Microeconomic Theory

The Course:

- This is the first rigorous course in microeconomic theory
- This is a course on economic methodology.
- The main goal is to teach analytical tools that will be useful in other economic and business courses

Microeconomic Theory

Microeconomics analyses the behavior of individual decision makers such as consumers and firms.

Three key elements:

1. Household choices (consumption, labor supply)
2. Firm choices (production)
3. Market interaction $\longrightarrow$ determines prices/quantities

Everyday Economics

Economics provides a universal framework

- Applies to different countries
- Applies to different goods

Examples:

- Should you come to class or stay in bed?
- What should you eat for breakfast?
- Should you commit a crime?

Economics:

- Maximizing behavior

Example: Your trip to class

How do you get here?

- Car - cost of car, gas, parking.
- Bus – cost of fare, time cost.

What if gas prices rise?

- I don’t care about prices: full tank
- Take bus instead.

Economics:

- Income effect.
- Substitution effect.

Example: How do you finance college?

How raise money?

- Job – graduate without debt.
- Borrow – higher wages later, perform better.

What if fees rise?

- Start job – have to borrow less
- Quit job – take econ classes instead of art

<u>Economics</u>:

- Labor supply decision.
- Engel curves

Example: At Home

Puzzle:

- In the winter, why are houses in LA colder than those in Chicago?

Heating decision:

- Chicago – insulate house, buy giant heater.
- Los Angeles – small space heater.

Economics:

- Technology choice.
- Fixed vs. marginal costs.

Microeconomic Models

A **model** is a simplification of the real world.

- Highlights key aspects of problem
- Use different simplifications for different problems.

Example: consumer’s choose between

- Two consumption goods
- Consumption and leisure
- Consumption in two time periods.

Aspects of Models

Qualitative vs. Quantitative

- Qualitative – isolates key effects
- Quantitative – estimate size of effects

Positive vs. Normative

- Positive – make predictions
- Normative – evaluate outcomes, make predictions.

How to evaluate a model?

- Test assumptions – are premises reasonable?
- Test predictions – is model accurate?

Ingredients in Economic Models

1. Agents have well specified objectives

- Consumers, Firms

2. Agents face constraints

- Money, technology, time.

3. Equilibrium

- Agents maximize given behavior of others.

Chapter 2

THE MATHEMATICS OF OPTIMIZATION

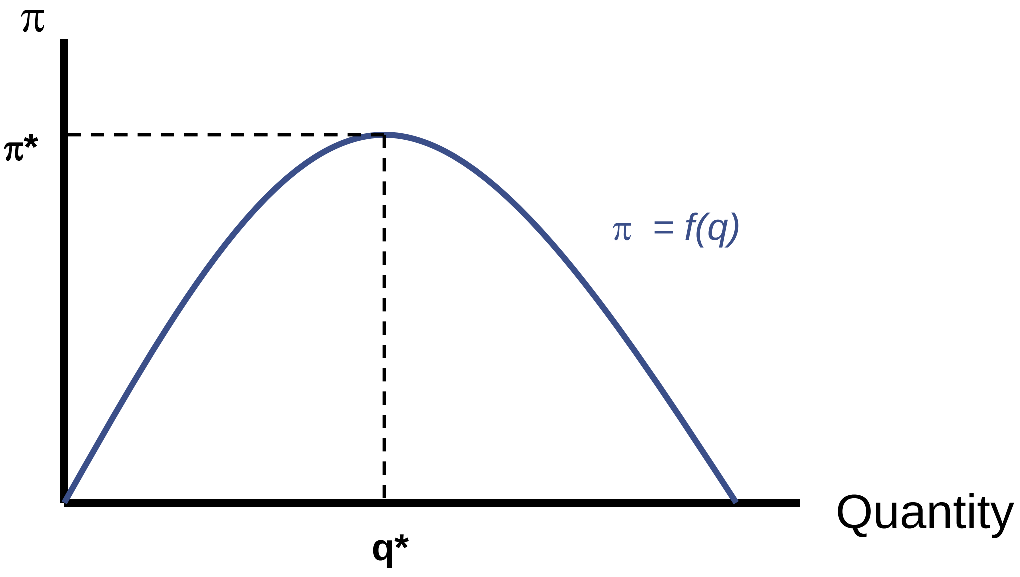
The Mathematics of Optimization

- Why do we need to know the mathematics of optimization?
- Consumers attempt to maximize their welfare/utility when making decisions.
- Firms attempt to maximizing their profit when choosing inputs and outputs.

Maximization of a Function of One Variable

- The manager of a firm wishes to maximize profits:

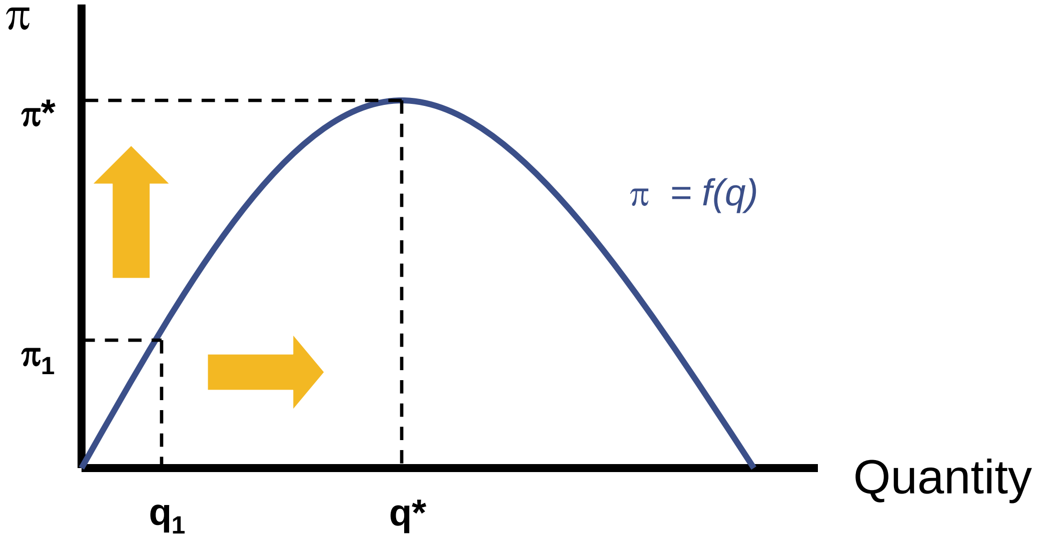
$$\pi = f(q)$$



Maximum profits of **π*** occur at q*

Maximization of a Function of One Variable

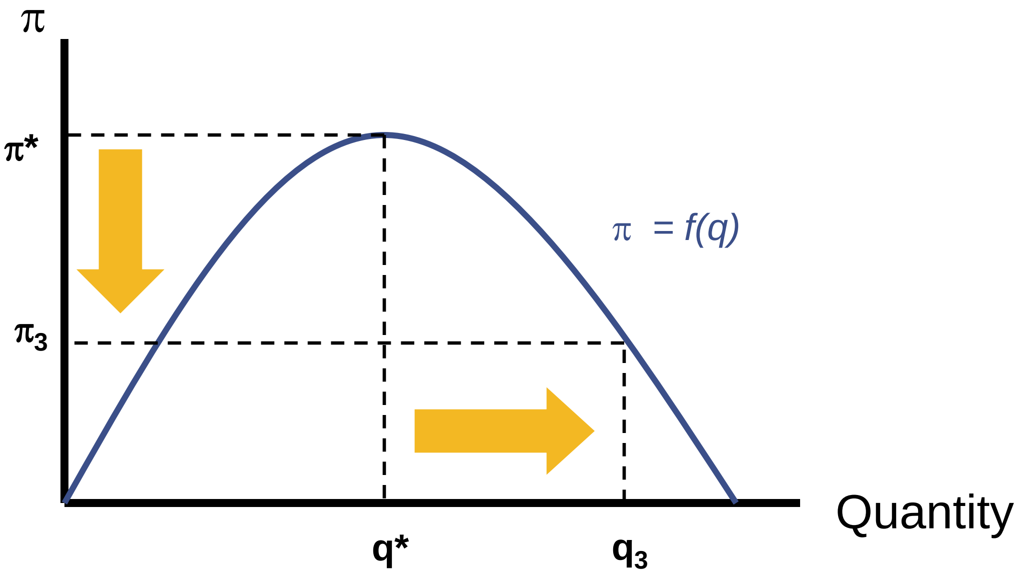
- If the manager produces less than q*, profits can be increased by increasing q:
  - A change from *q*1 to q* leads to a rise in π



$$\frac{\Delta \pi}{\Delta q} > 0$$

Maximization of a Function of One Variable

- If output is increased beyond q*, profit will decline
  - an increase from q^* to q_3 leads to a drop in π



$$\frac{\Delta\pi}{\Delta q} < 0$$

Derivatives

- The derivative of π = *f(q)* is the limit of Δπ/Δ*q* for very small changes in *q*

$$\frac{d\pi}{dq} = \frac{df}{dq} = \lim_{h \to 0} \frac{f(q+h) - f(q)}{h}$$

- The value of this ratio depends on the value of *q*

Value of a Derivative at a Point

- The evaluation of the derivative at the point $q = q_1$ can be denoted

$$\left.\frac{d\pi}{dq}\right|_{q=q_1}$$

- In our previous example,

$$\boxed{\left.\frac{d\pi}{dq}\right|_{q=q_1} > 0} \qquad \boxed{\left.\frac{d\pi}{dq}\right|_{q=q_3} < 0} \qquad \boxed{\left.\frac{d\pi}{dq}\right|_{q=q^*} = 0}$$

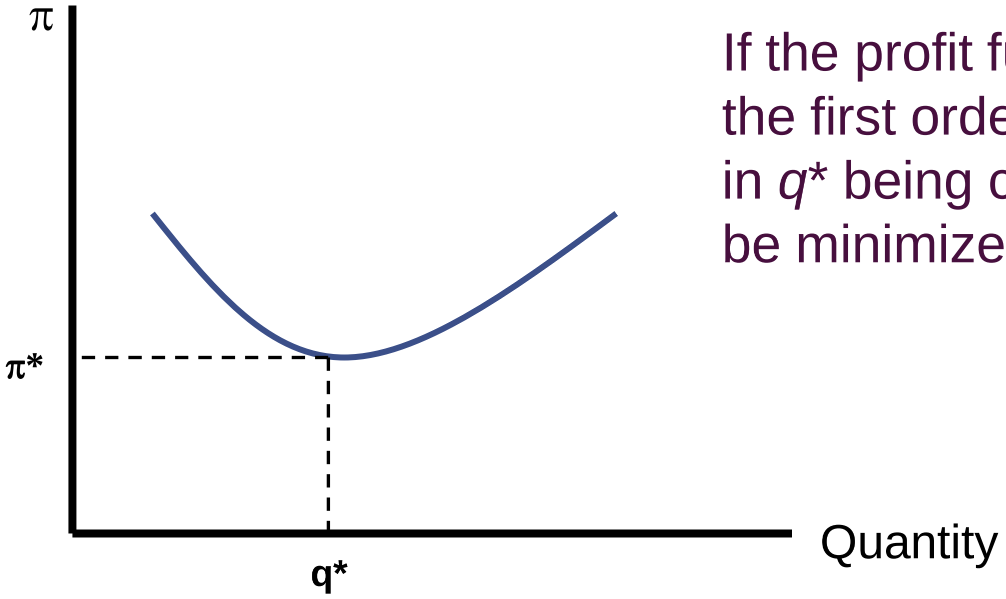
First Order Condition

- For a function of one variable to attain its maximum value at some point, the derivative at that point must be zero

$$\left.\frac{df}{dq}\right|_{q=q*} = 0$$

Second Order Conditions

- The first order condition ($d\pi/dq$) is a necessary condition for a maximum, but it is not a sufficient condition



If the profit function was u-shaped, the first order condition would result in q^* being chosen and π would be minimized

Second Order Condition

- The second order condition to represent a maximum is

$$\left.\frac{d^2\pi}{dq^2}\right|_{q=q*} = f''(q)\Big|_{q=q*} < 0$$

- The second order condition to represent a minimum is

$$\left.\frac{d^2\pi}{dq^2}\right|_{q=q*} = f''(q)\Big|_{q=q*} > 0$$

Functions of Several Variables

- Most goals of economic agents depend on several variables

- In this case we need to find the maximum and minimum of a function of several variables:

$$y = f(x_1, x_2, \ldots, x_n)$$

Partial Derivatives

- The partial derivative of the function *f* with respect to x_1 measures how *f* changes if we change x_1 by a small amount and we keep all the other variables constant.
- The partial derivative of *y* with respect to x_1 is denoted by

$$\frac{\partial y}{\partial x_1} \text{ or } \frac{\partial f}{\partial x_1} \text{ or } f_{x_1} \text{ or } f_1$$

Partial Derivatives

- A more formal definition of the partial derivative is

$$\frac{\partial f}{\partial x_1} = \lim_{h \to 0} \frac{f(x_1 + h, \overline{x}_2, \ldots, \overline{x}_n) - f(x_1, \overline{x}_2, \ldots, \overline{x}_n)}{h}$$

Second-Order Partial Derivatives

- The partial derivative of a partial derivative is called a <u>second-order partial derivative</u>

$$\frac{\partial(\partial f / \partial x_i)}{\partial x_j} = \frac{\partial^2 f}{\partial x_i \partial x_j} = f_{ij}$$

Young’s Theorem

- Under general conditions, the order in which partial differentiation is conducted to evaluate second-order partial derivatives does not matter

$$f_{ij} = f_{ji}$$

Total Differential

- Suppose that $y = f(x_1,x_2,\ldots,x_n)$
- We want to know by how much *f* changes if we change all the variables by a small amount $(dx_1,dx_2,\ldots,dx_n)$
- The total effect is measured by the total differential

$$dy = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 + \ldots + \frac{\partial f}{\partial x_n} dx_n$$

$$dy = f_1 dx_1 + f_2 dx_2 + \ldots + f_n dx_n$$

First-Order Conditions

- A necessary condition for a maximum (or minimum) of the function $f(x_1,x_2,\ldots,x_n)$ is that $dy = 0$ for any combination of small changes in the x's
- The only way for this to be true is if

$$f_1 = f_2 = \ldots = f_n = 0$$

First-Order Conditions

- To find a maximum (or minimum) we have to find the first order conditions:

$$\partial f/\partial x_1 = f_1 = 0$$

$$\partial f/\partial x_2 = f_2 = 0$$

$$\vdots$$

$$\partial f/\partial x_n = f_n = 0$$

Second Order Conditions - Functions of Two Variables

- The second order conditions for a maximum are:
  - $f_{11} < 0$
  - $f_{22} < 0$
  - $f_{11} f_{22} - f_{12}^2 > 0$

CONSTRAINED MAXIMIZATION

Constrained Maximization

- What if all values for the *x*’s are not feasible?
  - the values of *x* may all have to be positive
  - our choices are limited by the amount of resources/income available
- One method used to solve constrained maximization problems is the <u>Lagrangian multiplier method</u>

Lagrangian Multiplier Method

- Suppose that we wish to find the values of $x_1, x_2,\ldots, x_n$ that maximize

$$y = f(x_1, x_2,\ldots, x_n)$$

subject to a constraint that permits only certain values of the *x*’s to be used

$$g(x_1, x_2,\ldots, x_n) = 0$$

In our example:

$$m-p_1x_1-p_2x_2-p_3x_3\ldots-p_nx_n = 0$$

Lagrangian Multiplier Method

- First, set up the following expression

$$\mathbf{L} = f(x_1, x_2, \ldots, x_n) + \lambda g(x_1, x_2, \ldots, x_n)$$

- where λ is an additional variable called a Lagrangian multiplier

- L is often called the *Lagrangian*

- Then apply the method used in absence of the constraint to *L*

Lagrangian Multiplier Method

- Find the first-order conditions of the new objective function *L:*

$$\partial \mathbf{L}/\partial x_1 = f_1 + \lambda g_1 = 0$$

$$\partial \mathbf{L}/\partial x_2 = f_2 + \lambda g_2 = 0$$

$$\vdots$$

$$\partial \mathbf{L}/\partial x_n = f_n + \lambda g_n = 0$$

$$\partial \mathbf{L}/\partial \lambda = g(x_1, x_2, \dots, x_n) = 0$$

Lagrangian Multiplier Method

- The first-order conditions can generally be solved for $x_1, x_2, \ldots, x_n$ and λ

- The solution will have two properties:
  - the *x*’s will obey the constraint:

$$g(x_1, x_2, \ldots, x_n) = 0$$

  - these *x*’s will make the value of **L** as large as possible
  - since the constraint holds, **L** = *f* and *f* is also as large as possible

Interpretation of the Multiplier

- λ measures how much the objective function *f* increases if the constraint is relaxed slightly
- Consumption example:
  - λ represents the increase in utility if we increase income a little.
  - Poor people - high λ
  - Rich people – low λ
  - If $\lambda=0$ then the constraint is not binding

Interpretation of Lagrangian

- The Lagrangian is

$$\mathbf{L} = f(x_1, x_2) + \lambda[\mathrm{m}-p_1x_1-p_2x_2]$$

- Second term is penalty for exceeding budget by $1.
- Set penalty high enough so don’t go over.
- Penalty is same for each good, so at optimum we have

$$\lambda = \mathrm{MU}_1/p_1 = MU_2/p_2$$

Inequality Constraints

- Suppose constraint takes form

$$g(x_1, x_2,\ldots, x_n) \geq 0$$

- The Kuhn-Tucker conditions are

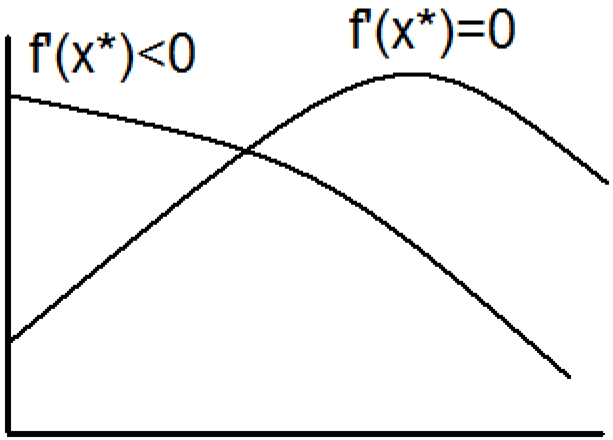
1. FOCs: $\partial \mathbf{L}/\partial x_i = f_i + \lambda g_i = 0$
2. Penalty is positive: $\lambda \geq 0$
3. Constraint holds: $g(x_1, x_2,\ldots, x_n) \geq 0$
4. Complimentary slackness: $\lambda g(x_1, x_2,\ldots, x_n)=0$

Example: Boundary Constraints

Suppose we maximise f(x) subject to x ≥ 0.

KT conditions imply that:

- If x*>0 then f’(x*)=0.
- If x*=0 then f’(x)≤0.



OTHER USEFUL RESULTS

Implicit Function Theorem

- Usually we write the dependent variable y as a function of one or more independent variable:

$$y = f(x)$$

- This is equivalent to:

$$y - f(x)=0$$

- Or more generally:

$$g(x,y)=0$$

Implicit Function Theorem

- Consider the implicit function:

$$g(x,y)=0$$

- The total differential is:

$$dg = g_x dx + g_y dy = 0$$

- If we solve for dy and divide by dx, we get the implicit derivate:

$$dy/dx=-g_x/g_y$$

- Providing $g_y \neq 0$

Implicit Function Theorem

- The implicit function theorem establishes the conditions under which we can derive the implicit derivative of a variable

- In our course we will always assume that this conditions are satisfied.

The Envelope Theorem

- Suppose we choose (x_1,x_2) to maximize

$$u(x_1,x_2,t)$$

where *t* is exogenous (e.g. t=time to enjoy goods).

- The total differential is

$$dv = v_1\, dx_1 + v_2\, dx_2 + v_m\, dm$$

- But at the optimum:

$$v_1 = 0 \quad \text{and} \quad v_2 = 0$$

- Hence $dv = v_m\, dm$ and $dv/dm = f_m$

The Envelope Theorem

Suppose time *t* increases.

1. Changes goods consumer buys
   - Spend more money on vacations.
2. Time also valuable in itself, holding consumption fixed.

- Envelope theorem says that only second effect matters.
- First effect is second-order since consumption chosen optimally.

Homogeneous Functions

- A function $f(x_1,x_2,\ldots x_n)$ is said to be homogeneous of degree k if

$$f(tx_1,tx_2,\ldots tx_n) = t^k f(x_1,x_2,\ldots x_n)$$

  - when a function is homogeneous of degree one, a doubling of all of its arguments doubles the value of the function itself
  - when a function is homogeneous of degree zero, a doubling of all of its arguments leaves the value of the function unchanged

Homogeneous Functions

- If a function is homogeneous of degree *k*, the partial derivatives of the function will be homogeneous of degree *k*-1

Euler’s Theorem

- Euler’s theorem shows that, for homogeneous functions, there is a special relationship between the values of the function and the values of its partial derivatives.
- If a function $f(x_1,\ldots,x_n)$ is homogeneous of degree k we have:

$$kf(x_1,\ldots,x_n) = x_1 f_1(x_1,\ldots,x_n) + \ldots + x_n f_n(x_1,\ldots,x_n)$$

Euler’s Theorem

- If the function is homogeneous of degree 0:

$$0 = x_1 f_1(x_1,\dots,x_n) + \dots + x_n f_n(x_1,\dots,x_n)$$

- If the function is homogeneous of degree 1:

$$f(x_1,\dots,x_n) = x_1 f_1(x_1,\dots,x_n) + \dots + x_n f_n(x_1,\dots,x_n)$$

Duality

- Any constrained maximization problem has associated with it a dual problem in constrained minimization that focuses attention on the constraints in the original problem

Duality

- Individuals maximize utility subject to a budget constraint
  - dual problem: individuals minimize the expenditure needed to achieve a given level of utility
- Firms maximize output for a given cost of inputs purchased
  - dual problem: firms minimize the cost of inputs to produce a given level of output