}{2r}$

60. $B.E. = \frac{GmM}{R} = mgR = 100 \times 10 \times 6.4 \times 10^6$

$= 6.4 \times 10^9$ J

61. $g' = g - R\omega^2\cos^2\phi$. Hence value of g′ changes with ϕ.

62. $g' = g - \omega^2R\cos^2\phi$

Rotation of the earth results in the decreased weight apparently. This decrease in weight is not felt at the poles as the angle of latitude is 90°.

63. An object of mass m_1 placed at the equator of the star, will experience two forces: (i) an attractive force due to gravity towards the centre of the star and (ii) an outward centrifugal force due to rotation of the star. The centrifugal force arises because the object is in a rotating (non-inertial) frame; this force is equal to the inward centripetal force but opposite in direction. Force on object due to gravity

$F_g = \frac{GmM}{R^2}$

Force on object is

$F_c = mR\omega^2$

The object with remain stuck to the star and not fly off if

$F_g > F_c$

i.e., $\frac{GmM}{R^2} > mR\omega^2$ or $M > \frac{R^3\omega^2}{G}$

64. i. Going down from surface towards centre –

$g_{depth} = \frac{g}{\left(1+\frac{d}{R}\right)}$

As d increases, g decreases.

ii. Going up from surface –

$$g_{height} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

As h increases, g decreases.

iii. Going from equator to pole –

g is less at equator and more at poles owing to bulge at equator and flattening at poles. Thus g increases in moving towards poles.

iv. Changing rotational velocity –

$g' = g - R\omega^2 \cos^2 \phi$

As ω increases, g decreases.

65. Inside the earth, $g = \frac{4}{3}\pi R\rho G$

∴ $g \propto r$

66. $g \propto \rho$

67. $g' = g\left(1-\frac{d}{R}\right)$

∴ $\frac{g}{n} = g\left(1-\frac{d}{R}\right)$ ∴ $d = \left(\frac{n-1}{n}\right)R$

(Note: Refer to Shortcut 3.ii.)

68. $g_h = g\left(\frac{R}{R+h}\right)^2$

∴ $\frac{g}{4} = g\left(\frac{R}{R+h}\right)^2$

∴ $\frac{1}{4} = \left(\frac{R}{R+h}\right)^2$

∴ $\frac{1}{2} = \frac{R}{R+h}$

∴ $R + h = 2R$

∴ $h = R$

69. Gravity at height h,

$g_h = g\left(1-\frac{2h}{R}\right)$

Gravity at depth d,

$g_d = g\left(1-\frac{d}{R}\right)$

Given : $g_h = g_d$

$\Rightarrow d = 2h$

70. Given: $g_d = g_h$ ….(i)

But, $g_d = g\left(1-\frac{d}{R}\right)$ and

$g_h = g\left(1-\frac{2h}{R}\right)$

∴ $g\left(1-\frac{d}{R}\right) = g\left(1-\frac{2h}{R}\right)$ ….[From (i)]

∴ $d = 2h$

∴ $d = 2 \times 1$ ….($\because$ h = 1 km)

∴ $d = 2$ km

71. $g_d = g\left(1-\frac{d}{R}\right) = 9.8\left(1-\frac{1600}{6400}\right)$

$g_d = 9.8 \times \frac{3}{4}$

$g_d = 7.35\ ms^{-2}$

72. Acceleration due to gravity at h = 5 km above

$g_h = g\left(1-\frac{2h}{R}\right) = 9.8\left(1-\frac{2\times5}{6400}\right) \approx 9.78\ m/s^2$

OR

$g_h = \frac{GM}{(R+h)^2} = \frac{GM}{(R+5)^2} = \frac{GM}{R^2} \times \frac{R^2}{(R+5)^2}$

$= \frac{gR^2}{(R+5)^2} = \frac{9.8\times(6400)^2}{(6400+5)^2} = 9.78\ m/s^2$

Acceleration due to gravity at depth = 5 km,

$g_d = g\left(1-\frac{d}{R}\right) = 9.8\left(1-\frac{5}{6400}\right) = 9.79\ m/s^2$

74. $\frac{\rho_1}{\rho_2} = 1 : 2$, $\frac{d_1}{d_2} = 4 : 1$

$g = \frac{4}{3}G\pi R\rho$

∴ $\frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{2} \times \frac{4}{1} = \frac{2}{1}$

75. $h = 3R \Rightarrow r = 4R$

$g = \frac{Gm}{R^2}$, $g_h = \frac{Gm}{(4R)^2} = \frac{Gm}{16R^2} = \frac{g}{16}$

∴ $\frac{g_h}{g} = \frac{1}{16}$

76. Acceleration due to gravity at a depth x below surface of earth is

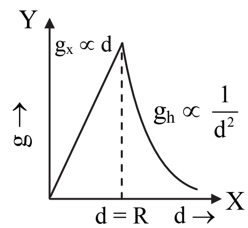
$g' = \frac{GM}{R^2}\left(1-\frac{x}{R}\right) = g\left(1-\frac{x}{R}\right)$

at the depth x, distance of point from centre of the earth is (R – x) i.e., d = R – x

In this case,
$g_x \propto R - x$
$\therefore$ $g_x \propto d$
At height h distance from centre of the earth is (R + h) i.e., d = R + h

In this case, $g_h = g\left(\dfrac{R}{R+h}\right)^2 = \dfrac{gR^2}{d^2}$

$\Rightarrow g_h \propto \dfrac{1}{d^2}$



78. h = R ⇒ r = 2R
$G = \dfrac{Gm}{R^2}$, $g_h = \dfrac{Gm}{(2R)^2} = \dfrac{1}{4}\dfrac{Gm}{R^2} = \dfrac{g}{4}$

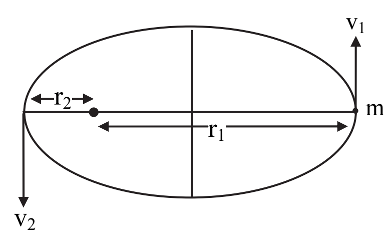
79. $M_p = 2M_E$, $D_P = 2D_E \Rightarrow R_P = 2R_E$
$T_E = 2$ s
$g_E = \dfrac{GM_E}{R_E}$, $g_P = \dfrac{GM_P}{R_P^2}$
$\therefore$ $g_P = g_E \times \dfrac{M_P}{M_E} \times \left(\dfrac{R_E}{R_P}\right)^2$
$= g_E \times 2 \times \left(\dfrac{1}{2}\right)^2 = \dfrac{g_E}{2} \Rightarrow \dfrac{g_E}{g_P} = 2$
Now, $T \propto \dfrac{1}{\sqrt{g}}$
$\therefore$ $T_P = T_E \times \sqrt{\dfrac{g_E}{g_P}} = T_E\sqrt{2} = 2\sqrt{2}$ s

80. $g' = g - \omega^2 R \cos^2 \lambda$, $\lambda = 60°$
$\therefore$ $0 = 1 - \omega^2 \times 6400 \times 10^3 \times \dfrac{1}{4}$
$\therefore$ $\omega^2 = \dfrac{10^{-4}}{16}$
$\Rightarrow \omega = \dfrac{10^{-2}}{4}$
$\therefore$ $\omega = 2.5 \times 10^{-3}$ rad/s

81. Gravitational acceleration of earth, $g = \dfrac{GM}{R^2}$
Where, M is mass of the earth.
As g is independent of mass of the Sun, increase in G will increase value of g. Hence, statement (D) is incorrect.
Also terminal velocity of raindrop depends on g therefore increase in g will cause raindrops to fall faster.
Hence, statement (A) is correct.
Increased value of g will make walking on ground more difficult. Hence, statement (B) is correct.
Time period of simple pendulum will decrease as $T \propto \dfrac{1}{\sqrt{g}}$. Hence, statement (C) is correct.

82. $F_{CP} = F_G$
$\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$
$v = \sqrt{\dfrac{GM}{r}}$
$T = \dfrac{2\pi r}{v}$
$T^2 = \dfrac{4\pi^2 r^3}{GM}$ ….(i)
$T^2 = Kr^3$ ….(ii)
$K = \dfrac{4\pi^2}{GM}$
$GMK = 4\pi^2$

83.



From law of conservation of angular momentum,
$mv_1r_1 = mv_2r_2$
$\Rightarrow$ $v_2 = \dfrac{v_1 r_1}{r_2}$ ….(i)
From law of conservation of energy,
$\dfrac{-GMm}{r_1} + \dfrac{1}{2}mv_1^2 = \dfrac{-GMm}{r_2} + \dfrac{1}{2}mv_2^2$ … (ii)
From equations (i) and (ii),
$v_1 = \sqrt{\dfrac{2GMr_2}{(r_1+r_2)r_1}}$
Angular momentum,
$L = mv_1r_1$
$= m\sqrt{\dfrac{2GMr_1r_2}{r_1+r_2}}$

84. $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$ also r = R + h
$\therefore$ $v = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{GMR^2}{R^2 r}} = \sqrt{\dfrac{g}{r}}\,R$

$\therefore \quad v = \left(\sqrt{\dfrac{9.8}{.25\times10^6+6.38\times10^6}}\right) \times 6.38 \times 10^6$

$= \sqrt{\dfrac{1.47}{10^6}} \times 6.38 \times 10^6$

$= 7.76 \times 10^3$ m/s = 7.76 km/s

85. We know that, $F \propto m_1m_2$

$\therefore \quad F \propto (xm)\times(1-x)\,m = xm^2(1-x)$

For maximum force, $\dfrac{dF}{dx} = 0$

$\therefore \quad \dfrac{dF}{dx} = m^2 - 2xm^2 = 0$

$\Rightarrow x = \dfrac{1}{2}$

86. $\dfrac{g_1}{g_2} = \dfrac{5}{2}$

$\therefore \quad \dfrac{\dfrac{G\rho_1\frac{4}{3}\pi R_1^3}{R_1^2}}{\dfrac{G\rho_2\frac{4}{3}\pi R_2^3}{R_2^2}} = \dfrac{5}{2}$

$\therefore \quad \dfrac{R_1}{R_2} = \dfrac{5}{2}\times\dfrac{\rho_2}{\rho_1} = \dfrac{5}{2}\times\dfrac{1}{2} = \dfrac{5}{4}$

$\dfrac{v_1}{v_2} = \sqrt{\dfrac{g_1R_1}{g_2R_2}} = \sqrt{\dfrac{5}{2}\times\dfrac{5}{4}}$

$\therefore \quad \dfrac{v_1}{v_2} = \dfrac{5}{2\sqrt{2}}$

87. $g = \dfrac{GM}{R^2}$ and K.E. $= \dfrac{L^2}{2I}$

If mass of the earth and its angular momentum remains constant then $g \propto \dfrac{1}{R^2}$ and K.E. $\propto \dfrac{1}{R^2}$

i.e. if radius of earth decreases by 2% then g and K.E. both increases by 4%.

Alternate Method:

$g = \dfrac{GM}{R^2}$ $\qquad \therefore \quad g \propto \dfrac{1}{R^2}$

$\therefore \quad dg \propto \dfrac{-2dR}{R^3}$ $\qquad \therefore \quad \dfrac{dg}{g} = \dfrac{-2dR}{R}$

$\therefore \quad \dfrac{dg}{g} \times 100 = -2\left(\dfrac{dR}{R} \times 100\right) = -2 \times -2\% = 4\%$

i.e. g increases by 4%

Now, K.E. $= \dfrac{L^2}{2I} \Rightarrow$ K.E. $\propto \dfrac{1}{I}$

$I = \dfrac{2}{5}MR^2 \Rightarrow I \propto R^2$

$\therefore \quad$ K.E. $\propto \dfrac{1}{R^2}$ $\qquad \therefore \quad dK \propto -\dfrac{2dR}{R^3}$

$\therefore \quad \dfrac{dK.E.}{K.E.} \times 100 = -2 \times \left(\dfrac{dR}{R}\times100\right)$

$= -2 \times -2\% = 4\%$

K.E. increased by 4%

88. By energy conservation

$U_i = U_f$

$\therefore \quad 0 + \dfrac{-GMm}{nR+R} = \dfrac{1}{2}mv^2 - \dfrac{GMm}{R}$

$\therefore \quad v = \sqrt{\dfrac{2gnR}{n+1}}$

89. $g' = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$ $\qquad \therefore \quad \dfrac{g}{16} = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

$\therefore \quad \left(1+\dfrac{h}{R}\right)^2 = 16$

$\therefore \quad 1+\dfrac{h}{R} = 4 \Rightarrow \dfrac{h}{R} = 3 \Rightarrow h = 3R$

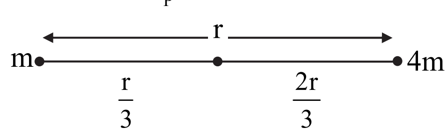
90. Gravitational attraction force on particle B

$F_g = \dfrac{GM_Pm}{\left(\dfrac{D_P}{2}\right)^2}$

Acceleration of particle due to gravity

$a = \dfrac{F_g}{m} = \dfrac{4GM_P}{D_P^2}$

91.



$\dfrac{Gm}{x^2} = \dfrac{G(4m)}{(r-x)^2}$

$\dfrac{1}{x} = \dfrac{2}{r-x}$

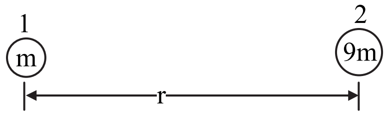
$\Rightarrow r - x = 2x \Rightarrow 3x = r$

$\Rightarrow x = \dfrac{r}{3}$

$\therefore \quad$ The gravitational potential $= -\dfrac{Gm}{r/3} - \dfrac{G(4m)}{2r/3}$

$= -\dfrac{3Gm}{r} - \dfrac{6Gm}{r} = -\dfrac{9Gm}{r}$

92.



Let at distance x from m gravitational field be zero.

$\therefore \quad \dfrac{Gm}{x^2} = \dfrac{G(9m)}{(r-x)^2}$

$\therefore \quad (r-x)^2 = 9x^2$

$\therefore \quad r - x = \pm 3x$

As r being distance, cannot be negative. Hence, negative value is neglected.

$\therefore \quad r = 4x$

$\therefore \quad x = \dfrac{r}{4}$

Net potential at x, $V = V_1 + V_2$

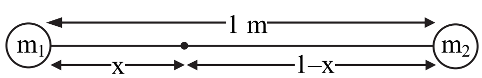
$= \dfrac{-Gm}{(r/4)} + \dfrac{-G(9m)}{[r-(r/4)]}$

$= \dfrac{-4Gm}{r} - \dfrac{9Gm}{3r/4}$

$= \dfrac{-4Gm}{r} - \dfrac{12Gm}{r}$

$= \dfrac{-16Gm}{r}$

93.



$-\dfrac{Gm_1}{x^2} = -\dfrac{Gm_2}{(1-x)^2}$

$\therefore \quad (1-x)^2 m_1 = m_2 x^2$

$\therefore \quad (1-x)\sqrt{m_1} = \sqrt{m_2}\, x$

$\therefore \quad \sqrt{m_1} - x\sqrt{m_1} = \sqrt{m_2}\, x$

$\therefore \quad \sqrt{m_1}\, x + \sqrt{m_2}\, x = \sqrt{m_1}$

$x + \sqrt{\dfrac{m_2}{m_1}}\, x = 1$

$\therefore \quad x\left(1 + \sqrt{\dfrac{m_2}{m_1}}\right) = 1$

$\therefore \quad x = \dfrac{1}{1+\sqrt{\dfrac{m_2}{m_1}}} = \dfrac{1}{1+\sqrt{\dfrac{8100}{100}}} = 0.1$

$\therefore$ Gravitational potential $= -\dfrac{Gm_1}{x^2}$

$= \dfrac{-6.67\times10^{-11}\times100}{0.1^2}$

$= -6.67 \times 10^{-7}$ J/kg

94. (T.E.) on surface = (T.E.) at height 'h'

$\therefore \quad (K.E.)_1 + (P.E.)_1 = (K.E.)_2 + (P.E.)_2$

$\therefore \quad \dfrac{1}{2}mu^2 + \left(-\dfrac{GMm}{R}\right) = 0 + \left(-\dfrac{GMm}{R+h}\right)$

$\dfrac{1}{2}mu^2 = \left(-\dfrac{GMm}{R+h}\right) - \left(-\dfrac{GMm}{R}\right)$

$= \dfrac{GMm}{R} - \dfrac{GMm}{R+h}$

$= GMm\left[\dfrac{1}{R} - \dfrac{1}{R+h}\right]$

$\therefore \quad u^2 = 2GM\left[\dfrac{1}{R} - \dfrac{1}{R+h}\right]$

$u^2 = 2gR^2\left[\dfrac{R+h-R}{R(R+h)}\right]$ ....($\because$ $GM = gR^2$)

$u^2 = 2gR\left[\dfrac{h}{R+h}\right]$

$\therefore \quad \dfrac{u^2}{2gR} = \left[\dfrac{h}{R+h}\right]$ $\qquad \therefore \quad \dfrac{R+h}{h} = \dfrac{2gR}{u^2}$

$\dfrac{R}{h} + 1 = \dfrac{2gR}{u^2}$

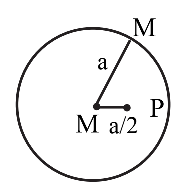
$\therefore \quad \dfrac{R}{h} = \dfrac{2gR}{u^2} - 1$

$\dfrac{R}{h} = \dfrac{2gR - u^2}{u^2}$

$\therefore \quad h = \dfrac{u^2 R}{2gR - u^2}$

95. $V_P = V_{sphere} + V_{partical}$

$= \dfrac{GM}{a} + \dfrac{GM}{a/2} = \dfrac{3GM}{a}$



96. Applying law of conservation of energy for asteroid at a distance 10 R_e and at earth's surface,

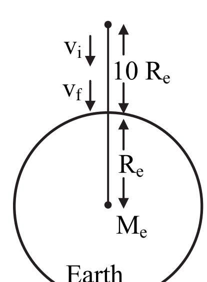
$K_i + U_i = K_f + U_f$ ....(i)

Now, $K_i = \dfrac{1}{2}mv_i^2$

and $U_i = \dfrac{GM_e m}{10R_e}$

$K_f = \dfrac{1}{2}mv_f^2$ and

$U_f = \dfrac{GM_e m}{R_e}$



Substituting these values in eq.(i), we get

$$\frac{1}{2}mv_i^2 - \frac{GM_e m}{10R_e} = \frac{1}{2}mv_f^2 - \frac{GM_e m}{R_e}$$

$$\Rightarrow v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 - \frac{1}{10}\right)$$

97. $F = \frac{Gm(M-m)}{r^2}$

For maximum force $\frac{dF}{dm} = 0$

$$\Rightarrow \frac{d}{dm}\left(\frac{GmM}{r^2} - \frac{Gm^2}{r^2}\right) = 0$$

$$\Rightarrow M - 2m = 0 \Rightarrow \frac{m}{M} = \frac{1}{2}$$

98. $m\omega^2 R \propto \frac{1}{R^n} \Rightarrow m\left(\frac{4\pi^2}{T^2}\right)R \propto \frac{1}{R^n} \Rightarrow T^2 \propto R^{n+1}$

$\therefore$ $T \propto R^{\left(\frac{n+1}{2}\right)}$

99. For the planet to orbit around the star, the centripetal force must be provided by gravitational force. Hence, $F_G = F_a$

$F_a \propto - r^{-5/2}$ ....(Given)

(–ve sign indicates force is towards centre of orbit)

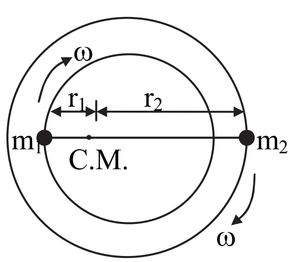
Hence, $a \propto - r^{-5/2}$

$\therefore$ $-\omega^2 r \propto - r^{-5/2}$ $\quad\therefore$ $\omega^2 \propto r^{-7/2}$

$\therefore$ $\frac{4\pi^2}{T^2} \propto r^{-7/2}$ or $T^2 \propto r^{7/2}$

100. Both the stars rotate with same angular velocity ω around the centre of mass (CM) in their respective orbits as shown in figure.

The magnitude of gravitational force m_1 exerts on m_2 is $|F| = \frac{Gm_1m_2}{(r_1 + r_2)^2}$



101. $\frac{Gm_A m_B}{(r_A + r_B)^2} = \frac{m_A r_A 4\pi^2}{T_A^2} = \frac{m_B r_B 4\pi^2}{T_B^2}$

$\Rightarrow m_A r_A = m_B r_B$

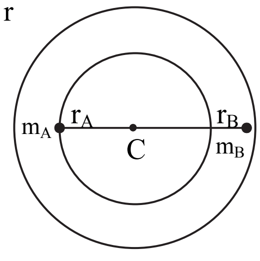
$\therefore$ $T_A = T_B$

102. If r < R then $F = \frac{GMm}{R^3}.r$

$\therefore$ $\frac{mv^2}{r} = \frac{GMm}{R^3}r \Rightarrow v \propto r$

If r > R then $F = \frac{GMm}{r^2}$

$\therefore$ $\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v \propto \frac{1}{r}$



104. Gravitational field = $Gm\left[\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + ...\right]$

Gravitational field = $\frac{4Gm}{3}$

105. Gravitational potential is given as,

$$V = \frac{-GM}{R}$$

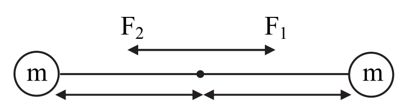
$\therefore$ $V = -GM\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \infty\right]$

$$= -G \times 2\left[1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \infty\right]$$

$$= -2G\frac{1}{\left(1 - \frac{1}{2}\right)}$$

$\therefore$ $V = -4G$

106.



We know gravitational force is always attractive in nature

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$

The two equal fields in opposite directions give a net field at the centre as zero.

Gravitational potential at the midpoint,

$\therefore$ $F_{net} = 0$

$V_{net} = V_{n_1} + V_{n_2}$

$$V_{net} = \left(\frac{-Gm}{r}\right) + \left(\frac{-Gm}{r}\right)$$

$$V_{net} = \frac{-2Gm}{r}$$

$\therefore$ At midpoint of the line joining the centre of sphere, gravitational field is zero and gravitational potential is $\frac{-2Gm}{r}$.

107. $V = -\dfrac{GM}{(R+h)}$ and $g = \dfrac{GM}{(R+h)^2}$

Taking ratio of both,

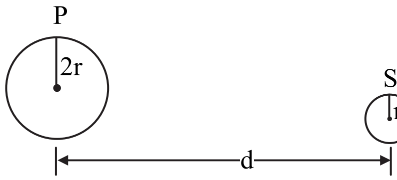
$\dfrac{|V|}{g} = R + h$

$\therefore \quad \dfrac{5.4\times10^7}{6.0} = R + h \qquad \therefore \quad 9 \times 10^6 = R + h$

$\therefore \quad h = (9 - 6.4) \times 10^6 = 2.6 \times 10^6 = 2600$ km

108. *Refer Mind bender 5.*

109.



Here, work has to be done to displace a body from distance $\left(\dfrac{d}{2}\right)$ to ∞. Let mass of the body be m and mass of planet and satellite be M_P and M_S respectively.

$\therefore$ Total work $W = W_P + W_S$

$= -GM_Pm\left[\dfrac{1}{\infty} - \dfrac{1}{(d/2)}\right] - GM_Sm\left[\dfrac{1}{\infty} - \dfrac{1}{(d/2)}\right]$

$= \dfrac{GM_Pm}{d/2} + \dfrac{GM_Sm}{d/2} = \dfrac{2Gm}{d}(M_P + M_S)$

Escape velocity should be such that it can perform work W.

i.e., $\dfrac{1}{2}mv_e^2 = \dfrac{2Gm}{d}(M_P + M_S)$

But, $M_P = \dfrac{4}{3}\pi(2r)^3\rho$ and $M_S = \dfrac{4}{3}\pi r^3(2\rho)$

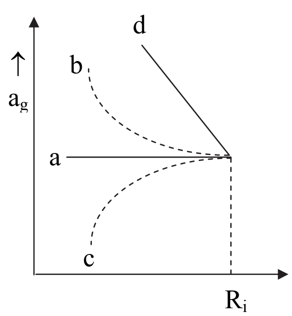
$\therefore \quad v_e^2 = \dfrac{4G}{d}\left[\dfrac{4}{3}\pi(2r)^3\rho + \dfrac{4}{3}\pi r^3(2\rho)\right]$

$= \dfrac{4G}{d} \times 10 \times \dfrac{4}{3}\pi r^3\rho$

$\therefore \quad v_e = 4\sqrt{\dfrac{10G\pi r^3\rho}{3d}}$

Evaluation Test

1.



As the star collapses,

its mass remains the same and radius decreases.

$a_g = \dfrac{GM}{R^2} \propto \dfrac{1}{R_i^2}$

a_g increases as radius decreases. Hence, option (B).

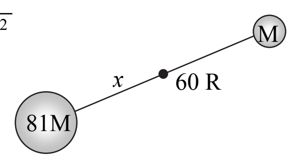
2. $a_1 = \dfrac{F_g}{m_1}$

$a_2 = \dfrac{F_g}{m_2}$

Since there is no external force, centre of mass remains at rest and energy remains same.

3. At point P,

$\dfrac{G(81M)}{x^2} = \dfrac{G(M)}{(60R - x)^2}$



$\therefore \quad (60R - x)^2 = \dfrac{x^2}{81}$

$\therefore \quad 60R - x = \dfrac{x}{9}$

$\therefore \quad x = 54R$ and $(60R - x) = 6$ R

4. $F_g = \dfrac{Gm_1m_2}{r^2}$

and $M = m_1 + m_2$

($\because$ m_1 and m_2 are made from M)

$\therefore \quad F_g = \dfrac{G(m_1)(M - m_1)}{r^2}$

….[Using product rule of derivation]

$\therefore \quad \dfrac{dF}{dm_1} = \dfrac{G}{r^2}[m_1(-1) + (M - m_1)(1)]$

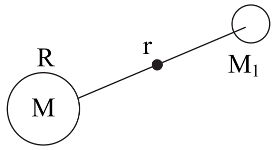
For F to be maximum, $\dfrac{dF}{dm} = 0$

$\therefore \quad -m_1 + (M - m_1) = 0$

$\therefore \quad M = 2m_1 \qquad \therefore \quad m_1 = \dfrac{M}{2}$

$\therefore \quad m_2 = M - m_1 = \dfrac{M}{2} \qquad \therefore \quad m_1 = m_2$

5. $F_g = \frac{GMm_1}{r^{n-1}} = \frac{m_1 v^2}{r}$



$\therefore$ $v = \sqrt{\frac{GM}{r^{n-1}}}$

$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r^{n-1}}}} = \frac{2\pi r}{\sqrt{GM}}\sqrt{r^{n-1}} \propto r^{(n+1)/2}$

6. We know that gravitational field inside a shell is zero.

$\therefore$ a_g at P = 0

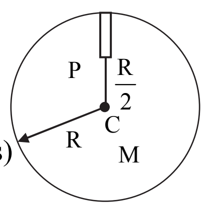
$\therefore$ $(a_g)_{\text{due to } I_1} + (a_g)_{\text{due to } I_2} = 0$

$\therefore$ $I_1 - I_2 = 0$

$\therefore$ $I_1 = I_2$

7. Here, they are talking about the escape velocity of parcel. But, now the launching is done from beneath the surface

$-\frac{GM(m)}{\left(\frac{R}{2}\right)} + \frac{1}{2}mv_e^2 = 0$



$\therefore$ $v_e = \sqrt{\frac{4GM}{R}} = \sqrt{2}$ (11.2 km/s)

= 15.84 km/s.

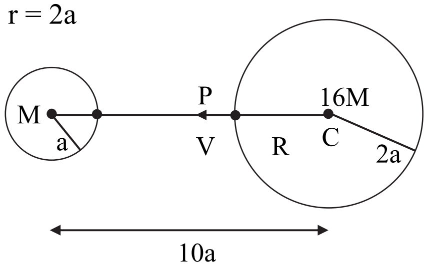
8. Let P be on the line joining the centres of the two stars and r be distance of P from the centre of smaller star.

$\frac{GM}{r^2} - \frac{G(16M)}{(10a - r)^2} = 0$

$\therefore$ $(10a - r)^2 = 16r^2$

$\therefore$ $10a - r = 4r$

$\therefore$ $r = 2a$



Now, if the particle projected from the larger planet has enough velocity (energy) to cross this point, it will reach the smaller planet. For this, the K.E. imparted to the body must be just enough to raise its total mechanichal energy to a value which is equal to P.E. at P, i.e.,

$\frac{1}{2}mv^2 - \frac{G(16M)m}{2a} - \frac{G(M)m}{8a}$

$= \frac{-G(M)m}{2a} - \frac{G(16M)m}{8a}$

$\therefore$ $\frac{v^2}{2} - \frac{65GM}{8a} = \frac{-5}{2}\frac{GM}{a}$

$\therefore$ $v_{min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$

9. $T^2 = \frac{(2\pi)^2}{GM}R^3$

$\therefore$ $\log_{10}T = \frac{3}{2}\log_{10}R + \log_{10}\left(\frac{4\pi^2}{GM}\right)$

$\therefore$ $\log_{10} R = \frac{2}{3}\log_{10}T - \frac{1}{3}\log_{10}\left(\frac{4\pi^2}{GM}\right)$

$\therefore$ $\frac{4\pi^2}{GM} = 10^{-18} \Rightarrow M = 6 \times 10^{29}$ kg

10. $g' = g\left(1 - \frac{d}{R}\right)$ (gravity at a depth d) $\qquad g' = g\left(1 - \frac{\omega^2 R}{g}\right)$, (gravity at the equator)

$\therefore$ $\frac{gd}{R} = \frac{g\omega^2 R}{g}$ $\qquad \therefore$ $d = \frac{\omega^2 R^2}{g}$

11. If G starts to decrease, the force between sun and earth will also start to decrease. Earth will try to follow a path of larger radius. Hence, its period of revolution round the sun will increase. But rotation of earth around its own axis will remain unchanged. The radius of the circular path of the earth will increase or the earth will follow a path of increasing radius. Thus, P.E. will increase so K.E. decreases.

12. $E = -\frac{GMm}{2r}$

$\therefore$ $-\frac{dE}{dt} = \frac{GMm}{2}\frac{1}{r^2}\frac{dr}{dt}$

$\int_0^t dt = \frac{GMm}{2C}\int_r^R \frac{dr}{r^2}$ ....$\left[\because \frac{dE}{dt} = C \text{ J/s}\right]$

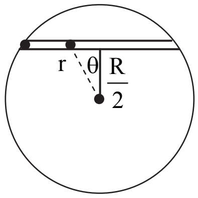
$\therefore$ $t = \frac{GMm}{2C}\left(\frac{1}{R} - \frac{1}{r}\right)$

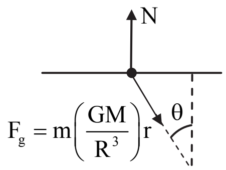
13. $g' = g\left(1 - \frac{2h}{R}\right)$

$w_2 - w_1$ = error in weighing

$= 2mg\left(\frac{h_1}{R} - \frac{h_2}{R}\right) = 2m\frac{GM}{R^2}\frac{h}{R}$

$\therefore$ $w_2 - w_1 = \frac{2mG}{R^2} \times \frac{4}{3}\pi R^3 \rho \times \frac{h}{R} = \frac{8}{3}\pi G m \rho h$

14. 



Pressing force = N

$$= \left(\frac{GMm}{R^3}\right) r\cos\theta$$

$$= \frac{GMm}{R^3} r \times \left(\frac{R/2}{r}\right)$$

$$= \frac{GMm}{2R^2} = \text{constant.}$$

15. $F = -\frac{k}{r^2} \Rightarrow E = -\frac{k}{r}$

Energy conservation implies,

$K_1 + U_1 = K_2 + U_2$

$\frac{1}{2}mv_1^2 - \frac{k}{a} = \frac{1}{2}mv_2^2 - \frac{k}{b}$ where $v_1 = \sqrt{\frac{k}{2ma}}$

and, $mv_1a = mv_2b$

∴ $v_2 = \frac{a}{b}v_1 = \frac{a}{b}\sqrt{\frac{k}{2ma}}$

∴ $\frac{1}{2}m\left(\frac{k}{2ma}\right) - \frac{k}{a} = \frac{1}{2}m\left(\frac{a}{b}\right)^2\left(\frac{k}{2ma}\right) - \frac{k}{b}$

∴ $\frac{a}{b} = 3$ or $\frac{a}{b} = 1$

16. During total eclipse, total attraction due to sun and Moon,

$F_1 = \frac{GM_sM_e}{r_1^2} - \frac{GM_mM_e}{r_2^2}$

When moon goes on opposite side, effective force of attraction is

$F_2 = \frac{GM_sM_e}{r_1^2} - \frac{GM_mM_e}{r_2^2}$

∴ $\Delta F = F_1 - F_2 = \frac{2GM_mM_e}{r_2^2}$

∴ $\Delta a = \frac{2GM_m}{r_2^2}$

Average force on earth,

$F_{av} = \frac{F_1 + F_2}{2} = \frac{GM_sM_e}{r_1^2}$

$a_{av} = \frac{GM_s}{r_1^2}$

∴ Percentage change in acceleration is

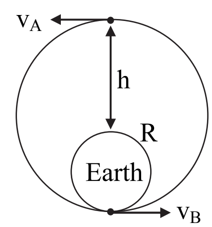
$$\frac{\Delta a}{a_{avg}} \times 100 = \frac{2GM_m}{r_2^2} \times \frac{r_1^2}{GM_s} \times 100$$

$$= 2\left(\frac{r_1}{r_2}\right)^2 \frac{M_m}{M_s} \times 100$$

17. Change in energy $= \frac{GMm}{2R} = \frac{1}{2}mv^2$

∴ Escape velocity is independent of the angle of projection as gravitational field is a conservative one.

18. Suppose the velocity of projection at A is v_A and at B is v_B.



∴ $\frac{mv_A^2}{\rho_A} = \frac{GM_em}{(R+h)^2}$ and

$\frac{mv_B^2}{\rho_B} = \frac{GM_em}{R^2}$

$\rho_A = \rho_B = \rho$ are the radii of curvatures at A, B.

Energy conservation gives,

$$\frac{-GM_em}{R+h} + \frac{1}{2}mv_A^2 = -\frac{GM_em}{R} + \frac{1}{2}mv_B^2$$

$$GM_em\left(\frac{1}{R} - \frac{1}{(R+h)}\right) = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

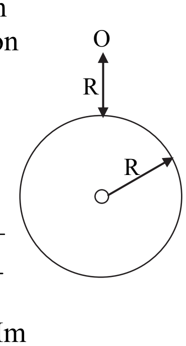
$$= \frac{1}{2}\rho GM_em\left(\frac{1}{R^2} - \frac{1}{(R+h)^2}\right)$$

∴ $\rho = \frac{2Rr}{R+r}$ ….[∵ r = R + h]

∴ $v_A^2 = \frac{\rho GM_e}{(R+h)^2} = 2GM_e\frac{R}{r(R+r)}$

19. Let v_{app} = velocity of approach

v_{sep} = velocity of separation



$e = \frac{v_{sep}}{v_{app}} = \sqrt{\frac{2}{3}}$

$\frac{GMm}{2R} = \frac{1}{2}mv_{app}^2$

∴ $v_{app} = \sqrt{\frac{GM}{R}} \Rightarrow v_{sep} = \sqrt{\frac{2GM}{3R}}$

Also, $\frac{-GMm}{R+h} = \frac{1}{2}mv_{sep}^2 - \frac{GMm}{R}$

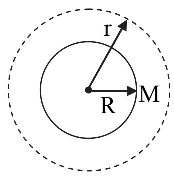
∴ $\frac{1}{2}v_{sep}^2 = GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$

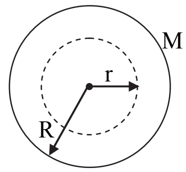
∴ $\frac{GM}{3R} = \frac{GM}{R}\left(1 - \frac{R}{R+h}\right)$

$\therefore \quad \frac{1}{3} = 1 - \frac{R}{R+h}$

$\therefore \quad \frac{R}{R+h} = \frac{2}{3}$

$\therefore \quad h = \frac{R}{2}$

20. 



For point P:

$E(4\pi r^2) = M(4\pi G)$

$\overline{E} = \frac{GM}{r^2}$

$\frac{g}{4} = \frac{GM}{r^2}$

$\therefore \quad \frac{1}{4R^2} = \frac{1}{r^2}$

For point Q:

$\overline{E} = \left(\frac{M}{\frac{4}{3}\pi R^3}\right) r \frac{(4\pi G)}{3}$

$= \left(\frac{GM}{R^3}\right) r$

$\therefore \frac{1}{4}\left(\frac{GM}{R^2}\right) = \left(\frac{GM}{R^3}\right) r$

$\therefore r = \frac{R}{4}$

$\therefore \quad r = 2R$

$\therefore \quad$ Separation $= 2R - \frac{R}{4}$ and $2R + \frac{R}{4}$

$= \frac{7R}{4}$ and $\frac{9R}{4}$

$\therefore \quad$ Maximum separation $= \frac{9R}{4}$

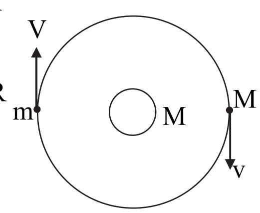
22. $\frac{1}{2}mv_1^2 = \frac{GMm}{(R+h_1)}$ $\qquad \frac{1}{2}mv_2^2 = \frac{GMm}{R+h_2}$

$\frac{1}{2}m.\frac{5}{7}\frac{GM}{R^2}R = \frac{GMm}{R+h_1}$ $\qquad \frac{1}{2}m.\frac{3}{5}\frac{GM}{R^2}R = \frac{GMm}{R+h_2}$

$\therefore \quad h_1 = \frac{2}{5}R$ $\qquad h_2 = \frac{2}{3}R$

$\therefore \quad h_1 : h_2 = 3 : 5$

23. $\frac{GMm}{R^2} + \frac{Gm}{4R^2} = m\left(\frac{2\pi}{T}\right)^2 R$



$\frac{Gm}{R^2}\left[M + \frac{m}{4}\right] = m\left(\frac{2\pi}{T}\right)^2 R$

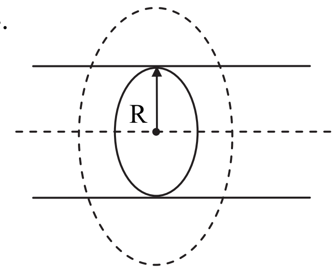
$\therefore \quad M + \frac{m}{4} = \frac{4\pi^2}{T^2}\frac{R^3}{G}$

$\therefore \quad M + \frac{m}{4} = \frac{4\times10}{30\times10^{14}} \times \frac{8\times10^{33}}{\frac{20}{3}\times10^{-11}}$

$\therefore \quad 10\times10^{30} + \frac{m}{4} = \frac{200}{15}\times10^{30}$

$\therefore \quad \frac{m}{4} = \frac{10}{3}\times10^{30}$

$\therefore \quad m = \frac{40}{3}\times10^{30}$ kg

24. 

$m\left(\frac{2G\rho}{r}\right)(\pi R^2) = \frac{mv^2}{r}$

where, λ = mass per unit length of the planet

$\therefore \quad v = \sqrt{2(G\rho)(\pi R^2)} = R\sqrt{2\pi G\rho}$

(**Note:** The orbital velocity is independent of the radial distance)

25. $T_s = \left(\frac{4\pi^2 r^3}{GM_{earth}}\right)^{\frac{1}{2}} = 6831$ s and $T_e = 86400$ s

Relative angular velocity $= \omega_{satellite} - \omega_{earth}$

$T = \frac{2\pi}{\omega_s - \omega_e} = \frac{2\pi}{\left[\frac{2\pi}{T_s} - \frac{2\pi}{T_e}\right]}$

$T = \frac{T_s T_e}{T_e - T_s} = 7417$ s

Textbook Chapter No.

03 Rotational Motion

Hints

Classical Thinking

2. Location of centre of mass does not depend upon choice of reference frame.

6. Moment of Inertia of a given body is $I = MR^2$
Thus, M.I. of a body depends on position of the axis of rotation and hence is not constant.

7. As axis of rotation changes, distribution of mass about the axis of rotation is changed.
$I = MR^2 \Rightarrow$ 'I' will change.

9. M.I. depends on the distribution of mass about the axis of rotation. Also, M.I. is proportional to the mass.

18. $K.E_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{2\pi}{T}\right)^2 = \frac{2I\pi^2}{T^2}$

$\Rightarrow K.E._{rot} \propto T^{-2}$

19. $E = \frac{1}{2}I\omega^2$

$\therefore \quad \omega = \sqrt{\frac{2E}{I}} = \sqrt{\frac{2\times 9}{2}} = 3$ rad/s

20. $E = \frac{1}{2}\ I\omega^2$

$\therefore \quad I = \frac{2E}{\omega^2} = \frac{2\times 360}{(30)^2} = \frac{2\times 360}{900} = 0.8\ \text{kg m}^2$

21. $MK^2 = I$

$\therefore \quad MK^2 = MR^2 \Rightarrow K^2 = R^2$
i.e. K is independent of M.

24. $I_{disc} = \frac{5MR^2}{4} = MK^2 \Rightarrow K^2 = \frac{5R^2}{4} \Rightarrow K = \frac{\sqrt{5}R}{2}$

28. $\tau = I\alpha = \text{kg m}^2\ \text{s}^{-2} = [M^1L^2T^{-2}]$

31. $\tau = I\alpha = MK^2\alpha$

32. $\tau = I\alpha = 2.5 \times 18 = 45$ Nm

33. $\tau = I\alpha$

$\therefore \quad \alpha = \frac{\tau}{I} = \frac{500}{100} = 5$

$\therefore \quad \alpha = \frac{\omega}{t} \Rightarrow \omega = \alpha.t = 5 \times 2 = 10$ rad/s

34. $I = \frac{\tau}{\alpha} = \frac{2000}{20} = 100\ \text{kg m}^2$

35. $P = \tau.\omega$

$\therefore \quad \tau = \frac{P}{\omega} = \frac{50\ \text{W}}{120\ \text{rad/s}} \approx 0.42$ Nm

36. $P = \tau\omega = 60 \times 2\pi \times 25 = 3000\ \pi$ W

39. $E_{total} = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$

$= \frac{1}{2} \times 10 \times 25 \times 10^{-4} \times \left(1+\frac{2}{5}\right)$

$= 0.0175\ \text{J} = 175 \times 10^{-4}$ J

40. For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

$\therefore \quad v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}} = \sqrt{\frac{2gh}{1+\frac{2}{5}}}$

$\therefore \quad v = \sqrt{\frac{10gh}{7}}$

$= \sqrt{\frac{10\times 9.8\times 0.6}{7}}$

$= \sqrt{8.4} \approx 2.9$ m/s

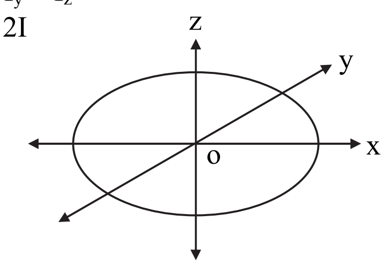
41. For a ring,

$a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)} = \frac{g\sin\theta}{1+1}$

$\therefore \quad a = \frac{g\sin\theta}{2} = \frac{g\sin 30°}{2} = \frac{g}{4}$

45. $I_x = I_y = I$
According to principle of perpendicular axes,
$I_x + I_y = I_z$

$\therefore \quad I_z = 2I$



46. $I_C = \frac{MR^2}{2}$

∴ $I_0 = \frac{MR^2}{2} + M\left(\frac{R}{2}\right)^2$

$= \frac{MR^2}{2} + \frac{MR^2}{4} = \frac{3MR^2}{4}$

47. For a solid cylinder, M. I. about axis $= \frac{MR^2}{2}$

∴ According to the theorem of parallel axes,

M. I. about line of contact $= \frac{MR^2}{2} + MR^2$

$= \frac{3}{2}MR^2$

49. M.I. of a rod about an axis passing through its edge and perpendicular to the rod $= \frac{ML^2}{3}$

∴ $I_x = \frac{ML^2}{3} + \frac{ML^2}{3} = \frac{2\times1\times(\sqrt{3})^2}{3} = 2 \text{ kg m}^2$

50. $\frac{2}{5}MR_s^2 = \frac{2}{3}MR_h^2$

∴ $\frac{R_s}{R_h} = \frac{\sqrt{5}}{\sqrt{3}}$

54. Unit of angular momentum, L = kg m^2 / s

$= \frac{\text{kg m}^2}{\text{s}} \frac{\text{s}}{\text{s}}$

$= \frac{\text{kg m}^2}{\text{s}^2} \text{s}$

= J-s

55. $\tau = \frac{dL}{dt} = \frac{4L-0}{4} = L$

56. Angular momentum $L = I\omega = \frac{Ml^2}{3}.\omega$

57. Consider two perpendicular diameters, one along the X-axis and the other along the Y-axis. Then, $I_x = I_y = \frac{1}{4}MR^2$

According to the perpendicular axes theorem, the moment of inertia of the disc about an axis passing through the centre is,

$I_c = I_x + I_y = \frac{1}{4}MR^2 + \frac{1}{4}MR^2 = \frac{1}{2}MR^2$

58. Additional rotational K.E. = 800 J

∴ $\frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2 = 800$

As $\omega_0 = 0 \Rightarrow \frac{1}{2}I\omega^2 = 800$

∴ $\omega = \sqrt{\frac{1600}{I}} = \sqrt{\frac{1600}{3.6}} \approx 21 \text{ rads}^{-1}$

From $\omega = \omega_0 + \alpha t$

∴ $21 = 0 + 15t, t = \frac{21}{15} = 1.4$ s

59. Acceleration of an object rolling down an inclined plane,

$a = \frac{g\sin\theta}{1+\left(\frac{K^2}{R^2}\right)}$

For a ring, $\frac{K^2}{R^2} = 1$

∴ $a_{ring} = \frac{g\sin\theta}{1+1} = 0.5g\sin\theta$

For a solid cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

∴ $a_{cyl.} = \frac{g\sin\theta}{\left(1+\frac{1}{2}\right)} \approx 0.67\ g\sin\theta$

For a solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

∴ $a_{sph} = \frac{g\sin\theta}{\left(1+\frac{2}{5}\right)} \approx 0.71g\sin\theta$

As acceleration of the solid sphere is maximum, hence the sphere will reach the ground with maximum velocity.

60. The disc rolls about the point of contact with the horizontal surface, therefore speed of centre of mass is v = r ω and that of topmost point is 2 r ω = 2 v.

Critical Thinking

1. $I \propto R^2$

∴ $\frac{dI}{I} = \frac{2RdR}{R^2} = \frac{2dR}{R}$

$= 2 \times 1\% = 2\%$

2. Earth is solid sphere, so M.I. = $\frac{2}{5}MR^2$

where M = $\frac{4}{3}\pi R^3\rho$

$\therefore$ M.I. = $\frac{2}{5}$ $(\frac{4}{3}\pi R^3\rho)R^2 = \frac{8}{15}\pi R^5\rho$

3. Moment of inertia of the system about the given axis $I = I_A + I_B + I_C$

As rod is thin,

$I_A = \Sigma m \times 0^2 = 0$

Rod B is rotating about one end

$\therefore$ $I_B = \frac{ML^2}{3}$

For rod C, all points are always at distance L from the axis of rotation, so

$I_C = \Sigma mL^2 = ML^2$

$\therefore$ $I = 0 + \frac{ML^2}{3} + ML^2 = \frac{4ML^2}{3}$

4. Hard boiled egg acts just like a rigid body while rotating. It is not in the case of a raw egg because of liquid matter present in it. In case of a raw egg, the liquid matter tries to go away from the centre, thereby increasing its moment of inertia i.e., $\frac{(I)_{raw\,egg}}{(I)_{boiled\,egg}} > 1$

As moment of inertia is more, raw egg will take more time to stop as compared to boiled egg (Law of Inertia).

5. $R^2 = \frac{I}{M} = \frac{0.25}{1}$

$\therefore$ R = 0.5 m $\Rightarrow$ d = 1 m

6. $E = \frac{1}{2}I\omega^2 = 1500$

$\frac{1}{2}I(\alpha t)^2 = 1500$

$\therefore$ $(1.2)(25)^2 t^2 = 3000$

$\therefore$ $t^2 = 4 \Rightarrow t = 2$ s

7. $E = \frac{1}{2}I\omega^2$

$\therefore$ $\frac{E_1}{E_2} = \frac{\frac{1}{2}I_1\omega_1^2}{\frac{1}{2}I_2\omega_2^2}$

$I_1 = I_2$ ….[Given]

$\therefore$ $\frac{E_1}{E_2} = \left(\frac{\omega_1}{\omega_2}\right)^2 = \left(\frac{\omega_1}{2\omega_1}\right)^2 = \frac{1}{4}$

$\therefore$ $E_2 = 4E_1$

8. For a uniform thin rod suspended from one end, $I = \frac{ml^2}{3}$, $\omega = 2\pi f$

$\therefore$ $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{ml^2}{3}\times(2\pi f)^2$

$= \frac{1}{2}\times\frac{ml^2}{3}\times 4\pi^2 f^2 = \frac{2}{3}\pi^2 f^2 ml^2$

9. $E = \frac{1}{2}I\omega^2$

$L = I\omega \Rightarrow L^2 = I^2\omega^2$

$\therefore$ $E = \frac{1}{2}\frac{L^2}{I}$

But $I = MR^2$

$\therefore$ $E = \frac{1}{2}\frac{L^2}{MR^2} = \frac{L^2}{2MR^2}$

10. n = 240 r.p.m. = $\frac{240}{60}$ = 4 r.p.s.

$\therefore$ $I = MR^2 = (10)\times(0.1)^2 = 0.1$ kg m^2

$\therefore$ $E = \frac{1}{2}I\omega^2 = \frac{1}{2}I(2\pi n)^2 = 2\pi^2 In^2$

$= 2\pi^2(0.1)\times 16 = 3.2\pi^2$ J

11. $I_1\omega_1 = I_2\omega_2 \Rightarrow MK_1^2\omega_1 = MK_2^2\omega_2$

$\therefore$ $\frac{K_1}{K_2} = \sqrt{\frac{\omega_2}{\omega_1}}$

12. Moment of inertia of solid sphere about its diameter, $I = \frac{2}{5}MR^2$

$\therefore$ $K = \sqrt{\frac{I}{M}} = \sqrt{\frac{2}{5}\frac{MR^2}{M}} = \sqrt{0.4}\,R$

13. M.I. of thin rod about axis passing through centre perpendicular to length is

Using, $I = MK^2 = \frac{ML^2}{12}$

$\therefore$ $K = \frac{L}{\sqrt{12}} = \frac{L}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$ m

14. $I = \sum_i m_i r_i^2 = 4\,Mb^2$ ….(i)

If K = radius of gyration of the system then,

$I = \left(\sum_i m_i\right)K^2 = 4\,MK^2$ ….(ii)

$\therefore$ Comparing equations (i) and (ii),

K = b

16. $\tau = I\alpha = I\dfrac{d\omega}{dt}$

where ω = constant

$\therefore \dfrac{d\omega}{dt} = 0 \Rightarrow \tau = 0$

17. $n_1 = 300$ r.p.m.

$= \dfrac{300}{60} = 5$ r.p.s;

$\omega = 2\pi(5) = 10\pi$ rad/s

$\tau = I\alpha = \left(\dfrac{2}{5}MR^2\right)\cdot\left(\dfrac{\omega-\omega_0}{t}\right)$

$= \dfrac{2}{5} \times 2000 \times 25 \times \left(\dfrac{2\pi-10\pi}{2}\right)$

$= -2 \times 10^4 \times 4 \times \pi = -2.5 \times 10^5$ dyne cm

Negative sign shows that it is a retarding torque.

18. $\tau = I\alpha$

$\therefore \tau' = \left(I + \dfrac{50I}{100}\right)\alpha = 1.5I\alpha = 1.5\tau$

19. $\alpha = \dfrac{\omega_f - \omega_i}{t}$

$\omega_i = 2\pi n = 2\pi \times 20 = 40\pi$ rad/s

$\therefore \alpha = \dfrac{0-40\pi}{10} = -4\pi$ rad/s^2 (retardation)

$\therefore \tau = I\alpha = 5 \times 10^{-3} \times (-4\pi)$

$= -2\pi \times 10^{-2}$ Nm

Negative sign shows that it is a retarding torque.

$\therefore |\tau| = 2\pi \times 10^{-2}$ Nm

20. $\omega_0 = \dfrac{2\pi\times 240}{60} = 8\pi = 25.12$ rad/s,

Using, $\tau = I\alpha$,

$\alpha = \dfrac{\tau}{I} = -\dfrac{0.81}{0.16} = -5.06$

$\therefore \omega = \omega_0 + \alpha t = 25.12 - (5.06 \times 2) = 15$ rad/s

21. n = 1800 rev/min = 30 rev/s

$\omega = 2\pi n = 60\pi$ rad/s

$\therefore \tau = \dfrac{P}{\omega} = \dfrac{100000}{60\pi} \approx 531$ Nm

22. $n_1 = 20$ r.p.m. $= \dfrac{20}{60} = \dfrac{1}{3}$ r.p.s.,

$n_2 = 60$ r.p.m. $= \dfrac{60}{60} = 1$ r.p.s.,

Work done by torque is the change in its rotational K.E.

$W = (K.E.)_f - (K.E.)_i$

$= \dfrac{1}{2}I\omega_f^2 - \dfrac{1}{2}I\omega_i^2 = \dfrac{1}{2}I\left(\omega_f^2 - \omega_i^2\right)$

$= \dfrac{1}{2}MK^2\left[(2\pi n_f)^2 - (2\pi n_i)^2\right]$

$= \dfrac{1}{2} \times 1 \times \dfrac{9}{\pi^2} \times 4\pi^2\left[(1)^2 - \left(\dfrac{1}{3}\right)^2\right]$

$= \dfrac{1}{2} \times 1 \times \dfrac{9}{\pi^2} \times 4\pi^2 \times \dfrac{8}{9}$

$\therefore W = 16$ J

23. Total K.E. of the loop $= \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}Mv^2$

$= \dfrac{1}{2}MR^2\omega^2 + \dfrac{1}{2}Mv^2$

$= Mv^2 = 8$ J

….(i) [$\because R^2\omega^2 = v^2$]

$\therefore$ Total K.E. of the disc $= \dfrac{1}{2}Mv^2 + \dfrac{1}{2}I\omega^2$

$= \dfrac{1}{2}Mv^2 + \dfrac{1}{2} \times \dfrac{1}{2}MR^2 \times \dfrac{v^2}{R^2}$

$= \dfrac{1}{2}Mv^2 + \dfrac{1}{4}Mv^2$

$= \dfrac{3}{4}Mv^2 = \dfrac{3}{4} \times 8 = 6$ J ….[From (i)]

24. In this case, $\dfrac{1}{2}mv^2\left(1+\dfrac{K^2}{R^2}\right) = mgh$

$\therefore \dfrac{1}{2}mv^2\left(1+\dfrac{K^2}{R^2}\right) = mg\,\dfrac{3v^2}{4g}$

$\therefore 1 + \dfrac{K^2}{R^2} = \dfrac{3}{2} \Rightarrow K^2 = \dfrac{R^2}{2}$

$\therefore MK^2 = \dfrac{MR^2}{2} \Rightarrow$ The body is a disc.

25. In the case of rolling, as K.E.,

$E = \dfrac{1}{2}Mv^2\left(1+\dfrac{I}{MR^2}\right)$ ….(i)

For ring, $I = MR^2$

$\therefore E_{ring} = M_{ring}v_{ring}^2$

$\therefore v_{ring} = \sqrt{\dfrac{E_{ring}}{0.3}}$ ….(ii)

∴ For cylinder, $I = \frac{1}{2}MR^2$

∴ $E_{cylinder} = \frac{3}{4}M_{cylinder}v^2_{cylinder}$ ….[from (i)]

∴ $v_{cylinder} = \sqrt{\frac{4E_{cylinder}}{3\times 0.4}}$

$= \sqrt{\frac{E_{cylinder}}{0.3}}$ ….(iii)

According to problem,

$E_{ring} = E_{cylinder}$

∴ $v_{ring} = v_{cylinder}$ ….[From (ii) and (iii)]

As the motion is uniform, both will reach the wall simultaneously.

26. $E_T = \left(1+\frac{K^2}{R^2}\right)\frac{1}{2}Mv^2$

$E_R = \left(\frac{K^2}{R^2}\right)\frac{1}{2}Mv^2$

∴ The fraction of total energy associated with rotation is $\frac{E_R}{E_T} = \frac{K^2/R^2}{1+K^2/R^2}$

For solid sphere, $K^2/R^2 = 2/5$

∴ $\frac{E_R}{E_{total}} = \frac{2}{7}$

27. For solid sphere, $I = \frac{2}{5}MR^2$

$E_T = \frac{1}{2}Mv^2$

∴ $E_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$

$= \frac{1}{5}MR^2\omega^2 = \frac{1}{5}Mv^2$

∴ $E = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$

28. $E_1 = \frac{1}{2}Mv^2$,

$E_2 = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\omega^2$

$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$

∴ $\frac{E_1}{E_2} = \frac{\frac{1}{2}Mv^2}{Mv^2} = \frac{1}{2}$

29. Total energy = K.E. of translation + K.E. of rotation

$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\times\frac{2}{5}MR^2\omega^2$

$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$

∴ $\frac{\text{K.E. of rotation}}{\text{Total energy}} = \frac{\left(\frac{1}{2}\right)I\omega^2}{\left(\frac{7}{10}\right)Mv^2} = \frac{\left(\frac{1}{5}\right)Mv^2}{\left(\frac{7}{10}\right)Mv^2} = \frac{2}{7}$

∴ Percentage of $(K.E.)_R = \frac{2}{7}\times 100\% = 28.57\%$

30. $E_T = \frac{1}{2}mv^2$ and

$E_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(MK^2)\frac{v^2}{R^2} = \frac{1}{2}Mv^2\frac{K^2}{R^2}$

∴ $R = \frac{E_T}{E_R} = \frac{\frac{1}{2}Mv^2}{\frac{1}{2}Mv^2\frac{K^2}{R^2}} = \frac{R^2}{K^2} = \frac{5}{2}$

31. For slipping or sliding without rolling,

$a = g\sin\theta$ and $v = \sqrt{2gh}$

For rolling without slipping,

∴ $a' = \frac{g\sin\theta}{(1+K^2/R^2)}$

∴ $v' = \sqrt{\frac{2gh}{(1+K^2/R^2)}}$

As $a' < a$ and $v' < v$, slipping cylinder reaches the bottom first with greater speed.

32. $a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)} = \frac{g\sin 30^\circ}{\left(1+\frac{2}{5}\right)}$

∴ $a = \frac{5g}{7}\times\left(\frac{1}{2}\right) = \frac{5g}{14}$

33. According to theorem of parallel axes, moment of inertia of a rod about one of its ends,

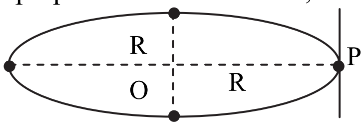
$I = \frac{ML^2}{12} + M\frac{L^2}{4} = \frac{ML^2}{3} = I_x = I_y$

∴ Moment of inertia of two rods about Z-axis

$= I_z = I_x + I_y$

= Moment of inertia of 2 rods placed along X and Y-axis $= \frac{2ML^2}{3}$

34. According to the theorem of parallel axes, M.I. of disc about an axis passing through P and perpendicular to the disc,



$I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

Total M.I. of the system,

$= \frac{3}{2}MR^2 + m(2R)^2 + m(\sqrt{2}R)^2 + m(\sqrt{2}R)^2$

$= (3M + 16m)\frac{R^2}{2}$

35. By the principle of parallel axes, $I_P = I_G + Mh^2$

$I_P = MK_P^2$, $I_G = MK_G^2$

$\therefore$ $MK_P^2 = MK_G^2 + Mh^2$

$\therefore$ $K_P^2 = K_G^2 + h^2$

$\therefore$ $100 = K_G^2 + 36$

$\therefore$ $K_G^2 = 64 \Rightarrow K_G = 8$ cm

36. $I_0 = \frac{1}{12}ML^2$

By applying theorem of parallel axes,

$I = I_0 + M\left(\frac{L}{2}\right)^2$

$= \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = 4 \times \left(\frac{1}{12}ML^2\right)$

$\therefore$ $I = 4I_0$

37. $I = \frac{2}{5}MR^2$

$\therefore$ According to the theorem of parallel axes,

$I' = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

$= \frac{7}{2}\left(\frac{2}{5}MR^2\right) = 3.5\ I$

38. M.I. at end of rod $= \frac{ML^2}{3} = 0.33ML^2$

M.I. at its centre $= \frac{ML^2}{12} = 0.083ML^2$

M.I. at a point midway between end and centre $= \frac{7ML^2}{48} = 0.145ML^2$

M.I. at a point $\frac{1}{8}$ length from centre

$= \frac{67ML^2}{768} = 0.087ML^2$

39. $I_A = \frac{MR^2}{2} = 0.5\ MR^2$

$2I_B = I_A$

$\therefore$ $I_B = \frac{I_A}{2} = 0.25\ MR^2$

$\therefore$ $I_C = I_A + MR^2 = \frac{MR^2}{2} + MR^2$

$= \frac{3}{2}MR^2 = 1.5\ MR^2$

$\therefore$ $I_D = I_B + MR^2 = 0.25\ MR^2 + MR^2 = 1.25\ MR^2$

$\therefore$ $I_B < I_A < I_D < I_C$

40. $I_A = \frac{ML^2}{12}$, $I_B = 0$

$\therefore$ $I_C = \frac{ML^2}{12} + M\left(\frac{L}{2} - \frac{L}{4}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{16}$

$\therefore$ $I_D = \frac{ML^2}{12} + M\left(\frac{L}{2} - \frac{L}{3}\right)^2$

$= \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2$

$= \frac{ML^2}{12} + \frac{ML^2}{36}$

41. $M = V\rho = \pi R^2 t\rho$

$\therefore$ $M_X = \pi R_X^2 t_X \rho$ and $M_Y = \pi R_Y^2 t_Y \rho$

Let $I = \frac{MR^2}{2}$

$\therefore$ $I_X = \frac{\pi R_x^4 t_x \rho}{2}$ and $I_Y = \frac{\pi R_y^4 t_y \rho}{2}$

$\therefore$ $\frac{I_Y}{I_X} = \frac{R_y^4 t_y}{R_x^4 t_x} = \frac{(4R)^4 (t/4)}{R^4 t} = \frac{(4)^4}{4} = 64$

$\therefore$ $I_Y = 64\ I_X$

42. The moment of inertia of ring about a tangent in its plane $= \frac{MR^2}{2} + MR^2 = \frac{3MR^2}{2}$

The moment of inertia of disc about its diameter $= \frac{MR^2}{4}$

$\therefore$ Ratio $= \frac{3MR^2/2}{MR^2/4} = \frac{6}{1}$

43. M.I. of ring (A) $\perp$ to plane $= MR^2$

M.I. of ring (B) passing through plane $= \frac{MR^2}{2}$

$\therefore$ M.I. of system $= \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

44. M.I. of thin rod, $I_1 = \frac{ML^2}{12}$ ....(i)

M.I. of ring, $I_2 = MR^2$ ....(ii)

The rod is bend to form a ring $\Rightarrow L = 2\pi R$

$\therefore$ Dividing equation (i) by (ii),

$$\frac{I_1}{I_2} = \frac{ML^2}{12} \times \frac{1}{MR^2}$$

$$= \frac{M(2\pi R)^2}{12} \times \frac{1}{MR^2} = \frac{4M\pi^2R^2}{12MR^2} = \frac{\pi^2}{3}$$

45. Let mass of the ring = mass of the disc = M

M.I. of the ring about the diameter = $\frac{MR_1^2}{2}$

M.I. of disc about the diameter = $\frac{MR_2^2}{4}$

Since M.I.s are equal,

$\therefore$ $\frac{MR_1^2}{2} = \frac{MR_2^2}{4}$

$\therefore$ $\frac{R_1^2}{R_2^2} = \frac{2}{4} \Rightarrow \frac{R_1}{R_2} = \frac{1}{\sqrt{2}}$

46. $I = \frac{2}{5}MR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2$

$$= \frac{8}{15} \times \frac{22}{7} \times R^5\rho = \frac{176}{105}R^5\rho$$

47. M.I. of sphere about the diameter = $\frac{2}{5}MR^2$

$\frac{2}{5}MR^2 = 20$ or $MR^2 = 50$

According to theorem of parallel axes,

M.I. about the tangent

$$= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2 = \frac{7}{5} \times 50 = 70 \text{ kg m}^2$$

48. $I_1 = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$

$\therefore$ $I_1 = \frac{1}{2}MR^2 + \frac{1}{12}M(4R^2)$

$$= \frac{1}{2}MR^2 + \frac{1}{3}MR^2 = \frac{5}{6}MR^2$$

$\therefore$ $I_2 = \frac{1}{2}MR^2 + \frac{1}{3}M(4R^2)$

$$= \frac{1}{2}MR^2 + \frac{4}{3}MR^2 = \frac{11}{6}MR^2$$

$\therefore$ $\frac{I_1}{I_2} = \frac{5}{11}$ and $I_2 > I_1$

$\therefore$ $I_2 - I_1 = \frac{11}{6}MR^2 - \frac{5}{6}MR^2 = MR^2$

50. $E = \frac{1}{2}I\omega^2 = \frac{(I\omega)^2}{2I} = \frac{L^2}{2I}$

51. According to conservation of angular momentum, $L' = L$

$\therefore$ $I'\omega' = I\omega$

$\therefore$ $\frac{I}{n}\omega' = I\omega \Rightarrow \omega' = n\omega$

52. $E = \frac{1}{2} \times L \times \omega$

$\therefore$ $225 = \frac{1}{2} \times L \times 25$

$\therefore$ $L = 9 \times 2 = 18$ J s

53. $R = 6400 \times 10^3 \text{m} = 6.4 \times 10^6$ m, $T = 24 \times 3600$ s

$$L = I\omega = \frac{2}{5}MR^2 \times \frac{2\pi}{T}$$

$$= \frac{2}{5} \times 6 \times 10^{24} \times (6.4 \times 10^6)^2 \times \left(\frac{2\pi}{24 \times 3600}\right)$$

$L = 7.145 \times 10^{33}$ kg m^2 s^{-1}

54. $\left(\frac{1}{2}MR^2 + M_bR^2\right)\omega = \frac{1}{2}MR^2\omega'$

(Since the boy reaches the centre, the final angular momentum of boy is zero).

$\therefore$ $\left(\frac{1}{2}M + M_b\right)\omega = \frac{1}{2}M\omega'$

$\therefore$ $(100 + 50)\omega = 100\omega'$

$\therefore$ $\omega' = \frac{3\omega}{2} = 15$ r.p.m.

55. According to principle of conservation of angular momentum,

$I_1\omega_1 = I_2\omega_2$

$\therefore$ $\frac{2}{5}MR^2 \times \frac{2\pi}{24} = \frac{2}{5}M\left(\frac{R}{n}\right)^2 \frac{2\pi}{T'}$

$\therefore$ $T' = \frac{24}{n^2}$ hours

56. $I = MR^2 = 1 \times (0.5)^2 = 0.25$ kg m^2

$\omega = 2\pi n = 2 \times \pi \times \frac{100}{\pi} = 200$ rad/s

$L = I\omega = 0.25 \times 200 = 50$ kg m^2/s

57. $L_i = \frac{1}{2}ma^2\omega$

$\therefore$ $L_f = \frac{1}{2}ma^2\omega' + ma^2\omega' = \frac{3}{2}ma^2\omega'$

As $L_i = L_f$,

$\frac{1}{2}ma^2\omega = \frac{3}{2}ma^2\omega'$

$\therefore \quad \omega = 3\omega' \quad$ or $\quad \omega' = \frac{\omega}{3}$

58. $E = \frac{1}{2}I\omega^2 = \frac{L^2}{2I} \Rightarrow E \propto L^2$

$\therefore \quad \frac{E_f}{E_i} = \left(\frac{L_f}{L_i}\right)^2 = \left(\frac{150}{100}\right)^2 = \frac{9}{4}$

$\therefore \quad \frac{E_f - E_i}{E_i} \times 100 = \left(\frac{E_f}{E_i} - 1\right) \times 100$

$= \left(\frac{9}{4} - 1\right) \times 100 = \frac{500}{4} = 125\%$

59. As kinetic energy is same,

$\frac{1}{2}I_R\omega_R{}^2 = \frac{1}{2}I_d\omega_d{}^2$

$\therefore \quad \frac{I_R\omega_R}{I_d\omega_d} = \frac{\omega_d}{\omega_R} \quad$ ….(i)

As same torque is applied,

$I_R\alpha_R = I_d\alpha_d$

$\frac{I_R\omega_R}{t_R} = \frac{I_d\omega_d}{t_d}$

$\therefore \quad \frac{I_R\omega_R}{I_d\omega_d} = \frac{t_R}{t_d} \quad$ ….(ii)

From equations (i) and (ii),

$\frac{\omega_d}{\omega_R} = \frac{t_R}{t_d}$

$\therefore \quad \omega_d t_d = \omega_R t_R \Rightarrow \theta_d = \theta_R = n$

60. $L = I\omega \Rightarrow L' = I'\omega$

$\therefore \quad \frac{L'}{L} = \frac{I'}{I} = \frac{M(R/2)^2}{MR^2} = \frac{1}{4} \Rightarrow L' = \frac{L}{4}$

61. Torque producing acceleration α_1,

$\tau = I_1\alpha_1 = 2mD^2\alpha_1$

Same torque produces α_2

$\therefore \quad \tau = I_2\alpha_2 = 2m(2D)^2\alpha_2$

$\therefore \quad 4(2mD^2)\alpha_2 = 2mD^2\alpha_1$

$\therefore \quad \alpha_2 = \frac{1}{4}\alpha_1$

62. As the body rolls the inclined plane, it loses potential energy. However, in rolling, it acquires both linear and angular speeds and hence gains the kinetic energy of translation and that of rotation. So, by conservation of mechanical energy,

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

But for rolling, $v = R\omega$

$\therefore \quad Mgh = \frac{1}{2}Mv^2\left[1 + \frac{I}{MR^2}\right]$

Let $1 + \frac{I}{MR^2} = \beta$

$\therefore \quad Mgh = \frac{1}{2}\beta Mv^2$

Hence $v = \sqrt{\frac{2gh}{\beta}}$

63. $v = \sqrt{\frac{2gh}{1 + K^2/R^2}}$, where $h = l \sin\theta$

For solid sphere, $v = \sqrt{\frac{10}{7}gh}$

$\therefore \quad v = \sqrt{\frac{10}{7} \times g \times l \sin\theta}$

$= \sqrt{\frac{10 \times 10 \times 3.5 \times \sin 30^\circ}{7}} = \sqrt{25}$

$\therefore \quad v = 5$ m/s

64. Initial moment of Inertia $I_1 = 1$ kg-m^2

Moment of Inertia of lump of wax = MR^2

$= 50 \times 10^{-3} \times (20 \times 10^{-2})^2$

$= 2 \times 10^{-3}$ kg m^2

Final moment of inertia,

$I_2 = 1 + 2 \times 10^{-3} = 1.002$ kg m^2

$\therefore \quad$ % Increase in M.I. $= \left(\frac{1.002 - 1}{1}\right) \times 100\ \%$

$= 0.002 \times 100\ \% = 0.2\ \%$

65. M.I. of disc of central zone,

$I_1 = \frac{4 \times (0.2)^2}{2} = 0.08$ kgm^2

M.I. of wooden annular disc,

$I_2 = \frac{3}{2}[(0.2)^2 + (0.5)^2] = \frac{3}{2}[0.04 + 0.25]$

$= 1.5 \times 0.29 = 0.435$ kg m^2

$\therefore \quad$ M.I. of whole disc $= I_1 + I_2 = 0.08 + 0.435$

$= 0.515$ kgm^2

66. Moment of inertia of complete disc about O is $I = \frac{1}{2}MR^2$. Mass of the cut - out part is $m = \left(\frac{M}{4}\right)$. The moment of inertia of the cut-out portion about its own centre,

$$I_0 = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{32}MR^2$$

because r = R/2. From the parallel axes theorem, the moment of inertia of the cut out portion about O is

$$I_c = I_0 + mr^2 = \frac{1}{32}MR^2 + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{3}{32}MR^2$$

∴ Moment of inertia of the shaded portion about O is

$$I_s = I - I_c = \frac{1}{2}MR^2 - \frac{3}{32}MR^2 = \frac{13}{32}MR^2$$

67. $E_1 = \frac{1}{2}I\omega^2$

In second case, $I' = 3I$

∴ According to conservation of angular momentum,

$I\omega = I'\omega'$

$$\omega' = \frac{I\omega}{I'} = \frac{I\omega}{3I} = \frac{\omega}{3}$$

Now, $E_2 = \frac{1}{2}I'\omega'^2$

$$= \frac{1}{2}\times 3I \times \frac{\omega^2}{9} = \frac{1}{3}\left(\frac{1}{2}I\omega^2\right) = \frac{1}{3}E$$

$$\therefore \quad \frac{E_1 - E_2}{E_1} = \frac{E - \frac{1}{3}E}{E} = \frac{2}{3}$$

68. $L_1 = I_1\omega_1$, $L_2 = I_2\,\omega_2$

Let $I_1 = MR^2$

$\omega_1 = 500$ r.p.m.

∴ $I_2 = MR^2 + MR^2 = 2MR^2$

From conservation of angular momentum,

$L_1 = L_2 \Rightarrow I_1\omega_1 = I_2\omega_2$

∴ $MR^2\,(500) = 2MR^2\,(\omega_2)$

∴ $\omega_2 = \frac{500}{2} = 250$ r.p.m.

69. By principle of conservation of angular momentum, $I\omega = I_1\omega_1$ …..(i)

Assuming earth to be a uniform solid sphere,

$I = \frac{2}{5}\,MR^2$

Then equation (i) becomes, $\frac{2}{5}MR^2$

$$\therefore \quad \omega = \frac{2}{5}M\left(\frac{R}{2}\right)^2\omega_1 \Rightarrow \frac{\omega}{\omega_1} = \frac{1}{4}$$

$$\therefore \quad \frac{T_1}{T} = \frac{1}{4} \quad \ldots\left[\because \omega = \frac{2\pi}{T}\right]$$

$$\therefore \quad T_1 = \frac{T}{4} = \frac{24}{4} = 6 \text{ hours}$$

70. The angular frequency of the composite system can be obtained by using the principle of conservation of angular momentum.

Total initial angular momentum of the two discs $I_1\omega_1 + I_2\omega_2$

Since the two discs are brought into contact face to face (one on top of the other) and their axes of rotation coincide, the moment of inertia I_c of the composite system will be equal to the sum of their individual moments of inertia, i.e. $I_c = I_1 + I_2$

If ω_c is the angular frequency of the composite system, the final angular momentum of the system is

$I_c\omega_c = (I_1 + I_2)\omega_c$

Since no external torque acts on the system,

Final angular momentum = Initial angular momentum

or $(I_1 + I_2)\omega_c = I_1\omega_1 + I_2\omega_2$

or $\omega_c = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

71. $\theta = \frac{3}{2}\times 2\pi$

∴ Work done $W = \tau\theta = Fr\theta$

$$= 200\times 3\times\left(\frac{3}{2}\times 2\pi\right)$$

$$= 5652 \text{ J}$$

72. From the law of conservation of energy, we have

Potential energy = Translational kinetic energy + Rotational kinetic energy

or $\quad mgH = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

or $\quad mgH = \frac{1}{2}mv^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = \frac{3}{4}mr^2\omega^2$

or $\quad \omega^2 = \frac{4gH}{3r^2}$

Now the rotational kinetic energy = $\frac{1}{2}I\omega^2$

∴ Substituting for ω^2 and I, we have,

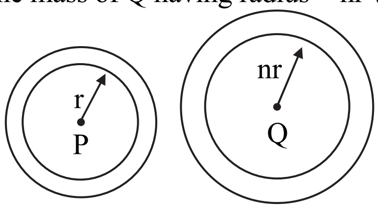
Rotational kinetic energy = $\frac{1}{2}\left(\frac{1}{2}mr^2\right)\frac{4gh}{3r^2}$

$$= \frac{mgH}{3}$$

Competitive Thinking

5. As the mass of disc is negligible, only the moment of inertia of five particles will be considered.
$I = \sum mr^2 = 5\ mr^2 = 5 \times 2 \times (0.1)^2 = 0.1\ kg\text{-}m^2$

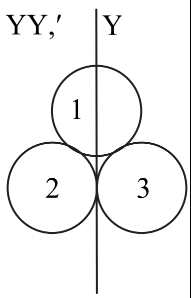
6. Let the mass of loop P having radius r be m
So the mass of Q having radius = nr will be nm



Moment of inertia of loop P, $I_p = mr^2$
Moment of inertia of loop Q, $I_Q = nm(nr)^2 = n^3mr^2$

∴ $\frac{I_Q}{I_P} = n^3 = 8 \Rightarrow n = 2$

7. Moment of inertia of system about YY,′



$I = I_1 + I_2 + I_3$

$= \frac{1}{2}MR^2 + \frac{3}{2}MR^2 + \frac{3}{2}MR^2$

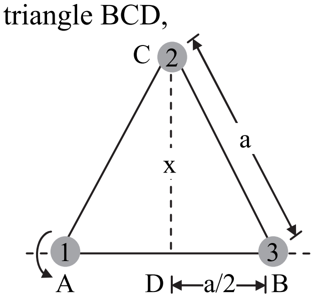
$= \frac{7}{2}MR^2$

8. M.I. of ring about diameter $I = \frac{MR^2}{2}$ ….(i)

∵ $L = \pi R \Rightarrow R = L/\pi$

∴ From equation (i), $I = \frac{ML^2}{2\pi^2}$

9. From triangle BCD,



$CD^2 = BC^2 - BD^2 = a^2 - \left(\frac{a}{2}\right)^2$

∴ $x^2 = \frac{3a^2}{4}$ ….(i)

Moment of inertia of system along the side AB,
$I_{system} = I_1 + I_2 + I_3$
$= m \times (0)^2 + m \times (x)^2 + m \times (0)^2$
$= mx^2 = \frac{3ma^2}{4}$ ….[From (i)]

10. Through bending, weight of opponent is made to act through the hip of the judo fighter to make its torque zero.

11. M.I. of thin Rod about one end, $I = \frac{ML^2}{3}$

Now, $L = 2\pi R \Rightarrow R = \frac{L}{2\pi}$

M.I. of ring about diameter,

$I_1 = \frac{MR^2}{2} = \frac{M\left(\frac{L^2}{4\pi^2}\right)}{2} = \frac{ML^2}{8\pi^2}$

∴ $\frac{I}{I_1} = \frac{ML^2}{3} \times \frac{8\pi^2}{ML^2} = \frac{8\pi^2}{3}$

13. $E = \frac{L^2}{2I} \Rightarrow E \propto \frac{1}{I}$ when L is constant

∴ As $I_1 > I_2 \Rightarrow E_1 < E_2$

15. $\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$

∴ $v^2 = \frac{I\omega^2}{m} = \frac{3 \times 2^2}{12} = 1 \Rightarrow v = 1\ m/s$

16. $K.E._{trans.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.4 \times 2^2 = 0.8\ J$

$K.E._{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right) \times \frac{v^2}{R^2}$

$= \frac{1}{4}Mv^2 = \frac{1}{4} \times 0.4 \times 2^2 = 0.4\ J$

∴ $K.E._{tot} = 0.8 + 0.4 = 1.2\ J$

17. $K.E._{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{ML^2}{12} \times \omega^2$

$= \frac{1}{2} \times A \times L \times D \times \frac{L^2}{12} \times \omega^2$

∴ $K.E._{rot} = \frac{1}{24}DAL^3\omega^2$

18. Total $(K.E)_{ring} = Mv^2$ ….(i)

Total $(K.E)_{disc} = \frac{3}{4}Mv^2$ ….(ii)

Divide equation (ii) by equation (i)

$$\frac{(K.E)_{disc}}{(K.E)_{ring}} = \frac{\frac{3}{4}Mv^2}{Mv^2}$$

$\therefore$ $(K.E)_{disc} = (K.E)_{ring} \times \frac{3}{4} = 4 \times \frac{3}{4} = 3J$

19. $I_{sphere} = I_s = \frac{2}{5}mR^2$

Let ω_s be angular speed of sphere,

$\therefore$ $E_{sphere} = \frac{1}{2}I_s\omega_s^2$

$$= \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega_s^2 \quad ....(i)$$

Similarly,

$I_{cylinder} = I_c = \frac{1}{2}mR^2$

Let ω_c be the angular speed of cylinder,
Then it is given
$\omega_c = 2\omega_s$

$\therefore$ $E_{cylinder} = \frac{1}{2}I_c\omega_c^2$

$$= \frac{1}{2}\left(\frac{1}{2}mR^2\right)(2\omega_s)^2 \quad ...(ii)$$

$\therefore$ $$\frac{E_{sphere}}{E_{cylinder}} = \frac{\frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega_s^2}{\frac{1}{2}\left(\frac{1}{2}mR^2\right)(4\omega_s^2)}$$

...[From (i) and (ii)]

$$= \frac{1}{5}$$

20. Initial K.E., $(K.E.)_i = \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2$

Final K.E., $(K.E.)_f = \frac{1}{2} \times (2I\omega^2)$

$$= I\left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

$\therefore$ Loss in K.E.= $(K.E.)_i - (K.E.)_f$

$$= \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2 - I\left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

$$= \frac{I}{4}(2\omega_1^2 + 2\omega_2^2 - \omega_1^2 - 2\omega_1\omega_2 - \omega_2^2)$$

$$= \frac{I}{4}(\omega_1 - \omega_2)^2$$

23. Radius of gyration of circular disc $k_{disc} = \frac{R}{\sqrt{2}}$

Radius of gyration of circular ring $k_{ring} = R$

$\therefore$ Ratio $= \frac{k_{disc}}{k_{ring}} = \frac{1}{\sqrt{2}}$.

24. M.I. of rod about an axis passing through centre,

$$I_C = \frac{ML^2}{12} = MK_1^2 \quad ...(i)$$

M.I. of rod about an axis passing through one end,

$$I_E = \frac{ML^2}{3} = MK_2^2 \quad ...(ii)$$

Divide equation (i) by equation (ii)

$$\frac{MK_1^2}{MK_2^2} = \frac{ML^2}{12} \times \frac{3}{ML^2}$$

$\therefore$ $\frac{K_1^2}{K_2^2} = \frac{1}{4}$ $\quad\therefore$ $\frac{K_1}{K_2} = \frac{1}{2}$

25. For disc, $K = \frac{R}{\sqrt{2}}$

…. [$\because$ axis passes through centre of disc and perpendicular to its plane]

$$= \frac{5}{\sqrt{2}} \approx 3.54 \text{ cm}$$

26. $I = MK^2 = 2 \times (50 \times 10^{-2})^2$
$= 2 \times 2500 \times 10^{-4}$
$= 50 \times 10^{-2}$ kg m^2
$= 0.5$ kgm^2

28. Power $= \vec{\tau}.\vec{\omega} = (\vec{r} \times \vec{F}).\vec{\omega}$

29. $\tau = I\,\alpha \Rightarrow I = \frac{\tau}{\alpha} = \frac{2000}{2} = 1000$ kg-m^2

30. $n_1 = 300$ r.p.m. $= \frac{300}{60} = 5$ r.p.s.,

$n_2 = 600$ r.p.m. $= \frac{600}{60} = 10$ r.p.s.

$\therefore$ Work done = Change in K.E.$_{rot}$

$= \frac{1}{2}I(\omega_2^2 - \omega_1^2)$

$= \frac{1}{2} \times \frac{MR^2}{2} \times 4\pi^2(n_2^2 - n_1^2)$

$= MR^2\pi^2(n_2^2 - n_1^2)$

$= 2 \times (1)^2 \times (3.14)^2 \times (10^2 - 5^2)$

$= 2 \times (3.14)^2 \times 75$

≈ 1479 J

31. Work done = increase in kinetic energy

$W = \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2 = \frac{1}{2}\left(\omega_2^2 - \omega_1^2\right)$

$= 2\pi^2 I \left(v_2^2 - v_1^2\right)$

$\therefore \quad I = \frac{W}{2\pi^2\left(v_2^2 - v_1^2\right)}$

32. As $\omega = \omega_0 + \alpha\tau$,

$\therefore \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 4.6}{t} = -\frac{4.6}{t}$ rad s^{-2}

Negative sign is for retarding Torque

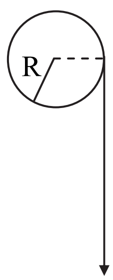
Using $\tau = I\alpha$,

$6.9 \times 10^2 = 3 \times 10^2 \times \frac{4.6}{t}$

….(Considering magnitude only)

$\therefore \quad t = \frac{3 \times 10^2 \times 4.6}{6.9 \times 10^2} = 2\,s$

33. $R = 20$ cm $= \frac{1}{5}$ m



Moment of inertia of flywheel about its axis,

$I = \frac{1}{2}MR^2$

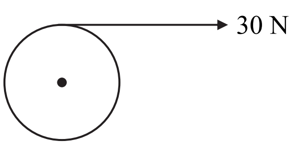
$= \frac{1}{2} \times 20 \times \left(\frac{1}{5}\right)^2 = 0.4$ kg m^2

F = 25 N

Using $\tau = I\alpha$,

$\alpha = \frac{\tau}{I} = \frac{FR}{I} = \frac{25 \times \frac{1}{5}}{0.4} = \frac{5\text{ Nm}}{0.4\text{ kgm}^2} = 12.5\ s^{-2}$

34.



$\tau = I\alpha$

$\therefore \quad \alpha = \frac{\tau}{I} = \frac{RF}{mR^2} = \frac{F}{mR} = \frac{30}{3 \times 0.4} = 25$ rad/s^2

35. Torque zero $\Rightarrow \alpha$ is zero

$\theta = 2t^3 - 6t^2$

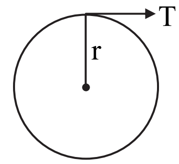
$\therefore \quad \frac{d\theta}{dt} = 6t^2 - 12t$

$\therefore \quad \frac{d^2\theta}{dt^2} = 0 \Rightarrow 12t - 12 = 0$

$\therefore \quad t = 1$ second

36. Using,

$Tr = I\,\alpha$,



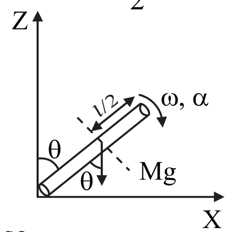
$T = \frac{I\alpha}{r} = \frac{mr^2}{2} \times \frac{\alpha}{r} = \frac{mr\alpha}{2}$

$= \frac{50 \times 0.5 \times 2 \times 2\pi}{2}$ N

$= 157$ N

37. Torque at angle θ

$\tau = Mg \sin\theta \frac{l}{2}$ ….(i)



Also,

$\tau = I\alpha$ ….(ii)

$\therefore \quad I\alpha = Mg \sin\theta\ \frac{l}{2}$ ….[from (i) and (ii)]

M.I. of rod here is,

$I = \frac{Ml^2}{3}$

$\therefore \quad \frac{Ml^2}{3}\alpha = Mg \sin\theta \frac{l}{2}$

$\therefore \quad \frac{l\alpha}{3} = \frac{g\sin\theta}{2} \qquad \therefore \quad \alpha = \frac{3g\sin\theta}{2l}$

39. Acceleration of a rolling body on an inclined plane is given by

$a = \frac{g\sin\theta}{1 + \frac{K^2}{r^2}}$

$\left(\frac{K^2}{r^2}\right)_{sphere} = \frac{2}{5}; \qquad \left(\frac{K^2}{r^2}\right)_{disc} = \frac{1}{2}$

$\therefore \quad a_{sphere} > a_{disc}$

$\therefore$ sphere will reach the bottom of the plane first.

41. For solid sphere:

$K_t = \frac{1}{2}Mv^2$

and $(K_t + K_r) = \frac{1}{2}Mv^2\left(1 + \frac{K^2}{R^2}\right)$

$= \frac{1}{2}Mv^2\left(1 + \frac{2}{5}\right)$

$\ldots\left[\left(\frac{K^2}{R^2}\right)_{\substack{solid\\sphere}} = \frac{2}{5}\right]$

$\therefore \quad \frac{K_t}{(K_t + K_r)} = \frac{\frac{1}{2}Mv^2}{\frac{1}{2}Mv^2\left(1+\frac{2}{5}\right)} = \frac{1}{7/5} = \frac{5}{7}$

42. The acceleration is given by,

$a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$

$\therefore \quad a = \frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)}$ ....($\because$ I = MK^2)

44. $E_T = \left(1+\frac{K^2}{R^2}\right)\frac{1}{2}Mv^2$

$E_R = \frac{K^2}{R^2}\ \frac{1}{2}Mv^2$

$\therefore$ The fraction of total energy associated with rotation is, $\frac{E_R}{E_{Total}} = \frac{K^2/R^2}{1+K^2/R^2}$

$\therefore$ For a ring, $\frac{K^2}{R^2} = 1$

$\therefore \quad \frac{E_R}{E_T} = \frac{1}{1+1} = \frac{1}{2}$

45. $a_{slipping} = g \sin\theta$

$a_{rolling} = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)} = \frac{g\sin\theta}{\left(1+\frac{2}{5}\right)} = \frac{5}{7}g\sin\theta$

$\therefore \quad \frac{a_{rolling}}{a_{slipping}} = \frac{5}{7}$

46. $K_{rolling} = K_f + U_r$

$K_{trans} + K_{rot} = 0 + Mgh$

$\therefore \quad \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mg \times \frac{3v^2}{4g}$

$\therefore \quad Mv^2 + I\frac{v^2}{R^2} = M.\frac{3}{2}v^2$

$\therefore \quad M + \frac{I}{R^2} = \frac{3}{2}.M \Rightarrow I = \frac{MR^2}{2}$

47. Hollow cylinder will take more time to reach the bottom because it possesses larger moment of inertia.

48. According to perpendicular axis theorem,

$I_z = I_x + I_y = 20 + 25 = 45$ kg m^2

51. M.I. of the circular disc will be

$2I = \frac{(2M)R^2}{2}$

$\therefore$ M.I. of the semicircular disc, $I = \frac{1}{2}MR^2$

53. M.I. of disc, $I = \frac{1}{2}MR_d^2$ ...(i)

M.I. of sphere, $I_{sphere} = \frac{2}{5}MR_S^2$ ...(ii)

$\because$ volume of disc = volume of sphere

$\therefore \quad \pi R_d^2\left(\frac{R_d}{6}\right) = \frac{4}{3}\pi R_S^3$

$\therefore \quad R_d^3 = 8R_S^3$

$\therefore \quad R_S = \frac{R_d}{2}$ ...(iii)

Substitute equation (iii) in equation (ii)

$\therefore \quad I_{sphere} = \frac{2}{5}M\left(\frac{R_d}{2}\right)^2 = \frac{2}{5}\times\frac{1}{4}MR_d^2$

$= \frac{1}{5}\left(\frac{1}{2}MR_d^2\right) = \frac{I}{5}$ ...from (i)

56. M.I. of the solid sphere about a diameter

$I = \frac{2}{5}MR^2$

M.I. of the disc about an axis through its edge and perpendicular to its plane is

$I = \frac{Mr^2}{2} + Mr^2$

$\therefore \quad \frac{2}{5}MR^2 = \frac{Mr^2}{2} + Mr^2 = \frac{3}{2}Mr^2$

$\therefore \quad r = \frac{2}{\sqrt{15}}R$

57. $I = \frac{ML^2}{12}$

Applying the theorem of parallel axes,

$\therefore \quad I_1 = I + M\times\left(\frac{L}{4}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$

58. $I_c = \frac{MR^2}{2} \Rightarrow$ M.I. of disc about any diameter,

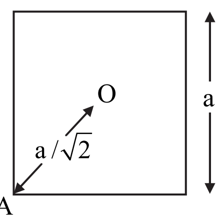
$I_d = \frac{1}{2}\frac{MR^2}{2} = \frac{MR^2}{4}$

$\therefore$ Applying theorem of parallel axes,

$I_t = I_d + MR^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$

59. $I_c = 4 \text{ kg m}^2 = MR^2$
Using theorem of perpendicular axes,
M.I. of ring about any diameter,
$I_d = \frac{I_c}{2} = \frac{4}{2} = 2 \text{ kg m}^2$
Applying theorem of parallel axes,
M.I. about tangent in its plane.
$I_t = I_d + MR^2 = 2 + 4 = 6 \text{ kg m}^2$

60. M.I. of the plate about an axis perpendicular to its plane and passing through its centre

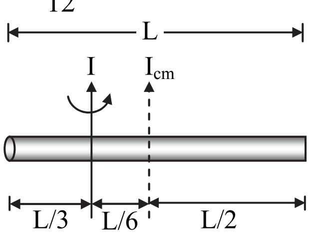


$I_0 = \frac{ma^2}{6}$
Applying parallel axes theorem,
$$I_A = I_0 + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2}{3}ma^2$$

61. Moment of inertia of a uniform circular disc about an axis through its centre and perpendicular to its plane is $I_C = \frac{1}{2}MR^2$
∴ Applying theorem of parallel axes, moment of inertia of a uniform circular disc about an axis touching the disc at its diameter and normal to the disc,
$$I = I_C + Mh^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

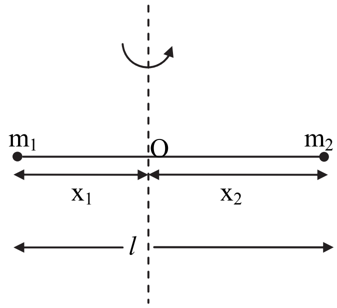
62. $I_1 = \frac{Ml^2}{12} + \frac{MR^2}{4}$ and $l = 2R$
$I_2 = \frac{Ml^2}{3} + \frac{MR^2}{4}$ and $l = 2R$
$$I_2 - I_1 = \frac{4MR^2}{3} + \frac{MR^2}{4} - \frac{MR^2}{3} - \frac{MR^2}{4}$$
$$= \frac{4MR^2}{3} - \frac{MR^2}{3} = \frac{MR^2}{3}(4-1)$$
∴ $I_2 - I_1 = MR^2$

63. $I_{cm} = \frac{ML^2}{12}$ (about middle point)



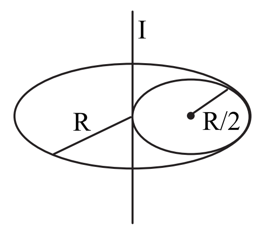
∴ Applying theorem of parallel axes,
$$I = I_{cm} + Mx^2 = \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9}$$

64.



Let O be the centre of mass of the system
∴ $x_1 = \frac{m_2 l}{m_1 + m_2}$ ...(considering m_1 as origin)
$x_2 = \frac{m_1 l}{m_1 + m_2}$ ...(considering m_2 as origin)
∴ M.I. of the system is given by,
$I = m_1x_1^2 + m_2x_2^2$
$$= m_1\left[\frac{m_2 l}{m_1 + m_2}\right]^2 + m_2\left[\frac{m_1 l}{m_1 + m_2}\right]^2$$
$$= \frac{m_1m_2^2l^2 + m_2m_1^2l^2}{(m_1 + m_2)^2} = \frac{m_1m_2(m_2 + m_1)l^2}{(m_1 + m_2)^2}$$
$$= \frac{m_1m_2l^2}{(m_1 + m_2)}$$

65.



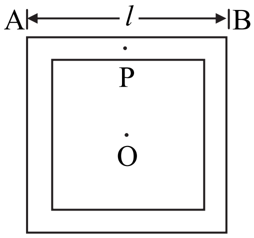
Moment of inertia of disc is given by
$I_{disc} = I_r + I_{hole}$ ….{I_r = M.I. of remaining part}
∴ $I_r = I_{disc} - I_{hole}$ ….(i)
$I_{disc} = \frac{MR^2}{2}$ ….(ii)
By parallel axes theorem we get,
$$I_{hole} = \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$$
….$\left\{\begin{array}{l} \because M_{hole} = \frac{M_{disc}}{4} \\ \because \text{the surface density is same} \end{array}\right\}$
∴ $I_{hole} = \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right]$ ….(iii)

Substituting eq (iii) and eq (ii) in eq (i) we get,

$$I_r = \frac{MR^2}{2} - \frac{MR^2}{32} - \frac{MR^2}{16}$$

$$= MR^2\left[\frac{1}{2} - \frac{1}{32} - \frac{1}{16}\right] = \frac{13}{32}MR^2$$

66. Moment of inertia of rod AB about point P and perpendicular to the plane = $\frac{Ml^2}{12}$



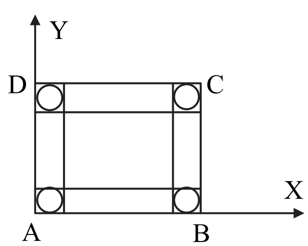
By applying parallel axes theorem,
M.I. of rod AB about point 'O'

$$= \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{3}$$

But the system consists of four rods of similar type. Hence by the symmetry,

$$I_{system} = 4\left(\frac{Ml^2}{3}\right)$$

67.



$I_{AB} = 0$ ….(i)

$I_{AD} = I_{BC} = \frac{ml^2}{3}$ ….(ii)

$I_{DC} = ml^2$ ….(iii)

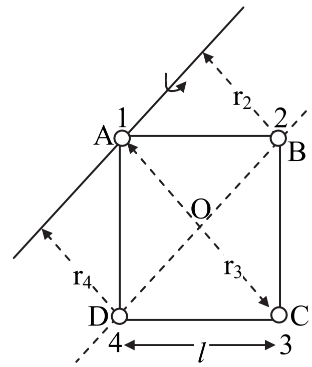
From equations (i), (ii) and (iii),

∴ Total moment of inertia

$$I = 0 + \frac{ml^2}{3} + \frac{ml^2}{3} + ml^2 = \frac{5}{3}ml^2$$

68. $r_2 = r_4 = OA = \frac{1}{\sqrt{2}}$ and $r_3 = l\sqrt{2}$

Moment of inertia of the system about given axis, $I = I_1 + I_2 + I_3 + I_4$

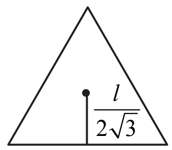


$\Rightarrow I = 0 + m(r_2)^2 + m(r_3)^2 + m(r_4)^2$

$\Rightarrow I = m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(l\sqrt{2}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$

∴ $I = 3ml^2$

69. Moment of inertia of a rod about an axis passing through centre and perpendicular to its length is = $\frac{ml^2}{12} = I_1$



Where l = length of the rod.
Using parallel axes theorem;
M.I about centroid = $(M.I)_{cm} + Mh^2$

Here $h = \frac{l}{2\sqrt{3}}$

∴ M.I about centroid = $\frac{ml^2}{12} + \frac{ml^2}{12}$

∴ M.I of each rod about centroid = $\frac{2ml^2}{12}$

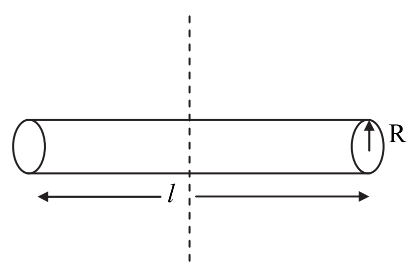
∴ M.I of system = $3 \times \frac{2ml^2}{12} = \frac{ml^2}{2} = I_2$

Given $I_2 = nI_1$

∴ $\frac{ml^2}{2} = n\left(\frac{ml^2}{12}\right)$

∴ $n = 6$

70.



$$I = \frac{mR^2}{4} + \frac{ml^2}{12}$$

$$I = \frac{m}{4}\left(R^2 + \frac{l^2}{3}\right)$$

$$= \frac{m}{4}\left(\frac{V}{\pi l} + \frac{l^2}{3}\right) \quad ....(\because V = \pi R^2 l)$$

Differentiating w.r.t. l on both sides,

$$\frac{dI}{dl} = \frac{m}{4}\left(\frac{-V}{\pi l^2} + \frac{2l}{3}\right)$$

But for moment of inertia to be minimum,

$$\frac{dI}{dl} = 0$$

$\therefore \quad \frac{V}{\pi l^2} = \frac{2l}{3} \qquad \therefore \quad V = \frac{2\pi l^3}{3}$

$\therefore \quad \pi R^2 l = \frac{2\pi l^3}{3} \qquad \therefore \quad \frac{l^2}{R^2} = \frac{3}{2}$

$\therefore \quad \frac{l}{R} = \sqrt{\frac{3}{2}}$

72. As no external torque acts on the body, its angular momentum will be conserved.

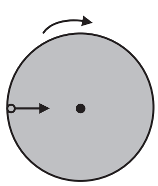
75. $L = I\omega$
$[L] = [I]\,[\omega] = [M^1L^2T^0]\,[M^0L^0T^{-1}]$
$= [M^1L^2T^{-1}]$

78. $L = I\omega = \frac{I.2\pi}{T} \Rightarrow L \propto \frac{1}{T}$

Hence, by doubling T, L becomes $\frac{1}{2}$ times.

79. Angular momentum acts always along the axis perpendicular to the plane of rotation.

80.



Here, the law of conservation of angular momentum is applied about vertical axis passing through centre. When insect is moving from circumference to centre, its moment of inertia will first decrease and then increase. Hence angular velocity will first increase and then decrease.

81. Angular momentum = linear momentum × Perpendicular distance of line of action of linear momentum from the axis of rotation = mv × l

82. We know,
$$K.E. = \frac{1}{2} I\omega^2$$
Here,
$(K.E.)_A = (K.E.)_B \quad$ ...(Given)

$\therefore \quad \frac{1}{2}I_A\omega_A{}^2 = \frac{1}{2}I_B\omega_B{}^2$

As $I_B > I_A$,
$\omega_B < \omega_A$

Also, $K.E. = \frac{1}{2}L\omega \quad ...(\because L = I\omega)$

$\therefore \quad \frac{1}{2}L_A\omega_A = \frac{1}{2}L_B\omega_B$

$\therefore$ as $\omega_B < \omega_A$
$L_B > L_A$

83. $\tau = \frac{dL}{dt} = \frac{4J - 1J}{4} = \frac{3J}{4}$

84. $L = I\omega = I \times 2\pi\,(n_2 - n_1)$
$= 0.06 \times 2\pi \times (5 - 0) = 0.6\,\pi$

85. We know that, $L = I\omega$

$\therefore \quad L_1 = I_1\omega_1$ and $L_2 = I_2\omega_2$

$\therefore \quad \frac{L_1}{L_2} = \frac{I_1\omega_1}{I_2\omega_2}$

$\Rightarrow \frac{L}{L} = \frac{2/5 M_1R^2\omega_1}{2/3 M_2R^2\omega_2}$

$(\because L_1 = L_2 = L$ and $R_1 = R_2 = R$ is given)

$\Rightarrow 1 = \frac{3}{5}\frac{M_1}{M_2}\frac{1}{2} \qquad \Rightarrow \frac{M_1}{M_2} = \frac{10}{3}$

86. $K.E. = \frac{1}{2}\frac{L^2}{I} \Rightarrow L^2 = 2 \times K.E. \times I$

$\therefore \quad L = 2 \times 4 \times 2 = 4$ kg m^2/s

87. $I_1\omega_1 = I_2\omega_2$
$I\omega = 2\,I\omega_2 \quad ....(\because I_2 = 2I)$

$\therefore \quad \omega_2 = \frac{\omega}{2}$

$\therefore \quad K.E._1 = \frac{1}{2}I\omega^2$

$K.E._2 = \frac{1}{2}I_2\omega_2^2$

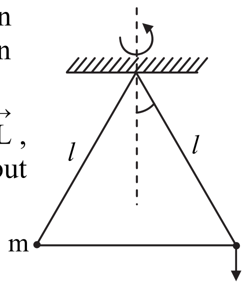
$= \frac{1}{2}(2I)\frac{\omega^2}{4} \quad ....\left(\because I_2 = 2I,\ \omega_2 = \frac{\omega}{2}\right)$

$= \frac{I\omega^2}{4}$

$\therefore \quad K.E._1 - K.E._2 = \frac{1}{2}I\omega^2\left[1 - \frac{1}{2}\right] = \frac{1}{2}I\omega^2 \times \frac{1}{2} = \frac{I\omega^2}{4}$

88. Initial angular momentum of ring, $L = I\omega = Mr^2\omega$
Final angular momentum of the system consisting of ring and four particles,
$L = (Mr^2 + 4mr^2)\omega'$
As there is no torque on the system, hence angular momentum remains constant.
$\therefore$ $Mr^2\omega = (Mr^2 + 4mr^2)\omega' \Rightarrow \omega' = \dfrac{M\omega}{M+4m}$

89. $\tau = mg \times l \sin\theta$. (Direction parallel to plane of rotation of particle)
as τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

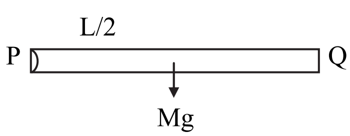


90. $I\omega = (I + I')\omega'$

$$\omega' = \left(\frac{I}{I+I'}\right)\omega = \left(\frac{I}{I+\frac{I}{8}}\right)\omega = \frac{8}{9}\omega$$

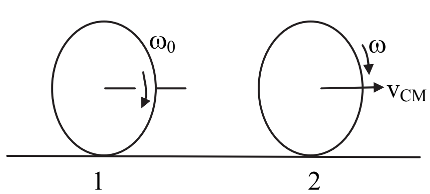
91. By conservation of angular momentum,
$$I_1\omega_1 = (I_1 + I_2)\omega_2 \Rightarrow \omega_2 = \left(\frac{I_1}{I_1+I_2}\right)\omega_1$$
$\therefore$ Loss in kinetic energy = $(K.E.)_i - (K.E.)_f$
$$= \frac{1}{2}I_1\omega_1^{\,2} - \frac{1}{2}(I_1 + I_2)(\omega_2^2) = \frac{1}{2}\left(\frac{I_1 I_2}{I_1+I_2}\right)\omega_1^2$$

92. For the rod PQ,
$$\frac{ML^2}{3}\alpha = T \times \frac{L}{2}$$



Now, T = Mg
$\therefore$ $\dfrac{ML^2}{3}\alpha = Mg \times \dfrac{L}{2}$
$$\alpha = \frac{3g}{2L}$$

93.



$L_1 = L_2$;
$L_1 = L_{CM} + mv_{CM}r = L_{CM} + mr^2\omega_0 = mr^2\omega_0$
[$\because$ $L_{CM} = 0$ initially]

$L_2 = L_{CM} + mv_{CM}r = mr^2\omega + mr^2\omega = 2mr^2\omega$
$2mr^2\omega = mr^2\omega_0$
$\therefore$ $\omega = \dfrac{\omega_0}{2}$
$\Rightarrow v_{CM} = r\omega = \dfrac{r\omega_0}{2}$

94. Velocity of the small object is given as,
$$v = \sqrt{\frac{2gh}{1+\frac{k^2}{r^2}}}$$
$\therefore$ $v^2 = \dfrac{2g\,3v^2}{4g\left(1+\frac{k^2}{r^2}\right)}$
$\therefore$ $1 + \dfrac{k^2}{r^2} = \dfrac{3}{2} \Rightarrow k^2 = \dfrac{1}{2}r^2$
But $k = \sqrt{\dfrac{I}{M}}$
$\therefore$ $\dfrac{I}{M} = \dfrac{1}{2}r^2 \Rightarrow I = \dfrac{1}{2}Mr^2 \rightarrow$ disc

95. $\omega_2 = 1.1\ \omega_1$, $E \propto \omega^2$
$\Rightarrow E_1 = K\omega_1^2,\ E_2 = K\omega_2^2$
$\therefore$ $E_2 - E_1 = K\left(\omega_2^2 - \omega_1^2\right) = K\omega_1^2\,(1.1^2 - 1^2)$
$= K\omega_1^2\,(0.21)$
$\therefore$ $\dfrac{E_2 - E_1}{E_1}\times 100 = \dfrac{K\omega_1^2 \times 0.21}{K\omega_1^2}\times 100 = 21\%$

96. K.E. possessed by rotating body,
$$(K.E.)_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(MK^2)\left(\frac{v^2}{R^2}\right)$$
$$= \frac{1}{2}Mv^2\left(\frac{K^2}{R^2}\right)$$
For M, R and ω same, v becomes constant.
Hence, as $\dfrac{K^2}{R^2}$ increases, K.E. i.e., work done in bringing body to rest increases.
$$\left(\frac{K^2}{R^2}\right)_A = \frac{2}{5}, \left(\frac{K^2}{R^2}\right)_B = \frac{1}{2} \text{ and } \left(\frac{K^2}{R^2}\right)_C = 1$$
$\therefore$ $W_C > W_B > W_A$

99. M.I. of disc about tangent in plane
$= \dfrac{5}{4}mR^2 = I$
$\therefore$ $mR^2 = \dfrac{4}{5}I$

M.I. of disc about tangent ⊥ to plane

$I' = \frac{3}{2}\ mR^2$

Substituting the value of MR^2 from equation (i), we get

$I' = \frac{3}{2}\left(\frac{4}{5}I\right) = \frac{6}{5}I$

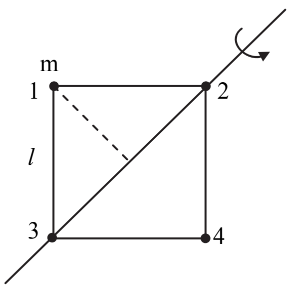
100. Torque: $\tau = I\alpha = \frac{MR^2}{2} \times \frac{\omega}{t}$

∴ $\tau = \frac{MR^2\omega}{2t}$

But $\tau = R \times F$

∴ $F = \frac{\tau}{R} = \frac{MR\omega}{2t}$

101. A distance of masses 2 and 3 from axis of rotation is zero, they don't contribute to moment of inertia.



$I_1 = I_4 = mR^2$

$= m\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{ml^2}{2}$

∴ $I_{Total} = I_1 + I_4 = ml^2$

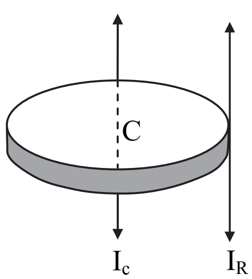
102. $I_c = \frac{1}{2}MR^2$

$\Rightarrow MR^2 = 6 \times 2 = 12$

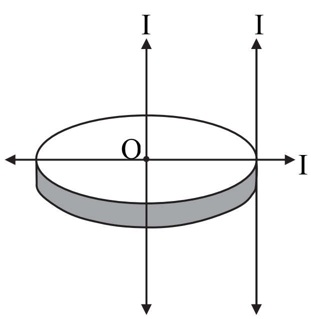
Using theorem of parallel axes,

$I_R = I_c + MR^2$

$= 6 + 12 = 18$ kgm^2



103.



From the figure,

$I_c = \frac{MR^2}{2}$ and $I = \frac{MR^2}{4} \Rightarrow MR^2 = 4I$ ….(i)

Using theorem of perpendicular axes,

$I_c = 2I_d = 2I$ ….(ii)

Now, using theorem of parallel axes,

$I_t = I_c + MR^2 = 2I + 4I = 6I$

….[from (i) and (ii)]

104. $\omega_0 = 0$, $\omega = 24$ rad/s, $t = 8$ s

∴ $\alpha = \frac{\omega - \omega_0}{t} = \frac{24}{8} = 3$ rad/s^2

From kinematical equations for rotational motion,

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2} \times 3 \times (8)^2 = 96$ rad.

105. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2 = 10$ rad

106. $I = 2$ kg m^2

$\omega_0 = 60$ rad/s

We know,

$\alpha = \frac{\omega - \omega_0}{t}$

After time t = 5 min = 300 s,

$\omega = 0$

∴ $\alpha = \frac{0 - 60}{300} = -\frac{1}{5}$ rad/s^2

3 min before stopping i.e., 2 min from starting,

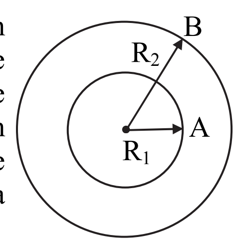
$\omega = \omega_0 + \alpha t$

$= 60 + \left(-\frac{1}{5}\right) \times 120$

$= 36$ rad/s

Now, $L = I\omega = 2 \times 36 = 72$ kg m^2/s

107. Let particle A be situated on the inner part and B on the outer part of the ring. As the ring is moving with uniform angular speed, both the particles will experience a centrifugal force



$\Rightarrow \frac{F_1}{F_2} = \frac{F_A}{F_B} = \frac{m\omega^2 R_1}{m\omega^2 R_2} \Rightarrow \frac{F_1}{F_2} = \frac{R_1}{R_2}$

108. K.E. $= \frac{L^2}{2I}$

From conservation of angular momentum about centre, L has to remain constant

K.E. $= \frac{L^2}{2(mr^2)}$

∴ K.E.′ $= \frac{L^2}{2\left(m.\frac{r^2}{4}\right)} = 4 \times \frac{L^2}{2(mr^2)}$

$\Rightarrow$ K.E.′ = 4 K.E.

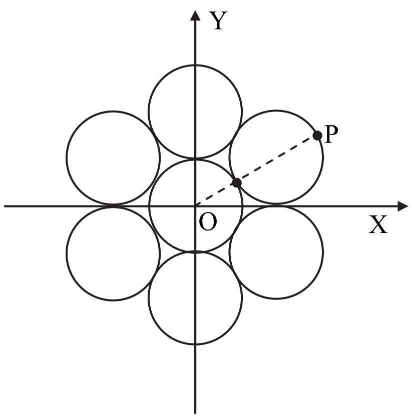
∴ K.E. is increased by a factor of 4.

109. Using principle of conservation of angular momentum

$$mv_0R_0 = mv\frac{R_0}{2} \Rightarrow v = 2v_0$$

$$K.E. = \frac{1}{2}mv^2 = 2mv_0^2$$

110.



Using parallel axes theorem,
M.I. about origin O,

$$I_O = \frac{MR^2}{2} + 6\left[\frac{MR^2}{2} + M(2R)^2\right]$$

$$\therefore \quad I_O = \frac{MR^2}{2} + \frac{54MR^2}{2}$$

$$I_O = \frac{55}{2}MR^2$$

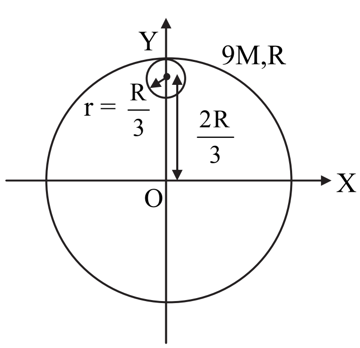
Similarly, using parallel axes theorem,
M.I. about the point P will be,

$$I_P = I_O + 7M(3R)^2$$

$$I_P = \frac{55}{2}MR^2 + 63MR^2$$

$$I_P = \frac{181}{2}MR^2$$

111.



Mass of portion removed will be,

$$m = \frac{M_0}{(\pi R_0^2)} \times (\pi r)^2 = \frac{9M}{R^2}\times\left(\frac{R}{3}\right)^2 = M$$

M.I. of the remaining part of the disc,

$$I = \frac{9MR^2}{2} - \left[\frac{M\left(\frac{R}{3}\right)^2}{2} + M\left(\frac{2R}{3}\right)^2\right]$$

$$\therefore \quad I = \frac{9MR^2}{2} - \left[\frac{MR^2}{18} + \frac{4MR^2}{9}\right]$$

$$\therefore \quad I = \frac{9MR^2}{2} - \left[\frac{9MR^2}{18}\right] = \frac{9MR^2}{2} - \frac{MR^2}{2}$$

$$\therefore \quad I = 4MR^2$$

112. Using principle of energy conservation,
K.E. of rotation + K.E of translation of falling mass = loss in P.E.

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$\therefore \quad mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}m\omega^2r^2 \qquad [\because \ v = \omega r]$$

$$\therefore \quad \omega^2 = \frac{2mgh}{(I + mR^2)} \qquad \therefore \quad \omega = \left[\frac{2mgh}{I + mr^2}\right]^{\frac{1}{2}}$$

113. $a = R\alpha$

$mg - T = ma$ ….(i)

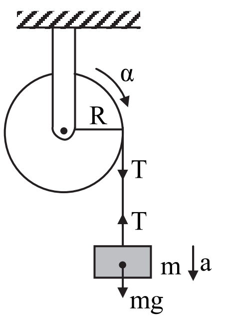
Also, $T \times R = mR^2\alpha$

or $T = ma$ ….(ii)

$\therefore$ Solving eq. (i) and (ii),

$mg = 2\ ma$

$$\therefore \quad a = \frac{g}{2}$$



114. $l_P > l_Q$

$$a_P = \frac{g\sin\theta}{l_P + mR^2} \text{ and } \left[a_Q = \frac{g\sin\theta}{l_Q + mR^2}\right]$$

$$\therefore \quad a_P < a_Q \Rightarrow v = u + at \Rightarrow t \propto \frac{1}{a} \Rightarrow t_P > t_Q$$

$$\therefore \quad v^2 = u^2 + 2as \Rightarrow v \propto a \Rightarrow v_P < v_Q$$

$$\therefore \quad \text{Translational K.E.} = \frac{1}{2}mv^2$$

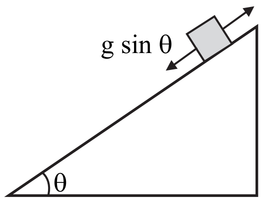
$\therefore$ (Translational K.E.)$_P$ < (Translational K.E.)$_Q$

$v = \omega R \Rightarrow \omega \propto v \Rightarrow \omega_P < \omega_Q$

Hence cylinder Q reaches the ground with larger angular speed.

115. Using $S = \frac{1}{2}at^2$

$$S = \frac{1}{2}g\sin\theta.(4)^2 \quad ….(i)$$



$\therefore \quad \frac{S}{4} = \frac{1}{2} g \sin\theta.(t)^2 \quad ....(ii)$

Dividing equation (ii) by (i),

$\frac{1}{4} = \frac{t^2}{16}$ or $t^2 = 4 \Rightarrow t = 2$ s

116. Rotational K.E.of sphere = $\frac{1}{2} I \omega^2$

75% of K.E. = Heat energy

$\therefore \quad \frac{1}{2} I\omega^2 \times \frac{75}{100} = MS\Delta\theta$

$\frac{1}{2} \times \frac{2}{3} MR^2\omega^2 \times \frac{75}{100} = MS\Delta\theta \quad \left[\because I_{sp} = \frac{2}{3} MR^2\right]$

$\frac{R^2\omega^2}{4S} = \Delta\theta$

117. $v = 54 \frac{km}{h} = 15$ m/s

$\omega_0 = \frac{v}{r} = \frac{15}{0.45} \frac{rad}{s}, \omega = 0$

$\omega = \omega_0 + \alpha t$

$0 = \frac{15}{0.45} + \alpha(15)$

$\alpha = -\frac{15}{0.45 \times 15} = -\frac{1}{0.45} \frac{rad}{s^2}$

$\therefore$ The magnitude of average torque

$\tau = I\alpha = -\frac{3}{0.45} = -\frac{300}{45} = -6.66$ kgm^2/s^2

118. The fan initially rotates with angular velocity ω_0.

$\therefore$ After switching off in time t, $\omega^2 = \omega_0^2 - 2\alpha\theta$

here, $\theta = \omega t$

and $\omega = 2\pi N$

as n revolutions are made in time t,

$N = \frac{n}{t} \Rightarrow \theta = 2\pi\left(\frac{n}{t}\right) \times t = 2\pi n$

$\therefore \quad \omega^2 = \omega_0^2 - 2\alpha(2\pi n)$

$\therefore \quad \left(\frac{\omega_0}{4}\right)^2 = \omega_0^2 - 2\alpha(2\pi n)$

$\therefore \quad 2\pi n(2\alpha) = \omega_0^2 - \frac{\omega_0^2}{16}$

$\therefore \quad 2\pi n = \frac{15}{16}\left(\frac{\omega_0^2}{2\alpha}\right) \quad ....(i)$

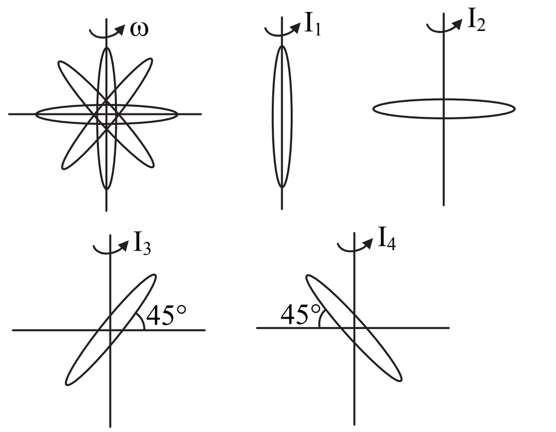
when the fan stops rotating, $0 = \omega_0^2 - 2\alpha(2\pi n')$

$\therefore \quad 2\pi n' = \frac{\omega_0^2}{2\alpha} \quad ....(ii)$

Comparing equations (i) and (ii), $n' = \frac{16}{15} n$

Evaluation Test

1.



$I_1 = \frac{MR^2}{2}, I_2 = MR^2$

$I_3 = \frac{\left(\frac{MR^2}{2}\right)}{2} = \frac{MR^2}{4} = I_4$

$\therefore \quad I = I_1 + I_2 + I_3 + I_4$
$= 2MR^2$

2. The concept is that I will be minimum when the rotation happens about the centre of mass.

I is minimum $\Rightarrow \frac{dI}{dx} = 0$

$\therefore \quad 6x - 24 = 0$

$\therefore \quad x = 4$

$\therefore$ X-coordinate of CM = 4.

3. $X_{CM} = \frac{\int x dm}{\int dm} = \frac{\int x \mu(dx)}{\int \mu dx} = \frac{\int \mu_0 x^2 dx}{\int \mu_0 x dx} = \frac{2}{3} l$

$I_{pivot} = \int x^2 dm = \int x^2 \mu(dx) = \int \mu_0 x^3 dx = \frac{\mu_0 l^4}{4}$

Now, $\tau = I\alpha$

$\therefore \quad \left(\frac{\mu_0 l^2}{2}\right)\left[\frac{2}{3} l\right] g = \left(\frac{\mu_0 l^4}{4}\right)\alpha$

$\therefore \quad \alpha = \frac{4g}{3l}$

4. $d\tau = \mu(dM)\,gr$

$$= \mu\left(\frac{M}{\pi R^2}(2\pi r dr)\right)gr$$

$\therefore$ $d\tau = \frac{2\mu Mg}{R^2} r^2 dr$

$\therefore$ $\tau = \frac{2\mu Mg}{R^2}\int_0^R r^2 dr = \frac{2}{3}\mu Mg\,R$

$\therefore$ $\frac{2}{3}\mu Mg\,R = \left(\frac{1}{2}MR^2\right)\alpha$

$\therefore$ $\alpha = \frac{4g\mu}{3R}$

5. $v = r\omega$ and $a = r\alpha$

$\therefore$ $\omega = \frac{4}{3}$ rad/s and $\alpha = 2$ rad/s^2

$\therefore$ $\theta = \frac{\alpha t^2}{2} + \omega t$

$\therefore$ $\theta = \frac{2\times(3)^2}{2} + \frac{4}{3}(3)$

$= 9 + 4$

$= 13$ rad

$= \left(\frac{13}{2\pi}\right)$ revolutions

≈ 2 revolutions

6. $I_O = I_{CM} + Md^2$

$$= I_C + M\left(\frac{2R}{\pi}\right)^2$$

$I_P = I_{CM} + Md^2$

$$= I_C + M\left(R - \frac{2R}{\pi}\right)^2$$

$\therefore$ $I_O - M\left(\frac{2R}{\pi}\right)^2 = I_P - \left(R - \frac{2R}{\pi}\right)^2$

$\therefore$ $I_P = MR^2 + M\left(R^2 + \left(\frac{2R}{\pi}\right)^2 - \frac{4R^2}{\pi}\right) - M\left(\frac{2R}{\pi}\right)^2$

$$= 2MR^2\left(1 - \frac{2}{\pi}\right)$$

7. $\tau_{net} = I\,\alpha$

$(Mg)\,R = \left(\frac{MR^2}{2} + 3mR^2\right)\alpha$

Also, $(Mg)R = \frac{1}{2}\left(\frac{MR^2}{2} + 3mR^2\right)\omega^2$

$\therefore$ $\omega^2 = \frac{4mgR}{(M+6m)R^2}$

$\therefore$ $\omega = \sqrt{\frac{4mg}{R(M+6m)}}$

8. $\rho = \frac{M}{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3} = \frac{M}{\frac{7}{8}\left(\frac{4}{3}\pi R^3\right)}$

$M_{\text{entire sphere}} = \rho V$

$$= \frac{M}{\left(\frac{7}{8}\left(\frac{4}{3}\pi R^3\right)\right)} \times \left(\frac{4}{3}\pi R^3\right)$$

$$= \frac{8}{7}M = M_1$$

$M_{\text{rem sphere}} = \frac{M}{7} = M_2$

$\therefore$ $I_{\text{system}} = \frac{2}{5}M_1R^2 - \left(\frac{2}{5}M_2R^2 + M_2\left(\frac{R}{2}\right)^2\right)$

$$= \frac{2}{5}M_1R^2 - \frac{13}{20}M_2R^2$$

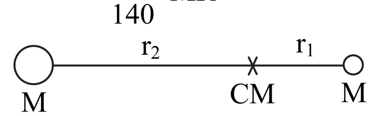
$$= \frac{2}{5}\left(\frac{8}{7}M\right)R^2 - \frac{13}{20}\left(\frac{M}{7}\right)R^2$$

$$= \frac{16}{35}MR^2 - \frac{13MR^2}{140}$$

$$= \frac{(64-13)}{140}MR^2$$

$$= \frac{51}{140}MR^2$$

9.



$mr_1 = Mr_2$, $r_1 + r_2 = d$

$\therefore$ $r_1 = \frac{Md}{M+m}$, $r_2 = \frac{md}{M+m}$

$I_{CM} = Mr_2^2 + m.r_1^2$, $\omega = \frac{v_1}{r_1} = \frac{v_2}{r_2}$

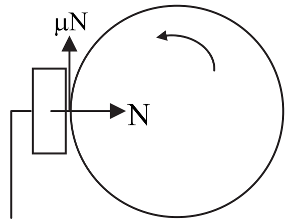
Now, (K.E.) $= \frac{1}{2}I\omega^2$, $\omega = 2\pi n = \frac{v_1}{r_1} = \frac{v_2}{r_2}$

$$= \frac{1}{2}\left(Mr_2^2 + mr_1^2\right)\omega^2$$

$$= \frac{1}{2}\left(M\frac{m^2d^2}{(M+m)^2} + m\frac{M^2d^2}{(M+m)^2}\right)(2\pi\nu)^2$$

$$= \frac{2\pi^2\nu^2 mMd^2}{(M+m)}$$

10.



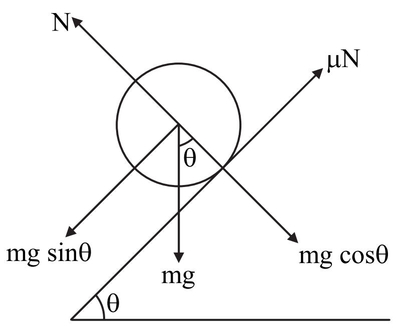
$\tau = I\alpha$

$\therefore \quad (\mu N)r = \left(\frac{Mr^2}{2}\right)\alpha \qquad \therefore \quad \alpha = \left(\frac{2\mu N}{Mr}\right)$

$\omega = 0 + \alpha t \qquad \therefore \quad \omega = \alpha t$

$\therefore \quad \omega = \left(\frac{2\mu N}{Mr}\right)t \qquad \therefore \quad N = \frac{Mr\omega}{2\mu t}$

11.



$mg\sin\theta - \mu\, mg\cos\theta = Ma$

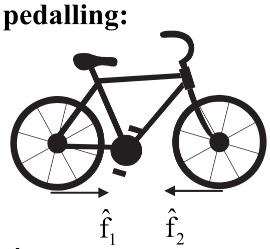
$\mu(mg\cos\theta)R = \frac{2M}{5}R^2\alpha$

$a = \frac{5}{7}g\sin\theta$ ….[Given]

$\therefore \quad \mu mg\cos\theta = \frac{2}{7}\, mg\sin\theta$

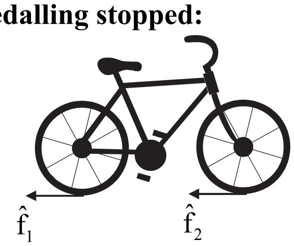
$\therefore \quad \mu = \frac{2}{7}\tan\theta = \frac{2}{7}\sqrt{\sec^2\theta - 1}$

12. **While pedalling:**



$a = \hat{f}_1 \cdot \hat{f}_2 = -1$

Pedalling stopped:



$b = \hat{f}_1 \cdot \hat{f}_2 = +1$

$a \times b = -1.$

13. $I_0 = I_{CM} + Md^2$

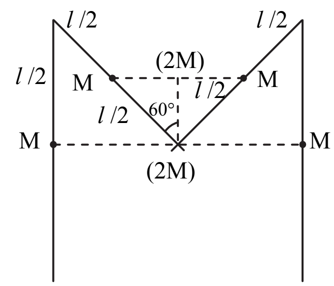
$\therefore \quad I_0 = I_A + Md^2$

$I_P = I_A + M(x^2 + y^2)$

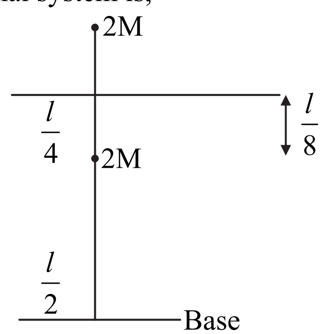
$\therefore \quad x^2 + y^2 = d^2 \quad [\because I_0 = I_P]$

It is an equation of a circle.

14. The object will not rotate if the force F is applied on the centre of mass of the system as the net torque will be zero.
So the question just boils down to find the centre of Mass of the system.

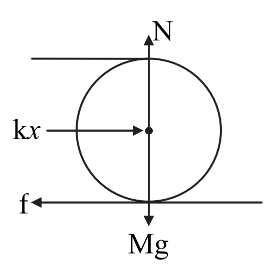


The calculations are shown in the diagram. Final system is,



$\therefore$ Force must be applied at a height $\frac{5l}{8}$ from base.

15. I.



$kx - f = Ma$

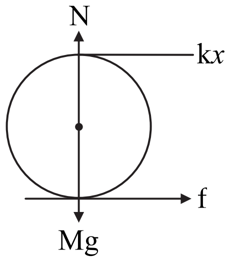
$fR = \left(\frac{2}{5}MR^2\right)\alpha$

$\therefore \quad f = \frac{2}{5}Ma$

$kx = \frac{7}{5}Ma$

$\therefore \quad f = \frac{2}{5}Ma$ in the direction considered.

II.



$kx + f = Ma$

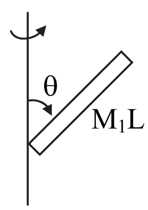
$(kx - f)R = \left(\frac{2}{5}MR^2\right)\alpha$

$\therefore$ $kx - f = \frac{2}{5}Ma$ $\qquad \therefore$ $kx = \frac{7}{10}Ma$

$\therefore$ $f = \frac{3}{10}Ma$ in the direction as considered.

$\therefore$ $\frac{\vec{f}_I}{\vec{f}_{II}} = \frac{\frac{-2}{5}Ma\,\hat{i}}{\frac{+3}{10}Ma\,\hat{i}} = \frac{-4}{3}$

16.



$MI = \frac{ML^2}{3}\sin^2\theta$

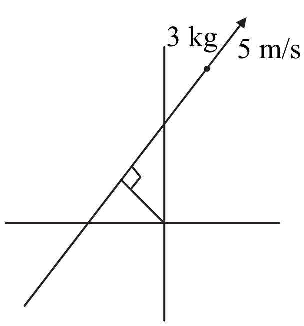
$\therefore$ For the given system,

$$I = \frac{\left(\frac{M}{3}\right)\left(\frac{L}{3}\right)^2}{3}\sin^2\theta + \frac{\left(\frac{2M}{3}\right)\left(\frac{2L}{3}\right)^2}{3}\cos^2\theta$$

$$= \frac{ML^2}{27}(\sin^2\theta + 8\cos^2\theta)$$

$$= \frac{ML^2}{27}(1 + 7\cos^2\theta)$$

17.



$L = mvr$

$= (3\text{ kg})\ (5\text{m/s})\left|\frac{39}{\sqrt{12^2+5^2}}\right|$

$= 45\text{ kg m}^2\text{/s}$

18. As the force applied is below the centre, the torque of friction exceeds that of force hence the thread winds and yo-yo rotates clockwise.

19. Friction will act upwards in both the cases.

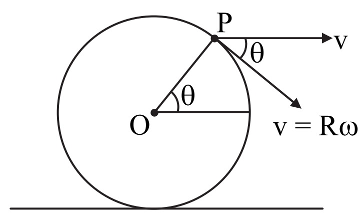
20. Since there will be no external torque about the point P, the angular momentum P will be conserved.

$\therefore$ $mvr = I\omega$

$\therefore$ $mvR = \frac{2}{5}mR^2\omega$

$\therefore$ $\omega = \frac{5v}{2R}$

21.

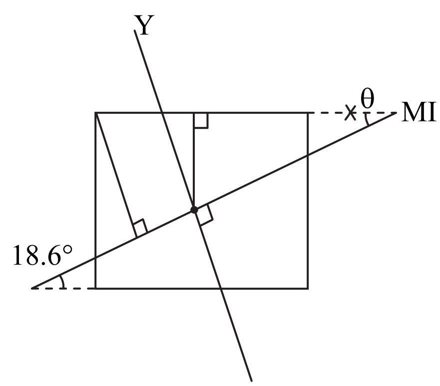


$v_R = \sqrt{v^2 + v^2 + 2v^2\cos\theta}$

$= \sqrt{2v^2(1+\cos\theta)}$

$= 2\,v\sin\left(\frac{\theta}{2}\right)$

22.



M.I. of a square plate about an axis perpendicular to the plane and passing through the centre would be $\frac{Ma^2}{12}$

Now, $I_X + I_Y = \frac{Ma^2}{12}$

and $I_X = I_Y$ by symmetry.

$\therefore$ $I_X = I_Y = \frac{Ma^2}{24}$

$\tan(\theta) = \frac{\left(\frac{a}{2}\right)}{\left(\frac{a}{2}\right) + x}$

$\therefore \quad \left(\frac{a}{2}+x\right)=\frac{a}{2}\cot\theta$

$\therefore \quad (a + x) = \frac{a}{2}(1 + \cot \theta)$

$d = (a + x) \sin \theta = \frac{a}{2}(\sin \theta + \cos \theta)$

$\therefore \quad I = \frac{Ma^2}{24} + Ma^2$

$= \frac{Ma^2}{24} + \frac{Ma^2}{4}(\sin \theta + \cos \theta)^2$

$= \frac{Ma^2 + 6Ma^2(1+\sin 2\theta)}{24}$

$\therefore \quad I = \frac{2\times(0.2)^2 + 6(2)(0.2)^2\left(1+\frac{3}{5}\right)}{24}$

$= \frac{0.08 + 0.48\times\left(\frac{8}{5}\right)}{24}$

$= \frac{0.848}{24} = 0.035 \text{ kg-m}^2$

23. $P_i = P_f$

$mv = Mv' \Rightarrow v' = \frac{mv}{M}$

About the centre of the rod,

$L_i = L_f$

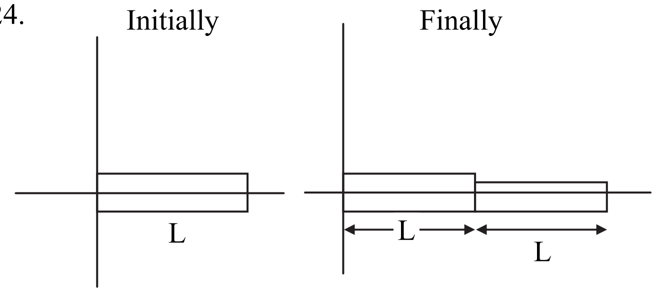
$\therefore \quad mv\left(\frac{1}{2}\right) + 2mv\left(\frac{1}{2}\right) = \left(\frac{ML^2}{12}\right)\omega$

$\therefore \quad \frac{3mvL}{2} = \frac{ML^2}{12}\omega$

$\therefore \quad \omega = 18\frac{mv}{ML}$

$\therefore \quad \frac{v'}{\omega} = \frac{\left(\frac{mv}{M}\right)}{18\left(\frac{mv}{M}\right)}L = \frac{L}{18}$

24. Initially Finally



$I_i = \frac{ML^2}{6} + \frac{ML^2}{2} = \frac{2}{3}ML^2$

$\therefore \quad I_f = \left[\frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 + \frac{ML^2}{12} + M\left(\frac{3L}{2}\right)^2\right]$

$= \frac{8}{3}ML^2$

$\therefore \quad I_i\,\omega_i = I_f\,\omega_f$

$\therefore \quad \frac{2}{3}ML^2\,\omega = \frac{8}{3}ML^2\,\omega_f$

$\therefore \quad \omega_f = \frac{\omega}{4}$

25. The catch here is that the incline is smooth/ frictionless. Hence, the rotational K·E. of the sphere will not be affected.

$\therefore$ Conserving Energy,

$\frac{1}{2}mv^2 = mgh$

$\Rightarrow v = \sqrt{2gh}$

Textbook Chapter No.

04 Oscillations

Hints

Classical Thinking

3. Linear S.H.M and its equation

5. $F = -kx \Rightarrow ma = -kx$

$\therefore$ $\frac{x}{a} = \left(-\frac{m}{k}\right)$ = constant

6. Unit of k = N/m = $\frac{kg\,m}{s^2\,m}$ = kg/s^2 = $\frac{[M^1]}{[T^2]}$ $= [M^1L^0T^{-2}]$

7. For S.H.M., $F = -kx$

$\therefore$ Force = Mass × Acceleration $\propto -x$

F = $-Akx$; where A and k are positive constants

8. $f = F = -kx$ and

P.E. = V = $\frac{1}{2}m\omega^2x^2$

For option A : $\frac{V}{F} + x = \frac{m\omega^2A^2}{-2kx} + x \neq 0$

Hence option (A) is incorrect.

For option B : $\frac{F}{V} + x = \frac{-kx}{m\omega^2x^2} + x \neq 0$

Hence option (B) is incorrect.

For option C : $\frac{2V}{F} + x = \frac{2\times\frac{1}{2}m\omega^2x^2}{-kx} + x$

$= \frac{m\omega^2x^2}{-m\omega^2x} + x$

$= -x + x = 0$

Hence option (C) is correct.

For option D : $\frac{F}{2V} + x = \frac{-Kx}{2\times\frac{1}{2}m\omega^2x^2} + x \neq 0$

Hence option (D) is incorrect.

9. The standard differential equation is satisfied by only the function sinωt − cosωt. Hence it represents S.H.M.

10. As $F = -kx \Rightarrow |F| \propto x$

11. Displacement and force (ma) are out of phase ($\Delta\alpha = \pi$) in S.H.M. Therefore, the correct graph will be (D)

12. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{200\times10^{-3}}{80}}$

$= 2\pi\sqrt{25\times10^{-4}} = 2\pi \times 5 \times 10^{-2}$

$= 10\pi \times 10^{-2} = 0.31$ s

13. $T = 2\pi\sqrt{\frac{m}{k}}$

$\therefore$ $\frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{4m}{m}} = 2$

$\therefore$ $T_2 = 2 \times 2 = 4$ s

14. $a = -\omega^2x$, at mean position x = 0

So acceleration is minimum (zero)

15. Acceleration = ω^2A is maximum at extreme position.

17. $a = -\omega^2x \Rightarrow \left|\frac{a}{x}\right| = \omega^2$

18. $-A\omega^2$ is the acceleration of the particle when it is at one extreme point.

19. A = 10 cm, T = 4 sec

$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

x = 5 cm when t = 0

$\therefore$ 5 cm = 10 cm sin ($\omega t + \phi$)

$\therefore$ $\sin\phi = \frac{1}{2}$

$\therefore$ $\phi = \frac{\pi}{6}$

$\therefore$ Equation of displacement is

$x = 10 \text{ cm} \sin\left(\frac{\pi t}{3} + \frac{\pi}{2}\right)$

20. Velocity is same. So by using v = Aω,

$A_1\omega_1 = A_2\omega_2 = A_3\omega_3$

22. Comparing given equation with $\frac{d^2x}{dt^2} + \omega^2x = 0$

we get,

$\therefore$ $\omega^2 = \alpha \Rightarrow \omega = \sqrt{\alpha}$

$\therefore$ $2\pi n = \sqrt{\alpha} \Rightarrow n = \frac{\sqrt{\alpha}}{2\pi}$

23. Maximum acceleration of S.H.M.,
$\alpha = \omega^2 A$
Maximum velocity of S.H.M.,
$\beta = A\omega$

$\therefore \quad \alpha = \frac{\omega^2 A \times A}{A} = \frac{\omega^2 A^2}{A} = \frac{\beta^2}{A}$

$\therefore$ Amplitude of oscillation is,
$A = \frac{\beta^2}{\alpha}$

24. $a_{max} = A\omega^2 = \frac{A \times 4\pi^2}{T^2} = \frac{1 \times 4 \times (3.14)^2}{0.2 \times 0.2}$

$F_{max} = m \times a_{max} = \frac{0.1 \times 4 \times (3.14)^2}{0.2 \times 0.2}$

$\therefore \quad F_{max} = 98.596$ N

25. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4.84}{0.98}} = 2.22$ rad/s

26. $\omega = \sqrt{\frac{k}{m}} \Rightarrow \frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}}$

$\therefore \quad 2 = \sqrt{\frac{m_1}{m_2}} \Rightarrow m_2 = \frac{m_1}{4}$

27. On comparing with standard equation
$\frac{d^2y}{dt^2} + \omega^2 y = 0$, we get $\omega^2 = k$,

$\therefore \quad \omega = \frac{2\pi}{T} = \sqrt{k} \Rightarrow T = \frac{2\pi}{\sqrt{k}}$

28. Here, Assertion is false because, the direction of velocity in S.H.M. can be towards or away from mean position whereas the displacement is always away from mean position.

29. Since the particle start from x = 0 and have the same amplitude but different time periods, they will meet again at x = 0 where their velocities are maximum equal to $A\omega_1$ and $A\omega_2$, i.e.

$\frac{v_1}{v_2} = \frac{\omega_1}{\omega_2} = \frac{2\pi}{T_1} \times \frac{T_2}{2\pi} = \frac{6}{3} = 2$

30. $v_{max} = A\omega = 0.20 \times 100 = 20$ cm/s

31. $v_{max} = A\omega$ where $\omega = 2\pi n = 2 \times \pi \times 100$
$\therefore \quad v_{max} = 0.5 \times 2\pi\,(100) = 100\,\pi$ m/s

32. $v^2 = 9(16 - x^2)$

$\therefore \quad v = 3\sqrt{16 - x^2}$

Comparing with $v = \omega\sqrt{A^2 - x^2}$, we get

$\omega = 3$, $A = 4$

$\therefore \quad v_{max} = A\omega = 4 \times 3 = 12$ unit

40. For S.H.M., displacement $x = a \sin \omega t$ and acceleration $A = -\omega^2 x \sin \omega t$ are maximum at $\omega t = \frac{\pi}{2}$.

41. In simple harmonic motion,
$y = A \sin\omega t$ and $v = A\omega \cos\omega t$. From these equations, we obtain $\frac{y^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$, which is an equation of ellipse.

44. Phase change = $2 \times 2\pi = 4\pi$ radian

45. $y = A\sin(2\pi nt + \alpha)$.
Its phase at time $t = 2\pi nt + \alpha$

46. $\alpha = \omega t \Rightarrow \frac{\pi}{2} = \omega \times 4$

$\therefore \quad \frac{\pi}{8} = \omega = \frac{2\pi}{T} \Rightarrow T = 16$ s

47. $\phi = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{6}{8}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

53. $F = -kx$

$\therefore \quad dW = Fdx = -kxdx$

$\therefore \quad \int_0^W dW = \int_0^x -kx\,dx$

$\therefore \quad W = U = -\frac{1}{2}kx^2$

54. $K.E_{max} = \frac{1}{2}m\omega^2 A^2$

$= \frac{1}{2} \times 1 \times (100)^2 \times (6 \times 10^{-2})^2 = 18$ J

55. K.E. = 3 × P.E.

$K.E. = \frac{1}{2}m\omega^2(A^2 - x^2) = 3 \times \frac{1}{2}\, m\omega^2 x^2$

$\therefore \quad A^2 = 4x^2 \Rightarrow A = 2x$

$\therefore \quad x = \frac{8}{2} = 4$ mm

56. Kinetic energy at mean position,

$$K.E_{max} = \frac{1}{2}mv_{max}^2$$

$\therefore$ $$v_{max} = \sqrt{\frac{2K.E_{max}}{m}}$$

$\therefore$ $$v_{max} = \sqrt{\frac{2\times16}{0.32}} = \sqrt{100} = 10 \text{ m/s}$$

57. $x = \frac{3}{4}A \Rightarrow \frac{A^2}{x^2} = \frac{16}{9}$ ….(i)

$\therefore$ $$\frac{T.E.}{P.E.} = \frac{\frac{1}{2}m\omega^2A^2}{\frac{1}{2}m\omega^2x^2} = \frac{A^2}{x^2} = \frac{16}{9}$$ ….[From (i)]

$\therefore$ $$\frac{80}{P.E.} = \frac{16}{9} \Rightarrow P.E. = 45 \text{ J}$$

58. $A = 10 \times 10^{-2}\text{m} = 10^{-1}\text{m}$

$$K.E_{max} = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2$$

$\therefore$ $$5 = \frac{1}{2} \times k \times (10^{-1})^2$$

$\therefore$ $$\frac{10}{10^{-2}} = k \Rightarrow k = 1000 \text{ N/m}$$

59. Comparing given equations with standard form, $A_1 = 10$ and $A_2 = 25$

$\therefore$ $$\frac{A_1}{A_2} = \frac{10}{25} = \frac{2}{5}$$

60. Phase difference between two S.H.M.s,

$$\left(\frac{2\pi}{3}t - \frac{\pi}{2}t\right) = \frac{\pi}{6}t = \frac{\pi}{6}(1) = \frac{\pi}{6}$$

61. Two equations are,
$y_1 = A_1 \sin(\omega t + 2\pi)$ and
$y_2 = A_2 \sin(\omega t + 4\pi)$
The phase difference, $\phi = 4\pi - 2\pi = 2\pi$
Resultant amplitude,

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos 2\pi} = \pm(A_1 + A_2)$$

62. $$T = 2\pi\sqrt{\frac{l}{g}}$$

$\therefore$ $$\frac{l}{T^2} = \frac{g}{4\pi^2} = \text{constant}$$

64. In the given case, effective acceleration $g_{eff.} = 0$

$\therefore$ $T = \infty$

65. When the pendulum is falling freely with acceleration g,

$$T' = 2\pi\sqrt{\frac{l}{g-g}} = \infty$$

66. $$T = 2\pi\sqrt{\frac{l}{g}}$$

$\therefore$ $T \propto \sqrt{l}$, hence if l is made 9 times then T becomes 3 times.

67. For seconds pendulum, T = 2 s

$\therefore$ $$2 = 2\pi\sqrt{\frac{l}{g}}$$

$\therefore$ $$l = \frac{g}{\pi^2} = \frac{4.9}{\pi^2} \approx 50 \text{ cm}$$

68. $a = \omega^2x$

$\therefore$ $$\omega^2 = \frac{a}{x} = \frac{2}{0.02} = 100$$

$\therefore$ $\omega = 10$ rad/s

72. As mg produces extension x, hence

$$k = \frac{mg}{x}$$

$\therefore$ $$T = 2\pi\sqrt{\frac{(M+m)}{k}} = 2\pi\sqrt{\frac{(M+m)x}{mg}}$$

73. With respect to the block, the springs are connected in parallel combination

$\therefore$ Combined stiffness $k = k_1 + k_2$

$\therefore$ $$n = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

74. When the springs are stretched by the same force F, the extensions in springs A and B are x_1 and x_2 respectively which are given by,

$F = k_1x_1 = k_2x_2$

$$\frac{x_1}{x_2} = \frac{k_2}{k_1}$$ ….(i)

Work done, $W_1 = \frac{1}{2}k_1x_1^2$ and $W_2 = \frac{1}{2}k_2x_2^2$

$\therefore$ $$\frac{W_1}{W_2} = \frac{k_1}{k_2}.\frac{x_1^2}{x_2^2}$$ ….(ii)

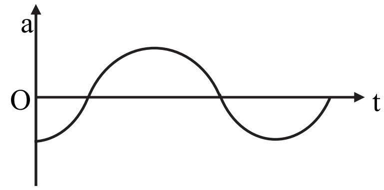
Using equation (i) in equation (ii) we get,

$$\frac{W_1}{W_2} = \frac{k_1}{k_2}.\frac{k_2^2}{k_1^2} = \frac{k_2}{k_1}$$

75. Force of friction = $\mu mg = m\omega^2 A = m(2\pi n)^2 A$

$\therefore$ $n = \frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$

76. $x = A\cos\omega t$



$v = \frac{dx}{dt} = -A\omega\sin\omega t$

$a = \frac{d^2x}{dt^2} = -A\omega^2\cos\omega t$

This is correctly depicted by graph in (C).

77. $\omega^2 = \frac{k}{m}$, $r = \frac{p}{2m}$

Angular frequency,

$\omega' = \sqrt{(\omega^2 - r^2)} = \sqrt{\frac{k}{m} - \frac{p^2}{4m^2}}$

Critical Thinking

1. $\omega = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}\frac{rad}{s}$

$\therefore$ $2 = 4\left(\sin\frac{\pi}{6}t_1\right)$ ....(For x = 2 cm)

$\therefore$ $\frac{2}{4} = \sin\frac{\pi}{6}t_1 \Rightarrow \frac{\pi}{6} = \frac{\pi}{6}t_1$

$\therefore$ $t_1 = 1$ s

Similarly, for x = 4 cm, it can be shown that $t_2 = 3$ s

So time taken by particle in going from 2 cm to extreme position is $t_2 - t_1 = 2$ s. Hence required ratio will be $\frac{1}{2}$.

2. In S.H.M., velocity of particle also oscillates simple harmonically. Speed is more when the particle is near the mean position than when it is near the extreme position. Therefore, the time taken for the particle to go from 0 to $\frac{A}{2}$ will be less than the time taken to go from $\frac{A}{2}$ to A. Hence, $T_1 < T_2$.

3. $F = kx$

$\therefore$ $mg = kx \Rightarrow m \propto kx$

$\therefore$ $\frac{m_1}{m_2} = \frac{k_1}{k_2}\times\frac{x_1}{x_2}$

$\therefore$ $\frac{4}{6} = \frac{k}{k/2}\times\frac{1}{x_2} \Rightarrow x_2 = 3$ cm

4. $k \propto \frac{1}{l}$. Since one fourth length is cut away, the remaining length is $\left(\frac{3}{4}\right)^{th}$. Hence k becomes $\frac{4}{3}$ times i.e., $k' = \frac{4}{3}k$.

5. Comparing given equation with standard equation,

$y = A\sin(\omega t + \alpha)$, we get, A = 2 cm, $\omega = \frac{\pi}{2}$

$\therefore$ $a_{max} = \omega^2 A = \left(\frac{\pi}{2}\right)^2 \times 2 = \frac{\pi^2}{2}$ cm/s^2

6. $y = 5\sin(\pi t + 4\pi)$.

Comparing it with standard equation

$y = A\sin(\omega t + \alpha)$ we get,

A = 5 m and $\frac{2\pi t}{T} = \pi t \Rightarrow T = 2$ s

7. $v_{max} = \omega A$

$\therefore$ $100 = \omega \times 10 \Rightarrow \omega = 10$ rad/s

$\therefore$ $v = \omega^2(A^2 - x^2)$

$\therefore$ $(50)^2 = (10)^2(10^2 - x^2)$

$\therefore$ $25 = 10^2 - x^2$

$\therefore$ $x^2 = 100 - 25 = 75 \Rightarrow x = 5\sqrt{3}$ cm

8. When particle starts from extreme position,

$x = A\cos\omega t$ ...(i)

$n = 60$ r.p.m. $= \frac{60}{60} = 1$ r.p.s.

$\omega = 2\pi n = 2\pi \times 1 = 2\pi$

$x = 0.1\cos(2\pi \times 2)$ ...[From (i)]

$= 0.1\cos 4\pi = 0.1$ m ....[$\because \cos 4\pi = 1$]

9. $x = A\sin\frac{2\pi}{T}t$

$\therefore$ $\frac{A}{\sqrt{2}} = A\sin\frac{2\pi}{T}\cdot t$ .... $\left(\because x = \frac{A}{\sqrt{2}}m\right)$

$\therefore$ $\sin\frac{2\pi}{T}t = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\therefore$ $\frac{2\pi}{T}t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8}$

10. Comparing with x = A sin (ωt + α) we get,

$\omega = \pi \Rightarrow 2\pi n = \pi \Rightarrow n = \frac{1}{2}$

∴ n per min $= \frac{1}{2} \times 60 = 30$ per min

11. $v = \frac{dx}{dt} = 4 \times \pi \times \cos\left(\pi t + \frac{\pi}{3}\right)$

$= 4\pi \cos\left(4\pi + \frac{\pi}{3}\right) = 4\pi \cos\left(\frac{\pi}{3}\right)$

$= 4\pi \times \frac{1}{2} = 2\pi$ cm/s

12. $a = \frac{dv}{dt} = -4\pi^2 \sin\left(4\pi + \frac{\pi}{3}\right)$

$= -4\pi^2 \sin\frac{\pi}{3} = -4\pi^2 \times \frac{\sqrt{3}}{2} = -2\sqrt{3}\,\pi^2$ cm/s^2

13. Velocity, $v = \omega\sqrt{a^2 - x^2}$

At x = s, let $v = v_0$

∴ $v_0 = \omega\sqrt{a^2 - s^2}$

∴ $v_0^2 = \omega^2(a^2 - s^2)$ ….(i)

Due to blow, the new velocity at x = s,

$v = \frac{v_0}{2}$

∴ $v^2 = \omega^2(a'^2 - s^2)$

∴ $\left(\frac{v_0}{2}\right)^2 = \omega^2(a'^2 - s^2)$

∴ $\frac{v_0^2}{4} = \omega^2(a'^2 - s^2)$ ….(ii)

Dividing equation (ii) by equation (i)

$\frac{1}{4} = \frac{a'^2 - s^2}{a^2 - s^2}$

∴ $a^2 - s^2 = 4a'^2 - 4s^2$

$a^2 + 3s^2 = 4a'^2$

∴ $a' = \frac{\sqrt{a^2 + 3s^2}}{2}$

14. We have,

$v^2 = \omega^2(A^2 - x^2)$ and $\alpha = -\omega^2 x$

∴ $v^2 = \omega^2A^2 - \omega^2x^2$ and $\alpha^2 = \omega^4 x^2 = \omega^2(\omega^2 x^2)$

∴ $v^2 = \omega^2A^2 - \frac{\alpha^2}{\omega^2}$

∴ $v^2 + \frac{\alpha^2}{\omega^2} = \omega^2A^2$

∴ $\frac{v^2}{\omega^2A^2} + \frac{\alpha^2}{\omega^4A^2} = 1$

∴ $\left(\frac{v}{\omega A}\right)^2 + \left(\frac{\alpha}{\omega^2 A}\right)^2 = 1$

which is an equation of an ellipse.

15. x = A sin ωt

∴ 6.5 = 13 sin ωt

$\sin\omega t = \frac{1}{2}$

∴ $\omega t = \sin^{-1}\left(\frac{1}{2}\right)$

∴ $\omega t = \frac{\pi}{6}$

∴ $\frac{2\pi t}{T} = \frac{\pi}{6}$

∴ $t = \frac{T}{12} = \frac{12}{12} = 1$ s

∴ time required for travelling from x = 6.5 to x = 0 is t = 1 s

∴ time required for x = 6.5 to x = – 6.5 is 2 t = 2 s

16. Comparing the equation with x = A sin ωt, we get,

$\omega = 20\pi \Rightarrow 2\pi n = 20\pi \Rightarrow n = 10$ Hz

17. $x = 6\cos\left(3\pi t + \frac{\pi}{3}\right)$

∴ $\frac{dx}{dt} = -6\sin\left(3\pi t + \frac{\pi}{3}\right)3\pi$ and

∴ $\frac{d^2x}{dt^2} = -6(9\pi^2)\cos\left(3\pi t + \frac{\pi}{3}\right)$

∴ $\frac{d^2x}{dt^2} = -9\pi^2 x$

18. $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.02} = 100\,\pi$ rad/s,

A = 2.5 m at t = 0

Equation of particle performing S.H.M. is given by,

x = A sin (ωt + α)

∴ 2.5 = 5 sin (100π × 0 + α)

∴ $\frac{2.5}{5} = \sin\alpha \Rightarrow \alpha = 30°$ or $\frac{\pi}{6}$

Hence, the correct equation is,

$x = 5\sin\left(100\pi t + \frac{\pi}{6}\right)$

19. As it starts from rest, we have,
$x = A\cos\omega t$. At $t = 0$, $x = A$
When $t = \tau$, $x = A - a$ and
when $t = 2\tau$, $x = A - 3a$
$\Rightarrow A - a = A\cos\omega\tau$ ….(i)
$\therefore \cos\omega\tau = \dfrac{A-a}{A}$
$A - 3a = A\cos 2\omega\tau$ ….(ii)
$\therefore \cos 2\omega = \dfrac{A-3a}{A}$
As, $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$,
$\Rightarrow \dfrac{A-3a}{A} = 2\left(\dfrac{A-a}{A}\right)^2 - 1$
$\therefore \dfrac{A-3a}{A} = \dfrac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$
$\therefore A^2 - 3aA = A^2 + 2a^2 - 4Aa$
$\therefore a^2 = 2aA \Rightarrow A = 2a$
Now, $A - a = A\cos\omega\tau$ ….[From (i)]
$\Rightarrow \cos\omega\tau = \dfrac{1}{2}$
$\therefore \dfrac{2\pi}{T}\tau = \dfrac{\pi}{3} \Rightarrow T = 6\tau$

21. $\alpha = 10\pi t + \dfrac{\pi}{2}$
substituting $t = 2 = 20\pi + \dfrac{\pi}{2} = \dfrac{41}{2}\pi$

22. Equation of linear S.H.M.,
$x = 8\cos(12\pi t)$
$\therefore x = 8\sin(12\pi t + \dfrac{\pi}{2})$
$\therefore$ Initial phase angle $= \dfrac{\pi}{2}$ rad

23. $x = A\sin(\omega t + \alpha)$
$\therefore +5 = 10\sin(2\pi \times 0 + \alpha) = 10\sin\alpha$
$\therefore \alpha = \sin^{-1}\left(\dfrac{5}{10}\right) = \sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$

24. $y = 10\sin(20t + 0.5)$
Comparing with equation $y = A\sin(\omega t + \alpha)$ we get,
initial phase $\alpha = 0.5$ rad

25. $y = 5\sin(\pi t + 4\pi)$
Comparing with standard equation,
$y = A\sin(\omega t + \alpha)$
$\therefore A = 5, \alpha = 4\pi$

26. $x = A\sin\omega t$
$\therefore 2.5 = 5\sin\dfrac{2\pi t}{6}$
$\therefore \dfrac{2\pi t}{6} = \dfrac{\pi}{6}$ or $t = \dfrac{1}{2}$ s
Phase difference corresponding to 6 s is 2π.
So, phase difference corresponding to $\dfrac{1}{2}$ s
is $\dfrac{2\pi}{12}$ i.e. $\dfrac{\pi}{6}$

27. For a particle performing S.H.M.,
$x = A\sin\omega t$ and
$v = A\omega\cos\omega t$
$a = -A\omega^2\sin\omega t = A\omega^2\cos(90 + \omega t)$
$\therefore a = A\omega^2\cos(\omega t + \dfrac{\pi}{2})$
$\therefore$ The acceleration shows a phase lead of $\dfrac{\pi}{2}$

28. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$
Using $v = A\omega\cos(\omega t + \alpha)$ we get,
$6.28 = 24\dfrac{\pi}{6}\cos\left(\dfrac{2\pi}{6} + \alpha\right)$
$\therefore \dfrac{1}{2} = \cos\left(\dfrac{\pi}{3} + \alpha\right)$
$\therefore \dfrac{\pi}{3} + \alpha = \cos^{-1}\left(\dfrac{1}{2}\right)$
$\therefore \dfrac{\pi}{3} + \alpha = \dfrac{\pi}{3}$
$\therefore \alpha = 0$

29. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{6} = \dfrac{\pi}{3}$
$t = \dfrac{1}{2}$ s, $\alpha = \left(\dfrac{\pi}{6}\right)^c$
Equation of S.H.M. is,
$x = A\sin(\omega t + \alpha)$
$= 10\sin\left(\dfrac{\pi}{3} \times \dfrac{1}{2} + \dfrac{\pi}{6}\right) = 10\sin\left(\dfrac{2\pi}{6}\right)$
$= 10\sin 60^\circ$
$= 10 \times \dfrac{\sqrt{3}}{2}$
$= 5\sqrt{3}$ cm

30. $\alpha = \frac{\pi}{2}$ rad

$y = A\sin(\omega t + \alpha)$

$\therefore \quad y = A\sin\left(\frac{2\pi}{T}t + \alpha\right)$

$\therefore \quad y = 0.5 \sin\left(\frac{2\pi}{0.4}t + \frac{\pi}{2}\right)$

$\therefore \quad y = 0.5 \sin\left(5\pi t + \frac{\pi}{2}\right) = 0.5 \cos 5\pi t$

31. In S.H.M., $a = -\omega^2 x$
Acceleration is always opposite to displacement.

32. P.E. = $\frac{1}{2}$ $m\omega^2 x^2 = 2.5$ J

$\therefore \quad \frac{1}{2} m\omega^2 \left(\frac{A}{2}\right)^2 = 2.5 \qquad \ldots[\because x = \frac{A}{2}]$

$\therefore \quad \frac{1}{2} m\omega^2 \frac{A^2}{4} = 2.5$

$\therefore \quad \frac{1}{2} m\omega^2 A^2 = 10$

$\therefore$ Total energy of system = $\frac{1}{2} m\omega^2 A^2 = 10$ J

33. K.E. = $\frac{2E}{3}$

$$\frac{\text{K.E.}}{\text{T.E.}} = \frac{\frac{1}{2}m\omega^2(A^2 - x^2)}{\frac{1}{2}m\omega^2 A^2} = \frac{A^2 - x^2}{A^2} = 1 - \frac{x^2}{A^2}$$

$\therefore \quad \frac{\left(\frac{2E}{3}\right)}{E} = 1 - \frac{x^2}{A^2}$

$\therefore \quad \frac{x^2}{A^2} = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow x = \frac{A}{3}$

34. P.E.$_1$ = $\frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{2\text{P.E}_1}{k}}$

P.E.$_2$ = $\frac{1}{2}ky^2 \Rightarrow y = \sqrt{\frac{2\text{P.E}_2}{k}}$

and P.E. = $\frac{1}{2}k(x + y)^2$

$\therefore \quad x + y = \sqrt{\frac{2\text{P.E}}{k}}$

$\therefore \quad \sqrt{\frac{2\text{P.E.}_1}{k}} + \sqrt{\frac{2\text{P.E.}_2}{k}} = \sqrt{\frac{2\text{P.E.}}{k}}$

$\sqrt{\text{P.E.}_1} + \sqrt{\text{P.E.}_2} = \sqrt{\text{P.E.}}$

35. K.E. = P.E. $\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\therefore \quad \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$

$\therefore \quad A^2 - x^2 = x^2$

$\therefore \quad x^2 = \frac{A^2}{2} \Rightarrow \frac{x}{A} = \frac{1}{\sqrt{2}}$

36. $y = 0.05 \sin 4\pi(5t + 0.4)$

$\therefore \quad y = 0.05 \sin(20\pi t + 1.6\pi)$

Comparing this with standard equation,
$y = A \sin(\omega t + \alpha)$ we get,
$A = 0.05$, $\omega = 20\pi$

T.E. = $\frac{1}{2}m\omega^2 A^2 = \frac{1}{2} \times 0.1 \times (20\pi)^2 \times (0.05)^2$

$= \frac{1}{2} \times 10^{-1} \times 4 \times 10^2 \times \pi^2 \times 25 \times 10^{-4} = 0.05\pi^2$ J

37. Comparing the given equations with the standard form we get,
$A_1 = 4$, $A_2 = 5$, $\omega_1 = 10$

$E = \frac{1}{2} mA^2\omega^2 \Rightarrow E \propto (A\omega)^2$

$\therefore \quad (A_1\omega_1)^2 = (A_2\omega_2)^2 \Rightarrow A_1\omega_1 = A_2\omega_2$

$\therefore \quad 4 \times 10 = 5 \times \omega \Rightarrow \omega = 8$ unit

38. $\alpha = 30° = \frac{\pi}{6}$

Using $F = -kx$, we get
$|F_{max}| = kA = m\omega^2 A$

$\therefore \quad E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}|F_{max}| \times A$

$\therefore \quad A = \frac{2E}{|F_{max}|} = \frac{2 \times 3 \times 10^{-5}}{1.5 \times 10^{-3}} = 4 \times 10^{-2} = 0.04$ m

$\therefore \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s

$\therefore$ The equation of motion is, $x = A \sin(\omega t + \alpha)$

$= 0.04 \sin\left(\pi t + \frac{\pi}{6}\right)$

39. $$\frac{\text{K.E.}}{\text{P.E.}} = \frac{\frac{1}{2}m\omega^2\left(A^2 - x^2\right)}{\left(\frac{1}{2}m\omega^2 x^2\right)} = \frac{\left(A^2 - \frac{A^2}{n^2}\right)}{\left(\frac{A^2}{n^2}\right)} = n^2 - 1$$

40. $\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{4}\left(\frac{1}{2}m\omega^2 A^2\right)$

$\therefore \quad A^2 - x^2 = \frac{A^2}{4}$

$\therefore \quad x^2 = \frac{3A^2}{4} \Rightarrow x = \frac{\sqrt{3}A}{2}$

41. K.E. = $\frac{1}{2}m\omega^2A^2\cos^2\omega t$,

P.E. = $\frac{1}{2}m\omega^2A^2\sin^2\omega t$

K.E. – P.E. = $\frac{1}{2}m\omega^2A^2[\cos^2\omega t - \sin^2\omega t]$

$= \frac{1}{2}m\omega^2A^2 . \cos 2\omega t$

∴ Angular frequency = 2ω

∴ $T' = \frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{\pi \times T}{2\pi} = 2$ s

42. Force increases linearly. i.e. $F \propto -x$

∴ $\frac{F'}{F} = \frac{x'}{x}$

∴ $\frac{F'}{F} = \frac{A}{2} \times \left(-\frac{4}{A}\right) = -2$

∴ $F' = -2F \Rightarrow \frac{x'}{x} = -2$

Potential energy, P.E. $\propto x^2$

∴ $\frac{P.E.'}{P.E.} = \left(\frac{x'}{x}\right)^2 = (-2)^2 = 4$

∴ P.E.′ = 4P.E.

Speed of particle is given by

$v = \omega\sqrt{A^2 - x^2} \Rightarrow v \propto \sqrt{A^2 - x^2}$

At $x = \frac{-A}{4}$,

$v \propto \sqrt{A^2 - \left(\frac{A}{4}\right)^2} = \sqrt{\frac{15}{16}}A$

∴ At $x = \frac{A}{2}$,

$v \propto \sqrt{A^2 - \left(\frac{A}{2}\right)^2} = \sqrt{\frac{3}{4}}A$

∴ $\frac{v'}{v} = \sqrt{\frac{3}{4}} \times \sqrt{\frac{16}{15}} = \sqrt{\frac{4}{5}}$

∴ Velocity at x = A/2 may be $\pm\sqrt{\frac{4}{5}}v$

Kinetic energy will be

$\frac{K.E.'}{K.E.} = \left(\frac{v'}{v}\right)^2 = \frac{4}{5} = 0.8$

∴ K.E.′ = 0.8 K.E.

43. Total energy of a particle executing simple harmonic motion is constant.

44. $x_1 = A_1 \sin\omega t$ and

$x_2 = A_2 \sin(\omega t + \alpha)$

∴ $x = x_1 + x_2$

$= A_1 \sin\omega t + A_2(\sin\omega t \cos\alpha + \cos\omega t \sin\alpha)$

$= A_1 \sin\omega t + (A_2 \sin\omega t \cos\alpha + A_2 \cos\omega t \sin\alpha)$

$= \sin\omega t(A_1 + A_2\cos\alpha) + \cos\omega t(A_2 \sin\alpha)$

Let $R\cos\delta = A_1 + A_2\cos\alpha$

$R\sin\delta = A_2\sin\alpha$

R = amplitude of resultant

∴ $R^2\cos^2\delta + R^2\sin^2\delta$

$= (A_1 + A_2\cos\alpha)^2 + (A_2\sin\alpha)^2$

∴ $R^2(\cos^2\delta + \sin^2\delta)$

$= A_1^2 + A_2^2\cos^2\alpha + 2A_1A_2\cos\alpha + A_2^2\sin^2\alpha$

∴ $R^2(1) = A_1^2 + A_2^2 + 2A_1A_2\cos\alpha$

∴ $R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\alpha}$

45. $x_1 = A_1\sin(\omega t + \alpha_1)$ and $x_2 = A_2\sin(\omega t + \alpha_2)$

∴ $x = x_1 + x_2$

$= A_1\sin(\omega t + \alpha_1) + A_2\sin(\omega t + \alpha_2)$

$= A_1[\sin\omega t\cos\alpha_1 + \cos\omega t\sin\alpha_1] + A_2[\sin\omega t\cos\alpha_2 + \cos\omega t\sin\alpha_2]$

$= \sin\omega t(A_1\cos\alpha_1 + A_2\cos\alpha_2) + \cos\omega t(A_1\sin\alpha_1 + A_2\sin\alpha_2)$

Put $A_1\cos\alpha_1 + A_2\cos\alpha_2 = A\cos\phi$

$A_1\sin\alpha_1 + A_2\sin\alpha_2 = A\sin\phi$

∴ $x = A\cos\phi\sin\omega t + A\sin\phi\cos\omega t$

$= A\sin(\omega t + \phi)$

Hence resultant is S.H.M. with same period T.

46. $R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right)}$

$= \sqrt{25 + 12\sqrt{3}}$

47. Initial phase of resultant motion is given by,

$\delta = \tan^{-1}\left(\frac{a_1\sin\alpha_1 + a_2\sin\alpha_2}{a_1\cos\alpha_1 + a_2\cos\alpha_2}\right)$

$= \tan^{-1}\left(\frac{3 \times \frac{1}{2} + \frac{4 \times \sqrt{3}}{2}}{3 \times \frac{\sqrt{3}}{2} + 4 \times \frac{1}{2}}\right)$

$= \tan^{-1}\left(\frac{3 + 4\sqrt{3}}{4 + 3\sqrt{3}}\right)$

48. In vacuum, the bob will not experience any frictional force. Hence, there shall be no dissipation. Therefore, it will oscillate with a constant amplitude.

49. The stone executes S.H.M. about centre of earth with time period T = $2\pi\sqrt{\frac{R}{g}}$; where R = Radius of earth.

50. The rotation of earth about its axis is periodic but not to and fro about a fixed point, hence not a simple harmonic motion.

51. T cos θ = mg

$\therefore$ $T = m\omega^2 l = \frac{mg}{\cos\theta} = \frac{50\times10^{-3}\times10}{0.5} = 1\text{ N}$

52. Restoring force = |– mg sin θ|

$= 200 \times 10^{-3} \times 10 \times \sin 30°$

$= \frac{200\times10^{-2}}{2}$

= 1 N

53. Period of simple pendulum,

$T = 2\pi\sqrt{\frac{l}{g}}$

Now, $2T = 2\pi\sqrt{\frac{l'}{g}}$

$\therefore$ $\frac{T}{2T} = \frac{\sqrt{l}}{\sqrt{l'}} \Rightarrow l' = 4l$

54. $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore$ $\frac{T_e}{T_m} = \sqrt{\frac{g_m}{g_e}} = \sqrt{\frac{g_e/6}{g_e}} = \frac{1}{\sqrt{6}}$

$\therefore$ $T_m = \sqrt{6}T_e$ ⇒ clock becomes slower.

55. h = 10 cm = 10 × 10^{-2} m = 0.1 m

According to the principle of conservation of energy, $\frac{1}{2}mv^2 = mgh$

or $v = \sqrt{2gh} = \sqrt{2\times9.8\times0.1} = 1.4$ m/s

56. $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{98}{980}} = \frac{2\pi}{\sqrt{10}}$

$\therefore$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi/\sqrt{10}} = \sqrt{10}$

$\therefore$ $v_{max} = \omega A = \sqrt{10} \times 2\sqrt{10} = 20$ cm/s

57. Linear momentum will be maximum, if velocity of bob is maximum.

In S.H.M, $v_{max} = \omega A$ ….(i)

T.E. = $\frac{1}{2}m\omega^2A^2 = E$

$\frac{2E}{m} = \omega^2A^2 = v^2_{max}$ [From equation (i)]

$\therefore$ $v_{max} = \sqrt{\frac{2E}{m}}$

Linear momentum,

$P_{max} = mv_{max} = m\sqrt{\frac{2E}{m}} = \sqrt{2mE}$

58. $T' = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

$= 2\pi\sqrt{\frac{1}{9.8\times\cos60°}} = 2\pi\sqrt{\frac{1}{9.8\times1/2}}$

$= \sqrt{\frac{2}{9.8}} = \sqrt{\frac{1}{4.9}} = \sqrt{\frac{10}{49}}$

$= \frac{1}{7}\times3.16 = 0.45$ s

59. Period of a second's pendulum is 2 s.

It will perform 100 oscillations in 200 s

60. Function of wrist watch depends upon spring action so it is not affected by gravity but pendulum clock has time period, $T = 2\pi\sqrt{\frac{l}{g}}$.

During free fall, effective acceleration becomes zero. Hence time period comes out to be infinity i.e. the clock stops.

61. Let T_1 and T_2 be the time period of vibrations of pendulum A and B respectively.

Then, $T_1 = 2\pi\sqrt{\frac{l_1}{g}}$ and $T_2 = 2\pi\sqrt{\frac{l_2}{g}}$

$\therefore$ $\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1.69}{1.44}} = \frac{13}{12}$

If the two pendulums go out of phase in time t, then in time t, if pendulum A completes n vibrations, the pendulum B will complete (n + ½) vibrations.

$\therefore$ $t = nT_1 = (n + ½)T_2$

$\therefore$ $\frac{T_1}{T_2} = \frac{(n+1/2)}{n} = \frac{13}{12}$

$\therefore$ 12n + 6 = 13n or n = 6

$\therefore$ $n + \frac{1}{2} = 6 + \frac{1}{2} = 6.5$

62. $T_1 = T$

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{16}} = \frac{1}{4} \quad ....(i)$$

$x_1 = A \sin \omega_1 t$ and $x_2 = B \sin \omega_2 t$

They are in phase after time t and phase difference is 2π

$\therefore \quad \omega_1 t - \omega_2 t = 2\pi$

$\therefore \quad \left(\frac{2\pi}{T_1} - \frac{2\pi}{T_2}\right)t = 2\pi$

$\therefore \quad \left(\frac{1}{T_1} - \frac{1}{T_2}\right)t = 1$

$\therefore \quad \frac{t}{T_1}\left(1 - \frac{T_1}{T_2}\right) = 1$

$\therefore \quad \frac{t}{T}\left(1 - \frac{1}{4}\right) = 1 \quad ....[\text{From (i)}]$

$\therefore \quad \frac{t}{T} \times \frac{3}{4} = 1 \Rightarrow t = \frac{4}{3}T$

63. $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \quad T' = 2\pi\sqrt{\frac{l}{g - \frac{g}{5}}} = 2\pi\sqrt{\frac{l}{\frac{4g}{5}}}$

$\therefore \quad T' = \sqrt{\frac{5}{4}}\ 2\pi\sqrt{\frac{l}{g}} = \frac{\sqrt{5}}{2}\ T$

64. $T \propto \sqrt{l}$. Time period depends only on effective length. Density has no effect on time period. If length is made 4 times, then time period becomes 2 times.

65. $n_1 : n_2 = 7:8$

Suppose at t = 0, pendulums begins to swing simultaneously.

If $n_1T_1 = n_2T_2$,

$\therefore \quad \frac{n_1}{n_2} = \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$

$\therefore \quad \frac{l_1}{l_2} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{8}{7}\right)^2 = \frac{64}{49}$

66. $l_e = 1$ m, $g_m = g/6$

Time period of second's pendulum is 2 s

$T_e = T_m$

$\therefore \quad 2\pi\sqrt{\frac{l_e}{g_e}} = 2\pi\sqrt{\frac{l_m}{g_m}}$

$\therefore \quad l_m = \frac{l_e}{g_e} \times g_m = \frac{1}{g} \times \frac{g}{6} = \frac{1}{6}$ m

67. $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \quad T^2 = \frac{4\pi^2 l}{g}$ where $\frac{4\pi^2}{g}$ = constant

$\Rightarrow T^2 \propto l$

$\therefore \quad 2\left(\frac{dT}{T} \times 100\right) = \frac{dl}{l} \times 100$

$\therefore \quad \frac{dT}{T} \times 100 = \frac{1}{2}\left(\frac{dl}{l} \times 100\right) = \frac{1}{2} \times (2) = 1\ \%$

$\therefore$ There is change of 1% per second

$\therefore$ In a day, there are $24 \times 60 \times 60 = 24 \times 3600$ s

$\therefore \quad \frac{24 \times 3600 \times 1}{100} = 24 \times 36 = 864$ s

$\therefore$ There will be change of 864 s per day.

68. $\frac{dT}{T} = \frac{1}{2}\frac{dl}{l}$

$\therefore \quad \frac{dl}{l} = \alpha(t_2 - t_1) = \alpha(40 - 20) = \alpha(20)$

$\therefore \quad dT = T \times \frac{1}{2}\left(\frac{dl}{l}\right)$

$= T \times \frac{1}{2} \times \alpha \times 20$

$= 86400 \times \frac{1}{2} \times 12 \times 10^{-6} \times 20$

...[$\because$ 1 day = 86400 s]

$= 86400 \times 10^{-5} \times 12$

$= 0.864 \times 12 \approx 10.4$ seconds

69. $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l} \quad ....(i)$

$l_2 = l_1 + 69\%\ l_1 = \frac{169\ l_1}{100} \quad ....[\text{Given}]$

$\therefore \quad \frac{l_2}{l_1} = \frac{169}{100}$

$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{169}{100}} \quad ....[\text{From (i)}]$

$\therefore \quad \frac{T_2}{T_1} = \frac{13}{10}$

$\therefore \quad \frac{T_2 - T_1}{T_1} \times 100 = \frac{3}{10} \times 100$

$= 30\ \%$

70. When they are in phase again, the phase difference is 2π.

$\therefore$ $2\pi\left(\frac{1}{4}-\frac{1}{4.25}\right)t = 2\pi$

$\therefore$ $\frac{0.25}{4\times 4.25}t = 1$

$\therefore$ $t = \frac{17.00}{0.25} = 68$ s

71. $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto g^{1/2}$

$\therefore$ $dT \propto -\frac{1}{2}g^{-1/2}$

$\therefore$ $\frac{dT}{T} = -\frac{1}{2}\frac{dg}{g} = -\frac{1}{2}\times(-2\%) = 1\%$

$\therefore$ As acceleration due to gravity decreases, the time period increases.

72. $l_2 = l_1 + 300$ % of $l_1 = 4l_1$ ….[Given]

$\therefore$ $\frac{l_1}{l_2} = \frac{1}{4}$

Now, $T \propto \sqrt{l}$

$\therefore$ $\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{4}}$ $\therefore$ $T_2 = 2\ T_1$

Hence % increase = $\frac{T_2 - T_1}{T_1}\times 100 = 100$ %

73. Amplitude of damped oscillator,
$A = A_0e^{-\lambda t}$; λ = constant, t = time

For t =1 min., $\frac{A_0}{2} = A_0e^{-\lambda t} \Rightarrow e^{\lambda} = 2$

For t = 3 min., $A = A_0e^{-\lambda\times 3} = \frac{A_0}{(e^{\lambda})^3} = \frac{A_0}{2^3} = \frac{1}{x}$

$\therefore$ $x = 2^3$

74. In the first case, $A_1 = \frac{A_0}{3}$ and $t_1 = 100$ T

$\therefore$ $\frac{A_0}{3} = a_0e^{-100bT}$

$\therefore$ $e^{-100bT} = \frac{1}{3}$

In the second case,
$A_2 = A_0e^{-bt_2} = A_0e^{-200bt} = A_0(e^{-100bt})^2$

$\therefore$ $A_2 = A_0\left(\frac{1}{3}\right)^2 = \frac{A_0}{9}$

$\therefore$ The amplitude will be reduced to 1/9th of its initial value.

75. The initial mechanical energy of a harmonic oscillator at time t = 0 is $E_1 = \frac{1}{2}kA^2$

But because of damping, its energy at time t becomes $E_2 = \frac{1}{2}KA^2e^{\frac{-bt}{m}}$ where b is the damping constant. It is given that at time t, $E_2 = \frac{E_1}{2}$

$\therefore$ $\frac{E_1}{E_2} = \frac{1}{\left(e^{\frac{-bt}{m}}\right)} \Rightarrow \frac{E_1}{\left(\frac{E_1}{2}\right)} = 2 = e^{\frac{bt}{m}}$

$\therefore$ $\frac{bt}{m} = \log_e 2$

$\therefore$ $t = \frac{m\log_e 2}{b} = \frac{0.25\times\log_e 2}{0.05}$

$\therefore$ $t = 5\log_e 2$

76. For a damped oscillator, the amplitude after time t is, $A = A_0e^{-\lambda t}$, where λ is the damping constant.

$\therefore$ $\frac{A_0}{27} = A_0e^{-6\lambda}$ ….[$\because$ $A = \frac{A_0}{27}$]

$\therefore$ $e^{-6\lambda} = \frac{1}{27}$ ….(i)

Let A′ be the amplitude after 2 minutes
Then $A' = A_0e^{-2\lambda} = A_0[e^{-6\lambda}]^{1/3}$

$\therefore$ $A' = A_0\left(\frac{1}{27}\right)^{1/3} = \frac{A_0}{3}$

77. $U = k|x|^3$

$\therefore$ $F = -\frac{d(P.E.)}{dx} = -3k|x|^2$ ….(i)

Also, for S.H.M., $x = A\sin\omega t$ and
$\frac{d^2x}{dt^2} + \omega^2x = 0$

Acceleration, $a = \frac{d^2x}{dt^2} = -\omega^2x \Rightarrow F = ma$

$= m\frac{d^2x}{dt^2} = -m\omega^2x$ ....(ii)

From equation (i) and (ii) we get, $\omega = \sqrt{\frac{3kx}{m}}$

$\therefore$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3kx}} = 2\pi\sqrt{\frac{m}{3k(A\sin\omega t)}}$

$\therefore$ $T \propto \frac{1}{\sqrt{A}}$

78. $\sigma = 1, T' = \sqrt{2}\,T$

The effective acceleration of a bob in water $= g' = g\left(1-\dfrac{\sigma}{\rho}\right)$ where σ and ρ are the density of water and the bob respectively. Since the period of oscillation of the bob in air and water are given as,

$T = 2\pi\sqrt{\dfrac{l}{g}}$ and $T' = 2\pi\sqrt{\dfrac{l}{g'}}$ respectively,

∴ $\dfrac{T}{T'} = \sqrt{\dfrac{g'}{g}} = \sqrt{\dfrac{g(1-\sigma/\rho)}{g}} = \sqrt{1-\dfrac{\sigma}{\rho}} = \sqrt{1-\dfrac{1}{\rho}}$

Substituting, $\dfrac{T}{T'} = \dfrac{1}{\sqrt{2}}$, we obtain,

$\dfrac{1}{2} = 1-\dfrac{1}{\rho} \Rightarrow \rho = 2$

79. $k_1x_1 = k_2x_2 = F$

∴ $W_1 = \dfrac{1}{2}k_1x_1^2 = \dfrac{1}{2}k_1\left(\dfrac{F}{k_1}\right)^2 = \dfrac{F^2}{2k_1}$

Similarly, $W_2 = \dfrac{F_2^2}{2k_2} \Rightarrow W \propto \dfrac{1}{k}$

∴ $W_1 > W_2 \Rightarrow k_1 < k_2 \Rightarrow$ Reason is true.

∴ Assertion, $W_1 = \dfrac{1}{2}k_1x^2$ and $W_2 = \dfrac{1}{2}k_2x^2$

$\Rightarrow W_2 > W_1$

∴ Assertion is false.

80. Here, Assertion is false because, the direction of velocity in S.H.M. can be towards or away from mean position whereas the displacement is always away from mean position.

81. Time period of simple pendulum ($T = 2\pi\sqrt{l/g}$) is independent of the amplitude of vibration, when amplitude is small.

82. $K.E. = \dfrac{1}{2}k(A^2 - x^2)$

As $x = A\sin(\omega t + \alpha)$

∴ $K.E. = \dfrac{1}{2}k[A^2 - A^2\sin^2(\omega t + \alpha)]$

$= \dfrac{1}{2}kA^2[(1 - \sin^2(\omega t + \alpha)]$

$= \dfrac{1}{2}kA^2\cos^2(\omega t + \alpha)$ ….(i)

As $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$,

$\cos^2(\omega t + \alpha) = \dfrac{1+\cos 2(\omega t+\alpha)}{2}$

∴ Eq. (i) becomes

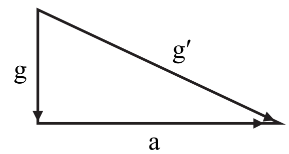
$K.E. = \dfrac{1}{2}kA^2\left(\dfrac{1+\cos 2(\omega t+\alpha)}{2}\right)$

∴ Kinetic energy of particle varies with two times of frequency of particle.

∴ If frequency of particle is 10 then the kinetic energy of the particle will vary with frequency $2 \times 10 = 20$

83. Potential energy of particle performing S.H.M. is given by, $P.E. = \dfrac{1}{2}m\omega^2x^2$, i.e., it varies parabolically such that at mean position, it becomes zero and maximum at extreme positions.

84. $T = 2\pi\sqrt{\dfrac{L}{g'}}$



Let $a' = (g^2 + a^2)^{1/2}$

∴ $T = 2\pi\sqrt{\dfrac{L}{(g^2+a^2)^{1/2}}}$

Competitive Thinking

1. $F = -kx$

3. Acceleration ∝ – displacement and acceleration is always directed towards the equilibrium position.

4. $v_{1max} = v_{2max}$

∴ $A_1\omega_1 = A_2\omega_2 \Rightarrow \dfrac{A_1}{A_2} = \dfrac{\omega_2}{\omega_1} = \sqrt{\dfrac{k_2}{m}\times\dfrac{m}{k_1}}$

$\dfrac{A_1}{A_2} = \left(\dfrac{k_2}{k_1}\right)^{\frac{1}{2}}$

5. $W_1 = \dfrac{1}{2}kx^2$

$W_2 = \dfrac{1}{2}(2k)x^2 = 2.\left(\dfrac{1}{2}kx^2\right)$

$\Rightarrow W_2 = 2W_1$

6. $T = 2\pi\sqrt{\dfrac{m}{K}}$

∴ $m = \dfrac{KT^2}{4\pi^2}$

∴ weight $= mg = \dfrac{KT^2}{4\pi^2}\times g = \dfrac{KT^2g}{4\pi^2}$

7. $T' = \frac{5T}{4} \Rightarrow \frac{T'}{T} = \frac{5}{4}$

Here, the hanging mass performs S.H.M.

With $T = 2\pi\sqrt{\frac{M}{k}}$ and

$T' = 2\pi\sqrt{\frac{M+m}{k}}$

$\therefore \frac{T'}{T} = \sqrt{\frac{M+m}{k} \times \frac{k}{M}}$

$\therefore \frac{5}{4} = \sqrt{\frac{M+m}{M}}$

$\therefore \frac{M+m}{M} = \frac{25}{16}$

$\therefore$ 9 M = 16 m $\Rightarrow \frac{m}{M} = \frac{9}{16}$

8. $T = 2\pi\sqrt{\frac{m}{k}}$

$\therefore T \propto \sqrt{m}$

i.e. $\frac{T_1}{T_2} = \sqrt{\frac{m_1}{m_2}}$

$m_1 = m$, $m_2 = m + 1$

$\therefore \frac{3}{5} = \sqrt{\frac{m}{m+1}}$

$\therefore \frac{m}{m+1} = \frac{9}{25}$

$\therefore$ 25m = 9m + 9

$m = \frac{9}{16}$

10. F = k x (in magnitude)

$\Rightarrow k = \frac{f}{x} = \frac{0.1 \times 10}{0.1} = 10$ N/m

Now, period of oscillations of the system,

$T = 2\pi\sqrt{\frac{m}{k}} = 2 \times 3.14 \times \sqrt{\frac{0.1}{10}} = 6.28 \times \frac{1}{10}$

$\therefore$ T = 0.628 s

11. Amplitude of resultant S.H.M.

$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos 90°}$

$R = \sqrt{A_1^2 + A_2^2} = \sqrt{a^2 + b^2}$

12. Standard equation of S.H.M., is of the type y = a sin ω t, y = a cos ωt or combination of the two.

But the equation, y = a tan ω t does not belong to any of these types.

13. $x = a\sin^2\omega t = \frac{a}{2}(1 - \cos 2\omega t)$

14. $y = A\sin\omega t$

$\therefore \frac{A}{2} = \frac{A\sin 2\pi}{T} \cdot t$

$\therefore \frac{2\pi t}{T} \Rightarrow \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore t = \frac{T}{12}$

(Note: Refer to Note 4.)

16. $y = a\sin\frac{2\pi}{T}t$

$\therefore \frac{a}{2} = a\sin\frac{2\pi t}{3}$

$\therefore \frac{1}{2} = \sin\frac{2\pi t}{3}$

$\therefore \sin\frac{2\pi t}{3} = \sin\frac{\pi}{6} \Rightarrow \frac{2\pi t}{3} = \frac{\pi}{6} \Rightarrow t = \frac{1}{4}$ s

17. $\frac{x}{a} = \sin\omega t$ and $\frac{y}{a} = \cos\omega t$

$\therefore \frac{y^2}{a^2} + \frac{x^2}{a^2} = 1 \Rightarrow y^2 + x^2 = a^2 \Rightarrow$ a circle

18. On comparing with standard equation $\frac{d^2y}{dt^2} + \omega^2 y = 0$ we get,

$\omega^2 = K \Rightarrow \omega = \frac{2\pi}{T} = \sqrt{K} \Rightarrow T = \frac{2\pi}{\sqrt{K}}$

19. $T \propto \sqrt{l}$,

$\therefore$ The effective lengths have the relation, $l_{sitting} > l_{standing} \Rightarrow (T)_{Sitting} > (T)_{Standing}$

20. For the given figure

$f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ ….(i)

If one spring is removed, then $k_{eq} = k$

$\therefore f' = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ ….(ii)

$\therefore$ From equations (i) and (ii), $\frac{f}{f'} = \sqrt{2} \Rightarrow f' = \frac{f}{\sqrt{2}}$

21. $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{4m}{m}} = 2$

$\Rightarrow T_2 = 2 \times 2 = 4$ s

22. $T \propto \frac{1}{\sqrt{k}} \Rightarrow T_1 : T_2 : T_3$

$= \frac{1}{\sqrt{k}} : \frac{1}{\sqrt{k/2}} : \frac{1}{\sqrt{2k}} = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}$

23. From the graph, T = 0.04 s

∴ $f = \frac{1}{T} = \frac{1}{0.04} = 25$ Hz

24. From graph, slope $K = \frac{F}{x} = \frac{8}{2} = 4$

$T = 2\pi\sqrt{\frac{m}{K}}$

∴ $T = 2\pi\sqrt{\frac{0.01}{4}} = 0.3$ s

25. In S.H.M., at mean position, velocity is maximum
So v = Aω (maximum)

26. $a_{max} = \omega^2 A$

27. Acceleration in S.H.M. is directly proportional to displacement and is always directed to its mean position.

28. Particle velocities are

$v_1^2 = \omega^2(A^2 - x_1^2)$

$v_2^2 = \omega^2(A^2 - x_2^2)$

On subtracting the relations

$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$

$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$

As $\omega = \frac{2\pi}{T}$ we get,

$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

29. the given equation can be written as,

$v^2 = \frac{1}{4}(25 - x^2)$

Comparing with general equation,

$v^2 = \omega^2(A^2 - x^2)$

∴ $\omega = \frac{1}{2} \Rightarrow T = \frac{2\pi}{\omega} = 4\pi$

30. For S.H.M., $v = \omega\sqrt{A^2 - x^2}$

$v_1 = v_0 = \omega_1\sqrt{A_1^2 - 0} = \omega_1 A_1 = \frac{2\pi A_1}{T_1}$

$v_2 = \omega_2\sqrt{A_2^2 - 0} = \omega_2 A_2 = \frac{2\pi A_2}{T_2}$

Given that, $A_2 = 2A_1$ and $T_2 = \frac{1}{3}T_1$

∴ $\frac{v_2}{v_1} = \frac{2\pi A_2}{T_2} \times \frac{T_1}{2\pi A_1} = \frac{T_1}{T_2} \times \frac{A_2}{A_1}$

∴ $\frac{v_2}{v_0} = 3 \times 2 = 6 \Rightarrow v_2 = 6v_0$

31. $v = \frac{v_{max}}{2}$ ….(Given)

x = a sin ωt

∴ v = aω cosωt and $v_{max} = a\omega$

∴ $a\omega \cos \omega t = \frac{a\omega}{2}$

∴ $\cos \omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$

∴ $x = a \sin\frac{\pi}{3} = \frac{\sqrt{3}a}{2}$

32. When velocity is u and acceleration is α, let the position of particle be x_1.
When velocity is v and acceleration is β, let the position of particle be x_2.
If ω is the angular frequency then,

$\alpha = \omega^2 x_1$

and $\beta = \omega^2 x_2$

∴ $\alpha + \beta = \omega^2(x_1 + x_2)$ ….(i)

Also, velocity of particle at particular instant can be given as,

$u^2 = \omega^2A^2 - \omega^2 x_1^2$

and $v^2 = \omega^2A^2 - \omega^2 x_2^2$

i.e., $v^2 - u^2 = \omega^2\left(x_1^2 - x_2^2\right)$

$v^2 - u^2 = \omega^2(x_1 - x_2)(x_1 + x_2)$ ….(ii)

from equation (i) we get

$v^2 - u^2 = (x_1 - x_2)(\alpha + \beta)$

∴ $x_1 - x_2 = \frac{v^2 - u^2}{\alpha + \beta}$

or $x_2 - x_1 = \frac{u^2 - v^2}{\alpha + \beta}$

33. At mean position, velocity is maximum.

∴ $v_{max} = A\omega$

∴ $v_1 = A\omega$

$v_2 = A_1\omega_1$

From conservation of linear momentum,

$m_1 v_1 = m\, v_2$

$\therefore \quad m_1 A\,\omega = (m_1 + m_2)\, A_1\, \omega_1$

$\therefore \quad \dfrac{A_1}{A} = \left(\dfrac{m_1}{m_1 + m_2}\right)\dfrac{\omega}{\omega_1}$

But $\omega = \sqrt{\dfrac{k}{m_1}}$; $\omega_1 = \sqrt{\dfrac{k}{m_1 + m_2}}$

$\therefore \quad \dfrac{A_1}{A} = \left(\dfrac{m_1}{m_1 + m_2}\right)\sqrt{\dfrac{k}{m_1}\dfrac{(m_1 + m_2)}{k}}$

$= \left(\dfrac{m_1}{m_1 + m_2}\right)\left(\dfrac{m_1 + m_2}{m_1}\right)^{1/2}$

$\therefore \quad \dfrac{A_1}{A} = \sqrt{\dfrac{m_1}{m_1 + m_2}}$

34. $x = 8 \sin\omega t + 6 \cos\omega t$

$= 8 \sin \omega t + 6 \sin\left(\omega t - \dfrac{\pi}{2}\right)$

$\therefore \quad R = \sqrt{8^2 + 6^2} = 10$ cm

35. $A = 50$ mm $= 50 \times 10^{-3}$m

$\therefore \quad v_{max} = A\omega = A \times \dfrac{2\pi}{T}$

$= (50 \times 10^{-3}) \times \dfrac{2\pi}{2} \approx 0.16$ m/s

36. Maximum acceleration is given as,

$\alpha = A\omega^2$ ....(i)

Maximum velocity is given as,

$\beta = A\omega$ ....(ii)

Dividing equation (i) by equation (ii), we get

$\dfrac{\alpha}{\beta} = \omega \Rightarrow \dfrac{\alpha}{\beta} = \dfrac{2\pi}{T}$

$T = 2\pi\dfrac{\beta}{\alpha}$

37. Given,

$A = 2$ m ; $x = 1$ m

$a_{max} - v_{max} = 4$

$\therefore \quad \omega^2 A - \omega A = 4$

$\therefore \quad (\omega^2 - \omega)A = 4$

$\therefore \quad (\omega^2 - \omega)\, 2 = 4$

$\therefore \quad \omega^2 - \omega - 2 = 0$

$\therefore \quad \omega^2 - 2\omega + \omega - 2 = 0$

$\omega(\omega - 2) + 1\, (\omega - 2) = 0$

$\therefore \quad (\omega + 1)\, (\omega - 2) = 0$

$\therefore \quad \omega = 2$ rad/s

{$\omega \neq -1$, $\because$ Angular velocity cannot be negative}

Time period, $T = \dfrac{2\pi}{\omega}$

$\therefore \quad T = \dfrac{2\pi}{2} = \pi = \dfrac{22}{7}$ s

velocity of particle at x = 1 is given by

$v = \omega\sqrt{A^2 - x^2} = 2\sqrt{(2)^2 - (1)^2} = 2\sqrt{3}$ m/s

38. Using $v = \omega\sqrt{A^2 - x^2}$

$\therefore \quad v^2 = \omega^2 (A^2 - x^2)$

$\therefore \quad \dfrac{v^2}{\omega^2} = A^2 - x^2$

$\therefore \quad \dfrac{v^2}{\omega^2} + x^2 = A^2$

$\therefore \quad$ Case 1: $\dfrac{13^2}{\omega^2} + 3^2 = A^2$ ….(i)

Case 2: $\dfrac{12^2}{\omega^2} + 5^2 = A^2$ ….(ii)

From equation (i) and (ii)

$\dfrac{13^2}{\omega^2} + 3^2 = \dfrac{12^2}{\omega^2} + 5^2$

$\therefore \quad \dfrac{1}{\omega^2}(13^2 - 12^2) = 5^2 - 3^2$

$\therefore \quad \dfrac{1}{\omega^2} = \dfrac{25 - 9}{169 - 144}$

$\therefore \quad \dfrac{1}{\omega^2} = \dfrac{16}{25}$

$\therefore \quad \omega = \dfrac{5}{4}$ rad/s

$\therefore \quad$ But $f = \dfrac{\omega}{2\pi} = \dfrac{5}{4} \times \dfrac{1}{2\pi} = \dfrac{5}{8\pi}$

39. K.E. = P.E.

$\therefore \quad \dfrac{1}{2} m\omega^2 (A^2 - x^2) = \dfrac{1}{2} m\omega^2 x^2$

$\therefore \quad A^2 - x^2 = x^2 \Rightarrow 2x^2 = A^2 \Rightarrow x = \dfrac{A}{\sqrt{2}}$

$\therefore \quad x = 0.71A$

40. From the given equation, A = 5 and ω = 4,

x = 3

$\therefore \quad v = \omega\sqrt{a^2 - x^2} = 4\sqrt{(5)^2 - (3)^2} = 16$

41. $x = 0.25 \sin (200\, t)$

Comparing with $x = A \sin\omega t$,

$A = 0.25$ m, $\omega = 200$ rad/s

$\therefore \quad v_{max} = A\omega = 0.25 \times 200 = 50$ m/s

42. Acceleration, $a = \omega^2 x$

$\therefore$ $16 \times 10^{-2} = \omega^2(4 \times 10^{-2})$

$\omega = 2$ rad/s

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi = 3.142$ s

43. Acceleration, $a = \omega^2 x$

$\omega = \sqrt{\frac{a}{x}} = \sqrt{\frac{20}{5}}$ ....($\because$ a = 20 m/s², x = 5 m)

$\omega = 2$ rad/s

Period, $T = \frac{2\pi}{\omega} = \pi$ s

44. $a = \omega^2 x$

$\therefore$ $\omega = \sqrt{a/x} = \sqrt{\frac{8}{2}} = 2$ rad/s

$\therefore$ $v_{max} = A\omega = 6 \times 2 = 12$ cm/s

45. $v_{max} = A\omega$ and $a_{max} = A\omega^2$

$\therefore$ $\omega = \frac{a_{max}}{v_{max}} = \frac{4}{2} = 2$ rad/s

46. Given, ($a_{max} = 1.0$ m/s² $v_{max} = 0.5$ ms⁻¹)

$a_{max} = \omega^2 A = \omega(\omega A) = \omega v_{max}$

$\therefore$ $\omega = \frac{a_{max}}{v_{max}} = \frac{1}{0.5}$

$\therefore$ $\omega = 2$ rad/s

47. $v_{max} = A\omega$ and $a_{max} = A\omega^2$

$\therefore$ $\frac{a_{max}}{v_{max}} = \frac{A\omega^2}{A\omega} = \frac{0.64}{0.16} \Rightarrow \omega = 4$ rad/s

$\therefore$ $0.16 = A \times 4 \Rightarrow A = 0.04$ m $= 4 \times 10^{-2}$ m

48. $a_{max} = A\omega^2$

$\therefore$ $A = \frac{a_{max}}{\omega^2} = \frac{7.5}{(3.5)^2} = 0.61$ m

49. $v_{max} = A\omega$

$\therefore$ $\omega = \frac{v_{max}}{A} = \frac{10}{4}$

Now, $v = \omega\sqrt{A^2 - x^2}$

$\therefore$ $v^2 = \omega^2(A^2 - x^2)$

$\therefore$ $x^2 = A^2 - \frac{v^2}{\omega^2}$

$\therefore$ $x = \sqrt{A^2 - \frac{v^2}{\omega^2}} = \sqrt{4^2 - \frac{5^2}{(10/4)^2}} = 2\sqrt{3}$ cm

50. Displacement of the particle, $x = A \sin \omega t$

Velocity of the particle,

$v = \frac{dx}{dt} = A\omega \cos \omega t$ ....(i)

Given that,

$v = \pi$ m/s, T = 16 s,

$\therefore$ $\omega = \frac{2\pi}{T} = \frac{\pi}{8}$ rad/s

Substituting in equation (i), we get,

$\pi = A \times \frac{\pi}{8} \times \cos\left(\frac{\pi}{8} \times 2\right)$

$\therefore$ $1 = \frac{A}{8} \cos\left(\frac{\pi}{4}\right) = \frac{A}{8} \times \frac{1}{\sqrt{2}}$

$\therefore$ $A = 8\sqrt{2}$ m

51. Velocity, $v = \omega\sqrt{A^2 - x^2}$ and

acceleration $= \omega^2 x$

Now given that, $\omega^2 x = \omega\sqrt{A^2 - x^2}$

$\therefore$ $\omega^2.1 = \omega\sqrt{2^2 - 1^2} \Rightarrow \omega = \sqrt{3}$

$\therefore$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$

52. Given: A = 3 cm

when x = 2 cm, v = a

i.e., $\omega\sqrt{A^2 - x^2} = \omega^2 x$

$\therefore$ $\frac{\omega^2}{\omega} = \frac{\sqrt{A^2 - x^2}}{x}$

$\therefore$ $\omega = \frac{\sqrt{A^2 - x^2}}{x} = \frac{\sqrt{3^2 - 2^2}}{2}$

$\omega = \frac{\sqrt{5}}{2}$ rad/s

$\therefore$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\sqrt{5}/2\right)} = \frac{4\pi}{\sqrt{5}}$ s

53. The coin will leave contact when it is at the highest point and for that condition

Maximum acceleration = Acceleration due to gravity

$\therefore$ $\omega^2 A = g \Rightarrow A = \frac{g}{\omega^2}$

54. For S.H.M., $\frac{d^2y}{dt^2} \propto -y$

55. Velocity of a particle executing S.H.M. is given by

$$v = \omega\sqrt{A^2 - x^2}$$

$$= \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{2\pi}{T}\sqrt{\frac{3A^2}{4}} = \frac{\pi A\sqrt{3}}{T}$$

56. Velocity of particle performing SHM is given by, $v = \omega\sqrt{A^2 - x^2}$

When the particle is at a distance $\frac{2A}{3}$ from equilibrium position it's speed is,

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$$

$$= \omega\sqrt{A^2 - \frac{4A^2}{9}} = \omega\sqrt{\frac{5A^2}{9}}$$

$\therefore \quad v = \frac{\omega\sqrt{5}A}{3}$

Now, $v' = 3v = 3 \times \frac{\omega\sqrt{5}}{3}A = \omega\sqrt{5}\,A$

But $v' = \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$

Where A' is new amplitude of motion,

$\therefore \quad \omega\sqrt{5}\,A = \omega\sqrt{(A')^2 - \left(\frac{4A^2}{9}\right)}$

$\therefore \quad 5A^2 = (A')^2 - \frac{4A^2}{9}$

$\therefore \quad (A')^2 = 5A^2 + \frac{4A^2}{9}$

$(A')^2 = \frac{49A^2}{9}$

$\therefore \quad A' = \frac{7}{3}A$

57. $v = \omega\sqrt{(A^2 - x^2)} = 2\sqrt{60^2 - 20^2} \approx 113\,\text{mm/s}$

58. Acceleration, $a = \omega^2 x$

$\therefore \quad \frac{aT}{x} = \frac{\omega^2 xT}{x} = \omega^2 T = \left(\frac{2\pi}{T}\right)^2 T = \frac{4\pi^2}{T}$

It is a constant term for S.H.M. i.e., it does not change with time.

59. Maximum acceleration,

$\omega^2 A = A \times 4\pi^2 n^2$

$= 0.01 \times 4 \times (\pi)^2 \times (60)^2$

$= 144\pi^2 \text{m/s}^2$

60. Maximum acceleration, $a_{max} = \omega^2 A$

Amplitude remaining constant, $a_{max} \propto \omega^2$

$$\frac{(a_{max})_1}{(a_{max.})_2} = \left(\frac{\omega_1}{\omega_2}\right)^2 = \left(\frac{100}{1000}\right)^2 = \left(\frac{1}{10}\right)^2$$

$\therefore$ Ratio of max. accelerations $= \frac{1}{10^2}$

61. $2A = 4\text{ cm} \Rightarrow A = \frac{4}{2} = 2\text{ cm}$

$a_{max} = A\omega^2 = A\cdot\frac{4\pi^2}{T^2}$

$\therefore \quad T = 2\pi\sqrt{\frac{A}{a_{max}}} = 2\times\pi\times\sqrt{\frac{2}{2\pi^2}} = 2\pi\times\frac{1}{\pi} = 2\text{ s}$

62. $T = \frac{2\pi}{\sqrt{3}}\text{s}$, $2A = 4\text{ cm} \Rightarrow A = 2\text{ cm}$

$v = A$ …(Given)

$\therefore \quad \omega\sqrt{A^2 - x^2} = \omega^2 x$ …(Numerically)

$\therefore \quad A^2 - x^2 = \omega^2 x^2 \Rightarrow x^2 = \frac{A^2}{\omega^2 + 1}$

$\therefore \quad x^2 = \frac{A^2}{\left(\frac{4\pi^2}{T^2} + 1\right)} = \frac{(2)^2}{\left(\frac{4\pi^2 \times 3}{4\pi^2} + 1\right)} = \frac{4}{4} = 1$

$\Rightarrow x = 1\text{ cm}$

63. r = 10 cm for the particle performing U.C.M. Now, projection of U.C.M. along any diameter of the circle is an S.H.M.
Hence, in the given example,
A = r = 10 cm

64. $a_{max} = A\omega^2 = A\cdot\frac{4\pi^2}{T^2} = \frac{3\times4\times(3.14)^2}{(2\times3.14)^2}$

$= \frac{12}{4} = 3\text{ cm/s}^2$

65. As the body starts from mean position,

$v = A\omega\cos\omega t$

$\therefore \quad v = A\times\frac{2\pi}{T}\times\cos\left(\frac{2\pi t}{T}\right)$

$\therefore \quad \pi = A\times\frac{2\pi}{24}\times\cos\left(\frac{2\pi\times4}{24}\right)$

$= \frac{A\pi}{12}\times\cos\left(\frac{\pi}{3}\right) = \frac{A\pi}{12}\times\frac{1}{2}$

$\therefore \quad A = \frac{24\pi}{\pi} = 24\,\text{m}$

$\therefore$ Path length = 2A = 48 m

67. Wavelength = velocity of wave × Time period
$\lambda = 300 \times 0.05 \Rightarrow \lambda = 15$ metre
According to problem, path difference between two points = 15 – 10 = 5m

$\therefore$ Phase difference = $\frac{2\pi}{\lambda} \times$ Path difference

$$= \frac{2\pi}{15} \times 5 = \frac{2\pi}{3}$$

69. From the graph of velocity (v) v/s distance (x), we see that the particle executes S.H.M. whose time is recorded from the extreme position.

70. $E = \frac{1}{2} m\omega^2 A^2 \Rightarrow E \propto A^2$

71. Total energy = $\frac{1}{2} m\omega^2 A^2$ = constant

75. K.E. = $\frac{1}{2} m\omega^2 (A^2 - x^2)$

P.E. = $\frac{1}{2} m\omega^2 x^2$

At extreme position, x = A

$\Rightarrow$ K.E. = 0 and P.E. = $\frac{1}{2} m\omega^2 A^2$

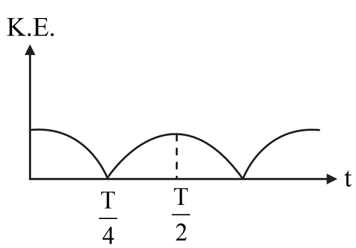
At mean position, x = 0

K.E. = $\frac{1}{2} m\omega^2 A^2$ and P.E. = 0

$\Rightarrow$ K.E. increases and P.E. decreases.

76. K.E. = $\frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$

K.E. is maximum at mean position and minimum at extreme position and extreme position is reached at every $\frac{T}{4}$. This is best depicted by graph (B).



77. T.E. = $\frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m \left(\frac{2\pi}{T}\right)^2 A^2$

$$= \frac{1}{2} m \times \frac{4\pi^2 A^2}{T^2} = \frac{2\pi^2 m A^2}{T^2}$$

78. $x = \frac{A}{2}$

$W = \frac{1}{2} m\omega^2 A^2$

$\therefore$ K.E. $= \frac{1}{2} m\omega^2 (A^2 - x^2)$

$$= \frac{1}{2} m\omega^2 \left(A^2 - \frac{A^2}{4}\right) = \frac{3}{8} m\omega^2 A^2$$

$$= \frac{3}{4}\left(\frac{1}{2} m\omega^2 A^2\right) = \frac{3W}{4}$$

P.E. $= \frac{1}{2} m\omega^2 x^2$

$$= \frac{1}{2} m\omega^2 \times \frac{A^2}{4} = \frac{1}{8} m\omega^2 A^2 = \frac{1}{4} W$$

79. K.E. = $\frac{1}{2} m\omega^2 (A^2 - x^2)$

P.E. = $\frac{1}{2} m\omega^2 x^2$

$\therefore$ $\frac{\text{K.E.}}{\text{P.E.}} = \frac{A^2 - x^2}{x^2}$

80. K.E. = $\frac{1}{2} m\omega^2 (A^2 - x^2)$,

P.E. = $\frac{1}{2} m\omega^2 x^2$

$\frac{\text{K.E.}}{\text{P.E.}} = \frac{A^2 - x^2}{x^2}$

Here $x = \frac{A}{2}$

$\therefore$ $\frac{\text{K.E.}}{\text{P.E.}} = \frac{A^2 - \frac{A^2}{4}}{\frac{A^2}{4}} = \frac{3A^2}{4} \times \frac{4}{A^2} = \frac{3}{1}$

81. $U = \frac{1}{2} kx^2$ but T = kx

So energy stored = $\frac{1}{2} \frac{(kx)^2}{k} = \frac{1}{2} \frac{T^2}{k}$

82. K.E. = $\frac{1}{2} mv^2 = \frac{1}{2} mA^2\omega^2 \cos^2 \omega t$

$= \frac{1}{2} m\omega^2 A^2 \left(\frac{1 + \cos 2\omega t}{2}\right)$ hence kinetic energy varies periodically with double the frequency of S.H.M. i.e. 2f.

83. T.E. = $\frac{1}{2}m\omega^2A^2$,
(where A = amplitude) Potential energy
K.E. = $\frac{1}{2}m\omega^2(A^2 - x^2)$
$= \frac{1}{2}m\omega^2\left[A^2 - \left(\frac{A}{2}\right)^2\right]$
$= \frac{1}{2}m\omega^2 \times \frac{3A^2}{4} = \frac{1}{2}\ m\omega^2A^2\left(\frac{3}{4}\right)$
∴ K.E. = $\frac{3}{4}$ T.E.

84. K.E. at mean position
$= \frac{1}{2}m\omega^2(A^2 - 0) = \frac{1}{2}m\omega^2A^2$
P.E. at x = $\frac{A}{2} = \frac{1}{2}m\omega^2\left(\frac{A}{2}\right)^2 = \frac{1}{8}m\omega^2A^2$
∴ The required ratio
$= \frac{\left(\frac{1}{2}m\omega^2A^2\right)}{\left(\frac{1}{8}m\omega^2A^2\right)} = 4:1$

85. T.E. in S.H.M. = $K.E._{max}$ = $P.E._{max}$. Here, the maximum kinetic energy of the oscillator.
$K.E._{max}$ is $\frac{1}{2}kA^2$
$= \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = 100$ J
But T.E. ≠ 100J.
∴ P.E. at equilibrium position = 160–100 = 60 J.
∴ $P.E._{max}$ = 100 + 60 = 160 J

86. $\frac{P.E.}{P.E._{max}} = \frac{\frac{1}{2}m\omega^2x^2}{\frac{1}{2}m\omega^2A^2}$
∴ $\frac{1}{4} = \frac{x^2}{A^2} \Rightarrow x = \frac{A}{2}$

87. x = 0 at mean position,
T.E. of S.H.M. = $\frac{1}{2}m\omega^2A^2$
∴ $25 = \frac{1}{2} \times 0.5 \times \omega^2A^2$
∴ $\omega^2A^2 = 100 \Rightarrow \omega A = 10 = v_{max}$
∴ The particle in S.H.M. has maximum velocity when it passes through mean position.
∴ v = 10 m/s

88. K.E. = $\frac{3}{4} \times$ T.E.
$\Rightarrow \frac{1}{2}m\omega^2(a^2 - x^2) = \frac{3}{4} \times \frac{1}{2}m\omega^2a^2$
$\Rightarrow 4(a^2 - x^2) = 3a^2$ which on solving gives
$a = \pm 2x$ or $x = \pm\frac{a}{2}$

89. $K.E_{max} = \frac{1}{2}m\omega^2\ a^2$
Comparing with standard equation
a = 8 cm, ω = 100 rad/s^2
∴ $K.E_{max} = \frac{1}{2} \times 4 \times 10^4 \times 64 \times 10^{-4} = 128$ J

90. $W_1 = \frac{1}{2}\ kx^2$ and
$W_2 = \frac{1}{2}\ k(x + y)^2$
$W_2 - W_1 = \frac{1}{2}\ k\ (x^2 + 2xy + y^2) - \frac{1}{2}kx^2$
$= \frac{ky}{2}\ (2x + y)$

91. $W_1 = \frac{1}{2}\ kx^2$ ….(i)
$W_2 = \frac{1}{2}\ k(2x)^2$ ….(ii)
Dividing equation (i) by (ii),
$\frac{W_1}{W_2} = \frac{1}{4}$
∴ $W_2 = 4W_1$
$\Delta W = W_2 - W_1$
$= 4W_1 - W_1$
$= 4 \times 10 - 10$
= 30 J

92. $v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$
$v_2 = \frac{dy_2}{dt} = -0.1\pi\sin\pi t = 0.1\pi\cos\left(\pi t + \frac{\pi}{2}\right)$
Phase difference of velocity of first particle with respect to the velocity of 2nd particle at t = 0 is
$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$

93. Resultant amplitude = $\sqrt{3^2 + 4^2} = 5$

94. If first equation is $x_1 = A_1 \sin \omega t$,

$\frac{x_1}{A_1} = \sin\omega t$ ….(i)

then second equation will be

$x_2 = A_2 \sin\left(\omega t + \frac{\pi}{2}\right)$

$= A_2\left[\sin\omega t\cos\frac{\pi}{2} + \cos\omega t\sin\frac{\pi}{2}\right]$

$= A_2 \cos\omega t$

$\cos \omega t = \frac{x_2}{A_2}$ ….(ii)

By squaring and adding equation (i) and (ii)

$\sin^2 \omega t + \cos^2 \omega t = \frac{x_1^2}{A_1^2} + \frac{x_2^2}{A_2^2}$

$\frac{x_1^2}{A_1^2} + \frac{x_2^2}{A_2^2} = 1$; This is the equation of ellipse.

95. If $x_1 = A_1 \sin \omega t$ and $x_2 = A_2 \sin(\omega t + 0)$
$= A_2 \sin \omega t$

But $A_1 = A_2$

∴ $x_2 = x_1$

This represents a straight line.

96. For a simple pendulum,

$T \propto \sqrt{l}$ or $T^2 \propto l$

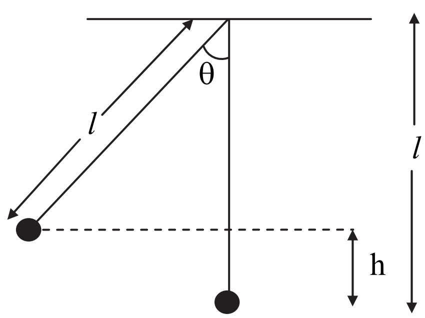
∴ $E \propto \omega^2 \propto \frac{1}{T^2} \Rightarrow E \propto \frac{1}{l}$

Hence energy will become two times if length is halved.

97. Inside the mine, g decreases.

Hence from $T = 2\pi\sqrt{\frac{l}{g}}$, we conclude that T increases.

99.



From figure,

$\cos\theta = \frac{l - h}{l}$

∴ $h = l - l\cos\theta = l(1 - \cos\theta)$

P.E = mgh

∴ P.E = $mgl(1 - \cos\theta)$

K.E. is maximum at mean position, which is equal to maximum P.E. at extreme position.

∴ $(K.E.)_{max} = mgl(1 - \cos\theta)$

100. Potential energy of particle at extreme position is, P.E. $= \frac{1}{2}M\omega^2A^2$

$= \frac{1}{2}M \times \frac{g}{L} \times A^2$ ….$\left(\because \omega = \sqrt{\frac{g}{l}}\right)$

101. When a little mercury is drained off, the position of c.g. of ball falls (w.r.t. fixed end) so that effective length of pendulum increases hence T increases.

103. $T = 2\pi\sqrt{\frac{l}{g}}$

∴ $\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{g + \frac{g}{4}}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

104. $T = 2\pi\sqrt{\frac{l}{g}}$ ….(i)

When the lift moves upwards with acceleration a,

$T' = 2\pi\sqrt{\frac{l}{g + a}}$

∴ $\frac{T}{2} = 2\pi\sqrt{\frac{l}{g + a}}$ ….(ii)

∴ Dividing equation (ii) by equation (i) we get,

a = 3g

105. $T = 2\pi\sqrt{\frac{l}{g}}$

∴ $T \propto \frac{1}{\sqrt{g}}$

$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{\left(\frac{g}{2}\right)}} = \sqrt{\frac{2}{1}}$

∴ $T' = \sqrt{2}\,T$

106. On earth's surface, $g = \frac{GM}{R^2}$

$\therefore$ At a height R, $g_R = \frac{GM}{(R+R)^2} = \frac{GM}{4R^2} = \frac{1}{4}.\frac{GM}{R^2}$

$\therefore$ $g_R = \frac{1}{4}g$

Now, $T \propto \frac{1}{\sqrt{g}} \Rightarrow T_1 \propto \frac{1}{\sqrt{g}}$ and $T_2 \propto \frac{1}{\sqrt{g_R}}$

$\therefore$ $\frac{T_1}{T_2} = \sqrt{\frac{g_R}{g}} = \sqrt{\frac{1}{4}} = 0.5$

107. $l_2 = 44\%$ of $l_1 \Rightarrow l_2 = 1.44l$

$T \propto \sqrt{l} \Rightarrow T_1 \propto \sqrt{l_1}$ and $T_2\ \sqrt{l_2}$

$\therefore$ $\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{T_2}{T_1} = \sqrt{1.44} = 1.2$

$\therefore$ % change in $T = \frac{T_2 - T_1}{T_1} \times 100 = \frac{(1.2-1)}{1} \times 100$

$= 20\%$

108. At B, the velocity is maximum. Using conservation of mechanical energy,

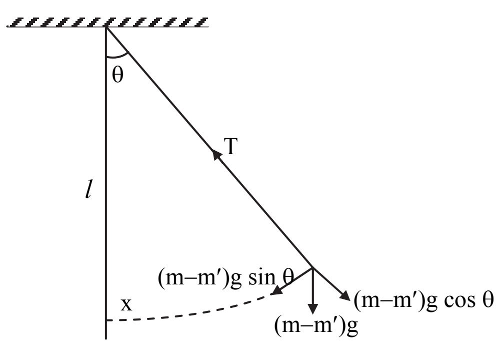
ΔP.E. = ΔK.E.

$\therefore$ $mgH = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gH}$

109. Time period is independent of mass of bob of pendulum.

110. As pendulum is immersed in liquid, its apparent weight is $mg - m'g$.

It is evident from the figure that restoring force on bob is-



$F = -(mg - m'g)\sin\theta$

for small θ, $\sin\theta \simeq \theta$

Hence,

$F = -(mg - m'g)\theta$

But $\theta = \frac{x}{l}$

$\therefore$ $F = -(mg - m'g)\ \frac{x}{l}$

Now, $mg = \rho vg$ and $m'g = \sigma vg$

ρ = density of brass bob, σ = density of liquid

But $\sigma = \frac{1}{10}\rho$

$\therefore$ $F = -\left(\rho vg - \frac{1}{10}\rho vg\right)\frac{x}{l} = \frac{-9}{10}\rho vg\ \frac{x}{l}$

$\therefore$ $F = \frac{-9}{10}mg\frac{x}{l}$

$\therefore$ $ma = \frac{-9}{10}mg\frac{x}{l}$

$\Rightarrow a = \frac{-9}{10}g\frac{x}{l}$ but $a = -\omega^2 x$

$\Rightarrow \omega^2 = \frac{9}{10}\frac{g}{l}$

$\Rightarrow \omega = \sqrt{\frac{9}{10}\frac{g}{l}}$

$\Rightarrow \omega = \frac{2\pi}{T'} = \sqrt{\frac{9}{10}\frac{g}{l}}$

$\Rightarrow T' = 2\pi\sqrt{\frac{l}{g}}\sqrt{\frac{10}{9}}$

$\Rightarrow T' = T\sqrt{\frac{10}{9}}$

111. $T = 2\pi\sqrt{\frac{l}{g}}$

Time period of pendulum of bob with material density 'σ' oscillating in liquid of density 'ρ' is

$T_1 = 2\pi\sqrt{\frac{l}{\left(1 - \frac{\rho}{\sigma}\right)g}}$

$\therefore$ $\frac{T_1}{T} = \frac{1}{\left(1 - \frac{\rho}{\sigma}\right)^{1/2}}$

Given $\rho = 1$ g/cc $= 10^3$ kg/m^3

$\sigma = \frac{9}{8} \times 10^3$ kg/m^3

$\frac{T_1}{T} = \frac{1}{\left(1 - \frac{10^3}{\frac{9}{8} \times 10^3}\right)^{1/2}}$

$\therefore$ $T_1 = 3T$

112. If t is the time taken by pendulums to come in same phase again first time after t = 0.
and N_S = Number of oscillations made by shorter length pendulum with time period T_S.
N_L = Number of oscillations made by longer length pendulum with time period T_L.
Then $t = N_S T_S = N_L T_L$

$\therefore \quad N_S \times 2\pi\sqrt{\frac{5}{g}} = N_L \times 2\pi\sqrt{\frac{20}{g}} \qquad (\because T = 2\pi\sqrt{\frac{l}{g}})$

$\therefore \quad N_S = 2N_L$ i.e. if $N_L = 1$, then $N_S = 2$

113. $T = 2\pi\sqrt{l/g} = 2\pi\sqrt{\frac{1}{\pi^2}} = 2$ s

114. Time period of simple pendulum,

$T = 2\pi\sqrt{\frac{l}{g'}}$

$= 2\pi\sqrt{\frac{1}{(g+2)}}$

$= 2\pi\sqrt{\frac{1}{12}}$

$= \frac{\pi}{\sqrt{3}}$

115. $T_A = \frac{20}{10} = 2$ sec

$T_B = \frac{10}{8} = 1.25$ sec

But $T \propto \sqrt{l}$

$\therefore \quad \frac{T_A}{T_B} = \sqrt{\frac{l_A}{l_B}}$

$\therefore \quad \frac{l_A}{l_B} = \frac{T_A^2}{T_B^2} = \frac{2^2}{1.25^2}$

$\frac{l_A}{l_B} = \frac{64}{25}$

116. For simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \quad T \propto \sqrt{l}$

Now, $T_2 = \frac{T_1}{2}$, $l_2 = l_1 - 0.6$ ….(given)

$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_2}{l_1} = \frac{T_2^2}{T_1^2}$

$\Rightarrow \frac{l_1 - 0.6}{l_1} = \frac{T_1^2}{4T_1^2}$

$\Rightarrow 4l_1 - 2.4 = l_1$

$\Rightarrow 3l_1 = 2.4 \Rightarrow l_1 = 0.8$ m

$\Rightarrow l_1 = 800$ mm

117. Given $l_2 = (l_1 + 0.36)$ m ; $T_2 = \left(T_1 + \frac{25}{100}T_1\right)$

Time period of simple pendulum is given by,

$T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \quad T \propto \sqrt{l}$

$\therefore \quad l \propto T^2$

$\therefore \quad \left(\frac{l_1}{l_2}\right) = \left(\frac{T_1}{T_2}\right)^2$

$\therefore \quad \left(\frac{l_1}{l_1 + 0.36}\right) = \left(\frac{T_1}{T_1 + 0.25\,T_1}\right)^2$

$\therefore \quad \left(\frac{l_1}{l_1 + 0.36}\right) = \left(\frac{T_1}{1.25\,T_1}\right)^2$

$\therefore \quad (1.25)^2\, l_1 = l_1 + 0.36$

$1.56\, l_1 = l_1 + 0.36$

$\therefore \quad 0.56\, l_1 = 0.36$

$\therefore \quad l_1 = \frac{0.36}{0.56}$

$l_1 = 0.64$ m

$\therefore \quad l_1 = 64$ cm

119. A particle oscillating under a force $\vec{F} = -k\vec{x} - b\vec{v}$ is a damped oscillator. The first term $-k\vec{x}$ represents the restoring force and second term $-b\vec{v}$ represents the damping force.

120. The given relation can be written as,
$x = 4\cos\pi t + 4\sin\pi t$
Resultant amplitude $\sqrt{4^2 + 4^2} = 4\sqrt{2}$

121. Total distance covered in one oscillation = 4a

Total time for one oscillation = $\frac{1}{n}$

Average speed = $\frac{4a}{\left(\frac{1}{n}\right)} = 4$ an

122. For body to remain in contact $a_{max} = g$

$\therefore \quad \omega^2 A = g \Rightarrow 4\pi^2 n^2 A = g$

$\therefore \quad n^2 = \frac{g}{4\pi^2 A} = \frac{10}{4\times(3.14)^2\times 0.01} = 25$

$\therefore \quad n = 5$ Hz

123. $mg = kx \Rightarrow \frac{m}{k} = \frac{x}{g}$

$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{9.8\times10^{-2}}{9.8}} = \frac{2\pi}{10}$ s

124. System is equivalent to parallel combination of springs

$\therefore$ $k_{eq} = k_1 + k_2 = 400$

$\therefore$ $T = 2\pi\sqrt{\dfrac{m}{k_{eq}}} = 2\pi\sqrt{\dfrac{0.25}{400}} = \dfrac{\pi}{20}$ s

125. $x_1 = A\sin(\omega t + \phi_1)$, $x_2 = A\sin(\omega t + \phi_2)$

$\therefore$ $x_1 - x_2 = A\left[2\sin\left(\omega t + \dfrac{\phi_1+\phi_2}{2}\right)\sin\left(\dfrac{\phi_1-\phi_2}{2}\right)\right]$

$\therefore$ $A = 2A\sin\left(\dfrac{\phi_1-\phi_2}{2}\right)$

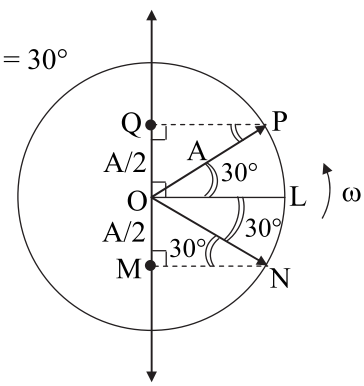
$\therefore$ $\sin\left(\dfrac{\phi_1-\phi_2}{2}\right) = \dfrac{1}{2}$

$\therefore$ $\dfrac{\phi_1-\phi_2}{2} = \dfrac{\pi}{6} \Rightarrow \phi_1 - \phi_2 = \dfrac{\pi}{3}$

126. OP = A = 25 cm and OQ = $\dfrac{A}{2}$ = 12.5 cm

$\Rightarrow \angle OPQ = 30°$

Similarly $\angle MNO = 30°$



$\therefore$ $\angle PON = 60° = \dfrac{\pi}{3}$

$\therefore$ $\omega t = \dfrac{\pi}{3}$

$\dfrac{2\pi}{T} \times t = \dfrac{\pi}{3}$

$\therefore$ $t = \dfrac{T}{6}$

$= \dfrac{3}{6}$ (Given: Period = 3s) = 0.5 s

127. For the graph given, amplitude (A) = 1 cm

Time period (T) = 8 s

$\therefore$ $\omega = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$ Hz

Acceleration, $a = -\omega^2 A\sin\omega t$

At $t = \dfrac{4}{3}$ s, $a = -\dfrac{\pi^2}{16}\times 1 \times \sin\left(\dfrac{\pi}{4}\times\dfrac{4}{3}\right)$

$\therefore$ $a = -\dfrac{\pi^2}{16}\sin\left(\dfrac{\pi}{3}\right) \Rightarrow A = \dfrac{-\sqrt{3}}{32}\pi^2 \text{ cm/s}^2$

128. Given: l = 1 m,

Path length (2A) = 16 cm

$\therefore$ Amplitude (A) = $\dfrac{16}{2}$ = 8 cm

Time period of simple pendulum,

$T = 2\pi\sqrt{\dfrac{l}{g}}$

but $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2\pi\sqrt{\dfrac{l}{g}}} = \sqrt{\dfrac{g}{l}} = \sqrt{\dfrac{\pi^2}{1}} = \pi$

For maximum velocity;

$v_{max} = A\omega = 8\pi$ cm/s

129. n = 5 Hz, $T = \dfrac{1}{5}$ s

$T = 2\pi\sqrt{\dfrac{m}{k}}$

The restoring force is equal to the weight of the spring.

$\therefore$ $kx = mg$

$\therefore$ $\dfrac{m}{k} = \dfrac{x}{g}$

$\therefore$ $T = 2\pi\sqrt{\dfrac{x}{g}}$

$\therefore$ $T = 2\pi\sqrt{\dfrac{A}{g}}$ ....($\because$ At highest position, x = A)

$\dfrac{1}{5} = 2\pi\sqrt{\dfrac{A}{g}}$

$\dfrac{1}{25} = 4\pi^2 \times \dfrac{A}{g}$

$\therefore$ $A = \dfrac{g}{100\pi^2} = \dfrac{10}{100\pi^2} = \dfrac{1}{10\pi^2}$

$\therefore$ $v_{max} = \omega A = 2\pi \times 5 \times \dfrac{1}{10\pi^2} = \dfrac{1}{\pi}$ m/s

130. $T = 2\pi\sqrt{\dfrac{l}{g}}$ (when stationary)

$T' = 2\pi\sqrt{\dfrac{l}{g+2}}$

(When lift is accelerating upwards)

$\because$ $y = t^2$

$v_y = \dfrac{dy}{dt} = 2t$

$g_y = \dfrac{dv_y}{dt} = 2 \text{ m/s}^2$

$T = 2\pi\sqrt{\dfrac{l}{10}}$, $T' = 2\pi\sqrt{\dfrac{l}{12}}$

$\Rightarrow \dfrac{T}{T'} = \sqrt{\dfrac{12}{10}} \Rightarrow T' = \sqrt{\dfrac{5}{6}}\,T$

131. The relation for kinetic energy of S.H.M. is given by

$= \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$ ....(i)

Potential energy is given by

$= \frac{1}{2}m\omega^2 x^2$ ....(ii)

Now, for the condition of question and from equations (i) and (ii),

$\frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{3}\times\frac{1}{2}m\omega^2 x^2$

or $\frac{4}{6}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$ or $x^2 = \frac{3}{4}A^2$

so, $x = \frac{A}{2}\sqrt{3} = 0.866$ a = 87% of amplitude.

132. Total energy of particle performing S.H.M. = $\frac{1}{2}m\omega^2 A^2$. Kinetic energy of particle performing S.H.M. = $\frac{1}{2}m\omega^2 A^2 \cos^2\left(\frac{2\pi}{T}\right)t$

According to problem, kinetic energy = 75% of total energy

$\Rightarrow \frac{1}{2}m\omega^2 A^2 \cos^2\left(\frac{2\pi}{T}\right)t = \frac{3}{4}\left(\frac{1}{2}m\omega^2 A^2\right)$

$\Rightarrow \cos^2\left(\frac{2\pi}{T}\right)t = \frac{3}{4} \Rightarrow \cos\left(\frac{2\pi}{T}\right)t = \frac{\sqrt{3}}{2}$

$\Rightarrow \left(\frac{2\pi}{T}\right)t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12}$ s

$\therefore$ $t = \frac{1}{6}$ s

133. the total energy of particle performing SHM is

$E = \frac{1}{2}ka^2 \Rightarrow E = \frac{1}{2}m\omega^2 a^2$

$\Rightarrow \omega = \sqrt{\frac{2E}{ma^2}} \quad \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{2E}{ma^2}}$

$\Rightarrow T = 2\pi\sqrt{\frac{ma^2}{2E}} = 2\pi\sqrt{\frac{0.2\times(2\times10^{-2})^2}{2\times4\times10^{-5}}}$

$\Rightarrow T = 2\pi\sqrt{\frac{0.2\times4\times10^{-4}}{2\times4\times10^{-5}}} = 2\pi$ seconds

134. $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$

$v = \frac{dx}{dt} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right)$ ....(i)

We know that $v_{max.} = a\omega$

$\therefore$ By substituting $v = \frac{a\omega}{2}$ in equation (i) we get time (t)

$\frac{a\omega}{2} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right)$

$\Rightarrow \frac{\pi}{3} = \omega t + \frac{\pi}{6} \Rightarrow \frac{\pi}{6} = \frac{2\pi}{T}.t \Rightarrow t = \frac{T}{12}$

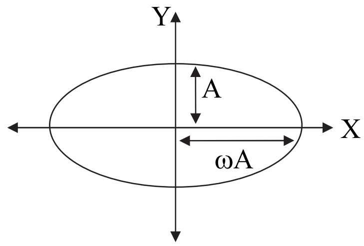
135. Relation between 'v' and 'x' in SHM is

$\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$ $\rightarrow$ Ellipse

Major axis = $2\omega A$

Minor axis = 2A

Given: $\frac{2\omega A}{2A} = 20\pi$



$\therefore$ $\omega = 20\pi$

$\therefore$ $2\pi f = 20\pi$

f = 10 Hz

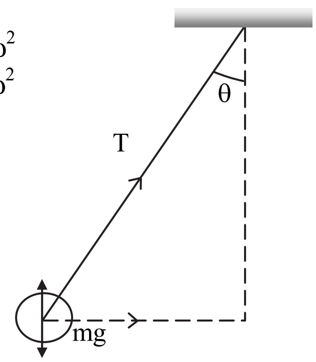
136. $T\sin\theta = mL\sin\theta\omega^2$

$324 = 0.5\times0.5\times\omega^2$

$\therefore$ $\omega^2 = \frac{324}{0.5\times0.5}$

$\therefore$ $\omega = \sqrt{\frac{324}{0.5\times0.5}}$

$\therefore$ $\omega = \frac{18}{0.5} = 36$ rad/s



137. $T = 2\pi\sqrt{\frac{m}{K}}$.

Also, spring constant (K) $\propto \frac{1}{\text{Length}(l)}$

When the spring is half in length, then K becomes twice.

$\therefore$ $T' = 2\pi\sqrt{\frac{m}{2K}} \Rightarrow \frac{T'}{T} = \frac{1}{\sqrt{2}} \Rightarrow T' = \frac{T}{\sqrt{2}}$

138. Extensions in springs are x_1 and x_2 then

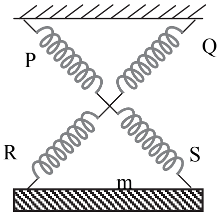
$k_1x_1 = k_2x_2$ and $x_1 + x_2 = A$

$\Rightarrow x_2 = \frac{k_1x_1}{k_2}$

$\Rightarrow x_1 + \frac{k_1x_1}{k_2} = A$

$\Rightarrow x_1 = \frac{k_2A}{k_1 + k_2}$

139.



Springs P and Q, R and S are in parallel

Then, x = k + k = 2k ….[for P, Q]

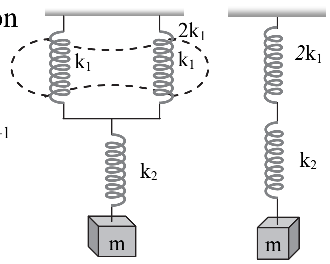
and y = k + k = 2k ….[for R, S]

x and y both in series

∴ $\frac{1}{k'} = \frac{1}{x} + \frac{1}{y} = \frac{1}{k}$

∴ Time period $T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{k}}$

140. In series combination



$\frac{1}{k_s} = \frac{1}{2k_1} + \frac{1}{k_2}$

$\Rightarrow k_s = \left[\frac{1}{2k_1} + \frac{1}{k_2}\right]^{-1}$

141. As $k \propto \frac{1}{l}$,

length of spring segments = $\frac{l}{6}, \frac{l}{3}, \frac{l}{2}$

∴ $k_1 = 6k$

$k_2 = 3k$

$k_3 = 2k$

when connected in series combination,

$\frac{1}{k'} = \frac{1}{6k} + \frac{1}{3k} + \frac{1}{2k}$

∴ $k' = k$ ….(i)

when connected in parallel combination,

$k'' = 6k + 3k + 2k$

∴ $k'' = 11k$ ….(ii)

Dividing equation (i) by equation (ii),

$\frac{k'}{k''} = \frac{k}{11k} = \frac{1}{11}$

142. With mass m_2 alone, the extension of the spring l is given by,

$m_2g = kl$ ….(i)

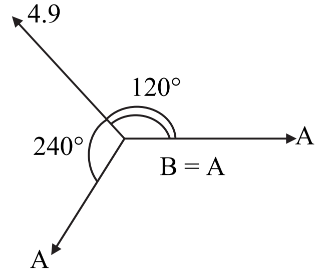
With mass $(m_1 + m_2)$, the extension l' is given by,

$(m_1 + m_2)g = kl' = k(l + \Delta l)$ ….(ii)

The increase in extension is Δl which is the amplitude of vibration. Subtracting equation (i) from equation (ii), we get,

$m_1g = k\Delta l \Rightarrow \Delta l = \frac{m_1 g}{k}$

143.



∴ B = A, $\phi = 240° = \frac{4\pi}{3}$

144. Frequency of oscillation is, $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

∴ $k = m(2\pi f)^2$

Mole weight (i.e., atomic mass) of silver is given 108.

∴ Mass of 1 atom,

$m = \frac{108}{6.02\times10^{23}} = 18 \times 10^{-23}$ g $= 18 \times 10^{-26}$ kg

∴ $k = 18 \times 10^{-26} \times (2\pi \times 10^{12})^2$

$= 4\pi^2 \times 18 \times 10^{-2}$

∴ k = 7.1 N/m

145. At maximum compression, the solid cylinder will stop.

So loss in K.E. of cylinder = Gain in P.E. of spring

∴ $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2$

∴ $\frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{1}{2}kx^2$

∴ $\frac{3}{4}mv^2 = \frac{1}{2}kx^2$

∴ $\frac{3}{4}\times3\times(4)^2 = \frac{1}{2}\times200\times x^2$

∴ $\frac{36}{100} = x^2 \Rightarrow x = 0.6$ m

146. At maximum compression,

Gain in P.E. of spring = loss in K.E. of sphere

∴ $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2$ ….(∵ v = rω)

$= \frac{7}{10}mv^2$

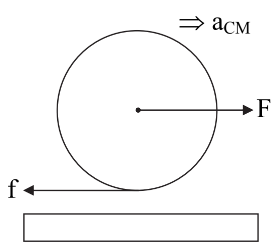
$\therefore \quad x^2 = \frac{14\,mv^2}{10\,k} = \frac{14 \times 2 \times (6)^2}{10 \times 36} = 2.8$

i.e., $x = \sqrt{2.8}$ m

147. K.E. is maximum at mean position and P.E. is minimum at mean position.

Evaluation Test

1.



$x = 4 \cos(2\pi t)$

$\therefore \quad a = -16\pi^2 \cos(2\pi t)$

$F - f = Ma_{CM}$ ….(1)

$\therefore \quad 16\pi^2 M \cos(2\pi t) - f = Ma_{CM}$

($F \rightarrow$ pseudo force due to acceleration of platform)

$f \cdot R = \left(\frac{1}{2}MR^2\right)\alpha$ ….(2)

$\therefore \quad f = \frac{Ma_{CM}}{2}$

$\therefore \quad \frac{3}{2}Ma_{CM} = 16\pi^2 M \cos(2\pi t)$

$\therefore \quad a_{CM} = \frac{32}{3}\pi^2 \cos(2\pi t)$

This is the acceleration w.r.t. the platform.

Acceleration w.r.t. ground,

$a = \left(\frac{32}{3} - 16\right)\pi^2 \cos(2\pi t)$

$= \frac{-16}{3}\pi^2 \cos(2\pi t)$

$= -\frac{16}{3}\pi^2\left(\frac{1}{2}\right)$

$= -\frac{8}{3}\pi^2$

2. $x = \cos(\pi t)$, $y = \cos\left(\frac{\pi t}{2}\right)$

$y = \sqrt{\frac{1 + \cos(\pi t)}{2}}$ i.e. $2y^2 - 1 = \cos(\pi t)$

$\therefore \quad 2y^2 = x + 1$ represents a parabola.

3. Since the amplitudes of the SHM is small,

$\theta_1 = \theta_0 \sin(\omega_1 t)$, (taking first one as reference)

$\theta_2 = \theta_0 \sin(\omega_2 t \pm \pi)$

For the two to be in same phase,

$\omega_1 t = \omega_2 t \pm \pi$

Substituting, $\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$ we get,

$\therefore \quad \frac{2\pi}{3}t = \frac{2\pi}{7}t + \pi \Rightarrow t = \frac{21}{8}$ s

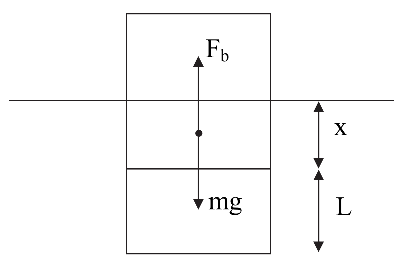
4. The concept is that projection of a circle on its diameter where the circular motion is uniform, is an SHM.

$\therefore$ Amplitude of motion = 0.5 m

$\omega = 60$ rev/min $= 2\pi$ rad/s

$\therefore \quad T = \frac{2\pi}{\omega} = 1$ s

5.



$F_b - F_g = -ma$

$\therefore \quad m\frac{d^2x}{dt^2} = -(\rho\omega g\,A(L + x) - mg)$

At equilibrium, $mg = \rho\omega g\,AL$

$m\frac{d^2x}{dt^2} = -(\rho\omega g\,A)x$

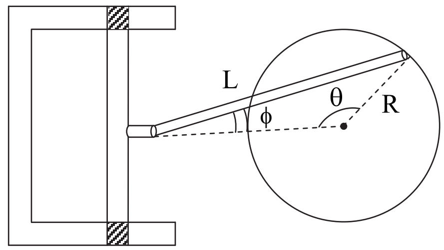
$\therefore \quad \omega_n = \sqrt{\frac{\rho(\omega)gA}{m}} = \sqrt{\frac{\rho\omega g\left(\frac{\pi D^2}{4}\right)}{m}}$

$= \sqrt{\frac{1000 \times 9.81 \times \pi \times 8 \times 8}{4 \times 350}}$

$= 40\sqrt{\frac{9.81 \times \pi}{35}}$

$= 37.52$ rad/s

6.



$x = L\cos\phi + R\cos\theta$

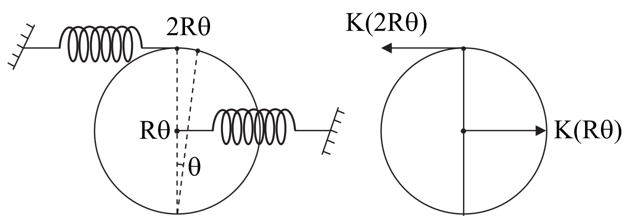
$L\sin\phi = R\sin\theta$

$$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{R}{L}\right)^2\sin^2\theta}$$

$$\therefore \quad x = R\cos\theta + L\sqrt{1-\left(\frac{R}{L}\right)^2\sin^2\theta}$$

Since the angular velocity is a constant, ($\theta = \omega t$) first term shows S.H.M. and second term does not.

7. If the sphere is displaced by a small θ,

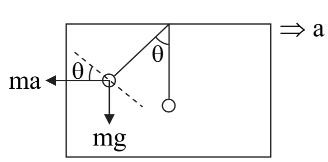


Net Restoring Torque = $2KR\theta\,(2R) - K(R\theta)\,R$

$$= 3KR^2\theta = \left(\frac{7}{5}MR^2\right)\alpha$$

$$\therefore \quad \omega^2 = \frac{15K}{7M} \Rightarrow \omega = \sqrt{\frac{15K}{7M}}$$

8.



In equilibrium,

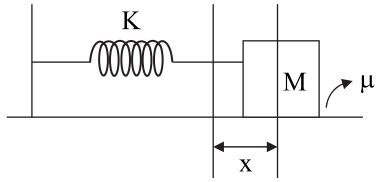
$ma\cos\theta = mg\sin\theta$

$$\therefore \quad \tan\theta = \frac{a}{g}$$

Now, in case of oscillation, the body goes x more than that at equilibrium because of gain in velocity.

$$\therefore \quad \text{Maximum displacement} = 2\tan^{-1}\left(\frac{a}{g}\right)$$

9.

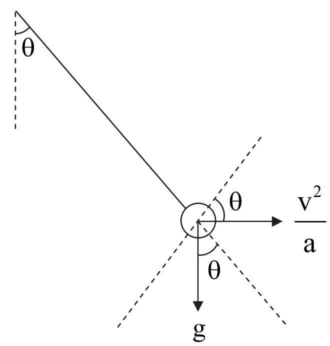


Initial momentum $\vec{P}$ is in negative direction.

Towards the end of one cycle, it will not come back to its original position as there are some frictional losses.

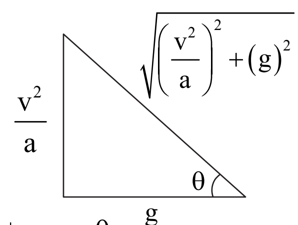
This is a case of damped oscillation.

10.



$$\frac{v^2}{a}\cos\theta = g\sin\theta$$

$$\therefore \quad \tan\theta = \frac{v^2}{ag}$$



$$\text{Now, } g' = \frac{v^2}{a}\sin\theta + g\cos\theta$$

$$= \cos\theta\left(\frac{v^2}{a}\tan\theta + g\right)$$

$$= \sqrt{\left(\frac{v^2}{a}\right)^2 + g^2}$$

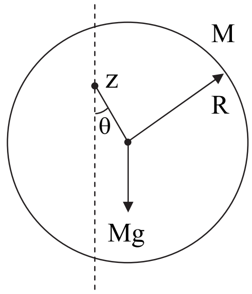
$$n_1^4 = \frac{g^2}{(4\pi^2 l)^2} \quad n^4 = \frac{\left(\frac{v^2}{a}\right)^2 + g^2}{(4\pi^2 l)^2}$$

$$\therefore \quad n^4 - n_1^4 = \frac{\left(\frac{v^2}{a}\right)^2}{(4\pi^2 l)^2}$$

$$\therefore \quad \left(\frac{v^2}{a}\right)^2 = \left(n^4 - n_1^4\right)\left(\frac{g^2}{n_1^4}\right)$$

$$\therefore \quad v^2 = ag\left(\frac{n^4}{n_1^4} - 1\right)^{1/2}$$

11.



Net torque = Iα

$\therefore$ $(Mg)\, z \sin\theta = \left(\dfrac{MR^2}{2} + Mz^2\right)\alpha$

$\therefore$ $\alpha = \left(\dfrac{Mgz}{\dfrac{MR^2}{2} + Mz^2}\right)\theta = -\omega^2\theta$

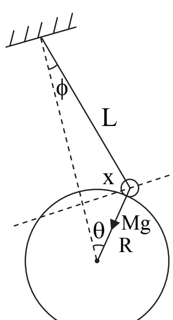
$\therefore$ Time period $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R^2 + 2z^2}{2gz}}$

$= 2\pi\sqrt{\dfrac{R^2}{2gz} + \dfrac{z}{g}}$

Time period is minimum when, $\dfrac{R^2}{2gz} = \dfrac{z}{g}$

i.e. $z = \dfrac{R}{\sqrt{2}}$

12.



$F_{rest} = -mg\sin(\theta + \phi)$

$\therefore$ $ma = -mg\,\phi\left(1 + \dfrac{\theta}{\phi}\right)$

$\therefore$ $\alpha = \dfrac{-g}{L}\left(1 + \dfrac{\theta}{\phi}\right)\phi$

$\theta = \dfrac{x}{R}$, $\phi = \dfrac{x}{L}$

….[$\because$ For small θ and ϕ, $\sin\theta \approx \theta$ and $\sin\phi \approx \phi$]

$\therefore$ $\alpha = \dfrac{-g}{L}\left(1 + \dfrac{L}{R}\right) = -g\left(\dfrac{1}{L} + \dfrac{1}{R}\right)$

$\therefore$ Time period $= 2\pi\sqrt{\dfrac{1}{\left(\dfrac{1}{L} + \dfrac{1}{R}\right)g}}$

13. $k\, n_1 = 2k(x_2) = 3k(x_3)$

as tension in the spring remains the same.

Also, $x_1 + x_2 + x_3 = A$

$\therefore$ $x_1 + \dfrac{x_1}{2} + \dfrac{x_1}{3} = A$

$\therefore$ $\dfrac{(6+3+2)x_1}{6} = A$

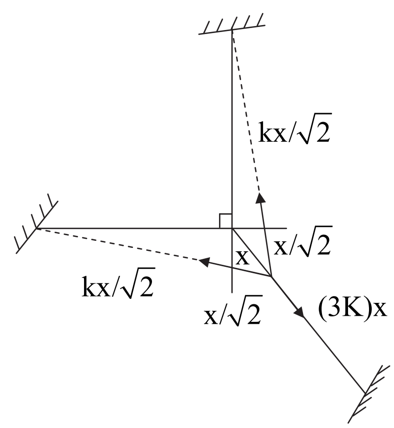
$\therefore$ $x_1 = \dfrac{6A}{11}$

$\therefore$ $x_2 = \dfrac{x_1}{2} = \dfrac{3A}{11}$

$\therefore$ Ratio of amplitudes $= \dfrac{x_1}{x_1 + x_2}$

$= \dfrac{\left(6\dfrac{A}{11}\right)}{\left(9\dfrac{A}{11}\right)} = \dfrac{2}{3}$.

14.

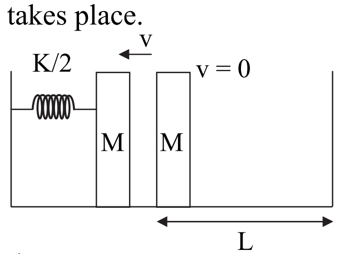


Net restoring force = –2Kx

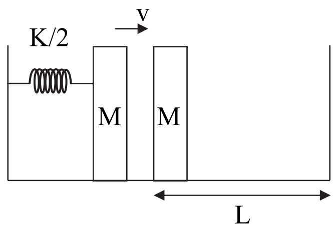
$\therefore$ Time period $= 2\pi\sqrt{\dfrac{M}{2K}}$

$\therefore$ Frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{2K}{M}}$

15. When the two blocks collide, velocity transfer takes place.



then

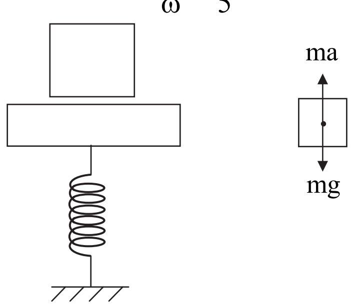


$\therefore$ Time period = $\dfrac{2\pi\sqrt{\dfrac{2M}{K}}}{2} + \dfrac{2L}{V} = \pi\sqrt{\dfrac{2M}{K}} + \dfrac{2L}{V}$

16. At the mean position,
$M_1v = (M_1 + M_2)v'$
$3v = 9v'$
$\therefore$ $v = 3v'$
Also, $\dfrac{1}{2}Kx^2 = \dfrac{1}{2}Mv^2$
$\therefore$ $v = \sqrt{\dfrac{K}{M}}\,x = \dfrac{10}{\sqrt{3}}(0.1) = \dfrac{1}{\sqrt{3}}$ m/s
$\therefore$ $v' = \dfrac{1}{3\sqrt{3}}$ m/s
$\therefore$ $\dfrac{1}{2}Kx^2 = \dfrac{1}{2}(M_1 + M_2)v'^2$
$\therefore$ $\dfrac{1}{2}(100)x^2 = \dfrac{1}{2}(9)\left(\dfrac{1}{27}\right)$
$\therefore$ $x = \sqrt{\dfrac{1}{300}} = \dfrac{1}{10\sqrt{3}}$ m $= \dfrac{10}{\sqrt{3}}$ cm
≈ 5.8 cm

17. $U = 5x(x - 4)$
$= 5(x^2 - 4x)$
$= 5[(x - 2)^2 - 4]$
$\therefore$ The particle executes SHM about x = 2.
$F = \dfrac{-dU}{dx} = 5[x + (x - 4)]$
$ma = 5(2x - 4)$
$\therefore$ $a = 100x - 200 = 100(x - 2)$
$\therefore$ $\omega^2 = 100 \Rightarrow \omega = 10$ rad/s
$\therefore$ Time period = $\dfrac{2\pi}{\omega} = \dfrac{\pi}{5}$ s

19.



The block will lose contact when N = 0
i.e. mg = ma
$g = A\omega^2$
$\therefore$ $A = \dfrac{g}{\omega^2} = \dfrac{g}{\left(\dfrac{4\pi^2}{T^2}\right)}$
$\therefore$ $A = \dfrac{10}{\pi^2}$

20. $B = \dfrac{-\left(\dfrac{F}{A}\right)}{\left(\dfrac{Ax}{V_0}\right)}$
$\therefore$ $F = -\left(\dfrac{BA^2}{V_0}\right)x$
$\therefore$ Time period = $2\pi\sqrt{\dfrac{BA^2}{MV_0}}$

21. For an SHM, Total Energy of a system is constant
$\therefore$ $\dfrac{1}{2}m(r^2\omega^2)\left(\dfrac{7}{5}\right) + Mg(R - r)(1 - \cos\theta) = \text{constant}$
$\therefore$ $\left(\dfrac{7}{10}mr^2\right)\omega^2 + Mg\,(R - r)\,\dfrac{Q^2}{2} = \text{constant}$
$\therefore$ $\left(\dfrac{7}{5}mr^2\right)\omega d\omega + Mg\,(R - r)\,\theta d\theta = 0$
$\therefore$ $\omega\dfrac{d\omega}{d\theta} = \dfrac{-5Mg(R - r)}{7Mr^2}\theta$
$\therefore$ $\alpha = \dfrac{-5}{7}\,\dfrac{(R - r)g}{r^2}\,\theta$
$\therefore$ Time Period $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{7r^2}{5(R - r)g}}$
$= 2\pi\sqrt{\dfrac{7 \times \dfrac{1}{4}}{5 \times 4.5 \times 10}} = \dfrac{\pi}{15}\sqrt{7}$ s
$= 0.55$ s

22. If the cart does not move, $T_1 = 2\pi\sqrt{\dfrac{l}{g}}$
If the cart is moving,
the centre of mass of the system does not move.
$ml_1 = M(l - l_1)$ or $l_1 = \dfrac{Ml}{M + m}$
$\therefore$ The effective length of the oscillation of pendulum would be l_1.

$T_2 = 2\pi\sqrt{\dfrac{Ml}{(M+m)g}}$

$\therefore \quad \dfrac{T_2}{T_1} = \sqrt{\dfrac{M}{M+m}}$

23. Time of ascent = Time of descent

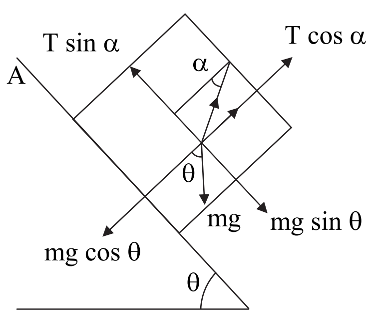
$S = at + \dfrac{1}{2}at^2$

$\therefore \quad 80 \text{ cm} = \dfrac{1}{2} \times (10 \sin 30)\, t^2$

$\therefore \quad t = \sqrt{\dfrac{0.80}{2.5}} \text{ m/s} = \sqrt{\dfrac{1.6}{5}} = \dfrac{4}{5\sqrt{2}}$

$\therefore \quad \text{Time period of oscillation } = 2 \times \left(\dfrac{2\times 4}{5\sqrt{2}}\right) = \dfrac{8\sqrt{2}}{5} \text{ s}$

24.



$T \sin\alpha + F_P = mg \sin\theta$

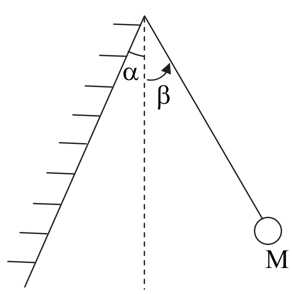
$T \sin\alpha + ma = mg \sin\theta$

But $a = g \sin\theta$

$\therefore \quad \sin\alpha = 0 \Rightarrow \alpha = 0$

$\therefore \quad TP = 2\pi\sqrt{\dfrac{l}{g_{eff}}} = 2\pi\sqrt{\dfrac{l}{g\cos\theta}}$

25.



Since the collision is elastic,
this system can be considered as a partial SHM system.

$\theta = \beta \sin(\omega t + \phi)$

at $t = 0,\ \theta = \beta$

$\therefore \quad \phi = \dfrac{\pi}{2}$

$\therefore \quad \theta = -\beta \cos(\omega t)$

$-\alpha = -\beta \cos(\omega t)$

$\therefore \quad t = \dfrac{1}{\omega}\cos^{-1}\left(\dfrac{\alpha}{\beta}\right)$ and $\omega = \sqrt{\dfrac{l}{g}}$

$\therefore \quad \text{Time Period} = \dfrac{2}{\omega}\cos^{-1}\left(\dfrac{\alpha}{\beta}\right) = 2\sqrt{\dfrac{g}{l}}\cos^{-1}\left(\dfrac{\alpha}{\beta}\right)$

26. At mean position,

P.E. $= \dfrac{1}{2}kx^2 = 0$

i.e., P.E. is minimum.

Also, velocity is maximum at mean position.

$\Rightarrow$ K.E. is maximum.

Textbook Chapter No.

05 Elasticity

Hints

Classical Thinking

15. Breaking force ∝ Area of cross-section of wire i.e. load held by the wire does not depend upon the length of the wire.

18. This is because strain is a dimensionless and unitless quantity.

21. Fluids have no shape of their own but occupy the volume of the vessel in which they are contained. Therefore, the fluids can have volume strain only.

25. Stress = $\frac{F}{A} = \frac{10}{(50\times10^{-2})^2} = 40\ N/m^2$

26. Stress (S) $\propto \frac{1}{Area(A)}$

$\frac{S_1}{S_2} = \frac{r_2^2}{r_1^2} = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$

27. Strain = $\frac{l}{L} = \frac{0.001}{L}$

28. Shearing strain $\phi = \frac{x}{h} = \frac{0.02}{10}$

∴ $\phi = 0.002$

33. $Y = \frac{Stress}{Strain}$ = Constant

It depends only on nature of the material.

37. $Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

For a perfectly rigid body, $\Delta L = 0$, so longitudinal strain is zero. Hence, Y is infinite.

38. Liquids don't have a definite length

Here, $L = 0 \Rightarrow Y = 0$

39. Young's modulus, $Y = \frac{F}{A\times strain}$

∴ $A = \frac{10^4}{7\times10^9\times(0.2/100)} = 7.1\times10^{-4}\ m^2$

40. $Y = \frac{FL}{Al} = \frac{2\times10\times2}{0.05\times10^{-4}\times0.04\times10^{-3}} = \frac{40}{2\times10^{-10}}$

$= 20\times10^{10}\ N/m^2$

47. $K = \frac{dP}{dV/V} = \frac{1.2\times10^7}{3\times10^{-3}/4}$

$= \frac{4.8\times10^{10}}{3} = 1.6\times10^{10}\ N/m^2$

48. $K = \frac{1}{Compressibility}$

∴ Bulk Modulus = $\frac{1}{0.5\times10^9} = 2\times10^{-9}\ N/m^2$

49. Isothermal elasticity,

$K_i = P = 1\ atm = 1.013\times10^5\ N/m^2$

56. $\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$

∴ Shearing strain = $\frac{Stress}{\eta} = \frac{10^8}{8\times10^{10}} = 1.25\times10^{-3}$

57. $\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$ = Poisson's ratio

63. $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{25\times10^{-6}}{5\times10^{-5}}$

∴ $\sigma = 0.5$

73. Loss of elastic strength produces more strain for a given stress.

78. This is due to increase in intermolecular distance.

81. Work done = $\frac{1}{2}Fl = \frac{Mgl}{2}$

82. $l \propto L$ i.e. if length is reduced to half, then increase in length will be $\frac{l}{2}$.

83. Strain = $\frac{\text{Extension}}{\text{Original length}} = \frac{(2L-L)}{L} = \frac{L}{L} = 1$

84. $Y = \frac{FL}{Al} \Rightarrow Y \propto \frac{1}{l}$

i.e. More elongation implies less elasticity.

85. At point b, yielding of material starts.

86. Elastic energy per unit volume

$= \frac{1}{2}$ stress × strain

$= \frac{1}{2}$ stress $\times \left(\frac{\text{stress}}{Y}\right) = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \frac{1}{2}\frac{S^2}{Y}$

Critical Thinking

5. Stress ∝ Strain $\propto \frac{F}{A}$

∴ Ratio of strain $= \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$

6. Total force at height 3L/4 from its lower end
= Weight suspended+Weight of 3/4 of the wire
= $W_1 + (3W/4)$

∴ Stress $= \frac{W_1 + (3W/4)}{A}$

7. Stress $= F/\pi r^2$

∴ $\frac{F_2}{F_1} = \frac{r_2^2}{r_1^2} = \frac{(0.75)^2}{(1.5)^2} = 0.25$

$F_2 = 0.25\ F_1 = 0.25 \times 1.5 \times 10^5$ N $= 0.375 \times 10^5$ N

8. Breaking force ∝ Area of cross-section

$\frac{F_1}{F_2} = \frac{A_1}{A_2}$

∴ $\frac{400}{F_2} = \frac{A_1}{2A_1} \Rightarrow F_2 = 800$ kg-wt

9. Breaking force = breaking stress × area of cross-section = weight of wire

$S \times A = A\, l \times \rho\, g$

∴ $l = \frac{S}{\rho g} = \frac{10^6}{3\times10^3\times9.8} \approx 34$ m

10. Breaking force $\propto r^2$

If diameter becomes double, then breaking force will become four times

i.e. 1000 × 4 = 4000 N

12. Stress $= \frac{F}{\text{Area}}$

∴ F = Stress × Area

$= 4.8 \times 10^7 \times 10^{-6} = 48$ N

This tension is balanced by centripetal force

$T = F = mr\omega^2$

∴ $\omega^2 = \frac{T}{mr} = \frac{48}{10\times0.3} = 16$

∴ ω = 4 rad/s

13. $K = \frac{\Delta P}{(\Delta V/V)}$

∴ $\frac{1}{K} \propto \frac{\Delta V}{V}$ ….[∵ ΔP = constant]

14. Compressibility $= \frac{1}{K} = \frac{dV}{V\times dP}$

∴ $5 \times 10^{-10} = \frac{dV}{dP\times V}$

∴ $dV = 5 \times 10^{-10} \times 10 \times 10^5 \times (10 \times 10^3 \text{ cc})$

$= 5$ cc

15. $K = \frac{dP}{(dV/V)}$

$P = h\rho g = 200 \times 10^3 \times 9.8$

∴ $\frac{dV}{V} = \frac{0.1}{100} = 10^{-3}$

∴ $K = \frac{200\times10^3\times9.8}{10^{-3}} = 19.6 \times 10^8$ N/m^2

16. $\frac{\Delta V}{V} = \frac{0.1}{100} = 1 \times 10^{-3}$

$K = \frac{h\rho g}{\left(\frac{\Delta V}{V}\right)} = \frac{200\times1\times10^3\times10}{1\times10^{-3}} = 2 \times 10^9$

17. $K = \frac{dP}{dV/V}$; $K = \frac{h\rho g}{dV/V}$

∴ $9.8 \times 10^8 = \frac{h\times10^3\times9.8}{0.1\times10^{-2}} \Rightarrow h = 100$ m

18. Bulk Modulus,

$B = \frac{-PV}{\Delta V} \Rightarrow P = -\frac{\Delta V\times B}{V}$

Given that $-\frac{\Delta V}{V} = 1\% = \frac{1}{100}$

∴ $P = \frac{7.5\times10^{10}}{100} = 7.5 \times 10^8$ N/m^2

19. $C = \frac{1}{K} = \frac{\Delta V/V}{\Delta P}$

$\Rightarrow \Delta V = C \times \Delta P \times V$

$= 4 \times 10^{-5} \times 100 \times 100 = 0.4$ cc

20. $\frac{r_2}{r_1} = 2$, $F_1 = F_2$, $\frac{l_1}{L_1} = 4$

Both the wires are made up of same material

$\Rightarrow Y_1 = Y_2$

∴ $\frac{F_1L_1}{\pi r_1^2 l_1} = \frac{F_2L_2}{\pi r_2^2 l_2}$

$\therefore \quad \frac{l_1}{L_1} . r_1^2 = \frac{l_2}{L_2} . r_2^2$

$\therefore \quad 4 = \frac{l_2}{L_2} \times \left(\frac{r_2}{r_1}\right)^2$

$\therefore \quad \frac{l_2}{L_2} = \frac{4}{\left(\frac{r_2}{r_1}\right)^2} = \frac{4}{(2)^2} = 1$

22. Strain = $\frac{\Delta L}{L} = \frac{F}{AY}$
Since F, A and Y are the same for the two wires, the strains in them are equal.

24. $L_2 = 2L_1$

$\therefore \quad \frac{l}{L} = \frac{L_2 - L_1}{L_1} = \frac{2L_1 - L_1}{L_1} = 1$

$\therefore$ Strain = 1, Stress = Y

25. $Y = \frac{MgL}{\pi r^2 l}$

$\therefore \quad l \propto \frac{1}{r^2}$

$\therefore \quad \frac{l_1}{l_2} = \frac{r_2^2}{r_1^2} = \left(\frac{2r_1}{r_1}\right)^2 = 4 \Rightarrow l_2 = \frac{l_1}{4}$

$\therefore$ When radius of wire is doubled, the elongation of the wire becomes $\frac{1}{4}$.

26. $Y = \frac{F/A}{l/L} = \frac{FL}{Al}$

$\therefore \quad l = \frac{FL}{AY}$

$\therefore \quad l \propto L$

$\frac{l_1}{l_2} = \frac{L_1}{L_2} = \frac{L}{L/3} = 3$

$\therefore \quad l_2 = \frac{l_1}{3} = \frac{l}{3}$

27. When the length of wire is doubled, then $l = L$ and strain = 1

$\therefore$ Y = stress = $\frac{F}{A}$

$\therefore$ Force = Y × A = $2 \times 10^{11} \times 0.1 \times 10^{-4} = 2 \times 10^6$ N

28. $Y = \frac{FL}{Al}$

$\therefore \quad Y \propto \frac{1}{l}$ ....[$\because$ F, L and A are constant]

$\therefore \quad \frac{l_1}{l_2} = \frac{Y_2}{Y_1} = \frac{1}{2}$

29. Young's Modulus, $Y = \frac{FL}{Al}$

$\therefore \quad A = \frac{FL}{Yl} = \frac{F}{Y\left(\frac{l}{L}\right)}$

$\therefore \quad A = \frac{20}{2 \times 10^{11} \times \left(\frac{1}{100}\right)} = 10^{-8}\ m^2 = 10^{-2}\ mm^2$

30. $Y = \frac{FL}{Al}$ $\qquad \therefore \quad l = \frac{FL}{AY}$

$\therefore \quad \frac{l_1}{l_2} = \frac{L_1}{L_2} \times \frac{A_2}{A_1} \times \frac{Y_2}{Y_1} = \frac{2}{1} \times \frac{4}{1} \times \frac{5}{3}$

$\therefore \quad \frac{l_1}{l_2} = \frac{40}{3}$

31. $Y = \frac{FL}{\pi r^2 l}$

$Y_1 = Y_2$ ....[Given]

$\therefore \quad \frac{FL}{\pi r_1^2 l_1} = \frac{FL}{\pi r_2^2 l_2}$

$\therefore \quad \frac{l_1}{l_2} = \frac{r_2^2}{r_1^2} = \frac{r_1^2/4}{r_1^2} = \frac{1}{4}$

$\therefore \quad l_2 = 4l_1 = 4 \times 1 = 4$ cm

32. $Y = \frac{F}{\pi r^2} \frac{L}{l}$

$\therefore \quad l = \frac{FL}{\pi r^2 Y} \Rightarrow l \propto \frac{1}{r^2}$

$\therefore \quad \frac{l_2}{l_1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{r_1}{2r_1}\right)^2 = \frac{1}{4}$

$\therefore \quad l_2 = \frac{l_1}{4} = \frac{0.01}{4} = 0.0025$

33. $Y = \frac{FL}{Al} \Rightarrow Y \propto \frac{1}{l}$

Let
l_C = increase in length of copper wire
l_S = increase in length of steel wire

$\therefore \quad \frac{l_C}{l_S} = \frac{Y_S}{Y_C} = \frac{2 \times 10^{11}}{1.2 \times 10^{11}} = \frac{5}{3}$

$\therefore \quad l_S = \frac{3}{5} l_C$

$4 = l_C + \frac{3}{5} l_C$ ....[Given]

$\therefore \quad 4 = \frac{8}{5}\, l_C$

$\therefore \quad l_C = \frac{4 \times 5}{8} = \frac{5}{2}$ mm = 2.5 mm

34. $Y_1 = Y_2$

$\frac{F_1 L_1}{l_1 A_1} = \frac{F_2 L_2}{l_2 A_2}$

$\therefore \quad \frac{l_1}{l_2} = \frac{F_1}{F_2} \times \frac{L_1}{L_2} \times \frac{A_2}{A_1}$

$\therefore \quad \frac{l_1}{l_2} = \frac{2}{3} \times \frac{2}{3} \times \frac{9}{4} \qquad (\because A = \pi r^2)$

$\therefore \quad \frac{l_1}{l_2} = 1 : 1$

35. $l_1 = L_1 - L$, $F_1 = 5$ N, $l_2 = L_2 - L$, $F_2 = 7$ N

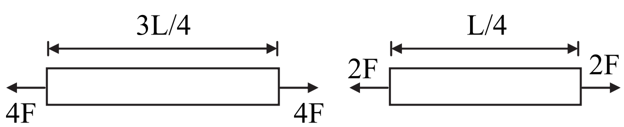
Here,

$l \propto F$ or $l \propto T$

$\therefore \quad \frac{l_1}{l_2} = \frac{T_1}{T_2}$

$\therefore \quad \frac{5}{7} = \frac{L_1 - L}{L_2 - L} \Rightarrow L = \frac{7L_1 - 5L_2}{2}$

36. The free body diagram of two parts are shown in figure.



Both the parts of body are stretched by forces shown. Therefore, total elongation is

$\Delta l = \Delta l_1 + \Delta l_2 = \frac{4F(3L/4)}{AY} + \frac{2F(L/4)}{AY} = \frac{7FL}{2AY}$

37. When there is increase in temperature, the change in length is

$l = L\,\alpha\,\Delta t$, $\quad \Delta t$ = change in temperature

$\therefore \quad$ Strain $= \frac{l}{L} = \alpha\,\Delta t = 12 \times 10^{-6} \times (10 - 0) = 12 \times 10^{-5}$

38. $\frac{l}{Y} = \frac{\text{Stress}}{Y} = \frac{X \times 9.8}{Y}$

$\therefore \quad \%\Delta l = \frac{9.8\,X \times 100}{Y} = \frac{980\,X}{Y}$

39. $Y = \frac{TL}{Al}$

$\therefore \quad T = YA\,\frac{l}{L} \qquad \ldots.(i)$

$l = \alpha\, L\Delta t$

$\therefore \quad \frac{l}{L} = \alpha\,\Delta t \qquad \ldots.(ii)$

$\therefore \quad$ From equations (i) and (ii),

$T = YA\alpha\Delta t$

$= 2.1 \times 10^{11} \times \frac{22}{7} \times 10^{-6} \times 11 \times 10^{-6} \times 10$

$\therefore \quad T = 72.5$ N

40. Modulus of rigidity is the property of material.

47. Energy per unit volume $= \frac{1}{2} \times$ stress $\times$ strain

$= \frac{1}{2} \times \frac{F}{A} \times \frac{dl}{l} = \frac{Fdl}{2Al}$

48. Work done in stretching a wire,

$W = \frac{1}{2} Fl = \frac{1}{2} \times 10 \times 0.5 \times 10^{-3}$

$= 2.5 \times 10^{-3}$ J

$\therefore \quad$ Work done to displace it through 1.5 mm

$W = F \times l = 5 \times 10^{-3}$ J

$\therefore \quad$ The ratio of above two work = 1 : 2

49. $W = \frac{1}{2}\, Fl$

But $Y = \frac{F/A}{l/L} = \frac{FL}{Al}$ or $F = \frac{YAx}{L}$

$\therefore \quad$ Work done $= \frac{1}{2}\,\frac{YAx^2}{L} = \frac{YAx^2}{2L}$

50. Strain = 0.06 % $= \frac{0.06}{100}$

$\therefore \quad$ Energy per unit volume $= \frac{1}{2} \times Y \times (\text{strain})^2$

$= \frac{1}{2} \times 2 \times 10^{10} \times \left(\frac{0.06}{100}\right)^2$

$= 10^{10} \times 3.6 \times 10^{-7}$

$= 3.6 \times 10^3$

$= 3600$ J/m^3

51. From the ideal gas equation, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$\therefore \quad \frac{E_2}{E_1} = \frac{P_2}{P_1} = \frac{V_1}{V_2} \times \frac{T_2}{T_1} = \left(\frac{1}{4}\right) \times \left(\frac{400}{300}\right) = \frac{1}{3}$

$\therefore \quad E_2 = \frac{E_1}{3}$

Hence elasticity will become $\frac{1}{3}$ times.

52. $Y = \frac{FL}{Al}$

$\therefore$ $F = \frac{YAl}{L} = \frac{2\times10^{11}\times4\times10^{-6}\times2\times10^{-3}}{5} = 320$ N

$\therefore$ Energy $= \frac{1}{2}\times$ load $\times$ extension

$= \frac{1}{2}\times 320\times 2\times 10^{-3}$

$= 320\times 10^{-3}$

$= 0.320$ J

53. For loaded wire, extension $= \frac{F.L}{AY}$

$\therefore$ $\frac{\text{Force}}{\text{extension}} = \frac{AY}{L} = K_1$ (spring constant)

So the wire may be regarded as a spring of force constant K_1. Now, the spring are in series. Their effective force constant is given by $KK_1 / (K + K_1)$

Let $T = 2\pi\sqrt{\frac{M}{(\text{force constant})}}$

$= 2\pi\sqrt{\frac{M(K+K_1)}{KK_1}}$

$= 2\pi\sqrt{\frac{M(K+(AY/L))}{K(AY/L)}}$

$T = 2\pi\sqrt{\frac{M(KL+YA)}{YAK}}$

54. K.E. of missile = Elastic P.E.

$\therefore$ $\frac{1}{2}mv^2 = \frac{1}{2}F\times l$

$\therefore$ $v = \sqrt{\frac{Fl}{m}}$ ….(i)

Let $Y = \frac{FL}{Al}$

$\therefore$ $F = \frac{YAl}{L} = \frac{5\times10^8\times10^{-6}\times2\times10^{-2}}{10\times10^{-2}} = 100$ N

$\therefore$ From (i), $v = \sqrt{\frac{100\times0.02}{5\times10^{-3}}} = \sqrt{400} = 20$ m/s

55. The pressure exerted by a 2500 m column of water on the bottom layer,

$P = h\rho g = 2500\text{ m}\times 1000\text{ kgm}^{-3}\times 10\text{ ms}^{-2}$
$= 2.5\times 10^7\text{ Nm}^{-2}$

Fractional compression $\frac{\Delta V}{V}$,

$\frac{\Delta V}{V} = \frac{P}{B} = \frac{2.5\times10^7\text{ Nm}^{-2}}{2.2\times10^9\text{ Nm}^{-2}} \approx 1.14\times10^{-2} = 1.14\%$

56. Young's modulus is defined only in elastic region and $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{8\times10^7}{4\times10^{-4}} = 2\times10^{11}$ N/m^2

57. $\eta = \frac{FL}{Al} = \frac{100\times20\times10^{-2}}{400\times10^{-4}\times0.25\times10^{-2}}$
$= 20\times10^{-2+6} = 2\times10^5$ N/m^2

58. Breaking load depends on the area of cross-section and is independent of length of the rod i.e., breaking load = breaking stress × cross-sectional area.

Competitive Thinking

2. When body is stretched by applying a load to its free end, longitudinal and shear strains both are produced in the spring.
(**Note:** If spring is spiral, then answer would be longitudinal.)

3. Longitudinal stress $= \frac{mg}{\pi r^2}$

$= \frac{100\times10^{-3}\times9.8}{3.14\times(1\times10^{-3})^2}$

$= \frac{9.8\times10^{-1}}{3.14\times10^{-6}}$

$= 3.1\times10^5$ N/m^2

4. Stress $= \frac{\text{force}}{\text{Area}} \Rightarrow$ Stress $\propto \frac{1}{A}$

$\therefore$ $\frac{S_B}{S_A} = \frac{A_A}{A_B} = (2)$

$\therefore$ $S_B = 2\ S_A$

5. Breaking stress is property of the material

$\therefore$ $\frac{T_1}{\pi r_1^2} = \frac{T_2}{\pi r_2^2}$

$\frac{500}{1^2} = \frac{T_2}{2^2}$

$\Rightarrow T_2 = 2000$ N

6. Stress $= \frac{F}{A} = \frac{mg}{A}$

But, $m = \rho V$

Representing volume and area in linear dimensions,

Stress $= \frac{L^3\rho g}{L^2}$

$\Rightarrow$ stress $\propto L$ ….($\because$ density is constant)

Given: Linear dimensions increase by factor of 9. Therefore, stress will also increase by factor of 9.

7. As length is constant, Strain = $\frac{\Delta L}{L} = \alpha\Delta Q$

$\therefore$ Pressure = Stress = Y × strain

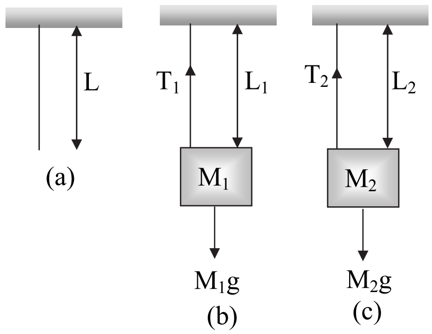
$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^{8}$ Pa

12. This is because due to increase in temperature, the intermolecular forces decrease.

13. For the wire of same material, Young's modulus remain same.

14. L be original length of the wire.



When a mass M_1 is suspended from the wire, change in length of wire,

$\Delta L_1 = L_1 - L$

When a mass M_2 is suspended from it, change in length of wire,

$\Delta L_2 = L_2 - L$

From figure (b), $T_1 = M_1g$ ….(i)

From figure (c), $T_2 = M_2g$ ….(ii)

Young's modulus, $Y = \frac{T_1L}{A\Delta L_1} = \frac{T_2L}{A\Delta L_2}$

$\frac{T_1}{\Delta L_1} = \frac{T_2}{\Delta L_2} \Rightarrow \frac{T_1}{L_1 - L} = \frac{T_2}{L_2 - L}$

$\therefore$ From equations, (i) and (ii)

$\frac{M_1g}{L_1 - L} = \frac{M_2g}{L_2 - L}$

$\Rightarrow M_1(L_2 - L) = M_2(L_1 - L)$

$M_1L_2 - M_1L = M_2L_1 - M_2L$

$\therefore$ $L(M_2 - M_1) = L_1M_2 - L_2M_1$

$\Rightarrow L = \frac{L_1M_2 - L_2M_1}{M_2 - M_1}$

15. $Y = \frac{F_1 \times l}{A \times (L_1 - l)}$ and $Y = \frac{F_2 \times l}{A \times (L_2 - l)}$

$\Rightarrow F_2(L_1 - l) = F_1(L_2 - l)$

$\therefore$ $l = \frac{F_2L_1 - F_1L_2}{F_2 - F_1}$

16. $Y = \frac{\text{stress}}{\text{strain}}$

$\therefore$ strain = $\frac{\text{stress}}{Y}$

$\therefore$ $\frac{l}{L} = \frac{\text{stress}}{Y}$

$\therefore$ Elongation $l = \frac{\text{stress}}{Y} \times L = \frac{1 \times 10^8}{2 \times 10^{11}} \times 1 = 0.5$ mm

17. Extension $l = \frac{MgL}{\pi r^2 Y}$

and the same wire is stretched hence,

$\frac{l_1}{l_2} = \frac{M_1}{M_2}$

here $l_1' = l_1 + L = 101$ mm = 10.1 cm

also, $l_2' = l_2 + L = 102$ mm = 10.2 cm

$\therefore$ $l = l' - L$

$\therefore$ $\frac{l_1' - L}{l_2' - L} = \frac{M_1}{M_2}$

$\therefore$ $\frac{10.1 - L}{10.2 - L} = \frac{80}{100} = \frac{4}{5}$

$\therefore$ $50.5 - 5L = 40.8 - 4L$

$\therefore$ L = 9.7 cm

Similarly, $\frac{l_3' - L}{l_1' - L} = \frac{M_3}{M_1} = \frac{160}{80}$

$\therefore$ $\frac{l_3' - 9.7}{0.4} = \frac{160}{80} = 2$

$\therefore$ $l_3' = 10.5$ cm

18. $l = \frac{FL}{AY} \Rightarrow l \propto \frac{L}{d^2}$

$\therefore$ $\frac{l_1}{l_2} = \frac{L_1}{L_2} \times \left(\frac{d_2}{d_1}\right)^2 = \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$

19. As material is same, Young's modulus of two wires is same.

Also, volume of both wires is same.

$V_1 = V_2$

$\therefore$ $A \times L = 3A \times L'$ $\Rightarrow L' = L/3$

To stretch second wire through same length (Δl), let force needed be F′

$\therefore$ $Y = \frac{F/A}{\Delta l/L} = \frac{F'/3A}{\Delta l/L'}$

$\therefore$ $FL = \frac{F'L'}{3} = \frac{F'L}{3 \times 3}$

$\therefore$ $F = \frac{F'}{9}$

$\therefore$ F′ = 9F

20. Here, $l \propto \frac{L}{A} \propto \frac{L}{\pi r^2}$

For option (A): $l \propto \frac{100}{\pi(0.1)^2} = \frac{1}{\pi}\times 10^4$ cm

For option (B): $l \propto \frac{200}{\pi(0.2)^2} = \frac{2\times10^2}{\pi\times4\times10^{-2}}$

$= \frac{0.5}{\pi}\times10^{-4}$ cm

For option (C): $l \propto \frac{300}{\pi(0.3)^2} = \frac{3\times10^2}{\pi\times9\times10^{-2}}$

$= \frac{0.33}{\pi}\times10^{4}$ cm

For option (D): $l \propto \frac{400}{\pi(0.4)^2} = \frac{4\times10^2}{\pi\times16\times10^{-2}}$

$= \frac{0.25}{\pi}\times10^{-4}$ cm

Thus, we see that highest elongation will be for option (A).

21. Young's Modulus for a wire is given as,

$Y = \frac{Mg\,L}{\Delta L\ A} \Rightarrow \Delta L = \frac{Mg\,L}{Y\,A}$

$\therefore$ $\Delta L \propto \frac{L}{A}$

Now, $\left(\frac{L}{A}\right)$ is maximum for L = 50 cm and diameter = 0.5 mm.

Hence, option (A) is correct.

22. $l = \frac{FL}{AY}$

$\therefore$ $l \propto \frac{1}{r^2}$ ....($\because$ F, L and Y are constant)

$\therefore$ $\frac{l_2}{l_1} = \left(\frac{r_1}{r_2}\right)^2 = (2)^2$

$\therefore$ $l_2 = 4l_1 = 4 \times 3 = 12$ mm

23. $Y = \frac{\text{stress}}{\text{strain}}$

Maximum strain $= \frac{\text{Maximum stress}}{Y} = \frac{mg/A}{Y}$

$\therefore$ $m = \frac{Y\times\text{strain}\times A}{g} = \frac{2\times10^{11}\times10^{-3}\times3\times10^{-6}}{10}$

$= 60$ kg

24. Breaking stress = strain × Young's modulus

$= 0.15 \times 2 \times 10^{11} = 3 \times 10^{10}$ Nm^{-2}

25. $Y_{steel} = 2\,Y_{brass}$

$L_s = L_b$

$A_s = A_b$

$\Delta L_s = \Delta L_b$

$Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} = \frac{WL}{A\Delta L}$

$W = \frac{YA\Delta L}{L}$

$\therefore$ $W \propto Y$

$\frac{W_s}{W_b} = \frac{Y_s}{Y_b} = 2 : 1$

26. $F = \frac{YAl}{L} = \frac{9\times10^{10}\times\pi\times4\times10^{-6}\times0.1}{100} = 360\,\pi$ N

27. $A_1 = 4$ mm^2

Under the same load,

lA = constant

$\therefore$ $\frac{A_2}{A_1} = \frac{l_1}{l_2} \Rightarrow A_2 = 4 \times \frac{0.1}{0.05} = 8$ mm^2

28. $l \propto \frac{1}{Y.A} \propto \frac{1}{Yr^2}$

$\therefore$ $l_A \propto \frac{1}{Y_A r_A^2}$ and $l_B \propto \frac{1}{Y_B r_B^2}$

$\therefore$ $\frac{l_B}{l_A} = \frac{Y_A r_A^2}{T_B r_B^2} = \frac{Y_A}{Y_B}\times\left(\frac{r_A}{r_B}\right)^2 = \frac{2}{1}\times\left(\frac{1}{2}\right)^2 = \frac{1}{2}$

$\therefore$ $l_A : l_B = 2 : 1$

29. Given: $L_P = L_Q$, $F_P = F_Q$ ....($\because$ same load)

$m_P : m_Q = m_1 : m_2$

$Y = \frac{FL}{A\Delta l}$

$\therefore$ $\Delta l = \frac{FL}{AY}$

$\Delta l \propto \frac{1}{A}$

Since $m = \rho V$

$= \rho \times A \times t$

$\therefore$ $m \propto A$

$\therefore$ $\Delta l \propto \frac{1}{m}$

$\therefore$ $\frac{\Delta l_P}{\Delta l_Q} = \frac{m_2}{m_1}$

31. Bulk Modulus

$K = \frac{\text{Hydraulic stress}}{\text{strain}}$

32. Bulk modulus $B = \frac{\Delta P}{-\Delta V / V}$

Expressing volume in terms of radius and change in radius,

$\frac{\Delta V}{V} = \frac{3\Delta R}{R}$

As negative sign indicates decrease, neglecting it,

$B = \frac{P}{3\Delta R / R}$

$\therefore \quad \frac{\Delta R}{R} = \frac{P}{3B}$

33. Bulk modulus is given as,

$K = \left(\frac{-dP}{dV / V}\right)$

where negative sign indicates volume decreases with increase in pressure.

$\therefore$ Fractional decrease in volume will be,

$\frac{dV}{V} = \frac{dP}{K}$

If area of cross-section of cylinder is a, then,

$dP = \frac{mg}{a}$

$\therefore \quad \frac{dV}{V} = \frac{mg}{Ka}$ .... (i)

Also, $V = \frac{4}{3}\pi r^3$

$\therefore \quad \frac{dV}{V} = 3\frac{dr}{r}$ .... (ii)

Equating equations (i) and (ii),

$\frac{dr}{r} = \frac{mg}{3Ka}$

34. $K = \frac{VdP}{dV} = \frac{1}{\left(\frac{dV}{V}\right)} \times dP = \frac{1}{\left(\frac{10}{100}\right)} \times 2 \times 10^5$

$\therefore \quad K = 2 \times 10^6$ N/m^2

35. $K = \frac{VdP}{dV}$

$\therefore \quad dP = \frac{KdV}{V} = 6 \times 10^3 \times \frac{10}{100} = 600$ N/m^2

36. $K = -\frac{P}{\Delta V / V} \Rightarrow \frac{P}{K} = -\frac{\Delta V}{V} = \frac{\Delta\rho}{\rho}$

....$\left(\text{As } \rho \propto \frac{1}{V}\right)$

$\therefore \quad P = \frac{K\Delta\rho}{\rho} = \frac{K \times 0.01}{100} = \frac{K}{10000}$

37. Compressibility $= \frac{1}{K}$

$|K| = \frac{dP}{\left(\frac{dV}{V}\right)} = \frac{h\rho g}{\left(\frac{dV}{V}\right)}$

$\frac{dV}{V} = 2.7 \times 10^3 \times 10^3 \times 9.8 \times 45.4 \times 10^{-11}$

$= 1.2 \times 10^{-2}$

39. $\sigma = \frac{\Delta r / r}{l / L}$

$\therefore \quad \frac{\Delta r}{r} \times 100 = \sigma \times \frac{l}{L} \times 100$

$= 0.5 \times \frac{0.04}{100} \times 100$

$= 0.5 \times \frac{4}{100}$

$= \frac{2}{100} = 0.02$ %

40. $\sigma = \frac{\Delta r / r}{l / L}$

$\therefore \quad \frac{\Delta r}{r} = \sigma \times \frac{l}{L} = 0.5 \times \frac{0.1}{100} = 5 \times 10^{-4}$

$\therefore \quad \Delta r = 5 \times 10^{-4} \times r = 5 \times 10^{-4} \times \frac{2}{2} = 5 \times 10^{-4}$

$\therefore \quad \Delta D = 10 \times 10^{-4} = 10^{-3}$

$\therefore \quad D_1 - D_2 = 10^{-3}$

$\therefore \quad D_2 = 2 - 10^{-3} = 1.999$ mm

41. $\sigma = \frac{\Delta r / r}{\Delta l / l}$

r is radius of wire, l is its length, Δr is change in r and Δl is the change in l when the wire is subjected to tension.

$V_1 = \pi r^2 l$

Volume of wire after elongation is,

$V_2 = \pi (r - \Delta r)^2 (l + \Delta l)$

Given $V_1 = V_2$

$\therefore \quad \pi r^2 l = \pi (r - \Delta r)^2 (l + \Delta l)$

$= \pi [r^2 - 2r(\Delta r) + (\Delta r)^2] (l + \Delta l)$

$= \pi r^2 (l + \Delta l) - 2\pi r \Delta r (l + \Delta l) + \pi(\Delta r)^2 (l + \Delta l)$

$\because$ Δr and Δl are very small, terms of order $(\Delta r \times \Delta l)$ and $(\Delta r)^2$ and higher can be ignored. Then, we have,

$\pi r^2 l = \pi r^2 l + \pi r^2 \Delta l - 2\pi r l \Delta r$

$\therefore \quad r\Delta l = 2l\Delta r \Rightarrow \frac{\Delta l}{l} = 2\frac{\Delta r}{r}$

$\therefore \quad \sigma = \frac{\Delta r / r}{\Delta l / l} = \frac{1}{2} = 0.5$

44. Value of Poisson's ratio lie in range of -1 to $\frac{1}{2}$

45. $Y = 2\eta(1+\sigma)$ we get,

$\sigma = \frac{Y}{2\eta} - 1 = \frac{2.4\eta}{2\eta} - 1 = 0.2$

50. $U = \frac{1}{2} \times F \times l$

$= \frac{1}{2} \times F \times \frac{FL}{AY}$

$= \frac{F^2L}{2AY}$

52. $E \propto l^2$

∴ $E_1 \propto l_1^2$ and $E_2 \propto l_2^2$

∴ $\frac{E_2}{E_1} = \frac{l_2^2}{l_1^2} \Rightarrow E_2 = \frac{E_1 l_2^2}{l_1^2} = E \times \left(\frac{10}{2}\right)^2 = 25\ E$

53. $E \propto l^2$

∴ $E_1 \propto kl_1^2$ and $E_2 \propto kl_2^2$

∴ $\frac{E_2}{E_1} = \frac{l_2^2}{l_1^2} \Rightarrow E_2 = E_1 \times \left(\frac{l_2}{l_1}\right)^2$

$= 0.25 \times \left(\frac{1}{0.2}\right)^2$

$= \frac{1}{4} \times 5^2 = \frac{25}{4}\ J/m^3$

54. $u = \frac{Y}{2} \times (\text{strain})^2 = \frac{1.1\times10^{11}}{2} \times \left(\frac{0.1}{100}\right)^2$

$= 0.55 \times 10^{11} \times (10^{-3})^2 = 5.5 \times 10^4\ J/m^3$

55. $dW = F.dl$

∴ $W = \int_0^L ax\,dx + \int_0^L bx^2\,dx = \frac{aL^2}{2} + \frac{bL^3}{3}$

56. Here, $k_Q = \frac{k_p}{2}$

By Hooke's law,

∴ $F_p = -k_p x_p$

$F_Q = -k_Q x_Q \Rightarrow \frac{F_p}{F_Q} = \frac{k_p}{k_Q}\frac{x_p}{x_Q}$

Given that,

$F_p = F_Q$

∴ $\frac{x_p}{x_Q} = \frac{k_Q}{k_p}$

∴ Energy stored in a spring is $U = \frac{1}{2}kx^2$

$= \frac{k_p x_p^2}{k_Q x_Q^2} = \frac{k_p}{k_Q} \times \frac{k_Q^2}{k_p^2} = \frac{1}{2}$ .... $\left[\because k_Q = \frac{k_p}{2}\right]$

$\Rightarrow U_p = \frac{U_Q}{2} = \frac{E}{2}$ ....[∴ $U_Q = E$]

57. Given,

$\frac{d_1}{d_2} = \frac{1}{3}$

∴ $\frac{r_1}{r_2} = \frac{1}{3}$

$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{9}$

Strain energy per unit volume is given by,

$u = \frac{1}{2} \times \text{stress} \times \text{strain}$

∴ $u = \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L}$

∴ $\frac{u_1}{u_2} = \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2$

∴ $\frac{u_1}{u_2} = \frac{9}{1}$

58. $\frac{Y_A}{Y_B} = \frac{\tan\theta_A}{\tan\theta_B} = \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3$

$Y_A = 3Y_B$

59. $L_2 = l_2(1 + \alpha_2\Delta\theta)$ and $L_1 = l_1(1 + \alpha_1\Delta\theta)$

$\Rightarrow (L_2 - L_1) = (l_2 - l_1) + \Delta\theta(l_2\alpha_2 - l_1\alpha_1)$

Given that, $(L_2 - L_1) = (l_2 - l_1)$

∴ $l_2\alpha_2 - l_1\alpha_1 = 0$

60. For the wires of same material and same thickness, Y and A are the same

$\Rightarrow \frac{FL}{l} = \text{constant}$ or $\frac{L}{\left(\frac{l}{F}\right)} = \text{constant}$

From the graph, $\frac{l}{F} = \text{slope}$

$\Rightarrow L \propto \text{slope}$

⇒ wire A has the largest length.

61. $W_1 = \frac{1}{2}Kl^2$

$W_1 + W_2 = \frac{1}{2}K(l + l_1)^2$

$W_2 = (W_1 + W_2) - W_1$

$= \frac{1}{2}K(l+l_1)^2 - \frac{1}{2}Kl^2$

$= \frac{1}{2}K\left[l^2 + l_1^2 + 2ll_1 - l^2\right]$

$= \frac{1}{2}K\left[l_1 + 2l\right]l_1$

62. Elongation $l = \alpha\Delta\theta L = \alpha tL$ $(\because \Delta\theta = t)$

Force = Y α t A

Work done = $\frac{1}{2}$ × Force × elongation

$\therefore$ $W = \frac{1}{2}\ Y\alpha t A \times \alpha t L = \frac{1}{2}\ Y\alpha^2 t^2 AL$

63. Change in length of rod due to change in temperature is,

$\Delta L = \alpha LT$ ....(i)

Also $Y = \frac{F}{A}\frac{L(1+\alpha T)}{\Delta L}$

$\therefore$ $\Delta L = \frac{FL(1+\alpha T)}{AY}$ ....(ii)

Equating equations (i) and (ii),

$\frac{FL(1+\alpha T)}{AY} = \alpha LT$

$\therefore$ $F = \frac{AY\alpha T}{(1+\alpha T)}$

64. When external pressure is applied on the cube, the compression produced in volume is

$\frac{\Delta V}{V} = \frac{P}{K}$ ....(i)

When heated, the cube will expand through,

$\Delta V = V(\gamma\,\Delta T)$

$\therefore$ $\frac{\Delta V}{V} = 3\alpha\Delta T$ ....(ii) $(\because \gamma = 3\alpha)$

Hence, equating equations (i) and (ii),

$3\alpha\Delta T = \frac{P}{K}$

$\therefore$ $\Delta T = \frac{P}{3\alpha K}$

65. As the lift is moving upward, the maximum tension in the rope = m(g + a)

Stress in the rope = $\frac{F}{A} = \frac{m(g+a)}{\pi r^2}$

$\therefore$ $T = \frac{m(g+a)}{\pi r^2} = \frac{m(g+a)}{\pi\left(\frac{d}{2}\right)^2}$

$\therefore$ $T = \frac{4m(g+a)}{\pi d^2}$

$\therefore$ $d^2 = \frac{4m(g+a)}{\pi T}$

$\therefore$ $d = \left[\frac{4m(g+a)}{\pi T}\right]^{\frac{1}{2}}$

66. As the force acting on both the wires is same,

$l_1 = \frac{FL_1}{A_1Y} = \frac{FL}{4\pi R^2 Y}$ and $l_2 = \frac{F(2L)}{4\pi(2R)^2 Y}$

$\therefore$ $\frac{l_1}{l_2} = \frac{L}{R^2} \times \frac{4R^2}{2L} = 2$

67. The stretching force on the wire due to its own weight is not uniform throughout its length. It is zero at the bottom and maximum at the point of suspension. Thus, average of the two must be taken.

So stretching force is equal to half the weight of wire

$F = \frac{W}{2} = \frac{mg}{2} = \frac{\rho Vg}{2}$

Now, $Y = \frac{F/A}{l/L}$

$\therefore$ $l = \frac{FL}{AY}$ $\therefore$ $l = \frac{\rho Vg}{2}\frac{L}{\pi r^2 Y}$

But $V = AL = \pi r^2 L$

$l = \frac{\rho\pi r^2 Lg L}{2\pi r^2 Y}$ $\therefore$ $l = \frac{\rho L^2 g}{2Y}$

68. $T = \frac{YAl}{L}$

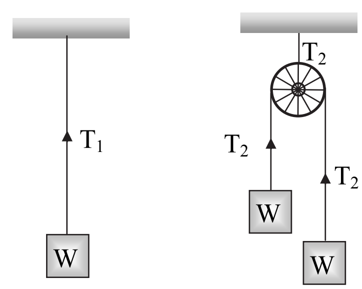
Increase in length of one segment of wire,

$l = \left(L + \frac{1}{2}\frac{d^2}{L}\right) - L = \frac{1}{2}\frac{d^2}{L}$

So, $T = \frac{Y\pi r^2 d^2}{2L^2}$

69. Elongation in the wire $l = \frac{TL}{AY}$

$\therefore$ Elongation in the wire ∝ Tension in the wire



In first case, $T_1 = W$ and in second case,

$$T_2 = \frac{2W \times W}{W + W} = W$$

As tension in the wire in both the cases are equal, the elongations in the wire will be equal.

70. $T = 2\pi\sqrt{\frac{l}{g}}$

Let the length of string change to l_1 due to additional mass

$\therefore \quad T_M = 2\pi\sqrt{\frac{l_1}{g}}$

$\therefore \quad \frac{T}{T_M} = \frac{\sqrt{\frac{l}{g}}}{\sqrt{\frac{l_1}{g}}} \Rightarrow \frac{T^2}{T_M^2} = \frac{l}{l_1}$

$\therefore \quad \frac{T_M^2}{T^2} - 1 = \frac{l_1}{l} - 1$

$\therefore \quad \left[\left(\frac{T_M}{T}\right)^2 - 1\right] = \frac{\Delta l}{l} \qquad$ ....(i)

Now, $Y = \frac{F/A}{\Delta l / l} = \frac{mg}{A} \times \frac{l}{\Delta l}$

or $\quad \frac{1}{Y} = \frac{A \times \Delta l}{Mg \times l} \qquad$ ....(ii)

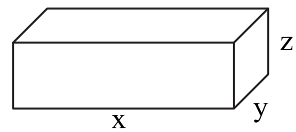
substituting $\Delta l / l$ from equation (i)

$$\frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

71. Thermal stress = $Y\alpha\Delta\theta$

As thermal stress and rise in temperature are equal, $\Rightarrow Y \propto \frac{1}{\alpha} \Rightarrow \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$

72.



Volume of the rod, $V = xyz$

Poisson's ratio for breadth, $\sigma = \frac{-dy/y}{dx/x}$

$\therefore \quad \frac{dy}{y} = -\sigma\frac{dx}{x}$

Similarly, $\frac{dz}{z} = -\sigma\frac{dx}{x}$

$\therefore \quad \frac{dV}{V} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$

$$= \frac{dx}{x} - \sigma\frac{dx}{x} - \sigma\frac{dx}{x} = (1 - 2\sigma)\frac{dx}{x}$$

Now, Stress = Y × strain

Given, Stress = 0.01 Y

$\therefore \quad 0.01\,Y = Y\frac{dx}{x} \qquad \Rightarrow \frac{dx}{x} = 0.01$

$\therefore \quad \frac{dV}{V} = 0.01 \times (1 - 0.6) = 0.01 \times 0.4$

Percentage volume change = 0.4%

Evaluation Test

1. Shear area = πdt (of the plate)

$\therefore$ Maximum shear force = $\sigma_s \pi dt$

Area of cross-section of punch = $\frac{\pi d^2}{4}$

$\therefore$ Maximum normal force of punch = $\frac{\pi d^2}{4} \times \sigma_c$

$\therefore \quad \sigma_s \pi dt = \sigma_c \frac{\pi d^2}{4}$

$$t = \frac{\sigma_c d}{4\sigma_s} = \frac{4 \times 10^8 \times 10 \times 10^{-2}}{4 \times 2 \times 10^8}$$

$$= 0.5 \times 10^{-1} \times 10^2 \text{ cm} = 5 \text{ cm}$$

2. In the first case, the net force is zero. So, the extension is $\frac{FL}{AY}$ but in the other, the body has an acceleration because of which T is a function of distance and hence Δl.

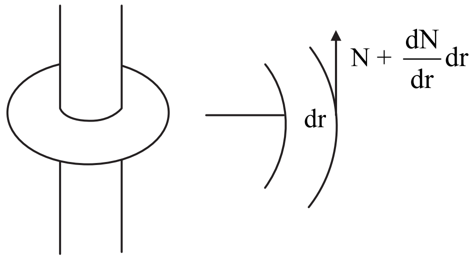
3. $K.E = \frac{1}{2}m\omega^2R^2 = \pi R^3 a\rho\omega^2$

Stressing in ring = $\frac{T}{a} = \rho R^2\omega^2$

$P.E. = \frac{1}{2}\frac{(\text{Stress})^2}{Y} \times \text{volume}$

$P.E. = \frac{\pi\rho^2R^5a\omega^4}{Y}$

$\therefore \quad \frac{K.E}{P.E.} = \frac{\pi R^3 a\rho\omega^2}{\pi\rho^2R^5a\omega^4} \times Y = \frac{Y}{\rho R^2\omega^2}$

4. 

Consider an elementary ring of width dr at a distance r from the axis. The part outside exerts couples N + $\frac{dN}{dr}$ dr on this ring while the part inside exerts a couple N on the opposite direction. We have for equilibrium,

$$\frac{dN}{dr}dr = -dI\beta$$

While dI is the moment of inertia of the elementary ring, β is the angular acceleration and minus sign is needed because the couple (Nr) decreases, with distance, vanishing at the outer radius, N (r_2) = 0, Now,

$$dI = \frac{m}{\pi\left(r_2^2 - r_1^2\right)}\ 2\pi r\ drr^2$$

Thus, $dN = \frac{2m\beta}{r_2^2 - r_1^2} r^3 dr$

On integration, $N = \frac{1}{2}\frac{m\beta}{\left(r_2^2 - r_1^2\right)}\ \left(r_2^4 - r_1^4\right)$

$$= \frac{m\beta\left(r_2^2 + r_1^2\right)}{2}$$

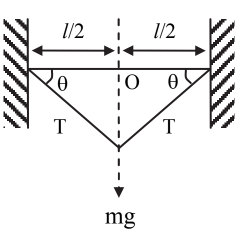
5. If area of cross-section is different, the breaking loads are different for same material.

6. Maximum restoring force develops at the end where force is applied. This force decreases linearly such that it becomes zero at the other end so stress also decreases linearly.

7. Equal Strains ⇒ Equal D$l \propto \frac{F}{AY}$

∴ $\frac{F_1}{F_2} = \frac{A_1}{A_2} \times \frac{Y_1}{Y_2} = 1$

∴ $F_1 = F_2 \Rightarrow$ x = 1 m

8. 

Let the point O descend by distance x

From the condition of equilibrium of point O,

2 T sin θ = mg

or $T = \frac{mg}{2\sin\theta} = \frac{mg}{2x}\sqrt{\left(\frac{l}{2}\right)^2 + x^2}$ ….(i)

Now, $\frac{T}{\pi\left(\frac{d}{2}\right)^2} = \sigma = \varepsilon E$ or $T = \varepsilon E\pi\frac{d^2}{4}$ ….(ii)

(Here, σ is stress and ε is strain)

In addition to this,

$$\varepsilon = \frac{\sqrt{\left(\frac{l}{2}\right)^2 + x^2} - \frac{l}{2}}{\frac{l}{2}} = \sqrt{1+\left(\frac{2x}{l}\right)^2} - 1 \quad ….(iii)$$

From equation (i), (ii) and (iii),

$$x - \frac{x}{\sqrt{1+\left(\frac{2x}{l}\right)^2}} = \frac{mgl}{\pi E d^2}$$

or $x = 1\left(\frac{mg}{2\pi E d^2}\right)^{\frac{1}{3}}$ = 2.5 cm

9. $\Delta l = \frac{Fl}{AY}, \frac{\Delta l}{(F/A)} = \frac{l}{Y}$ = Slope of curve

∴ $\frac{l}{Y} = \frac{(4-2)\times 10^{-3}}{4000 \times 10^3}$

Given, l = 1m → $Y = \frac{4000\times 10^3}{2\times 10^{-3}} = 2\times 10^9\ N/m^2$

10. The change in length of the rod due to increase in temperature in absence of walls is,

$\Delta l = l \propto \Delta T = 1000 \times 10^{-4} \times 20$ mm

= 2 mm

But rod can expand upto 100 mm only.

At that temperature, its natural length is = 1002 mm

∴ Mechanical stress = $Y\frac{\Delta l}{l} = 10^{11} \times \frac{1}{1000}$

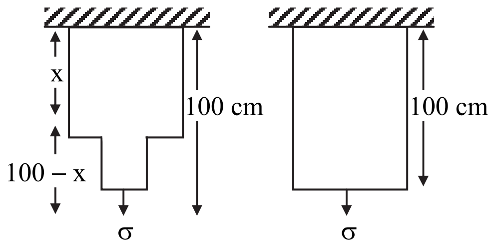
$= 10^8\ N/m^2$

11. The force F_1 causes extension in rod.

F_2 causes compression in left half of rod and an equal extension in right half of rod.

Hence, F_2 does not effectively change length of the rod.

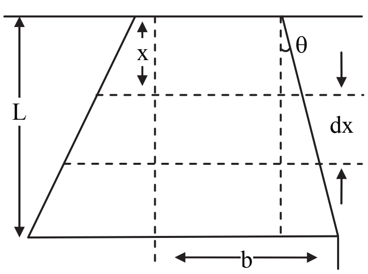
12. Maximum stress lies in stepped bar in the portion of lesser area (5 cm^2)
For the stress σ in lesser area,
the stress in larger cross-section $= \frac{\sigma A/2}{A} = \frac{\sigma}{2}$
Strain energy of stepped bar
$= \frac{\sigma^2}{2Y} \times 5 \times (100 - x) + \left(\frac{\sigma}{2}\right)^2 \times \frac{1}{2Y} \times 10 \times x$



$= \frac{\sigma^2}{2Y}(500 - 5x + 2.5x) = \frac{\sigma^2}{2Y}[500 - 2.5x]$
Strain energy of uniform bar,
$= \frac{\sigma^2}{2Y} \times 10 \times 100$
As per given condition,
$\frac{\sigma^2}{2Y}[500 - 2.5x] = \frac{40}{100} \times \frac{\sigma^2}{2Y} \times 10 \times 100$
∴ $500 - 2.5x = 400$
∴ $2.5x = 100 \Rightarrow x = \frac{100}{2.5} = 40$ cm

13. Atmospheric pressure is same in every direction
Hence, F = PA = 2P

14. Consider an element of length dx at distance dx from the fixed end, then the change in length of element will be.



(a + L tan θ = b)

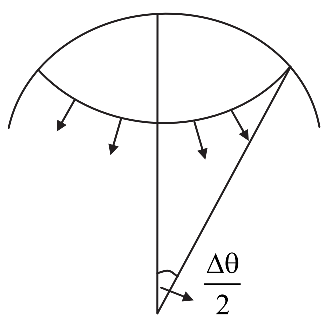
$dy = \frac{Fdx}{YA}$
But, $A = \pi r^2 = \pi(a + x\tan\theta)^2$
∴ $\Delta L = \int_0^L dy = \frac{F}{\pi y}\int_0^L \frac{dx}{(a + x\tan\theta)^2}$

∴ $\Delta L = \frac{FL}{\pi a(a + L\tan\theta)Y} = \frac{FL}{\pi abY}$
$\Rightarrow \Delta L = \frac{6.28 \times 9.8 \times 10}{3.14 \times (19.6 \times 10^{-4}) \times (10 \times 10^{-4}) \times (2 \times 10^{11})}$
∴ $\Delta L = 5 \times 10^{-4}$ m = 0.5 mm

15. In case of punching, shear elasticity is involved, the hole will be punched, if
$\left[\frac{F}{A}\right]$ > ultimate shear stress.
∴ F > (shear stress) × (area)
∴ $F_{min} = (3.45 \times 10^8)(2\pi rl)$
$= (3.45 \times 10^8)(2 \times 3.14 \times 0.73 \times 10^{-2} \times 1.27 \times 10^{-2})$
= 200 kN

16. For a wire, $k = \frac{YA}{l}$
and for the series of combination,
$k_e = \frac{k_1 \times k_2}{k_1 \times k_2} = \frac{(Y_1Y_2)A}{Y_1L_2 + Y_2L_1}$

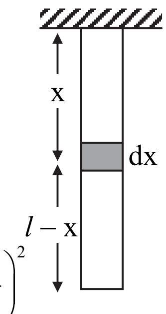
17. We have,
$\eta = \frac{Fl}{A\Delta l}$
$\Rightarrow \Delta l = \frac{Fl}{A\eta} = \frac{9 \times 10^4 \times 0.5}{(0.5)^2 \times 2 \times 10^9}$
$= 9 \times 10^{-5}$ m

18.


Consider an element of area $dS = \pi(r\Delta\theta / r)^2$ about z-axis chosen arbitrarily. There are tangential tensile forces all around the ring of the cap. Their resultant is
$S\left[2\pi\left(\frac{r\Delta\theta}{2}\right)\Delta r\right]\sin\frac{\Delta\theta}{2}$
Hence, in the limit,
$P_m\pi\left(\frac{r\Delta\theta}{2}\right)^2 = S\pi\left(\frac{r\Delta\theta}{2}\right)\Delta r\,\Delta\theta$
or $P_m = \frac{2S\Delta r}{r} = 39.5$ atm.

19. When a rod is deformed by its own weight, the stress increases as one moves up, the stressing force being the weight of the portion below the element considered.

$\therefore$ Stress on element dx is,
$\rho\pi r^2 (l - x)g / \pi r^2 = \rho g (l - x)$
Extension of the element is
$\Delta dx = d\Delta dx = \rho g (l - x)\, dx/E$
Integrating, we get the extension of the whole rod as,



$$\Delta l = \frac{1}{2}\frac{\rho g l^2}{E}$$

Elasitc energy of the element is

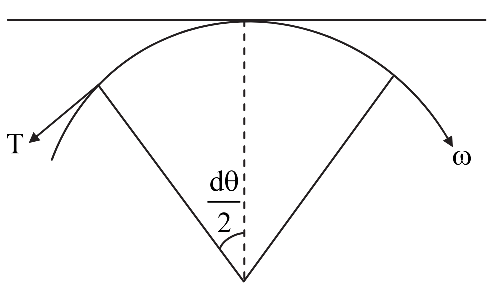
$$\frac{1}{2}\rho g (l - x)\ \frac{\rho g(l - x)}{E}\ \pi r^2\, dx$$

Integrating,

$$\Delta U = \frac{1}{6}\frac{\pi r^2\rho^2 g^2 l^3}{E} = \frac{2}{3}\pi r^2 l E\left(\frac{\Delta l}{l}\right)^2$$

20. $2T \sin\dfrac{d\theta}{2} = (Rd\theta)a\rho\omega^2 R \quad \ldots\left[\sin\dfrac{d\theta}{2} \approx \dfrac{d\theta}{2}\right]$

$\therefore$ $T = a\rho R^2\omega^2$



21. $\left(\dfrac{\rho V g}{2A}\right) = \text{stress} = \sigma$

$\therefore$ $\dfrac{\rho L g}{2} = \sigma \Rightarrow L = \dfrac{2\sigma}{\rho g}$

22. $T - W = mv^2/r$

or $T = W + \dfrac{mv^2}{r}$

$$= 10\text{ N} + \frac{(1\text{kg})(2\text{m}^{-1})^2}{0.2\text{m}} = 30\text{ N}$$

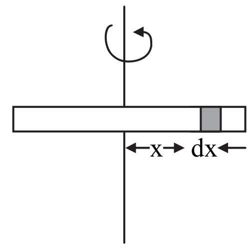
We have $Y = \dfrac{T/A}{l/L}$

or $l = \dfrac{TL}{AY}$

$$= \frac{30\text{N}\times(20\text{cm})}{(3\times10^{-5}\text{m}^2)\times(2\times10^{11}\text{Nm}^{-2})}$$

$= 5 \times 10^{-5} \times 20\text{ cm} = 10^{-3}\text{ cm} = 10\ \mu\text{m}$

23.



Let is consider an element of rod at a distance n from its rotation axis. (From Netwon's second law in projection from directed towards the rotation axis,

$$-dT = (dm)\omega^2 x = \frac{m}{l}\omega^2 x\,dx$$

On integrating. $-T = \dfrac{m\omega^2}{l}\dfrac{x^2}{2} + c$ (constant)

But at, $x = \pm\dfrac{1}{2}$ or free end, $T = 0$

Thus at, $0 = \dfrac{m\omega^2}{2}\dfrac{R}{4} + c$ or $c = \dfrac{m\omega^2 l}{8}$

Hence, $T = \dfrac{m\omega^2}{2}\left(\dfrac{1}{4} - \dfrac{x^2}{l}\right)$

Thus, $T_{max} = \dfrac{m\omega^2 l}{8}$ (at mid-point)

Condition required for problem is,

$T_{max} = 5\sigma_m$

So, $\dfrac{m\omega^2 l}{8} = 5\sigma_m$ or $\omega = \dfrac{2}{l}\sqrt{\dfrac{2\sigma_m}{\rho}}$

Hence the number of r.p.s.,

$$n = \frac{\omega}{2\pi} = \frac{1}{\pi l}\sqrt{\frac{2\sigma_m}{\rho}}$$

24. Suppose that the steel band was made into a loop of radius R, then length the loop $l = 2\pi R$

Consider, an infinitesimally thin section of radius ρ and thickness dρ in the loop. The length of this section of loop is 2πρ. Hence, the longitudinal strain corresponding to this section is,

$$\varepsilon = \frac{2\pi\rho - 2\rho R}{2\pi R} = \frac{P}{R} - 1$$

So, elastic energy density is,

$$u = \frac{1}{2}E\varepsilon^2 = \frac{1}{2}E\left(\frac{P}{R} - 1\right)^2$$

$\therefore \quad U = \int u \, dV = \int_{R+\frac{\delta}{2}}^{R+\frac{\delta}{2}} \frac{1}{2}E\left(\frac{x}{R}-1\right)^2 2\pi \, \rho \, d\rho \, b$

$= \frac{\pi^2 E b \delta^3}{l}$ ….(on integrating)

25. By Energy Conservation,

$\frac{1}{2} F\Delta l = \frac{1}{2} MV^2$

$\therefore \quad V = \sqrt{\frac{F\Delta l}{M}} = \sqrt{\frac{100 \times 4cm}{1kg}} = 2 \text{ m/s}$

Textbook Chapter No.

06 Surface Tension

Hints

Classical Thinking

20. Weight = $2\pi rT$

Hence, radius remaining constant, $W \propto T$

$\therefore \frac{W_1}{W_2} = \frac{T_1}{T_2} = \frac{30}{60} = \frac{1}{2}$

38. Waterproofing agents are used so that the material does not get wet. This means angle of contact is obtuse.

53. Excess pressure inside soap bubble, $P = \frac{4T}{r}$

Smaller bubble has more excess pressure.

55. $P \propto \frac{1}{r}$

$\therefore \frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{1}{2} \Rightarrow P_1 : P_2 = 1 : 2$

56. $P = \frac{4T}{r} = \frac{4\times0.04}{5\times10^{-3}} = \frac{4\times40\times10^{-3}}{5\times10^{-3}} = 32$ Pa

57. $P = \frac{2T}{r} = \frac{2\times7.2\times10^{-2}}{10^{-3}} = 14.4 \times 10^1$

$= 144$ N/m^2

58. $P = \frac{4T}{r} = \frac{4\times30}{3\times10^{-1}} = 400$ dyne/cm^2

62. $h = \frac{2T\cos\theta}{r\rho g} \Rightarrow h \propto T$

64. $h \propto \frac{1}{r} \propto \frac{1}{D}$

75. Using $T = F/l$ we get,

$T = \frac{1N}{m} = \frac{1\times10^5 \text{ dyne}}{10^2 \text{ cm}} = 10^3$ dyne / cm

Critical Thinking

3. $F_A < \frac{F_C}{\sqrt{2}}$ or $F_C > \sqrt{2}\ F_A$

Clearly, the cohesive force dominates.

5. Surface tension of oil is less than that of water. So oil spreads on water.

6. Surface Tension = 70 dyne/cm = $\frac{70\times10^{-5}}{10^{-2}}$

$= 7 \times 10^{-2}$ N/m

7. A membrane has two free surfaces, therefore total force acting on each side = $T \times 2L$

Force per unit length of the frame = $\frac{T\times2L}{L}$

$= 2T$

8. $T = \frac{F}{2l} = \frac{720}{2\times5} = 72$ dyne/cm

9. The force on disc = T × circumference

$= 7 \times 10^{-2} \times 2 \times \pi \times r$

$= 7 \times 10^{-2} \times 2 \times \frac{22}{7} \times (20 \times 10^{-2})$

$= 8.8 \times 10^{-2}$ N

10. $F = T \times (2\pi R)$

$\therefore (2\pi R) = \frac{F}{T} = \frac{75\times10^{-4}}{6\times10^{-2}} = 12.5 \times 10^{-2}$ m

11. $F = T \times l = 2 \times 2\pi r \times T = 0.0616 \times 10^5$ dyne

$\therefore T = \frac{6160\times7}{4\times22\times7}$ dyne cm^{-1}

$= 70$ dyne cm^{-1}

12. Force due to S.T. $= 2\,(2\pi r)\,T$

$\therefore$ Force required to lift the ring $= 2\,(2\pi r)\,T$

$= 2 \times 2 \times \frac{22}{7} \times \frac{3}{4} \times 10^{-2} \times 0.07$

$= 22 \times 3 \times 10^{-2} \times 0.01$

$= 66 \times 10^{-4}$ N

13. $F = T \times (2\pi r_1 + 2\pi r_2)$

$= T \times 2\pi \times (1.75 + 2.25) \times 10^{-2}$

$= 0.074 \times 2 \times 3.14 \times 4 \times 10^{-2}$

$= 1.86 \times 10^{-2}$ N

14. $F = \frac{2AT}{t} = \frac{2\times8\times75}{0.12\times10^{-1}} = 10^5$ dyne

15. Pull due to surface tension $= T \times 2 \times (l + t)$

$= 0.07 \times 2(9.8 + 0.2) \times 10^{-2}$

$= 14 \times 10^{-3}$ N

16. Surface energy should remain constant by law of conservation of energy. Hence, total surface area should be conserved, i.e.
$4\pi r_1^2 + 4\pi r_2^2 = 4\pi r^2$
Let $r_1 = r_2 = r \Rightarrow r^2 + r^2 = R^2$
$\therefore$ $R = \sqrt{2}\, r = 1.4\, r$

17. Here, Assertion is false but Reason is true. As work done is,
$W = \text{S.T.} \times \text{increase in area}$
or $\text{S. T.} = \dfrac{W}{\text{increase in area}}$
$= \dfrac{2\times 10^{-4}}{(10\times 8 - 10\times 4)10^{-4}}$
$= 5 \times 10^{-2}$ N/m.

18. $dW = T \times 8\pi\ (R_2^2 - R_1^2)$
$= T \times 8\pi(25R^2 - 9R^2)$
$= T \times 8\pi\ (16\ R^2) = 128\ \pi R^2 T$

19. W = T × Surface area of bubble
Since the soap bubble has two surfaces,
$W = T \times 2 \times 4\pi R^2 = 8\pi R^2 T$

20. $W = 2 \times 4\pi R^2 \times \sigma$; R is increased by a factor of 2, so W is increased by a factor of 4.

21. Increase in surface area $= n \times 4\pi r^2 - 4\pi R^2$
Required energy is equal to the product of surface tension and increase in surface area.
$= (4\pi n r^2 - 4\pi R^2) \times T$

22. $T = \dfrac{\text{Work done}}{\text{Change in area}}$
$\therefore$ $T = \dfrac{3\times 10^{-4}}{2\times(10\times 11 - 10\times 6)\times 10^{-4}} = 3 \times 10^{-2}$ N/m

23. Effective area $= 2 \times 0.02\ m^2 = 0.04\ m^2$
Surface energy, $T\Delta A = 5\ N\ m^{-1} \times 0.04\ m^2$
$= 2 \times 10^{-1}$ J

24. $\Delta P = \dfrac{4T}{r} \Rightarrow \Delta P \propto \dfrac{1}{r}$
Further, as radius of soap bubble increases with time, $\Delta P \propto \dfrac{1}{t}$

25. Work done = S.T. × increase in surface area
$= 25 \times 10^{-3} \times 2 \times 4\pi \times [(9 \times 10^{-2})^2 - (6 \times 10^{-2})^2]$
$= 200 \times 10^{-3} \times \pi \times [45 \times 10^{-4}]$
$= 9000\ \pi \times 10^{-7}$
$= 90\pi \times 10^{-5}$ J

26. Work done in blowing a soap bubble of radius R is given by, $W = 8\pi R^2 T$
$= 8 \times 3.14 \times \left(\dfrac{6\times 10^{-2}}{2}\right)^2 \times 2.1 \times 10^{-2}$
$= 47.4 \times 10^{-5}$ J

27. Since conditions are isothermal, therefore, energy will be conserved.
$\therefore$ $2[2 \times 4\pi r^2 T] = 2 \times 4\pi R^2 T$
$R^2 = 2r^2$ $\therefore$ $R = 2^{1/2}\, r$

28. $V = \dfrac{4}{3}\pi r^3 \Rightarrow V \propto r^3 \Rightarrow r \propto V^{1/3}$
Now,
$W = 4\ \pi\ r^2\ T \Rightarrow W \propto r^2 \propto V^{2/3}$
$\therefore$ $\dfrac{W'}{W} = \left(\dfrac{r'}{r}\right)^2 = \left(\dfrac{2V}{V}\right)^{2/3} = (2)^{2/3} = 4^{1/3}$
$\therefore$ $W' = 4^{1/3}\ W$

29. Work done = surface tension × change in surface area
$= T \times (2A - A)$
$= T \times A$
$= 3 \times 10^{-3} \times 1.3 \times 10^{-4}$
$= 3.9 \times 10^{-7}$ J

30. $2 \times \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$ or $R = 2^{1/3} r$
Final surface area $= 4\pi R^2 = 4\pi 2^{2/3}\, r^2$
Initial surface area $= 2 \times 4\pi r^2$
$\therefore$ Energy released $= [8\pi r^2 - 4 \times 2^{2/3}\ \pi r^2]T$

31. Work done $= T \times \Delta A$
$= 0.072 \times [(20 \times 0.2 \times 10^{-4}) - (20 \times 0.1 \times 10^{-4})]$
$= 0.072 \times 0.1 \times 20 \times 10^{-4}$
$= 0.072 \times 2 \times 10^{-4}$
$= 1.44 \times 10^{-5}$ J

32. Initial surface area = 2 × length × separation
$= 2 \times 10 \times 0.5$
$= 10\ cm^2$
$= 10 \times 10^{-4}\ m^2$
Final surface area
$= 2 \times 10 \times (0.5 + 0.1) \times 10^{-4} = 12 \times 10^{-4}\ m^2$
Work done $= W = T \times \Delta A$
$= 0.070 \times [12 \times 10^{-4} - 10 \times 10^{-4}] = 14 \times 10^{-6}$ J

33. Area of film $= 2\ (10 \times 10^{-2} \times 5 \times 10^{-2})$
$= (50 \times 10^{-4}\ m^2) \times 2$
$W = T\Delta A$
$= 0.035 \times (50 \times 10^{-4}) \times 2$
$= 0.035 \times 100 \times 10^{-4}$
$= 0.035 \times 10^{-2} = 3.5 \times 10^{-4}$ J

34. Let r = radius of each small drop and R = radius of a big single drop.

Then $n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\therefore$ $R = n^{1/3} r$

Initial surface energy

$= E_1 = n \times 4\pi r^2 \times T = nE$

Final surface energy

$= E_2 = 4\pi R^2 \times T = 4\pi r^2 n^{2/3} \times T = n^{2/3} E$

Energy released $= E_1 - E_2 = E(n - n^{2/3})$

35. $u = T \times 4\pi R^2$

When drop is sprayed into 1000 droplets each of radius r, then

$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{10}$

$u' = 1000 \times T \times 4\pi r^2$

$= 1000 \times T \times 4\pi \frac{R^2}{100} = 10 \times 4\pi R^2 T = 10\ u$

36. Volume of small droplet $= \frac{4}{3}\pi r^3$

Volume of big drop $= \frac{4}{3}\pi R^3$

Due to volume conservation,

$\frac{4}{3}\pi R^3 = 64 \times \left(\frac{4}{3}\pi r^3\right)$

$\therefore$ $R^3 = (4)^3 r^3 \Rightarrow R = 4r$

$\therefore$ $r = \frac{R}{4} = \frac{1}{4} = 0.25$ mm

Work done $= T \times \Delta A = T[n4\pi r^2 - 4\pi R^2]$

$= 4\pi T[nr^2 - R^2]$

$= 4\pi \times 72 \times 10^{-3}[64 \times (0.25 \times 10^{-3})^2 - (10^{-3})^2]$

$= 288\pi \times 10^{-3}[4 \times 10^{-6} - 10^{-6}]$

$= 2.7 \times 10^{-6}$ J

37. $\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

$\therefore$ $R^3 = 8r^3 \Rightarrow R = 2r$

Work done $= T(n \times 4\pi r^2 - 4\pi R^2)$

$= T\left(8 \times 4\pi \times \frac{R^2}{4} - 4\pi R^2\right)$

$= T\,4\pi(2R^2 - R^2) = 4\pi R^2 T$

38. $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \Rightarrow R^3 = nr^3$

$\therefore$ $R = n^{1/3} r \Rightarrow 1.4 = 5r$

$\therefore$ $r = \frac{1.4}{5} = 0.28$ mm

Change in energy $= T \times \Delta A$

$= 75 \times [n4\pi r^2 - 4\pi R^2]$

$= 75 \times 4 \times \pi[125(0.28 \times 10^{-1})^2 - (1.4 \times 10^{-1})^2]$

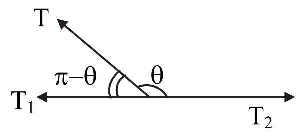
$= 300 \times 3.14[5(1.4 \times 10^{-1})^2 - (1.4 \times 10^{-1})^2]$

$= 300 \times 3.14 \times 4 \times 1.96 \times 10^{-2}$

$= 9.42 \times 7.84 \approx 74$ erg

39. $T_1 + T\cos(\pi - \theta) = T_2$

$\therefore$ $\cos(\pi - \theta) = \frac{T_2 - T_1}{T}$



$\therefore$ $-\cos\theta = \frac{T_2 - T_1}{T}$

$\therefore$ $\cos\theta = \frac{T_1 - T_2}{T}$

40. Excess pressure P for a soap bubble is

$P = 2 \times \frac{4T}{r}$ ....($\because$ bubble has two surfaces)

$= \frac{2 \times 4 \times 0.02}{4 \times 10^{-2}} = 4$ N/m^2

41. $P_1 = 4P_2$

$\therefore$ $\frac{4T}{r_1} = 4 \times \frac{4T}{r_2} \Rightarrow r_2 = 4r_1$

$\therefore$ $V = \frac{4}{3}\pi r^3 \Rightarrow V \propto r^3$

$\therefore$ $\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$

42. $\frac{V_1}{V_2} = \frac{8}{1}$

$\therefore$ $\frac{\left(\frac{4}{3}\pi r_1^3\right)}{\left(\frac{4}{3}\pi r_2^3\right)} = \frac{8}{1}$

$\therefore$ $\left(\frac{r_1}{r_2}\right)^3 = \frac{8}{1} \Rightarrow \frac{r_1}{r_2} = \frac{2}{1}$

But $P \propto \frac{1}{r}$

$\therefore$ $\frac{P_1}{P_2} = \frac{r_2}{r_1} = \frac{1}{2}$

43. $\Delta P = T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

As $r_1 = r$ and $r_2 = \infty$,

$\Delta P = \frac{T}{r}$ But $r = d/2$

$\therefore$ $\Delta P = \frac{2T}{d}$

$\therefore \quad F = P.A = \frac{2T}{d}A$

$= \frac{2 \times 75 \times 10}{0.01}$

$= 150 \times 10^3$ dyne

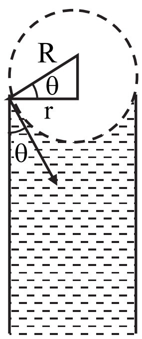
$= 150$ gm-wt

44. $h = \frac{2T\cos\theta}{r\rho g}$

$\therefore \quad h\rho g = \frac{2T\cos\theta}{r}$

45. $h = \frac{2T\cos\theta}{r\rho g} \Rightarrow h \propto \frac{1}{r}$

46.



From figure, $R = \frac{r}{\cos\theta}$

47. $h = \frac{2T}{r\rho g}$

$\therefore \quad r = \frac{2T}{h\rho g}$ (where r = radius of curvature)

$= \frac{2 \times 547}{1.356 \times 13.59 \times 980} = 0.06$ cm

48. Rise in capillary $= h = \frac{2T\cos\theta}{r\rho g}$

As angle of contact $\theta = 0° \Rightarrow \cos\theta = 1$ and $\rho = 1$ g/cc

$\therefore \quad h = \frac{2T}{r\rho g} = \frac{2 \times 70}{(1/42) \times 1 \times 980}$

$\therefore \quad h = \frac{140 \times 42}{980} \Rightarrow h = 6$ cm

49. $h = \frac{2T\cos\theta}{r\rho g} \Rightarrow T = \frac{hr\rho g}{2\cos\theta}$

$\therefore \quad \frac{T_w}{T_m} = \frac{h_w}{h_m} \times \frac{\cos\theta_m}{\cos\theta_w} \times \frac{\rho_w}{\rho_m}$

$= \frac{10}{3.42} \times \frac{\cos 135°}{\cos 0°} \times \frac{1}{13.6} = \frac{1}{6.5}$

50. $h = \frac{2T}{r\rho g}$

$\therefore \quad \frac{h_m}{h_e} = \frac{g_e}{g_m} = 6 \quad \ldots.\left[\because g_m = \frac{g_e}{6}\right]$

$\therefore \quad h_m = 6\,h_e = 6\,h$

51. In an artificial satellite, there is a state of weightlessness. So, water will rise up to full length of tube and will form a new surface of higher radius of curvature but will not come out.

52. $h = \frac{2T\cos\theta_1}{r\rho g} = \frac{2T\cos 0°}{r\rho g} = 4$

$\Rightarrow \frac{2T}{r\rho g} = 4$

$\therefore \quad \frac{2T\cos\theta_2}{r\rho g} = 2$

$\therefore \quad 4 \times \cos\theta_2 = 2 \Rightarrow \cos\theta_2 = \frac{1}{2}$

$\therefore \quad \theta_2 = 60°$

53. $h = \frac{2T\cos\theta}{r\rho g} \Rightarrow T = \frac{hr\rho g}{2\cos\theta}$

$\therefore \quad \frac{T_l}{T_w} = \frac{\rho_l}{\rho_w} \times \frac{h_l}{h_w} = \frac{850}{1000} \times 3.0 = 2.55$

$\therefore \quad T_l = 7.0 \times 10^{-2} \times 2.55 = 0.18$ N/m

54. $h_2 - h_1 = \frac{2T\cos\theta}{\rho g}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

$= \frac{4T\cos\theta}{\rho g}\left[\frac{1}{D_2} - \frac{1}{D_1}\right]$

$= \frac{4 \times 7 \times 10^{-2} \times \cos 0^{o}}{10^3 \times 10 \times 10^{-3}}\left[\frac{1}{3} - \frac{1}{6}\right]$

$= \frac{28 \times 10^{-2}}{10}\left[\frac{1}{3} - \frac{1}{6}\right]$ m

$= 4.66 \times 10^{-3}$ m $= 4.66$ mm

55. $h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 0.072 \times \cos 0°}{0.024 \times 10^{-2} \times 1000 \times 10}$

$= 6$ cm $\quad \ldots.[\because \cos 0° = 1]$

56. $h = \frac{2T\cos\theta}{r\rho g} \Rightarrow h \propto \frac{1}{r}$

$\therefore \quad \frac{h_A}{h_B} = \frac{r_B}{r_A} = \frac{r_B}{2r_B} = \frac{1}{2}$

57. $l \cos 60° = 2$ or $l = 2 \times 2$ cm $= 4$ cm

58. $l = \dfrac{h}{\sin(90-\theta)}$

$= \dfrac{h}{\sin 60°} = \dfrac{6}{\sqrt{3}/2} = \dfrac{12}{\sqrt{3}} = 4\sqrt{3}$ cm

59. $\dfrac{r_1}{r_2} = \dfrac{h_2}{h_1} = \dfrac{6.6}{2.2} = \dfrac{3}{1}$

60. $\dfrac{2T}{R} = h\rho g \Rightarrow 4T/2R = h\rho g$

∴ $\dfrac{4T}{D} = h\rho g$

∴ $D = \dfrac{4T}{h\rho g} = \dfrac{4\times0.07}{0.40\times10^3\times9.8} = \dfrac{1}{14}\times10^{-3}$ m

∴ $D = \dfrac{1}{14}$ mm

61. $P_1 = \dfrac{4T}{r_1}$, $P_2 = \dfrac{4T}{r_2} \Rightarrow P_1 = 2P_2$

∴ $\dfrac{1}{r_1} = \dfrac{2}{r_2} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{1}{2}$

Now, $V_1 = \dfrac{4}{3}\pi r_1^3$, $V_2 = \dfrac{4}{3}\pi r_2^3$

∴ $V_1 = nV_2$

∴ $\dfrac{4}{3}\pi r_1^3 = n\dfrac{4}{3}\pi r_2^3 \Rightarrow r_1^3 = n r_2^3$

∴ $n = \left(\dfrac{r_1}{r_2}\right)^3 = \left(\dfrac{1}{2}\right)^3 = \dfrac{1}{8} = 0.125$

62. As volume is conserved,

$\dfrac{4}{3}\pi R^3 = n\dfrac{4}{3}\pi r^3$

∴ $n = \dfrac{R^3}{r^3} = \left(\dfrac{0.5\times10^{-2}}{1\times10^{-3}}\right)^3 = (5)^3 = 125$

∴ $R^3 = 125r^3 \Rightarrow R = 5r$

$W = n4\pi r^2T - 4\pi R^2T$

$= n4\pi r^2T - 4\pi(25r^2)T$

$= 4\pi r^2T\,(125 - 25)$

$= 400 \times \dfrac{22}{7} \times 10^{-6} \times 7 \times 10^{-2}$

$= 88 \times 10^{-6}$

∴ $W = 8.8 \times 10^{-5}$ J

63. Let r be the radius of each droplet and R be the radius of the big drop.
Since the total volume is the same, we have

$10^6 \times \dfrac{4\pi r^3}{3} = \dfrac{4\pi R^3}{3}$

∴ $R^3 = 10^6 r^3 \Rightarrow R = 100\,r$

∴ The surface energy of one million drops,
$E_1 = 4\pi r^2T \times 10^6$
The surface energy of one big drop,
$E_2 = 4\pi R^2T$

∴ $\dfrac{E_2}{E_1} = \left(\dfrac{R}{r}\right)^2 \times \dfrac{1}{10^6} = \left(\dfrac{100r}{r}\right)^2 \times \dfrac{1}{10^6} = \dfrac{1}{10^2}$

64. External pressure
= atmospheric pressure + ρgh
where ρ is density of water = 1000 kg/m^3

∴ External pressure = $10^5 + 1000 \times 10 \times 20$
= $10^5 + 2 \times 10^5 = 3 \times 10^5$ N/m^2

Competitive Thinking

9. Force required to separate the plates,

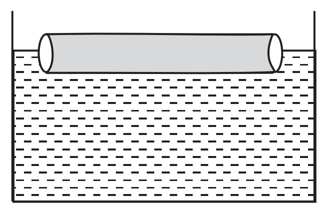
$F = \dfrac{2TA}{t} = \dfrac{2\times70\times10^{-3}\times10^{-2}}{0.05\times10^{-3}} = 28$ N

10. $\dfrac{F_{flat}}{F_{curved}} = \dfrac{T\times2r}{T\times\pi r} = \dfrac{2}{\pi}$

11. $2Tl = mg$

∴ $T = \dfrac{mg}{2l} = \dfrac{1.5\times10^{-2}}{2\times30\times10^{-2}} = \dfrac{1.5}{600} = 0.025$ N/m

12.



For wire to float into water, its weight should be balanced by the surface tension of the water.

∴ $mg = Tl$ ....(where, l = length of the wire)

∴ $V\rho g = Tl$

∴ $\pi r^2 l\rho g = Tl$

∴ $r^2 = \dfrac{T}{\pi\rho g}$

∴ $r = \sqrt{\dfrac{T}{\pi\rho g}}$

16. *Refer Shortcut 9*

18. Work done in increasing the radius of soap bubble is $W = 8\pi T[r_2^2 - r_1^2] = 8\pi T\,(4r^2 - r^2)$
$= 24\pi r^2T$

19. $W \propto r^2$

∴ $W_1 \propto r_1^2$ and $W_2 \propto r_2^2$

∴ $\dfrac{W_1}{W_2} = \left(\dfrac{r_1}{r_2}\right)^2 = \left(\dfrac{4}{3}\right)^2 = 16 : 9$

20. $W = 4\pi r^2 T (n - n^{2/3})$

$W = 4\pi \times \frac{(2\times10^{-3})^2}{4} \times 0.072\left[1000 - (10^3)^{2/3}\right]$

$W = 8.146 \times 10^{-4}$ J

21. $W = 8\pi r^2 T = 8 \times 3.14 \times (5 \times 10^{-2})^2 \times 30 \times 10^{-2}$

$= 1.88 \times 10^{-2}$ J

22. $W = 8\pi r^2 T = 8 \times 3.14 \times (1 \times 10^{-2})^2 \times 3 \times 10^{-2}$

$= 7.54 \times 10^{-5}$ J

23. $W = 8\pi\left(r_2^2 - r_1^2\right)T$

$= 8 \times 3.14 \times [(6 \times 10^{-2})^2] - (4 \times 10^{-2})^2] \times 0.035$

$= 17.58 \times 10^{-4} \approx 1.8 \times 10^{-3}$ J

24. Net force on stick $= F_1 - F_2 = (T_1 - T_2)l$

$= (0.07 - 0.06) \times 2 = 0.01 \times 2 = 0.02$ N

25. The rectangular film of liquid has two surfaces. Hence, the increase in surface area is,

$\Delta A = [(5 \times 4)\ cm^2 - (4 \times 2)\ cm^2] \times 2$

$= (20 - 8) \times 2\ cm^2$

$= 24 \times 10^{-4} m^2$

Also,

W = T.ΔA

∴ $T = \frac{W}{\Delta A} = \frac{3\times10^{-4}}{24\times10^{-4}} = 0.125\ Nm^{-1}$

26. Surface energy = surface tension × surface area

E = T × 2A

∴ New surface energy, $E_1 = T \times 2(A/2) = T \times A$

∴ % decrease in surface energy $= \frac{E - E_1}{E} \times 100$

$= \frac{2TA - TA}{2TA} \times 100 = 50\%$

27. Surface area of drop, $A_1 = 4\pi R^2$

Surface area of 512 droplets, $A_2 = 512\ (4\pi r^2)$

∵ volume of drop = n × (volume of droplet)

∴ $\frac{4}{3}\pi R^3 = 512 \times \frac{4}{3}\pi r^3$

∴ R = 8r

∴ $A_2 = \frac{512(4\pi R^2)}{64}$

∴ $A_2 = 8\ (4\pi R^2)$

Surface energy ∝ Area

∴ $\frac{E_2}{E_1} = \frac{A_2}{A_1} = \frac{8(4\pi R^2)}{4\pi R^2}$

∴ $E_2 = 8E_1 = 8E$ ... {∵ $E_1 = E$}

28. Let R be the radius of bigger drop and r be the radius of single small water drop.

Volume of big drop = n(Volume of small drop)

∴ $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$

$\Rightarrow R^3 = nr^3$

$R = n^{\frac{1}{3}} r$

Surface energy of n drops $(E_n) = n \times 4\pi r^2 \times T$

Surface energy of big drop $(E) = 4\pi R^2 T$

∴ $\frac{E_n}{E} = \frac{nr^2}{R^2} = \frac{nr^2}{\left(n^{\frac{1}{3}}r\right)^2} = \frac{nr^2}{n^{\frac{2}{3}}r^2} = n^{\frac{1}{3}} = \sqrt[3]{n} : 1$

29. As volume remains constant,

$R^3 = 8000r^3 \Rightarrow R = 20r$

∴ $\frac{\text{Surface energy of one big drop}}{\text{Surface energy of 8000 small drop}} = \frac{4\pi R^2 T}{8000\ 4\pi r^2 T}$

$= \frac{R^2}{8000r^2} = \frac{(20r)^2}{8000r^2} = \frac{1}{20}$

30. n = 1000, R = 1 cm,

By applying conservation of volume

initial volume = final volume

$(1)^3 = nr^3$

$r = \frac{1}{n^{1/3}} = \frac{1}{(1000)^{1/3}}$

$r = \frac{1}{10}$ cm

r = 0.1 cm

r = 0.001 meter

Gain in surface energy

$= T\Delta S$

$= 0.075\ \{4\pi\ [1000 \times (0.001)^2 - (0.01)^2]\}$

Gain in surface energy $= 8.5 \times 10^{-4}$ J

31. $W = T\Delta A$

$= 0.03\ [\ 2 \times 4\pi \times (5^2 - 3^2) \times 10^{-4}]$

$= 24\pi\ (16) \times 10^{-6}$

$= 0.384\ \pi \times 10^{-3}$ J

$\approx 0.4\pi$ mJ

32. $r = \sqrt{r_1^2 + r_2^2} = \sqrt{9+16} = 5$ cm

(Note: Refer to Mindbender 1.)

33. $\Delta P \propto \frac{1}{r} \Rightarrow \frac{\Delta P_1}{\Delta P_2} = \frac{r_2}{r_1} = \frac{r}{4r} = \frac{1}{4}$

34. $r = \frac{r_1 r_2}{r_1 - r_2} = \frac{5\times4}{5-4} = 20$ cm

(Note: Refer to Mindbender 2.)

40. Since for such liquid (Non-wetting), angle of contact is obtuse.

41. Cohesive force decreases; so angle of contact decreases.

42. Angle of contact is acute.

44. Since the soap bubble has two surfaces, excess pressure is

$P = \frac{2 \times 2T}{r} = \frac{4T}{r}$

46. $\Delta P \propto \frac{1}{r} \Rightarrow \frac{r_1}{r_2} = \frac{\Delta P_2}{\Delta P_1} = \frac{1}{3}$

∴ $\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{27}$

47. $P_1V_1 = P_2V_2$

∴ $(H+h)\rho g \times \frac{4}{3}\pi r^3 = H\rho g \times \frac{4}{3}\pi(2r)^3$

∴ $H + h = 8H \Rightarrow h = 7H$

(Note: Refer to Mindbender 3.)

48. ΔP_1 = pressure difference between smaller bubble and larger bubble

ΔP_2 = pressure difference between inside and outside the larger bubble

Now, $\Delta P_1 = \frac{4T}{R_1}$, $\Delta P_2 = \frac{4T}{R_2}$

As required pressure difference $\Delta P = \frac{4T}{R}$

$\Delta P = \Delta P_1 + \Delta P_2$

∴ $\frac{4T}{R} = \frac{4T}{R_1} + \frac{4T}{R_2}$

$R = \frac{R_1R_2}{R_1 + R_2}$

$= \frac{2 \times 1 \times 10^{-4}}{(2+1) \times 10^{-2}}$

$= 6.67 \times 10^{-3}$ m

50. Height of water column > length of tube.

So liquid will rise to the top of capillary tube but will not overflow.

52. $l = \frac{h}{\sin\theta} = \frac{3}{\sin 30°} = \frac{3}{\left(\frac{1}{2}\right)} = 6$ cm

53. $h = \frac{2T\cos\theta}{r\rho g}$

∴ $h \propto \frac{1}{r}$

∴ $h_1r_1 = h_2r_2$

∴ $\frac{h_1}{h_2} = \frac{r_2}{r_1}$ ….(i)

Also, area $A = \pi r^2$

∴ $r \propto \sqrt{A}$

∴ $\frac{r_2}{r_1} = \sqrt{\frac{A_2}{A_1}}$ ….(ii)

From equations (i) and (ii),

$\frac{h_1}{h_2} = \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{A/9}{A}} = \frac{1}{3}$

∴ $h_2 = 3h_1 = 3h$

54. Rise in capillary tube,

$h = \frac{2T.\cos\theta}{r\rho g}$

Given that, h, T, r and g are constant.

∴ $\frac{\cos\theta}{\rho}$ = constant

i.e. $\frac{\cos\theta_1}{\rho_1} = \frac{\cos\theta_2}{\rho_2} = \frac{\cos\theta_3}{\rho_3}$

as $\rho_1 > \rho_2 > \rho_3$

$\cos\theta_1 > \cos\theta_2 > \cos\theta_3$

∴ $\theta_1 < \theta_2 < \theta_3$

As the liquids rise in capillary tube,

$\theta < \frac{\pi}{2}$

∴ $0 \le \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$

55. From $h = \frac{2T\cos\theta}{r\rho g}$, the rise in capillary depends upon the surface tension of the liquid and surface tension of soap water solution is less than water. Hence, height will be less in second case. Also, as the soap solution wets the surface of capillary in contact, the shape of meniscus will be concave.

56. Rise of water in capillary tube is given by

$h = \frac{2T\cos\theta}{R\rho g}$

For water, $\cos\theta = 1$

Also, the radius of capillary tube becomes (R – r) after inserting wire of radius r.

∴ $h = \frac{2T}{(R - r)\rho g}$

57. The length of the water column will be equal to full length of capillary tube.

59. $h = \dfrac{2T\cos\theta}{r\rho g}$

Here, $h \propto \dfrac{1}{r} \Rightarrow h_1r_1 = h_2r_2$

∴ $r_2 = \dfrac{h_1r_1}{h_2} = \dfrac{4\times2}{8} = 1$ cm

60. The angle of contact is given by,

$\cos\theta = \dfrac{\rho ghr}{2T}$

ρ = density of water
h = height of water in capillary
r = radius of capillary
T = surface tension of water

∴ $\cos\theta = \dfrac{1000\times10\times5\times10^{-2}\times0.2\times10^{-3}}{2\times7\times10^{-2}}$

∴ $\cos\theta = \dfrac{5}{7} \quad \Rightarrow \theta = \cos^{-1}\left[\dfrac{5}{7}\right]$

61. $h_1r_1 = h_2r_2$

∴ $h_2 = \dfrac{h_1r_1}{r_2} = \dfrac{1.8\times r}{0.9\,r} = 2$ cm

62. $h \propto \dfrac{1}{r}$

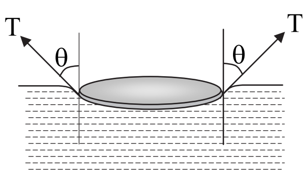
∴ $r_1h_1 = r_2h_2 \Rightarrow h_2 = \dfrac{r_1h_1}{r_2} = \dfrac{r_1\times1.2}{\left(\dfrac{r_1}{2}\right)} = 2.4$ mm

65. $W \propto r^2$

∴ $W_1 \propto R^2$ and $W_2 \propto (3R)^2$

$\Rightarrow \dfrac{W_1}{W_2} = \dfrac{R^2}{9R^2} = 1:9$

66.



Here, Weight of metal disc = total upward force
= upthrust force + force due to surface tension
= weight of displaced water + T cos θ (2π r)
= W + 2π rT cos θ

67. $T_{water} = \dfrac{rhg\rho}{2}$

(Assuming water is pure and angle of contact zero)

∴ $h = \dfrac{2T_{water}}{r\rho g}$ ....(i)

Weight of water = $Mg = \rho\pi r^2hg$

Substituting for h ....[From (i)]

∴ $Mg = \dfrac{2T_{water}}{r\rho g}\times\rho\pi r^2 g$

$= 2\pi r\, T_{water}$

$= 2\times3.142\times0.1\times10^{-3}\times0.07$

$= 4.4\times10^{-5}$ N = 44 μN

68. Using, $h = \dfrac{2T\cos\theta}{rdg}$,

Mass of the water in the first tube,

$m = \pi r^2hd = \pi r^2\times\left(\dfrac{2T\cos\theta}{rdg}\right)\times d = \dfrac{\pi r2T\cos\theta}{g}$

$\Rightarrow m \propto r$

∴ $\dfrac{m'}{m} = \dfrac{r'}{r} = \dfrac{2r}{r} = 2$

$\Rightarrow m' = 2m = 2\times5g = 10$ g

69. F = 105 dyne = 105×10^{-5} N,

$T = 7\times10^{-2}$ N/m

Now the force due to surface tension on the circular cross-section of capillary with inner radius r will be,

$F = 2\pi rT$

∴ $2\pi r = \dfrac{F}{T} = \dfrac{105\times10^{-5}}{7\times10^{-2}} = 15\times10^{-3}$ m = 1.5 cm

70. Excess pressure inside the soap bubble = $\dfrac{4S}{r}$

Hence the pressure inside the soap bubble $= P_{atm} + \dfrac{4S}{r}$

From ideal gas equation, PV = nRT

$$\frac{P_AV_A}{P_BV_B} = \frac{n_A}{n_B} \Rightarrow \frac{\left(8+\dfrac{4S}{r_A}\right)\dfrac{4}{3}\pi(r_A)^3}{\left(8+\dfrac{4S}{r_B}\right)\dfrac{4}{3}\pi(r_B)^3} = \frac{n_A}{n_B}$$

Substituting S = 0.04 N/m, r_A = 2 cm,

r_B = 4 cm we get, $\dfrac{n_A}{n_B} = \dfrac{1}{6}$

∴ $\dfrac{n_B}{n_A} = 6.$

71. Energy released = $(A_f - A_i)T$

$\therefore$ $A_f = 4\pi R^2 = \frac{3}{3}4\pi\frac{R^3}{R} = \frac{3V}{R}$ and

$\therefore$ $A_i = n \times 4\pi r^2 = \frac{V}{\frac{4}{3}\pi r^3}4\pi r^2 = \frac{3V}{r}$

$\Rightarrow$ Energy released = $T(A_i - A_f) = 3VT\left(\frac{1}{r} - \frac{1}{R}\right)$

72. Outside pressure = 1 atm

Pressure inside first bubble = 1.01 atm

Pressure inside second bubble = 1.02 atm

$\therefore$ Excess pressures will be

$\Delta P_1 = 1.01 - 1 = 0.01$ atm and

$\Delta P_2 = 1.02 - 1 = 0.02$ atm

Now, $\Delta P \propto \frac{1}{r} \Rightarrow r \propto \frac{1}{\Delta P}$

$\therefore$ $\frac{r_1}{r_2} = \frac{\Delta P_2}{\Delta P_1} = \frac{0.02}{0.01} = \frac{2}{1}$

Now, $V = \frac{4}{3}\pi r^3$

$\Rightarrow V \propto r^3$

$\therefore$ $\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$

73. Pressure inside tube = $P = P_0 + \frac{4T}{r}$

Let hemispherical radius be r_1 and sub-hemispherical radius be r_2

Hence pressure on side 1 will be greater than side 2. So, air from end 1 flows towards end 2.

74. Excess pressure inside soap bubble is given as

$P_i - P_o = \frac{4T}{r}$;

P_i = Pressure inside soap bubble

P_o = Pressure outside soap bubble

Let excess pressure inside for 1st bubble and 2nd bubble be P_1 and P_2 respectively.

$\therefore$ $P_1 = \frac{4T}{r_1}$, $P_2 = \frac{4T}{r_2}$

$\therefore$ $\frac{P_1}{P_2} = \frac{r_2}{r_1}$

$\Rightarrow \frac{r_1}{r_2} = \frac{P_2}{P_1} = \frac{1}{3}$

As volume $\propto$ radius^3

$\therefore$ $\frac{V_1}{V_2} = \left(\frac{1}{3}\right)^3$

$\therefore$ $\frac{V_1}{V_2} = \frac{1}{27}$

75. As, $\frac{4}{3}\pi b^3 = N \times \frac{4}{3}\pi a^3$

$\therefore$ $b^3 = Na^3$

Energy released,

$\Delta U = T \times 4\pi a^2 \times N - T \times 4\pi b^2$

$= T \times 4\pi\frac{b^3}{a} - T \times 4\pi b^2$

This energy is converted into K.E.

$\therefore$ $\frac{1}{2}mv^2 = T \times 4\pi b^3\left[\frac{1}{a} - \frac{1}{b}\right]$

$\Rightarrow \frac{1}{2}\rho \times \frac{4}{3}\pi b^3 \times v^2 = T \times 4\pi b^3\left(\frac{1}{a} - \frac{1}{b}\right)$

$v = \left[\frac{6T}{\rho}\left(\frac{1}{a} - \frac{1}{b}\right)\right]^{1/2}$

76. In equilibrium,

For air inside capillary,

$P_0(lA) = P'(l - x)A$

Where, P' is pressure in capillary after being submerged into water.

$\therefore$ $P' = \frac{P_0 l}{l - x}$

Now since level of water inside capillary coincides with outside, the excess pressure,

$\Delta P = P' - P_0 = \frac{2\gamma}{r}$

$\therefore$ $\frac{P_0 l}{l - x} - P_0 = \frac{2\gamma}{r}$

Solving above equation, we get,

$x = \frac{l}{\left(1 + \frac{P_0 r}{2\gamma}\right)}$

Evaluation Test

1. $\frac{4}{3}\pi r^3\rho g = 2\pi rT + \frac{1}{2}\times\frac{4}{3}\pi r^3\sigma g$

∴ $2\pi rT = \frac{4}{3}\pi r^3\rho g - \left(\frac{4}{3}\pi r^3\sigma g\right)\times\frac{1}{2}$

$2\pi T = \frac{4}{3}\pi r^2 g\left(\rho - \frac{\sigma}{2}\right)$

∴ $r^2 = \frac{2\pi T}{\frac{4}{3}\pi g\left(\rho-\frac{\sigma}{2}\right)}$

∴ $r^2 = \frac{3T}{g(2\rho-\sigma)} \Rightarrow r = \sqrt{\frac{3T}{g(2\rho-\sigma)}}$

2. The pressures are

$P_{atm} - \frac{2T}{r}$, $P_{atm} + \frac{2T}{R}$, P_{atm} respectively.

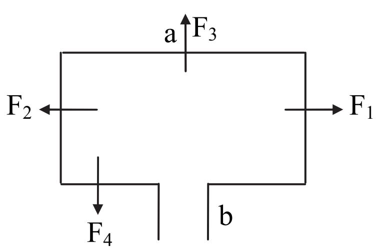
3. Air flows from high pressure to low pressure region. Thus the smaller bubble will be engulfed.

4. Balancing forces on the edge,

$(T\cos\theta)\ 2\pi r = mg$

∴ $r = \frac{0.157\times10\times10^{-3}}{2\times3.14\times0.075\times1}$ m = 3.3 mm

5. F_1 and F_2 are balanced.



Resultant force = $F_3 - F_4$

$= \alpha_1 l - \alpha_2 l$

$= (\alpha_1 - \alpha_2)l$

6. If an bubble is formed, its radius is equal to that capillary

∴ Required pressure = $P_0 + \rho gh + \frac{2s}{r}$

7. $h = \frac{2\alpha}{dgr}$

where, h = rise of liquid in capillary tube

Work done by surface tension

$= Fh - (2\pi\alpha)\left(\frac{2\alpha}{dgr}\right) = \frac{4\pi\alpha^2}{dg}$.

Hence option (A) is correct.

P.E. $= mg\left(\frac{h}{2}\right) = (d\pi r^2 hg)\left(\frac{\alpha}{dgr}\right) = \frac{2\pi\alpha^2}{dg}$

Hence option (C) is correct.

Remaining energy $\frac{2\pi\alpha^2}{dg}$ is liberated as heat.

Hence option (D) is correct.

8. The surface area is given by (S.T.) × Area

Work Done = Final surface energy – Initial surface energy.

$= \sigma 4\pi\,(2r)^2 - \sigma 4r^2 = 12\pi\sigma r^2$

9. The correct reason would be that the soap bubble has an extra force due to the force of surface tension. Which has magnitude $2T(2\pi r)$.

10. The two statements are not related. The first statement is false and the length of tube and vertical direction are one and the same.

11. As there is no weight to bring equilibrium, the liquid level will keep rising due to the force of surface tension.

12. The atmospheric pressure from sides of two plates presses them towards each other.

13. rh = constant $\Rightarrow r \propto \frac{1}{h}$

Hence, if h is halved, then r is doubled.

14. This is same as saying the there is no gravity in space as the weight will cancel the pseudo force of the lift. Thus the force of surface tension will take it to the maximum possible height.

15. $P_1V_1 + P_2V_2 = PV$

or $\frac{4T}{r_1}\times\frac{4}{3}\pi r_1^3 + \frac{4T}{r_2}\times\frac{4}{3}\cdot\pi r_2^3 = \frac{4T}{R}\times\frac{4}{3}\pi R^3$

or $R = \sqrt{r_1^2 + r_2^2}$

16. $h_1r_1 = h_2r_2$ or $h_2 = \frac{h_1r_1}{r_2}$ ….(i)

Here $\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$ where $A_1 = A$ and $A_2 = \frac{A}{16}$

∴ $\frac{r_1^2}{r_2^2} = \frac{16}{1} \Rightarrow \frac{r_1}{r_2} = 4$ ….(ii)

∴ From (i) and (ii), $h_2 = 5 \times 4 = 20$ cm

17. $\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi(4r)^3$

$\therefore$ R = 4r

$S_1 = 64 \times 4\pi r^2 \times T$ and $S_2 = 4\pi R^2 T$

$\therefore \frac{S_1}{S_2} = \frac{64 \times 4\pi r^2 \times T}{4\pi R^2 \times T} = 64\left(\frac{r}{R}\right)^2 = \frac{64}{16} = 4$

18. $\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2}$

$R = \frac{r_1 r_2}{r_1 - r_1} = 4$ mm

19. Then $P = P_0 + \frac{4S}{r}$

Now $P \times \frac{4}{3}\pi r^3 = nRgT$

$\Rightarrow \left(P_0 + \frac{4S}{r}\right)\frac{4}{3}\pi r^3 = 2RgT$

For 2 bubbles,

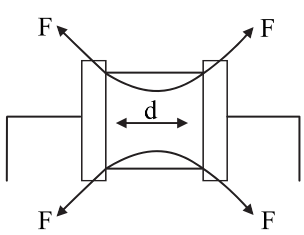
$$\frac{\left(P_0 + \frac{4S}{r_A}\right)\pi_A{}^3}{\left(P_0 + \frac{4S}{r_B}\right)\pi_B{}^3} = \frac{n_A}{n_B}$$

$$\therefore \frac{\left(8 + \frac{4 \times 0.004}{2 \times 10^{-2}}\right)(2 \times 10^{-2})^3}{\left(8 + \frac{4 \times 0.004}{4 \times 10^{-2}}\right)(4 \times 10^{-2})^3} = \frac{n_A}{n_B}$$

$\Rightarrow \frac{n_B}{n_A} \approx 8$

20. The air pressure is greater inside the smaller bubble (4 S/r). Hence, air flows from the smaller to larger bubble.

21.



The weight will be balanced by the force of surface tension.

$\therefore$ $(2T\, l \cos\theta) = \rho g(h(dl))$

$\therefore$ $h = \frac{2T}{\rho g d}$

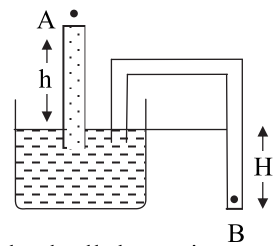
22. Force of surface tension balances the weight of liquid raised

$\therefore$ $\pi\,(d_2 + d_1)S = \rho\frac{\pi(d_2{}^2 - d_1{}^2)}{4}hg$

$\therefore$ $h = \frac{4s}{\rho(d_2 - d_1)g} = \frac{4 \times 0.075}{10^3 \times (2 - 1.5) \times 10^{-3} \times 10}$

= 0.06 m = 6 cm

23.



To check all the options, we just need to apply Bernoulli's principle at two points A and B.

B is just inside the tube.

$P_A + \rho gh = P_B + \rho gH$

$\therefore$ $P_{atm} + \rho gh = P_B + \rho gH$

$\therefore$ $P_B = P_{atm} + \rho g\,(h - H)$

For option (A),

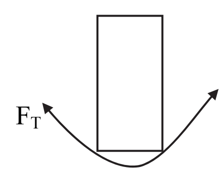
Since H > h, $P_B < P_{atm}$

Hence water flows out.

For option (B),

$0 < H \le h$, $P_B > P_{atm}$

$\therefore$ We can see that the weight of a part of water above is balanced down. Now since H < h, the force due to surface tension has to balance some part of the weight; hence convex meniscus.



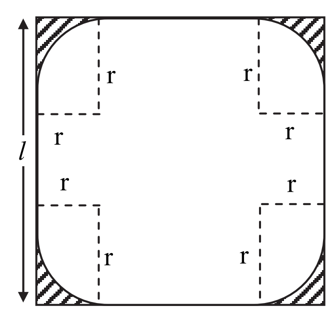
For option (C),

the weight will be just balanced by the pressure force at H = 0

For option (D),

Same explanation as in (B).

24.



Corresponding to the given figure, area of pricked region would be,

$A = \pi r^2 + 4(l - 2r)r + (l - 2r)^2$

$= \pi r^2 + (l - 2r)\,(4r + l - 2r)$

$= \pi r^2 + l^2 - (2r)^2 \Rightarrow (\pi - 4)r^2 + l^2$

Now, given that $l = 4$ units and L = 15 units

But $L = 4(l - 2r) + 2\pi r$

$= 4l + (2\pi - 8)r$

$\therefore$ $15 = 16 + (2\pi - 8)r$

$\therefore$ $r = \left(\frac{1}{8 - 2\pi}\right) = 0.58$ units

Total surface area of soap film

$= l^2 -$ (Area of pricked region)

$= (4 - \pi)r^2$

= 0.289 sq. units

[**Note:** If loop would have taken the shape of a circle, then

$L = \pi d$

$\therefore$ $d = \frac{L}{\pi} = \frac{15}{\pi} = 4.775 >$ Length of the side of the square loop

Thus, it would not form a circle but will take shape as shown in the figure.]

25. Tension in the thread is uniform. We can find the tension in any portion of thread as follows:

Force = Surface Tension × length

i.e. Tension in the wire $= (S) \times r$

$$= S \times \left(\frac{1}{8 - 2\pi}\right)$$

$$= \left(\frac{S}{8 - 2\pi}\right)$$

Textbook Chapter No.

07 Wave Motion

Hints

Classical Thinking

32. If the observer is receding from a stationary source, then

Apparent frequency = $\left(\frac{v - v_0}{v}\right)n$

35. $y = A \sin(\omega t - kx)$

Wave speed, $v = \frac{\omega}{k}$

Maximum particle speed, $v_p = A\omega$

According to given condition, $v_p < v$

$\therefore \quad A\omega < \frac{\omega}{k} \Rightarrow A < \frac{1}{k}$

$\therefore \quad A < \frac{\lambda}{2\pi} \qquad \ldots\left[\because k = \frac{2\pi}{\lambda}\right]$

37. When pulse is reflected from a rigid support, the pulse is inverted both lengthwise and sidewise.

41. Phase difference between the two waves is

$\phi = (\omega t - \beta_2) - (\omega t - \beta_1) = (\beta_1 - \beta_2)$

Resultant amplitude,

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\beta_1 - \beta_2)}$

Critical Thinking

1. $n = \frac{1}{T} = \frac{1}{0.2} = \frac{10}{2} = 5$ Hz

2. Comparing the given equation with standard equation $y = A \sin(\omega t - kx)$ we get,

$\frac{2\pi}{\lambda} = 0.01\,\pi \Rightarrow \lambda = 200$ m

Phase difference = $\frac{2\pi}{\lambda} \times$ (Path difference)

$= \frac{2\pi}{200} \times 25 = \frac{\pi}{4}$

3. Comparing the given equation with standard form,

$y = A \sin(\omega t - \frac{2\pi x}{\lambda} + \phi)$ we get,

$\frac{2\pi}{T} = 20\pi \Rightarrow T = \frac{1}{10}$

$\therefore \quad n = \frac{1}{T} = 10$ Hz and

$\frac{2\pi}{\lambda} = 5\pi \Rightarrow \lambda = \frac{2}{5} = 0.4$ m

Using, $v = n\lambda = 10 \times 0.4 = 4$ m/s

4. $A = 0.5$ m, $\lambda = 1$ m, $n = 2$ Hz

General equation of wave travelling in negative x-direction,

$y = A \sin\left(\omega t + \frac{2\pi}{\lambda}x\right)$

$\therefore \quad y = 0.5 \sin(2\pi 2t + \frac{2\pi}{1}x) \qquad \ldots[\because \omega = 2\pi n]$

$\therefore \quad y = 0.5 \sin(4\pi t + 2\pi x)$

5. $y = 4 \sin\left(\pi t + \frac{\pi x}{16}\right)$

Comparing with standard form,

$y = A \sin(2\pi nt + \frac{2\pi}{\lambda}x)$ we get,

$A = 4$ cm, $2\pi n = \pi \Rightarrow n = 0.5$ Hz and

$\lambda = 32$ cm $\qquad \ldots\left[\because \frac{2\pi}{\lambda} = \frac{\pi}{16}\right]$

Using,

$v = n\lambda = 16$ cm/s $\qquad$ ….(negative x-direction)

6. $\Delta\phi = \frac{2\pi t}{T} = 2\,\pi nt = 2 \times \pi \times 0.5 \times 0.4 = 0.4\,\pi$

7. $n = \frac{1}{T} = \frac{1}{0.04} = 25$ Hz, $v = 25$ m/s,

Using, $\lambda = \frac{v}{n} = \frac{25}{25} = 1$ m

Equation of the wave is,

$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$= 0.02 \sin 2\pi(25t - x)$

8. $y_1 = A_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and

$y_2 = A_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$

So phase difference, $\delta = \phi + \frac{\pi}{2}$ and

Using, $\Delta x = \frac{\lambda}{2\pi}.\delta$ we get,

$\Delta x = \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

9. The given equation is $y = 10 \sin (0.01\, \pi x - 2\pi t)$
Hence ω = coefficient of $t = 2\pi$
Maximum speed of the particle $v_{max} = a\omega$
$= 10 \times 2\pi = 10 \times 2 \times 3.14 = 62.8 \approx 63$ cm/s

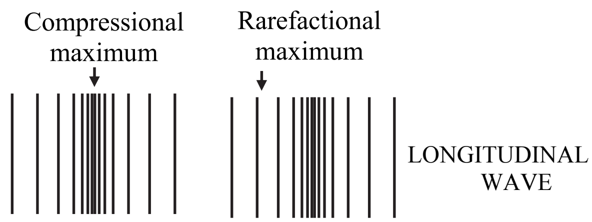
10. At $t = 0$ and $x = \frac{\pi}{2k}$, the displacement
$y = A_0 \sin\left(\omega(0) - k \times \frac{\pi}{2k}\right) = -A_0 \sin\frac{\pi}{2} = -A_0$
Point of maximum displacement (A_0) in negative direction is Q.

11. $x = 5 \sin\left(\frac{t}{0.04} - \frac{x}{4}\right)$ cm
$\therefore$ $x = 5 \sin 2\pi\left[\frac{t}{2\pi \times 0.04} - \frac{x}{2\pi \times 4}\right]$
Comparing with standard form,
$x = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ we get,
$T = 2\pi \times 0.04,\ \lambda = 2\pi \times 4$
$\therefore$ $v = \frac{\lambda}{T} = \frac{4}{0.04} = 100$ cm/s $= 1$ m/s

12. Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference
$\therefore$ $\pi = \frac{2\pi}{\lambda} \times x \Rightarrow \frac{\lambda}{2} = x$
From equation, $y = 0.04 \sin (500\pi t + 1.5\pi x)$
Compare with standard wave equation,
$y = A \sin\left(\frac{2\pi t}{T} + \frac{2\pi x}{\lambda}\right)$ we get,
$\frac{2\pi}{\lambda} = 1.5\,\pi \Rightarrow \frac{\lambda}{2} = \frac{1}{1.5} = 0.66$
$\therefore$ $x = 0.66$ m

13. $y = 0.5\,(314\,t - 12.56\,x)$
Compare this equation with standard wave equation,
$y = A \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$ we get,
$\frac{2\pi}{\lambda} = 12.56 \Rightarrow \lambda = \frac{2 \times 3.14}{12.56} = 0.5$ m

14. $n = 400 \Rightarrow T = 1/400$
$\phi_1 = \omega t_1 - kx$
$\phi_2 = \omega t_2 - kx$ (at same point)
$\Delta\phi = \phi_2 - \phi_1 = \omega(t_2 - t_1) = 2\pi n \times (t_2 - t_1)$
$= 2\pi \times 400 \times 10^{-3} = 0.8\,\pi$
$\therefore$ $0.8\,\pi = 180 \times 0.8 = 144°$ ….[$\because \pi = 180°$]

15.


$T = 0.2$ sec $\Rightarrow n = \frac{1}{T} = 5$ Hz
Time interval between two consecutive compressional maxima, $T = \frac{1}{n} = \frac{1}{500}$ s
Time interval between compressional maxima and rarefactional maxima, $\frac{T}{2} = \frac{1}{2n} = \frac{1}{1000}$ s

16. Here, $A = 0.05\,\text{m},\ \frac{5\lambda}{2} = 0.25 \Rightarrow \lambda = 0.1\,\text{m}$
Now using standard equation of wave,
$y = A \sin\frac{2\pi}{\lambda}(vt - x)$ we get,
$y = 0.05 \sin 2\pi(3300t - 10x)$

17. Amplitude of reflected wave = 0.9 A
On reflection at free end (rarer medium), no phase change is introduced.
$\therefore$ Equation of reflected wave is
$y = 0.9\,A \sin (2\pi nt)$

18. Frequency remains constant in both media
$n = 100$ kHz $= 10^5$ Hz
$v_{air} = 340$ m/s, $v_w = 1450$ m/s
Reflected wave travels in air and its wavelength is
$\lambda_{air} = \frac{v_{air}}{n} = \frac{340}{10^5}$
$= 3.4 \times 10^{-3}$ m $= 3.4$ mm
Transmitted wave travels in water and its wavelength is
$\lambda_w = \frac{v_w}{n} = \frac{1450}{10^5}$
$= 1.45 \times 10^{-2}$ m $= 1.45$ cm

19. $y_1 = A \sin\left(\omega t + \frac{\pi}{6}\right)$,

$y_2 = A \cos \omega t = A \sin\left(\omega t + \frac{\pi}{2}\right)$

Using, $A_R = \sqrt{A^2 + A^2 + 2A^2 \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)}$

$= \sqrt{A^2 + A^2 + 2A^2 \times \frac{1}{2}} = \sqrt{3}\,A$

20. $\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \Rightarrow \frac{I_1}{I_2} = \frac{25}{100} = \frac{1}{4}$

21. Resultant amplitude

$A = \sqrt{A^2 + A^2 + 2AA\cos\phi} = \sqrt{4A^2\cos^2\left(\frac{\phi}{2}\right)}$

As $I \propto A^2$, in this case, $I \propto 4A^2$

22. When the waves are in same phase,
$I_1 = (A + A)^2 = 4A^2$
When the waves are 90° out of phase,
$I_2 = A^2 + A^2 + 2A^2 \cos 90° = 2A^2$

$\therefore \frac{I_1}{I_2} = \frac{4A^2}{2A^2} = 2 : 1$

23. In case of interference of two waves, resultant intensity

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

If ϕ varies randomly with time,
$(\cos\phi)_{av} = 0$

$\therefore$ $I = I_1 + I_2$
For n identical waves,
$I = I_0 + I_0 + \ldots\ldots = nI_0$

$\therefore$ $I = 10\, I_0$

24. $a^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$ ….(i)
Here, $\phi = \phi_1 - \phi_2$,
$a_1 = a_2 = a$
Substituting these values in equation (i) we get,

$\cos\phi = -\frac{1}{2} \Rightarrow \phi = 2\pi/3$

25. $y = \frac{1}{\sqrt{a}}\sin\omega t \pm \frac{1}{\sqrt{b}}\sin\left(\omega t + \frac{\pi}{2}\right)$

Here, phase difference $= \frac{\pi}{2}$

The resultant amplitude

$A = \sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2} = \sqrt{\frac{1}{a} + \frac{1}{b}} = \sqrt{\frac{a+b}{ab}}$

26. $x_1 = A \sin(\omega t - 0.1x)$ and
$x_2 = A \sin(\omega t - 0.1x - \phi/2)$

$\therefore$ $x_1 + x_2 = A \sin(\omega t - 0.1x) + A \sin(\omega t - 0.1x - \phi/2)$

$= A\left[\sin(\omega t - 0.1x) + \sin\left(\omega t - 0.1x - \frac{\phi}{2}\right)\right]$

$= A \times 2 \sin\left[\frac{\omega t - 0.1x + \omega t - 0.1x - (\phi/2)}{2}\right]$

$\cos\left[\frac{\omega t - 0.1x - \omega t + 0.1x + (\phi/2)}{2}\right]$

$= 2A \sin\left[\omega t - 0.1x - \frac{\phi}{4}\right] \cos\left(\frac{\phi}{4}\right)$

$= 2A \cos\left(\frac{\phi}{4}\right) \sin\left(\omega t - 0.1x - \frac{\phi}{4}\right)$

$\therefore$ Required amplitude $= 2A \cos\frac{\phi}{4}$

27. Given that $y_1 = 3 \sin 2\pi(50)t$ and
$y_2 = 4 \sin 2\pi(75)t$

$\therefore$ Comparing given equations with standard form,
$y = A \sin 2\pi nt$ we get,
$n_1 = 50$ and $A_1 = 3$ and $n_2 = 75$ and $A_2 = 4$
Now, $I \propto A^2 n^2$

$\therefore \frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 \times \left(\frac{n_1}{n_2}\right)^2$

$= \left(\frac{3}{4}\right)^2 \times \left(\frac{50}{75}\right)^2 = \frac{9}{16} \times \frac{4}{9} = \frac{1}{4}$

28. $\frac{A_1}{A_2} = \frac{5}{3} \Rightarrow A_1 = \frac{5}{3}A_2$

$\therefore \frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\frac{5}{3}A_2 + A_2\right)^2}{\left(\frac{5}{3}A_2 - A_2\right)^2}$

$= \left(\frac{\frac{8A_2}{3}}{\frac{2A_2}{3}}\right)^2 = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$

$\therefore$ $I_{max} : I_{min} :: 16 : 1$

29. $\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2}$

$\therefore \frac{A_{max}}{A_{min}} = \frac{\left(\frac{A_1}{A_2} + 1\right)}{\left(\frac{A_1}{A_2} - 1\right)}$

Given that, $\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} = \frac{9}{1}$

∴ $\frac{A_1}{A_2} = \frac{3}{1} \Rightarrow \frac{A_{max}}{A_{min}} = \frac{3+1}{3-1} = \frac{4}{2}$

30. $n - 4 = 250$, or $n + 4 = 250$

∴ $n = 254$ or $n = 246$

31. Beat frequency = 258 – 256 = 2 Hz

∴ Time interval between two maxima

$= \frac{1}{\text{beat frequency}}$

$= \frac{1}{2} = 0.5$ s

32. Time interval between a maxima and consecutive minima is

$\Delta t = \frac{1}{2(n_1 - n_2)} = \frac{1}{2\times 4} = \frac{1}{8}$ s

33. $n_{56} = n_1 + (56 - 1)4$

Also, $n_{56} = 2n_1$

∴ $2n_1 = n_1 + 55 \times 4$

∴ $n_1 = 220$ Hz

34. Forks arranged in a series of increasing frequency from n_1 to n_{32}

∴ $n_{32} = n_1 + 31(6) = n_1 + 186$ ….(i)

Given condition is $n_{32} = 2n_1$ ….(ii)

From (i) and (ii),

$2n_1 = n_1 + 186 \Rightarrow n_1 = 186$ Hz

35. Here, $\omega_1 = 2\pi n_1 = 500\,\pi$, $n_1 = 250$ Hz

∴ $\omega_2 = 2\pi n_2 = 506\,\pi$, $n_2 = 253$ Hz

No. of beats / s = $n_2 - n_1 = 253 - 250 = 3$ Hz

No. of beats / minute = 3 × 60 = 180

36. $n_1 = \frac{v}{\lambda_1}$, $n_2 = \frac{v}{\lambda_2}$

$\Rightarrow \lambda_1 = 2$ m, $\lambda_2 = 2.02$ m

Since $\lambda_1 < \lambda_2$,

∴ $n_1 > n_2$

∴ $n_1 - n_2 = 2$

∴ $\frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 2 \Rightarrow v\left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right) = 2$

∴ $v = \frac{2\times \lambda_1\lambda_2}{\lambda_2 - \lambda_1} = \frac{2\times 2\times 2.02}{2.02 - 2} = 404$ m/s

37. Frequency of fork A = f_A = 200 Hz

No. of beats per second = 4

Hence, frequency of fork B is either

200 + 4 = 204 Hz or 200 – 4 = 196 Hz.

When B is loaded with wax, the beats stop. On loading, the number of beats per second has decreased. Hence, the answer should be 204 Hz. This is because after loading with wax, the frequency will decrease to 200 Hz (i.e. to frequency of fork A) and beats disappear.

38. Beats per second = $\frac{10}{3}$

$\lambda_1 = 100$ cm, $\lambda_2 = 101$ cm ….[Given]

Suppose the velocity is v

∴ Frequency of first wave = $n_1 = \frac{v}{\lambda_1} = \frac{v}{100}$ and

frequency of second wave = $n_2 = \frac{v}{\lambda_2} = \frac{v}{101}$

∴ $n_1 - n_2 = \frac{10}{3}$

∴ $\frac{v}{100} - \frac{v}{101} = \frac{10}{3} \Rightarrow v = \frac{101\times 100\times 10}{3}$

∴ v = 33667 cm/s = 336.67 m/s

39. Let the frequencies of the 28 forks be

$n_1 \ldots\ldots n_i \ldots\ldots n_{28}$

Such that $n_{i-1} - n_i = 4$ Hz

∴ $n_1 - n_{28} = 108$ Hz

$\frac{n_1}{n_{28}} = 2 \Rightarrow n_1 = 2n_{28}$

∴ $2n_{28} - n_{28} = 108$ Hz

$n_{28} = 108$ Hz and $n_1 = 216$ Hz

40. $n_1\lambda_1 = n_2\lambda_2$

∴ $\frac{110}{177} < \frac{110}{175}$ ….[∵ $n_1 > n_2$]

∴ $n_1 = n_2 + 6$

∴ $(n_2 + 6)\lambda_1 = n_2\lambda_2$

∴ $(n_2 + 6)\frac{110}{177} = n_2 \times \frac{110}{175}$

∴ $175\,(n_2 + 6) = 177\,n_2$

∴ $n_2 = 3 \times 175 = 525$ Hz

∴ $n_1 = n_2 + 6 = 525 + 6 = 531$ Hz

41. v = 340 m/s,

$v_s = 72\ \frac{km}{hr} = \frac{72\times 10^3}{3600}$ m/s = 20 m/s

Using, $n' = \frac{vn}{v - v_s} = \frac{340\times 640}{340 - 20}$

$= \frac{340\times 640}{320} = 680$ Hz

42. v_0 = 720 km/hr = 200 m/s

Using, $n' = \left(\frac{v - v_o}{v + v_o}\right) n$

$\therefore$ $n' = \left(\frac{340 - 200}{340 + 200}\right) n = \frac{140}{540} \times 1080 = 280$ Hz

43. $n' = \frac{v + v_o}{v - v_s} \times n = \frac{340 + 60}{340 - 60} \times 133$

$\therefore$ $n' = 190$ Hz

44. There is no relative motion between source and listener.

45. Let n = actual frequency of sound produced by source.

$\therefore$ $n' = n\left(\frac{v - v_l}{v - v_s}\right)$ $\therefore$ $\frac{n}{n'} = \frac{v - v_s}{v - v_l}$

46. $n' = \left(\frac{v}{v - v_s}\right) n$

$\therefore$ $n' - n = \left(\frac{v}{v - v_s}\right) n - n = \frac{vn - vn + v_s n}{v - v_s}$

$\therefore$ $\frac{n' - n}{n} = \frac{v_s}{v - v_s} = \frac{25}{100} = \frac{1}{4}$

$\therefore$ $4v_s = v - v_s \Rightarrow 5v_s = 332 \Rightarrow v_s = 66.4$ m/s

47. $v = 108$ km/hr $= 108 \times \frac{5}{18} = 30$ m/s

If observer moves towards stationary source, then the apparent frequency

$n' = \left(\frac{v + v_o}{v}\right) n \Rightarrow n = \frac{n'v}{v + v_o}$

$\therefore$ $n = \frac{504 \times 330}{330 + 30} = \frac{504 \times 330}{360} = 462$ Hz

48. $n' = \left(\frac{v + v_o}{v - v_s}\right) n$

$\therefore$ $\frac{1}{T'} = \left(\frac{v + v_o}{v - v_s}\right) \times \frac{1}{T}$ ....$\left[\because n = \frac{1}{T}\right]$

$\therefore$ $\frac{1}{T'} = \left(\frac{340 + 20}{340 - 20}\right) \times \frac{1}{10} = \frac{360}{3200}$

$\therefore$ $T' = \frac{3200}{360} = 8.9$ s

49. Since there is no relative motion between the source and listener, the apparent frequency equals original frequency.

50. Frequency of the note reflected by the wall is

$n_1 = n\left(\frac{v}{v - v_o}\right)$

$\therefore$ Frequency of the note heard by the engine driver will be

$n' = \frac{(v + v_o)}{v} n_1 = \frac{v + v_o}{v} \times \frac{n\,v}{v - v_o}$

$= \left(\frac{v + v_o}{v - v_o}\right) n$

$= \left(\frac{340 + 60}{340 - 60}\right) \times 1400$ ....[$\because$ n = 1400 Hz]

$= \frac{400}{280} \times 1400 = 2000$ Hz

51. When source is moving towards listener,

$n_1 = \frac{v \times n}{v - v_s} = \frac{300 \times 600}{300 - 200} = 1800$ Hz

When source is moving away from listener,

$n_2 = \frac{v \times n}{v + v_s} = \frac{300 \times 600}{300 + 200} = 360$ Hz

$\therefore$ Change in frequency $= n_1 - n_2 = 1800 - 360$

$= 1440$ Hz

52. $n' = \left(\frac{v + u_l}{v - u_s}\right) n = \left(\frac{v + v/2}{v - v/2}\right) n = \frac{3v/2}{v/2} n$

$\therefore$ $n' = 3n \Rightarrow \frac{n'}{n} = 3$

$\therefore$ $\frac{n' - n}{n} = 2$

$\therefore$ Percentage change $= \frac{n' - n}{n} \times 100$

$= 2 \times 100 = 200$ %

53. The apparent frequency, when observer is approaching source is

$n_1 = \left(\frac{300 + v}{300}\right) n$

The apparent frequency, when observer is moving away from the source is

$n_2 = \left(\frac{300 - v}{300}\right) n$

According to given question,

$n_1 - n_2 = \frac{2}{100}$ n

$\therefore$ $\frac{300 + v}{300} - \frac{300 - v}{300} = \frac{2}{100}$

$\therefore$ $2v = 2 \times 3 \Rightarrow v = 3$ m/s

54. $y = 0.5 \sin [\pi (0.01x - 3t)]$
$= 0.5 \sin [0.01 \pi x - 3\pi t]$
Comparing with standard wave equation,
$y = A \sin \left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$ we get,
$\frac{2\pi}{T} = 3\pi \Rightarrow T = \frac{2}{3}$
$\therefore$ $n = \frac{1}{T} = \frac{3}{2}$ Hz
$\frac{2\pi}{\lambda} = 0.01\pi \Rightarrow \lambda = 200$ m
$\therefore$ Velocity $= n\lambda = \frac{3}{2} \times 200 = 300$ m/s

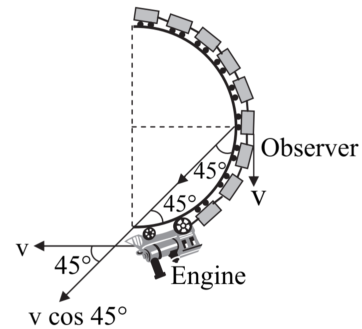
55. $\lambda = \frac{v}{n} = \frac{350}{350} = 1$ m $= 100$ cm
Also, path difference (Δx) between the waves at the point of observation is AP – BP = 25 cm
$\therefore$ $\Delta\phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{1} \times \left(\frac{25}{100}\right) = \frac{\pi}{2}$
$\therefore$ $A = \sqrt{(A_1)^2 + (A_2)^2} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5$ mm

56. Let n be a frequency of given fork.
We have following possibilities for n:
Case I: When 2 beats/s are produced, oscillator reads 514 Hz.
$\therefore$ $n - 2 = 514$ or $n + 2 = 514$
$\therefore$ $n = 516$ Hz or $n = 512$ Hz ….(i)
Case II: When 6 beat/s are produced, oscillator reads 510 Hz
$\therefore$ $n - 6 = 510$ or $n + 6 = 510$
$\therefore$ $n = 516$ Hz or $n = 504$ Hz ….(ii)
$\therefore$ From equations (i) and (ii),
$\therefore$ $n = 516$ Hz

57. $y_1 = 4 \sin (400 \pi t)$, $y_2 = 3 \sin (404 \pi t)$
Comparing with standard form, $y = A \sin 2\pi nt$ we get,
$A_1 = 4$, $A_2 = 3$, $n_1 = 200$, $n_2 = 202$
$\therefore$ Beat frequency $= n_2 - n_1$
$= 202 - 200 = 2$ beats/second
$\therefore$ Intensity ratio $= \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(7)^2}{(1)^2} = \frac{49}{1}$

58. Since source is moving towards a stationary observer,
$n' = \left(\frac{v}{v - v_s}\right) n = \frac{v}{\left(v - \frac{v}{10}\right)} \times 90 = 100$ Hz

59. The situation is shown in the figure
Both the source (engine) and the observer (Person in the middle of the train) have the same speed but their direction of motion is at right angle to each other. The component of velocity of observer towards source is v cos 45° and that of source along the time joining the observer and source is also v cos 45°. There is no relative motion between them, so there is no change in frequency heard. So frequency heard is 200 Hz.



61. Energy density (E) $= \frac{I}{v} = 2\pi^2\rho n^2 A^2$
$v_{max} = \omega A = 2\pi n A \Rightarrow E \propto (v_{max})^2$
i.e., graph between E and v_{max} will be a parabola symmetrical about E axis.

Competitive Thinking

3. Comparing with standard equation we get
$\frac{2\pi}{\lambda} = 10\pi \Rightarrow \lambda = 0.2$ m
$\omega = 2\pi$
$\therefore$ $n = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$ Hz
and the wave is travelling along the positive direction.

4. Here, $\frac{ct}{\lambda}$ is dimensionless and unit of ct is same as that of x. Also unit of λ is same as that of A, which is also the unit of x.

5. Given equation is,
$y = 3 \sin \pi\left(\frac{t}{0.02} - \frac{x}{20}\right) = 3 \sin 2\pi\left(\frac{t}{0.04} - \frac{x}{40}\right)$
Comparing with the standard form,
y = A sin 2p we get,
$T = 0.04 \text{ s} \Rightarrow n = \frac{1}{T} = \frac{1}{0.04} = \frac{100}{4} = 25$ Hz

6. $n = \frac{\omega}{2\pi} = \frac{400\pi}{2\pi} = 200$ Hz ....[$\because \omega = 400\pi$]

7. Given equation is

$y = 5 \sin 2\pi \left(\frac{t}{0.04} - \frac{x}{40}\right)$.

Comparing with the standard form,

$y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$ we get,

$\lambda = 40$ cm

8. Comparing the given equation with $y = A \cos(\omega t - kx)$ we get,

$k = \frac{2\pi}{\lambda} = \pi \Rightarrow \lambda = 2$ cm

9. Comparing the given equation with standard equation,

$y = A \sin 2\pi \left(nt - \frac{x}{\lambda}\right)$ we get,

$\omega = 2\pi n = 200\pi \Rightarrow n = 100$ Hz

Also, $k = \frac{20\pi}{17}$

$\therefore \lambda = \frac{2\pi}{k} = \frac{2\pi}{20\pi/17} = 1.7$ m

and $v = \frac{\omega}{k} = \frac{200\pi}{20\pi/17} = 170$ m/s

10. $v = \frac{\omega}{k} = \frac{600}{2} = 300$ m/s

11. $y = a \sin\left(2\pi nt - \frac{2\pi}{5}x\right)$

For particle velocity v_p,

$\frac{dy}{dt} = a \times 2\pi n \cos\left(2\pi nt - \frac{2\pi}{5}x\right)$

$(v_p)_{max} = 2\pi na$

Comparing with standard equation progressive constant,

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{5} \Rightarrow \lambda = 5$

Wave velocity $v = n\lambda = 5$ n

$\therefore \frac{(v_p)_{max}}{v} = \frac{2\pi na}{5n} = \frac{2\pi a}{5}$

12. Given equation of the wave can also be written as,

$Y = 3 \sin\left[2\pi\left(\frac{t}{6} - \frac{x}{10}\right) + \frac{\pi}{4}\right]$

Comparing with $y = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \frac{\pi}{4}\right]$

(where, x and y are in metre)

we get,

$A = 3$ m, $F = \frac{1}{T} = 0.17$ Hz, $\lambda = 10$ m and

$v = F\lambda = 1.7$ m/s

Hence, option (D) is correct.

13. Comparing the given equation with $y = A \sin(\omega t - kx)$ we get, $\omega = 3000\pi$

$\therefore n = \frac{\omega}{2\pi} = 1500$ Hz

and $k = \frac{2\pi}{\lambda} = 12\pi \Rightarrow \lambda = \frac{1}{6}$ m

Using $v = n\lambda$,

$v = 1500 \times \frac{1}{6} = 250$ m/s

14. $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$ ....(i)

and $y_2 = 5\left[\sin 3\pi t + \sqrt{3}\cos 3\pi t\right]$

$= 5 \times 2\left[\frac{1}{2} \times \sin 3\pi t + \frac{\sqrt{3}}{2} \times \cos 3\pi t\right]$

$= 10\left[\cos\frac{\pi}{3}\sin 3\pi t + \sin\frac{\pi}{3}\cos \pi t\right]$

$y_2 = 10\left[\sin\left(3\pi t + \frac{\pi}{t}\right)\right]$ ....(ii)

($\because$ sin (A + B) = sinA cosB + cosA sinB)

Comparing equation (i) and (ii), we get ratio of amplitudes as 1 : 1.

15. From, $y = 60 \cos (1800 t - 6x)$

$A = 60$, $\omega = 1800$, $k = 6$

Velocity of wave propagation is

$v_w = n\lambda$; $n = \frac{\omega}{2\pi} = \frac{1800}{2\pi}$,

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{6}$

$\therefore v_w = \frac{1800}{2\pi} \times \frac{2\pi}{6} = 300$ m/s

Velocity of particle is

$v_p = \frac{dy}{dt} = 1800 \times 60 \cos (1800 t - 6x)$

$\therefore v_{p_{max}} = 1800 \times 60$ μm/s

$\therefore v_{p_{max}} = 1800 \times 60 \times 10^{-6}$ m/s

$\therefore \frac{v_{p_{max}}}{v_w} = \frac{1800 \times 60 \times 10^{-6}}{300}$

$= 360 \times 10^{-6} = 3.6 \times 10^{-4}$

16. According to given information,
$5\lambda = 4 \Rightarrow \lambda = 0.8$ m
Hence frequency,
$n = \frac{v}{\lambda} = \frac{128}{0.8} = 160$ Hz
and Angular frequency
$\omega = 2\pi n = 2 \times 3.14 \times 160 = 1005$ rad/s
Also, propagation constant,
$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.8} = 7.85\ m^{-1}$
On substituting these values in standard equation we get,
$y = (0.02)$ m $\sin(7.85x - 1005\,t)$

17. Comparing the given equation with standard equation,
$k = \frac{2\pi}{\lambda} = \pi \times 10^{-2} \Rightarrow \lambda = 200$ m and
$\omega = 2\pi n = 2\pi \times 10^6 \Rightarrow v = 10^6$ Hz

18. Speed $= n\lambda = n(4ab) = 4n \times ab \quad \ldots\left(\because ab = \frac{\lambda}{4}\right)$
$\therefore$ Path difference between b and e is $\frac{3\lambda}{4}$
Now, Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference
$= \frac{2\pi}{\lambda}.\frac{3\lambda}{4} = \frac{3\pi}{2}$

19. Points B and F are in same phase as they are λ distance apart.

20. Given, $y = 12 \sin(5t - 4x)$ cm
$\therefore$ $y = 12 \sin 2\pi \left(\frac{5t}{2\pi} - \frac{4x}{2\pi}\right)$
Comparing above eq. with,
$y = A \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$
We get, $\lambda = \frac{2\pi}{4}$ cm
Relation between phase difference and path difference is
$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$
$\therefore$ $\frac{\pi}{2} = \frac{2\pi}{\left(\frac{2\pi}{4}\right)}\Delta x$ $\qquad \therefore$ $\Delta x = \frac{\pi}{8}$ cm

23. $A_{max} = \sqrt{A^2 + A^2} = A\sqrt{2}$, frequency will remain same i.e. ω.

24. Since $\phi = \frac{\pi}{2}$,
$\therefore$ $A = \sqrt{A_1^2 + A_2^2} = \sqrt{(4)^2 + (3)^2} = 5$

25. $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ and
$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$
$\therefore$ Sum of maximum and minimum intensities $= 2\,(I_1 + I_2)$

27. For producing beats, there must be small difference in frequency.

28. Using, $v = n\lambda$ or $n = \frac{v}{\lambda}$ we get,
$n_1 = \frac{v}{\lambda_1} = \frac{330}{5} = 66$ Hz
and $n_2 = \frac{v}{\lambda_2} = \frac{330}{5.5} = 60$ Hz
Number of beats per second,
$n_1 - n_2 = 66 - 60 = 6$

29. From the given equations of progressive waves, $\omega_1 = 500\pi$ and $\omega_2 = 506\pi$
$\therefore$ $n_1 = 250$ Hz and $n_2 = 253$ Hz
Hence, beat frequency $= n_2 - n_1 = 253 - 250 = 3$ beats per second
$\therefore$ Number of beats per minute = 180.

30. $2\pi\, f_1 = 600\pi \Rightarrow f_1 = 300$ and
$2\pi\, f_2 = 608\pi \Rightarrow f_2 = 304$
$\therefore$ $|f_1 - f_2| = 4$ beats
$\therefore$ $\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(5+4)^2}{(5-4)^2} = \frac{81}{1}$

31. Using, $\frac{I_{max}}{I_{min}} = \left(\frac{\frac{A_1}{A_2} + 1}{\frac{A_1}{A_2} - 1}\right) = \left(\frac{\frac{4}{3} + 1}{\frac{4}{3} - 1}\right)^2 = \frac{49}{1}$

32. Using $v = n\lambda$,
$n_1 = \frac{v}{\lambda_1} = \frac{v}{0.50}$ and $n_2 = \frac{v}{\lambda_2} = \frac{v}{0.51}$
$\therefore$ $\Delta n = n_1 - n_2 = v\left[\frac{1}{0.5} - \frac{1}{0.51}\right] = 12$
$\therefore$ $v = \frac{12 \times 0.51 \times 0.50}{0.01} = 306$ m/s

33. Frequency of string $= 440 \pm 5$
When frequency of tuning fork is decreased, beat frequency is increased.
$\therefore$ Frequency of string = 445 Hz

34. Comparing given equation with standard form, $y = A \sin 2\pi nt$ we get,
$n_1 = \frac{316}{2\pi}$ and $n_2 = \frac{310}{2\pi}$
Number of beats heard per second,
$n_1 - n_2 = \frac{316}{2\pi} - \frac{310}{2\pi} = \frac{3}{\pi}$

35. n_A = Known frequency = 288 c.p.s
x = 4 b.p.s.,
After loading of wax on tuning fork B, n_B decreases. If we consider $n_A > n_B$ then,
after loading, $n_A - n_B$ will increase. But it contradicts the given data that x decreases to 2 b.p.s.
∴ $n_B = n_A + x = 288 + 4 = 292$ c.p.s.

36. $n_A = 512$ Hz
Given that, $n_A - n_B = 8$
When B is loaded with wax, the number of beats reduces to 4 per second.
⇒ $n_B - n_A = 8$ is the correct equation.
⇒ $n_B = n_A + 8 = 512 + 8 = 520$ Hz

37. $n_x = 300$ Hz
x = beat frequency = 4 Hz, which is decreasing after increasing the tension of the string Y.
Also, ∵ $n \propto \sqrt{T}$, tension of wire Y increases so n_y increases
Hence, if $n_y > n_x$
beat frequency increases, which contradicts the data.
∴ $n_y < n_x$
∴ $n_x - n_y = x$
$n_y = n_x - x = 300 - 4 = 296$ Hz

38. Suppose n_p = frequency of piano
n_f = Frequency of tuning fork = 256 Hz
x = Beat frequency = 5 b.p.s., which is decreasing after changing the tension of piano wire.
Now, $n_p \propto \sqrt{T}$
Also, tension of piano wire is increasing so n_p increases.
Hence, if $n_p > n_f$ then beat frequency increases with increase in tension, which contradicts the given data.
∴ $n_f > n_p$
⇒ $n_P = n_f - x = 256 - 5$ Hz.

39. Let n be frequency of tuning fork.
Let n_1, n_2 be frequency of wire at tension T_1, T_2 respectively.
$n \propto \sqrt{T}$ …(i)
$n_1 = n - 6$ …(ii)
$n_2 = n + 6$ …(iii)
∴ $\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{225}{256}} = \frac{15}{16}$
∴ $\frac{n - 6}{n + 6} = \frac{15}{16}$ …from (i), (ii), (iii)
∴ $16n - 96 = 15n + 90$
∴ $n = 186$ Hz

40. If m frequencies are arranged in increasing order, then,
$n_m = n_1 + (m - 1)X$
where X = no. of beats produced.
∴ here,
$n_3 = n_1 + (2)X$
∴ $n + 1 = n - 1 + 2X$
$2X = 2$
∴ $X = 1$

41. Let the frequency of first fork be 'n' then frequency of 56th fork will be
$n' = n + 4 \times 55$
this is because each successive tuning fork is separated by 4 Hz in frequency from the previous one.
Also, $n' = 3n$ ….(given)
∴ $3n = n + 4 \times 55$
⇒ $n = 110$ Hz

42. Apparent frequency for source moving towards the stationary observer is given by,
$n' = n\left[\frac{v}{v - v_s}\right]$
As the source moves towards the observer, frequency increases, hence wavelength decreases.

43. $n' = n\left(\frac{v}{v - v_S}\right) \Rightarrow \frac{n'}{n} = \frac{v}{v - v_S} \Rightarrow \frac{v}{v - v_S}$
$= 3 \Rightarrow v_S = \frac{2v}{3}$

44. Apparent frequency is given by,
$F' = \left[\frac{V \pm V_0}{V \mp V_S}\right]F$
∵ source is stationary,
∴ $V_S = 0$; $V_0 = V_1$
∴ $F_1 = \left[\frac{V + V_1}{V}\right]F$
$F_2 = \left[\frac{V - V_1}{V}\right]F$

$\therefore \quad \frac{F_1}{F_2} = \frac{V + V_1}{V - V_1}$

$\therefore \quad 2 = \frac{V + V_1}{V - V_1}$

$\therefore \quad 2V - 2V_1 = V + V_1$

$\therefore \quad V = 3V_1$

$\therefore \quad \frac{V}{V_1} = 3$

45. Apparent frequency heard by observer while moving towards the source of sound is,

$n' = n\left(\frac{v + v_0}{v}\right)$

Apparent frequency heard by observe while moving away from the source is,

$n'' = n\left(\frac{v - v_0}{v}\right)$

$\therefore \quad n' - n'' = \frac{n}{v}(v + v_0 - v + v_0) = \frac{2nv_0}{v}$

46. Using $\frac{n_{approaching}}{n_{receding}} = \frac{v + v_s}{v - v_s}$

$\therefore \quad \frac{1000}{n_r} = \frac{350 + 50}{350 - 50} \Rightarrow n_r = 750$ Hz

47. Frequency of sound heard by the man from approaching train

$n_a = n\left(\frac{v}{v - v_s}\right) = 240\left(\frac{320}{320 - 4}\right) = 243$ Hz

Frequency of sound heard by the man from receding train $n_r = n\left(\frac{v}{v + v_s}\right)$

$= 240\left(\frac{320}{320 + 4}\right) = 237$ Hz

Hence, number of beats heard by man per second $= n_a - n_r = 243 - 237 = 6$

Alternate method :

$\therefore$ Number of beats heard per second $= \frac{2nvv_s}{v^2 - v_s^2}$

$= \frac{2nvv_s}{(v - v_s)(v + v_s)} = \frac{2 \times 240 \times 320 \times 4}{(320 - 4)(320 + 4)} = 6$

48. $n_1 = n\left[\frac{v}{v - v_s}\right] = n\left[\frac{320}{320 - 20}\right] = n \times \frac{320}{300}$ Hz

$n_2 = n\left[\frac{v}{v + v_s}\right] = n \times \frac{320}{340}$ Hz

Percentage change in frequency

$\left|\frac{n_2 - n_1}{n_1}\right| \times 100 = \left|\frac{n_2}{n_1} - 1\right| \times 100$

$= 100\left[\frac{300}{340} - 1\right] \approx 12\ \%$

49. $n' = n\left(\frac{v}{v + v_s}\right)$

$\frac{n'}{n} = \left(\frac{v}{v + v_s}\right)$

$\frac{5}{6} = \frac{350}{350 + v_s}$

$v_s = 70$ m/s

50. n_1 = Frequency of the police car's horn heard by motorcyclist

n_2 = Frequency of the siren heard by motorcyclist.

v = Speed of motor cyclist

$n_1 = \frac{330 - v}{330 - 22} \times 176$ and $n_2 = \frac{330 + v}{330} \times 165$

$\because \quad n_1 - n_2 = 0$

$\therefore \quad \frac{330 - v}{308} \times 176 = \frac{330 + v}{330} \times 165$

$\therefore \quad v = 22$ m/s

51. The frequency of reflected sound heard by the driver,

$n' = n\left(\frac{v - (-v_0)}{v - v_s}\right) = n\left(\frac{v + v_0}{v - v_s}\right)$

$= 124\left[\frac{330 + (72 \times 5/18)}{330 - (72 \times 5/18)}\right] = \frac{124 \times 35}{31}$

= 140 vibration/s

52. $n' = n\left(\frac{v + v_0}{v - v_s}\right) = 1000\left(\frac{333 + 33}{333 - 33}\right) = 1220$ Hz.

53. Both source and observer are moving towards each other,

$\therefore \quad n = n_0\left(\frac{v + v_0}{v - v_s}\right) = 400\left(\frac{340 + 16.5}{340 - 22}\right)$

$n = 448$ Hz

54. As siren moves towards cliff, frequency incident on cliff is,

$n' = \left(\frac{v}{v - v_s}\right)n = \frac{330}{330 - 15} \times 800 \approx 838$ Hz

As listener is stationary, he will hear sound of same frequency after reflection.

55. the expression for apparent frequency is

$$n' = n\left(\frac{v \pm v_o}{v \pm v_s}\right)$$

the frequency received by the wall from moving car is

$$n'_{wall} = 620\left(\frac{330+0}{330-20}\right) = 660 \text{ Hz}$$

this frequency is reflected as an echo towards car. Hence, frequency of echo heard by the driver is

$$n'_{driver} = 660\left(\frac{330+20}{330-0}\right) = 700 \text{ Hz}$$

56. $f_{incident} = f_{reflected} = \frac{v}{v - v_s} n = \frac{320}{320-10} \times 8 \text{ kHz}$

$\therefore$ $f_{observed} = \frac{320+10}{320} f_{reflected} = 8 \times \frac{330}{310}$

$= 8.51 \text{ kHz} \approx 8.5 \text{ kHz}.$

57. $n' = \frac{v+v_0}{v} n = \frac{v+\frac{v}{5}}{v}.f = \frac{6}{5}f = 1.2f$

$\therefore$ Source is stationary, wavelength remains unchanged for observer.

58. As source crosses stationary listener then, ratio of apparent frequencies before crossing (n_1) and after crossing (n_2) is,

$$\frac{n_1}{n_2} = \frac{v+v_s}{v-v_s}$$

$\therefore$ $n_2 = \frac{n_1(v-v_s)}{v+v_s} = \frac{500(350-50)}{350+50}$

$\therefore$ $n_2 = 375$ Hz

59. As observer is at rest, frequency heard by observer

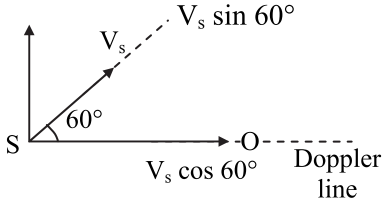
Case I: $n' = n\left(\frac{v}{v-v_s}\right)$

Case II: $n' = n\left(\frac{v}{v+v_s}\right)$

As speed v_s is constant, $n' = \text{constant} \times n$.

Thus, as engine approaches observer, apparent frequency heard is higher and as source moves away, apparent frequency heard is lesser. Hence, the graph (D) represents the situation best.

60.



$$n' = n\left(\frac{V}{V - V_s \cos 60^\circ}\right)$$

$$= 100\left(\frac{330}{330 - 19.4 \times \frac{1}{2}}\right)$$

$$= 100\left(\frac{330}{330-9.7}\right) = 100\left(\frac{330}{320.3}\right)$$

$= 103.02$ Hz

61. Frequency of sound remains constant.

62. Resultant amplitude $A_R = 2A \cos\left(\frac{\theta}{2}\right)$

$= 2 \times (2A)\cos\left(\frac{\theta}{2}\right) = 4A \cos\left(\frac{\theta}{2}\right)$

63. Wave velocity = v

Particle velocity,

$v_{max} = \frac{dy}{dt} = y_0\left(\frac{2\pi v}{\lambda}\right) \cos\left\{\frac{2\pi}{\lambda}(vt - x)\right\}$

$\therefore$ $v_{max} = y_0\left(\frac{2\pi v}{\lambda}\right)$

Let, $v_{max} = 2v$

$y_0\left(\frac{2\pi v}{\lambda}\right) = 2v \Rightarrow \lambda = \pi y_0$

64. $\nu_a = 250 \pm 4 = 254$ Hz or 246 Hz

$\nu_b = 513 \pm 5 = 518$ Hz or 508 Hz

Now, $\nu_b = 2\nu_a$

Which is 508 = 2(254)

$\therefore$ $\nu = 254$ Hz

65. Phase difference of 90° or $\frac{\pi}{2}$ rad

corresponds to a path difference of $\frac{\lambda}{4}$

$\therefore$ $\lambda = 4 \times 0.8 \text{ m} = 3.2 \text{ m}$

Using, $v = n\lambda = 120 \times 3.2 = 12 \times 32 = 384$ m/s

66. As the source and the observer move away from each other, using formula,

$n' = \left(\frac{v - v_L}{v + v_S}\right)n$ we get,

$n = \left(\frac{v+v_S}{v-v_L}\right)n' = \frac{340+10}{340-10} \times 1950$

$= \frac{35}{33} \times 1950$

$= 2068$ Hz

67. We know that, $n' = \left(\frac{v \pm v_0}{v \mp v_s}\right) n$

As siren is at rest, $v_s = 0$

$\therefore$ $n_A = \left(\frac{v + v_A}{v}\right) n$

$\Rightarrow$ $4.5 = \frac{340 + v_A}{340} \times 4$

$\Rightarrow$ $v_A = 42.5$ m/s

and $n_B = \left(\frac{v + v_B}{v}\right) n$

$\Rightarrow$ $5 = \frac{340 + v_B}{340} \times 4$

$\Rightarrow$ $v_B = 85$ m/s

68. As student walks to the wall, frequency incident on wall be n_1.

$\therefore$ $n_1 = \left(\frac{v}{v - v_s}\right) n$ ....(i)

where, v_s is velocity of student.
Now, wall will reflect sound of frequency n_1. But as the student is moving towards the wall, apparent frequency heard by student,

$n' = \left(\frac{v + v_s}{v}\right) n_1$

$= \left(\frac{v + v_s}{v}\right) \times \left(\frac{v}{v - v_s}\right) n$ ....[Using equation (i)]

$= \left(\frac{v + v_s}{v - v_s}\right) \times n = \left(\frac{342 + 2}{342 - 2}\right) \times 170$

$= 172$ Hz

Beat frequency = 172 – 170 = 2 Hz

69. Given equation is,

$y = 0.03 \sin 8\pi \left(\frac{t}{0.016} - \frac{x}{1.6}\right)$

$= 0.03 \sin 2\pi \left(\frac{t}{0.004} - \frac{x}{0.4}\right)$

$\therefore$ Comparing with the standard form,

$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$ we get,

$T = 0.004 \text{ s} = n = \frac{1}{T} = \frac{1}{0.004} = \frac{1000}{4} = 250$ Hz,

$\lambda = 0.4$ m

$\therefore$ Using, $v = n\lambda = 250 \times 0.4 = 100$ m/s

70. Here, $n_{11} = n_1 + (11 - 1) \times 8 = n_1 + 80$
and $n_{11} = 2n_1$

$\therefore$ $2n_1 = n_1 + 80 \Rightarrow n_1 = 80$ Hz

$\therefore$ $n_{10} = 80 + (10 - 1) \times 8 = 152$ Hz

71. $T = \frac{1}{n_2 - n_1} = \frac{1}{325 - 320} = \frac{1}{5} = 0.2$ s

72. Using, $v = n\lambda$ we get, $n = \frac{v}{\lambda}$

Given that, $n_2 - n_1 = 5$

$\therefore$ $v\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = 5$

$\therefore$ $v\left(\frac{1}{52} - \frac{1}{52.5}\right) = 5 \Rightarrow v = \frac{5 \times 52 \times 52.5}{0.5}$

$= 10 \times 52 \times 52.5 = 273$ m/s

$\therefore$ $n_1 = \frac{273}{52.5 \times 10^{-2}} = 520$ Hz and

$n_2 = \frac{273}{52 \times 10^{-2}} = 525$ Hz

73. $n_A = 305$ Hz
Given that, $n_A \sim n_B = 5$
When B is filed, the number of beats reduce to 3 beats/s.

$\therefore$ The correct equation is,
$n_B - n_A = 5 \Rightarrow n_B = n_A + 5 = 305 + 5 = 310$ Hz

74. $n_B = 384$ Hz
Given that $n_A \sim n_B = 4$
When A is filed, the number of beats reduce to 3 per second $\Rightarrow$ The correct equation is,
$n_B - n_A = 4 \Rightarrow n_A = n_B - 4 = 384 - 4 = 380$ Hz

75. Given that, phase difference of $\frac{\pi}{6}$ rad
Corresponds to a path difference of x m.

$\because$ A phase difference of 2π rad corresponds to path difference of λ, we get,

Now, $\lambda = \frac{v}{n} = \frac{100}{50} = 2$ m

$\therefore$ $x = \frac{2}{12} = \frac{1}{6}$ m

76. Given that, $v_{max} = 4 v_p$

$\therefore$ $A\omega = 4 \times n\lambda$

$\therefore$ $A \times \frac{2\pi}{T} = 4 \times \frac{1}{T} \times \lambda$

$\therefore$ $A \times \pi = 2\lambda$ or $\lambda = \frac{\pi A}{2}$

77. Given equations are,
$y_1 = a \sin (2000\pi t) = a \sin 2\pi (1000 t)$ and
$y_2 = a \sin (2008\pi t) = a \sin 2\pi (1004 t)$

$\therefore$ Comparing with the standard form,
$y = A \sin 2\pi nt$ we get,
$n_1 = 1000$ Hz and $n_2 = 1004$ Hz

$\therefore$ Number of beats = 1004 – 1000 = 4 beats/s

78. Given equation is,

$y = A \sin(100\pi t + 3x)$

$= A \sin 2\pi \left(50t + \dfrac{3x}{2\pi}\right)$

$= A \sin 2\pi \left[\dfrac{t}{\left(\frac{1}{50}\right)} + \dfrac{x}{\left(\frac{2\pi}{3}\right)}\right]$

∴ Comparing with the standard form,

$y = A \sin 2\pi \left(\dfrac{t}{T} + \dfrac{x}{\lambda}\right)$ we get, $\lambda = \dfrac{2\pi}{3}$

A phase difference of $\dfrac{\pi}{3}$ rad corresponds to a path difference of $\dfrac{x}{6}$ m $= \dfrac{1}{6} \times \dfrac{2\pi}{3} = \dfrac{\pi}{9}$ m

79. By comparing given equation of progressive wave with standard equation

$y = a \cos(kx - \omega t)$ we get,

$k = \dfrac{2\pi}{\lambda} = \alpha \Rightarrow \alpha = \dfrac{2\pi}{0.08} = 25\pi$

and $\omega = \dfrac{2\pi}{T} = \beta \Rightarrow \beta = \dfrac{2\pi}{2} = \pi$

80. Waves travelling to the right can be given by

$y_1 = A \sin(\omega t - kx)$ ….(i)

When getting reflected from the fixed end of the string, there is an additional phase difference of π. The reflected wave is

$y_2 = A \sin(\omega t + kx + \pi)$

$\Rightarrow y_2 = -A \sin(\omega t + kx)$ ….(ii)

Superposing, (i) + (ii) is the same as $y = \sin C - \sin D$

∴ $y = 2A \cos \dfrac{C+D}{2} \sin \dfrac{C-D}{2}$

∴ $y = 2A \cos \omega t \sin kx$

∴ The stationary wave is given as

$y = 0.06 \sin \dfrac{2\pi x}{3} \cos(120\pi t)$

Here, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{3}$ and $\omega = 120\pi$

∴ $\lambda = 3$ m, $n = \dfrac{120\pi}{2\pi} = 60$ Hz

81. Let n be the frequency of fork C

∴ $n_A = n + \dfrac{3n}{100} = \dfrac{103n}{100}$ and $n_B = n - \dfrac{2n}{100} = \dfrac{98n}{100}$

But $n_A - n_B = 5 \Rightarrow \dfrac{5n}{100} = 5 \Rightarrow n = 100\,\text{Hz}$

∴ $n_A = \dfrac{(103)(100)}{100} = 103$ Hz

82. When listener is moving away from the stationary source,

then apparent frequency, $\dfrac{n'}{n} = \left[\dfrac{v - v_L}{v}\right]$

∴ $0.94 = \left[\dfrac{330 - v_L}{330}\right] \Rightarrow v_L = 19.8$ m/s

Initial velocity of listener is zero, and it becomes 19.8 m/s after covering distance s

$v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2as$

$\Rightarrow s = \dfrac{v^2}{2a} = \dfrac{(19.8)^2}{2 \times 2} = 98$ m

83. Listener moves from A to B with velocity (u). Let the apparent frequency of sound from source A by listener be

$n' = n\dfrac{v - v_0}{v + v_s} = 680\left(\dfrac{340 - u}{340 + 0}\right)$

The apparent frequency of sound from source B by listener is,

$n'' = n\dfrac{v - v_0}{v + v_s} = 680\left(\dfrac{340 + u}{340 - 0}\right)$

Given that, listener hears 10 beats per second.

Hence, $n'' - n' = 10$

$\Rightarrow 680\left(\dfrac{340 + u}{340}\right) - 680\left(\dfrac{340 - u}{340}\right) = 10$

$\Rightarrow 2(340 + u - 340 + u) = 10 \Rightarrow u = 2.5\ \text{ms}^{-1}$

84. $dB = 10 \log_{10}\left(\dfrac{I}{I_0}\right)$; where $I_0 = 10^{-12}\ \text{Wm}^{-2}$

Since, $40 = 10 \log_{10}\left(\dfrac{I_1}{I_0}\right) \Rightarrow \dfrac{I_1}{I_0} = 10^4$

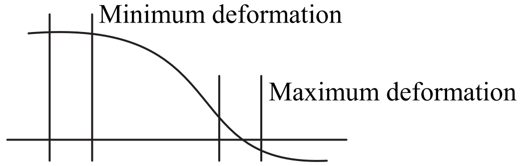
Also, $20 = 10 \log_{10}\left(\dfrac{I_2}{I_0}\right) \Rightarrow \dfrac{I_2}{I_0} = 10^2$

∴ $\dfrac{I_2}{I_1} = 10^{-2} = \dfrac{d_1^2}{d_2^2} \Rightarrow d_2^2 = 100\,d_1^2$

$\Rightarrow d_2 = 10$ m ….[∵ $d_1 = 1$ m]

Evaluation Test

1.



For a travelling wave,
$y = A \sin(\text{cot} \pm kx + \theta)$
at a given position (x) : $y = A \sin(\omega t + \phi)$
Thus, the particle performs SHM
At a given position,
deformation w.r.t. mean position is minimum, therefore its deformation potential energy is minimum.

2. Total energy radiated per unit time i.e. power will be equal to the energy reaching the surface of radius x per second

$\therefore$ Intensity $= \dfrac{\text{power}}{\text{area}} = \dfrac{P}{\pi x^2} \Rightarrow I \propto \dfrac{1}{x^2}$

3. Direction reverses after reflection and phase difference introduced after each reflection depending upon nature of support.

4. For the given situation,

$$A_y = \frac{2v_2}{v_1 + v_2} A_l$$

But $v = \sqrt{\dfrac{T}{\mu}}$ $\therefore$ $v_1 = \sqrt{\dfrac{T}{\mu_l}}$, $v_2 = \sqrt{\dfrac{T}{\mu_y}}$

$$\therefore \quad A_y = \frac{2\sqrt{\dfrac{T}{\mu_y}}}{\left(\sqrt{\dfrac{T}{\mu_l}} + \sqrt{\dfrac{T}{\mu_y}}\right)} A_1 = \frac{\dfrac{2}{\sqrt{\mu_y}}}{\left(\dfrac{1}{\sqrt{\mu_l}} + \dfrac{1}{\sqrt{\mu_y}}\right)} A_l$$

$$= \frac{2\sqrt{\dfrac{\mu_l}{\mu_y}}}{\left(1 + \sqrt{\dfrac{\mu_l}{\mu_y}}\right)} \qquad \ldots.[\because A_l = 1]$$

6. From the figure,

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \;;\; T_2 = 2\,T_1$$

where, T_1 = tension in string AB
and T_2 = tension in string CD

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{2T_1}} = \frac{1}{\sqrt{2}}$$

7. $\dfrac{\text{Coefficient of t}}{\text{Coefficient of x}} = \dfrac{l}{t} \Rightarrow \dfrac{30}{0.01} = \dfrac{45 \times 10^{-2}}{t}$

$$\Rightarrow t = \frac{45 \times 10^{-4}}{30} = 150\ \mu s$$

8. Given equation is,

$$y = y_0 \sin 2\pi \left(f\,t - \frac{x}{\lambda}\right)$$

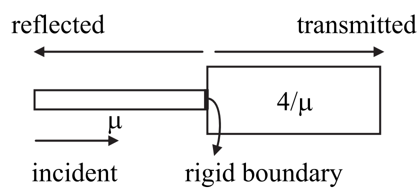
$$A\,\omega = \frac{\omega}{k} \Rightarrow A\,k = 1$$

$$\therefore \quad y_0 \frac{2\pi}{\lambda} = 1 \Rightarrow \lambda = 2\,\pi y_0$$

9. In general, to find the equation of a travelling wave of a given curve, replace x by $x \pm vt$ in the equation of curve. If the wave is travelling in +x direction, use $x - vt$ and otherwise.

10. For the given situation, the relation between pulse speed and height is governed by,
$v^2 = gh \Rightarrow$ The graph is as shown in (D).

11.



Reflected wave will have a phase inversion of π while the transmitted wave will not.
Hence, $y_t = (4\ \text{mm}) \sin(5\,t + 40\,x)$

12. If x is taken from the end of about which rope is rotated then,

$$T(x) = \frac{M\omega^2}{2L}\left(L^2 - x^2\right)$$

$$\therefore \quad v(x) = \sqrt{\frac{T(x)}{\mu}} = \frac{\omega}{\sqrt{2}}\sqrt{L^2 - x^2}$$

$$\therefore \quad \frac{dx}{dt} = \frac{\omega}{\sqrt{2}}\sqrt{L^2 - x^2}$$

$$\therefore \quad v(x) = \int_0^L \frac{dx}{\sqrt{L^2 - x^2}} = \frac{\omega}{\sqrt{2}}\int_0^T dt$$

$$\frac{\omega T}{\sqrt{2}} = \left[\sin^{-1}\left(\frac{x}{L}\right)\right]_0^L = \frac{\pi}{2}$$

$$\therefore \quad \theta = \omega\,T = \frac{\pi}{\sqrt{2}}$$

13. $y = \dfrac{10}{10x - \pi t}$

$A = 10$ cm, $\lambda = \dfrac{\pi}{5}$ cm

$\therefore$ $f = \dfrac{1}{2}$ Hz

$\therefore$ Assertion is false but Reason is true.

14. In the given case, the wave must be bounded.

15. $\Psi = \sin\left[\omega t - \dfrac{2\pi}{\lambda}(x\cos\alpha + y\cos\beta)\right]$

represents a wave travelling along a line in x-y plane through origin making an angle α with x-axis and β with y-axis.

$\Delta\phi = \dfrac{2\pi}{\lambda}\left[(x_2 - x_1)\cos\alpha + (y_2 - y_1)\cos\beta\right]$

Comparing with the given equation, we get

$\alpha = 30°$, $\beta = 60°$, $\lambda = 1$ m, $\omega = 30$/s

Let $(x_1, y_1) \equiv (2\sqrt{3}\,\text{m}, 2\text{m})$ and

$(x_2, y_2) \equiv (3\sqrt{3}\,\text{m}, 3\text{m})$

On substituting the values and simplifying we get,

$\rightarrow \Delta\phi = 4\pi = n\pi \Rightarrow n = 4$

16. The apparent wavelength after reflection is,

$\lambda' = \lambda + v_w\left(\dfrac{\lambda}{v_S - v_W}\right)$,

v_w = Velocity of reflecting surface

$= \left(\dfrac{v_s}{v_s - v_w}\right)\lambda$

$\therefore$ $\nu' = \left(\dfrac{v_s - v_w}{v_s}\right)\nu = \left(\dfrac{334 - 2}{334}\right)334 = 332$ Hz

17. As B is moving away from A,

the frequency heard by B has to decrease.

$f = f_0\left(\dfrac{v - v_A}{v - v_B}\right)$

Thus the graph will shift by some amount but the bandwidth would remain constant.

Note: We cannot comment about the magnitude of intensity heard.

18. Let f = 250 Hz, then f – 2 = 248 Hz, f + 2 = 252 Hz

At x = 0,

$y = y_1 + y_2 + y_3 = A\sin 2\pi(f + 2)t + A\sin 2\pi(f - 2)t + A\sin 2\pi ft$

$\Rightarrow y = 2A\sin 2\pi ft\cos 4\pi t + A\sin 2\pi ft$

$\Rightarrow y = A(2\cos 4\pi t + 1)\sin 2\pi ft$

Intensity, $I \propto R^2$, $I = KA^2(2\cos 4\pi t + 1)^2$

For maximum and minimum intensity,

$\dfrac{dI}{dt} = 0 \Rightarrow 2KA^2(1 + 2\cos 4\pi t)(-\theta\,\pi\sin 4\pi t)$

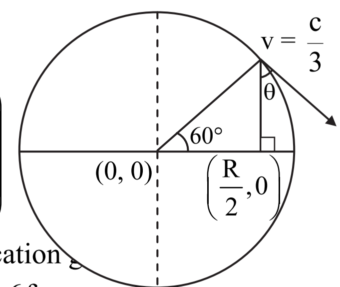
$\Rightarrow t = 0, \dfrac{1}{4}, \dfrac{1}{2}, \dfrac{3}{4}, 1 \Rightarrow \Delta t = \dfrac{1}{4}$

Beat frequency $= \dfrac{1}{\Delta t} = 4$ Hz

19. Quality (Wave form) of sound distinguish the different sources of sound from each other.

20. Frequency will be maximum when the approach velocity is maximum.

Approach velocity is maximum, when θ is maximum and θ is maximum when body is just above point $\left(\dfrac{R}{2}, 0\right)$



$y = y_0\left(\dfrac{v}{v \pm v_s}\right)$

$= f_o\left(\dfrac{c}{c \pm \dfrac{c}{3}\cos\theta}\right)$

which on simplification g

$I_{max} = \dfrac{6f_o}{5}$, $I_{min} = \dfrac{6f_o}{7}$ …[$\because \theta = 60°$]

21. $v = \sqrt{\dfrac{T}{\mu}}$, $v_1 = v$, $v_r = \dfrac{v}{2}$

$A_1 = A$

$A_r = \left(\dfrac{2v_2}{v_1 + v_2}\right)A_1 = \dfrac{2\left(\dfrac{v}{2}\right)}{\left(3\dfrac{v}{2}\right)}A = \dfrac{2}{3}A$

$\therefore$ $E = \dfrac{1}{2}\mu\omega^2A^2(\lambda)$

Frequency remains same for both cases,

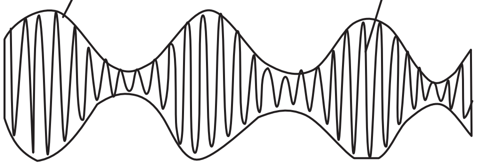
$= \dfrac{1}{2}\mu\omega^2A^2\left(\dfrac{v}{f}\right)$

$\therefore$ $E \propto \mu v A^2$

$\therefore$ $\dfrac{E_1}{E_2} = \dfrac{(\mu_1)}{(4\mu_1)}\dfrac{(v)}{\left(\dfrac{v}{2}\right)}\dfrac{(A)^2}{\left(\dfrac{2}{3}A\right)^2} = \dfrac{9}{8}$

$\therefore$ Fraction transmitted $= \dfrac{8}{9}E_1$

22.



For beats,

$$y = 2A\cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right)\sin\left(2\pi\left(\frac{f_1+f_2}{2}\right)t\right)$$

Beat frequency remains constant and frequency of vibration of particles is $\frac{f_1+f_2}{2}$.

23. Higher pressure $\rightarrow$ higher density

24. The loudness of sound is measured on decibel scale which is logarithmic.

Loudness or sound level = $10\log\left(\frac{I}{I_0}\right)$. Each increase in intensity by a power of 10 increases decibel reading of 10 units.

Hence, to increase the decibel reading by 20, there needs to be an increase in intensity by $10 \times 10 = 100$.

25. Frequency observed by man is same as "observed" by the wall and it reflects the same and as man and wall are relatively at rest, hence man hears same frequency of reflected sound. Hence, beat frequency is zero.

Textbook Chapter No.

08 Stationary Waves

Hints

Classical Thinking

17. Frequency of p^{th} overtone is
$n_p = pn_1$
where p = no. of segments or loops
n_1 = Fundamental frequency
(given) p = 1

$\therefore$ $n_p = n_1$
i.e., fundamental mode or 1^{st} harmonic

18. Comparing given equation with the standard form,
$y = A \sin\left(\frac{2\pi x}{\lambda}\right).\cos(2\pi nt)$ we get,
$2\pi tn = 8\pi t \Rightarrow n = \frac{8}{2} = 4$ cycles / s

25. In open organ pipe, both even and odd harmonics are produced.

26. In an open organ pipe, all harmonics are present. For p^{th} overtone, we have $(p + 1)^{th}$ harmonic

28. In closed pipes, only odd harmonics are present.

39. When two bodies have the same frequency, then one is excited and other vibrates with its natural frequency due to resonance.

42. $n \propto \frac{1}{l} \Rightarrow \frac{n_1}{n_2} = \frac{l_2}{l_1}$

$\therefore$ $l_2 = \frac{300}{400} \times 30 = 22.5$ cm

43. For closed pipe, in general,
$n = \frac{v}{4l}(2N - 1) \Rightarrow n \propto \frac{1}{l}$

$\therefore$ If length of air column decreases, then frequency increases.

44. $n_{closed} = \frac{v}{4L}$, $n_{open} = \frac{v}{2L}$

$\therefore$ $n_{open} = 2n_{closed} = 2n$

Critical Thinking

2. In closed organ pipe, if
$y_{incident} = A \sin(\omega t - kx)$, then
$y_{reflected} = A \sin(\omega t + kx + \pi) = - A \sin(\omega t + kx)$
Superimposition of these two waves gives the required stationary wave.

3. $\cos\alpha + \cos\beta = 2\cos\frac{(\alpha+\beta)}{2}\cos\frac{(\alpha-\beta)}{2}$

$\therefore$ $y = y_1 + y_2 = 2 \times 0.05 \times \cos(\pi x)\cos(4\pi t)$
For node, $\cos(\pi x) = 0$
$\Rightarrow \pi x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$
$\Rightarrow x = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots. \quad \Rightarrow x = 0.5$ m

4. Using, $\frac{2\pi}{\lambda}$ = coefficient of x in the argument of the sine function $= k \Rightarrow \lambda = \frac{2\pi}{k}$
Distance between adjacent nodes = $\lambda/2$.

$\therefore$ The distance between adjacent nodes = $\frac{\pi}{k}$

5. Velocity, $v = n\lambda$,
$\lambda = \frac{v}{n} = \frac{1200}{300} = 4$ m

$\therefore$ The distance between a node and the neighbouring antinode is $\frac{\lambda}{4} = 1$ m.

6. $y = 6\sin\frac{\pi x}{6}\cos 8\pi t$
Comparing with the standard wave equation
$y = A\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{T}\right)$ we get,
$\frac{2\pi x}{\lambda} = \frac{\pi x}{6} \Rightarrow \lambda = 12$

$\therefore$ The distance between two consecutive nodes,
$\frac{\lambda}{2} = \frac{12}{2} = 6$

7. Energy is not carried by stationary waves.

8. The given equation can be written as,
$y = 4\sin\left(4\pi t - \frac{\pi x}{16}\right)$

$\therefore$ $v = \frac{\text{Co-efficient of } t(\omega)}{\text{Co-efficient of } x(k)}$

$\therefore$ $v = \frac{4\pi}{\pi/16} = 64$ cm/s along + x direction.
(Note: Refer Shortcut 9. iii.)

9. $y = A \sin(100t) \cos(0.01x)$
Comparing with standard wave equation,
$y = 2A \sin\left(\frac{2\pi t}{T}\right) \cos\left(\frac{2\pi x}{\lambda}\right)$ we get,
$\frac{2\pi t}{T} = 100\, t$
$\therefore$ $T = \frac{2\pi}{100} \Rightarrow n = \frac{1}{T} = \frac{100}{2\pi}$
Also, $\frac{2\pi x}{\lambda} = 0.01\, x \Rightarrow \lambda = \frac{2\pi}{0.01}$
Velocity of wave,
$v = n\lambda = \frac{100}{2\pi} \times \frac{2\pi}{0.01} = 10^4$ mm/s
$= 10 \times 10^3$ mm/s $= 10$ m/s

10. Minimum time interval between two instants when the string is flat $= \frac{T}{2} = 0.5$ s
$\therefore$ $T = 1$ s
Hence $\lambda = v \times T = 10 \times 1 = 10$ m

11. For a vibrating string, $\lambda = \frac{2L}{p}$
where p = Number of loops = Order of vibration or mode
$\therefore$ For fourth mode p = 4, $\lambda = \frac{2(2)}{4} = 1$ m
$\therefore$ $v = n\lambda = 500 \times 1 = 500$ m/s

12. $y = 0.021\sin(x + 30t)$
Comparing this equation with the standard form we get,
$\omega = 30$ rad/s and $k = 1$
$\therefore$ $v = \frac{\omega}{k} = \frac{30}{1} = 30$ m/s
Using, $v = \sqrt{\frac{T}{m}}$ we get,
$30 = \sqrt{\frac{T}{1.3 \times 10^{-4}}} \Rightarrow T = 0.117$ N
(Note: Refer Shortcut 9. iii.)

13. Here, $\lambda = 2 \times 8 = 16$ m
$n = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2}\sqrt{\frac{T}{L^2 m}}$
$= \frac{1}{2}\sqrt{\frac{T}{L^2\left(\frac{M}{L}\right)}} = \frac{1}{2}\sqrt{\frac{T}{ML}}$
$= \frac{1}{2}\sqrt{\frac{96}{0.120 \times 8}} = 5$ Hz

14. Stretched wire produces integral number of harmonics
Let $420 = 6 \times 70$ Hz
$490 = 7 \times 70$ Hz
$\therefore$ Fundamental frequency of wire is 70 Hz
$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$
$\therefore$ $L = \frac{1}{2n}\sqrt{\frac{T}{m}}$
$\therefore$ $L = \frac{1}{2 \times 70}\sqrt{\frac{450}{5 \times 10^{-3}}}$
$= \frac{1}{2 \times 70} \times 3 \times 100$
$= \frac{30}{14} = 2.1$ m

15. Using, $n = \frac{1}{2L}\sqrt{\frac{T}{m}} \Rightarrow$ For given m, $n \propto \frac{\sqrt{T}}{L}$
$\therefore$ $\frac{n_1}{n_2} = \frac{L_2}{L_1}\sqrt{\frac{T_1}{T_2}} = \frac{1}{4}\sqrt{\frac{1}{4}} = \frac{1}{8}$
$\therefore$ $n_2 = 8n_1 = 8 \times 200 = 1600$ Hz

16. $L_1 + L_2 + L_3 = 110$ cm and $n_1L_1 = n_2L_2 = n_3L_3$
$n_1 : n_2 : n_3 :: 1 : 2 : 3$
$\therefore$ $\frac{n_1}{n_2} = \frac{1}{2} = \frac{L_2}{L_1} \Rightarrow L_2 = \frac{L_1}{2}$ and
$\frac{n_1}{n_3} = \frac{1}{3} = \frac{L_3}{L_1} \Rightarrow L_3 = \frac{L_1}{3}$
$\therefore$ $L_1 + \frac{L_1}{2} + \frac{L_1}{3} = 110$
$\therefore$ $L_1 = 60$; $L_2 = 30$ cm, $L_3 = 20$ cm

17. $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow v \propto \frac{\sqrt{T}}{r}$
$\therefore$ $\frac{v_A}{v_B} = \sqrt{\frac{T_A}{T_B}} \cdot \frac{r_B}{r_A} = \sqrt{\frac{1}{2}} \cdot \frac{1}{2} = \frac{1}{2\sqrt{2}}$

18. $n \propto \frac{1}{L}$
$\therefore$ $\frac{\Delta n}{n} = -\frac{\Delta L}{L}$
If length is decreased by 2%, then frequency increases by 2% i.e., $\frac{n_2 - n_1}{n_1} = \frac{2}{100}$
$\therefore$ $n_2 - n_1 = \frac{2}{100} \times n_1 = \frac{2}{100} \times 392 = 7.8 \approx 8$

19. $n \propto \sqrt{T}$

$\therefore \quad \frac{T_2}{T_1} = \frac{n_2^2}{n_1^2}$

$\therefore \quad T_2 = \frac{n_2^2}{n_1^2} \times T_1 = \left(\frac{320}{256}\right)^2 \times 16 = 25$ kg-wt

$\therefore \quad \Delta T = T_2 - T_1 = 25 - 16 = 9$ kg-wt

20. $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Let $T' = 2T$, $A' = \frac{1}{2}A$

Now, $m = AL\rho$

$\therefore \quad m' = A'L\rho = \frac{1}{2}AL\rho = \frac{m}{2}$

$\therefore \quad n' = \frac{1}{2l}\sqrt{\frac{T'}{m'}} = \frac{1}{2l}\sqrt{\frac{2T}{(m/2)}} = 2\left(\frac{1}{2l}\sqrt{\frac{T}{m}}\right) = 2n$

21. $n \propto \sqrt{T}$

$\therefore \quad n_1 \propto \sqrt{T_1}$ and $n_2 \propto \sqrt{T_2}$

But $T_2 > T_1 \Rightarrow n_2 > n_1$

$\therefore \quad n - n_1 = 5$

$\therefore \quad n - k\sqrt{T_1} = 5 \qquad \therefore \quad n - k\sqrt{100} = 5$

$\therefore \quad n - 10k = 5 \qquad$ ….(i)

$\therefore \quad n_2 - n = 5$

$\therefore \quad k\sqrt{T_2} - n = 5 \qquad \therefore \quad k\sqrt{121} - n = 5$

$\therefore \quad 11k - n = 5 \qquad$ ….(ii)

Adding equations (i) and (ii),

$k = 10$

Substituting in equation (i),

$n - 100 = 5 \Rightarrow n = 105$ Hz

22. On earth:

$n = \frac{1}{2L}\sqrt{\frac{Mg}{m}} = \frac{1}{2L}\sqrt{\frac{g}{m}}$, Since M = 1 kg

On moon: $n' = \frac{1}{2L}\sqrt{\frac{Mg/6}{m}} = \frac{1}{2L}\sqrt{\frac{Mg}{6m}}$

For resonance: $n = n'$

$\frac{1}{2L}\sqrt{\frac{g}{m}} = \frac{1}{2L}\sqrt{\frac{Mg}{6m}}$

which gives M = 6 kg

23. $m = \frac{0.01}{0.5} = 2 \times 10^{-2}$ kg/m

$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$\therefore \quad n = \frac{1}{2\times 0.5}\sqrt{\frac{800}{2\times 10^{-2}}} = \frac{2\times 10^2}{1} = 200$ Hz

24. $n_A = 324$ Hz, $n_b = 6$ Hz

The frequency of string B is

$n_B = n_A \pm n_b = 324 \pm 6 = 330$ or 318 Hz

Now, the frequency of a string is proportional to the square root of tension. Hence, if the tension in A is slightly decreased, its frequency will be slightly reduced, i.e., it will become less than 324 Hz. If the frequency of string B is 330 Hz, the beat frequency would increase to a value greater than 6 Hz if the tension in A is reduced. But the beat frequency is found to reduce to 3 Hz. Hence, the frequency of B cannot be 330 Hz. It is therefore 318 Hz. When the tension in A is reduced, its frequency becomes 324 – 3 = 321 Hz which will produce beats of frequency 3 Hz with string B of frequency 318 Hz.

25. Probable frequencies of tuning fork be n + 4 or n – 4

Now, $n \propto \frac{1}{l}$

$\therefore \quad \frac{n+4}{n-4} = \frac{100}{95}$ or $95(n+4) = 100(n-4)$

$\therefore \quad 95n + 380 = 100n - 400$

$\therefore \quad 5n = 780 \Rightarrow n = 156$ Hz

27. $L_2 = 3L_1 = 3 \times 24.7 = 74.1$ cm

28. For closed organ pipe, only odd harmonics are present. Hence note of frequency 100 Hz will not be emitted as $100 = 2 \times 50$.

29. For a closed pipe,

2[nd] overtone = 5[th] harmonic

$\therefore$ 5[th] harmonic = 5 × fundamental frequency

= 5 × 50 = 250 Hz

30. For closed pipe, $n = \frac{v}{4l} \Rightarrow n = \frac{332}{4\times 42} \approx 2$ Hz

31. $n_1 = \frac{v}{4(L+e)}$

$\therefore \quad n_2 = \frac{v}{4\left(\frac{L}{2}+e\right)} = \frac{2v}{4(L+2e)}$

Clearly, n_2 is less than double of n_1.

32. $n_1 - n_2 = 10 \qquad$ ….(i)

Using $n_1 = \frac{v}{4L_1}$ and $n_2 = \frac{v}{4L_2}$ we get,

$\frac{n_1}{n_2} = \frac{L_2}{L_1} = \frac{26}{25} \qquad$ ….(ii)

On solving these equations,

$n_1 = 260$ Hz, $n_2 = 250$ Hz

33. $n = \frac{v}{4L} \Rightarrow n \propto \frac{1}{L}$

$\therefore \quad \frac{L_1}{L_2} = \frac{100}{101} = \frac{n_2}{n_1}$

As $L_2 > L_1$, hence $n_2 < n_1$

$\therefore \quad n_1 - n_2 = 5$

$\therefore \quad \frac{100}{101} = \frac{n_2}{n_2 + 5}$

$\therefore \quad 101\ n_2 - 100\ n_2 = 5 \times 100$

$\therefore \quad n_2 = 500$ Hz

$\therefore \quad n_1 = n_2 + 5 = 500 + 5 = 505$ Hz

34. According to problem,

$\frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{v}{4L}$ ....(i)

and $\frac{1}{2l}\sqrt{\frac{T+8}{m}} = \frac{3v}{4L}$ ....(ii)

Dividing equation (i) by equation (ii),

$\sqrt{\frac{T}{T+8}} = \frac{1}{3} \Rightarrow 9T = T + 8 \Rightarrow T = 1$ N

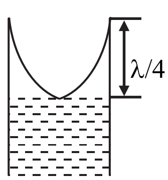
35. For closed pipe, $n_1 = \frac{v}{4L}$

$\therefore \quad 250 = \frac{v}{4 \times 0.2}$

$\therefore \quad v = 200$ m/s

36. Fundamental frequency of a closed pipe is given by $n_0 = \frac{v}{4L}$

Length l of air column first decreases and then becomes constant (when rate of inflow = rate of outflow). Therefore, f_0 will first increase and then become constant.

37. 

The first resonance will occur at length $L = \frac{\lambda}{4}$

For closed pipe, only odd frequencies are present.

So next resonance will be obtained at length $\frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$

38. $e = \frac{l_2 - 3l_1}{2} = \frac{49 - 3 \times 16}{2} = 0.5$ cm

39. $\lambda = (15 + 1) \times 4 = 64$ cm

For second resonance, $L = \frac{3\lambda}{4}$

$\therefore \quad L = \frac{3}{4} \times 64 = 48$ cm

$\therefore \quad$ Length of the tube = L – e = 48 – 1 = 47 cm

40. Here, $L_2 - L_1 = \frac{\lambda}{2}$ or $\lambda = 2(L_2 - L_1)$

Using, $v = n\lambda$,

$n = \frac{v}{\lambda} = \frac{v}{2(L_2 - L_1)} = \frac{330}{2(49-16) \times 10^{-2}} = 500$ Hz

41. Fundamental frequency of open pipe,

$n_1 = \frac{v}{2L} = \frac{350}{2 \times 0.5} = 350$ Hz

42. For open organ pipe,

$n_0 = \frac{v}{2L} = \frac{320}{2 \times 40 \times 10^{-2}} = 400$ Hz

n = 1200 Hz = 3 × 400 Hz

$\therefore \quad$ The mode of vibration is 3^{rd} harmonic $\Rightarrow 2^{nd}$ overtone

43. $L_1 = 50$ cm, $L_2 = 50.5$ cm

As $L_2 > L_1$, so $n_2 < n_1$

For open pipe,

$n = \frac{v}{2L}$

$n_1 - n_2 = 3$ beats/s

$\therefore \quad \frac{v}{2}\left(\frac{1}{L_1} - \frac{1}{L_2}\right) = 3$

$\therefore \quad \frac{v}{10^{-2}}\left(\frac{1}{50} - \frac{1}{50.5}\right) = 6$

$\therefore \quad v = \frac{6 \times 50 \times 50.5 \times 10^{-2}}{0.5} = 303$ m/s

44. $\lambda_1 = 2L$, $\lambda_2 = 2L + 2\Delta L$

$n_1 = \frac{v}{2L}$ and $n_2 = \frac{v}{2L + 2\Delta L}$

$\therefore \quad$ No. of beats $= n_1 - n_2 = \frac{v}{2}\left(\frac{1}{L} - \frac{1}{L + \Delta L}\right)$

$= \frac{v\Delta L}{2L^2}$

45. It is given that

First overtone of closed pipe = First overtone of open pipe

$\therefore \quad 3\left(\frac{v}{4L_1}\right) = 2\left(\frac{v}{2L_2}\right);$

where L_1 and L_2 are the lengths of closed and open organ pipes.

∴ $\frac{L_1}{L_2} = \frac{3}{4}$

46. Given frequencies are 425, 595, 765 Hz
v = 340 m/s
Option A : For a closed pipe having L = 1 m,
$n_c = \frac{v}{4L} = \frac{340}{4} = 85$ Hz
Option B: For L = 2 m, $n_c = 42.50$ Hz
Option C: For open pipe having, L = 1 m,
$n_0 = \frac{v}{2L} = \frac{340}{2} = 170$ Hz
Option D: For L = 2 m, $n_0 = 85$ Hz
Open pipe has all the harmonics, which is not possible.
Closed pipe has only odd harmonics. Hence L = 2 m is not possible.
∴ Correct option is (A).

47. First overtone frequency for closed pipe = $\frac{3v}{4L}$
Fundamental frequency for open pipe = $\frac{v}{2L}$
First overtone frequency for open pipe $= 2\left(\frac{v}{2L}\right) = \frac{v}{L} = \frac{4}{3} \times \frac{3v}{4L}$

49. Since they are turned to same pitch, fundamental frequencies are same, $n_o = n_c$
∴ $\frac{v}{2L_o} = \frac{v}{4L_c}$
∴ $\frac{L_o}{L_c} = \frac{4}{2} = 2 : 1$ ∴ $L_o : L_c : : 2 : 1$

50. Let L_1 and L_2 be the lengths of open and closed pipes respectively. (Neglecting end correction)
∴ $\lambda_1 = 2L_1$, $\lambda_2 = 4L_2$
Given that, $\lambda_1 = \lambda_2$
∴ $2L_1 = 4L_2$
∴ $\frac{L_1}{L_2} = \frac{1}{2}$

51. For open pipe, $n_o = 4n_f = \frac{4v}{2L_o} = \frac{2v}{L_o}$
For closed pipe, $n_c = 7n_f = \frac{7v}{4L_c}$
$n_o = n_c$ ....[Given]
∴ $\frac{2v}{L_o} = \frac{7v}{4L_c} \Rightarrow \frac{L_o}{L_c} = \frac{8}{7}$

52. Fundamental frequency of closed pipe,
$n' = \frac{v}{4L} \Rightarrow L = \frac{v}{4n'}$
Fundamental frequency of open pipe,
$n = \frac{v}{2L} \Rightarrow L = \frac{v}{2n}$
∴ $\frac{v}{4n'} = \frac{v}{2n} \Rightarrow n' = \frac{n}{2}$

53. When one end is closed, $n_1 = 100/2 = 50$ Hz
$n_2 = 3\ n_1 = 150$ Hz,
$n_3 = 5\ n_1 = 250$ Hz and so on

54. Let 'L' be the length of the pipe,
$n = \frac{v}{2L}$ ....(i)
When the pipe having a length of $\frac{2}{5}L$ is inside water, then length of the air column,
$L_1 = L - \frac{2L}{5} = \frac{3L}{5}$
∴ $n' = \frac{v}{4L_1} = \frac{v}{4 \times \frac{3L}{5}} = \frac{5v}{12L}$
$= \frac{5}{6}\left(\frac{v}{2L}\right)$ ....[From (i)]
∴ $n' = \frac{5}{6}n$

55. For open pipe, fundamental frequency, $n = \frac{v}{2L}$
For closed pipe, $n' = \frac{v}{4L} = \frac{1}{2}\cdot\left(\frac{v}{2L}\right) = \frac{1}{2}n$
∴ $\frac{n}{2} = 512$ Hz $\Rightarrow n = 2 \times 512 = 1024$ Hz

56. L = 45 = 5 × 9
L′ = 99 = 11 × 9
Hence other lengths between these values are,
$L_1 = 7 \times 9 = 63$ cm
$L_2 = 9 \times 9 = 81$ cm
So fundamental length is 9 cm
∴ $9 = \frac{\lambda}{4} \Rightarrow \lambda = 9 \times 4 = 36$ cm

57. For open pipe, $n_1 = \frac{v}{2L_1} \Rightarrow L_1 = \frac{v}{2n_1}$
For closed pipe, $n_2 = \frac{v}{4L_2} \Rightarrow L_2 = \frac{v}{4n_2}$
After joining, $L = L_1 + L_2$

Since it is a closed pipe,

$$n = \frac{v}{4L} = \frac{v}{4(L_1 + L_2)}$$

$$= \frac{v}{4\left(\frac{v}{2n_1} + \frac{v}{4n_2}\right)}$$

$$= \frac{8n_1\ n_2}{4(4n_2 + 2n_1)}$$

$$= \frac{n_1 n_2}{2n_2 + n_1}$$

$$= \frac{500 \times 450}{(2 \times 450) + 500}$$

$= 160.7$ Hz ≈ 161 Hz

58. $n_p = \frac{v}{4L}$, $n_q = \frac{v}{2L}$, $n_r = \frac{2v}{2L}$, $n_s = \frac{3v}{4L}$

$n_p : n_q : n_r : n_s$

$\frac{v}{4L} : \frac{2v}{4L} : \frac{4v}{4L} : \frac{3v}{4L}$

$\therefore$ $n_p : n_q : n_r : n_s :: 1 : 2 : 4 : 3$

59. $N = \frac{p}{2L}\sqrt{\frac{T}{m}}$

$\therefore$ $p = 2NL\sqrt{\frac{m}{T}}$

$\therefore$ $p^2 = \frac{\text{constant}}{T} \Rightarrow p^2T = \text{constant}$

60. In perpendicular position, $N = \frac{p_\perp}{2l}\sqrt{\frac{T}{m}}$

In parallel position, $N = \frac{p_\parallel}{l}\sqrt{\frac{T}{m}}$

$\therefore$ $\frac{p_\perp}{2} = p_\parallel \Rightarrow p_\perp = 2p_\parallel$

61. For perpendicular position, N = n

$m = \frac{1.0 \times 10^{-3}}{8 \times 10^{-1}} = \frac{1}{8} \times 10^{-2}$, $T = 0.4 \times 9.8$ N

Using, $N = \frac{p}{2L}\sqrt{\frac{T}{m}}$

$$N = \frac{4}{2 \times 0.8} \times \sqrt{\frac{0.4 \times 9.8}{\frac{1}{8} \times 10^{-2}}}$$

$$= \frac{4}{2 \times 8 \times 10^{-1}} \times \sqrt{8 \times 4 \times 2 \times 49} = \frac{4 \times 8 \times 7}{2 \times 8 \times 10^{-1}}$$

$= 140$ Hz

62. $n \propto \sqrt{T} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$

$n_2 = 3n_1$ ….[Given]

$\therefore$ $\frac{n_1}{n_2} = \frac{1}{3}$

$\therefore$ $\frac{1}{3} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{T_1}{T_1 + 8}}$

$\therefore$ $\frac{T_1}{T_1 + 8} = \frac{1}{9}$

$\therefore$ $9\ T_1 = T_1 + 8 \Rightarrow T_1 = 1$ kg-wt

63. Maximum pressure at closed end will be atmospheric pressure added to acoustic wave pressure.

$\therefore$ $p_{max} = p_A + p_0$ and $p_{min} = p_A - p_0$

$\therefore$ $\frac{p_{max}}{p_{min}} = \frac{p_A + p_0}{p_A - p_0}$

64. When a musical instrument is played, it produces a fundamental note which is accompanied by a number of overtones called harmonics. The number of harmonics is not the same for all instruments. It is the number of harmonics which distinguishes the note produced by a sitar from that produced by a violin.

65. Mass per unit lengths are constant.

If p is the number of loops and T is the tension, then

$Tp^2 = \text{constant}$

$\therefore$ $T_1p_1^2 = T_2p_2^2$

$\therefore$ $6 \times 10^{-3} \times 10 \times (3)^2 = T_2 \times (2)^2$

$\therefore$ $6 \times 10^{-3} \times 90 = 4\ T_2$

$\therefore$ $T_2 = \frac{540 \times 10^{-3}}{4}$ N

$\therefore$ $T_2 = 135 \times 10^{-3}$ N $= m \times 10$

$\therefore$ $m = 13.5 \times 10^{-3}$ kg $= 13.5$ g

66. $L_1 = 40$ cm, $L_2 = 30$ cm

$n = \frac{1}{2L}\sqrt{\frac{T}{m}} \Rightarrow \frac{\sqrt{T}}{L} = \text{constant}$

$\therefore$ $\frac{\sqrt{T_1}}{L_1} = \frac{\sqrt{T_2}}{L_2} \Rightarrow \frac{L_1}{L_2} = \sqrt{\frac{T_1}{T_2}}$

$\therefore$ $\frac{T_2}{T_1} = \left(\frac{L_2}{L_1}\right)^2 = \left(\frac{30}{40}\right)^2 = \frac{9}{16}$

Let T_1 = Vdg and density of fluid in which weight will be immersed is ρ

∴ $T_1 - T_2 = V\rho g$

∴ $\frac{T_1 - T_2}{T_1} = \frac{\rho}{d}$

∴ $1 - \frac{T_2}{T_1} = \frac{\rho}{d} = 1 - \frac{9}{16} = \frac{7}{16}$

∴ $\frac{d}{\rho} = \frac{16}{7}$

67. $n = \frac{1}{2L}\sqrt{\frac{T}{\pi r^2 d}}$

$n' = \frac{1}{2L'}\sqrt{\frac{9T}{\pi r'^2 d}}$

∴ $\frac{n'}{n} = \frac{3L}{L'} \cdot \frac{r}{r'}$ ….(i)

∵ mass remains the same

$\frac{r}{r'} = \sqrt{\frac{L'}{L}}$

Substituting in eq. (i)

$\frac{n'}{n} = 3\sqrt{\frac{L}{L'}}$

∵ $L' > L$

∴ $n' < 3n$

68. $n \propto \frac{\sqrt{T}}{l}$

∴ $l \propto \sqrt{T}$ (∵ n = constant)

∴ $\frac{l_2}{l_1} = \sqrt{\frac{T_2}{T_1}}$

∴ $l_2 = l_1\sqrt{\frac{169}{100}}$

∴ $l_2 = 1.3 l_1 = l_1 + 0.30\, l_1 = 30\%$ of l_1

69. According to law of tension,

$N \propto \sqrt{T}$

Therefore, when the tension is doubled, the frequency becomes $\sqrt{2}$ times.

70. Let v_1 be the speed of sound at 27 °C and v_2 at 31 °C then,

$\frac{v_2}{v_1} = \left(\frac{273+31}{273+27}\right)^{\frac{1}{2}} = \left(\frac{304}{300}\right)^{\frac{1}{2}} = \left(1 + \frac{4}{300}\right)^{\frac{1}{2}}$

$= 1 + \frac{1}{2} \times \frac{4}{300} = \frac{151}{150}$

Now, frequency ∝ speed of sound

∴ $\frac{n_2}{n_1} = \frac{151}{150}$

∴ $n_2 = n_1 \times \frac{151}{150} = \frac{300 \times 151}{150} = 302$ Hz

Hence beat frequency = 302 – 300 = 2

71. For 1st resonance, $L_1 + e = \frac{\lambda}{4}$

For 2nd resonance, $L_2 + e = \frac{3\lambda}{4}$

∴ $L_2 - L_1 = \frac{\lambda}{2}$

Speed of sound, $v = n\lambda = 500 \times 2\,(L_2 - L_1)$

$= 500 \times 2\,(52 - 17) \times 10^{-2}$

$= 350$ m/s

72. $n \propto \sqrt{T} = \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$ ….(i)

∴ $n_2 = n_1 + \frac{50}{100} \times n_1 = \frac{150 n_1}{100}$

∴ $\frac{n_1}{n_2} = \frac{100}{150} = \frac{2}{3}$ ….[From (i)]

∴ $\frac{T_1}{T_2} = \frac{4}{9}$

∴ % increase $= \frac{T_2 - T_1}{T_1} \times 100 = \left(\frac{T_2}{T_1} - 1\right) \times 100$

$= \left(\frac{9}{4} - 1\right) \times 100 = \frac{500}{4} = 125\%$

Competitive Thinking

2. Progressive waves propagate energy while stationary waves do not propagate energy.

3. Waves $z_1 = A\sin(kx - \omega t)$ is travelling towards positive x-direction.

Wave $z_2 = A\sin(kx + \omega t)$ is travelling towards negative x-direction.

Wave $z_3 = A\sin(ky - \omega t)$ is travelling towards positive y direction.

Since waves z_1 and z_2 are travelling along the same line, so they will produce stationary wave.

10. $n = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}} \Rightarrow n \propto \frac{1}{r}$ and $v \propto n \Rightarrow v \propto \frac{1}{r}$

11. Velocity of transverse wave on string,

$V \propto \frac{1}{r}$

$\therefore$ $V_A \propto \frac{1}{r_A}$ ….(i)

$V_B \propto \frac{1}{r_B}$ ….(ii)

Divide equation (i) by equation (ii)

$\therefore$ $\frac{V_A}{V_B} = \frac{r_B}{r_A} = \frac{r_B}{2r_B}$ ….{$\because r_A = 2r_B$}

$\therefore$ $\frac{V_A}{V_B} = \frac{1}{2}$

12. $n \propto \frac{1}{l r}$

$\therefore$ $n_1 l_1 r_1 = n_2 r_2 l_2$

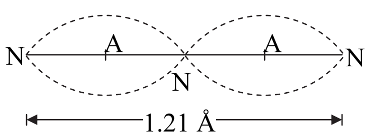
$n_1 l_1 r_1 = n_2\, 2r_1 \times 2l_1$

$\therefore$ $n_1 = 4n_2$

$\therefore$ $n_2 = \frac{n_1}{4}$

14. Particles have kinetic energy maximum at mean position.

16.



17. In fundamental mode of vibration, wavelength is maximum

$\therefore$ $L = \frac{\lambda}{2} = 40 \text{ cm} \Rightarrow \lambda = 80 \text{ cm}$

18. At fixed end, node is formed and distance between two consecutive nodes,

$\frac{\lambda}{2} = 10 \text{ cm} \Rightarrow \lambda = 20 \text{ cm}$

$\therefore$ $v = n\lambda = 100 \times 20 \times 10^{-2} = 20 \text{ m/s}$

19. In fifth overtone, number of loops = 6

$\therefore$ Length of 6 loops = 2.4 m

$\therefore$ Length of each loop = $\frac{2.4}{6}$ = 0.4 m

$\therefore$ Distance between a node and antinode is half of length of loop = $\frac{0.4}{2}$ = 0.2 m

20. $n \propto \sqrt{T}$

21. $n \propto \sqrt{T}$

$\frac{n'}{n} = \sqrt{\frac{T'}{T}}$

$\frac{n'}{n} = \sqrt{\frac{2T}{T}}$

$n' = \sqrt{2}\, n$

22. Here, $\lambda = 2l$

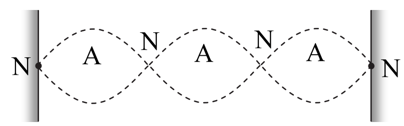
$\therefore$ $v = n\lambda = 480 \times 2 \times 0.3 = 288 \text{ m/s}$

23. Here, $\lambda = \frac{v}{n} = \frac{36}{72} = 0.5 \text{ m} = \frac{1}{2} \text{ m}$

$\therefore$ Distance between wall and first antinode

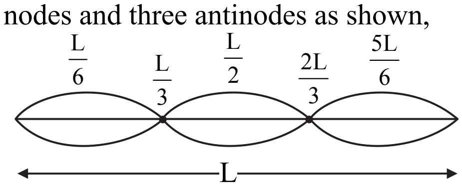
$= \frac{\lambda}{4} = \frac{1}{8} \text{m}$

24. The sonometer wire vibrates in second overtone as shown in the figure

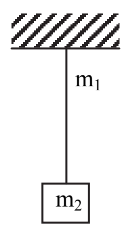


$\Rightarrow$ 4 Nodes and 3 Antinodes

25. String vibrating in second overtone forms four nodes and three antinodes as shown,



26.



Let velocity of pulse at lower end be v_1 and at top be v_2

$\therefore$ $\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1}$ ($\because \lambda = \frac{v}{n}$ and n = constant)

velocity of transverse wave on string

$v = \sqrt{\frac{T}{m}}$

where, m is linear density.

In this case, $v \propto \sqrt{T}$

$\therefore$ $\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{(m_2 + m_1)}{m_2}}$

Where, T_2 is tension at upper end of rope and T_1 is tension at lower end of rope.

27. Using, $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n \propto \sqrt{T}$

As $T_1 > T_2 \Rightarrow n_1 > n_2$

$\therefore$ $n_1 - n_2 = 6$

The beat frequency will remain fixed at 6 if

i. n_1 remains same but n_2 is increased to a new value $(n_2' - n_2 = 12)$ by increasing tension T_2.

ii. n_2 remains same but n_1 is decreased to a new value $(n_1 - n_1' = 12)$ by decreasing tension T_1.

28. $n \propto \frac{1}{L} \Rightarrow \frac{L_2}{L_1} = \frac{n_1}{n_2}$

$\therefore$ $L_2 = L_1\left(\frac{n_1}{n_2}\right) = 50 \times \frac{270}{1000} = 13.5$ cm

29. $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{60.5}{\left(\frac{0.035}{7}\right)}} = 110$ m/s

30. $\nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$

….(where 'm' is mass per unit length)

But, $m = \frac{M}{L}$

$\therefore$ $\nu = \frac{n}{2}\sqrt{\frac{T}{\left(\frac{M}{L}\right)L^2}} = \frac{n}{2}\sqrt{\frac{T}{ML}}$

31. We have, $v = \sqrt{\frac{T}{m}}$

$T = v^2 m$ ….(i)

$\therefore$ $m = 2 \times 10^{-2}$ $\therefore$ $K = \frac{\omega}{v}$

$v = \frac{\omega}{K}$

$v = \frac{120\pi}{2\pi/3}$

$v = 180$ m/l_E

From equation (i)

$T = (180)^2 \times 2 \times 10^{-2}$

$T = 648$ N

32. At resonance, frequency of A.C. will be equal to natural frequency of wire,

$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times1}\sqrt{\frac{10\times9.8}{9.8\times10^{-3}}} = \frac{100}{2} = 50$ Hz

33. String vibrates in five segments

$\therefore$ $\frac{5}{2}\lambda = l \Rightarrow \lambda = \frac{2l}{5}$

$\therefore$ $n = \frac{v}{\lambda} = 5\times\frac{v}{2l} = 5\times\frac{20}{2\times10} = 5$ Hz

34. $n = \frac{1}{2L}\sqrt{\frac{T}{\pi r^2 d}}$

$n' = \frac{1}{2L'}\sqrt{\frac{4T}{\pi r'^2 d}}$

$\therefore$ $\frac{n'}{n} = \frac{2L}{L'}\cdot\frac{r}{r'}$ ….(i)

$\because$ mass remains the same

$\frac{r}{r'} = \sqrt{\frac{L'}{L}}$

Substituting in eq. (i)

$\frac{n'}{n} = 2\sqrt{\frac{L}{L'}}$

$\because$ $L' > L$

$\therefore$ $n' < 2n$

35. $n = \frac{1}{2L}\sqrt{\frac{T}{m}} \Rightarrow n \propto \sqrt{T}$

For octave, $n' = 2n$

$\therefore$ $\frac{n'}{n} = \sqrt{\frac{T'}{T}} = 2$

$\therefore$ $T' = 4T = 16$ kg-wt

36. $n \propto \sqrt{T} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$

$\therefore$ $\frac{n}{2n} = \sqrt{\frac{10}{T_2}} \Rightarrow T_2 = 40$ N

37. $n \propto \frac{1}{l} \Rightarrow nl = \text{constant}$

$\therefore$ $n_1 l_1 = n_2 l_2$

$\therefore$ $n_1 l_1 = (n_1 - 2)l_2$ ....[$n_2 < n_1$ as length increases]

$\therefore$ $\frac{l_2}{l_1} = \frac{n_1}{n_1 - 2} = \frac{250}{248} = \frac{125}{124}$

38. $l_1 : l_2 : l_3 = \frac{1}{n_1} : \frac{1}{n_2} : \frac{1}{n_3} = 6 : 3 : 2$

$l_1 = \frac{6}{11}\times 99 = 54$

$l_2 = \frac{3}{11}\times 99 = 27$

$l_3 = \frac{2}{11}\times 99 = 18$

39. Here, nl = constant

∴ $n_1l_1 = n_2l_2 \Rightarrow 110\,(l_1) = (l_1 - 5)\,n_2$

∴ $\frac{110\times 60}{55} = n_2 \Rightarrow n_2 = 120$ Hz

∴ Number of beats = 120 − 110 = 10

40. When the length of sonometer wire increases by 4%, the new length,

$l_2 = 1.04\,l_1$

Now, nl = constant

∴ $n_1\,l_1 = n_2\,(1.04\,l_1) \Rightarrow n_1 = 1.04\,n_2$

∴ $n_2 = n_1 - 8 \qquad \ldots(\because n_2 < n_1)$

∴ $n_2 = 1.04\,n_2 - 8$

∴ $0.04\,n_2 = 8 \Rightarrow n_2 = 200$ Hz

41. $n \propto \sqrt{T}$

∴ $\frac{\Delta n}{n} = \frac{1}{2}\frac{\Delta T}{T}$

∴ Beat frequency, $\Delta n = \left(\frac{1}{2}\frac{\Delta T}{T}\right)n$

$= \frac{1}{2}\times\frac{2}{100}\times 400 = 4$

42. Let the frequency of tuning fork be N.

As the frequency of vibrating string $\propto \frac{1}{\text{length of string}}$

For sonometer wire of length 20 cm, frequency must be (N + 5) and that for the sonometer wire of length 21cm, the frequency must be (N − 5) as in each case, the tuning fork produces 5 beats/s with sonometer wire

∴ $n_1l_1 = n_2l_2 \Rightarrow (N + 5)\times 20 = (N - 5)\times 21$

∴ N = 205 Hz

43. Using, $n = \frac{1}{2}\sqrt{\frac{T}{m}}$

Number of beats $= \frac{1}{2}\sqrt{\frac{T}{m}}\left[\frac{1}{l_2} - \frac{1}{l_1}\right]$

$= \frac{1}{2}\sqrt{\frac{20}{1\times 10^{-3}}}\left[\frac{1}{49.1\times 10^{-2}} - \frac{1}{51.6\times 10^{-2}}\right] = 7$

44. Fundamental frequency $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2\rho}}$

$\Rightarrow n \propto \frac{1}{lr} \Rightarrow \frac{n_1}{n_2} = \frac{r_2}{r_1}\times\frac{l_2}{l_1} = \frac{r}{2r}\times\frac{2L}{L} = \frac{1}{1}$

45. Fundamental frequency of the first wire is

$n = \frac{1}{2l_1}\sqrt{\frac{T}{m}} = \frac{1}{2l_1}\sqrt{\frac{T}{\pi r_1^2\rho}} = \frac{1}{2l_1r_1}\sqrt{\frac{T}{\pi\rho}}$

The first overtone $n_1 = 2n = \frac{1}{l_1r_1}\sqrt{\frac{T}{\pi\rho}}$

Similarly, the second overtone of the second wire will be,

$n_2 = \frac{3}{2l_2r_2}\sqrt{\frac{T}{\pi\rho}}$

Given that $n_1 = n_2$

∴ $\frac{1}{l_1r_1}\sqrt{\frac{T}{\pi\rho}} = \frac{3}{2l_2r_2}\sqrt{\frac{T}{\pi\rho}}$

∴ $3l_1r_1 = 2l_2r_2$

$\frac{l_1}{l_2} = \frac{2r_2}{3r_1}$

$= \frac{2r_2}{3(2r_2)} \qquad \ldots.(\because r_1 = 2r_2)$

$= \frac{1}{3}$

46. $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2\rho}} \propto \sqrt{\frac{T}{r^2\rho}}$

∴ $\frac{n_1}{n_2} = \sqrt{\left(\frac{T_1}{T_2}\right)\left(\frac{r_2}{r_1}\right)^2\left(\frac{\rho_2}{\rho_1}\right)} = \sqrt{\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)^2\left(\frac{1}{2}\right)} = 1$

∴ $n_1 = n_2$

47. $n = \frac{1}{2l}\sqrt{\frac{T}{M}} \Rightarrow n \propto l^{-1}$

∴ $\%\,\frac{\Delta n}{n} = -\frac{\Delta l}{l}\times 100$

$= -\Delta l = -1\% = 1\%$ (In magnitude)

48. Mass per unit length of the string

$m = \frac{1.0\times 10^{-3}}{20\times 10^{-2}} = 5\times 10^{-3}\ \text{kg m}^{-1}$

speed of waves in string

$v = \sqrt{\frac{I}{m}} = \sqrt{\frac{0.5}{5\times 10^{-3}}} = 10\ \text{ms}^{-1}$

Now, $v = n\lambda$

∴ $\lambda = \frac{v}{n} = \frac{10}{100} = 0.1$ cm = 10 cm

∴ separation between successive nodes $= \frac{\lambda}{2}$

= 5 cm

49. $\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow \nu \propto \frac{1}{l}$

$l = l_1 + l_2 + l_3$ ....(Given)

$\therefore \frac{1}{\nu} = \frac{1}{\nu_1} + \frac{1}{\nu_2} + \frac{1}{\nu_3}$

50. Let the length of original string is l

$l = l_1 + l_2 + l_3$ ....(i)

$\because n = \frac{V}{2l}$

$n_1 = \frac{V}{2l_1}$

$n_2 = \frac{V}{2l_2}$

$n_3 = \frac{V}{2l_3}$

From equation (i),

$\frac{V}{2n} = \frac{V}{2n_1} + \frac{V}{2n_2} + \frac{V}{2n_3}$

$\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

52. For stationary waves, the distance between successive nodes and antinodes is always $\frac{\lambda}{4}$.

54. For a closed organ pipe,

$n_1 : n_2 : n_3 .... = 1 : 3 : 5 ....$

55. Given: $l = 83 \times 10^{-2}$ cm, v = 332 m/s

$n_0 = \frac{v}{4L} = \frac{332}{4 \times 83 \times 10^{-2}} = 100$

$n_0 : n_1 : n_2 : n_3 : n_4 = 1 : 3 : 5 : 7 : 9$

$= 100 : 300 : 500 : 700 : 900$

$\therefore$ Number of possible natural frequency = 5.

56. n = 100 Hz and n′ = 500 Hz = 5 × 100

$\therefore$ n′ = 5 n $\Rightarrow$ Pipe is closed at one end.

57. In closed organ pipe, for p[th] mode corresponding frequency is

$(n_{p-1})_c = (2p - 1)\, n_c$

where, $n_c = \frac{v}{4L}$

In open organ pipe, for p[th] mode corresponding frequency is

$(n_{p-1})_0 = p\, n_0$

where, $n_0 = \frac{v}{2L}$

....(Given length and medium is same for both the pipes)

$\therefore \frac{(n_{p-1})_0}{(n_{p-1})_c} = \frac{p\left(\frac{v}{2L}\right)}{(2p-1)\left(\frac{v}{4L}\right)} = \frac{2p}{2p-1}$

58. In a closed pipe, odd harmonics are observed so lengths for resonance are also in sequence of l_1, $3l_1$, $5l_1$,..., where, l_1 is the minimum length of the column for which resonance occurs.

$\therefore$ Next length = $3l_1 = 3 \times 50 = 150$ cm

59. For a closed pipe, frequency of second note

$= \frac{3v}{4l} = \frac{3 \times 330}{4 \times 1.5} = 165$ Hz

60. Frequency of 2[nd] overtone $n_3 = 5n_1 = 5 \times 50 = 250$ Hz.

61. For a pipe closed at one end,

$n = \frac{v}{4l} = \frac{340}{4 \times 34 \times 10^{-2}}$

$\therefore$ Frequency of 5[th] overtone

$n' = 11\, n = 11 \times \frac{340 \times 10^2}{4 \times 34} = 2750$ Hz

62. For a closed pipe,

Frequency of 1[st] overtone,

$n' = 3n \Rightarrow n = \frac{n'}{3} = \frac{480}{3} = 160$ Hz

63. For a closed pipe, $n = \frac{v}{4l} = \frac{330}{4 \times 1}$

$\therefore$ Frequency of second note = 3 n = $\frac{3 \times 330}{4 \times 1}$ Hz

64. For a pipe closed at one end,

Fundamental frequency,

$n = \frac{v}{4L} = \frac{L/t}{4L} = \frac{1}{4t} = \frac{1}{4 \times 0.01} = 25$ Hz

65. As tube is closed at one end and open at other end.

$\frac{(2n+1)v}{4l} = 260$ Hz ....(i)

$\frac{(2n-1)v}{4l} = 220$ Hz ....(ii)

Subtracting equation (ii) from equation (i),

$\frac{2v}{4l} = 40$

$\therefore$ Fundamental frequency = $\frac{v}{2l} = 20$ Hz

66. Number of beats per second,

$$n = \frac{16}{20} = \frac{4}{5} \Rightarrow n = n_1 - n_2 = \frac{v}{4}\left(\frac{1}{l_1} - \frac{1}{l_2}\right)$$

∴ $\frac{4}{5} = \frac{v}{4}\left(\frac{1}{1} - \frac{1}{1.01}\right) = \frac{0.01v}{4 \times 1.01} = \frac{v}{4 \times 101}$

∴ $v = \frac{16 \times 101}{5} = 323.2 \text{ ms}^{-1}$

67. For 1st resonance,

$$l_0 = \frac{v}{4n} = \frac{340}{4 \times 340} = 0.25 = 25 \text{ cm}$$

Next resonance will occur at a distance of $3l_0 = 75$ cm and further at $5l_0 = 125$ cm (which is not possible).

Hence, $h = 120 - 3l_0 = 120 - 75 = 45 \text{ cm} = 0.45 \text{ m}$

68. For the second resonance, $x = 3L_1 = 54$ but during summer, temperature increases and hence velocity of sound increases.

∴ $x > 3L_1$ i.e., $x > 54$ cm

69. $\frac{v}{4(l + e)} = n \Rightarrow l + e = \frac{v}{4n}$

∴ $l = \frac{v}{4n} - e$

Here, $e = (0.6)r = (0.6)(2) = 1.2$ cm

∴ $l = \frac{336 \times 10^2}{4 \times 512} - 1.2 = 15.2 \text{ cm}$

70. Let e be the end correction then according to the information given,

$$\frac{v}{4(l_1 + e)} = \frac{3v}{4(l_2 + e)} \Rightarrow 0.35 + e = 3(0.1 + e)$$

$e = 2.5 \text{ cm} = 0.025 \text{ m}$.

71. $e = \frac{l_2 - 3l_1}{2} = \frac{48 - 3(15)}{2} = 1.5 \text{ cm}$

72. $e = 0.3\ d$

$d = \frac{l_2 - 3l_1}{2}$

∴ $d = \frac{l_2 - 3l_1}{0.6} = \frac{0.62 - 3 \times 0.2}{0.6} = \frac{6.2 - 6}{6} = 0.033 \text{ m}$

$= 3.33 \text{ cm}$

73. Fundamental frequency of open tube, $n = \frac{v}{2L}$

where v is the velocity of sound in air and L is the length of the tube

∴ $n = \frac{330}{2 \times 0.25} = 660 \text{ Hz}$

The emitted frequencies are n, 2n, 3n, 4n, …

i.e., 660 Hz, 1320 Hz, 1980 Hz, 2640 Hz, …

74. For a closed pipe, fundamental frequency

$$n_1 = \frac{v}{4L} = 100 \text{ Hz}$$

For an open pipe, fundamental frequency

$$n'_1 = \frac{v}{2L} = 2n_1 = 200 \text{ Hz}$$

In an open pipe all multiples of the fundamental are produced. Hence, frequencies produced can be 200 Hz, 400 Hz and so on.

75. The air column in a pipe open at both ends can vibrate in a number of different modes subjected to the boundary condition that there must be an antinode at the open end.

Hence option (A) is correct.

The ratio of frequencies when pipe is open at both the ends is given as,

n : 2 n : 3 n : 4 n : 5 n

where $n = \frac{v}{2L}$

∴ Both odd as well even i.e., All harmonics are present.

Hence, option (B) and (C) are correct

Pressure variation is minimum at antinode

∴ Option (D) is incorrect.

76. For an open pipe,

$e = 0.6\ d$

∴ $d = \frac{e}{0.6}$

∴ $2r = \frac{e}{0.6}$

∴ $r = \frac{0.8}{1.2} = \frac{2}{3} \text{cm}$

77. Fundamental frequency $n = \frac{v}{2L}$

∴ $350 = \frac{350}{2L} \Rightarrow L = \frac{1}{2} \text{ m} = 50 \text{ cm}$

78. For a pipe open at both ends,

$$n = \frac{v}{2l} = \frac{333}{2 \times 33.3 \times 10^{-2}} = 500 \text{ Hz}$$

∴ Frequency of 5th overtone,

$n = 6n = 6 \times 500 = 3000$ Hz

79. Fundamental frequency of closed organ pipe $= \frac{v}{4L}$

∴ $\frac{v}{4L} = \frac{3v}{2l_0}$

$l_0 = \frac{12 \times 20}{2} = 120 \text{ cm}$

80. $n_c = \frac{3v}{4L_1}$ and $n_0 = \frac{4v}{2l_2}$

∴ $n_c = n_0$ gives,

$3l_2 = 8l_1 \Rightarrow \frac{l_1}{l_2} = \frac{3}{8}$

81. First overtone frequency of a closed pipe = second harmonic frequency of an open pipe

$\frac{3v}{4l_1} = \frac{2v}{2l_2}$

$\frac{l_1}{l_2} = \frac{3}{4}$

82. For resonance,

∴ $n_c = n_0$

∴ $\frac{v}{4L_1} = \frac{v}{2L_2} \Rightarrow \frac{L_1}{L_2} = \frac{1}{2}$

83. Frequency of 5th overtone of closed organ pipe = Frequency of fifth overtone of open organ pipe.

∴ $11\ n = 6\ n'$

∴ $11 \times \frac{v}{4L} = 6 \times \frac{v}{2L'}$ ∴ $\frac{L}{L'} = \frac{11}{12}$

84. Difference between successive resonance frequencies $\Delta n = 170$ Hz

If pipe is open, air column will vibrate with all harmonics i.e $n_1, 2n_1, 3n_1, \ldots$

$\Delta n = n_1 = 170$ Hz

But in that case, successive resonance frequencies will be multiples of 170 Hz which contradicts the data given in question.

If pipe is closed, air column will vibrate with only odd harmonics , i.e., $n_1, 3n_1, 5n_1$

$\Rightarrow \Delta n = 2n_1$

∴ $n_1 = \frac{170}{2} = 85$ Hz.

In this case $5n_1$, $7n_1$ and $9n_1$ resonance frequencies will correspond to 425, 595 and 765 Hz respectively as given in the question. Hence, given pipe is closed pipe and

length of pipe $l_c = \frac{v}{4n_1} = \frac{340}{4 \times 85} = 1$ m.

85. Distance between six successive nodes,

$= \frac{5\lambda}{2} = 85$ cm

∴ $\lambda = \frac{2 \times 85}{5} = 34$ cm $= 0.34$ m

∴ Speed of sound in gas,

$= n\lambda = 1000 \times 0.34 = 340$ m/s

86. Difference between two successive resonance frequencies

$\Delta n = 595 - 425 = 170$ Hz

Similarly $\Delta n = 425 - 255 = 170$ Hz

If pipe is open at both ends, air column will vibrate with all harmonics i.e. $n_1, 2n_1, 3n_1, \ldots$

∴ $\Delta n = n_1 = 170$ Hz

But in that case, successive resonance frequencies will be multiples of 170 Hz which contradicts the given data.

If pipe is closed, air column will vibrate with only odd harmonics i.e., $n_1, 3n_1, 5n_1, \ldots$

∴ $\Delta n = 2n_1$

∴ $n_1 = \frac{170}{2} = 85$ Hz

In this case, $3n_1$, $5n_1$, $7n_1$ corresponds to frequencies 255, 425 and 595 Hz.

87. Before dipping in water,

Fundamental frequency, $f = \frac{v}{2l}$

After dipping in water, pipe will get filled with water partially and will act as closed organ pipe of length $\frac{l}{2}$.

∴ After dipping in water,

Fundamental frequency $f' = \frac{v}{4\left(\frac{l}{2}\right)} = \frac{v}{2l} = f$

88. For a closed pipe,

$n_3 = \frac{7v}{4l}$ ....(i)

For an open pipe

$n_2 = \frac{3v}{2l}$ ....(ii)

According to given condition, we have

$\frac{7v}{4l} = \frac{3v}{2l} + 150$ ....[from (i) and (ii)]

∴ $\frac{7v}{4l} - \frac{3v}{2l} = 150$

$\frac{7v - 6v}{4l} = 150$

$\frac{v}{4l} = 150$

∴ Fundamental frequency of pipe open at both ends is

$\frac{v}{2l} = 2(150) = 300$ Hz

89. $n_o = \frac{v}{2L_{open}}$

$n_c = 3 \times \frac{v}{4L_{closed}}$

$n_c = 3 \times \frac{v}{4 \times \frac{L_{open}}{2}} = 3 \times \left(\frac{v}{2L_{open}}\right)$

$= 3 \times 100 = 300$ Hz

90. Open pipe resonance frequency, $f_1 = \frac{2v}{2L}$

Closed pipe resonance frequency, $f_2 = \frac{nv}{4L}$

∴ $f_2 = \frac{n}{4} f_1$ where, n is odd

As $f_2 > f_1 \Rightarrow n = 5$

91. Frequency of first overtone of closed pipe = Frequency of first overtone of open pipe

∴ $\frac{3v_1}{4L_1} = \frac{v_2}{L_2} \Rightarrow \frac{3}{4L_1}\sqrt{\frac{\gamma P}{\rho_1}} = \frac{1}{L_2}\sqrt{\frac{\gamma P}{\rho_2}}$

$\left[\because v = \sqrt{\frac{\gamma P}{\rho}}\right]$

∴ $L_2 = \frac{4L_1}{3}\sqrt{\frac{\rho_1}{\rho_2}} = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$

92. For a pipe open at both ends,

$f = \frac{V}{2L}$

∴ $f_1 = \frac{V}{2L}$, $f_2 = \frac{V}{2(L+d)}$

∴ beat frequency $f_b = f_1 - f_2 = \frac{V}{2L} - \frac{V}{2(L+d)}$

∴ $f_b = V\left[\frac{2(L+d) - 2L}{4L(L+d)}\right] = V\frac{2d}{4L(L+d)}$

∴ $f_b = \frac{Vd}{2L(L+d)}$

93. Second overtone of open pipe is third harmonic,

∴ $n_3 = \frac{3v}{2l}$

First overtone of closed pipe is third harmonic,

$n_2 = \frac{3v}{4l}$

here, L′ be length of open pipe,

∴ $\frac{3v}{2L'} = \frac{3v}{4L}$

∴ $L' = \frac{4L}{2} = 2L$

94. Fundamental frequency of open organ pipe,

$n_1 = \frac{v}{2l_o}$

Frequency of third harmonic for closed organ pipe,

$n_2 = \frac{3v}{4l_c}$

Given: $n_1 = n_2$

∴ $\frac{v}{2l_o} = \frac{3v}{4l_c}$

∴ $l_o = \frac{2l_c}{3} = \frac{2 \times 20}{3} = 13.33$ cm

95. Fundamental frequency of a pipe closed at one end = Frequency of 2nd overtone of pipe open at both ends × $\frac{1}{2}$

∴ $\frac{v}{4nL_1} = \frac{1}{2} \times \frac{3v}{2nL_2} \Rightarrow \frac{1}{L_1} = \frac{3}{L_2}$

∴ $L_2 = 3L_1 = 30$ cm

96. $t_1 - t_2 = 1$

∴ $\frac{L}{340} - \frac{L}{3740} = 1$

∴ $3400\ L = 340 \times 3740$

∴ $L = \frac{34 \times 374}{34} = 374$ m

97. For a pipe closed at one end,

$n_1 = \frac{v}{4L_1}$ and for a pipe open at both ends,

$n_2 = \frac{v}{2L_2} \Rightarrow L_1 = \frac{v}{4n_1}$ and $L_2 = \frac{v}{2n_2}$

For the new pipe, $L = L_1 + L_2 = \frac{v}{4L_1} + \frac{v}{2L_2}$

$n = \frac{v}{4L} = \frac{v}{4 \times \left(\frac{v}{4n_1} + \frac{v}{2n_1}\right)} = \frac{n_1 n_2}{2n_1 + n_2}$

98. $n_1 = \dfrac{v}{2(l_1 + 2e)}$

$\therefore$ $v = 2n_1 (l_1 + 2e)$ ....(i)

$n_2 = \dfrac{v}{2(l_2 + 2e)}$

$\therefore$ $v = 2n_2 (l_2 + 2e)$ ....(ii)

From equation (i) and (ii), we get

$e = \dfrac{n_2 l_2 - n_1 l_1}{2(n_1 - n_2)}$

99. Plucking distance from one end $= \dfrac{l}{2p}$

$\therefore$ $25 = \dfrac{100}{2p} \Rightarrow p = 2$

$\therefore$ $n = \dfrac{p}{2L}\sqrt{\dfrac{T}{m}} = \dfrac{2}{2\times 1}\sqrt{\dfrac{20}{5\times 10^{-4}}} = 200$ Hz

100. $\lambda = \dfrac{L}{p} = \dfrac{80}{4} = 20$ cm

101. Here, Tp^2 = constant

$\therefore$ $T p_1^2 = (T - 0.011)\ p_2^2$

$\therefore$ $T (25) = (T - 0.011) (36)$

$\therefore$ $11\ T = 0.011 \times 36 \Rightarrow T = 0.036$ kg-wt

102. $v = 4nl$ ....(i)

$v = \sqrt{\dfrac{\gamma P}{\rho}}$ ....(ii)

$\therefore$ $\sqrt{\dfrac{\gamma P}{\rho}} = 4nl$ ....[From equation (i) and (ii)]

$\therefore$ $\gamma = \dfrac{(84 \times 4)^2 \times 1.2}{1.0 \times 10^5} = 1.354 \approx 1.4$

103. $n_a = 250 \pm 4 = 254$ Hz or 246 Hz

$n_b = 513 \pm 5 \to 518$ Hz or 508 Hz

Now, $n_b = 2n_a$

Which is $508 = 2(254)$

$\therefore$ $n = 254$ Hz

104. The frequency of vibration of a string $n = \dfrac{p}{2l}\sqrt{\dfrac{T}{m}}$

Also number of loops = Number of antinodes.

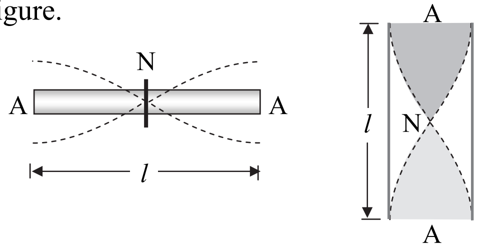
$\therefore$ With 5 antinodes and hanging mass of 9 kg, we have p = 5 and T = 9g $\Rightarrow$ $n_1 = \dfrac{5}{2l}\sqrt{\dfrac{9g}{m}}$

With 3 antinodes and hanging mass M, we have p = 3 and T = Mg $\Rightarrow$ $n_2 = \dfrac{3}{2l}\sqrt{\dfrac{Mg}{m}}$

$\because$ $n_1 = n_2 \Rightarrow \dfrac{5}{2l}\sqrt{\dfrac{9g}{m}} = \dfrac{3}{2l}\sqrt{\dfrac{Mg}{m}}$

$\therefore$ $25 \times 9\ g = 9 \times Mg \Rightarrow M = 25$ kg.

105. If a rod clamped in the middle, then it vibrates similar to an open organ pipe as shown in the figure.

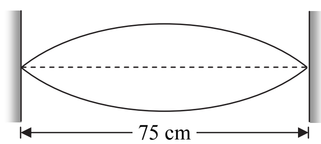


$\therefore$ Fundamental frequency of vibrating rod is, given by $n_1 = \dfrac{v}{2l} \Rightarrow 2.53 = \dfrac{v}{2\times 1}$

$\therefore$ v = 5.06 km/s.

106. In a stretched string, all multiples of fundamental frequencies can be obtained.

i.e., if fundamental frequency is 'n', then higher frequencies will be 2n, 3n, 4n, 5n …



$\therefore$ Any two successive frequencies will differ by 'n'

Given that, n = 420 – 315 = 105 Hz.

$\therefore$ The lowest resonant frequency of the string is 105 Hz.

107. $n_1 - n_2 = 6$

$\therefore$ $\dfrac{1}{2l}\sqrt{\dfrac{T'}{m}} - \dfrac{1}{2l}\sqrt{\dfrac{T}{m}} = 6$

$\therefore$ $\dfrac{1}{2l}\sqrt{\dfrac{T'}{m}} - 600 = 6$

$\therefore$ $\dfrac{1}{2l}\sqrt{\dfrac{T'}{m}} = 606$ ....(i)

also, $\dfrac{1}{2l}\sqrt{\dfrac{T}{m}} = 600$ ....(ii)

Dividing Equation (i) by Equation (ii), we get

$$\left(\dfrac{\dfrac{1}{2l}\sqrt{\dfrac{T'}{m}}}{\dfrac{1}{2l}\sqrt{\dfrac{T}{m}}}\right) = \dfrac{606}{600}$$

$\therefore$ $\sqrt{\frac{T'}{T}} = (1.01) \Rightarrow \frac{T'}{T} = (1.02)$

$\therefore$ $T' = T(1.02)$

Increase in tension,

$\Delta T' = T \times 1.02 - T = (0.02T)$

$\therefore$ Fractional increase in the tension, $\frac{\Delta T'}{T} = 0.02$

108. $y = 0.02 \sin \left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$

Using, $v = \sqrt{\frac{T}{m}} = \frac{\omega}{k} \Rightarrow \sqrt{\frac{T}{0.04}} = \frac{\left(\frac{1}{0.04}\right)}{\left(\frac{1}{0.50}\right)}$

$\therefore$ $T = \left(\frac{0.50}{0.04}\right)^2 \times 0.04 = (12.5)^2 \times 0.04 = 6.25$ N.

109. As string is clamped resulting wave is a standing wave of equation $y = 2A \sin kx \cos\omega t$

Comparing with given equation,

$\omega = 60\pi$ and $k = \frac{2\pi}{3}$

Now velocity $v = \frac{\omega}{k} = \frac{60\pi}{\frac{2\pi}{3}} = 90$ m/s

Also, velocity of transverse wave,

$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{M/L}}$

$\therefore$ $T = v^2 \times \frac{M}{L} = \frac{90^2 \times 3 \times 10^{-2}}{1.5} = 162$ N

110. Velocity of transverse string $v = \sqrt{\frac{T}{m}}$, where, m is linear density.

Tension $T = Mg = mxg$

$\therefore$ $v = \sqrt{\frac{mxg}{m}}$

$\frac{dx}{dt} = \sqrt{xg}$

For string of length L, integrating over,

$\int_0^L \frac{dx}{\sqrt{xg}} = \int_0^t dt$

$\therefore$ $\int_0^t dt = \frac{1}{\sqrt{g}} \int_0^L x^{-1/2}\, dx$

$\therefore$ $t = \frac{1}{\sqrt{g}}\left[\frac{x^{1/2}}{1/2}\right]_0^{20}$ $\quad (\because L = 20 \text{ m})$

$= \frac{2}{\sqrt{10}} \times \sqrt{20} = 2\sqrt{2}$ s

111. Using $\lambda = 2(l_2 - l_1) \Rightarrow v = 2n(l_2 - l_1)$

$\therefore$ $2 \times 512\ (63.2 - 30.7) = 33280$ cm/s

Actual speed of sound,

$v_0 = 332$ m/s $= 33200$ cm/s

$\therefore$ Error $= 33280 - 33200 = 80$ cm/s

112. For a resonance tube experiment, difference between lengths of column for two successive resonances is given by,

$L_{n+1} - L_n = \frac{\lambda}{2} = \frac{v}{2n}$

$\therefore$ $v = 2n(L_{n+1} - L_n) = 2 \times 320 \times (0.73 - 0.20)$
$= 339.2$ m/s

113. For a closed organ pipe, the frequency of fundamental mode is $n_c = \frac{v}{4L_c}$

For an open organ pipe, the frequency of fundamental mode is $n_0 = \frac{v}{2L_o}$

$L_c = L_o$ ….[Given]

$\therefore$ $n_0 = 2n_c$ …(i)

$n_0 - n_c = 2$ [Given] …(ii)

$\therefore$ Solving equations (i) and (ii), we get,

$n_0 = 4$ Hz, $n_c = 2$ Hz

When the length of the open pipe is halved, its frequency of fundamental mode is

$n'_0 = \frac{v}{2\left(\frac{L_0}{2}\right)} = 2n_0 = 2 \times 4 \text{ Hz} = 8 \text{ Hz}$

When the length of the closed pipe is doubled, its frequency of fundamental mode is

$n'_c = \frac{v}{4(2L_c)} = \frac{1}{2}n_c = \frac{1}{2} \times 2\text{Hz} = 1 \text{ Hz}$

Hence, number of beats produced per second $= n'_0 - n'_c = 8 - 1 = 7$.

114. $m = \frac{M}{L} = \frac{AL\rho}{L} = A\rho$

$Y = \frac{T/A}{l/L} \Rightarrow T = \frac{YlA}{L}$.

Hence lowest frequency of vibration,

$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Y\left(\frac{l}{L}\right)A}{A\rho}} = \frac{1}{2l}\sqrt{\frac{Yl}{L\rho}}$

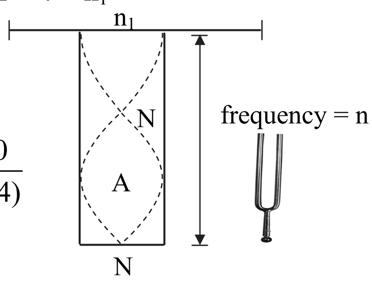
$\therefore$ $n = \frac{1}{2 \times 1}\sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{1 \times 9 \times 10^{3}}} = 35$ Hz.

115. As string and tube are in resonance, $n_1 = n_2$

$|n_1 - n| = 4$ Hz

When T increases, n_1 also increases. It is given that beat frequency decreases to 2 Hz.

$\Rightarrow n - n_1 = 4 \Rightarrow n = 4 + n_1$



Given that,

$n_1 = n_2$

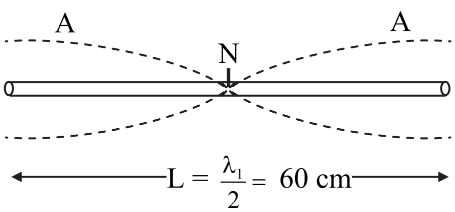
$\therefore \quad n = 4 + n_2$

$\therefore \quad n_2 = \frac{3v}{4l} = \frac{3\times 340}{4\times(3/4)}$

$= 340$ Hz

$\therefore \quad n = 344$ Hz

116.



Fundamental frequency, $\nu_0 = \frac{v}{\lambda_1}$

here, $\lambda_1 = 2L$

Also, $v = \sqrt{\frac{Y}{\rho}}$

$\therefore \quad \nu_0 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$

$\therefore \quad \nu_0 = \frac{1}{2\times 60\times 10^{-2}} \times \sqrt{\frac{9.27\times 10^{10}}{2.7\times 10^{3}}}$

$= 4.88 \times 10^3$ Hz ≈ 5 kHz

117. The waves 1 and 3 reach out of phase. Hence resultant phase difference between them is π.

$\therefore$ Resultant amplitude of 1 and 3 = 10 − 7 = 3 μm

This wave has phase difference of $\frac{\pi}{2}$ with 4 μm

$\therefore$ Resultant amplitude = $\sqrt{3^2 + 4^2} = 5$ μm

118. Because the tuning fork is in resonance with air column in the pipe closed at one end, the frequency is $n = \frac{(2N-1)v}{4l}$ where N = 1, 2, 3 .... corresponds to different modes of vibration

Substituting n = 340 Hz, v = 340 m/s, the length of air column in the pipe can be

$l = \frac{(2N-1)340}{4\times 340} = \frac{(2N-1)}{4}$ m $= \frac{(2N-1)\times 100}{4}$ cm

For N = 1, 2, 3, ... we get l = 25 cm, 75 cm, 125 cm ... etc.

As the tube is only 120 cm long, length of air column after water is poured in it may be 25 cm or 75 cm only. Hence, the corresponding length of water column in the tube will be (120 − 25) cm = 95 cm or (120 − 75) cm = 45 cm.

Thus minimum length of water column is 45 cm.

119. Critical hearing frequency for a person is 20,000 Hz.

For a closed pipe vibrating in N^{th} mode, frequency of vibration

$n_1 = \frac{(2N-1)v}{4l} = (2N-1)n$

$\therefore \quad 20{,}000 = (2N - 1) \times 1500$

$\therefore \quad N = 7.1 \approx 7$

Also, in closed pipe,

Number of overtones = (Number of mode of vibration) − 1

= 7 − 1 = 6.

120. $A^2 = A^2 + A^2 + 2A^2 \cos\theta$

$\therefore \quad \cos\theta = -\frac{1}{2}$

$\therefore \quad \theta = \cos^{-1}\left[-\frac{1}{2}\right] = \frac{2\pi}{3}$

121. For both the positions in Melde's experiment, Tp^2 = constant.

$\therefore \quad T_1p_1^2 = T_2p_2^2$

$\therefore \quad (m_0 + m_1)g\,p_1^2 = (m_0 + m_2)g\,p_2^2$

$\therefore \quad m_0p_1^2 + m_1p_1^2 = m_0p_2^2 + m_2p_2^2$

$\therefore \quad m_0\left(p_1^2 - p_2^2\right) = m_2p_2^2 - m_1p_1^2$

$\therefore \quad m_0 = \frac{m_2p_2^2 - m_1p_1^2}{p_1^2 - p_2^2}$

122. Here $n = \frac{(2n-1)v}{4L} \le 1250$

$\therefore \quad \frac{(2n-1)\times 340}{0.85\times 4} \le 1250$

$\therefore \quad 2n - 1 \le 12.5 \Rightarrow n \le 6.75$

$\therefore$ Number of possible oscillations is 6.

123. For open pipe first overtone, $n_1 = \frac{v}{L}$

For closed pipe first overtone, $n'_1 = \frac{3v}{4L}$

$\therefore$ Number of beats produced are,

$$n_1 - n'_1 = \frac{v}{L} - \frac{3v}{4L} = 3$$

$\therefore$ $\frac{v}{4L} = 3$

$\therefore$ $\frac{v}{L} = 12$ ….(i)

When length of open pipe is made $\frac{L}{3}$, the fundamental frequency becomes,

$$n = \frac{v}{2\left(\frac{L}{3}\right)} = \frac{3v}{2L}$$

When length of closed pipe is made 3 times, the fundamental frequency becomes,

$$n' = \frac{v}{4(3L)} = \frac{v}{12L}$$

$\therefore$ Beats produced = n − n′

$$= \frac{3v}{2L} - \frac{v}{12L}$$

$$= \frac{17}{12} \times \frac{v}{L} = \frac{17}{12} \times 12 \quad \text{….[From (i)]}$$

$= 17$

124. $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$$= \frac{1}{2l}\sqrt{\frac{\text{stress} \times A}{M/L}}$$

$$= \frac{1}{2l}\sqrt{\frac{\text{stress}}{M/V}} = \frac{1}{2l}\sqrt{\frac{\text{stress}}{\text{density}}}$$

$$= \frac{1}{2l}\sqrt{\frac{\gamma \times \text{strain}}{\text{density}}} = \frac{1}{2\,(1.5)}\sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^{3}}}$$

≈ 178.2 Hz

Evaluation Test

1. For the number of beats to increase from 5/s to 6/s, the frequency of the fork with smaller frequency must decrease. This is achieved by putting wax to its prongs. Hence (D) is the correct option.

2. A node will be formed in the middle with two antinodes at the ends of the pipe. Pressure antinodes are displacement nodes.

3. $k = \frac{3\pi}{2}$ and $\omega = 300\pi$

$\therefore$ $\lambda = \frac{4}{3}$ m and f = 150 Hz

…[$\because \lambda = \frac{2\pi}{K}$ and $f = \frac{2\pi}{\omega}$]

x = 0 is pressure maximum, hence a node.

$\therefore$ It is closed at x = 0

For a pipe closed at one end, $L = (2n+1)\frac{\lambda}{4}$

For a pipe closed at both ends, $L = \frac{n\lambda}{2}$

Let us check for x = 2m,

$\frac{n\lambda}{2} = 2$

$\therefore$ n = 3 which is valid.

$\Rightarrow$ The pipe is closed at x = 2m

4. $v = \sqrt{\frac{\gamma RT}{M}}$

$$\lambda F = \sqrt{\frac{\gamma RT}{M}} \Rightarrow F = \frac{1}{\lambda}\sqrt{\frac{\gamma RT}{M}} = \frac{1}{2l}\sqrt{\frac{\gamma RT}{M}}$$

($\because \lambda = 2l$ for fundamental frequency)

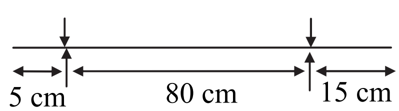
$\therefore$ $$\frac{F_C}{F_D} = \frac{1}{2l_c}\sqrt{\frac{\gamma RT}{M_C}} \times 2l_D\sqrt{\frac{M_D}{rRT}} = \left(\frac{l_D}{l_C}\right) \times \sqrt{\frac{M_D}{M_C}}$$

Now, $l_C = \frac{2l}{3}$ $l_D = \frac{l}{3}$

$M_C = 14$ $M_D = 44$

Thus, $\frac{F_C}{F_D} = \left(\frac{l_D}{l_C}\right) \times \sqrt{\frac{M_D}{M_C}} = \sqrt{\frac{11}{14}}$

5. For minimum natural frequency,
5 cm part should have antinode at end.



Hence, 5 cm = $\frac{\lambda_1}{4}$

(for minimum natural frequency)

$\therefore$ $\lambda_1 = 20$ cm $= \frac{1}{5}$ m

5 cm $= \frac{\lambda_2}{4} + \frac{\lambda_2}{2}$ (for next natural frequency)

$\therefore \quad \lambda_2 = \frac{20}{3}\text{ cm} = \frac{1}{15}\text{ m}$

Also, $v = \sqrt{\frac{Y}{\rho}}$, $Y = 1.6 \times 10^{11}$ N/m,

$\rho = 2500$ kg/m^3

$\therefore \quad v = 8000$ m/s

$f_1 = \frac{v}{\lambda_1}$ and $f_2 = \frac{v}{\lambda_2}$

$\therefore \quad f_1 = 40$ kHz, $\quad f_2 = 120$ kHz

6. The total mechanical energy between adjacent antinodes,

$$E = \frac{1}{2}\left(\rho\omega^2 A^2 \frac{\lambda s}{2}\right) \text{ of the two waves}$$

$$= \frac{1}{2}\left[\frac{1}{2}(\rho s)\omega^2 a^2\left[\frac{2\pi}{k}\right] + \frac{1}{2}(\rho s)\omega^2 (2a)^2\left[\frac{2\pi}{k}\right]\right]$$

$$= \frac{5}{2}\frac{\pi\rho\omega^2 a^2 s}{k}$$

7. $\frac{A_{max}}{A_{min}} = \frac{A_1 + A_2}{A_2 - A_2} = X, \ \frac{A_2}{A_1} = \frac{X-1}{X+1}$

As Energy $\propto A^2 \propto \left(\frac{X-1}{X+1}\right)^2$

8. $n_0 = \frac{v}{2l}$

$n_1 = \frac{v}{2(l/2 - \Delta l)}$ $\quad n_2 = \frac{v}{2(l/2 + \Delta l)}$

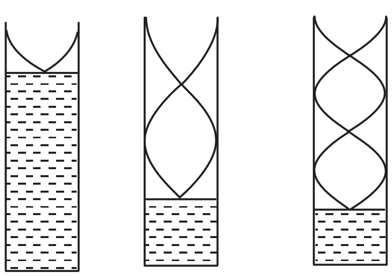
Beat frequency $= n_1 - n_2$

$$= v\left[\frac{1}{l - 2\Delta l} - \frac{1}{l + 2\Delta l}\right]$$

$$= v\left[\frac{(l + 2\Delta l) - (l - 2\Delta l)}{l^2 - 4\Delta l^2}\right]$$

$$= v\frac{4\Delta l}{l^2 - \Delta l^2} \approx \frac{4\Delta l v}{l^2}$$

$$\approx \frac{8\Delta l v}{l(2l)} \approx \frac{8\Delta l\, n_0}{l}$$

9. 

Fundamental frequency $(2n + 1)\frac{\lambda}{4} = L$

$\therefore \quad f = \frac{(2n+1)}{4L} \times v$

For 1st case, $l = \frac{3}{8}$ m

$\therefore \quad (2n + 1) = \frac{f}{v} \times 4l = \frac{680}{340} \times 4 \times \frac{3}{8} = 3$

$\therefore \quad n = 1$

$\Rightarrow$ Next overtone is for n = 2

Thus,

$L = \frac{5\lambda}{4} = \frac{5}{4} \times \frac{1}{2} = \frac{5}{8}$ m

$\therefore \quad X = \frac{5}{8} - \frac{3}{8} = \frac{1}{4}$ m = 25 cm

10. For minima,

$\Delta X = (2n + 1)\frac{\lambda}{2}$ and $\lambda = \frac{v}{f}$

$\therefore \quad \Delta X = \frac{(2n+1)}{2}\frac{v}{f}$

$\therefore \quad 0.5 = \frac{(2n+1)}{2}\frac{300}{f}$

$\therefore \quad f = (2n + 1)300$

All odd multiples of 300 are silenced.

Hence correct option is (A).

11. $\lambda = \frac{v}{n} = \frac{330}{482} = 0.685$

Here, second resonance occurs at $l_2 = \frac{3\lambda}{4}$

$\therefore \quad \frac{3\lambda}{4} < 0.75$ m

Hence it is possible to perform experiment.

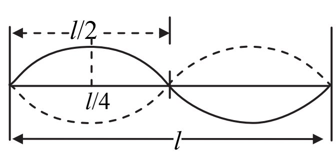
12. Options (C) and (D) will not form a standing wave.

(A) At x = 0, it has amplitude = 0

$\therefore$ Sum of the two amplitudes will be 'a' which is not the condition of the problem.

(B) At x = 0, it has amplitude = – a which will cancel out to give zero.

Hence, option (B) is correct.

13. 

String vibrates with two loops. (Second Harmonic)

The point where we touch the string becomes a node and where we pluck it becomes an antinode.

14. $v = f\lambda,\ l = \dfrac{5\lambda}{2}$

$\therefore\quad v = \left(\dfrac{2l}{5}\right)f = \dfrac{2}{5}\times\left(\dfrac{82.5}{100}\right)\times 1000$

≈ 330 m/s

15. By comparing the given equation with standard form, we get

$A = 0.05$ m, $\omega = 40\ \pi$ rad/s

$\left(v_{max}\right)_{x=0.375} = A\omega = 0.05 \times 40\pi$

$= 2\pi$ m/s

16. In this case, $n(2) = (n + 1)(1.6)$

$\therefore\quad \dfrac{n+1}{n} = \dfrac{2}{1.6} = \dfrac{5}{4}$

$\therefore\quad 5n = 4n + 4$

$\therefore\quad n = 4$

$\therefore\quad L = 8.0$ cm

17. If x is at an angle θ.

The $\Delta\phi$ between x and 1 = 2θ,

the $\Delta\phi$ between x and 2 = 2θ and

the $\Delta\phi$ between x and 3 = 2π

$\Rightarrow$ points x and 3 are in phase.

18. $L = (n + 1)\dfrac{\lambda}{2}$ and $\dfrac{\lambda}{4} = d$

$\therefore\quad L = 2(n + 1)d$

19. The frequency of the wire remains the same.

$n = \dfrac{p}{2L}\sqrt{\dfrac{T}{\mu}}$

$\therefore\quad \dfrac{p_1}{l\sqrt{\mu}} = \dfrac{p_2}{4l\sqrt{4\mu}}$

$\therefore\quad \dfrac{p_1}{p_2} = \dfrac{1}{8}$

$\therefore\quad \lambda = \dfrac{2l}{p} = \dfrac{2(4l)}{8} = l$

20. String crosses mean position simultaneously.

21. $\nu = \dfrac{p}{2L}\sqrt{\dfrac{T}{\mu}}$, p = mode of vibration

$T = \dfrac{YA\Delta L}{L}$

$\therefore\quad \nu \propto p\sqrt{\dfrac{Y}{\mu}}$

$\Rightarrow$ Frequency of second mode is 2ν.

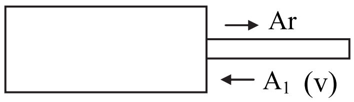
22. $I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

where, δ = phase difference

$\therefore\quad I \propto \dfrac{I}{d^2} \Rightarrow I_0$ for d then $\dfrac{I_0}{4}$ for 2d

$\therefore\quad I_{net} = I_0 + \dfrac{I_0}{4} + 2\sqrt{I_0 . \dfrac{I_0}{4}}\cos(2\pi)$

$= \dfrac{9I_0}{4}$

23. 

where, n = number of strands

A_1 and A_r are amplitudes of incident and reflected waves respectively.

$A_r = \left[\dfrac{v_2 - v_1}{v_1 + v_2}\right]A_1$

$\therefore\quad 0.45 = \left[\dfrac{v - \dfrac{v}{\sqrt{n}}}{v + \dfrac{v}{\sqrt{n}}}\right] \times 1$

$\therefore\quad \dfrac{0.45}{1} = \dfrac{\sqrt{n} - 1}{\sqrt{n} + 1}$

$\therefore$ On solving the above equation, we get n = 7

24. Wave frequency is given as average of frequencies of interfereing waves.

The waveform on the left has low average than right one

But looking at beats (i.e. difference in frequencies), graph on the left has the higher difference than the right one.

Textbook Chapter No.

09 Kinetic Theory of Gases and Radiation

Hints

Classical Thinking

6. $R = \frac{PV}{nT} = \frac{W}{nT} = \frac{[M^1L^2T^{-2}]}{[mol][K]}$
$= [M^1L^2T^{-2}K^{-1}mol^{-1}]$

32. $c_{rms} = \sqrt{\frac{3P}{\rho}}$, $v = \sqrt{\frac{\gamma P}{\rho}}$

$\therefore \frac{c_{rms}}{v} = \sqrt{\frac{3}{\gamma}} = \sqrt{\frac{3}{1.41}} \approx 1.46$

36. $c_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\times1.013\times10^5}{0.09}} \approx 1838$ m/s

37. $\frac{c_{O_2}}{c_{H_2}} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$

38. $P = \frac{c_{rms}^2\rho}{3} = \frac{(500)^2\times6\times10^{-2}}{3}$
$= 25\times10^4\times2\times10^{-2} = 50\times10^2$
$= 5\times10^3$ N/m^2

39. $P = \frac{1}{3}\frac{mn}{V}c_{rms}^2$

$\therefore n = \frac{3PV}{mc_{rms}^2} = \frac{3\times10^5\times100\times10^{-6}}{4.556\times10^{-25}\times350^2} \approx 5.4\times10^{20}$

47. $\frac{K.E.}{vol} = \frac{3}{2}P$. Here P is constant.

48. $c_{rms} = \sqrt{\frac{3RT}{M_0}}$

Now, $K.E._{(\text{gram molecule})} = \frac{1}{2}\times M_0 c_{rms}^2$
$= \frac{1}{2}\times M_0\times\frac{3RT}{M_0}$
$= \frac{3}{2}RT$

51. $K.E. = \frac{3}{2}k_BT = \frac{3}{2}\frac{R}{N}T = \frac{3}{2}\times\frac{2}{1}\times300$

$\therefore$ K.E. = 900 cal

52. Energy = 300 J/litre = 300×10^3 J/m^3

Using, $P = \frac{2}{3}E = \frac{2\times300\times10^3}{3} = 2\times10^5$ N/m^2

53. $\frac{3}{2}k_BT = 1$ eV

$\therefore T = \frac{1.6\times10^{-19}\times2}{3\times1.38\times10^{-23}} \approx 7730$ K

54. $c_{rms} \propto \sqrt{T}$

$\therefore \frac{300}{c_{rms}} = \sqrt{\frac{27+273}{927+273}} = \sqrt{\frac{300}{1200}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore c_{rms} = 2\times300 \Rightarrow c_{rms} = 600$ m/s

55. Using Boyle's law,
$P_1V_1 = P_2V_2$
$\therefore 5\times(0.2) = 1\times V_2 \Rightarrow V_2 = 1$ m^3

56. Applying the method of partial pressures (Dalton's Law),
$P' = P_1 + P_2 \Rightarrow P' = 2P$

68. $\frac{C_p}{C_v} = \frac{7}{2}R\times\frac{2}{5R} = \frac{7}{5} = 1.4$

81. For ideal monatomic gas, $C_p = \frac{5}{2}R$

$\therefore R = \frac{2}{5}C_p = 0.4C_p \Rightarrow n = 0.4$

87. $dQ = dU + dW$ where $dW = PdV$

88. $\Delta W = P\Delta V$; here ΔV is negative. Hence ΔW will be negative

91. The process is very fast; so the gas fails to gain or lose heat. Hence, this process is adiabatic.

92. In adiabatic process, no transfer of heat takes place between system and surrounding.

93. In isothermal process, temperature remains constant.

94. In isothermal expansion, temperature remains constant; hence no change in internal energy.

95. In isothermal process, heat is released by the gas to maintain the constant temperature.

102. A refrigerator acts as a heat pump as it sends heat from sink at lower temperature to source at higher temperature.

105. As the change is sudden, the process is adiabatic

$\therefore$ PV^{γ} = constant $\Rightarrow P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\therefore$ $P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 10^6 \times \left(\frac{300}{150}\right)^{1.4} = 10^6\ (2)^{1.4}$

$= 2.6 \times 10^6$ dyne / cm^2

106. As the change is sudden, the process is adiabatic

$\therefore$ $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\therefore$ $\frac{P_2}{P_1} = \left[\frac{V_1}{V_2}\right]^{\gamma} = \left[\frac{4}{1}\right]^{3/2} = \frac{8}{1}$

107. $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$\therefore$ $Q_2 = 500 \times \frac{300}{260} \approx 577$ calorie

109. By the first law of thermodynamics,
$\Delta Q = \Delta U + \Delta W$
In adiabatic process, $\Delta Q = 0 \Rightarrow \Delta U = -\Delta W$

110. In isochoric process, volume remains constant.

115. Highly polished mirror-like surfaces are good reflectors but not good radiators.

119. Perfectly black body is black in colour because it does not reflect or transmit the radiation.

122. When light incident on pin hole enters into the box and suffers successive reflections at the inner wall, at each reflection some energy is absorbed. Hence the ray once enters the box can never come out and pin hole acts like a perfect black body.

127. A black body has a continuous emission spectrum

133. $\lambda_m = \frac{b}{T}$

$\therefore$ $T = \frac{b}{\lambda_m} = \frac{2.93\times10^{-3}}{4000\times10^{-10}} = 7325$ K

$= 7.325 \times 10^3$ K

134. $T = \frac{b}{\lambda_m} = \frac{2.93\times10^{-3}}{2.93\times10^{-10}} = 10^7$ K

135. According to Wien's law, $\lambda_{m_1}T_1 = \lambda_{m_2}T_2$

$\therefore$ $\lambda_{m_2} = \frac{\lambda_{m_1}T_1}{T_2} = 4.08 \times \frac{700}{1400} = 2.04$ µm

136. According to Wien's law,

$\frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} = \frac{4800}{3600} = \frac{48}{36} = \frac{4}{3}$

137. As $\lambda_m \propto \frac{1}{T}$

$\therefore$ Temperature of other star must be $\frac{T}{2}$

138. According to Wien's law,
$\lambda_m T = \lambda'_m T'$

$\therefore$ $\lambda_0 T = \lambda' \times 2T \Rightarrow \lambda' = \frac{\lambda_0}{2}$

139. According to Wien's law, $\lambda_m T$ = constant
$\lambda_r > \lambda_y > \lambda_b$

$\therefore$ $T_r < T_y < T_b$ or $T_A < T_C < T_B$

146. $E = \frac{Q}{At}$

$\therefore$ $[E] = \frac{[Q]}{[A].[t]} = \frac{[M^1L^2T^{-2}]}{[L^2][T^1]} = [M^1L^0T^{-3}]$

152. Kirchhoff's law of radiation

159. In M.K.S. system, unit of σ is $\frac{J}{m^2 \times s \times K^4}$

$\therefore$ $1\frac{J}{m^2 \times s \times K^4} = \frac{10^7\,erg}{10^4\,cm^2 \times s \times K^4}$

$= 10^3 \frac{erg}{cm^2 \times s \times K^4}$

160. Rate of loss of heat $\propto$ Area.

161. Rate of cooling $= \frac{80-60}{5} = \frac{20}{5} = 4$ °C/min

164. Temperature of the body decreases exponentially with time.

$\therefore$ $\frac{d\theta}{dt}$ also decreases exponentially.

170. Average K.E. of molecules per mole of ideal gas $= \frac{3}{2}RT$

where R = universal gas constant
T = same for all gases
Average K.E. of molecules for one mole of all ideal gases at same temperature is same.

172. Black cloth is a good absorber of heat. Therefore, ice covered by black cloth melts more as compared to that covered by white cloth.

175. For an isothermal change, PV = constant.

∴ On differentiating, PdV + VdP = 0

$\Rightarrow \frac{dP}{P} = -\frac{dV}{V}$

Critical Thinking

1. Gas constant = $\frac{8.3\times10^3}{28}$ = 2.96×10^2 J/kg K

3. $P_1V_1 = \mu_1RT_1$

∴ $V_1 = \frac{\mu_1RT_1}{P_1} = \frac{1}{2}\frac{R(300)}{2}$ = 75 R

$P_2V_2 = \mu_2RT_2$

∴ $V_2 = \mu_2\frac{RT_2}{P_2} = 1.5\frac{R(350)}{5}$ = 105 R

∴ $P(V_1 + V_2) = (\mu_1 + \mu_2)RT$

∴ P (75 R + 105 R) = (0.5 + 1.5) R (273 + 69)

∴ P × 180 R = 2 × R × 342

∴ $P = \frac{342}{90}$ = 3.8 atm

4. The equation of state is, PV = nRT

$\Rightarrow P = \frac{nRT}{V}$ (ideal gas condition)

Let for M mass there is μ moles, then for mass 3M, there are $\frac{3Mn}{M}$ = 3μ moles

Let n′ = 3n, T′ = T/3 and V′ = $\frac{V}{3}$

Then $P' = \frac{n'RT'}{V'} = \frac{3nR\frac{T}{3}}{V/3} = \frac{3nRT}{V}$ = 3P

7. According to the gas equation, $PV = Nk_BT$

For the gas A, we have,

$PV = N_1k_BT$ ....(i)

For the gas B, we have, $(2P)\left(\frac{V}{8}\right) = N_2k_B(2T)$

$\Rightarrow PV = 8N_2k_BT$ ....(ii)

∴ From equations (i) and (ii),

$N_1 = 8N_2 \Rightarrow \frac{N_1}{N_2} = 8$

9. Using, $c_{rms} \propto \sqrt{T}$,

$\frac{(c_{rms})_1}{(c_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$

Given that,

$T_2 = 273$ k, $(c_{rms})_1 = \frac{(c_{rms})_2}{2}$ or $\frac{(c_{rms})_1}{(c_{rms})_2} = \frac{1}{2}$

∴ $\frac{1}{2} = \sqrt{\frac{T_1}{273}} \Rightarrow T_1 = \frac{273}{4}$ = 68.25 K

11. Mean free path = $\frac{3+7+1+2+4+3}{6} = \frac{20}{6}$

12. The R.M.S. velocity of the molecule of a gas is given by, $c_{rms} = \sqrt{\frac{3kT}{m}}$, where k is the Boltzmann's constant and m is the mass of a molecule.

∴ $c_{rms} \propto \frac{1}{\sqrt{m_{rms}}} \Rightarrow c \propto m^{-1/2}$

13. Mean square velocity = $\frac{3^2+4^2+5^2}{3}$

$= \frac{50}{3} \approx 16.7$ m/s

14. Average velocity = $\frac{3+4+5}{3}$ = 4 m/s

15. Mean square velocity = $\frac{(5)^2+(6)^2+(7)^2}{3}$

$= \frac{25+36+49}{3} = \frac{110}{3}$

$= 36.7\ m^2/s^2$

16. $c_{avg} = \frac{5+(-5)}{2}$ = 0 cm/s

$c_{r.m.s.} = \sqrt{\frac{5^2+(-5)^2}{2}}$ = 5 cm/s

17. $c_{mean} = \frac{2+3+4}{3}$ = 3 m/s

$c_{r.m.s.} = \sqrt{\frac{2^2+3^2+4^2}{3}} = \sqrt{\frac{4+9+16}{3}} = \sqrt{\frac{29}{3}}$

= 3.109 m/s

∴ $\frac{c_{mean}}{c_{r.m.s.}} = \frac{3}{3.109} < 1$

18. $\bar{c} = \frac{c_1 + c_2 + c_3 + c_4 + c_5}{5}$

$= \frac{10+20+30+40+50}{5} = \frac{150}{5} = 30$ m/s

$c_{r.m.s.} = \sqrt{\frac{10^2 + 20^2 + 30^2 + 40^2 + 50^2}{5}}$

$= \sqrt{\frac{100+400+900+1600+2500}{5}}$

$= \sqrt{\frac{5500}{5}} = \sqrt{1100} = 33.16$ m/s

$\therefore \frac{c_{r.m.s.}}{\bar{c}} = \frac{33.16}{30} = 1.105$

$\therefore c_{r.m.s.} : \bar{c} = 1.105 : 1$

19. $\frac{T_A}{M_A} = 4\frac{T_B}{M_B}$

$\Rightarrow \sqrt{\frac{T_A}{M_A}} = 2\sqrt{\frac{T_B}{M_B}}$

$\Rightarrow \sqrt{\frac{3RT_A}{M_A}} = 2\sqrt{\frac{3RT}{M_B}} \Rightarrow (c_{rms})_A = 2(c_{rms})_B$

$\Rightarrow \frac{(c_{rms})_A}{(c_{rms})_B} = 2$

20. $\frac{(c_{rms})_1}{(c_{rms})_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{9}{8}}$

$\therefore (c_{rms})_1 : (c_{rms})_2 : \sqrt{9} : \sqrt{8}$

21. $P = \frac{1}{3}\rho c_{rms}^2$

Since mass and volume is same, the density is constant.

$\therefore P \propto c_{rms}^2$ But $c_{rms}^2 \propto \frac{1}{M} \Rightarrow P \propto \frac{1}{M}$

$\therefore \frac{P_O}{P_H} = \frac{M_H}{M_O} = \frac{2}{32} = \frac{1}{16}$

$\therefore P_O = \frac{1}{16} \times 4 = 0.25$ atm

22. $P = \frac{1}{3}\rho c_{rms}^2 = \frac{1}{3}\frac{M}{V} c_{rms}^2$

$\therefore P' = \frac{1}{3}\frac{(M/2)}{V}(2c_{rms})^2 = \frac{1}{3}\frac{M}{V}\frac{1}{2}(4c_{rms}^2)$

$= 2\left(\frac{1}{3}\frac{M}{V}c_{rms}^2\right) = 2P$

$\therefore \frac{P}{P'} = \frac{1}{2}$

23. $P = \frac{1}{3}\rho c_{rms}^2$ ....(i)

Let $c_{rms}^2 = \frac{3RT}{M}$

From equation (i) we get,

$P = \frac{1}{3}\rho\left(\frac{3RT}{M}\right) = \frac{\rho RT}{M}$

$\therefore \rho = \frac{PM}{RT}$

$\therefore \rho \propto \frac{P}{T}$ and $\rho' \propto \frac{P'}{T'}$

....[$\because$ M and R are constant]

Given that, P′ = 2P

$\therefore \frac{P'}{P} = 2$ and $\frac{T'}{T} = \frac{1}{3}$

Then, $\frac{\rho'}{\rho} = \frac{P'}{P} \times \frac{T}{T'} = 2 \times 3 = 6$

$\therefore \rho' = 6\rho$

24. Using, $P_1V_1 = P_2V_2$

$\frac{P_1}{P_2} = \frac{V_2}{V_1} = \frac{V_1 - \frac{10}{100}V_1}{V_1} = \frac{90}{100}$

$\therefore \frac{P_2}{P_1} = \frac{100}{90}$

$\therefore \frac{P_2 - P_1}{P_1} \times 100 = \frac{10}{90} \times 100 = 11.11\%$

25. Mean kinetic energy of molecule depends upon temperature only. For O_2, it is same as that of H_2 at the same temperature of –73 °C.

26. In the mixture, gases will acquire thermal equilibrium (same temperature). Hence, their kinetic energies will also be same.

27. K.E. $= \frac{1}{2}MN(c_{rms}^2)_1 = \frac{1}{2}M(2N)(c_{rms}^2)_2$

$\therefore \frac{(c_{rms}^2)_1}{(c_{rms}^2)_2} = \frac{2}{1}$

$\therefore \frac{(c_{rms}^2)_1}{(c_{rms}^2)_2} = \frac{T_1}{T_2}$ ....($\because c \propto \sqrt{T}$)

$\therefore T_1 = \frac{T}{2}$

28. $K.E._{av} = \frac{3}{2} k_B T$

$\therefore$ $K.E._{av} \propto T$

$\therefore$ $\frac{K.E._2}{K.E._1} = \frac{T_2}{T_1} = \frac{600}{300} = 2$

$\therefore$ $K.E._2 = 2K.E._1 = 2K.E.$

29. Kinetic energy $= \frac{3}{2} k_B T = \frac{3RT}{2N_A} = \frac{3}{2} \frac{PV}{N_A \mu}$

$= \frac{3}{2} \frac{PV}{N}$

$\therefore$ $\frac{K.E.}{V} = \frac{3}{2} \frac{P}{N}$

As N = 1,

$\frac{K.E.}{V} = \frac{3}{2} \times P = \frac{3}{2} \times 10^5 = 1.5 \times 10^5$ J

30. $K.E. \propto T$

$\therefore$ $\frac{K.E._1}{K.E._2} = \frac{T_1}{T_2} = \frac{27+273}{T+273}$

But $K.E._2 = 2K.E._1$

$\therefore$ $\frac{K.E._1}{2K.E._1} = \frac{300}{T+273}$

$\therefore$ $T + 273 = 600 \Rightarrow T = 327$ °C

31. $c_{r.m.s.} \propto \sqrt{\frac{T}{M}}$

$\frac{(c_{rms})_{He}}{(c_{rms})_H} = \sqrt{\frac{T_{He}}{T_H} \times \frac{M_H}{M_{He}}}$

$\therefore$ $\frac{1}{2} = \sqrt{\frac{T_{He}}{273} \times \frac{2}{4}}$ $\therefore$ $\frac{1}{4} = \frac{T_{He}}{273} \times \frac{1}{2}$

$\therefore$ $T_{He} = \frac{273}{2}$ K $= 136.5$ K

32. $c_{r.m.s.} = \sqrt{\frac{3RT}{M}}$

$\therefore$ $\frac{(c_{rms})_2}{(c_{rms})_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{(273+90)}{(273+30)}} \approx 1.1$

$\therefore$ % increase $= \left[\frac{(c_{rms})_2}{(c_{rms})_1} - 1\right] \times 100$

$= 0.1 \times 100 = 10\%$

33. $c_H = \sqrt{\frac{3RT}{M_H}}$, $c_O = \sqrt{\frac{3RT}{M_O}}$

As T = constant, $c \propto \frac{1}{\sqrt{M}}$

$M_H < M_O \Rightarrow c_H > c_O$

34. $(c_{r.m.s.})_N = \sqrt{\frac{3RT_N}{M_N}}$ and $(c_{r.m.s.})_O = \sqrt{\frac{3RT_O}{M_O}}$

Given that, $(c_{r.m.s.})_H = (c_{r.m.s.})_O$

$\therefore$ $\frac{3RT_N}{M_N} = \frac{3RT_O}{M_O}$

$\therefore$ $\frac{T+273}{28} = \frac{127+273}{32}$

$\therefore$ $T + 273 = \frac{400}{32} \times 28 = 350$ K

$\therefore$ $T = 350 - 273 = 77$ °C

35. $s = \frac{Q}{m\theta}$

Since there is no change of temperature,

$\theta = 0 \Rightarrow s = \infty$

36. $C_p = \frac{Q}{n\Delta T} = \frac{294}{2 \times 5} = 29.4$

37. $\Delta Q = nC_p\Delta T$ (at constant pressure)

$\Delta U = nC_v\Delta T$

$\therefore$ $\frac{\Delta U}{\Delta Q} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{3}{5}$

....($\because$ γ for monatomic gas $= \frac{5}{3}$)

38. $C_p - C_v = 300$

$\frac{C_p}{C_v} = 1.4 \Rightarrow C_v = \frac{C_P}{1.4}$

$\therefore$ $C_p - \frac{C_p}{1.4} = 300$

$\therefore$ $C_p\left(1 - \frac{1}{1.4}\right) = 300$

$\therefore$ $0.4\ C_p = 300 \times 1.4$

$\therefore$ $C_p = \frac{300 \times 1.4}{0.4} = 1050$ J/kg K

39. $dU = C_v dT = \left(\frac{5}{2}R\right) dT$

$\therefore$ $dT = \frac{2(dU)}{5R}$

From first law of thermodynamics,

$dU = dQ - dW = Q - \frac{Q}{4} = \frac{3Q}{4}$

$\therefore$ Molar heat capacity, $c = \frac{dQ}{dT} = \frac{Q}{\left(\frac{2(dU)}{5R}\right)}$

$= \frac{5RQ}{2\left(\frac{3Q}{4}\right)} = \frac{10}{3}R$

40. $dQ = dE + dW$ But $dW = 0$

∴ $dQ = dE = C_v dT$

For monatomic gas, $C_v = \frac{3}{2}R$

∴ $dQ = nC_v dT = 3 \times \frac{3}{2} R \times 100 = 450\ R$

41. From first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

Work done at constant pressure,

$(\Delta W)_p = (\Delta Q)_p - \Delta U$

∴ $(\Delta W)_p = (\Delta Q)_p - (\Delta Q)_v$

$(\Delta Q)_v = \Delta U$

Also, $(\Delta Q)_p = mc_p\Delta T$ and $(\Delta Q)_v = mc_v\Delta T$

∴ $(\Delta W)_p = m(c_p - c_v)\Delta T$

∴ $(\Delta W)_p = 1 \times (3.4 \times 10^3 - 2.4 \times 10^3) \times 10$

$= 10^4$ cal

43. Differentiating the equation,

PV = constant w.r.t. V,

$P\Delta V + V\Delta P = 0 \Rightarrow \frac{\Delta P}{P} = -\frac{\Delta V}{V}$

44. For isothermal process,

$PV = RT \Rightarrow P = \frac{RT}{V}$

∴ $W = PdV = \int_{V_1}^{V_2} \frac{RT}{V} dV = RT \log_e \frac{V_2}{V_1}$

45. It is an isothermal process. Hence, work done

$= P(V_2 - V_1)$

$= 1 \times 10^5 \times (1.091 - 1) \times 10^{-6} = 0.0091$ J

46. In case of adiabatic expansion, ΔW = positive and $\Delta Q = 0$

∴ Using 1st law of thermodynamics,

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W$

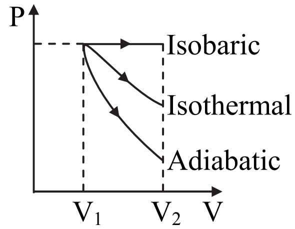
$\Rightarrow \Delta U$ will be negative.

47. Work done $= P\Delta V = P(V_2 - V_1)$

48. In thermodynamic process, work done is equal to the area bound by the PV curve with volume axis.

∴ According to graph shown, we have

$W_{adiabatic} < W_{isothermal} < W_{isobaric}$



49. A quasi-static process like a slow isothermal expansion or compression of an ideal gas is reversible process while the other given processes are irreversible in nature.

50. $\Delta Q = \Delta U + \Delta W$

$\Rightarrow \frac{\Delta W}{\Delta Q} = 1 - \frac{\Delta U}{\Delta Q} = 1 - \frac{\mu C_V dT}{\mu C_P dT}$

$\Rightarrow \frac{\Delta W}{\Delta Q} = 1 - \frac{C_v}{C_p} = 1 - \frac{3}{5} = \frac{2}{5} = 0.4$

∴ Percentage of heat utilised $= 0.4 \times 100 = 40\%$

51. As internal energy is a state function, the change in internal energy does not depend upon the path followed i.e. $\Delta U_I = \Delta U_{II}$

52. We know that, slopes of isothermal and adiabatic curves are always negative and slope of adiabatic curve is always greater than that of isothermal curve.

Hence, in the given graph, curve A and B represent adiabatic and isothermal changes respectively.

53. Process CD is isochoric as volume is constant; process DA is isothermal as temperature constant and process AB is isobaric as pressure is constant.

54. In first case, $\eta_1 = \frac{T_1 - T_2}{T_1}$

In second case, $\eta_2 = \frac{2T_1 - 2T_2}{2T_1} = \frac{T_1 - T_2}{T_1} = \eta$

55. $\eta = 1 - \frac{T_2}{T_1}$

∴ $\frac{30}{100} = 1 - \frac{350}{T_1}$

∴ $\frac{350}{T_1} = 1 - \frac{30}{100} = \frac{7}{10}$

∴ $T_1 = 500\ K = 227\ °C$

57. $a = \frac{Q_a}{Q} \Rightarrow 0.75 = \frac{Q_a}{200}$

∴ $Q_a = 0.75 \times 200 = 150$ cal

58. $Q = Q_a + Q_r + Q_t$

∴ $10 = 2 + 7 + Q_t \Rightarrow Q_t = 1$ J

∴ Coefficient of transmission, $t = \frac{Q_t}{Q} = \frac{1}{10} = 0.1$

59. For athermanous body, $Q_t = 0$

$\therefore$ If $\frac{Q_a}{Q} = 20\ \%$ then $\frac{Q_r}{Q} = 80\ \%$

$\therefore$ Coefficient of reflection,

$r = \frac{Q_r}{Q} = 80\ \% = \frac{80}{100} = 0.8$

60. $Q = p$, $Q_r + Q_t = q$

Let, $Q = Q_a + Q_r + Q_t$

$\therefore$ $p = Q_a + q \Rightarrow Q_a = p - q$

$\therefore$ Coefficient of absorption, $a = \frac{Q_a}{Q} = \frac{p-q}{p}$

61. Initially, the black body at room temperature is darkest and when placed in furnace, it absorbs heat till its temperature becomes that of furnace. After this, it emits the radiation of all wavelengths and appears bright.

64. $E \propto A \propto r^2$

$\therefore$ $\frac{E_2}{E_1} = \left(\frac{r_2}{r_1}\right)^2$

$\therefore$ $E_2 = \left(\frac{5}{10}\right)^2 \times (10) = \frac{1}{4} \times 10 = 2.5\ J/m^2\,s$

65. $E = \frac{Q}{At} = \frac{(Q/t)}{A}$

$\therefore$ $A = \frac{(Q/t)}{E} = \frac{60}{1000} = 6 \times 10^{-2}$

But area of cube, $A = 6l^2$

$\therefore$ $6l^2 = 6 \times 10^{-2} \Rightarrow l^2 = 10^{-2}$

$\therefore$ $l = 10^{-1} = 0.1\ m = 10\ cm$

66. $E = \frac{Q}{At} = \frac{0.3}{15 \times 10^{-3} \times 40} = 0.50\ kcal/m^2\,s$

67. In vacuum, heat flows by the radiation mode only.

68. By Stefan's law, $\frac{dQ}{dt} = A\,\sigma\,T^4$

$\therefore$ $\sigma = \frac{dQ}{dt} \times \frac{1}{AT^4} = \left(\frac{J}{S}\right) \times \frac{1}{m^2 \times K^4} = W/m^2K^4$

69. $E_1 \propto T_1^4$ and $E_2 \propto T_2^4$

$\therefore$ $\frac{E_2}{E_1} = \frac{T_2^4}{T_1^4}$ But $T_2 = \frac{T_1}{2}$

$\therefore$ $\frac{E_2}{E_1} = \frac{\left(\frac{T_1}{2}\right)^4}{T_1^4} = \frac{1}{16} \Rightarrow E_2 = \frac{E_1}{16}$

70. $Q \propto T^4 \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{T_1}{T_2}\right)^4$

$\therefore$ $\frac{Q_1}{Q_2} = \left(\frac{T}{T + \frac{T}{2}}\right)^4 = \frac{16}{81}$

$\therefore$ $Q_2 = \frac{81}{16} Q_1$

$\therefore$ % increase in energy $= \frac{Q_2 - Q_1}{Q_1} \times 100 \approx 400\%$

71. $\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{727+273}{127+273}\right)^4$

$= \frac{(1000)^4}{(400)^4} = \frac{10^4}{4^4} = \frac{625}{16}$

72. Radiated power by black body,

$P = \frac{Q}{t} = A\sigma T^4 \qquad \Rightarrow P \propto AT^4 \propto r^2T^4$

$\therefore$ $\frac{P_1}{P_2} = \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{T_1}{T_2}\right)^4$

$\therefore$ $\frac{440}{P_2} = \left(\frac{20}{10}\right)^2 \times \left(\frac{500}{1000}\right)^4$

$\therefore$ $P_2 = 440 \times \frac{1}{4} \times 16$

$\therefore$ $P_2 = 1760\ W$

73. $\frac{dQ}{dt} = \sigma\,T^4\,Ae$

$\therefore$ $\frac{300}{60} = 5.67 \times 10^{-8} \times (727 + 273)^4 \times 50 \times 10^{-4} \times e$

$\therefore$ $\frac{300}{60} = 5.67 \times 10^{-8} \times 10^{12} \times 50 \times 10^{-4} \times e$

$\therefore$ $e = \frac{300}{283.50 \times 60} = 0.0176 \approx 0.018$

74. Energy radiated from a body,

$Q = Ae\sigma T^4 t$

$\therefore$ $\frac{Q_2}{Q_1} = \left(\frac{T_2}{T_1}\right)^4$

$\therefore$ $\frac{T_2}{T_1} = \left(\frac{Q_2}{Q_1}\right)^{1/4} = \left(\frac{4.32 \times 10^6}{2.7 \times 10^{-3}}\right)^{1/4}$

$= \left(\frac{16 \times 27}{27} \times 10^8\right)^{1/4}$

$= 2 \times 10^2$

$\therefore$ $T_2 = 200 \times T_1 = 200 \times 400 = 80000\ K$

75. Rate of heat loss $\propto (T^4 - T_0^4)$

$\therefore \frac{R_1}{R_2} = \frac{(T_1^4 - T_0^4)}{(T_2^4 - T_0^4)}$

$\therefore \frac{R_1}{R_2} = \frac{(600)^4 - (300)^4}{(900)^4 - (300)^4} = \frac{1215}{6480}$

$\therefore R_2 = \frac{16}{3} R$

76. Rate of loss of heat per sec $= \sigma A (T^4 - T_0^4)$
$= \sigma (4\pi R^2)(T^4 - T_0^4)$

$\therefore \left(\frac{dQ}{dt}\right)_1 = \sigma 4\pi R_1^2 (T^4 - T_0^4)$ and

$\left(\frac{dQ}{dt}\right)_2 = \sigma 4\pi R_2^2 (T^4 - T_0^4)$

$\therefore \frac{(dQ/dt)_1}{(dQ/dt)_2} = \frac{R_1^2}{R_2^2}$

77. Heat radiated per second per unit area $\propto T^4$
Here, $T_1 = 127$ °C = 400 K
$T_2 = 527$ °C = 800 K
Since $T_2 = 2T_1$ and $E \propto T^4$,

$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{2T_1}{T_1}\right)^4 = (2)^4 = 16$

$\therefore E_2 = 16 E_1 = 16 \times 6 = 96$ J

78. $\frac{dQ}{dt} \propto A\theta^4 \propto r^2\theta^4 \propto m^{2/3}\theta^4$

$\therefore \frac{\left(\frac{dQ_1}{dt}\right)}{\left(\frac{dQ_2}{dt}\right)} = \left(\frac{m_1}{m_2}\right)^{2/3} \times \left(\frac{\theta_1}{\theta_2}\right)^4$

$= \left(\frac{8}{1}\right)^{2/3} \times \left(\frac{2000}{1000}\right)^4$

$= 4 \times 16 = 64 : 1$

79. According to Newton's law,

$\frac{d\theta}{dt} = - K(\theta - \theta_0)$

$\therefore \frac{d\theta}{\theta - \theta_0} = - K.dt$

Upon integration, we get
$\log(\theta - \theta_0) = - Kt + c$
This is a equation of straight line.

80. Rate of cooling = k.(excess temperature)

$\therefore 0.2 = k(20) \Rightarrow k = \frac{0.2}{20} = 0.01$

81. $\frac{(d\theta_1/dt_1)}{(d\theta_2/dt_2)} = \frac{(\theta_1' - \theta_0)}{(\theta_2' - \theta_0)}$

$\therefore \frac{0.75}{(d\theta_2/dt_2)} = \frac{50}{30}$

$\therefore \left(\frac{d\theta_2}{dt_2}\right) = \frac{0.75 \times 30}{50} = 0.45$ °C/s

82. $\frac{dQ}{dt} = - K(T - T_0)$

$0.6 = - K(40)$ ….(i)

$\frac{dQ_2}{dt} = -K(20)$ ….(ii)

Dividing equation (i) by (ii) we get,

$\frac{0.6}{\left(\frac{dQ_2}{dt}\right)} = \frac{40}{20} = 2$

$\therefore \frac{dQ_2}{dt} = \frac{0.6}{2} = 0.3$ °C/s

83. In first case,

$\frac{50 - 40}{5} = K\left[\frac{50 + 40}{2} - \theta_0\right]$ ….(i)

In second case,

$\frac{40 - 33.33}{5} = K\left[\frac{40 + 33.33}{2} - \theta\right]$….(ii)

By solving equations (i) and (ii), $\theta_0 = 20$ °C

84. Rate of cooling (R) $\propto$ Fall in temperature of body $(\theta - \theta_0)$

$\therefore \frac{R_1}{R_2} = \frac{\theta_1 - \theta_0}{\theta_2 - \theta_0} = \frac{100 - 40}{80 - 40} = \frac{3}{2}$

85. $\frac{61 - 59}{4} = K\left[\frac{61 + 59}{2} - 30\right] = K[30]$ ….(i)

$\frac{51 - 49}{t} = K\left[\frac{51 + 49}{2} - 30\right] = K[20]$ ….(ii)

$\therefore$ By dividing equation (i) by equation (ii) we get,

$\therefore \frac{t}{4} = \frac{30}{20} \Rightarrow t = 6$ min

86. $\frac{0.1}{5} = 49.95 - \theta$

$\therefore 0.1 = 249.75 - 5\theta$ ....(i)

Also,

$\frac{0.1}{10} = 39.95 - \theta$

$\therefore 0.1 = 399.5 - 10\theta$ ....(ii)

By subtracting equation (i) from equation (ii),
$249.75 - 5\theta - 399.5 + 10\,\theta = 0$

$\therefore$ $5\theta = 150 \Rightarrow \theta = 30$ °C

87. $\left(\dfrac{d\theta}{dt}\right)_1 = \dfrac{64-50}{10} = \dfrac{14}{10}$

$\left(\dfrac{d\theta}{dt}\right)_2 = \dfrac{50-42}{10} = \dfrac{8}{10}$

$\therefore$ Ratio $= \dfrac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \dfrac{14/10}{8/10} = \dfrac{7}{4}$

88. $\dfrac{80-60}{1} = K\left(\dfrac{80+60}{2} - 30\right) \Rightarrow K = \dfrac{1}{2}$

Again, $\dfrac{60-50}{t} = \dfrac{1}{2}\left(\dfrac{60+50}{2} - 30\right) = \dfrac{25}{2}$

$\therefore$ $t = 0.8$ min $= 0.8 \times 60 = 48$ s

89. In first case,

$\dfrac{50-40}{10} = K\left[\dfrac{50+40}{2} - 20\right]$ ....(i)

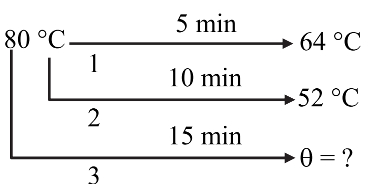
In second case,

$\dfrac{40-\theta_2}{10} = K\left[\dfrac{40+\theta_2}{2} - 20\right]$ ....(ii)

By solving equations (i) and (ii), $\theta_2 = 33.3$ °C.

90. According to Newton law of cooling,

$\dfrac{\theta_1 - \theta_2}{t} = K\left[\dfrac{\theta_1+\theta_2}{2} - \theta_0\right]$



For first process: $\dfrac{(80-64)}{5}$

$= K\left[\dfrac{80+64}{2} - \theta_0\right]$ ....(i)

For second process: $\dfrac{(80-52)}{10}$

$= K\left[\dfrac{80+52}{2} - \theta_0\right]$ ....(ii)

For third process: $\dfrac{(80-\theta)}{15}$

$= K\left[\dfrac{80+\theta}{2} - \theta_0\right]$ ....(iii)

On solving equation (i) and (ii) we get $K = \dfrac{1}{15}$ and $\theta_0 = 24$ °C. Substituting these values in equation (iii) we get $\theta = 42.7$ °C

92. Density of water is maximum at 4 °C. In both heating and cooling of water from this temperature, level of water rises due to decrease in density, i.e., water will overflow in both A and B.

93. For A, $e_A = \dfrac{E_A}{(E_b)_A} \Rightarrow E_A = e_A(E_b)_A$

For B, $e_B = \dfrac{E_B}{(E_b)_B} \Rightarrow E_B = e_B(E_b)_B$

$\therefore$ $e_A(E_b)_A = e_B(E_b)_B$ ....[$\because E_A = E_B$]

$\therefore$ $\dfrac{(E_b)_A}{(E_b)_B} = \dfrac{e_B}{e_A} = \dfrac{0.6}{0.3} = 2$

Now, $E_b \propto T^4$

$\therefore$ $\dfrac{(E_b)_A}{(E_b)_B} = \dfrac{T_A^{\,4}}{T_B^{\,4}} = 2 \Rightarrow \dfrac{T_A}{T_B} = (2)^{1/4}$

$\therefore$ $T_A = (2)^{1/4}\, T_B$

94. According to Wien's law,

$\lambda_m T$ = constant $\therefore$ $\lambda_{m_1} T_1 = \lambda_{m_2} T_2$

$\therefore$ $T_2 = \dfrac{\lambda_{m_1}}{\lambda_{m_2}} T_1 = \dfrac{\lambda_0}{\left(\frac{3\lambda_0}{4}\right)} \times T_1 = \dfrac{4}{3} T_1$

Now, $P \propto T^4$ $\therefore$ $\dfrac{P_2}{P_1} = \left(\dfrac{T_2}{T_1}\right)^4$

$\therefore$ $\dfrac{P_2}{P_1} = \left(\dfrac{4/3\,T_1}{T_1}\right)^4 = \dfrac{256}{81}$

95. Specific heat for diatomic gas, $C_v = \dfrac{7}{2} R$

$(\Delta Q)_v = nC_v\Delta T$

$\therefore$ $\Delta Q = \dfrac{7}{2} \times 2 \times 100 \times R = 700$ R

96. Black cloth is a good absorber of heat. Therefore, ice covered by black cloth melts more as compared to that covered by white cloth.

97. $mc\dfrac{d\theta}{dt} = \sigma A(T^4 - T_0^{\,4})$

$\therefore$ $\dfrac{d\theta}{dt} = \dfrac{\sigma 4\pi r^2 (T^4 - T_0^{\,4})}{\left(\frac{4}{3}\pi r^3 \rho c\right)}$

$\therefore$ $\dfrac{d\theta}{dt} \propto \dfrac{1}{r\rho c}$

98. $\frac{d\theta}{dt} = K\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)$

In first case,

$\frac{3}{1} = K\,(64 - 22.5) = 41.5\ K$

$\therefore\quad K = \frac{3}{41.5}$

In second case,

$\frac{6}{t} = \frac{3}{41.5}\ (43.5 - 22.5)$

$= \frac{3}{41.5} \times 21 \approx 1.5$

$\therefore\quad t = \frac{6}{1.5} = 4$ min

99. Using, $W = \mu RT \log_e \frac{V_2}{V_1}$

$= \left(\frac{m}{M}\right) RT \log_e\left(\frac{V_2}{V_1}\right)$

$= 2.3 \times \frac{m}{M} RT \log_{10}\left(\frac{V_2}{V_1}\right)$

$= 2.3 \times \frac{96}{32} R\,(273 + 27) \log_{10}\left(\frac{140}{70}\right)$

$= 2.3 \times 900\ R \log_{10}2$

100. $\eta = 1 - \frac{T_2}{T_1}$

$\therefore\quad \frac{1}{2} = 1 - \frac{500}{T_1} \Rightarrow \frac{500}{T_1} = \frac{1}{2}$ ….(i)

$\frac{60}{100} = 1 - \frac{T_2'}{T_1} \Rightarrow \frac{T_2'}{T_1} = \frac{2}{5}$ ….(ii)

Dividing equation (i) by (ii),

$\frac{500}{T_2'} = \frac{5}{4} \Rightarrow T_2' = 400$ K

101. $PV^\gamma = K$

$\therefore\quad P\gamma V^{\gamma-1}\,dV + dP.V^\gamma = 0$

….[On differentiation]

$\therefore\quad \frac{dP}{P} = -\gamma\frac{dV}{V}$ or $\frac{dP}{P} \times 100$

$= -\gamma\left(\frac{dV}{V}\times 100\right)$

$= -1.4 \times 5$

$= 7\ \%$ ….[considering magnitude only]

102. The cyclic process 1 is clockwise where as process 2 is anticlockwise. Clockwise area represents positive work and anticlockwise area represents negative work. Since negative area (2) > positive area (1), hence net work done is negative.

103. From the given VT diagram,

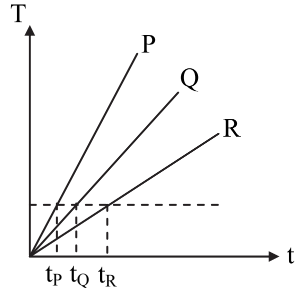
For process AB, $V \propto T \Rightarrow$ Pressure is constant

($\because$ Quantity of the gas remains same)

For process BC, V = Constant and for process CA, T = constant

These processes are correctly represented on PV diagram by graph (C).

104. Substances having higher specific heat take more time to get heated to a higher temperature and longer time to get cooled.



If line is drawn parallel to the time axis, it cuts the given graphs at three distinct points. Corresponding points on the time axis shows that

$t_R > t_Q > t_P \Rightarrow c_R > c_Q > c_P$

$t_P > t_Q > t_R \Rightarrow c_P > c_Q > c_R$

105. $PV = Nk_BT$

$\frac{N}{V} = \frac{P}{k_BT} = \frac{4\times10^{-10}}{1.38\times10^{-23}\times300}$

$= \frac{4\times10^{-10}}{4.14\times10^{-21}}$

$\approx 10^{11}$

$\therefore$ Number of molecules per $m^3 \approx 10^{11}$

$\therefore$ Number of molecules per cm^3 ($\approx 10^{-6}\ m^3$) $= 10^{11-6} = 10^5$

106. According to the first law of thermodynamics, dQ = dU + dW. In an isothermal change, temperature of the system is constant. So change in internal energy, dU = 0. Therefore, dQ = dW.

107. $c_{r.m.s.} = \sqrt{\frac{3RT}{M}}$

Let momentum of A, $p_A = M_Ac_A$

$= M_A\sqrt{\frac{3RT}{M_A}}$

$\therefore \quad 3\,RT = \dfrac{p_A^2 M_A}{M_A^2} = \dfrac{p_A^2}{M_A}$ ....(i)

Let momentum of B = $p_B = M_B c_P$

$= M_B \sqrt{\dfrac{3RT}{M_B}}$

$\therefore \quad 3\,RT = \dfrac{p_B^2 M_B}{M_B^2} = \dfrac{p_B^2}{M_B}$ ....(ii)

From equations (i) and (ii) we get,

$\dfrac{p_A^2}{M_A} = \dfrac{p_B^2}{M_B}$

$\therefore \quad p_A^2 = \left(\dfrac{M_A}{M_B}\right) p_B^2$

$\therefore \quad p_A = \left(\dfrac{M_A}{M_B}\right)^{1/2} p_B$

108. Using, $P_1V_1 = P_2V_2$ we get,

$PV = P' \times \dfrac{80V}{100} \Rightarrow \dfrac{P'}{P} = \dfrac{10}{8}$

$\therefore \quad \dfrac{P' - P}{P} \times 100 = \left(\dfrac{10}{8} - 1\right) \times 100$

$= \left(\dfrac{2}{8} \times 100\right)$

$= \dfrac{1}{4} \times 100 = 25\ \%$

109. For 1st case,

$\eta = \left(1 - \dfrac{T_1}{T_2}\right) \times 100$

$\therefore \quad \left(1 - \dfrac{T_1}{500}\right) \times 100 = 40 \Rightarrow T_1 = 300\ K$

For 2nd case,

$\eta = \left(1 - \dfrac{300}{T_2}\right) \times 100 = 60 \Rightarrow T_2 = 750\ K$

Competitive Thinking

2. Number of moles in 4 g of hydrogen,

$n = \dfrac{m}{M} = \dfrac{4}{2} = 2$

$\therefore \quad PV = nRT = 2RT$

4. Ideal gas equation gives,

PV = nRT

$\therefore$ For n = 1

$V = \dfrac{RT}{P}$ ....(i)

$\therefore \quad \text{density} = \dfrac{\text{molar mass}}{\text{volume}}$

$= \dfrac{m(N_A)P}{RT}$ ....[From (i)]

But, $\dfrac{R}{N_A} = k$

k = Boltzmann constant

$\therefore \quad \text{density} = \dfrac{mP}{kT}$

6. Since PV = nRT,

For 1 mole of gas, $50 \times 100 = 1 \times R \times T$

For 2 mole of gas, $100 \times V = 2 \times R \times T$

$\therefore \quad \dfrac{50 \times 100}{V \times 100} = \dfrac{1}{2}$

$\Rightarrow V = 100\ mL$

7. $PV = nRT = \dfrac{m}{M}RT$

$\therefore \quad \dfrac{m}{VP} \Rightarrow \dfrac{\text{density}}{P} = \dfrac{M}{RT}$

$\left(\dfrac{\text{density}}{P}\right)_{\text{At } 0^\circ C} = \dfrac{M}{R(273)} = x$ ....(i)

$\left(\dfrac{\text{density}}{P}\right)_{\text{At } 100^\circ C} = \dfrac{M}{R(373)}$ ....(ii)

$\therefore$ From equations (i) and (ii) we get,

$\therefore \quad \left(\dfrac{\text{density}}{P}\right)_{\text{At } 100^\circ C} = \dfrac{273x}{373}$

8. From PV = nRT as per given data,

$P \propto n \Rightarrow \dfrac{P_O}{P_H} = \dfrac{n_O}{n_H} = \dfrac{m/m_o}{m/m_H} = \dfrac{m_H}{m_o}$

$\therefore \quad P_O = P_H.\dfrac{M_H}{M_O} = 4.\dfrac{2}{4} = 2\ atm$

9. Using ideal gas equation,

$PV = nRT = \dfrac{m}{M}RT$

$\therefore \quad V = \dfrac{mRT}{MP} = \dfrac{2.8 \times 8300 \times (27 + 273)}{28 \times 0.821 \times 1.013 \times 10^5}$

$= \dfrac{2.99 \times 10^5}{10^5} \approx 3$ litre

10. Ideal gas equation is, PV = nRT

$\therefore \quad \dfrac{n}{V} = \dfrac{P}{RT} = \text{constant}$

Hence, at constant pressure and temperature, both balloons will contain equal number of molecules per unit volumes.

Note: This result is nothing but Avogadro's law.

11. By Dalton's law of partial pressures, the total pressure will be $P_1 + P_2 + P_3$.

12. By ideal gas equation,

$\therefore$ $PV = nRT \Rightarrow \frac{V}{T} = \frac{nR}{P}$

$\frac{V}{T}$ = constant ….[at constant P]

Hence, graph (A) is correct.

13. Using ideal gas equation,
before heating, at $T_1 = 17 + 273 = 290$ K,
$PV = n_1R \times 290$ ....(i)
After heating, at $T_2 = 27 + 273 = 300$ K,
$PV = n_2R \times 300$ ....(ii)
where, n_1 and n_2 are number of moles at T_1 and T_2 respectively.
From equations (i) and (ii),

$$n_2 - n_1 = \frac{PV}{R\times300} - \frac{PV}{R\times290}$$

But, $n_f - n_i = (n_2 - n_1)N_A$

i.e., $n_f - n_i = -\frac{PV}{R}\times\left(\frac{10}{290\times300}\right)\times6.023\times10^{23}$

Given: $P = 10^5$ Pa and $V = 30\ m^3$

$\Rightarrow$ Number of molecules $n_f - n_i$

$$= -\frac{10^5\times30\times10\times6.023\times10^{23}}{8.3\times290\times300}$$

$= -2.5 \times 10^{25}$

16. $\Delta p = mv - (-mv) = 2mv$

17. Using, $c \propto \sqrt{T}$,

$$\frac{(c_{rms})_1}{(c_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$$

Given that, $T_2 = 273$ K,

$(c_{rms})_1 = 4\ (c_{rms})_2$ or $\frac{(c_{rms})_1}{(c_{rms})_2} = 4$

$\therefore$ $4 = \sqrt{\frac{T_1}{273}}$

$\Rightarrow T_1 = 273 \times 16 = 4368$ K
$= 4368 - 273 = 4095$ °C

18. The rms velocity is related to Temperature as $c \propto \sqrt{T}$

$\therefore$ $\frac{c_1}{c_2} = \sqrt{\frac{T_1}{T_2}}$ $\Rightarrow$ $\frac{c_1}{\frac{1}{2}c_1} = \sqrt{\frac{0+273}{T_2}}$

$\Rightarrow T_2 = \frac{273}{4} = 68.25$ K

$\Rightarrow t_2 = T_2 - 273 = -204.75$ °C

19. $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$

$\therefore$ $\frac{v_S}{400} = \sqrt{\frac{(273+227)}{(273+27)}} = \sqrt{\frac{5}{3}}$

$\therefore$ $v_s = 400\sqrt{5/3} \approx 516$ m/s

20. $c_{r.m.s.} = \sqrt{\frac{3kT}{m}} \Rightarrow c_{r.m.s.} \propto \frac{1}{\sqrt{m}}$

21. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v'_{rms} = \sqrt{\frac{3R(2T)}{M/2}} = 2\ v_{rms}$

22. $v_{R.M.S.} = \sqrt{\frac{\gamma RT}{M}}$

$v \propto \frac{1}{\sqrt{M}}$

$\frac{v_1}{v_2} = \sqrt{\frac{M_1}{M_2}}$

$v_2 = 2$ km/s

23. $c_{rms} \propto \frac{1}{\sqrt{M}}$

let c_1 be the rms velocity of uranium of mass $M_1 = 235$ units and c_2 be the rms velocity of uranium of mass $M_2 = 238$ units

$\therefore$ $\frac{c_1 - c_2}{c_2} = \frac{\sqrt{M_2} - \sqrt{M_1}}{\sqrt{M_1}}$

$= \frac{\sqrt{238} - \sqrt{235}}{\sqrt{235}}$

$= 0.0064$

$\therefore$ % ratio $= \frac{c_1 - c_2}{c_2} \times 100 = 0.64$

24. $c_{rms} = \sqrt{\frac{3RT}{M}}$ $\therefore$ $c_{rms} \propto \sqrt{T}$

$\therefore$ $\frac{c_2}{c_1} = \sqrt{\frac{T_2}{T_1}}$ $\therefore$ $c_2 = c_1\sqrt{\frac{T_2}{T_1}}$

$\therefore$ $c_2 = 200 \times \sqrt{\frac{127+273}{27+273}}$

$\therefore$ $c_2 = 200 \times \sqrt{\frac{400}{300}}$ $\therefore$ $c_2 = \frac{400}{\sqrt{3}}$ m/s

25. $c_{rms} \propto \sqrt{T} \Rightarrow \frac{\Delta c}{c} = \frac{1}{2}\frac{\Delta T}{T} = \frac{1}{2}\times\frac{6}{300} = \frac{1}{100}$

$\therefore$ The rms velocity will increase nearly by 1%

26. Its known from kinetic theory of gases-

$\frac{c_1}{c_2} = \sqrt{\frac{T_1}{T_2}}$ but $c_2 = 2c_1$ (given)

$\therefore \quad \frac{c_1}{2c_1} = \sqrt{\frac{27+273}{T}} \Rightarrow \frac{1}{4} = \frac{300}{T}$

$\Rightarrow T = 1200\text{ K} = 927\text{ °C}$

27. Using, $c_{rms} \propto \sqrt{T}$,

$\frac{(c_{rms})_1}{(c_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$.

Given that, $(c_{rms})_2 = \frac{(c_{rms})_1}{2}$ or $\frac{(c_{rms})_1}{(c_{rms})_2} = 2$,

$T_1 = 327 + 273 = 600\text{ K}$

$\therefore \quad 2 = \sqrt{\frac{600}{T_2}}$ or

$T_2 = \frac{600}{4} = 150\text{ K} = 150 - 273 = -123°\text{ C}$

28. $\lambda = \frac{1}{\lambda d^2 n\sqrt{2}} = \frac{1}{4\pi r^2 n\sqrt{2}}$

$\Rightarrow \lambda \propto \frac{1}{r^2}$

29. Mean square speed

$= \frac{2^2+3^2+4^2+5^2+6^2}{5} = \frac{90}{5} = 18\text{ m}^2/\text{s}^2$

30. Average speed $= \frac{1+3+5+7}{4} = \frac{16}{4} = 4\text{ km/s}$

R.M.S. speed $= \sqrt{\frac{1^2+3^2+5^2+7^2}{4}} = \sqrt{\frac{84}{4}}$

$= 4.583\text{ km/s}$

$\therefore$ R.M.S. speed – average speed = 0.583 km/s

31. $v_{mean} = \frac{150+160+170+180+190}{5}$

$= \frac{850}{5} = 170\text{ m/s}$

$v_{r.m.s.} = \sqrt{\frac{150^2+160^2+170^2+180^2+190^2}{5}}$

$= \sqrt{\frac{144500}{5}} = \sqrt{29100} = 170.59\text{ m/s}$

$\therefore \quad \frac{v_{r.m.s.}}{v_{mean}} = \frac{170.59}{170} \approx 1$

33. $\frac{\text{K.E.}}{\text{Volume}} = E = \frac{3}{2}P \qquad \Rightarrow P = \frac{2}{3}E$

34. Using, $P = \frac{1}{3}\rho c_{rms}^2$,

$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} \times \frac{(c_{rms}^2)_1}{(c_{rms}^2)_2}$

$\therefore \quad \left(\frac{(c_{rms})_1}{(c_{rms})_2}\right)^2 = \left(\frac{P_1}{P_2}\right) \times \left(\frac{\rho_2}{\rho_1}\right) = \frac{3}{2} \times \frac{2}{3} = 1$

$\frac{(c_{rms})_1}{(c_{rms})_2} = 1$

37. Total translational kinetic energy

$= \frac{3}{2}nRT = \frac{3}{2}PV$

All the molecules in an ideal gas moving randomly in all direction collide and their velocity changes after collision.

38. Pressure exerted by the gas on wall of container is given by,

$P = \frac{1}{3}\rho c^2 \qquad \ldots\{c \equiv \text{r.m.s. speed}\}$

$\therefore \quad P = \frac{1}{3}\left(\frac{M}{V}\right)c^2$

$P = \frac{2}{3}\,\frac{1}{2}\left(\frac{M}{V}\right)c^2$

$\therefore \quad P = \frac{2}{3}\left(\frac{\text{K.E}}{V}\right) \qquad \ldots\{\because \text{K.E.} = \frac{1}{2}Mc^2\}$

39. Using Charles' law,

$\frac{P_1}{P_2} = \frac{T_1}{T_2}$

$\therefore \quad P_2 = \frac{P_1 T_2}{T_1} = \frac{P(273+927)}{(273+27)} = 4\text{ P}$

40. $c_{r.m.s.} = \sqrt{\frac{3RT}{M}} \Rightarrow c_{r.m.s.} \propto \sqrt{\frac{T}{M}}$

$\therefore \quad \frac{(c_{rms})_2}{(c_{rms})_1} = \sqrt{\frac{M_1}{M_2} \times \frac{T_2}{T_1}} = \sqrt{\frac{1}{2} \times \frac{1}{2}}$

$\therefore \quad (c_{rms})_2 = \frac{(c_{rms})_1}{2} = \frac{300}{2} = 150\text{ m/s}$

41. $E = \frac{3}{2}RT = \frac{3}{2} \times 8.31 \times 273 = 3.4 \times 10^3\text{ J}$

42. The average kinetic energy of monatomic gas molecule (K.E.) $= \frac{3}{2}k_B T$

$$K.E. = \frac{3}{2} \times (1.38 \times 10^{-23}\ J\ K^{-1}) \times (300\ K)$$
$$= \frac{3 \times (1.38 \times 10^{-23}\ JK^{-1}) \times (300\ K)}{2 \times (1.6 \times 10^{-19}\ J/eV)}$$
$$= 3.9 \times 10^{-2}\ eV = 0.039\ eV$$

43. Average kinetic energy per molecule for any kind of molecule of an ideal gas is

$K.E_{avg} = \frac{3}{2}kT$

∴ $(K.E_{avg})_{hydrogen} = \frac{3}{2}\ kT_1$ and

$(K.E_{avg})_{oxygen} = \frac{3}{2}\ kT_2$

But $T_1 = T_2$

∴ $(K.E_{avg})_O = (K.E_{avg})_H$

45. Average kinetic energy $= \frac{3}{2}RT$

i.e. K.E. ∝ T

As T is constant, K.E. remains same.

46. Using, K.E. ∝ T,

$\frac{K.E_1}{K.E_2} = \frac{T_1}{T_2}$.

Given that, $K.E._1 = 2K.E_2$, $T_2 = 273$ K

∴ $2 = \frac{T_1}{273} \Rightarrow T_1 = 546$ K

47. For 1 kg gas, energy, $E = \frac{f}{2}rT$

As $P = \rho rT$

$\Rightarrow rT = \frac{P}{\rho}$

∴ $E = \frac{5}{2} \times \frac{8 \times 10^4}{4}$ ....[∵ f = 5 for diatomic gas]

∴ $E = 5 \times 10^4$ J

48. Internal energy of a gas with f degrees of freedom,

$U = \frac{f}{2}nRT$

Now, $f_{O_2} = \frac{5}{2}$, $f_{Ar} = \frac{3}{2}$

∴ $U_{total} = \frac{5}{2}(2)\ RT + \frac{3}{2}(4)\ RT = 11RT$.

50. Let molar heat capacity at constant pressure $= s_P$ and molar heat capacity at constant volume $= s_V$

∴ $s_P - s_V = R$

Now, principal specific heat, $C = \frac{s}{M}$

∴ $C_P - C_V = \frac{R}{M}$ ∴ For H_2, $a = \frac{R}{2}$

For N_2, $b = \frac{R}{28}$ ∴ $\frac{a}{b} = 14$

$\Rightarrow a = 14b$

51. $C_p - C_v = R$

∴ $C_p = R + C_v$ ....(i)

also, $C_p = \gamma C_v$ ....(ii)

∴ substituting $C_v = \frac{3R}{2}$ in eq. (i) and (ii)

$R + \frac{3R}{2} = \gamma \times \frac{3R}{2}$

∴ $\gamma = \frac{5}{3}$

52. Molar specific heat at constant pressure $C_p = \frac{7}{2}R$

Using, $C_p - C_v = R$

$C_v = C_p - R = \frac{7}{2}R - R = \frac{5}{2}R$

∴ $\frac{C_p}{C_v} = \frac{(7/2)R}{(5/2)R} = \frac{7}{5}$

53. Given,

$\frac{R}{C_v} = 0.4$

∴ $\frac{C_p - C_v}{C_v} = 0.4$

∴ $\frac{C_p}{C_v} = 0.4 + 1$

∴ $\gamma = 1.4$

∴ the molecules of the gas are rigid diatomic.

57. Given: $\frac{C_P}{C_V} = \gamma$

∴ $\frac{C_P - C_V}{C_V} = \frac{\gamma - 1}{1}$

∴ $\frac{R}{C_V} = \gamma - 1$ ....(∵ $C_P - C_V = R$)

∴ $C_V = \frac{R}{\gamma - 1}$

58. For rigid diatomic molecule,

$\gamma = \frac{C_P}{C_V} = \frac{7}{5}$ $\therefore$ $C_V = \frac{5}{7}C_P$

Also for molar specific heats,

$C_P - C_V = R$ $\therefore$ $C_P - \frac{5}{7}C_P = R$

$\frac{2}{7}C_P = R$ $\therefore$ $n = \frac{2}{7} = 0.2857$

59. $dV = n \times C_v \times d\theta$

$= n \times \frac{R}{\gamma - 1} \times d\theta$ ...$\left(\because C_v = \frac{R}{\gamma-1}\right)$

$= 2000 \times \frac{8.314}{0.4} \times (-10)$

$= -4.2 \times 10^5$ J

60. $\theta = ms\Delta T$

$\frac{d\theta}{dt} = ms\frac{dT}{dt}$

$Pdt = msdT$

$dT = \frac{P}{ms}dt$

Rise in temperature (dT) $\propto \frac{1}{s}$

From graph we can observe that rise in temperature in graph A is more than B and C.

$\therefore$ dT is maximum for A and minimum for C and specific heat value is maximum for C and minimum for A.

61. State of a thermodynamic system cannot be determined by a single variable (P or V or T).

63. Heat supplied to a gas raises its internal energy and does some work against expansion, so it is a special case of law of conservation of energy.

64. In adiabatic process, PV^γ = constant

$\left(\frac{RT}{V}\right).V^\gamma$ = constant

$\therefore$ $TV^{\gamma-1}$ = constant

66. In adiabatic process, no heat transfer takes place between system and surrounding.

67. For an adiabatic process,

$P \propto T^{\gamma/\gamma-1}$

Given that, $P \propto T^3$

$\therefore$ $\frac{\gamma}{\gamma-1} = 3 \Rightarrow \gamma = 3\gamma - 3$

$\therefore$ $-2\gamma = -3 \Rightarrow \gamma = \frac{3}{2}$

68. In a refrigerator, the heat dissipated in the atmosphere is more than that taken from the cooling chamber, therefore the room is heated if the door of a refrigerator is kept open.

69. $\Delta Q = \Delta W + \Delta U$

$\therefore$ $35 = -15 + \Delta U \Rightarrow \Delta U = 50$ J

70. For an adiabatic process, $\Delta Q = 0$

$\because$ Work is done on the gas, $\Delta W = -90$ J

$\therefore$ From $\Delta Q = \Delta U + \Delta W$,

$0 = \Delta U - 90$

$\therefore$ $\Delta U = +90$ J

71. $dQ = dU + dW$

$mL = dU + PdV$

$\therefore$ $dU = mL - PdV$

$= (1 \times 540 \times 4.2) - (10^5 \times 1650 \times 10^{-6})$

$\therefore$ $dU = 2103$ J

72. In an adiabatic process, $\Delta Q = 0$

$\therefore$ $\Delta U + \Delta W = 0$ $(\because \Delta Q = \Delta U + \Delta W)$

73. $\Delta u = nC_v\Delta T = 1 \times \frac{5R}{2}\Delta T$

For BC, $\Delta T = 600 - 800 = -200$ K

$\therefore$ $\Delta u = \frac{5R}{2} \times (-200) = -500$ R

74. By 1st law of thermodynamics,

$\therefore$ $\Delta Q = \Delta U + \Delta W$

$2 \times 10^3 \times 4.2 = \Delta U + 500$

$\therefore$ $\Delta U = 7900$ J

75. In a closed cyclic process, the change in internal energy is always zero $\Rightarrow E = 0$

76. $\Delta Q = \Delta U + \Delta W$

$\therefore$ $\Delta W = \Delta Q - \Delta U = 110 - 40 = 70$ J

77. By 1st law of thermodynamics,

$\Delta Q = \Delta U + P(\Delta V)$

$\therefore$ $\Delta U = \Delta Q - P(\Delta V)$

$= 1500 - (2.1 \times 10^5)(2.5 \times 10^{-3}) = 975$ J

78. $\Delta U = nC_V\Delta T = n(C_P - R)\Delta T$

$= 5 \times \left(8 - \frac{8.36}{4.18}\right) \times 10 = 5 \times 6 \times 10$

$= 300$ calories

79. $\Delta Q = \Delta U + \Delta W$

$\therefore$ $\Delta Q = 0 - 150$ J

Thus, heat has been given by the system.

80. Using first law of thermodynamics,
$\Delta Q = \Delta U + \Delta W$
$\Rightarrow \Delta U = \Delta Q - \Delta W$
Given that, $\Delta Q = 35$ J, $\Delta W = -15$ J
$\therefore$ $\Delta U = 35 \text{ J} - (-15 \text{ J}) = 50$ J
Note: ΔW is negative because work is done on the system.

81. In an isothermal compression, there is always an increase of heat which needs to be given out
Using, $\Delta Q = \Delta U + \Delta W$
$\Delta Q = \Delta W$ ....[$\because$ $\Delta U = 0$]
$\therefore$ $\Delta Q = -1.5 \times 10^4 \text{ J} = -\dfrac{1.5\times10^4}{4.18}$ calories
$= -3.6 \times 10^3$ calories

82. $W = \mu RT \log_e\left(\dfrac{V_2}{V_1}\right)$
$= 0.2 \times 8.3 \times \log_e 2 \times (27 + 273)$
$= 0.2 \times 8.3 \times 300 \times 0.693 \approx 345$ J

83. $T_1 = 27 + 273 = 300$ K
$T_2 = 627 + 273 = 900$ K, $\gamma = 1.5$
For an adiabatic change, $\dfrac{T^\gamma}{P^{\gamma-1}}$ = constant
$\therefore$ $\left(\dfrac{P_2}{P_1}\right)^{1/2} = \left(\dfrac{T_2}{T_1}\right)^{3/2} \Rightarrow \left(\dfrac{P_2}{10^5}\right)^{1/2} = \left(\dfrac{900}{300}\right)^{3/2}$
$\therefore$ $P_2 = 27 \times 10^5 \text{ N / m}^2$

84. Using, $dQ = dU + dW$,
$0 = -2 + dW \Rightarrow dW = 2$ J
$\therefore$ Work done by the gas = 2 J
or Work done on the gas = –2 J

85. Due to compression the temperature of the system increases to a very high value. This causes the flow of heat from system to the surroundings, thus decreasing the temperature. The decrease in temperature results in decrease in pressure.

86. Using, $\dfrac{P_2}{P_1} = \left(\dfrac{V_1}{V_2}\right)^\gamma \Rightarrow \dfrac{P'}{P} = (8)^{5/2}$
$\therefore$ $P' = P \times (2)^{15/2}$

87. For isobaric process, work done,
$w_1 = P(V_2 - V_1)$
$= P(2V - V)$ ...($\because$ Volume is doubled)
For isothermal process,
$w_2 = nRT \log_e\left(\dfrac{V_2}{V_1}\right)$
$= PV \log_e\left(\dfrac{2V}{V}\right)$
$\therefore$ $\dfrac{w_2}{w_1} = \dfrac{PV \log_e(2)}{PV}$
$\therefore$ $w_2 = w_1 \log_e 2$

88. Using, $W = \dfrac{R(T_i - T_f)}{\gamma - 1} \Rightarrow 6R = \dfrac{R(T - T_f)}{\left(\dfrac{5}{3} - 1\right)}$
$\therefore$ $T_f = (T - 4)$K

89. Number of moles of He = $\dfrac{1}{4}$
Using, $T_1V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$,
$\therefore$ $T_1(5.6)^{\gamma-1} = T_2(0.7)^{\gamma-1}$
$\therefore$ $T_1 = T_2\left(\dfrac{1}{8}\right)^{2/3} \Rightarrow 4T_1 = T_2$
$\therefore$ Work done $= \dfrac{nR[T_2 - T_1]}{\gamma - 1} = \dfrac{\frac{1}{4}R[3T_1]}{\left(\dfrac{2}{3}\right)} = \dfrac{9}{8}RT_1$

90. Given: $T_1 = 27°\text{ C} = 273 + 27 = 300$ K,
$\dfrac{V_2}{V_1} = 2$
For adiabatic process,
$TV^{\gamma-1}$ = constant
$\therefore$ $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$
$\gamma = \dfrac{5}{3}$ for monatomic gas.
$\therefore$ $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = \left(\dfrac{1}{2}\right)^{\frac{5}{3}-1} = \left(\dfrac{1}{2}\right)^{\frac{2}{3}} = \left(\dfrac{1}{4}\right)^{\frac{1}{3}} = 0.63$
$\therefore$ $T_2 = T_1 \times 0.63$
$\therefore$ $T_2 = 300 \times 0.63 = 189$ K
Now, change in internal energy,
$\Delta U = \dfrac{f}{2} nR\Delta T$
Where,
f = degrees of freedom of a monatomic gas
= 3
As the gas expands adiabatically, the internal energy decreases.
$\therefore$ $\Delta U = -\dfrac{3}{2} \times 2 \times 8.3 \times 111$
$\therefore$ $\Delta U = -2.76$ kJ.

91. Change in internal energy,

$$\Delta U = \frac{P_2V_2 - P_1V_1}{\gamma - 1} = \frac{2\times6 - 5\times4}{\frac{7}{5} - 1}$$

$$\left(\because \ \gamma = \frac{7}{5} \text{ for ideal diatomic gas}\right)$$

$$= -20 \text{ kJ}$$

92. $\Delta W = P\Delta V = 10^3 \times 0.25 = 250$ J

93. $V = \frac{AT - BT^2}{P}$

$W = P\,\Delta\,V = P[V_2 - V_1]$

$$= P\left[\frac{AT_2 - BT_2^2}{P} - \left(\frac{AT_1 - BT_1^2}{P}\right)\right]$$

$$= \left[A(T_2 - T_1) - B\left(T_2^2 - T_1^2\right)\right]$$

94. As the room works as a source here, the heat delivered will be more. Hence, the amount of heat delivered to the room by refrigerator is given by,

$$\frac{Q_1}{W} = \frac{T_1}{T_1 - T_2}$$

Where, T_1 = room temperature = t_1°C

T_2 = temperature inside the refrigerator = t_2°C

$$\therefore \quad \frac{Q_1}{W} = \frac{t_1 + 273}{(t_1 + 273) - (t_2 + 273)}$$

$$\therefore \quad \frac{Q_1}{W} = \frac{t_1 + 273}{t_1 - t_2}$$

95. $\alpha = 5$;

T_1 = temp. of surrounding

T_2 = temp. of source (inside temp.)

$T_2 = -20$ºC

$= -20 + 273$

$T_2 = 253$K

$T_1 = ?$

$$\alpha = \frac{T_2}{T_1 - T_2}$$

$$\Rightarrow 5 = \frac{253}{T_1 - 253}$$

$\Rightarrow 5T_1 - 1265 = 253$

$$T_1 = \frac{1518}{5} = 303.6\text{K}$$

$= 30.6$ºC ≈ 31ºC

96. $\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

here, $T_2 = 4$ °C = 277 K

$T_1 = 303$ K

$Q_2 = 600$ cal

$$\therefore \quad \frac{600}{W} = \frac{277}{303 - 277}$$

$$\therefore \quad W = \frac{600}{10.65} = 56.31 \text{ cal}$$

$$\therefore \quad P = \frac{W}{t} = \frac{56.31}{1\text{ s}} \times 4.2$$

$\therefore \quad P = 236.5$ W

97. Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

$$= \frac{T_1 - T_2}{T_1} = \frac{100}{373} = 0.268$$

$= 26.8\%$

98. $\eta = \left[1 - \frac{T_2}{T_1}\right] \times 100 = \left[1 - \frac{300}{500}\right] \times 100 = 40\%$

99. $\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{400} = \frac{1}{4} = 25\%$

$\Rightarrow$ 26% efficiency is impossible

100. $\eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q}$

$$\therefore \quad W = \left(1 - \frac{T_2}{T_1}\right)Q = \left\{1 - \frac{(273 + 27)}{(273 + 627)}\right\} \times Q$$

$$\Rightarrow W = \left(1 - \frac{300}{900}\right) \times 3 \times 10^6$$

$= 2 \times 10^6 \times 4.2$ J

$= 8.4 \times 10^6$ J

101. For a monatomic gas, $\gamma = \frac{C_P}{C_V} = \frac{5}{3}$

Using $\Delta Q = \mu C_P \Delta T$ and $\Delta U = \mu C_V \Delta T$

we get, $\frac{\Delta U}{\Delta Q} = \frac{C_V}{C_P} = \frac{3}{5}$

$\therefore$ Fraction of heat energy to increase the internal energy be 3/5.

102. To raise the temperature of a gas, the amount of heat that must be supplied

At constant volume

$Q_v = mC_v\Delta T$

At constant pressure

$Q_p = mC_p\Delta T$

$\therefore \quad \dfrac{Q_v}{Q_p} = \dfrac{C_p}{C_v}$

For diatomic gas,

$\dfrac{C_p}{C_v} = 1.4$ or $\dfrac{7}{5}$

$\therefore \quad \dfrac{Q_v}{Q_p} = \dfrac{1}{1.4} = \dfrac{5}{7}$

103. Fraction of energy used in doing external work is given by

$\dfrac{\Delta W}{\Delta Q} = 1 - \dfrac{C_v}{C_p}$

but $\gamma = \dfrac{C_p}{C_v} = 1.4$

$\therefore \quad \dfrac{300}{\Delta Q} = 1 - \dfrac{1}{1.4}$

$\therefore \quad \Delta Q = \dfrac{300 \times 1.4}{0.4} = 1050$ J

104. Work done by the system = Area of shaded portion on P-V diagram

$= (300 - 100)10^{-6} \times (100 - 200) \times 10^3 = -20$ J

105. AB is isobaric process; BC is isothermal process; CD is isometric process and DA is isothermal process.

These processes are correctly represented by graph (A).

106. Work done = Area of PV graph (here trapezium)

$= \dfrac{1}{2}(1\times10^5 + 5\times10^5)\times(5-1)$

$= 12 \times 10^5$ J

107. $Q_{ABC} = Q_{AC} + W_{ABCA}$

In this case,

W_{ABCA} = Area of PV graph = $\dfrac{1}{2}\times AB\times BC$

$\Rightarrow 500 = Q_{AC} + \dfrac{1}{2}\times(4\times10^4\times2\times10^{-3})$

$\Rightarrow Q_{AC} = 500 - 40 = 460$ J

108. For both the paths, ΔU remains same.

For path iaf : $\Delta U = \Delta Q - \Delta W = 50 - 20 = 30$ J.

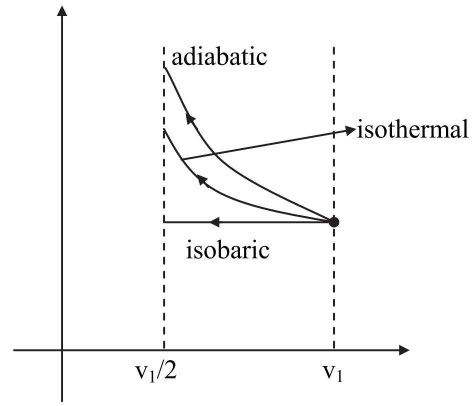
For path fi : $\Delta U = -30$J and $\Delta W = -13$J

$\therefore \quad \Delta Q = -30 - 13 = -43$ J.

109. 1st process is isothermal expansion which is correctly shown in option (D)

2nd process is isobaric compression which is correctly shown in option (D).

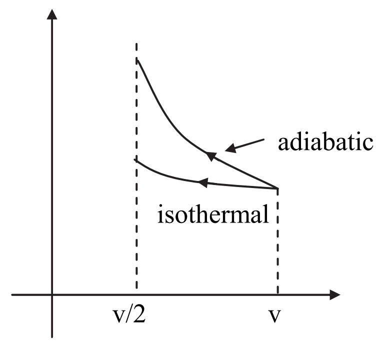
110. $v_1 \rightarrow v_1/2$



Work done = area under curve

$W_{adiabatic} > W_{isothermal} > W_{isobaric}$

111.



Work done = area under curve

While compressing the gas adiabatically, the area under the curve is more than that for isothermal compression.

113. Open window behaves like a perfectly black body.

115. Using, a + r + t = 1,

t = 1 – (a + r) = 1 – (0.74 + 0.22)

= 1 – 0.96 = 0.04

116. Using, a + r + t = 1,

a + 0.74 + 0.22 = 1 ⇒ a = 0.04

By Kirchhoff's law, a = e ⇒ e = 0.04

117. Using $r = \dfrac{Q_r}{Q}$,

$r = \dfrac{15}{150} = 0.1$

Using a + r + t = 1,

t = 1 – (a + r) = 1 – (0.6 + 0.1) = 0.3

Now using, $t = \dfrac{Q_t}{Q}$ we get,

$Q_t = Q_t = 150 \times 0.3 = 45$ J

118. $r + a + t = 1$

$\therefore$ $t = 1 - r - a = 1 - 0.8 - 0.1 = 1 - 0.9 = 0.1$

$Q = 1000$ J/min

$\therefore$ Heat energy transmitted per minute

$Q_t = Q \times t = 1000 \times 0.1 = 100$ J

$\therefore$ Heat energy transmitted in 5 minutes

$= 100 \times 5 = 500$ J

120. From Wien's displacement law,

$\lambda \propto \frac{1}{T}$

$\Rightarrow \nu \propto T$

This means more the temperature higher will be the corresponding frequency

Given $T_2 > T_1$, hence frequency corresponding to maximum energy is more at T_2.

121. As $\lambda_{Red} > \lambda_{Green} > \lambda_{Violet}$,

$\lambda_Q > \lambda_R > \lambda_P$.

According to Wien's law, $T_Q < T_R < T_P$

122. By Wien's law, $\lambda_m \propto \frac{1}{T}$ and from the figure,

$(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$

$\therefore$ $T_1 > T_3 > T_2$.

123. From Wien's displacement law,

$\lambda_m \propto \frac{1}{T}$

$\therefore$ $\lambda_m T = \text{constant}$

126. From Wien's displacement law

$T = \frac{b}{\lambda_{max}}$

b = Wien's constant

$\therefore$ $T = \frac{2892 \times 10^{-6}}{14.46 \times 10^{-6}} = 200$ K

127. By Wien's law, $T \propto \frac{1}{\lambda_m}$

$\therefore$ $\frac{T_S}{T_N} = \frac{(\lambda_N)_{max}}{(\lambda_S)_{max}} = \frac{350}{510} \approx 0.69$

128. $\lambda_{m_2} = \frac{T_1}{T_2} \times \lambda_{m_1} = \frac{2000}{3000} \times \lambda_{m_1} = \frac{2}{3}\lambda_{m_1} = \frac{2}{3}\lambda_m$

129. By Wien's law, $\frac{\lambda_{m2}}{\lambda_{m1}} = \frac{T_1}{T_2}$

$\therefore$ $\lambda_{m_2} = \frac{T_1}{T_2} \times \lambda_{m_1} = \frac{1500}{2500} \times 5000 = 3000$ Å

130. According to Wien's displacement law,

$\lambda_{max} \propto \frac{1}{T}$

$\therefore$ $\lambda_{max} T = b$

also T = 5760 K

$\therefore$ $\lambda_{max} = \frac{2.88 \times 10^6 \text{nmK}}{5760 \text{ K}} = 500$ nm

$\therefore$ wavelength of maximum energy = 500 nm

i.e. U_2 is maximum energy.

131. Black body has maximum radiated energy at same temperature.

132. From Wien's displacement law-

$\lambda_{max} T = \text{constant}$

If T is also same, λ_{max} = constant

Hence, $\lambda'_{max} = \lambda''_{max}$

133. From Stefan's law,

$E \propto AT^4$ ...(i)

$\therefore$ $E_1 \propto A_1 T_1^4$ ...(ii)

$E_2 \propto A_2 T_2^4$ ...(iii)

Divide equation (iii) by equation (ii)

$$\frac{E_2}{E_1} = \left(\frac{A_2}{A_1}\right)\left(\frac{T_2}{T_1}\right)^4 = \left(\frac{\frac{l}{3} \times \frac{b}{3}}{l \times b}\right)\left(\frac{327 + 273}{27 + 273}\right)^4$$

$$= \left(\frac{1}{9}\right)\left(\frac{600}{300}\right)^4$$

$\therefore$ $\frac{E_2}{E_1} = \frac{1}{9} = (16)$

$\therefore$ $E_2 = \frac{16}{9}E$ ...{$\because E_1 = E$}

134. $E \propto T^4$

$\therefore$ $\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{T_1}{2T_1}\right)^4 = \frac{1}{16}$

$\therefore$ $E_2 = 16E_1$

135. $Q \propto r^2T^4$

$\Rightarrow \frac{Q_2}{Q_1} = \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{T_2}{T_1}\right)^4 = (2)^2 \times (2)^4 = 64$

137. For black body, $P = A\varepsilon\sigma T^4$.

For same power, $A \propto \frac{1}{T^4}$

$\therefore$ $\left(\frac{r_1}{r_2}\right)^2 = \left(\frac{T_2}{T_1}\right)^4$

$\therefore$ $\frac{r_1}{r_2} = \left(\frac{T_2}{T_1}\right)^2$ i.e., $\frac{r_2}{r_1} = \left(\frac{T_1}{T_2}\right)^2$

138. For a black body, $\frac{Q}{t} = P = A\sigma T^4$

$\therefore \quad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4$

$\therefore \quad \frac{P_2}{20} = \left(\frac{273+727}{273+227}\right)^4$

$\therefore \quad \frac{P_2}{20} = (2)^4 \Rightarrow P_2 = 320$ W

139. Rate of energy $\frac{Q}{t} = P = A\varepsilon\sigma T^4 \Rightarrow P \propto T^4$

$\Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{927+273}{127+273}\right)^4$

$\therefore \quad P_2 = 405$ W

140. By Stefan's law,
Rate of loss of heat $\propto$ Area
For sphere, $A = 4\pi r^2$
$\Rightarrow A \propto r^2$

$\therefore \quad R_1 \propto r_1^2$ and $R_2 \propto r_2^2$

$\therefore \quad \frac{R_1}{R_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

141. By Stefan's law,

$R \propto T^4 \Rightarrow \frac{R_1}{R_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{27+273}{927+273}\right)^4$

$= \left(\frac{300}{1200}\right)^4 = \frac{1}{256}$

142. By Stefan's law,

$E \propto T^4 \Rightarrow \frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)$

$\therefore \quad \frac{7}{E_2} = \left(\frac{273+227}{273+727}\right)^4 = \frac{1}{16}$

$\therefore \quad E_2 = 112 \frac{\text{cal}}{\text{cm}^2 \times \text{s}}$

143. $Q \propto T^4$

$\Rightarrow \frac{H_A}{H_B} = \left(\frac{273+727}{273+327}\right)^4 = \left(\frac{10}{6}\right)^4 = \left(\frac{5}{3}\right)^4 = \frac{625}{81}$

144. By Stefan's law, $R \propto T^4$

$\therefore \quad \frac{R_2}{R_1} = \left(\frac{T_2}{T_1}\right)^4$

$\therefore \quad R_2 = R_1\left(\frac{T_2}{T_1}\right)^4$

$= 5 \times \left(\frac{727+273}{227+273}\right)^4 = 5 \times \left(\frac{1000}{500}\right)^4$

$= 80$ cal/m^{2}s

145. For perfectly black body,
$Q = \sigma AT^4 t$
$= 5.7 \times 10^{-8} \times 1 \times (727 + 273)^4 \times 60$
$= 3.42 \times 10^6$
$= 34.2 \times 10^5$ J

146. Rate of loss of heat, $E = \sigma eA(T^4 - T_0^4)$
$= 5.67 \times 10^{-8} \times 0.4 \times 200 \times 10^{-4} \times [(273 + 527)^4 - (273 + 27)^4]$
$= 5.67 \times 10^{-8} \times 0.4 \times 200 \times 10^{-4} \times (800)^4 - (300)^4 \approx 182$ J/s

147. For the black body,
Using, $E_b = \sigma T^4$,

$\therefore \quad 81 = \sigma(300)^4$ ....(i)

For ordinary body, Using,
$E = e\sigma T^4$,

$\therefore \quad E = 0.8 \times \sigma \times (500)^4$

$= 0.8 \times \frac{81}{(300)^4} \times (500)^4$ ....From (i)

$\therefore \quad E = \frac{64.8 \times 625}{81} = 500$ J/m^{2}s

148. $P = \sigma AT^4$

$\therefore \quad P \propto AT^4$

i.e., $P \propto r^2T^4$

$\therefore \quad \frac{P_2}{P_1} = \left(\frac{r_2}{r_1}\right)^2 \left(\frac{T_2}{T_1}\right)^4$

Now, $r_2 = \frac{r_1}{2}$ and $T_2 = 2T_1$

$\therefore \quad \frac{P_2}{P_1} = \frac{1}{4} \times 16$

$\therefore \quad P_2 = 4 \times 450 = 1800$ W

149. $\sigma \times 4\pi R^2.(T^4 - T_0^4) = 912 \times \pi R^2$

$\therefore \quad T^4 - T_0^4 = \frac{912}{4 \times \sigma} = \frac{912}{4 \times 5.7 \times 10^{-8}} = 40 \times 10^8$

$\therefore \quad T^4 = 40 \times 10^8 + (300)^4 = (40 + 81) \times 10^8$

$\therefore \quad T \approx 330$ K

150. The rate of radioactive energy emission from a hot surface is given by Stefan-Boltzmann Law-

$$R = \frac{dE}{dt} = \varepsilon\sigma A\left(T_{hot}^4 - T_{ambient}^4\right)$$

Hence, $\frac{R'}{R} = \frac{\left(400^4 - 200^4\right)}{\left(600^4 - 200^4\right)} = \frac{3}{16}$

151. Rate of loss of heat by radiation is given as –

$$\frac{dQ}{dt} = \sigma A\left(T_{hot}^4 - T_{cold}\right) = R$$

$\therefore \frac{R_A}{R_B} = \frac{\left(T_{hot}^4 - T_{cold}^4\right)_A}{\left(T_{hot}^4 - T_{cold}^4\right)_B}$

$\therefore \frac{R_A}{R_B} = \frac{\left[(327+273)^4 - (27+273)^4\right]}{\left[(227+273)^4 - (27+273)^4\right]}$

$= \frac{(600^4 - 300^4)}{(500^4 - 300^4)}$

$= 2.23$ or $\frac{9}{4}$

152. Using Stefan's law,

$R \propto AT^4 \propto r^2T^4$

$\therefore \frac{R_1}{R_2} = \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{T_1}{T_2}\right)^4$

$= \left(\frac{8}{2}\right)^2 \times \left(\frac{127+273}{527+273}\right)^4$

$= 16 \times \left(\frac{400}{800}\right)^4 = \frac{16}{16} = 1$

153. When water leaves the body through perspiration energy content of molecules remained in body decreases, therefore temperature also decreases.

154. According to Newton's law

Rate of cooling $\propto$ temperature difference $\Delta\theta$

155. According to Newton's law of cooling,

Rate of cooling $\propto$ Mean temperature difference

$\therefore \frac{\text{Fall in temperature}}{\text{Time}} \propto \left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$

$\because \left(\frac{\theta_1 + \theta_2}{2}\right)_1 > \left(\frac{\theta_1 + \theta_2}{2}\right)_2 > \left(\frac{\theta_1 + \theta_2}{2}\right)_3$

$\Rightarrow T_1 < T_2 < T_3$

156. According to Newton's law of cooling,

In first case, $\frac{70-60}{5} = K\left[\frac{70+60}{2} - 30\right]$

$\Rightarrow K = \frac{2}{35}$ °C/min

In 2^{nd} case, $\frac{60-50}{t} = K\left[\frac{60+50}{2} - 30\right]$

$\therefore \frac{10}{t} = \frac{2}{35}[55-30]$

$\therefore t = \frac{10 \times 35}{2 \times 25} = 7$ min

157. According to Newton's law of cooling,

In first case, $\frac{75-65}{t}$

$= K\left[\frac{75+65}{2} - 30\right]$ ….(i)

In second case, $\frac{55-45}{t}$

$= K\left[\frac{55+45}{2} - 30\right]$ ….(ii)

$\therefore$ Dividing equation (i) by (ii) we get,

$\frac{5t}{10} = \frac{40}{20} \Rightarrow t = 4$ minutes

158. By Newton's law of cooling,

$\frac{d\theta}{dt} = K(\theta - \theta_0)$

For 1^{st} case, $\frac{100-70}{8} = K(100-15)$

$\therefore \frac{30}{8} = K(85) \Rightarrow K = \frac{30}{8 \times 85}$ ….(i)

For 2^{nd} case, $\frac{70-40}{t} = K(70-15)$

$\therefore \frac{30}{t} = \frac{30 \times 55}{8 \times 85}$ ….[From (i)]

$\therefore t = \frac{8 \times 85}{55} = 12.36 \text{ s} \approx 14 \text{ s}$

159. By Newton's law of cooling,

$\frac{\theta_1 - \theta_2}{\Delta t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$

$\therefore \frac{3T - 2T}{10} = K\left[\frac{3T + 2T}{2} - T\right]$

$\frac{T}{10} = K\left[\frac{3T}{2}\right]$

$\therefore K = \frac{2}{30}$

Hence, let x be the temperature of the body at the end of next 10 minutes.

∴ $\frac{2T-x}{10}=\frac{2}{30}\left[\frac{2T+x}{2}-T\right]=\frac{2}{30}\left[\frac{x}{2}\right]$

∴ $2T-x=\frac{x}{3}\Rightarrow x=\frac{3T}{2}$

160. From Newton's law of cooling,

$\frac{\theta_1-\theta_2}{\Delta t}=K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$

For 1st case: $\frac{70-60}{5}=K[65-\theta_0]$ ….(i)

$\Rightarrow 2=K[65-\theta_0]$

For 2nd case: $\frac{60-54}{5}=K[57-\theta_0]$ ….(ii)

∴ Dividing equation (i) by equation (ii) we get,

$\frac{5}{3}=\frac{65-\theta_0}{57-\theta_0}$

∴ $285-5\theta_0=195-3\theta_0$

∴ $2\theta_0=90\Rightarrow\theta_0=45\ °C$

161. Using Newton's law of cooling.

$\frac{d\theta}{dt}=K(\theta-\theta_0)$

For 1st case, $0.5=K(50)$

$\Rightarrow K=\frac{0.5}{50}$

$\left(\frac{d\theta}{dt}\right)_2=K(30)=\frac{0.5}{50}\times 30=0.3\ °C/min$

162. As for a black body, rate of absorption of heat is more, thermometer A shows faster rise in temperature but finally both will acquire the atmospheric temperature.

163. By 1st law of thermodynamics,

$dU=dQ-dW\Rightarrow dU=dQ\ (<0)$

….[∵ dW = 0]

∴ $dU<0\Rightarrow$ Temperature will decrease.

164. For the first process, using $\Delta Q=\Delta U+\Delta W$.

$\Rightarrow 8\times10^5=\Delta U+6.5\times10^5$

$\Rightarrow \Delta U=1.5\times10^5$ J

Since final and initial states are same in both processes, ΔU will be same in both processes

For second process, using $\Delta Q=\Delta U+\Delta W$,

$\Rightarrow 10^5=1.5\times10^5+\Delta W$

$\Rightarrow \Delta W=-0.5\times10^5$ J

165. Given $\Delta t=60-30=30\ °C$

As the pressure remains constant,

For isobaric process, work done is

$W=P(\Delta V)$

Due to thermal expansion,

$\Delta V=V_0(\gamma\Delta t)=\frac{M}{\rho}(\gamma\Delta t)$

$=\frac{1.5}{9\times10^3}\times5\times10^{-5}\times30$

$=250\times10^{-9}\ m^3$

∴ $W=10^5\times250\times10^{-9}=25\times10^{-3}$ J

166. Using, $\Delta Q=mC_P\Delta T$,

$\Delta Q=100\times10^{-3}\times4184\times20\approx8.4\times10^3$

$\Delta Q\approx8.4$ kJ, $\Delta W=0$

∴ Using, $\Delta Q=\Delta U+\Delta W$,

$\Delta Q=\Delta U\approx8.4$ kJ

167. In adiabatic process, ΔQ = 0 and work is done on the system ⇒ internal energy of the system increases

∴ $\Delta U=\Delta W\Rightarrow\mu\times C_v\times\Delta T=\Delta W$

∴ $\mu\times\left(\frac{R}{\gamma-1}\right)\times7=146\times10^3$

$\Rightarrow 10^3\times\frac{8.3}{(\gamma-1)}\times7=146\times10^3$

⇒ On solving we get, γ = 1.4

⇒ The gas is diatomic.

168. Here, $\frac{1}{2}Mv^2=C_v.\Delta T$

$\Rightarrow\frac{1}{2}Mv^2=\frac{R}{\gamma-1}\Delta T$

∴ $\Delta T=\frac{M.v^2(\gamma-1)}{2R}=\frac{(\gamma-1)Mv^2}{2R}$

169. Given $T_1=27\ °C=300$ K

$T_2=0\ °C=273$ K

coefficient of performance $=\beta=\frac{Q_2}{W}=\frac{T_2}{T_1-T_2}$

here, $Q_2=mL$

∴ $\frac{mL}{W}=\frac{T_2}{T_1-T_2}$

∴ $W=\frac{2\times333\times10^3\times(300-273)}{273}$

∴ $W=\frac{2\times27\times333}{273}\times10^3=65.87\times10^3$ J

170. $\eta = 1 - \frac{T_2}{T_1}$ i.e., $\frac{1}{10} = 1 - \frac{T_2}{T_1}$

$\therefore \quad \frac{T_2}{T_1} = 1 - \frac{1}{10} = \frac{9}{10} \Rightarrow \frac{T_1}{T_2} = \frac{10}{9}$

$\therefore \quad W = Q_2\left(\frac{T_1}{T_2} - 1\right)$,

i.e., $10 = Q_2\left(\frac{10}{9} - 1\right)$ $10 = Q_2\left(\frac{1}{9}\right) \Rightarrow Q_2 = 90$ J

171. Given, $\eta' = \eta + \frac{8\eta}{100} = \frac{180\,\eta}{100}$

$\eta' = \frac{9}{5}\eta$

also, $T_1 = 100$ K (say)

it is increased by 25 %

$\therefore \quad T_1' = 125$ K

$T_2' = T_2$

$\therefore \quad \frac{\eta}{\eta'} = \frac{T_1 - T_2}{T_1} \times \frac{T_1'}{T_1' - T_2}$

$\frac{5}{9} = \frac{100 - T_2}{100} \times \frac{125}{125 - T_2} = \frac{100 - T_2}{125 - T_2} \times \frac{125}{100}$

$\therefore \quad 625 - 5T_2 = (900 - 9T_2) \times 1.25$

$= 1125 - 11.25\, T_2$

$\therefore \quad 6.25\, T_2 = 500$

$\therefore \quad T_2 = 80$ K

$\therefore \quad \eta' = \frac{T_1' - T_2}{T_1'} \times 100 = \frac{125 - 80}{125} \times 100$

$\therefore \quad \eta' = 36$ %

172. $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{390}{590} = \frac{20}{59}$

Heat used in work, $W = \eta Q = \frac{20}{59} \times 500$

$= 169.49$ kcal

Heat delivered to sink $= Q - W = 330.51$ kcal

173. $T_1 = 400$ K, $T_2 = 300$ K

For heat engine

$\eta = \frac{W}{Q} = \frac{T_1 - T_2}{T_1}$

$\therefore \quad W = Q \times \frac{400 - 300}{400}$

$\therefore \quad Q = \frac{W \times 400}{100} = 800 \times 4 = 3200$ J

174. $\eta = \frac{W}{Q}$ for heat engine

For carnot engine, $\eta = \frac{T_{Hot} - T_{Cold}}{T_{Hot}}$

$\therefore \quad W = Q \times \frac{T_H - T_C}{T_H} = Q \times \frac{500 - 200}{500}$

$\Rightarrow W = Q \times \frac{3}{5}$ $\qquad \Rightarrow Q = W \times \frac{5}{3}$

$\Rightarrow Q = \frac{800 \times 5}{3} = \frac{4000}{3}$ J

175. $\eta = 1 - \frac{T_2}{T_1} = \frac{1}{6}$

$\therefore \quad \frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6}$ ....(i)

When T_2 is reduced by 62 °C,

$\eta' = 2 \times \eta = \frac{2}{6} = \frac{1}{3}$

$\therefore \quad \frac{1}{3} = 1 - \frac{(T_2 - 62)}{T_1}$

$\therefore \quad \frac{T_2 - 62}{T_1} = \frac{2}{3}$

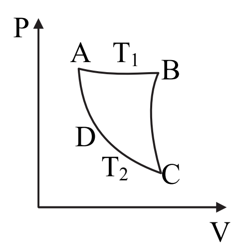
$\therefore \quad \frac{5(T_2 - 62)}{6 \times T_2} = \frac{2}{3}$ ....[From (i)]

$\therefore \quad T_2 = 310$ K and $T_1 = \frac{6 \times 310}{5} = 372$ K

176. $T_B = T_1$, $T_C = T_2$, $\gamma = 1.4$

$V_B = V$, $V_C = 32V$

Using, $T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$



$\therefore \quad \frac{T_C}{T_B} = \frac{T_2}{T_1} = \left(\frac{V_B}{V_C}\right)^{\gamma-1}$

$= \left(\frac{1}{32}\right)^{\gamma-1} = \frac{1}{4}$

$\therefore \quad \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75.$

177. According to Avogadro's law,

1 mole = 22.4 L of any gas

$\therefore \quad 67.2$ L = 3 mole $\qquad \therefore \quad n = 3$

$c_v = \frac{3}{2}R$ for monatomic gas

$\therefore \quad \Delta Q = n\, c_v\, \Delta T$

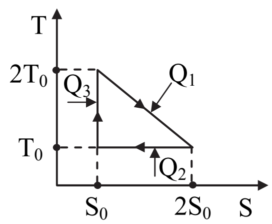
$= 3 \times \frac{8.31 \times 3}{2} \times 20$

$= 748$ J

178. From the graph, $W_{AB} = 0$ and
$W_{BC} = 8 \times 10^4[5 - 2] \times 10^{-3} = 240$ J
$\therefore$ $W_{AC} = W_{AB} + W_{BC} = 0 + 240 = 240$ J
$\therefore$ $\Delta Q_{AC} = \Delta Q_{AB} + \Delta Q_{BC} = 600 + 200 = 800$ J
By 1st law of thermodynamics,
$\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC}$
$\Rightarrow 800 = \Delta U_{AC} + 240$
$\Rightarrow \Delta U_{AC} = 560$ J

179. For the given cyclic process,
total work done = $W_{AB} + W_{BC} + W_{CA}$
$\Delta W_{AB} = P\Delta V = 10(2 - 1) = 10$ J and $\Delta W_{BC} = 0$
[$\because$ V = constant]
$\therefore$ By 1st law of thermodynamics,
$\Delta Q = \Delta U + \Delta W$
$\Delta U = 0$ (Process ABCA is cyclic)
$\therefore$ $\Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$
$\therefore$ $5 = 10 + 0 + \Delta W_{CA} \Rightarrow \Delta W_{CA} = -5$ J

180. $Q_1 = T_0S_0 + \frac{1}{2}T_0S_0 = \frac{3}{2}T_0S_0$,
$Q_2 = T_0S_0$ and $Q_3 = 0$



$\therefore$ $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$
$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$

181. For a cyclic process, $\Delta U = 0$
$\therefore$ By 1st law of thermodynamics,
$\Delta Q = \Delta U + \Delta W = 0 + \Delta W$
= Area of closed curve
$\therefore$ $\Delta Q = \pi r^2 = \pi\left(\frac{20}{2}\right)^2$ kPa $\times$ litre
$= 100\pi \times 10^3 \times 10^{-3}$ J $= 100\,\pi$J

182. Rate of cooling, $R = \frac{\Delta\theta}{t} = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$
$\Rightarrow R \propto \frac{A}{m} \propto \frac{\text{Area}}{\text{Volume}}$
$\Rightarrow$ For the same surface area, $R \propto \frac{1}{\text{Volume}}$
$\because$ Volume of cube < Volume of sphere
$\Rightarrow$ $R_{Cube} > R_{Sphere}$ i.e. cube cools down at a faster rate.

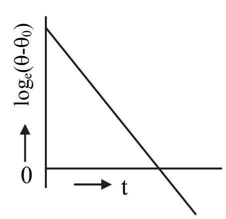
183. Rate of cooling (R) $= \frac{\Delta\theta}{t} = \frac{A \in \sigma(T^4 - T_0^4)}{mc}$
$\Rightarrow R \propto \frac{A}{m} \propto \frac{\text{Area}}{\text{volume}} \propto \frac{r^2}{r^3} \propto \frac{1}{r}$
$\Rightarrow$ Rate (R) $\propto \frac{1}{r} \propto \frac{1}{m^{1/3}}$
$\left[\because m = \rho \times \frac{4}{3}\pi r^3 \Rightarrow r \propto m^{1/3}\right]$
$\therefore$ $\frac{R_1}{R_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$

184. Rate of cooling $\left(-\frac{dT}{dt}\right) \propto$ emissivity (e)
From graph, $\left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y \Rightarrow e_x > e_y$
Further emissivity (e) $\propto$ Absorptive power (a)
$\Rightarrow a_x > a_y$
($\because$ good absorbers are good emitters).

185. According to Wien's displacement law,
$\lambda_m \propto \frac{1}{T} \Rightarrow \lambda_{m_2} < \lambda_{m_1}$ ($\because T_1 < T_2$)
Therefore I–λ graph for T_2 has lesser wavelength (λ_m) and so curve for T_2 will shift towards left side.

186. From Wien's displacement law,
$\lambda_{max} T = b$
Hence, $\frac{T_A}{T_B} = \frac{(\lambda_{max})_B}{(\lambda_{max})_A} = \frac{500}{300} = \frac{5}{3}$
Now, power Ratio, $\frac{Q_A}{Q_B} = \frac{\sigma A_A e T_A^4}{\sigma A_B e T_B^4} = \frac{r_A^2\, T_A^4}{r_B^2\, T_B^4}$
where, $A = 4\pi r^2$
i.e. $\frac{Q_A}{Q_B} = \frac{3^2}{5^2} \times \frac{5^4}{3^4} = \left(\frac{5}{3}\right)^2$

187. $\frac{d\theta}{dt} = -k(\theta - \theta_0)$
$\therefore$ $\int_{\theta_0}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k\int_0^t dt$
$ln\,(\theta - \theta_0) = kt + C$
So graph is straight line.



188. $\gamma_{mixture} = \frac{\left(\frac{n_1\gamma_1}{\gamma_1 - 1} + \frac{n_2\gamma_2}{\gamma_2 - 1}\right)}{\left(\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}\right)}$
Here, n_1 = moles of helium $= \frac{16}{4} = 4$
n_2 = moles of oxygen $= \frac{16}{32} = \frac{1}{2}$

$$\therefore \quad \gamma_{mix} = \frac{\left(\frac{4\times5/3}{\frac{5}{3}-1}+\frac{1/2\times7/5}{\frac{7}{5}-1}\right)}{\left(\frac{4}{\frac{5}{3}-1}+\frac{1/2}{\frac{7}{5}-1}\right)} = 1.62$$

189. i. The dotted line in the diagram shows that there is no change in the value of $\frac{PV}{nT}$ for different temperatures T_1 and T_2 for increasing pressure. Hence this gas behaves ideally. Hence, dotted line corresponds to 'ideal' gas behaviour.

ii. At high temperatures, the deviation of the gas is less and at low temperature the deviation of gas is more. In the graph, deviation for T_2 is greater than for T_1.

$\Rightarrow T_1 > T_2$

iii. The two curves intersect at dotted line. Hence, the value of $\frac{PV}{nT}$ at that point on the y-axis is same for all gases.

190. From ideal gas equation

$PV = nRT$

$PV = n_1RT$

After leakage,

$P'V = n_2RT$

No. of moles of gas leaked is given by $n_1 - n_2$

i.e. $n_1 - n_2 = \frac{PV}{RT} - \frac{P'V}{RT}$

$\Rightarrow n_1 - n_2 = \frac{V}{RT}(P - P')$

191. For step-1: Isothermal Expansion

$PV = P_2(2V)$ or $P_2 = \frac{P}{2}$

For step-2: Adiabatic Expansion

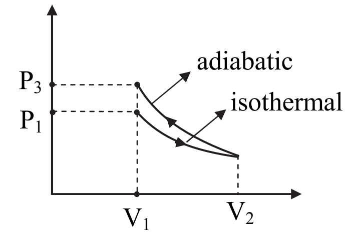
$P_2V_2^\gamma = P_3V_3^\gamma$

$\therefore \quad \frac{P}{2}(2V)^{\frac{5}{3}} = P_3(16V)^{\frac{5}{3}}$

$\therefore \quad P_3 = \frac{P}{2}\left(\frac{2V}{16V}\right)^{\frac{5}{3}} = \frac{P}{2}\times\left(\frac{1}{8}\right)^{\frac{5}{3}} = \frac{P}{64}$

192. The work done is negative.

Pressure $P_3 > P_1$



193. The temperature of the metal will decrease exponentially with time to θ_0.

194. $A \equiv (V_0, 2P_0)$; $B \equiv (2V_0, P_0)$

Equation of line AB in slope-point form is,

$y - y_1 = m(x - x_1)$

$\therefore \quad P - 2P_0 = \left(\frac{P_0 - 2P_0}{2V_0 - V_0}\right)(V - V_0)$

$\ldots\left\{\because m = \frac{y_2 - y_1}{x_2 - x_1}\right\}$

$\therefore \quad P = 2P_0 - \frac{P_0}{V_0}(V - V_0)$

$\therefore \quad P = 2P_0 - \frac{P_0}{V_0}V + P_0$

$\therefore \quad P = \frac{-P_0}{V_0}V + 3P_0$

From ideal gas equation we have;

$P = \frac{nRT}{V}$

$\therefore \quad \frac{nRT}{V} = \frac{-P_0}{V_0}V + 3P_0$

$\therefore \quad T = \frac{1}{nR}\left[\frac{-P_0}{V_0}V^2 + 3P_0V\right] \quad \ldots.(i)$

For $T = T_{max}$;

$\frac{dT}{dV} = 0$

$\therefore \quad \frac{1}{nR}\left[\frac{-2P_0}{V_0}V + 3P_0\right] = 0$

$\therefore \quad 3P_0 = \frac{2P_0}{V_0}V$

$\therefore \quad V = \frac{3}{2}V_0 \quad \ldots.(ii)$

substituting equation (ii) in equation (i)

$$T = \frac{1}{nR}\left[-\frac{P_0}{V_0}\left(\frac{3}{2}V_0\right)^2 + 3P_0\left(\frac{3}{2}V_0\right)\right]$$

$$\therefore \quad T = \frac{1}{nR}\left[-\frac{9}{4}P_0V_0 + \frac{9}{2}P_0V_0\right]$$

$$\therefore \quad T = \frac{9P_0V_0}{4nR}$$

195. Amount of energy required is given as,

$$E = \frac{f}{2}nRT = \frac{f}{2}NK(T_2 - T_1)$$

$$\therefore \quad E = \frac{f}{2}(n \cdot N_A)\cdot k_B \cdot (T_2 - T_1)$$

where $N = n.\ N_A$ and k_B = Boltzmann constant

$$\therefore \quad E = \frac{3}{2}nN_A k_B(T_2 - T_1)\cdot \quad ....[\because f = 3 \text{ for } H_e]$$

$$\text{Now, } n = \frac{m}{M} = \frac{1}{4}$$

$$\therefore \quad E = \frac{3}{2}\times\frac{1}{4}N_A k_B(T_2 - T_1) = \frac{3}{8}N_A k_B(T_2 - T_1)$$

196. Using, $c_{rms} = \sqrt{\frac{3RT}{M}}$,

$$(c_{rms})_O = \sqrt{\frac{3RT_1}{M_O}} \text{ and } (c_{rms})_H = \sqrt{\frac{3RT_2}{M_H}}$$

Given that,

$(c_{rms})_O = (c_{rms})_H$, $T_H = 127 + 273 = 400$ K

$$\therefore \quad \sqrt{\frac{3RT_1}{M_O}} = \sqrt{\frac{3RT_2}{M_H}}$$

$$\therefore \quad \frac{T_1}{M_O} = \frac{T_2}{M_H}$$

$$\therefore \quad T_2 = T_1 \times \frac{M_H}{M_O} = 400 \times \frac{2}{32}$$

$$= 25 \text{ K} = 25 - 273 = -248\ ^\circ\text{C}$$

197. Escape velocity at the surface of the earth

$= 11.2$ km/s $= 11.2 \times 10^3$ m/s

Oxygen will escape when rms speed of its molecules,

$c_{rms} = 11.2 \times 10^3$ m/s

$$\therefore \quad \sqrt{\frac{3k_BT}{m_0}} = 11.2 \times 10^3$$

$$\therefore \quad T = \frac{(11.2)^2\times10^6\times2.76\times10^{-26}}{3\times1.38\times10^{-23}} = 8.363 \times 10^4 \text{ K}$$

199. The entropy in an isolated system increases in accordance with second law of thermodynamics.

200. $Q_p = m.C_p\,\Delta\theta$ and $Q_v = m.C_v.\Delta\theta$

$$\therefore \quad \frac{Q_v}{Q_p} = \frac{C_v}{C_p}$$

Using, $C_p - C_v = R$ we get,

$$\frac{C_v}{C_p} = 1 - \frac{R}{C_p} = 1 - \frac{8.3}{20.7} \approx 0.6$$

$$\therefore \quad Q_v = Q_p \cdot \frac{C_v}{C_p} = 207 \times 0.6 = 124.2 \text{ J}$$

201. $P = \frac{1}{3}\left(\frac{U}{V}\right) = \frac{1}{3}kT^4$

($\because \frac{u}{V} \propto T^4$ and k is constant of proportionality)

$PV = nRT$

$$\frac{nRT}{V} = \frac{1}{3}kT^4$$

$\Rightarrow V \propto T^{-3}$

Volume of spherical shell of radius R = $\frac{4}{3}\pi R^3$

i.e., $V \propto R^3$

$$\Rightarrow R \propto \frac{1}{T}$$

202. According to Wien's displacement law,

$\lambda_m T$ = constant

$$\therefore \quad \frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} = \frac{3/4\lambda_0}{\lambda_0} = \frac{3}{4}$$

Power radiated for a black body, $P = \sigma AT^4$

$$\therefore \quad \frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$\therefore \quad \frac{P}{nP} = \left(\frac{3}{4}\right)^4 = \frac{81}{256}$$

$$\therefore \quad n = \frac{256}{81}$$

Evaluation Test

1. Before heating, let the pressure of gas be P.
PA = kx_1
$\therefore$ $x_1 = \frac{PA}{k} = \left(\frac{nRT}{V}\right)\frac{A}{k} = \frac{1\times8.3\times100\times10^{-2}}{0.83\times10} \approx 1$ m
During heating process,
the spring is compressed further by 0.1 m
$\therefore$ $x_2 = 1.1$ m
$\therefore$ Work done by gas = $\frac{1}{2}10(1.1^2 - 1^2) = 5 \times 0.21$
$= 1.05 \approx 1.0$ J

2. Since coefficient of linear expansion of bolt is more than that of pipe, the bolt will expand more. It implies that the bolt will become loose and hence will be free from stress.

3. Since molar specific heat is proportional to cube of temperature, the correct plot is B. At a particular temperature, the molar specific heat becomes almost constant.

4. Thermal expansion of isotropic object does not depend upon shape, size and presence of hole or cavity.

5. Black is a good absorber and also a good emitter as per Kirchhoff's radiation law.

6. Rate of flow of water = 2 litre min^{-1}
$= 2 \times 10^{-3}\ m^3\ min^{-1}$
Mass of water flowing per min,
$m = 2 \times 10^{-3} \times 10^3 = 2$ kg min^{-1}
$\Delta T = 77 - 27 = 50°C$
$c = 4.2 \times 10^3$ J $kg\ °C^{-1}$
Using $Q = mc\Delta T$, we get,
$Q = 2 \times 4.2 \times 10^3 \times 50 = 4.2 \times 10^5$ J min^{-1}
Rate of consumption of fuel
$= \frac{Q}{\text{heat of combination}} = \frac{4.2\times10^5\ J\ min^{-1}}{4\times10^7\ J/kg}$
$= 10.5 \times 10^{-3}$ kg min^{-1} = 10.5 g min^{-1}

7. Change in internal energy,
$\Delta U = nC_v\Delta T = nC_v(T_2 - T_1)$ …(i)
$\therefore$ Option (A) is correct
Using dQ = dU + dW
(1^{st} law of thermodynamics)
$\therefore$ dU = –dW
...[$\because$ dQ = 0 in adiabatic process]
Option (B) is correct.
In equation (i) if $(T_2 - T_1) = 0$, then $\Delta U = 0$
$\therefore$ Option (C) is also correct.
$\therefore$ (D) is correct.

8. Here, $\eta = \frac{T_1 - T_2}{T_1}$
$\therefore$ $\frac{1}{3} = \frac{T_1 - T_2}{T_1}$
$\therefore$ $3T_1 - 3T_2 = T_1$
$\therefore$ $T_1 = \frac{3}{2}T_2$
and $\frac{3}{3} = \frac{T_1 - (T_2 - 335)}{T_1}$
$\therefore$ $1 = \frac{3/2T_2 - T_2 + 335}{3/2T_2}$
i.e. $T_2 = 335$ K i.e. 62 °C
and $T_1 = \frac{3}{2}\times335 \approx 502$ K
$\therefore$ $T_1 = 502 - 273 = 229$ °C

9. Since power radiated is same for body A and body B,
$\therefore$ $\frac{T_A^4}{T_B^4} = \frac{0.49}{0.01}$ $\left(\because \frac{1}{\text{emissivity}} \propto T^4\right)$
or $\frac{T_A}{T_B} = \left(\frac{0.49}{0.01}\right)^{\frac{1}{4}} = 2.6$
or $T_B = \frac{T_A}{2.6} = \frac{5200}{2.6} = 2000$ K
Using Wien's displacement law
i.e., $\lambda_m T$ = constant
we get, $\lambda_A T_A = \lambda_B T_B$
or $\lambda_A = \lambda_B\left(\frac{T_B}{T_A}\right) = \frac{\lambda_B}{2.6}$
But $\lambda_B - \lambda_A = 1\ \mu m$ (given)
$\Rightarrow$ $\lambda_B - \frac{\lambda_B}{2.6} = 1\ \mu m$
or $\frac{1.6}{2.6}\lambda_B = 1\ \mu m$
or $\lambda_B = \frac{2.6}{1.6} \Rightarrow \lambda_B = 1.6\ \mu m$

10. For the given line AB, V and T both increase.

∴ Using PV = nRT, we get

$P(k'T) = nRT$ $\quad (\because V = k'T \text{ here})$

or $P = \dfrac{nR}{k'} = \text{constant}$

Therefore, in P-V diagram the corresponding line will be a straight line parallel to X-axis (V-axis) such that V is increasing.

For the given line BC, volume is constant but temperature is decreasing.

∴ $P = \dfrac{nRT}{\text{constant}}$

or $P \propto T$ (decreasing)

In P-V diagram, the corresponding line will be a straight line parallel to Y axis (P axis) with decreasing P.

For the given line CA, temperature is constant with volume decreasing

∴ $P = \dfrac{nRT}{V}$ i.e., PV = constant

∴ In P-V diagram, corresponding line is a hyperbola with P increasing.

11. As a and d are two points on the same adiabatic path,

∴ $T_1 (V_a)^{\gamma-1} = T_2 (V_d)^{\gamma-1}$

i.e. $\dfrac{T_1}{T_2} = \dfrac{(V_d)^{\gamma-1}}{(V_a)^{\gamma-1}}$

Similarly, $T_1 (V_b)^{\gamma-1} = T_2 (V_c)^{\gamma-1}$

i.e., $\dfrac{T_1}{T_2} = \dfrac{(V_c)^{\gamma-1}}{(V_b)^{\gamma-1}}$

∴ $\dfrac{(V_d)^{\gamma-1}}{(V_a)^{\gamma-1}} = \dfrac{(V_c)^{\gamma-1}}{(V_b)^{\gamma-1}}$

i.e. $\dfrac{V_d}{V_a} = \dfrac{V_c}{V_b}$ or $\dfrac{V_a}{V_d} = \dfrac{V_b}{V_c}$

12. Here, PV = constant

∴ $PdV = -VdP$

i.e. $\dfrac{dP}{dV} = -\dfrac{P}{V}$

Bulk Modulus, $K = \dfrac{-dP}{dV/V} = \dfrac{-dP}{dV}V$

$= -\left(\dfrac{-P}{V}V\right) = P$

13. For A, $dQ_A = nC_P dT_A$ $\quad (\because$ A is free to move)

For B, $dQ_B = nC_v dT_B$ $\quad (\because$ B is fixed)

Since, $dQ_A = dQ_B$

∴ $nC_p dT_A = nC_v dT_B$

or $dT_B = \left(\dfrac{C_p}{C_v}\right) dT_A = \gamma dT_A$

$= 1.4 \times 40 = 56$ K

15. $P_0 = \left(\dfrac{m_0}{M}\right)\dfrac{RT}{V} = \dfrac{10 \times R \times 293}{MV}$ ….(i)

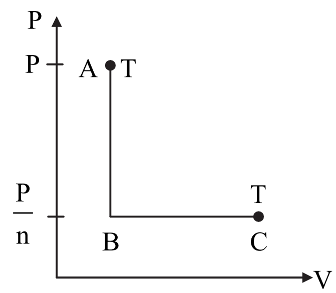
Gas is heated to 50 °C and x gram of gas escapes, pressure is still P_0

∴ $P_0 = \dfrac{(10-x)g}{M} \times R \times \dfrac{(273+50)}{V}$ ….(ii)

∴ $10(293) = (10-x)(323) \Rightarrow x \approx 0.92$ g

….[From (i) and (ii)]

16.



AB is an isochoric process

∴ $\dfrac{P_A}{T_A} = \dfrac{P_B}{T_B}$ or $\dfrac{P}{T} = \left(\dfrac{P}{n}\right)\dfrac{1}{T_B} \Rightarrow T_B = \left(\dfrac{T}{n}\right)$

For 1 mole of the gas,

$Q_{AB} = C_V \Delta T = C_V\left(\dfrac{T}{n} - T\right) = C_V T\left(\dfrac{1}{n} - 1\right)$

$= C_V T\left(\dfrac{1-n}{n}\right)$

$Q_{BC} = C_P \Delta T$ for 1 mole of the gas

$= C_P\left(T - \dfrac{T}{n}\right)$

$Q_{BC} = C_P T\left(\dfrac{n-1}{n}\right)$ $\quad Q_{net} = Q_{AB} + Q_{BC}$

$= C_V T\left(\dfrac{1-n}{n}\right) + C_P T\left(\dfrac{n-1}{n}\right)$

$= \dfrac{T}{n}(C_V - nC_V + nC_P - C_P)$

$= \dfrac{T}{n}\{(n(C_P - C_V) - (C_P - C_V)\}$

$= \dfrac{T}{n}(nR - R) = \dfrac{T}{n}(n-1)R$

$= RT(1 - n^{-1})$

17. Assertion is false, Reason is true.

$P_1V_1^{\gamma} = P_1V_2^{\gamma}$

$\therefore \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma}$

$\therefore V_2 = V_1\left[\frac{P_1}{P_2}\right]^{\frac{1}{\gamma}} = V_1C^{1/\gamma}$ ...(C > 1)

$\therefore V_2' = V_1'\left[\frac{P_1}{P_2}\right]^{\frac{1}{\gamma'}} = V_1C^{1/\gamma}$

$\because \gamma > \gamma'$

$\therefore \begin{bmatrix}\gamma \rightarrow \text{Monotonic} \\ \gamma' \rightarrow \text{Polyatomic}\end{bmatrix} \Rightarrow V_2' > V_2$

18. Isothermal compression $\Rightarrow$ T = constant

$v_{rms} = \sqrt{\frac{3RT}{M}}$

$\therefore$ Mean momentum = $m\bar{v} = m\sqrt{\frac{8RT}{\pi M}}$

Mean kinetic energy = $\frac{3}{2}RT$

All the above equations are functions of temperature, which is a constant.

19. According to Kirchhoff's law, good absorbers are good emitters and bad reflectors. While at lower temperature, a black-body absorbs all the incident radiations. It does not reflect any radiation incident upon it when it is thrown into the furnace. Initially, it is the darkest body.

At later times, the black body attains the temperature of the hot furnace and so it radiates maximum energy. It becomes the brightest of all.

Option (A) represents the answer.

20. $3PV = n_HRT$ ....(i)

$P(2V) = n_0R(3T)$ ....(ii)

Dividing equation (i) by (ii),

$\frac{3}{2} = \frac{n_H}{n_O}\frac{1}{3} \Rightarrow \frac{n_H}{n_O} = \frac{9}{2}$

Using Avogadro's principle,

$\frac{\rho_H}{\rho_O} = \frac{(2n_HN_A)/V}{(32n_ON_A)/2V} = \frac{n_H}{n_O}\frac{1}{8} = \frac{9}{16}$

Textbook Chapter No.

10 Wave Theory of Light

Hints

Classical Thinking

7. Light is electromagnetic in nature. It does not require any material medium for its propagation.

35. Frequency remains same, i.e. $n = n'$

36. $\sin r = \frac{\sin 30°}{1.6} = \frac{1}{3.2} = 0.3125$

$\therefore$ $r = 18°$

37. $\mu_g = \frac{\lambda_a}{\lambda_g}$

$\therefore$ $\lambda_g = \frac{\lambda_a}{\mu} = \frac{5460}{1.5} = 3640$ Å

38. $v_g = \frac{c}{\mu_g} = \frac{(3 \times 10^8)}{1.8} = 1.67 \times 10^8$ m/s

39. When a wave passes from one medium to another, its frequency remains unchanged

40. $v_g = 2 \times 10^8$ m/s, $v_w = 2.25 \times 10^8$ m/s

$\therefore$ ${}_g\mu_w = \frac{v_g}{v_w} = \frac{2\times10^8}{2.25\times10^8} = 0.89$

44. The magnitude of electric field vector varies periodically with time because it is the form of electromagnetic wave.

52. $\mu = \sqrt{3} = \tan i_p$

$\therefore$ $i_p = \tan^{-1}\left(\sqrt{3}\right) = 60°$

53. According to Brewster's law, when a beam of ordinary light (i.e. unpolarised) is reflected from a transparent medium (like glass), the reflected light is completely plane polarised at angle of polarisation.

54. At polarizing angle, the reflected and refracted rays are mutually perpendicular.

62. According to Doppler effect, wherever there is a relative motion between source and observer, the frequency observed is different from that given out by source.

65. When the source and observer approach each other, apparent frequency increases and hence wavelength decreases.

66. As the star is accelerated towards earth, its apparent frequency increases, hence apparent wavelength decreases. Therefore, colour of light changes gradually to violet.

67. When source is receding away, apparent wavelength increases. Displacement is towards red region.

68. Doppler effect does not apply to shock waves.

69. Radius of earth cannot be calculated from Doppler effect.

70. Wavelength of light decreases as same number of waves are not contained in a smaller distance.

71. Red shift implies that apparent wavelength λ' increases and hence apparent frequency ν' decreases.

Critical Thinking

6. $\bar{\nu}_m = \mu\bar{\nu} = \frac{4}{3} \times 3 \times 10^6 = 4 \times 10^6$/m

7. $i = 60°$, $r = (60° - 15°) = 45°$

$\therefore$ $\mu = \frac{\sin i}{\sin r} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{1} = 1.22$

8. Velocity of light in window, $v = \frac{c}{\mu} = \frac{3\times10^8}{1.5}$

$\therefore$ $v = 2 \times 10^8$ m/s

$\therefore$ time $= \frac{\text{distance}}{\text{velocity}} = \frac{4\times10^{-3}}{2\times10^8} = 2 \times 10^{-11}$ s

9. Distance that light travelled through glass slab

$d = v \times t$ (v = velocity of sunlight)

$\therefore$ Time taken by sunlight to penetrate,

$t = \frac{d}{v} = \frac{d}{c/\mu} = \frac{3\times10^{-3}\times1.5}{3\times10^8} = 1.5 \times 10^{-11}$ s

10. Distance travelled in medium,

$d = \frac{ct}{\mu} = \frac{3\times10^{10}\times10^{-9}}{3/2} = 20$ cm

11. Refractive index of medium, $\mu = \frac{c}{v}$

$\therefore$ $v = \frac{c}{\mu} = \frac{3\times10^8}{1.25} = 2.4 \times 10^8$ m/s

12. No. of waves = $\dfrac{\text{thickness}}{\text{wavelength}}$

$\therefore$ Thickness of glass piece $= \dfrac{18\times1.33}{1.5} = 15.96 \approx 16$ cm

13. Number of waves,

$N = \dfrac{t}{\lambda} \Rightarrow \dfrac{t}{\lambda}$ = constant for same N

$\therefore \dfrac{X}{\lambda_\omega} = \dfrac{4}{\lambda_g}$

$\therefore X = 4 \times \dfrac{\lambda_\omega}{\lambda_g} = 4 \times \dfrac{\mu_g}{\mu_\omega} = 4 \times \dfrac{5/3}{4/3} = 5$ cm

14. $\mu_g = \dfrac{(\lambda_r)_{air}}{(\lambda_r)_{glass}} = \dfrac{6400}{4000} = 1.6$

$\therefore \mu_g = \dfrac{(\lambda_v)_{air}}{(\lambda_v)_{glass}}$

$\therefore (\lambda_v)_{glass} = \dfrac{(\lambda_v)_{air}}{\mu_g} = \dfrac{4400}{1.6} = 2750$ Å

15. $\mu_1 \sin\alpha = \mu_2 \sin\beta = \mu_3 \sin\gamma = \mu_4 \sin\delta$

As AB and CD are parallel, $\alpha = \delta$

$\therefore \mu_1 = \mu_4$

16. $\dfrac{AB}{AD} = \cos i$, $\dfrac{CD}{AD} = \cos r$

$\therefore \dfrac{AB}{CD} = \dfrac{\cos i}{\cos r}$

(Note: Refer Mindbender 3.)

17. $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin i}{\sin(i/2)} = \dfrac{2\sin\frac{i}{2}\cdot\cos\frac{i}{2}}{\sin\left(\frac{i}{2}\right)}$

$\therefore \dfrac{\mu}{2} = \cos\dfrac{i}{2}$ $\qquad \therefore \dfrac{i}{2} = \cos^{-1}\left(\dfrac{\mu}{2}\right)$

$\therefore i = 2\cos^{-1}\left(\dfrac{\mu}{2}\right)$

18. Let λ_g (in cm) and λ_w (in cm) be the wavelengths in glass and water. By definition, in a distance λ there is one wave. Therefore,

Number of waves in 8 cm of glass = $8/\lambda_g$,

Number of waves in 10 cm of water = $10/\lambda_w$

Thus, $\dfrac{8}{\lambda_g} = \dfrac{10}{\lambda_w} \Rightarrow \dfrac{\lambda_w}{\lambda_g} = \dfrac{5}{4}$

Now, $\mu_g = \dfrac{c}{v_g}$ and $\mu_w = \dfrac{c}{v_w}$

$\therefore \dfrac{\mu_g}{\mu_w} = \dfrac{v_w}{v_g} = \dfrac{n\lambda_w}{n\lambda_g} = \dfrac{\lambda_w}{\lambda_g}$

$\therefore \mu_g = \dfrac{\lambda_w}{\lambda_g} \times \mu_w = \dfrac{5}{4} \times \dfrac{4}{3} = \dfrac{5}{3}$

19. $v_d = \dfrac{5}{12}c \Rightarrow \dfrac{v_d}{c} = \dfrac{5}{12}$; $v_w = \dfrac{3}{4}c \Rightarrow \dfrac{v_w}{c} = \dfrac{3}{4}$

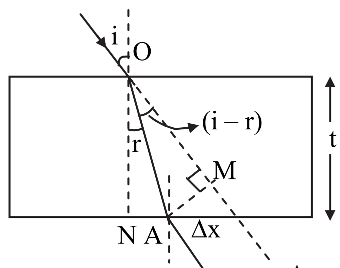
${}_w\mu_d = \dfrac{\mu_d}{\mu_w} = \dfrac{c/v_d}{c/v_w} = \dfrac{12/5}{4/3} = \dfrac{12}{5} \times \dfrac{3}{4} = \dfrac{9}{5}$

Using, ${}_w\mu_d = \dfrac{\sin i}{\sin r}$

$\therefore \sin i = {}_w\mu_d \times \sin r = \dfrac{9}{5} \times \sin 30° = \dfrac{9}{5} \times \dfrac{1}{2} = \dfrac{9}{10}$

$\therefore i = \sin^{-1}\left(\dfrac{9}{10}\right)$

20.



In ΔOAM, $\sin(i - r) = \dfrac{\Delta x}{OA}$

$\therefore \Delta x = OA \sin(i - r)$ ....(i)

ΔOAN, $\cos r = \dfrac{ON}{OA}$

$\therefore OA = \dfrac{t}{\cos r}$ ....(ii)

From (i) and (ii),

$\Delta x = \dfrac{t\sin(i-r)}{\cos r}$

21. ${}_w\mu_g = \dfrac{v_w}{v_g}$

$\therefore v_w = {}_w\mu_g \times v_g = \dfrac{9}{8} \times 2 \times 10^8 = 2.25 \times 10^8$ m/s

22. $I' = \dfrac{I}{2}\cos^2\theta = \dfrac{I}{6}$

$\therefore \cos\theta = \dfrac{1}{\sqrt{3}}$ $\qquad \therefore \theta = 55°$

23. Let i, r and r′ be the angle of incidence, reflection and refraction respectively.

Let $r + r' = 90°$

$\therefore r = 90° - 30° = 60°$ $\qquad \therefore i = r = 60°$

24. From Brewster's law, $\mu = \tan i_p$

$\mu = \frac{c}{v} = \tan 60° = \sqrt{3}$

$\therefore \quad v = \frac{c}{\sqrt{3}} = \frac{3\times10^8}{\sqrt{3}} = \sqrt{3} \times 10^8$ m/s

25. $\tan i_p = \mu = 1.55$

$\therefore \quad i_p = 57°10'$

$r = 90° - i_p = 90° - 57°17' = 32°49'$

26. From the figure,

$i + r = 90° \Rightarrow r = 90° - i$

$\therefore \quad \mu = \frac{\sin i}{\sin r} = \tan i = \frac{\sin i}{\sin(90 - i)}$

$\therefore \quad \sin i_c = \frac{1}{\mu} = \cot i$

28. When unpolarised light is made incident at polarising angle, the reflected light is plane polarised in a direction perpendicular to the plane of incidence.

Therefore, $\vec{E}$ in reflected light will vibrate in vertical plane with respect to plane of incidence.

31. $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$; $\Delta\lambda = \frac{0.5}{100}$

$\therefore \quad v = \frac{0.5}{100} \times c = \frac{0.5}{100} \times 3 \times 10^8 = 1.5 \times 10^6$ m/s

32. Doppler shift is given by

$\Delta\lambda = \frac{v\lambda}{c} = \frac{5000\times6000}{3\times10^8} = 0.1$ Å

33. $\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c} = \frac{6\times10^7}{3\times10^8} = 0.2$

$\therefore \quad \Delta\lambda = 0.2\,\lambda_0$

$\therefore \quad \lambda' = 1.2\,\lambda_0 = 1.2 \times 4600 = 5520$ Å

34. $\Delta\lambda = 5200° - 5000° = 200$ Å

$\therefore \quad \frac{\Delta\lambda}{\lambda_0} = \frac{v}{c} \Rightarrow v = \frac{c\Delta\lambda}{\lambda_0}$

$\therefore \quad v = \frac{3\times10^8\times200}{5000} = 1.2 \times 10^7$ m/s.

35. Observed frequency, $\nu' = \nu\left(1 - \frac{v}{c}\right)$

$\therefore \quad \nu' = 6 \times 10^{14}\left(1 - \frac{0.8\,c}{c}\right) = 1.2 \times 10^{14}$ Hz

37. Time required for light to reach from source to slab is $t_1 = \frac{x}{c}$ where c = velocity of light in air.

Time required for light to pass through slab is $t_2 = \frac{d}{v}$ where v = velocity of light in glass

According to given condition, $t_1 = t_2$

$\therefore \quad \frac{x}{c} = \frac{d}{v} \Rightarrow d = \frac{x.v}{c} = \frac{x}{(c/v)} = \frac{x}{\mu}$

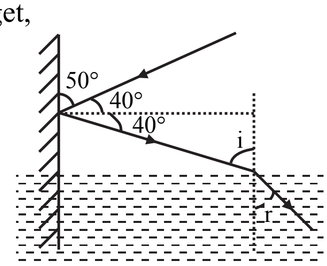
38. $i + i' = 90°$

$\therefore \quad i = 45° \quad (\because i = i')$

$\mu = \frac{\sin i}{\sin r} = \frac{3}{2}$

$\therefore \quad \sin r = \frac{2}{3} \times \sin i = \frac{2}{3} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{3}$

39. Using, $\mu = \frac{\sin i}{\sin r}$ we get,



$\sin r = \frac{\sin 50°}{\mu}$

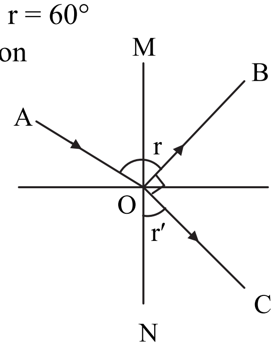
$= \frac{0.76}{1.33} = 0.57$

$\therefore \quad r = \sin^{-1}(0.57)$

$\Rightarrow r = 35°$

40. $i = 60°$, reflecting angle, $r = 60°$

Let r' = angle of refraction



$\therefore \quad \angle BOC = 90°$

$\therefore \quad r + r' = 90°$

$\therefore \quad r' = 90° - 60° = 30°$

From Snell's law,

$\mu = \frac{\sin i}{\sin r}$

$\therefore \quad \mu = \frac{\sin 60°}{\sin 30°}$

$\therefore \quad \mu = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} \Rightarrow \mu = \sqrt{3}$

41. According to Doppler effect,

$\lambda' = \lambda\sqrt{\frac{1 - v/c}{1 + v/c}}$ for v = c

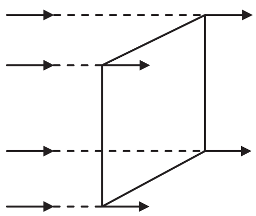
$\lambda' = 5500\sqrt{\frac{(1 - 0.8)}{1 + 0.8}} = 1833.3$ Å

$\therefore \quad$ Shift = 5500 – 1833.3 ≈ 3667 Å

Competitive Thinking

2. Huygens' wave theory fails to explain the particle nature of light (i.e. photoelectric effect)

5. When the point source or linear source of light is placed at very large distance, a small portion of spherical or cylindrical wavefront appears to be plane and is called a plane wavefront.



Among the given options none of the sources generates plane wavefront, it can be artificially produced by reflection from a mirror or by refraction through a lens.

6. Direction of wave is perpendicular to the wavefront.

8. Origin of spectra is not explained by Huygens' theory.

9. The locus of all particles in a medium vibrating in the same phase is called wavefront.

10. On the wavefront, all the points are in same phase.

11. From Huygens' principle, if the incident wavefront be parallel to the interface of the two media (i = 0), then the refracted wavefront will also be parallel to the interface (r = 0). In other words, if light rays fall normally on the interface, then on passing to the second medium, they will not deviate from their original path.

13. $\mu = \frac{\sin i}{\sin r}$ and $\mu = \frac{c}{v}$

$\therefore$ For same i, as r increases, value of μ decreases.

But $\mu \propto \frac{1}{v}$, hence as value of μ decreases v increases.

This means as sin r increases v increases. Therefore, speed of light is minimum where angle of refraction is minimum.

14. $\mu = \frac{c}{v} = \frac{\sin i}{\sin r}$

$\therefore \quad v = c\frac{\sin r}{\sin i} = 3 \times 10^8 \times \frac{\sin 30^\circ}{\sin 45^\circ}$

$= 3 \times 10^8 \times \frac{\sqrt{2}}{2} = 2.12 \times 10^8$ m/s

15. $\mu = \frac{\sin i}{\sin r} = \frac{i}{r}$ $\quad (\because \ i <<, \sin i \approx i)$

$\mu = \frac{c}{v} = \frac{c}{0.75c} = \frac{i}{r}$

$\therefore \quad r = 0.75\ i = \frac{3}{4}i$

$\delta = i - r = i - \frac{3}{4}i = \frac{i}{4}$

16. $\mu = \frac{c_a}{c_g} = \frac{c_a}{0.8c_a}$

$\therefore \quad \frac{\sin i}{\sin r} = \frac{1}{0.8}$

for small angle i, sin i ≈ i and sin r ≈ r

$\therefore \quad \frac{i}{r} = \frac{1}{0.8}$

$\therefore \quad r = 0.8\ i$

Angle of deviation,

$\delta = i - r = i - 0.8\ i = 0.2\ i$

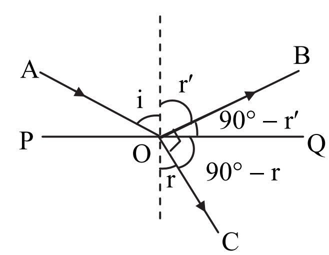
$\delta = \frac{i}{5}$

17. ${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.5}{1.3}$

18. From the figure,

$\angle BOC = 90^\circ$

$\therefore \quad \mu_g = \frac{\lambda a}{\lambda g} = 1.5$



19. Using c = n λ,

$\lambda_a = \frac{c}{n_a} = \frac{3 \times 10^8}{4 \times 10^{14}} = 0.75 \times 10^{-6}$ m

$\therefore \quad \lambda_a = 7500$ Å

Now, ${}_a\mu_g = \frac{c}{v_g} = \frac{n_a\lambda_a}{n_g\lambda_g} = \frac{\lambda_a}{\lambda_g}$

$\therefore \quad \lambda_g = \frac{\lambda_a}{{}_a\mu_g} = \frac{7500}{1.5} = 5000$ Å

$\therefore \quad \lambda_a - \lambda_g = 7500 - 5000 = 2500$ Å

$= 2500 \times 10^{-10} = 2.5 \times 10^{-7}$ m

21. $\bar{v} = \frac{1}{\lambda} = \frac{1}{5000 \times 10^{-10}}$

$= \frac{10^7}{5} = 0.2 \times 10^7$

$= 2 \times 10^6$

22. No. of wavelengths in a meter is called as wave number.

$\therefore \quad \bar{v} = \frac{1}{\lambda} = \frac{1}{4000 \times 10^{-10}}$

$= 25 \times 10^5 \text{ m}^{-1}$

$= 2500 \text{ mm}^{-1}$

23. $f = 9 \text{ GHz} = 9 \times 10^9 \text{ Hz}$

Velocity of radiation in air,

$c = 3 \times 10^8 \text{ m/s}$

$\therefore \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^9} = \frac{10^{-1}}{3} \text{ m}$

Wave number for the wavelength,

$\bar{v} = \frac{1}{\lambda}$

$\therefore$ Here, number of waves in 1 m,

$\bar{v} = \frac{\text{length}}{\lambda} = \frac{1}{\frac{10^{-1}}{3}} = 30$

24. ${}_a\mu_m = \frac{\lambda_a}{\lambda_m}$

$\therefore \quad \frac{1}{\lambda_m} = \bar{v}_m = \frac{{}_a\mu_m}{\lambda_a} = \frac{4}{3 \times 6000 \times 10^{-10}}$

$= 0.222 \times 10^7$

$= 2.2 \times 10^6$

25. $\mu_g = \frac{c}{v}$

$\therefore \quad v = \frac{c}{\mu_g} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$

$v = \frac{\text{thickness of glass plate (d)}}{\text{time require to travel through it (t)}}$

$\therefore \quad t = \frac{d}{v} = \frac{2 \times 10^{-3}}{2 \times 10^8} = 10^{-11} \text{ s}$

26. $\mu = \frac{c}{v}$

$\Rightarrow v = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$

$t = \frac{\text{distance}}{\text{speed}} = \frac{4 \times 10^8}{2 \times 10^8} = 2 \text{ s}$

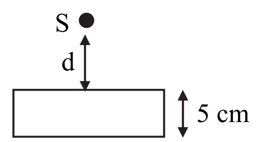
27. ${}_g\mu_w = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{v_g}{v_w}$

$\therefore \quad \frac{v_g}{v_w} = \frac{1.33}{1.5} = 0.8867 : 1$

28. ${}_a\mu_g = \frac{v_a}{v_g} = \frac{d_a / t}{d_g / t} = \frac{d_a}{d_g} = \frac{x}{5}$

$\therefore \quad x = 5 \times 1.5 = 7.5 \text{ cm}$

29.



thickness of slab (t) = 5 cm

$\mu = 1.6$

Now, $\mu_g = \frac{c}{v} = \frac{d/T}{t/T} = \frac{d}{t}$

$\therefore \quad d = \mu t = 1.6 \times 5 = 8 \text{ cm}$

30. ${}_a\mu_g = \frac{\sin i}{\sin r} \Rightarrow 1.5 = \frac{\sin 45}{\sin r}$

$\therefore \quad \sin r = \frac{1}{1.5\sqrt{2}} \Rightarrow r = 28°7'$

Ratio of widths $= \frac{\cos i}{\cos r} = \frac{\cos 45°}{\cos 28°7'}$

$\therefore$ Ratio of widths $= 0.801 = \frac{1}{1.2475}$

31. $\mu = \frac{c}{v} = \frac{c}{t/T} = \frac{cT}{t}$

$\therefore \quad T = \frac{\mu t}{c}$

32. Speed of light in medium of ref. index (4/3) is

$v = \frac{c}{\mu} = \frac{3 \times 10^8}{4/3} = \frac{9 \times 10^8}{4}$

$\Rightarrow t = \frac{d}{v} = \frac{4.5 \times 4}{9 \times 10^8} = 2 \times 10^{-8} = 20 \text{ ns}$

33. Given: $N_g = N_w$

But number of waves $N = \frac{d}{\lambda}$;

where d = thickness of the medium

$\therefore \quad \frac{d_g}{\lambda_g} = \frac{d_w}{\lambda_w}$

But $\lambda_g = \frac{\lambda_{air}}{\mu_g}$ and $\lambda_w = \frac{\lambda_{air}}{\mu_w}$

$\therefore \quad \mu_g d_g = \mu_w d_w$

$\therefore \quad \mu_w = \frac{\mu_g d_g}{d_w} = \frac{1.5 \times 6}{7} = \frac{9}{7} = 1.286$

34. In double refraction, light rays always splits into two rays (O-ray and E-ray). O-ray has same velocity in all direction but E-ray has different velocity in different direction.
For calcite $\mu_E < \mu_o \Rightarrow v_E > v_o$
For quartz $\mu_E > \mu_o \Rightarrow v_o > v_E$

35. Polarisation is not shown by sound waves.

36. Ultrasonic waves are longitudinal waves.

39. In the figure shown, the unpolarised light is incident at polarising angle of 90° – 33° = 57°. Hence, the reflected light is plane polarised. When plane polarised light is passed through Nicol prism (a polariser or analyser), the intensity gradually reduces to zero and finally increases.

40. When the plane-polarised light passes through certain substance, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle.

42. Given: Reflected ray and refracted ray are perpendicular to each other. This implies that the angle of incidence equals polarising angle (i_p). For $i = i_p$, reflected light is completely plane polarised i.e., its electric vector is perpendicular to the plane of incidence.

43. $\delta = i - r$
but $i = i_p$
∴ $i_p - r = \delta = 24°$ ....(i)
$i_p + r = 90°$ ....(ii)
Solving equations (i) and (ii),
$i_p = 57°$

44. $\theta_P + r = 90°$
$r = 90° - 57° = 33°$

45. $\mu = \tan i_p = \tan 54.74° = \sqrt{2}$
∵ $\sqrt{2} = \dfrac{\sin 45°}{\sin r}$
∴ $\sin r = \dfrac{1}{2} \Rightarrow r = 30°$

46. $\tan i = \mu$ (by Brewster's Law)

47. For polarising angle,
$\tan\theta = \mu = \dfrac{c}{v}$
∴ $\cot\theta = \dfrac{v}{c}$ ∴ $\theta = \cot^{-1}\left(\dfrac{v}{c}\right)$

49. $\mu = \tan i_P$ ∴ $i_P = \tan^{-1}\mu$

51. By using $\mu = \tan i_p$
∴ $\mu = \tan 60° = \sqrt{3}$,
Also, $C = \sin^{-1}\left(\dfrac{1}{\mu}\right)$
∴ $C = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

52. Given that,
reflected ray is plane polarised.
Using Brewster's law,
∴ $\dfrac{\mu_g}{\mu_w} = \tan i$
∴ $\mu_g = \tan(51°) \times \mu_w = 1.235 \times 1.4 = 1.73$

53. Shifting towards violet region shows that apparent wavelength has decreased. Hence we conclude that the source is moving towards the earth.

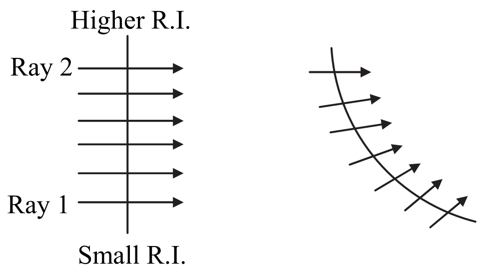
54. $\Delta\lambda = \lambda\dfrac{v}{c} = 5700 \times \dfrac{100\times10^3}{3\times10^8} = 1.90$ Å

55. Here, $\Delta\lambda = 0.5$ nm $= 0.5 \times 10^{-9}$ m
$v = 300$ km s^{-1} $= 300 \times 10^3$ ms^{-1}
$\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$
∴ $\lambda = \dfrac{\Delta\lambda c}{v}$
∴ $\lambda = \dfrac{0.5\times10^{-9}\times3\times10^8}{300\times10^3}$
$= 5 \times 10^{-7}$ m
$= 5000 \times 10^{-10}$ m
$= 5000$ Å

56. $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$
∴ $v = \dfrac{\Delta\lambda}{\lambda}c = \dfrac{5}{6563}\times3\times10^8\,\text{km/s} = 2.29\times10^5\,\text{m/s}$

57. As speed of observer is comparable to speed of light, given motion is relativistic.
∴ Apparent frequency,
$\nu = \nu_0\sqrt{\dfrac{c+v}{c-v}}$
$= 10\sqrt{\dfrac{c+\frac{c}{2}}{c-\frac{c}{2}}}$
$= 10\sqrt{3}$ GHz $= 17.3$ GHz.

58. Consider a plane wavefront travelling horizontally. When it moves, its different parts move with different speeds (as $\mu \propto \frac{1}{v}$). Ray 1 will travel faster than Ray 2. So, its shape will change as shown and beam will bend upward.



59. The amplitude will be A cos 60° = $\frac{A}{2}$

60. $_m\mu_g = \frac{\mu_g}{\mu_m} = \frac{v_m}{v_g} = \frac{4}{3}$

$\therefore \quad \frac{v_m - v_g}{v_g} = \frac{4-3}{3} = \frac{1}{3}$ ….(i)

Given that, $v_m - v_g = 6.25 \times 10^7$

Substituting in equation (i),

$\therefore \quad v_g = 3 \times 6.25 \times 10^7$ m/s

$\therefore \quad v_m = 6.25 \times 10^7 + 3 \times 6.25 \times 10^7$

$\therefore \quad = 4 \times 6.25 \times 10^7$

$= 25 \times 10^7$

$= 2.5 \times 10^8$ m/s

61. $_a\mu_m = \frac{v_a}{v_m} = \frac{\lambda_a}{\lambda_m}$

$\therefore \quad \frac{\lambda_a}{\lambda_m} = 1.5 = \frac{3}{2}$

$\therefore \quad \frac{\lambda_m}{\lambda_a} = \frac{2}{3} \Rightarrow \frac{\lambda_m - \lambda_a}{\lambda_a} = \frac{2-3}{3} = \frac{-1}{3}$

$\therefore$ Percentage change = $\frac{1}{3} \times 100$

= 33.33% (in magnitude)

62. $i = 2r$

$\therefore \quad r = i/2$

$\mu = \frac{\sin\ i}{\sin\ r}$

$= \frac{\sin i}{\sin(i/2)}$

$= \frac{2\sin(i/2)\cdot\cos(i/2)}{\sin(i/2)}$

$\therefore \quad \frac{\mu}{2} = \cos\frac{i}{2} \Rightarrow \frac{i}{2} = \cos^{-1}\left(\frac{\mu}{2}\right)$

$\therefore \quad i = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

63. Doppler shift when the source is moving towards observer, $\lambda' = \lambda\left(1 - \frac{v}{c}\right)$

$\therefore \quad 5400$ Å = 6200 Å $\left(1 - \frac{v}{c}\right)$

$\therefore \quad v = \left[1 - \frac{54}{62}\right]c \approx 3.9 \times 10^7$ m/s

Evaluation Test

1. $_a\mu_m = \frac{v_a}{v_m} = \frac{\nu\lambda_a}{\nu\lambda_m} = \frac{\lambda_a}{\lambda_m}$

Also, $_a\mu_m = \tan i_p$

$\therefore \quad \tan i_p = \frac{\lambda_a}{\lambda_m}$

$\therefore \quad \lambda_m = \lambda_a\left(\frac{1}{\tan i_p}\right)$

or $\lambda_a = \lambda_m \tan i_p$

2. Let I_0 be the intensity of unpolarised light. The intensity transmitted by the first sheet is $\frac{I_0}{2}$.

Therefore transmitted intensity = $\left(I_0 - \frac{I_0}{2}\right) = \frac{I_0}{2}$

This will be the intensity of incident light on the second polaroid. The intensity transmitted by the second polaroid will be $\left(\frac{I_0}{2}\right)\cos^2\theta$

where θ is the angle between their axes.

$\sin\theta = \frac{4}{5} \Rightarrow \cos\theta = \frac{3}{5}$

∴ $\left(\frac{I_0}{2}\right)\cos^2\theta = \left(\frac{I_0}{2}\right)\left(\frac{3}{5}\right)^2 = \frac{9}{25}I_0$

Ratio of intensity of emergent light to that of unpolarised light = $\frac{9}{25}$

3. Let θ be the angle between the first two polarisers and ϕ be the angle between the next two. Here,

$\theta + \phi = 90°$

If I_0 is the intensity of the incident unpolarised light, then the intensity after passing the first polariser,

$I_1 = I_0 (\cos^2\theta)_{Av} = \frac{I_0}{2}$

$I_2 = I_1 \cos^2\theta$ and

$I_3 = I_2 \cos^2\phi = I_2\cos^2(90-\theta) = I_2\sin^2\theta$

∴ $I_3 = (I_1\cos^2\theta)\sin^2\theta$

∴ $I_3 = \frac{I_1}{4}\sin^2 2\theta = \frac{I_0}{8}\sin^2 2\theta$

Now, $I_3 = 2\ Wm^{-2}$ and $I_0 = 32\ Wm^{-2}$

∴ $2 = \frac{32}{8}\sin^2 2\theta$

∴ $\sin^2 2\theta = \frac{1}{2}$ or $\sin 2\theta = \frac{1}{\sqrt{2}}$

∴ $2\theta = 45°$ or $\theta = 22.5°$

4. As $\nu\lambda = c$,

∴ $\ln\nu + \ln\lambda = \ln c$

∴ $\frac{d\nu}{\nu} + \frac{d\lambda}{\lambda} = 0$

∴ $\frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda}$ (for small changes in ν and λ)

∴ $\frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda} = \frac{v_{radial}}{c}$

or $v_{radial} = c\left[\frac{0.4}{674}\right] = \frac{3\times10^8\times0.4}{674}$

…..[∵ $\Delta\lambda = 0.4$ nm]

$= 1.78 \times 10^5$ m/s

$= 640\ kms^{-1}$

5. $I = I_0\cos^2\theta$

$IA = (I_0 A)\cos^2\theta$, where A is the area of the polariser.

$P = P_0\cos^2\theta$, where P represents power.

∴ $P_{Average} = P_0(\cos^2\theta)_{Average} = \frac{P_0}{2}$

….[∵ average of $\cos^2\theta$ over a cycle is $\frac{1}{2}$]

$T = \frac{2\pi}{\omega} = \frac{2\times\pi}{\pi} = 2$ s

∴ Energy passing through per revolution $= P_{average} \times 2$ s

$= P_0\left(\frac{1}{2}\right)\times 2 = (10^{-2}\ W)\times\left(\frac{1}{2}\right)\times 2 = 10^{-2}$ J

6. Assertion is false, Reason is true.

If light is polarised by reflection, then the angle between reflected and refracted rays is 180°.

7. $\Delta\lambda = \lambda - \lambda' = 6820 - 6800 = 20$ Å

Also, $\Delta\lambda = -\frac{v\lambda}{c}$

i.e., $v = \frac{\Delta\lambda}{\lambda}c = \frac{-20}{6820}\times(3\times10^8)$

$= -8.79\times10^5\ ms^{-1}$

(The negative sign indicates receding speed).

8. Intensity $= \frac{\text{energy}}{\text{time}\times\text{area}}$

$= \frac{E}{t\times 2\pi d l}$

⇒ Intensity $\propto \frac{1}{d}$

But intensity ∝ Amplitude^2

∴ Amplitude$^2 \propto \frac{1}{d}$

or Amplitude $= \frac{1}{\sqrt{d}} = \frac{1}{d^{1/2}}$

9. Here for minima,

$a\sin\theta = n\lambda$

For first dark band, n = 1

∴ $\sin\theta = \frac{\lambda}{a}$ or $\theta = \frac{\lambda}{a}$

….(∵ $\sin\theta \simeq \theta$ for small angles)

Let distance of first dark band from axis be y then angle of diffraction θ is given by $\frac{x}{f}$

∴ $\frac{x}{f} = \frac{\lambda}{a}$ or $x = \frac{\lambda}{a}f$

10. $\mu = \frac{\sin i}{\sin r}$

$\sin r = \frac{\sin i}{\mu} = \frac{\sin 35°}{1.5} = \frac{0.5736}{1.5}$

$\therefore$ $\sin r = 0.3824$

$\therefore$ $r = 22.48° = 22° 29'$

$\therefore$ Required ratio = $\frac{W_2}{W_1} = \frac{\cos 22.48'}{\cos 35°} \approx 1.13$

11. Angle made with surface = 60°

$\therefore$ $i = 90° - 60° = 30°$

$1.5 = \frac{\sin i}{\sin r}$

$\therefore$ $\sin r = \frac{\sin i}{1.5} = \frac{\sin 30}{1.5} = 0.3333$

$\therefore$ $r = 19° 28'$

Ratio of the width

$= \frac{\cos r}{\cos i} = \frac{\cos 19° 28'}{\cos 30°} = \frac{0.9428}{0.8661} = 1.088 \approx 1 : 1$

12. $v_d = \frac{2}{5}c$ $\quad$ $v_w = \frac{3}{4}c$

$\therefore$ $\frac{c}{v_d} = \frac{5}{2} = \mu_d$ $\quad$ $\frac{c}{v_w} = \frac{4}{3} = \mu_w$

$\therefore$ ${}_w\mu_d = \frac{\mu_d}{\mu_w} = \frac{5/2}{4/3} = \frac{15}{8}$

$\therefore$ ${}_w\mu_d = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin 30°}$

$\therefore$ $\frac{15}{8} = \frac{\sin i}{\sin 30°}$

$\therefore$ $\sin i = \frac{15}{8} \times \frac{1}{2} = \frac{15}{16}$

$\therefore$ $i = \sin^{-1}\left(\frac{15}{16}\right)$

13. In polar regions, magnetic compass becomes inoperative hence sunlight which is easily available and scattered by earth's atmosphere gives plane polarised light when scattered through 90°. This is used for navigation purpose.

14. The plane wavefront with the ray at the periphery has to travel least distance through the lens whereas the ray along the principal axis has to travel thickness of the lens hence this is delayed than the peripheral ray. This results in a spherical converging wavefront.

15. For spherical wavefront, radius = r

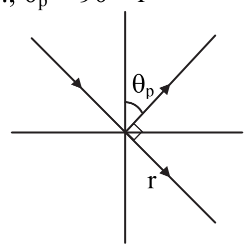
Also, $I \propto a^2$ but $I \propto \frac{1}{r^2}$

$\therefore$ $a \propto \frac{1}{r}$

16. Speed of light in glass depends upon the colour of the light. Violet colour travels faster than the red light in a glass prism.
This is because refractive index of glass for violet colour is less than that for red.

18. In the propagation of e.m. waves, plane of polarisation contains the direction of propagation.

19. Here $\theta_p + 90° + r = 180°$
i.e., $\theta_p = 90 - r$



As $\theta_p - r = 34°$

$\therefore$ $90 - r - r = 34$

i.e., $2r = 56 \Rightarrow r = 28°$

20. If the intensity of the unpolarised light in the incident beam = I_0, then the intensity of the unpolarised component transmitted is same for all orientation of the polarising sheet

$\Rightarrow I_0' = \left(\frac{I_0}{2}\right)$

The transmitted intensity of the polarised light component

$I_p' = I_p \cos^2 \theta$

$\therefore$ $(I_p')_{max} = I_p$ for $\theta = 0$ and

$(I_p')_{min} = 0$ for $\theta = \frac{\pi}{2}$

Now the maximum transmitted intensity = $I_p + \frac{I_0}{2}$ and the minimum transmitted intensity $= \frac{I_0}{2}$

It is given that,

$I_p + \frac{I_0}{2} = 5\left(\frac{I_0}{2}\right)$

$I_p = I_0 \Rightarrow \frac{I_p}{I_o} = 1 : 1$

Textbook Chapter No.

11 Interference and Diffraction

Hints

Classical Thinking

1. For interference, frequency must be same and phase difference must be constant.

3. For interference, phase difference must be constant.

4. $I \propto a^2$

6. For destructive interference, path difference is odd multiple of $\frac{\lambda}{2}$.

10. $X = \frac{\lambda D}{d}$

12. $I = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$
Substituting $a_1^2 + a_2^2 = A$ and $a_1a_2 = B$,
$I = A + B \cos \phi$

23. If one of the slit is closed, then interference fringes are not formed on the screen but a fringe pattern is observed due to diffraction from slit.

24. $X \propto \lambda$
$\therefore$ λ_v = minimum

25. In Young's double slit experiment,
$\because$ $X = \frac{D\lambda}{d}$
$\therefore$ The fringe width can be increased by decreasing the separation between two slits.

26. Fringe width $(X) = \frac{D\lambda}{d}$
$\therefore$ $X \propto \lambda$
As $\lambda_{red} > \lambda_{yellow}$, hence fringe width will increase.

27. For interference, λ of both the waves must be same.

34. $n_1\lambda_1 = n_2\lambda_2$
$\therefore$ $62 \times 5893 = n_2 \times 4358$
$\therefore$ $n_2 \approx 84$
(Note: Use shortcut 4.)

47. Fringe width, $(X) \propto \frac{1}{\text{Prism angle } (\alpha)}$

50. $2\theta = \frac{2\lambda}{d}$ (where d = slit width)
$\therefore$ As d decreases, θ increases.

56. For a diffraction pattern, $x \propto \frac{1}{a}$

67. Due to difference in frequencies of two waves, interference is not possible.

Critical Thinking

1. $y_1 = a \sin \omega t$ and
$y_2 = b \cos \omega t = b \sin\left(\omega t + \frac{\pi}{2}\right)$
So phase difference, $\phi = \pi/2$

2. Two independent light sources cannot be coherent because they cannot generate waves having a constant phase difference.

3. Interference occurs in longitudinal as well as transverse waves. The choices (A), (B) and (D) are conditions for sustained or permanent interference.

4. Path difference $= 12.5\ \lambda = 25\left(\frac{\lambda}{2}\right)$
$\Rightarrow$ odd multiple of $\frac{\lambda}{2}$
$\Rightarrow$ destructive interference

5. Path difference $= 29\lambda$
$= 58\ \frac{\lambda}{2}$
= even multiple of $\frac{\lambda}{2}$
$\Rightarrow$ point is bright

6. $\Delta x = 260\frac{\lambda}{4} = 130\frac{\lambda}{2}$ = even multiple of $\frac{\lambda}{2}$
$\Rightarrow$ point is bright.

7. Path difference $= 65\lambda = 650 \times 10^{-5}$ cm
$\therefore$ $\lambda = \frac{650 \times 10^{-5}}{65} = 10000$ Å

8. $\phi = \frac{\pi}{3}$, $a_1 = 4$, $a_2 = 3$

So, $a = \sqrt{a_1^2 + a_2^2 + 2a_1.a_2\cos\phi}$

$\therefore$ $a = \sqrt{37} \approx 6$

9. For maxima, $2\pi n = \frac{2\pi}{\lambda}(XO) - 2\pi l$

$\therefore$ $\frac{2\pi}{\lambda}(XO) = 2\pi(n + l)$ or $(XO) = \lambda(n + l)$

10. Let the amplitudes of the two waves be a_1 and a_2

$\therefore$ $a_1^2 \propto 4I$ and $a_2^2 \propto I$

Let amplitude of the new wave = a

$\Rightarrow a^2 \propto 3I$

Let K be the constant of proportionality

$\therefore$ $a_1^2 = K(4I)$, $a_2^2 = K(I)$

$a^2 = K(3I)$

$\therefore$ $a^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\theta$

(where θ is the phase angle)

$K(3I) = K(4I) + KI + 2\sqrt{K(4I)}\,.\sqrt{KI}\cos\theta$

$\therefore$ $3 = 4 + 1 + 4\cos\theta$

$\therefore$ $\cos\theta = \frac{-1}{2}$

$\Rightarrow \theta = 120°$

11. $I \propto \frac{1}{r^2} \Rightarrow I = Kr^{-2}$

$\therefore$ $dI = K(-2)r^{-3}\,dr$

$\therefore$ $\frac{dI}{I} = \frac{(-2)dr}{r}$

$\therefore$ $\frac{dI}{I} = -2 \times 1\,\%$

$= -2\,\%$

$\therefore$ Intensity must decrease by 2%.

12. In interference between waves of equal amplitudes 'a', the minimum intensity is zero and the maximum intensity is proportional to $4a^2$. For waves of unequal amplitudes 'a' and A(A > a), the minimum intensity is non-zero and the maximum intensity is proportional to $(a + A)^2$, which is greater than $4a^2$.

14. Contrast between the bright and dark fringes will be reduced.

15. $X = \frac{\lambda D}{d} = \frac{6000\times10^{-10}\times25\times10^{-2}}{1\times10^{-3}}$

$= 6 \times 25 \times 10^{-6} = 150 \times 10^{-6}$ m

$= 0.015 \times 10^{-2}$ m $= 0.015$ cm

16. $X = \frac{\lambda D}{d} \Rightarrow X \propto \lambda$ for the same set-up.

$\therefore$ $\frac{X_1}{X_2} = \frac{\lambda_1}{\lambda_2}$

$\Rightarrow \frac{1.0}{X_2} = \frac{5000}{6000}$

$\therefore$ $X_2 = \frac{6000}{5000} = 1.2$ mm

17. $X = \frac{\lambda D}{d}$

$\therefore$ $X \propto \lambda \propto \frac{1}{\mu}$

$\therefore$ $\frac{X'}{X} = \frac{\mu}{\mu'} = \frac{1}{\left(\frac{4}{3}\right)}$

$\therefore$ $X' = 0.4 \times \frac{3}{4}$

$\Rightarrow X' = 0.3$ mm

18. $X = \frac{D\lambda}{d}$

$\therefore$ $X = \frac{L\lambda}{d} \Rightarrow \lambda = \frac{Xd}{L}$

19. Distance of third maxima from central maxima is

$x = \frac{3\lambda D}{d} = \frac{3\times5000\times10^{-10}\times(200\times10^{-2})}{0.2\times10^{-3}}$

$= 1.5$ cm

20. $D_1 - D_2 = 4 \times 10^{-2}$ m, $X_1 - X_2 = 2 \times 10^{-5}$ m

$d = 10^{-3}$ m

Let, $X = \frac{\lambda D}{d}$

$\therefore$ $X_1 - X_2 = \frac{\lambda}{d}(D_1 - D_2)$

$\therefore$ $2 \times 10^{-5} = \frac{\lambda}{10^{-3}}(4 \times 10^{-2})$

$\therefore$ $\lambda = \frac{2\times10^{-5}\times10^{-3}}{4\times10^{-2}}$

$= 5000 \times 10^{-10}$ m

$= 5000$ Å

21. Distance between successive fringes

= fringe width

$X = \frac{\lambda D}{d} = \frac{8\times10^{-5}\times200}{0.05} = 0.32$ cm

22. Fringe width of maximum just opposite to slit,

$X_n = \frac{n\lambda D}{d} = \frac{d}{2}$

$\Rightarrow n = \frac{d^2}{2\lambda D}$

23. Fringe width,

$X = \frac{\lambda D}{d} \Rightarrow \frac{X}{D} = \frac{\lambda}{d}$

For sharp fringes, S < X

$\therefore \quad \frac{S}{D} < \frac{X}{D} = \frac{\lambda}{d}$

$\therefore \quad \frac{S}{D} < \frac{\lambda}{d}$

24. $X_8 = \frac{8\lambda_1 D}{d}$ and $X_6 = \frac{6\lambda_2 D}{d}$

$\therefore \quad \frac{X_8}{X_6} = \frac{d_1}{d_2} = \frac{8\lambda_1}{6\lambda_2} = \frac{4\lambda_1}{3\lambda_2}$

25. $X \propto \frac{\lambda}{d}$

$\Rightarrow X_1 \propto \frac{\lambda}{d}, X_2 \propto \frac{2\lambda}{d/2}$

$\therefore \quad \frac{X_2}{X_1} = \frac{4\lambda}{d} \times \frac{d}{\lambda} = 4$

26. Let $X_1 = \frac{n_1\lambda_1 D_1}{d_1}$ and $X_2 = \frac{n_2\lambda_2 D_2}{d_2}$

Given that, $X_1 = X_2$, $D_1 = D_2$, $d_1 = d_2$

$n_1 \lambda_1 = n_2 \lambda_2$

$\therefore \quad n_1(2500) = n_2(3500)$

$\Rightarrow \frac{n_1}{n_2} = \frac{3500}{2500} = \frac{7}{5}$

So we can say, 7^{th} order of 1^{st} source coincides with 5^{th} order of 2^{nd} source.

27. Using relation, $d\sin\theta = n\lambda$ we get,

$\sin\theta = \frac{n\lambda}{d}$

$\therefore$ For n = 3,

$\sin\theta = \frac{3\lambda}{d} = \frac{3\times 589\times 10^{-9}}{0.589} = 3 \times 10^{-6}$

$\therefore \quad \theta = \sin^{-1}(3 \times 10^{-6})$

28. In Young's double slit experiment,

$\sin\theta = \theta = (y/D)$, so $\Delta\theta = (\Delta y/D)$

Hence, angular fringe width $\theta_0 = \Delta\theta$ (with $\Delta y = X$) will be

$\theta_0 = \frac{X}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$

Here $\theta_0 = 1° = (\pi/180)$ rad and

$\lambda = 6000$ Å $= 6 \times 10^{-7}$ m

$\therefore \quad d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times 6 \times 10^{-7}$

$= 3.44 \times 10^{-5}$ m = 0.03 mm

29. $X = n\frac{\lambda D}{d}$

$X_3 = X_4$ ....[Given]

$\therefore \quad 3\lambda_1 = 4\lambda_2 \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$

30. Using,

$X = \frac{\lambda D}{d} = \frac{5000\times 10^{-10}\times 1.2}{0.5\times 10^{-3}}$

$= 12 \times 10^{-4}$ m

$= 1.2$ mm

$\therefore$ Number of fringes $= \frac{3}{1.2} = 2.5$

$\therefore$ Phase difference,

$\Delta\phi = 2n\pi = 2 \times 2.5\ \pi = 5\pi$ radian

31. For dark fringes,

$x_n = (2n-1)\frac{\lambda D}{2d}$

For n = 2, $3 = \frac{3}{2}\frac{\lambda D}{d}$ ...(i)

For bright fringe, $x_n = n\frac{\lambda D}{d}$

$\therefore \quad x_4 = 4\frac{\lambda D}{d}$ ...(ii)

From equations (i) and (ii),

$\frac{x_4}{3} = \frac{4\lambda D}{d}\frac{2d}{3\lambda D}$

$\therefore \quad \frac{x_4}{3} = \frac{8}{3} \Rightarrow x_4 = 8$ mm

32. $x_3 = n\lambda\frac{D}{d} = 3\frac{\lambda D}{d}$ ....(bright fringe)

$x_5 = (2n-1)\frac{\lambda}{2}\frac{D}{d}$ ....(dark fringe)

$= (2 \times 5 - 1)\frac{\lambda}{2}\frac{D}{d} = \frac{9}{2}\frac{\lambda D}{d}$

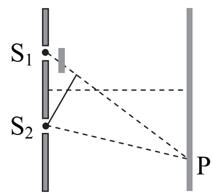
$\therefore \quad x_5 - x_3 = \frac{9}{2}\frac{\lambda D}{d} - \frac{3\lambda D}{d}$

$= \frac{3}{2}\frac{\lambda D}{d} = \frac{3}{2} \times \frac{6.5\times 10^{-7}\times 1}{10^{-3}}$

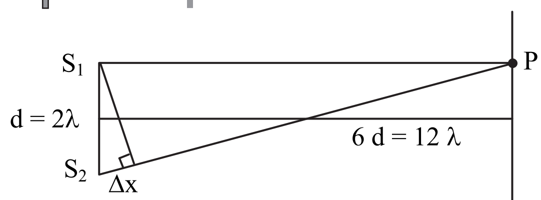
$= 9.75 \times 10^{-4}$ m = 0.975 mm

33. For dark fringe at P,
$S_1P - S_2P = \Delta = (2n - 1)\lambda/2$
Here, n = 3 and $\lambda = 6000$
$\therefore \quad \Delta = \frac{5\lambda}{2} = 5 \times \frac{6000}{2} = 15000$ Å = 1.5 micron

34. Path difference at P, $\Delta = (S_1P + (\mu - 1)t) - S_2P$
$= (S_1P - S_2P) + (\mu - 1)t$



35.



Let $\Delta x = S_2P - S_1P$
$\therefore \quad (S_2P)^2 = (S_1P)^2 + (S_1S_2)^2$
$= (12\lambda)^2 + (2\lambda)^2$
$= 144\lambda^2 + 4\lambda^2$
$= 148\lambda^2$
$\therefore \quad S_2P = 12.17\lambda$
$\therefore \quad \Delta x = 12.17\lambda - 12\lambda = 0.17\lambda = \frac{\lambda}{6}$
$\therefore \quad \Delta\phi = \frac{2\pi\Delta x}{\lambda} = \frac{2\pi\lambda}{6\lambda} = \frac{\pi}{3}$
$\therefore \quad I = I_{max}\cos^2\frac{\phi}{2} = I_0\cos^2\left(\frac{60°}{2}\right)$
$\therefore \quad I = I_0\cos^2 30° = \frac{3}{4}I_0$

36. $d = \sqrt{d_1d_2} = \sqrt{4.5 \times 2 \times 10^{-6}} = 3 \times 10^{-3}$ m
$\therefore \quad X = \frac{\lambda D}{d} = \frac{6000 \times 10^{-10} \times 1}{3 \times 10^{-3}}$
$= 2 \times 10^{-4}$ m = 0.2 mm

37. $d = \sqrt{d_1d_2} = \sqrt{(1.6)(3.6)} = 2.4$ mm

38. D = 1 m, d = 1 mm, v = 40 cm, u = 60 cm
$\therefore \quad d_1 = \frac{v}{u}d = \frac{40}{60} \times 1$ mm = 0.67 mm

39. $X = \frac{\lambda D}{d} = \frac{6000 \times 10^{-10} \times 1}{6 \times 10^{-3}} = 10^{-4}$ m
$\therefore$ Fringe width = 10^{-4} m
$\therefore$ No. of fringes formed per mm = $\frac{10^{-3}}{10^{-4}} = 10$

40. $d = d_1\frac{u}{v} = 1.2 \times \frac{20}{80} = 0.3$ cm = 3 mm

41. The distance of 10^{th} bright band from central bright band,
$x_{10} = \frac{10\lambda D}{d} = \frac{10 \times 6000 \times 10^{-10} \times 1}{0.5 \times 10^{-3}}$
$= \frac{6 \times 10^{-6}}{5 \times 10^{-4}}$
$= 1.2 \times 10^{-2}$ m = 1.2 cm

42. u = 5 cm, v = 75 cm, D = 80 cm
$\therefore \quad X = \frac{\lambda D}{d} = \frac{5890 \times 10^{-8} \times 80}{0.05}$
$= 9424 \times 10^{-5}$ cm

43. $\frac{x_{30}}{x_{20}} = \frac{30}{20} \Rightarrow x_{30} = \frac{3}{2} \times 8 = 12$ mm

44. $x = n\lambda$
$\therefore \quad n = \frac{0.005 \times 10^{-2}}{5000 \times 10^{-10}} = 100$

45. The fringe width between first and seventh bright fringes is
$X = (7 - 1)\frac{\lambda D}{d}$
$= 6 \times \frac{6000 \times 10^{-10}}{1.2 \times 10^{-3}} \times 1.0$
$= \frac{36 \times 10^{-7}}{12 \times 10^{-4}} = 3 \times 10^{-3} = 0.003$ m

46. From given data,
$(12 - 3)\lambda\frac{D}{d} = (14 - 4)\lambda_1\frac{D}{d}$
$\therefore \quad 9 \times 6000 = 10\lambda_1$
$\therefore \quad \lambda_1 = \frac{9 \times 6000}{10}$
$\Rightarrow \lambda_1 = 5400$ Å

47. $(n + 1)\lambda_g = n\lambda_r$
$\therefore \quad (n + 1) \times 5200 = n \times 6500$
$\therefore \quad 52n + 52 = 65n$
$\Rightarrow n = 4$

48. $\frac{X_1}{X_2} = \frac{\lambda_1}{\lambda_2}$
$\therefore \quad X_2 = X_1\frac{\lambda_2}{\lambda_1} = 0.32 \times \frac{4000}{6400} = 0.20$ mm
$\therefore$ Percentage decrease = $\frac{0.32 - 0.20}{0.32} \times 100$
= 37.5%

49. Band width $\propto \lambda$

$\because$ $\lambda_{yellow} < \lambda_{red}$, hence for red light, the diffraction bands become broader and further apart.

51. For diffraction, size of the obstacle must be of the order of wavelength of wave i.e., $a \approx \lambda$

53. $a \sin\theta = n\lambda$

For n = 1,

$$\sin\theta = \frac{\lambda}{a} = \frac{550\times10^{-9}}{0.55\times10^{-3}} = 10^{-3} = 0.001 \text{ rad}$$

54. Position of first minima = Position of third maxima

$\therefore$ $$\frac{1\times\lambda_1 D}{d} = \frac{(2\times3+1)}{2}\frac{\lambda_2 D}{d}$$

$\therefore$ $\lambda_1 = 3.5\,\lambda_2$

55. Position of n^{th} minima, $x_n = \dfrac{n\lambda D}{d}$

For n = 1,

$$5\times10^{-3} = \frac{1\times5000\times10^{-10}\times1}{d}$$

$\therefore$ $d = 10^{-4}$ m = 0.1 mm

56. Diffraction is obtained when the slit width is of the order of wavelength of EM waves (or light). Wavelength of X-rays (1-100 Å) is very less than slit width (0.6 mm). Therefore, no diffraction pattern will be observed.

57. Linear diameter of second maximum,

$$2x = \frac{2(2n+1)f\lambda}{2a}$$

$\lambda = 5\times10^{-7}$ m, $a = 5\times10^{-4}$ m, f = 0.8 m

$\therefore$ $$2x = \frac{2\times5\times5\times10^{-7}\times0.8}{2\times5\times10^{-4}\text{m}}$$

$= 4\times10^{-3}$ m = 4 mm

58. Angular width, $\theta = \dfrac{2\lambda}{a} \Rightarrow \theta \propto \lambda$

$\therefore$ $$\frac{\theta_1}{\theta_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{\theta_1}{\left(\frac{70}{100}\theta_1\right)} = \frac{6000}{\lambda_2}$$

$\therefore$ $\lambda_2 = 4200\,\text{Å}$

59. In a single slit diffraction experiment, position of minima is given by, $d\sin\theta = n\lambda$

So, for first minima of red, $\sin\theta = 1\times\left(\dfrac{\lambda_R}{d}\right)$

and as first maxima is midway between first and second minima, for wavelength λ', its position will be

$$d\sin\theta' = \frac{\lambda' + 2\lambda'}{2} \Rightarrow \sin\theta' = \frac{3\lambda'}{2d}$$

According to given condition $\sin\theta = \sin\theta'$

$\therefore$ $\lambda' = \dfrac{2}{3}\lambda_R$

$\therefore$ $\lambda' = \dfrac{2}{3}\times589 = 392.6$ nm = 3926 Å

62. $d \propto \lambda$

$\therefore$ $$\frac{d_1}{d_2} = \frac{\lambda_1}{\lambda_2}$$

$\therefore$ $$\frac{0.1}{d_2} = \frac{6000}{4800} \Rightarrow d_2 = 0.08 \text{ mm}$$

63. Limit of resolution,

$$d = \frac{1.22\lambda}{2\mu\sin\alpha} = \frac{0.61\lambda}{\mu\sin\alpha}$$

Numerical aperture = $\mu\sin\alpha$ = 0.12

$\therefore$ $$d = \frac{0.61\times6\times10^{-7}}{0.12} = 30.5\times10^{-7}\text{ m}$$

64. $$\text{R.P.} = \frac{1.22\lambda}{a} = \frac{d}{x}$$

$\therefore$ $$x = \frac{ad}{1.22\lambda} = \frac{10^{-3}\times0.1}{1.22\times5\times10^{-7}} = 163.9 \text{ m}$$

65. $$d\theta = \frac{1.22\lambda}{a}$$

$$\Rightarrow a = \frac{1.22\lambda}{d\theta}$$

$\therefore$ $$a = \frac{1.22\times5\times10^{-5}\times180}{10^{-3}\times3.14} \approx 3.5 \text{ cm}$$

66. $d\theta$ = angle of the cone of light from the objects

$$d\theta = \frac{\text{diameter of the telescope}}{\text{distance of the moon}}$$

$$= \frac{5}{4\times10^5\times10^3}$$

$\therefore$ $$d\theta = \frac{1.22\lambda}{D} \Rightarrow D = \frac{1.22\lambda}{d\theta} = \frac{1.22\times5000\times10^{-10}}{5/(4\times10^8)}$$

$\therefore$ D = 48.8 m $\approx$ 50 m

67. In case of an excessively thin film, the path difference is $\frac{\lambda}{2}$. As the path difference between two rays is $\frac{\lambda}{2}$, the film appears dark.

68. Fringe visibility, $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

69. $\mu = \frac{\lambda_{air}}{\lambda_{water}}$

$\therefore$ $\frac{3}{2} = \frac{6000}{\lambda_w} \Rightarrow \lambda_w = 4000$ Å

$X = \frac{\lambda D}{d} \Rightarrow X \propto \lambda$

$\therefore$ $\frac{X'}{X} = \frac{4000}{6000}$

$\therefore$ $X' = \frac{2}{3} \times 3 = 2$ mm

$\therefore$ Change in fringe width $= X - X'$
$= 3 - 2 = 1$ mm

70. $(\mu_1 - 1)\frac{tD}{d} - (\mu_2 - 1)\frac{tD}{d} = \frac{5\lambda D}{d}$

$\therefore$ $[(1.7 - 1) - (1.4 - 1)]\frac{tD}{d} = \frac{5\lambda D}{d}$

$\therefore$ $\frac{0.3tD}{d} = \frac{5\lambda D}{d}$

$\therefore$ $t = \frac{5\lambda}{0.3} = \frac{5 \times 4800 \times 10^{-10}}{0.3}$
$= 8 \times 10^{-6}$ m $= 8 \times 10^{-3}$ mm

71. $\lambda = 600$ nm $= 600 \times 10^{-9}$ m
$t = 18$ μm $= 18 \times 10^{-6}$ m

$S = (\mu - 1)\frac{tD}{d}$ ….(i)

Fringe width,

$X = \frac{\lambda D}{d}$ ….(ii)

$\therefore$ From equations (i) and (ii), $S = \frac{(\mu - 1)t.X}{\lambda}$

$\therefore$ No. of fringes $= \frac{S}{X} = \frac{(\mu - 1)t}{\lambda}$

$= \frac{(1.6 - 1) \times 18 \times 10^{-6}}{600 \times 10^{-9}}$

$= \frac{0.6 \times 18 \times 10^{3}}{600} = 18$

72. For the first minimum,

$a \sin \theta_1 = \lambda \approx a\theta_1 = \frac{ad_1}{D}$ ....(i)

For the sixth minimum,

$a \sin \theta_6 = 6\lambda \approx a\theta_6 = \frac{ad_6}{D}$ ....(ii)

$\therefore$ By subtracting equation (i) from equation (ii),

$(6\lambda - \lambda) = \frac{a}{D}(d_6 - d_1)$

$\therefore$ $a = \frac{5D\lambda}{(d_6 - d_1)} = \frac{5 \times 0.5 \times 5000 \times 10^{-10}}{0.5 \times 10^{-3}}$

$\therefore$ $a = 25 \times 10^{-4}$ m $= 2.5$ mm

73. For 5th dark fringe, $x_1' = (2n - 1)\frac{\lambda}{2}\frac{D}{d} = \frac{9\lambda D}{2d}$

For 7th bright fringe, $x_2 = n\lambda\frac{D}{d} = \frac{7\lambda D}{d}$

$\therefore$ $x_2 - x_1' = (\mu - 1)t\frac{D}{d}$

$\therefore$ $\frac{7\lambda D}{d} - \frac{9\lambda D}{d} = (\mu - 1)t\frac{D}{d}$

$\therefore$ $t = \frac{2.5\lambda}{(\mu - 1)}$

74. For 10th order fringes (for λ_1),

$\frac{10\lambda_1 D}{d} = 2.37 - 1.25 = 1.12$ mm

$\therefore$ $\frac{1.12\,mm}{\lambda_1} = \frac{10\,D}{d}$ ….(i)

For λ_2, 10th order fringes,

$\frac{10\lambda_2 D}{d} = \lambda_2\left(\frac{1.12\,mm}{\lambda_1}\right)$ ….[From (i)]

$= \frac{7000}{6000} \times 1.12 \times 10^{-3}$

$= 1.30$ mm

$\therefore$ Reading for 10th order would be 1.25 mm + 1.30 mm = 2.55 mm.

$\therefore$ The zero order reading would be same for both wavelengths.

Competitive Thinking

1. The refractive index of air is slightly more than 1. When chamber is evacuated, refractive index decreases and hence the wavelength increases and fringe width also increases.

2. Colours of thin film are due to interference of light.

3. For constructive interference, path difference is even multiple of $\frac{\lambda}{2}$.

4. $\frac{I_1}{I_2} = \frac{1}{25}$

$\therefore \frac{a_1^2}{a_2^2} = \frac{1}{25}$ ....[$\because I \propto a^2$]

$\therefore \frac{a_1}{a_2} = \frac{1}{5}$

5. $\frac{I_1}{I_2} = n$

We know, $I \propto a^2$

$\therefore \frac{a_1}{a_2} = \sqrt{n}$ ....(i)

Now, $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{\left(\frac{a_1}{a_2} + 1\right)^2}{\left(\frac{a_1}{a_2} - 1\right)^2}$

Substituting equation (i) above, we get

$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{n} + 1\right)^2}{\left(\sqrt{n} - 1\right)^2}$

6. $\frac{a_1}{a_2} = \frac{4}{3}$

$\therefore \frac{a_1 + a_2}{a_1 - a_2} = \frac{4+3}{4-3} = \frac{7}{1}$

$\therefore \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \frac{49}{1}$

7. $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{100}{1}$

$\therefore \frac{a_1}{a_2} = \frac{10}{1} \Rightarrow \frac{a_1 + a_2}{a_1 - a_2} = \frac{11}{9}$

$\therefore \frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{11}{9}\right)^2 = \frac{121}{81}$

8. $\frac{\text{Intensity of bright band}}{\text{Intensity of dark band}} = \frac{16}{1}$

But $I \propto a^2$

$\Rightarrow$ amplitude of bright band $a_b = 4$ and amplitude of dark band $a_d = 1$

$\therefore$ Intensity of individual sources,

$I_{max} = (a_b + a_d)^2 = (4 + 1)^2 = 25$

$I_{min} = (a_b - a_d)^2 = (4 - 1)^2 = 9$

9. Resultant intensity,

$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

For maximum I_R, $\phi = 0°$

$\therefore I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

10. Ratio of slit widths = 4 : 9 $\Rightarrow I_1 : I_2 = 4 : 9$

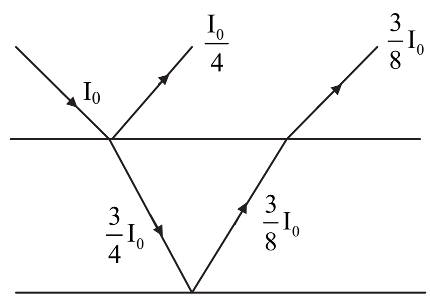
$\therefore \frac{a_1^2}{a_2^2} = \frac{4}{9} \Rightarrow \frac{a_1}{a_2} = \frac{2}{3}$

$\therefore \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{25}{1}$

11. $\frac{I_{max}}{I_{min}} = \frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1} = \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2$

$= \left(\frac{\sqrt{\frac{1}{25}} + 1}{\sqrt{\frac{1}{25}} - 1}\right)^2 = \left(\frac{5+1}{5-1}\right)^2 = \frac{36}{16} = \frac{9}{4}$

12.



Given that, 25% of total intensity of incident light is reflected from upper surface. This implies, if intensity of incident light is I_0, the intensity of light reaching the lower surface of plate will be $\frac{3}{4}I_0$.

As 50% of this intensity is reflected, the final intensity of light emerging from glass plate will be $\frac{3}{8}I_0$.

$\therefore I_1 = \frac{I_0}{4}$

$I_2 = \frac{3}{8}I_0$

Now, $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \left(\frac{\frac{1}{2} + \sqrt{\frac{3}{8}}}{\frac{1}{2} - \sqrt{\frac{3}{8}}}\right)^2$

13. Since, superimposing waves have Intensity I_0

$\because$ $I_1 = I_2 = I_0$

So $I_{max} = 4I_0$

and $I_{min} = 0$

Hence, $I_{average} = \frac{I_{max} + I_{min}}{2}$

$\Rightarrow I_{average} = 2I_0$

14. $a_1 = \sqrt{I_1}$, $a_2 = \sqrt{I_2}$

Maximum intensity: $I_{max} = (a_1 + a_2)^2$

$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Minimum intensity: $I_{min} = (a_1 - a_2)^2$

$= \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$\therefore$ $I_{max} + I_{min} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 + \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$= I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} + I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2}$

$= 2(I_1 + I_2)$

15. Resultant intensity is given by,

$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

At point P, $\phi = \frac{\pi}{2}$

$\therefore$ $(I_R)_P = I + 9I + 0$ ...$\{\because \cos 90° = 0\}$

$(I_R)_P = 10I$

At point Q, $\phi = \pi$

$\therefore$ $(I_R)_Q = I + 9I - 2\sqrt{I \times 9I}$ ...$\{\because \cos 180° = -1\}$

$= 10I - 6I$

$(I_R)_Q = 4I$

$\therefore$ Difference between resultant intensities at point P and Q is = 10I – 4I = 6I

16. $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{4I} + \sqrt{9I})^2 = 25I$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{4I} - \sqrt{9I})^2 = I$

17. $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{I} + \sqrt{4I})^2 = 9I$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{I} - \sqrt{4I})^2 = I$

18. $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2 = \left(\frac{\sqrt{\frac{9}{1}} + 1}{\sqrt{\frac{9}{1}} - 1}\right)^2 = \frac{4}{1}$

19. $\frac{a_1}{a_2} = \frac{3}{5}$

$\therefore$ $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(3+5)^2}{(3-5)^2} = \frac{16}{1}$

20. $A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

Given that, $a_1 = a_2 = a$

$\therefore$ $A^2 = 2a^2(1 + \cos\phi) = 2a^2\left(1 + 2\cos^2\frac{\phi}{2} - 1\right)$

$\Rightarrow A^2 \propto \cos^2\frac{\phi}{2}$

Now, $I \propto A^2$

$\therefore$ $I \propto A^2 \propto \cos^2\frac{\phi}{2} \Rightarrow I \propto \cos^2\frac{\phi}{2}$

21. Given,

$\frac{I_1}{I_2} = n$

$\therefore$ $I_1 = n I_2$

$\therefore$ $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$= \left(\sqrt{nI_2} + \sqrt{I_2}\right)^2$

Say $I_2 = I$

$\therefore$ $I_{max} = \left(\sqrt{n} + 1\right)^2 I$

Similarly, $I_{min} = \left(\sqrt{n} - 1\right)^2 I$

$\therefore$ $\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\left(\sqrt{n}+1\right)^2 - \left(\sqrt{n}-1\right)^2}{\left(\sqrt{n}+1\right)^2 + \left(\sqrt{n}-1\right)^2}$

$= \frac{n + 1 + 2\sqrt{n} - n - 1 + 2\sqrt{n}}{n + 1 + 2\sqrt{n} + n + 1 - 2\sqrt{n}}$

$= \frac{4\sqrt{n}}{2n + 2} = \frac{2\sqrt{n}}{n + 1}$

22. Two coherent sources must have a constant phase difference otherwise they cannot produce interference.

25. Path difference for all wavelengths at the central point will be zero.

26. The two sources of light emitting different wavelengths will not form interference fringes.

27. $X \propto \lambda \Rightarrow \lambda \propto \frac{1}{\mu}$

28. In the presence of thin glass plate, the fringe pattern shifts but no change in fringe width occurs.

29. $\beta = \frac{\lambda D}{d}$

λ increases from violet to red

$\therefore$ $\lambda_R > \lambda_G > \lambda_B \Rightarrow \beta_R > \beta_G > \beta_B$

31. For maxima, path difference, $\Delta x = n\lambda$

$\therefore$ For n = 1, $\Delta x = \lambda = 6320$ Å

32. $X \propto \lambda$

$\therefore$ $\frac{X_2}{X_1} = \frac{\lambda_2}{\lambda_1} \Rightarrow X_2 = X_1 \times \frac{\lambda_2}{\lambda_1}$

$= 0.32 \times \frac{4800}{6400}$

$= 0.24$ mm

$\therefore$ Change in X = 0.32 − 0.24

$= 0.08$ mm $= 8 \times 10^{-5}$ m

33. $X = \frac{\lambda D}{d} \Rightarrow d = \frac{\lambda D}{X}$

$\therefore$ $d = \frac{6000 \times 10^{-10} \times (40 \times 10^{-2})}{0.012 \times 10^{-2}} = 0.2$ cm

34. Using, $X = \frac{\lambda D}{d} = \frac{6000 \times 10^{-7}\ \text{mm} \times 25 \times 10\ \text{mm}}{1\ \text{mm}}$

$= 15 \times 10^{-2} = 0.15$ mm

35. We know that, $\frac{Xd}{D} = n\lambda$

as X, d and D are same, $n\lambda$ = constant

$\therefore$ $n_1 \lambda_1 = n_2\lambda_2$

9×5896 Å $= 11 \times \lambda_2$

$\Rightarrow \lambda_2 = \frac{9 \times 5896}{11}$

$\therefore$ $\lambda_2 = 4824$ Å

36. Path difference $= 5\lambda = 10 \times \frac{\lambda}{2}$

$\Rightarrow$ Point is bright.

$\therefore$ Using, $X_n = nX$ we get,

$0.5 = 5X \Rightarrow X = 0.1$ mm

37. $X = \frac{\lambda D}{d}$ and $X' = \frac{\lambda D'}{d'}$

But $d' = \frac{d}{2}$ and $D' = 2D$

$\therefore$ $X' = \frac{\lambda(2D)}{(d/2)} = 4\frac{\lambda D}{d} = 4X$

$\therefore$ Fringe width will become four-times.

38. Distance of n[th] bright fringe, $x_n = \frac{n\lambda D}{d} \Rightarrow x_n \propto \lambda$

$\therefore$ $\frac{x_{n_1}}{x_{n_2}} = \frac{\lambda_1}{\lambda_2}$

$\therefore$ $\frac{x(\text{Blue})}{x(\text{Green})} = \frac{4360}{5460}$

$\therefore$ x (Green) > x (Blue)

39. $X \propto D$

$\therefore$ % change in fringe width = 25%

40. $X \propto \frac{1}{d}$

$\therefore$ If d becomes thrice, then X becomes $\frac{1}{3}$ times.

41. Second minimum is exactly in front of one slit indicates, $y_2' = \frac{d}{2}$

But $y_2' = \frac{(2n-1)\lambda D}{2d}$

For n = 2

$\therefore$ $\frac{d}{2} = \frac{(2 \times 2 - 1)\lambda D}{2d}$

$\therefore$ $\lambda = \frac{d^2}{3D}$

42. Fringe width is independent of the order of fringe.

43. $X = \frac{\lambda D}{d} \Rightarrow X \propto D$

$\therefore$ $\frac{X_1}{X_2} = \frac{D_1}{D_2} \Rightarrow \frac{X_1 - X_2}{\beta_2} = \frac{D_1 - D_2}{D_2}$

$\therefore$ $\frac{\Delta X}{\Delta D} = \frac{X_2}{D_2} = \frac{\lambda_2}{d_2}$

$\therefore$ $\lambda_2 = \frac{3 \times 10^{-5}}{5 \times 10^{-2}} \times 10^{-3} = 6 \times 10^{-7}\,\text{m} = 6000\,\text{Å}$

44. $n_1\lambda_1 = n_2\lambda_2 \Rightarrow \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$

$\therefore$ $\frac{n_2}{92} = \frac{5898}{5461} \Rightarrow n_2 = 99$

45. Using,

$n_1\lambda_1 = n_2\lambda_2 \Rightarrow n_2 = n_1\frac{\lambda_1}{\lambda_2} = 60 \times \frac{5600}{4800} = 70$

46. $n\lambda_1 = (n+1)\lambda_2$

$\therefore$ $n(6750) = (n+1)(5400)$

$\therefore$ $n \times 5 = (n+1) \times 4 \Rightarrow n = 4$

47. $n_1\lambda_1 = n_2\lambda_2$

$\therefore$ $n\lambda_{Red} = (n + 1)\lambda_{Green}$

$\therefore$ $\frac{n+1}{n} = \frac{\lambda_{Red}}{\lambda_{Green}} = \frac{6000}{5000} = \frac{6}{5}$

$\therefore$ $6n = 5n + 5$

$\therefore$ $n = 5$

48. $d \sin\theta = \pm n\lambda$

But, $d = \lambda$ ....(given)

$\therefore$ $\sin\theta = \pm n$

Where n is the order of maxima

As maximum value of $\sin\theta = 1$

$n = \pm 1$

i.e., the number of bright fringes formed include, central maxima and first order maxima on either side of central maxima.

So maximum number of bright fringes = 3

49. $n_1\lambda_1 = n_2\lambda_2 \Rightarrow 3 \times 590 = 4 \times \lambda_2$

$\therefore$ $\lambda_2 = 442.5$ nm

50. $X = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times2}{10^{-3}} = 10^{-3}$ m = 1.0 mm

51. P is the position of 11^{th} bright fringe from Q. From central position O, P will be the position of 10^{th} bright fringe.

Path difference between the waves reaching $P = S_1B = 10\lambda = 10 \times 6000 \times 10^{-10} = 6 \times 10^{-6}$m.

52. The dark band formed at point A is of the order n = 5.

Path difference of n^{th} dark band is given by,

$$\Delta x_n = (2n - 1)\frac{\lambda}{2}$$

$\therefore$ $\Delta x_5 = \frac{[2(5) - 1]\ 6 \times 10^{-7}}{2}$

$= 2.7 \times 10^{-6}$m

$\therefore$ $\Delta x_5 = 2.7 \times 10^{-4}$cm

53. Distance between 1^{st} order dark fringes = width of principal maximum

$\therefore$ $x = \frac{2\lambda D}{d} = \frac{2\times600\times10^{-9}\times2}{10^{-3}}$

$= 2400 \times 10^{-6}$

$= 2.4 \times 10^{-3}$ m

$= 2.4$ mm

54. $x = (2n + 1)\frac{\beta}{2}$

For 5^{th} dark fringe, n = 5

$\therefore$ $x_5 = \frac{9}{2}\beta = \frac{9}{2}\times2\times10^{-3} = 9 \times 10^{-3}$ cm

55. Distance of 6^{th} bright fringe,

$X_6 = \frac{n\lambda D}{d} = \frac{6\lambda D}{d}$

Distance of 4^{th} dark fringe,

$X'_4 = \frac{(2n-1)\lambda D}{2d} = \frac{7}{2}\frac{\lambda D}{d}$

$\therefore$ $X_6 - X'_4 = \frac{\lambda D}{d}\left(6-\frac{7}{2}\right) = \frac{5}{2}\frac{\lambda D}{d}$

$= \frac{5}{2}\times\frac{4\times10^{-7}\times1}{1\times10^{-3}}$

$= 10^{-3}$ m = 1 mm

56. Fringe width,

$X = \frac{n\lambda D}{d}$

For fourth bright fringe,

$X_4 = \frac{4D\lambda}{d}$

and $X'_4 = \frac{4D\lambda'}{d}$

$\therefore$ $X_4 - X'_4 = \frac{4D}{d}(\lambda - \lambda')$

$= \frac{4 \times 1.2 \times \left[(6500 - 5200) \times 10^{-10}\right]}{2\times10^{-3}}$

$= 3.12 \times 10^{-4}$ m = 0.312 mm

57. $\frac{I}{I_0} = \cos^2\left(\frac{\phi}{2}\right)$; $\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\times\frac{\lambda}{6}$

$\therefore$ $I = I_0\cos^2\left(\frac{\pi}{3}\right) = \frac{3I_0}{4}$

58. Shift in the fringe pattern $X_o = \frac{(\mu-1)t.D}{d}$

$= \frac{(1.5-1)\times2.5\times10^{-5}\times100\times10^{-2}}{0.5\times10^{-3}} = 2.5$ cm

59. For 5^{th} dark fringe in air

$(x'_5)_a = \frac{(2\times5-1)\lambda D}{2d} = \frac{9}{2}\frac{\lambda D}{d}$

For 8^{th} bright fringe in medium,

$(x_8)_m = \frac{8\lambda D}{\mu d}$, where μ is refractive index of medium

$(x'_5)_a = (x_8)_m$

$\therefore$ $\frac{9}{2}\frac{\lambda D}{d} = \frac{8\lambda D}{\mu d}$

$\therefore$ $\mu = \frac{8\times2}{9} \approx 1.78$

60. Distance of n^{th} dark fringe from central fringe,

$x_n = \frac{(2n-1)\lambda D}{2d}$

$\therefore \quad x_2 = \frac{(2\times 2-1)\lambda D}{2d} = \frac{3\lambda D}{2d}$

$\therefore \quad 1\times 10^{-3} = \frac{3\times\lambda\times 1}{2\times 0.9\times 10^{-3}} \Rightarrow \lambda = 6\times 10^{-5}$ cm

61. $\theta = \frac{\lambda}{d}$; θ can be increased by increasing λ, so λ has to be increased by 10%

$\therefore$ Increase in $\lambda = \frac{10}{100}\times 5890 = 589$ Å

62. Distance of 5^{th} bright fringe from central fringe,

$X_{5B} = \frac{5\lambda D}{d}$ ….(i)

Distance of 3^{rd} dark fringe from central fringe,

$X_{3D} = \frac{(2\times 3-1)\lambda D}{2d} = \frac{5}{2}\frac{\lambda D}{d}$ ….(ii)

From equations (i) and (ii), required distance,

$X_{5B} - X_{3D} = \left(5-\frac{5}{2}\right)\frac{\lambda D}{d} = \frac{5}{2}\times\frac{5\times 10^{-7}\times 1}{1\times 10^{-3}}$

$= 1.25$ mm.

63. For $y_1 = y_2$

$n_1\lambda_1 = n_2\lambda_2$

$\therefore \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{520}{650} = \frac{4}{5}$

$\therefore \quad n_1\lambda_1 = n_2\lambda_2 = 520\times 5 = 650\times 4 = 2600$ nm

$\therefore \quad y_1 = \frac{n_1\lambda_1 D}{d} = \frac{2600\times 10^{-9}\times 1.5}{0.5\times 10^{-3}}$

$= 7.8\times 10^{-3}$ m $= 7.8$ mm.

64. $\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$

$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{10000\,\text{Å}}{12000\,\text{Å}} = \frac{5}{6}$

$n_1\lambda_1 = n_2\lambda_2 = 5\times 12000 = 6\times 10000 = 60000$

Therefore, $x = \frac{n_1\lambda_1 D}{d} = \frac{60000\times 10^{-10}\times 2}{2\times 10^{-3}}$

$= 6\times 10^{-3}$ m $= 6$ mm

65. Given: $X_n = X_{n+1}$

$\therefore \quad \frac{n\lambda_1 D}{d} = \frac{(n+1)\lambda_2 D}{d}$

$\therefore \quad n\times 780\times 10^{-9} = (n+1)\times 520\times 10^{-9}$

$\therefore \quad 780n - 520n = 520$

$\therefore \quad 260n = 520$

$\therefore \quad n = \frac{520}{260} = 2$

66. Using, $\Delta x = (2n-1)\frac{\lambda}{2}$

$\therefore \quad 0.05 = (2n-1)\times\frac{5000\times 10^{-8}}{2}$

$\therefore \quad \frac{0.1}{5\times 10^{-5}} = (2n-1)$

$\therefore \quad 2n-1 = \frac{10000}{5} \Rightarrow n \approx 1000$

67. $120\lambda = 72\times 10^{-6}$

$\Rightarrow \lambda = 6000$ Å.

The point is bright as path difference is even multiple of $\frac{\lambda}{2}$.

68. Fringe shift,

$X_0 = \frac{X}{\lambda}(\mu-1)\,t$

$= \frac{\beta}{(5000\times 10^{-10})}(1.5-1)\times 2\times 10^{-6}$

$= 2\beta$

i.e., The central bright maximum will shift 2 fringes upwards.

69. *Using shortcut 6,*

$t = \frac{N\lambda}{\mu-1}$

$t = \frac{7\lambda}{(\mu-1)} = \frac{7\times 600\times 10^{-9}}{(1.6-1)} = 7\ \mu$m

71. Using,

$X_0 = X_2 - X_1 = \frac{D}{d}(\mu-1)t$

$X_0 = \frac{n\lambda D}{d} = \frac{D}{d}(\mu-1)t$

$\therefore \quad t = \frac{n\lambda}{\mu-1} = \frac{3\times 5.45\times 10^{-5}}{1.5-1} = 32.7\times 10^{-5}$ cm

72. Using,

$X = \frac{\lambda D}{d}$

$= \frac{(5000\times 10^{-7}\times 10^{3}\,\text{mm})\times(100\times 10\,\text{mm})}{0.2\,\text{mm}}$

$= 2.5$ mm

$\therefore$ The distance between the consecutive bright and dark bands $= \frac{2.5}{2} = 1.25$ mm

73. $\frac{\text{Path difference}}{\lambda} = \frac{1.8\times10^{-5} - 1.23\times10^{-5}}{6000\times10^{-10}}$

$= \frac{(1.80-1.23)\times10^{-5}}{6000\times10^{-10}}$

$= \frac{57}{6} = 9.5$

$\therefore$ Path difference = 9.5 λ

As path difference is odd multiple of $\frac{\lambda}{2}$, point is dark.

74. $\lambda \approx d$, size of the obstacle.

76. $\lambda_{blue} < \lambda_{yellow}$
Hence diffraction bands become narrower.

80. For first minima in diffraction pattern,
$a \sin\theta = 1 \times \lambda_{Red}$
For first maxima in diffraction pattern,

$a \sin\theta = \frac{3}{2}\lambda$

As both coincide, $\lambda_{Red} = \frac{3}{2}\lambda$

$\therefore$ $\lambda = \lambda_{Red} \times \frac{2}{3} = 6600 \times \frac{2}{3} = 4400$ Å

81. For n^{th} secondary minimum, p.d. = $a \sin\theta_n = n\lambda$
and for n^{th} secondary maximum,

p.d. = $a \sin\theta_n = (2n+1)\frac{\lambda}{2}$

$\therefore$ For 1^{st} minimum, $a \sin 30° = \lambda$ ….(i)

For 2^{nd} maximum, $a \sin\theta_n = (2+1)\frac{\lambda}{2}$ ….(ii)

$\therefore$ Dividing equations (i) by equation (ii),

$\frac{1/2}{\sin\theta_n} = \frac{2}{3} \Rightarrow \theta_n = \sin^{-1}\left(\frac{3}{4}\right)$

82. For first minima, $\theta = \frac{\lambda}{a}$ or $a = \frac{\lambda}{\theta}$

$\therefore$ $a = \frac{6500\times10^{-8}\times6}{\pi}$ $\left[\because 30° = \frac{\pi}{6} \text{ radian}\right]$

$= 1.24 \times 10^{-4}$ cm
$= 1.24 \times 10^{-6}$ m
$= 1.24$ micron

83. The angular half width of the central maxima is given by,

$\sin\theta = \frac{\lambda}{a} \approx \theta$

$\therefore$ $\theta = \frac{6328\times10^{-10}}{0.2\times10^{-3}}$ rad

$= \frac{6328\times10^{-10}\times180}{0.2\times10^{-3}\times\pi}$ degree = 0.18°

$\therefore$ Total width of central maxima = 2θ = 0.36°

84. Given : λ = 600 nm = 600×10^{-9} m

Total angular width, $2\theta = \frac{2\lambda}{a} = \frac{2\times600\times10^{-9}}{0.2\times10^{-3}}$

$= 6 \times 10^{-3}$ rad

85. Distance between the first dark fringes on either side of central maxima = width of central maxima

$= \frac{2\lambda D}{d} = \frac{2\times600\times10^{-9}\times2}{1\times10^{-3}}$

= 2.4 mm

86. Distance of n^{th} minima from the centre of the screen is, $y_n = \frac{n\lambda D}{a}$

here, n = 1

$\therefore$ $y = \frac{\lambda D}{a} = \frac{5\times10^{-5}\times60}{0.02} = 0.15$ cm

87. Distance of 1^{st} minima from central maxima

$x_1 = \frac{\lambda D}{a}$

Distance between two minima on either side of the central maxima is

$2x_1 = \frac{2\lambda D}{a} = \frac{2\times5000\times10^{-10}\times2}{0.2\times10^{-3}} = 10^{-2}$ m

88. For secondary maxima, $\theta = \frac{(2n+1)\lambda}{2a} = \frac{5\lambda}{2a}$

$\therefore$ $\frac{x}{f} = \frac{5\lambda}{2a} \Rightarrow 2x = \frac{5\lambda f}{d} = \frac{5\times0.8\times6\times10^{-7}}{4\times10^{-4}}$

$= 6 \times 10^{-3}$ m = 6 mm

89. In single slit diffraction, for small angle,

$d\theta = 2n\frac{\lambda}{2}$ is the condition for minimum.

$\therefore$ $d = \frac{n\lambda}{\theta} = \frac{1\times698\times10^{-9}}{\left(2°\times\frac{\pi}{180}\right)^c}$

$\therefore$ $d = 2 \times 10^{-5}$ m

$\therefore$ d = 0.02 mm

90. Angular width of central maxima

$= \frac{2\lambda}{d} = \frac{2\times589.3\times10^{-9}}{0.1\times10^{-3}}$ rad

$= 0.0117 \times \frac{180}{\pi} = 0.68°$

91. In diffraction of light by single slit, the width of central maximum is given as -

width of central maxima = $\frac{2\lambda D}{d}$

$\therefore$ $W = \frac{2\lambda D}{d}$

But W = d ….(given)

$\therefore$ $d = \frac{2\lambda D}{d}$

$\Rightarrow D = \frac{d^2}{2\lambda}$

92. Here, wavelength, $\lambda = 625$ nm $= 625 \times 10^{-9}$ m

Number of lines per meter, $N = 2 \times 10^5$

For principal maxima in grating spectra $\frac{\sin\theta}{N} = n\lambda$,

where n(= 1, 2, 3) is the order of principal maxima and θ is the angle of diffraction.

The maximum value of sin θ is 1.

$\therefore$ $n = \frac{1}{N\lambda} = \frac{1}{2\times10^5\times625\times10^{-9}} = 8$

$\therefore$ Number of maxima = 2n + 1 = 2 × 8 + 1 = 17

93. $d\theta = \frac{1.22\lambda}{a} = \frac{y}{d}$

$\therefore$ $y = \frac{1.22\lambda d}{a} = \frac{1.22\times5\times10^{-7}\times10^3}{10\times10^{-2}} = 6.1\times10^{-3}$ m

= 6.1 mm ≈ 5 mm ≈ 0.5 cm

94. $N.A = \frac{\lambda}{2d}$

$\therefore$ $N.A \propto \frac{1}{d}$ ….(at λ = constant)

96. Angular magnification ∝ focal length of objective lens.

Angular resolution ∝ aperture (diameter) of objective lens.

97. R. P. of telescope $= \frac{a}{1.22\lambda}$

$\therefore$ $R.P. \propto \frac{1}{\lambda}$

As λ decreases, R. P. increases.

98. Resolving power of a microscope is,

$R.P. \propto \frac{1}{\lambda}$

$\therefore$ $\frac{R.P._1}{R.P._2} = \frac{\lambda_2}{\lambda_1} = \frac{6000}{4000} = \frac{3}{2}$

99. Limit of resolution $\theta = \frac{1.22\lambda}{D}$

$= \frac{1.22\times6000\times10^{-10}}{0.1}$

$= 7.32\times10^{-6}$ rad

100. $R.P. = \frac{D}{1.22\lambda} = \frac{2}{1.22\times0.5\times10^{-6}}$

$= \frac{4}{1.22}\times10^6 = 3.28\times10^6$

101. Resolving power of telescope,

$R.P. = \left(\frac{d}{1.22\lambda}\right) = \frac{1.22}{1.22\times5000\times10^{-10}}$

$\therefore$ $R.P. = 2\times10^6$

102. When a beam of light is used to determine the position of an object, the maximum accuracy is achieved if the light is of shorter wavelength, because

Accuracy $\propto \frac{1}{\text{Wavelength}}$

103. Distance between n^{th} bright fringe and m^{th} dark fringe (n > m)

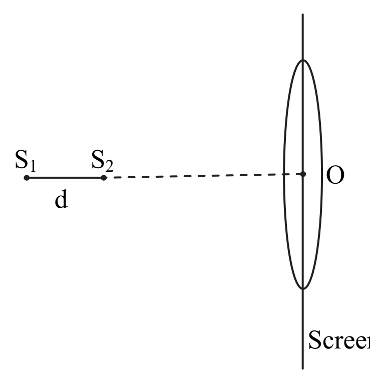
$\Delta x = \left(n - m + \frac{1}{2}\right)X$

$= \left(n - m + \frac{1}{2}\right)\frac{\lambda D}{d}$

$= \left(5 - 3 + \frac{1}{2}\right)\times\frac{6.5\times10^{-7}\times1}{1\times10^{-3}}$

≈ 1.63 mm

104.



Amongst the options only for a circle with centre as O, path difference will be constant, giving steady interference.

105. From formula, $I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$\therefore \quad \cos^2\left(\frac{\phi}{2}\right) = \frac{I}{I_{max}} = \frac{1}{2}$

$\Rightarrow \cos^2\phi = 0, \qquad \therefore \quad \cos\phi = 0$

$\therefore \quad \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

Corresponding path difference,

$\Delta x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{4\lambda}{4}$

$\therefore \quad \Delta x = (2n+1)\frac{\lambda}{4}$

106. Using, $X = \frac{\lambda D}{d}$, $X_1 = \frac{6000\times10^{-7}\times D}{d} = 2$ mm

$\therefore \quad \frac{D}{d} = \frac{2}{6000\times10^{-7}} = \frac{1}{3}\times10^4$

When the apparatus is dipped in water, wavelength and hence fringe width decreases by a factor of μ.

$\therefore \quad X_2 = \frac{X}{\mu} = \frac{2}{1.33} = 1.5$ mm

$\therefore$ Change in fringe width = 2 – 1.5 = 0.5 mm

107. Fringe width, $X = \frac{\lambda D}{d}$

$\therefore \quad X \propto \frac{D}{d}$ ….(i)

$X' \propto \frac{D'}{d'}$ ….(ii)

Dividing equation (ii) by equation (i),

$\frac{X'}{X} = \frac{D'd}{d'D}$

But $D' = 1.25$ D; $d' = \frac{d}{2}$

$\therefore \quad \frac{X'}{X} = \frac{(1.25\,D)(d)}{(d/2)D}$

$\therefore \quad X' = 2.5$ X

108. Let $A_1 = A_0$. Then $A_2 = 2A_0$

Intensity $I \propto A^2$

Hence $I_1 = I_0$, $I_2 = 4I_0$

We have $I = I_0 + 4I_0 + 2\sqrt{I_0\times4I_0}\cos\phi$

For I_{max} . $\cos\phi = 1$

$\therefore \quad I_m = 9I_0$ or $I_0 = \frac{I_m}{9}$

For a phase difference of ϕ,

$I = I_0 + 4I_0 + 2\sqrt{4I_0^2}\cos\phi$

$= I_0 + 4I_0(1+\cos\phi)$

$= I_0\left(1+8\cos^2\frac{\phi}{2}\right)$ ....$\left[\because 1+\cos\phi = 2\cos^2\frac{\phi}{2}\right]$

$= \frac{I_m}{9}\left(1+8\cos^2\phi/2\right)$

109. Phase difference, $\phi = \frac{2\pi}{\lambda}(\Delta)$

For path difference λ, phase difference $\phi_1 = 2\pi$ for path difference $\lambda/4$, phase difference $\phi_2 = \pi/2$.

Using, $I = 4I_0\cos^2\frac{\phi}{2}$

$\therefore \quad \frac{I_1}{I_2} = \frac{\cos^2(\phi_1/2)}{\cos^2(\phi_2/2)}$

$\therefore \quad \frac{K}{I_2} = \frac{\cos^2(2\pi/2)}{\cos^2\left(\frac{\pi/2}{2}\right)} = \frac{1}{1/2} \Rightarrow I_2 = \frac{K}{2}$.

110. phase difference = $\frac{2\pi}{\lambda}\times$ path difference

$\therefore \quad \phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{6} = \frac{\pi}{3}$

We know that for double slit interference

$I = 4I'\cos^2\phi/2$

(I' is intensity of each slit)

$\therefore \quad I = 4I'\cos^2\pi/6$

$\therefore \quad I = 4I'\left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \quad I = 3I'$

Also, the maximum intensity in interference is $I_o = 4I'$

$\therefore \quad \frac{I}{I_o} = \frac{3}{4}$

111. Resultant intensity,

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

At central position with coherent source,

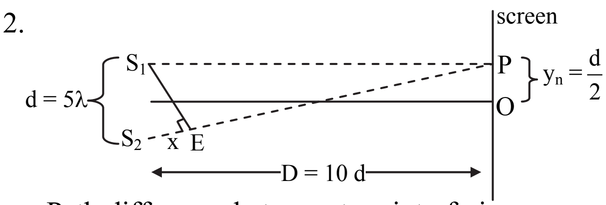
$I_{coh} = 4I_0$ $[\because I_1 = I_2 = I_0]$ ….(i)

In case of incoherent at a given point, ϕ varies randomly with time $\Rightarrow (\cos\phi)_{av} = 0$

$\therefore \quad I_{Incoh} = I_1 + I_2 = 2I_0$ ….(ii)

$\therefore \quad \frac{I_{coh}}{I_{Incoh}} = \frac{2}{1}$ ….[from (i) and (ii)]

112.



Path difference between two interfering waves arriving at point P is,

$x = \frac{yd}{D} = \frac{\left(\frac{d}{2}\right)d}{(10d)} = \frac{d}{20}$

∴ $x = \frac{5\lambda}{20} = \frac{\lambda}{4}$

$\Rightarrow$ phase difference, $\phi = \frac{\pi}{2} = 90°$

$I = I_0 \cos^2 \frac{\phi}{2}$

$= I_0 \cos^2 45°$

∴ $I = \frac{I_0}{2}$

113. Let n^{th} minima of 400 nm coincide with m^{th} minima of 560 nm

∴ $(2n - 1)400 = (2m - 1)560$

∴ $\frac{2n-1}{2m-1} = \frac{7}{5} = \frac{14}{10} = \frac{21}{15}$

4^{th} minima of 400 nm coincides with 3rd minima of 560 nm.

∴ The location of this minima is

$= \frac{7(1000)(400 \times 10^{-6})}{2 \times 0.1}$

= 14 mm

Next, 11^{th} minima of 400 nm will coincide with 8^{th} minima of 560 nm

∴ Location of this minima is

$= \frac{21(1000)(400 \times 10^{-6})}{2 \times 0.1}$

= 42 mm

∴ Required distance = 42 mm – 14 mm = 28 mm

114. $I = 4I_0 \cos^2 (\phi/2) \Rightarrow \phi = 2\pi/3$

∴ $\Delta x \times (2\pi/\lambda) = 2\pi/3 \Rightarrow \Delta x = \frac{\lambda}{3}$

∴ $\sin \theta = \Delta x/d$

$\Rightarrow \sin \theta = \lambda/3d$

115. $\frac{10\lambda D}{d} = \frac{2\lambda D}{a}$

$\Rightarrow a = \frac{2d}{10} = 0.2d = 0.2 \times 1 \text{ mm} = 0.2 \text{ mm}$

116. Angular width of fringe: $\theta = \frac{\lambda}{d}$

For λ = constant,

$\theta \propto \frac{1}{d}$

∴ $\frac{\theta}{\theta'} = \frac{d'}{d}$

∴ $\frac{0.20}{0.21} = \frac{d'}{2}$

∴ $d' = \frac{2 \times 0.2}{0.21} = 1.9 \text{ mm}$

117. Given: 2θ = 60°

Considering condition for minima in diffraction,

Path difference $(\Delta x) = a \sin\theta = n\lambda$

As a = 1μm, θ = 30° and n = 1,

∴ $\lambda = \frac{a \sin\theta}{n} = 1 \times 10^{-6} \times \frac{1}{2}$

∴ λ = 0.5 μm

If same setup is used for YDSE,

Fringe width $\beta = \frac{\lambda D}{d}$

As, β = 1 cm and D = 50 cm,

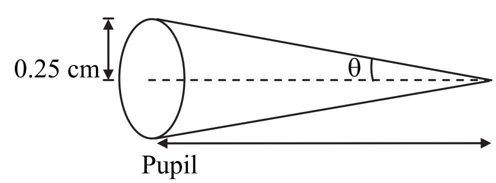
∴ $d = \frac{\lambda D}{\beta} = \frac{0.5 \times 10^{-6} \times 0.5}{0.01} = 25 \text{ μm}$

118. $d\theta = \frac{1.22\lambda f}{D}$

$= \frac{1.22 \times 5000 \times 10^{-10} \times 5}{2.5 \times 10^{-3}}$

$= 1.22 \times 10^{-3}$ m

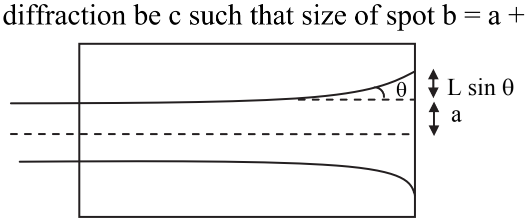
119.



$R.P. = \frac{1.22\lambda}{2\mu \sin\theta} = \frac{1.22 \times (500 \times 10^{-9} \text{ m})}{2 \times 1 \times \left(\frac{1}{100}\right)}$

$= 3.05 \times 10^{-5}$ m ≈ 30 μm

120. Let geometrical spread be a and spread due to diffraction be c such that size of spot b = a + c



From the figure,
$c = L \sin\theta$
For $\theta < c$, $\sin\theta \approx \theta$

$\therefore \quad c = L\theta = \dfrac{L\lambda}{a}$

$\therefore \quad b = a + \dfrac{L\lambda}{a}$ ….(i)

For minimum value of b,

$\therefore \quad 0 = \dfrac{a^2 + L\lambda}{a}$

$\therefore \quad a^2 = L\lambda$ [considering magnitude]

$\therefore \quad a = \sqrt{L\lambda}$

Substituting value of a in equation (i)

$b_{min} = \sqrt{L\lambda} + \dfrac{L\lambda}{\sqrt{L\lambda}} = 2\sqrt{L\lambda} = \sqrt{4L\lambda}$

Evaluation Test

1. The n^{th} bright fringe of the λ pattern and the n'^{th} bright fringe of the λ' pattern are situated at $y_n = n\dfrac{D\lambda}{d}$ and $y'_n = n'\dfrac{D\lambda'}{d}$

As these coincide, $y_n = y'_n$

$\therefore \quad \dfrac{nD\lambda}{d} = \dfrac{n'D\lambda'}{d} \qquad \therefore \quad \dfrac{n}{n'} = \dfrac{\lambda'}{\lambda} = \dfrac{900}{750}$

Hence the first position where overlapping occurs is,

$y' = y_6 = \dfrac{nD\lambda}{d} = \dfrac{6(1.5\,m)(750\times10^{-9}\,m)}{(2\times10^{-3}\,m)} \approx 3.4$ mm

2. For n^{th} maxima in Young's double slit experiment,

$y = \dfrac{nD\lambda}{d}$ or $\lambda = \dfrac{yd}{nD} = \dfrac{(10^{-3}\,m)(2\times10^{-3}\,m)}{n(2\,m)}$

$\therefore \quad \lambda = \dfrac{10000\times10^{-10}\,m}{n} = \dfrac{10000}{n}$ Å

But 3500 Å < λ < 7000 Å
For n = 1, 2, 3
λ = 10000 Å, 5000 Å, (3333.3) Å
For n = 2, λ = 5000 Å lies between 3500 Å to 7000 Å. The other wavelengths cannot fulfill this condition.

3. For Young's double slit experiment, the position of minima is;

$y = \left(n + \dfrac{1}{2}\right)\dfrac{D\lambda'}{d}$

Adjacent minima is the Ist minima or n = 0

$\therefore \quad y_1 = \left(0 + \dfrac{1}{2}\right)\dfrac{D\lambda'}{d} = \dfrac{D\lambda'}{2d}$

When immersed in liquid, $\lambda = \dfrac{\lambda'}{\mu_m}$

$\therefore \quad y_1 = \left(\dfrac{D\lambda'}{2\mu_m d}\right)$

Now fringe shift due to introduction of sheet on the path of one of the beams is β.

$\beta = \dfrac{D}{d}(\mu - 1)t$

The requirement is, mimina must appear on the axis.

$\therefore \quad \beta = y_1$ or $\dfrac{D}{d}\left(\dfrac{\mu_p}{\mu_m} - 1\right)t = \dfrac{D\lambda'}{2\mu_m d}$

$\therefore \quad t = \dfrac{\lambda'}{2(\mu_p - \mu_m)}$

4. Applying $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$, at central fringe (where $\theta = 0$) we get,
$I_R = I_1 + I_1 + 2I_1 = 4I_1$
Phase difference at a distance x when path difference becomes $\dfrac{xd}{D}$, is given by

$\theta' = \dfrac{2\pi}{\lambda}\dfrac{xd}{D}$

$\therefore \quad I_R' = I_1 + I_1 + 2I_1\cos\left(\dfrac{2\pi xd}{\lambda d}\right)$

$= \dfrac{I}{4} + \dfrac{I}{4} + 2\dfrac{I}{4}\cos\left(\dfrac{2\pi xd}{\lambda D}\right)$

or $I_R' = \dfrac{I}{2}\left(1 + \cos\dfrac{2\pi xd}{\lambda D}\right)$

$= I\cos^2\left(\dfrac{\pi xd}{\lambda D}\right)$

5. Using, $I = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$
At central point i.e., for maximum
$I_{max} = (A_1 + A_2)^2 = I_0$
$= (A + 2A)^2 = I_0$
or $I_0 = 9A^2$ or $A^2 = I_0/9$
For other points,
path difference = d sin θ

Again, $I_0 = A^2 + (2A)^2 + 4A\cos\left(\frac{2\pi}{\lambda}d\sin\alpha\right)$

$= A^2\left[5+4\cos\frac{2\pi}{\lambda}d\sin\alpha\right]$

$= \frac{I_0}{9}[5 + 8\cos^2 \pi/\lambda \times d\sin\alpha - 1]$

or $I_\alpha = \frac{I_0}{9}\left[1+8\cos^2\frac{\pi}{\lambda}d\sin\alpha\right]$

6. For minima, $d\sin\theta = n\lambda$

Here $n = 1$, $d\left(\frac{y}{D}\right)$

$= 1(5400 \text{ Å})$

$y_1 = \frac{D}{d}(5400 \text{ Å})$

Now, first maximum is approximately between the first minima and second minima.

$y_I = \left(\frac{y_1+y_2}{2}\right) = \left(\frac{1+2}{2}\right)\frac{D\lambda'}{d}$

As $y_1 = y_I \Rightarrow \frac{D}{d}(5400 \text{ Å}) = \left(\frac{3}{2}\right)\frac{D}{d}\lambda'$

$\therefore \quad \lambda' = \frac{2\times5400\text{ Å}}{3} = 3600 \text{ Å}$

7. For diffraction at circular aperture,

$\theta = \frac{1.22\lambda}{d} = \frac{1.22\times(6\times10^{-7}\text{ m})}{(2\times10^{-3}\text{ m})} = 3.66 \times 10^{-4}$ rad

If r is the radius of the image formed by the lens at its focus, then $\theta = \left(\frac{r}{f}\right)$

$\therefore \quad r = f\theta = (6 \times 10^{-2}\text{ m})(3.66 \times 10^{-4}\text{ rad})$

$= 21.96 \times 10^{-6}$ m

$A = \pi r^2 = (3.14)(21.96 \times 10^{-6}\text{ m})^2$

$= 15.14 \times 10^{-10}\text{ m}^2$

$I = \frac{P}{S}$

$= \frac{8\times10^{-3}\text{ W}}{15.14\times10^{-10}\text{ m}^2} \approx 5.2\ \frac{\text{kW}}{\text{m}^2}$

8. As $\theta_R = 1.22\frac{\lambda}{d}$

The angle subtended by the object at the human eye is $\theta = \frac{y}{D}$

where, y is the separation between the marks and D is the distance of the marks from the eye.

Now for clarity of vision, $\theta > \theta_R$

$\therefore \quad \frac{y}{D} > \frac{1.22\lambda}{d} \Rightarrow D < \frac{yd}{1.22\lambda}$

$\therefore \quad D_{greatest} = \frac{yd}{1.22\lambda} = \frac{(1\times10^{-3}\text{ m})(1.8\times10^{-3}\text{ m})}{1.22\times5550\times10^{-10}\text{ m}}$

$= 2.66 \approx 2.7$ m

9. For no appreciable diffraction effects, the distance must be less than Fresnel distance. The distance of the hill is $\frac{60\text{ km}}{2} = 30$ km.

The aperture can be taken as a = 100 m.

30 km < Z_f

$Z_f = \frac{a^2}{\lambda} = \frac{(100\text{ m})^2}{\lambda} \Rightarrow 30\text{ km} < \frac{(100\text{ m})^2}{\lambda}$

or $\lambda < \frac{(100\text{ m})^2}{30\text{ km}} \Rightarrow \lambda_{max} = \frac{(100\text{ m})^2}{30000\text{ m}} = 0.333$ m

$= 33.3$ cm

10. The gap between successive wavefronts is λ.

Hence the required time, $t = \frac{(3\lambda)}{c}$

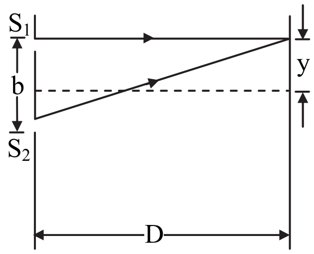
11. The interference patterns due to different component colors of white light overlap. The central bright fringes for different colors are at the same position. Hence, the central fringe is white. For a point P for which $S_2P - S_1P = \lambda_b/2$ where λ_b(= 4000 Å) represents wavelength of blue light, the blue component will be absent and the fringe will appear red in color. Slightly farther away where $S_2Q - S_1Q = \lambda_b = \frac{\lambda_r}{2}$ where λ_r (= 8000 Å) is the wavelength for the red colour, the fringe will be predominantly blue. Thus, the fringe closest on either side of the central white fringe is red and the farthest will appear blue.

12. In the given situation,

$y = (2n-1)\frac{\lambda}{2}\frac{D}{d}$

$= (2n-1)\frac{\lambda}{2}\frac{D}{b}$

….(∵ 'missing wavelength' ⇒ minima and here, d = b)



But y = b/2

∴ $\frac{b}{2} = (2n-1)\frac{\lambda}{2}\frac{D}{b}$

∴ $\lambda = \frac{b^2}{(2n-1)D}$

∴ For n = 1,2,…; $\lambda = \frac{b^2}{D}, \frac{b^2}{3D}...$

13. Distance of m[th] bright fringe of λ pattern and m'th bright fringe of λ' pattern are at

$y = \frac{mD\lambda}{d}$ and $y' = \frac{m'D\lambda'}{d}$

Since y = y′

∴ $\frac{m}{m'} = \frac{\lambda'}{\lambda} = \frac{750}{600} = \frac{5}{4}$

∴ m = 5 and m′ = 4

Now the position where 5[th] bright fringe of λ pattern will coincide with 4[th] bright fringe of λ′ pattern,

$y = \frac{5 \times 1 \times 600 \times 10^{-9}}{1 \times 10^{-3}}$

$= 0.3 \times 10^{-3}$ m

= 0.3 mm

14. Fringe width, $x = \frac{\lambda D}{d}$

Half-angular width of central bright portion,

$\theta = \frac{\lambda}{a}$

Overlapping length,

$y = (2\theta)D - d = \frac{2\lambda D}{a} - d$

Number of bright fringes

$= \frac{y}{x} = \frac{\left(\frac{2\lambda D}{a} - d\right)}{\lambda D / d}$

$= \frac{(2\lambda D - da)d}{a\lambda D}$

15. Distance covered between two consecutive maxima = λ/2

Total distance covered = $(n-1)\frac{\lambda}{2} = S$

∴ $\lambda = \frac{2S}{n-1}$

Using c = λν we get, $\nu = \frac{c}{\lambda}$

Assuming velocity of TV waves in air to be c we get,

$\nu = \frac{c}{2S/n-1} = \frac{(n-1)c}{2S}$

16. Visible light has wavelength (λ) 6000 Å. The least marking on metre scale is 1 mm. If D is the required distance then angle subtended by 1 mm at distance D, $\theta = \frac{1\,mm}{D\,m} = \frac{1}{D \times 1000}$ rad

In order to see the marking clearly, this angle must be equal to or greater than $\frac{\lambda}{a}$ of the instrument.

$\frac{1}{1000D} \geq \frac{\lambda}{a}$ or $D \leq \frac{a}{1000\lambda}$

∴ $D \leq \frac{2 \times 10^{-3}\,m}{1000 \times 6 \times 10^{-7}}$

∴ D = 3.3 m

17. $\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{v}$

x-rays are fast moving high-energy electrons.
As speed of electron increases, its de-Broglie wavelength decreases.
Angular width for central maximum is given as,

$\omega = \frac{2\lambda}{d}$ ∴ $\omega \propto \lambda \propto \frac{1}{v}$

∴ If speed of electron increases, angular width of central maximum will decrease.

18. $d\theta = \frac{\text{diameter of the telescope}}{\text{distance of the moon}}$

∴ $d\theta = \frac{5}{4 \times 10^5 \times 10^3}\,m = \frac{5}{4 \times 10^8}\,m$

∴ $d\theta = \frac{1.22\lambda}{d}$

∴ $d = \frac{1.22\lambda}{d\theta} = \frac{1.22 \times 6 \times 10^{-7}}{\left(\frac{5}{4 \times 10^8}\right)}$

∴ d = 58.5 m ≈ 59 m

19. For the first minimum on either side of the maximum,

$a \sin \theta = \lambda$ or $\sin \theta = \dfrac{\lambda}{a}$

$\therefore \quad \sin \theta = \dfrac{3}{5} = 0.6$

$\therefore \quad \theta = 36° \, 52'$

Since central maximum spreads on both sides

Angular spread = $\pm 36° \, 52'$

20. Position of first minima in diffraction pattern of λ_1 is given by, $a \sin \theta = n\lambda$

$\therefore \quad a \sin \theta_1 = 1\lambda_1$

$\therefore \quad \sin \theta_1 = \dfrac{\lambda_1}{a}$

For the first maxima of wavelength λ_2,

$a \sin \theta_2 = \dfrac{3}{2}\lambda_2$

$\therefore \quad \sin \theta_2 = \dfrac{3\lambda_2}{2a}$

But $\theta_1 = \theta_2$ or $\sin \theta_1 = \sin \theta_2$

$\therefore \quad \dfrac{\lambda_1}{a} = \dfrac{3\lambda_2}{2a} \Rightarrow \lambda_2 = \dfrac{2}{3}\lambda_1 = \dfrac{2}{3} \times 600$

$\therefore \quad \lambda_2 = 400$ nm

21. $y'_n = \dfrac{(2n-1)\lambda D}{2d}$ where n = 1, 2, 3, ….

$y'_3 = \dfrac{5}{2}\dfrac{\lambda D}{d}$ and $y'_{10} = \dfrac{19}{2}\dfrac{\lambda D}{d}$

Since the bands are on opposite sides of the central bright band, the distance between these bands is $y'_3 + y'_{10}$

$\therefore \quad y'_3 + y'_{10} = \dfrac{5}{2}\dfrac{\lambda D}{d} + \dfrac{19}{2}\dfrac{\lambda D}{d}$

$= \dfrac{12 \times 5896 \times 10^{-10} \times 0.60}{0.4 \times 10^{-3}}$

$\approx 1.1 \times 10^{-2}$ m ≈ 1.1 cm

22. Interference effects are commonly observed in thin flims when their thickness is comparable to wavelength of incident light.

For excessively thin film, as compared to wavelength of light, it appears dark and for a film which is too thick, it results into uniform illumination of the film. In thin film, interference takes place between the waves reflected from its two surfaces and waves refracted through it.

Textbook Chapter No.

12 Electrostatics

Hints

Classical Thinking

9. $\text{T.N.E.I} = \oint \varepsilon \vec{E} \cdot d\vec{s}$

$= \oint \varepsilon E ds \cos\theta$

$\therefore$ T.N.E.I. is maximum when,

$\cos\theta = 1$

$\Rightarrow \theta = 0°$

12. As no charge is enclosed within the cylinder,

$\therefore$ T.N.E.I. = q = 0

13. Electric field is zero at any interior point as there is no line of force.

14. S.I. unit of electric flux is $\frac{N \times m^2}{C} = \frac{J \times m}{C}$

= volt × m

19. If σ is the surface density, then charge on the surface S, q = σ.S

Electric intensity E due to remaining charges on the surface, $E = \frac{\sigma}{2k\varepsilon_0}$

Force experienced due to other charges on surface S,

$F = E \times q = \frac{\sigma}{2k\varepsilon_0} \sigma S = \frac{\sigma^2}{2k\varepsilon_0} S$

21. Energy density $\propto E^2$

Now, $E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$

$\therefore$ Energy density $\propto E^2 \propto \frac{1}{r^4}$

22. $u = \frac{1}{2} k\varepsilon_0 E^2$

$= \frac{1}{2} \times 4 \times 8.85 \times 10^{-12} \times (200)^2$

$= 7.08 \times 10^{-7}$ J/m^3

23. $u \propto E^2$

$\therefore \frac{u'}{u} = \left(\frac{E'}{E}\right)^2 = \left(\frac{E}{2E}\right)^2 = \frac{1}{4}$

$\therefore u' = \frac{u}{4}$

38. $C = \frac{Q}{V}$

But, $V = \frac{W}{Q}$ ….(W = work done)

$\therefore C = \frac{Q^2}{W} = \frac{(It)^2}{W}$

$\therefore [C] = \frac{[A^2T^2]}{[M^1L^2T^{-2}]} = [M^{-1}L^{-2}T^4A^2]$

39. $C = \frac{\varepsilon A}{d} \Rightarrow C \propto \varepsilon$

$\therefore C \propto A$ and $C \propto \frac{1}{d}$

40. $E = \frac{V}{d} \Rightarrow d = \frac{V}{E} = \frac{20}{400} = \frac{1}{20}$ m = 5 cm

41. $C = \frac{Ak\varepsilon_0}{d}$

$= \frac{(5 \times 10^{-4}) \times 5 \times 8.85 \times 10^{-12}}{2 \times 10^{-3}}$

$= 1.10 \times 10^{-11} = 11 \times 10^{-12}$ F = 11 pF

50. $q = CV$ and $U = \frac{1}{2}CV^2 = \frac{q^2}{2C}$

51. $U = \frac{1}{2} CV^2$

$= \frac{1}{2} \times 10 \times 10^{-6} \times (1000)^2$

$= 0.5 \times 10 \times 10^{-6} \times 10^6 = 5$ J

52. $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 8 \times 10^{-6} \times (100)^2$

$= 4 \times 10^{-2} = 0.04$ J

53. $U = \frac{1}{2} \times QV$

$= \frac{1}{2} \times 6 \times 10^{-6} \times 500$

$= 15 \times 10^{-4}$ J

56. $C_1 = \frac{C}{4}$ (for series); $C_2 = 4C$ (for parallel)

∴ $\frac{C_1}{C_2} = \frac{1}{16}$

57. $\frac{1}{C_s} = \frac{1}{3} + \frac{1}{9} + \frac{1}{18} = \frac{1}{2} \Rightarrow C_s = 2\ \mu F$

$C_p = 3 + 9 + 18 = 30\ \mu F$

∴ $\frac{C_s}{C_p} = \frac{2}{30} = \frac{1}{15}$

58. We will arrange the capacitors such that three of them are in parallel and the fourth one is in series with the combination,

∴ $\frac{1}{C_{eff.}} = \frac{1}{(4+4+4)} + \frac{1}{4} = \frac{1}{12} + \frac{1}{4} = \frac{4}{12} = \frac{1}{3}$

∴ $C_{eff.} = 3\ \mu F$

59. Let C be capacitance of each capacitor connected in parallel.

∴ $C_{eff.} = 3C$

Now, 3C and C are in series.

∴ $\frac{1}{C'_{eff.}} = \frac{1}{3C} + \frac{1}{C} = \frac{4}{3C}$

∴ $C'_{eff.} = \frac{3C}{4} = 3.75$

∴ $C = \frac{3.75 \times 4}{3} = 1.25 \times 4 = 5\ \mu F$

63. As density of line is more at A than B, $E_A > E_B$

64. $u = \frac{1}{2}\varepsilon_0 E^2$

∴ $E = \sqrt{\frac{2u}{\varepsilon_0}} = \sqrt{\frac{2 \times 44.25 \times 10^{-8}}{8.85 \times 10^{-12}}} = 316.2$ N/C

65. Potential of both spheres will be same.

66. $C = \frac{Q}{V} = \frac{40}{10-(-10)} = \frac{40}{20} = 2$ F

67. $u = \frac{1}{2} CV^2$

∴ $V = \sqrt{\frac{2u}{C}} = \sqrt{\frac{2 \times 50}{100 \times 10^{-6}}} = \sqrt{10^6}$

∴ $V = 10^3$ V = 1000 V

68. Two capacitors of 1.5 μF each are in parallel.

∴ $C_{eff} = 1.5 + 1.5 = 3\ \mu F$

Now, 3 μF, 3 μF and 3 μF are in series,

∴ $\frac{1}{C'_{eff}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$

∴ $C'_{eff} = 1\ \mu F$

69. There are two loops, each having two capacitors of 20 μF each in parallel.

∴ $C_{eff.} = 20 + 20 = 40\ \mu F$ for each loop.

Now, these two capacitors of 40 μF each are in series.

∴ $C_{eff.} = \frac{40 \times 40}{40 + 40} = \frac{1600}{80} = 20\ \mu F$

Critical Thinking

2. As there is no charge residing inside the cube, the net flux is zero.

3. T.N.E.I. does not depend upon shape or the size of Gaussian surface but depends only upon charge enclosed within the surface.

4. Total number of surfaces = 6

Total charge, Q = 24 C

Total flux, $\phi = \frac{Q}{\varepsilon_0}$

So, flux through each surface $= \frac{\phi}{6} = \frac{24}{6\varepsilon_0}$

$= \frac{4}{\varepsilon_0}$ V-m

5. Electric intensity at a distance r from the centre of a charged spherical conductor of radius R,

$E = \frac{q}{4\pi k\varepsilon_0 r^2}$ ....(i)

Since the charge is uniformly distributed on A, the surface density of charge on A will be

$\sigma = \frac{q}{4\pi R^2} \Rightarrow q = 4\pi R^2 \sigma$

Substituting in eq. (i), we get

$E = \frac{4\pi R^2 \sigma}{4\pi k\varepsilon_0 r^2} = \frac{\sigma R^2}{k\varepsilon_0 r^2}$

6. Electric field near the surface of the conductor is given by, $\frac{\sigma}{\varepsilon_0}$ and it is perpendicular to surface.

7. $V = \frac{kq}{R}$ i.e. $V \propto \frac{1}{R}$

∴ Potential on smaller sphere will be more.

8. $E \propto \frac{1}{k} \Rightarrow E_1 \propto \frac{1}{k_1}$ and $E_2 \propto \frac{1}{k_2}$

∴ $E_2 = \frac{E_1}{k_2} = \frac{6}{3} = 2$ N/C

9. $E = \frac{\sigma R}{k\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0} \times \frac{\sigma R \times 4\pi}{kr}$

$= \frac{9\times10^9 \times 2\times10^{-6} \times 5\times10^{-3} \times 4\times3.14}{6.28\times2}$

= 90 V/m

10. $E = \frac{\lambda}{2\pi k\varepsilon_0 r} \Rightarrow \lambda = 2\pi\varepsilon_0 rE$

∴ $\lambda = 4\pi\varepsilon_0 \left(\frac{r}{2}\right) E = \frac{1}{9\times10^9} \times \left(\frac{2}{2}\right) \times 4.5 \times 10^4$

$= \frac{1}{2} \times 10^{-5} = 5\ \mu$C/m

11. Electric potential inside a conductor is constant and it is equal to that on the surface of conductor.

12. $E = \frac{\lambda}{2\pi K\varepsilon_0 r}$ i.e. $E \propto \frac{1}{r}$

∴ $\frac{E'}{E} = \frac{r}{r'} = \frac{20}{40} = \frac{1}{2}$

∴ $E' = \frac{E}{2} = \frac{0.4}{2} = 0.2$ N/C

13. $E = \frac{\sigma R^2}{k\varepsilon_0 r^2}$

Just outside the conductor, $R \le r \Rightarrow \frac{R^2}{r^2} \approx 1$

∴ $E = \frac{\sigma}{k\varepsilon_0} = \frac{\sigma 4\pi}{4\pi\varepsilon_0 k}$

$= \frac{12\times10^{-12} \times 4\times3.14\times9\times10^9}{3.14}$

$= 43.2 \times 10^{-2} = 0.43$ V/m

14. $E = \frac{Q}{4\pi\varepsilon_0 R^2}$

∴ $E_{max} = \frac{Q_{max}}{4\pi\varepsilon_0 R^2}$

∴ $Q_{max} = 4\pi\varepsilon_0 R^2 \times E_{max}$

$= \frac{1}{9\times10^9} \times (10 \times 10^{-2})^2 \times 2 \times 10^6$

$= \frac{2}{9} \times 10^{-5}$ C

15. $E_1 + -(E_2) = 0$

∴ $E_1 = E_2$

Let x be the distance of the point from centre of A where electric field is zero.

∴ $\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(80-x)^2}$

∴ $\frac{(80-x)^2}{x^2} = \frac{15}{5} = 3$

∴ $\frac{80-x}{x} = \sqrt{3}$

....[Retaining positive square root]

∴ $80 - x = \sqrt{3}\,x$

∴ $80 = \sqrt{3}\,x + x \Rightarrow 80 = (1+\sqrt{3})x$

∴ $x = \frac{80}{1+\sqrt{3}} \approx 29$ cm

16. Charge density $\sigma = \frac{q}{A}$

∴ $q = \sigma.A = \sigma\,(4\,\pi R^2)$

∴ Distance of point from centre

$r = R + 0.2 = 0.1 + 0.2 = 0.3$ m

∴ $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma(4\pi R^2)}{r^2} = \frac{\sigma}{\varepsilon_0}\left(\frac{R}{r}\right)^2$

$= \frac{1.8\times10^{-6}\times(0.1)^2}{(0.3)^2\varepsilon_0} = \frac{2\times10^{-7}}{\varepsilon_0}$

17. As $\sigma_1 = \sigma_2$,

∴ $\frac{Q_1}{4\pi r_1^2} = \frac{Q_2}{4\pi r_2^2}$

∴ $\frac{Q_1}{4\pi\varepsilon_0 r_1^2} = \frac{Q_2}{4\pi\varepsilon_0 r_2^2}$

∴ $E_1 = E_2 \Rightarrow \frac{E_1}{E_2} = \frac{1}{1}$ or $E_1 : E_2 = 1 : 1$

18. The cube has six surfaces and as the charge is at its centre. Hence, it will produce equal number of lines of forces through each surface.

The charge of Q will produce in all $\frac{Q}{\varepsilon_0}$ lines of force.

∴ Each surface will allow $\left(\frac{Q}{6\varepsilon_0}\right)$.

19. Flux linked with the given sphere $\phi = \frac{Q}{\varepsilon_0}$;

where Q = Charge enclosed by the sphere.
Hence $Q = \phi\varepsilon_0 = (E \times \text{Area})\varepsilon_0$

$\therefore$ $Q = \left(\frac{A}{\gamma_0}\right)(4\pi\gamma_0^2)\,\varepsilon_0 = 4\pi\varepsilon_0 A\gamma_0$.

20. $\lambda = \frac{q}{2\pi r l} = \frac{10\times10^{-3}}{2\times3.14\times1\times10^{-3}\times10^{3}}$

$= 1.59\times10^{-3}\ \text{C/m}^2$

21. $F = \frac{\sigma^2 ds}{2\varepsilon_0 k} = \frac{q^2}{2\varepsilon_0 k ds}$

$= \frac{\left(\sqrt{8.85}\times10^{-6}\right)^2}{2\times8.85\times10^{-12}\times1\times1} = 0.5\ \text{N}$

22. $\frac{dF}{ds} = \frac{\sigma^2}{2k\varepsilon_0}$

$= \frac{0.885\times0.885\times10^{-12}}{2\times8.85\times10^{-12}}$

$= 4.425\times10^{-2}\ \text{N/m}^2$

23. $f = \frac{\sigma^2}{2\varepsilon_0 k} = \frac{q^2}{2\varepsilon_0 k ds^2}$

$= \frac{q^2}{32\varepsilon_0 k\pi^2 R^4}$ ....($\because ds = 4\pi R^2$)

$= \frac{\left(12\times10^{-6}\right)^2}{32\times8.85\times10^{-12}\times1\times9.87\times\left(10^{-1}\right)^4}$

$\approx 5.15\times10^2\ \text{N/m}^2$.

24. $u = \frac{1}{2}k\varepsilon_0 E^2$

$= \frac{1}{2}\times2\times8.85\times10^{-12}\times(400)^2$

$= 1.416\times10^{-6}\ \text{J/m}^3$

25. $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\frac{V^2}{R^2}$

$= \frac{1}{2}\times\frac{8.85\times10^{-12}\times100\times10^{6}}{10^{-4}} = 4.425\ \text{J/m}^3$

29. $Q = VC \Rightarrow V = Q/C$
As V is constant,

$\therefore$ $\frac{Q_g}{C_g} = \frac{Q_o}{C_o} \Rightarrow \frac{Q_g}{Q_o} = \frac{C_g}{C_o}$ where C_g is the new capacitance and Q_g is new charge.

$\because$ $C_g > C_o \Rightarrow Q_g > Q_o$

30. Since d decreases, so C increases.

$\because$ battery is disconnected $\Rightarrow$ Q is constant .

$\therefore$ $V \propto \frac{1}{C}$

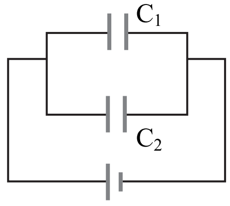
Since V decreases, so C will increase.

32. Total charge on capacitors connected in parallel is,

$Q_0 = \frac{C_0}{V_0}$ ….(i)

Where C_0 = effective capacitance of parallel combination.

$= C + C = 2C$….($\because C_1 = C_2 = C$) (ii)



Let C_2 be kept in a dielectric medium,
then, $C_2' = kC$

$\Rightarrow C_0' = C + kC = (1+k)C$

Hence, total charge on the capacitors,

$Q_0' = \frac{C_0'}{V_0} = \frac{(1+k)C}{V_0}$ ….(iii)

Dividing equation (iii) by equation (i)

$\frac{Q_0'}{Q_0} = \frac{(1+k)C}{V_0}\times\frac{V_0}{C_0} = \frac{(1+k)}{2}$ ….from (ii)

$\therefore$ $Q_0' = \frac{(1+k)Q_0}{2}$

33. $C = \frac{Q}{V}$ ….(i)

$V = \frac{Q}{4\pi\varepsilon_0 r}$ ….(ii)

From equation (i) and (ii)

$4\pi\varepsilon_0 r = \frac{\varepsilon_0 A}{d}$

$\therefore$ $d = \frac{A}{4\pi r} = \frac{\pi(20\times10^{-3})^2}{4\pi\times1} = 0.1\ \text{mm}$

34. $C = \frac{\varepsilon_0 A}{d} = 8\ \text{pF}$ and $C' = \frac{\varepsilon_0 kA'}{d'}$

But $A' = A$, $d' = d/2$

$\therefore$ $C' = \frac{\varepsilon_0 k\times A}{d/2} = \frac{2\times5\times\varepsilon_0 A}{d}$

$\therefore$ $C' = 10\times8\ \text{pF} = 80\ \text{pF}$

35. Without dielectric, $C_0 = \frac{\varepsilon_0 A}{d}$

With dielectric, $C_1 = \frac{k_1\varepsilon_0 A}{d/2} = 2k_1 C_0$

and $C_2 = \frac{k_2\varepsilon_0 A}{d/2} = 2k_2 C_0$

As C_1, C_2 are in series,

$$C_s = \frac{C_1C_2}{C_1+C_2}$$

$$= \frac{2k_1C_0 \times 2k_2C_0}{2C_0(k_1+k_2)} = \frac{2k_1k_2C_0}{k_1+k_2}$$

$\therefore$ $\frac{C_s}{C_0} = \frac{2k_1k_2}{k_1+k_2}$

36. Capacity of capacitor = C

$Q = CV = \frac{\varepsilon_0 AV}{d}$ ....(i)

After inserting a slab, capacitance becomes C_1 and charge remains same, $Q = C_1V_1$

By increasing the distance, we get same potential difference as in first case.

$Q = C_2V$ ....(ii)

$C = \frac{\varepsilon_0 A}{d-t}$

$\therefore$ $\frac{1}{C_2} = \frac{d-3+2.4}{\varepsilon_0 A} + \frac{3}{k\varepsilon_0 A} = \frac{d-0.6}{\varepsilon_0 A} + \frac{3}{k\varepsilon_0 A}$

From equations (i) and (ii),

$C = C_2$

$\therefore$ $\frac{1}{C} = \frac{1}{C_2}$

$\therefore$ $\frac{d}{\varepsilon_0 A} = \frac{d-0.6}{\varepsilon_0 A} + \frac{3}{k\varepsilon_0 A}$

$\therefore$ $d = d - 0.6 + \frac{3}{k}$

$\therefore$ $k = \frac{3}{0.6} = 5$

37. Capacity of plate in medium,

$C_m = \frac{k\varepsilon_0 A}{d}$ ....(i)

If medium is removed,

$C = \frac{\varepsilon_0 A}{d}$ ....(ii)

From equations (i) and (ii),

$C_m = kC$

$\therefore$ $C = \frac{C_m}{k} = \frac{16\mu F}{8} = 2\ \mu F$

38. Potential difference across the condenser,

$V = V_1 + V_2 = E_1t_1 + E_2t_2 = \frac{\sigma}{k_1\varepsilon_0}t_1 + \frac{\sigma}{k_2\varepsilon_0}t_2$

$V = \frac{\sigma}{\varepsilon_0}\left(\frac{t_1}{k_1}+\frac{t_2}{k_2}\right) = \frac{Q}{A\varepsilon_0}\left(\frac{t_1}{k_1}+\frac{t_2}{k_2}\right)$

39. If length of the foil is l, then

$C = \frac{k\varepsilon_0(l \times b)}{d}$ ....[$\because$ $A = l \times b$]

$\therefore$ $2\times10^{-6} = \frac{2.5\times8.85\times10^{-12}(l\times400\times10^{-3})}{0.15\times10^{-3}}$

$\therefore$ $l = \frac{2\times10^{-6}\times0.15\times10^{-3}}{2.5\times8.85\times10^{-12}\times400\times10^{-3}} = 33.9$ m

40. While drawing the dielectric plate outside, the capacitance decreases till the entire plate comes out and then becomes constant. So, V increases and then becomes constant.

41. $U_1 = \frac{1}{2}CV_1^2$, $U_2 = \frac{1}{2}CV_1^2$

$\therefore$ $\frac{U_1}{U_2} = \frac{V_1^2}{V_2^2}$

$\therefore$ $U_2 = \frac{V_2^2}{V_1^2}U_1 = \frac{900}{100} \times U_1 = 9\ U_1$

42. Increase in energy $= \frac{1}{2}C_1V_1^2 - \frac{1}{2}C_0V_0^2$

$= \frac{1}{2}C(V_1^2 - V_0^2)$ ....($\because$ $C_1 = C_0 = C$)

$= \frac{1}{2} \times 10 \times 10^{-6}(121 - 100)$

$= \frac{1}{2} \times 10 \times 21 \times 10^{-6} = 105 \times 10^{-6}$ J $= 105\ \mu$J

43. $U_1 + U_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

$= \frac{1}{2}[4 \times 10^{-6} \times 50 \times 50 + 2 \times 10^{-6} \times 100 \times 100]$

$= \frac{1}{2}[10^{-2} + 2 \times 10^{-2}] = \frac{3}{2} \times 10^{-2}$ J

44. Initial energy of the system,

$U_i = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2 = \frac{1}{2}C\left(V_1^2 + V_2^2\right)$

When the capacitors are joined, common potential, $V = \frac{CV_1+CV_2}{2C} = \frac{V_1+V_2}{2}$

$\therefore$ Final energy of the system,

$$U_f = \frac{1}{2}(2C)V^2 = \frac{1}{2}2C\left(\frac{V_1+V_2}{2}\right)^2$$

$$= \frac{1}{4}C(V_1+V_2)^2$$

$\therefore$ Decrease in energy $= U_i - U_f = \frac{1}{4}C(V_1 - V_2)^2$

45. $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

Here, Q in both cases is same

$\therefore$ $U_1 = \frac{Q^2}{2C_1}$ and

$U_2 = \frac{Q^2}{2C_2} = \frac{Q^2}{2kC_1}$

Now, $C \propto k \Rightarrow C_2 = kC_1$

$\therefore$ Decrease in energy $= U_1 - U_2$

$$= \frac{Q^2}{2C_1} - \frac{Q^2}{2kC_1} = \frac{Q^2}{2C_1} \times \left(1 - \frac{1}{k}\right)$$

$\therefore$ Fractional decrease in energy $= \frac{U_1 - U_2}{U_1}$

$$= \frac{\frac{Q^2}{2C_1}\times\left(1-\frac{1}{k}\right)}{\left(\frac{Q^2}{2C_1}\right)} = 1 - \frac{1}{k}$$

46. $U = \frac{1}{2}CV^2 = \frac{1}{2}\times 4\times 10^{-6} \times (10^3)^2 = 2$ J

47. Since charge remains same in series combination,

$\therefore$ $C_1V_1 = C_2V_2$

$\therefore$ $\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{4}{1} = 4$

48. Potential difference in the circuit = 24 – 12 = 12 volt. This potential difference is divided among two capacitors C_1 and C_2 in the inverse ratio of their capacities (as they are joined in series)

$\therefore$ $V_1 = \frac{C_2}{C_1 + C_2}V = \frac{4}{2+4} \times 12 = 8$ volt

As plate of capacitor C_1 towards point B will be at positive potential, hence

$V_B - V_A = 8$ volt

$\therefore$ $V_A - V_B = -8$ V

49. On connecting O at A, 4 μF capacitor is charged to a constant potential (E).

As connection of O is switched over to B, the total charge on 4 μF capacitor that will be shared between 4 μF and 2 μF capacitors is $\frac{4}{4+2} = \frac{2}{3}$ of original charge.

50. The effective capacitance is C_1 when three capacitors are connected in series

$\therefore$ $\frac{1}{C_1} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{37}{60}$

$\therefore$ $C_1 = 60 / 37$ μF ....(i)

When three capacitors are connected in parallel mode, the effective capacitance is C_2

$\therefore$ $C_2 = 4 + 5 + 6 = 15$ μF ....(ii)

From (i) and (ii),

$\frac{C_2}{C_1} = \frac{15}{60/37} = \frac{37}{4}$

51. The capacitors of capacitance 2 μF and 6 μF are connected in series. Hence, their effective capacitance, $C_s = \frac{6\times2}{6+2} = 1.5$ μF.

These two branches are connected in parallel.

$\therefore$ Equivalent capacitance (C′) = 1.5 + 1.5 = 3 μF

Now 4 μF, 4 μF and C′ are connected in series.

$\therefore$ Relation for the capacitance between P and Q,

$\frac{1}{C''} = \frac{1}{4} + \frac{1}{4} + \frac{1}{3} = \frac{5}{6}$ or $C'' = \frac{6}{5}$ μF

52. Given six capacitors are in parallel

$\therefore$ $C_{eq} = 6C = 6 \times 2$ μF = 12 μF

53. $C_{eff} = C + \frac{C}{2} = \frac{3C}{2}$

$\therefore$ Work done $= \frac{1}{2}\left(\frac{3C}{2}\right)V^2 = \frac{3CV^2}{4}$

54. Capacitance of first capacitor (C_1)

$= 30$ μF $= 30 \times 10^{-6}$ F and its voltage (V_1)

$= 500$ V

Capacitance of the second capacitor (C_2)

$= 15$ μF $= 15 \times 10^{-6}$ F and its voltage (V_2)

$= 300$ V

$\therefore$ Common potential (V) $= \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$= \frac{(30\times10^{-6}\times500)+(15\times10^{-6}\times300)}{(30\times10^{-6})+(15\times10^{-6})}$$

≈ 433 V

55. $\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3}$

$= \frac{1}{2} + \frac{1}{2+1}$

$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

∴ $C_{eff} = \frac{6}{5}$ μF

∴ Total charge, $Q = \frac{6}{5} \times 10^{-6} \times 120 = 144 \times 10^{-6}$ C

∴ Potential difference across C_1,

$V_1 = \frac{Q}{C_1} = \frac{144 \times 10^{-6}}{2 \times 10^{-6}} = 72$ V

56. Charge on capacitor,
$Q = CV = 8 \times 10^{-6} \times 12 = 96$ μC

∴ $V = \frac{Q}{C} \Rightarrow C = \frac{Q}{V}$

Total capacity, $C = C_1 + C_2 = \frac{96}{3} = 32$ μF

∴ $C_2 = 32$ μF $- C_1 = 32 - 8 = 24$ μF

57. $C_1 = 2$ μF, $V = 100$ V

∴ $Q_1 = C_1V = 2 \times 100 = 200$ μC
If we connect this condenser to uncharged condenser, then total charge, $Q = Q_1$
$Q = 200$ μC (due to parallel combination)
Total capacitance $C = C_1 + C_2$
$= 2$ μF $+ 3$ μF $= 5$ μF

∴ Common potential $= \frac{\text{Total charge}}{\text{Total capacitance}}$

$= \frac{200 \times 10^{-6}}{5 \times 10^{-6}} = 40$ V

58.

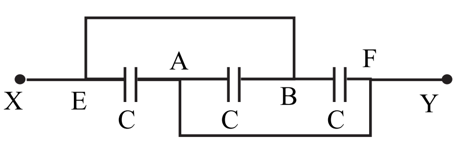


Figure (a)

Join B and E together. Similarly, join A and F. Then the given circuit becomes as shown in figure (b)

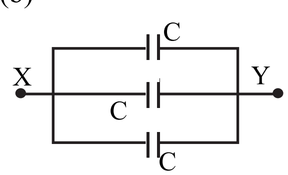
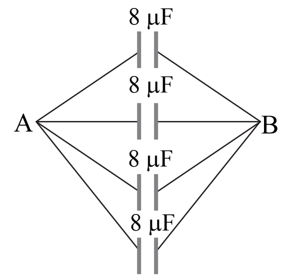


Figure (b)

∴ $C_{eq} = C + C + C = 3C = 3 \times 2 = 6$ μF

59. Given circuit can be drawn as,



∴ Equivalent capacitance between A and B
$= C_p = 4 \times 8 = 32$ μF

60. $V = \frac{Q}{C}$ But $Q = C_{eff}V$ ....(i)

$C_p = 3 + 6 + 3 = 12$ μF

∴ $C_s = C_{eff} = \frac{12 \times 2}{12 + 2} = \frac{24}{14} = \frac{12}{7}$ μF

∴ $Q = \frac{12}{7} \times 70 = 120$ μC ....[From (i)]

∴ $V = \frac{120}{2} = 60$ V

61. When two capacitors are connected in series combination,

$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1C_2}$

∴ $C_S = \frac{C_1C_2}{C_1 + C_2} = \frac{15}{4}$ μF

$15(C_1 + C_2) = 4C_1C_2$ ….(i)
When two capacitors are connected in parallel combination,
$C_1 + C_2 = 16$ ….(ii)
Substituting eq. (ii) in eq. (i),
$15 \times 16 = 4C_1C_2$

∴ $15 \times 4 = C_1C_2$
$60 = C_1C_2$

∴ $C_1(16 - C_1) = 60$ ….[From (ii)]
$C_1^2 - 16C_1 + 60 = 0$

∴ $C_1^2 - 10C_1 - 6C_1 + 60 = 0$

∴ $(C_1 - 10)(C_1 - 6) = 0$

∴ $C_1 = 10$ μF or $C_1 = 6$ μF
Hence, values of capacitors are 6 μF and 10 μF.

62. In parallel combination,
$C_{eff} = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 = 6 \times 1$
$= 6$ μF and $V = 2$ V

∴ $Q = CV = 6 \times 2 = 12$ μC

∴ $Q_1 = \frac{Q}{6} = \frac{12}{6} = 2$ μC

In series, $V_1 = \frac{Q_1}{C_1} = \frac{2\ \mu C}{1\ \mu F} = 2\ V$

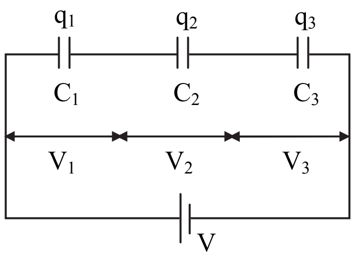
$\therefore$ $V_T = 6V_1 = 12\ V$

C_{eff} (in series) $= \frac{C}{6} = \frac{1}{6}\ \mu F$

Using $E = \frac{1}{2}CV^2$,

$\therefore$ $E = \frac{1}{2} \times \frac{1}{6} \times 10^{-6} \times 12 \times 12 = 12 \times 10^{-6}\ J = 12\ \mu J$

63.



In series grouping of condensers, the charge on each plate is same, $\Rightarrow q_1 = q_2 = q_3 = q$

Let $q = CV \Rightarrow V = \frac{q}{C}$

$\therefore$ $V_1 : V_2 : V_3 = \frac{q_1}{C_1} : \frac{q_2}{C_2} : \frac{q_3}{C_3}$

$= \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

64. Equivalent capacitance $= \frac{2 \times 3}{2+3} = \frac{6}{5}\ \mu F$

$\therefore$ Total charge by $Q = CV = \frac{6}{5} \times 1000 = 1200\ \mu C$

$\therefore$ Potential (V) across 2 μF is

$V = \frac{Q}{C} = \frac{1200}{2} = 600\,\text{volt}$

$\therefore$ Potential on internal plates $= 1000 - 600 = 400\,V$

65. $\frac{1}{C_{eff.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$

$\therefore$ $C_{eff.} = 2\ pF = 2 \times 10^{-12}\ F$

66. If C is the capacitance of each capacitor then,

$\frac{1}{C_{eff}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$

$\therefore$ $C_{eff} = \frac{C}{3} = 2\mu F \Rightarrow C = 6\ \mu F$

Now, for parallel combination,

$C_{eff} = 3C = 3 \times 6 = 18\ \mu F$

$\therefore$ $U = \frac{1}{2}C_{eff}V^2 = \frac{1}{2} \times 18 \times 10^{-6} \times (200)^2$

$= 9 \times 10^{-6} \times 4 \times 10^4 = 36 \times 10^{-2}$

$= 0.36\ J$

69. $U = \frac{1}{2}\frac{Q^2}{C}$ and $C = \frac{Ak\varepsilon_0}{d}$

$\therefore$ $U \propto \frac{1}{C} \propto \frac{1}{k}$

$\therefore$ $\frac{U_1}{U_2} = \frac{k_2}{k_1}$

If $k_1 = 1$ and $k_2 = 2$ then,

$U_2 = \frac{U_1}{k_2} = \frac{U_1}{2}$

70. $E = \frac{\sigma R}{k\varepsilon_0 r} = \frac{\sigma R 4\pi}{4\pi\varepsilon_0 \times k \times r}$

$= \frac{0.25 \times 10^{-6} \times 4 \times 10^{-3} \times 4 \times 3.14 \times 9 \times 10^9}{6.28 \times 2}$

$= 9\ V/m$

71. $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$\therefore$ $\lambda = 2\pi\ \varepsilon_0 r\ E = \frac{4\pi\varepsilon_0 rE}{2}$

$= \frac{1}{2 \times 9 \times 10^9} \times 4 \times 10^{-2} \times 9 \times 10^4 = 2 \times 10^{-7}\ C\ m^{-1}$

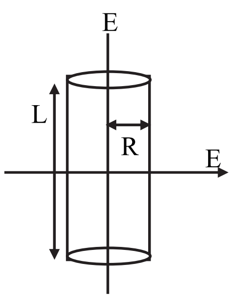
72. Total flux ϕ

$= \vec{E}.\Delta\vec{A} = E\ \Delta A \cos\theta$

(where θ is an angle between E and ΔA)

For top and bottom faces of the cylinder,

$\theta = 90°$



$\therefore$ $\phi = E\Delta A \cos 90°$ ....($\because \cos 90° = 0$)

$\therefore$ $\phi = 0$

73. Volume = 1 litre = $1 \times 10^{-3}\ m^3$

$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2} \times 8.85 \times 10^{-12} \times (10^3)^2$

$= 4.425 \times 10^{-6}$

$\therefore$ Energy stored in $10^{-3}\ m^3$ of air

$= 4.425 \times 10^{-6} \times 10^{-3} = 4.425 \times 10^{-9}\ J$

74. Presence of proton will not affect field between the plates (since proton charge is quite small compared to the charges on the plate)

$\therefore$ $E = \frac{V}{d} = \frac{200}{2\times10^{-2}} = \frac{20000}{2} = 10000$ V/m

75. $C_1 = 4 \times 10^{-6}$ F, $V_1 = 50$ volt, $C_2 = 2 \times 10^{-6}$ F, $V_2 = 100$ volt

$\therefore$ Total energy before connection

$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

$= \frac{1}{2}(4 \times 10^{-6} \times 50 \times 50 + 2 \times 10^{-6} \times 100 \times 100)$

$= 1.5 \times 10^{-2}$ J

Equivalent capacity in parallel combination,

$C_P = C_1 + C_2 = 4 \times 10^{-6} + 2 \times 10^{-6} = 6 \times 10^{-6}$ F

Common potential in parallel combination of capacitors, $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$= \frac{4 \times 10^{-6} \times 50 + 2 \times 10^{-6} \times 100}{4 \times 10^{-6} + 2 \times 10^{-6}}$

$= \frac{4\times10^{-4}}{6\times10^{-6}} = \frac{2}{3} \times 10^2$ volt

$\therefore$ Total energy after connection

$= \frac{1}{2}C_pV^2$

$= \frac{1}{2} \times 6 \times 10^{-6} \times \frac{2}{3} \times \frac{2}{3} \times (10^2)^2$

$= \frac{4}{3} \times 10^{-2} = 1.33 \times 10^{-2}$ J

76. $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$

At any instant, let the separation between plates be x

$\therefore$ $U = \frac{1}{2}\frac{\varepsilon_0 A}{x}V^2$

$\therefore$ $\frac{dU}{dt} = \frac{1}{2}\varepsilon_0 AV^2(-1)\frac{1}{x^2}\frac{dx}{dt} = -\frac{1}{2}\frac{\varepsilon_0 AV^2}{x^2}(v)$

i.e., potential energy decreases as $(1/x^2)$.

77. The two condensers filled with k and with air are in parallel.

With air: $C_1 = \frac{\varepsilon_0}{d}\left(\frac{3A}{4}\right) = \frac{3\varepsilon_0 A}{4d}$

With medium: $C_2 = \frac{\varepsilon_0 K}{d}\left(\frac{A}{4}\right) = \frac{\varepsilon_0 Ak}{4d}$

$\therefore$ $C_{eq} = C_1 + C_2$

$\therefore$ $C_{eq} = \frac{3\varepsilon_0 A}{4d} + \frac{\varepsilon_0 Ak}{4d} = \frac{\varepsilon_0 A}{4d}\left[\frac{3}{4} + \frac{k}{4}\right] = \frac{C}{4}(k+3)$

$\therefore$ $C_{eq} = \frac{C}{4}(k+3)$

78. $C = \frac{\varepsilon_0 A}{d}$

$\therefore$ $C = \frac{C_1C_2}{C_1 + C_2} = \frac{\left(\frac{\varepsilon_0 A}{d/2}\right)\cdot\left(\frac{\varepsilon_0\varepsilon_r A}{d/2}\right)}{\left(\frac{\varepsilon_0 A}{d/2} + \frac{\varepsilon_0\varepsilon_r A}{d/2}\right)}$

$= \frac{2\varepsilon_0 A\varepsilon_r}{d(1+\varepsilon_r)} = \frac{2C\varepsilon_r}{(1+\varepsilon_r)}$

79. As separation between plates is reduced, C increases but charge on it remains same. Hence, from the relation $U = \frac{1}{2}\frac{q_0^2}{C}$, U decreases. Also, work done in charging the capacitor is stored as potential energy.

Competitive Thinking

1. From dimension we can check the answer, only $\varepsilon_0(\phi_1 + \phi_2)$ having the same dimension q_y to the charge

$\because$ $\phi = \frac{q_{net}}{\varepsilon_0}$

$q_{net} = \phi\,\varepsilon_0$

all other options don't having the dimension equal to charge

So answer is

$\varepsilon_0(\phi_1 + \phi_2)$

2. Electric flux $(\phi) = \frac{q_{Inc}}{\varepsilon_0}$

$\phi = \frac{10\mu C}{\varepsilon_0}$

If more 10 μC charge is placed.

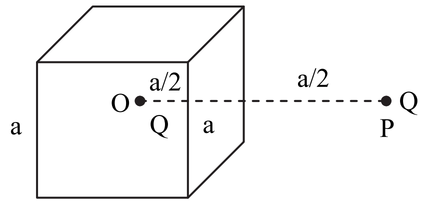
Electric flux $= \frac{20\mu C}{\varepsilon_0} = 2\phi$

4. $\phi = \frac{\Sigma q}{\varepsilon_0} = 0 \Rightarrow$ [$\because$ charge on dipole is zero.]

5. Total flux = $\frac{Q}{\varepsilon_0}$ using Gauss' law

∴ flux through one face = $\frac{Q}{6\varepsilon_0}$

6.



Flux due to charge at O,

$\phi_1 = 5 \times \frac{Q}{6\varepsilon_0}$

Flux due to charge at P

$\phi_2 = \frac{Q}{6\varepsilon_0}$

∴ $\phi = \phi_1 + \phi_2 = \frac{Q}{\varepsilon_0}$

7. Charge enclosed by cylindrical surface is, $Q_{enc} = 100\ Q$. By applying Gauss' law,

$\phi = \frac{1}{\varepsilon_0}(Q_{enc.}) = \frac{1}{\varepsilon_0}(100Q)$

8. Total flux = $(-14 + 78.85 - 56)\text{nC}/\varepsilon_0$

$= 8.85 \times 10^{-9}\,\text{C} \times \frac{4\pi}{4\pi\varepsilon_0}$

$= 8.85 \times 10^{-9} \times 9 \times 10^9 \times 4\pi$

$= 1000\ \text{Nm}^2\text{C}^{-1}$

10. Flux = $\frac{\text{Total charge enclosed}}{\varepsilon_0}$

i.e. for first surface,

$\phi_1 = \frac{-q}{\varepsilon_0}$

For second surface,

$\phi_2 = \frac{q}{\varepsilon_0}$

11. $\phi_E = \frac{Q_{enclosed}}{\varepsilon_0}$; $Q_{enclosed}$ remains unchanged.

12. ∵ Charge 8q is placed at one corner of the cube, we can imagine it to be placed at the centre of a large cube which can be formed using an arrangement of 8 similar cubes.
Charge 8q is at centre of the 8 cubes arranged to form a closed box.

∴ By using Gauss's law,

total flux through the bigger cube = $\frac{8q}{\varepsilon_0}$

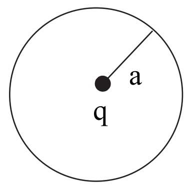
∴ Flux through one small cube = $\frac{1}{8} \times \frac{8q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$.

13. By Gauss' law,

$\phi = \frac{1}{\varepsilon_0}\ (Q_{enclosed})$

∴ $Q_{enclosed} = \phi\varepsilon_0 = (-8 \times 10^3 + 4 \times 10^3)\varepsilon_0$
$= -4 \times 10^3\ \varepsilon_0\ \text{C}$

14.



Let charge enclosed in the sphere of radius a be q. According to Gauss' theorem,

$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$

$E.4\pi r^2 = \frac{q}{\varepsilon_0}$

$4\pi Ar^3 = \frac{q}{\varepsilon_0}$ ….(∵ E = Ar)

$\Rightarrow q = 4\pi\varepsilon_0 Aa^3$ ….(∵ r = a)

15. $E = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow E \propto \frac{1}{r}$

16. Electric field intensity at a point outside uniformly charged thin plane sheet is given by,

$E = \frac{\sigma}{2\varepsilon_0}$

∴ It is independent of 'd'.

17. Relation for electric field is given by, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

∴ $\lambda = 2\pi\varepsilon_0 rE = \frac{2 \times 2\pi\varepsilon_0 rE}{2}$

$= \frac{1 \times 2 \times 10^{-2} \times 7.182 \times 10^8}{2 \times 9 \times 10^9} = 7.98 \times 10^{-4}$ C/m

18. Electric field in vacuum $E_v = \frac{\sigma}{\varepsilon_0}$ and

in medium, $E = \frac{\sigma}{\varepsilon_0 k}$

If k > 1, then $E < E_0$.

19. The electric field is always perpendicular to the surface of a conductor. On the surface of a metallic solid sphere, the electrical field is oriented normally (*i.e.* directed towards the centre of the sphere).

20. Given that,

$\sigma_s = \sigma_c$

Now, $E_s = \dfrac{\sigma_s R^2}{\varepsilon r^2}$ and $E_c = \dfrac{\sigma_c R}{\varepsilon r}$

$\therefore \quad E_s = \dfrac{\sigma R}{\varepsilon r} \times \dfrac{R}{r} = E_c \dfrac{R}{r}$

21. $E = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{r^2}$

$= \dfrac{9\times10^9 \times \left(4\times10^{10}\times1.6\times10^{-19}\right)}{\left(20\times10^{-2}\right)^2}$

$= 1440$ N/C

22. $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{ne}{r^2}$

$\therefore \quad n = \dfrac{Er^2}{e}4\pi\varepsilon_0$

$\therefore \quad n = \dfrac{0.036\times0.1\times0.1}{9\times10^9\times1.6\times10^{-19}}$

$= \dfrac{360}{144}\times10^5$

$= 2.5 \times 10^5$

23. If charge acquired by the smaller sphere is Q, then it's potential, $V = \dfrac{kQ}{r}$

$\therefore \quad 120 = \dfrac{kQ}{2} \Rightarrow kQ = 240$ ..... (i)

Whole charge resides on the outer sphere,

$\therefore$ Potential of the outer sphere,

$V' = \dfrac{kQ}{6}$

$\therefore \quad V' = \dfrac{240}{6}$ ….[From (i)]

$\therefore \quad V' = 40$ V

24. At any point inside the sphere, the potential is same and is equal to that at the surface.

25. After redistribution, the new charges on spheres are $Q_1' = \left(\dfrac{10}{10+20}\right)\times10 = \dfrac{10}{3}\mu C$

and $Q_2' = \left(\dfrac{20}{10+20}\right)\times10 = \dfrac{20}{3}\mu C$

Ratio of charge densities,

$\dfrac{\sigma_1}{\sigma_2} = \dfrac{Q_1'}{Q_2'}\times\dfrac{r_2^2}{r_1^2}$ ….$\left[\because \sigma = \dfrac{Q}{4\pi r^2}\right]$

$= \dfrac{10/3}{20/3}\times\left(\dfrac{20}{10}\right)^2 = \dfrac{2}{1}$

26. There will be zero charge inside closed surface

27. T.N.E.I. = $\sum q_{enclosed}$

$\therefore$ T.N.E.I. for A = zero

T.N.E.I. for B = $(2q - q) = q \Rightarrow (0, q)$

28. T.N.E.I. over the closed surface

$= \sum q = 5 + 7 - 4 = 8\,C$

29. Electric potential, $V = \dfrac{Q}{4\pi\varepsilon_0 R}$

Electric field inside a charged conductor is zero.

30. $V_1 + V_2 = 0$

$\therefore \quad \dfrac{kq'}{r_1} + \dfrac{kq}{r_2} = 0 \Rightarrow q' = -\left(\dfrac{r_1}{r_2}\right)q$

31. $V = \dfrac{q}{4\pi\varepsilon_0 r} + \dfrac{q'}{4\pi\varepsilon_0 R}$

Now, $q = \sigma.4\pi r^2$ and $q' = \sigma.4\pi R^2$

$\therefore \quad V = \dfrac{\sigma.4\pi r^2}{4\pi\varepsilon_0 r} + \dfrac{\sigma.4\pi R^2}{4\pi\varepsilon_0 R} \Rightarrow V = \dfrac{\sigma(R+r)}{\varepsilon_0}$

32. $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{r^2}$

$\therefore \quad E = \dfrac{9\times10^9\times3\times10^{-9}}{(3\times10^{-2})^2} = 3\times10^4$ V/m

33. $E = \dfrac{kq}{r^2}$

$q = \dfrac{Er^2}{k} = \dfrac{2\times(0.3)^2}{9\times10^9} = \dfrac{2\times9\times10^{-2}\times10^{-9}}{9}$

$\therefore \quad q = 2\times10^{-11}$ C

34. $\vec{AB} = \left(\vec{B} - \vec{A}\right) = a\left(-4\hat{j} + 3\hat{k}\right)$

$\therefore$ Work done = $\vec{F}\cdot\vec{AB} = q\left(\dfrac{\sigma}{2\varepsilon_0}\right)\hat{k}\cdot a\left(-4\hat{j}+3\hat{k}\right)$

$= \dfrac{3q\sigma a}{2\varepsilon_0}$

35. For a charged conductor of any shape (assuming air medium),

$E_1 = \dfrac{\sigma}{\varepsilon_0}$ ….(i)

For a infinite thin plane sheet (assuming air medium),

$E_2 = \dfrac{\sigma}{2\varepsilon_0}$ ….(ii)

Comparing (i) and (i)

$E_1 = 2E_2$.

36. Initially, F = qE and $E = \dfrac{\sigma}{\varepsilon_0}$

$\therefore \quad F = \dfrac{q\sigma}{\varepsilon_0}$ ….(i)

When one plate is removed, then E becomes $\dfrac{\sigma}{2\varepsilon_0}$

$\therefore \quad F' = \dfrac{q\sigma}{2\varepsilon_0} = \dfrac{F}{2}$ ….[From (i)]

37. Electric field intensity is given by,

$E = \dfrac{\sigma}{\varepsilon_0}$

Between plates $E = E_1 - E_2$

But electric field intensity inside the sheet is zero

$\therefore \quad E_1 - E_2 = 0$

Outside plates $E = E_1 + E_2$

i.e., $E_1 + E_2 = \dfrac{\sigma}{\varepsilon_0}$

38. For the soap bubble,

$P_m - P_{out} = P_{excess} = P_{ST} - P_{electro}$

$= \dfrac{4T}{r} - \dfrac{q^2}{2A^2\varepsilon_0}$

$= \dfrac{4T}{r} - \dfrac{q^2}{2(4\pi r^2)^2\varepsilon_0}$

$= \dfrac{4T}{r} - \dfrac{q^2}{32\pi^2 r^4\varepsilon_0}$

For equilibrium,

$P_m = P_{out}$

$\therefore \quad \dfrac{4T}{r} = \dfrac{q^2}{32\pi^2 r^4\varepsilon_0}$

$\therefore \quad q = \sqrt{\dfrac{128\pi^2 r^4\varepsilon_0 T}{r}} = \sqrt{\dfrac{16\times 8\pi^2 r^4\varepsilon_0 T}{r}}$

$\therefore \quad q = 4\pi r^2\sqrt{\dfrac{8\varepsilon_0 T}{r}}$

39. Electrostatic energy density,

$\dfrac{du}{dV} = \dfrac{1}{2}k\varepsilon_0 E^2 \qquad \therefore \quad \dfrac{du}{dV} \propto E^2$

40. Energy $= \dfrac{1}{2}\varepsilon_0 E^2 \times (A\times d) = \dfrac{1}{2}\varepsilon_0\left(\dfrac{V^2}{d^2}\right)Ad$

$= \dfrac{1}{2}\times\dfrac{8.85\times10^{-12}\times(10^5)^2\times25\times10^6}{0.75\times10^3} = 1475J$

41. When put 1 cm apart in air, the force between Na and Cl ions = F. When put in water, the force between Na and Cl ions $= \dfrac{F}{k}$

42. $k = \dfrac{E_{\text{without dielectric}}}{E_{\text{with dielectric}}} = \dfrac{2\times10^5}{1\times10^5} = 2$

43. $C = \dfrac{Ak\varepsilon_0}{d}$

44. For a spherical capacitor,

$C = 4\pi\varepsilon_0 k\left(\dfrac{ab}{b-a}\right) \Rightarrow C \propto k$

(Note: Refer Mindbender 1.)

45. By inserting the dielectric slab, capacitance (*i.e.* ability to hold the charge) increases. In the presence of battery more charge is supplied from battery.

46. Electric field between plates, $E = \dfrac{q}{\varepsilon_0 A}$

Electrostatic force, $F = qE = \dfrac{q^2}{\varepsilon_0 A}$

Thus, F is independent of distance between the plates.

47. $C = \dfrac{\varepsilon_0 A}{d}$

Hence as d increases, C decreases.

Q is constant ⇒ V increases.

48. After separation:

i. charge = constant

ii. capacity $C = \dfrac{\varepsilon_0 A}{d}$

Capacity decreases with increase in distance.

iii. $V = \dfrac{Q}{C}$

Potential increases as capacitance decreases.

49. For spherical conductor, $C = 4\pi\varepsilon_0 R$

Now, for a sphere,

$V = \dfrac{4}{3}\pi R^3$ and $A = 4\pi R^2$

Also, $R = \dfrac{3V}{A} \qquad \therefore \quad C = 12\pi\varepsilon_0\dfrac{V}{A}$

50. Volume of 8 drops will be same as volume of 1 large drop formed by combining smaller drops.

$\therefore$ $8\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$\therefore$ R = 2r

the capacitance of bigger drop is

$C' = 4\pi\varepsilon_0 R = 4\pi\varepsilon_0\, 2r = 2C$

51. $C = 4\pi\varepsilon_0 k\left[\frac{ab}{b-a}\right] = \frac{1}{9\times10^9}\cdot 6\left[\frac{12\times9\times10^{-4}}{3\times10^{-2}}\right]$

$= 24 \times 10^{-11} = 240$ pF

(*Note: Refer Mindbender 1.*)

52. Initial charge on the capacitor Q = 10 × 12 = 120 μC

Final charge on the capacitor Q′ = (5 × 10) × 12 = 600 μC

$\therefore$ Charge supplied by the battery later = Q′ – Q = 480 μC

53. Using, $C = \frac{\varepsilon_0 A}{d}$

$\therefore$ $A = \frac{Cd}{\varepsilon_0} = \frac{3\times5\times10^{-3}}{8.85\times10^{-12}} \approx 1.695\times10^9\ m^2$

54. The required ratio, $\frac{\left(\frac{1}{2}qV\right)}{qV} = \frac{1}{2}$

55. $V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 kA} \Rightarrow V \propto d$

56. $C \propto \frac{1}{d} \Rightarrow \frac{C_1}{C_2} = \frac{d_2}{d_1} \Rightarrow \frac{15}{C_2} = \frac{2}{6}$

$\therefore$ $C_2 = 45$ μF

57. Common potential = $\frac{C_1V_1 + C_2V_2}{C_{eff.}}$

$= \frac{20\times10^{-6}\times500 + 10\times10^{-6}\times200}{20\times10^{-6}+10\times10^{-6}}$

$= \frac{12000}{30} = 400$ V

(Note: Refer Shortcut 8.)

58. $Q = CV = 6 \times 10^{-6} \times 18 = 108$ μC

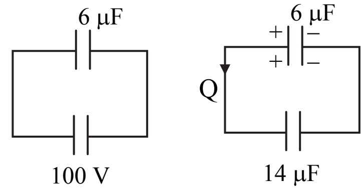
59. Initial charge on 6 μF condensor is –

$Q = CV = 6 \times 10^{-6} \times 100 = 6 \times 10^{-4}$ C.

When 6 μF and 14 μF are joined, the total charge in circuit must remain same. Also, the potential on both condensor will be finally same as they are connected at ends with each other.

$\therefore$ $V' = \frac{Q}{C_{parallel}}$

$\therefore$ $V' = \frac{6\times10^{-4}}{(6+14)\times10^{-6}} = 30$ V



Now, $\frac{Q_1}{Q_2} = \frac{C_1V_1}{C_2V_2} = \frac{6\,\mu F\times V'}{14\,\mu F\times V'}$

$\therefore$ $\frac{Q_1}{Q_2} = \frac{6}{14}$

60. $C = \frac{A\varepsilon}{d} \Rightarrow C_1 = \frac{A_1\varepsilon}{d_1}$ and $C_2 = \frac{A_2\varepsilon}{d_2}$

$\therefore$ $\frac{C_2}{C_1} = \frac{A_2\varepsilon}{d_2}\times\frac{d_1}{A_1\varepsilon} = \frac{A_2}{A_1}\times\frac{d_1}{d_2} = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

$\therefore$ $C_2 = \frac{C_1}{4} = \frac{1}{4}\times12 = 3$ μF

61. $E = \frac{\sigma}{k\varepsilon_0} \Rightarrow \sigma = k\varepsilon_0 E$

$\therefore$ $\sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4 \approx 6 \times 10^{-7}\ C/m^2$

62. Force between plates of capacitor

$F = qE = q\left(\frac{q}{2A\epsilon_0}\right)$

$F = \frac{q^2}{2A\epsilon_0}$

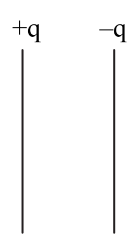
$\therefore$ q = CV

$F = \frac{C^2V^2}{2A\epsilon_0}$

$F = \frac{\left(\frac{A\epsilon_0}{d}\right)CV^2}{2A\epsilon_0}$

$F = \frac{CV^2}{2d}$

63.

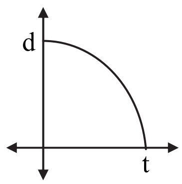


As separation between the plates are decreasing as they approach each other and V = E.d

Electric field remains constant between the plates, so $V \propto d$

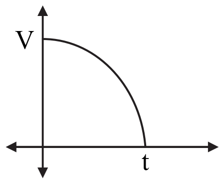
Now, force on each plate = $\frac{q^2}{2A\varepsilon_0}$ But, F = ma

acceleration (a) = $\frac{F}{m}$



i.e., acceleration (a) = $\frac{q^2}{2A\varepsilon_0(m)}$

a = constant
So V-t curve



64. $C_{medium} = k\,C_{air}$

$\therefore$ $k = \frac{C_{medium}}{C_{air}} = \frac{110}{50} = 2.2$

65. Aluminium being a metal, the field inside it will be zero. Hence it would not affect the field in between the two plates. Hence capacity $= \frac{q}{V} = \frac{q}{Ed}$ remains unchanged.

66. $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 (5A)}{2d}$

$= \frac{6\varepsilon_0 A}{2d} = \frac{3\varepsilon_0 A}{d}$

$\therefore$ $\Delta C = C' - C = \frac{3\varepsilon_0 A}{d} - \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}$

Percentage change in capacitance,

$$\frac{\Delta C}{C} = \frac{\left(\frac{2\varepsilon_0 A}{d}\right)}{\left(\frac{\varepsilon_0 A}{d}\right)} \times 100\,\% = 200\%$$

67. When a dielectric is introduced between the plates, as battery remains connected, E or V remains unchanged.
Charge on plates before introduction of dielectric medium is, $q_0 = C_0V$
After inserting the medium, $q = kC_0V$
Induced charge, $q' = q - q_0$
$= C_0V\,(k - 1)$
$= 90 \times 10^{-12} \times 20 \left(\frac{5}{3} - 1\right) = 1.2$ nC

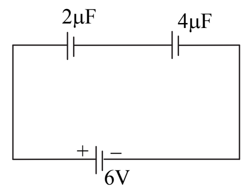
68. Electric field inside parallel plate capacitor having charge Q at place where dielectric is absent $= \frac{Q}{A\varepsilon_0}$ and where dielectric is present $= \frac{Q}{kA\varepsilon_0}$

69. Energy, $U = \frac{1}{2}\frac{Q^2}{C}$.
For a charged capacitor, charge Q is constant and with the increase in separation, C will decrease ($C \propto \frac{1}{d}$).
Hence overall U will increase.

70. $U = \frac{Q^2}{2C} = \frac{(40\times10^{-6})^2}{2\times10\times10^{-6}} = \frac{16\times10^{-10}}{2\times10^{-5}} = 8\times 10^{-5}$ J
$= 8 \times 10^{-5} \times 10^7 = 800$ erg

71. $U = \frac{1}{2}CV^2 = \frac{1}{2}\times12\times10^{-12} \times (50)^2 = 1.5 \times 10^{-8}$ J

72.



$C_{eff} = \frac{c_1c_2}{c_1 + c_2} = \frac{2\times4}{2+4} = \frac{8}{6}\mu F$

$Q = C_{eff}\,V = \frac{8}{6}\times10^{-6}\times6 = 8 \times 10^{-6}$ C

$\therefore$ $Q = 8\mu C$
Now ,
$U = \frac{1}{2}CV^2 = \frac{1}{2}\times\left(\frac{8}{6}\times10^{-6}\right)\times6^2 = 24\mu J$

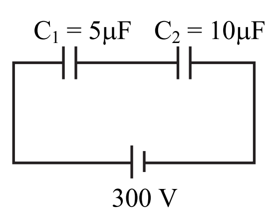
73. Total capacitance of given system,
$\frac{1}{C_{eff}} = \frac{1}{4} + \frac{1}{(4+4)} + \frac{1}{4}$

$\therefore$ $\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{4}$

$\therefore$ $C_{eq} = \frac{8}{5}\,\mu F$

$\therefore$ $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\times\frac{8}{5}\times 10^{-6} \times 225$
$= 180 \times 10^{-6}$ J
$= 180 \times 10^{-6} \times 10^7$erg $= 1800$ erg

74.



$$C_{eq} = \left[\frac{1}{C_1}+\frac{1}{C_2}\right]^{-1} = \frac{C_1C_2}{C_1+C_2}$$

$$= \frac{5\times10}{15} = \frac{10}{3}\ \mu F$$

$$= \frac{10}{3}\times 10^{-6}\ F$$

$$U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\times\frac{10}{3}\times 10^{-6}\times 300^2 = 0.15\ J$$

75. Energy stored in fully charged capacitor,

$$U = \frac{1}{2}CV^2$$

But work done by battery W = QV or

$U = W = CV.V = CV^2$

Energy required to charge the capacitor,

$$\therefore\quad U = CV^2 = \frac{\varepsilon_0 A}{d}.V^2 = \frac{\varepsilon_0 Ad}{d^2}\cdot V^2$$

$$= \varepsilon_0 E^2 Ad \qquad \ldots\left[\because E = \frac{V}{d}\right]$$

76. $U = \frac{1}{2}CV^2 = \frac{1}{2}\times 6\times 10^{-6}(100)^2 = 0.03\,J$

77. Work done in placing the charge = Energy stored in the condenser

$$\therefore\quad W = \frac{Q^2}{2C} = \frac{(8\times10^{-18})^2}{2\times100\times10^{-6}} = 32\times10^{-32}\ J$$

78. Work done $= \frac{1}{2}qV = \frac{1}{2}\times 4\times 4\times 10^{-6}$

$$= 8\times 10^6\ J$$

$$\therefore\quad \text{Power} = \frac{\text{work}}{\text{time}} = \frac{8\times10^6}{0.1} = 80\ MW$$

79. $U = \frac{1}{2}QV =$ Area of triangle OAB

80. Heat produced = Energy stored in capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2}\times 4\times10^{-6}\times(400)^2 = 2\times10^{-6}\times16\times10^4$$

$= 0.32$ J

81. $U = \frac{1}{2}CV^2 = \frac{1}{2}\times 700\times10^{-12}\times(50)^2$

$$= 350\times 10^{-12}\times 2500 = 8.75\times 10^{-7}\ J$$

82. $U = \frac{1}{2}\frac{Q^2}{C}$

$\therefore$ Increase in energy $= \frac{1}{2C}\left[Q_2^2 - Q_1^2\right]$

$$= \frac{1}{2\times48\times10^{-6}}[0.5^2 - 0.1^2]$$

$$= \frac{10^6}{96}[24\times10^{-2}]$$

$$= 0.25\times 10^4 = 2500\ J$$

83. $W = \int_q^Q \frac{q}{C}\,dq$

$$\therefore\quad W = \int_{q=5C}^{q=10C}\frac{q}{C}\,dq = \frac{\left[q^2\right]_{5C}^{10C}}{2C} = \frac{75C}{2}$$

Let W′ be the work done in increasing the voltage across capacitor from 10V to 15V.

$$\therefore\quad W' = \int_{q=10C}^{q=15C}\frac{q}{C}\,dq = \frac{\left[q^2\right]_{10C}^{15C}}{2C} = \frac{125C}{2}$$

$$\therefore\quad \frac{W'}{W} = \frac{125}{75} \qquad \therefore\quad W' = 1.67W$$

84. When connected in series,

$$\left(C_{eq}\right)_1 = \frac{C_1}{N_1};\ V_1 = 3V$$

When connected in parallel,

$$\left(C_{eq}\right)_2 = N_2C_2;\ V_2 = V$$

$$U = \frac{1}{2}CV^2$$

$$\therefore\quad \frac{1}{2}\left(C_{eq}\right)_1 V_1^2 = \frac{1}{2}\left(C_{eq}\right)_2 V_2^2$$

$$\frac{1}{2}\frac{C_1}{N_1}9V^2 = \frac{1}{2}N_2C_2V^2$$

$$C_1 = \frac{C_2N_1N_2}{9}$$

85. $\frac{1}{C_R} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$$\therefore\quad C_R = \left(C_1^{-1}+C_2^{-1}+C_3^{-1}\right)^{-1}$$

86. $\frac{1}{C_s} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$

$$\frac{1}{C_s} = \frac{1}{1}+\frac{1}{2}+\frac{1}{4}$$

$$C_s = \frac{4}{7}\ pF$$

87. Capacitance of parallel plate capacitor is given by,

$C = \dfrac{KA\varepsilon_0}{d}$

$\therefore$ $C \propto \dfrac{A}{d}$; K = 1 for air

Equivalent capacitance is given by,

$C_p = C_1 + C_2 + C_3$

$C_p = \dfrac{A\varepsilon_0}{3d} + \dfrac{A\varepsilon_0}{3(2d)} + \dfrac{A\varepsilon_0}{3(3d)}$

$= A\varepsilon_0\left(\dfrac{1}{3d} + \dfrac{1}{6d} + \dfrac{1}{9d}\right)$

$= \dfrac{A\varepsilon_0}{d}\left(\dfrac{11}{18}\right)$

$\therefore$ $C_p = \dfrac{11\varepsilon_0 A}{18d}$

88. The two capacitors thus formed are in parallel.

$\therefore$ $C = \dfrac{\varepsilon_0 A}{t \times 2}(k_1 + k_2)$

89. The given arrangement is effectively an arrangement of (n − 1) capacitors connected in parallel.

$\therefore$ $C_R = (n - 1)\,C$

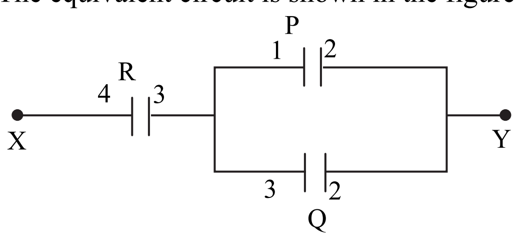
90. $\dfrac{1}{C_{eff}} = \dfrac{1}{C} + \dfrac{1}{C} + \dfrac{1}{C} = \dfrac{3}{C} \Rightarrow C_{eff} = \dfrac{C}{3}$

$\therefore$ $V' = V_1 + V_2 + V_3 = V + V + V = 3V$

91. The given arrangement is equivalent to the parallel combination of three identical capacitors.

Hence equivalent capacitance $= 3C = 3\dfrac{\varepsilon_0 A}{d}$

92. The equivalent circuit is shown in the figure.



The condensers P and Q are in parallel. Hence their equivalent capacitance is 2C. This combination is in series with capacitor R. Hence the equivalent capacitance between X and Y is given by

$C_{PQ} = \dfrac{C \times 2C}{C + 2C} = \dfrac{2}{3}C = \dfrac{2}{3}\dfrac{\varepsilon_0 A}{d}$.

93. For series combination, $V_1 = \dfrac{Q}{C_1}$ and $V_2 = \dfrac{Q}{C_2}$

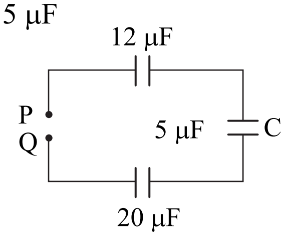
$\therefore$ $\dfrac{V_2}{V_1} = \dfrac{C_1}{C_2} = 4 : 1$

94. The given circuit can be redrawn as shown in figure where, C = (3 + 2) μF = 5 μF

$\therefore$ $\dfrac{1}{C_{PQ}} = \dfrac{1}{5} + \dfrac{1}{20} + \dfrac{1}{12}$

$= \dfrac{20}{60} = \dfrac{1}{3}$

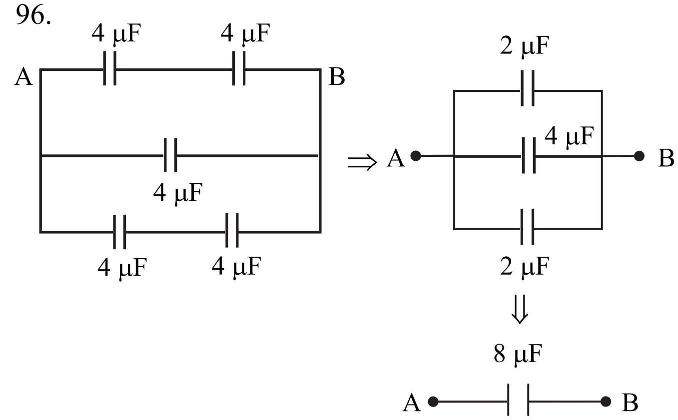
$\therefore$ $C_{PQ} = 3\ \mu F$



95. $C_{PR} = \dfrac{C}{2} + \dfrac{C}{3} = \dfrac{5C}{6}$

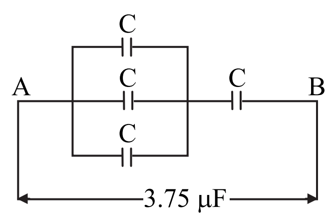
$C_{PQ} = C + \dfrac{C}{4} = \dfrac{5C}{4} \Rightarrow \dfrac{C_1}{C_2} = \dfrac{2}{3}$

96.

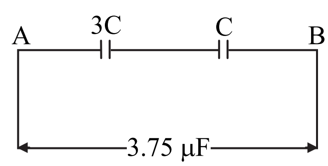


$\therefore$ $C_{AB} = 8\ \mu F$

97.



$C_P = C + C + C = 3C$

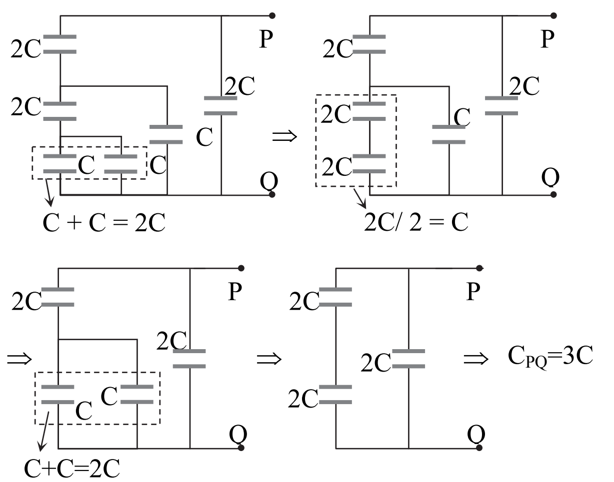


$C_{eq} = \dfrac{3C \times C}{3C + C}$ $\therefore$ $3.75 = \dfrac{3C^2}{4C}$

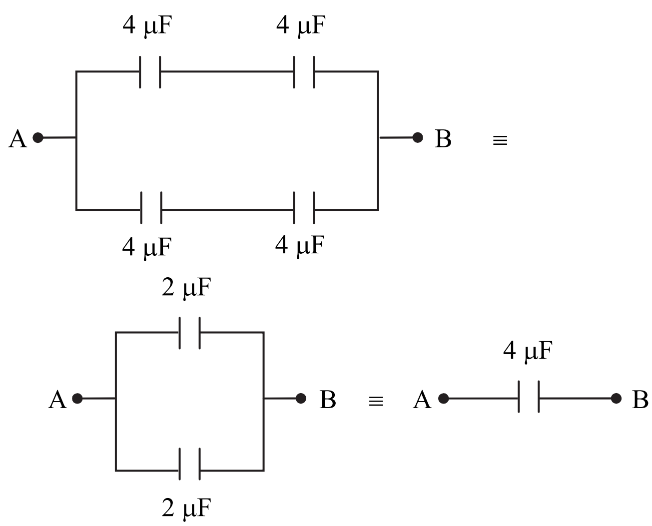
$\therefore$ $3.75 = \dfrac{3C}{4}$

$\therefore$ $C = \dfrac{3.75 \times 4}{3} = 5\ \mu F$

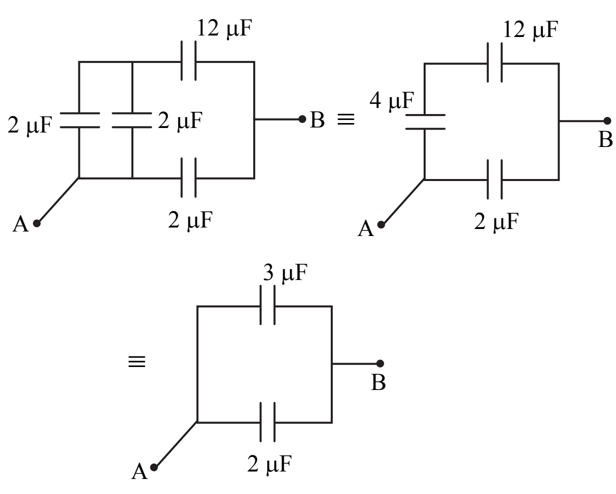
98.



99. The circuit resembles Wheatstone's balanced network



100. The Given circuit is



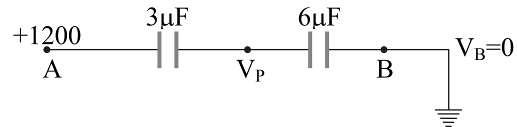
101. $\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{10} + \frac{1}{15}$

$\therefore$ $C_{eq} = 2\ \mu F$

$\therefore$ Charge on each capacitor,

$Q = C_{eq} \times V = 2 \times 100 = 200\ \mu C$

102. Given circuit can be reduced as follows:



In series combination, charge on each capacitor remains same.

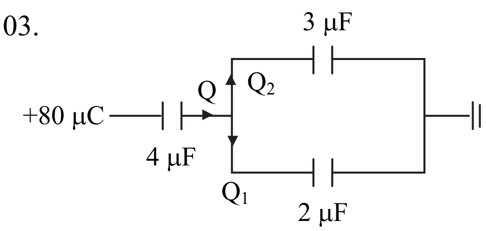
So using Q = CV,

$C_1V_1 = C_2V_2 \Rightarrow 3(1200 - V_p) = 6(V_P - V_B)$

$\therefore$ $1200 - V_p = 2V_p$ ....($\because$ $V_B = 0$)

$\therefore$ $3V_p = 1200 \Rightarrow V_p = 400$ volt

103.



As $C = \frac{Q}{V}$

$\therefore$ $C \propto Q$

$\therefore$ $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$

i.e., $\frac{Q_1}{Q_2} = \frac{2}{3}$

$Q = Q_1 + Q_2$

i.e., $Q_1 = Q - Q_2$

i.e., $Q_1 = 80 - Q_2$

$\therefore$ $\frac{80 - Q_2}{Q_2} = \frac{2}{3}$

$3(80 - Q_2) = 2Q_2$

$240 - 3Q_2 - 2Q_2 = 0$

$240 - 5Q_2 = 0$

$Q_2 = \frac{240}{5}$

$Q_2 = 48\ \mu C$

104. $\frac{1}{C_{eff.}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15} = \frac{3+2+4}{60} = \frac{9}{60}$

$\therefore$ $C_{eff.} = \frac{60}{9}\ \mu F \Rightarrow Q = \frac{60}{9} \times 90 = 600$ e.s.u.

$\therefore$ $V_2 = \frac{Q}{C_2} = \frac{600}{30} = 20$ e.s.u.

105. Effective capacity when connected in parallel
$= C + C = 2C$

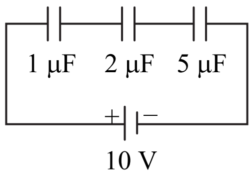
Effective capacity when connected in series $= \frac{C}{2}$

$\therefore \quad 2C - \frac{C}{2} = 6$

$\frac{3C}{2} = 6$

$C = 4\ \mu F$

106.



Equivalent capacitance of capacitor is given by,

$\frac{1}{C_s} = \frac{1}{1} + \frac{1}{2} + \frac{1}{5}$

$C_s = \frac{10}{17} \mu F$

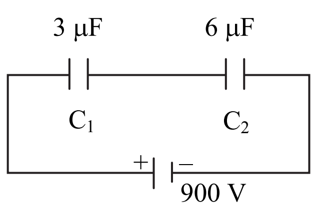
Now, charge is given by,

$Q = C_s V = \frac{10}{17} \times 10 = \frac{100}{17} \mu C$

$\therefore$ Potential difference across $2\mu F$ capacitor

$= \frac{100/17}{2} = \frac{50}{17} V$

107.



$Q = CV$, Here Q is a constant

$\therefore \quad C \propto \frac{1}{V}$

$\therefore \quad \frac{C_1}{C_2} = \frac{V_2}{V_1} \Rightarrow \frac{3}{6} = \frac{V_2}{V_1} \Rightarrow V_1 = 2V_2$

Also $V_1 + V_2 = 900$ V

$\therefore \quad 2V_2 + V_2 = 900$ V

$V_2 = 300$ V and $V_1 = 600$ V

Common potential $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$= \frac{3 \times 10^{-6} \times 600 + 6 \times 10^{-6} \times 300}{3 \times 10^{-6} + 6 \times 10^{-6}}$

$= \frac{1800 \times 10^{-6} + 1800 \times 10^{-6}}{9 \times 10^{-6}} = \frac{3600}{9} = 400$ V

108. Given that net F_E and F_G is zero.
i.e., $F_E = F_G$

$\therefore \quad \frac{1}{4\pi\varepsilon_0} \times \frac{(\Delta e)^2}{d^2} = \frac{Gm^2}{d^2}$ ….(i)

In case of hydrogen atoms, net charge on one H-atom will be Δe

$\therefore \quad \Delta e = m\sqrt{\frac{G}{\left(\frac{1}{4\pi\varepsilon_0}\right)}}$ ….[from (i)]

$= 1.67 \times 10^{-27} \sqrt{\frac{6.67 \times 10^{-11}}{9 \times 10^9}} = 1.438 \times 10^{-37}$ C

109. Force on charged particle in electric field,
$F = eE$

$\therefore$ Acceleration experienced by it, $a = \frac{eE}{m}$

For electron, $a_e = \frac{eE}{m_e}$ and for proton $a_p = \frac{eE}{m_p}$

As, $m_p > m_e$, $a_e > a_p$

As electron is pulled with greater acceleration, it will take lesser time to cover height h.

110. When $\frac{Q_1}{R_1} \neq \frac{Q_2}{R_2}$; current will flow in connecting wire so that energy decreases in the form of heat through the connecting wire.

111. In air, the potential difference between the plates,

$V_{air} = \frac{\sigma}{\varepsilon_0}.d$

In the presence of partially filled medium, potential difference between the plates,

$V_m = \frac{\sigma}{\varepsilon_0}(d - t + \frac{t}{k})$

Potential difference between the plates with dielectric medium and increased distance is,

$V_m' = \frac{\sigma}{\varepsilon_0}\left\{(d + d') - t + \frac{t}{k}\right\}$

According to question, $V_{air} = V_m'$ which gives

$k = \frac{t}{t - d'} \Rightarrow k = \frac{2}{2 - 1.6} = 5$

112. As it is evident from symmetry of figure, plates 2 and 4 have charges + Q/2 each.

We know that, $C = \frac{q}{V} \Rightarrow C = \frac{(Q/2)}{V}$

$\Rightarrow Q = 2CV = 2\frac{\varepsilon_0 A}{d} V$ ….$\left(\because C = \frac{\varepsilon_0 A}{d}\right)$

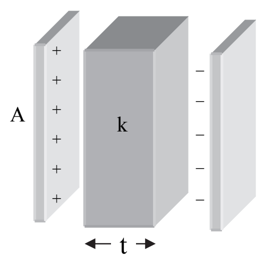
113. $F' = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\left(r-t+t\sqrt{k}\right)^2}$

$$\frac{F}{F'} = \frac{\left[r-\left(\frac{r}{2}\right)+\left(\frac{r}{2}\right)\sqrt{4}\right]^2}{r^2} = \frac{\left(\frac{3r}{2}\right)^2}{r^2} = \frac{\frac{9}{4}r^2}{r^2}$$

$$\frac{F}{F'} = \frac{9}{4}$$

$$F' = \frac{4}{9}F$$

114.



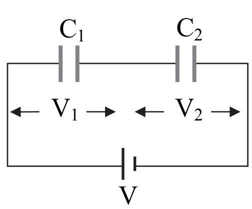
Potential difference between the plates,

$$V = V_{air} + V_{medium} = \frac{\sigma}{\varepsilon_0} \times (d-t) + \frac{\sigma}{k\varepsilon_0} \times t$$

$\therefore$ $V = \frac{\sigma}{\varepsilon_0}(d - t + \frac{t}{k}) = \frac{Q}{A\varepsilon_0}(d - t + \frac{t}{k})$

$\therefore$ $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{\left(d-t+\frac{t}{k}\right)} = \frac{\varepsilon_0 A}{d-t\left(1-\frac{1}{k}\right)}$

115.



Given,
$C_1 = C_2 = C$ (say)
We have,
$V = V_1 + V_2$
When capacitor C_1 is completely filled with dielectric material of constant K,

$V_1 = \frac{V_2}{K}$ ...{$\because$ initially $V_1 = V_2$; $V_2 = \frac{q}{C_2}$ }

$\therefore$ $V = \frac{V_2}{K} + V_2$

$\therefore$ $KV = V_2 + KV_2$

$\therefore$ $KV = V_2(1 + K)$

$\therefore$ $V_2 = \frac{KV}{1 + K}$

116. When two air capacitors are connected in series, their effective capacity is,

$$C_1 = \frac{C\times C}{C+C} = \frac{C^2}{2C} = \frac{C}{2}$$

When one of them is filled with dielectric material, effective capacity becomes,

$\frac{1}{C_2} = \frac{1}{C} + \frac{1}{KC}$ ….(where K is dielectric constant)

$\therefore$ $\frac{1}{C_2} = \frac{1}{C}\left[1+\frac{1}{K}\right]$

$\therefore$ $C_2 = \frac{C}{\left[1+\frac{1}{K}\right]} = \frac{CK}{(K+1)}$

Change in effective capacities,

$$C_2 - C_1 = \frac{CK}{K+1} - \frac{C}{2}$$
$$= C\left[\frac{K}{K+1} - \frac{1}{2}\right]$$
$$= C\left[\frac{2K-(K+1)}{2(K+1)}\right]$$
$$= C\left[\frac{K-1}{2(K+1)}\right]$$
$$= \frac{C}{2}\left[\frac{K-1}{K+1}\right]$$

117. Work done, $W = U_f - U_i$

$U_i = \frac{1}{2}CV_0^2$ and $U_f = \frac{1}{2}\frac{(C)}{3}.(3V_0)^2 = 3 \times \frac{1}{2}CV_0^2$

$\therefore$ $U_f - U_i = \frac{1}{2}CV_0^2(3-1) = CV_0^2$

$\therefore$ $W = \frac{\varepsilon_0 A V_0^2}{d}$ ….$\left[\because C = \frac{\varepsilon_0 A}{d}\right]$

118. The electric field is due to all charges present whether inside or outside the given surface.

119. $\phi_{Total} = \phi_A + \phi_B + \phi_C = \frac{q}{\varepsilon_0}$;

$\therefore$ $\phi_B = \phi$ and $\phi_A = \phi_C = \phi'$ [assumed]

$\therefore$ $2\phi' + \phi = \frac{q}{\varepsilon_0} \Rightarrow \phi' = \frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi\right)$.

120. Electric flux, $\phi_E = \int \vec{E}.d\vec{S} = \int EdS\cos\theta$

$$= \int EdS\cos 90° = 0$$

The lines are parallel to the surface.

121. According to Gauss's law, total flux coming out of a closed surface enclosing charge q is given by, $\phi = \oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$

From this expression, it is clear that total flux linked with a closed surface only depends on the enclosed charge and independent of the shape and size of the surface.

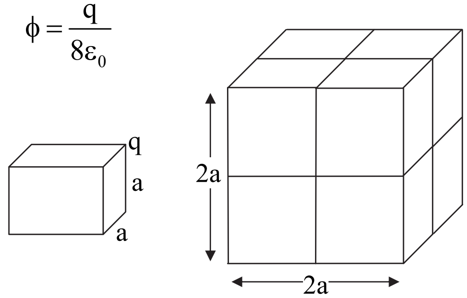
$\phi = \oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0} = 20$ Vm ....[Given]

Thus, $\frac{q}{\varepsilon_0}$ is constant as long as the enclosed charge is constant

$\Rightarrow$ The flux over a concentric sphere of radius 20 cm = 20 Vm.

122. Eight identical cubes are required to arrange so that this charge is at centre of the cube formed.

$\therefore \quad \phi = \frac{q}{8\varepsilon_0}$



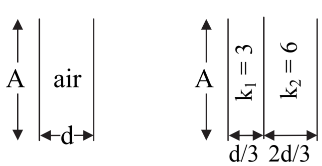
123. Using, $U = \frac{1}{2}\frac{Q^2}{C}$,

$1.21U = \frac{1}{2}\frac{(Q+2)^2}{C}$

$\therefore \quad \frac{1.21}{1} = \frac{(Q+2)^2}{Q^2} \Rightarrow \sqrt{\frac{1.21}{1}} = \frac{Q+2}{Q}$

$\therefore \quad 1.1Q = Q + 2 \Rightarrow Q = 20$ C

124.



$C_{air} = \frac{\varepsilon_0 A}{d} = 9.$

$\frac{1}{C_{med}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{k_1\varepsilon_0 A} + \frac{d_2}{k_2\varepsilon_0 A}$

$\therefore \quad C_{med} = \frac{k_1 k_2 \varepsilon_0 A}{k_1 d_2 + k_2 d_1} = \frac{3\times 6\times \varepsilon_0 A}{3\times\frac{2d}{3} + 6\times\frac{d}{3}} = \frac{18\varepsilon_0 A}{4d}$

$\therefore \quad \frac{C_{med}}{C_{air}} = \frac{18\varepsilon_0 A}{4d}\times\frac{d}{\varepsilon_0 A} = \frac{18}{4}$

$\therefore \quad C_{med} = \frac{18}{4}\times 9 = 40.5$ pF

126. When another capacitor is connected in parallel, then capacitance increases by a factor 2 and potential difference becomes half.

$\therefore$ Final energy (U) = $\frac{1}{2}CV^2$

$= \frac{1}{2}\times 2C\times\left(\frac{V}{2}\right)^2$

$= \frac{CV^2}{4}$

$\therefore$ Total electrostatic energy of resulting system decreases by a factor 2.

127. Capacitance of a cylindrical capacitor = $\frac{2\pi\varepsilon_0 L}{ln\left(\frac{b}{a}\right)}$

Energy stored in the capacitor,

$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2 ln\left(\frac{b}{a}\right)}{2\pi\varepsilon_0 L} = \frac{Q^2}{L}\times k$

where k is a constant.

If the charge and length are doubled,

$\frac{Q'^2}{L'}\times k = \frac{4}{2}\left(\frac{Q^2}{L}\right) = 2$ times the energy.

[*Note: Refer Mindbender 2.*]

129. Heat produced = Energy of charged capacitor

$= \frac{1}{2}CV^2$

$= \frac{1}{2}\times(2\times10^{-6})\times(100)^2$

$= 0.01$ J

130. Power $= \frac{\frac{1}{2}CV^2}{t} = \frac{40\times10^{-6}\times(3000)^2}{2\times2\times10^{-3}} = 90$ kW

131. Let r be radius of each small drop and R be radius of bigger drop.

The volume remains constant

$\therefore \quad \frac{4}{3}\pi R^3 = n\times\frac{4}{3}\pi r^3$

$\therefore \quad R = n^{1/3}r$

For the small drop,

Capacitance, $C_0 = 4\pi\varepsilon_0 r$ and

charge $q_0 = C_0 V = 4\pi\varepsilon_0 rV$

For the bigger drop,
Capacitance, $C = 4\pi\varepsilon_0 R$ and
charge $Q = nq_0$

$\therefore$ Potential of bigger drop $= \dfrac{Q}{C} = \dfrac{nq_0}{4\pi\varepsilon_0 R}$

$= \dfrac{n(4\pi\varepsilon_0 rV)}{4\pi\varepsilon_0 R} = nV\left(\dfrac{r}{R}\right) = n\left(\dfrac{1}{n^{1/3}}\right)V$

$= n^{2/3}V$

132. Let the charge of each drop is g

$\because$ $C = \dfrac{g}{V} \Rightarrow g = CV$

$\therefore$ charge of final drop $\Rightarrow Q = ng$

Let ratio of each small drop is r and big drop is R

$V = nv$

$\dfrac{4}{3}\pi R^3 = n.\dfrac{4}{3}\pi r^3$

$R^3 = nr^3$

$R = (n)^{1/3} r$

Potential on big drop

$V' = \dfrac{kQ}{R}$

$V' = \dfrac{kng}{n^{1/3}r}$

Ratio of energy start in big drop to small drop

$\dfrac{U'}{U} = \dfrac{\frac{1}{2}QV'}{\frac{1}{2}gV} \Rightarrow \dfrac{QV'}{gV} \Rightarrow \dfrac{ng.\frac{kg}{r}n^{2/3}}{g.\frac{kg}{r}}$

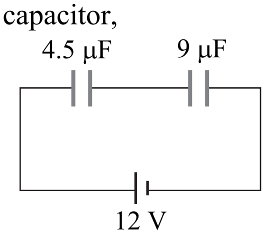
$U' = \dfrac{n^{5/3}}{1}$

133. Using, $U = \dfrac{1}{2}CV^2$,

$U = \dfrac{1}{2}\times\dfrac{A\varepsilon_0}{d}\times(Ed)^2$ $\quad\ldots\left[\because E = \dfrac{V}{d}\right]$

$= \dfrac{1}{2}\varepsilon_0 E^2 Ad$

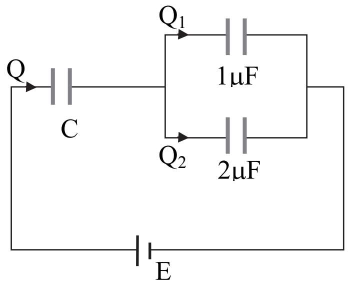
134. The given circuit can be redrawn as follows.
The P.D. across 4.5 μF capacitor,



$V = \dfrac{9}{\left(\frac{9}{2}+9\right)}\times 12$

$= 8$ V

135.



$Q_2 = \dfrac{2}{2+1}Q = \dfrac{2Q}{3}$ ….(i)

$Q = C_R V$

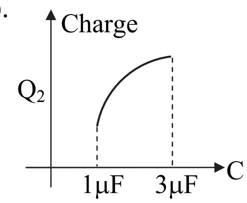
C_R = (1 μF||2 μF) series with C

$\therefore$ $C_R = \dfrac{3C}{C+3}$

$\therefore$ $Q = E\left(\dfrac{C\times 3}{C+3}\right)$

$\Rightarrow Q_2 = \dfrac{2}{3}\left(\dfrac{3CE}{C+3}\right) = \dfrac{2CE}{C+3}$ …. using (i)

This shows as C increases Q increases but not linearly. Also the given relation does not correspond to exponential graph. Hence correct choice is (B).



136. 12 μF and 6 μF are in series and again are in parallel with 4 μF.

$\therefore$ Effective capacitance resultant of these three capacitor will be

$= \dfrac{12\times 6}{12+6} + 4 = 4 + 4 = 8$ μF

This system is in series with 1 μF capacitor.

$\therefore$ Its equivalent capacitance $= \dfrac{8\times 1}{8+1} = \dfrac{8}{9}$ μF ....(i)

Now, equivalent of 8μF, 2μF and 2μF

$= \dfrac{4\times 8}{4+8} = \dfrac{32}{12} = \dfrac{8}{3}$ μF .....(ii)

Combinations (i) and (ii) are in parallel and are in series with C

$\therefore$ $\dfrac{8}{9}+\dfrac{8}{3} = \dfrac{32}{9}$ and $C_{eq} = 1 = \dfrac{\frac{32}{9}\times C}{\left(\frac{32}{9}+C\right)}$

$\therefore$ $C = \dfrac{32}{23}$ μF

137. The given figure is equivalent to a balanced Wheatstone's bridge.

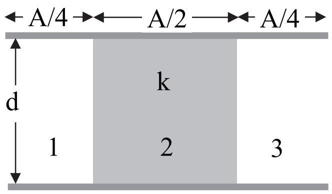
$\therefore$ $C_{eq} = 6\ \mu F$

138. $C_p = 4C_s$

$\Rightarrow (C_1 + C_2) = 4\dfrac{C_1C_2}{(C_1 + C_2)}$

$\Rightarrow (C_1 - C_2)^2 = 0 \qquad \Rightarrow C_1 = C_2$

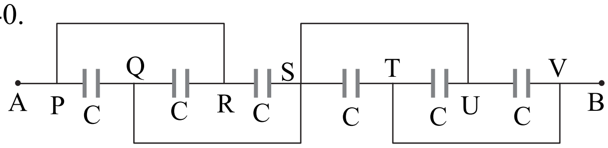
139. $C_1 = \dfrac{\varepsilon_0\left(\dfrac{A}{4}\right)}{d}, C_2 = \dfrac{k\varepsilon_0\left(\dfrac{A}{2}\right)}{d}, C_3 = \dfrac{\varepsilon_0\left(\dfrac{A}{4}\right)}{d}$



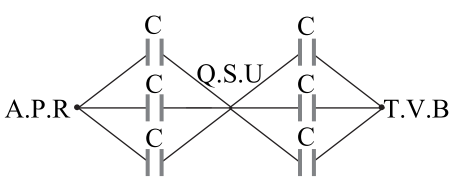
$\therefore$ $C_{eq} = C_1 + C_2 + C_3$

$= \left(\dfrac{k+1}{2}\right)\dfrac{\varepsilon_0 A}{d} = \left(\dfrac{4+1}{2}\right)\times 10 = 25\ \mu F$

140.



$\Rightarrow$



$\Rightarrow$

The equivalent capacitance between A and B is $\dfrac{1}{C_{eq}} = \dfrac{3C + 3C}{(3C)(3C)} \qquad \Rightarrow C_{eq} = 1.5C$

141. C_1 and C_2 are in parallel,

$\therefore$ $C_{eq1} = C_1 + C_2 = 18$ pF

C_{eq2} and C_3 are in series,

$\therefore$ $C_{eq2} = \dfrac{C_{eq1} \times C_3}{C_{eq1} + C_3} = 6$ pF

C_{eq2} and C_4 are in parallel,

$\therefore$ $C_{eq2} + C_4 = 6 + 9 = 15$ pF

142. The capacitance of a parallel plate capacitor in the absence of the dielectric is

$C_0 = \dfrac{\varepsilon_0 A}{d}$ ….(i)

The capacitance of a parallel plate capacitor in the presence of dielectric slab of thickness t and dielectric constant k, is

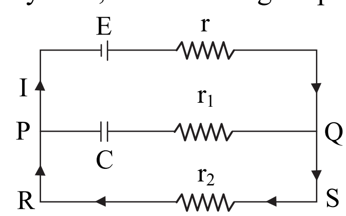
$C = \dfrac{\varepsilon_0 A}{(d-t) + \left(\dfrac{t}{k}\right)} = \dfrac{\varepsilon_0 A}{\left(d - \dfrac{3}{4}d\right) + \left(\dfrac{3d}{4k}\right)}$

$C = \dfrac{\varepsilon_0 A}{\left(\dfrac{d}{4} + \dfrac{3d}{4k}\right)} = \dfrac{4k\varepsilon_0 A}{d(k+3)}$ ….(ii)

Dividing equation (ii) by equation (i) we get,

$\dfrac{C}{C_0} = \dfrac{4k\varepsilon_0 A}{d(k+3)} \times \dfrac{d}{\varepsilon_0 A} = \dfrac{4k}{k+3}$

143. In steady state, current through capacitor is zero



$\therefore$ $V_{PQ} = V_{RS}$

Also, $I = \dfrac{E}{r + r_2}$

$\therefore$ $V_{PQ} = \left(\dfrac{E}{r + r_2} \times r_2\right) = V_{RS}$

$\therefore$ Charge on capacitor is, $Q_C = CV_{PQ} = CE\dfrac{r_2}{(r + r_2)}$

144. The charge flowing through C_4 is

$q_4 = C_4 \times V = 4$ CV ….(i)

For the series combination of C_1, C_2 and C_3,

$\dfrac{1}{C_{eq}} = \dfrac{1}{C} + \dfrac{1}{2C} + \dfrac{1}{3C}$

$\therefore$ $\dfrac{1}{C_{eq}} = \dfrac{6+3+2}{6C} = \dfrac{11}{6C} \Rightarrow C_{eq} = \dfrac{6C}{11}$

Now, C_{eq} and C_4 form parallel combination giving,

$C' = C_{eq} + C_4 = \dfrac{6C}{11} + 4C = \dfrac{50C}{11}$

$\therefore$ Net charge, $q = C'V = \dfrac{50}{11}CV$

Total charge flowing through C_1, C_2, C_3 will be

$q' = q - q_4 = \dfrac{50}{11}CV - 4CV = \dfrac{6CV}{11}$ ….(ii)

As C_1, C_2, C_3 are in series combination, the charge flowing through them will be same.

$\therefore$ From equations (i) and (ii),

the required ratio $= \dfrac{6CV/11}{4CV} = \dfrac{3}{22}$

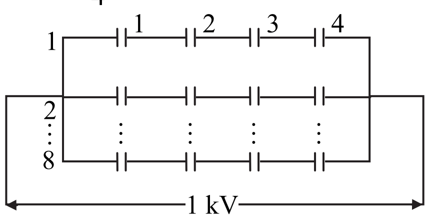
145. The ±q charges appearing on the inner surfaces of A, are bound charges. B is uncharged initially and as it is isolated, the charges on A will not be affected on closing the switch S. No charge will flow into B.

146. To hold 1 kV P.D., minimum four capacitors, which can withstand P.D. upto 300 V, connected in series are required.

∴ $C_s = \frac{1}{4}\,\mu F$

Now, to get capacitance of 2 μF, 8 such series combinations should be connected in parallel.

i.e. $C_{eq} = \frac{8}{4} = 2\ \mu F$.



Hence, the minimum number of capacitors required are 8 × 4 = 32.

147. $E_{inside} = \frac{\rho}{3\varepsilon_0} r$ ….(r < R)

$E_{outside} = \frac{\rho R^3}{3\varepsilon_0 r^2}$ ….(r ≥ R)

i.e. inside the uniformly charged sphere, field varies linearly ($E \propto r$) with distance and that outside it varies according to $E \propto \frac{1}{r^2}$

148. Work done = Energy stored in condenser

∴ $mgh = \frac{1}{2}CV^2$

∴ $h = \frac{CV^2}{2mg} = \frac{10\times10^{-6}\times(6\times10^3)^2}{2\times10\times10^{-3}\times10} = 1800\ m$

149. Electric field E is given by,

$E = \frac{V}{d}$ ….$\left\{\begin{matrix} V \equiv \text{potential difference} \\ d \equiv \text{plate separation}\end{matrix}\right\}$

∴ $E = \frac{V}{h}$ ….{∵ d = h}

….(i)

But $V = \frac{Q}{C}$

$V = \frac{it}{C}$ ….(ii)

Substituting equation (ii) in equation (i) we get,

$E = \frac{it}{Ch}$

150. $C_{net} = 5\ \mu F$

$Q_{net} = 5 \times 8 = 40\ \mu C$

We know,

$Q_{2\mu F} = 2 \times 8 = 16 \mu C$

∴ $Q_{4\mu F} = Q_{12\mu F} = Q_{net} - Q_{2\mu F}$

…{∵ 9μF|| 3μF = 12μF}

$= 40 - 16 = 24\mu C$

Voltage across 4μF and 12μF can be given as,

$V_{4\mu F} + V_{12\mu F} = V$

∴ $V_{4\mu F} = V - \frac{Q_{12\mu F}}{C_{12\mu F}} = 8 - \frac{24}{12} = 6V$

∴ $V_{12\mu F} = 2V$

i.e. $V_{9\mu F} = 2V$

∴ $Q_{9\ \mu F} = 9 \times 2 = 18\ \mu C$

∴ $Q = Q_{4\ \mu F} + Q_{9\ \mu F} = 42\ \mu C$

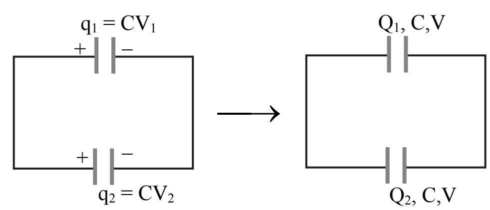
∴ $E = \frac{kQ}{r^2} = \frac{9\times10^9 \times 42 \times 10^{-6}}{30 \times 30} = 420\ N/C$

151. Work done in compression

= Energy stored in condenser

⇒ Ratio of energies = 1

152.



Initially from charge conservation

$q_1 + q_2 = Q_1 + Q_2$

$CV_1 + CV_2 = CV + CV$

$C(V_1 + V_2) = 2CV$

$V = \frac{V_1 + V_2}{2}$

Charge in potential energy of system

$\Delta U = U_i - U_f$

$= \left\{\frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2\right\} - \left\{\frac{1}{2}CV^2 + \frac{1}{2}CV^2\right\}$

$= \frac{1}{2}C\left\{V_1^2 + V_2^2 - 2V^2\right\}$

$= \frac{1}{2}C\left\{V_1^2 + V_2^2 - 2\left(\frac{V_1 + V_2}{2}\right)^2\right\}$

$\Rightarrow \frac{1}{2}C\left\{V_1^2 + V_2^2 - \frac{1}{2}V_1^2 - \frac{1}{2}V_2^2 - V_1V_2\right\}$

$\Rightarrow \frac{1}{2}C\left\{\frac{V_1^2}{2}+\frac{V_2^2}{2}-V_1V_2\right\}$

$\Rightarrow \frac{1}{4}C\left\{V_1^2+V_2^2-2V_1V_2\right\} \Rightarrow \frac{1}{4}C\left(V_1-V_2\right)^2$

$\Delta U = \frac{1}{4}C\left(V_1-V_2\right)^2$

153. Initial energy stored, $U = \frac{1}{2}(2\mu F)\times V^2$

$\therefore$ Energy dissipated on connection across 8 μF,

$\Delta U = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)V^2 = \frac{1}{2}\times\frac{2\mu F\times 8\mu F}{10\mu F}\times V^2$

$= \frac{1}{2}\times(1.6\mu F)V^2$

$\therefore$ % loss of energy $= \frac{\Delta U}{U}\times 100 = \frac{1.6}{2}\times 100$

$= 80\ \%$

155. Potential at O, $V = \int \frac{kdq}{L+r}$

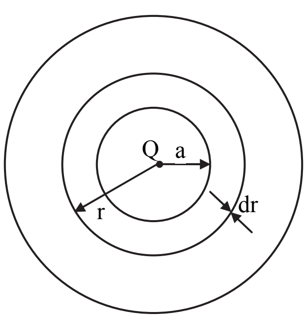
$\because$ $dq = \frac{Q}{L}\,dL$

$\therefore$ $V = \frac{kQ}{L}\int_0^L \frac{dL}{L+r}$

$= \frac{kQ}{L}\,ln\,[L+r]_0^L = \frac{kQ}{L}\,ln2$

$= \frac{Q}{4\pi\varepsilon_0 L}\,ln\,2$

156.



Electric field due to charge Q at r = a is,

$E_a = \frac{kQ}{a^2}$ ....(i)

Consider a shell of thickness dr in the region $a \le r \le b$.

charge on shell, $dq = \text{Area}\times\rho = 4\pi r^2 d r\ \frac{A}{r}$

$\therefore$ total charge in the region $a \le r \le b$ is, $q = \int_a^b dq$

$= 4\pi A\int_a^b r\ dr = 4\pi A\left[\frac{r^2}{2}\right]_a^b$

$\therefore$ $q = 2\pi A\,[b^2 - a^2]$

Electric field at r = b is,

$E_b = \frac{k\left[2\pi A\left[b^2-a^2\right]+Q\right]}{b^2}$ ....(ii)

For electric field to be constant in the region $a \le r \le b$ we must have, $E_a = E_b$

from equation (i) and (ii)

$\frac{kQ}{a^2} = k\frac{\left[2\pi A\left[b^2-a^2\right]+Q\right]}{b^2}$

$\therefore$ $\frac{Qb^2}{a^2}-Q = 2\pi\ A\left(b^2-a^2\right)$

$\frac{Qb^2-Qa^2}{a^2} = 2\pi A\left(b^2-a^2\right)$

$\frac{Q\left(b^2-a^2\right)}{a^2} = 2\pi A\left(b^2-a^2\right)$

$\therefore$ $A = \frac{Q}{2\pi a^2}$

157. To make potential zero net charge on two capacitors must be made zero. Hence, capacitors must be connected such that

$Q = Q_1 - Q_2 = 0$

$\therefore$ $C_1V_1 - C_2V_2 = 0$ $\therefore$ $C_1V_1 = C_2V_2$

$\therefore$ $120\ C_1 = 200\ C_2$ $\therefore$ $3\ C_1 = 5\ C_2$

158. $C = \frac{Q}{V}$

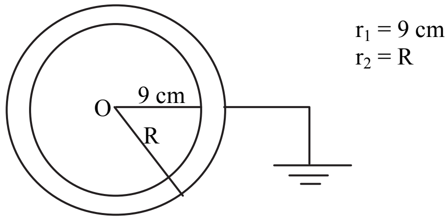
but $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ for a spherical body.

$\therefore$ $C = 4\pi\varepsilon_0 R$

$\therefore$ $C = 4\pi\times 8.85\times 10^{-12}\times 6400\times 10^3$

$\therefore$ $C = 7.1\times 10^{-4}$ F

159.



Capacity of isolated sphere, $C = \frac{q}{V}$

$C = 4\pi\varepsilon_0 r_1$ ....(i)

Capacity of earthed concentric hollow sphere is; $C_H = 4\pi\varepsilon_0\left(\frac{r_1 r_2}{r_2 - r_1}\right)$

$\therefore$ $10C = 4\pi\varepsilon_0\left(\frac{r_1 R}{(R - r_1)}\right)$ ….(ii)

Dividing equation (ii) by equation (i),

$\frac{10C}{C} = \left(\frac{r_1 R}{R - r_1}\right)\frac{1}{r_1}$

$\therefore$ $10 = \frac{9R}{(R-9)}\frac{1}{9}$

$\therefore$ $10R - 90 = R$

$\therefore$ $9R = 90$

$\therefore$ $R = 10$ cm

160. Here, V is constant.

$U = \frac{1}{2}CV^2$

$C' \to kC \Rightarrow$ energy stored will become k times

$q = CV \Rightarrow$ q will become k times

$\therefore$ Surface charge density, $\sigma' = \frac{kq}{A} = k\sigma_0$

161. A charged cloud induces opposite charge on pointed conductors. At sharp points of the conductor, surface density of charge is very high and charge begins to leak from the pointed ends by setting up oppositely charged electic wind. When this wind comes in contact with the charged cloud, it neutralizes some of the charge on it. Hence, the potential difference between the cloud and the building is reduced. This in turn reduces the chance of lightening striking the building (if the lightening strikes the building, then the charge is conducted to the earth and the building remains safe).

Evaluation Test

1. $\phi_E = \frac{q_{in}}{\varepsilon_0} = 0 \Rightarrow q_{in} = 0$

Now,

q_{IN} for $S_1 = -3q - q + q = -3q$

q_{IN} for $S_2 = +q - q = 0$

q_{IN} for $S_3 = -3q + q = -2q$

q_{IN} for $S_4 = -3q$

2. $V_P - V_Q = \int_{\vec{r}_q}^{\vec{r}_p} \vec{E}.d\vec{l}$

If $\vec{E}$ is constant, then

$V_P - V_Q = -\vec{E}\cdot\int_{\vec{r}_Q}^{\vec{r}_p} d\vec{l}$

$V_P - V_Q = -\vec{E}\cdot(\vec{r}_P - \vec{r}_Q)$

$= -(2\hat{i} + \hat{j}).\left((1-2)\hat{i} + (2-1)\hat{j} + (0-1)\hat{k}\right)$

$= -(2\hat{i} + \hat{j}).\left(-1\hat{i} + 1\hat{j} - 1\hat{k}\right) = -(-2 + 1) = 1$ V

3. The initial potential of the outer shell,

$V_2 = \frac{KQ}{R_2} + \frac{K(2Q)}{R_2} = \frac{K(Q+2Q)}{R_2}$

After connecting the shells, by a wire, the potentials of the shells,

$V_1' = \frac{Kq}{R_1} + \frac{K(3Q-q)}{R_2}$ and

$V_2' = \frac{Kq}{R_2} + \frac{K(3Q-q)}{R_2}$

where 'q' is the remnant charge on inner shell.

As inner and outer shell are connected, $V_1' = V_2'$

$\Rightarrow \frac{Kq}{R_1} = \frac{Kq}{R_2}$ $\Rightarrow q = 0$ or $R_1 = R_2$

The later is not possible $\Rightarrow q = 0$

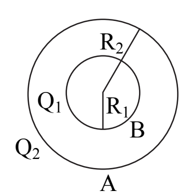
Thus, $V_2' = \frac{K(3Q)}{R_2} \Rightarrow V_2 = V_2'$

So the potential of the outer shell does not change after connecting with wire.

$\Rightarrow$ (A) is correct.

4. Assertion is true, Reason is true and Reason is a correct explanation for Assertion.

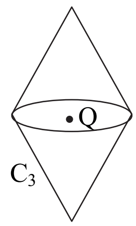
$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 + Q_2}{R_2}\right)$



$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R_1} + \frac{Q_2}{R_2}\right)$

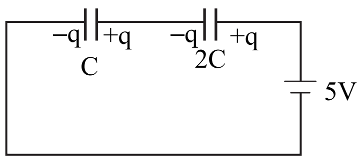
$\therefore$ $V_B - V_A = \frac{1}{4\pi\varepsilon_0}Q_1\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

5. Let us enclose the charge at the mouth of the conical flask with another identical flask. Flux through the closed surface = $\frac{2Q}{\varepsilon_0}$.



By symmetry, flux through either flask is $\frac{1}{2}\left(\frac{2Q}{\varepsilon_0}\right) = \frac{Q}{\varepsilon_0}$

6.



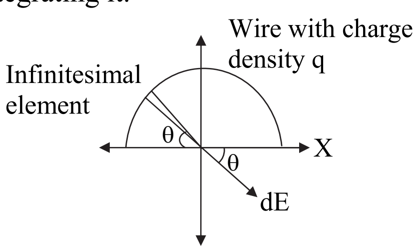
As both the earthed points are at 0 V, we can redraw the circuit as,

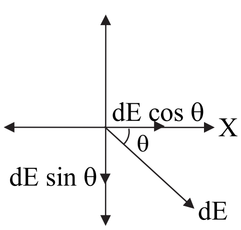
C and 2C in series = $\frac{2C\,C}{2C+C} = \frac{2}{3}C$

$q = C_{eq}V = \left(\frac{2}{3}C\right)(5\text{ V})$

$= \left(\frac{2}{3}\times 6\,\mu\text{F}\right)$ (5 V) = 20 μC

7. The electric field at the center of the semicircle can be found by calculating the field due to an infinitesimal element and integrating it.





Charge on the infinitesimal element

$(\lambda q) = \lambda dx = \lambda(Rd\theta) = \lambda Rd\theta$ ….(i)

Electric field at O due to this charge

$(dE) = k(dq)/R^2$ ….(ii)

Where $k = \frac{1}{4\pi\varepsilon_0}$

Substituting (i) in (ii),

Electric field $dE = k\lambda d\theta/R$ ….(iii)

X component of electric field

$= dE_x = dE\cos\theta = \frac{(k\lambda\cos\theta)d\theta}{R}$ (from iii)

Y component of the electric field

$= dE_y = dE\sin\theta = \frac{(k\lambda\sin\theta)d\theta}{R}$ (from iii)

$E_x = \int_0^{\pi}\frac{(k\lambda\cos\theta)}{R}d\theta = 0$

Net electric field due to the wire at point O along Y axis

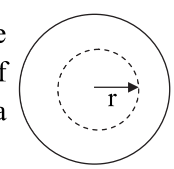
$E_y = -\int_0^{\pi}\frac{(k\lambda\sin\theta)}{R}d\theta = \frac{k\lambda}{R}(2) = -2k\lambda/R$

Resultant electric field (E)

$= \sqrt{E_x^2 + E_y^2} = 2k\lambda/R = \frac{\lambda}{2\pi\varepsilon_0 R}$

The resultant electric field at the center of the circle = $\frac{\lambda}{2\pi\varepsilon_0 R}$

8. Consider the Gaussian surface to be a spherical shell of negligible thickness at a distance of r from the centre.



The net flux through the surface is $E(4\pi r^2)$

The net charge (Q) enclosed

$= \int_v \rho dV = \int \rho_0[1 - x^2/9]\times 4\pi x^2 \times dx$

where, the limits of x varies from 0 to r.

∴ $Q_{net} = \int_0^r \rho_0\left(1 - \frac{x^2}{9}\right)(4\pi x^2)dx$

$= 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^5}{5\times 9}\right]$

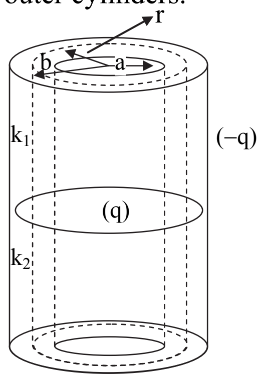
Applying Gauss's law $E(4\pi r^2) = Q_{net}/\varepsilon_0$

$E = \frac{\rho_0}{k\varepsilon_0}\left[\frac{r}{3} - \frac{r^3}{5\times 9}\right]$

Hence electric field (E) at a distance r from the centre = $\frac{\rho_0}{2\varepsilon_0}\left[\frac{r}{3} - \frac{r^3}{5\times 9}\right]$

$= \frac{\rho_0}{2\varepsilon_0}\left[\frac{15r - r^2}{45}\right] = \frac{\rho_0}{90\varepsilon_0}(15r - r^3)$

9. The capacitance is to be found between the inner and outer cylinders.



Consider a Gaussian cylinder of radius r. Applying Gauss's law,

$E(2\pi r)\,L = q(L/\varepsilon)$

Hence, $E = q/2\pi\varepsilon r$

Potential difference, $\Delta V = \int E\cdot dr$

$= (q/2\pi\varepsilon)\, ln(b/a)$

Hence, $q = [2\pi\varepsilon/ln(b/a)]\Delta V$

∴ $q_{net} = q\cdot L = [2\pi\varepsilon L/ln(b/a)]\,\Delta V$

$C = q/\Delta V$

∴ $C = 2\pi\varepsilon L/ln(b/a)$

Now we can consider the top and bottom parts of the cylinder as two capacitors in parallel.

∴ Net capacitance,

$C_{net} = C_1 + C_2$

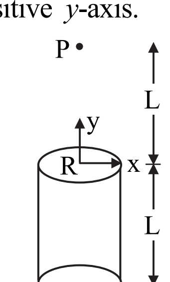
$$C_{net} = \frac{2\pi(L/2)}{l\,n(b/a)}(\varepsilon_1 + \varepsilon_2)$$

where $\varepsilon_1 = k_1\varepsilon_0$ and $\varepsilon_2 = k_2\varepsilon_0$

Capacitance of the arrangement

$$= \frac{\pi L}{ln(b/a)}(k_1\varepsilon_0 + k_2\varepsilon_0) = \frac{3\pi L\varepsilon_0 k}{2\,ln(2)}$$

10. Consider a unit charge as shown at P and the co-ordinate frame is chosen as shown in the figure. The force on P acts along the positive *y*-axis.



Let us imagine that the cylinder can be broken into a number of thin disks. Now the field at P due to one such disk at a distance *x* from P is

$$dE = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

where, *x* varies from L to 2L.

Hence, the total field is given by

$$E = \int dE = \int_L^{2L} \frac{Q}{2\varepsilon_0(\pi R^2 L)}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)dx$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2 L}\left(L - \sqrt{4L^2 + R^2} + \sqrt{L^2 + R^2}\right)$$

Therefore, electric field at a point at a distance L from one end of the cylinder

$$= \frac{Q}{2\pi\varepsilon_0 R^2 L}\left(L - \sqrt{4L^2 + R^2} + \sqrt{L^2 + R^2}\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 (2)^2 4}\left(4 - \sqrt{4(4)^2 + 2^2} + \sqrt{4^2 + 2^2}\right)$$

$$= \frac{Q}{16\pi\varepsilon_0}\left(2 - \sqrt{17} + \sqrt{5}\right)$$

11. The circuit with the switch 1 in 'ON' position is shown in figure (i). We apply the Kirchoff's 2nd law. Consider the closed loop through the 6C capacitors.

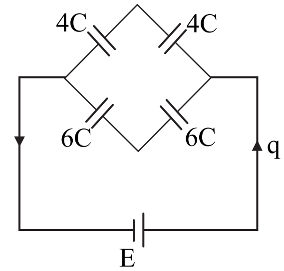


Figure (i)

Potential drop across the capacitors

$$= -\left[\frac{q_1}{6C} + \frac{q_1}{6C}\right]$$

We are traversing the loop from negative to positive. Therefore, potential drop due to battery can be taken as positive.

Writing the equation for net potential drop along the loop,

$$E - \left[\frac{q_1}{6C} + \frac{q_1}{6C}\right] = 0 \qquad \Rightarrow q_1 = 3CE$$

∴ Charge flow along the 6C capacitor = 3CE

∴ Energy stored in the capacitor

$$= \frac{1}{2}QE = \frac{1}{2}q_1(E/2) = 3/4\ CE^2$$

In the second case, when the switch 2 is 'ON', the circuit diagram would be as given in figure (ii)

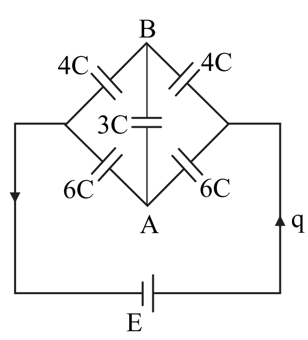
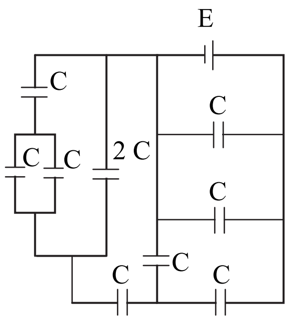


Figure (ii)

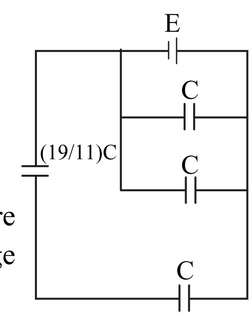
The circuit is symmetric about AB. Therefore, we can say that charge entering the 4C capacitor to B would be the same as charge leaving B through the other 4C capacitor. Therefore, there would be no charge flow along 3C capacitor. Hence, Energy in the 3C capacitor = 0.

12. The given circuit can be redrawn and reduced to the following:



Now, the potential across the two capacitors in parallel in E. Hence the charge stored in each is

$q_1, q_2 = CE$ ....(i)



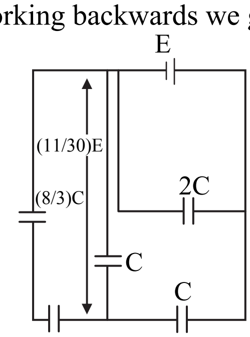
The other two capacitors are in series. Hence the charge in each of them is

$q_3 = (19/30)\ CE$ ....(ii)

Therefore the potential across the (19/11) C capacitor is

$V_1 = q_3 / [(19/11)\ C] = (11/30)E$

Now working backwards we get the circuit,



Since the potential is (11/30)E, the charge on the parallel capacitor,

$q_4 = (11/30)CE$ ....(iii)

For the two series capacitors, net C = (8/11)C

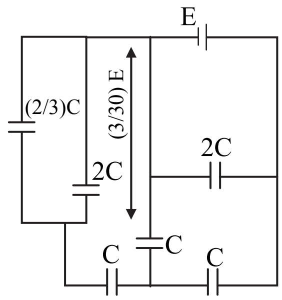
Hence, charge in the capacitors

$q_5 = (8/30)CE$

The potential across the (8/3)C capacitor,

$V_2 = (3/30)E$

We now consider the following circuit:



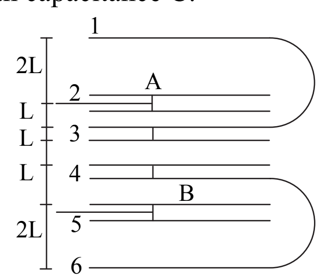
The charge on the X = 2 C capacitor is

$Q_6 = (3/15)CE = (1/5)CE$

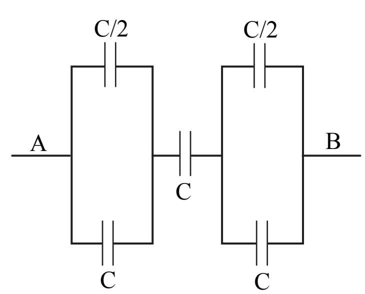
13. Let the plates be numbered as shown below.

Plates 2, 3, 4 and 5 may be treated as a collection of two plates as shown in the diagram.

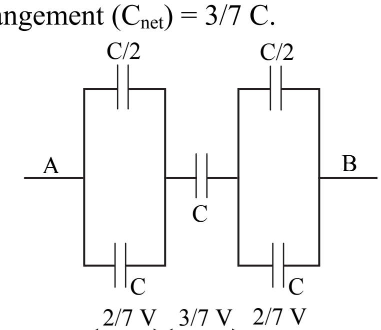
We get five capacitors with top and bottom capacitors having a capacitance C/2 and the rest with capacitance C.



Hence the circuit gets reduced as shown in the figure below.



The equivalent capacitance of the above arrangement (C_{net}) = 3/7 C.

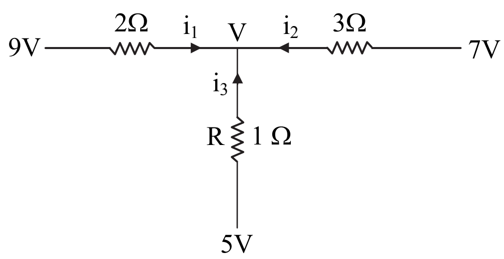


If the potential applied across AB is V, the charge on the capacitors (q)

q = CV

Hence the charges on plate X = q = (–1/7) CV

14. The current flow in different segments can be found considering different open loops and applying Kirchoff's junction law.



There are no direct series and parallel connections which can be directly identified.

This circuit consists of only resistors. So elements need not be removed from the circuit.

Let us mark different nodes and loops in the circuit and consider the node B.

Current entering the node = current leaving the node.

Current enters through AB, CB and DB.

Let us assume the potential at the nodal point B to be V.

Current entering the node = $i_1 + i_2 + i_3$

$$= \frac{V-9}{2} + \frac{V-7}{3} + \frac{V-5}{1}$$

$$= \frac{11V-71}{6} \quad \text{….(i)}$$

Current leaving the node = 0 ….(ii)

Equating equations (i) and (ii), $\frac{11V-71}{6} = 0$

∴ Voltage at node B = V = $\frac{71}{11}$

∴ Current flowing through wire AB (i_1) = $\frac{V-9}{2}$

$$= -\frac{14}{11}$$

$$= -1.27 \text{ A}$$

15. Net charge inside the sphere

$= \int \rho dV$

Due to spherical symmetry, we get,

$$Q = \int_0^R 4\pi r^2 \rho dr$$

$$= 4\pi A \int_0^R r^2 (R-r) dr$$

$$= 4\pi A \left(\frac{R^4}{3} - \frac{R^4}{4} \right)$$

∴ $A = \frac{3Q}{\pi R^4}$

16. Electric flux through:

i.	X-Y plane	3 × 100 = 300
ii.	Y-Z plane	8 × 100 = 800
iii.	X-Z plane	4 × 100 = 400

Hence, the required ratio = 3 : 8 : 4

17. Potential of the bigger drop

$= n^{2/3} \times$ potential on each droplet

$= 64^{2/3} \times 10$

$= 4^2 \times 10$

$= 16 \times 10$

$= 160$ V

18. Redistribution of charges takes place and during flow of charges some energy is lost as heat.

19. $$5\int_0^{20} (10V+4)\,dV = 5\left[\frac{10V^2}{2} + 4V\right]_0^{20}$$

$$= 5[5 \times 400 + 80]$$

$$= 5[2000 + 80]$$

$$= 5 \times 2080$$

$$= 10400 \text{ J}$$

20. $C = \frac{Q}{V}$, $C = \frac{k\varepsilon_0}{d}$

$C \propto \frac{1}{V} \Rightarrow C \propto \frac{1}{d}$

∴ As 'd' increases, C decreases

Hence 'V' increases.

Textbook Chapter No.

13 Current Electricity

Hints

Classical Thinking

18. $\frac{V}{L} = 10$ V/m

$\therefore$ $V = 10 \times L = 10 \times 25 \times 10^{-2} = 2.5$ V

20. Potentiometer is said to be more sensitive if it gives large change in the balancing length for a small change in p. d. (i.e., $\frac{dE}{dl}$ should be small)

$E = \frac{V}{L} \times l$

$\therefore$ $\frac{dE}{dl} = \frac{V}{L} \Rightarrow \frac{V}{L}$ be small

22. $r = R\left(\frac{l}{l_1} - 1\right) = 5\left(\frac{120}{80} - 1\right) = 5 \times \frac{1}{2} = 2.5\ \Omega$

23. Zero (No Potential difference across voltmeter).

24. $r = R\left(\frac{l}{l_1} - 1\right) = 10\left(\frac{75}{60} - 1\right)$

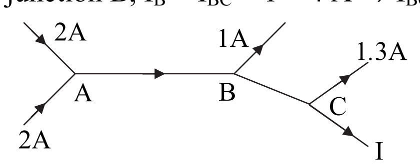
$= 10\left(\frac{15}{60}\right) = 2.5\ \Omega$

Critical Thinking

1. At a junction,
Current entering = Current leaving

$\therefore$ $I + 4 + 2 = 5 + 3 \Rightarrow I = 2$ A

2. According to Kirchhoff's first law,
At junction A, $I_A = 2 + 2 = 4$ A
At junction B, $I_B = I_{BC} + 1 = 4\text{ A} \Rightarrow I_{BC} = 3$A



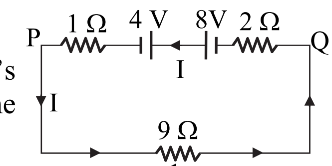
At junction C, $I = I_{BC} - 1.3 = 3 - 1.3 = 1.7$ A

3. $V = I(R + r)$

$\therefore$ $50 = 4.5\ (10 + r)$

$\therefore$ $4.5\ r = 5 \Rightarrow r = \frac{5}{4.5} = 1.1\ \Omega$

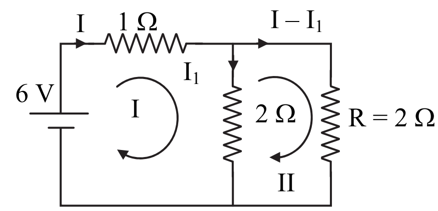
4. Applying Kirchhoff's voltage law to the given loop QPQ,



$-2I + 8 - 4 - 1 \times I - 9I = 0 \Rightarrow I = \frac{1}{3}$A

$\therefore$ Potential difference across PQ $= \frac{1}{3} \times 9 = 3$ V

5.



Applying Kirchhoff's law to loop I,
$6 - I - 2I_1 = 0$ ….(i)
Applying it to loop II,
$-2(I - I_1) + 2I_1 = 0$ ….(ii)

$\therefore$ $-2I = -4I_1 \Rightarrow I = 2I_1$

Substituting in equation (i),
$6 - 2I_1 - 2I_1 = 0$

$\therefore$ $I_1 = \frac{6}{4} = 1.5$ A

6. Applying Kirchhoff law,
$(2 + 2) = (0.1 + 0.3 + 0.2)I$

$\therefore$ $I = \frac{20}{3}$ A

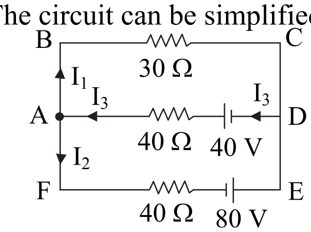
$\therefore$ Potential difference across A

$= 2 - 0.1 \times \frac{20}{3} = \frac{4}{3}$ V (less than 2 V)

Potential difference across B

$= 2 - 0.3 \times \frac{20}{3} = 0$

7. The circuit can be simplified as follows:



Applying Kirchhoff's current law to junction A,

$I_3 = I_1 + I_2$ ….(i)

Applying Kirchhoff's voltage law for the loop ABCDA,

$-30I_1 + 40 - 40I_3 = 0$

$\therefore$ $-30I_1 - 40(I_1 + I_2) + 40 = 0$ ….[From (i)]

$\therefore$ $7I_1 + 4I_2 = 4$ ….(ii)

Applying Kirchhoff's voltage law for the loop ADEFA,

$-40I_2 + 80 + 40 - 40I_3 = 0$

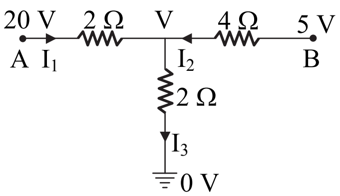
$\therefore$ $-40I_2 - 40(I_1 + I_2) = -120$ ….[From (i)]

$\therefore$ $I_1 + 2I_2 = 3$ ….(iii)

On solving equations (ii) and (iii),

$I_1 = -0.4$ A

8.



Let V be the potential of the junction as shown in figure. Applying junction law, we have

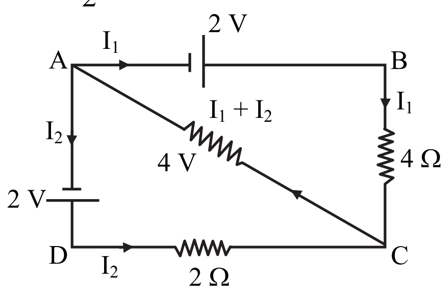
$$\frac{20-V}{2} + \frac{5-V}{4} = \frac{V-0}{2}$$

$\therefore$ $40 - 2V + 5 - V = 2V$

$\therefore$ $5V = 45 \Rightarrow V = 9$ V

$\therefore$ $I_3 = \dfrac{V}{2} = 4.5$ A

9.



Applying Kirchhoff's voltage law to ABCA,

$2 - 4I_1 - 4(I_1 + I_2) = 0$ ….(i)

Applying Kirchhoff's voltage law to ADCA,

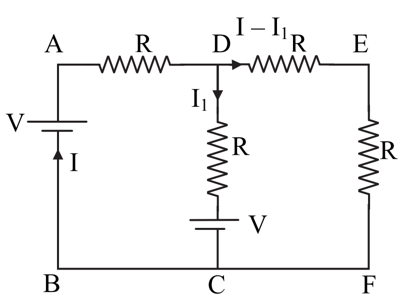
$2 - 2I_2 - 4(I_1 + I_2) = 0$ ….(ii)

Subtracting equation (ii) from equation (i),

$-4I_1 + 2I_2 = 0$

$\therefore$ $2I_2 = 4I_1 \Rightarrow I_2 / I_1 = 2$

10.



For loop ABCDA,

$IR + I_1R + V - V = 0$

$\therefore$ $(I + I_1)R = 0 \Rightarrow I_1 = -I$

Now, In loop ABFEA,

$IR + (I - I_1)R + (I - I_1)R - V = 0$

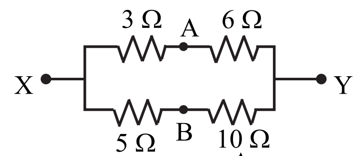
$\therefore$ $IR + IR - I_1R + IR - I_1R = V$

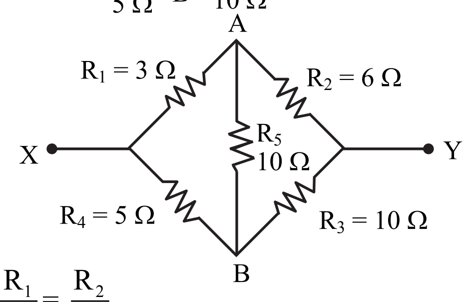
$\therefore$ $3IR - 2I_1R = V$

$\therefore$ $3IR - 2(-I)R = V$

$\therefore$ $5IR = V \Rightarrow I = \dfrac{V}{5R}$

11.





$\because$ $\dfrac{R_1}{R_4} = \dfrac{R_2}{R_3}$

Wheatstone's bridge network is balanced. Hence there is no current flowing through AB (through R_5).

$\therefore$ The given circuit is equivalent to

$R_{xy} = (3 + 6) \,||\, (5 + 10)$

$\therefore$ $R_{xy} = \dfrac{9\times15}{15+9} = \dfrac{9\times15}{24} = \dfrac{45}{8}\ \Omega$

12. The bridge is balanced.

The balance condition after replacing 10 Ω resistor by 20 Ω resistor will remain the same.

$\therefore$ $R_{eq.} = 4\Omega \parallel 28\ \Omega = \dfrac{4\times28}{4+28} = \dfrac{4\times28}{32} = \dfrac{7}{2}\ \Omega$

$\therefore$ $I = \dfrac{V}{R_{eq.}} = \dfrac{12\times2}{7} = 3.4$A

13. As the bridge is balanced, $\dfrac{R_{AB}}{R_{BC}} = \dfrac{R_{AD}}{R_{DC}}$

$\therefore$ $\dfrac{15+6}{(X\parallel8)+3} = \dfrac{15+(6\parallel6)}{4+(4\parallel4)}$

$\therefore$ $\dfrac{21}{\left(\dfrac{8X}{8+X}\right)+3} = \dfrac{18}{4+2}$

$\therefore$ $168 + 21X = 33X + 72$

$\therefore$ $12X = 96 \Rightarrow X = \dfrac{96}{12} = 8\ \Omega$

14. As the bridge is balanced,

$$\frac{R_{AB}}{R_{AD}} = \frac{R_{BC}}{R_{CD}}$$

$\therefore$ $$\frac{4+4}{\left(\frac{4}{3}+X\right)} = \frac{10 \| 5}{5 \| 5}$$

$\therefore$ $$\frac{8}{\left(\frac{4}{3}+X\right)} = \frac{50/15}{25/10}$$

$\therefore$ $$\frac{8}{\left(\frac{4}{3}+X\right)} = \frac{50}{15}\times\frac{10}{25} = \frac{4}{3}$$

$\therefore$ $\frac{4}{3} + X = 6 \Rightarrow X = 6 - \frac{4}{3} = \frac{14}{3}\ \Omega$

15. For the balance condition,

$\frac{P}{Q} = \frac{R}{S \| X}$ where X is the resistance with which S is shunted,

$\therefore$ $$\frac{3}{3} = \frac{4}{\left(\frac{6\times X}{6+X}\right)}$$

$\therefore$ $6X = 24 + 4X \Rightarrow X = 12\ \Omega$

16. $$\frac{X}{100-X} = \frac{2}{3}$$

$\therefore$ $3X = 200 - 2X$

$\therefore$ $5X = 200 \Rightarrow X = 40$ cm

17. 1st case: $\frac{R_1}{X} = \frac{2}{3}$ ....(i)

2nd case: $\frac{R_2}{X} = \frac{3}{2}$ ....(ii)

Adding equations (i) and (ii),

$$\frac{R_1}{X} + \frac{R_2}{X} = \frac{2}{3} + \frac{3}{2}$$

$$\frac{R_1 + R_2}{X} = \frac{13}{6}$$

Let l be the distance of null point from left.

$$\frac{l}{100 - l} = \frac{13}{6}$$

$\therefore$ $6\,l = 1300 - 13\,l$

$\therefore$ $19\,l = 1300 \Rightarrow l = \frac{1300}{19} = 68.4$ cm from left.

18. Let X be the smaller resistance in the metre bridge $l_X = 20$ cm

$\therefore$ $l_R = 100 - 20 = 80$ cm

As the bridge is balanced,

$$\frac{l_X}{l_R} = \frac{X}{R}$$

$\therefore$ $$\frac{20}{80} = \frac{X}{R}$$

$\therefore$ $$\frac{X}{R} = \frac{1}{4}$$

$\therefore$ $R = 4X$ ....(i)

From second condition,

$$\frac{X+15}{R} = \frac{40}{100-40}$$

$\therefore$ $$\frac{X+15}{R} = \frac{40}{60}$$

$\therefore$ $$\frac{X+15}{R} = \frac{2}{3}$$

$\therefore$ $2R = 3X + 45$

$\therefore$ $R = \frac{3X+45}{2}$ ....(ii)

Equating (i) and (ii) we get,

$$\frac{3X+45}{2} = 4X$$

$\therefore$ $8X = 3X + 45$

$\therefore$ $5X = 45 \Rightarrow X = 9\ \Omega$

19. 1st case: $\frac{R_P}{R_Q} = \frac{2}{3}$

$R_P = \frac{2}{3} R_Q$ ....(i)

2nd case: Resistance, instead of R_Q is

$R_Q \| 10 = \frac{10R_Q}{10} + R_Q = R'$

Now, $R_P/R' = 1 \Rightarrow R_P = R'$

$\therefore$ $R_P = \frac{10R_Q}{10+R_Q}$ ....(ii)

From equations (i) and (ii),

$$\frac{2}{3} R_Q = \frac{10R_Q}{10+R_Q}$$

$\therefore$ $$\frac{1}{3} = \frac{5}{10+R_Q}$$

$\therefore$ $10 + R_Q = 15$

$\therefore$ $R_Q = 5\ \Omega$ and $R_P = 10/3\ \Omega$

20. Manganin or constantan is used for making the potentiometer wire.

21. G is a sensitive galvanometer and to protect it from damage of heavy currents, some resistance R′ is introduced.

22. $R_{AB} = 2 \times 10 = 20\ \Omega$

∴ $I = \frac{3}{10+20} = \frac{3}{30} = \frac{1}{10}$

∴ $V = I\,R_{AB} = \frac{1}{10} \times 20 = 2\ V$

∴ $\frac{V}{L} = \frac{2}{10} = 0.2\ V/m$

23. Potential gradient $= \frac{I\rho}{A}$

$= \frac{10^{-2} \times 10^{-3} \times 10^{9} \times 10^{-2}}{10^{-2} \times 10^{-4}}$

$= \frac{10^{2}}{10^{-6}} = 10^{8}\ V/m$

24. $I = \frac{E}{R+r} = \frac{2}{8+2} = 0.2\,A$

∴ $V = IR = 0.2 \times 8 = 1.6\ V$

∴ Potential gradient $= \frac{V}{L} = \frac{1.6}{4} = 0.4\ V/m$

25. $I = \frac{E}{R+r} = \frac{2}{990+10} = \frac{2}{1000}\,A$

∴ $V = IR = \frac{2}{1000} \times 10$

∴ Potential gradient $= \frac{V}{L} = \frac{2}{100} \times \frac{1}{2} = 0.01\ V/m$

26. $I = \frac{E}{R+r_2} = \frac{5}{40+10} = \frac{5}{50} = 0.1\,A$

27. Resistance per unit length is 1 Ω/m
Balancing length = 2.9 m
Resistance across balancing length = 2.9 Ω
e.m.f. = 1.45 V
Current, $I = \frac{1.45}{2.9} = 0.5\ A$

28. $E \propto l$

∴ $\frac{E}{1.02} = \frac{75}{50}$

∴ $E = \frac{3}{2} \times 1.02 = 3 \times 0.51 = 1.53\ V$

29. $I = \frac{2}{R+10}$

∴ $V = I\,R_{AB} = \frac{2}{R+10} \times 10 = \frac{20}{R+10}$

∴ $\frac{V}{L} = \frac{20}{(R+10)1} = \frac{20}{R+10}$

∴ $E_1 = l\left(\frac{V}{L}\right)$

∴ $10 \times 10^{-3} = 0.4\left(\frac{20}{R+10}\right)$

∴ $R + 10 = \frac{8}{10^{-2}} = 800 \Rightarrow R = 790\ \Omega$

30. $\frac{V}{L} = \frac{E}{L}\left(\frac{R_{AB}}{R+R_{AB}}\right)$

∴ $E_1 = \frac{V}{L}\,l = \frac{E}{L}\left(\frac{R_{AB}}{R+R_{AB}}\right).l$

∴ $\frac{E_1}{E} = \frac{20}{(20+20).10} \times 5 = \frac{1}{4}$

∴ $E : E_1 :: 4 : 1$

31. Using,

$\frac{E_1+E_2}{E_1-E_2} = \frac{l_1}{l_2}$

∴ $\frac{1.5+1.1}{1.5-1.1} = \frac{260}{l_2}$

∴ $\frac{2.6}{0.4} = \frac{260}{l_2} \Rightarrow l_2 = \frac{260}{2.6} \times 0.4 = 40\ cm$

32. When null point is obtained on potentiometer wire, the cell whose potential difference is to be measured does not supply current to potentiometer wire since galvanometer deflection is zero. Therefore current through the potentiometer wire is due to auxiliary battery.

33. As current through G is zero, it balances Wheatstone's bridge.

∴ $\frac{6}{12} = \frac{9}{R} \Rightarrow R = 18\ \Omega$

34. $\frac{l_P}{l_Q} = \frac{P}{Q} = \frac{1}{3} \Rightarrow 3P = Q$

∴ $3P - Q = 0$ ....(i)

$\frac{P+40}{Q+40} = \frac{3}{5}$

∴ $5P + 200 = 3Q + 120$

∴ $5P - 3Q = -80$ ....(ii)

Solving equations (i) and (ii) we have,
$P = 20\ \Omega,\ Q = 60\ \Omega$

35. P.D. across potentiometer wire = 2 V

Potential gradient = $\frac{V}{L} = \frac{2}{100}$ V/cm

Now, $E = \left(\frac{V}{L}\right)l$

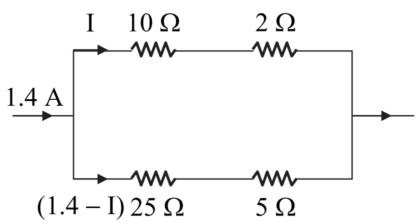
$\therefore$ $E = \frac{2}{100} \times 75 = 2 \times \frac{3}{4} = 1.5$ V

36. $I = \frac{E}{R+r} = \frac{4}{30+30} = \frac{1}{15}$ A

$\therefore$ $V = I \times R = \frac{1}{15} \times 30 = 2$ V

$\Rightarrow K = \frac{2}{10} = \frac{1}{5}$ V/m

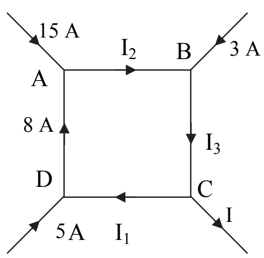
37. Since it's a balanced Wheatstone's bridge, the circuit can be redrawn as



$\therefore$ $12 I = 30 (1.4 - I)$

$\therefore$ $12 I = 42 - 30 I \Rightarrow I = 1$ A

38. By Kirchhoff's current law,



Let the currents I_1, I_2 and I_3 be as shown in the figure.

Applying Kirchhoff's junction law to D,

$I_1 + 5 = 8 \Rightarrow I_1 = 3$ A

Applying it to A,

$I_2 = 8 + 15 = 23$ A

Applying it to B,

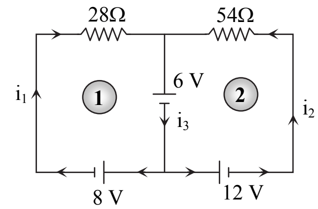
$I_2 + 3 = I_3 \Rightarrow I_3 = 26$ A

Applying it to C,

$I_1 + I = I_3$

$\Rightarrow I = I_3 - I_1 = 26 - 3 = 23$ A

39. Suppose current through different paths of the circuit is as follows:



Applying Kirchhoff's voltage law to loop (1) and loop (2) we get,

$28i_1 = -6 - 8 \Rightarrow i_1 = -\frac{1}{2}$ A and

$54i_2 = -6 - 12 \Rightarrow i_2 = -\frac{1}{3}$ A

$\therefore$ $i_3 = i_1 + i_2 = -\frac{5}{6}$ A

40. Potential gradient $(x) = \frac{I\rho}{A} = \frac{0.1\times10^{-7}}{10^{-6}} = 10^{-2}$ V/m

41. Let E_A, E_B and E_C be the e.m.f. of three cells A, B and C respectively.
For the given potentiometer,
$E_A + E_B + E_C = kl_1 = k \times 740$ ....(i)
$E_A + E_B = kl_2 = k \times 440$ ....(ii)
$E_B + E_C = kl_3 = k \times 540$ ....(iii)
From eq. (i) and (ii), we get
$E_C = 300k$
From equations (i) and (iii) we get,
$E_A = 200k$
Substituting value of E_A into equation (ii) we get,
$E_B = 240k$

$\therefore$ $E_A : E_B : E_C = 200k : 240k : 300k$
$= 10 : 12 : 15 = 1 : 1.2 : 1.5$

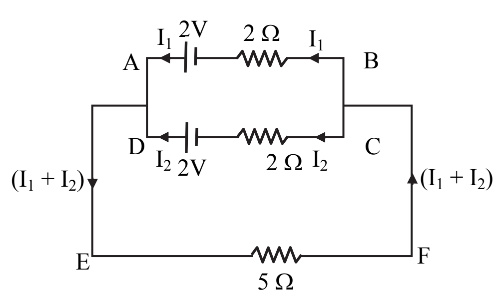
$\therefore$ $E_A = 1V$, $E_B = 1.2$ V, $E_V = 1.5$ V

42. $K = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$

$\therefore$ $\frac{0.2\times10^{-3}}{10^{-2}} = \frac{2}{(R + 490 + 0)} \times \frac{R}{1}$

$\therefore$ $R = 4.9\ \Omega$

43.



Applying Kirchhoff's second law for closed loop AEFBA we get,

$-(I_1 + I_2) \times 5 - I_1 \times 2 + 2 = 0$ or

$7I_1 + 5I_2 = 2$ ....(i)

Again, applying Kirchhoff's second law for a closed loop DEFCD we get,

$-(I_1 + I_2) \times 5 - I_2 \times 2 + 2 = 0$

or $5I_1 + 7I_2 = 2$ ....(ii)

Multiplying (i) by 5 and (ii) by 7 we get,

$35I_1 + 25I_2 = 10$ ....(iii)

$35I_1 + 49I_2 = 14$ ....(iv)

Subtracting (iv) from (iii) we get,

$-24I_2 = -4 \Rightarrow I_2 = \frac{1}{6}$ A

Substituting the value of I_2 in equation (i) we get,

$7I_1 = 2 - 5 \times \frac{1}{6} \Rightarrow 7I_1 = \frac{7}{6} \Rightarrow I_1 = \frac{1}{6}$ A

The current through the 5 Ω,

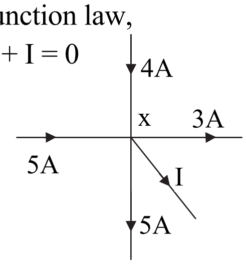
$= I_1 + I_2 = \frac{1}{6}\text{ A} + \frac{1}{6}\text{ A} = \frac{1}{3}\text{ A}$

Competitive Thinking

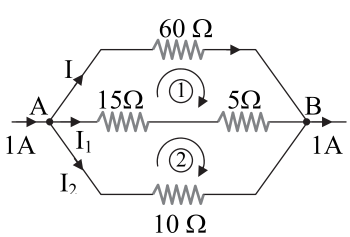
2. According to Kirchhoff's voltage law, the correct equation is $\varepsilon_1 - (i_1 + i_2)R - i_1r_1 = 0$

3. According to Kirchhoff's law,
$I_{CD} = I_2 + I_3$

4. According to Kirchhoff's junction law,
$(+5) + (+4) + (-3) + (-5) + I = 0$
$\therefore$ $I = -1$ A
[–ve sign shows that current is flowing away from x.]



5.



Applying Kirchhoff's law

At junction A:

$i + i_1 + i_2 = 1$ ....(i)

For Loop (1)

$-60i + (15+5)i_1 = 0$

$\therefore$ $i_1 = 3i$ ....(ii)

For loop (2)

$-(15 + 5)\, i_1 + 10\, i_2 = 0$

$\therefore$ $i_2 = i_1 = (3\,i) = 6i$ ....(iii)

On solving equations (i), (ii) and (iii) we get

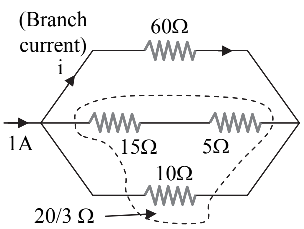
i = 0.1 A

Alternate Method:

Branch current =

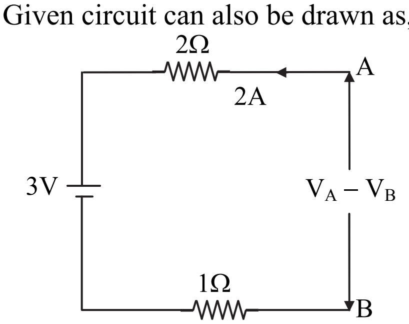
$$\text{Main current} \times \left(\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right)$$

$$\Rightarrow i = 1 \times \left[\frac{\frac{20}{3}}{\frac{20}{3} + 60}\right] = 0.1 \text{ A}$$



(Note: Use shortcut 3.)

6. Given circuit can also be drawn as,



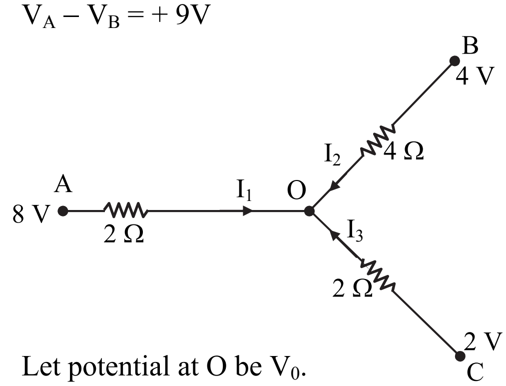
By Kirchhoff's law,

$V_A - (2 \times 2) - (3) - (2 \times 1) - V_B = 0$

$\therefore$ $V_A - 4 - 3 - 2 - V_B = 0$

$\therefore$ $V_A - V_B = +9$V

7.



Let potential at O be V_0.

According to Kirchhoff's current law,

$I_1 + I_2 + I_3 = 0$

$$\frac{8 - V_0}{2} + \frac{4 - V_0}{4} + \frac{2 - V_0}{2} = 0$$

∴ $2(8 - V_0) + 4 - V_0 + 2(2 - V_0) = 0$

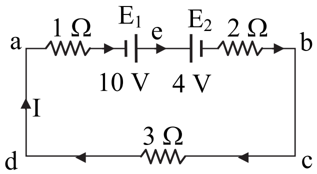
∴ $16 - 2V_0 + 4 - V_0 + 4 - 2V_0 = 0$

∴ $5V_0 = 24$

∴ $V_0 = \dfrac{24}{5}$

$= 4.8$ V

8. Since $E_1(10\text{ V}) > E_2(4\text{ V})$, hence current in the circuit will be clockwise.

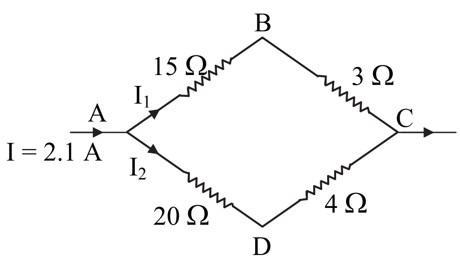


Applying Kirchhoff's voltage law,

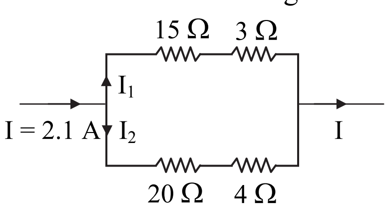
$-1 \times I + 10 - 4 - 2 \times I - 3I = 0$

∴ I = 1A (a to b via e)

9. No current flows through the 6 Ω resistor as the Wheatstone network is balanced.



In parallel combination voltage remains same.



∴ $I_1 \times (15 + 3) = I_2 \times (20 + 4)$

$I_1 \times 18 = I_2 \times 24$

∴ $3I_1 = 4I_2$

∴ $I_2 = \dfrac{3}{4}I_1$

According to KCL,

$I_1 + I_2 = 2.1$

$I_1 + \dfrac{3}{4}I_1 = 2.1$

$\dfrac{7}{4}I_1 = 2.1$

$I_1 = \dfrac{2.1 \times 4}{7}$

$= 1.2$ A

10. Applying Kirchhoff's voltage law to loop containing V_{CC}, R_L and Transistor -

$+V_{CC} - I_CR_L - V_{CE} = 0$

$\Rightarrow V_{CC} = V_{CE} + I_CR_L$

Applying Kirchhoff's voltage law to loop containing V_{BB}, R_B and Transistor-

$+V_{BB} - I_BR_B - V_{BE} = 0 \Rightarrow V_{BB} = V_{BE} + I_BR_B$

12. For balancing the bridge

$\dfrac{P}{Q} = \dfrac{R}{S}$

∴ $S = \dfrac{S_1S_2}{S_1 + S_2}$ ....($\because$ S_1, S_2 are in parallel)

∴ $\dfrac{P}{Q} = \dfrac{R(S_1 + S_2)}{S_1S_2}$

13. Four resistances forming a Wheatstone's network are 8 Ω, 12 Ω, 6 Ω and 27 Ω. After shunting the 27 Ω resistance with say, S, the balance condition will be,

$\dfrac{8}{12} = \dfrac{6}{\left(\dfrac{27S}{27+S}\right)} \Rightarrow \dfrac{1}{3} = \dfrac{3(27+S)}{27S}$

∴ $27S = 243 + 9S \Rightarrow 13.5\ \Omega$

14. Let S be shunted with resistance X.

At balanced condition,

$\dfrac{P}{Q} = \dfrac{P}{\dfrac{SX}{S+X}} \Rightarrow \dfrac{2}{2} = \dfrac{2}{\dfrac{3X}{3+X}} \Rightarrow \dfrac{3X}{3+X} = 2$

$3X = 6 + 2X \Rightarrow X = 6\ \Omega$

15. The resistances in four arms of a Wheatstone's bridge are, 10 Ω, 10 Ω, 10 Ω and 20 Ω.

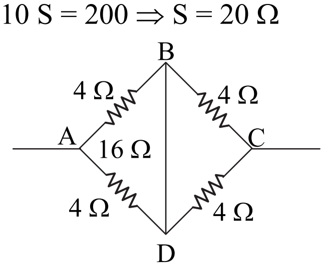
Let S be the resistance to be connected across 20Ω.

∴ Balance condition is,

$\dfrac{10}{10} = \dfrac{10}{\left(\dfrac{20S}{20+S}\right)} \Rightarrow 20S = 10(20 + S)$

∴ $10S = 200 \Rightarrow S = 20\ \Omega$

16.



According to the principle of Wheatstone's bridge, the effective resistance between the given points is

$= (4 + 4)\ \Omega \parallel (4 + 4)\ \Omega$

$= 8\Omega \parallel 8\Omega = 4\Omega$

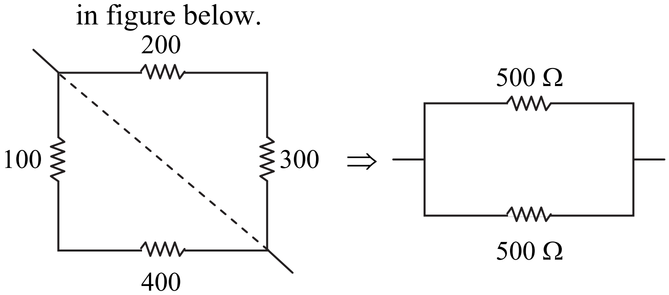
18. This is a balanced Wheatstone's bridge circuit. Hence potentials at B and D will be same and no current flows through the resistance 4R.

19. This is a balanced Wheatstone bridge. Hence no current will flow from the diagonal resistance 10 Ω.

$\therefore$ Equivalent resistance $= \dfrac{(10+10)\times(10+10)}{(10+10)+(10+10)}$

$= 10\ \Omega$

20. Considering resistors are connected as shown in figure below.



$\therefore\ R_{\text{eff max}} = \dfrac{500\times500}{500+500} = 250\ \Omega$

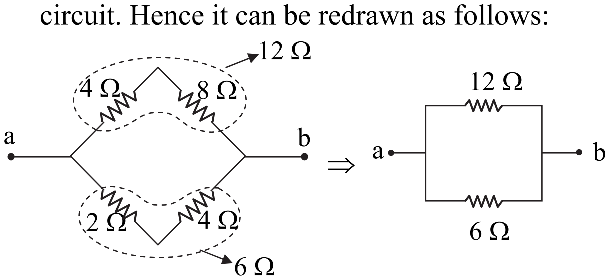
21. Wheatstone's network is balanced as $\dfrac{P}{R} = \dfrac{Q}{S}$

$\therefore$ No current flows through galvanometer.

$\therefore\ R_{\text{eff.}} = \dfrac{25\times50}{25+50} = \dfrac{25\times50}{75} = \dfrac{50}{3}\ \Omega$

$\therefore\ I = \dfrac{V}{R} = \dfrac{6}{50/3} = 0.36\,\text{A}$

22. Given circuit is a balanced Wheatstone bridge circuit. Hence it can be redrawn as follows:



$\therefore\ R_{AB} = \dfrac{12\times6}{(12+6)} = 4\ \Omega$

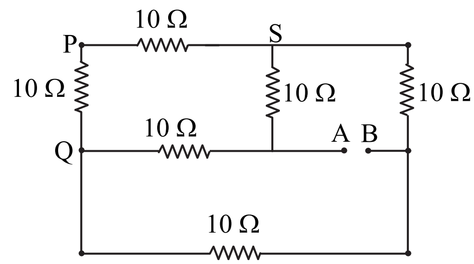
23. The given circuit is a balanced Wheatstone's bridge circuit. Hence potential difference between A and B is zero.

24. For balanced Wheatstone bridge, $\dfrac{P}{Q} = \dfrac{R}{S}$

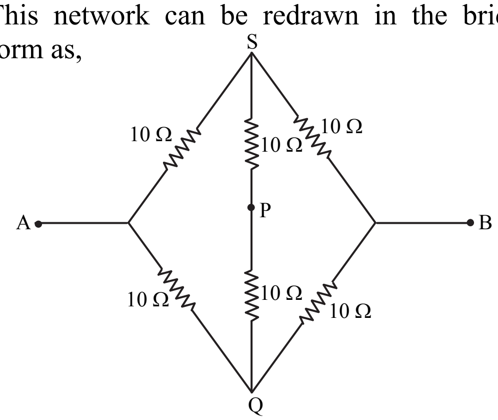
$\therefore\ \dfrac{12}{(1/2)} = \dfrac{x+6}{(1/2)}$

$\Rightarrow x = 6\ \Omega$

25.

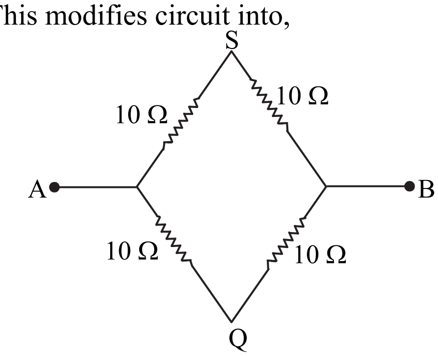


This network can be redrawn in the bridge form as,



In this case, $\dfrac{AS}{SB} = \dfrac{AQ}{QB}$ Hence, bridge is balanced and no current will flow through SPQ branch and thus, is neglected.

This modifies circuit into,



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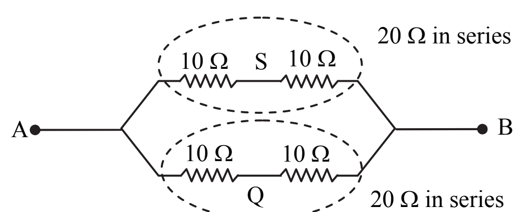
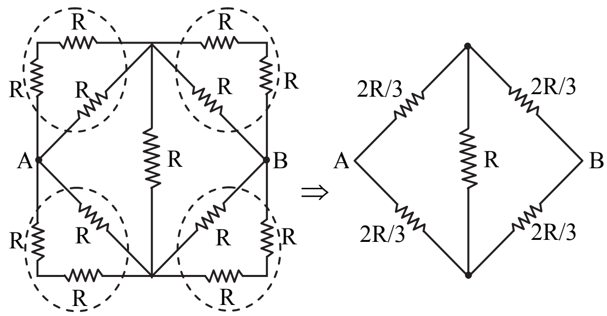


$R_{AB} = [20\ \Omega \parallel 20\ \Omega]$

$= \left[\frac{1}{20}+\frac{1}{20}\right]^{-1}$

$= 10\ \Omega$

26.



As the bridge is balanced,

$\therefore \quad R_{eq} = \frac{2R}{3}$

27. $\frac{X}{1} = \frac{20}{80} \Rightarrow X = \frac{1}{4}\ \Omega = 0.25\ \Omega$

29. $S = \left(\frac{100-1}{1}\right) R$

Initially, $30 = \left(\frac{100-l}{l}\right) \times 10$

$\therefore \quad l = 25$ cm

Finally, $10 = \left(\frac{100-l}{l}\right) \times 30$

$\therefore \quad l = 75$ cm

So, shift 75 cm – 25 cm = 50 cm

30. $\frac{X}{R} = \frac{l_x}{l_R}$

$\frac{20}{30} = \frac{l_x}{l_R}$

$\Rightarrow \frac{l_x}{l_R} = \frac{40}{60}$

as for metrebridge, $l_x + l_R = 100$ cm

$\Rightarrow l_x = 40$ cm

After reducing resistance,

$\frac{X'}{R} = \frac{l'_x}{100-l'_x}$

$\therefore \quad \frac{10}{30} = \frac{l'_x}{100-l'_x}$

$\therefore \quad l'_x = 25$ cm

The distance through which balance point is shifted $l_x - l'_x = 40 - 25 = 15$ cm to the left

31. For first case, the balancing condition is

$\frac{10+R_1}{R_2} = \frac{50}{50}$

$\therefore \quad R_2 = 10 + R_1$.

For second case, the balancing condition is

$\frac{R_1}{R_2} = \frac{40}{60}$

$\frac{R_1}{10+R_1} = \frac{2}{3} \Rightarrow R_1 = 20\ \Omega$

32. Let l_X be balancing length obtained in front of smaller resistance.

$\therefore \quad l_X = 40$ cm, $l_R = 60$ cm

When the bridge is balanced,

$\frac{X}{R} = \frac{l_X}{l_R} = \frac{40}{60} = \frac{2}{3}$ ....(i)

when 30 Ω is connected in series with X, effective resistance becomes (X + 30) Ω

Also, length shifts by 20 cm

$\Rightarrow l_{X+30} = 40 + 20 = 60$ cm

$\therefore \quad \frac{X+30}{R} = \frac{60}{40} = \frac{3}{2}$

$R = \frac{2(X+30)}{3}$ ....(ii)

From equations (i) and (ii),

$\frac{X}{2\frac{(X+30)}{3}} = \frac{2}{3}$

$\therefore \quad \frac{3X}{2(X+30)} = \frac{2}{3}$

$\therefore \quad 5X = 120$

$\therefore \quad X = 24\ \Omega$

33. Initially, $\frac{5}{l_1} = \frac{R}{100-l_1}$ ....(i)

Finally, $\frac{5}{1.6\,l_1} = \frac{R/2}{(100-1.6l_1)}$ ....(ii)

$\therefore \quad \frac{R}{1.6(100-l_1)} = \frac{R}{2(100-1.6l_1)}$

$\therefore \quad 160 - 1.6\,l_1 = 200 - 3.2\,l_1$

$\therefore \quad 1.6\,l_1 = 40$

$\therefore \quad l_1 = 25$

From equation (i),

$\frac{5}{25} = \frac{R}{75} \Rightarrow R = 15\ \Omega$

34. In balancing condition, $\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{l_1}{100 - l_1}$

$\Rightarrow \frac{X}{Y} = \frac{20}{80} = \frac{1}{4}$ ....(i)

and $\frac{4X}{Y} = \frac{l}{100 - l}$ ....(ii)

$\Rightarrow \frac{4}{4} = \frac{l}{100 - l}$

$\Rightarrow l = 50$ cm

35. Initially,

$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{60}{40} = \frac{3}{2}$ ….(i)

When, wire is stretched by 20 % i.e., becomes 1.2 L

(Using shortcut 4),

Resistance will increase to $1.44R_2$

Hence, after stretching wire,

$\frac{R_1'}{R_2'} = \frac{l}{100 - l}$

But $R_1' = R$ and $R_2' = 1.44\ R_2$

$\therefore \quad \frac{R_1}{1.44 R_2} = \frac{l}{100 - l}$

From (i),

$\frac{3}{1.44 \times 2} = \frac{l}{100 - l}$

$\therefore \quad 300 - 3l = 2.88\ l$

$\therefore \quad l = \frac{300}{5.88} \approx 51$ cm

36. Let balancing length be l,

$\therefore \quad \frac{R_1}{R_2} = \frac{l}{100 - l}$ ....(i)

If R_1 and R_2 are interchanged balancing length becomes, (l – 10)

$\therefore \quad \frac{R_2}{R_1} = \frac{l - 10}{[100 - (l - 10)]} = \frac{l - 10}{110 - l}$ ....(ii)

From equations (i) and (ii),

$\frac{l}{100 - l} = \frac{110 - l}{l - 10}$

$\therefore \quad l^2 - 10l = (110 \times 100) + (l^2 - 210l)$

$\therefore \quad 200\ l = 110 \times 100$

$\therefore \quad l = 55$ cm

Substituting in equation (i), we get,

$\frac{R_1}{R_2} = \frac{55}{45} = \frac{11}{9}$ ....(iii)

When R_1 and R_2 are connected in series,

$R_1 + R_2 = 1000\ \Omega$ . ....(iv)

On solving equations (iii) and (iv), we get,

$R_1 = 550\ \Omega$ and $R_2 = 450\ \Omega$

37. Balancing length is independent of the area of cross-section of the wire.

38. Metrebridge is balanced,

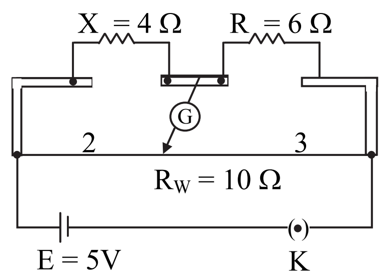
$\therefore \quad \frac{R}{80} = \frac{AC}{BC} = \frac{20}{80}$

$\therefore \quad R = 20\ \Omega$

39. Unknown resistance, $X = R\ \frac{l_1}{l_2} = 6 \times \frac{2}{3}$

$\therefore \quad X = 4\ \Omega$

Resistance of bridge wire $R_W = 0.1\ \Omega$/cm = 10 Ω



Equivalent resistance,

$R_{eq} = (X + R) \parallel R_W = (10\ \Omega) \parallel (10\ \Omega)$

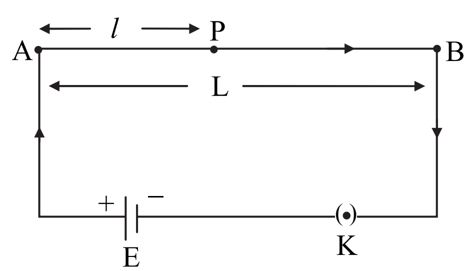
$R_{eq} = 5\ \Omega$

Current drawn from the battery is, $I = \frac{E}{R_{eq}}$

$= \frac{5}{5}$

$\therefore \quad I = 1A$

41.



For a potentiometer wire AB of length L,

$V_{AP} = \left(\frac{V_{AB}}{L}\right) l$

$\therefore \quad \frac{V_{AP}}{V_{AB}} = \frac{l}{L}$

The ratio $\left(\frac{V_{AP}}{V_{AB}}\right)$ would remain constant if the length of the wire is increased.

∴ $L \propto l$

Hence balancing length 'l' will increase if length of potentiometer wire is increased.

42. Potential difference per unit length,

$$\frac{V}{L} = \frac{2}{4} = 0.5 \text{ V/m}$$

43. EMF of Cell,

$E = kl'$

$$E = \frac{E'}{l} \times l'$$

$$E = \frac{E'}{10} \times 2.5 \quad \text{....(i)}$$

After increasing the length by 1m,

$$E = \frac{E'}{11} \times x$$

Substituting for E from equation (i)

$$\frac{E'}{10} \times 2.5 = \frac{E'}{11} \times x$$

∴ $x = \frac{2.5 \times 11}{10} = 2.75$ m

44. $I = \frac{e}{(R + R_h + r)}$

∴ $\frac{V}{L} = \frac{2}{(15 + 5 + 0)} \times \frac{5}{1}$

∴ K = 0.5 V/m = 0.005 V/cm

45. $R = \frac{\rho l}{A}$

∴ $\frac{R}{l} = \frac{\rho}{A} = \frac{40 \times 10^{-8}}{8 \times 10^{-6}} = 5 \times 10^{-2}$ Ω/m

Potential gradient is given by,

$$\frac{V}{l} = \frac{IR}{l} = 0.2 \times 5 \times 10^{-2} = 10^{-2} \text{ V/m}$$

46. P.D. across the wire

= Potential gradient × length

V_0 = 1mV/cm × 400 cm

= 0.4 V

Current in the wire, $I = \frac{0.4}{8} = 0.05$ A

$$R = \frac{V - V_0}{I} = \frac{2 - 0.4}{0.05} = 32 \ \Omega$$

47. $K = \frac{e}{(R + R_h + r)} \frac{R}{L}$

∴ $\frac{10^{-3}}{10^{-2}} = \frac{2}{(3 + R_h + 0)} \times \frac{3}{1}$

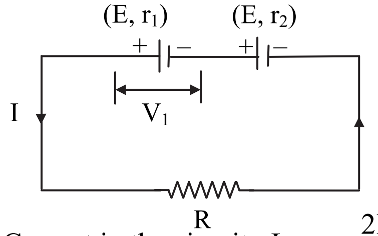
∴ $R_h = 57$ Ω

48. $V = I.R = \frac{e}{(R + R_h + r)} R$

∴ $10^{-3} = \frac{2}{(10 + R + 1)} \times 10$

∴ R = 19,989 Ω

49.



Current in the circuit: $I = \frac{2E}{R + r_1 + r_2}$

Terminal p.d across 1st cell is $V_1 = E - Ir_1$

Given: $V_1 = 0$

⇒ $E - Ir_1 = 0$

$$E - \left(\frac{2E}{R + r_1 + r_2}\right) r_1 = 0$$

$$E = \frac{2Er_1}{R + r_1 + r_2} = 2r_1$$

$R + r_1 + r_2 = 2r_1$

Or $R = r_1 - r_2$

50. $\frac{E}{l} = \frac{e}{(R + R_h + r)} \frac{R}{L}$

∴ $0.4 = \frac{5}{(5 + 45 + 0)} \times \frac{5}{10} \times l$

∴ $l = 8$ m

51. Current drawn when resistors are in series,

$$I_s = I = \frac{E}{nR + R} = \frac{E}{(n+1)R} \quad \text{....(i)}$$

Current drawn when resistors are in parallel,

$$I_p = 10\, I = \frac{E}{\frac{R}{n} + R} \quad \text{....(ii)}$$

Substituting for I using equation (i) in equation (ii),

$$\frac{10E}{(n+1)R} = \frac{E}{\left(1 + \frac{1}{n}\right)R}$$

∴ $n + 1 = 10\left(1 + \frac{1}{n}\right)$

∴ $n - \frac{10}{n} = 9$

∴ $n^2 - 9n - 10 = 0$

∴ $n^2 - 10n + n - 10 = 0$

$\therefore$ n (n – 10) + 1 (n – 10) = 0

$\therefore$ (n + 1) (n – 10) = 0

Neglecting negative value of n,

n = 10

52. $r = \left(\dfrac{l_1 - l_2}{l_2}\right) \times R'$

$\therefore \quad r = \left(\dfrac{55-50}{50}\right) \times 10 = 1\ \Omega$

53. $r = \left(\dfrac{l_1}{l_2} - 1\right)R$

$= \left(\dfrac{3}{2.85} - 1\right)\ 9.5\ \Omega$

$= \dfrac{0.15}{2.85} \times 9.5\Omega = 0.5\ \Omega$

54. $r = \left(\dfrac{l_1 - l_2}{l_2}\right)R$

$\therefore \quad R = \left(\dfrac{25}{100}\right)2 = 0.5\ \Omega$

55. Internal resistance,

$r = \left(\dfrac{l_1}{l_2} - 1\right)\ R$

$= \left(\dfrac{52}{40} - 1\right) \times 5$

$= \dfrac{12 \times 5}{40} = 1.5\ \Omega$

56. $E_1 \propto L_1$ and $E_1 \propto L_2$

$\therefore \quad \dfrac{E_1}{E_2} = \dfrac{L_1}{L_2} \Rightarrow \dfrac{1.25}{E_2} = \dfrac{30}{40}$

$\Rightarrow E_2 = \dfrac{5}{3} \approx 1.67$ V

57. $\dfrac{E_1}{E_2} = \dfrac{l_1 + l_2}{l_1 - l_2} = \dfrac{58+29}{58-29} = \dfrac{87}{29} = \dfrac{3}{1}$

58. $\dfrac{E_1}{E_2} = \dfrac{l_1 + l_2}{l_1 - l_2} = \dfrac{(6+2)}{(6-2)} = \dfrac{2}{1}$

59. While assisting net E.M.F = $E_1 + E_2$

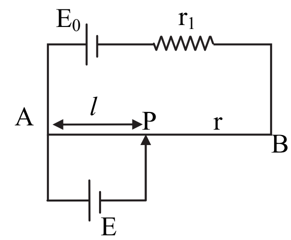
opposing net E.M.F = $|E_1 - E_2|$

for potentiometer $E \propto l$

$\therefore \quad \dfrac{E_1 + E_2}{E_1 - E_2} = \dfrac{50}{10} = \dfrac{5}{1}$

$\therefore \quad \dfrac{E_1}{E_2} = \dfrac{5+1}{5-1} = \dfrac{6}{4} = \dfrac{3}{2}$

60.



Current in wire AB = $\dfrac{E_0}{r_1 + r}$

Potential gradient (K) = $\dfrac{ir}{L} = \left(\dfrac{E_0}{r_1 + r}\right) \cdot \dfrac{r}{L}$

$E = Kl$

$\therefore \quad E = \left(\dfrac{E_0}{r_1 + r}\right)\dfrac{r}{L} \times l$

61. Resistance: $R = \rho\dfrac{L}{A}$

But, $A = \dfrac{\pi d^2}{4}$ and $L_1 = \dfrac{V_1}{A_1}$

$\therefore \quad \dfrac{R_1}{R_2} = \dfrac{L_1}{A_1} \times \dfrac{A_2}{L_2} = \dfrac{L_1}{L_2} \times \dfrac{d_2^2}{d_1^2}$

$= \dfrac{V}{A_1} \times \dfrac{A_2}{V} \times \dfrac{d_2^2}{d_1^2}$

$= \dfrac{d_2^2}{d_1^2} \times \dfrac{d_2^2}{d_1^2} = \dfrac{d_2^4}{d_1^4}$

62. S_1 is open and S_2 is closed

So, $I = \dfrac{12}{(6+4)} = \dfrac{12}{10}$

$\Rightarrow$ I = 1.2 A

63.

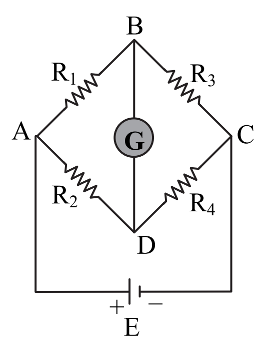


Figure (a)

In the Wheatstone bridge shown in figure (a), null point is obtained when,

$\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$ ….(i)

When the positions of galvanometer and cell (E) are interchanged, we get circuit shown in figure (b).

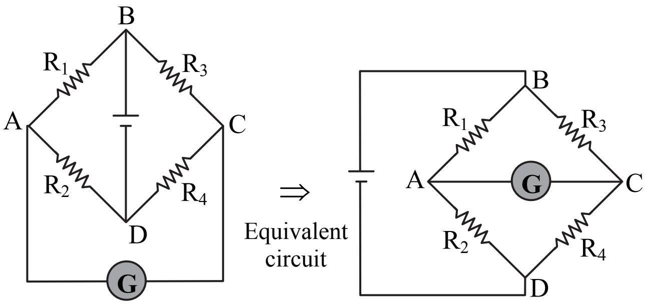


Figure (b)

From figure (b), null point is obtained when,

$\frac{R_1}{R_3} = \frac{R_2}{R_4}$

i.e., $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ ….(ii)

∴ From equations (i) and (ii), we can say that null point is not disturbed when galvanometer and cell are interchanged.

64. Maximum resistance is obtained when resistors are connected in series.

$R_s = 10 \times (2)$

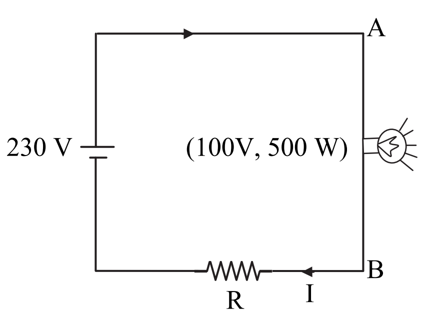
$R_s = 20\ \Omega$

Minimum resistance is obtained when resistors are connected in parallel.

$R_p = \frac{2}{10}\ \Omega$

∴ $\frac{R_s}{R_p} = \frac{20}{2/10} = 100$

65.



Power P = IV

∴ $I = \frac{P}{V} = \frac{500}{100} = 5A$

By Kirchhoff's law,

$230\ V = IR + V_{AB}$

∴ $230\ V = IR + 100\ V$

∴ $IR = 130\ V$

∴ $R = \frac{130}{5}\Omega = 26\ \Omega$

66. Voltage across 10 Ω = voltage across 40 Ω

∴ $I_1 (10) = I_2 (40)$

∴ $I_2 = \frac{2.5 \times 10}{40} = 0.625\ A$

∴ $R = \frac{50}{I_1 + I_2} = \frac{50 - 25}{2.5 + 0.625} = 8\ \Omega$

67. Resistance between P and Q,

$R_{PQ} = R \parallel \left(\frac{R}{3} + \frac{R}{2}\right) = \frac{R \times \frac{5}{6}R}{\left(R + \frac{5}{6}R\right)} = \frac{5}{11}R$

Resistance between Q and R,

$R_{QR} = \frac{R}{2} \parallel \left(R + \frac{R}{3}\right) = \frac{\frac{R}{2} \times \frac{4R}{3}}{\left(\frac{R}{2} + \frac{4R}{3}\right)} = \frac{4}{11}R$

Resistance between P and R,

$R_{PR} = \frac{R}{3} \parallel \left(\frac{R}{2} + R\right) = \frac{\frac{R}{3} \times \frac{3R}{2}}{\left(\frac{R}{3} + \frac{3R}{2}\right)} = \frac{3}{11}R$

Hence it is clear that R_{PQ} is maximum.

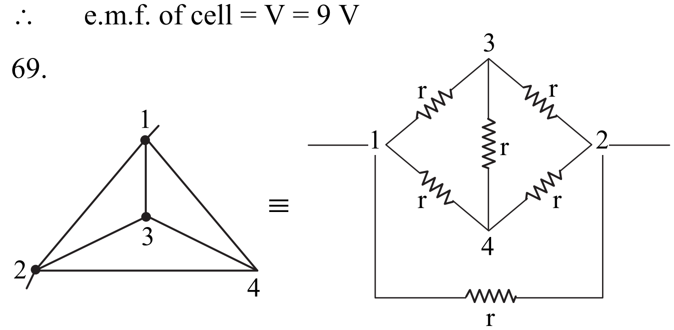
68. $F = qE$

∴ $E = \frac{F}{q}$ $\Rightarrow \frac{F}{q} = \frac{V}{L}$

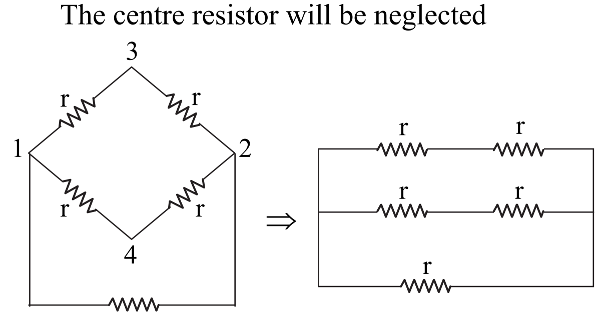
∴ $V = \frac{FL}{q} = \frac{2.4 \times 10^{-19} \times 6}{1.6 \times 10^{-19}} \Rightarrow V = 9\ V$

∴ e.m.f. of cell = V = 9 V

69.



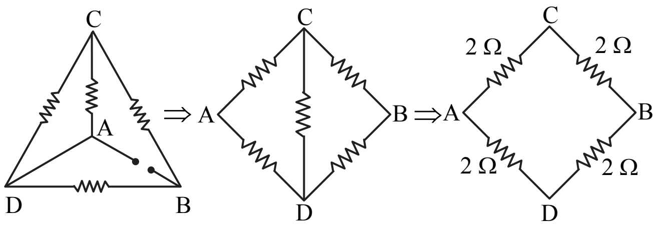
The centre resistor will be neglected



$\therefore \quad \frac{1}{R_p} = \frac{1}{2r} + \frac{1}{2r} + \frac{1}{r}$

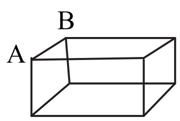
$\therefore \quad R_p = \frac{r}{2}$

70. The equivalent circuits are as shown below



The circuit is a balanced Wheatstone's bridge. Hence effective resistance between A and B
$= 4\ \Omega \parallel 4\ \Omega = 2\ \Omega$

71.



Assuming, x – as an equivalent of the remaining without link

$\frac{7}{12} = \frac{1(x)}{1+x} = \frac{x}{1+x}$

$7(1 + x) = 12x$

$7 + 7x = 12x$

$7 = 5x$

$x = \frac{7}{5}\Omega$

72. The given network is a balanced Wheatstone bridge. Its equivalent resistance will be

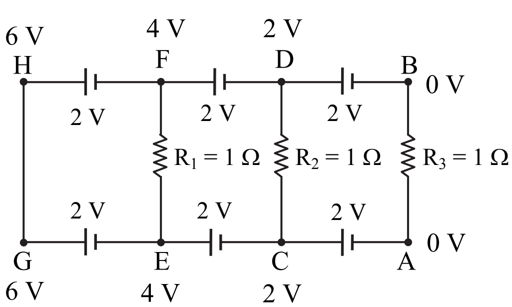
$R = \frac{18}{5}\ \Omega$

$\therefore \quad i = \frac{V}{R} = \frac{V}{18/5} = \frac{5V}{18}$

73. Since the current coming out from the positive terminal is equal to the current entering the negative terminal, the current in the respective loop will remain confined to the loop.

$\therefore$ current through 2 Ω resistor is zero

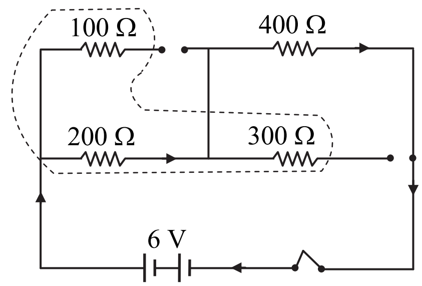
74.



Consider potential at points A and B be zero. Hence, potential at points C and D will be 2 V. Similarly, potential at E and F is 4 V.
This implies, potential drop across each resistor R_1, R_2 and R_3 is zero.

$\therefore$ current through each resistor is zero.

75. Equivalent circuit is given by



Capacitors behave as infinite resistance in steady state

$I_{steady} = \frac{\text{Voltage}}{\text{Resistance}} = \frac{6}{(100 + 200 + 300)}$

$= \frac{6}{600} = \frac{1}{100}\text{ A} = 10\text{ mA}$

76. Current from D to C = 1 A

$\therefore \quad V_D - V_C = 2 \times 1 = 2$ V

$V_A = 0 \Rightarrow V_C = 1$ V,

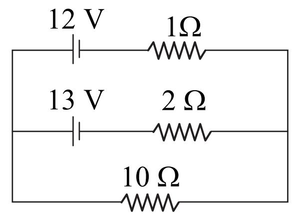
$\therefore \quad V_D - V_C = 2$

$\therefore \quad V_D - 1 = 2 \Rightarrow V_D = 3$ V

$\therefore \quad V_D - V_B = 2$

$\Rightarrow 3 - V_B = 2 \Rightarrow V_B = 1$ V

77.



$E_{eq} = \frac{E_1r_2 + E_2r_1}{r_1 + r_2} = \frac{(12\times2)+(13\times1)}{1+2} = \frac{37}{3}\text{V}$

Also, $r_{eq} = \frac{r_1r_2}{r_1 + r_2} = \frac{1\times2}{1+2} = \frac{2}{3}\Omega$

Current in the circuit will be,

$I = \frac{E_{eq}}{R + r_{eq}} = \frac{\frac{37}{3}}{10 + \frac{2}{3}} = \frac{37}{32}$ A

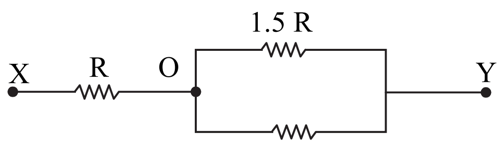
The voltage across the load,

$V = IR = \frac{37}{32} \times 10 = 11.56$ V

78. With increase in temperature, the value of unknown resistance will increase.
For balanced Wheatstone bridge condition,
$\frac{R}{X} = \frac{l_1}{l_2}$
To take null point at same point or $\frac{l_1}{l_2}$ to remain unchanged, $\frac{R}{X}$ should also remain unchanged. Therefore, if X is increasing R should also increase.

79. $l_1 = 52 + 1 = 53$ cm, $l_2 = 48 + 2 = 50$ cm
As the bridge is balanced,
$\frac{l_1}{l_2} = \frac{X}{R} = \frac{53}{50} = \frac{X}{10}$
$\Rightarrow X = 10.6\ \Omega$

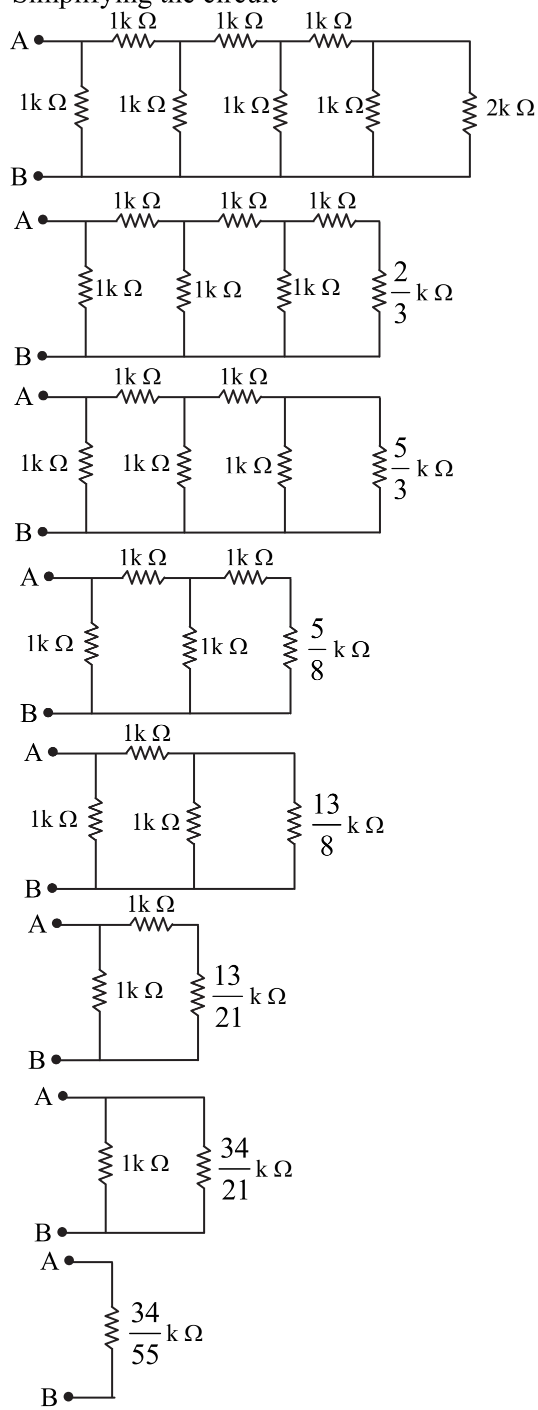
80. As I is independent of R_6 , no current flows through R_6. This implies that the junction of R_1 and R_2 is at the same potential as the junction of R_3 and R_4. This must satisfy the condition $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, as in the Wheatstone's bridge.

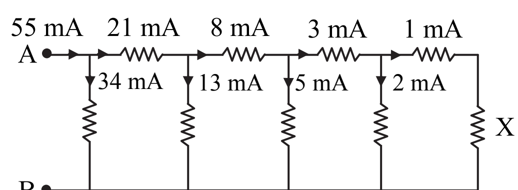
81.


$R_{OY} = \frac{1.5R \times 3R}{(1.5+3)R} = R$
$R_{XO} = R_{OY} = R$
$\Rightarrow V_{XO} = V_{OY}$ $\Rightarrow V_A = V_B = V_C$

82. $I = \frac{3}{6\times10^3} = 0.5\times10^{-3}$
$V_{AD} = IR = 0.5 \times 10^{-3} \times 3 \times 10^3 = 1.5$ V
$Q = \frac{2\times1.5}{3} = 2 \times 0.5 = 1\ \mu C$
Applying KVL from B to C,
$V_B - 0.5 \times 10^{-3} \times 2 \times 10^3 + \frac{1}{2} = V_C$
$V_B - V_C = 1 - \frac{1}{2} = 0.5$ V

83. $I = \frac{110}{20\times10^3 + R}$ ….(i)
Now, V = IR
$\therefore$ $5 = \left(\frac{110}{20\times10^3 + R}\right) \times 20 \times 10^3$ ….[From (i)]
$\therefore$ $10^5 + 5R = 22 \times 10^5$
$\therefore$ $R = 21 \times \frac{10^5}{5} = 420\ k\Omega$

84. Simplifying the circuit


$\therefore$


$\therefore$ $V_{AB} = 55 \times 10^{-3} \times \frac{34}{55} \times 10^3 = 34$ V

Evaluation Test

1. The given circuit is a balanced Wheatstone's network as shown in figure (ii). Hence, points Q and S are at the same potential
$\Rightarrow V_Q - V_S = 0$ V

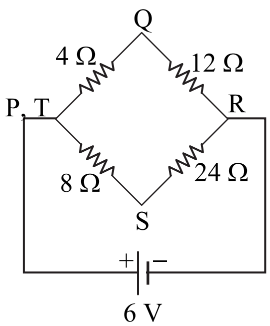
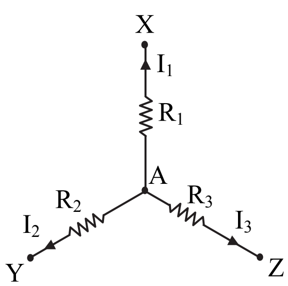


Figure (ii)

2. Applying Kirchoff's junction rule to point A, (see figure)
$-I_1 - I_2 - I_3 = 0$
$\Rightarrow I_1 + I_2 + I_3 = 0$ ….(i)
If V_A is the potential at A, by applying Ohm's law to R_1, R_2 and R_3 then we get,
$V_A - V_1 = I_1 R_1$,
$V_A - V_2 = I_2 R_2$ and
$V_A - V_3 = I_3 R_3$



$\therefore$ $I_1 = \dfrac{V_A - V_1}{R_1}$,

$I_2 = \dfrac{V_A - V_2}{R_2}$,

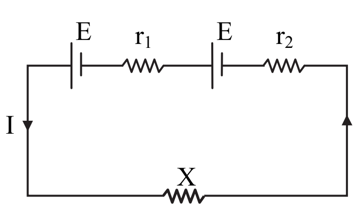
$I_3 = \dfrac{V_A - V_3}{R_3}$

Substituting for I_1, I_2 and I_3 in equation (i) we get,

$$V_A\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\left[\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right]=0$$

$$\Rightarrow V_A = \left[\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right]\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]^{-1}$$

3.



From the figure,

$$I = \frac{E+E}{r_1+r_2+X} = \frac{2E}{r_1+r_2+X} \quad ….(i)$$

P.D. across first cell, $V_1 = E - Ir_1$

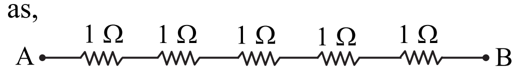
$$= E - \frac{2E}{r_1+r_2+X} r_1$$

Given that, $V_1 = 0$

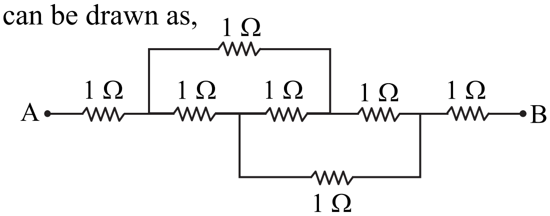
$\therefore$ $E = \dfrac{2Er_1}{r_1+r_2+X}$

$\Rightarrow X + r_1 + r_2 = 2r_1$ or $X = r_1 - r_2$

4. The circuit for the dashed lines can be drawn as,



$\therefore$ $R_{eq} = 5 \times 1 = 5\ \Omega$
The circuit obtained by adding dashed lines can be drawn as,



R'_{eq} for this combination after simplifying the circuit,
$R'_{eq} = 3\ \Omega$

$\therefore$ Difference in the final and initial values of R_{eq} is 2 Ω.

5.

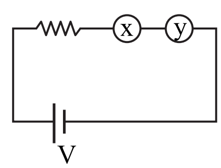


Figure (i)

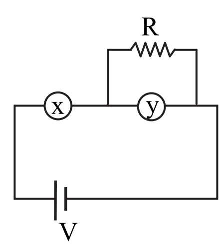


Figure (ii)

Equaivalent resistance decreases. Hence current will increase
$\therefore$ $V_x + V_y = V$
Due to the change, V_x increases
$\Rightarrow$ voltmeter reading will decrease.

6. P = 60 W, h = 12 m, V = 100 litre, η = 80%

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P}{\left(\frac{w}{t}\right)}$$

$\therefore$ $\dfrac{mgh}{t} \times \eta = P$

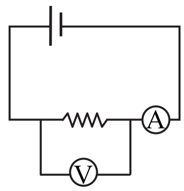
$\therefore \quad \frac{m}{t} = \frac{60}{10 \times 12 \times 0.8} = 0.625 \approx 0.63$ kg/s

$\therefore \quad t = \frac{m}{0.63}$

$= \frac{100 \times 10^3 \times 10^{-3}}{0.63} \approx 159$ s

7. $i = 40$ mA

$= 40 \times 10^{-3}$ A



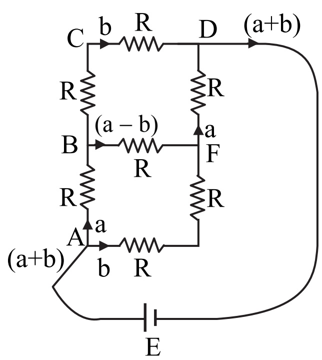
Using, $I = \frac{E}{R_{net} + r}$

$40 \times 10^{-3} = \frac{3}{2 + 2 + \frac{100 R_V}{100 + R_V}}$

$\Rightarrow 4 + \frac{100 R_V}{100 + R_V} = \frac{3}{0.04} = 75$

$\Rightarrow R_V \approx 245\ \Omega$

8. Let the currents through various branches be as shown



Applying Kirchoff's voltage law in loop ABCDEA and loop ABFA we get,

$E - aR - 2Rb = 0$ ….(i)

$- aR - (a - b)R + 2Rb = 0$ ….(ii)

$2aR = 3Rb \Rightarrow 2a = 3b$

$E = R \times \frac{3b}{2} + 2Rb = (a + b) R_{eq}$

$\frac{7Rb}{2} = \left(\frac{3b}{2} + b\right) R_{eq}$

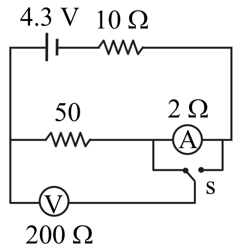
$\Rightarrow \frac{7Rb}{2} = \frac{5b}{2} R_{eq}$

$R_{eq} = \frac{7R}{5}$

Entering current, $(a + b) = \frac{3b}{2} + b = \frac{5b}{2} = I$

Current in common side, $(a - b) = \frac{b}{2} = \frac{I}{5}$.

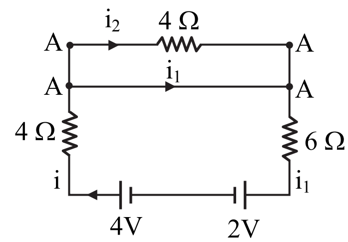
9.



$R_{eq} = 10 + \frac{52 \times 200}{252} = \frac{2520 + 10400}{252} = 51.269\ \Omega$

$\therefore \quad I = \frac{4.3}{51.269} = 0.08$ A

10.



Potential difference across upper 4 Ω resistance is zero

$\therefore$ current is zero $\Rightarrow i_2 = 0$

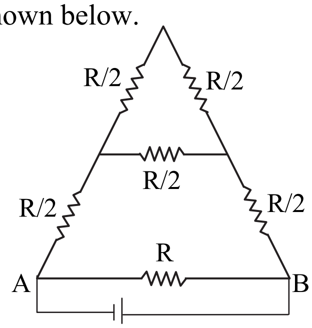
Other two resistors are in series combination.

Hence current is same.

$= \frac{4 - 2}{4 + 6} = 0.2$ A

$\therefore \quad i = i_1 = 0.2$ A, $i_2 = 0$

11. From symmetry of network, it follows that the circuit can be replaced by an equivalent one as shown below.



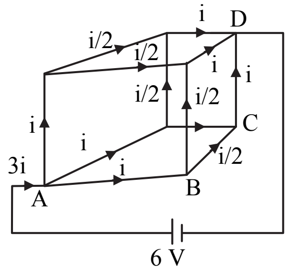
We replace the inner triangle consisting of an infinite number of elements by a resistor of resistance $R_{AB}/2$, where the resistance $R_{AB} = R_x$ and $R_{AB} = A\rho$. After simplification, the circuit becomes a system of conductors connected in series and parallel. In order to find R_x, we write the equation,

$R_x = R\left(R + \frac{RR_x/2}{R + R_x/2}\right) \div \left(R + R + \frac{RR_x/2}{R + R_x/2}\right)$

Solving the equation, we obtain

$R_{AB} = R_x = \frac{R(\sqrt{7} - 1)}{3} = \frac{A\rho(\sqrt{7} - 1)}{3}$

13. Using symmetry and junction rule, we can arrange the currents as shown. Applying loop rule along ABCD and battery to A, we get



$$-iR - \frac{i}{2}R - iR + 6 = 0$$

$$6 = \frac{5iR}{2} = \frac{5i}{2}\times 2 \text{ or } I = \frac{6}{5} = 1.2 \text{ A}$$

....[$\because$ R = 2 Ω]

Current through the battery, 3i = 3.6 A

14. $1.5 \text{ V} = k.l_1 = k(76.3)$ ….(i)

$E - ir = i(9.5\ \Omega) = kl_2$

$\therefore$ $i = \frac{E}{9.5+r} = \frac{1.5}{9.5+r}$

$\frac{(1.5)}{9.5+r}(9.5) = kl_2$ ….(ii)

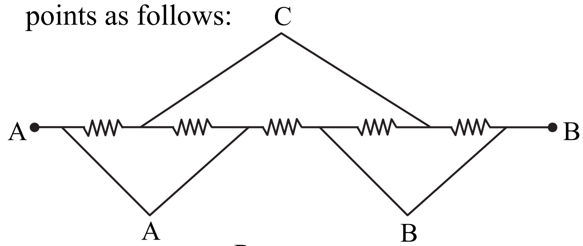
Dividing (ii) by (i), we get,

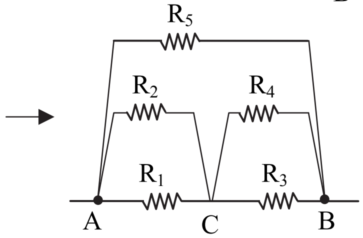
$$\frac{9.5}{9.5+r} = \frac{l_2}{l_1} = \frac{64.8 \text{ cm}}{76.3 \text{ cm}} \Rightarrow \frac{9.5+r}{9.5} = \frac{76.3}{64.8}$$

$\therefore$ $\frac{r}{9.5} = \left(\frac{76.3}{64.8} - 1\right)$

$\therefore$ $r = \left(\frac{76.3}{64.8} - 1\right)(9.5)\Omega = 1.7\ \Omega$

15. We relabel the circuit in terms of potential points as follows:





$R_1 = R_2 = R_3 = R_4 = R_5 = 2\ \Omega$

$\therefore$ $R_{eq} = 1\ \Omega$

16. $i_{AD} = i_{DB} + i_{DC}$

Let potential at D be V

$$\frac{(7-V)}{10} = \frac{(V-0)}{20} + \frac{(V-1)}{30}$$

On solving the above equation, we get $V_D \approx 4$V

Hence option (A) is correct.

Currents through the sections DB and DC are,

$\frac{7-4}{10} = 0.3$ A,

$\frac{4}{20} = 0.2$ A,

$\frac{4-1}{30} = 0.1$ A

Hence option (B) is correct

Total power drawn $= (0.3)^2 \times 10 + (0.2)^2 \times 20 + (0.1)^2 \times 30$

$= 0.9 + 0.8 + 0.3$

$= 2.00$ W

Hence option (D) is correct.

So, only incorrect option is (C)

17. When the diameter of wire AB is increased, its resistance will decrease. Therefore, the potential difference between A and B due to battery B_1 will decrease. So, the null point will be obtained at a smaller value of x.

18. Here for the minor arc AB,

$$R_{AB} = \frac{R}{2\pi r}\times(r\beta) = \frac{R\beta}{2\pi}$$

$\left(\because \beta = \frac{l}{r}\right)$

and for the major arc,

$$R_{AB} = \frac{R}{2\pi r}\times r\,(2\pi - \beta)$$

$$= \frac{R}{2\pi}(2\pi - \beta)$$

$\therefore$ $$R_{eq} = \frac{R_{AB(minor)}\,R_{AB(major)}}{\left(R_{AB(minor)} + R_{AB(major)}\right)}$$

$$= \frac{\frac{R\beta}{2\pi}\times\frac{R}{2\pi}(2\pi-\beta)}{\frac{R\beta}{2\pi} + \frac{R(2\pi-\beta)}{2\pi}}$$

$$= \frac{R\beta}{4\pi^2}(2\pi - \beta)$$

19. Let R be the resistance of each resistor.
Since these three resistors are in parallel so their equivalent resistance is R/3.
Current in circuit,

$$I = \frac{E}{R_1 + r} = \frac{2}{R/3 + 0.2}$$

Heat produced,

$$H = I^2 \frac{R}{3}$$

$$= \frac{R}{3}\left(\frac{4}{\left(0.2 + \frac{R}{3}\right)^2}\right) \quad ....(i)$$

For maximum heat, $\frac{dH}{dR} = 0$

$\therefore \quad \frac{4}{3}\left(\frac{1}{(0.2 + R/3)^2} - \frac{2R}{(0.2 + R/3)^3} \times \frac{1}{3}\right) = 0$

$\therefore \quad 3\left(0.2 + \frac{R}{3}\right) = 2R$

or $R = 0.6\ \Omega$

20. In the given network, points x and P will be equipotential, when effective resistance across YP is equal to resistance across WP

$\therefore \quad \frac{(1+r)(1)}{(1+r)+1} = 1 - r$

(where r is resistance of ZP and (1 – r) is resistance or YP)

$\therefore \quad r = \sqrt{2} - 1$

Then $(1 - X) = 1 - (\sqrt{2} - 1)$

$= \sqrt{2} - (\sqrt{2} - 1)$

$\therefore \quad \frac{YP}{PZ} = \frac{\sqrt{2}}{1}$

Textbook Chapter No.

14 Magnetic Effect of Electric Current

Hints

Classical Thinking

2. B represents the magnetic field.

6. From Ampere's circuital law, $\oint_c \vec{B}.\ \vec{dl} = \mu_0 I$

   where I is the current in the closed path.

7. $B = \mu_0 n I \Rightarrow$ B does not depend upon radius

9. Magnetic field due to solenoid is independent of diameter ($\because B = \mu_0 nI$).

15. $\tau = NBIA = 100 \times 0.2 \times 2 \times (0.08 \times 0.1) = 0.32\,N\,m$

    Direction is given by Fleming's left hand rule.

21. The resistance of an ideal voltmeter is considered as infinite so that it does not change the current in the circuit.

24. The voltmeter is a high resistance galvanometer.

29. For motion of a charged particle in a magnetic field, we have $r = \frac{mv}{qB}$ i.e. $r \propto v$

33. Particles are entering perpendicularly. Hence, they will describe circular path. Since their masses are different, they will describe paths of different radii.

34. $r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

    where, K = K.E. of the charged particle.

    $\Rightarrow r \propto \sqrt{\frac{m}{q}}$

    $\therefore \frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2\left(\frac{q_1}{q_2}\right) = \left(\frac{R}{R/2}\right)^2\left(\frac{q}{4q}\right)$

    $\therefore \frac{m_1}{m_2} = 1$

Critical Thinking

1. $\vec{F} = I\vec{l} \times \vec{B}$

   $\therefore F = IlB \sin\theta$

   $\therefore F = 1.6 \times 0.5 \times 2 \times \sin(90°) = 1.6$ N

2. $F = BIl\sin\theta$

   $\therefore \sin\theta = \frac{F}{BIl} = \frac{15}{2\times10\times1.5} = \frac{1}{2}$

   $\therefore \theta = 30°$

3. $\vec{F} = I\vec{l} \times \vec{B} = IlB\sin\theta$

   $\therefore F = 0$ when $\sin\theta = 0 \Rightarrow \theta = 0$

4. $B = \frac{\mu_0}{4\pi}\frac{2I}{r}$

   New distance = $\frac{r}{2}$

   $\therefore$ New magnetic field = $\frac{\mu_0}{4\pi}\frac{2I}{\left(\frac{r}{2}\right)} = 2B$

5. Using, $B = \frac{\mu_0}{4\pi}\frac{2I}{r}$,

   $B = 10^{-7} \times \frac{2I}{r} = 10^{-7} \times \frac{2\times2}{5} = 8 \times 10^{-8}$ T

6. $B_1 = 10^{-3}$ T, $x_1 = 4$ cm $= 4 \times 10^{-2}$ m,

   $x_2 = 12$ cm $= 12 \times 10^{-2}$ m

   $B_1 = \frac{\mu_0}{4\pi}.\frac{2I}{x_1}$

   $\therefore 10^{-3} = 10^{-7} \times \frac{2I}{4\times10^{-2}}$

   $\therefore I = 2 \times 10^2$ A = 200 A

   $B_2 = \frac{\mu_0}{4\pi}.\frac{2I}{x_2} = 10^{-7} \times \frac{2\times200}{12\times10^{-2}} = 3.33 \times 10^{-4}$ T

7. $B = \mu_0 nI = \frac{\mu_0 nI}{2\pi r} = 4\pi \times 10^{-7} \times \frac{3000 \times 5}{2\pi \times 12}$

   $= 2.5 \times 10^{-4}$ T

8. $B = \mu_0 nI = 4\pi \times 10^{-7} \times 10 \times 5 = 2\pi \times 10^{-5}$ T

9. $B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r} = 10^{-7} \times \frac{2\pi nI}{r}$

   $= \frac{10^{-7}\times2\pi\times250\times(20\times10^{-3})}{(40\times10^{-3})}$

   $= 7.85 \times 10^{-5}$ T $\approx 7.9 \times 10^{-5}$ T

10. $B = \frac{\mu_0 n_1 I_1}{2r_1} + \frac{\mu_0 n_2 I_2}{2r_2}$

$= \frac{\mu_0}{2}\left[\frac{5\times0.20}{0.20} + \frac{5\times0.30}{0.30}\right] = 5\mu_0$

11. $B_A = \frac{\mu_0}{4\pi}\times\frac{2\pi I}{R}$

$\therefore$ $B_B = \frac{\mu_0}{4\pi}\times\frac{2\pi(2I)}{2R} = \frac{\mu_0}{4\pi}\times\frac{2\pi I}{R}$

$\therefore$ $\frac{B_A}{B_B} = \frac{1}{1}$

12. Magnetic field at the centre of coil $B = \frac{\mu_0 nI}{2R}$

$n = 1$ and $2\pi R = L \Rightarrow R = \frac{L}{2\pi}$

$\therefore$ $B = \frac{\mu_0 2\pi I}{2L} = \frac{\pi\mu_0 I}{L}$

13. $B_1 = \frac{\mu_0}{4\pi}\frac{I\theta}{a}$, $B_2 = \frac{\mu_0}{4\pi}\frac{I\theta}{b}$

$\therefore$ Field due of ABCD $= B_1 - B_2 = \frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

14. $B = 0.4\times10^{-4}$ T $= 4\times10^{-5}$ T

Using $B = \frac{\mu_0 nI}{2r}$ we get,

$n = \frac{2Br}{\mu_0 I} = \frac{2\times4\times10^{-5}\times200\times10^{-3}}{4\times3.14\times10^{-7}\times0.25}$

$\therefore$ $n = 50.9 \approx 51$

15. $B = \frac{\mu_0}{4\pi}\frac{(2\pi-\theta)I}{r} = \frac{\mu_0}{4\pi}\frac{\left(2\pi-\frac{\pi}{2}\right)\times I}{r}$

$\therefore$ $B = \frac{3\mu_0 I}{8r}$

16. Using, $B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi nI}{r}$,

$B = 10^{-7}\times\frac{2\pi nI}{r} = 10^{-7}\times\frac{2\times\pi\times25\times4}{5\times10^{-2}}$

$\therefore$ $B = 1.256\times10^{-3}$ T

17. $r_1 : r_2 = 1 : 2$ and $B_1 : B_2 = 1 : 3$

$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi nI}{r} \Rightarrow I \propto Br$

$\therefore$ $\frac{I_1}{I_2} = \frac{B_1 r_1}{B_2 r_2} = \frac{1\times1}{3\times2} = \frac{1}{6}$

18. $B = \frac{\mu_0 NI}{l}$

$\therefore$ $0.2 = \frac{4\pi\times10^{-7}\times N\times10}{0.8}$

$\therefore$ $N = \frac{4\times10^4}{\pi}$

Since N turns are made from the winding wire, so length of the wire (L)

$= 2\pi r\times N$ [$2\pi r$ = length of each turns]

$\therefore$ $L = 2\pi\times3\times10^{-2}\times\frac{4\times10^4}{\pi}$

$= 2.4\times10^3$ m

19. $d = 9$ cm $= 9\times10^{-2}$ m

$\therefore$ $A = \pi r^2 = \pi(4.5\times10^{-2})^2$

$\therefore$ $B = nIA = 30\times1\times\pi\times(4.5\times10^{-2})^2$

$\therefore$ $B = 19.08\times10^{-2}$ Am^2

20. It oscillates with the decreasing amplitude as current is passed. Coil oscillates but current is momentary (it is for small time) and current decreases and becomes zero. So, oscillation of the coil is of decreasing amplitude.

21. $B = \frac{\tau}{InA} = \frac{5}{5\times100\times50\times10^{-4}} = 2$ T

22. Field is radial (plane of coil parallel to magnetic field)

$\therefore$ $\tau = nIAB$

$= 100\times100\times10^{-6}\times(5\times2\times10^{-4})\times0.1$

$= 10^{-6}$ N m

23. $\tau = nIAB\cos60° = nIAB\sin(90° - 60°)$

$= 500\times0.2\times4\times10^{-4}\times10^{-3}\times\frac{1}{2}$

$\therefore$ $\tau = 2\times10^{-5}$ N-m

24. $I = \frac{C\theta}{nAB} = \frac{5\times10^{-7}\times45}{200\times0.02\times0.08\times0.2}$

$\therefore$ $I = 3.5\times10^{-4}$ A

25. $B = 80$ gauss $= 80\times10^{-4}$ tesla

For equilibrium of coil,

$nBIA = C\theta$

$\therefore$ $C = \frac{nBIA}{\theta}$

$= \frac{40\times80\times10^{-4}\times0.2\times10^{-3}\times5\times10^{-4}}{20}$

$= 1.6\times10^{-9}$ Nm/degree

26. $C = \frac{IAB}{\theta} = \frac{2\times10^{-5}}{10} = 2 \times 10^{-6}$ Nm/deg

27. $C\theta = nIAB$

$\therefore\ I = \frac{C\theta}{nAB} = \frac{1.5\times10^{-9}\times10}{100\times15\times10^{-4}\times0.025}$

$= 4 \times 10^{-6}$ A = 4 μA

28. The coils are placed perpendicular to each other. The magnetic field due to current through each at the centre is B.

Then resultant magnetic field due to current through both the coils will be

$= \sqrt{B^2+B^2} = \sqrt{2}\ B$

$\therefore$ The ratio $= \frac{\sqrt{2}B}{B} = \frac{\sqrt{2}}{1}$

29. $n = \frac{60}{30} = 2$

Now, $S = \frac{G}{n-1} = \frac{G}{2-1}$

$\therefore\ S = \frac{G}{1} \Rightarrow S = G = 12\ \Omega$

30. $S = 12\ \Omega = \frac{G}{n-1},\ n = \frac{50}{10} = 5$

$\therefore\ S = \frac{G}{n-1} = \frac{G}{5-1} = \frac{G}{4}$

$\therefore\ G = 4S = 4 \times 12 = 48\ \Omega$

31. $\frac{I_g}{I} = \frac{1}{34} = \frac{S}{S+3663}$

$\therefore\ S = \frac{3663}{33} = 111\ \Omega$

32. $I_g = \frac{S}{S+G}.I \qquad \therefore\ 2 = \frac{S}{S+12}\times 5$

$\therefore$ S = 8 Ω in parallel

33. $S = \frac{I_g G}{I-I_g} = \frac{15\times10^{-3}\times5}{1.5-15\times10^{-3}} = 0.0505\ \Omega$

34. $\frac{I_G}{I} = \frac{S}{S+G} = \frac{2.5}{2.5+25} = \frac{2.5}{27.5} = \frac{25}{275} = \frac{1}{11}$

35. $I_g = 5.4 \times 10^{-6}$ A,

$\frac{I_g}{I} = \frac{S}{S+G}$

$\therefore\ I = I_g\left(\frac{S+G}{S}\right) = 5.4 \times 10^{-6} \times \left(\frac{1+30}{1}\right)$

$= 5.4 \times 10^{-6} \times 31 = 1.67 \times 10^{-4}$ A

36. $\frac{I_g}{I} = \frac{S}{S+G} = \frac{4}{40}$

$\therefore\ \frac{I_g}{I} \times 100 = 10\ \%$

37. $I_g = \left(\frac{S}{S+G}\right)I$

$\frac{10}{100}I = \left(\frac{S}{S+G}\right)I$

$\therefore\ \frac{1}{10} = \left(\frac{10}{10+G}\right)$

$\therefore\ 10 + G = 100$

$\therefore\ G = 90\ \Omega$

38. Shunt resistances $S = \frac{I_g G}{(I-I_g)} = \frac{10\times99}{(100-10)} = 11\ \Omega$

39. $I_g = 10 \times 10^{-6}$ A

Using, $S = \frac{I_g}{I-I_g}G$

$= \frac{10\times10^{-6}\times1000}{1-10\times10^{-6}} \approx 10^{-2}\ \Omega$

$= 0.01\ \Omega$

$\therefore$ S = 0.01 Ω is parallel

40. $(R_{eff} = 30 \parallel 30 = 15\ \Omega = G)$

$\therefore\ S = \frac{I_g G}{I-I_g} = \frac{I_g(15)}{2I_g - I_g} = 15\ \Omega \quad ....[\because I = 2I_g]$

41. Fraction of current passing through the galvanometer is $\frac{I_g}{I}$

$= \frac{S}{S+G} = \frac{10}{10+90} = \frac{10}{100} = \frac{1}{10}$

Fraction of current passing through shunt is

$\frac{I_s}{I} = 1 - \frac{I_g}{I} = 1 - \frac{1}{10} = \frac{9}{10}$

42. $G = 6000 \times 3 = 18000\ \Omega = 18$ kΩ

Using, $I_g = \frac{V}{G} = \frac{3}{18\times10^3} = \frac{1}{6}\times10^{-3}$ A,

$\therefore$ Value of series resistance $R = \frac{V}{I_g} - G$

$= \frac{12}{\left(\frac{1}{6}\times10^{-3}\right)} - 18\times10^3$

$= 72 \times 10^3 - 18 \times 10^3$

$= 54 \times 10^3 = 5.4 \times 10^4\ \Omega$

43. For the actual measurement of potential difference, it is necessary that the current between two points of the conductor should remain the same after putting the measuring device across two points. This is the case when resistance of device is very high (i.e., infinite).

44. $R_S = \frac{V}{I_g} - G = \frac{10}{0.25\times10^{-3}} - 40 = 39960\ \Omega$

45. $I_g = 5 \times 10^{-3}$ A

Using, $R = \frac{V}{I_g} - G$

$\therefore$ $3960 = \frac{20}{5\times10^{-3}} - G$

$\therefore$ $G = 4000 - 3960 = 40\ \Omega$

46. $R = \frac{V}{I_g} - G$

$\therefore$ $0 = \frac{V}{3\times10^{-3}} - 100$

$\therefore$ $V = 100 \times 3 \times 10^{-3} = 0.3$ V

47. $I_g = \frac{V}{R} = \frac{3}{200} = 15$ mA

In (A), 10 mA < 15 mA $\Rightarrow I < I_g$

$\therefore$ $I \neq 10$ mA

48. The current through the galvanometer

$= \frac{3}{2950+50}$

$= 10^{-3}$ A

$\therefore$ To reduce the deflection from 30 divisions to 20 divisions, the current required

$= \frac{20}{30} \times 10^{-3} = \frac{2}{3} \times 10^{-3}$ A

$\therefore$ The required resistance, $R = \frac{3}{R+50} = \frac{2}{3} \times 10^{-3}$

$\therefore$ $R + 50 = \frac{3\times3}{2} \times 10^{3}$

$\therefore$ $R + 50 = 4.5 \times 10^{3}$

$\therefore$ $R = 4500 - 50 = 4450\ \Omega$

49. A voltmeter always has high resistance as R is in series.

To increase the range of ammeter i.e. to increase I, its resistance must decrease.

$\therefore$ High range $\Rightarrow$ low resistance.

50. $V_1 = 80$ volt,

$R_1 = 200 \times 80 = 16000\ \Omega = 16\ k\Omega$,

$I_1 = \frac{V_1}{R_1} = \frac{80}{16000} = 5 \times 10^{-3}$ A

Current in series connection of voltmeter remains constant.

$\therefore$ $I_2 = 5 \times 10^{-3}$ A, $R_2 = 32 \times 10^{3}\ \Omega$,

$V_2 = I_2R_2 = 5 \times 10^{-3} \times 32 \times 10^{3} = 160$ V

$\therefore$ Line voltage $= V_1 + V_2 = 80 + 160 = 240$ V

51. $S_I = \frac{d\theta}{dI}$, $S_v = \frac{d\theta}{dV}$

$\therefore$ $\frac{S_I}{S_v} = \frac{dV}{dI} = G \Rightarrow S_v = \frac{S_I}{G}$

52. $\frac{I_g}{I} = \frac{1}{5}$

$\therefore$ $\frac{S}{S+G} = \frac{I_g}{I} = \frac{1}{5}$

$\therefore$ $5S = S + G$

$\therefore$ $4S = 20 \Rightarrow S = 5\ \Omega$

53. Voltage sensitivity $= \frac{\text{Current sensitivity}}{G}$

$\therefore$ 2 div. per mV $= \frac{S_i}{5}$

$\therefore$ $S_i = 10$ div. per mA

54. $I = \frac{V}{R} = \frac{2}{20\ k\Omega} = 0.1$ mA $= 100\ \mu$A

$\therefore$ Sensitivity , $S = \frac{d\theta}{dI} = \frac{50}{100} = \frac{1}{2}$ div/μA

55. Current sensitivity (S)

$= \frac{d\theta}{dI} = \frac{1}{K} = \frac{nAB}{C}$

$\therefore$ $S \propto n$

$\therefore$ $\frac{S'}{S} = \frac{n'}{n}$

$\therefore$ $n' = \frac{125}{100} \times 28 = 35$

56. Current sensitivity $= \frac{d\theta}{dI} = \frac{nAB}{C}$

$= \frac{80\times5\times10^{-4}\times5}{10^{-8}} = 20 \times 10^{6}$ rad/A

$= 20$ rad/μA

57. 5 div = 1 mA

$\therefore$ 30 div = 6 mA $= 6 \times 10^{-3}$ A,

1 div = 1 mV

$\therefore$ 30 div = 30 mV $= 30 \times 10^{-3}$ V

Now, $G = \frac{V}{I_g} = \frac{30\times10^{-3}}{6\times10^{-3}} = 5\ \Omega$

Also, since $\frac{S}{S+G} = \frac{I_g}{I}$

$\therefore \quad \frac{S}{S+5} = \frac{6\times10^{-3}}{6} = 10^{-3}$

$\therefore \quad S = \frac{5\times10^{-3}}{1-10^{-3}} = \frac{5\times10^{-3}}{\left(\frac{1000-1}{1000}\right)} = \frac{5\times10^{-3}\times10^{3}}{999}$

$\therefore \quad S = \frac{5}{999}\,\Omega$

58. $\frac{I_g}{I} = \frac{S}{S+G}$

$\therefore \quad \frac{I_g}{I} = \frac{G/10}{(G/10)+G} = \frac{G/10}{11G/10} = \frac{1}{11}$

Initially, the sensitivity, $\alpha_i = \frac{d\theta}{dI_g}$

Finally, after the shunt is used, $\alpha_f = \frac{d\theta}{dI}$

$\therefore \quad \frac{\alpha_f}{\alpha_i} = \frac{\theta/I}{\theta/I_g} = \frac{I_g}{I} = \frac{1}{11}$

59. Current due to motion of α particle = $\frac{2e}{T}$

$\therefore$ Magnetic moment = $I \times A = \frac{2e}{T}\times \pi r^2$

$= \frac{e(2\pi r)r}{T} = evr$

60. $R = \frac{mv}{eB}$

Now, $v \to 2v$

$\therefore \quad R \to 2R = 2 \times 2$ cm = 4 cm

61. Cyclotron frequency, $f = \frac{qB}{2\pi m}$

where, q = charge of proton

$\therefore \quad f = \frac{1.6\times10^{-19}\times1.4}{\left(2\times\frac{22}{7}\times1.6\times10^{-27}\right)} = \frac{49}{22}\times10^{7}$ Hz

62. $t = 2.3 \times 10^{-8}$ s

$\therefore \quad T = 2t = 4.6 \times 10^{-8}$

$\therefore \quad f = \frac{1}{T} = \frac{1}{4.6\times10^{-8}} = 2.1 \times 10^{7}$ Hz

63. $K.E = \frac{q^2B^2R^2}{2m}$

$= \frac{\left(1.6\times10^{-19}\right)^2\times(0.5)^2\times\left(4\times10^{-1}\right)^2}{2\times1.67\times10^{-27}}$

$= \frac{(1.6)^2\times10^{-38}\times25\times10^{-2}\times16\times10^{-2}}{2\times1.67\times10^{-27}}$

$= \frac{1024\times10^{-42}}{3.34\times10^{-27}} = 306.58\times10^{-15}$

$= 3.06 \times 10^{-13}$ J

$= \frac{3.06\times10^{-13}}{1.6\times10^{-19}}$ eV

$= 1.9 \times 10^{6}$ eV

= 1.9 MeV

64. 10 div = 1 mA and 2 div = 1 mV

$\therefore$ 150 div = 15 mA and 150 div = 75 mV

$\therefore \quad R_o = G = \frac{V}{I} = \frac{75}{15} = 5\,\Omega$

$\therefore \quad \frac{S}{S+G} = \frac{I_g}{I}$

$\therefore \quad \frac{S}{S+5} = \frac{15\times10^{-3}}{6} = \frac{5\times10^{-3}}{2}$

$\therefore \quad 2S = 5\times10^{-3}\,S + 25\times10^{-3}$

$\therefore \quad S = 0.0125\,\Omega$

65. Magnetic field at the centre of circular coil, $(L = 2\pi r_1)$

$B_{circular} = \frac{\mu_0}{4\pi}\frac{2\pi i}{r_1} = \frac{\mu_0}{4\pi}\frac{4\pi^2 i}{L}$ and

Magnetic field at the centre of semi-circular coil, $(L = \pi r_2)$

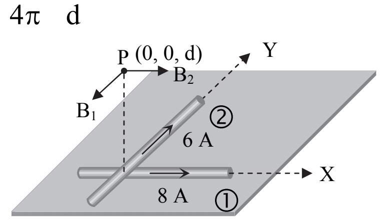
(Using shortcut 5),

$B_{semi-circular} = \frac{\mu_0}{4\pi}\frac{\pi i}{r_2} = \frac{\mu_0}{4\pi}\frac{\pi^2 i}{L}$

$\therefore \quad \frac{B_{circular}}{B_{semi-circular}} = 4$

66. Magnetic field at P due to wire (1),

$B_1 = \frac{\mu_0}{4\pi}\frac{2(8)}{d}$



and that due to wire (2), $B_2 = \frac{\mu_0}{4\pi}\frac{2(6)}{d}$

∴ $B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0}{4\pi}\frac{16}{d}\right)^2 + \left(\frac{\mu_0}{4\pi}\frac{12}{d}\right)^2}$

$= \frac{\mu_0}{4\pi} \times \frac{2}{d} \times 10 = \frac{5\mu_0}{\pi d}$

67. $I_g = \frac{4}{100} I$

Using, $S = \frac{I_g G}{I - I_g}$ we get,

∴ $G = \frac{S(I - I_g)}{I_g} = \frac{S\left(I - \frac{4I}{100}\right)}{\left(\frac{4I}{100}\right)}$

$= \frac{96 IS}{4I} = 24\,S = 24 \times 5 = 120\ \Omega$

68. Potential drop across galvanometer = Potential drop across the shunt

i.e., $I_g G = (I - I_g) S$

$\Rightarrow S = \frac{I_g}{I - I_g} G$

For $I_g = \frac{I}{10}$

$S = \frac{I/10}{(I - I/10)} G = \frac{G}{9}$

Competitive Thinking

1. $F = BIl = 2 \times 1.2 \times 0.5 = 1.2$ N

2. By Ampere's circuital law,

$\oint \vec{B}.\vec{dl} = \mu_0 I_{enclosed} = \mu_0(2-1) = \mu_0$

3. For the same distance, field will remain the same $\left[\because B = \frac{\mu_0}{4\pi}\frac{2I}{r}\right]$

4. Because inside the pipe, I = 0

∴ $B = \frac{\mu_0 I}{2\pi r} = 0$

5. $B = \frac{\mu_0}{4\pi}\frac{2I}{r} \Rightarrow B \propto \frac{1}{r}$

∴ $\frac{B_1}{B_2} = \frac{r_2}{r_1} \Rightarrow \frac{10^{-8}}{B_2} = \frac{12}{4} \Rightarrow B_2 = 3.33 \times 10^{-9}$ tesla

6. $B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{1}{r}$

∴ When r is doubled, B is halved.

7. $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

∴ $5 \times 10^{-5} = \frac{\mu_0}{2\pi} \times \frac{\pi}{r}$

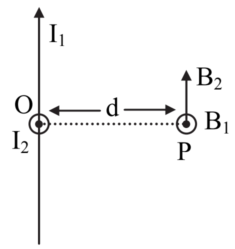
∴ $r = \frac{\mu_0 \times \pi}{5 \times 10^{-5} \times 2\pi}$

∴ $r = 10^4 \mu_0$ metre

8. $B = \frac{\mu_0}{4\pi}\frac{2I}{r} \Rightarrow 10^{-5} = 10^{-7} \times \frac{2I}{(10 \times 10^{-2})}$

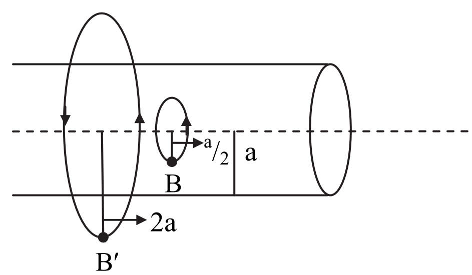
∴ I = 5A

9.



$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)^{1/2}$

10.



According to Ampere's Circuital law,
For inside loop,

$B = \frac{\mu_0 r' I}{2\pi r^2}$ ....$\left(\text{as } I' = \frac{I \times A'}{A}\right)$

∴ $B = \frac{\mu_0 I\left(\frac{a}{2}\right)}{2\pi a^2}$

$B = \frac{\mu_o I}{4\pi a}$ ....(i)

For outside loop,

$B' \times (2\pi r') = \mu_0 I$

∴ $B' = \frac{\mu_0 I}{2\pi(2a)} = \frac{\mu_0 I}{4\pi a}$ ....(ii)

From equations (i) and (ii),

$\frac{B}{B'} = 1$

11. Applying Ampere's law, $\oint B.dl = \mu_0 I$, to any closed path inside the pipe, we find no current is enclosed. Hence, B = 0.

12. $B \propto \frac{1}{r}$

$\therefore \frac{B_1}{B_2} = \frac{r_2}{r_1} = \frac{2r}{r} = 2$

13. the magnetic induction at centre of a coil is

$B = \frac{\mu_0 Ni}{2r}$

$\therefore B_1 = \frac{\mu_0 \times 10 \times 0.2}{2 \times 20 \times 10^{-2}}$, $B_2 = \frac{\mu_0 \times 10 \times -0.3}{2 \times 40 \times 10^{-2}}$

$\therefore B = B_1 + B_2 = \mu_0 \left(5 - \frac{15}{4}\right) = \frac{5}{4}\mu_0$

14. Magnetic field at the centre of a circular loop of radius R carrying current I,

$B = \frac{\mu_0 2\pi I}{4\pi R} = \frac{\mu_0 I}{2R}$ and $M = IA = I(\pi R^2)$

$\therefore \frac{B}{M} = \frac{\mu_0 I}{2R} \times \frac{1}{I\pi R^2} = \frac{\mu_0}{2\pi R^3} = x$ [Given]

When both the current and radius are doubled, the ratio becomes

$\frac{B'}{M'} = \frac{\mu_0}{2\pi(2R)^3} = \frac{1}{8}\left(\frac{\mu_0}{2\pi R^3}\right) = \frac{x}{8}$

15. Let the wire of length l be bent into circle of radius R.

$\therefore B = \frac{\mu_0 nI}{2R}$

here, n = 1

$R = \frac{l}{2\pi}$ $\therefore B = \frac{\mu_0 I}{2\left(\frac{l}{2\pi}\right)}$

$\therefore B = \frac{\mu_0 \pi I}{l}$ ….(i)

When the same wire is bent into coil of n turns, let R′ be the radius of the coil,

$\therefore 2\pi nR' = l$ $\therefore R' = \frac{l}{2\pi n}$

$\therefore B' = \frac{\mu_0 nI}{2R'} = \frac{\mu_0 nI}{2\left(\frac{l}{2\pi n}\right)} = \frac{\mu_0 \pi I}{l} n^2$

$\therefore B' = n^2 B$ ….[From (i)]

16. Magnetic field at the centre of current carrying coil is given by $B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi Ni}{r} \Rightarrow B \propto \frac{N}{r}$

$\therefore \frac{B_1}{B_2} = \frac{N_1}{N_2} \times \frac{r_2}{r_1}$ ….(i)

Given that,

$N_1 = 1, N_2 = 2, r_1 = r, r_2 = r/2,$

$B_1 = B$

$\therefore \frac{B}{B_2} = \frac{1}{2} \times \frac{r/2}{r} = \frac{1}{4} \Rightarrow B_2 = 4B$ ….[From (i)]

Alternate Method:

$B_2 = n^2 B_1 = 2^2 B = 4B$

[*Note: Refer Mindbender 1.*]

17. $B_0 = \frac{\mu_0 NI}{2a} = \frac{\mu_0 I \times 1}{2a} = \frac{\mu_0 I}{2a}$ for 1 turn.

For rewinding the coil in three turns, new radius a/3, number of turns (N′) = 3.

$\therefore$ New magnetic field $= \frac{\mu_0 I \times 3}{2 \times (a/3)} = \frac{9\mu_0 I}{2a} = 9 B_0$

[*Note: Refer Mindbender 1.*]

18. $\frac{\mu_0 I_c}{2R} = \frac{\mu_0 I_e}{2\pi H}$

$\therefore H = \frac{I_e R}{\pi I_c}$

19. The magnetic field in the solenoid along its axis (i) at an internal point $= \mu_0 nI$

$= 4\pi \times 10^{-7} \times 5000 \times 4 = 25.1 \times 10^{-3}$ Wb/m^2

(Here, n = 50 turns/cm = 5000 turns/m)

(ii) at one end

$B_{end} = \frac{1}{2} B_{in} = \frac{\mu_0 nI}{2} = \frac{25.1 \times 10^{-3}}{2}$

$= 12.6 \times 10^{-3}$ Wb/m^2

20. $M = 2000 \times 1.5 \times 10^{-4} \times 2 = 0.6$

$\therefore \tau = MB \sin 30 = 0.6 \times 5 \times 10^{-2} \times \frac{1}{2}$

$\tau = 1.5 \times 10^{-2}$ N.m

21. The proton is moving parallel to the axis of solenoid. The magnetic field inside the solenoid is uniform hence it doesn't affect the velocity of proton.

22. $B = \mu_0 nI$ ....(n = N/L)

$= 4 \times 3.14 \times 10^{-7} \times \frac{400}{0.4 \times 10^{-2}} \times 5$

$= 0.628$ T

24. $I = \frac{C\theta}{nAB} \Rightarrow I \propto \theta$

25. $\tau = NBIA = 100 \times 0.5 \times 1 \times 400 \times 10^{-4} = 2$ N-m

26. $\tau \propto A$

For square coil,

Area (A_1) = length^2 = a^2

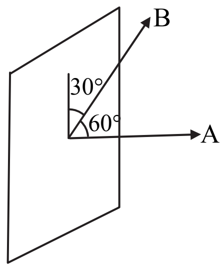
For circular coil,

Area (A_2) = $\pi \times (\text{radius})^2 = \pi\left(\frac{a}{\sqrt{\pi}}\right)^2$

$= \pi \times \frac{a^2}{\pi} = a^2$

$\therefore \quad \frac{\tau_1}{\tau_2} = \frac{A_1}{A_2} = \frac{a^2}{a^2} = 1$

27.



τ = NAIB sin θ
= 50 × .012 × 2 × 0.2 × sin 60°
= 0.20 Nm

29. Assertion is incorrect as shunt is added in parallel. Reason is correct as, to increase range additional shunt is connected across it.

30. $n = \frac{I}{I_g} = \frac{100}{0.2} = 500$

$\therefore \quad R = \frac{G}{n} = \frac{G}{500}$

31. I_g = 10% of I = $\frac{I}{10}$

$\therefore \quad S = \frac{G}{(n-1)} = \frac{90}{(10-1)}$ = 10 Ω in parallel

32. $R_{eff.} = \frac{SG}{S+G} \Rightarrow 25 = \frac{S \times 500}{S+500}$

$\therefore$ 500 S = 25 S + 12500 $\Rightarrow S = \frac{500}{19}\Omega$

33. Resistance of shunted ammeter = $\frac{GS}{G+S}$

Also, $\frac{I}{I_g} = 1 + \frac{G}{S}$

$\therefore \quad \frac{GS}{G+S} = \frac{I_g.G}{I} = \frac{0.05 \times 120}{10} = 0.6\ \Omega$

34. $S = \frac{I_g G}{I - I_g} = \frac{5 \times 10^{-3} \times 10^2}{1 - 5 \times 10^{-3}} = \frac{0.5}{1 - 5 \times 10^{-3}}$

$= \frac{5}{10 - 0.05} = \frac{5}{9.95}\Omega$

35. $S = \frac{I_g G}{I - I_g} = \frac{5 \times 10^{-3} \times 99}{(0.5 - 5 \times 10^{-3})} = 1\ \Omega$

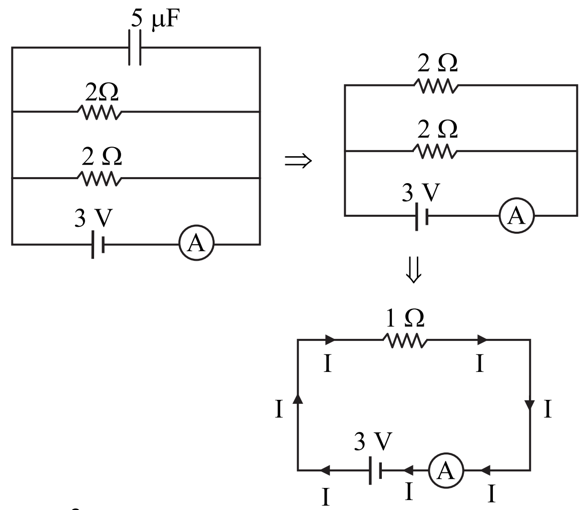
36. To convert an ammeter to range nI, $S = \frac{G}{n-1}$

Here, I = 1 mA = 10^{-3} A
nI = 10 A

$\therefore$ n = 10^4

$\therefore \quad S = \frac{100\Omega}{10^4} = 10^{-2}\Omega = 0.01\ \Omega$

37. We know current through the capacitor will be zero at steady state and ammeter is ideal.



$I = \frac{3}{1}$

I = 3 A

40. Using, $R_s = \frac{V}{I_g} - G$ we get,

for 1st case, $100 = \frac{V}{I_g} - R$ ....(i) and

for 2nd case, $1000 = \frac{2V}{I_g} - R$ ....(ii)

By subtracting equation (i) from (ii) we get,

$\frac{V}{I_g} = 900$

$\therefore$ R = 900 Ω

41. Using, $R_s = \frac{V}{I_g} - G$ we get,

for 1st case, $50 = \frac{V}{I_g} - G$ ....(i) and

for 2nd case, $500 = \frac{2V}{I_g} - G$ ....(ii)

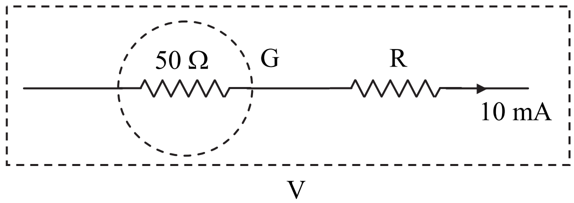
By subtracting equation (i) from (ii) we get,

$\frac{V}{I_g} = 450\ \Omega$

Substituting this value in equation (i),

$\therefore$ G = 450 – 50 = 400 Ω

42.



$V = I_g (G + R)$

$\therefore$ $100 = 10 \times 10^{-3} \times (50 + R)$

$\therefore$ $50 + R = 10000$

$\therefore$ $R = 9950\ \Omega$

43. $R = \frac{V}{I_g} - G$

$R = \frac{3}{5\times10^{-4}} - 50 = \frac{3\times10^4}{5} - 50$

$= 6000 - 50 = 5950\ \Omega$

44. Given: $I_g = 5 \times 10^{-3}$ A and G = 15 Ω

Let series resistance be R.

$\therefore$ $V = I_g (R + G)$

$\therefore$ $10 = 5 \times 10^{-3} (R + 15)$

$\therefore$ $R = 2000 - 15 = 1985 = 1.985 \times 10^3\ \Omega$.

45. As the galvanometer is to be converted into voltmeter, the resistance should be connected in series.

$R = \frac{V}{I_g} - G = \frac{20}{5\times10^{-3}} - 50 = 3950\ \Omega$

46. As the voltmeter has full scale deflection of 6V and is graded as 3000 Ω/V, hence total resistance of voltmeter is G = 6 × 3000 Ω

$\Rightarrow$ G = 18000 Ω

The full scale deflection current of voltmeter is

$\therefore$ $I_g = \frac{6}{18000} = \frac{1}{3000}$ A

The resistance in series that must be connected for 12 V full scale deflection is

$R_S = \frac{V}{I_g} - G = \frac{12}{\frac{1}{3000}} - 18000$

$\therefore$ $R_S = 36000 - 18000 = 18000\ \Omega$

47. $S = \frac{nBA}{C}$

$S \propto n$

48. $S_i = \frac{d\theta}{dI} = \frac{nAB}{C}$

$\therefore$ $S_i \propto \frac{1}{C}$ $\Rightarrow$ A has maximum sensitivity.

49. In a moving coil galvanometer,

$I = \frac{k}{nBA}\theta$

$\therefore$ $\theta = \frac{nBA}{k} I$

For the given value of current I, θ increases if B increases. The use of iron core increases the magnetic field, so the deflection θ increases for the same value of current I making the galvanometer more sensitive. Hence, Assertion is correct.

But soft iron can be easily magnetised or demagnetised, hence Reason is wrong.

50. $S_i = \frac{nAB}{C}$

Here, $S_i \propto n \Rightarrow \frac{S_1}{S_2} = \frac{n_1}{n_2}$

$\therefore$ $\frac{S_1}{\left(\frac{125\ S_1}{100}\right)} = \frac{28}{n_2} \Rightarrow \frac{4}{5} = \frac{28}{n_2}$

$\therefore$ $n_2 = 35$

51. $S_i = x$ div/mm $= \frac{x\ \text{div}}{10^{-3}\ \text{m}} = x \times 10^3$ div/m

$S_v = y$ div/m

Now, $S_v = \frac{S_i}{G} \Rightarrow G = \frac{S_i}{S_v}$

$\therefore$ $G = \frac{x}{y} \times 10^3$

52. Resistance of the galvanometer,

$G = \frac{S_i}{S_v} = \frac{5\,\text{div/mA}}{20\,\text{div/V}} = \frac{5 \times 10^3\ \text{div/A}}{20\ \text{div/V}}$

$= \frac{5000}{20}$ V/A $= 250\ \Omega$

53. Current sensitivity, $\frac{d\theta}{dI} = \frac{NBA}{C}$

$\therefore$ $\frac{d\theta}{dI} = \frac{100 \times 5 \times 10^{-4}}{10^{-8}} = 5$ rad/μA

54. $S = \frac{G}{n-1}$

Given: $S = \frac{G}{8}$

$\therefore \quad \frac{G}{8} = \frac{G}{n-1}$

$\Rightarrow n = 9$

As the range of galvanometer is increased 9 times, its sensitivity will become $\frac{1}{9}$.

$\therefore \quad s' = \frac{s}{9}$

56. $r = \frac{mv}{qB} = \frac{\sqrt{2Em}}{qB}$

57. $r = \frac{mv}{Bq} \Rightarrow r \propto v$

58. Radius of circular path:

$r = \frac{mv}{qB}$

$\therefore \quad r \propto \frac{1}{B}$

When B is reduced to $\frac{B}{2}$, r is doubled

$\therefore$ New radius of circular path is 2r.

59. In cyclotron,

$v = \frac{\pi r}{t}$

$\therefore \quad v \propto r$

While, $\omega = \frac{Bq}{m}$

i.e., ω is independent of r.

60. $\omega = \frac{2\pi}{T} = \frac{qB}{m} \Rightarrow \omega \propto v^0 \quad \left(\because T = \frac{2\pi m}{qB}\right)$

61. $K.E. = \frac{q^2B^2r^2}{2m}$

But here K.E. = qV

$\therefore \quad r^2 = \frac{qv \times 2m}{q^2B^2}$

$r \propto \sqrt{m}$

$\therefore \quad \frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^2$

62. Radius of circular path: $r = \frac{\sqrt{2mqV}}{qB}$

$r \propto \sqrt{V}$ where B is constant

i.e., $V \propto r^2$

$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^2$

$\therefore \quad \frac{V_2}{V} = \left(\frac{2r}{r}\right)^2 = 4$

$\therefore \quad V_2 = 4V$

63. $r = \frac{\sqrt{2mK}}{qB} \Rightarrow r \propto \sqrt{K}$

$\therefore \quad \frac{R}{R_2} = \sqrt{\frac{K}{2K}} \Rightarrow R_2 = R\sqrt{2}$

64. $r = \frac{mv}{qB}$

i.e., $B = \frac{mv}{qr}$

$B = \frac{9.1 \times 10^{-31} \times 10^6}{1.6 \times 10^{-19} \times 0.2} = \frac{9.1}{1.6 \times 2} \times 10^{-5}$

$= 2.84 \times 10^{-5}$ T

65. $r = \frac{mv}{qB} \Rightarrow r = \frac{v}{\left(\frac{q}{m}\right)B}$

$r = \frac{10^9}{(10^{11})\, 10^{-4}} = 10^2$ m

66. $T = \frac{2\pi m}{Bq} \Rightarrow \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p} \cdot \frac{q_p}{q_\alpha} = \frac{2}{1}$

67. $T = \frac{2\pi m}{qB}$

i.e., T is independent of v.

$\therefore$ Time period will remain the same.

68. Here, f = 10 MHz = 10^7 Hz

r = 50 cm = 50×10^{-2} m

$v = 2\pi rf = 2\pi \times 50 \times 10^{-2} \times 10^7$

$= 3.14 \times 10^7$ ms^{-1}

69. Cyclotron frequency, $f = \frac{Bq}{2\pi m}$

$\therefore \quad f = \frac{1 \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$

$= 2.79 \times 10^{10}$ Hz

$= 27.9 \times 10^9$ Hz ≈ 28 GHz

70. Radius of circular path: $r = \frac{\sqrt{2mE}}{qB}$

$E = \frac{q^2 B^2 r^2}{2m}$ ....(i)

Cyclotron frequency is $f = \frac{qB}{2\pi m}$

∴ $q^2B^2 = 4\pi^2m^2f^2$ ....(ii)

Using equation (ii) in equation (i),

$E = \frac{1}{2m}(4\pi^2m^2f^2)r^2$

∴ $E = 2\pi^2mf^2r^2$ ....(in joule)

∴ $E = \frac{2\pi^2mf^2r^2}{e}$ ....(in eV)

$= \frac{2\times 10\times(1.67\times10^{-27})\times(10\times10^6)^2\times(0.6)^2}{1.6\times10^{-19}}$ eV

$= 7.5 \times 10^6$ eV

= 7.5 MeV

The closest value in the option is 7 MeV

∴ Option (C) is correct.

71. Frequency of revolution is,

$f = \frac{Be}{2\pi m} = \frac{3.57\times10^{-2}\times1.76\times10^{11}}{2\times3.14}$

$\approx 1 \times 10^9$ Hz = 1 GHz

72. Initially $F_E = F_m$

∴ qE = qvB

∴ $B = \frac{E}{v} = \frac{2\times10^4}{10^6} = \frac{2}{100} = 2\times10^{-2}$ T

Now when E is switched off,

$r = \frac{mv}{qB} = \frac{mv}{eB} = \frac{v}{B\times\left(\frac{e}{m}\right)}$

$= \frac{10^6}{2\times10^{-2}\times10^8} = \frac{1}{2} = 0.5$ m

73. The oscillator frequency must be same as proton's cyclotron frequency.

$f = \frac{qB}{2\pi m}$

∴ $B = \frac{2\pi mf}{q} = \frac{2\times3.14\times1.67\times10^{-27}\times12\times10^6}{1.6\times10^{-19}}$

$= 78.6\times10^{-2}$ T ≈ 0.8 T

74. For a charged particle inside a magnetic field, radius of path is,

$r = \frac{mv}{qB} = \frac{p}{qB}$

$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

As K.E. for all the particles is given to be same,

$p \propto \sqrt{m}$

Also, the magnetic field is same,

∴ $r \propto \frac{p}{q}$ or $\frac{\sqrt{m}}{q}$

For given particles,

$q_p = q_e$ $q_\alpha = 2q_p$

$m_p = 1836\ m_e$ $m_\alpha = 4m_p$

∴ $r_p \propto \frac{\sqrt{m_p}}{q_p}$, $r_e \propto \frac{\sqrt{\frac{m_p}{1836}}}{q_p}$, $r_\alpha \propto \frac{\sqrt{4m_p}}{2q_p} \propto \frac{\sqrt{m_p}}{q_p}$

∴ $r_e < r_p = r_\alpha$

75. $r = \frac{\sqrt{2m(K.E.)}}{Bq}$

$r_p : r_d : r_\alpha :: \frac{\sqrt{m}}{q} : \frac{\sqrt{2m}}{q} : \frac{\sqrt{4m}}{2q} = 1 : \sqrt{2} : 1$

76. $F = IlB \sin\theta$

$\theta = 90°$

∴ $\sin 90° = 1$

∴ $F = IlB$

∴ $mg = IlB$

$m = \frac{IlB}{g} = \frac{2.5\times50\times10^{-2}\times0.5}{10}$

$= \frac{1}{16} = 62.5$ g

77. We know

$F_B = i\left(\vec{l}_{eq}\times\vec{B}\right) \Rightarrow i\,l_{eff}B$ ($\because l_{eff}\perp B$)

For PQ $\vec{l}_{eq} \parallel \vec{B}$

$(\vec{F}_B)_{PQ} = 0$

For PR

$l_{PR} = \frac{\sqrt{3}}{2}l$ (which is perpendicular to $\vec{B}$)

$(F_B)_{PR} = i\,l_{eff}B = i\left(\frac{\sqrt{3}}{2}l\right)B$

$(F_B)_{PR} = \frac{\sqrt{3}}{2}ilB$

Similarly for QR

$(F_B)_{PR} = \frac{\sqrt{3}}{2}ilB$

78. $B = \frac{\mu_0 I}{2r}$

$I = \frac{q}{t} = e \times n$

$B = \frac{\mu_0 e \times n}{2r}$

79. Kinetic energy in magnetic field remains constant and it is K.E. = qV

$\therefore$ K.E $\propto$ q

(V = constant)

$\therefore$ $K.E_p : K.E_d : K.E_a = q_p : q_d : q_a = 1 : 1 : 2$

80. $I = \frac{q}{t} = 100 \times e$

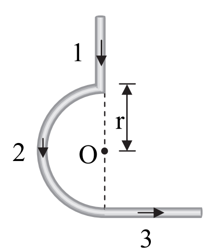
$B_{centre} = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0}{4\pi}\frac{2\pi \times 100e}{r}$

$= \frac{\mu_0 \times 200 \times 1.6 \times 10^{-19}}{4 \times 0.8} = 10^{-17}\ \mu_0$

81. $I = \frac{q}{t} = \frac{2 \times 1.6 \times 10^{-19}}{2} = 1.6 \times 10^{-19}$ A

$\therefore$ $B = \frac{\mu_0 I}{2r} = \frac{\mu_0 \times 1.6 \times 10^{-19}}{2 \times 0.8} = \mu_0 \times 10^{-19}$

82.



Magnetic fields due to different portions 1, 2 and 3 are respectively,

$B_1 = 0$,

$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi I}{r}$ (directed outside the paper)

$B_3 = \frac{\mu_0}{4\pi}.\frac{I}{r}$ (directed outside the paper)

$\therefore$ $B_{net} = B_2 + B_3 = \frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}$

83. Magnetic field at point O,

$\vec{B} = \frac{\mu_0 I}{4\pi R}(-2\hat{k}) + \frac{\mu_0 I}{4\pi R}\pi(-\hat{i}) = \frac{-\mu_0 I}{4\pi R}[\pi\hat{i} + 2\hat{k}]$

84. If a wire of length l is bent in the form of a circle of radius r then $2\pi r = l$

$\therefore$ $r = \frac{l}{2\pi} = \frac{\pi^2}{2\pi} = \frac{\pi}{2}$

Magnetic field due to straight wire

$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r} = \frac{\mu_0}{4\pi} \times \frac{2 \times 2}{1 \times 10^{-2}}$

Also, magnetic field due to circular loop,

$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{r} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi \times 2}{\pi/2}$

$\therefore$ $\frac{B_2}{B_1} = \frac{1}{50}$

85. Magnetic field due to one side of the square at centre O

$B_1 = \frac{\mu_0}{4\pi}.\frac{2I\sin 45°}{a/2} = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\,I}{a}$

Hence magnetic field at centre due to all sides,

$B = 4B_1 = \frac{\mu_0(2\sqrt{2}I)}{\pi a}$

Magnetic field due to n turns

$B_{net} = nB = \frac{\mu_0 2\sqrt{2}nI}{\pi a}$

$= \frac{\mu_0 2\sqrt{2}nI}{\pi(2l)}$ ....($\because$ a = $2l$)

$= \frac{\sqrt{2}\mu_0 nI}{\pi l}$

86. Case I,

Wire is bent to circle,

$L = 2\pi r$

$\Rightarrow r = \frac{L}{2\pi}$

$\therefore$ magnetic induction at centre,

$\therefore$ $B_{circle} = \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{2\left[\frac{L}{2\pi}\right]}$

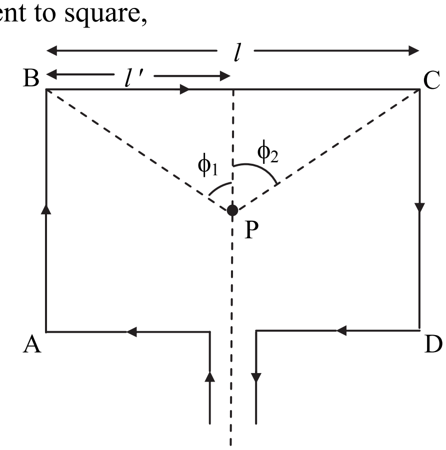
$\therefore$ $B_A = \frac{\mu_0 \pi I}{L}$ ....(i)

Case II

Wire is bent to square,

$L = 4l$

$\therefore$ $l = \frac{L}{4}$



Magnetic induction at P due to side BC

$B_{BC} = \frac{\mu_0}{4\pi}\frac{I}{l'}(\sin\phi_1 + \sin\phi_2)$

$\because$ $\phi_1 = \phi_2 = 45^o$ here

$\therefore$ $B_{BC} = \frac{\mu_0}{4\pi}\frac{I}{l'}\left[\frac{2}{\sqrt{2}}\right] = \frac{\mu_o I}{2\sqrt{2}\pi l'}$

As $l' = \frac{l}{2} = \frac{L}{8}$

$\therefore$ $B_{BC} = \frac{4\mu_0 I}{\sqrt{2}\pi L}$

$\therefore$ By all four sides

$B_B = \frac{16\mu_0 I}{\sqrt{2}\pi L}$

$\therefore$ $\frac{B_A}{B_B} = \frac{\mu_0\pi I}{L}\times\frac{\sqrt{2}\pi L}{16\mu_0 I} = \frac{\sqrt{2}\times\pi^2}{16} = \frac{\pi^2}{8\sqrt{2}}$

87. Case I,
Wire is bent to circle,
$L = 2\pi r$

$\Rightarrow r = \frac{L}{2\pi}$

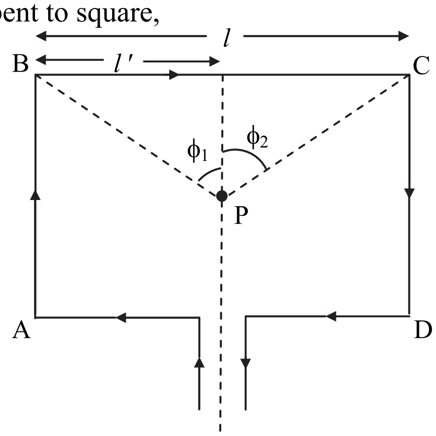
$\therefore$ magnetic induction at centre,

$\therefore$ $B_{circle} = \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{2\left[\frac{L}{2\pi}\right]}$

$\therefore$ $B_{circle} = \frac{\mu_0\pi I}{L}$ ....(i)

Case II
Wire is bent to square,
$L = 4l$

$\therefore$ $l = \frac{L}{4}$



Magnetic induction at P due to side BC

$B_{BC} = \frac{\mu_0}{4\pi}\frac{I}{l'}(\sin\phi_1 + \sin\phi_2)$

$\because$ $\phi_1 = \phi_2 = 45^o$ here

$\therefore$ $B_{BC} = \frac{\mu_0}{4\pi}\frac{I}{l'}\left[\frac{2}{\sqrt{2}}\right] = \frac{\mu_o I}{2\sqrt{2}\pi l'}$

As $l' = \frac{l}{2} = \frac{L}{8}$

$\therefore$ $B_{BC} = \frac{4\mu_0 I}{\sqrt{2}\pi L}$

$\therefore$ By all four sides

$B_{sq} = \frac{16\mu_0 I}{\sqrt{2}\pi L}$

$\therefore$ $\frac{B_{circle}}{B_{sq}} = \frac{\mu_0\pi I}{L}\times\frac{\sqrt{2}\pi L}{16\mu_0 I} = \frac{\sqrt{2}\times\pi^2}{16} = 0.87$

But $B_{sq} > B_{circle}$

$\Rightarrow \frac{B_{sq}}{B_{circle}} = 1.15$

89. For sides AD and BC, force acting on them is equal and opposite. Hence the net force is zero.

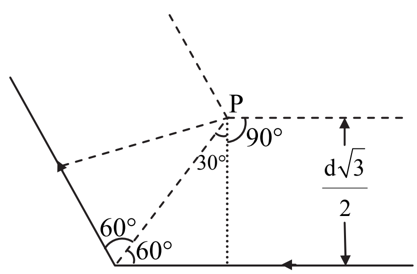
$\therefore$ $F_{net} = F_{BA} - F_{CD}$

here, $F_{BA} = \frac{\mu_0}{4\pi}\frac{2IiL}{\frac{L}{2}}$

for $F_{CD} = \frac{\mu_0}{4\pi}\frac{2IiL}{3\frac{L}{2}}$

$\therefore$ $F_{net} = \frac{\mu_0 IiL}{2\pi}\left[\frac{1}{\frac{L}{2}} - \frac{1}{3\frac{L}{2}}\right] = \frac{\mu_0 IiL}{2\pi}\left[\frac{4L}{3L^2}\right] = \frac{2\mu_0 Ii}{3\pi}$

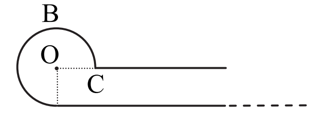
90.



$B_{net} = 2\left[\frac{\mu_0}{4\pi}\times\frac{I}{\left(\frac{d\sqrt{3}}{2}\right)}\times[1+\sin 30°]\right]$

$= 2\left[\frac{\mu_0}{4\pi}\times\frac{2I}{d\sqrt{3}}\times\frac{3}{2}\right] = \frac{\sqrt{3}\mu_0 I}{2\pi d}$

91. The angle subtended by the circular part ABC at the centre is $3\pi/2$.



Field due to ABC,

$B_1 = \frac{\mu_0}{4\pi}\frac{I}{r}\left(\frac{3\pi}{2}\right)$

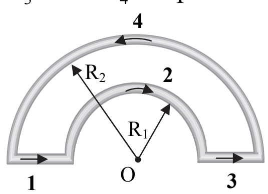
Field due to AD at O,

$B_2 = \frac{\mu_0}{2\pi r} \times \frac{1}{2} = \frac{\mu_0 I}{4\pi r}$

…[∵ A is at the end of the wire]

∴ Total induction = $\frac{\mu_0 I}{4\pi r}\left(\frac{3\pi}{2} + 1\right)$

92. In the figure, magnetic fields at O due to sections 1, 2, 3 and 4 are considered as B_1, B_2, B_3 and B_4 respectively.



$B_1 = B_3 = 0$

$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{R_1}$ (directed into the paper)

$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{R_2}$ (directed out of the paper)

As $|B_2| > |B_4|$

∴ $B_{net} = B_2 - B_4$

$\Rightarrow B_{net} = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (directed into the paper)

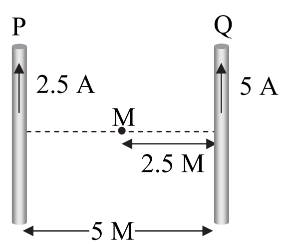
93. In the figure, magnetic field at mid point M is given by,

$B_{net} = B_Q - B_P$

$= \frac{\mu_0}{4\pi}\cdot\frac{2}{r}(I_Q - I_P)$

$= \frac{\mu_0}{4\pi} \times \frac{2}{2.5}(5 - 2.5)$

$= \frac{\mu_0}{2\pi}$



94. $M = I \times \text{Area of loop } \hat{k}$

$= I \times \left[a^2 + \frac{\pi a^2}{4 \times 2} \times 4\right]\hat{k}$

$= I \times a^2\left[\frac{\pi}{2} + 1\right]\hat{k}$

95. $|\vec{B}| = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r}$ ∴ $|\vec{B}| \propto \frac{1}{r}$

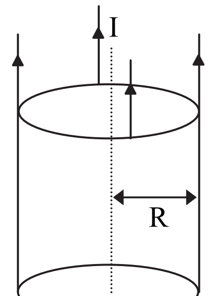
96. Magnetic field inside the conductor,

$B_{in} \propto r$ and magnetic field outside the conductor,

$B_{out} \propto \frac{1}{r}$

(where r is the distance of observation point from axis).

97.



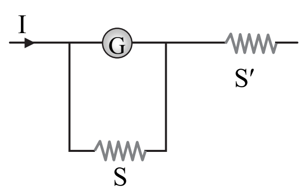
As magnetic field inside conductor is zero,
For d < R, B = 0
However, for d > R,

$B = \frac{\mu_0 I}{2\pi d}$

i.e., $B \propto \frac{1}{d}$

Hence, the variation is best depicted by graph (C).

98.

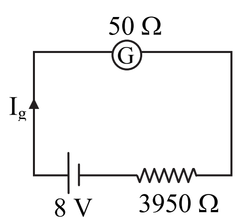


Now, $G = \left(\frac{GS}{G+S}\right) + S'$

∴ $G - \frac{GS}{G+S} = S'$

∴ $S' = \frac{G^2}{G+S}$

99.

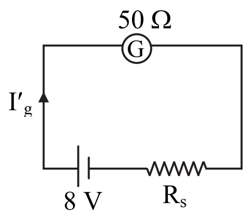


the current in the circuit for which galvanometer shows full scale deflection of 30 divisions is

$I_g = \frac{V}{R} = \frac{8}{3950 + 50} = 2 \text{ mA}$

For deflection to become 15 divisions, the current through galvanometer must be halved.

$\therefore \quad I'_g = \frac{I_g}{2} = 1 \text{ mA}$



but, $I'_g = \frac{V}{R'} = \frac{8}{R_s + 50}$

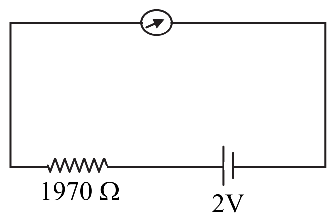
$= 1 \text{ mA}$

$\therefore \quad \frac{8}{R_s + 50} = 10^{-3}$

$\Rightarrow R_s + 50 = 8 \times 10^3$

$\Rightarrow R_s = 7950 \ \Omega$

100.



$$I = \frac{V}{(R + r)} = \frac{2}{(1970 + 30)} = \frac{2}{2000} = 1 \text{ mA}$$

$\therefore$ for 10 divisions of deflection, I = 0.5 mA

$\therefore \quad 0.5 \times 10^{-3} = \frac{2}{(R' + r)}$

$\therefore \quad R' + r = \frac{2}{0.5 \times 10^{-3}}$

$\therefore \quad R' = (4 \times 10^3) - 30$

$\therefore \quad R' = 3970 \ \Omega$

101. $V = I_g (G + R)$

i.e., $I_g = \frac{V}{(G + R)} = \frac{3}{50 + 2950}$

$= \frac{3}{3000}$

$= \frac{1}{1000} = 10^{-3} \text{ A}$

Now, 30 divisions represent 10^{-3} A

Let 20 divisions represent I A

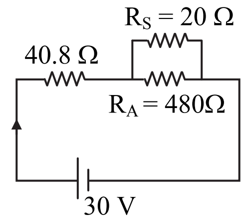
$\therefore \quad I = \frac{2}{3} \times 10^{-3} \text{ A}$

Also, $I = \frac{V}{(R_{eq} + r)} = \frac{3}{3000 + r}$

$\therefore \quad \frac{2}{3} \times 10^{-3} = \frac{3}{3000 + r}$

$\therefore \quad r = 1500 \ \Omega$

102.



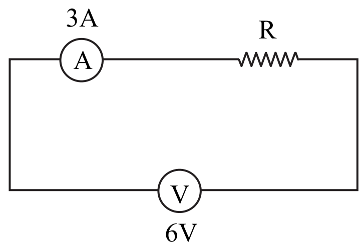
$$R_{eq} = 40.8 + \frac{480 \times 20}{500} = 40.8 + 19.2 = 60 \ \Omega$$

$I = \frac{30}{60} = 0.5 \text{ A}$

So reading of ammeter is 0.5 A

103. With ideal ammeter, V = IR

$R = \frac{6}{3} = 2 \ \Omega$



but the ammeter has resistance of its own hence, external resistance has to be less than 2 Ω.

104. $\frac{I_G}{I} = \frac{S}{S + G}$

$\frac{1}{4} = \frac{3}{3 + G}$

$\therefore \quad 3 + G = 12$

$\therefore \quad G = 9 \ \Omega$

If additional shunt of 2 Ω is connected then total shunt resistance becomes,

$\frac{1}{S'} = \frac{1}{2} + \frac{1}{3}$

$\therefore \quad S' = \frac{2 \times 3}{2 + 3} = \frac{6}{5} \ \Omega = 1.2 \ \Omega$

Now, $\frac{I_G}{I} = \frac{S'}{S' + G} = \frac{1.2}{1.2 + 9} = \frac{1.2}{10.2} = \frac{1}{8.5}$

105. For full scale deflection, $I_g = \frac{250 \text{mV}}{G}$ ampere

Value of shunt required for converting it into ammeter of range 250 ampere is,

$$S = \frac{G}{\left(\frac{I}{I_g} - 1\right)}$$

$\therefore \quad S = \frac{I_g G}{I - I_g} \approx \frac{250 \text{mV}}{250 \text{mA}} \approx 1 \ \Omega$

106. Kinetic energy,

$$\frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

For an α-particle, the charge is two times that of the proton but mass is 4 times that of the proton. Hence compared to kinetic energy of a proton, for the same conditions in the cyclotron, energy of alpha particle is E.

107. For cyclotron,

$$B = \frac{mv}{er} = \frac{m\omega}{e} \quad \ldots(\because v = R\omega)$$

$$= \frac{m\times 2\pi\nu}{e} = \frac{2\pi m\nu}{e}$$

$$\text{K.E.} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}m(R\omega)^2$$

$$= \frac{1}{2}mR^2(4\pi^2\nu^2)$$

$$= 2\ mR^2\pi^2\nu^2$$

108. Radius in magnetic field

$$R = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB}$$

$$E = \frac{q^2B^2R^2}{2m}$$

For proton

$$E_1 = \frac{e^2\times B^2\times R^2}{2\times m_p}$$

For α-particle

$$E_2 = \frac{(2e)^2\times B^2\times R^2}{2\times 4m_p}$$

∴ $E_1 = E_2$

109. The electron is revolving along a circular path

∴ K.E. = qV ….(i)

also, we know,

$$\text{K.E.} = \frac{1}{2}mv^2$$

but, v = rω

∴ $\text{K.E.} = \frac{1}{2}mr^2\omega^2$ ….(ii)

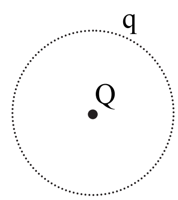
Equating (i) and (ii)

$$\frac{1}{2}mr^2\omega^2 = qV$$

∴ $V = \frac{mr^2\omega^2}{2q} = \frac{9.1\times10^{-31}\times(0.20)^2\times(120)^2}{2\times1.6\times10^{-19}}$

∴ $V = 1.638\times10^{-9}$ V

110.



Electrostatic force of attraction, $F = \frac{KqQ}{r^2}$

But, centripetal force is given by, $F = \frac{mv^2}{r}$

∴ $\frac{mv^2}{r} = \frac{KqQ}{r^2}$

$$v \propto \frac{1}{\sqrt{r}}$$

Time taken by charge to complete a circular path is given by, $T = \frac{2\pi r}{v}$

∴ $T \propto \frac{r}{v}$

∴ $T \propto r^{3/2}$ ….$\left(\because v \propto \frac{1}{\sqrt{r}}\right)$

But, for circular loop, $B = \frac{\mu_0 I}{2r}$

∴ $B \propto \frac{I}{r}$

As current $I = \frac{Q}{T}$

$I \propto \frac{1}{T} \propto \frac{1}{r^{\frac{3}{2}}}$ ∴ $B \propto \frac{r^{-3/2}}{r}$

∴ $B \propto r^{-5/2}$ i.e., $B \propto \frac{1}{r^{5/2}}$

Evaluation Test

1. Here, net field,
B = Field due to circular portion
– Field due to straight portion

$= \left(\frac{\mu_0 I}{2r} - \frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 I}{2r}\left(1 - \frac{1}{\pi}\right) = \frac{\mu_0 I(\pi - 1)}{2\pi r}$

(perpendicular to the plane of page and directed into it)
Field due to circular portion is directed into the plane of the paper and that due to straight portion is directed outward and perpendicular to the plane of paper. Thus net field is directed into the plane of the paper.

2. Magnetic field inside a solenoid, $B \propto I$
Energy density,

$E = \frac{1}{2\mu_0}B^2 \Rightarrow E \propto B^2 \Rightarrow E \propto I^2$

Hence curve should be a parabola symmetric about E axis passing through (0, 0).

3. The coil is made up of tiny current elements. Force acting on each current element is directed outwards. As a result of this the coil expands.

4. Magnetic field due to AB is zero because C lies on the extended wire itself.
Magnetic field due to infinite wire CD is

$B_1 = \frac{\mu_0}{4\pi r}(\sin 0° + \sin 90°) = \frac{\mu_0 i}{4\pi r}$

Magnetic field due to circular portion,

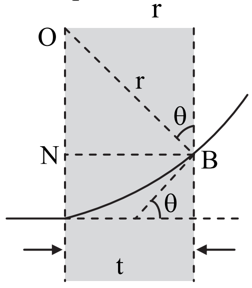
$B_2 = \frac{\mu_0}{4\pi}\frac{i\left(\frac{3}{4}2\pi r\right)}{r^2} = \frac{\mu_0 i}{4\pi r}\frac{3\pi}{2}$

$\therefore \quad B = B_1 + B_2 = \frac{\mu_0 i}{4\pi r}\left(\frac{3}{2}\pi + 1\right)$

5. Using $qV = \frac{1}{2}mv^2$, we get

$v = \sqrt{2Vq/m}$

Again, $Bqv = \frac{mv^2}{r}$ i.e., $r = \frac{mv}{Bq}$



From Figure, $t = r\sin\theta$

or $\sin\theta = \frac{t}{r} = \frac{tBq}{mv} = \frac{tBq}{m\left(\frac{2Vq}{m}\right)^{\frac{1}{2}}}$

$= \frac{Bt\sqrt{q}}{\sqrt{m}\sqrt{2V}} = Bt\sqrt{\frac{q}{2Vm}}$

6. Change in momentum = Impulse

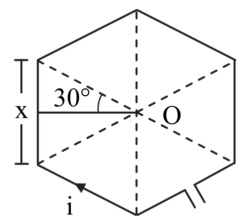
i.e., $mv = \int_0^t F\,dt = \int_0^t BIl\,dt$

$= Bl\int_0^t I\,dt = Blq$

Or $\quad v = \frac{Blq}{m} \quad$ But $v = \sqrt{2gh}$

$\therefore \quad \sqrt{2gh} = \frac{Blq}{m}$ or $q = \frac{m\sqrt{2gh}}{Bl}$

7.



Here, $B = 6 \times \frac{\mu_0}{4\pi}\frac{i}{r}(\sin\theta_1 + \sin\theta_2)$

$= 6\frac{\mu_0}{4\pi}\frac{i(2\sin 30)}{\left(\frac{\sqrt{3}}{2}x\right)} = \frac{\sqrt{3}\mu_0 i}{\pi x}$

$\left(\because \text{Here, } r = \frac{\sqrt{3}}{2}x\right)$

8. Here, magnetic field due to straight portion,

$B_{PQ} = \frac{\mu_0 I}{4\pi R\cos\theta}(\sin\theta + \sin\theta)$

$(\because OM = R\cos\theta)$

$= \frac{\mu_0 I}{4\pi R}\frac{2\sin\theta}{\cos\theta} = \frac{\mu_0 I}{2\pi R}\tan\theta$

and magnetic field due to circular portion,

$B'_{PQ} = \frac{\mu_0 I}{2R}\left(\frac{2\pi - 2\theta}{2\pi}\right)$

$= \frac{\mu_0 I}{2\pi R}(\pi - \theta)$

$\therefore \quad B = B_{PQ} + B'_{PQ} = \frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta)$

9. Energy, $E = \frac{p^2}{2m} = \frac{(mv)^2}{2m} = \frac{(qBR)^2}{2m}$

$$\left(\because \frac{mv^2}{R} = qvB\right)$$

Then, $E_\alpha = \frac{(2eBR)^2}{2 \times 4m_p}$

where m_p is mass of proton.

and $E_d = \frac{(2eBR)^2}{2 \times 2m_p} \Rightarrow \frac{E_d}{E_\alpha} = \frac{2}{1}$

or $E_d = 2E_\alpha = 2 \times 2 = 4$ MeV

10. Magnetic induction at 'a',

$B = \frac{\mu_0 nIr^2}{2(r^2 + a^2)^{3/2}}$ and at centre

$\therefore$ $B_C = \frac{\mu_0 nI}{2r}$, we get

$$B_C - B = \frac{\mu_0 nI}{2}\left[\frac{1}{r} - \frac{r^2}{r^3\left(1 + \frac{a^2}{r^2}\right)^{3/2}}\right]$$

$$= \frac{\mu_0 nI}{2}\left[\frac{1}{r} - \frac{1}{r}\left(1 - \frac{3}{2}\frac{a^2}{r^2}\right)\right] \quad (\because a << r)$$

$\therefore$ Fractional decrease

$$= \frac{B_C - B}{B_C} = \frac{\mu_0 nI}{2}\left[\frac{3}{2}\frac{a^2}{r^3}\right] \Big/ \frac{\mu_0 nI}{2r}$$

$$= \frac{3}{2}\frac{a^2}{r^2}$$

11. Considering a ring of radius r and width dr, charge on ring, $dq = (2\pi RdR)\sigma$

Current, $dI = \frac{dq}{dt} = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \sigma\omega RdR$

$$(\because T = 2\pi)$$

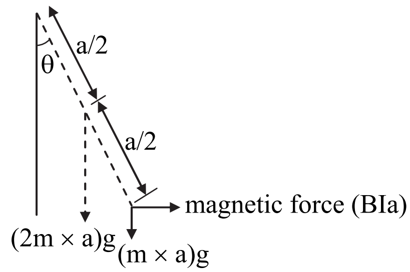
Using, $dB = \frac{\mu_0 dIR^2}{2(R^2 + y^2)^{3/2}}$

$\therefore$ $B = \int dB = \frac{\mu_0 \sigma\omega}{2}\int_0^R \frac{R^3\, dr}{(R^2 + y^2)^{3/2}}$

$$= \frac{\mu_0 \sigma\omega}{2}\left[\frac{R^2 + y^2}{\sqrt{R^2 + y^2}} - 2y\right]$$

12. Here magnetic force = BIa

Weight of a side is mag, where m is mass per unit length, and that of two sides i.e., 2 mag is effective at the centre.



Then taking moments,

$2 \text{ mag} \times \frac{a}{2}\sin\theta + \text{mag} \times a \sin\theta = \text{BIa}\, a\cos\theta$

i.e. $2ma^2g \sin\theta = BIa^2\cos\theta$

or $\tan\theta = \frac{BI}{2mg}$ But $m = A\rho$

$\therefore$ $\tan\theta = \frac{BI}{2A\rho g}$

$\therefore$ $B = \frac{2A\rho g}{I}\tan\theta$

13. Since $R_1 < r < R_2$,

$B = \frac{\mu_0 I}{2\pi r}$ where r is distance

Now, electric field, $E = \frac{q}{2\pi\varepsilon_0 r l}$

$\therefore$ $V = \int_{R_1}^{R_2} E\, dr = \frac{q}{2\pi\varepsilon_0 l}\int_{R_1}^{R_2}\frac{dr}{r}$

$$= \frac{q}{2\pi\varepsilon_0 l}\log\left(\frac{R_2}{R_1}\right)$$

i.e., $\frac{q}{2\pi\varepsilon_0 l} = \frac{V}{\log(R_2/R_1)}$

$\therefore$ $E = \frac{V}{r\log(R_2/R_1)}$

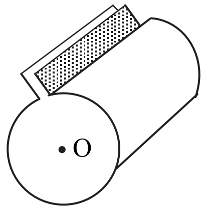
For no deflection,

$F_E = F_M$ i.e., $eE = evB$

$\therefore$ $\frac{eV}{r\log(R_2/R_1)} = \frac{ev\mu_0 I}{2\pi r}$

i.e., $v = \frac{2\pi V}{\mu_0 I \log(R_2/R_1)}$

14. The structure can be compared to solenoid having a single turn.
Using Ampere's circuital law,



$\oint \vec{B} \cdot \vec{dx} = \mu_0 I \Rightarrow Bx = \mu_0 I$

or $\quad B = \dfrac{\mu_0 I}{x}$

15. Magnetic induction, $B = \dfrac{\mu_0 I}{2r}$

For the coil,
$2\pi r = 4(2\pi r') \Rightarrow r' = r/4$

$\therefore$ New magnetic induction, $B' = \dfrac{4\mu_0 I}{2r'}$

$\therefore \ B' = \dfrac{4\mu_0 I}{2r} \times 4 = 16B$

16. Magnetic moment, M = IA
and magnetic field at the centre of a loop carrying current $= \dfrac{\mu_0 I}{2r} = X$ or $I = \dfrac{X(2r)}{\mu_0}$

So, $M = \dfrac{X.2r}{\mu_0} \times \pi r^2$

$\therefore \quad M = \dfrac{2\pi X r^3}{\mu_0}$

17. For voltmeter,

$R = \dfrac{V}{I_g} - G$

$= \dfrac{50}{50 \times 10^{-6}} - 100$

$= 10^6 - 10 \approx 10^3 \ k\Omega$

$\therefore$ Option (A) is not correct.

$R = \dfrac{V}{I_g} - G$

$= \dfrac{10}{50 \times 10^{-6}} - 100$

$= 199.9 \ k\Omega \approx 200 \ k\Omega.$

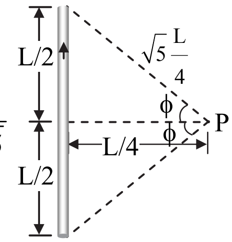
$\therefore$ Option (B) is correct.
Option (C) is not possible as for a voltmeter, resistance should be connected in series.
For ammeter,

$S = \left(\dfrac{I_g}{I - I_g}\right) G$

$= \left[\dfrac{50 \times 10^{-6}}{(10 \times 10^{-3}) - (50 \times 10^{-6})}\right] \times 100 = 0.5 \ \Omega.$

$\therefore$ Option (D) is not correct.

18. By using, $B = \dfrac{\mu_0}{4\pi} \dfrac{I}{r}(\sin\phi_1 + \sin\phi_2)$



$\therefore \quad B = \dfrac{\mu_0}{4\pi} \dfrac{I}{(L/4)}(2\sin\phi)$

Also, $\sin\phi = \dfrac{L/2}{\sqrt{5}L/4} = \dfrac{2}{\sqrt{5}}$

$\therefore \quad B = \dfrac{4\mu_0 I}{\sqrt{5}\pi L}$

19. Magnetic field at centre, $B = \dfrac{\mu_0}{4\pi} \dfrac{2\pi I}{r}$

Magnetic field at a point on the axis,

$B' = \dfrac{\mu_0}{4\pi} \times \dfrac{2\pi I r^2}{(r^2 + x^2)^{\frac{3}{2}}}$

Given, $B' = \dfrac{B}{27} \Rightarrow \dfrac{B}{B'} = 27$

$\therefore \quad \dfrac{\dfrac{\mu_0}{4\pi} \dfrac{2\pi I}{r}}{\dfrac{\mu_0}{4\pi} \times \dfrac{2\pi I r^2}{(r^2 + x^2)^{\frac{3}{2}}}} = 27$

$\therefore \quad \dfrac{(r^2 + x^2)^{\frac{3}{2}}}{r^3} = 27$

$\therefore \quad \dfrac{(r^2 + x^2)^{\frac{1}{2}}}{r} = 3$

$\therefore \quad \dfrac{r^2 + x^2}{r^2} = 9$

$\therefore \quad r^2 + x^2 = 9r^2$

$\therefore \quad 8r^2 = x^2$

$\therefore \quad x = 2\sqrt{2}\, r$

20. Here, the wire does not produce any magnetic field at O because the conductor lies on the line through O. Also, the loop does not produce magnetic field at O.

Textbook Chapter No.

15 Magnetism

Hints

Classical Thinking

5. Gyromagnetic ratio = $\frac{M_0}{L_0}$

9. $\mu_r < 1$ and $\varepsilon_r > 1$.

15. With rise in temperature, their magnetic susceptibility decreases, i.e.,

$\chi_m \propto \frac{1}{T}$

42. As every atom of a diamagnetic material is not a complete magnet in itself, its susceptibility is not affected by the temperature.

43. Iron is ferromagnetic in nature. Lines of force due to external magnetic field prefer to pass through iron.

Critical Thinking

1. Magnetic induction is defined as the force exerted on a fictitious dipole of unit pole strength

$\therefore$ $B = \frac{F}{m} \Rightarrow F = mB$

2. Magnetic field intensity = $\frac{\mu_0}{4\pi}\frac{M}{x^3} \propto Mx^{-3}$

$\therefore$ n = –3

3. The magnetic dipole moment of the earth

$M = IA = I\,\pi R^2$

$\therefore$ $I = \frac{M}{\pi R^2} = \frac{6.4\times10^{21}}{3.14\times6.4\times6.4\times10^{12}} = \frac{10^9}{6.4\times3.14}$

$\therefore$ $I \approx 5 \times 10^7$ A

4. Magnetic dipole moment,

$M = nIA = nI\,(\pi r^2)$

$= 5 \times 10 \times \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} = 0.77\ \text{Am}^2$

The direction of M is perpendicular to the plane of the coil. Hence, it is along the Z axis.

5. $l_{eff} = \left[\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2\right]^{\frac{1}{2}} = \left[\frac{l^2}{4}+\frac{l^2}{4}\right]^{\frac{1}{2}}$

$= \left[\frac{l^2}{2}\right]^{\frac{1}{2}} = \frac{l}{\sqrt{2}}$

$\therefore$ $M' = ml_{eff} = \frac{ml}{\sqrt{2}} = \frac{M}{\sqrt{2}}$

6. The magnetic moment of the revolving electron is

$M = IA = \frac{e}{T} \times \pi r^2$ But $T = \frac{2\pi r}{v}$

$\therefore$ $M = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$

$\therefore$ $M = \frac{1.6\times10^{-19}\times2.5\times10^6\times0.5\times10^{-10}}{2} = 10^{-23}\ \text{Am}^2$

7. $r = 0.5\ \text{Å} = 0.5 \times 10^{-10}$ m,

$f = 10^{10}$ MHz $= 10^{16}$ Hz

The revolving electron is equivalent to a current

$M = IA = (ef)\,\pi r^2$

$\therefore$ $M = 1.6 \times 10^{-19} \times 10^{16} \times 3.14 \times (0.5 \times 10^{-10})^2$

$= 1.256 \times 10^{-23}\ \text{Am}^2$

8. time(t) = $\frac{\text{Distance travelled}}{\text{Velocity}}$

$\therefore$ $t = \frac{2R+\pi R}{v} = \frac{R(\pi+2)}{v}$

$\therefore$ $I = \frac{q}{t} = \frac{qv}{R(\pi+2)}$

$\therefore$ $M = I \times A = \frac{qv}{R(\pi+2)} \times \frac{\pi R^2}{2} = \frac{\pi Rqv}{2(\pi+2)}$

9. Net magnetic induction $B = B_0 + B_m$

$= \mu_0 H + \mu_0 M_z$

10. $\chi = (\mu_r - 1)$

$\therefore$ $\chi = (600 - 1) = 599$

11. Relative permeability,

$\mu_R = \frac{\mu}{\mu_0} = \frac{0.1256}{4\pi\times10^{-7}}$

$= \frac{0.1256}{4\times3.14\times10^{-7}} = \frac{1256\times10^{-4}}{12.56\times10^{-7}} = 10^5$

12. $M_z = \frac{M_{net}}{V} = \frac{M}{Al} = \frac{1}{5\times10^{-4}\times6\times10^{-2}}$

$= 3.3 \times 10^4$ A/m

13. % increase in magnetic field

$= \frac{B - B_0}{B_0} \times 100 = \frac{\mu_0\chi H\times100}{\mu_0 H}$

$= \chi \times 100 = 6.8 \times 10^{-5} \times 100 = 6.8 \times 10^{-3}$

14. Volume of the magnet,

$V = \frac{\text{mass}}{\text{density}} = \frac{75\times10^{-3}}{75\times10^{2}} = 10^{-5}\ m^3$

∴ Magnetization, $M_z = \frac{M_{net}}{V} = \frac{3}{10^{-5}}$

∴ $M_z = 3 \times 10^5$ A/m

15. From Curie's law, $\chi \propto \frac{1}{T}$

∴ $\frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$ but it is given that $\frac{\chi_2}{\chi_1} = \frac{1}{2}$

and $T_1 = 273 + 127 = 400$ K

∴ $\frac{1}{2} = \frac{400}{T_2}$

∴ $T_2 = 800\ K = (800 - 273) = 527\ °C$

16. When $\chi = 0.5$, $\frac{1}{T} = 5 \times 10^{-3}$/K

∴ $T = \frac{1}{5\times10^{-3}} = \frac{1000}{5} = 200$ K

According to Curie's law, $\chi = \frac{C}{T}$

∴ $C = \chi T = 0.5 \times 200 = 100$ K

17. As temperature of a ferromagnetic material is raised, its susceptibility χ remains constant first and then decreases.

18. For paramagnetic substance, magnetisation M is proportional to magnetising field H, and M is positive.

20. Magnetism of a magnet falls with rise of temperature and becomes practically zero at Curie temperature.

21. The volume of the cubic domain is

$V = (10^{-6}\ m)^3 = 10^{-18}\ m^3$

Net dipole moment

$M_{net} = 8\times10^{10}\times9\times10^{-24}\ A\ m^2 = 72\times10^{-14}\ A\ m^2$

∴ Magnetization, $M_Z = \frac{M_{net}}{\text{Domain volume}}$

$= \frac{72\times10^{-14}\ A\ m^2}{10^{-18}\ m^3}$

$= 72 \times 10^4\ A\ m^{-1}$

$= 7.2 \times 10^5\ A\ m^{-1}$

22. Magnetic intensity,

$H = nI = 500 \times 1 = 500\ Am^{-1}$

$\mu_r = 1 + \chi \Rightarrow \chi = (\mu_r - 1)$

∴ $M = \chi H = (\mu_r - 1)H = (500 - 1) \times 500$

$= 2.495 \times 10^5\ Am^{-1} \approx 2.5 \times 10^5\ Am^{-1}$

23. $B = \mu_0\mu_r H \Rightarrow \mu_r \propto \frac{B}{H}$ = slope of B- H curve

According to the given graph, slope of the graph is highest at point Q.

24. $B \propto A$

∴ $\frac{B_{cir}}{B_{sq}} = \frac{A_{cir}}{A_{sq}} = \frac{\pi r^2}{l^2}$ ….(i)

Now, $2\pi r = 4l \Rightarrow l = \frac{\pi r}{2}$ ….(ii)

∴ From equations (i) and (ii),

$\frac{B_{cir}}{B_{sq}} = \frac{\pi r^2}{(\pi r/2)^2} = \pi r^2 \times \frac{4}{\pi^2 r^2} = \frac{4}{\pi}$

Competitive Thinking

1. $2\pi r = L \Rightarrow r = \frac{L}{2\pi}$

∴ $M = IA = I\pi\frac{L^2}{4\pi^2}$ $\Rightarrow M = \frac{IL^2}{4\pi}$

2. $M = nIA$

For a coil, $A = \pi r^2$

∴ $M \propto r^2 n$

but as radius becomes $\left(\frac{1}{4}\right)^{th}$, n becomes 4 times

∴ $\frac{M_1}{M_2} = \frac{n_1 r_1^2}{n_2 r_2^2} = \frac{n_1}{n_2}\times\left(\frac{r_1}{r_2}\right)^2 = \frac{1}{4}\times4^2 = 4$

∴ $M_2 = \frac{M_1}{4}$

3. $M = nIA = nI\pi r^2 = 10^2 \times 1\times 3.142 \times 10^{-2}$

$= 3.142\ Am^2$

4. Torque,

$\tau = MB_H \sin\theta = 0.1\times10^{-3}\times4\pi\times10^{-3}\times\sin30^o$

$= 10^{-7}\times4\pi\times\frac{1}{2} = 2\pi\times10^{-7}\ N\times m$

5. $\tau = MB\sin\theta \Rightarrow \tau \propto \sin\theta$

$\Rightarrow \frac{\tau_1}{\tau_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \frac{\tau}{\tau/2} = \frac{\sin 90}{\sin\theta_2}$

$\Rightarrow \sin\theta_2 = \frac{1}{2} \Rightarrow \theta_2 = 30^\circ$

$\Rightarrow$ angle of rotation $= 90° - 30° = 60°$

6. $L = \frac{nh}{2\pi}$ and $I = \frac{q}{T} = \frac{e\omega}{2\pi}$

Now, M = IA

$\therefore$ $M = \frac{e\omega}{2\pi}\pi R^2 = \frac{e\omega R^2}{2}$

$\therefore$ $M = \frac{e}{2}R^2 \times \frac{nh}{2\pi m R^2} = \frac{enh}{4\pi m}$

$\therefore$ $\frac{M}{L} = \frac{enh}{4\pi m} \times \frac{2\pi}{nh} = \frac{e}{2m}$

7. The magnetic moment of the revolving electron is given by, $M = \frac{neh}{4\pi m} = n\left(\frac{eh}{4\pi m}\right)$

Thus, $M \propto n$ (the principal quantum number).

8. $\frac{M}{L} = \frac{nIA}{mvr} = \frac{q}{2m}$

9. Gyromagnetic ratio, $\frac{M}{L} = \frac{e}{2m}$

$\therefore$ $m = \frac{e}{2(M/L)}$

$= \frac{1.6\times10^{-19}}{2\times8.8\times10^{10}} = \frac{1}{11}\times 10^{-29}$ kg

10. As we know for circulating electron magnetic moment

$M = \frac{1}{2}evr$ ....(i)

and angular momentum $J = mvr$ ....(ii)

From equations (i) and (ii) $M = \frac{eJ}{2m}$

11. Magnetization is given by, $M_Z = \frac{CB_{ext}}{T}$

12. Intensity of magnetization $= \frac{M_{net}}{Volume}$

$= \frac{M_{net}}{\text{length}\times\text{area of cross-section}}$

$= \frac{3}{3\times10^{-2}\times2\times10^{-4}} = 5\times10^5$ A/m

14. Magnetic field inside a solenoid is given by,

$B = \mu n I$

$= \mu_0 \mu_r n I = \mu_0 (1+\chi) n I$

15. $\chi_m = (\mu_r - 1) \Rightarrow \chi_m = (5500-1) = 5499$

16. The bar magnet has coercivity $4\times10^3\,Am^{-1}$ i.e., it requires a magnetic intensity $H = 4\times10^3\ Am^{-1}$ to get demagnetised. Let i be the current carried by solenoid having n number of turns per metre length, then by definition H = ni. Here, $H = 4\times10^3\ A\ m^{-1}$

$n = \frac{N}{l} = \frac{60}{0.12} = 500$ turn metre^{-1}

$\Rightarrow i = \frac{H}{n} = \frac{4\times10^3}{500} = 8$ A

17. $\mu = \frac{B}{H} = \frac{(\phi/A)}{H} = \frac{\phi}{HA} = \frac{6\times10^{-4}}{2000\times3\times10^{-4}}$

$\therefore$ $\mu = 10^{-3}$ Wb/A–m

18. $B = \mu M_z$ Also, $B = \frac{\phi}{A}$

$\therefore$ $\mu = \frac{B}{M_z} = \frac{\phi}{AM_z}$

$\therefore$ $\mu = \frac{5\times10^{-5}}{0.5\times10^{-4}\times5000}$

$= 2\times10^{-4}$ Wb/Am

19. Neon atom is diamagnetic. Hence its net magnetic moment is zero.

22. $B = (1+\chi)H$

For paramagnetic materials, χ is small and positive.

For diamagnetic materials, χ is small and negative.

23. $\chi \propto \frac{1}{T}$

$\therefore$ $\frac{\chi_1}{\chi_2} = \frac{T_2}{T_1} \Rightarrow \chi_1 T_1 = \chi_2 T_2$

25. Repelled due to induction of similar poles.

29. Diamagnetic substances are repelled by magnetic field.

33. Needle N_1 is ferromagnetic. Ferromagnetic materials are strongly attracted by magnet.

Needle N_2 is paramagnetic. Paramagnetic materials are weakly attracted by magnet.

Needle N_3 is diamagnetic. Diamagnetic materials are weakly repelled by magnetic.

35. Diamagnetic will be feebly repelled.

Paramagnetic will be feebly attracted.

Ferromagnetic will be strongly attracted.

36. $\chi \propto \frac{1}{T} \Rightarrow \frac{\chi_1}{\chi_2} = \frac{T_2}{T_1}$

$\Rightarrow T_2 = \frac{1.0\times10^{-5}}{1.5\times10^{-5}} \times (273 + 27) = 200$

K = –73 °C

37. $\chi \propto \frac{1}{T} \Rightarrow \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$

$\frac{\chi_2}{0.0075} = \frac{273-73}{273-173}$

$\frac{\chi_2}{0.0075} = \frac{200}{100}$

$\chi_2 = 0.0150$

39. On heating, different domains have net magnetization in them which are randomly distributed. Thus, the net magnetisation of the substance due to various domains decreases to minimum.

40. Soft iron is highly ferromagnetic.

42. Diamagnetic material is repelled by magnetic field. This magnetic field energy of current sources will be converted into potential energy of the rod which is set up by switching on the current source.

43. Magnetic induction at a point inside the solenoid,

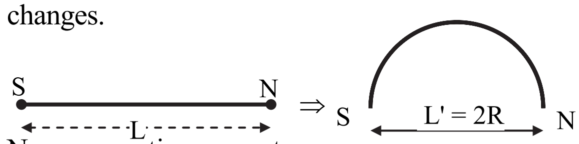
$B = \mu_0 ni = \mu_0 \frac{N}{l} i$

Magnetic flux $\phi = BA = \frac{\mu_0 NiA}{l}$

Magnetic moment

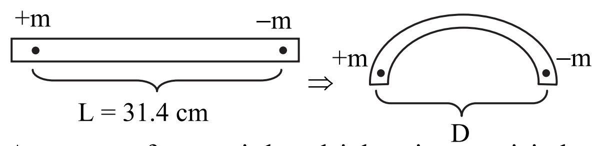
$= NiA = \frac{\phi l}{\mu_0} = \frac{1.57\times10^{-6}\times0.6}{4\pi\times10^{-7}} = 0.75\ Am^2$

46. On bending a rod, its pole strength remains unchanged whereas its magnetic moment changes.



New magnetic moment

$M' = m(2R) = m\left(\frac{2L}{\pi}\right) = \frac{2M}{\pi}$

47. 

As magnet of magnetic length is bent into semicircle,

$L = \pi R \Rightarrow R = \frac{L}{\pi}$

$\therefore$ $D = 2R = \frac{2L}{\pi}$

Magnetic moment (M) = (pole strength) × (Distance between poles)

$\therefore$ $M = mD = m\frac{2L}{\pi}$

$\therefore$ $M = \frac{0.8\times2\times31.4\times10^{-2}}{3.14}$

$\therefore$ $M = 0.16\ Am^2$

48. When a ferromagnetic material is heated above its Curie temperature, then it behaves like paramagnetic material.

49. From the relation, susceptibility of the material is

$\chi = \frac{I}{H}$ $\Rightarrow \chi \propto I$

Thus, greater the value of susceptibility of a material greater will be the value of intensity of magnetisation i.e., more easily it can be magnetised.

Evaluation Test

1. Magnetic field lines avoid passing through diamagnetic materials. Due to this reason, the bar of diamagnetic material aligns perpendicular to the magnetic field

Magnetic field lines prefer passing though the paramagnetic materials. So, the bar of paramagnetic material aligns parallel to the magnetic field.

2. χ (susceptibility) = $\frac{1}{H}$

For paramagnetic substances,

$0 < \chi < E$, where E is a small positive number.

Hence I vs H graph is a straight line with a small positive slope i.e., graph III.

3. Magnetic intensity H = nI = (500)(1)

$= 5 \times 10^2\ Am^{-1}$

Magnetization $M_Z = (B - \mu_0 H)/\mu_0$

$= (\mu_r\mu_0 H - \mu_0 H)/\mu_0$

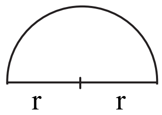
$= (\mu_r - 1)H = (350 - 1)(5 \times 10^2)\ Am^{-1}$

$= 1.75 \times 10^5$

$\approx 1.8 \times 10^5\ Am^{-1}$

4. Time (t) = $\frac{\text{Distance travelled}}{\text{Velocity}}$

$$= \frac{2r + \pi r}{v} = \frac{r(\pi + 2)}{2X}$$



$\therefore$ $I = \frac{q}{t} = \frac{2Q(2X)}{r(\pi + 2)}$

$\therefore$ $M = I \times A = \frac{4QX}{r(\pi + 2)} \times \frac{\pi r^2}{2}$

$\therefore$ $M = \frac{2Q \times \pi r X}{\pi + 2}$

5. The magnetic field inside the toroid in the absence of tungsten, $B_0 = \mu_0 H$

When filled with tungsten, $B = \mu_0(1 + \chi)H$

The increase in field = $B - B_0$

$= \mu_0 \chi H$

The percent increase in the magnetic field

$$= \frac{B - B_0}{B_0} \times 100$$

$$= \frac{\mu_0 \chi H \times 100}{\mu_0 H}$$

$= \chi \times 100$

$= 4.6 \times 10^{-5} \times 100$

$= 4.6 \times 10^{-3}$

Hence, the closest option is (B).

6. The relative permeability of the rod is given by,

$\mu_R = 1 + \chi_m = 1 + 599 = 600$

$\therefore$ The permeability of iron = $\mu = \mu_0 \mu_R$

$\therefore$ $\mu = 4\pi \times 10^{-7} \times 600$

$B = \mu H = 4\pi \times 10^{-7} \times 600 \times 800$

$\therefore$ $B = 192\pi \times 10^{-3}$

$\therefore$ The magnetic flux produced in the coil,

$\phi = BA = 192\pi \times 10^{-3} \times 1 \times 10^{-5}$

$\therefore$ $\phi = 192 \times 3.14 \times 10^{-8} \approx 6 \times 10^{-5}$ Wb

7. The bar magnet has coercivity 4×10^3 Am^{-1} i.e., it requires a magnetic intensity $H = 4 \times 10^3$ Am^{-1} to get demagnetised. Let i be the current carried by solenoid having n number of turns per metre length, then by definition H = ni.

Here, $H = 4 \times 10^3$ Ampere turn metre^{-1}

$n = \frac{N}{l} = \frac{50}{0.10} = 500$ turn metre^{-1}

$\therefore$ $i = \frac{H}{n} = \frac{4 \times 10^3}{500} = 8.0$ A

8. Net dipole moment is, $M_{net} = M_Z \times V$.

Volume of the cylinder $V = \pi r^2 l$, where r is the radius and l is the length of the cylinder, then dipole moment,

$M_{net} = M_Z \pi r^2 l$

$= (4.2 \times 10^3) \times \frac{22}{7} \times (0.6 \times 10^{-2})^2 \times (4 \times 10^{-2})$

$\therefore$ $M_{net} = 1.9 \times 10^{-2}$ J/T

9. In paramagnetic substances, intrinsic magnetic moment is not zero. Further, in the absence of external magnetic field, spin exchange interaction is present.

10. Mean radius = $r = \frac{6 + 8}{2} = 7$ cm

$= 7 \times 10^{-2}$ m

$\therefore$ Number of turns/length,

$n = \frac{N}{2\pi r} = \frac{1500}{2\pi \times 7 \times 10^{-2}} = 3412.19$

As $B = \mu n i$, where B = 2 T and i = 0.5 A

$\therefore$ $\mu = \frac{B}{ni} = \frac{2}{3412.19 \times 0.5}$

$\therefore$ $\mu = 11.7 \times 10^{-4}$ Tm A^{-1}

$\mu_r = \frac{\mu}{\mu_0} = \frac{11.7 \times 10^{-4}}{4\pi \times 10^{-7}} = 931.5$

11. $B = \mu_0 (H + I)$ where, I be intensity of magnetization.

$\therefore$ $I = \frac{B}{\mu_0} - H = \frac{\mu H}{\mu_0} - H$

$= \mu_r H - H = (\mu_r - 1) H$

For a solenoid of n turns per unit length carrying current i; H = ni.

$\therefore$ $I = (\mu_r - 1)$ ni

Here, n = 6 turns/cm = 600 turns/m

$I = (900 - 1) \times 600 \times 0.4$

$\therefore$ $I \approx 2.16 \times 10^5$ Am^{-1}

As magnetic moment, $M = I \times V$

$\therefore$ $M = 2.16 \times 10^5 \times 10^{-4} = 21.6$ Am^2

12. On passing current through the coil, it acts as a magnetic dipole. Torque acting on magnetic dipole is counter balanced by the moment of additional weight about position O. Torque acting on a magnetic dipole,

$\tau = MB \sin\theta = (NiA)B \sin 90° = NiAB$

Again, τ = Force × Lever arm = $\Delta mg \times l$

$\therefore$ $NiAB = \Delta mgl$

$\therefore$ $B = \frac{\Delta mgl}{NiA}$

$= \frac{40 \times 10^{-6} \times 9.8 \times 20 \times 10^{-2}}{100 \times 18 \times 10^{-3} \times 1 \times 10^{-4}}$

$\therefore$ B = 0.44 T

13. From the relation susceptibility of the material is

$$\chi = \frac{I}{H}$$

$\Rightarrow$ $\chi \propto I$

Thus, greater the value of susceptibility of a material greater will be the value of intensity of magnetisation i.e., more easily it can be magnetised.

14. Iron is ferromagnetic in nature. Lines of force due to external magnetic field prefer to pass through iron.

Textbook Chapter No.

16 Electromagnetic Induction

Hints

Classical Thinking

6. $\phi = BA\cos\theta = 5 \times 10^{-2} \times 0.2 \times \cos 60°$
$= 5 \times 10^{-3}$ Wb

13. Since $e \propto B$, so by reducing magnetic field to half, induced e.m.f. will also be reduced to half.

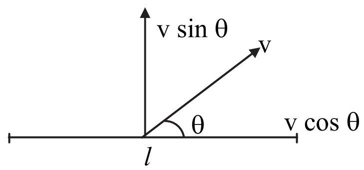
15. $|e| = \frac{d\phi}{dt} = \frac{240}{2\times60} = 2$ V

16. $|e| = \frac{d\phi}{dt} = \frac{3\times10^{-3} - 2\times10^{-3}}{25}$
$= 0.04 \times 10^{-3} = 0.04$ mV

17. e.m.f. induced between ends of conductor,
$e = Blv = 5 \times 10^{-3} \times 1.5 \times 5 = 37.5 \times 10^{-3}$ V

18. $|e| = nA\frac{dB}{dt} = 100 \times 10^{-2} \times 10^3 = 10^3$ volt

19. $E = Blv \sin\theta$



32. $\phi = LI \Rightarrow L = \frac{\phi}{I} = \frac{y}{x}$ henry

33. $\phi = LI = 5 \times 10^{-3} \times 2 = 0.01$ weber

34. $L = \frac{\phi}{I} = \frac{10\times10^{-6}}{2.5\times10^{-3}} = 4 \times 10^{-3}$ H = 4 mH

35. $\phi = LI = 2 \times 5.8 = 11.6$ Wb

36. $e = -L\frac{dI}{dt} = -(2) \times (-0.5) = +1$ V

37. $e = L\frac{dI}{dt} = 1 \times 10^{-3} \times \frac{(5-3)}{10^{-3}} = 2$ V

38. Inductance of coil,
$L = \frac{e}{\left(\frac{dI}{dt}\right)} = \frac{8}{\left(\frac{8-4}{0.1}\right)} = 0.2$ H

41. $M = \frac{\phi}{I} = \frac{200}{5} = 40$ H

42. $M = \frac{e}{\left(\frac{dI}{dt}\right)} = \frac{e.dt}{dI} = \frac{1000\times0.01}{2} = 5$ H

60. $e = 100 \sin(100\pi t + 0.6)$
Comparing with the standard form,
$e = e_0 \sin(\omega t + \theta)$ we get,
Peak volt $= e_0 = 100$ V

61. $e_0 = e_{rms} \times \sqrt{2} = 220 \times \sqrt{2} \approx 311$ volt

62. $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$ A

68. $X_L = \omega L$
$\therefore \quad \omega = \frac{X_L}{L} = \frac{1}{10^{-3}} = 1000$

69. Impedance of circuit, $Z = X_C$
$\therefore \quad Z = \frac{1}{2\pi fC} = \frac{1}{2\pi\times50\times50\times10^{-6}} \approx 63.7\ \Omega$

70. The impedance of combination,
$Z = \left(2\pi fL - \frac{1}{2\pi fC}\right)$
$= 2\pi \times 50 \times 1.2 - \frac{1}{2\pi\times50\times10^{-5}}$
$= 376.8 - 318.5 = 58.3\ \Omega$

71. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$
$\therefore \quad X_C = \infty$ ....[f = 0 for D.C]

72. $Z = \sqrt{R^2 + X_L^2} = 10\sqrt{2}\ \Omega$
$V_0 = \sqrt{2}\ V = 220\sqrt{2}$ V
$\therefore \quad I_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{10\sqrt{2}} = 22$ A

79. e = 100 sin (100t) and I = 100 sin(100t)
Comparing these equations with the standard forms,
$e = e_0 \sin\omega t$ and
$I = I_0 \sin\omega t$ we get,
$e_0 = 100$ V and
$I_0 = 100 \times 10^{-3}$ A

$$P = e_{rms} I_{rms} = \frac{e_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}}$$

$$= \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} = \frac{10}{2} = 5 \text{ W}$$

87. $|e| = M\frac{dI}{dt}$

$\therefore$ $15 \times 10^{-3} = M \times \frac{3}{10} \Rightarrow M = 0.05$ H

88. $\phi = nBA \cos\theta$
$\because$ Plane of the loop is at right angles to the field.
$\Rightarrow \theta = 90°$
$\therefore$ $\phi = 1 \times 4 \times 10^{-3} \times 0.4 \times \cos 90° = 0$

89. $e = -M\frac{dI}{dt} = 4 \times \frac{5}{\left(\frac{1}{1500}\right)} = 6000 \times 5 = 30$ kV

Critical Thinking

1. $\phi = nBA = 10^3 \times 10^{-2} \times 10^{-4} = 10^{-3}$ weber

2. $\phi = nAB \cos\theta = 1 \times 0.5 \times 4 \times \cos 60°$
$= 2 \times \frac{1}{2} = 1$ weber

3. $|e| = \frac{d\phi}{dt} = \frac{1 - 0.1}{0.1} = \frac{0.9}{0.1} = 9$ V

$\therefore$ $I = \frac{e}{R} = \frac{9}{100} = 0.09$ A

4. $\phi = BA \cos\omega t = \frac{\mu_0 IA}{2R} \cos\omega t$

$$= \frac{4 \times 3.14 \times 10^{-7} \times 10 \times 10^{-4}}{2 \times 0.628} \cos\omega t$$

$= 10^{-9} \cos\omega t$

5. $I = \frac{e}{R} = \frac{\left(\frac{d\phi}{dt}\right)}{R} = \frac{1}{R}\frac{d}{dt}(4t^2 - 4t + 1)$

$\therefore$ $I = \frac{8t - 4}{R} = \frac{8 \times (1/2) - 4}{10} = 0$

6. $|e| = \frac{\phi_2 - \phi_1}{t} = \frac{B_2A_2 - B_1A_1}{t}$

$$= \frac{1.8 \times (100 \times 10^{-4}) - 1.0 \times (\frac{22}{7} \times 49 \times 10^{-4})}{0.1}$$

= 26 mV

7. Since the magnetic field is uniform, the flux ϕ through the square loop at any time t is constant, because
$\phi = B \times A = B \times L^2$ = constant.
$\therefore$ $e = -\frac{d\phi}{dt}$ = zero

8. $e = Blv \Rightarrow IR = Blv$

$\therefore$ $v = \frac{IR}{Bl}$

$= \frac{I\rho l}{BA.l}$ ....$\left(R = \frac{\rho l}{A}\right)$

$= \frac{I\rho}{BA} = \frac{3 \times 10^{-3} \times 9 \times 10^{-6}}{2 \times 1.8 \times 10^{-7}}$

$= \frac{27 \times 10^{-9}}{36 \times 10^{-8}} = \frac{3}{4} \times 10^{-1} = 0.075$

$\therefore$ $v = 7.5 \times 10^{-2}$ m/s

9. $\theta = 90° - 30° = 60°$, $1\text{ T} = 10^4$ G
$\phi = nAB \cos\theta$
$\therefore$ $\phi = 100 \times (\pi \times 10^{-4}) \times (10^6 \times 10^{-4}) \times \cos 60°$
$= 100 \times \pi \times 10^{-2} \times \frac{1}{2} = 0.5\pi$ Wb

10. $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(6t^2 - 5t + 1)$
$= -(12t - 5)$
As t = 0.25 s, e = – [12 (0.25) – 5]
= – (3 – 5) = 2 V

$\therefore$ $I = \frac{e}{R} = \frac{2}{10} = 0.2$ A

11. $e = \frac{-n(\phi_2 - \phi_1)}{t}$

$$= \frac{-50(1 \times 10^{-6} - 31 \times 10^{-6})}{0.02}$$

$= 7.5 \times 10^{-2}$ V

12. $e = -\frac{n(B_2 - B_1)A\cos\theta}{t}$

$\therefore$ $t = \frac{-50 \times (0 - 2 \times 10^{-2}) \times 100 \times 10^{-4} \times \cos 0°}{0.1}$

$\therefore$ t = 0.1 s

13. $d\phi = n\,AB = 10 \times 4 \times 10^{-2} \times 10^{-2} = 4 \times 10^{-3}$ Wb

∴ $|e| = \frac{d\phi}{dt} = \frac{4\times10^{-3}}{0.5} = 8 \times 10^{-3}$ V = 8 mV

14. $e = v_t Bl = (v \sin\theta) Bl = vBl \sin 30°$

$= 10 \times 0.5 \times 1 \times \frac{1}{2} = 2.5$ V

15. $|e| = nA\frac{dB}{dt}$

$= 100 \times 50 \times 10^{-4} \times \frac{(0.1 - 0.05)}{0.05} = 0.5$ V

16. $e = -\frac{d\phi}{dt} = -nA\frac{dB}{dt}$

Now, $\frac{dB}{dt} = 10^8\ \frac{\text{gauss}}{\text{s}} = 10^4\ \frac{\text{tesla}}{\text{s}}$

∴ $e = -10 \times 10^{-3} \times 10^4 = -100$ V

∴ e = 100 volt (numerically)

∴ $I = \frac{e}{R} = \frac{100}{20} = 5$ ampere

18. As ϕ through coil is constant and there is no relative motion between magnet and coil, neither e.m.f. nor current is induced in coil.

19. As I increases, ϕ increases

∴ I_i is such that it opposes the increase in ϕ. Hence ϕ decreases (By Right Hand Rule). The induced current will be counter clockwise.

20. $|e| = \frac{d\phi}{dt} = \frac{BdA}{dt} = B \times \frac{(\pi r^2 - L^2)}{dt}$

$= 6.6 \times 10^{-3}$ V

22. $|e| = L\frac{dI}{dt} \Rightarrow L = e\frac{dt}{dI}$

$dI = 2 - (-2) = 4$ A

∴ $L = \frac{8 \times 0.05}{4} = 0.1$ H

23. $|e| = L\frac{dI}{dt}$ or $L \propto dt$

∴ $\frac{L_1}{L_2} = \frac{dt_1}{dt_2} = \frac{5}{50\times10^{-3}} = 100 : 1$

24. $L = \frac{\mu_0 N^2 A}{l}$

where N is the total number of turns.

As $L \propto N^2$

∴ $\frac{L_2}{L_1} = \left(\frac{N_2}{N_1}\right)^2 = (2)^2$

∴ $L_2 = 4L_1$

25. $L = \frac{\mu_0 N^2 A}{l}$ or $L \propto N^2$

∴ $\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2$

∴ $\frac{108}{L_2} = \left(\frac{600}{500}\right)^2$

∴ $L_2 = 75$ mH

26. Let $\phi_1 = \phi_2 = \phi$

∵ $L = \frac{\phi}{I} \Rightarrow I = \frac{\phi}{L}$

∴ $I_1 = \frac{\phi}{L_1}, I_2 = \frac{\phi}{L_2}$

∴ $\frac{I_1}{I_2} = \frac{\left(\frac{\phi}{L_1}\right)}{\left(\frac{\phi}{L_2}\right)} = \frac{L_2}{L_1} = \frac{2\times10^{-3}}{8\times10^{-3}} = \frac{1}{4}$

27. $L = \mu_0 nI$

∴ $\frac{L_2}{L_1} = \frac{\mu}{\mu_0}$ ....(∵ n and I are same)

∴ $L_2 = \mu_r L_1 = 900 \times 0.18 = 162$ mH

28. $\tan\theta = \frac{2\pi fL}{R} = \frac{2\pi \times 200 \times 1}{300 \times \pi} = \frac{4}{3}$

∴ $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

29. $e = -M\frac{dI}{dt} = -M \times \frac{I_2 - I_1}{t}$

$= -4 \times \frac{0-5}{10^{-3}} = 2 \times 10^4$ V

30. $V_P = V_I = 300$ volt,

$V_S = V_o = 15$ kV $= 15 \times 10^3$ volt

∴ $\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{300}{15\times10^3} = \frac{2}{100} = \frac{1}{50}$

31. $\frac{V_S}{V_P} = \frac{N_S}{N_P} \Rightarrow V_S = \frac{N_S}{N_P} \times V_P$

$= \frac{500}{100} \times 220 = 1100$ volt

32. $\frac{V_P}{V_S} = \frac{N_P}{N_S}$

∴ $N_P = \left(\frac{220}{2200}\right) \times 2000 = 200$

33. $\frac{N_S}{N_P} = \frac{V_S}{V_P}$

∴ $\frac{200}{100} = \frac{V_S}{120} \Rightarrow V_S = 240$ V

$\frac{V_S}{V_P} = \frac{I_P}{I_S}$

∴ $\frac{240}{120} = \frac{10}{I_S} \Rightarrow I_S = 5$ A

34. $\frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{500}{2500} = \frac{1}{5}$

∴ $V_P = \frac{200}{5} = 40$ V

Also, $I_PV_P = I_SV_S$

∴ $I_P = I_S\frac{V_S}{V_P} = 8 \times 5 = 40$ A

35. $\frac{N_S}{N_P} = \frac{V_S}{V_P}$

∴ $\frac{1}{20} = \frac{V_S}{2400} \Rightarrow V_s = 120$ V

For 100% efficiency, $V_SI_S = V_PI_P$

∴ $120 \times 80 = 2400\ I_P \Rightarrow I_P = 4$ A

36. For 100% efficient transformer, $V_SI_S = V_PI_P$

∴ $\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P}$

∴ $\frac{I_P}{4} = \frac{25}{100}$

∴ $I_P = 1$ A

38. $P_P = P_S = I_SE_S$

∴ $I_S = \frac{P_S}{E_S} = \frac{2000}{200} = 10$ A

Now, $\frac{N_S}{N_P} = \frac{I_P}{I_S}$

∴ $N_S = \frac{N_PI_P}{I_S} = \frac{1000 \times 0.1}{10} = 10$

39. $\phi_B = BA\cos\theta$

where θ is the angle between normal to the plane of the coil and magnetic field.

Induced e.m.f.,

∴ $e = BA\sin\theta$

$\theta = 0°$ ….[Given]

∴ Magnetic flux is maximum and induced e.m.f. is zero.

40. $e_0 = nAB\omega = nAB.2\pi f = 2(nA) \times B \times \pi \times f$

$= 2 \times 2 \times 7 \times 10^{-5} \times \frac{22}{7} \times 100 = 88$ mV

41. $e = \frac{2nAB}{t} = \frac{2 \times 10^3 \times 0.05 \times 4 \times 10^{-3}}{0.01} = 40$ V

42. $e_0 = nAB\omega = 2\pi fnAB$

$= 2 \times \pi \times \frac{2000}{60} \times 50 \times 80 \times 10^{-4} \times 0.05$

$= \frac{4\pi}{3}$ V

43. $e_0 = 2\pi f\,nAB$

$= 2 \times \pi \times \left(\frac{600}{60}\right) \times (5000) \times (50 \times 10^{-4}) \times 8 \times 10^{-4}$

$= 12560 \times 10^{-4} = 1.256$ V

44. $e_0 = \omega nBA = (2\pi f)nBA$

$= 2 \times 3.14 \times 100 \times 5000 \times 0.2 \times 0.25$

$= 157$ kV

45. $\theta = \omega t = 90°$, $n = \frac{400}{60} = \frac{20}{3}$ r.p.s.

Alternating current induced in the coil is given by,

$I = I_0 \sin\omega t = \frac{2\pi fnBA}{R} \times \sin 90°$

$= \frac{2 \times \pi \times 20 \times 1 \times 10^{-3} \times \pi(0.4)^2}{3 \times \pi^3} \times 1$

$= 6.79 \times 10^{-4}$ A

$= 0.68$ mA

46. General equation for instantaneous e.m.f. is,

$e = e_0 \sin(\omega t + \phi) = 200 \sin(2\pi\, 50t)$

$= 200 \sin(100\,\pi t)$

47. The instantaneous current in a circuit is,

$I = \sin(\omega t + \phi)$

As $I = I_0 \sin(\omega t + \phi)$

∴ $I_0 = 1$ A

∴ $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ A

48. $I_{rms} = \frac{e_{rms}}{X_C} = \frac{e_0\omega C}{\sqrt{2}} = \frac{100\sqrt{2} \times 100 \times 0.5 \times 10^{-6}}{\sqrt{2}}$

$= 5 \times 10^{-3}$ A $= 5$ mA

49. $I_{peak} = I_0 = I_{r.m.s} \times \sqrt{2} = 10\sqrt{2}$ A

50. $I_{rms} = \frac{V_{rms}}{R} = \frac{200}{40} = 5$ A

$I_0 = \sqrt{2}\ I_{rms} = 1.414 \times 5 \approx 7.1$ A

51. $V_L^2 = V^2 - V_R^2 = (20)^2 - (12)^2$
$= 400 - 144 = 256$
$\therefore$ $V_L = 16$ V

52. Phase difference relative to the current,
$\phi = \left(314t - \frac{\pi}{6}\right) - 314t = -\frac{\pi}{6}$ rad

53. Time taken by the current to reach the maximum value $t = \frac{T}{4} = \frac{1}{4f} = \frac{1}{4 \times 50} = 5 \times 10^{-3}$ s
and $I_0 = I_{rms}\sqrt{2} = 10\sqrt{2} = 14.14$ A

54. $e_{rms} = \frac{e_0}{\sqrt{2}} = \frac{2\pi fnAB}{\sqrt{2}} = \sqrt{2}\,\pi fnAB$
$= \sqrt{2} \times \pi \times \left(\frac{1000}{60}\right) \times 50 \times (30 \times 10^{-4}) \times 5 \times 10^{-4}$
≈ 5.55 mV

55. At resonance, $V_L = V_C$
$\therefore$ $V_T = V_R = 100$ V

56. $X = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 10 \times 10^{-6}}$
$\approx 318.5\ \Omega$
$I_{rms} = \frac{V}{X_C} = \frac{200}{318.5} = 0.6$ A
$I_p = I_{rms} \times \sqrt{2} = 0.6 \times \sqrt{2}$ A

57. $X_C = \frac{1}{2\pi fC} \Rightarrow X_C \propto \frac{1}{f}$
$\therefore$ $\frac{X_C'}{X_C} = \frac{f}{f'} = \frac{50}{200} = \frac{1}{4}$
$\therefore$ $X_C' = \frac{X_C}{4} = \frac{10}{4} = 2.5\ \Omega$

58. $X_C \propto \frac{1}{f}$
$\therefore$ $\frac{X_{C_2}}{X_{C_1}} = \frac{f_1}{f_2} = \frac{1}{2}$
$\therefore$ $X_{C_2} = \frac{10}{2} = 5\ \Omega$

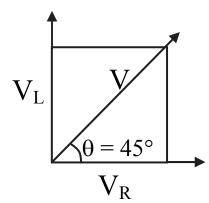
59. $\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$
$= \tan^{-1}\left(\frac{2\pi fL - \frac{1}{2\pi fC}}{R}\right)$
$= \tan^{-1}\left(\frac{2\pi \times 50 \times \frac{2}{\pi} - \frac{\pi}{2\pi \times 50 \times 10^{-6}}}{10}\right)$
$= -90°$ (approx)

60. $X_L \propto f_L \Rightarrow \frac{X_{L_2}}{X_{L_1}} = \frac{f_2L_2}{f_1L_1} = \frac{2f_1 \times 2L_1}{f_1L_1} = 4$
$\therefore$ $X_{L_2} = 4 \times 1000 = 4000\ \Omega$

61. $\tan\phi = \frac{X_L - X_C}{R} = \frac{300 - 200}{100} = \frac{100}{100}$
$\therefore$ $\tan\phi = 1 \Rightarrow \phi = 45°$

62. $E = \sqrt{V_R^2 + (V_C - V_L)^2}$
$= \sqrt{(40)^2 + (80 - 40)^2}$
$= \sqrt{1600 + 1600}$
$= \sqrt{2(1600)} = 40\sqrt{2}$ V

63. Figure below shows the graph for the given case
$\tan 45° = \frac{X_L}{X_R} = \frac{V_L}{V_R} = \frac{\omega L}{R}$
$\therefore$ $\omega L = R$
$\therefore$ $L = \frac{R}{\omega} = \frac{R}{2\pi f}$
$= \frac{100}{2 \times 3.14 \times 1000}$
$= \frac{100}{6.28} \times 10^{-3}$
$\approx 16 \times 10^{-3}$ H $= 16$ mH



64. $I = \frac{E}{Z}$
$\therefore$ $Z = \frac{E}{I} = \frac{50}{2} = 25$
$Z^2 = R^2 + (X_C \sim X_L)^2$
$\therefore$ $25^2 = 20^2 + (X_C - X_L)^2$
$\therefore$ $(X_C - X_L)^2 = 625 - 400 = 225$
$\therefore$ $X_C - X_L = 15$
$\therefore$ $X_C = X_L + 15 = 10 + 15 = 25\ \Omega$

65. For D.C., $R = \frac{100}{1} = 100\ \Omega$
For A.C. $Z = \frac{100}{0.5} = 200\ \Omega$
Now, $Z^2 = R^2 + X_L^2$
$\therefore$ $X_L^2 = (200)^2 - (100)^2$
$= 40000 - 10000 = 30000$

$\therefore\quad X_L = \sqrt{30000} = 173.2\ \Omega$

$X_L = 2\pi fL$

$\therefore\quad L = \dfrac{X_L}{2\pi f}$

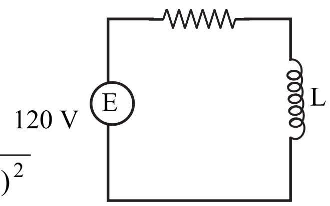
$= \dfrac{173.2}{2\times3.14\times50} = 0.55$ H

66. $R = \dfrac{120}{0.5}$

$= 240\ \Omega$

Effective impedance for A.C. source,

$Z = \dfrac{120}{0.40} = 300\ \Omega$



Using, $Z^2 = R^2 + X_L^2$

$X_L = \sqrt{Z^2 - R^2}$

$= \sqrt{(300)^2 - (240)^2}$

$= 180\ \Omega$

$\therefore\quad 2\pi fL = 180$

$\therefore\quad L = \dfrac{180}{2\pi f}$

$= \dfrac{180}{2\pi(60)} = \dfrac{1.5}{\pi} \approx 0.48$ H

67. $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + \left[\omega L - \dfrac{1}{\omega C}\right]^2}}$

As resistance is negligible, $R \to 0$

$\therefore\quad I = \dfrac{V}{\left(\omega L - \dfrac{1}{\omega C}\right)}$

Now, $\omega L - \dfrac{1}{\omega C} = \dfrac{V}{I} = \dfrac{100}{5}$

$= 20\ \Omega$ ….(i)

If the value of capacitor is decreased to half then,

$\omega L - \dfrac{1}{\left(\omega \dfrac{C}{2}\right)} = \dfrac{100}{10} = 10\ \Omega$

$\omega L - \dfrac{2}{\omega C} = 10\ \Omega$ ….(ii)

By equation (i) – equation (ii), we get

$\dfrac{1}{\omega C} = 10\ \Omega$

$\therefore$ Voltage across capacitor

= I × Resistance across capacitor

= 5 × 10

= 50 V

68. $e = 5 \sin(\omega t + 90)$ and

$I = 2 \sin\omega t$

There is phase difference of $\dfrac{\pi}{2}$ between E and $I \Rightarrow P = 0$

69. Average power lost / cycle

$= e_{rms}I_{rms} \cos\theta = \dfrac{e_0}{\sqrt{2}}\dfrac{I_0}{\sqrt{2}} \cos\theta = \dfrac{1}{2} e_0 I_0 \cos\theta$

70. Power dissipation in pure inductive and capacitive circuit is zero.

71. $R = \dfrac{V}{I} = \dfrac{12}{4} = 3\ \Omega$

$\therefore\quad Z = \dfrac{E_{rms}}{I_{rms}} = \dfrac{12}{2.4} = 5$

$Z^2 = R^2 + X_L^{\,2}$

$\therefore\quad X_L^{\,2} = 25 - 9 = 16$

$\therefore\quad X_L = \omega L = 4$

$\therefore\quad L = \dfrac{4}{50} = 0.08\ \text{H} = 8 \times 10^{-2}$ H

72. $R = 40 + 40 = 80\ \Omega$

$\therefore\quad X_L - X_C = 100 - 40 = 60\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{80^2 + 60^2} = 100$

$\therefore$ Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{80}{100} = 0.8$

73. $P_{avg} = \dfrac{e_0 I_0 \cos\theta}{2} = \dfrac{100\times10^{-1}\times\cos\left(\dfrac{\pi}{2}\right)}{2} = 0$

74. $I_{WL} = I_{rms} \sin\phi$

$\therefore\quad \sqrt{3} = 2 \sin\phi \Rightarrow \sin\phi = \dfrac{\sqrt{3}}{2} \Rightarrow \phi = 60°$

$\therefore$ Power factor $= \cos\phi = \cos 60° = \dfrac{1}{2}$

75. $\cos\phi = \dfrac{1}{2}$

$\phi = 60°,\ \tan 60° = \dfrac{\omega L}{R}$

$\therefore\quad L = \dfrac{\sqrt{3}R}{\omega} = \dfrac{\sqrt{3}\times100}{2\pi\times50} = \dfrac{\sqrt{3}}{\pi}$ H

76. $Z = \sqrt{R^2 + \left(\dfrac{1}{2\pi fC}\right)^2}$

$= \sqrt{(3000)^2 + \dfrac{1}{\left(2\pi\times50\times\dfrac{2.5}{\pi}\times10^{-6}\right)^2}}$

$\therefore\quad Z = \sqrt{(3000)^2 + (4000)^2} = 5 \times 10^3\ \Omega$

$\therefore$ Power factor, $\cos\phi = \frac{R}{Z} = \frac{3000}{5\times10^3} = 0.6$

Power dissipated, $P = e_{rms} I_{rms} \cos\phi = \frac{v_{rms}^2 \cos\phi}{Z}$

$\therefore$ $P = \frac{(200)^2 \times 0.6}{5\times10^3}$

$= 4.8$ W

77. The current will lag behind the voltage when reactance of inductance is more than the reactance of condenser. Thus, $\omega L > \frac{1}{\omega C}$ or $\omega > \frac{1}{\sqrt{LC}}$ or $n > \frac{1}{2\pi\sqrt{LC}}$ or $n > n_r$ where n_r = resonant frequency.

78. Given that, $X_L = X_C$

$\therefore$ $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times10^{-3}\times10\times10^{-6}}}$

$= \frac{1}{\sqrt{4\times10^{-8}}} = \frac{1}{2\times10^{-4}}$

$= \frac{10^4}{2} = 5\times10^3$

79. $f = \frac{1}{2\pi\sqrt{LC}}$

$f = \frac{1}{2\pi\sqrt{2\times2\times10^{-6}}} \approx 80$ Hz

80. Using, $f_r = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\times\pi\sqrt{100\times10^{-6}\times4\times10^{-8}}}$

$= \frac{1}{2\pi\times2\times10^{-6}} = \frac{10^6}{4\pi} = \frac{25}{\pi}\times10^4$ Hz

81. $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{9\times10^{-3}\times10\times10^{-6}}}$

$= \frac{1}{2\pi\times3\times10^{-4}} = \frac{10000}{6\times3.14} = 0.530$ kHz

82. $e = Blv = 0.15\times0.5\times2 = 0.15$ V

$\therefore$ $I = \frac{e}{R} = \frac{0.15}{3} = 0.05$

$\therefore$ $F = BIl = 0.15\times0.05\times0.5 = 3.75\times10^{-3}$ N

83. Component of the length perpendicular to the field $l' = l\sin60°$

$= 1.0\times\left(\frac{\sqrt{3}}{2}\right) = 0.5\sqrt{3}$

$\therefore$ $e = l'Bv = 0.5\sqrt{3}\times0.5\times10$

$= 4.3$ volt

84. $e = e_0\sin(\omega t+\phi)$

$\therefore$ $e_{rms} = \frac{e_0}{\sqrt{2}} = \frac{200}{\sqrt{2}}$

$\therefore$ Power, $P = e_{rms} I_{rms}\cos\phi$

$\therefore$ $I_{rms} = \frac{P}{e_{rms}\times\cos\phi} = \frac{1000\sqrt{2}}{200\times\cos60^\circ}$

$= 10\sqrt{2}$ A

85. $X_L = \omega L = 2\pi fL = 2\pi\times50\times0.7 \approx 220\ \Omega$

$Z = \sqrt{R^2+X_L^2} = \sqrt{220^2+220^2} = 220\sqrt{2}$ ohm

$\therefore$ $I_v = \frac{e_v}{Z} = \frac{220}{220\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707$ A

86. Heat produced by A.C. = 3 × Heat produced by D.C

$\therefore$ $I_{rms}^2 Rt = 3\times I^2Rt$

$I_{rms}^2 = 3\times2^2$

$\therefore$ $I_{rms} = 2\sqrt{3} = 3.46$ A

87. $dq = \frac{d\phi}{R} = I\,dt$ = Area under I – t graph

$\therefore$ $d\phi = R\times$ (Area under I – t graph)

$= 10\times\frac{1}{2}\times4\times0.1 = 2$ weber

88. If a horizontal straight conductor placed along N-S falls under gravity, then there is no induced e.m.f. along the length of the conductor as there is no change in flux.

89. When magnet falls through ring, there is change of flux associated with the ring. It produces induced e.m.f. and hence induced current. By Lenz's law, the current flows in such a direction so as to produce an induced e.m.f. which opposes the falling magnet. Acceleration of magnet is less than acceleration due to gravity.

90. When there is a cut in the ring, e.m.f. will be induced in it but there is no induced current in the ring. Hence there is no opposition to falling magnet. Therefore, acceleration is equal to 'g'.

91. $\phi = nBA\cos\theta = 10\ Ba^2\cos\omega t$

$\therefore$ $e = -\frac{d\phi}{dt} = \frac{d}{dt}(10Ba^2\cos\omega t) = 10\ Ba^2\ \omega\sin\omega t$

92. Comparing given equation with the standard form,

$e = e_0\sin\omega t$ we get,

$e = 200\sin 100\pi$

$e_0 = 200,\ \omega = 100\ \pi$

Now, $e_0 = nAB\omega$

$\therefore$ $B = \frac{e_0}{An\omega}$

$= \frac{200}{(0.25\times0.25)\times1000\times100\pi} = 0.01\ T$

93. $Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$

From above equation at $f = 0 \Rightarrow z = \infty$

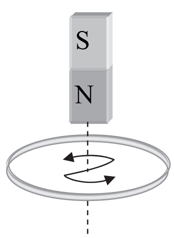
When $f = \frac{1}{2\pi\sqrt{LC}}$ (resonant frequency)

$\Rightarrow Z = R$

For $f > \frac{1}{2\pi\sqrt{LC}} \Rightarrow Z$ starts increasing.

i.e., for frequency $0 - f_r$, Z decreases and for f_r to ∞, Z increases. This is justified by graph C.

94. It is evident that when viewed from the magnet side, the induced current will be anticlockwise.



95. $B = 1.25\ mT = 1.25 \times 10^{-3}\ T$

$e = Blv$

$\therefore$ The mechanical power required,

$P = eI = BlvI$

$= 1.25 \times 10^{-3} \times 0.1 \times 1 \times 50$

$= 6.25 \times 10^{-3}\ W$

$= 6.25\ mW$

96. $e = nBA\omega\sin\omega t$

Given that, $n = 1,\ B = \frac{\mu_0 I}{2b},\ A = \pi a^2$

$\therefore$ $e = \frac{\mu_0 I}{2b}(\pi a^2)\omega\sin(\omega t)$

Competitive Thinking

1. The energy of the field increases with the magnitude of the field. Lenz's law infers that there is an opposite field created due to increase or decrease of magnetic flux around a conductor so as to hold the law of conservation of energy.

2. Considering that the electron is moving from left to right, the flux linked with the loop (directed into the page) will first increase and then decrease as the electron passes by. Hence the induced current in the loop will be first anticlockwise and will change its direction as the electron passes by.

3. When e^- is coming towards the loop, magnetic flux of one type increases and when going away, the same magnetic flux decreases. So induced current opposite will reverse its direction as e^- goes past the coil.

4. If the current increases with time in loop A, then magnetic flux in B will increase. By Lenz's law, loop-B will be repelled by loop-A.

5. $\phi \propto I$

If solenoid is pulled out then flux decreases resulting into decrease in the value of current.

6. $\phi = BA = 10^3 \times 10^{-2} = 10$ weber

7. $e = -\frac{d\phi}{dt}$

$= -(10t - 4)$

$\therefore$ $e = -(10 \times 0.2 - 4) = 2$ volt

8. $|e| = \frac{d\phi}{dt} = \frac{d}{dt}(5t^2 + 3t + 16) = (10t + 3)$

When t = 3 s, $e_3 = (10 \times 3 + 3) = 33$ V

When t = 4 s, $e_4 = (10 \times 4 + 3) = 43$ V

Hence e.m.f. induced in fourth second

$= e_4 - e_3 = 43 - 33 = 10$ V

9. $|e| = \frac{d\phi}{dt} = \frac{d}{dt}(3t^2 + 4t + 9) = 6t + 4$

at t = 2 sec,

$|e| = 16$ V

10. $e = -\frac{d\phi}{dt} = -(100t)$

At t = 2 s,

$I = \left|\frac{e}{R}\right| = \frac{100\times2}{400} = 0.5$ A

11. $|e| = \frac{d\phi}{dt} = A\frac{dB}{dt}$

∴ $|e| = A\left(\frac{\frac{3}{4}B}{2}\right) = \frac{3AB}{8}$

12. $\phi_1 = 4 \times 10^{-4}$ Wb

$\phi_2 = 0.1\ \phi_1 = 0.4 \times 10^{-4}$ Wb

∴ $d\phi = |\phi_2 - \phi_1| = 3.6 \times 10^{-4}$ Wb

dt = t second

$e = \frac{d\phi}{dt}$

∴ $0.72 \times 10^{-3} = \frac{3.6 \times 10^{-4}}{t}$

∴ $t = \frac{3.6 \times 10^{-4}}{0.72 \times 10^{-3}}$ ∴ t = 0.5 second

13. $\phi = n \times A \times B$

∴ $e = \frac{d\phi}{dt} = nA\frac{dB}{dt}$

$= 200 \times 0.15 \times \frac{(0.6-0.2)}{0.4} = 30$ V

14. $|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\cdot\frac{dB}{dt} = 1 \times \frac{B}{0.2} = 5B$

15. $e = -\frac{d\phi}{dt}$

$\phi = B.A$

Here $A = \pi r^2$ as magnetic field is restricted to region of radius r.

∴ $e = -\pi r^2 . \frac{d\vec{B}}{dt}$ in loop 1

As the loop 2 is outside the region of magnetic field, e = 0 for loop 2.

16. $I = \frac{e}{R} = \frac{-n(\phi_2 - \phi_1)}{R\ \ t}$ ...($\because e = -n\frac{d\phi}{dt}$)

$= \frac{-n(\phi_2 - \phi_1)}{(R+4R)t} = \frac{-n(\phi_2 - \phi_1)}{5Rt}$

17. $|e| = \frac{d\phi}{dt}$

$iR = \frac{d\phi}{dt}$

∴ $\int d\phi = R\int I\,dt$

This means

$|d\phi|$ = Resistance × area under current – time graph

$= 100 \times \frac{1}{2} \times 10 \times 0.5 = 250$ Wb

18. The magnitude of induced e.m.f. is given by

$|e| = Blv$

v = 300 m/min = 5 m/s

∴ $B = \frac{|e|}{lv} = \frac{2}{0.5 \times 5} = 0.8$ tesla.

19. $e \propto \omega$

20. $e = Bvl = 0.1 \times 15 \times 0.1 = 0.15$ V

(Considering B, l and v are mutually perpendicular.)

21. Induced emf, $e = Blv$

$= 5 \times 10^{-4} \times 0.1 \times 5$

$= 2.5 \times 10^{-4}$ V/s

22. $e = B \times v \times l$

$= 5.0 \times 10^{-5} \times 1.50 \times 2$

$= 10.0 \times 10^{-5} \times 1.5$

$= 15 \times 10^{-5}$

= 0.15 mV

23. $e = Blv \sin\alpha$

$= B(2r)v \sin\alpha$

24. $v = 1080$ km/hr $= 1080 \times \frac{5}{18} = 300$ m/s

Induced emf

$e = B\,l\,v = 1.75 \times 10^{-5} \times 40 \times 300 = 0.21$ V

25. The e.m.f is induced when there is change of flux. As in this case there is no change of flux, hence no e.m.f. will be induced in the wire.

26. The e.m.f. induced is directly proportional to rate at which flux is intercepted which in turn varies directly as the speed of rotation of the generator.

∴ $e \propto f \Rightarrow \frac{e_2}{e_1} = \frac{f_2}{f_1}$

∴ $f_2 = \frac{120}{100} \times 1500$ r.p.m. = 1800 r.p.m.

27. Given

ω = constant

∴ $v_{avg} = \left(\frac{0+v}{2}\right)$

Emf induced between axle and rim of the wheel is;

$e = Blv_{avg}$

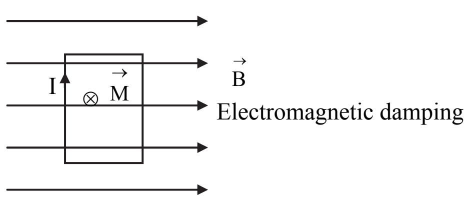
$= \frac{Blv}{2} = \frac{Bl\omega r}{2}$

∴ $e = \frac{Bl^2\omega}{2}$ ....($\because r = l$)

28. $e = Bl_{eff}\,v$ (where l_{eff} = Diameter)
$= B(2\ r)v = 2r\ Bv$ and R is at higher potential by Fleming's right hand rule.

29. Time varying magnetic field gives rise to eddy currents in accordance with Lenz's law.

31.



33. $e = L\dfrac{dI}{dt} \Rightarrow L$ = volt-s/ampere

34. $L = \dfrac{e}{\left(\dfrac{dI}{dt}\right)} = \dfrac{5}{\left(\dfrac{(3-2)}{10^{-3}}\right)} = \dfrac{5}{1} \times 10^{-3}\ H = 5\ mH$

35. $|e| = L\dfrac{dI}{dt} \Rightarrow L = \dfrac{|e|}{\left(\dfrac{dI}{dt}\right)} = \dfrac{220}{\left(\dfrac{10-0}{0.5}\right)} = 11\ H$

36. $e = -L\dfrac{dI}{dt} = -5 \times 2 = -10\ V$

37. $N\phi = LI$

$\therefore$ $\phi = \dfrac{LI}{N} = \dfrac{8\times10^{-3}\times5\times10^{-3}}{400} = 10^{-7} = \dfrac{\mu_0}{4\pi}\ Wb$

38. $n\phi = LI$

$\Rightarrow L = \dfrac{n\phi}{I} = \dfrac{500\times4\times10^{-3}}{2} = 1$ henry

39. Given: N = 1000; I = 4A; $\phi = 4 \times 10^{-3}$ Wb.

$\therefore$ total magnetic flux linked with solenoid = $N\phi$

Self inductance, $L = \dfrac{N\phi}{I}$ ....($\because \phi = LI$)

$\therefore$ $L = \dfrac{1000\times4\times10^{-3}}{4} = 1\ H$

40. $|e| = L\dfrac{dI}{dt} = 5\times2 = 10\,V$

41. $|e| = L\dfrac{dI}{dt}$

$\therefore$ $10 = L\times\dfrac{10}{1} \Rightarrow L = 1\ H$

42. $|e| = L\dfrac{dI}{dt}$

$\therefore$ $1 = \dfrac{L\times\left[10-(-10)\right]}{0.5} \Rightarrow L = 25\ mH$

43. $e = \dfrac{MdI}{dt}$

$e = M\dfrac{d}{dt}(I_m \sin \omega t)$

Now, $\dfrac{d}{dt}(I_m \sin \omega t) = I_m\omega \cos \omega t$

For maximum value of emf, $\dfrac{dI}{dt}$ is maximum

$\Rightarrow \cos \omega t = 1$

$\therefore$ $\dfrac{dI}{dt} = I_m\omega$

$\therefore$ $e = 0.005 \times 10 \times 100\,\pi = 5\,\pi$

44. $\phi_Q = M\,I_P$

But

$|e_p| = M\dfrac{dI_Q}{dt}$

$M = \dfrac{e_P \times dt}{dI_Q}$

$\therefore$ $\phi_Q = \dfrac{e_P \times dt}{dI_Q} \times I_P$

$\phi_P = \dfrac{15\times10^{-3}}{10} \times 1.8$

$\phi_P = 2.7 \times 10^{-3}$ Wb = 2.7 mWb

45. For R-C series circuit,

$Z = \sqrt{R^2 + X_C^2}$

$= \sqrt{(100)^2 + (100)^2}$

$= 100\sqrt{2}\ \Omega$

Peak value of displacement current,

$i_0 = \dfrac{V_0}{Z} = \dfrac{V_{rms}\sqrt{2}}{Z} = \dfrac{220\sqrt{2}}{100\sqrt{2}} = 2.2\ A$

46. As efficiency is always less than unity in practice, output power is less than the input power

47. We know that for step-down transformer,

$V_P > V_S$ but $\dfrac{V_P}{V_S} = \dfrac{I_S}{I_P}$;

$\therefore$ $I_S > I_P$

Current in the secondary coil is greater than the primary.

48. $\eta = \dfrac{P_{output}}{P_{input}} \times 100 = \dfrac{100}{220\times\dfrac{1}{2}} = \dfrac{100}{110} = \dfrac{10}{11} = 90\%$

49. Given: $V_p = 220$ V, $V_s = 3.3 \times 10^3$ V
$N_p = 600$, $P = 4.4 \times 10^3$ W
Power, $P = V_S I_s$

$\therefore$ $I_S = \dfrac{P}{V_s} = \dfrac{4.4\times10^3}{3.3\times10^3} = \dfrac{4}{3}$ A

50. Using, $\dfrac{V_s}{V_p} = \dfrac{I_p}{I_s}$

$\therefore$ $I_p = \dfrac{11000\times 2}{220} = 100$ A

51. $\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{V_sI_s}{V_pI_p} = 0.8$

$\therefore$ $I_p = \dfrac{(440)(2)}{(0.8)(220)} = 5$ A

52. $\eta = \dfrac{P_o}{P_i}$

$\therefore$ $P_o = \eta P_i = \dfrac{80}{100}\times 4\times10^3$ W
But $P_o = e_s I_s$
$\therefore$ $e_s I_s = 0.8 \times 4000$
$\therefore$ $I_s = \dfrac{0.8\times4000}{240}$
$\therefore$ $I_s = 13.33$ A

53. Transformer works on A.C. alone which changes in magnitude as well as in direction.

54. $\dfrac{N_s}{N_p} = \dfrac{V_s}{V_p}$

$\therefore$ $\dfrac{50}{1000} = \dfrac{V_s}{220}$
$\Rightarrow V_s = 11$ V
Now, $V_sI_s = V_pI_p$
$\therefore$ $11 \times I_s = 220 \times 1$
$\Rightarrow I_s = 20$ A

55. $\dfrac{V_S}{V_P} = \dfrac{N_S}{N_P} = \dfrac{I_P}{I_S}$

i.e., $I_P = \dfrac{N_SI_S}{N_P}$

$= \dfrac{25}{1} \times 2 = 50$ A

56. Power output $= 3 \times \dfrac{90}{100} = 2.7$ kW

$I_p = 6$ A

$\therefore$ $V_S = \dfrac{2.7\times10^3}{6} = 450$ V and $I_p = \dfrac{3\times10^3}{200} = 15$ A

57. $P = V_{rms}I_{rms}$
$12 = 48 \times I_{rms}$

$I_{rms} = \dfrac{12}{48} = \dfrac{1}{4}$ A

$I_{rms} = \dfrac{I_0}{\sqrt{2}}$

$I_0 = I_{rms} \times \sqrt{2}$

$I_0 = \dfrac{1}{4} \times \sqrt{2}$

$I_0 = \dfrac{1}{2\sqrt{2}}$ A

58. $e = -\dfrac{NBA(\cos\theta_2 - \cos\theta_1)}{\Delta t}$

$= -2000 \times 0.3 \times 70 \times 10^{-4}\dfrac{(\cos180° - \cos0°)}{0.1}$

$= -42 \times (-1 - 1)$

$\therefore$ $e = 84$ V

59. Using, $I_0 = \dfrac{e_0}{R} = \dfrac{\omega nBA}{R} = \dfrac{2\pi fnB(\pi r^2)}{R}$

$I_0 = \dfrac{2\pi\times\left(\dfrac{200}{60}\right)\times1\times10^{-2}\times\pi(0.3)^2}{\pi^2}$

$= 6 \times 10^{-3}$ A $= 6$ mA

60. In D.C. ammeter, a coil is free to rotate in the magnetic field of a fixed magnet. If an alternating current is passed through such a coil, the torque will reverse its direction each time the current changes direction and the average value of the torque will be zero.

63. Alternating voltage: $e = 200\sqrt{2}\ \sin(100\ t)$ volt
Comparing with $e = e_0 \sin \omega t$
$\omega = 100$ rad/s, $e_0 = 200\sqrt{2}$
Capacitive reactance,

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{100\times10^{-6}}\ \Omega = 10^4\ \Omega$

$I_0 = \dfrac{e_0}{X_C}$

$I_0 = \dfrac{200\sqrt{2}}{10^4}$

$I_0 = 2\sqrt{2}\times10^{-2}$ A

$I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{2\sqrt{2}\times10^{-2}}{\sqrt{2}} = 2 \times 10^{-2}$ A $= 20$ mA

64. Ammeter measures the rms value of current

$\therefore$ $I_{rms} = \dfrac{V_{rms}}{X_C} = \dfrac{V_0}{\sqrt{2}}(\omega C)$

$= \dfrac{50\sqrt{2}}{\sqrt{2}} \times 100 \times 10 \times 10^{-6}$

$= 5 \times 10^{-2}$ A = 50 mA

67. e = 200 sin 50 t
Comparing this equation with the standard form,
$e = e_0 \sin \omega t$ we get, $e_0 = 200$ V

$\therefore$ $e_{rms} = \dfrac{200}{\sqrt{2}}$ V

Now, $I_{rms} = \dfrac{e_{rms}}{R} = \dfrac{200}{\sqrt{2}\times 50} = 2\sqrt{2} = 2.828$

68. Comparing the given equation with the standard form, $I = I_0 \sin \omega t$ we get, $I_0 = 4$ A

$I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{4}{\sqrt{2}} = 2\sqrt{2}$ ampere

69. Given,
I = 50 cos(100t + 45°) A
Comparing the equation by $I = I_0 \cos(\omega t + \phi)$
$I_0 = 50$ A

$\therefore$ $I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{50}{\sqrt{2}} = 25\sqrt{2}$ A

70. $V_0 = \sqrt{2}\, V_{rms} = 1.414 \times 100 = 141.4$ V

71. Induced emf e = NBAω sin ωt
But sin ωt = 1
So, $e_0 = NBA\omega$
$e_0 = 100 \times 0.3 \times 2.5 \times 60$
$e_0 = 4500$
$e_0 = 4.5 \times 10^3$ volt
$e_0 = 4.5$ kV

72. $e_{r.m.s.} = \dfrac{e_0}{\sqrt{2}} = \dfrac{423}{\sqrt{2}} \approx 300$ V

73. Comparing the given equation with standard form,
$e = e_0 \sin \omega t$ we get, ω = 120, $e_0 = 240$ V

$\therefore$ $f = \dfrac{\omega}{2\pi} = \dfrac{120 \times 7}{2 \times 22} \approx 19$ Hz

$\therefore$ $e_{rms} = \dfrac{240}{\sqrt{2}} = 120\sqrt{2} \approx 170$ V

74. V = 5 cos 1000t volt
$V = V_0 \cos \omega t$
$V_0 = 5$ volt
ω = 1000 rad/s
L = 3mH = 3×10^{-3} H, R = 4 Ω

Maximum current,

$10 = \dfrac{V_0}{Z}$

$10 = \dfrac{5}{\sqrt{R^2 + \omega^2 L^2}} = \dfrac{5}{\sqrt{4^2 + (1000 \times 3 \times 10^{-3})^2}}$

$= \dfrac{5}{5} = 1$ A

75. Comparing given equation with the standard form,
$e = e_0 \sin \omega t$ we get, ω = 2πf

$\therefore$ 2πf = 377 ⇒ f = 60 Hz

76. $e_{rms} = \dfrac{e_0}{\sqrt{2}} = \dfrac{141.4}{1.414} = 100$ V

77. Comparing the given equation with standard form,
$e = e_0 \sin \omega t$ we get, $E_0 = 200\sqrt{2}$ v, ω = 100

$I_{rms} = \dfrac{V_{rms}}{X_C} = \dfrac{V_0 \omega C}{\sqrt{2}}$

$= \dfrac{200\sqrt{2} \times 100 \times (1 \times 10^{-6})}{\sqrt{2}}$

$= 2 \times 10^{-2}$ A = 20 mA

78. $X_C = \dfrac{1}{\omega C}$,

$\because$ angular frequency (ω) for D.C. source is Zero

$\therefore$ Capacitive reactance becomes infinite.

81. In LCR circuit power is always dissipated through resistor.

82. $Z = \sqrt{R^2 + X_L^2}$, $X_L = \omega L$ and ω = 2πf

$\therefore$ $Z = \sqrt{R^2 + 4\pi^2 f^2 L^2}$

83. $\tan \phi = \dfrac{X_C - X_L}{R} \Rightarrow \tan 45° = \left(\dfrac{\dfrac{1}{2\pi f C} - 2\pi f L}{R}\right)$

$\therefore$ $C = \dfrac{1}{2\pi f (2\pi f L + R)}$

84. $E = M\dfrac{di}{dt}$

$E = 2 \times 10^{-2} \dfrac{d(5 \sin 10\pi t)}{dt}$

$= 2 \times 10^{-2}\, 5(\cos 10\pi t) \times 10\pi$

$E_{max} = 2 \times 10^{-2} \times 5 \times 1 \times 10\pi = \pi$

85. $X_C = \frac{1}{2\pi fC} \Rightarrow C = \frac{1}{2\pi f X_C}$

$\therefore$ $C = \frac{1}{2\times\pi\times\frac{400}{\pi}\times 25} = 0.5 \times 10^{-4} = 50\ \mu F$

86. As the current I leads the voltage by $\frac{\pi}{4}$, it is an RC circuit $\Rightarrow \tan\phi = \frac{X_C}{R}$

$\therefore$ $\tan\frac{\pi}{4} = \frac{1}{\omega CR}$

$\therefore$ $\omega CR = 1$

Given that, $\omega = 100$ rad/s

$\therefore$ $CR = \frac{1}{100}\ s^{-1}$

$\therefore$ From all the given options, only option (A) is correct.

87. $\tan\phi = \frac{X_L}{R} = \frac{\sqrt{3}R}{R} = \sqrt{3}$

$\therefore$ $\phi = 60^\circ = \pi/3$

88. $\tan\phi = \frac{X_L}{R} = 1 \Rightarrow \phi = 45^\circ$ or $\pi/4$

89. $\tan\phi = \frac{X_L - X_C}{R} = \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

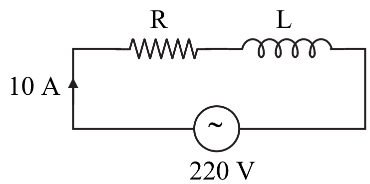
$$= \left(\frac{2\times 3.14\times 50\times\frac{100}{\pi}\times 10^{-3} - \frac{1}{2\times 3.14\times\frac{10^{-3}}{2\pi}\times 50}}{10}\right)$$

i.e. $\tan\phi = 1$

$\phi = \tan^{-1}(1)$

$\therefore$ $\phi = 45^\circ$

90. Resistance, $R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$



For a RL circuit;

$V = \sqrt{V_R^2 + V_L^2}$

$\therefore$ $V_L^2 = V^2 - V_R^2$

$I^2(2\pi fL)^2 = V^2 - (IR)^2$

$\therefore$ $L^2 = \frac{V^2 - (IR)^2}{I^2 4\pi^2 f^2} = \frac{(220)^2 - (10\times 8)^2}{10^2\times 4\times(3.14)^2\times(50)^2}$

$L^2 = 0.425 \times 10^{-2}$

$\therefore$ $L = 0.065$ H

91. Given, $V_L = 40$ V; $V_C = 120$ V; $V_R = 60$ V

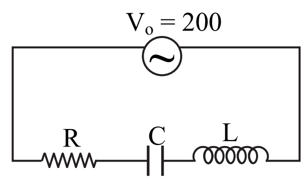
$\therefore$ Source voltage, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$= \sqrt{(60)^2 + (40-120)^2}$

$= \sqrt{(60)^2 + (80)^2}$

$\therefore$ $V = 100$ volt

92.



$R = 100\ \Omega$, $V_o = 200$, $f = 50$ Hz

C-I: When capacitance is removed then circuit is L – R circuit

$\therefore$ $Q = \tan^{-1}\left(\frac{X_L}{R}\right)$

$60 = \tan^{-1}\left(\frac{X_L}{100}\right)$

$\tan 60^\circ = \frac{X_L}{100}$

$\sqrt{3} = \frac{X_L}{100}$

$X_L = 100\sqrt{3}$

C-II : when inductor is removed then circuit is R – C circuit

$Q = \tan^{-1}\left(\frac{X_C}{R}\right)$

$60^\circ = \tan^{-1}\left(\frac{X_C}{100}\right)$

$\tan 60 = \frac{X_C}{100}$

$X_C = 100\sqrt{3}$

Now the current in L – C – R circuit is,

$V_o = I_o Z$

$i_o = \frac{V_o}{Z}$

$i_o = \frac{200}{100}$

$i_o = 2$ A

$V_o = 200$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{R^2 + \left(100\sqrt{3} - 100\sqrt{3}\right)^2}$

$Z = R \Rightarrow Z = 100$

93. $E_{rms} = 10$ V, $\omega = 200$, $R = 50\ \Omega$,
$L = 400\text{mH} = 400 \times 10^{-3}$ H,
$c = 200\mu F = 200 \times 10^{-6}$ F

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}$

$= \sqrt{50^2 + \left[\left(200 \times 400 \times 10^{-3} - \frac{1}{200 \times 200 \times 10^{-6}}\right)\right]^2}$

$= \sqrt{50^2 + (80 - 25)^2}$

$Z = 74.3\ \Omega$

$I_{rms} = \frac{E_{rms}}{Z} = \frac{10}{74.3} = 0.13459$ A

$E_L = I_{rms} X_L = 0.1345 \times 80 = 10.8$ V

94. $Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(3)^2 + (14 - 10)^2}$

∴ $Z = 5\ \Omega$

95. For series LCR circuit,

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(300)^2 + \left(1000 \times 0.9 - \frac{10^6}{1000 \times 2}\right)^2}$

$= 500\ \Omega$

96. $I_o = \frac{V_o}{X_L}$ (∵ $Z = X_L$ for pure inductive circuit)

$I_o = \frac{\sqrt{2}\,V_{rms}}{X_L} = \frac{\sqrt{2} \times 200}{2\pi f L}$

$I_o = \frac{\sqrt{2} \times 200}{2\pi \times 50 \times 1} = 0.9$ A

97. $I = \frac{e}{\sqrt{R^2 + X_L^2}}$

$I = \frac{220}{\sqrt{(20)^2 + (2 \times \pi \times 50 \times 0.2)^2}} = \frac{220}{66} = 3.33$ A

98. $X_L = \omega L$
$X_L = 2\pi f L$
$X_L \propto f$

∴ The graph will be a linear graph

99. We have, $X_C = \frac{1}{C \times 2\pi f}$ and $X_L = L \times 2\pi f$

100. Power $= I^2R = \left(\frac{I_p}{\sqrt{2}}\right)^2 R = \frac{I_p^2 R}{2}$

101. For purely resistive circuit Power (P) = $\frac{e_{rms}^2}{R}$

When inductance is connected in series with resistance

$P' = e_{rms}\, i_{rms} \cos\phi$

$= e_{rms}\left(\frac{e_{rms}}{Z}\right)\left(\frac{R}{Z}\right) = \frac{e_{rms}^2}{Z^2} R$

$P' = \frac{(PR)}{Z^2} R$ $\quad (\because e_{rms}^2 = PR)$

$P' = \frac{PR^2}{Z^2}$

102. $P = VI$

$I = \frac{P}{V} = \frac{100}{220} = \frac{5}{11}$ A

103. $P = e_{rms}\, I_{rms} \cos\phi$

But, $\cos\phi = \frac{R}{Z}$ and $e_{rms} = I_{rms} \times Z$

$P = e_{rms} \times \frac{e_{rms}}{Z} \times \frac{R}{Z} = \frac{220 \times 220 \times 18}{33 \times 33} = 800$ W

104. $P_{avg} = e_{rms} \times I_{rms} \times \cos\phi$

$\cos\phi = \frac{P_{avg}}{e_{rms} \times I_{rms}} = \frac{63}{210 \times 3} = 0.1$

105. Average power dissipated $= e_{rms} \times I_{rms}$

$= I_{rms} \times R \times I_{rms}$

$= \frac{I_0}{\sqrt{2}} \times R \times \frac{I_0}{\sqrt{2}}$

$= \frac{I_0^2 R}{2} = \frac{(2)^2 \times 10}{2}$

$= \frac{4}{2} \times 10 = 20$ watt

106. Comparing the given equations with the standard forms,

$e = e_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$

we get,

$e_0 = 100$ V, $I_0 = 100$ mA and $\phi = \frac{\pi}{3}$ rad

$P = e_{rms} \times I_{rms} \times \cos\phi$

$= \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} \times \cos\frac{\pi}{3}$

$= \frac{10^4 \times 10^{-3}}{2} \times \frac{1}{2} = \frac{10}{4} = 2.5$ watt

107. Phase angle, $\phi = 90°$

∴ $P = e.\ I.\ \cos\phi = 0$

108. $P_{avg} = V_{rms}I_{rms}\cos\phi$

$$= \left(\frac{v_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\left(\cos\frac{\pi}{3}\right) = \frac{v_0 I_0}{4}$$

109. $P = e_{rms}I_{rms}\cos\phi$ and $P_{max} = e_{rms}I_{rms}$

Since P = 50 % P_{max} = 0.5 P_{max}

$$\Rightarrow \cos\phi = 0.5 \Rightarrow \phi = \frac{\pi}{3}$$

110. Using, $P = VI\cos\phi = I^2 Z\cos\phi$ we get,

$$\cos\phi = \frac{P}{I^2 Z} = \frac{2}{4\times 1} = 0.5$$

111. Comparing given equations with the standard forms,

$e = e_0 \sin\omega t$ and $I = I_0\sin(\omega t + \alpha)$

$e_0 = 200$ V, we get, I = 1 A, $\phi = \frac{\pi}{3}$ rad

$$e_{rms} = \frac{200}{\sqrt{2}},\ I_{rms} = \frac{1}{\sqrt{2}}$$

∴ $P = e_{rms}I_{rms}\cos\phi$

$$= \frac{200}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\cos\frac{\pi}{3} = 50 \text{ watt}$$

112. Comparing given equations with the standard forms,

$e = e_0\sin\omega t$ and $i = i_0\sin(\omega t + \phi)$ we get,

$e_0 = 100$ V, $I_0 = 100$ mA

e = 100 sin (100 t) V and

$$I = 100\sin\left(100t + \frac{\pi}{3}\right)\text{mA}$$

∴ $$\text{Power} = \frac{e_0}{\sqrt{2}}\cdot\frac{I_0}{\sqrt{2}}\cos\phi$$

$$= \frac{100\times100}{2}\times\cos\left(\frac{\pi}{3}\right)\times10^{-3}$$

$$= \frac{100\times100}{2}\times\frac{1}{2}\times10^{-3}$$

= 2.5 W

113. $$P = \frac{V_{rms}^2}{Z}\cos\phi = \frac{V_{rms}^2}{Z}\left(\frac{R}{Z}\right)$$

∴ $$P = \frac{V_0^2}{2}\frac{R}{Z^2} \quad \text{....(i)}$$

Given $V_0 = 10$ V; ω = 340 rad/s; L = 20 mH; C = 50 μF; R = 40 Ω

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

∴ $$P = \frac{(10)^2}{2}\times(40)$$

$$\times\frac{1}{\left[(40)^2 + \left(340\times20\times10^{-3} - \frac{1}{340\times50\times10^{-6}}\right)^2\right]}$$

$$= \frac{2000}{1600 + [6.8 - 58.8]^2} = \frac{2000}{1600 + [2704]}$$

$$= \frac{2000}{4304} \approx 0.46 \text{ W}$$

Nearest answer is option (C).

114. Given: L = 20mH = 20×10^{-3} H

C = 100 μF = 100×10^{-6} F, R = 50 Ω

V = 10 sin 314t,

But, $V = V_0\sin\omega t$

On comparision we get,

ω = 314 rad/s and V_0 = 10 V

Inductive reactance,

$X_L = \omega L = 314\times20\times10^{-3} = 6.28\ \Omega$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{314\times100\times10^{-6}} = 31.85\ \Omega$$

Impedance,

$Z^2 = R^2 + (X_L - X_c)^2$

$Z^2 = 50^2 + (6.28 - 31.85)^2$

$Z^2 = 3153\ \Omega$

Average power,

$$P_{av} = \frac{V_{rms}^2 R}{Z^2} = \frac{V_0^2 R}{2\times Z^2} = \frac{100\times50}{2\times3153} = 0.79 \text{ W}$$

115. $$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$= \sqrt{(8)^2 + (31-25)^2}$$

$$= \sqrt{64+36}$$

= 10 Ω

∴ Power factor, $\cos\phi = \frac{R}{Z} = \frac{8}{10} = 0.8$

116. Power factor = $\cos\phi = \frac{R}{Z}$

as current remains same, we can write,

$$\cos\phi = \frac{R}{Z} = \frac{V_R}{\sqrt{(V_R)^2 + (V_L - V_C)^2}}$$

$$= \frac{80}{\sqrt{(80)^2 + (60)^2}} = 0.8$$

117. For CR circuit, power factor is given by

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$$

$\therefore$ $(\cos\phi)_1 = \dfrac{R}{\sqrt{R^2 + \frac{1}{(\omega_1 C)^2}}}$ ….(i)

$\therefore$ $\dfrac{1}{\sqrt{2}} = \dfrac{R}{\sqrt{R^2 + \frac{1}{(\omega_1 C)^2}}}$

$\therefore$ $\dfrac{1}{2} = \dfrac{R^2}{R^2 + \frac{1}{(\omega_1 C)^2}}$

$\therefore$ $R^2 + \dfrac{1}{(\omega_1 C)^2} = 2R^2$

$\therefore$ $R^2 = \dfrac{1}{(\omega_1 C)^2}$ ….(ii)

Now,

$(\cos\phi)_2 = \dfrac{R}{\sqrt{R^2 + \frac{1}{(\omega_2 C)^2}}}$

But, $\omega_2 = \dfrac{\omega_1}{2}$

$\therefore$ $(\cos\phi)_2 = \dfrac{R}{\sqrt{R^2 + \frac{4}{(\omega_1 C)^2}}}$ ….(iii)

Dividing equation (iii) by equation (i)

$$\frac{(\cos\phi)_2}{(\cos\phi)_1} = \frac{R}{\sqrt{R^2 + \frac{4}{(\omega_1 C)^2}}} \times \frac{\sqrt{R^2 + \frac{1}{(\omega_1 C)^2}}}{R}$$

$\therefore$ $(\cos\phi)_2 = \cos\phi_1 \times \sqrt{\dfrac{R^2 + \frac{1}{(\omega_1 C)^2}}{R^2 + \frac{4}{(\omega_1 C)^2}}}$

Using eq(ii),

$(\cos\phi)_2 = \dfrac{1}{\sqrt{2}}\sqrt{\dfrac{R^2 + R^2}{R^2 + 4R^2}}$

$= \dfrac{1}{\sqrt{2}}\sqrt{\dfrac{2R^2}{5R^2}}$

$\therefore$ $(\cos\phi)_2 = \dfrac{1}{\sqrt{5}}$

118. e = 100 sin 30 t

$\therefore$ $e_{rms} = \dfrac{100}{\sqrt{2}}$

$I = 20 \sin\left(30t - \dfrac{\pi}{4}\right)$

$\therefore$ $I_{rms} = \dfrac{20}{\sqrt{2}}$

Also, $\phi = \dfrac{\pi}{4}$

$\therefore$ Average power consumed,

$P = e_{rms} \times I_{rms} \times \cos\dfrac{\pi}{4}$

$= \dfrac{100}{\sqrt{2}} \times \dfrac{20}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = \dfrac{2000}{2\sqrt{2}} = \dfrac{1000}{\sqrt{2}}$ W

Wattless current, $I = I_{rms} \sin\dfrac{\pi}{4}$

$\therefore$ $I = \dfrac{20}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = \dfrac{20}{2} = 10$ A

121. Using, $f_r = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow L \propto \dfrac{1}{C}$ for fixed f_r

$\therefore$ $\dfrac{L_2}{L_1} = \dfrac{C_1}{C_2} = \dfrac{C}{2C} \Rightarrow L_2 = \dfrac{L}{2}$

122. Impedance of LCR circuit will be minimum at resonant frequency

$\therefore$ $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{1\times10^{-3}\times0.1\times10^{-6}}} = \dfrac{10^5}{2\pi}\ s^{-1}$

123. $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{5\times10^{-3}\times2\times10^{-6}}}$

$= \dfrac{10^4}{2\pi} = \dfrac{5\times10^3}{\pi}$ Hz

124. Given that, $V_L = V_C$

$f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{3\times10^{-3}\times30\times10^{-6}}}$

$= \dfrac{10^4}{2\pi\times3} \approx 530$ Hz

125. $\omega L = \dfrac{1}{\omega C}$ $\omega = \dfrac{1}{\sqrt{LC}}$

$\Rightarrow X_L$ and X_C will get interchanged.

$\Rightarrow 200\ L = \dfrac{1}{800C}$

$\Rightarrow \dfrac{1}{\sqrt{LC}} = \sqrt{200\times800} = 400$ Hz

126. $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times3.14\sqrt{5\times10^{-4}\times20\times10^{-6}}}$

∴ $f_0 = \frac{10^4}{6.28} \approx 1592$ Hz

127. $f_r = \frac{1}{2\pi\sqrt{LC}} \Rightarrow f_r \propto \frac{1}{\sqrt{LC}}$

∴ $\frac{(f_r)_2}{(f_r)_1} = \frac{1}{\sqrt{L_2C_2}}\sqrt{L_1C_1} = \left(\frac{L_1C_1}{L_2C_2}\right)^{1/2}$

$= \left(\frac{L\times C}{2L\times4C}\right)^{1/2} = \frac{1}{(8)^{1/2}}$

∴ $\frac{(f_r)_2}{(f_r)_1} = \frac{1}{2\sqrt2}$

$\Rightarrow (f_r)_2 = \frac{f_1}{2\sqrt2}$

∴ $(f_r)_2 = \frac{f}{2\sqrt2}$ ....[∵ $(f_r)_1 = f$]

128. According to condition of parallel resonance for LC circuit, at resonant frequency (f_r) impedance of circuit is maximum and current is minimum.

130. the voltage equation in going from point A to B is

$-IR + E - L\frac{di}{dt} - V_{AB} = 0$

∴ $V_{BA} = -2\times2 + 12 - (5\times10^{-3}\times10^2)$

$\left(\because \frac{di}{dt} \text{is decreasing hence rate is negative}\right)$

∴ $V_{BA} = -4 + 12 + 0.5 = 8.5$ Volt

131. $V_{AB} - IR + E + L\frac{dI}{dt} = 0$

∴ $V_{AB} = (2)(7) - 4 - (9\times10^{-3})(10^3)$

$= 14 - 4 - 9$

∴ $V_{AB} = 1$V

132. $I_{rms} = \sqrt{(I^2)} = \sqrt{(8+6\sin\omega t)^2}$

$I_{rms} = \sqrt{(64+96\sin\omega t+36\sin^2\omega t)}$

$I_{rms} = \sqrt{(64)+96(\sin\omega t)+36(\sin^2\omega t)}$

Since $(\sin^2\omega t) = 0.5$ and $(\sin\omega t) = 0$

$I_{rms} = \sqrt{64+0+36\times0.5} = 9.05$ A

133. $e = e_0\cos\omega t = e_0\cos(2\pi ft)$

$= 10\cos\left(\frac{2\pi\times50\times1}{600}\right) = 10\cos\frac{\pi}{6} = 5\sqrt3$ V

134. Comparing given equation with the standard form, $I = I_0\sin\omega t$ we get,

$\frac{2\pi}{T} = 200\pi \Rightarrow T = \frac{1}{100}$ s

The current takes $\frac{T}{4}$ s to reach the peak value.

∴ Time to reach the peak value = $\frac{1}{400}$ s

135. $e = e_0\sin\theta$

e will be maximum when θ is 90°

∴ Plane of the coil will be horizontal.

136. $|e| = \frac{d\phi}{dt} = B\frac{dA}{dt} = B\frac{d}{dt}(\pi r^2) = 2\pi Br\frac{dr}{dt}$

∴ $|e| = 2\pi\times0.04\times2\times10^{-2}\times2\times10^{-3} = 3.2\pi\ \mu V$

137. $\phi = (5t^2 - 4t + 1)$ Wb

∴ $\frac{d\phi}{dt} = (10t - 4)\ Wbs^{-1}$

$e = \frac{-d\phi}{dt} = -(10t-4)$

At, t = 0.2 s, $e = -(10\times0.2-4) = 2$ V

∴ $I = \frac{e}{R} = \frac{2}{10} = 0.2$ A

138. $E = -\frac{d\phi}{dt}$

$E = -\frac{d(B.A)}{dt}$

$E = -A\frac{dB}{dt}$

$= -A\frac{d}{dt}\frac{\mu_0 I}{2\pi(vt)}$

$\Rightarrow -AI\frac{\mu_0}{2\pi v}\frac{d}{dt}(t^{-1})$

$\Rightarrow AI\frac{\mu_0}{2\pi v}t^{-2}$

$E \propto \frac{1}{t^2}$

139. Area of square loop, A = 10 cm × 10 cm

$A = 100\ cm^2 = 100\times10^{-4}\ m^2 = 10^{-2}\ m^2$

Initial magnetic flux linked with loop,

$\phi_1 = B_1A\cos\phi = 0.1\times10^{-2}\times\cos45^\circ$

$= \frac{0.1\times10^{-2}\times1}{\sqrt2} = \frac{10^{-3}}{\sqrt2}$ Wb

Final magnetic flux linked with loop,

$\phi_2 = 0$ Wb ....[∵ $B_2 = 0$]

$\therefore$ The induced e.m.f. in the loop,

$$e = -\frac{d\phi}{dt} = -\frac{(\phi_2 - \phi_1)}{t} = -\frac{\left(0 - \frac{10^{-3}}{\sqrt{2}}\right)}{0.7}$$

$$= \frac{10^{-3}}{0.7 \times \sqrt{2}} \approx 10^{-3}\text{ V}$$

$\therefore$ $I = \frac{e}{R} = \frac{10^{-3}}{1} = 10^{-3}\text{ A} = 1.0\text{ mA}$

140. $\frac{n d\phi}{dt} = \frac{L dI}{dt} \Rightarrow nB\frac{dA}{dt} = \frac{L dI}{dt}$

$\therefore$ $\frac{1 \times 1 \times 5}{10^{-3}} = L \times \left(\frac{2-1}{2 \times 10^{-3}}\right) \Rightarrow L = 10\text{ H}$

141. Using, $\frac{e_S}{e_p} = \frac{N_S}{N_P} = \frac{1500}{50}$ we get,

$e_S = 30\ e_P$

Now, $|e_p| = \frac{d\phi}{dt} = 4$ volt

$\Rightarrow e_S = 30 \times 4 = 120$ V

142. $e_o = i_o \times X_L$

$X_L = \omega L = 2\pi f L = 2\pi(50) = 100\pi$

$i_o = \frac{2}{\pi}$ ampere

$\therefore$ $e_o = \frac{2}{\pi} \times 100\pi = 200$ V

144. $\phi = BA$

$\phi = (B)(\pi r^2)$

$\therefore$ $e = \frac{d\phi}{dt} = (B)(2\pi r)\left(\frac{dr}{dt}\right)$

$= (0.025)(2\pi)(2 \times 10^{-2})(10^{-3})$

$= \pi\ \mu V$

145. At B, flux is maximum, which means $\frac{d\phi}{dt} = 0$

As, $|e| = \frac{d\phi}{dt} \Rightarrow |e| = 0$

146. $e = \int_{2l}^{3l} Bv\,dl = \int_{2l}^{3l} B(\omega l)\,dl = B\omega\left(\frac{l^2}{2}\right)_{2l}^{3l} = \frac{5B\omega l^2}{2}$

147. The induced EMF is given by

$e = \vec{l} \cdot \left(\vec{v} \times \vec{B}\right)$.

but since $\vec{l}, \vec{v}$ and $\vec{B}$ are mutually perpendicular to each other, hence

$e = Blv$

Now, rate of increase of induced EMF is

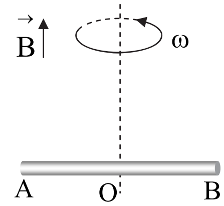
$\therefore$ $\frac{de}{dt} = Bl\frac{dv}{dt} = Bla$

$\therefore$ $\frac{de}{dt} = 5 \times 10^{-4} \times 0.1 \times 5 = 2.5 \times 10^{-4}\text{ Vs}^{-1}$

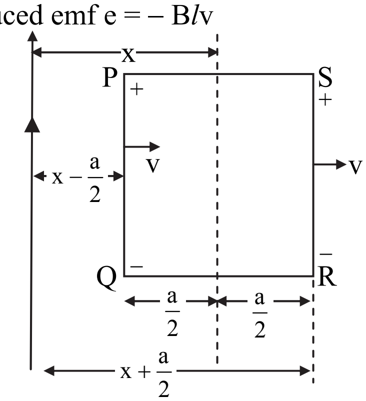
148. Potential difference between

O and A is $V_0 - V_A = \frac{1}{2}Bl^2\omega$

O and B is $V_0 - V_B = \frac{1}{2}Bl^2\omega$



149. Induced emf $e = -Blv$



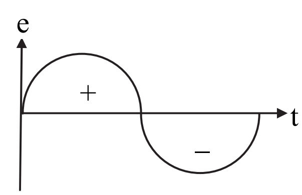
$\varepsilon_{PQRS} = \varepsilon_{PQ} + \varepsilon_{RS}$

$$= \frac{\mu_0 I}{2\pi\left(x - \frac{a}{2}\right)}av - \frac{\mu_0 I}{2\pi\left(x + \frac{a}{2}\right)}av$$

$$= \frac{\mu_0 Iav}{2\pi}\left[\frac{2}{2x-a} - \frac{2}{2x+a}\right]$$

$$= \frac{\mu_0 Iav}{\pi}\left[\frac{2x+a-2x+a}{(2x-a)(2x+a)}\right] = \frac{2\mu_0 Ia^2 v}{\pi(2x-a)(2x+a)}$$

150.



$e = n\omega AB\sin\omega t$

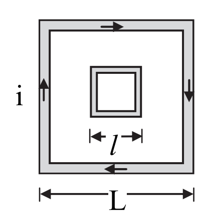
$\therefore$ e changes direction twice per revolution.

151. Magnetic field produced due to large loop

$B = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}I}{L}$

Flux linked with smaller loop

$\phi = B(l^2) = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}Il^2}{L}$

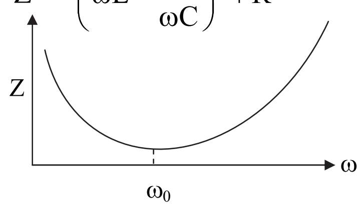


$\therefore$ $\phi = MI \Rightarrow M = \frac{\phi}{I} = \frac{\mu_0}{4\pi}.\frac{8\sqrt{2}l^2}{L} \Rightarrow M \propto \frac{l^2}{L}$

152. As the magnet moves towards the coil, the magnetic flux increases (nonlinearly). Also there is a change in polarity of induced emf when the magnet passes on to the other side of the coil.

153. According to I – t graph, in the first half, current increases uniformly so a constant negative e.m.f. get induced in the circuit.
In the second half, current decreases uniformly so a constant positive e.m.f. gets induced.
Hence graph (C) is correct.

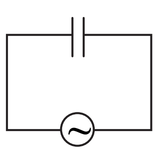
154. $Z^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 + R^2$



∴ As we gradually increase frequency, Z first decreases and then increases

155. Induced electric field is non-conservative.
Also we have,
$$\oint \vec{e} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{e} \cdot \vec{ds} \neq 0$$

156. In pure capacitive circuit, let an A.C. voltage be supplied of the form
$e = e_0 \sin \omega t$ ….(i)



we know that, $C = \frac{q}{e}$
$\Rightarrow q = Ce = Ce_0 \sin \omega t$
∴ $I = \frac{dq}{dt} = Ce_0\, \omega \cos \omega t$
∴ $I = I_0 \cos \omega t$ ….(taking $I_0 = Ce_0\omega$)
∴ $I = I_0 \sin (\pi/2 + \omega t)$ ….(ii)
Thus, on comparing (i) and (ii), we see that current leads the voltage by a phase angle of π/2.

157. Let $\omega_1 = 50 \times 2\pi \Rightarrow \omega L = 20\ \Omega$
∴ $\omega_2 = 100 \times 2\pi \Rightarrow \omega' L = 40\ \Omega$
∴ $I = \frac{200}{Z} = \frac{200}{\sqrt{R^2 + (\omega' L)^2}} = \frac{200}{\sqrt{(30)^2 + (40)^2}}$
∴ $I = 4$ A

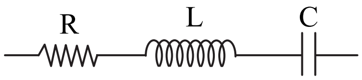
158. From $V = 200\sqrt{2} \sin \omega t$, $V_0 = 200\sqrt{2}$
∴ $I_0 = \frac{V_0}{Z} = \frac{200\sqrt{2}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{200\sqrt{2}}{\sqrt{20^2 + (15-15)^2}}$
∴ $I_0 = \frac{200\sqrt{2}}{20} = 10\sqrt{2}$

159. For inductor,
$I \propto \frac{1}{X_L} \propto \frac{1}{f}$
Hence, as frequency increases, current decreases.
For capacitor,
$I \propto \frac{1}{X_C} \propto f$
Hence, as frequency increases, current increases.

160. $I = \frac{P}{V} = \frac{1000}{100} = 10$ A
The voltage drop across heater must remain same and the current it draws must be same.
Hence, voltage across coil is
$V_C = 200\sqrt{2} - 100 = 182$ V
We know that $I = \frac{V_C}{\omega L}$
$\Rightarrow L = \frac{V_C}{I\omega} = \frac{182}{10 \times 2\pi \times 50}$
∴ L = 0.057 henry

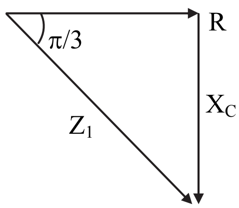
161. Quantity of heat liberated in the ammeter of resistance R
i. due to direct current of 3 ampere
$= [(3)^2 R/J]$
ii. due to alternating current of 4 ampere
$= [(4)^2 R/J]$
∴ Total heat produced per second
$= \frac{(3)^2 R}{J} + \frac{(4)^2 R}{J} = \frac{25R}{J}$
Let the equivalent alternating current be I virtual ampere; then
$\frac{I^2 R}{J} = \frac{25R}{J}$ or $I = 5$ A

162. $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{300}$
$= 10^6$ Hz
Now, $f_r = \frac{1}{2\pi\sqrt{LC}} \Rightarrow \sqrt{LC} = \frac{1}{2\pi f_r}$
∴ $L = \frac{1}{4\pi^2 f_r^2 C}$
∴ $L = \frac{1}{4\pi^2 (10^6)^2 \times 2.4 \times 10^{-6}}$
$\approx 10^{-8}$ H

163. 

When L is removed,

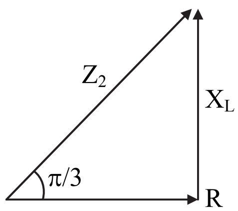
$\frac{X_C}{R} = \tan\frac{\pi}{3}$



$\therefore$ $X_C = R\tan\frac{\pi}{3}$

When C is removed,

$\frac{X_L}{R} = \tan\frac{\pi}{3}$



$X_L = R\tan\frac{\pi}{3}$

$\therefore$ $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

$\therefore$ $\cos\phi = \frac{R}{Z} = 1$

164. $I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

$\therefore$ As ω increases, I_{rms} increases and hence the bulb glows brighter.

165. Brightness $\propto P_{consumed} \propto \frac{1}{R}$ for Bulb

$\therefore$ $R_{ac} = R_{dc}$

$\Rightarrow$ brightness will be equal in both the cases.

166. For pure inductor $\phi = \frac{\pi}{2}$

$P_{av} = VI\cos\phi = VI\cos\frac{\pi}{2}$

$\therefore$ $P_{av} = 0$

167. Impedance is given as,

$Z = \frac{\sqrt{R^2 + X_L^2}}{R^2 + (L \times 2\pi f)^2}$

$\therefore$ If frequency is decreased, impedance decreases.

If number of turns decreases, self inductance decreases and thus impedance decreases.

At resonance, $X_C = X_L$ and impedance decreases.

When iron rod is inserted, impedance increases. Hence current decreases. Hence option (D) is correct.

168. Phase difference $\Rightarrow \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

For pure L, R circuit;

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$\phi = \tan^{-1}\left(\frac{2\pi f L}{R}\right)$

$\phi = \tan^{-1}\left(\frac{2\pi \times \frac{25}{\pi} \times 2}{100}\right)$

$\phi = \tan^{-1}(1)$

$\phi = 45°$

169. $\tan\phi = \frac{X_L}{R} = \frac{1/\sqrt{3}}{1}$

$\tan\phi = \frac{1}{\sqrt{3}}$

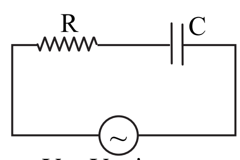
$\phi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\phi = 30° = \frac{\pi^C}{6}$

But,

$\phi = \omega t$

$\therefore$ $t = \frac{\phi}{\omega} = \frac{\pi/6}{2\pi(50)} = \frac{1}{600}$ s

170. 

$V = V_0 \sin\omega t$

$X_c = \frac{1}{2\pi f C}$

current in ciruit

$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}$

Or $I = \frac{2\pi f C}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}} \times V$

Voltage drop across capacitor

$V_c = I \times X_c = \frac{2\pi f C}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}} \times \frac{1}{2\pi f C}$

$V_c = \frac{V}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}}$

When mica is introduced capacitance will increase, hence voltage across capacitor gets decreased.

171. $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ $\therefore$ $f \propto \frac{1}{\sqrt{C}}$

172. As the electric and magnetic fields share energy equally in an LC circuit,

$\frac{1}{2}Li^2 = \frac{1}{2}CV^2$

$\therefore$ $I = \left(\frac{CV^2}{L}\right)^{1/2} = \left(\frac{16\times10^{-6}\times20^2}{40\times10^{-3}}\right)^{1/2} = 0.4$ A

173. Frequency of oscillation, $f = \frac{1}{2\pi\sqrt{LC}}$

$\therefore$ $f \propto \frac{1}{\sqrt{C}}$

$\therefore$ $\frac{f_{air}}{f_{dielectric}} = \sqrt{\frac{C_{dielectric}}{C_{air}}}$

But $\frac{C_{dielectric}}{C_{air}} = k$ ….(k = dielectric constant)

$\therefore$ $\frac{f_{air}}{f_{dielectric}} = \sqrt{k}$

$\frac{125}{100} = \sqrt{k}$

$\therefore$ $k = \left(\frac{5}{4}\right)^2$

$\therefore$ $k = 1.56$

Evaluation Test

1. Magnetic field at the centre

$= \frac{\mu_0 I}{L}\left(\frac{3+\sqrt{3}}{3}+\frac{1}{2}\right)$

Emf, $|e| = \left|\frac{d\phi}{dt}\right| = A\frac{dB}{dt}$

$= (\pi r^2)\left[\frac{d}{dt}\left(\frac{\mu_0 I}{L}\left(\frac{3+\sqrt{3}}{3}+\frac{1}{2}\right)\right)\right]$

$\therefore$ $e = \pi r^2\,\frac{\mu_0}{L}\left(\frac{3+\sqrt{3}}{3}+\frac{1}{2}\right)\frac{d}{dt}(I_0 e^{\alpha t})$

$= \frac{\mu_0\,\pi r^2}{L}\left(\frac{3+\sqrt{3}}{2}+\frac{1}{2}\right)I_0\,\alpha\,e^{\alpha t}$

$= \frac{\mu_0 I_0 \alpha \pi r^2}{L}\left(\frac{3+\sqrt{3}}{3}+\frac{1}{2}\right)e^{\alpha t}$

2. Here, B is constant and radius r is linearly changing only during time interval 5 to 10 units.

Using, $e = \frac{d}{dt}(B\pi r^2) = (B\pi)\left(2r\frac{dr}{dt}\right)$

Hence during this period, the emf is as shown in (D).

3. Assertion and Reason both are correct and reason is correct explanation of assertion because $e = -L\left(\frac{di}{dt}\right)$

4. $\tau_{restoring} = mgl \sin\theta \approx -mgl\theta$

$\alpha = \frac{\tau}{I} = \frac{-mgl}{ml^2}\theta = -\left(\frac{g}{l}\right)\theta$

$T = 2\pi\sqrt{\frac{l}{g}}$, $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{2}} = \sqrt{5}$ rad/s

$\theta = \theta_0 \sin\omega t$ ….(i)

Now, $e = -\frac{d\phi}{dt} = \frac{-d}{dt}(BA)\cos\omega t = BA\,\omega\sin\omega t$

$\therefore$ $e = BA\,\omega\sin(\omega t)$

$\therefore$ $e = BA\left(\frac{d\theta}{dt}\right)\sin\omega t$

$\therefore$ $e = BA(\theta_0\omega\cos\omega t)\sin\omega t$ ….(ii)

Since $\sin\omega t = \sin\theta$ and taking $\sin\theta \approx \theta$.

Substituting value of θ from equation (i), we get

$e = BA(\theta_0\omega\cos\omega t)(\theta_0\sin\omega t)$

$\therefore$ $e = \frac{BA\omega\sin(2\omega t)\left(\theta_0^2\right)}{2}$

$\therefore$ $e = \frac{1\times4\times10^{-4}\times\sqrt{5}\times\sin(2\sqrt{5}t)}{2}\times\frac{10^{-4}}{4}$

$= \frac{\sqrt{5}}{2}\sin(2\sqrt{5}\,t)\times10^{-8}$

$= 5\sqrt{5}\,\sin(2\sqrt{5}\,t)\times10^{-9}$ volt

5. The emf induced in the rod of length 0.5 m is $e = Bnvl = 0.50\times4\times0.5 = 1$ volt

The free electrons of rod experience force along $\overrightarrow{BA}$ therefore end A becomes negative and end B becomes positive. That is the direction of the induced emf is from B towards A.

The current in the circuit ABCD,

$$i = \frac{e}{R} = \frac{1}{0.2} = 5 \text{ A}$$

The force required to maintain the motion

$= i l B = 5 \times 0.5 \times 0.5 = 1.25$ N

Mechanical work done by the force per second or mechanical power

$= Fv = 1.25 \times 4 \times 1 = 5$ watts

6. The two loops are connected in such a way that the currents induced in the loops are always equal in magnitude but opposite in direction. That is, if the current in the left loop is clockwise, it is anticlockwise in right loop and vice-versa. Thus, the emfs induced in the two loops will oppose each other.

The emf induced in first loop,

$$e_1 = \frac{d}{dt}(a^2B) = a^2 \frac{dB}{dt}$$

$$= a^2 \frac{d}{dt}(B_0 \sin \omega t) = a^2 B_0\omega \cos \omega t$$

The emf induced in second loop,

$$e_2 = \frac{d}{dt}(d^2B) = b^2 \frac{dB}{dt}$$

$$= b^2 \frac{d}{dt}(B_0 \sin \omega t) = b^2 B_0 \omega \cos \omega t$$

Net emf induced,

$e = e_1 - e_2 = (a^2 - b^2) B_0\omega \cos \omega t$

Total resistance of the loops, $R = 4 (a + b) r$

where, r = resistance per unit length

∴ Instantaneous current at time t,

$$i = \frac{e}{R} = \frac{(a^2 - b^2)B_0\omega \cos\omega t}{4(a+b)r}$$

For maximum value of current induced,

$\cos \omega t = 1$

$$\therefore \quad i_0 = \frac{(a^2 - b^2)B_0\omega}{4(a+b)r} = \frac{(a-b)B_0\omega}{4r}$$

Here, a = 0.20 m, b = 0.10 m, $B_0 = 10^{-3}$ T,

Resistance per unit length $r = 50 \times 10^{-3}$ Ω/m,

ω = 100 rad/s

$$\therefore \quad i_0 = \frac{[(0.20) - (0.10)] \times 10^{-3} \times 100}{4 \times 50 \times 10^{-3}} = 0.05 \text{ A}$$

$$\therefore \quad \frac{1}{n} = 0.05 \Rightarrow n = 20$$

7. Rate of work done by external agent is:

$\frac{dW}{dt} = \frac{BIL(dx)}{dt} = BIL\,v$ and thermal power dissipated in resistor = eI = (BvL) I

Clearly both are equal. Hence (A) is correct.

If applied external force is doubled, the rod will experience a net force and hence acceleration. As a result, velocity increases, hence (B) is correct.

Since, $I = \frac{e}{R}$

On doubling 'R', current and hence required power becomes half. Hence (D) is correct.

Since $P = BIl\,v$ and $I \propto \frac{1}{R}$

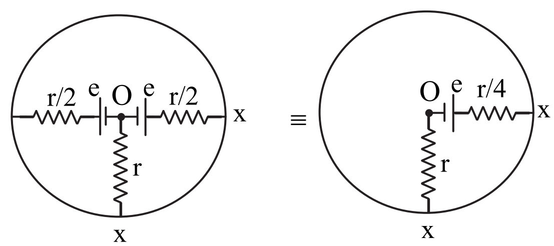
Hence option (C) is incorrect.

8. Induced emf, $e = \frac{B\omega r^2}{2} = \left(\frac{B\omega a^2}{2}\right)$

(∵ radius = a)

By nodal equation, $4\left(\frac{x-e}{r}\right) + \left(\frac{x-0}{r}\right) = 0$

$5x = 4e$



(i) (ii)

$$\Rightarrow x = \frac{4e}{5} = \frac{2B\omega a^2}{5}$$

$$\therefore \quad I = \frac{x}{r} = \frac{2B\omega a^2}{5r}$$

Also, direction of current in 'r' will be towards negative terminal of cell. i.e. from rim towards centre.

Alternatively, we can obtain the same result by considering the equivalence of cells (fig. ii)

9. $\int \vec{E} \cdot \vec{dx} = -\frac{d\phi}{dt}$ and taking the sign of flux according to right hand rule we get,

$\int \vec{E} \cdot \vec{dx} = -[-(-2\alpha A) + (-\alpha A)] = -\alpha A$

10. The emf induced,

$e = -M\frac{di}{dt}$

$e = 40,000$ V

$\therefore \frac{di}{dt} = \frac{i_2 - i_1}{t_2 - t_1} = \frac{0-4}{10\times10^{-6}} = -4\times10^5$ A/s

$\therefore$ Mutual inductance,

$M = \frac{e}{(di/dt)} = \frac{40000}{(-4\times10^5)} = 0.1$ Henry

$\therefore \frac{n}{10} = 0.1 \Rightarrow n = 10$

11. $\Delta\phi = 2\pi R^2 B$

Initially current was zero. So self-linked flux was zero.

$\therefore$ Finally, $Li = 2\pi R^2 \times B \Rightarrow i = \frac{2\pi R^2 B}{L}$

12. $E = \frac{B^2}{2\mu_0}$. Hence a graph between E and B will be a parabola symmetric about E axis and passing through origin.

13. $d\phi = \vec{B}\times\vec{A} = BA\cos 60° = \frac{1}{2500}$

$\therefore E = \frac{d\phi}{dt} = \frac{1}{2500\times0.2} = 2\times10^{-3}$ V

14. $E(2\pi l) = \pi R^2\left(\frac{dB}{dt}\right)$

$\therefore E = \frac{R^2}{2l}\left(\frac{dB}{dt}\right)$

Now, $qE + mg = kx$

$\therefore x = \frac{qR^2}{k2l}\left(\frac{dB}{dt}\right) + \frac{mg}{k}$

$\therefore x = \frac{1}{k}\left[mg + \frac{qR^2}{2l}\frac{dB}{dt}\right]$

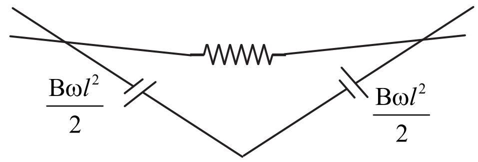
15. $e_{AB} = \left(\frac{dB}{dt}\right)\times$ area of Δ AOB

$= 4\times\frac{1}{2}\times\left(4\times\frac{\sqrt{3}}{2}\times2\right)\times2$

$\therefore$ Total emf of loop $= 3\times\left(4\times\frac{1}{2}\times4\times\frac{\sqrt{3}}{2}\times2\right)\times2$

$= 2\times24\sqrt{3} = 48\sqrt{3}$ volt

16. $i = \frac{\left(\frac{B\omega l^2}{2}+\frac{B\omega l^2}{2}\right)}{R} = \frac{B\omega l^2}{R}$



17. $\left|\int\vec{E}\cdot\vec{dl}\right| = \left|\frac{d\phi}{dt}\right|$

$\therefore E(2\pi)\frac{l}{2} = \pi\frac{l^2}{4}\times\frac{dB}{dt}$

$\therefore E = \frac{l}{4}\alpha$

Now, $F = qE = \frac{ql\alpha}{4}$

$\therefore$ The forces cancel out to give $F_{net} = 0$

18. Total charge flowing through the wire is

$q = \int Idt = \frac{I}{R}\int\left(\frac{d\phi}{dt}\right)dt$

$\Rightarrow q = -\left(\frac{1}{R}\Delta\phi\right)$

Since the current in the coil before and after the rotation remains the same so,

$\Delta I = 0$

$\Rightarrow q = \frac{-1}{R}\Delta\phi$

Further,

$\Delta\phi = \int d\phi = \int Badr = \frac{\mu_0 2Ia}{2\pi}\int_{a-b}^{a+b}\frac{dr}{r}$

$\Rightarrow \Delta\phi = \frac{\mu_0}{4\pi}2Ia\log_e\left(\frac{a+b}{b-a}\right)$

$= \text{constant}\times\frac{aI}{R} \Rightarrow m = 1, n = 1, p = -1$

$\therefore m + n + p = 1$

19. $\phi = \vec{B}\times\vec{A} = B\left(\frac{\pi a^2}{2}\right)\cos(\omega t)$

Since, $e = \left|\frac{-d\phi}{dt}\right| = B\left(\frac{\pi a^2}{2}\right)\omega\sin(\omega t)$

Induced current, $I = \frac{e}{R} = \frac{B\pi a^2}{2R}\omega\sin(\omega t)$

At any moment t, the thermal power generated in circuit,

$$P_t = e \times I = \left(\frac{B\pi a^2\omega}{2}\right)^2 \frac{1}{R}\sin^2(\omega t)$$

Mean power,

$$\langle P\rangle = \frac{\left(\frac{B\pi a^2\omega}{2}\right)^2 \frac{1}{R}\int_0^T \sin^2 \omega t}{\int_0^T dt} = \frac{1}{2R}\left(\frac{B\pi\omega a^2}{2}\right)^2$$

$\Rightarrow p = 2$

20. $E_{avg}\,(2\pi r) = \dfrac{B\pi a^2}{\Delta t}$

$E_{avg}\,(\lambda 2\pi r)\, r = \dfrac{B\pi a^2}{\Delta t} \times \lambda \times r = I\,\alpha_{avg}$

$\therefore$ $B_0\pi a^2\,\lambda r = mr^2\,\alpha_{avg}\,\Delta t$

$\therefore$ $\omega = \dfrac{B_0\pi a^2\lambda}{mr} = \dfrac{1\times\pi\times(10^{-2})^2\times\frac{4}{\pi}}{0.5\times(2\times10^{-2})}$

$= 4 \times 10^{-2}$ rad/s

21. $q = CBvl$

$l = \dfrac{dq}{dt} = CBla$

Now, $ma = mg - Bl\,(CBla)$

$\therefore$ $a = \dfrac{mg}{m + B^2l^2C}$

Substituting the values given,

$a = 5\ m/s^2$

22. $e = \dfrac{d\phi}{dt} = \dfrac{d}{dt}(B\cdot A) = \dfrac{d}{dt}(KIA) = K'\,\dfrac{dI}{dt}$

$\therefore$ $e = 0$ if $\dfrac{dI}{dt} = 0$ and $e = K$ if $\dfrac{dI}{dt} = K$.

Now, for the first portion of the given i vs t graph, $\dfrac{dI}{dt} = 0$ and for the remaining two sections,

$\dfrac{dI}{dt}$ = constant

Hence the correct option is (C).

23. Induced electric field

$E\,(2\,\pi\, r) = \dfrac{d\phi}{dt}$

$E = \dfrac{\pi a^2\left(2B_0 t\right)}{2\pi r}$

Torque due to field about centre of ring,

$\tau_1 = (qE)\, r = \lambda(2\pi r)\left(\dfrac{2\pi a^2 B_0 t}{2\pi r}\right) r$

Ring starts rotating when,

τ due to electric field = τ due to friction

$\tau_1 = (\mu mg)\, r$

On Solving, we get, $t = \dfrac{\mu mg}{2\pi a^2 B_0\lambda}$

$$= \frac{\left(\frac{\pi}{4}\right)4\times10}{2\pi\times\left(5\times10^{-2}\right)^2\times125\times4} = 4\ s$$

24. $emf = L = \dfrac{di}{dt} = \dfrac{2-0.5}{0.03} = 50$

$E_{stored} = \dfrac{1}{2}\,Li^2 = \dfrac{1}{2}\times 50 \times 0.5^2$

$= 25 \times 0.25 = 6.25$ J

Textbook Chapter No.

17 Electrons and Photons

Hints

Classical Thinking

19. The maximum velocity or the kinetic energy of photoelectrons depends on frequency and not on intensity.

22. If the frequency of incident radiation is kept constant at a value greater than ν_0 (threshold frequency), then the rate of emission of photoelectrons from emitter is directly proportional to intensity of incident radiation.

24. Photoelectric effect is one photon, one electron phenomenon, i.e., one photon can not eject more than one photoelectron.

28. Photoelectric effect shows the particle nature of light and not the wave nature of light.

35. The maximum kinetic energy with which an electron is emitted from a metal surface is independent of the intensity of the light and depends only upon its frequency.

36. $\lambda = \dfrac{hc}{W_0}$

$\therefore \quad \lambda' = \dfrac{hc}{2W_0} = \dfrac{\lambda}{2}$

52. $\dfrac{1}{2}mv^2_{max} = h\nu - h\nu_0$

$= h(3\nu_0) - h\nu_0$

$= 3h\nu_0 - h\nu_0 = 2h\nu_0$

53. Explanation of photoelectric effect is possible with quantum (particle) nature of light. In photoelectric effect, during interaction of radiation with matter, radiation behaves as if it is made up of particles viz. photons.

Critical Thinking

1. For no emission of photoelectron, energy of incident light < work function

$\therefore \quad h\nu < \phi \Rightarrow < \dfrac{\phi}{h}$

2. Retarding potential, $V_0 = \dfrac{h}{e}(\nu - \nu_0)$

3. If threshold frequency is ν_0, then light frequency becomes $1.5\ \nu_0$.
If we make it half it becomes $0.75\ \nu_0$, which is smaller than threshold frequency, therefore photoelectric current is zero.

5. Minimum kinetic energy is always zero.

6. The velocity of the photoelectron ejected from near the surface is larger than those coming from interior of metal because for the given energy of the incident photon, less energy is spent in ejecting the electron near the surface than that from the interior of the surface.

7. $eV_0 = h\nu - W_0$

$eV_0 = (2.4 - 1.6)$ eV

$\therefore \quad V_0 = 0.8$ V

8. $V_0 = \dfrac{(E - W_0)}{e} = \dfrac{(2\text{ eV} - 0.6\text{ eV})}{e} = 1.4$ V

9. $W_0 = \dfrac{hc}{\lambda_0} = \dfrac{6.6\times10^{-34}\times3\times10^{8}}{6600\times10^{-10}} = 3\times10^{-19}$ J

10. $\phi_0 = \dfrac{hc}{\lambda_{max}}$

$\Rightarrow \lambda_{max} = \dfrac{hc}{\phi_0} = \dfrac{12400\text{ eV Å}}{5\text{ eV}} = 2480$ Å

11. $\dfrac{hc}{\lambda_{max}} = 2$ eV

$\lambda_{max} = \dfrac{hc}{2\text{eV}} = \dfrac{6.63\times10^{-34}\times3\times10^{8}}{2\times1.6\times10^{-19}}$

$= \dfrac{6.63\times3}{3.2}\times10^{-7}$

$= 6215$ Å

12. $W_0 = h\nu_0$

$\therefore \quad \nu_0 = \dfrac{W_0}{h} = \dfrac{3.3\times1.6\times10^{-19}}{6.63\times10^{-34}}$

$= \dfrac{5.28\times10^{-19}}{6.63\times10^{-34}} = 7.9\times10^{14}$

$\approx 8\times10^{14}$ Hz

13. $W_0 = h\nu_0$

$\therefore \quad W_0 = \frac{hc}{\lambda_0} = \frac{6.63\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}}$

$= 3.978 \times 10^{-19}$ J

$= \frac{3.978\times10^{-19}}{1.6\times10^{-19}} = 2.48$ eV

14. $W_0 = \frac{h\nu_0}{e}$ eV $= \frac{6.63\times10^{-34}\times1.6\times10^{15}}{1.6\times10^{-19}} = 6.63$ eV

K.E $= E - W_0 = 8 - 6.63 = 1.37$ eV

15. $\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W_0$

$\therefore \quad \frac{1}{2}mv^2 = \frac{6.63\times10^{-34}\times3\times10^{8}}{(3.6\times10^{-7})\times1.6\times10^{-19}} - 2.5 = 1$

$\therefore \quad v = \sqrt{\frac{2\times1\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 0.6 \times 10^6$ m/s

$= 6 \times 10^5$ m/s

16. $E = \frac{hc}{\lambda} - W_0$ and $2E = \frac{hc}{\lambda'} - W_0$

$\therefore \quad \frac{\lambda'}{\lambda} = \frac{E + W_0}{2E + W_0}$

$\therefore \quad \lambda' = \lambda\left(\frac{1 + W_0/E}{2 + W_0/E}\right)$

Since $\frac{(1 + W_0/E)}{(2 + W_0/E)} > \frac{1}{2}$, so $\lambda' > \frac{\lambda}{2}$

17. Einstein's photoelectric equation is,

$\frac{1}{2}mv^2 = h\nu - h\nu_0$ …(i)

Comparing with equation of straight line,

$y = mx + c$ …(ii)

where, m = slope and c = intercept for line on X axis.

Comparing equation (i) and equation (ii) we get,

Slope = h = Planck's constant.

19. $eV_1 = \frac{hc}{\lambda_1} - W_0$, $eV_2 = \frac{hc}{\lambda_2} - W_0$

$\therefore \quad \frac{hc}{\lambda_1} - eV_1 = \frac{hc}{\lambda_2} - eV_2$

$\therefore \quad e(V_2 - V_1) = hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$

$\therefore \quad hc\left(\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right) = e(V_2 - V_1)$

$\therefore \quad h = \frac{e}{c}(V_2 - V_1)\frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$

20. $\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0$

$= 9.2$ eV $- 4.2$ eV

$= 5$ eV

$= 5 \times 1.6 \times 10^{-19}$

$= 8 \times 10^{-19}$ J

21. Slope of $V_0 - \nu$ curve for all metals are same $\left(\frac{h}{e}\right)$ i.e. curves should be parallel.

22. Using Einstein photoelectric equation,

$E = W_0 + K.E_{max}$

$h\nu_1 = W_0 + \frac{1}{2}mv_1^2$ ….(i)

$h\nu_2 = W_0 + \frac{1}{2}mv_2^2$ ….(ii)

Subtracting equation (ii) from equation (i) we get,

$h(\nu_1 - \nu_2) = \frac{1}{2}m(v_1^2 - v_2^2)$

$\therefore \quad (v_1^2 - v_2^2) = \frac{2h}{m}(\nu_1 - \nu_2)$

But $\nu_2 = \frac{\nu_1}{2} = \frac{\nu}{2}$

$\therefore \quad (v_1^2 - v_2^2) = \frac{2h}{m}\left(\nu - \frac{\nu}{2}\right) = \frac{h\nu}{m}$

23. $\lambda_0 = \frac{c}{\nu_0} = \frac{3\times10^{8}}{5\times10^{14}} = 6\times10^{-7}$ m $= 6000$ Å

25. The intercept of the line on the ν-axis gives the threshold frequency ν_0 and work function $W_0 = h\nu_0$. Thus, work function = slope × intercept. The value of slope is the same for metals A and B but ν_0 for B is greater than that for A.

Competitive Thinking

2. Initially when the moving electron is very far away from stationary electron, it only has kinetic energy but as it approaches the stationary electron, its K.E. decreases due to repulsion and gets converted to P.E. according to law of conservation of energy. Hence, K.E. decreases and P.E. increases.

4. Intensity ∝ No. of photons

∝ No. of photoelectrons

6. Stopping potential does not depend on the relative distance between the source and the cell.

7. Intensity increases means that more photons of same energy will emit more electrons of same energy, hence only photoelectric current increases.

8. For photo emission $\nu \geq \nu_0$ or $\lambda \leq \lambda_0$

9. $\lambda_R > \lambda_y > \lambda_g$. Here threshold wavelength $< \lambda_y$

10. For work function of 5 eV,

$\lambda_{min} = \dfrac{4\times10^{-15} \times 3 \times 10^8}{5} = 240$ nm,

For work function of 2 eV,

$\lambda_{max} = \dfrac{4\times10^{-15} \times 3 \times 10^8}{2} = 600$ nm

This means wavelength of 650 nm cannot be used.

11. The saturation photoelectric current is directly proportional to the intensity of incident radiation but it is independent of its frequency. Hence, saturation photoelectric current becomes double, when both intensity and frequency of the incident light are doubled.

12. If the voltage given is V, then the energy of electron,

$\dfrac{1}{2}mv^2 = eV$

$\therefore$ $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2\times1.6\times10^{-19}\times1000}{9.1\times10^{-31}}}$

$= 1.875 \times 10^7$

$\approx 1.9 \times 10^7$ m/s

14. $\lambda = 1$ Å $= 10^{-10}$ m

$E = h\nu_{max} = \dfrac{hc}{\lambda} = \dfrac{6.63\times10^{-34}\times3\times10^8}{1\times10^{-10}} = 19.8\times10^{-16}$ J

15. $\dfrac{1}{2}mv_{max}^2 = eV$

$\therefore$ $v_{max} = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2\times1.6\times10^{-19}\times9}{9.1\times10^{-31}}}$

$= 1.8 \times 10^6$ m/s

16. The plate current reduces with increasing wavelength. When wavelength exceeds certain value, photo electric effect ceases, making current value zero.

17. Photoelectric current $\propto$ intensity of light

$\therefore$ $I_1 < I_2$

18. $K_{max} = eV_0 \Rightarrow 4$ eV $= eV_0$

$\therefore$ $V_0 = 4$ V

22. According to Einstein's equation,

$h\nu = h\nu_0 + K.E_{max}$

$\therefore$ $K.E_{max} = h\nu - h\nu_0'$. Comparing it with $y = mx + c$, we can say that, this is the equation of straight line having positive slope (h) and negative intercept ($h\nu_0$) on K.E. axis.

23. From Einstein's photoelectric equation,

$\dfrac{hc}{\lambda} = \dfrac{1}{2}mv_{max}^2 + W_0$

$\therefore$ $\dfrac{hc}{\lambda} = eV_0 + W_0$ ...($\because \dfrac{1}{2}mv_{max}^2 = eV_0$)

$\therefore$ $V_0 \propto \dfrac{1}{\lambda}$

Thus, if incident wavelength is decreased, then stopping potential will increase.

24. $eV_0 = h\nu - h\nu_0$

If ν increases, V_0 will increase.

25. Above threshold frequency (ν_0), the stopping potential increases with the increase in frequency.

26. $eV_0 = h\nu - W_0$

$= \dfrac{hc}{\lambda} - W_0$

$= \left[\dfrac{6.63\times10^{-34}\times3\times10^8}{332\times10^{-9}\times1.6\times10^{-19}} - 1.07\right]$ eV

$= 2.67$ eV

Nearest answer is (D)

27. $W_0 = \dfrac{hc}{\lambda}$

$\therefore$ $\lambda = \dfrac{hc}{W_0} = \dfrac{6.63\times10^{-34}\times3\times10^8}{4.2\times1.6\times10^{-19}}$

$\lambda = 2.959 \times 10^{-7}$ Å

$\therefore$ $\lambda = 2959$ Å

28. $K.E_{max}$ (eV) $= E(eV) - W_0(eV)$

$= 6.2 - 4.2$

$= 2$ eV

$\therefore$ $K.E_{max}$(joule) $= 2 \times 1.6 \times 10^{-19}$ J

$= 3.2 \times 10^{-19}$ J

29. Using, $E = h\nu - W$ for the two cases we get,

$0.5 = h\nu - W$ ....(i) and

$0.8 = 1.2\, h\nu - W$ ....(ii)

By equation (i) × 1.2 – equation (ii) we get,

$0.2\, W = 0.2$ or $W = 1$ eV

30. Number of photons emitted per second

$n = \dfrac{p}{h\nu} = \dfrac{10\times10^3}{6.6\times10^{-34}\times880\times10^3} = 1.72 \times 10^{31}$

31. $\lambda = 3300$ Å

$v_{max} = 0.4 \times 10^6$ m/s

$(K.E.)_{max} = \frac{1}{2} mv_{max}^2 = \frac{hc}{\lambda} - W_0$

$\therefore \quad \frac{1}{2} \times 9.1 \times 10^{-31} \times (0.4 \times 10^6)^2$

$= \frac{6.6\times10^{-34}\times3\times10^8}{3300\times10^{-10}} - W_0$

$\therefore \quad \frac{6.6\times10^{-34}\times3\times10^8}{3300\times10^{-10}} - 7.28\times10^{-20} = W_0$

$\therefore \quad W_0 = 5.3 \times 10^{-19}$ J

32. Energy of incident light

$E = \frac{12375}{2000} = 6.18\,eV$

Using, $E = W_0 + eV_0$

$V_0 = \frac{(E - W_0)}{e} = \frac{(6.18\,eV - 5.01\,eV)}{e}$

$= 1.17\,V \approx 1.2\,V$

33. According to Einstein's photoelectric equation

$E = W_0 + K_{max}$

$\therefore \quad V_0 = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$

$\therefore$ As λ decreases, V_0 increases.

34. $E = W_0 + K_{max}$;

$\therefore \quad E = \frac{12375}{5000} = 2.475\,eV$

$\therefore \quad K_{max} = E - W_0 = 2.475 - 1.9 = 0.58\,eV$

35. $E = W_0 + K.E_{max}$

$1 = 0.5 + (K.E_{max})_1 \Rightarrow (K.E_{max})_1 = 0.5$

$2.5 = 0.5 + (K.E_{max})_2 \Rightarrow (K.E_{max})_2 = 2$

$\frac{(K.E_{max})_1}{(K.E_{max})_2} = \frac{0.5}{2} = \frac{1}{4}$

$\left(\frac{v_1}{v_2}\right) = \sqrt{\frac{(K.E_{max})_1}{(K.E_{max})_2}} = \frac{1}{2}$

36. When, $\nu_1 = 2\nu_0$,

$\therefore \quad (K.E._1)_{max} = h(2\nu_0) - h\nu_0 = h\nu_0$....(i)

When, $\nu_2 = 5\nu_0$

$(K.E._2)_{max} = h(5\nu_0) - h\nu_0 = 4h\nu_0$ ....(ii)

Dividing equation (i) by equation (ii),

$\frac{(K.E._1)_{max}}{(K.E._2)_{max}} = \frac{1}{4}$

As, $(K.E.)_{max} = \frac{1}{2} mv_{max}^2$

$\therefore \quad \frac{v_1^2}{v_2^2} = \frac{1}{4}$

$\therefore \quad \frac{v_1}{v_2} = \frac{1}{2}$

37. $eV = \frac{hc}{\lambda} - W_0$

$\therefore \quad \frac{1}{2} mv_{max}^2 = eV$

$\therefore \quad V = \frac{v_{max}^2 \times m}{2e} = \frac{v_{max}^2}{2\left(\frac{e}{m}\right)} \frac{1}{2}$

$= \frac{1.2\times10^6\times1.2\times10^6}{2\times\left(1.8\times10^{11}\right)} = 4$ V

38. The work function has no effect on current as long as $h\nu > W_0$. The photoelectric current is proportional to the intensity of light. Since, there is no change in the intensity of light, therefore $I_1 = I_2$.

39. In photoelectric effect, energy is conserved.

$\therefore \quad V_S = \frac{h}{e}(\nu - \nu_0)$

$\therefore \quad V_S = \frac{6.6\times10^{-34}\times(8.2\times10^{14} - 3.3\times10^{14})}{1.6\times10^{-19}}$

$= \frac{6.6\times4.9}{1.6}\times10^{-1} = 2.0\,V$

40. Using photoelectric equation,

$\frac{hc}{\lambda} = W_0 + 3V_0$ ....(i)

$\frac{hc}{2\lambda} = W_0 + V_0$ ....(ii)

Subtracting equation (ii) from equation (i),

$\frac{hc}{2\lambda} = 2V_0$

$\therefore \quad V_0 = \frac{hc}{4\lambda}$

Substituting in equation (ii)

$\therefore \quad W_0 = \frac{hc}{2\lambda} - V_0 = \frac{hc}{2\lambda} - \frac{hc}{4\lambda}$

$\frac{hc}{\lambda_0} = \frac{hc}{4\lambda}$ $\quad \left(\because W_0 = \frac{hc}{\lambda_0}\right)$

$\Rightarrow \lambda_0 = 4\lambda$

41. From Einstein's equation,
$h\nu = eV_0 + h\nu_0$
$\therefore \quad \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = eV_0$
case (i) $\lambda = \lambda$; $V_0 = V$
$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = eV \quad$ ....(i)
case (ii) $\lambda = 3\lambda$; $V_0 = \frac{V}{6}$
$\frac{hc}{3\lambda} - \frac{hc}{\lambda_0} = \frac{eV}{6} \quad$ ....(ii)
dividing equation (i) by equation (ii)
$\therefore \quad \frac{\left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0}\right)}{\left(\frac{hc}{3\lambda} - \frac{hc}{\lambda_0}\right)} = 6$
$\therefore \quad \frac{1}{\lambda} - \frac{1}{\lambda_0} = 6\left(\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right)$
$\therefore \quad \frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{2}{\lambda} - \frac{6}{\lambda_0}$
$\therefore \quad \frac{-1}{\lambda_0} + \frac{6}{\lambda_0} = \frac{2}{\lambda} - \frac{1}{\lambda}$
$\therefore \quad \frac{5}{\lambda_0} = \frac{1}{\lambda}$
$\therefore \quad \lambda_0 = 5\lambda$

42. Using Einstein's photoelectric equation
Case I :
$eV = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$ ....(i)
Case II :
$e\frac{V}{4} = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$
$\therefore \quad eV = \frac{4hc}{2\lambda} - \frac{4hc}{\lambda_0} \quad$ ....(ii)
$= 4hc\left[\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right]$
Equating (i) and (ii),
$hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] = 4\,hc\left[\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right]$
$\frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{2}{\lambda} - \frac{4}{\lambda_0}$
$\therefore \quad \lambda_0 = 3\,\lambda$

43. Energy radiated as visible light
$= \frac{5}{100} \times 100 = 5$ J/s
If n be the number of photons emitted per second, then, $nh\nu = E = 5$
$\therefore \quad n = \frac{5\lambda}{hc} = \frac{5 \times 5.6 \times 10^{-7}}{(6.62 \times 10^{-34})(3 \times 10^{8})} = 1.4 \times 10^{19}$

44. K.E. = E – W_0
$\therefore \quad W_0 = 10.20 - 3.57$
$\therefore \quad v_0 = \frac{6.63 \times 1.6 \times 10^{-19}}{6.67 \times 10^{-34}} = 1.6 \times 10^{15}$ Hz

45. $W_0 = E - K.E.$
$\therefore \quad W_0 = E_1 - K.E_1$ and
$W_0 = E_2 - K.E_2 = E_2 - 2K.E_1$
$\therefore \quad E_2 - 2K.E_1 = E_1 - K.E_1$
$\therefore \quad K.E_1 = E_2 - E_1 = 4 - 2.5 = 1.5$ eV
$\therefore \quad W_0 = 2.5 - 1.5 = 1$ eV

46. Using photoelectric equation, $h\nu = K.E. + W_0$
Initially,
$h\nu = 0.4 + W_0 \quad$ ….(i)
After increasing incident frequency by 30%,
$h(1.3\,\nu) = 0.9 + W_0 \quad$ ….(ii)
multiplying equation (i) by 1.3 and then subtracting from equation (ii),
$0 = [0.9 - 1.3(0.4)] + [W_0 - 1.3\,W_0]$
$\therefore \quad 0.3\,W_0 = 0.9 - 0.52$
$\therefore \quad W_0 = \frac{0.38}{0.3} = 1.267$ eV

47. We know,
$(K.E.)_{max} = h\nu - W_0$
$\therefore \quad 2\text{ eV} = 5\text{ eV} - W_0$
$W_0 = 3$ eV
Hence, when $h\nu = 6$ eV,
$(K.E.)_{max} = 6\text{ eV} - 3\text{ eV} = 3$ eV
Also, $(K.E.)_{max} = eV_0 = 3$ eV
$\Rightarrow V_0 = 3$ V
As, stopping potential is a retarding potential, potential of A relative to C = – 3 V

48. Let K_1 and K_2 be the maximum kinetic energy of photoelectrons for incident light of frequency ν and 2ν respectively.
By Einstein's photoelectric equation,
$K_1 = h\nu - W_0 = K \quad$ ….(i)
and $K_2 = h(2\nu) - W_0 \quad$ ….(ii)
$= 2h\nu - W_0 = h\nu + h\nu - W_0$
$\therefore \quad K_2 = h\nu + K \quad$ ….[From (i)]

49. $\frac{hc}{\lambda} = W_0 + \frac{1}{2}mv_{max}^2$

Assuming W_0 to be negligible in comparison to $\frac{hc}{\lambda}$,

$v_{max}^2 \propto \frac{1}{\lambda} \Rightarrow v_{max} \propto \frac{1}{\sqrt{\lambda}}$

∴ On increasing wavelength from λ to 4λ, v_{max} becomes half.

50. Cut off frequency is given as ν

Work function $W_0 = h\nu$

Now, $E = K.E. + W_0$

$2h\nu = \frac{1}{2}mv^2 + h\nu$

∴ $\frac{1}{2}mv^2 = 2h\nu - h\nu$

∴ $\frac{1}{2}mv^2 = h\nu$

∴ $v = \sqrt{\frac{2h\nu}{m}}$

51. $\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W_0$ or

$\frac{hc}{\lambda} = \frac{1}{2}mv^2 + W_0$ and

$\frac{1}{2}mv_1^2 = \frac{hc}{\left(\frac{3\lambda}{4}\right)} - W_0 = \frac{4}{3}\left(\frac{1}{2}mv^2 + W_0\right) - W_0$

∴ $v_1^2 = \frac{4}{3}v^2 + \text{constant}$

So, $v_1 > v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

52. For ejected electron,

$\frac{1}{2}mv^2 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$

∴ $v = \sqrt{\frac{2hc}{m}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]}$

$= \sqrt{\frac{2\times 4.14\times 10^{-15}\times 3\times 10^{8}\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}\left[\frac{10^{10}}{2536} - \frac{10^{10}}{3250}\right]}$

$= 6.15 \times 10^5$ m/s $\approx 0.6 \times 10^6$ m/s^{-1}

53. From Einstein's photoelectric equation,

$h\nu_1 = W_0 + eV_1$

$h\nu_2 = W_0 + eV_2$

∴ $\frac{\nu_1}{\nu_2} = \frac{W_0 + eV_1}{W_0 + eV_2}$

∴ $W_0\nu_1 + eV_2\nu_1 = W_0\nu_2 + eV_1\nu_2$

∴ $e = \frac{W_0(\nu_2 - \nu_1)}{\nu_1 V_2 - V_1\nu_2}$

54. $K.E_{max} = (h\nu - W_0)$

where, ν = frequency of incident light

55. Velocity of photon $c = \nu\lambda$

56. $E \propto \frac{1}{\lambda}$

We know that, $\lambda_{infrared} > \lambda_{visible}$

∴ $E_{infrared} < E_{visible}$

57. $p = \frac{h}{\lambda}$, $E = \frac{hc}{\lambda}$

Thus, if λ decreases, both p and E will increase.

58. $\lambda = \frac{hc}{E} = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{35\times 10^{3}\times 1.6\times 10^{-19}} = 3.5 \times 10^{-11}$

$= 35 \times 10^{-12}$ m

59. $K_{max}(eV) = 12375\left[\frac{1}{\lambda(\text{Å})} - \frac{1}{\lambda_0(\text{Å})}\right]$

$= 12375\left[\frac{1}{1000} - \frac{1}{2000}\right] = 6.2$ eV

60. $p = \frac{h\nu}{c}$

∴ $\nu = \frac{pc}{h} = \frac{3.3\times 10^{-29}\times 3\times 10^{8}}{6.6\times 10^{-34}} = 1.5 \times 10^{13}$ Hz

61. $p = \frac{h}{\lambda} = \frac{6.6\times 10^{-34}}{4400\times 10^{-10}} = 1.5\times 10^{-27}$ kg.m / s

and mass $m = \frac{p}{c} = \frac{1.5\times 10^{-27}}{3\times 10^{8}} = 5\times 10^{-36}$ kg

62. $p = \frac{E}{c}$

∴ $E = p \times c = 2 \times 10^{-16} \times (3 \times 10^{10}) = 6 \times 10^{-6}$ erg.

63. $E = \frac{hc}{\lambda}$

∴ $\frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1}$

∴ $E_2 = \frac{E_1\lambda_1}{\lambda_2} = \frac{3.2\times 10^{-19}\times 6000}{4000} = 4.8 \times 10^{-19}$ J

64. Energy of photon, $E = \frac{hc}{\lambda}$ (joules) $= \frac{hc}{e\lambda}$(eV)

∴ $E = \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times\lambda_{(m)}\,\text{Å}} = \frac{12375}{\lambda(\text{Å})}$ eV

$= \frac{12.37}{\lambda(\text{Å})} \approx \frac{12.4}{\lambda}$ k eV

65. Energy of incident light

$E(\text{eV}) = \frac{12375}{3320} = 3.72\,\text{eV}$ (332 nm = 3320 Å)

According to the relation $E = W_0 + eV_0$

$\Rightarrow V_0 = \frac{(E - W_0)}{e} = \frac{3.72\,\text{eV} - 1.07\,\text{eV}}{e}$ =2.66 V

66. $E = W_0 + K_{max}$; $E = \frac{12375}{3000} = 4.125$ eV

∴ $K_{max} = E - W_0 = 4.125\text{ eV} - 1\text{ eV} = 3.125$ eV

∴ $\frac{1}{2}mv_{max}^2 = 3.125\times1.6\times10^{-19}$ J

∴ $v_{max} = \sqrt{\frac{2\times3.125\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 1\times10^6$ m/s

67. Energy received from the sun

$= 2$ cal cm^{-2} $(min)^{-1}$ = 8.4 J cm^{-2} $(min)^{-1}$

Energy of each photon received from sun,

$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{5500\times10^{-10}} = 3.6\times10^{-19}$ J

∴ Number of photons reaching the earth per cm^2 per minute will be

$n = \frac{\text{Energy received from sun}}{\text{Energy of one photon}}$

$= \frac{8.4}{3.6\times10^{-19}} = 2.3\times10^{19}$

68. The stopping potential gives maximum kinetic energy of the electron. It depends on the material as well as the frequency of incident light whereas the current depends on the number of incident photons. Hence, it is 0.5 V. By inverse square law, saturation current is inversely proportional to square of distance.

∴ $12 = \frac{K}{(0.2)^2}$ and $I = \frac{K}{(0.4)^2}$

∴ $\frac{I}{12} = \frac{(0.2)^2}{(0.4)^2} = \frac{1}{4}$ or I = 3 mA

∴ I = 3 mA ⇒ stopping potential = 0.5 V

69. Using $\frac{hc}{\lambda} = W_0 + \frac{1}{2}mv^2$

∴ $\frac{hc}{400\times10^{-9}} = W_0 + \frac{1}{2}mv^2$ ….(i)

and $\frac{hc}{250\times10^{-9}} = W_0 + \frac{1}{2}m(2v)^2$ ….(ii)

On solving equation (i) and equation (ii) we get,

$\frac{1}{2}mv^2 = \frac{hc}{3}\left[\frac{1}{250\times10^{-9}} - \frac{1}{400\times10^{-9}}\right]$ ….(iii)

From equations (i) and (iii),

$W_0 = 2hc\times10^6$ J

70. $K_{max} = \frac{1}{2}mv_{max}^2 = h\nu - W_0$ ….(i)

Now, when ν is doubled, then

$\frac{1}{2}m\,(2\,v_{max})^2 = 2h\nu - W_0$

∴ $4\times\frac{1}{2}mv_{max}^2 = 2h\nu - W_0$

∴ $4(h\nu - W_0) = 2h\nu - W_0$ ….[from equation (i)]

∴ $3W_0 = 2\,h\nu$

∴ $W_0 = \frac{2h\nu}{3}$

71. $\left(\frac{hc}{\lambda}\right)\times N = 200\times\frac{25}{100}$ ….[Given]

∴ $N = \frac{200\times25}{100}\times\frac{\lambda}{hc} = \frac{200\times25\times0.6\times10^{-6}}{100\times6.2\times10^{-34}\times3\times10^{8}}$

$= 1.5\times10^{20}$

72. Intensity of light

$I = \frac{\text{Watt}}{\text{Area}} = \frac{nhc}{A\lambda}$

∴ Number of photon $n = \frac{IA\lambda}{hc}$

∴ $n = \frac{1}{100}\times\frac{A\lambda}{hc} = \frac{1}{100}\times\frac{1\times10^{-4}\times300\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}}$

$= 1.5\times10^{12}$ /s

73. $P = \frac{n}{t}\frac{hc}{\lambda}$

But P = Fc

∴ $Fc = \frac{n}{t}\frac{hc}{\lambda}$

∴ $\frac{n}{t} = \frac{F\lambda}{h} = \frac{6.62\times10^{-5}\times5\times10^{-7}}{6.62\times10^{-34}} = 5\times10^{22}$

74. Using $h\nu - h\nu_0 = K_{max}$

$h(\nu_1 - \nu_0) = K_1$ and ….(i)

$h(\nu_2 - \nu_0) = K_2$ ….(ii)

$\therefore \quad \frac{\nu_1 - \nu_0}{\nu_2 - \nu_0} = \frac{K_1}{K_2} = \frac{1}{K}$,

$\therefore \quad \nu_0 = \frac{K\nu_1 - \nu_2}{K - 1}$.

75. $KE_1 = \frac{hc}{\lambda} - W_0$

$KE_2 = \frac{hc}{\lambda/2} - W_0 = \frac{2hc}{\lambda} - W_0$

$KE_2 = 3KE_1$

$\Rightarrow \frac{2hc}{\lambda} - W_0 = 3\left(\frac{hc}{\lambda} - W_0\right)$

$\Rightarrow 2\ W_0 = \frac{hc}{\lambda}$ $\quad \Rightarrow W_0 = \frac{hc}{2\lambda}$

76. We know that

$\frac{1}{2}mv^2_{max} = hf - W_0$

For 1st frequency,

$\frac{1}{2}mv_1^2 = hf_1 - W_0$ ….(i)

For 2nd frequency,

$\frac{1}{2}mv_2^2 = hf_2 - W_0$ ….(ii)

Subtracting equation (ii) from equation (i),

$\frac{1}{2}m\left(v_1^2 - v_2^2\right) = h\left(f_1 - f_2\right)$

$\Rightarrow v_1^2 - v_2^2 = \frac{2h}{m}\left(f_1 - f_2\right)$

77. Stopping potential is same for (a) and (b). Hence, their frequencies are same. Also maximum current values are different for (a) and (b). Hence, they will have different intensities.

78. $\frac{1}{2}mv_1^2 = \frac{hc}{\lambda_1} - W_0$

$\frac{1}{2}mv_2^2 = \frac{hc}{\lambda_2} - W_0$

$\left(\frac{v_1}{v_2}\right)^2 = \left(\frac{\frac{hc}{\lambda_1} - W_0}{\frac{hc}{\lambda_2} - W_0}\right) = \left(\frac{2}{1}\right)^2$ $\quad …. \left[\because \frac{v_1}{v_2} = 2\right]$

$\therefore \quad \frac{4hc}{\lambda_2} - 4W_0 = \frac{hc}{\lambda_1} - W_0$

$\therefore \quad \frac{4hc}{\lambda_2} - \frac{hc}{\lambda_1} = 3W_0$

$\therefore \quad \frac{4 \times 1240}{310} - \frac{1240}{248} = 3W_0$

$\therefore \quad 3\ W_0 = 11 \Rightarrow W_0 = 3.7$ eV

79. Using Einstein equation, $E = W_0 + \frac{1}{2}mv^2$

$\therefore \quad \sqrt{\frac{2(E - W_0)}{m}} = v$

A charged particle placed in uniform magnetic field experience a force

$F = \frac{mv^2}{r}$

$\Rightarrow evB = \frac{mv^2}{r}$ $\quad \Rightarrow r = \frac{mv}{eB}$

$\therefore \quad r = \frac{\sqrt{2m(E - W_0)}}{eB}$.

80. Using, $K = \frac{1}{2}mv^2 = h\nu - h\nu_0$ for same metal,

$K_A = \frac{hc}{\lambda_A} - h\nu_0$,

$K_B = \frac{2hc}{\lambda_A} - h\nu_0$

$\therefore \quad K_A < \frac{K_B}{2}$

81. $E = 15$ keV $= 15 \times 10^3$ eV

$\because \quad E = \frac{12400}{\lambda}$ eV Å

$\lambda = \frac{12400}{E}$ eV Å

$\lambda = \frac{12400}{15 \times 10^3\ eV}$ eV Å

$\lambda = 0.826$ Å ($\lambda < 0.01$ Å)

It belongs to X-rays.

82. $K.E_{max} = \left(\frac{hc}{\lambda}\right)$joules – 2.2 eV

$(\because K.E_{max} = \frac{hc}{\lambda} - \phi_0)$

$K.E_{max} = \left(\frac{hc}{\lambda \times 1.6 \times 10^{-19}}\right)$eV – 2.2 eV

$K.E_{max} = 2$ eV – 2.2 eV = –0.2 eV

As kinetic energy can never be negative, hence photo-emission doesn't occur.

Evaluation Test

2. We know that,

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W_0$$

$$= \frac{(6.63\times10^{-34})(3\times10^{8})}{120\times10^{-9}} - 3.0 \times 1.6 \times 10^{-19}$$

$$= 16.57 \times 10^{-19} - 4.8 \times 10^{-19}$$

$$= 11.77 \times 10^{-19}\ J$$

$\therefore$ $mv^2 = 2 \times (11.77 \times 10^{-19})$

or $mv^2 = 23.54 \times 10^{-19}$ J

$$v = \sqrt{\frac{23.54\times10^{-19}}{9.1\times10^{-31}}} = 1.61 \times 10^6$$

Now, $Bev = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Be}$

$$r = \frac{9.1\times10^{-31}}{(4\times10^{-5})(1.6\times10^{-19})} \times 1.61 \times 10^6$$

or $r = 0.228$ m ≈ 0.23 m

3. For the first wavelength:

$eV_{s_1} = h\nu_1 - W_0$ ....(i)

For the second surface:

$eV_{s_2} = h\nu_2 - W_0$ ....(ii)

Subtracting equation (i) from equation (ii),

$$V_{s_2} - V_{s_1} = \frac{h}{e}(\nu_2 - \nu_1)$$

or $$V_{s_2} = V_{s_1} + \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= V_{s_1} + \frac{hc}{e}\left[\frac{\lambda_1 - \lambda_2}{\lambda_2\lambda_1}\right]$$

$$= 0.2 + 1240\left[\frac{450-120}{120\times450}\right]$$

....$\left[\because \frac{hc}{e} \approx 1240\,eV - nm\right]$

$= 7.78$ V

From equation (i),

$$W_0 = \frac{hc}{\lambda_1} - eV_{s_1}\ J$$

$$\frac{W_0}{e} = \frac{hc}{e\lambda_1} - Vs_1\ eV = \frac{1240}{450} - 0.2\ eV$$

$= 2.56$ eV

$$\nu_0 = \frac{W_0}{h} = \frac{2.56 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$= 0.62 \times 10^{15} = 6.2 \times 10^{14}$ Hz

5. Stopping potential does not depend upon the distance of source from photocell but saturation current

$$\propto \left[\frac{1}{\text{square of distance of source}}\right]$$

$\therefore$ $I_1 \propto \frac{1}{(0.2)^2}$ and $I_2 \propto \frac{1}{(0.4)^2}$

$\therefore$ $\frac{I_2}{12} = \frac{(0.2)^2}{(0.4)^2}$

or $I_2 = 12\left(\frac{0.2}{0.4}\right)^2 = 3$ mA

6. $W_0 = \frac{hc}{\lambda_{max}} \Rightarrow \lambda_{max} = \frac{hc}{W_0} = \frac{12400\,eV\,Å}{4\,eV} \approx 3100$ Å

7. Energy of green photon,

$$E = \frac{hc}{\lambda}$$

$$= \frac{(6.6\times10^{-34})(3\times10^{8})}{(4000\times10^{-10})}$$

$= (4.95 \times 10^{-19})$J

Energy received per second

$= (4.95 \times 10^{-19})(5 \times 10^4)$

$= 2.48 \times 10^{-14}$ W/m^2

Sensitivity of eye in comparison to ear

$$= \frac{\text{Power per square metre detected by ear}}{\text{Energy received per second}}$$

$$= \frac{10^{-13}}{(2.48\times10^{-14})}$$

$= 4$

8. Here, $\frac{\lambda_2}{\lambda_1}\frac{(\lambda_0 - \lambda_1)}{(\lambda_0 - \lambda_2)} = \frac{2}{1}$

or $\frac{5.4}{3.4}\frac{(\lambda_0 - 3.4\times10^{-7})}{(\lambda_0 - 5.4\times10^{-7})} = \frac{2}{1}$

or $\lambda_0 = 12.7 \times 10^{-7}$ m

Now, $W_0 = \frac{hc}{\lambda_0}$

$$= \frac{(6.6\times10^{-34})(3\times10^{8})}{(12.7\times10^{-7})(1.6\times10^{-19})}$$

$= 0.98$ eV

9. Here $\frac{hc}{\lambda_1} = \frac{1}{2}mv_1^2 + W_0$

and $\frac{hc}{\lambda_2} = \frac{1}{2}mv_2^2 + W_0$

Then $\left(\frac{v_1}{v_2}\right)^2 = \frac{\frac{hc}{\lambda_1} - W_0}{\frac{hc}{\lambda_2} - W_0}$

or $n^2 = \frac{\frac{hc}{\lambda_1} - W_0}{\frac{hc}{\lambda_2} - W_0}$

or $n^2\left(\frac{hc}{\lambda_2} - W_0\right) = \left(\frac{hc}{\lambda_1} - W_0\right)$

$\therefore$ $W_0 = \frac{hc\left(n^2 - \frac{\lambda_2}{\lambda_1}\right)}{\lambda_2(n^2 - 1)}$

10. Saturation current depends on intensity. Hence B and C will have same intensity different from that of A. Stopping potential depends on frequency. So A and B will have the same frequency different from that of C.
Hence option (A) is correct.

11. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

$\therefore$ $\lambda_P = \frac{h}{\sqrt{2m_p(q_p)V}}$; $\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha(q_\alpha)V'}}$

Now, $\lambda_P = \lambda_\alpha \Rightarrow \frac{h}{\sqrt{2m_p q_p V}} = \frac{h}{\sqrt{2m_\alpha q_\alpha V'}}$

$\therefore$ $m_p q_p V = m_\alpha q_\alpha V'$

$\therefore$ $V' = \frac{m_P q_P V}{m_\alpha q_\alpha} = \frac{(1)(1)V}{(4)(2)} = \frac{V}{8}$ volt

12. $E_{photon} = \frac{hc}{\lambda} = \frac{1240\,eV - nm}{320\,nm} \approx 3.88$ eV

This is greater than the work functions of Na(2.75 eV) and K(2.30 eV) but lesser than the work functions of Mo (4.17 eV) and Ni(5.15 eV).
Hence Na and K will give photocurrent and Mo and Ni wouldn't.
In photoelectric effect, as intensity increases, photocurrent increases.

15. Gain in K.E. = Loss in P.E.

$\therefore$ $\frac{p^2}{2m} = qV \Rightarrow p = \sqrt{2mqV}$

$\therefore$ $\frac{p_p}{p_\alpha} = \frac{\sqrt{2m_p(e)V}}{\sqrt{2m_\alpha(2e)V}} = \sqrt{\frac{m_P}{m_\alpha}\left(\frac{e}{2e}\right)} = \sqrt{\frac{1}{4}.\frac{1}{2}}$

$= \frac{1}{2\sqrt{2}}$

16. Using $E = \frac{nhc}{\lambda}$, we get

$10^{-7} = \frac{n(6.6\times10^{-34})(3\times10^8)}{(3000\times10^{-10})}$

$\therefore$ $n = 1.5 \times 10^{11}$

17. $\lambda_0 = \frac{hc}{W} = \frac{(6.6\times10^{-34})(3\times10^8)}{4.125\times1.6\times10^{-19}} = 300$ nm

18. The maximum KE of ejected electron is given by

$(KE)_{max} = h\nu - W_0$

$= \frac{hc}{\lambda} - W_0$

For minimum B, $\lambda = 2000$ Å

$\therefore$ $(KE)_{max} = \frac{(6.63\times10^{-34})(3\times10^8)}{2000\times10^{-10}\times(1.6\times10^{-19})} - 2.22$ eV

$= 6.19$ eV $- 2.22$ eV

$= 3.97$ eV

Further, $(KE)_{max} = \frac{1}{2}mv^2$

$= 3.97$ eV

$= 3.97 \times 1.6 \times 10^{-19}$ J

$\therefore$ $v = \left[\frac{2\times3.97\times1.6\times10^{-19}}{9.1\times10^{-31}}\right]^{1/2}$

$= 11.8 \times 10^5$ m/s

For zero current,

$\frac{mv^2}{R} = evB$

or, $B = \frac{mv}{eR}$

$= \frac{(9.1\times10^{-31})(11.8\times10^5)}{(1.6\times10^{-19})\times0.1}$

$= 6.7 \times 10^{-5}$ T

19. Using $E = \frac{1}{2}mv^2 = h\nu_0 - W_0$ we get,

$E_1 = (1 - 0.6) = 0.4$ eV

and $E_2 = (2.5 - 0.6) = 1.9$ eV

$\therefore \quad \frac{E_1}{E_2} = \frac{v_1^2}{v_2^2} = \frac{0.4}{1.9} \approx 0.21$

or $\frac{v_1}{v_2} = 0.458 \approx 0.5$

20. Using Einstein's photoelectric equation,

$h\nu = W_0 + K_{max}$

$\therefore \quad \frac{hc}{\lambda} = W_0 + e(3V_0) \quad$ ….(i)

$(\because K_{max} = eV_s)$

Also, $\frac{hc}{2\lambda} = W + eV_0 \quad$ ….(ii)

Subtracting equation (i) from 3 × equation (ii) we get,

$\left(\frac{3}{2} - 1\right)\frac{hc}{\lambda} = 3W_0 - W_0$ or $W_0 = \frac{hc}{4\lambda}$

But $W_0 = \frac{hc}{\lambda_0}$, where λ_0 is the threshold wavelength, hence $\lambda_0 = 4\lambda$.

Hence, option (C) is correct.

Textbook Chapter No.

18 Atoms, Molecules and Nuclei

Hints

Classical Thinking

12. Energy increases from lower state to higher state.

17. In outermost stationary orbit, electron is at maximum distance from the nucleus. Hence the energy of electron is least negative.

18. $r \propto n^2 \Rightarrow r \propto (3)^2$

19. $r \propto n^2$

$\therefore \quad \frac{r_1}{r_2} = \frac{n_1^2}{n_2^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

21. $v \propto \frac{1}{n} \qquad \therefore \quad \frac{v_1}{v_2} = \frac{n_2}{n_1} = \frac{2}{1}$

22. $A \propto r^2$, but $r \propto n^2$

$\therefore \quad A \propto n^4$

32. $E_n \propto \frac{1}{n^2}$

$\therefore \quad \frac{E_3}{E_5} = \frac{(5)^2}{(3)^2} = \frac{25}{9}$

33. $K.E = \frac{e^2}{8\pi\varepsilon_0 r}$

$= \frac{\left(1.6\times10^{-19}\right)^2}{8(3.14)(8.854\times10^{-12})(0.529\times10^{-10})(1.6\times10^{-19})}$ eV

$= 13.6$ eV

34. $E_\infty = 0$ and

$E_5 = \frac{-13.6}{25} = -0.544$ eV

$\therefore \quad \Delta E = E_\infty - E_5$

$= 0 - (-0.54) = 0.54$ eV

35. $T.E. = \frac{1}{2}(P.E.) = -(K.E.)$

$\frac{K.E.}{P.E.} = -\frac{1}{2}$

36. Wave number $= \frac{1}{\lambda} = \frac{1}{6000\times10^{-10}}$

$= 1.66 \times 10^6\ m^{-1}$

38. As n increases, energy difference between adjacent energy levels decreases.

41. $R \propto m$. Thus, if mass is reduced to half, then Rydberg constant also becomes half.

43. Energy is absorbed when atom goes from lower state to higher state.

47. As difference between the levels increases, energy emitted increases and hence wavelength decreases. It means colour must change to violet.

51. For $_7N^{13}$, N = 13 – 7 = 6 and

for $_6C^{12}$, N = 12 – 6 = 6

As number of neutrons is same, they are isotones.

52. They have same mass number (A).

61. Actual mass of the nucleus is always less than total mass of nucleons

$\therefore \quad M < (NM_n + ZM_p)$.

64. In fusion, two lighter nuclei combines which is not the radioactive decay.

70. $_zX^A \xrightarrow{_{-1}\beta^0} {}_{z+1}Y^A \xrightarrow{_2He^4(\alpha)}$

$_{z-1}K^{A-4} \xrightarrow{_0\gamma^0} {}_{z-1}K^{A-4}$

71. $T = \frac{0.6931\times1}{\lambda} = \frac{0.6931\times1}{4.28\times10^{-4}} \approx 1620$ years

72. Fraction of sample after n-half-lives is given by

$\frac{N}{N_0} = \frac{1}{2^n}$

Where; n = t/T

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{15/5} = \frac{1}{8}$

$\therefore \quad$ Decayed fraction $= 1 - \frac{1}{8} = \frac{7}{8}$

73. Fraction of sample after n-half-lives is given by

$\frac{N}{N_0} = \frac{1}{2^n}$

Where; n = t/T

$$N = N_0\left(\frac{1}{2}\right)^{t/T}$$

$\therefore$ $$\frac{N_0}{64} = N_0\left(\frac{1}{2}\right)^{30/T}$$

$\therefore$ $$T = \frac{30}{6} = 5\ s$$

74. Fraction of sample after n-half-lives is given by
$$\frac{N}{N_0} = \frac{1}{2^n}$$
Where; n = t/T
$$N_t = N_0\left(\frac{1}{2}\right)^{t/T} = 50000\left(\frac{1}{2}\right)^{10/5} = 12500$$

77. According to Bohr's theory,
$$mvr = n\frac{h}{2\pi}$$
$\therefore$ Circumference, $2\pi r = n\left(\frac{h}{mv}\right) = n\lambda$

78. $$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-27}\ \text{erg - s}}{200\,g\times3\times10^{3}\,cm\,s^{-1}}$$
$$= 1.1 \times 10^{-32}\ cm$$

79. $$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{10^{-3}\times100} = 6.63 \times 10^{-33}\ m$$

80. $$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{2\times10^{-3}\times10^{-3}\times100\times10^{-2}}$$
$$= 3.32 \times 10^{-28}\ m$$

92. For $\lambda > 2D$, $\sin\theta > 1$
which is not possible.

93. If the energy radiated in the transition be E, then we have,
$$E_{R\to G} > E_{Q\to S} > E_{R\to S} > E_{Q\to R} > E_{P\to Q}$$
For getting blue line, the energy radiated should be maximum $\left(\because E \propto \frac{1}{\lambda}\right)$.

94. Using, $R \propto A^{1/3}$
$$\frac{R_{Li}}{R_{Fe}} = \left(\frac{Li^7}{Fe^{56}}\right)^{1/3} = \left(\frac{7}{56}\right)^{1/3} = \frac{1}{2}$$

95. Minimum energy required to excite from ground state
$$= 13.6\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 10.2\ eV$$

96. $r_n \propto n^2 \Rightarrow A_n \propto n^4$ where, A_n = area
$\therefore$ $$\frac{A_1}{A_0} = \left(\frac{2}{1}\right)^4 = \frac{16}{1}$$

97. The elements having atomic number greater than that of uranium (U-92) are called transuranic elements. Plutonium (Pu) with atomic number 94 is transuranic.

Critical Thinking

1. $L \propto n$ and $p \propto \frac{1}{n}$
$\Rightarrow L \times p \propto 1 \Rightarrow L \times p \propto n^0$

2. Angular momentum = mvr = $\frac{nh}{2\pi}$
$\therefore$ Angular momentum $\propto$ n
$\therefore$ Ratio = $\frac{n_1}{n_2} = \frac{1}{2}$

3. $r_n \propto n^2$
$\therefore$ $$\frac{r_n}{r_0} = \left(\frac{n}{1}\right)^2 = (4)^2 = 16$$
$\therefore$ $r_n = 16 \times 0.53 = 8.48$ Å

4. $r_n \propto n^2$
$\therefore$ $A \propto r^2 \propto n^4$
$\therefore$ $$\frac{A_2}{A_1} = \left(\frac{n_2}{n_1}\right)^4 = \left(\frac{3}{1}\right)^4 = 81$$
$\therefore$ $A_2 = 81\ A_1 = 81\ A$

6. $T = \frac{2\pi r}{v}$, $r \propto n^2$ and
$v \propto \frac{1}{n} \Rightarrow T \propto n^3$
$\therefore$ $$\frac{T_1}{T_2} = \frac{{n_1}^3}{(2n_1)^3} = \frac{1}{8}$$

7. $$\frac{mv^2}{r} = qvB \Rightarrow mv = qBr$$
Now, $mvr = \frac{nh}{2\pi}$
$\therefore$ $$qBr_n^2 = \frac{nh}{2\pi} \Rightarrow r_n^2 = \frac{nh}{2\pi qB}$$

8. Radius of electron in the hydrogen atom in the ground state = $r_1 = 5.3 \times 10^{-11}$ m. (n_1 = 1) Radius of electron in the hydrogen atom in the excited state = $r_2 = 13.25 \times 10^{-10}$ m.

For a hydrogen atom,
$r \propto n^2$

$\therefore \quad \frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2$

$\therefore \quad \frac{5.3\times10^{-11}}{13.25\times10^{-10}} = \frac{{n_1}^2}{{n_2}^2}$

$\therefore \quad n_2^2 = 25 \Rightarrow n_2 = 5$

9. Change in angular momentum of electron,

$L_5 - L_4 = \frac{h}{2\pi}[5-4] = \frac{6.64\times10^{-34}}{2(3.14)} = 1.05\times10^{-34}$ J-s

10. Using, $E_n = \frac{-13.6}{n^2}$,

$E_3 = \frac{-13.6}{3^2} = -1.51$ eV

11. Energy of electron, $E_n = \frac{-13.6}{n^2}$ eV

$\therefore \quad -0.544 \text{ eV} = \frac{-13.6}{n^2}$ eV

$\therefore \quad n^2 = 25 \Rightarrow n = 5$

Orbital velocity of electron in ground state,

$v_n = \frac{e^2}{2\varepsilon_0 hn} = \frac{e^2}{2\varepsilon_0 h(5)} = \frac{v}{5}$

12. $E_n = \frac{B}{n^2}$ where $B = 16 \times 10^{-18}$ J

$\therefore \quad E_4 = \frac{16\times10^{-18}}{(4)^2} = \frac{16\times10^{-18}}{16} = 1 \times 10^{-18}$ J

$\therefore \quad E_2 = \frac{16\times10^{-18}}{(2)^2} = \frac{16\times10^{-18}}{4} = 4 \times 10^{-18}$ J

Let $E_2 - E_4 = h\nu = \frac{hc}{\lambda}$

$\therefore \quad \lambda = \frac{hc}{E_2 - E_4} = \frac{h\times3\times10^8}{(4-1)\times10^{-18}} = \frac{h\times3\times10^8}{3\times10^{-18}}$

$= 10^{26}$ h

13. For Lyman series, $n_1 = 1$, $n_2 = \infty$

$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = R$

$\therefore \quad \lambda = \frac{1}{R}$

14. $\frac{1}{\lambda_L} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$, $\lambda_L = \frac{4}{3R}$

$R = 1.0967 \times 10^7 \text{ m}^{-1} = 1.0967 \times 10^5 \text{ cm}^{-1}$

$\therefore \quad \lambda_L = \frac{4}{3\times109670}$ cm

15. $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = \frac{15R}{16}$

$\therefore \quad \lambda = \frac{16}{15R} = \frac{16}{15}\times10^{-5}$ cm

$\therefore \quad n = \frac{c}{\lambda} = \frac{3\times10^{10}}{\left(\frac{16}{15}\times10^{-5}\right)} = 2.81\times10^{15}$ Hz

16. Given that, we get six wavelengths.
Maximum number of spectral lines,

$\frac{n(n-1)}{2} = 6$ which on solving gives n = 4

Using $\frac{1}{\lambda} = R\left(1 - \frac{1}{4^2}\right)$ we get,

$\frac{1}{\lambda} = R\left(1 - \frac{1}{16}\right) = \frac{15R}{16}$

$\therefore \quad \lambda = \frac{16}{15R} = \frac{16}{15\times1.097\times10^7}$

$= 9.72 \times 10^{-8} = 97.2 \times 10^{-9}$ m

$= 97.2$ nm ≈ 97 nm

(Note: Use shortcut 3.)

17. For Lyman series, $\frac{1}{\lambda} = R_H\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$,

where n = 2, 3, 4,….
For shortest wavelenth, $n = \infty$

$\therefore \quad \frac{1}{\lambda} = \frac{R_H}{1}$

$\therefore \quad \lambda = \frac{1}{R_H} = \frac{1}{109678 \text{ cm}^{-1}}$

$= 9.117 \times 10^{-6}$ cm = 911.7 Å.

18. $\frac{\lambda_{Br}}{\lambda_{Pf}} = \frac{\left(\frac{1}{5^2} - \frac{1}{6^2}\right)}{\left(\frac{1}{4^2} - \frac{1}{5^2}\right)} = \frac{11/9}{9/4} = \frac{44}{81}$

$\because \quad \nu \propto 1/\lambda$

$\therefore \quad \frac{\nu_{Br}}{\nu_{Pf}} = \frac{\lambda_{Pf}}{\lambda_{Br}} = \frac{81}{44}$

19. $\frac{\lambda_B}{\lambda_L} = \frac{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)}{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)} = \frac{3/4}{5/36} = \frac{27}{5}$

$\therefore \quad \lambda_L = \frac{5}{27}\lambda_B = \frac{5}{27}\times6563 = 1215.4$ Å ≈ 1215 Å

20. For Balmer series,

$$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

And for Paschen series,

$$\frac{1}{\lambda_P} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$$

Now, for series limit, $n = \infty$

$\therefore$ $$\frac{\left(\frac{1}{\lambda_B}\right)}{\left(\frac{1}{\lambda_P}\right)} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{9}\right)}$$

$\therefore$ $$\frac{\lambda_P}{\lambda_B} = \frac{9}{4}$$

$\therefore$ $$\lambda_p = \frac{9}{4} \times 6400 = 9 \times 1600 = 14400 \text{ Å}$$

21. Frequency of radiation emitted

$$\nu = Rc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] = 10^7 \times 3 \times 10^8 \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 3 \times 10^{15} \times \frac{5}{9 \times 4} = \frac{5}{12} \times 10^{15} \approx 4 \times 10^{14} \text{ Hz}$$

22. $$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$\therefore$ $$\frac{1}{\lambda} = R\left[\frac{n_2^2 - n_1^2}{n_1^2 n_2^2}\right]$$

$\therefore$ $$\lambda = \frac{1}{R}\left[\frac{n_1^2 n_2^2}{n_2^2 - n_1^2}\right]$$

$\therefore$ $$\frac{36}{5R} = \frac{1}{R}\left[\frac{n_1^2 n_2^2}{n_2^2 - n_1^2}\right]$$

$\Rightarrow$ $n_1^2\, n_2^2 = 36$ and $n_2^2 - n_1^2 = 5$

$\therefore$ On simplifying these two equations, we get $n_2 = 3$, $n_1 = 2$

23. For longest wavelength in Lyman series, $n_1 = 1$, $n_2 = 2$

$\therefore$ $$\frac{1}{\lambda_L} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \Rightarrow \lambda_L = \frac{4}{3R}$$

For shortest wavelength $n_1 = 1$, $n_2 = \infty$

$\therefore$ $$\frac{1}{\lambda_S} = R\left[\frac{1}{1^2} - 0\right] \Rightarrow \lambda_S = \frac{1}{R}$$

$\therefore$ $$\frac{\lambda_L}{\lambda_S} = \frac{4R}{3R} = \frac{4}{3}$$

$\therefore$ $$\lambda_L = \frac{4}{3} \times 912 = 1216 \text{ Å}$$

24. $$\frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[\frac{4-1}{4}\right] = \frac{3R}{4}$$

$\therefore$ $$\lambda_1 = \frac{4}{3R} = 121.6 \text{ nm} \quad \text{....(i)}$$

Let $$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = R\left[\frac{16-4}{64}\right] = \frac{12R}{64}$$

$\therefore$ $$\lambda_2 = \frac{64}{12R} \quad \text{....(ii)}$$

From equations (i) and (ii),

$$\frac{\lambda_2}{\lambda_1} = \frac{\lambda_2}{121.6} = \frac{64}{12R} \times \frac{3R}{4}$$

$\therefore$ $\lambda_2 = 4 \times 121.6 = 486.4$ nm

25. $$\frac{1}{\lambda_{AC}} = R_H\left[\frac{1}{C^2} - \frac{1}{A^2}\right] = \frac{1}{2000}$$

and $$\frac{1}{\lambda_{BC}} = R_H\left[\frac{1}{C^2} - \frac{1}{B^2}\right] = \frac{1}{6000}$$

$\therefore$ $$\frac{1}{\lambda_{AB}} = R_H\left[\frac{1}{B^2} - \frac{1}{A^2}\right]$$

$$= R_H\left[\frac{1}{B^2} - \frac{1}{C^2} + \frac{1}{C^2} - \frac{1}{A^2}\right]$$

$$= R_H\left[\frac{1}{C^2} - \frac{1}{A^2}\right] - R_H\left[\frac{1}{C^2} - \frac{1}{B^2}\right]$$

$$= \frac{1}{\lambda_{AC}} - \frac{1}{\lambda_{BC}} = \frac{1}{2000} - \frac{1}{6000}$$

$$= \frac{2}{6000} = \frac{1}{3000}$$

$\therefore$ $\lambda_{AB} = 3000$ Å

26. $$\bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$X = R$ ….(Lyman series)

$Z = R\left(\frac{1}{4}\right)$ ….(Balmer series)

$\therefore$ $$Y = R\left(1 - \frac{1}{4}\right) = \frac{3}{4} R$$

From above, $X = Y + Z \Rightarrow Z = X - Y$

27. In X-ray spectra, depending on the accelerating voltage and the target element, we may find sharp peaks superimposed on continuous spectrum. These are at different wavelengths for different elements. They form characteristic X-ray spectrum.

28. Density of nucleus = $\frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$

$= \frac{A \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi\left(1.1 \times 10^{-15}\right)^3 \times A}$

$= 2.97 \times 10^{17}$ kg m^{-3}.

Since, density of nucleus is independent of mass number, hence density of all nuclei is same.

29. ${}_4Be^9 + {}_2He^4 \rightarrow {}_6C^{12} + {}_0n^1$.

30. Using $R = R_0 A^{1/3}$,

$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} \Rightarrow \frac{R}{R_{He}} = \left(\frac{A}{4}\right)^{1/3}$

$\therefore$ $(14)^{1/3} = \left(\frac{A}{4}\right)^{1/3}$

$\therefore$ $A = 56 \Rightarrow Z = 56 - 30 = 26$

31. $R \propto (1)^{1/3}$

$\therefore$ $R_{80} \propto (80)^{1/3}$ and $R_{10} \propto (10)^{1/3}$

$\therefore$ $\frac{R_{80}}{R_{10}} = \left(\frac{80}{10}\right)^{1/3} = (8)^{1/3} = 2$

$\therefore$ $R_{80} = 2 \times R_{10} = 2 \times 3 \times 10^{-15} = 6 \times 10^{-15}$ m

32. The equation is $O^{17} \rightarrow {}_0n^1 + O^{16}$

$\therefore$ Energy required = B.E. of O^{17} – B.E. of O^{16}

$= 17 \times 7.75 - 16 \times 7.97$

$= 4.23$ MeV

33. Energy is released in a process when total binding energy (B.E.) of the nucleus is increased or we can say when total B.E. of products is more than the reactants. By calculation, we can see that only in case of option (C), this happens.

Given $W \rightarrow 2Y$

B.E. of reactants = $120 \times 7.5 = 900$ MeVand

B.E. of products = $2 \times (60 \times 8.5) = 1020$ MeV

i.e., B.E. of products > B.E. of reactants.

34. $n_\alpha = \frac{A - A'}{4} = \frac{232 - 208}{4} = 6$

$n_\beta = (2n_\alpha - Z + Z') = (2 \times 6 - 90 + 82) = 4$

35. Average life

$T = \frac{\text{Sum of all lives of all the atom}}{\text{Total number of atoms}} = \frac{1}{\lambda}$

$\therefore$ $T\lambda = 1$

36. Fraction that remains after n half lives,

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$

$\therefore$ $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{T/2}{T}} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}$

37. $\frac{dN}{dt} = -\lambda N$

$\therefore$ $\left|\frac{dN}{dt}\right| = \frac{0.693}{T_{1/2}} \times N$

$= \frac{0.693}{1.2 \times 10^7} \times 4 \times 10^{15}$

$= 2.3 \times 10^8$ atoms/s

38. Using $N = N_0\left(\frac{1}{2}\right)^{t/T}$

$\therefore$ $N = \left(1 - \frac{7}{8}\right)N_0 = \frac{1}{8}N_0$

$\therefore$ $\frac{1}{8}N_0 = N_0\left(\frac{1}{2}\right)^{t/T}$

$\therefore$ $\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/5} \Rightarrow t = 15$ days

39. $\frac{dN}{dt} = -\lambda N$

$\therefore$ $n = -\lambda N$ .... $\left(\because \frac{dN}{dt} = n\right)$

$\therefore$ $\lambda = -\frac{n}{N}$

$\therefore$ Half-life = $\frac{0.693}{\lambda} = \frac{0.693}{\lambda} = \frac{0.693\ N}{n}$ s

40. Using $N = N_0e^{-\lambda t}$,

$\frac{N_0}{2} = N_0e^{-\lambda T_{1/2}} \Rightarrow 2 = e^{\lambda T_{1/2}}$

$\therefore$ By taking $\log_e$ on both the sides,

$\log_e 2 = \lambda T_{1/2} \Rightarrow \lambda T_{1/2} = 0.693$

41. $A = A_0e^{-\lambda t}$

$\therefore$ $975 = 9750\ e^{-\lambda \times 5}$

$e^{5\lambda} = 10$

$\therefore$ $5\lambda = \log_e 10 = 2.303 \log_{10} 10 = 2.303$

$\therefore$ $\lambda \approx 0.461$

42. $p = \frac{h}{\lambda} = \frac{6.625 \times 10^{-34}}{10^{-17}}$

$= 6.625 \times 10^{-17}$ kg m s^{-1}

43. $\lambda = \dfrac{h}{mv} \Rightarrow v = \dfrac{h}{\lambda \times m}$

$= \dfrac{6.6\times10^{-34}}{(66\times10^{-9})\times(9\times10^{-31})}$

$= 0.011 \times 10^{6}$

$= 1.1 \times 10^{4}\ ms^{-1}$

44. $m_{He} = \dfrac{4/1000}{6.02\times10^{23}}$ kg

$= 6.64 \times 10^{-27}$ kg

$\lambda = \dfrac{h}{mv}$

$= \dfrac{6.63\times10^{-34}}{6.64\times10^{-27}\times2.4\times10^{2}}$

$= 0.416 \times 10^{-9}$ m

$= 0.416$ nm

45. For the ground state, $mvr = \dfrac{h}{2\pi}$

$\therefore$ $2\pi r = \dfrac{h}{mv} = \lambda$ = de-Broglie wavelength

$\therefore$ de-Broglie wavelength $\lambda = \dfrac{h}{mv}$

$\therefore$ $\lambda = \dfrac{h}{h/2\pi r} = 2\pi r$

46. $\lambda_A = \dfrac{h}{mv}$, $\lambda_B = \dfrac{h}{0.25m\times0.75v}$

$\dfrac{\lambda_B}{\lambda_A} = \dfrac{1}{0.25\times0.75} = 5.3$

$\therefore$ $\lambda_B = 5.3\ \lambda_A = 5.3$ Å

47. $\dfrac{h}{\sqrt{2mE}} = \dfrac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times50\times1.6\times10^{-19}}}$

$= \dfrac{6.63\times10^{-34}}{\sqrt{1.456\times10^{-47}}}$

$= 1.737$ Å

48. $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2mqV}}$

$\therefore$ $\dfrac{\lambda_e}{\lambda_p} = \sqrt{\dfrac{m_p q_p V}{m_e\times e\times V}} = \sqrt{\dfrac{m_p}{m_e}}$

[$\because$ V is the same and $q_p = e$ (in magnitude)]

$\therefore$ $\left(\dfrac{\lambda_e}{\lambda_p}\right) = \left(\dfrac{m_p}{m_e}\right)^{1/2}$

49. $\dfrac{mv^2}{a_0} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{a_0^2}$

$\therefore$ $v = \dfrac{e}{\sqrt{4\pi\varepsilon_0 a_0 m}}$

50. $E\left(=\dfrac{hc}{\lambda}\right) \propto \dfrac{Z^2}{n^2} \Rightarrow \lambda \propto \dfrac{1}{Z^2}$

$\therefore$ $\lambda_{He^+} = \dfrac{20.397}{4} = 5.099\,\text{cm}$

51. $T = \dfrac{2\pi r}{v}$; r = radius of n^{th} orbit $= \dfrac{n^2h^2}{\pi mZe^2}$

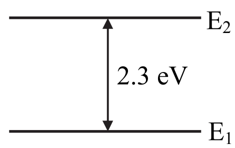
v = speed of e^- in n^{th} orbit $= \dfrac{ze^2}{2\varepsilon_0 nh}$

$\therefore$ $T = \dfrac{4\varepsilon_0^2 n^3 h^3}{mZ^2e^4} \Rightarrow T \propto \dfrac{n^3}{Z^2}$

52. $E_n = \dfrac{13.6}{n^2}\times Z^2$. For first excited state, n = 2 and for Li^{++}, Z = 3

$\therefore$ $E = \dfrac{13.6}{4}\times9 = 30.6$ eV

53.



Using, $E_2 - E_1 = h\nu$ we get,

$\nu = \dfrac{E_2 - E_1}{h} = \dfrac{2.3\times1.6\times10^{-19}\,J}{6.6\times10^{-34}\,Js}$

$= 0.56 \times 10^{15}\ s^{-1} = 5.6 \times 10^{14}$ Hz

54. $\dfrac{N_0}{2}$ is the new N_0

To reduce one fourth the time taken,

$t = 2(T_{1/2}) = 2 \times 40 = 80$ years.

$\therefore$ $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{40} = 0.0173$ years

55. Since electron and positron annihilate,

$\lambda = \dfrac{hc}{E_{Total}} = \dfrac{6.6\times10^{-34}\times3\times10^{8}}{(0.51+0.51)\times10^{6}\times1.6\times10^{-19}}$

$= 1.21\times10^{-12}\,\text{m} = 0.012$ Å .

56. $v = \dfrac{2\pi r}{T} \Rightarrow T = \dfrac{2\pi r}{v}$

But $r \propto n^2$ and $v \propto \dfrac{1}{n}$

$\therefore$ $T \propto \dfrac{r}{v} \propto \dfrac{n^2}{(1/n)} \propto n^3$

57. Since for n = 3,

$$E_3 = \frac{-13.6}{3^2} = -1.51 \text{ eV}$$

For n = 1, $E_1 = \frac{-13.6}{1^2} = -13.6$ eV

∴ The energy of the photon emitted in the transition from n = 3 to n = 1 is

$E_3 - E_1 = (-1.51) - (-13.6) = 12.09$ eV.

58. Ground state energy = –(Ionisation potential)

= –13.6 eV

$E_f = -13.6 + 12.1 = -1.5$ eV

∴ Energy state, $n^2 = \frac{E_i}{E_f} = \frac{-13.6}{-1.5} = 9$

∴ n = 3 i.e., second excited state.

∴ Number of spectral lines from

$$n = 3 \text{ to } n = 1 = \frac{n(n-1)}{2} = \frac{3(2)}{2} = 3$$

59. For n = 1, maximum number of states = $2n^2$ = 2 and for n = 2, 3, 4, maximum number of states would be 8, 18, 32 respectively, Hence number of possible elements

= 2 + 8 + 18 + 32 = 60.

60. Binding energy per nucleon,

$$E_{bn} = \frac{E_b}{A}$$

For deuteron, A = 2

∴ $1.115 \text{ MeV} = \frac{E_b}{A} \Rightarrow E_b = 2 \times 1.115$ MeV

Now, $E_b = \Delta mc^2$

Mass defect,

$$\Delta m = \frac{2 \times 1.115}{931.5} \text{u} \quad \text{....}[\because 1\text{u} = 931.5 \text{ MeV/c}^2]$$

$= 0.0024$ u

61. As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$; where, Number of half lives,

$$n = \frac{t}{T}$$

For sample X,

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{8/T_X} \text{ or } \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{8/T_X}$$

$$\Rightarrow 4 = \frac{8}{T_X} \quad \text{....(i)}$$

For sample Y,

$$\left(\frac{1}{256}\right) = \left(\frac{1}{2}\right)^{8/T_Y} \text{ or } \left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{8/T_Y}$$

$$\Rightarrow 8 = \frac{8}{T_Y} \quad \text{....(ii)}$$

Dividing equation (i) by (ii) we get

$$\frac{4}{8} = \frac{8}{T_X} \times \frac{T_Y}{8}$$

$$\Rightarrow \frac{1}{2} = \frac{T_Y}{T_X} \text{ or } \frac{T_X}{T_Y} = \frac{2}{1}$$

62. For Balmer series,

$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$, where, n = 3, 4, 5 ……

When, we put n = 3, 4, 5 …… and $R = 10^{-7}$ m^{-1} in the given formula, the value of λ calculated lies between 4000 Å and 8000 Å, which is visible region.

63. The binding energy per nucleon of the nuclei of high mass number is small as compared to that of stable nuclei. Such nuclei undergo radioactive decay so as to attain greater value of B. E. / A

64. Matter is not uniformly distributed inside the nucleus.

Competitive Thinking

5. Bohr radius, $r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$

6. K.E. of an electron revolving in nth orbit is,

$$\text{K.E.} = \frac{e^2}{8\pi\varepsilon_0 r_n} \Rightarrow \text{K.E.} \propto \frac{1}{r}$$

Hence, to double the K.E. of electron, its orbit radius should be halved.

9. $v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v}$

Now, $I = \frac{e}{T} = \frac{e}{\left(\frac{2\pi r}{v}\right)} = \frac{ev}{2\pi r}$

10. Using, $I = \frac{ev}{2\pi r} = \frac{1.6 \times 10^{-19} \times 2.18 \times 10^6}{2 \times 3.14 \times 0.53 \times 10^{-10}}$

$= 1.04 \times 10^{-3}$ A = 1.04 mA

11. $v_n = \frac{e^2}{2\varepsilon_0 hn} \Rightarrow v_1 = \frac{e^2}{2\varepsilon_0 h}$

∴ $\frac{v_1}{c} = \frac{e^2}{2\varepsilon_0 hc}$

12. $T \propto n^3 \Rightarrow \frac{T_2}{T_1} = \frac{2^3}{1^3} = \frac{8}{1}$

$\therefore$ $T_2 = 8\ T$

13. Angular momentum, $L = \frac{nh}{2\pi}$

$\therefore$ $\Delta L = L_4 - L_1 = \frac{4h}{2\pi} - \frac{h}{2\pi} = \frac{3h}{2\pi}$

14. $v_n \propto \frac{1}{n} \Rightarrow \frac{v_3}{v_1} = \frac{n_1}{n_3} = \frac{1}{3}$

$\therefore$ $v_3 = \frac{2.1 \times 10^6}{3} = 0.7 \times 10^6$ m/s

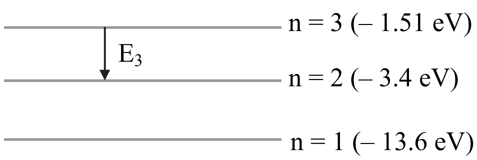
15. Ground state energy = –Ionisation potential

$\therefore$ $E_1 = -13.6$ eV

Now, $E_n = \frac{E_1}{n^2}$

$\Rightarrow E_2 = \frac{-13.6}{2^2} = -3.4$ eV

16.



$E_{3\to2} = -3.4 - (-1.51) = -1.89$ eV

$\therefore$ $|E_{3\to2}| \approx 1.9$ eV

17. 4^{th} excited state means n = 5 and 2^{nd} excited state means n = 3

$E_5 = \frac{E_1}{25}$ and $E_3 = \frac{E_1}{9}$

where, $E_1 = 13.6$ eV

$\therefore$ $|E_5 - E_3| = \left|\frac{13.6}{25} - \frac{13.6}{9}\right| = 0.967$ eV

18. Using $E_n \propto \frac{-13.6\ Z^2}{n^2}$ we get,

$E_1 = -\frac{13.6(3)^2}{(1)^2}$

$E_3 = -\frac{13.6(3)^2}{(3)^2}$

$\therefore$ $\Delta E = E_3 - E_1 = 13.6(3)^2\left[1 - \frac{1}{9}\right] = \frac{13.6 \times 9 \times 8}{9}$

$\therefore$ $\Delta E = 108.8$ eV

19. Energy required to remove electron in the n = 2 state $= +\frac{13.6}{(2)^2} = 3.4$ eV

20. Hydrogen atom takes ΔE amount of energy for excitation from ground state (n = 1) to n = 3 state.

$\therefore$ $\Delta E = E_3 - E_1 = \frac{-13.6}{(3)^2} - (-13.6) = 12.1$ eV

21. P.E. = 2 × Total energy

$= 2 \times (-13.6) = -27.2$ eV

22. For an electron in a Bohr orbit in H-atom,

K.E. = –T.E.

$\therefore$ $\frac{K.E.}{T.E.} = \frac{-1}{1}$

i.e., 1 : –1

23. For hydrogen and hydrogen-like atoms,

$(T.E)_n = -13.6\frac{Z^2}{n^2}$ eV

$(P.E.)_n = 2(T.E)_n = -27.2\frac{Z^2}{n^2}$ eV and

$(K.E)_n = |(T.E)_n| = 13.6\frac{Z^2}{n^2}$ eV

From these three relations, we can see that as n decreases, $(K.E)_n$ will increase but $(T.E)_n$ and $(P.E.)_n$ will decrease.

24. K.E. = – T.E = $-\frac{1}{2}$ P.E.

Also, Total energy of 4^{th} state of hydrogen atom is

$E_4 = \frac{-13.6}{4^2}$ eV = –0.85 eV

$\therefore$ P.E = –1.7 eV, K.E = 0.85 eV

25. K.E. = –T.E. = + 3.4 eV,

and T.E. = $\frac{1}{2}$ P.E. $\Rightarrow$ P.E. = – 6.8 eV

26. $\omega = 2\pi\nu = \frac{2\pi c}{\lambda} = 2\pi c\bar{\nu} \Rightarrow \omega \propto \bar{\nu}$.

30. Balmer series lies in the visible region.

31. $\frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$

a) For n = 5 to p = 4,

$\lambda = \frac{400}{9R}$

b) For n = 4 to p = 3

$\lambda = \frac{144}{7R}$

c) For n = 3 to p = 2

$\lambda = \frac{36}{5R}$

d) For n = 2 to p = 1

$\lambda = \frac{4}{3R}$

∴ λ is minimum for n = 2 to p = 1 transition.

32. $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For first number of Lyman series,

$\frac{1}{\lambda_L} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \Rightarrow \lambda_L = \frac{4}{3RZ^2}$

For first number of Paschen series,

$\frac{1}{\lambda_P} = RZ^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \Rightarrow \lambda_P = \frac{144}{7RZ^2}$

∴ $\frac{\lambda_L}{\lambda_P} = \frac{4/3RZ^2}{144/7RZ^2} = \frac{7}{108}$

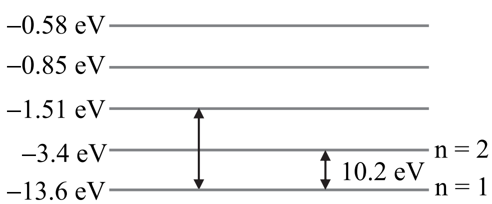
33. Given : $R = 10^7\ m^{-1}$

For the last line of Balmer series, $n_1 = 2$, $n_2 = \infty$

Wave number, $\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$= \frac{10^7}{4} = 0.25 \times 10^7\ m^{-1}$

35.



It is clear that difference of 11.1 eV is not possible to obtain.

36. The absorption lines are obtained when the electron jumps from ground state (n = 1) to the higher energy states. Thus only 1, 2 and 3 lines will be obtained.

37. $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

∴ $\frac{1}{970.6 \times 10^{-10}} = 1.097 \times 10^7\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$

∴ $n_2 \approx 4$

∴ Number of emission lines,

$N = \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$

(Note: Use shortcut 3.)

38. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n^2}\right]$

$\Rightarrow \lambda \propto \frac{1}{Z^2}$ for given n_1 and n_2

$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

39. $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$

∴ $\lambda = \frac{16}{3R} = \frac{16}{3} \times 10^{-5}$ cm

∴ $n = \frac{c}{\lambda} = \frac{3 \times 10^{10}}{\left(\frac{16}{3} \times 10^{-5}\right)}$

$= \frac{9}{16} \times 10^{15}$ Hz

40. For Lyman series,

$\frac{1}{\lambda_{max}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R$

$\frac{1}{\lambda_{min}} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = \frac{R}{1}$

∴ $\frac{\lambda_{max}}{\lambda_{min}} = \frac{4}{3}$

41. The wavelength of spectral line in Balmer series is given by $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

For first line of Balmer series, n = 3

∴ $\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$;

For second line, n = 4

∴ $\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{16}$

∴ $\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$

∴ $\lambda_1 = \frac{20}{27} \times 6561 = 4860$ Å

42. For Paschen series

$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$; n = 4, 5, 6....

For first member of Paschen series n = 4

$\frac{1}{\lambda_1} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] \Rightarrow \frac{1}{\lambda_1} = \frac{7R}{144}$

∴ $R = \frac{144}{7\lambda_1} = \frac{144}{7 \times 18800 \times 10^{-10}} = 1.1 \times 10^7$

For shortest wave length n = ∞

So $\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right] = \frac{R}{9}$

∴ $\lambda = \frac{9}{R} = \frac{9}{1.1\times10^{-7}} = 8.225\times10^{-7}\,m = 8225\,Å$

43. $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Lyman series : $n_1 = 1$, $n_2 = 2$

$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ …(i)

Balmer series:
$n_1 = 2$, $n_2 = 3$

$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$ …(ii)

Dividing equation (ii) by equation (i),

$$\frac{\lambda_1}{\lambda_2} = \frac{R\left(\frac{1}{4} - \frac{1}{9}\right)}{R\left(1 - \frac{1}{4}\right)} = \frac{\frac{5}{36}}{\frac{3}{4}} = \frac{5}{36}\times\frac{4}{3} = \frac{5}{27}$$

44. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For last line of Balmer: $n_1 = 2$ and $n_2 = \infty$

∴ $\frac{1}{\lambda_b} = RZ^2\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]$

$\lambda_b = \frac{4}{RZ^2}$

For last line of Lyman series: $n_1 = 1$ and $n_2 = \infty$

∴ $\frac{1}{\lambda_l} = RZ^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$

$\lambda_l = \frac{1}{RZ^2}$

$\frac{\lambda_b}{\lambda_l} = \frac{(4/RZ^2)}{(1/RZ^2)} = 4$

45. For Balmer series,

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

where, n = 3, 4, 5
For second line n = 4,

∴ $\frac{1}{\lambda} = RZ^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16}RZ^2$

Assuming atom to be hydrogen, Z = 1,

∴ $\lambda = \frac{16}{3R}$

46. For Balmer series,

$\frac{1}{\lambda} = RZ^2\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

∴ $\frac{1}{\lambda_\alpha} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5RZ^2}{36}$ ....(i)

$\frac{1}{\lambda_\beta} = RZ^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3RZ^2}{16}$ ....(ii)

∴ Dividing equation (ii) by equation (i),

$\frac{\lambda_\alpha}{\lambda_\beta} = \frac{3RZ^2}{16}\times\frac{36}{5RZ^2} = \frac{27}{20}$

47. $\nu = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Series limit of Balmer: $n_1 = 2$, $n_2 = \infty$

∴ $\nu_1 = \frac{RZ^2}{4}$

Series limit of Paschen: $n_1 = 3$, $n_2 = \infty$

∴ $\nu_2 = \frac{RZ^2}{9}$

1st line of Balmer series: $n_1 = 2$, $n_2 = 3$

∴ $\nu_3 = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{RZ^2}{4} - \frac{RZ^2}{9} = \nu_1 - \nu_2$

48. Series limit for Lyman series is,

$\lambda_L = \frac{1}{R}$

∴ $\nu_L = Rc$ ….($\because \nu = \frac{c}{\lambda}$)

Series limit for Pfund series is,

$\lambda_p = \frac{25}{R} \Rightarrow \nu_p = \frac{Rc}{25} = \frac{\nu_L}{25}$

49. $\frac{1}{\lambda_B} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$ $\frac{1}{\lambda_{Br}} = RZ^2\left[\frac{1}{4^2} - \frac{1}{5^2}\right]$

$= RZ^2\left[\frac{5}{36}\right]$ $= RZ^2\left[\frac{9}{400}\right]$

∴ $\lambda_B = \frac{36}{5RZ^2}$ ∴ $\lambda_{Br} = \frac{400}{9RZ^2}$

∴ $\frac{\lambda_B}{\lambda_{Br}} = \frac{36}{5RZ^2}\times\frac{9RZ^2}{400} = 0.162$

50. For Brackett series,

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{4^2}-\frac{1}{5^2}\right] = \frac{9}{25\times16}R \text{ and}$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{4^2}-\frac{1}{\infty^2}\right] = \frac{R}{16}$$

$$\therefore \quad \frac{\lambda_{max}}{\lambda_{min}} = \frac{25}{9}$$

51. For Lyman series,

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

$$\therefore \quad \frac{1}{\lambda_{min}} = RZ^2\left[1-\frac{1}{\infty}\right] = RZ^2 \qquad \text{….(i)}$$

For Paschen series,

$$\frac{1}{\lambda_{max}} = RZ^2\left[\frac{1}{3^2}-\frac{1}{4^2}\right] = \frac{7RZ^2}{144} \qquad \text{….(ii)}$$

∴ By dividing equation (i) by equation (ii),

$$\frac{\lambda_{max}}{\lambda_{min}} = RZ^2 \times \frac{144}{7RZ^2} = \frac{144}{7}$$

$$\therefore \quad \lambda_{max} = \frac{144}{7}\times 912 \approx 18761 \text{ Å}$$

52. $$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\therefore \quad \frac{1}{\lambda} = \left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{5}{36}R$$

$$\therefore \quad f = \frac{c}{\lambda} = \frac{5}{36}Rc$$

53. Case I:

n = 3 to p = 2

$$\frac{1}{\lambda} = R\left[\frac{1}{p^2}-\frac{1}{n^2}\right] = R\left[\frac{1}{4}-\frac{1}{9}\right]$$

$$\frac{1}{\lambda} = \frac{5}{36}R \qquad \text{….(i)}$$

Case II:

n′ = 4 to p′ = 3

$$\frac{1}{\lambda'} = R\left[\frac{1}{p'^2}-\frac{1}{n'^2}\right] = R\left[\frac{1}{9}-\frac{1}{16}\right]$$

$$\frac{1}{\lambda'} = \frac{7}{144}R \qquad \text{….(ii)}$$

∴ Dividing equation (i) and (ii)

$$\frac{\lambda'}{\lambda} = \frac{5}{36}\times\frac{144}{7} = \frac{20}{7}$$

$$\therefore \quad \lambda' = \frac{20}{7}\lambda$$

54. $$\frac{1}{\lambda_1} = \left[\frac{1}{2^2}-\frac{1}{3^2}\right]RZ^2 = \frac{5}{36}RZ^2 \Rightarrow \lambda_1 = \frac{36}{5}x$$

Let $RZ^2 = x$

$$\frac{1}{\lambda_2} = \left[\frac{1}{1^2}-\frac{1}{2^2}\right]RZ^2 = \frac{3}{4}RZ^2 \Rightarrow \lambda_2 = \frac{4}{3}x$$

$$\frac{1}{\lambda_3} = \left[\frac{1}{1^2}-\frac{1}{3^2}\right]RZ^2 = \frac{8}{9}RZ^2 \Rightarrow \lambda_3 = \frac{9}{8}x$$

Comparing with given combinations,

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2} = \frac{\frac{36}{5}x\times\frac{4}{3}x}{\frac{36}{5}x+\frac{4}{3}x} = \frac{\frac{48}{5}x^2}{\frac{108+20}{15}x}$$

$$= \frac{48}{5}x^2\times\frac{15}{128x} = \frac{36}{32}x = \frac{9}{8}x$$

55. $$\Delta E = \frac{hc}{\lambda}$$

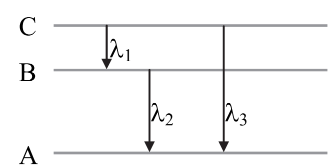
For energy level diagram,

$$\lambda_1 = \frac{hc}{[-E-(-2E)]} = \frac{hc}{E}$$

$$\lambda_2 = \frac{hc}{\left[-E-\left(\frac{-4E}{3}\right)\right]} = \frac{hc}{\left(\frac{E}{3}\right)}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$$

56. Let the energy in A, B and C state be E_A, E_B and E_C, then from the figure



$(E_C - E_A) = (E_C - E_B) + (E_B - E_A)$

$$\therefore \quad \frac{hc}{\lambda_1}+\frac{hc}{\lambda_2} = \frac{hc}{\lambda_3}$$

$$\therefore \quad \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

57. $$\text{K.E.} \propto \left(\frac{Z}{n}\right)^2 \text{ and}$$

K.E. = – (T.E.), P.E. = –2(K.E.)

This implies as K.E. increases and as K.E. increases, T.E., P.E. decreases.

58. Minimum wavelength in X-ray spectrum,

$$\lambda_{min} = \frac{hc}{eV}$$

Taking logarithm on both sides,

$$\log_e(\lambda_{min}) = \log_e\left(\frac{hc}{e}\right) - \log_e(V)$$

Comparing with, y = mx + c, relation has negative slope and positive Y-intercept.
This is satisfied by graph in option (C).

59. Wavelength of continuous X-rays does not depend on the material used. However, wavelength of characteristic X-ray depends on material used as the metal target (Z).

63. To balance the atomic number and mass number on both sides, ${}^{1}_{0}X$

∴ X represents neutron $\left({}^{1}_{0}n\right)$

65. Nuclear density is independent of the mass number so the required ratio will be 1 : 1.

67. Let X have atomic number Z and mass number A

∴ ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

But ${}^{A-4}_{Z-2}Y \rightarrow {}^{A'}_{Z'}Z + 2e^-$ i.e.,

${}^{A-4}_{Z-2}Y \rightarrow {}^{A'}_{Z'}Z + {}^{0}_{-1}e + {}^{0}_{-1}e$

$\Rightarrow A' = A - 4$ and $Z' = Z$

Since X and Z has same atomic number and different mass numbers, they are isotopes of each other.

68. $B = [ZM_p + NM_n - M(N, Z)]c^2$

∴ $M(N, Z) = ZM_p + NM_n - B/c^2$

70. $R = R_0A^{\frac{1}{3}}$

$$\frac{R_{Te}}{R_{Al}} = \left(\frac{A_{Te}}{A_{Al}}\right)^{\frac{1}{3}} = \left(\frac{125}{27}\right)^{\frac{1}{3}} = \frac{5}{3}$$

∴ $R_{Te} = \frac{5}{3}R_{Al}$

71. $R = R_oA^{\frac{1}{3}}$

∴ $\frac{R_{Te}}{R_{Al}} = \left(\frac{A_{Te}}{A_{Al}}\right)^{\frac{1}{3}}$

∴ $R_{Te} = 3.6 \times 10^{-15} \times \left(\frac{125}{27}\right)^{\frac{1}{3}}$

∴ $R_{Te} = 3.6 \times 10^{-15} \times \frac{5}{3}$

∴ $R_{Te} = 6 \times 10^{-15}$m = 6 Fermi.

72. $R = R_0A^{1/3}$

∴ $\frac{R_{Ge}}{R_{Be}} = \left(\frac{A_{Ge}}{A_{Be}}\right)^{1/3}$

∴ $\frac{2R_{Be}}{R_{Be}} \times (A_{Be})^{1/3} = (A_{Ge})^{1/3}$

∴ $2^3 \times 9 = A_{Ge}$

∴ $A_{Ge} = 72$

73. $R = R_0(A)^{1/3}$

∴ $\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{64}{27}\right)^{1/3} = \frac{4}{3}$

∴ $R_2 = 3.6 \times \frac{4}{3} = 4.8$

74. $X \rightarrow Y + Z$

Now, $P_y = P_z$ (P → linear momentum)

$m_yv_y = m_zy_z$

$\Rightarrow \frac{m_y}{m_z} = \frac{V_z}{V_y} \Rightarrow \frac{2}{1} = 2$

$\Rightarrow A_y = 2A_z$

Now, $\frac{R_z}{R_y} = \frac{(A_Z)^{1/3}}{(A_y)^{1/3}} = \left(\frac{1}{2}\right)^{1/3}$

$\Rightarrow 1 : 2^{1/3}$

75. B.E. per nucleon is maximum for Fe^{56}.

76. Binding energy per nucleon increases with atomic number. The greater the binding energy per nucleon, the stability of the nucleus will be more.
For ${}_{26}Fe^{56}$, number of nucleons is 56.
This is the most stable nucleus because maximum energy is needed to pull a nucleon away from it.

78. Since nuclear density is constant,

∴ mass ∝ volume.

79. $E = \Delta mc^2 = 3 \times (3 \times 10^8)^2 = 27 \times 10^{16}$ J

80. $E = \Delta mc^2 = 1.5 \times (3 \times 10^8)^2 = 13.5 \times 10^{16}$ J

81. Mass defect = Δm = 0.02866 u

Total energy = E $= \Delta mc^2$
= 0.02866 × 931 MeV
= 26.68 MeV

Energy liberated per nucleon $= \frac{E}{A} = \frac{26.68}{4}$
= 6.67 MeV

82. $\frac{\text{B.E.}}{\text{A}} = \frac{\Delta mc^2}{eA}$

But 1 u = 931 MeV/c^2

∴ $\frac{\text{B.E.}}{\text{A}} = \frac{0.03 \times 931}{4}$

$= 6.9825$ MeV/nucleon

83. B.E. = Δmc^2

$= [2(1.0087 + 1.0073) - 4.0015] \times 931$

$= 28.4$ MeV

84. $\Delta m = 1 - 0.993 = 0.007$ g

∴ $E = (\Delta m)c^2$

$= (0.007 \times 10^{-3})(3 \times 10^8)^2$

$= 63 \times 10^{10}$ J.

85. Energy required to remove one neutron

$\Delta E = (17 \times 7.75) - (16 \times 7.97)$

$= 131.75 - 127.52$

$= 4.23$ MeV

86. During fusion, binding energy of daughter nucleus is always greater than the total energy of the parent nuclei so energy released

$= c - (a + b) = c - a - b$

87. Q = 2(B.E. of He) – (B.E. of Li)

$= 2 \times (4 \times 7.06) - (7 \times 5.60)$

$= 56.48 - 39.2 \approx 17.3$ MeV

88. 1 curie = 3.7×10^{10} disintergration/s.;

1 rutherford = 2.7×10^{-5} curie

$= (2.7 \times 10^{-5})(3.7 \times 10^{10})$ disintegration/s

$\approx 1 \times 10^6$ disintegration/s

89. Penetration power of γ is 100 times of β, while that of β is 100 times of α.

90. ${}_{Z}X^{A} \xrightarrow{\alpha} {}_{Z-2}Y^{A-4} \xrightarrow{2\beta^-} {}_{Z}X^{A-4}$

92. ${}_{94}Pu^{239} \rightarrow {}_{92}U^{235} + {}_{2}He^{4}$

Hence, the particle emitted when Pu decays into U is, α-particle.

95. $A_1 = \lambda N_1$ and $A_2 = \lambda N_2$

∴ $N_1 - N_2 = \left[\frac{A_1 - A_2}{\lambda}\right]$

96. Time taken to reduce from 2/3rd to 1/3rd should also be one half life i.e., 20 days.

97. $\frac{N_0}{32} = N_0\left(\frac{1}{2}\right)^{60/T}$

∴ $5 = \frac{60}{T} \Rightarrow T = 12$ days

98. $\frac{N}{N_0} = \frac{1}{(1+7)} = \frac{1}{8} = \frac{1}{(2)^3}$

∴ n = 3 $\left[\because \frac{1}{2^n} = \frac{1}{(2)^3}\right]$

∴ $n = \frac{t}{T} \Rightarrow t = 3 \times 20$

(Half-life of X = T = 20 years)

∴ t = 60 years

99. $N = N_o e^{-\lambda t}$

$\Rightarrow \frac{N_o}{20} = N_o e^{-\lambda t}$

$\Rightarrow ln\,1 - ln20 = -\lambda t$

$\Rightarrow t = \frac{ln20}{ln2} \times 6.93$

∴ $t = \frac{2.99 \times 6.93}{0.693} = 29.9 \approx 30$ days.

100. $\frac{X}{Y} = \frac{1}{7}$

∴ $\frac{X}{X+Y} = \frac{1}{8} = \frac{1}{2^3}$

$\Rightarrow$ 3 half-lives

∴ $\Delta T = 3 \times 1.4 \times 10^9$ years $= 4.2 \times 10^9$ yrs.

101. Given: $\lambda_A = 8\lambda$, $\lambda_B = \lambda$, $(N_B)_0 = (N_A)_0 = N_0$

For, $N_B = \frac{N_A}{e}$,

$N_0 e^{-\lambda t} = \frac{N_0 e^{-8\lambda t}}{e}$

∴ $-\lambda t = -8\lambda t - 1$

∴ $7\lambda t = -1$

∴ $t = -\frac{1}{7\lambda}$

Negative sign here, indicates process of disintegration,

∴ $t = \frac{1}{7\lambda}$

102. Half life $T_{1/2} = 5$ min

Total time t = 20 min

∴ Number of half lives, $n = \frac{t}{T_{1/2}} = \frac{20}{5} = 4$

Now,

$\frac{N}{N_0} = \left[\frac{1}{2}\right]^n = \left[\frac{1}{2}\right]^4$

∴ $\frac{N}{N_0} = \frac{1}{16}$

Disintegrated nuclei of given element will be,

$$\left[\frac{N_0 - N}{N_0}\right]\times 100 = \left[1 - \frac{N}{N_0}\right]\times 100$$

$$= \left[1 - \frac{1}{16}\right]\times 100 = 93.75\%$$

103. Nuclei remaining (N) = 600 – 450 = 150

Comparing with $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$\therefore \quad \frac{150}{600} = \left(\frac{1}{2}\right)^n \qquad \therefore \quad \frac{1}{4} = \left(\frac{1}{2}\right)^n$

$\Rightarrow n = 2$

i.e., nuclei would disintegrate in two half-lives which in this case equals 20 minutes.

104. Number of nuclei remained after time t can be written as $N = N_0e^{-\lambda t}$

$N_1 = N_0e^{-5\lambda t}$ ….(i)

and $N_2 = N_0e^{-\lambda t}$ ….(ii)

Dividing equation (i) by equation (ii), we get,

$$\frac{N_1}{N_2} = e^{(-5\lambda+\lambda)t} = e^{-4\lambda t} = \frac{1}{e^{4\lambda t}}$$

$$\frac{N_1}{N_2} = \left(\frac{1}{e}\right)^2 = \frac{1}{e^2} \qquad \text{….[Given]}$$

$\therefore \quad \frac{1}{e^2} = \frac{1}{e^{4\lambda t}}$

$\therefore \quad 2 = 4\lambda t \Rightarrow t = \frac{2}{4\lambda} = \frac{1}{2\lambda}$

105. Using $\lambda = \lambda_1 + \lambda_2$

$\therefore \quad \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

$\therefore \quad T = \frac{T_1T_2}{T_1 + T_2} = \frac{810\times1620}{810+1620} = 540$ years

$\therefore \quad \frac{1}{4}$ th of material remains after 1080 years.

(*Note: Refer mindbender 2.*)

106. $T = \frac{T_1T_2}{T_1+T_2} = \frac{5\times10^3\times10^5}{5\times10^3+10^5} = 4762$ yrs

107. $T_{1/2} = \frac{0.693}{\lambda}$

Average life $\tau = \frac{1}{\lambda}$

$\therefore \quad T_{1/2} = 0.693\ \tau$

$\therefore \quad \tau = \frac{10}{0.693} = 14.43$ hours

108. $N_A = N_0e^{-\lambda t}$

$N_B = N_0 - N_0e^{-\lambda t}$

$\therefore \quad \frac{N_B}{N_A} = \frac{N_0 - N_0e^{-\lambda t}}{N_0e^{-\lambda t}} = 0.3$

$\therefore \quad e^{\lambda t} - 1 = 0.3$

$\therefore \quad e^{\lambda t} = 1.3$

$\therefore \quad \lambda t = ln(1.3)$

$\therefore \quad t = T\frac{ln(1.3)}{ln(2)} \qquad \text{….}\left[\because \lambda = \frac{ln(2)}{T}\right]$

$\Rightarrow t = T\frac{\log(1.3)}{\log 2}$

109. $t_2 - t_1 = \frac{T}{\log_e 2}\log_e\left(\frac{N_1}{N_2}\right)$

$$= \frac{20}{\log_e 2}\log_e\left(\frac{50}{12.5}\right)$$

$$= \frac{20}{\log_e 2}\log_e 4 = 40 \text{ minutes}$$

110. $t = \frac{T}{\log_e(2)}\left[\log_e\left(\frac{N_0}{N}\right)\right]$

$\therefore \quad t_1 = \frac{T}{\log_e(2)}\left[\log_e\left(\frac{N_0}{N_1}\right)\right]$

$t_2 = \frac{T}{\log_e(2)}\left[\log_e\left(\frac{N_0}{N_2}\right)\right]$

$\therefore \quad t_2 - t_1 = \frac{T}{\log_e(2)}\left[\log_e\left(\frac{N_1}{N_2}\right)\right]$

For 40% decay, $N_1 = 60$

For 85% decay, $N_2 = 15$

$\therefore \quad t_2 - t_1 = \frac{30}{\log_e(2)}\left[\log_e\left(\frac{60}{15}\right)\right]$

$$= \frac{30}{\log_e(2)}\times\log_e(4)$$

$= 30 \times 2 = 60$ min

111. we know for radioactive decay,

$N = N_0e^{-\lambda t}$ (or) $ln\frac{N_0}{N} = \lambda t$

For 20% decay

$t = \frac{1}{\lambda}ln\frac{N_0}{N}$

$\Rightarrow t = \frac{20}{0.693}\left(ln\frac{100}{20}\right) \qquad \text{….}\left(\because \lambda = \frac{0.693}{T}\right)$

$\Rightarrow t = \frac{20}{0.693}ln(5) \qquad \text{….(i)}$

For 80% decay

$t' = \frac{1}{\lambda}\ ln\ \frac{N_0}{N'}$

$\Rightarrow t' = \frac{20}{0.693}\ ln\left(\frac{100}{80}\right)$

$\Rightarrow t' = \frac{20}{0.693}\ ln\left(\frac{5}{4}\right)$ ….(ii)

thus, $\Delta t = t - t'$

$= \frac{20}{0.693}\left(ln 5 - ln\frac{5}{4}\right)$

$= \frac{20}{0.693}\ ln\ 4$

$= 40$ min.

112. Number of nuclei remaining $N = N_0\left(\frac{1}{2}\right)^n$

For element A, $T_A = 20$ min. Hence, 80 minutes, correspond to 4 half lives.

∴ No. of nuclei decayed of A(N'_A) $= N_0 - N$

$= N_0\left[1 - \frac{1}{2^4}\right]$

Similarly for element B, $T_B = 40$ min. Hence, 80 minutes correspond to 2 half lives.

∴ No. of nuclei decayed of B (N'_B) $= N_0 - N$

$= N_0\left[1 - \frac{1}{2^2}\right]$

∴ Taking ratio,

$$\frac{N'_A}{N'_B} = \frac{N_0\left(\frac{15}{16}\right)}{N_0\left(\frac{3}{4}\right)} = \frac{5}{4}$$

113. Remaining amount

$= 16 \times \left(\frac{1}{2}\right)^{32/2} = 16 \times \left(\frac{1}{2}\right)^{16} = \left(\frac{1}{2}\right)^{12} < 1$ mg

114. $\frac{m}{m_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$

Given: $T_{1/2} = 12.5$ years, $t = 50$ years

∴ $\frac{m}{m_0} = \left(\frac{1}{2}\right)^{50/12.5}$

$= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

∴ $m = \frac{64}{16} = 4$ mg

115. Half life T = 10 days, t = 5 days

∴ $n = \frac{t}{T} = \frac{5}{10} = \frac{1}{2}$

$N = \frac{N_0}{2^n} = \frac{1000X}{2^{1/2}} = \frac{1000X}{\sqrt{2}} = 0.707 \times 1000X$

$= 707X$

116. Let rate of disintegration 10,000 dis/min be taken as initial rate (N_0) and let N = 2500 dis/min.

$\frac{N}{N_0} = e^{-\lambda t}$

∴ $\frac{2500}{10000} = e^{-\lambda \times 4}$ ….(Given : t = 4 min)

∴ $\frac{1}{4} = e^{-4\lambda}$ ∴ $e^{4\lambda} = 4$

∴ $4\lambda = \log_e 4$ ∴ $4\lambda = \log_e 2^2$

∴ $4\lambda = 2\log_e 2$ ∴ $\lambda = \frac{2}{4}\log_e 2$

∴ $\lambda = 0.5\log_e 2$

117. $\frac{dN}{dt} = -\lambda N$

Where, negative sign indicates that nuclei disintegrate

Given: $\frac{dN}{dt} = -55.3 \times 10^{11}$

∴ $55.3 \times 10^{11} = (7.9 \times 10^{-10}) \times N$

∴ $N = 7 \times 10^{21}$

118. $M = M_0 e^{-\lambda t}$; Given $t = 2\left(\frac{1}{\lambda}\right)$

$\Rightarrow M = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 \Rightarrow M = 1.35$ g

119. The number of nuclei decayed in 2 days is,

$N_2 = N_0 e^{-2/\tau}$

Similarly, in 3 days, the number of nuclei decayed will be,

$N_3 = N_0 e^{-3/\tau}$

where $\tau = \frac{t_{1/2}}{ln 2} = \frac{3}{ln 2}$ ....(i)

∴ Fractional Decay on third day

$= \frac{N_2 - N_3}{N_0} = \frac{\left[N_0 e^{-2/\tau} - N_0 e^{-3/\tau}\right]}{N_0}$

$= e^{\frac{-2 ln 2}{3}} - e^{-ln 2}$ ....[using (i)]

$= 2^{\frac{-2}{3}} - 2^{-1} = 0.63 - 0.5 = 0.13$

120. $_{92}U^{238} \xrightarrow{\alpha} {}_{90}Th^{234} \xrightarrow{\beta} {}_{91}Pa^{234} \xrightarrow{({}_{-1}\beta^0)} {}_{92}U^{234}$

121. An element is represented as ${}^{A}_{Z}X$
where, A is atomic mass No. Z is atomic number.
when a β^- particle is emitted,
A does not change, Z increases by 1
When an α particle is emitted:
A decreases by 4, Z decreases by 2.

$${}^{180}_{72}X \xrightarrow{\beta^-} {}^{180}_{73}X_1 \xrightarrow{\alpha} {}^{176}_{71}X_2 \xrightarrow{\beta^-} {}^{176}_{72}X_3 \xrightarrow{\alpha} {}^{172}_{70}X_4$$

122. $_ZX^A \xrightarrow[3({}_2\alpha^4)]{} {}_{Z-6}(X)^{A-12} \xrightarrow[2({}_{+1}\beta^0)]{} {}_{Z-8}(X)^{A-12}$

$$\therefore \frac{\text{Number of neutrons}}{\text{Number of protons}} = \frac{(A-12)-(Z-8)}{Z-8}$$

$$= \frac{A-Z-4}{Z-8}$$

123. As emission of β^- doesn't affect the atomic mass no. A, hence No. of α particle emitted to decrease A from 238 to 206 is

$$\frac{238-206}{4} = \frac{32}{4} = 8$$

(As single α decreases A by 4)
Thus, 8α particles needs to be emitted to decrease (A) from 238 to 206
But emitting 8α will bring down the atomic No. (Z) from 92 to 76.
(As single α decrease Z by 2)
Thus 6 β^- needs to be emitted to raise (Z) from 76 to 82.
(As single β^- increases Z by 1)

124. Number of α-particles emitted $= \dfrac{238-222}{4} = 4$

This decreases atomic number to $90 - 4 \times 2 = 82$
Since atomic number of $_{83}Y^{222}$ is 83, this is possible if one β particle is emitted.

125. Rate disintegration, $R = \lambda N_0 e^{-\lambda t}$

$$\lambda = \frac{0.693}{T}$$

$$\therefore R = \frac{0.693}{T} N_0\, e^{-0.693\, t/T}$$

$$R_1 = \frac{0.693}{2} N_0\, e^{-0.693 \times 12/2} = \frac{0.693}{2} N_0\, e^{-6(0.693)}$$

$$R_2 = \frac{0.693}{4} N_0\, e^{-0.693 \times 12/4} = \frac{0.693}{4} N_0\, e^{-3(0.693)}$$

$$\therefore R_1 : R_2 = \frac{4}{2} \times e^{-3(0.693)} = 0.25 = 1:4$$

127. $L = mv\,r = \dfrac{nh}{2\pi}$

For n = 4, $mvr_4 = \dfrac{2h}{\pi} \Rightarrow h = \dfrac{mvr_4\pi}{2}$

But $r_4 = 16$ r

$$\therefore h = \frac{mv16r\pi}{2} \Rightarrow h = mvr8\pi$$

$$\therefore \lambda = \frac{h}{mv} = 8\pi r$$

128. $\lambda = \dfrac{h}{mv} \Rightarrow \dfrac{6.626\times10^{-34}}{0.1\times10} = 6.626 \times 10^{-34}$ m

129. $\lambda = \dfrac{h}{mv}$

$$\therefore \lambda \propto \frac{1}{v}$$

Now, $v \propto \sqrt{T}$

$$\therefore \lambda \propto \frac{1}{\sqrt{T}}$$

$$\Rightarrow \frac{\lambda_{27}}{\lambda_{927}} = \sqrt{\frac{T_{927}}{T_{27}}}$$

$$\Rightarrow \lambda_{27} = 2\,\lambda_{927}$$

$$\Rightarrow \lambda_{927} = \frac{\lambda_{27}}{2} = \frac{\lambda}{2}$$

130. $\lambda = \dfrac{h}{p} \Rightarrow p \propto \dfrac{1}{\lambda}$

131. For a charged particle, de-Broglie wavelength is,

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\therefore \lambda \propto \frac{1}{\sqrt{V}}$$

$$\therefore \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}}$$

132. $E = \dfrac{1}{2}mv^2 = \dfrac{m^2v^2}{2m}$

$$= \frac{1}{2m}\left(p^2\right)$$

....($\because$ momentum p = mv)

$$= \frac{1}{2m} \times \frac{h^2}{\lambda^2} \qquad ....\left(\because p = \frac{h}{\lambda}\right)$$

$$= \frac{h^2}{2m\lambda^2}$$

133. de-Broglie wavelength,

$\lambda = \frac{h}{p}$

But $p = \sqrt{2mE}$ $\therefore$ $\lambda \propto \frac{1}{\sqrt{E}}$

i.e., $E \propto \frac{1}{\lambda^2}$

$\therefore$ $\left(\frac{E_2}{E_1}\right) = \left(\frac{\lambda_1}{\lambda_2}\right)^2 = \left(\frac{1}{0.5}\right)^2 = 4$

As, $E_2 = 4E_1$

$\Delta E = 3E_1$

134. $\lambda = \frac{h}{p}$,

But, $p = \sqrt{2mE}$ $\therefore$ $\lambda = \frac{h}{\sqrt{2mE}}$,

$\therefore$ $\lambda \propto \frac{1}{\sqrt{E}}$ $\therefore$ $\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$

$\therefore$ $\frac{0.4 \times 10^{-10}}{1.0 \times 10^{-10}} = \sqrt{\frac{E}{1}}$ $\therefore$ E = 0.16 keV

135. Using $\lambda = \frac{h}{\sqrt{2mE}}$,

$E_{electron} = \frac{h^2}{(\lambda^2 \times 2m)}$ and $E_{photon} = \frac{hc}{\lambda}$

$\therefore$ $\frac{E_{photon}}{E_{electron}} = \left[\frac{hc}{\lambda} . \frac{\lambda^2 \times 2m}{h^2}\right]$

$= \frac{2mc^2}{\left(\frac{hc}{\lambda}\right)} = \frac{2 \times 5 \times 10^5}{(50 \times 10^3)} = \frac{20}{1}$

136. For electron, $\lambda_e = \frac{h}{\sqrt{2mE_e}}$ and for photon,

$\lambda_p = \frac{hc}{E_p}$

$\therefore$ $\frac{\lambda_e}{\lambda_p} = \frac{h}{\sqrt{2mE_e}} \times \frac{E_p}{hc}$

$\frac{\lambda_e}{\lambda_p} = \frac{1}{c}\left[\frac{E}{2m}\right]^{\frac{1}{2}}$ ....($\because E_p = E_e$)

137. For photon,

$E = \frac{hc}{\lambda}$

$\therefore$ For electron,

$\lambda' = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m\frac{hc}{\lambda}}} = \sqrt{\frac{h\lambda}{2mc}}$

138. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mk}} \Rightarrow \lambda \propto \frac{1}{\sqrt{k}}$

139. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

Now, $\lambda' = \frac{h}{\sqrt{2m(16E)}}$

$= \frac{1}{4} \times \frac{h}{\sqrt{2mE}} = \frac{\lambda}{4} = 0.25\lambda$

$\therefore$ % change = $\lambda - \lambda' = \lambda - 0.25\lambda$

$= 0.75\lambda \Rightarrow 75\%$

140. $\lambda = \frac{h}{p}$

K.E. $= \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}$

$= \frac{(6.6 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (5.5 \times 10^{-7})^2}$

$\approx 7.91 \times 10^{-25} \approx 8 \times 10^{-25}$ J

141. Let p be initial momentum of electron,

Given,

$p' = p + P_m$

as $\lambda \propto \frac{1}{p}$

increase in p, decreases λ

$\therefore$ $\lambda' = \lambda - \frac{0.5}{100}\lambda = \left(1 - \frac{0.5}{100}\right)\lambda$

$\therefore$ $\frac{\lambda'}{\lambda} = \frac{p}{p'}$

$\therefore$ $\left(1 - \frac{0.5}{100}\right) = \frac{p}{p + P_m}$

$0.995\ p + 0.995\ P_m = p$

$0.995 P_m = \frac{p}{200}$

$\therefore$ $p \approx 200\ P_m$

142. $\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{100}} = 1.227$ Å

143. $\lambda = \frac{12.27}{\sqrt{V}}$ Å $= \frac{1.227 \times 10^{-9}}{\sqrt{400}}$

$= 0.061 \times 10^{-9}$ m = 0.06 nm

144. $\lambda = \frac{h}{\sqrt{2meV}}$

$\therefore$ $V = \frac{h^2}{2me\lambda^2}$

$= \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times (1 \times 10^{-10})^2}$

≈ 150 volt

145. $\lambda_{db} \propto \frac{1}{\sqrt{m}\sqrt{V}}$

$\therefore$ $\lambda_e \propto \frac{1}{\sqrt{m_e}\sqrt{V}}$

$\lambda_p \propto \frac{1}{\sqrt{m_p}\sqrt{9V}}$

$\therefore$ $\frac{\lambda_p}{\lambda_e} = \sqrt{\frac{m_e}{m_p}} \cdot \sqrt{\frac{V}{9V}}$

$\therefore$ $\frac{\lambda_p}{\lambda_e} = \sqrt{\frac{m}{M}}\left(\frac{1}{3}\right)$ ...($\because$ $m_e = m$; $m_p = M$)

$\therefore$ $\lambda_p = \frac{\lambda_e}{3}\sqrt{\frac{m}{M}}$

$\therefore$ $\lambda_p = \frac{\lambda}{3}\sqrt{\frac{m}{M}}$ ...($\because$ $\lambda_e = \lambda$)

146. K.E. = 120 eV

$\therefore$ V = 120 V

$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{120}} = 1.12$ Å $= 1.12 \times 10^{-10}$ m

$= 112 \times 10^{-12}$ m = 112 pm

147. For electron, de-Broglie wavelength is,

$\lambda = \frac{1}{\sqrt{2meV}}$ $\therefore$ $\lambda \propto \frac{1}{\sqrt{V}}$

$\therefore$ $\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}}$

$\therefore$ $V_2 = \frac{V_1 \times \lambda_1^2}{\lambda_2^2} = \frac{10000 \times \lambda^2}{(2\lambda)^2} = \frac{10000}{4} = 2500$ V

148. $\lambda = \frac{h}{\sqrt{2mqV}}$

We know, $q_\alpha = 2q_P$

$m_\alpha = 4\ m_p$

$\therefore$ $\frac{\lambda_P}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_P q_P}} = \sqrt{\frac{4m_P \times 2q_P}{m_P \times q_P}} = \sqrt{8} = 2\sqrt{2}$

149. K.E. of electrons $= \frac{p^2}{2m}$

here, $p = \frac{h}{\lambda}$ ....(De-Broglie hypothesis)

$\therefore$ K.E. $= \frac{\left(\frac{h}{\lambda}\right)^2}{2m}$ ....(i)

Also, if λ_0 is cutoff wavelength, maximum K.E. of X- ray photons $= \frac{hc}{\lambda_0}$ ....(ii)

Maximum K.E. of X- ray will be equal to that of electrons.

$\therefore$ $\frac{hc}{\lambda_0} = \frac{h^2}{2\lambda^2 m}$ ....[from (i) and (ii)]

$\therefore$ $\lambda_0 = \frac{2\lambda^2 mc}{h}$

150. $\lambda \propto \frac{1}{\sqrt{V}}$

To decrease wavelength potential difference between anode and filament is increased.

152. From Bragg's law,

$2d \sin\theta = n\lambda$ or $\lambda = \frac{2d\sin\theta}{n}$

$\therefore$ For maximum wavelength, $n_{min} = 1$, $(\sin\theta)_{max} = 1$

$\therefore$ $\lambda_{max} = 2d$ or $\lambda_{max} = 2 \times 10^{-7}$ cm = 20 Å

153. As electron transits from higher energy level to lower, its n decreases, hence it K.E. increases. This implies its velocity increases. This means statements (A) and (B) are correct.

Also, de Broglie wavelength $\lambda \propto \frac{1}{v}$. Hence, as velocity increases, associated de Broglie wavelength decreases. Hence, statement (D) is correct. But, angular momentum $L \propto n$. This means, as energy level changes, associated angular, momentum changes. Hence, statement (C) is incorrect.

154. Number of lines, $N_E = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$

155. The hydrogen spectrum consists of different series of spectral lines and each series can have infinite lines within itself. Hence, No. of spectral line observed in hydrogen atom is ∞.

156. $\frac{1}{\lambda} = R\left(\frac{1}{n^2}-\frac{1}{p^2}\right)$

In this case, n = 1 and p = 4

$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{1^2}-\frac{1}{4^2}\right) = R\left(1-\frac{1}{16}\right) = \frac{15}{16}R$

Energy of photon is given by,

$E = h\nu = h\frac{c}{\lambda} = hcR\frac{15}{16}$ ….(i)

According to Einstein mass-energy relation,

$E = mc^2$ ….(ii)

From equations (i) and (ii),

$mc^2 = hcR\frac{15}{16}$

$\therefore \quad c^2 = \frac{15hRc}{16m}$

$c = \frac{15hR}{16m}$

157. $E_{photon} = \frac{hc}{\lambda}(\text{in eV}) = \frac{4\times10^{-15}\times3\times10^{8}}{300\times10^{-9}} = 4 \text{ eV}$

For an electron in the ground state of hydrogen atom first excitation energy is 10.2 eV. Since E_{photon} < 10.2 eV no excitation is possible.

158. $\Delta E = E_1 - E_2$

$\therefore \quad \Delta E = \frac{13.6}{1}-\frac{13.6}{2^2}$

$\Delta E = 13.6 \times \frac{3}{4} = 10.2 \text{ eV}$

This is the energy associated with emitted photon i.e., $h\nu$ = 10.2 eV

but according to photoelectric equations,

$h\nu = W_0 + eV_0$

$\therefore \quad 10.2 \text{ eV} = 4.2 \text{ eV} + eV_0$

$\therefore \quad eV_0 = 6 \text{ eV}$

159. For least energetic photon emitted in Lyman series, $E = E_2 - E_1 = 10.2$ eV

$\lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{10.2\times1.6\times10^{-19}}$

$= 1.2187 \times 10^{-7} \text{ m} \approx 122 \text{ nm}$

160. Let the percentage of B^{10} atoms be x.

$\therefore$ Hence percentage of B^{11} atom = (100 – x)

Average atomic weight

$= \frac{10x + 11(100-x)}{100} = 10.81 \Rightarrow x = 19$

$\therefore \quad \frac{N_{B^{10}}}{N_{B^{11}}} = \frac{19}{81}$

(Note: Refer Mindbender1.)

161. Mass of proton = mass of antiproton

$= 1.67\times10^{-27}\text{kg} = 1 \text{ amu}$

Energy equivalent to 1 amu = 931 MeV

So energy equivalent to 2 amu $= 2\times931$ MeV

$= 1862\times10^{6}\times1.6\times10^{-19} \approx 3\times10^{-10}$ J.

162. Using principle of momentum conservation,

$m_1v_1 = m_2v_2$

$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \left(\frac{R_2}{R_1}\right)^3 \qquad m \propto A \propto R^3$

$\frac{v_1}{v_2} = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$

163. Momentum of photon = $\frac{E}{c} = \frac{6\times1.6\times10^{-13}}{3\times10^{8}}$

$= 3.2 \times 10^{-21}$ kg m/s

As the momentum is conserved in nuclear reactions, momentum of nucleus

$= 3.2 \times 10^{-21}$ kg m/s

$\therefore \quad (K.E.)_{nucleus} = \frac{p^2}{2m} = \frac{\left(3.2\times10^{-21}\right)^2}{2\times20\times1.6\times10^{-27}}$

$= 1.6 \times 10^{-16}$ J = 1,000 eV = 1keV

164. Given that, A_0 = 8 count, A = 1 count, t = 3 hours

$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$, where n is the number of half lives

$\therefore \quad \frac{1}{8} = \left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n \Rightarrow n = 3$

Now, $n = \frac{t}{T_{1/2}}$, where $T_{1/2}$ is the half-life of a radioactive sample

$\therefore \quad T_{1/2} = \frac{t}{n} = \frac{3}{3} = 1$ hour.

165. 20 g substance reduces to 10 g

$\therefore \quad T_{1/2} = 4$ min

Using, $M = M_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

$\therefore \quad 10 = 80\left(\frac{1}{2}\right)^{t/4}$

$\Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/4}$

$\Rightarrow$ t = 12 min

166. $N = N_0\left(\frac{1}{2}\right)^2 \Rightarrow \frac{N}{N_0} = \frac{1}{4}$

Probability $= 1 - \frac{N}{N_0} = 1 - \frac{1}{4} = \frac{3}{4}$

167. By using $N = N_0 e^{-\lambda t}$ and average life time $t = \frac{1}{\lambda}$

$\therefore$ $N = N_0 e^{-\lambda \times 1/\lambda} = N_0 e^{-1}$

$\therefore$ $\frac{N}{N_0} = e^{-1} = \frac{1}{e}$

$\therefore$ Disintegrated fraction $= 1 - \frac{N}{N_0} = 1 - \frac{1}{e} = \frac{e-1}{e}$

168. $N_1 = \frac{N_{01}}{(2)^{t/20}}, N_2 = \frac{N_{02}}{(2)^{t/10}}$

$N_1 = N_2$ ....[Given]

$\therefore$ $\frac{40}{(2)^{t/20}} = \frac{160}{(2)^{t/10}} \Rightarrow 2^{-t/20} = 2^{\left(2-\frac{t}{10}\right)}$

$\therefore$ $\frac{-t}{20} = 2 - \frac{t}{10} \Rightarrow \frac{-t}{20} + \frac{t}{10} = 2$

$\therefore$ $\frac{t}{20} = 2 \Rightarrow t = 40$ s

169. $N = N_0 e^{-\lambda t}$

$\frac{N_0}{e} = N_0 e^{-\lambda(5)} \Rightarrow \lambda = \frac{1}{5}$

$\therefore$ $\frac{N_0}{2} = N_0 e^{-\lambda(t')} \Rightarrow t = 5 \log_e 2$

170. Although the beta spectrum is a continuous spectrum, the energy states of daughter nucleus are discrete.
Binding energy of Hydrogen nucleus is zero whereas for Helium it is 28.3 MeV.

171. $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{20} = 0.03465$

$t = \frac{2.303}{\lambda}\log\left(\frac{N_0}{N}\right)$

$\therefore$ $t_1 = \frac{2.303}{0.03465}\log\left(\frac{100}{67}\right) = 11.6\,\text{min}$

and $t_2 = \frac{2.303}{0.03465}\log\left(\frac{100}{33}\right) = 32\,\text{min}$

Hence time difference between points of time
$= t_1 - t_2 = 32 - 11.6$
$= 20.4$ min ≈ 20 min.

172. Half-life = 6 min. $= \frac{ln2}{\lambda}$

$\Rightarrow l = \frac{ln2}{6}$

$l = \frac{0.692}{6}$

$\because$ at t = 0, 1024 particles per minute
After 42 minute, 7 half-life is complete

$\Rightarrow$ no. of particles $= \frac{1024}{2^7}$

No. of particle = 8

173. By conservation of linear momentum,

$mv = mv_1 + \frac{m}{2}v_2$

where, v_1 and v_2 are velocities of particles A and B after collision.

$\therefore$ $2v = 2v_1 + v_2$ ....(i)

As collision is head on and elastic,

$e = \frac{v_2 - v_1}{u_1 - u_2} = 1$

$\therefore$ $v = v_2 - v_1$ ....(ii)

Solving equation (i) and (ii),

$v = 3v_1$ and $v = \frac{3v_2}{4}$

As, $\lambda \propto \frac{1}{p}$

$\therefore$ $\frac{\lambda_A}{\lambda_B} = \frac{p_2}{p_1} = \frac{\frac{m}{2}v_2}{mv_1} = \frac{(4/3)v}{(2/3)v} = 2$

174. $\nu \propto \left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) \propto \frac{n^2 - (n-1)^2}{n^2(n-1)} = \frac{2n-1}{n^2(n-1)^2}$

$\therefore$ For $n >> 1$, $\nu \propto \frac{1}{n^3}$

175. $E_{Ph} = \frac{1240}{500}\text{eV} = 2.48\,\text{eV}$

$K.E_{max} = E_{Ph} - \phi_0 = 2.48 - 2.28 = 0.2$ eV
For electron,

$\lambda_{min} = \frac{12.27}{\sqrt{K.E_{max}(eV)}}\,\text{Å} = \frac{12.27}{\sqrt{0.2}}\,\text{Å} = 27.436\,\text{Å}$

$= 27.436 \times 10^{-10}$ m

$\lambda_{min.} = 2.7436 \times 10^{-9}$ m

$\lambda \geq \lambda_{min}$

176. Using $r = \frac{mv}{qB}$ and $\frac{1}{2}mv^2 = eV_0$

$\therefore$ $r = \frac{\sqrt{2meV_0}}{eB} = \frac{1}{B}\sqrt{\frac{2m}{e}V_0}$

$\Rightarrow V_0 = \frac{B^2r^2e}{2m} = 0.8$ eV

For transition between 3 to 2,

$E = 13.6\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{13.6\times5}{36} = 1.88$ eV

$\therefore$ Work function = 1.88 eV – 0.8 eV
= 1.08 eV
≈ 1.1 eV

177. $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^8}{975\times10^{-10}\times1.6\times10^{-19}}$ eV

$\therefore$ Energy of the destination orbit = −13.6 + 12.5 = – 0.85 eV

$\therefore$ The Hydrogen atom will be excited to n = 4

$\therefore$ Number of spectral lines = $\frac{4(4-1)}{2} = 6$

178. Ground state energy = – 13.6 eV = E_1

Now, $E_n = \frac{-13.6}{n^2}$

$\therefore$ $E_2 = \frac{-13.6}{2} = -3.4$ eV

$\therefore$ Energy released = $-3.4 - (-13.6)^2 = 10.2$ eV

179. $(K.E.)_{initial} = (P.E.)_{closest\ approach}$

$\therefore$ $\frac{1}{2}mv^2 = \frac{2Ze^2}{4\pi\varepsilon_0 r_0}$ $\qquad \Rightarrow r_0 \propto \frac{1}{m}$

180. Kinetic energy of neutron in thermal equilibrium is $\frac{3}{2}kT$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m(K.E.)}} = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}} = \frac{h}{\sqrt{3mkT}}$$

181. de-Broglie's the formula is

$\lambda = \frac{h}{\sqrt{2m(K.E)}}$

But kinetic energy of thermal neutrons is k_BT

where, k_B = Boltzmann constant

$\therefore$ $\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.38\times10^{-23}\times(27+273)}}$

$\therefore$ $\lambda = 1.78\times10^{-10}$ m = 1.78 Å

182. Power = $\frac{\text{Energy released per fission(E)}}{\text{Time for one fission(T)}}$ = Ef

where,

f = frequency = No. of fissions per second.

$\therefore$ $f = \frac{P}{E} = \frac{6.4}{200\times10^6\times1.6\times10^{-19}} = \frac{6.4}{200\times1.6\times10^{-13}}$

$\therefore$ f = 2×10^{11} per second.

183. $\frac{B.E.}{A}$ starts at a small value, rises to a maximum at $_{26}Fe^{56}$, then decreases to 7.5 MeV for heavy nuclei.

184. $\lambda = \frac{h}{p}$ ....(De-Broglie formula)

$\lambda_\alpha = \frac{h}{p_\alpha} = \frac{h}{m_\alpha v_\alpha} = \frac{h}{4mv}$ ....(i)

[as mass of alpha is 4 times mass of proton/neutron and velocity given is v]

$\lambda_d = \frac{h}{p_d} = \frac{h}{m_d v_d} = \frac{h}{2m\,2v}$ ....(ii)

[as mass of deuteron is 2 times mass of proton/neutron and velocity given is 2v]

From (i) and (ii),

$\frac{\lambda_\alpha}{\lambda_d} = \frac{1}{1}$

185. We know that de-Broglie wavelength,

$\lambda = \frac{h}{mv}$

Velocity of a body falling from height H is given by

$v = \sqrt{2gH}$

$\therefore$ $\lambda = \frac{h}{m\sqrt{2gH}} = \frac{h}{m\sqrt{2g}\sqrt{H}}$

Here, $\frac{h}{m\sqrt{2g}}$ is a constant say 'K'.

$\therefore$ $\lambda \propto \frac{1}{\sqrt{H}}$

186. For photon: $E = \frac{hc}{\lambda_p}$

$\therefore$ $\lambda_p = \frac{hc}{E}$ ....(i)

For electron: $E = mc^2 = pc$

$\Rightarrow p = \frac{E}{c}$

$\therefore \quad \lambda_e = \frac{h}{p} = \frac{hc}{E}$ ....(ii)

Comparing equations (i) and (ii),

$\lambda_p \propto \lambda_e$

187. According to Bohr's second postulate,

$mvr_n = \frac{nh}{2\pi}$

$\therefore \quad 2\pi r_n = \frac{nh}{mv}$

But, de-Broglie wavelength $\lambda = \frac{h}{mv}$

$\therefore \quad 2\pi r_n = n\lambda$

No. of waves contained in the orbit $= \frac{\text{Circumference of the orbit}}{\text{wavelength}}$

$= \frac{2\pi r_n}{\lambda}$

$= n = 2$

Evaluation Test

1. We know

$N = N_0 e^{-\lambda t}$

For X_1, $\lambda = 5\lambda$

$\therefore \quad N_1 = N_0\, e^{-5\lambda t}$ ….(i)

For X_2,

$\therefore \quad N_2 = N_0\, e^{-\lambda t}$ ….(ii)

$\therefore \quad \frac{N_1}{N_2} = \frac{e^{-5\lambda t}}{e^{-\lambda t}} = e^{-4\lambda t}$

Given that, $\frac{N_1}{N_2} = \frac{1}{e}$

$\therefore \quad \frac{1}{e} = e^{-4\lambda t}$ or $4\lambda t = 1 \Rightarrow \quad t = \frac{1}{4\lambda}$

2. According to Bohr's postulate

$mvr = \frac{nh}{2\pi} = \frac{h}{2\pi}$ ….($\because$ n = 1)

or $v = \frac{h}{2\pi mr}$ ….(i)

We know that the rate of flow of charge is current.

Hence, $i = \frac{e}{t} = e\left(\frac{v}{2\pi r}\right) = \frac{e}{2\pi r} \times v$

$= \frac{e}{2\pi r} \times \frac{h}{2\pi mr} = \frac{eh}{4\pi^2 mr^2}$ ….(ii)

Magnetic dipole moment, $M = i \times A$

$\therefore \quad M = \frac{eh}{4\pi^2 mr^2} \times \pi r^2$

$M = \frac{eh}{4\pi m}$ ….(iii)

Torque, $\vec{\tau} = \vec{M} \times \vec{B}$

or $\tau = MB \sin 60°$

$\therefore \quad \tau = \frac{eh}{4\pi m} \times B \times \frac{\sqrt{3}}{2}$ $\quad\therefore \quad \tau = \frac{ehB}{8\pi m}\sqrt{3}$

3. We know that, de-Broglie wavelength

$\lambda = \frac{h}{mv}$ and $E = \frac{1}{2}mv^2$

$\therefore \quad \lambda = \frac{h}{\sqrt{2mE}}$

In first case,

$200 \times 10^{-12} = \frac{h}{\sqrt{2mE_1}}$ ….(i)

In second case,

$100 \times 10^{-12} = \frac{h}{\sqrt{2mE_2}}$ ….(ii)

Dividing equation (i) by equation (ii), we get

$2 = \sqrt{\left(\frac{E_2}{E_1}\right)}$

Or $E_2 = 4E_1$

So, energy to be added $= 4E_1 - E_1 = 3E_1$

Now, $\frac{h}{\sqrt{2mE_1}} = 200 \times 10^{-12}$

or $\sqrt{2mE_1} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}}$

or $\sqrt{2mE_1} = 3.315 \times 10^{-24}$

or $E_1 = \frac{(3.315 \times 10^{-24})^2}{2(9.1 \times 10^{-31})} = 0.6038 \times 10^{-17}$

$\therefore \quad$ Energy added $= 3E_1$

$= \frac{3 \times 0.6038 \times 10^{-17}}{(1.6 \times 10^{-19})}$ eV

$= 113$ eV

4. Power to be obtained from power house
= 200 megawatt

∴ Energy obtained per hour
= 200 megawatt × 1 hour
= (200 × 10^6 watt) × (3600 s)
= 72 × 10^{10} J

Here only 10% of output is utilized. In order to obtain 72 × 10^{10} J of useful energy, the output energy from the power house

$$= \frac{(72\times10^{10})\times100}{10}$$

= 72 × 10^{11} J

Let this energy be obtained from a mass-loss of Δm kg. Then

$(\Delta m)c^2 = 72 \times 10^{11}$

Or $\Delta m = \dfrac{72\times10^{11}}{(3\times10^8)^2} = 8 \times 10^{-5}$ kg

Δm = 0.08 g

Since 0.90 milligram (= 0.90 × 10^{-3} g) mass is lost in 1 g uranium, hence for a mass loss of 0.08 g the uranium required

$$= \frac{1\times0.08}{0.90\times10^{-3}}$$

= 88.88 ≈ 89 g

Thus, to run the power house, 89 g uranium is required per hour.

5. Lyman series belongs to the ultraviolet region.

6. K.E. = $\dfrac{13.6}{n^2}$ eV, P.E. = $\dfrac{-2(13.6)}{n^2}$ eV

For Hydrogen, Z = 1

∴ $\Delta K.E = K.E_f - K.E_i$

$$\Delta K.E. = 13.6\left[\frac{1}{(2)^2}-\frac{1}{(1)^2}\right]$$

= – 10.2 eV (decrease)

$$\Delta P.E. = -2(13.6)\left[\frac{1}{(2)^2}-\frac{1}{(1)^2}\right]$$

= 20.4 eV (increase)

Angular momentum, $L = \dfrac{nh}{2\pi}$

∴ $\Delta L = \dfrac{h}{2\pi}(2-1) = \dfrac{h}{2\pi}$

= 1.05 × 10^{-34} J-s (increase)

7. For Lyman series, $n_f = 1$ and $n_i = 2$
and Z = 2(He)

$$\Delta E = -13.6\, Z^2\left(\frac{1}{n_i^2}-\frac{1}{n_f^2}\right)$$

$$= -13.6\,(2)^2\times\left(\frac{-3}{4}\right) = 13.6\times3$$

∴ Total available energy = 3 × 13.6 Joule
Ionization energy of Hydrogen = 13.6 eV
Now energy available to an electron after the ionisation of hydrogen,

$$\Delta E = 3\times13.6 - 13.6 = 2\times13.6\text{ eV} = \frac{1}{2}m_e v^2$$

∴ $\dfrac{1}{2}m_e v^2 = 2 \times 13.6$ Ev

∴ $v^2 = \dfrac{2\times2\times13.6\times1.6\times10^{-19}}{9.1\times10^{-31}}$

∴ v = 3.1 × 10^6 m/s

8. Orbital frequency,

$$f = \frac{v_n}{2\pi r_n}$$

$$v_n = \frac{2.2\times10^6 Z}{n}\text{ m/s} = \frac{2.2\times10^6(1)}{2}$$

= 1.1 × 10^6 ms^{-1}

Now radius,

$$r_n = 0.53\times10^{-10}\frac{n^2}{Z} = 4\times0.53\times10^{-10}\text{ m}$$

….[∵ n = 2]

= 2.12 × 10^{-10} m

f = number of revolution in one second

$$= \frac{N}{t} = \frac{v_n}{2\pi r_n}$$

∴ Number of revolutions,

$$N = f\times t = \frac{1.1\times10^6}{2\pi\times2.12\times10^{-10}}\times10^{-8}$$

= 8.2 × 10^6 revolutions

∴ Period = $\dfrac{1}{8.2\times10^6} = 1.2\times10^{-7}$

9. $_1H^2 + {}_1H^2 \longrightarrow {}_2He^4$ + Energy
Binding energy (B.E.) of $_1H^2$ = 2 × 1.1
= 2.2 MeV

∴ B.E. of two $_1H^2$ = 2 × 2.2 = 4.4 MeV
B.E. of $_2He^4$ nucleus = 4 × 7.1 = 28.4 MeV

∴ Energy released when two $_1H^2$ fuse to form $_2He^4$ = 28.4 – 4.4 = 24 MeV

10. Total energy of C^{12} atom
= Number of Nucleons × 7.68
= 12 × 7.68 = 92.16 MeV
Similarly, energy for C^{13} atom
= 13 × 7.47 = 97.11 MeV
Energy required to remove 1 neutron from C^{13} = (97.11 – 92.16) = 4.95 MeV

11. Using, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

∴ For 33% decay, $\frac{N}{N_0} = \frac{67}{100}$

∴ $\left(\frac{67}{100}\right) = \left(\frac{1}{2}\right)^{t_1/10}$ ….(i)

For 67% decay, $\frac{N}{N_0} = \frac{33}{100}$

∴ $\frac{33}{100} = \left(\frac{1}{2}\right)^{t_2/10}$ ….(ii)

Dividing equation (ii) by equation (i) we get,

$\frac{33}{67} = \left(\frac{1}{2}\right)^{\frac{t_2 - t_1}{10}} \approx \left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^{(t_2 - t_1)/10}$

$\Rightarrow \frac{t_2 - t_1}{10} = 1$ or $t_2 - t_1 = 10$ min

12. From law of conservation of momentum,
mu = 2 mv or $v = \frac{u}{2}$
Excitation energy,
$\Delta E = \frac{1}{2} mu^2 - 2 \times \frac{1}{2} m\left(\frac{u}{2}\right)^2 = \frac{1}{4} mu^2$
Minimum excitation energy
$= 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ eV
$= \frac{3}{4} \times 13.6$
= 10.2 eV

∴ $(10.2)(1.6 \times 10^{-19}\text{ J}) = \frac{1}{4}(1.0078)(1.66 \times 10^{-27})u^2$

∴ $u = 6.25 \times 10^4\text{ ms}^{-1}$

13. Using magnetic moment,
M = current × area = $\frac{q}{t} \times A$

∴ $M = \frac{\omega}{2\pi} \times q \times \pi r^2 = \frac{1}{2}\omega q r^2$

But orbital angular momentum,
$L = m\omega r^2 = \frac{nh}{2\pi}$
For n = 1,
$\omega r^2 = h/2\pi m$

∴ $M = \frac{1}{4\pi}\frac{qh}{m}$
$= \frac{(1.6\times10^{-19})(1.05\times10^{-34})}{2\times9.1\times10^{-31}}$
$= 9.2 \times 10^{-24}\text{ Am}^2$

14. A photon is emitted when hydrogen atom comes to first excited state i.e., n = 2

∴ Energy transferred
$= -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right)$
$= \frac{3}{4} \times 13.6$ eV
= 10.2 eV
By conservation of momentum,
$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + 10.2$
or $v_1^2 - vv_1 + 10.2 = 0$ ….[eliminating v_2]

∵ v_1 is real $\Rightarrow v^2 \geq 4 \times 10.2$
or $v_{min} = \sqrt{\frac{4\times10.2}{m}}$

∴ $K.E_{min} = \frac{1}{2}m\left(v_{min}\right)^2$
$= \frac{1}{2}m\frac{4\times10.2}{m}$
= 20.4 eV

15. Sum of masses of deutron and lithium nuclei before disintegration
= 2.0147 + 6.0169
= 8.0316 amu
Mass of α particles
= 2 × 4.0039
= 8.0078 amu
Difference of mass
= 8.0316 – 8.0078
= 0.0238 amu
Mass converted into energy
= 0.0238 × 931.3 MeV
Energy given to each α particle
$= \frac{0.0238\times931.3}{2}$
= 11.08 MeV

16. For C^{14}, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730}$

$\Rightarrow \lambda = 1.21 \times 10^{-4}\ yr^{-1}$ since A = 0.144 Bq and A_0 = 0.28 Bq

Using, $A = A_0 e^{-\lambda t}$ or $t = \frac{1}{\lambda} ln\left(\frac{A_0}{A}\right)$

$t = \frac{1}{1.21\times10^{-4}} ln\left(\frac{0.28}{0.144}\right)$

≈ 5500 years

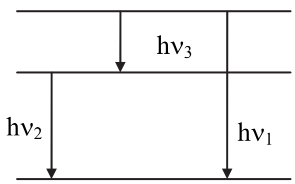
17. Assertion is false, reason is true. The reduced mass of atomic deuterium is greater than that of atomic hydrogen as

$\mu = \frac{m_e m_n}{m_e + m_n}$,

where m_e = mass of electron and

m_n = mass of nucleus.

18.



For three energy levels, the possible transition are as shown in the diagram.

It is given, $\lambda_1 < \lambda_2 < \lambda_3 \Rightarrow \nu_1 > \nu_2 > \nu_3$.

The largest gap will correspond to $h\nu_1$

$h\nu_1 = h\nu_2 + h\nu_3$ or $\frac{hc}{\lambda_1} = \frac{hc}{\lambda_2} + \frac{hc}{\lambda_3}$

$\Rightarrow \frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$

19. Angular momentum of nth orbit = $\frac{nh}{2\pi}$.

Again, $mvr = \frac{nh}{2\pi}$

∴ $v = \frac{nh}{2\pi mr}$ ….(i)

The time taken for completing an orbit

$T = \frac{2\pi r}{v} = \frac{2\pi r(2\pi mr)}{nh}$

Or $T = \frac{4\pi^2 mr^2}{nh}$ ….(ii)

Now, $r = r_0 n^2$ ….[∵ $r \propto n^2$]

∴ $T = \frac{4\pi^2 m r_0^2 n^4}{nh} = \frac{4\pi^2 m r_0^2 n^3}{h}$

Number of orbits completed in 10^{-6} s = $\frac{10^{-6}}{T}$

$= \frac{10^{-6}\times h}{4\pi^2 m r_0^2 n^3}$

$= \frac{10^{-6}\times(6.63\times10^{-34})}{4(3.14)^2(9.1\times10^{-31})(5.3\times10^{-11})^2(2)^3}$

$= 8.22 \times 10^8$

20. To ionize the H atom in ground state minimum K.E. of photoelectron needed = 13.6 eV.

∵ W_0 = 1.9 eV

∴ Minimum energy (or maximum wavelength) incident = 13.6 + 1.9 ≈ 16 eV

∴ $\lambda'_{max} = \frac{6.6\times10^{-34}\times3\times10^8}{16\times1.6\times10^{-19}}$ = 77.3 nm ≈ 77 nm

Textbook
Chapter No.

19 Semiconductors

Hints

Classical Thinking

94. At absolute zero, semiconductor behaves as an insulator.

95. The current gain,

$\beta = \frac{\Delta I_C}{\Delta I_B}$

$\therefore$ $\Delta I_C = \beta \times \Delta I_B = 4 \times 6 = 24$ mA

$\therefore$ Change in collector current = 24 mA

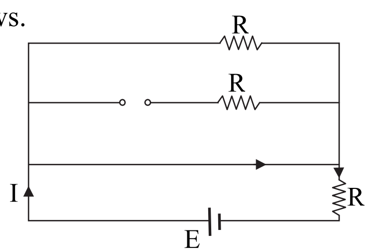
Critical Thinking

4. By using mass action law,

$n_i^2 = n_e n_h$

$\therefore$ $n_h = \frac{n_i^2}{n_e} = \frac{(10^{16})^2}{10^{21}} = 10^{11}$ per m^3

13. Diodes D_1 and D_3 are forward biased and D_2 is reverse biased. So the circuit can be redrawn as follows.



$\therefore$ $I = \frac{E}{R}$

15. $I = \frac{P}{V} = \frac{240\times10^{-3}}{5} = 48$ mA

16. $I_S = \frac{V_S - V_Z}{R_S} = \frac{30-10}{1.5\times10^3} = 13.33$ mA

$I_L = \frac{V_O}{R_L} = \frac{10}{2\times10^3} = 5$ mA

$I_Z = I_S - I_L = 13.33 - 5 = 8.33$ mA

20. In P-N-P transistors, majority charge carriers are holes while in case of N-P-N transistors, majority charge carriers are electrons which have greater mobility.

23. $I_E = I_B + I_C = I_B + \beta I_B$

$= I_B(\beta + 1) = 5\ \mu A\ (50 + 1)$

$= 255\ \mu A = 0.255$ mA

24. If forward bias is made large, the majority charge carriers would move from the emitter to the collector through the base with high velocity. This would give rise to excessive heat causing damage to transistor.

25. $\beta_{dc} = \frac{I_C}{I_B} \Rightarrow I_C = 99 \times 20 = 1980\ \mu A$

$\therefore$ $I_E = I_C + I_B = 1980 + 20 = 2000\ \mu A$

26. $I_C = \frac{80}{100} \times I_E$

$\therefore$ $24 = \frac{80}{100} \times I_E \Rightarrow I_E = 30$ mA

Using, $I_E = I_B + I_C$, $I_B = 30 - 24 = 6$ mA

27. α is the ratio of collector current and emitter current while β is the ratio of collector current and base current.

28. $V_b = I_b R_b$

$\therefore$ $R_b = \frac{9}{35\times10^{-6}} = 257\ k\Omega$

39. Time t = CR is known as time constant. It is time in which charge on the capacitor decreases to $\frac{1}{e}$ times of its initial charge (steady state charge).

In figure (i), PN junction diode is in forward bias, so current will flow in the circuit i.e., charge on the capacitor decreases and in time t it becomes $Q = \frac{1}{e}(Q_o)$; where $Q_o = CV$

$\therefore$ $Q = \frac{CV}{e}$

In figure (ii), p-n junction diode is in reverse bias, so no current will flow through the circuit hence charge on capacitor will not decay and it remains same i.e., CV after time t.

40. $I = \frac{P}{V} = \frac{100\times10^{-3}}{0.5} = 0.2$ A

$V_R = 1.5 - 0.5 = 1.0$ volt

$\therefore$ $R = \frac{1.0}{0.2} = 5\ \Omega$

41. This is because n-side is more positive as compared to p-side.

Competitive Thinking

1. With temperature rise, the conductivity of semiconductors increases.

3. At 0 K, semiconductor behaves as an insulator.

4. The conduction and valence bands in the conductors merge into each other.

8. Band gap of insulator is highest, while that of conductor is least. So,

$E_{g_1} > E_{g_3} > E_{g_2}$

i.e., $E_{g_1} > E_{g_2}$,

$E_{g_3} > E_{g_2}$

$\therefore$ $E_{g_1} > E_{g_2} < E_{g_3}$

10. The energy gap between valence band and conduction band in germanium is 0.76 eV and the energy gap between valence band and conduction band in silicon is 1.1 eV. Also, it is true that thermal energy produces fewer minority carriers in silicon than in germanium

11. Gallium is trivalent impurity.

12. Antimony and phosphorous are both pentavalent.

13. Extrinsic semiconductors (n-type or p-type) are neutral.

16. $n_i^2 = n_h n_e \Rightarrow n_e = \dfrac{\left(3 \times 10^{16}\right)^2}{4.5 \times 10^{22}} = \dfrac{9 \times 10^{32}}{4.5 \times 10^{22}}$

$= 2 \times 10^{10}\ m^{-3}$

17. $n_e n_h = n_i^2$

$\Rightarrow 4 \times 10^{10} \times n_h = 4 \times 10^{16}$

$\therefore$ $n_h = 10^6\ m^{-3}$

18. In p-type semiconductors, holes are the majority charge carriers

20. Boron is a trivalent impurity.

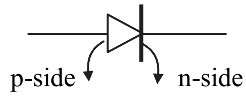
21. In n-type semiconductors, minority carriers are holes, majority carriers are electrons and pentavalent atoms are dopants.

22. Phosphorus is a pentavalent impurity.
Hence, $n_e >> n_h$.

24. When a free electron is produced, a hole is also produced at the same instant.

25. In reverse bias, no current flows.

27.

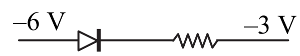


For forward bias, p-side must be at higher potential than n-side.

28. For diode to be in forward bias, p-side of diode needs to be connected at potential higher than potential to which n-side of diode is connected.
This condition is satisfied in option (A) only.

29. From the figure in option (A),
'p' is at low potential (–6 V) than 'n' (–3 V)

$\therefore$ diode is reverse biased.

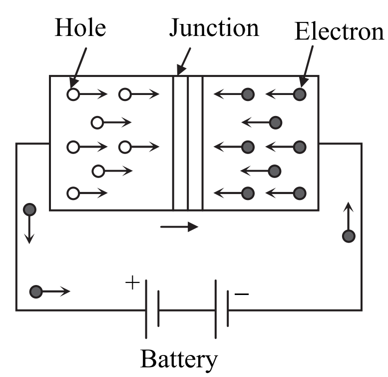


31. In forward bias, the diffusion current increases and drift current remains constant. Hence no current flows due to diffusion.
In reverse bias, diffusion becomes more difficult. Hence net current (very small) is due to drift.

32. As in both the figures the p-type material of diode is connected to positive terminal of battery and n-type to negative terminal, both are forward biased.

33. In case of p-n junction diode, width of the depletion region decreases as the forward bias voltage decreases.

34. When p-side of junction diode is connected to positive of battery and n-side to the negative, then junction diode is in forward biased mode.



In this mode, more number of electrons enter in n-side from battery thereby increasing the number of donors on n-side.

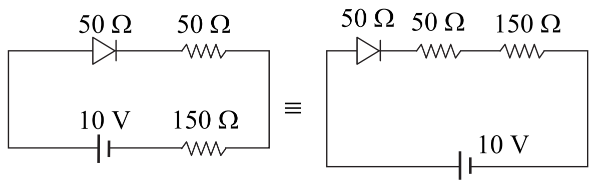
35. $R_d = \dfrac{\Delta V}{\Delta I} = \dfrac{0.6}{1.2 \times 10^{-3}} = 500\ \Omega$

36. Potential difference V = 4 –(–6) = 10 V

∴ $I = \frac{V}{R} = \frac{10}{10^3} = 10^{-2}$ A

37. $V_{AB} = 0.2 \times 10^{-3} (5 \times 10^{-3} + 5 \times 10^{-3}) + 0.2$
$= 2.2$ V

38. Since the diode in reverse bias offers infinite resistance, the equivalent circuit becomes.

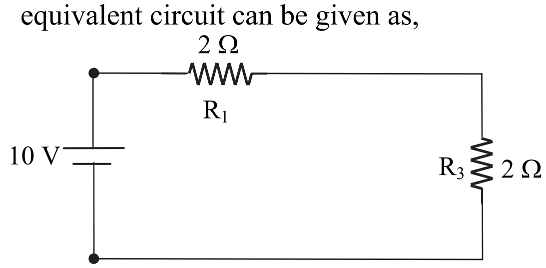


$I = \frac{10}{50+50+150} = \frac{10}{250} = 0.04$ A

39. For $V_A > V_B$:
Both diodes are forward biased so equivalent resistance $R_1 = \frac{50 \times 50}{50 + 50} = 25\ \Omega$

For $V_B > V_A$:
Both diodes are reverse biased so equivalent resistance is infinity.

40. In given circuit, the diode D_1 is connected in reverse biased. Hence, no current flows through resistance R_2. As diode D_2 is ideal, the equivalent circuit can be given as,



∴ $I = \frac{V}{R_1 + R_3} = \frac{10}{2+2} = 2.5$ A

41. Voltage drop across Si diode will be approximately 0.7 V.

∴ $I = \frac{V - V_{diode}}{R} = \frac{3 - 0.7}{200} = 0.0115$ A = 11.5 mA

46. When reverse bias is increased, the electric field at the junction also increases. At some stage, the electric field breaks the covalent bonds thus generating a large number of charge carriers. This is called zener breakdown.

48. For a considerable range of load resistance, the current in the zener diode may change but the voltage across it remains unaffected. Hence the output voltage across the zener diode is a regulated voltage.

49. For the breakdown condition of the zener diode, the applied voltage, $V_1 > V_Z$. In this case, for a wide range of values, the current will change but the voltage will remain constant.

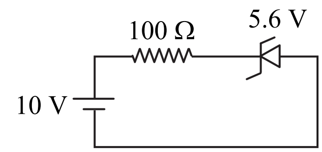
50. Zener breakdown voltage = 6 V

∴ Potential across 4 kΩ = 6 V
and potential across 6 kΩ = (10 – 6) = 4 V

Current through the 6 kΩ = $\frac{4}{6000}$

$= \frac{2}{3000}$ A $= \frac{2}{3}$ mA

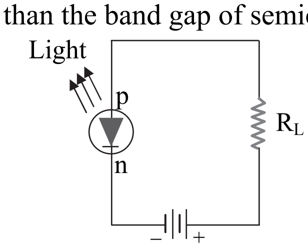
51.



$I = \frac{10 - 5.6}{100} = \frac{4.4}{100}$
$= 0.04 = 44 \times 10^{-3}$ A
$= 44$ mA

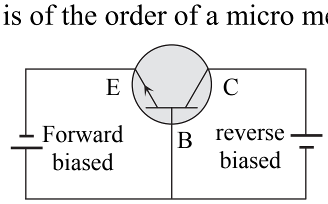
53. When a light (wavelength sufficient to break the covalent bond) falls on the junction, new hole-electron pairs are created. Number of electron-hole pairs produced depends upon number of photons. Hence photo e.m.f. or current is proportional to intensity of light.

55. When a junction diode is forward biased as shown in figure, energy is released at the junction due to recombination of electrons and holes. In the junction diode made of gallium arsenide or indium phosphide, the energy is released in visible region. Such a junction diode is called light emitting diode or LED. The radiated energy emitted by LED is equal or less than the band gap of semiconductor.



56. Base is thinnest layer in a transistor and has the width of about 3-5 μm. Thus, its thickness is of the order of a micro meter.

57.



58. When NPN transistor is used as an amplifier, majority charge carrier electrons of N-type emitter move from emitter to base and then base to collector.

59. As emitter (N) is common to both, the base (P) and collector (N), it is a CE amplifier circuit.

63. In active region of CE amplifier, the collector-base junction is reverse biased while emitter-base junction is forward biased.

64. During positive half cycle due to forward biasing, emitter current and consequently collector current increases.

As, $V_{CE} = V_{CC} - I_C R_L$, increase in collector current causes decrease in collector voltage. This, as collector is connected to positive terminal of V_{CC} battery, makes collector less positive, i.e., negative with respect to initial value.

Thus, during positive half cycle, unlike input signal voltage, output signal voltage at collector varies through a negative half cycle.

Similarly, it can be seen that, during negative half cycle, unlike input signal voltage, output signal voltage at collector varies through a positive half cycle.

This shows, in a CE amplifier, input and output voltages are in opposite (180°) phase.

Alternate method:

For a CE amplifier,

input signal voltage $V_i = \Delta I_B \times R_B$

where, ΔI_B = change in base current and

R_B = input resistance of emitter base circuit.

AC current gain $\beta_{ac} = \dfrac{\Delta I_C}{\Delta I_B}$

where, ΔI_C = change in collector current.

As, $V_{CE} = V_{CC} - I_C R_L$, considering change in V_{CE},

$\Delta V_{CE} = 0 - \Delta I_C R_L$

[Since change in base current ΔI_B changes collector current, but not V_{CC}]

$\therefore$ $\Delta V_{CE} = -(\beta_{ac} \times \Delta I_B)\, R_L$

Output voltage $V_o = \Delta V_{CE}$

$\therefore$ Voltage gain of CE amplifier,

$$A_V = \frac{V_o}{V_{in}} = \frac{-\beta_{ac}\Delta I_B R_L}{\Delta I_B R_B} = -\beta_{ac}\left(\frac{R_L}{R_B}\right)$$

Negative sign indicates that output voltage is out of phase (180)° with respect to input voltage.

66. For a transistor,

$$\beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}} \Rightarrow (1-\alpha_{dc}) = \frac{\alpha_{dc}}{\beta_{dc}} \quad \text{....(i)}$$

Simplifying the ratio given in the question,

$$\frac{\beta_{dc}-\alpha_{dc}}{\alpha_{dc}\beta_{dc}} = \frac{\beta_{dc}\left(1-\dfrac{\alpha_{dc}}{\beta_{dc}}\right)}{\alpha_{dc}\beta_{dc}} = \frac{1-\dfrac{\alpha_{dc}}{\beta_{dc}}}{\alpha_{dc}}$$

$$= \frac{1-(1-\alpha_{dc})}{\alpha_{dc}} \quad \text{....[Using equation (i)]}$$

$$\therefore \quad \frac{\beta_{dc}-\alpha_{dc}}{\alpha_{dc}\beta_{dc}} = \frac{1-1+\alpha_{dc}}{\alpha_{dc}} = 1$$

67. $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.98}{0.02} = 49$

68. $\beta_{dc} = \dfrac{\alpha_{dc}}{1-\alpha_{dc}} = \dfrac{69/70}{1-(69/70)} = 69$

69. $I_E = I_B + I_C$

$\therefore$ $\dfrac{I_E}{I_C} = \dfrac{I_B}{I_C} + 1$

$\therefore$ $\dfrac{1}{\alpha} = \dfrac{1}{\beta} + 1 = \dfrac{1}{19} + 1 = \dfrac{20}{19}$

$\therefore$ $\alpha = \dfrac{19}{20} = 0.95$

70. $\beta = \dfrac{I_C}{I_B} \Rightarrow I_C = \beta\, I_B = 100 \times 5$ mA

$= 500 \times 10^{-3} = 0.5$ A

71. $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.96}{1-0.96} = 24$

72. $\alpha = \dfrac{I_C}{I_E} = 0.96$ and $I_E = 7.2$ mA

$\therefore$ $I_C = 0.96 \times I_E = 0.96 \times 7.2 = 6.91$ mA

$\therefore$ $I_E = I_C + I_B$

$\therefore$ $7.2 = 6.91 + I_B$

$\therefore$ $I_B = 0.29$ mA

73. $A_V = \beta \dfrac{R_o}{R_{in}} = \dfrac{I_C}{I_B}\dfrac{R_o}{R_{in}} = \dfrac{I_C R_o}{V_{in}} = g_m R_o$

$\therefore$ $A_V \propto g_m$.

$\therefore$ $\dfrac{A_{V_1}}{A_{V_2}} = \dfrac{g_{m_1}}{g_{m_2}} = \dfrac{0.03}{0.02} = \dfrac{3}{2}$

$\therefore$ $A_{V_2} = \dfrac{2}{3} A_{V_1} = \dfrac{2}{3}$G.

74. $\alpha = \frac{I_C}{I_E} = 0.95$ ....[Given]

$\therefore$ $I_C = 0.95\ I_E$

Now, $I_E = I_C + I_B \Rightarrow 0.05\ I_E = I_B$

$\therefore$ $I_E = \frac{I_B}{0.05} = \frac{0.2\,\text{mA}}{0.05} = 4$ mA

75. $\alpha = 0.8 \Rightarrow \beta = \frac{0.8}{(1-0.8)} = 4$

$\therefore$ $\beta = \frac{\Delta i_C}{\Delta i_B} \Rightarrow \Delta i_C = \beta \times \Delta i_B = 4 \times 6 = 24$ mA.

76. $\beta = 45$

$\frac{I_c}{I_b} = 45$

$V = I_c R$

$\therefore$ $5 = I_c \times 1 \times 10^3$

$\therefore$ $I_c = 5 \times 10^{-3}$ A

$I_b = \frac{I_c}{45} = \frac{5 \times 10^{-3}}{45} = 0.111 \times 10^{-3}$ A = 111 μA

77. $\Delta i_C = \alpha\,\Delta i_E = 0.98 \times 2 = 1.96$ mA

$\therefore$ $\Delta i_B = \Delta i_E - \Delta i_C = 2 - 1.96 = 0.04$ mA.

78. $I_E = I_B + I_C \Rightarrow I_C = I_E - I_B$

79. In common emitter transistor amplifier current gain $\beta > 1$, so output current > input current, hence assertion is correct.
Also, input circuit has low resistance due to forward biasing to emitter base junction, hence reason is false.

80. Current gain,

$\beta = \frac{\Delta i_C}{\Delta i_B} \Rightarrow \Delta i_B = \frac{1 \times 10^{-3}}{100} = 10^{-5}\,\text{A} = 0.01$ mA

By using $\Delta i_E = \Delta i_B + \Delta i_C \Rightarrow \Delta i_E = 0.01 + 1 = 1.01$ mA.

81. $I = \frac{Q}{T} = \frac{ne}{T}$

where, n is the number of electrons entering the emitter, e is the charge on one electron

$\therefore$ $I = \frac{10^{10} \times 1.6 \times 10^{-19}}{2 \times 10^{-6}} = 800 \times 10^{-6}$ A

83. $\alpha = \frac{I_C}{I_E} = \frac{5.488}{5.60} = 0.98$

$\Rightarrow \beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$

84. Given:

$\Delta I_C = 0.49$ mA

$\Delta I_E = 0.50$ mA

We know,

$\beta = \frac{\Delta I_C}{\Delta I_B}$

$\Delta I_E = \Delta I_C + \Delta I_B$

$\therefore$ $\Delta I_B = \Delta I_E - \Delta I_C = 0.50 - 0.49 = 0.01$ mA

$\therefore$ $\beta = \frac{0.49}{0.01} = 49$

85. Current gain for common emitter is, $\beta = \frac{I_C}{I_B}$

$\therefore$ $\beta = \frac{95\%\text{ of } I_E}{5\%\text{ of } I_E}$

$\therefore$ $\beta = \frac{95}{5} = 19$

86. $\beta = \frac{I_C}{I_B} \Rightarrow I_C = \beta I_B = 2 \times 10^{-3}$ A

$V_{CC} - I_C R_L = V_{CE} \Rightarrow 10 - (2 \times 10^{-3})R_L = 4$

$\therefore$ $R_L = 3$ kΩ

88. The input characteristics of the CE mode transistor (common emitter mode) represents the variation of the input current (base current I_B) with input voltage (base emitter voltage V_{BE}) at constant output voltage (collector emitter voltage V_{CE}).

89. Voltage gain = $A_V = \beta\frac{R_2}{R_1}$ and

Current gain $\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$

$\therefore$ $A_V = (49)\left[\frac{500 \times 10^3}{R_1}\right]$

$\therefore$ Power gain = $\beta.A_V$

$\therefore$ $6.0625 \times 10^6 = 49 \times \left[\frac{500 \times 10^3}{R_1}\right] \times 49$

$\therefore$ $R_1 = \frac{49^2 \times 500 \times 10^3}{6.0625 \times 10^6}$

$\therefore$ $R_1 \approx 198\ \Omega$

90. The collector current is given by,

$I_C = \frac{V_C}{R_C} = \frac{0.6\,\text{V}}{600\,\Omega} = 1 \times 10^{-3}$ A = 1 mA

$\beta = \frac{I_C}{I_B} \Rightarrow I_B = \frac{I_C}{\beta} = \frac{1\,\text{mA}}{20} = 0.05$ mA

91. Here,
Collector current, $I_C = 25$ mA
Base current, $I_B = 1$ mA
As $I_E = I_B + I_C = (1 + 25)$ mA = 26 mA
As $\alpha = \frac{I_C}{I_E} = \frac{25}{26}$

92. $V_i = I_B \times R_B$
$\therefore$ $I_B = \frac{20}{500 \times 10^3} = 40 \times 10^{-6}$ A = 40 μA
$V_C = I_C \times R_C$
$\therefore$ $I_C = \frac{20}{4 \times 10^3} = 5 \times 10^{-3}$ A = 5 mA
$\beta = \frac{I_C}{I_B} = \frac{5 \times 10^{-3}}{40 \times 10^{-6}} = 125$

93. $\beta = \frac{I_C}{I_B} = 60$ $\therefore$ $I_C = 60\, I_B$
But $I_E = I_B + I_C$ $\therefore$ $I_B = I_E - I_C$
$\therefore$ $I_B = 6.6 - 60I_B$ $\therefore$ $61I_B = 6.6$
$I_B = 0.108$ mA

94. Given,
$R_{in} = R_B = 1$ kΩ
$R_{out} = R_C = 2$ kΩ
$V_{out} = 4$ V
$\beta = 100$
We know,
$A_V = \beta \times$ resistance gain
$\therefore$ $A_V = \beta \times \frac{R_C}{R_B} = 100 \times \frac{2\,k\Omega}{1\,k\Omega} = 200$
Also, $A_V = \frac{V_{out}}{V_{in}}$
$\therefore$ $\frac{V_{out}}{V_{in}} = 200$ $\therefore$ $\frac{4}{V_{in}} = 200$
$\therefore$ $V_{in} = \frac{4}{200} = 20$ mV

95. Given: $R_L = 2$ kΩ = 2000 Ω,
$R_i = 150$ Ω, $\Delta I_B = 20$ μA $= 20 \times 10^{-6}$ A,
$\Delta I_C = 1.5$ mA $= 1.5 \times 10^{-3}$A
Voltage gain is given by,
$A_v = \frac{V_0}{V_1}$
$= \frac{R_L\, \Delta I_C}{R_i\, \Delta I_B}$
$= \frac{2000 \times 1.5 \times 10^{-3}}{150 \times 20 \times 10^{-6}}$
$= 1000$

96. The input resistance is
$R_i = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{0.04}{20 \times 10^{-6}}$
$\therefore$ $R_i = 2 \times 10^3 = 2$ kΩ
the A.C current gain is
$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2\,mA}{20\,\mu A} = \frac{2 \times 10^{-3}}{20 \times 10^{-6}} = 100$

97. Given
$0.5 = I_C R$
$I_C = \frac{0.5}{800}$
$I_C = 0.625$ mA
$\because$ $I_C + I_B = I_E$
$\because$ $\alpha = \frac{I_C}{I_E}$
$\because$ $\alpha = 0.96$ (given)
$0.96 = \frac{0.625\,mA}{I_E}$
$I_E = 0.651$ mA
$I_B = I_E - I_C$
$I_B = 0.651 - 0.625$
$I_B = 0.026$ mA
$\Rightarrow V_{CE} = V_{CC} - I_C R_C$
$= 8 - 0.625 \times 10^{-3} \times 800$
$\therefore$ $V_{CE} = 7.5$ V

98. $I_C = \frac{V_{CE}}{R_C} = \frac{2}{4 \times 10^3} = 0.5 \times 10^{-3}$ A = 0.5 mA
$\therefore$ $\beta = \frac{I_C}{I_B}$
$\therefore$ $I_B = \frac{I_C}{\beta} = \frac{0.5 \times 10^{-3}}{50} = 10^{-5}$ A = 10 μA

100. Amplification with negative feedback is
$A' = \frac{A}{1 + \beta A}$
Where β = fraction of output feedback to input
$\therefore$ $\beta = \frac{9}{100} = 0.09$ and $A' = 10$
$\Rightarrow 10 = \frac{A}{1 + 0.09A} \Rightarrow A = 100$

101. $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi} \frac{1}{\sqrt{\frac{10}{\pi^2} \times 10^{-3} \times 0.04 \times 10^{-6}}}$
$= 25$ kHz

106. Gate shown in option (B) is a NOR gate. Output of NOR gate when both the inputs are 0, is 1.

107. For 'OR' gate, $X = A + B$
i.e., $0 + 0 = 0,\ 0 + 1 = 1,\ 1 + 0 = 1,\ 1 + 1 = 1$

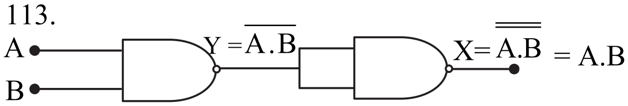
109. Truth table for the given circuit is

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

This belongs to AND gate

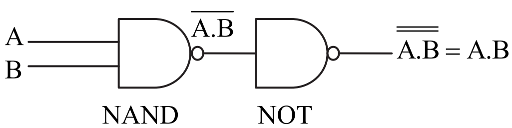
110. The Boolean expression for 'NOR' gate is
$Y = \overline{A + B}$
If A = B = 0(Low), $Y = \overline{0+0} = \overline{0} = 1$ (High)

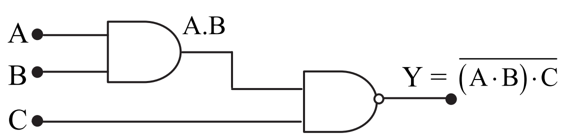
112. If inputs are A and B, then output for NAND gate is $Y = \overline{AB}$
∴ If A = B = 1, $Y = \overline{1.1} = \overline{1} = 0$

113.


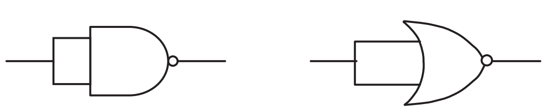
A	B	Y	X
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

∴ $X = \overline{\overline{A.B}} = A.B$

115. A single terminal NAND works as NOT.


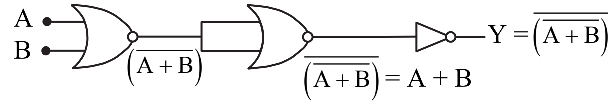
116.


A	B	(A·B)	C	$Y = \overline{(A\cdot B)\cdot C}$
0	0	0	0	1
1	1	1	1	0

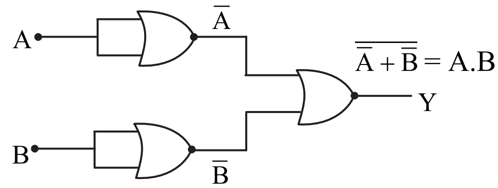
117.


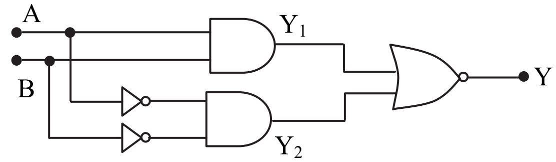
Shorted NAND　　Shorted NOR

Both shorted NAND and NOR gates act as a NOT gate.

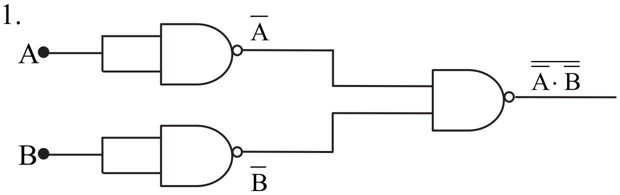
118.


Thus, given network is equivalent to NOR gate.

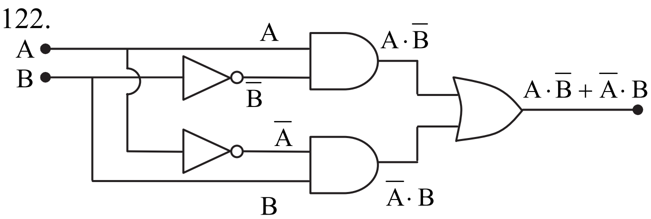
119.


120.


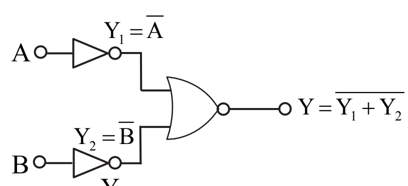
$Y_1 = A\cdot B$,
$Y_2 = \overline{A}\cdot\overline{B}$
$Y = \overline{Y_1 + Y_2}$
For A = B = 1,
$Y_1 = 1,\ Y_2 = 0$
∴ $Y = \overline{Y_1 + Y_2} = 0$
Similarly, for A = B = 0,
$Y_1 = 0,\ Y_2 = 1$
∴ $Y = \overline{Y_1 + Y_2} = 0$

121.


From figure,
Output $Y = \overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

122.


123. $Y = \overline{\overline{A}+\overline{B}}$
$= \overline{\overline{A}}.\overline{\overline{B}}$
$= A.B$



124. From time graph it is clear that output remains high when any of the input is high. This is represented by OR gate.

125. These gates are called digital building blocks because by using various combinations of these gates (either NAND or NOR) we can compile all other gates (like OR, AND, NOT, XOR).

126. p-n junction diode works only in forward bias and not in reverse bias.

127. Diode will be in forward bias only in 0-5 volt hence, it will conduct.

128. Majority charge carriers in n-type semiconductor are electrons.

129. For a Solar cell, Open circuit $\Rightarrow$ I = 0 and potential V = e.m.f.

Also, $\rightarrow$ Short circuit $\Rightarrow$ I = I and potential V = 0

131. The voltage-current curve for GaAs material is as shown in figure below.

Thus, there exists a region where increase in voltage leads to decrease in current which is a non-ohmic behaviour and is attributed to negative resistance.

132. Heating will have effect on number of minority as well as majority charge carriers. This change in charge carriers will affect overall V-I characteristics of p-n junction.

133. Resistivity of a semiconductor decreases with temperature. The atoms of a semiconductor vibrate with larger amplitudes of higher temperatures thereby increasing its conductivity.

134. With decrease in temperature, resistance of metal decreases and semi conductor increases.

135. $E_g = \dfrac{hc}{\lambda}$

$$\Rightarrow \lambda = \frac{hc}{E_g} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.9\times1.6\times10^{-19}} = 6.54\times10^{-7}\text{ m}$$

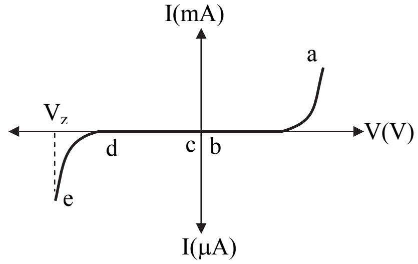
136. The electronic configuration of C and Si are:

$^{6}C = 1S^2, 2S^2\,2P^2$ and $^{14}Si = 1S^2, 2S^2\,2P^6, 3S^2\,3P^2$

Thus, the electrons in the outer most shell of carbon atoms are more tightly bound to the nucleus unlike for silicon and are not available for conduction. Hence it acts as an insulator.

137. $E = \dfrac{V}{d} = \dfrac{3}{300\times10^{-10}} = 10^{8}\,\dfrac{V}{m} = 10^{6}\,\dfrac{V}{cm}$

138.



When the reverse bias is greater than the V_z, it is breakdown condition. In breakdown region, ($V_i > V_z$) for a wide range of load; (R_L), the voltage across R_L remains constant though the current may change. Hence portion 'de' of the characteristics is relevant for the zener diode to operate as a voltage regulator.

139. During the positive half cycle of the input A.C. signal, diode D_1 is forward biased and D_2 is reverse biased. Hence in the output voltage signal, A and C are due to D_1. During negative half cycle of input A.C. signal, D_2 conducts. Hence output signals B and D are due to D_2.

140. Rectifier converts AC signal into pulsating DC signal. Filter circuit filters DC signal while regulator makes the DC value stable.

141. Given, R_L= 800 Ω, V_L = 0.8 V

$\therefore \quad I_C = \dfrac{V_L}{R_L} = 1\text{ mA} = 10^{-3}\text{ A}$

$r_i = 192\ \Omega$

Current amplification $= \alpha = \dfrac{I_C}{I_B} = 0.96$

$\therefore \quad I_B = \dfrac{10^{-3}}{0.96} = \dfrac{1}{960}$

Also, $A_V = \dfrac{V_L}{V_{in}} = \dfrac{V_L}{I_B r_i} = \dfrac{0.8}{192}\times 960 = 4$

$$A_P = \frac{I_C^2 R_L}{I_B^2 r_i} = \frac{\left(10^{-3}\right)^2\times800}{\left(\frac{1}{960}\right)^2\times192} = 3.84$$

142. $i = \dfrac{V_{net}}{R_{net}} = \dfrac{3.5-0.5}{100} = \dfrac{3}{100}\text{ A} = 30\text{ mA}$

143. The Boolean expression for the given combination is

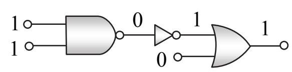
Y = (A + B).C

The truth table for the same is:

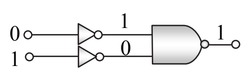
A	B	C	Y = (A+B).C
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
0	1	1	1
1	0	1	1
1	1	1	1

∴ A = 1, B = 0, C = 1.

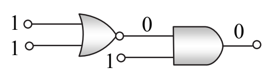
144. (i)



(ii)



(iii)



The outputs of (i), (ii) and (iii) are respectively 1, 1, 0.

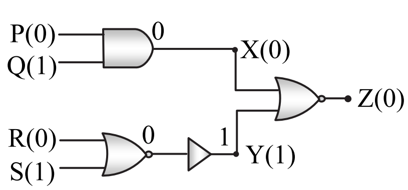
145.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

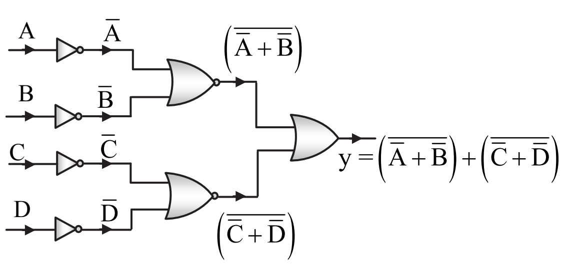
Y will be 1 when either of the inputs or both the inputs are 1.

146. $\left(\overline{A.B}\right)+\overline{C}=\left(\overline{A}+\overline{B}+\overline{C}\right)$

147.



148. $y=\left(\overline{\overline{A}+\overline{B}}\right)+\left(\overline{\overline{C}+\overline{D}}\right)$



For Option (A)

$y=\left(\overline{\overline{0}+\overline{0}}\right)+\left(\overline{\overline{1}+\overline{0}}\right)=0+0=0$

∴ Option (A) is incorrect.

For Option (B)

$y=\left(\overline{\overline{1}+\overline{0}}\right)+\left(\overline{\overline{1}+\overline{0}}\right)=0+0=0$

∴ Option (B) is incorrect.

For Option (C)

$y=\left(\overline{\overline{0}+\overline{1}}\right)+\left(\overline{\overline{0}+\overline{1}}\right)=0+0=0$

∴ Option (C) is incorrect.

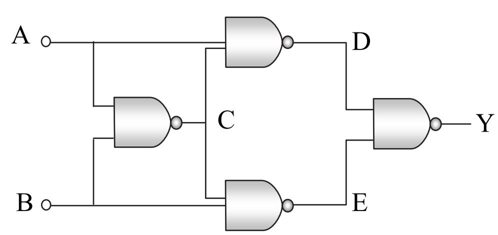
For Option (D)

$y=\left(\overline{\overline{0}+\overline{0}}\right)+\left(\overline{\overline{1}+\overline{1}}\right)=0+1=1$

Hence answer is option (D).

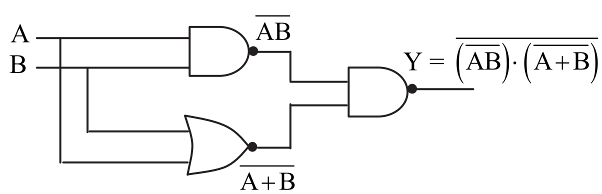
149. To get the output Y = 1 from the AND gate, both its inputs must be one. For this C = 1, and for the OR gate, either A or B or both must be = 1.

150.



A	B	C	D = $\overline{A.C}$	E = $\overline{C.B}$	Y
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0

151.



From Figure,

Output $Y=\overline{(\overline{A\cdot B})\cdot(\overline{A+B})}=\left(\overline{\overline{A\cdot B}}\right)+\left(\overline{\overline{A+B}}\right)$

$=(A\cdot B)+(A+B)$

A	B	A·B	A + B	Y
0	0	0	0	0
1	0	0	1	1
0	1	0	1	1
1	1	1	1	1

Evaluation Test

1. In semiconductor, the forbidden energy gap between valence band and conduction band is very small (almost equal to kT). Further, the valence band is completely filled and the conduction band is empty.

2. P.D. across series resistance,
$= 9V – 4V = 5V$
∴ Current through series resistance,
$i = \frac{4}{100} = 0.04$ A.
∴ Current through load resistance,
$i_L = \frac{V_L}{R_L} = \frac{4}{400} = 0.01$ A

3. $\alpha = \frac{\text{Change in collector current}}{\text{Change in emitter current}}$
∴ $\beta = \frac{\alpha}{1-\alpha} = \frac{0.94}{1 - 0.94} = 15.67$
∴ $\Delta I_C = \beta(\Delta I_B) = 15.67 \times 0.5 = 7.83$ mA

4. The energy of emission,
$E = h\nu = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{5780\times10^{-10}}$
$= 3.43 \times 10^{-19}$ J
$= \frac{3.43\times10^{-19}}{1.6\times10^{-19}} = 2.14$ eV
For, $\lambda = 5780$ Å, E = 2.14 eV
The condition for emission of electrons is,
$h\nu > E_g$.
But here, $h\nu < E_g$ [$E_g = 2.8$ eV]
∴ For emission of electrons, $\lambda < 5780$ Å is a must.

5. When a p-n junction diode is formed, n-side attains positive potential and p-side attains negative potential. When ends of p and n of a p-n junction are joined by a wire, there will be a steady conventional current from n-side to p-side through the wire and p-side to n-side through the junction.

6. $I = \frac{E-V}{R} = \frac{8-3}{60} = \frac{1}{12}$A
∴ $I_L = \frac{V_Z}{R_L} = \frac{3}{120} = \frac{1}{40}$A
∴ $I_Z = I - I_L = \frac{1}{12} - \frac{1}{40} = \frac{7}{120}$A

7. In a common emitter configuration, input impedence is given by
$\text{Impedence} = \left(\frac{\Delta v_{BE}}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$
The base current I_b is of order of few microampere. Hence, the input impedence of common emitter amplifier is low.
Therefore, assertion as well as reason are true statements but reason is not the correct explanation of assertion.

8. The base in a transistor is made thin because most of the holes coming from the emitter are able to diffuse through the base region to the collector retion. Hence, the assertion is true but reason is false.

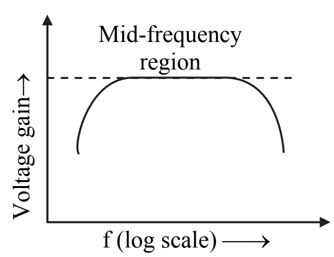
10. Voltage gain $= \frac{I_C\times R_C}{I_B\times R_B} = \frac{(2\times10^{-3})(4\times10^{3})}{(10\times10^{-6})(400)}$
$= 2000$

11. $n_e = \frac{n_i^2}{np} = \frac{(2\times10^{16})(2\times10^{16})}{(3.5\times10^{22})}$
$\approx 1.1 \times 10^{10}$ m^{-3}.

12. When A is V(0) or B is V(0) or both are 0, accordingly D_1 or D_2 or both are forward biased. Current flows via R, the potential at Y is 0. But when both A and B are at V(1), then D_1 and D_2 do not conduct current. So potential at Y is V(1). Y is 1 only when A and B are both 1.
Thus, this represents an AND gate.
⇒ Option (B) is correct.

13. For $0 < t < t_1$, Input = 0 ⇒ output = 1
For $t_1 < t < t_3$, Input = 1 ⇒ output = 0
For $t_3 < t < t_4$, Input = 0 ⇒ output = 1
Hence (B) is the correct option.

14. $P = \overline{A}$ and $Q = \overline{B}$
Now Y = 1 ⇒ both P and Q are 0
P = 0 ⇒ A = 1 amd Q = 0 ⇒ B = 1

15.


For a transistor in CE mode, the voltage gain vs frequency (log scale) looks as shown in the diagram.
As can be seen, the voltage gain is low at high and low frequencies and constant at mid frequencies.

16. Here, $R_i = 500\ \Omega$, $R_0 = 40 \times 10^3\ \Omega$, $\beta = 75$

Voltage gain = $\beta\left(\frac{R_0}{R_i}\right)$

$$= 75 \times \frac{40\times10^3\ \Omega}{500\ \Omega} = 6000$$

Power gain = Voltage gain × Current gain
$= 6000 \times 75 = 450000 \approx 4.5 \times 10^5$

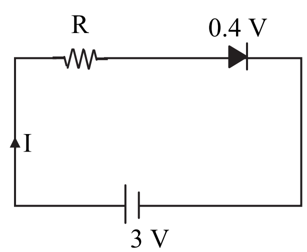
17. Since diode D_1 is reverse biased, therefore it will act like an open circuit.
Effective resistance of the circuit, $R = 5 + 3 = 8\ \Omega$.
Current in the circuit, $I = E/R = 10/8 = 1.25$ A.

18. Applying Kirchhoff's second law, we have
$I \times R + 0.7 = 4$

∴ $R = \frac{4-0.7}{I} = \frac{3.3}{2\times10^{-3}} = 1650\ \Omega$

Power dissipated across $R = I^2R$
$= \left(2\times10^{-3}\right)^2 \times 1650 = 6.6\times10^{-3}$ W

19.



The value of R should be such that the current in the circuit does not exceed 5 mA. By Ohm's law, we have
$I \times R + 0.4\ V = 3\ V$

∴ $5 \times 10^{-3} \times R = 2.6$

∴ $R = 520\ \Omega$

20. Given that, $\alpha = 0.96$ $I_E = 8$ mA,

$\alpha = \frac{I_C}{I_E}$

∴ $I_C = \alpha\ I_E = 0.96 \times 8 \approx 7.7$ mA.

The base current,
$I_B = I_E - I_C = 8 - 7.7 = 0.3$ mA

21. $I_E = \frac{ne}{t} = \frac{10^{10}\times1.6\times10^{-19}}{10^{-6}} = 1.6 \times 10^{-3}$ A

$= 1.6$ mA

$I_B = 3\%$ of $I_E = \frac{3\times1.6}{100} = 0.048$mA

The current transfer ratio, $\frac{I_C}{I_E} = \frac{1.552}{1.6} = 0.97$

22. A.C. current gain, $\beta = \frac{\alpha}{1-\alpha} = \frac{0.96}{1-0.96} = 24.$

Collector current,

$I_C = \frac{\text{Voltage drop across collector resistor}}{\text{Load resistance}}$

$= \frac{4V}{500\ \Omega} = 8\times10^{-3}$ A

Now, $\beta = \frac{I_C}{I_B}$

∴ Base current, $I_B = \frac{I_C}{\beta} = \frac{8\times10^{-3}\ A}{24} = 0.33$ mA.

23. $E = \frac{hc}{\lambda}$

∴ $\lambda = \frac{hc}{E}$

$= \frac{6.63\times10^{-34}\times3\times10^8}{0.74\times1.6\times10^{-19}}$

$= 16.798 \times 10^{-7}$

$= 1679.8 \times 10^{-9}$ m

≈ 1680 nm

Textbook Chapter No.

20 Communication Systems

Hints

Classical Thinking

1. A communication system is made up of three parts: transmitter, communication channel and receiver.
3. Quality of transmission is governed both by nature of signal and nature of communication channel/medium.
5. Sound produced by a fork is continuous. Therefore, it is a sort of analog signal.
8. A guided medium alone can provide point to point communication.
22. FM because modulation index $\propto$ B.W.
24. It mixes weak signals with carrier signals.
35. Ozone layer will absorb ultraviolet rays; reflect the infrared radiation and does not reflect back radiowaves.
40. $d = \sqrt{2hR} \Rightarrow d \propto h^{1/2}$
58. Both the assertion and the reason are true but reason is not correct explanation of assertion as UHF/VHF waves being of high frequency are not reflected by ionosphere.

Critical Thinking

1. All the three types of energy losses persist in transmission lines.
2. A transmitter is made up of message signal generator, modulator and antenna.
3. A communication link between a fixed base station and mobile units on a ship or aircraft works on 30 to 470 MHz.
12. Number of stations

$$= \frac{\text{B.W.}}{2\times\text{Highest modulating frequency}} = \frac{300{,}000}{2\times15000}$$

$= 10$

13. $\mu = \frac{A_m}{A_c} = \frac{15}{30} \times 100 = 50\%$
14. When $\mu > 1$, then carrier is said to be over modulated.
16. Here, $f_c = 1.5$ MHz $= 1500$ kHz, $f_m = 10$ kHz

$\therefore$ Low side band frequency $= f_c - f_m$

$= 1500$ kHz $- 10$kHz

$= 1490$ kHz

Upper side band frequency $= f_c + f_m$

$= 1500$ kHz $+ 10$kHz

$= 1510$ kHz

17. In space communication, the information can be passed from one place to another with the speed of light ($= 3\times10^8$ m/s). Hence time taken for a distance of 100 km $= \frac{100\times10^3}{3\times10^8} = 3.3\times10^{-4}$ s
18. MUF $= f_c \sec (I)$

$I = 74°$ for F-layer

$\therefore$ MUF $= 50 \times 10^6 \times 3.62 = 181$ MHz

20. $E = h\nu \Rightarrow E \propto \nu$
21. Sky wave propagation is suitable for radiowaves of frequency 3 MHz to 30 MHz.
23. A geosynchronous satellite is located at a height of about 36000 km from the surface of earth and its period of revolution around earth is 24 hours.
24. $d = \sqrt{2Rh} = \sqrt{2\times6400\times1000\times300} = \sqrt{3840\times10^6}$

$= 62 \times 10^3 = 62$ km

25. For ionosphere propagation, the critical frequency is given by $f_c = 9\sqrt{N_{max}}$

where, N_{max} is the maximum electron density in per m^3.

$\therefore$ $(N_{max})^{1/2} = \frac{f_c}{9} = \frac{9\sqrt{2}\times10^6}{9} = \sqrt{2} \times 10^6$

$\therefore$ $N_{max} = (\sqrt{2} \times 10^6)^2 = 2 \times 10^{12}/m^3$

27. The maximum distance of the line of sight is

$D_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

where, R is the radius of the earth

$= \sqrt{2\times64\times10^5\times50} + \sqrt{2\times64\times10^5\times32}$

$= 80 \times 10^2\sqrt{10} + 64 \times 10^2\sqrt{10}$

$= 144 \times 10^2\sqrt{10}$

$= 455 \times 10^2 = 45.5 \times 10^3$ m $= 45.5$ km

28. When an electromagnetic wave enters an ionised layer of earth's atmosphere, the motion of electron cloud produces space current which has a phase retardation of 90° with the sinusoidal electromagnetic wave. The electric field oscillations in electromagnetic wave also produces its own capacitive displacement current which leads the field by 90°. Thus, the space current lags behind the displacement current by a phase of 180°.

31. Optical source frequency,

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{800\times10^{-9}} = 3.75 \times 10^{14} \text{ Hz}$$

Bandwidth of channel (1% of above)
$= 3.75 \times 10^{12}$ Hz
Number of channels = (Total bandwidth of channel) / (Bandwidth needed per channel)
Number of channels for audio signal

$$= \frac{3.75\times10^{12}}{8\times10^3} \approx 4.7 \times 10^8$$

32. 1% of 10 GHz $= 10 \times 10^9 \times \frac{1}{100} = 10^8$ Hz

∴ Number of channels $= \frac{10^8}{5\times10^3} = 2 \times 10^4$

33. In AM modulation, the amplitude of the carrier signal varies in accordance with the information signal. AM signal is easily affected by external atmosphere and electrical disturbances. Thus it results in noisy reception. In FM modulation, amplitude of carrier wave is fixed while its frequency is changing. FM reception is quite immune to noise as compared to AM reception and gives better quality transmission. It is preferred for transmission of music.
Demodulation is the process in which the original modulating voltage is recovered from the modulated wave.

34. The modulation index determines the strength and quality of the transmitted signals.
If the modulation index is small the amount of variation in the carrier amplitude will be small. Consequently, the audio signal being transmitted will not be strong.
High modulation index offers greater degree of modulation hence the audio signal reception is clear and strong.

Competitive Thinking

5. VHF (Very High Frequency) band having frequency range 30 MHz to 300 MHz is typically used for TV and radar transmission.

6. A maximum frequency deviation of 75 kHz is permitted for commercial FM broadcast stations in the 88 to 108 MHz VHF band.

7. Optical fibres are not subjected to electromagnetic interference from outside.

8. Optical communication using fibres is performed in frequency range of 1 THz to 1000 THz and an optical fibre can offer a transmission band width in excess of 100 GHz.

9. A very small part of light energy is lost from an optical fibre due to absorption or due to light leaving the fibre as a result of scattering of light sideways by impurities in the glass fibre.

10. Bandwidth is equal to twice the frequency of modulating signals
∴ Bandwidth $= 2f_m = 2 \times 4000$ Hz $= 8$ kHz

12. Modulation index μ is kept ≤ 1 to avoid distortion

13. One carrier f_c and two side band frequencies $f_c \pm f_m$

14. In amplitude modulation, the amplitude of carrier wave is varied according to information signal.

16. For amplitude modulation,
Bandwidth $= f_{USB} - f_{LSB} = (f_c+f_m)-(f_c-f_m) = 2f_m$
∴ Bandwidth is equal to twice the frequency of modulating signal frequency.

18. Amplitude modulated signal contains frequencies $\omega_m + \omega_c$, ω_c and $\omega_c - \omega_m$.

19. $V_C = \mu V_0$
∴ $12 = \frac{75}{100} V_0$
∴ $V_0 = 16$ V

20. Frequencies of resultant signal are
$F_c + f_m$, f_c and $f_c - f_m$
i.e., (2000 + 5) kHz, 2000 kHz, (2000 – 5) kHz,
2005 kHz, 2000 kHz, 1995 kHz

21. Modulation index, $\mu = \frac{A_m}{A_c}$

$$A_m = \mu A_c = \left(\frac{50}{100}\right) \times 12 \quad \Rightarrow A_m = 6 \text{ V}$$

22. $\mu = \frac{A_m}{A_c} = \frac{15}{60} \times 100 = 25\%$

23. $\mu = \frac{A_m}{A_c} = \frac{20}{30} = 0.67$

24. $m_f = \frac{\delta}{f_m} = \frac{\text{Frequency variation}}{\text{Modulating frequency}} = \frac{10 \times 10^3}{2 \times 10^3} = 5$

25. Modulation Index $= \frac{A_m}{A_c} = \frac{\left(\frac{A_{max} - A_{min}}{2}\right)}{\left(\frac{A_{max} + A_{min}}{2}\right)}$

$= \frac{15-10}{15+10} \times 100\% = 20\%$

26. A general FM expression has a form
$e_{FM} = e_0 \sin(\omega_c t + \mu \sin \omega_m t)$
Thus, on comparison, $\omega_m = 10^3$

$\therefore$ $f = \frac{\omega_m}{2\pi} = \frac{10^3}{2\pi} = 159$ Hz

27. The distance of coverage of a transmitting antenna is $d = \sqrt{2Rh}$

$\therefore$ $h = \frac{d^2}{2R} = \frac{(12.8 \times 10^3 \text{ m})^2}{2 \times 6400 \times 10^3 \text{ m}} = 12.8$ m

28. Area covered, $A = \pi d^2 = \pi(2Rh)$
Given: h = 105 m
$\therefore$ $A = 3.142 \times 2 \times 6.4 \times 10^6 \times 105$
$= 4.223 \times 10^9 \text{ m}^2$
$= 4.223 \times 10^3 \text{ km}^2 = 4223 \text{ km}^2$

29. $d = \sqrt{2hR} = \sqrt{2 \times 500 \times 6.4 \times 10^6 \text{ m}}$
= 80,000 m = 80 km

30. Height of T.V. tower = h_T
Range $\propto \sqrt{h_T}$
and Area $\propto$ (Range)^2
so Area $\propto h_T$

31. Carrier frequency > audio frequency

32. Modem acts as the modulator as well as a demodulator. Modem acts as a modulator in the transmitting mode and it acts as a demodulator in the receiving mode.

38. The critical frequency of a sky wave for reflection from a layer of atmosphere is given by $f_c = 9(N_{max})^{1/2}$
$\therefore$ $10 \times 10^6 = 9(N_{max})^{1/2}$

$\therefore$ $N_{max} = \left(\frac{10 \times 10^6}{9}\right)^2 \approx 1.2 \times 10^{12} \text{ m}^{-3}$

39. $n_{eff} = n_0\sqrt{1 - \left(\frac{80.5 N}{\nu^2}\right)}$

$= 1\sqrt{1 - \frac{80.5 \times (400 \times 10^6)}{(55 \times 10^6)^2}} \approx 1$

Now, $n_{eff} = \frac{\sin i}{\sin r}$

$\therefore$ sin r = sin i $\Rightarrow$ r = i = 45°

40. $f = \frac{1}{2\pi RCm}$

$= \frac{1}{2 \times 3.14 \times 100 \times 10^3 \times 250 \times 10^{-12} \times 0.6}$

$= 1.0615 \times 10^{-3} \times 10^7 = 1.0615 \times 10^4$

$\therefore$ $f \approx 10.62$ k Hz

41. No. of channels $= \frac{\text{carrier frequency} \times 10\%}{\text{channel bandwidth}}$

$= \frac{10 \times 10^9}{5 \times 10^3} \times \frac{10}{100} = 2 \times 10^5$

42. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$\therefore$ $40 \times 1000 = \sqrt{2 \times 6.4 \times 10^6 \times h} + \sqrt{2 \times 6.4 \times 10^6 \times 45}$

$\therefore$ $40 \times 10^3 = \sqrt{2 \times 6.4 \times 10^6 \times h} + 24 \times 10^3$

$\therefore$ $h = \frac{\left(16 \times 10^3\right)^2}{2 \times 6.4 \times 10^6} = 20$ m

43. Optical source frequency $v = \frac{c}{\lambda}$

$\therefore$ $v = \frac{3 \times 10^8 \text{ ms}^{-1}}{1200 \times 10^{-9} \text{ m}} = 2.5 \times 10^{14}$ Hz

$\therefore$ Bandwidth of channel (2% of the source frequency) $= 5 \times 10^{12}$ Hz
Now, Number of channels

$= \frac{\text{Total bandwidth}}{\text{Bandwidth needed per channel}}$

$= \frac{5 \times 10^{12} \text{ Hz}}{5 \times 10^6 \text{ Hz}} = 10^6 = 1$ million

44. The ionosphere reflects the sky waves which are actually the radio waves (frequency range from 2 MHz to 30 MHz) back to earth during their propagation through atmosphere. The refractive index of ionosphere is less than its free space value. Thus, it behaves as a rarer medium and turns the wave away from wave normal during the entry of wave into ionosphere. The refraction of the beam continues until the critical angle is reached, beyond which reflection takes place. Beyond a very high value of frequency called critical

frequency, the reflection cannot take place. Beyond critical frequency, waves cross the ionosphere and never return back to earth as for these values of frequency, the refractive index of ionosphere becomes very high.

45. The expression for modulated carrier signal C_m (t) is –

$$C_m(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2} \cos (\omega_c - \omega_m) t - \frac{\mu A_c}{2} \cos (\omega_c + \omega_m) t$$

Where, $\mu = \frac{A_m}{A_c}$ is modulation index.

Hence, the three frequencies are ω_c, $\omega_c - \omega_m$, $\omega_c + \omega_m$.
Thus, one of the angular frequency of the AM wave is equal to the angular frequency of carrier wave.

46. It is true that the radio waves are polarised electromagnetic waves. The antenna of portable AM radio is sensitive to only magnetic components of electromagnetic waves. On account of this, the set should be placed horizontal and in proper situation so that the signals are received properly from radio station.

Evaluation Test

1. The critical frequency for sky wave propagation,
$f_c = 9\sqrt{N_{max}} = 9(10^{10})^{1/2}$
$= 9 \times 10^5$ Hz = 900 kHz

2. For sky wave propagation: the critical frequency
$f_c = 9(N_{max})^{\frac{1}{2}} \Rightarrow N_{max} = \frac{f_c^2}{81}$
$= \frac{(5\times10^6)^2}{81} = 0.3 \times 10^{12}$
$\approx 3 \times 10^{11}$ per cubic metre

3. $d = \sqrt{2hR}$
$d' = \sqrt{2h'R}$ but $d' = 2$ d ….[Given]
∴ $\sqrt{2h'R} = 2\sqrt{2hR}$
∴ $h' = 2$ h $= 4 \times 150 = 600$ m
Increase in height of tower
600 m – 150 m = 450 m

4. In space communication, the speed of information is equal to speed of light. Hence time taken for a distance of 60 km is
$= \frac{60\times10^3 \text{ m}}{3\times10^8 \text{ ms}^{-1}} = 2 \times 10^{-4}$ s

5. AM avoids receiver complexity.

6. Assertion is true but reason is false as UHF/VHF waves being of high frequency are not reflected by ionosphere.

8. Assertion is true but reason is false as a dipole antenna is omnidirectional.

9. The modulation index determines the strength and quality of the transmitted signals.
If the modulation index is small, the amount of variation in the carrier amplitude will be small. Consequently the audio signal being transmitted will not be strong.
High modulation index offers greater degree of modulation hence the audio signal reception is clear and strong.

10. In an amplitude modulated wave,
$\nu_{\text{carrier wave}} >> \nu_{\text{audio-wave}}$
⇒ For a 400 cycle/s audio wave, among the given frequencies, 40000 cycle/second carrier frequency will be appropriate.

11. Modulation index,
$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{11-3}{11+3} = \frac{8}{14} = 57.14$ %

12. $f_{SB} = f_c \pm f_m = 3000 \pm 0.5 = 3000.5$ kHz and 2999.5 kHz

13. Frequency of carrier, f_c = 1 MHz = 1000 kHz
Frequency of signal, f_s = 4 kHz
Modulation factor, m_a = 50% = 0.5
Amplitude of carrier, A_c = 100 V
The lower and upper side band frequencies are $f_c - f_s$ and $f_c + f_s$ respectively, hence they are 996 kHz and 1004 kHz
Hence option (B) is correct.

14. We know that,
height of T.V. tower = 200 m
Distance through which signal can be received
(d) $= \sqrt{2hR}$
$= \sqrt{2\times200\times6.4\times10^6}$
$\approx 50 \times 10^3$

Population density

$$= \frac{\begin{pmatrix}\text{Total population covered} \\ \text{by T.V. tower}\end{pmatrix}}{\text{Area}}$$

∴ Total population covered by T.V. tower

$$= \text{Population density} \times \pi d^2$$

$$= \frac{10^3}{(10^3)^2} \times 3.14 \times 2500 \times 10^6$$

$$= 78.50 \text{ lakh}$$

15. Number of stations

$$= \frac{\text{B.W.}}{2 \times \text{Highest modulating frequency}}$$

$$= \frac{200000}{2 \times 10000} = 10$$

MHT-CET 2019

6th May 2019 (Afternoon)

Hints

1. Mean (t) = $\dfrac{30+32+35+31}{4} = 32$

$\therefore$ Mean error (Δt) = $\dfrac{|\Delta t_1| + |\Delta t_2| + |\Delta t_3| + |\Delta t_4|}{4}$

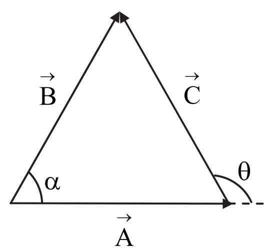
$= \dfrac{2+0+3+1}{4} = \dfrac{6}{4} = 1.5$

Hence rounding off,

$\Delta t = \pm 2$ s

$\therefore$ $t \pm \Delta t = 32 \pm 2$ s

2.



$\vec{A} + \vec{B} = \vec{C}$

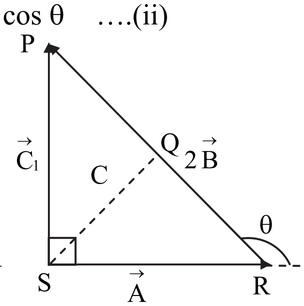
$C^2 = A^2 + B^2 + 2AB \cos\theta$ ….(i)

and $\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$

When B is doubled, resultant is perpendicular to $\vec{A}$

$\therefore$ $C_1^2 = A^2 + 4B^2 + 4AB\cos\theta$ ….(ii)

From right angled triangle PSR



$4B^2 = C_1^2 + A^2$

$C_1^2 = 4B^2 - A^2$

Substituting in (ii) and solving,

$A^2 + 2AB\cos\theta = 0$ ….(iii)

Substituting (iii) in (i),

C = B

3. $\bar{d}_1 = \bar{a} + \bar{b}$

In magnitude,

$d_1^2 = a^2 + b^2 + 2ab\cos\theta$ ….(i)

$\bar{d}_2 = \bar{b} + \bar{a}$

In magnitude,

$d_2^2 = b^2 + a^2 + 2ba\cos(180 - \theta)$

….(ii)

$d_2^2 = b^2 + a^2 - 2ba\cos\theta$

Adding equation (i) and (ii),

$d_1^2 + d_2^2 = a^2 + b^2 + 2ab\cos\theta + b^2 + a^2 - 2ba\cos\theta$

$d_1^2 + d_2^2 = 2(a^2 + b^2)$

$\Rightarrow a^2 + b^2 = \dfrac{d_1^2 + d_2^2}{2}$

5. $\dfrac{1}{2}mv^2 = F \times s$

For upward force, velocity = v

$\therefore$ $F = \dfrac{mv^2}{2s}$

$= \dfrac{\rho A s v^2}{2s}$ ….($m = \rho V = \rho As$)

$= \dfrac{1}{2}\rho v^2 A$

6. $\mu = \dfrac{\sin i}{\sin r}$

But i = 2r

$\therefore$ $r = \dfrac{i}{2}$

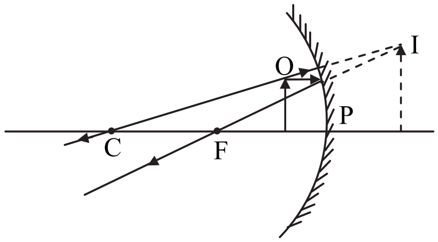
$\mu = \dfrac{\sin i}{\sin\left(\dfrac{i}{2}\right)}$

$\mu = \dfrac{2\sin\left(\dfrac{i}{2}\right)\cos\left(\dfrac{i}{2}\right)}{\sin\left(\dfrac{i}{2}\right)}$

$\therefore$ $\mu = 2\cos\dfrac{i}{2}$

$\Rightarrow i = 2\cos^{-1}\left(\dfrac{\mu}{2}\right)$

7.



For an object within the focal length, image formed is virtual, erect and magnified.

8. $\frac{F}{l} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

Here, r = b and $I_1 = I_2 = I$

∴ $\frac{F}{l} = \frac{\mu_0 2I^2}{4\pi b}$

9. At the magnetic north pole of the earth, $B_H = 0$

Angle of dip, $\delta = \tan^{-1}\left(\frac{B_v}{B_H}\right)$

∴ δ is maximum when B_H is zero.

10. M = IA

$M = I(\pi r^2)$

$M \times 4\pi = I\,\pi r^2 \times 4\pi$

$4\pi M = I\,(2\pi r)^2$

∴ $(2\pi r)^2 = \frac{4\pi M}{I}$

∴ $2\pi r = \sqrt{\frac{4\pi M}{I}}$

∴ length = $\sqrt{\frac{4\pi M}{I}}$

11. Degree moved by hour hand,

for 1 revolution = 360°

for 1 hour = $\frac{360°}{12}$ = 30°

for 1 min = $\frac{30}{60}$ = 0.5°

∴ for 20 mins = 20 × 0.5° = 10°

Hence, at 12.20 pm

Angular seperation = 120° – 10° = 110°

14. $g_h = g\left(\frac{R}{R+\frac{R}{2}}\right)^2 = g\left(\frac{2R}{3R}\right)^2$

$g_h = \frac{4g}{9}$ ….(i)

$(v_c)_h = \sqrt{g_h R_h}$

$= \sqrt{\frac{4g}{9}\times\frac{R}{2}}$

$= \frac{1}{\sqrt{3}}\sqrt{2gR}$ ….($v_e = \sqrt{2gR}$)

$(v_c)_h = \frac{1}{\sqrt{3}}v_e$

15. According to law of conservation of angular momentum,

$I_1\omega_1 = I_2\omega_2$

∴ $I_1\omega_1 = \left(I_1 - \frac{25I_1}{100}\right)\omega_2$

∴ $I_1\omega_1 = 0.75I_1\omega_2$

∴ $\omega_2 = \frac{I_1\omega_1}{0.75I_1}$

$= \frac{1.5\pi}{0.75}$ ….(Given: $\omega_1 = 1.5\,\pi$)

$= 2\pi$

But $f = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi}$ = 1 rps

In rpm; f = 60 rpm

16. Work done; $W = \tau\theta$

∴ $W = I\alpha\theta$ ….($\because \tau = I\alpha$)

∴ $I = \frac{W}{\alpha\theta}$

∴ $I = \frac{W}{2\pi^2\left(n_2^2 - n_1^2\right)}$ ….$\left(\begin{array}{l}\because \omega_f^2 = \omega_i^2 + 2\alpha\theta \\ \therefore \alpha\theta = 2\pi^2\left(n_2^2 - n_1^2\right)\end{array}\right)$

17. According to the given condition,

$\frac{MR^2}{2} = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$

∴ $\frac{R^2}{2} - \frac{R^2}{4} = \frac{L^2}{12}$

∴ $\frac{R^2}{4} = \frac{L^2}{12}$

∴ $L^2 = \frac{12R^2}{4}$

∴ $L^2 = 3R^2$

∴ $L = \sqrt{3}R$

18. When the spring gets compressed by length L, K.E. lost by mass m = P.E. stored in the compressed spring

$\frac{1}{2}mv_{max}^2 = \frac{1}{2}kx^2$

$\therefore \quad v_{max} = \sqrt{\frac{k}{m}}x$

Maximum momentum of the block,

$P_{max} = mv_{max} = \sqrt{mk}\,x$

19. $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\therefore \quad \frac{n}{n'} = \sqrt{\frac{k}{m}\times\frac{m'}{k'}} = \sqrt{\frac{k}{m}\times\frac{2m}{2k}} = 1$

or $n' = n$

21. $P_1 = \frac{1}{2}kx_1^2$

$\Rightarrow x_1^2 = \frac{2P_1}{k}$

$P_2 = \frac{1}{2}kx_2^2$

$\Rightarrow x_2^2 = \frac{2P_2}{k}$

$P = \frac{1}{2}k(x_1 + x_2)^2$

$= \frac{1}{2}k(x_1^2 + x_2^2 + 2x_1x_2)$

$= \frac{1}{2}k\left(\frac{2P_1}{k} + \frac{2P_2}{k} + 2\sqrt{\frac{2P_1}{k}\cdot\frac{2P_2}{k}}\right)$

$P = \frac{1}{2}k\times\frac{2}{k}\left(P_1 + P_2 + 2\sqrt{P_1P_2}\right)$

$P = P_1 + P_2 + 2\sqrt{P_1P_2}$

22. Bending of a beam $(\delta) = \frac{Wl^3}{4Ybd^3}$

$\therefore \quad \delta \propto \frac{1}{Y}$

23. $K = \frac{PV}{\Delta V}$

$\therefore \quad \Delta V = \frac{PV}{K}$

But $K = \frac{1}{\sigma}$

$\therefore \quad \Delta V = \sigma PV$

24. $\Delta E = 2(T \times \Delta A)$

$= 2T\left(4\pi r_2^2 - 4\pi r_1^2\right)$

$= 2 \times 0.035 \times 4 \times \frac{22}{7} \times\left[\left(6\times10^{-2}\right)^2 - \left(4\times10^{-2}\right)^2\right]$

$= 2 \times 0.035 \times 4 \times \frac{22}{7} \times (36 - 16) \times 10^{-4}$

$= 1.76 \times 10^{-3}$ J

25. $T = \frac{F}{2l} = \frac{mg}{2l}$

$\therefore \quad m = \frac{2Tl}{g}$

26. Comparing equation $y = 2\sin 2\pi\left(\frac{t}{0.01} - \frac{x}{50}\right)$

with $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

Here, A = 2 cm

$n = \frac{1}{T} = \frac{1}{0.01} = 100\,\text{Hz}$

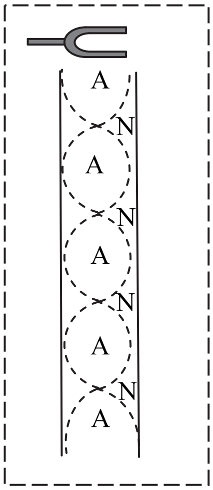
$\lambda = 50$ cm

$v = n\lambda = 100 \times 50 = 5000$ cm/s

Hence, option (C) is incorrect.

27. For stationary wave, the resultant particle velocity at all points is zero.

28. For an organ pipe open at both ends.



For third overtone, the pipe has 4 nodes and 5 antinodes

29. $e = \dfrac{l_2 - 3l_1}{2}$

$= \dfrac{74.1 - 3\times 24.1}{2}$

$e = 0.9$

But $e = 0.3d$

$\therefore \quad d = \dfrac{e}{0.3} = \dfrac{0.9}{0.3} = 3$ cm

30. The relation between α and η is given by,

$\alpha = \dfrac{1-\eta}{\eta}$

Only condition (B) satisfies the above equation.

31. Ratio of rate of emission $= \left(\dfrac{T_1}{T_2}\right)^4$

$= \left(\dfrac{367+273}{207+273}\right)^4$

$= \left(\dfrac{640}{480}\right)^4$

$= \left(\dfrac{4}{3}\right)^4$

$= 3.16 : 1$

33. For n^{th} bright band,

$X_n = n\dfrac{\lambda D}{d}$ ….(i)

For n^{th} dark band,

$x_n = (2n - 1)\dfrac{\lambda D}{2d}$

$\therefore \quad x_{n+1} = [2(n+1) - 1]\dfrac{\lambda D}{2d}$

$= (2n + 2 - 1)\dfrac{\lambda D}{2d}$

$= (2n + 1)\dfrac{\lambda D}{2d}$

Since, dark fringe is on the other side of bright fringe.

$\therefore \quad x_{n+1} + x_n = (2n + 1)\dfrac{\lambda D}{2d} + \dfrac{n\lambda D}{d}$

$= \dfrac{\lambda D}{2d}(2n+1+2n)$

$= (4n + 1)\dfrac{\lambda D}{2d}$

34. For any point in interference pattern,

$I = I_{max}\cos^2\dfrac{\phi}{2}$

$\therefore \quad \dfrac{I_{max}}{4} = I_{max}\cos^2\dfrac{\phi}{2}$

$\therefore \quad \cos^2\dfrac{\phi}{2} = \dfrac{1}{4}$

$\therefore \quad \cos\dfrac{\phi}{2} = \dfrac{1}{2}$

$\therefore \quad \dfrac{\phi}{2} = 60^\circ = \dfrac{\pi}{3}$

$\therefore \quad \phi = \dfrac{2\pi}{3}$

We know that,

$\phi = \left(\dfrac{2\pi}{\lambda}\right)\Delta x$

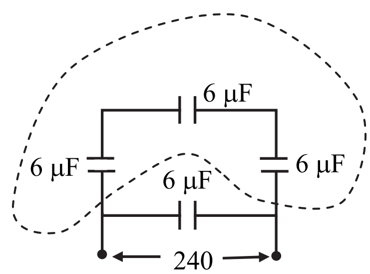
and $\Delta x = d\sin\theta$

$\therefore \quad \dfrac{2\pi}{3} = \dfrac{2\pi}{\lambda}(d\sin\theta)$

$\therefore \quad \dfrac{\lambda}{3d} = \sin\theta$

$\therefore \quad \theta = \sin^{-1}\left(\dfrac{\lambda}{3d}\right)$

35.

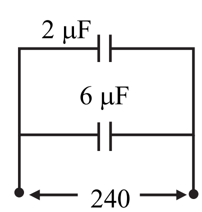


Here, 6μF, 6μF and 6μF are in series.

$\therefore \quad \dfrac{1}{C_s} = \dfrac{1}{6} + \dfrac{1}{6} + \dfrac{1}{6}$

$\therefore \quad C_s = 2\ \mu F$

The circuit can be drawn as,



Here, 2 μF and 6 μF are in parallel,

$\therefore \quad C_p = 2 + 6 = 8\ \mu F$

36. $U = \frac{1}{2}C\left(V_2^2 - V_1^2\right)$

$= \frac{1}{2} \times 15 \times 10^{-6} \times \left(25^2 - 15^2\right)$

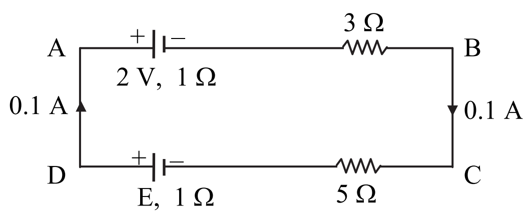
$= \frac{15 \times 10^{-6} \times (625 - 225)}{2}$

$= \frac{15 \times 10^{-6} \times 400}{2}$

$= 15 \times 10^{-6} \times 200$

$= 3 \times 10^{-3}$ J

37.



Applying KVL in loop ABCDA,

$-2 - 0.1(1) - 0.1\ (3) - 0.1\ (5) - 0.1\ (1) + E = 0$

$\therefore$ $-2 - 1 + E = 0$

$\therefore$ E = 3 V

41. Time period of revolution is given by,

$T = \frac{2\pi m}{qB}$

Hence, time period is independent of velocity of the particle.

42. Given: $\chi = 3 \times 10^{-4}$

$H = 4 \times 10^4$ A/m

$\therefore$ $M_z = \chi H$

$= 3 \times 10^{-4} \times 4 \times 10^4$

$= 12$ A/m

43. $E = iX_L$

$\therefore$ $i = \frac{E}{X_L} = \frac{E}{2\pi fL}$

44. $W_o = \frac{hc}{\lambda_o}$

Case (i) $\lambda_1 = \frac{hc}{W_1}$

Case (ii) $\lambda_2 = \frac{hc}{W_2}$

Dividing equation (i) by (ii),

$\therefore$ $\frac{\lambda_1}{\lambda_2} = \frac{W_2}{W_1} = \frac{6000 \times 10^{-10}}{4000 \times 10^{-10}} = \frac{3}{2}$

45. $\frac{1}{2}mv_{max}^2 = eV$

$\therefore$ $v_{max} = \sqrt{\frac{2eV}{m}}$

46. $R = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$ ....(i)

$V = \frac{e^2}{2\varepsilon_0 hn}$ ....(ii)

Multiply equation (i) and (ii),

$RV = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2} \times \frac{e^2}{2\varepsilon_0 hn}$

$RV = \frac{hn}{2\pi m}$

$\therefore$ $n \propto RV$ ....($\because$ h, π and m are constant)

47. ${}^{238}_{92}U \rightarrow {}^{230}_{90}Th + 2\left({}^{4}_{2}He\right) + 2\left({}^{0}_{-1}e\right)$

49.

A	B	C	A+B	Y = (A+B)·C
1	0	1	1	1
0	1	0	1	0
1	0	0	1	0
1	1	0	1	0